Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/29-1.1.3.8-P-x-c-x-^m-a+b-x^n-^p

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [594]. This is test number [29].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (594)	0.00 (0)
Mathematica	100.00 (594)	0.00 (0)
Maple	97.14 (577)	2.86 (17)
Fricas	90.57 (538)	9.43 (56)
Mupad	75.59 (449)	24.41 (145)
Sympy	72.39 (430)	27.61 (164)
Giac	70.71 (420)	29.29 (174)
Maxima	69.87 (415)	30.13 (179)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

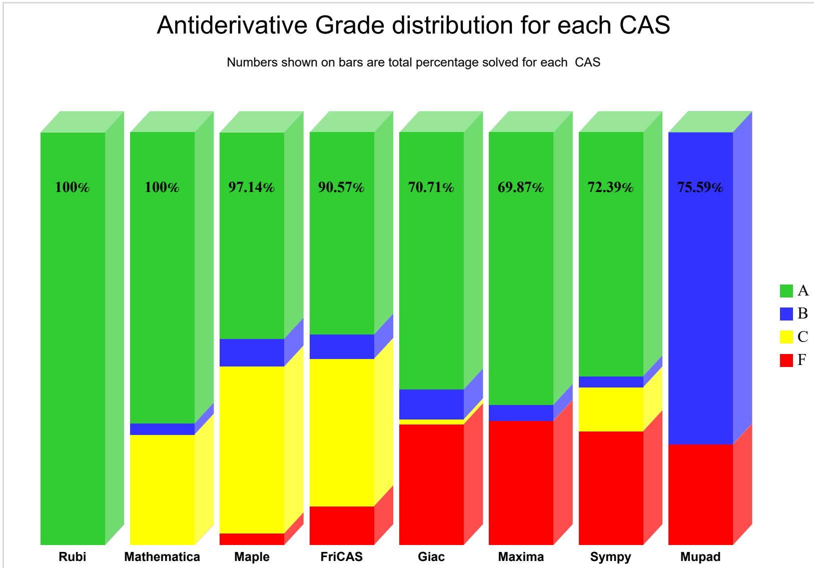
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

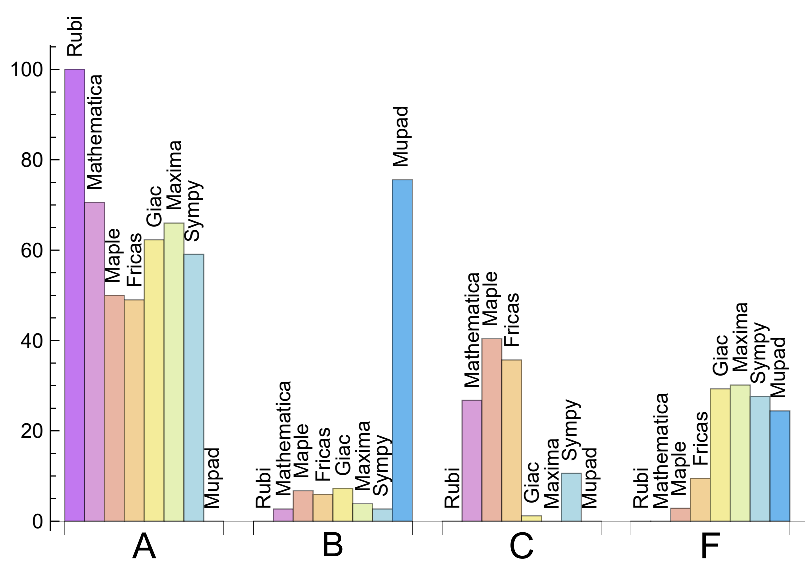
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	70.539	2.694	26.768	0.000
Maxima	65.993	3.872	0.000	30.135
Giac	62.290	7.239	1.178	29.293
Sympy	59.091	2.694	10.606	27.609
Maple	50.000	6.734	40.404	2.862
Fricas	48.990	5.892	35.690	9.428
Mupad	0.000	75.589	0.000	24.411

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	17	94.12	5.88	0.00
Fricas	56	55.36	41.07	3.57
Mupad	145	0.00	100.00	0.00
Sympy	164	1.22	96.95	1.83
Giac	174	92.53	2.30	5.17
Maxima	179	96.09	0.00	3.91

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.26
Giac	0.28
Rubi	0.52
Maple	1.67
Mathematica	2.80
Fricas	3.12
Sympy	4.78
Mupad	6.34

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	171.29	0.91	154.00	0.95
Sympy	185.59	1.15	128.00	0.93
Maxima	188.27	1.11	173.00	0.99
Giac	209.21	1.19	185.50	1.02
Maple	234.76	0.92	147.00	0.82
Rubi	249.34	1.02	223.00	1.00
Mupad	490.04	2.28	199.00	1.03
Fricas	15695.66	70.51	201.50	1.02

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

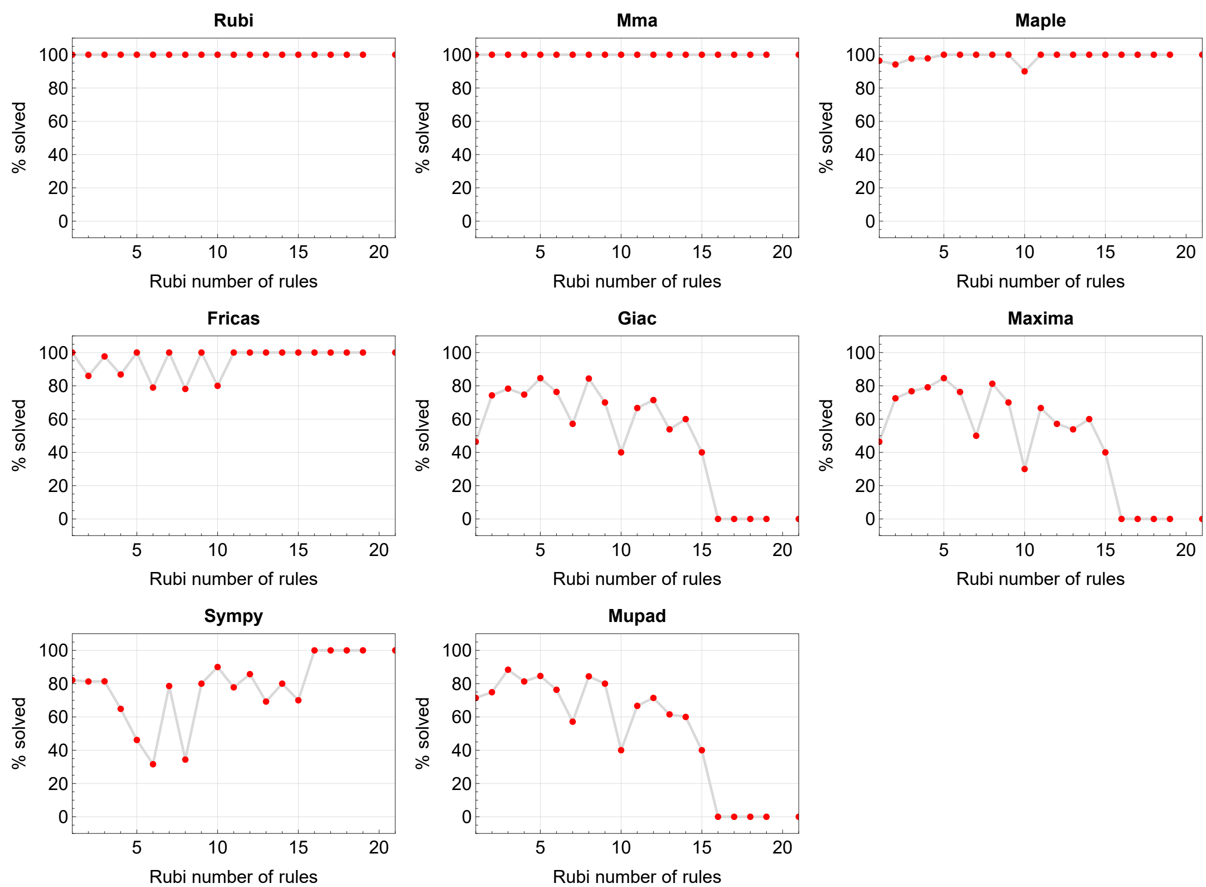


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

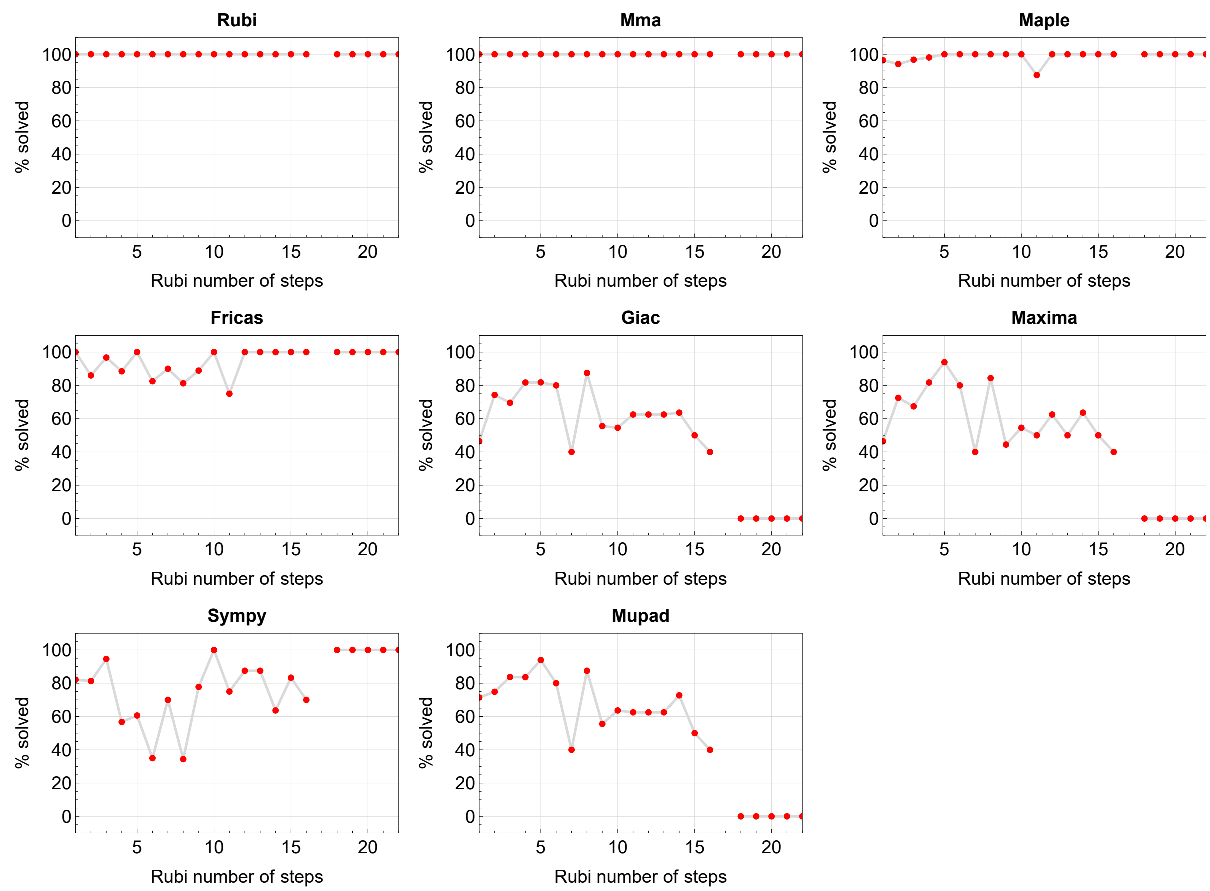


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

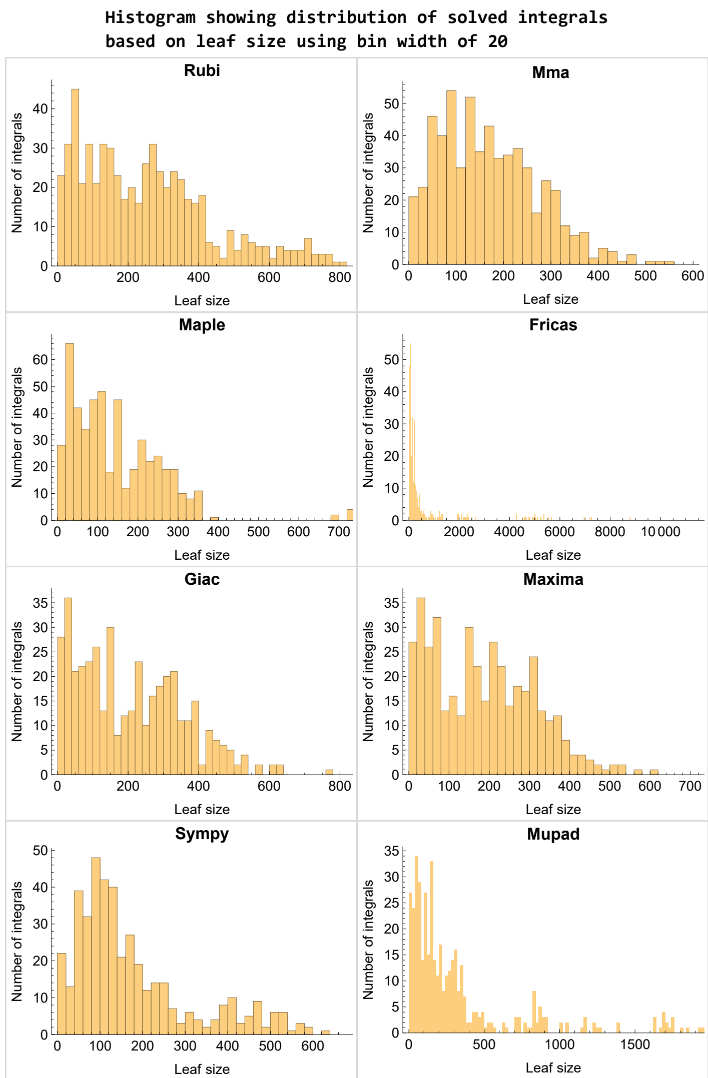


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

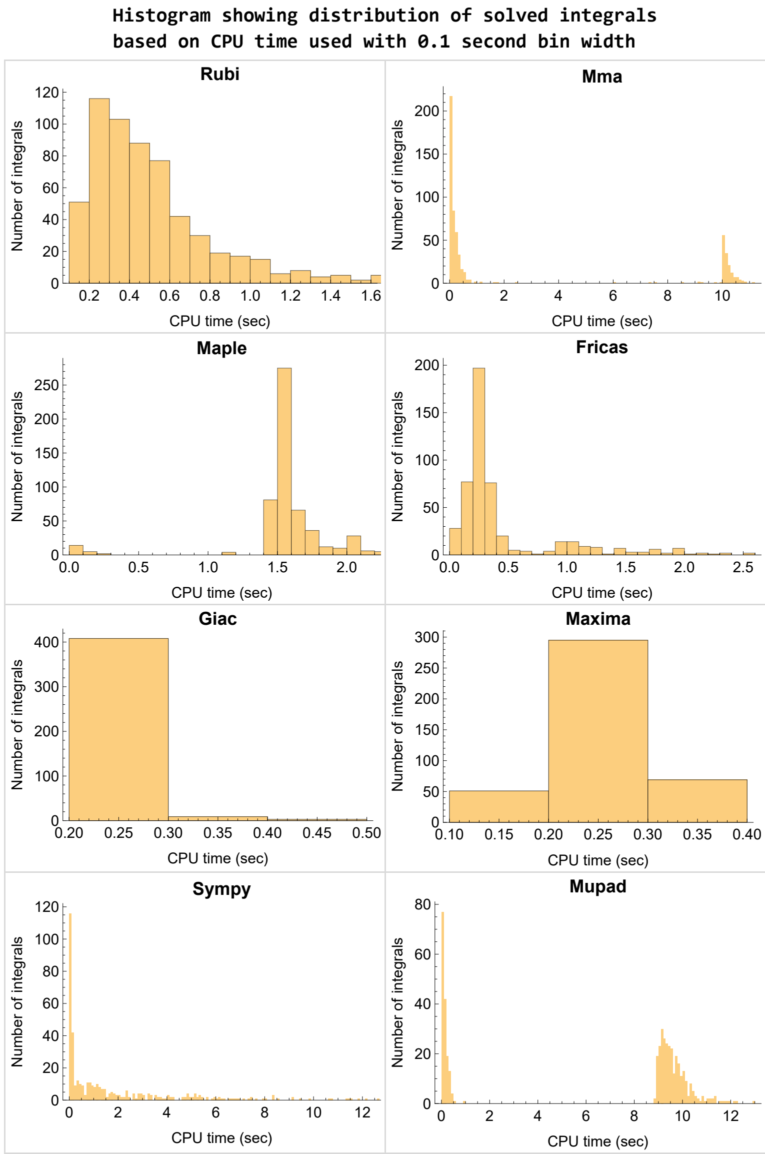


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

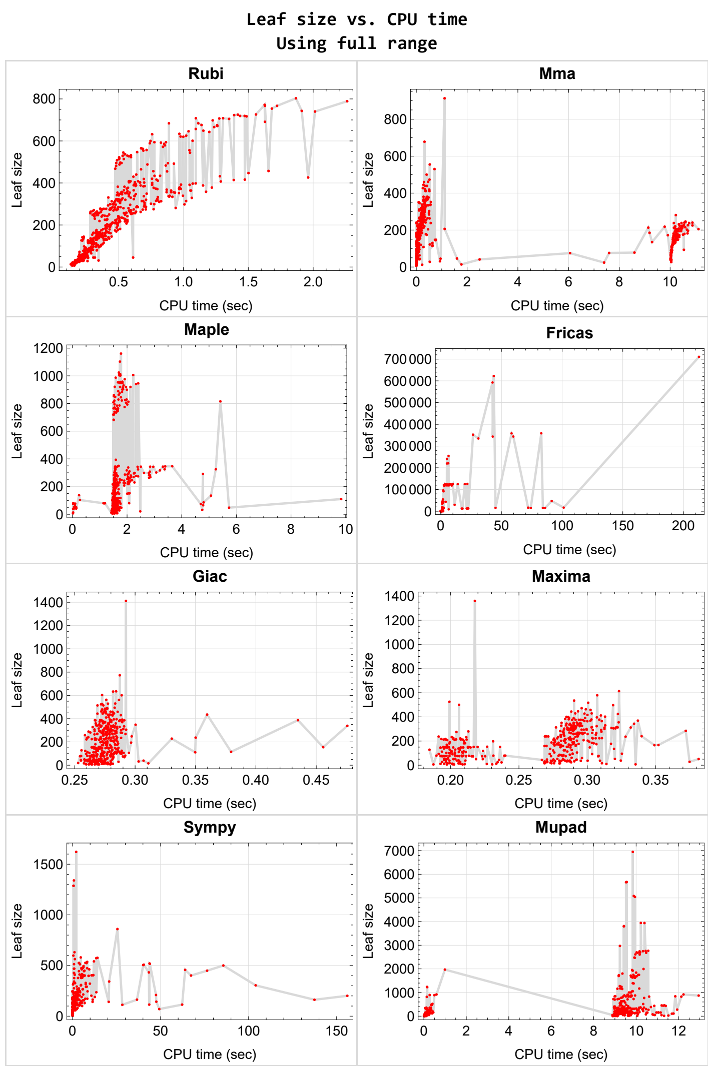


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{81, 93, 97, 113, 592, 594\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

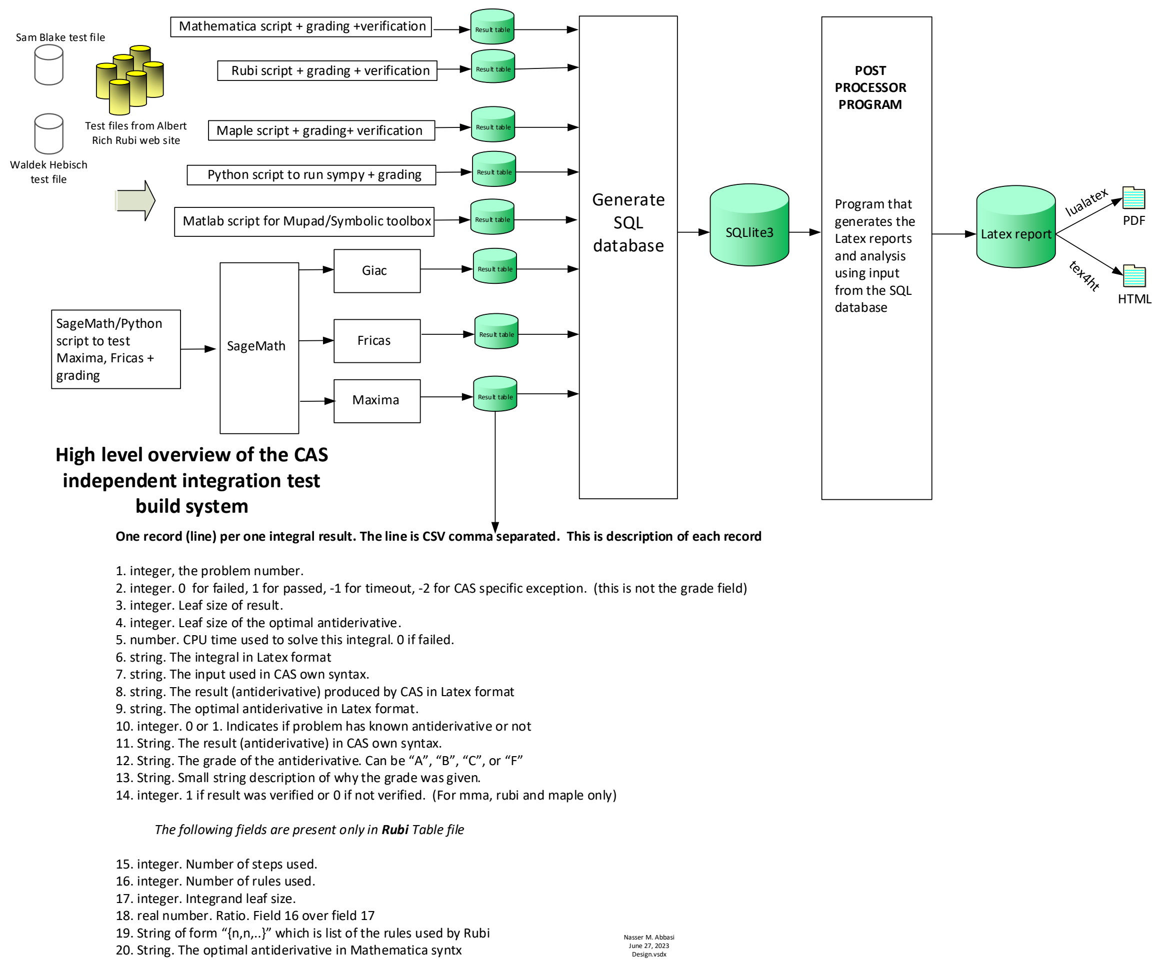
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 21
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2.1 List of integrals sorted by grade for each CAS

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2.1.8 Sympy 28

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37, 38, 39, 40, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 591, 592, 593, 594 }

B grade { 21, 32, 33, 34, 35, 36, 41, 44, 45, 46, 47, 369, 370, 371, 372, 557 }

C grade { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 124, 161, 210, 211, 212, 213, 214, 220, 221, 222, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 567, 590 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 19, 28, 39, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 76, 77, 78, 107, 108, 109, 110, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 159, 161, 167, 179, 180, 181, 182, 183, 184, 185, 211, 212, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 350, 357, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 408, 409, 410, 411, 417, 418, 419, 420, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 477, 478, 479, 480, 481, 482, 483, 484, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 589, 593 }

B grade { 20, 21, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 44, 45, 46, 47, 83, 84, 85, 86, 87, 88, 89, 90, 99, 100, 101, 102, 103, 104, 105, 106, 123, 369, 370, 371, 372, 557 }

C grade { 7, 8, 9, 10, 11, 12, 22, 23, 24, 25, 26, 27, 57, 58, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 91, 92, 93, 94, 95, 96, 97, 98, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 213, 214, 220, 221, 222, 233, 234, 235, 236, 237, 238, 239, 260, 261, 262, 263, 264, 265, 266, 286, 287, 288, 289, 290, 291, 292, 293, 294, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361, 362, 363, 403, 404, 405, 406, 407, 412, 413, 414, 415, 416, 421, 422, 423, 424, 425, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 592, 594 }

F normal fail { 474, 475, 476, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 587, 588, 591 }

F(-1) timedout fail { 590 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 76, 77, 78, 123, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 159, 161, 167, 180, 181, 182, 183, 184, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 477, 478, 479, 480, 481, 482, 483, 484, 495, 496, 497, 498, 499, 506, 507, 508, 509, 510, 511, 512, 513, 514, 525, 526, 527, 528, 529, 530, 531, 532, 533, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 571, 572, 573, 574, 575, 579, 580, 585, 589, 593, 594 }

B grade { 40, 41, 44, 45, 46, 47, 156, 160, 163, 164, 168, 179, 185, 221, 222, 283, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 557, 569, 570, 577, 578, 591, 592 }

C grade { 7, 8, 9, 10, 11, 12, 24, 25, 26, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 149, 150, 151, 152, 154, 155, 157, 158, 162, 165, 166, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 192, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494 }

F normal fail { 474, 475, 476, 500, 501, 502, 503, 504, 505, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 534, 535, 551, 552, 553, 576, 581, 582, 583, 586, 587, 588 }

F(-1) timedout fail { 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209 }

F(-2) exception fail { 584, 590 }

2.1.5 Maxima

A grade { 1, 2, 4, 5, 7, 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 30, 33, 36, 39, 42, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 76, 77, 78, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 589, 592, 593 }

B grade { 3, 6, 20, 21, 31, 32, 34, 35, 37, 38, 40, 41, 43, 45, 46, 115, 161, 179, 185, 370, 371, 557, 594 }

C grade { }

F normal fail { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 210, 211, 212, 213, 214, 220, 221, 222, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 587, 588, 590, 591 }

F(-1) timedout fail { }

F(-2) exception fail { 11, 12, 25, 26, 73, 74, 75 }

2.1.6 Giac

A grade { 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 32, 35, 36, 39, 42, 43, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 116, 118, 120, 122, 124, 126, 128, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 170, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 191, 195, 196, 197, 201, 202, 203, 204, 207, 208, 209, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 477, 478, 479, 480, 481, 482, 483, 484, 487, 488, 489, 490, 491, 492, 493, 494, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 579, 580 }

B grade { 3, 6, 29, 115, 117, 119, 121, 123, 125, 127, 129, 131, 149, 150, 151, 161, 169, 171, 172, 173, 179, 186, 187, 188, 192, 193, 194, 198, 199, 200, 205, 206, 254, 371, 485, 486, 557, 577, 578, 591, 592, 593, 594 }

C grade { 30, 33, 34, 44, 45, 369, 370 }

F normal fail { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 86, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 107, 108, 109, 110, 111, 112, 113, 114, 210, 211, 212, 213, 214, 220, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 576, 581, 582, 583, 584, 585, 586, 588, 589, 590 }

F(-1) timedout fail { 37, 38, 40, 41 }

F(-2) exception fail { 87, 88, 89, 90, 103, 104, 105, 106, 587 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 91, 92, 93, 94, 95, 96, 97, 98, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 435, 444, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 536, 548, 549, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 581, 582, 583, 589, 591, 592, 593, 594 }

C grade { }

F normal fail { }

F(-1) timedout fail { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 83, 84, 85, 86, 87, 88, 89, 90, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 210, 211, 212, 213, 214, 220, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 550, 551, 552, 553, 576, 584, 585, 586, 587, 588, 590 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 29, 31, 37, 38, 39, 42, 43, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 126, 128, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 159, 160, 162, 163, 164, 165, 167, 168, 169, 170, 180, 181, 182, 183, 184, 211, 212, 217, 218, 220, 223, 224, 225, 226, 227, 228, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 251, 252, 253, 254, 260, 262, 264, 266, 267, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 344, 345, 346, 351, 352, 353, 358, 359, 360, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 489, 495, 496, 497, 498, 499, 510, 511, 512, 513, 514, 515, 516, 517, 518, 529, 530, 531, 532, 533, 540, 541, 542, 543, 544, 545, 546, 552, 553, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 580, 581, 585 }

B grade { 3, 6, 125, 127, 140, 149, 158, 166, 179, 185, 221, 222, 557, 577, 578, 579 }

C grade { 18, 19, 20, 21, 27, 28, 30, 32, 33, 34, 35, 36, 49, 123, 161, 210, 213, 214, 215, 216, 219, 365, 366, 367, 368, 369, 370, 371, 372, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 534, 535, 536, 537, 538, 539, 547, 548, 549, 550, 576, 582, 584, 586 }

F normal fail { 589, 590 }

F(-1) timedout fail { 40, 41, 44, 45, 46, 47, 69, 72, 73, 74, 75, 129, 130, 131, 132, 150, 151, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 229, 230, 231, 232, 245, 246, 247, 248, 249, 250, 255, 256, 257, 258, 259, 261, 263, 265, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 341, 342, 343, 347, 348, 349, 350, 354, 355, 356, 357, 361, 362, 363, 364, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 485, 486, 487, 488, 490, 491, 492, 493, 494, 551, 583, 587, 588, 591 }

F(-2) exception fail { 592, 593, 594 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	53	48	77	53	102	77	58
N.S.	1	1.00	0.74	0.67	1.07	0.74	1.42	1.07	0.81
time (sec)	N/A	0.199	0.066	5.733	0.193	0.314	0.455	0.254	8.967

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	155	135	237	192	275	236	149
N.S.	1	1.00	0.96	0.84	1.47	1.19	1.71	1.47	0.93
time (sec)	N/A	0.303	0.148	5.071	0.191	0.270	0.799	0.260	0.069

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	355	325	525	457	631	526	299
N.S.	1	1.00	1.30	1.19	1.92	1.67	2.30	1.92	1.09
time (sec)	N/A	0.456	0.307	5.247	0.199	0.281	1.104	0.273	0.100

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	82	74	128	90	163	128	103
N.S.	1	1.00	0.72	0.65	1.12	0.79	1.43	1.12	0.90
time (sec)	N/A	0.281	0.106	4.699	0.185	0.309	0.634	0.256	8.997

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	320	292	500	417	563	514	316
N.S.	1	1.00	1.00	0.91	1.56	1.30	1.76	1.61	0.99
time (sec)	N/A	0.540	0.334	4.776	0.206	0.558	1.310	0.269	9.147

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	708	708	913	816	1360	1221	1622	1412	896
N.S.	1	1.00	1.29	1.15	1.92	1.72	2.29	1.99	1.27
time (sec)	N/A	1.064	1.114	5.415	0.218	0.499	2.069	0.292	0.274

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	156	124	32	135	1931	76	141	127
N.S.	1	0.97	0.77	0.20	0.84	11.99	0.47	0.88	0.79
time (sec)	N/A	0.348	0.059	4.752	0.273	1.919	0.339	0.270	9.239

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	191	180	65	169	2088	105	174	169
N.S.	1	1.01	0.95	0.34	0.89	11.05	0.56	0.92	0.89
time (sec)	N/A	0.401	0.157	4.784	0.281	0.931	0.449	0.273	9.056

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	225	205	86	203	2215	146	194	206
N.S.	1	1.05	0.95	0.40	0.94	10.30	0.68	0.90	0.96
time (sec)	N/A	0.471	0.161	4.801	0.283	0.931	0.597	0.274	0.283

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	258	229	110	238	2308	185	218	241
N.S.	1	1.08	0.95	0.46	0.99	9.62	0.77	0.91	1.00
time (sec)	N/A	0.551	0.201	9.837	0.269	0.940	0.709	0.268	9.336

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	156	125	32	0	1961	76	141	127
N.S.	1	0.97	0.78	0.20	0.00	12.18	0.47	0.88	0.79
time (sec)	N/A	0.350	0.059	1.504	0.000	1.003	0.337	0.285	9.199

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	154	125	34	0	1905	78	136	124
N.S.	1	0.96	0.78	0.21	0.00	11.83	0.48	0.84	0.77
time (sec)	N/A	0.333	0.055	1.680	0.000	0.985	0.343	0.268	0.229

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.163	0.007	1.454	0.276	0.279	0.044	0.258	9.002

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.166	0.006	1.469	0.280	0.274	0.045	0.268	0.031

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	17	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.77	0.73
time (sec)	N/A	0.169	0.007	1.514	0.269	0.322	0.041	0.264	9.011

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	17	19	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.86	0.82
time (sec)	N/A	0.169	0.006	1.457	0.269	0.267	0.037	0.253	0.127

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	42	41	33	32	32	44	33	46
N.S.	1	1.02	1.00	0.80	0.78	0.78	1.07	0.80	1.12
time (sec)	N/A	0.209	0.011	1.486	0.287	0.277	0.064	0.271	0.152

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	34	28	54	28	28
N.S.	1	1.00	1.07	1.00	1.17	0.97	1.86	0.97	0.97
time (sec)	N/A	0.171	0.014	1.522	0.276	0.290	0.072	0.268	0.055

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	29	33	26	53	26	28
N.S.	1	1.00	1.00	1.00	1.14	0.90	1.83	0.90	0.97
time (sec)	N/A	0.175	0.010	1.500	0.281	0.411	0.080	0.267	0.051

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	35	35	195	163	107	88	48	49
N.S.	1	0.90	0.90	5.00	4.18	2.74	2.26	1.23	1.26
time (sec)	N/A	0.178	0.016	1.540	0.276	0.566	0.125	0.281	9.076

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	37	129	202	174	114	105	57	49
N.S.	1	0.90	3.15	4.93	4.24	2.78	2.56	1.39	1.20
time (sec)	N/A	0.185	0.057	1.523	0.275	0.457	0.153	0.282	0.241

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	90	47	159	310	26	103	98
N.S.	1	1.00	0.76	0.40	1.35	2.63	0.22	0.87	0.83
time (sec)	N/A	0.301	0.013	1.566	0.269	0.912	0.063	0.272	9.277

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	90	49	159	305	22	115	96
N.S.	1	1.00	0.76	0.42	1.35	2.58	0.19	0.97	0.81
time (sec)	N/A	0.275	0.013	1.620	0.270	0.285	0.073	0.274	9.406

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	52	188	1961	76	147	127
N.S.	1	1.00	0.77	0.32	1.17	12.18	0.47	0.91	0.79
time (sec)	N/A	0.335	0.041	1.716	0.274	1.024	0.340	0.275	9.278

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	135	122	36	0	1344	75	118	158
N.S.	1	1.01	0.91	0.27	0.00	10.03	0.56	0.88	1.18
time (sec)	N/A	0.368	0.033	1.530	0.000	1.425	0.151	0.279	0.193

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	130	123	36	0	1040	70	124	178
N.S.	1	0.97	0.92	0.27	0.00	7.76	0.52	0.93	1.33
time (sec)	N/A	0.325	0.032	1.523	0.000	1.533	0.197	0.269	9.330

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	72	42	42	36	60	37	84
N.S.	1	1.00	1.95	1.14	1.14	0.97	1.62	1.00	2.27
time (sec)	N/A	0.216	0.024	1.545	0.282	0.336	0.138	0.263	9.146

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	71	38	44	36	60	38	86
N.S.	1	1.00	1.82	0.97	1.13	0.92	1.54	0.97	2.21
time (sec)	N/A	0.208	0.021	1.508	0.267	0.266	0.148	0.272	9.026

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	76	115	47	134	58	115	147
N.S.	1	1.00	1.58	2.40	0.98	2.79	1.21	2.40	3.06
time (sec)	N/A	0.204	0.026	1.515	0.275	0.311	0.161	0.379	9.269

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	43	72	96	36	40	85	76	145
N.S.	1	0.91	1.53	2.04	0.77	0.85	1.81	1.62	3.09
time (sec)	N/A	0.197	0.033	1.526	0.274	0.283	0.143	0.280	9.593

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	99	97	122	182	58	56	176
N.S.	1	1.00	1.74	1.70	2.14	3.19	1.02	0.98	3.09
time (sec)	N/A	0.234	0.034	1.486	0.275	0.483	0.187	0.277	9.585

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	106	98	93	43	95	51	142
N.S.	1	1.00	2.26	2.09	1.98	0.91	2.02	1.09	3.02
time (sec)	N/A	0.222	0.045	1.524	0.273	0.480	0.164	0.265	0.371

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	146	116	51	52	100	143	172
N.S.	1	1.00	2.92	2.32	1.02	1.04	2.00	2.86	3.44
time (sec)	N/A	0.241	0.058	1.506	0.282	0.472	0.175	0.267	9.566

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	150	119	167	53	110	109	172
N.S.	1	1.00	2.83	2.25	3.15	1.00	2.08	2.06	3.25
time (sec)	N/A	0.244	0.066	1.485	0.280	0.782	0.190	0.283	9.635

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	149	117	168	56	109	91	173
N.S.	1	1.00	2.76	2.17	3.11	1.04	2.02	1.69	3.20
time (sec)	N/A	0.225	0.050	1.476	0.287	0.316	0.184	0.269	9.381

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	147	118	52	53	102	85	171
N.S.	1	1.00	2.77	2.23	0.98	1.00	1.92	1.60	3.23
time (sec)	N/A	0.223	0.046	1.496	0.278	0.291	0.187	0.273	9.544

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	57	95	112	162	160	70	0	193
N.S.	1	0.93	1.56	1.84	2.66	2.62	1.15	0.00	3.16
time (sec)	N/A	0.206	0.021	1.560	0.281	0.297	0.176	0.000	9.448

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	66	116	117	173	205	73	0	221
N.S.	1	0.94	1.66	1.67	2.47	2.93	1.04	0.00	3.16
time (sec)	N/A	0.251	0.033	1.571	0.278	0.357	0.199	0.000	9.428

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	46	50	32	31	31	42	32	46
N.S.	1	1.15	1.25	0.80	0.78	0.78	1.05	0.80	1.15
time (sec)	N/A	0.219	0.012	1.466	0.269	0.290	0.060	0.303	0.157

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	76	122	217	236	430	0	0	386
N.S.	1	1.09	1.74	3.10	3.37	6.14	0.00	0.00	5.51
time (sec)	N/A	0.236	0.048	1.593	0.281	1.992	0.000	0.000	10.698

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	238	227	252	470	0	0	444
N.S.	1	1.00	2.70	2.58	2.86	5.34	0.00	0.00	5.05
time (sec)	N/A	0.282	0.567	1.587	0.306	1.665	0.000	0.000	11.273

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	12	12	12	12	7	13	12
N.S.	1	1.00	1.09	1.09	1.09	1.09	0.64	1.18	1.09
time (sec)	N/A	0.155	0.001	1.492	0.201	0.384	0.025	0.278	0.038

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	210	17	20	16	15
N.S.	1	1.00	1.00	0.86	10.00	0.81	0.95	0.76	0.71
time (sec)	N/A	0.163	0.003	1.497	0.275	0.407	0.068	0.311	9.660

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F(-1)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	247	219	78	429	0	194	436
N.S.	1	1.00	3.48	3.08	1.10	6.04	0.00	2.73	6.14
time (sec)	N/A	0.262	0.283	1.499	0.282	1.981	0.000	0.297	10.856

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	288	224	238	459	0	235	456
N.S.	1	1.00	3.79	2.95	3.13	6.04	0.00	3.09	6.00
time (sec)	N/A	0.276	0.194	1.524	0.271	1.021	0.000	0.288	11.331

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	253	222	239	450	0	133	453
N.S.	1	1.00	3.24	2.85	3.06	5.77	0.00	1.71	5.81
time (sec)	N/A	0.280	0.270	1.525	0.270	1.029	0.000	0.286	11.204

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	244	223	78	450	0	125	435
N.S.	1	1.00	3.25	2.97	1.04	6.00	0.00	1.67	5.80
time (sec)	N/A	0.279	0.278	1.521	0.279	1.780	0.000	0.286	11.394

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	28	26	26	24	27	35
N.S.	1	1.00	0.97	0.88	0.81	0.81	0.75	0.84	1.09
time (sec)	N/A	0.191	0.016	1.722	0.269	0.493	0.207	0.272	9.458

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	60	62	55	47	47	323	52	87
N.S.	1	1.09	1.13	1.00	0.85	0.85	5.87	0.95	1.58
time (sec)	N/A	0.239	0.039	1.496	0.271	0.369	0.462	0.275	10.083

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.147	0.001	1.455	0.187	0.274	0.021	0.269	0.027

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	32	32	5	33	63
N.S.	1	1.00	1.00	1.10	1.07	1.07	0.17	1.10	2.10
time (sec)	N/A	0.213	0.010	1.481	0.291	0.288	0.059	0.274	0.133

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	17	16
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	0.94	0.89
time (sec)	N/A	0.177	0.007	1.471	0.311	0.270	0.045	0.267	10.058

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	98	97	97	117	97	97
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.04	0.86	0.86
time (sec)	N/A	0.304	0.006	1.469	0.203	0.411	0.023	0.272	0.093

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	74	74	90	74	74
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.02	0.84	0.84
time (sec)	N/A	0.269	0.005	1.482	0.223	0.378	0.022	0.268	0.039

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	58	50	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.97	0.83	0.83
time (sec)	N/A	0.232	0.003	0.138	0.224	0.343	0.018	0.270	0.030

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	14	12	11	10	10	8	10	10
N.S.	1	1.17	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.159	0.001	1.425	0.226	0.652	0.022	0.280	0.020

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	156	124	32	135	1931	76	141	127
N.S.	1	0.97	0.77	0.20	0.84	11.99	0.47	0.88	0.79
time (sec)	N/A	0.356	0.046	1.464	0.320	0.900	0.344	0.270	0.213

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	191	180	65	169	2088	105	174	169
N.S.	1	1.01	0.95	0.34	0.89	11.05	0.56	0.92	0.89
time (sec)	N/A	0.413	0.148	1.468	0.279	0.923	0.448	0.285	10.158

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	585	594	78	786	0	117	265	0	0
N.S.	1	1.02	0.13	1.34	0.00	0.20	0.45	0.00	0.00
time (sec)	N/A	0.764	8.594	1.722	0.000	0.128	2.638	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	556	562	76	762	0	93	170	0	0
N.S.	1	1.01	0.14	1.37	0.00	0.17	0.31	0.00	0.00
time (sec)	N/A	0.656	7.596	1.591	0.000	0.107	1.827	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	525	530	75	744	0	69	163	0	0
N.S.	1	1.01	0.14	1.42	0.00	0.13	0.31	0.00	0.00
time (sec)	N/A	0.586	6.056	1.635	0.000	0.086	1.650	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	495	75	720	0	43	78	0	0
N.S.	1	1.01	0.15	1.47	0.00	0.09	0.16	0.00	0.00
time (sec)	N/A	0.506	10.041	1.515	0.000	0.089	1.615	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	527	96	765	0	94	163	0	0
N.S.	1	1.01	0.18	1.47	0.00	0.18	0.31	0.00	0.00
time (sec)	N/A	0.590	10.060	1.505	0.000	0.133	3.771	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	554	565	123	809	0	155	163	0	0
N.S.	1	1.02	0.22	1.46	0.00	0.28	0.29	0.00	0.00
time (sec)	N/A	0.650	10.095	1.500	0.000	0.112	11.466	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	600	138	853	0	214	163	0	0
N.S.	1	1.03	0.24	1.47	0.00	0.37	0.28	0.00	0.00
time (sec)	N/A	0.746	10.120	1.514	0.000	0.089	36.616	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	590	594	135	785	0	87	187	0	0
N.S.	1	1.01	0.23	1.33	0.00	0.15	0.32	0.00	0.00
time (sec)	N/A	0.863	10.146	1.632	0.000	0.082	1.738	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	589	130	821	0	153	189	0	0
N.S.	1	0.99	0.22	1.38	0.00	0.26	0.32	0.00	0.00
time (sec)	N/A	0.760	10.132	1.538	0.000	0.171	5.186	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	632	170	861	0	261	209	0	0
N.S.	1	1.01	0.27	1.37	0.00	0.42	0.33	0.00	0.00
time (sec)	N/A	0.790	10.193	1.517	0.000	0.151	47.416	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	676	684	196	921	0	373	0	0	0
N.S.	1	1.01	0.29	1.36	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.913	10.253	1.549	0.000	0.110	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	179	200	43	192	5014	156	175	357
N.S.	1	0.96	1.08	0.23	1.03	26.96	0.84	0.94	1.92
time (sec)	N/A	0.446	0.090	1.649	0.276	0.892	0.516	0.279	10.141

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	214	66	240	7245	245	214	370
N.S.	1	1.00	0.96	0.30	1.08	32.64	1.10	0.96	1.67
time (sec)	N/A	0.505	0.169	1.660	0.269	1.335	8.527	0.284	10.211

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	280	277	90	303	8787	0	294	513
N.S.	1	0.99	0.98	0.32	1.07	31.16	0.00	1.04	1.82
time (sec)	N/A	0.583	0.246	1.524	0.283	6.542	0.000	0.284	10.334

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	270	269	85	0	12827	0	282	769
N.S.	1	0.99	0.99	0.31	0.00	47.16	0.00	1.04	2.83
time (sec)	N/A	0.617	0.283	1.742	0.000	1.431	0.000	0.277	10.379

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	439	181	0	29479	0	462	1700
N.S.	1	1.00	1.06	0.44	0.00	70.86	0.00	1.11	4.09
time (sec)	N/A	0.843	0.419	1.690	0.000	11.406	0.000	0.272	9.180

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	645	643	678	350	0	47284	0	773	2971
N.S.	1	1.00	1.05	0.54	0.00	73.31	0.00	1.20	4.61
time (sec)	N/A	1.216	0.321	1.694	0.000	91.411	0.000	0.287	9.237

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	54	38	37	37	44	38	49
N.S.	1	1.00	1.26	0.88	0.86	0.86	1.02	0.88	1.14
time (sec)	N/A	0.268	0.016	1.594	0.309	0.432	0.062	0.261	0.107

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	54	36	37	37	46	38	51
N.S.	1	1.00	1.17	0.78	0.80	0.80	1.00	0.83	1.11
time (sec)	N/A	0.270	0.013	1.491	0.273	0.586	0.065	0.265	0.098

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	37	37	48	38	49
N.S.	1	1.00	1.00	0.86	0.84	0.84	1.09	0.86	1.11
time (sec)	N/A	0.212	0.010	1.450	0.300	0.321	0.061	0.267	9.379

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	47	47	0	21	92	0	312
N.S.	1	1.00	0.20	0.20	0.00	0.09	0.40	0.00	1.36
time (sec)	N/A	0.324	10.027	1.790	0.000	0.107	0.884	0.000	0.159

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	43	41	0	23	97	0	342
N.S.	1	1.00	0.17	0.16	0.00	0.09	0.38	0.00	1.33
time (sec)	N/A	0.336	10.023	1.614	0.000	0.152	1.207	0.000	9.597

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	144	144	63	95	0	21	82	0	326
N.S.	1	1.00	0.44	0.66	0.00	0.15	0.57	0.00	2.26
time (sec)	N/A	0.233	10.031	1.720	0.000	0.146	1.180	0.000	9.564

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	67	52	0	23	99	0	360
N.S.	1	1.00	0.50	0.39	0.00	0.17	0.73	0.00	2.67
time (sec)	N/A	0.229	10.030	1.748	0.000	0.188	0.984	0.000	0.274

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	468	90	1003	0	49	122	0	0
N.S.	1	1.00	0.19	2.14	0.00	0.10	0.26	0.00	0.00
time (sec)	N/A	0.479	10.069	1.749	0.000	0.098	1.712	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	481	481	91	949	0	55	128	0	0
N.S.	1	1.00	0.19	1.97	0.00	0.11	0.27	0.00	0.00
time (sec)	N/A	0.494	10.070	1.805	0.000	0.141	2.347	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	92	952	0	48	112	0	0
N.S.	1	1.00	0.34	3.51	0.00	0.18	0.41	0.00	0.00
time (sec)	N/A	0.313	10.048	1.758	0.000	0.105	2.378	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	93	1012	0	56	129	0	0
N.S.	1	1.00	0.35	3.80	0.00	0.21	0.48	0.00	0.00
time (sec)	N/A	0.302	10.054	1.723	0.000	0.088	1.948	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	520	516	89	1004	0	53	124	0	0
N.S.	1	0.99	0.17	1.93	0.00	0.10	0.24	0.00	0.00
time (sec)	N/A	0.523	10.057	1.742	0.000	0.100	1.256	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	533	529	89	950	0	56	129	0	0
N.S.	1	0.99	0.17	1.78	0.00	0.11	0.24	0.00	0.00
time (sec)	N/A	0.543	10.057	1.777	0.000	0.105	1.443	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	90	953	0	52	114	0	0
N.S.	1	1.00	0.35	3.72	0.00	0.20	0.45	0.00	0.00
time (sec)	N/A	0.327	10.042	1.667	0.000	0.121	1.395	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	92	1013	0	57	131	0	0
N.S.	1	1.00	0.37	4.04	0.00	0.23	0.52	0.00	0.00
time (sec)	N/A	0.317	10.041	1.727	0.000	0.197	1.342	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	49	48	0	21	92	0	313
N.S.	1	1.00	0.39	0.38	0.00	0.17	0.72	0.00	2.46
time (sec)	N/A	0.222	10.030	1.722	0.000	0.084	0.864	0.000	0.136

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	45	42	0	23	97	0	343
N.S.	1	1.00	0.32	0.30	0.00	0.16	0.68	0.00	2.42
time (sec)	N/A	0.233	10.020	1.613	0.000	0.120	1.220	0.000	9.148

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	264	264	63	96	0	21	82	0	327
N.S.	1	1.00	0.24	0.36	0.00	0.08	0.31	0.00	1.24
time (sec)	N/A	0.354	10.033	1.617	0.000	0.153	1.190	0.000	9.173

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	67	52	0	23	97	0	361
N.S.	1	1.00	0.27	0.21	0.00	0.09	0.39	0.00	1.46
time (sec)	N/A	0.348	10.031	1.641	0.000	0.254	0.971	0.000	0.118

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	47	48	0	21	92	0	312
N.S.	1	1.00	0.37	0.38	0.00	0.17	0.73	0.00	2.48
time (sec)	N/A	0.228	10.021	1.705	0.000	0.091	1.170	0.000	9.174

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	43	42	0	23	97	0	342
N.S.	1	1.00	0.30	0.29	0.00	0.16	0.68	0.00	2.39
time (sec)	N/A	0.237	10.016	1.646	0.000	0.161	1.003	0.000	0.056

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	263	263	63	96	0	21	82	0	326
N.S.	1	1.00	0.24	0.37	0.00	0.08	0.31	0.00	1.24
time (sec)	N/A	0.344	10.024	1.684	0.000	0.140	0.930	0.000	9.172

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	67	52	0	23	97	0	360
N.S.	1	1.00	0.27	0.21	0.00	0.09	0.39	0.00	1.45
time (sec)	N/A	0.335	10.025	1.615	0.000	0.253	1.189	0.000	9.153

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	90	1003	0	48	122	0	0
N.S.	1	1.00	0.35	3.92	0.00	0.19	0.48	0.00	0.00
time (sec)	N/A	0.315	10.074	1.757	0.000	0.101	1.762	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	90	949	0	56	128	0	0
N.S.	1	1.00	0.34	3.61	0.00	0.21	0.49	0.00	0.00
time (sec)	N/A	0.301	10.064	1.738	0.000	0.099	2.378	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	497	91	952	0	49	112	0	0
N.S.	1	1.00	0.18	1.92	0.00	0.10	0.23	0.00	0.00
time (sec)	N/A	0.525	10.045	1.733	0.000	0.154	2.402	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	488	488	93	1012	0	55	128	0	0
N.S.	1	1.00	0.19	2.07	0.00	0.11	0.26	0.00	0.00
time (sec)	N/A	0.535	10.052	1.739	0.000	0.236	1.985	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	89	1004	0	52	124	0	0
N.S.	1	1.00	0.37	4.17	0.00	0.22	0.51	0.00	0.00
time (sec)	N/A	0.318	10.054	1.715	0.000	0.095	1.233	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	89	950	0	57	129	0	0
N.S.	1	1.00	0.36	3.83	0.00	0.23	0.52	0.00	0.00
time (sec)	N/A	0.307	10.052	1.743	0.000	0.105	1.393	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	549	545	90	953	0	53	114	0	0
N.S.	1	0.99	0.16	1.74	0.00	0.10	0.21	0.00	0.00
time (sec)	N/A	0.570	10.040	1.694	0.000	0.119	1.384	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	536	92	1013	0	56	129	0	0
N.S.	1	0.99	0.17	1.88	0.00	0.10	0.24	0.00	0.00
time (sec)	N/A	0.559	10.043	1.748	0.000	0.283	1.378	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	495	75	720	0	43	78	0	0
N.S.	1	1.01	0.15	1.47	0.00	0.09	0.16	0.00	0.00
time (sec)	N/A	0.491	0.017	1.493	0.000	0.085	0.951	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	508	75	681	0	47	82	0	0
N.S.	1	1.01	0.15	1.35	0.00	0.09	0.16	0.00	0.00
time (sec)	N/A	0.494	10.037	1.510	0.000	0.093	1.052	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	522	76	683	0	43	73	0	0
N.S.	1	1.01	0.15	1.33	0.00	0.08	0.14	0.00	0.00
time (sec)	N/A	0.543	10.029	1.510	0.000	0.085	1.022	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	513	78	726	0	47	83	0	0
N.S.	1	1.01	0.15	1.43	0.00	0.09	0.16	0.00	0.00
time (sec)	N/A	0.517	10.032	1.526	0.000	0.132	1.017	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	247	42	33	0	18	61	0	373
N.S.	1	1.00	0.17	0.13	0.00	0.07	0.25	0.00	1.52
time (sec)	N/A	0.338	10.020	1.645	0.000	0.120	0.742	0.000	8.979

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	272	38	29	0	18	65	0	406
N.S.	1	1.00	0.14	0.11	0.00	0.07	0.24	0.00	1.50
time (sec)	N/A	0.369	10.019	1.661	0.000	0.301	0.848	0.000	9.001

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	275	278	58	65	0	18	56	0	374
N.S.	1	1.01	0.21	0.24	0.00	0.07	0.20	0.00	1.36
time (sec)	N/A	0.361	10.034	1.622	0.000	0.085	0.781	0.000	8.961

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	262	62	36	0	18	66	0	405
N.S.	1	1.00	0.24	0.14	0.00	0.07	0.25	0.00	1.55
time (sec)	N/A	0.360	10.032	1.582	0.000	0.084	0.820	0.000	8.965

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	134	34	126	39057	126	227	182
N.S.	1	1.00	1.54	0.39	1.45	448.93	1.45	2.61	2.09
time (sec)	N/A	0.242	0.035	1.473	0.289	1.803	0.451	0.285	9.250

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	184	32	207	41851	124	213	160
N.S.	1	1.00	0.84	0.15	0.95	191.10	0.57	0.97	0.73
time (sec)	N/A	0.357	0.088	1.471	0.297	1.214	0.434	0.283	9.063

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	115	168	69	157	40560	156	254	283
N.S.	1	1.05	1.53	0.63	1.43	368.73	1.42	2.31	2.57
time (sec)	N/A	0.282	0.154	1.498	0.281	1.878	0.594	0.274	9.190

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	246	224	66	238	43065	155	238	282
N.S.	1	1.02	0.93	0.27	0.99	178.69	0.64	0.99	1.17
time (sec)	N/A	0.393	0.198	1.479	0.293	1.492	0.585	0.279	9.464

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	150	193	89	186	40637	194	272	315
N.S.	1	1.10	1.42	0.65	1.37	298.80	1.43	2.00	2.32
time (sec)	N/A	0.349	0.141	1.518	0.284	1.211	1.151	0.283	9.566

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	280	249	86	269	43180	192	256	315
N.S.	1	1.05	0.94	0.32	1.01	162.33	0.72	0.96	1.18
time (sec)	N/A	0.456	0.198	1.480	0.294	2.189	1.137	0.277	9.415

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	184	217	113	223	40780	231	296	351
N.S.	1	1.14	1.34	0.70	1.38	251.73	1.43	1.83	2.17
time (sec)	N/A	0.419	0.179	1.484	0.286	1.265	0.890	0.283	9.441

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	313	274	110	304	43302	231	280	350
N.S.	1	1.08	0.94	0.38	1.04	148.80	0.79	0.96	1.20
time (sec)	N/A	0.534	0.290	1.508	0.288	4.058	0.864	0.288	0.317

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	42	40	35	35	313	37	100
N.S.	1	1.00	1.75	1.67	1.46	1.46	13.04	1.54	4.17
time (sec)	N/A	0.180	0.019	1.478	0.276	0.282	0.284	0.271	9.375

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	99	27	86	34376	83	86	71
N.S.	1	1.00	1.01	0.28	0.88	350.78	0.85	0.88	0.72
time (sec)	N/A	0.244	0.085	1.504	0.280	1.224	0.266	0.265	0.104

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	187	39	153	120560	471	259	725
N.S.	1	1.00	1.61	0.34	1.32	1039.31	4.06	2.23	6.25
time (sec)	N/A	0.276	0.050	1.508	0.286	2.582	5.327	0.276	9.881

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	229	37	257	121386	466	271	712
N.S.	1	1.00	0.83	0.13	0.93	438.22	1.68	0.98	2.57
time (sec)	N/A	0.388	0.100	1.517	0.283	2.586	5.313	0.273	9.597

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	151	211	83	191	116982	508	306	477
N.S.	1	1.03	1.45	0.57	1.31	801.25	3.48	2.10	3.27
time (sec)	N/A	0.336	0.193	1.620	0.290	3.010	40.489	0.280	9.538

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	313	305	80	294	124258	505	301	472
N.S.	1	1.02	0.99	0.26	0.95	403.44	1.64	0.98	1.53
time (sec)	N/A	0.452	0.324	1.655	0.297	3.375	40.164	0.280	0.357

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	193	244	114	230	118710	0	334	826
N.S.	1	1.08	1.36	0.64	1.28	663.18	0.00	1.87	4.61
time (sec)	N/A	0.426	0.202	1.489	0.287	5.142	0.000	0.291	9.735

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	355	337	111	336	124787	0	330	826
N.S.	1	1.04	0.99	0.33	0.99	365.94	0.00	0.97	2.42
time (sec)	N/A	0.530	0.336	1.489	0.287	7.020	0.000	0.285	9.634

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	233	276	150	279	118903	0	370	874
N.S.	1	1.10	1.31	0.71	1.32	563.52	0.00	1.75	4.14
time (sec)	N/A	0.504	0.264	1.505	0.282	9.179	0.000	0.289	9.951

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	394	369	147	383	124960	0	366	873
N.S.	1	1.06	0.99	0.40	1.03	335.91	0.00	0.98	2.35
time (sec)	N/A	0.627	0.415	1.520	0.284	13.972	0.000	0.275	9.763

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	27	27	24	25	24	27	25	25
N.S.	1	0.96	0.96	0.86	0.89	0.86	0.96	0.89	0.89
time (sec)	N/A	0.163	0.004	1.433	0.197	0.265	0.016	0.266	9.139

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	32	32	27	27	27	29	27	27
N.S.	1	0.97	0.97	0.82	0.82	0.82	0.88	0.82	0.82
time (sec)	N/A	0.167	0.003	1.472	0.207	0.245	0.018	0.268	0.037

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	58	50	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.97	0.83	0.83
time (sec)	N/A	0.238	0.003	1.486	0.197	0.245	0.024	0.266	0.026

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	32	33	27	27	27	31	27	27
N.S.	1	0.97	1.00	0.82	0.82	0.82	0.94	0.82	0.82
time (sec)	N/A	0.161	0.001	1.440	0.194	0.254	0.017	0.266	0.038

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	60	50	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.00	0.83	0.83
time (sec)	N/A	0.202	0.003	1.499	0.219	0.244	0.021	0.258	0.027

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	61	53	53
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.94	0.82	0.82
time (sec)	N/A	0.279	0.005	1.474	0.195	0.253	0.021	0.270	0.029

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	76	76	90	76	76
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83
time (sec)	N/A	0.245	0.005	1.476	0.193	0.243	0.022	0.272	0.042

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	33	16	15	27	29	15	26
N.S.	1	1.00	1.94	0.94	0.88	1.59	1.71	0.88	1.53
time (sec)	N/A	0.143	0.001	1.473	0.198	0.242	0.021	0.276	0.035

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	60	51	50	50	58	50	50
N.S.	1	1.00	1.33	1.13	1.11	1.11	1.29	1.11	1.11
time (sec)	N/A	0.199	0.003	1.499	0.202	0.250	0.022	0.275	0.026

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	53	53	60	53	53
N.S.	1	1.00	1.30	1.08	1.06	1.06	1.20	1.06	1.06
time (sec)	N/A	0.191	0.004	1.500	0.203	0.241	0.029	0.271	0.028

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	76	76	88	76	76
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.14	0.99	0.99
time (sec)	N/A	0.253	0.004	1.508	0.195	0.260	0.022	0.282	0.039

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	53	53	61	53	53
N.S.	1	1.00	1.30	1.08	1.06	1.06	1.22	1.06	1.06
time (sec)	N/A	0.191	0.004	1.535	0.198	0.261	0.027	0.272	0.029

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	76	76	90	76	76
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.17	0.99	0.99
time (sec)	N/A	0.233	0.005	1.483	0.196	0.255	0.024	0.263	0.039

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	79	79	92	79	79
N.S.	1	1.00	1.18	0.98	0.96	0.96	1.12	0.96	0.96
time (sec)	N/A	0.294	0.005	1.503	0.190	0.263	0.023	0.273	0.042

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	102	102	121	102	102
N.S.	1	1.00	1.14	0.94	0.94	0.94	1.11	0.94	0.94
time (sec)	N/A	0.295	0.008	1.461	0.209	0.255	0.022	0.274	9.133

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	150	150	180	150	150
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.19	0.99	0.99
time (sec)	N/A	0.343	0.005	1.501	0.197	0.268	0.025	0.276	9.256

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	160	220	89	200	117016	520	315	483
N.S.	1	1.03	1.42	0.57	1.29	754.94	3.35	2.03	3.12
time (sec)	N/A	0.343	0.151	1.505	0.283	2.996	43.720	0.280	0.367

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	202	253	120	249	118761	0	352	832
N.S.	1	1.07	1.35	0.64	1.32	631.71	0.00	1.87	4.43
time (sec)	N/A	0.416	0.212	1.499	0.281	6.660	0.000	0.278	9.519

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	242	286	156	297	118945	0	388	880
N.S.	1	1.10	1.30	0.71	1.35	540.66	0.00	1.76	4.00
time (sec)	N/A	0.500	0.285	1.534	0.285	8.984	0.000	0.288	9.638

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	134	78	25	123	120	88	97	36
N.S.	1	1.33	0.77	0.25	1.22	1.19	0.87	0.96	0.36
time (sec)	N/A	0.341	0.038	1.452	0.283	0.270	0.176	0.289	0.125

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	16	15	15	19	15	15
N.S.	1	1.00	1.00	0.73	0.68	0.68	0.86	0.68	0.68
time (sec)	N/A	0.146	0.012	1.472	0.280	0.262	0.039	0.275	9.062

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	107	29	147	12348	88	115	119
N.S.	1	1.00	0.87	0.24	1.20	100.39	0.72	0.93	0.97
time (sec)	N/A	0.285	0.058	1.468	0.302	1.044	0.310	0.279	0.210

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	134	78	25	123	128	88	97	32
N.S.	1	1.33	0.77	0.25	1.22	1.27	0.87	0.96	0.32
time (sec)	N/A	0.309	0.019	1.458	0.284	0.257	0.181	0.277	0.143

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	144	113	31	167	493	68	131	315
N.S.	1	1.02	0.80	0.22	1.18	3.50	0.48	0.93	2.23
time (sec)	N/A	0.325	0.052	1.487	0.292	0.279	0.219	0.281	9.212

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	99	33	147	12741	85	114	162
N.S.	1	1.00	0.80	0.27	1.20	103.59	0.69	0.93	1.32
time (sec)	N/A	0.304	0.048	1.485	0.288	1.046	0.309	0.287	0.218

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	129	34	187	46651	292	143	270
N.S.	1	1.00	0.79	0.21	1.15	286.20	1.79	0.88	1.66
time (sec)	N/A	0.314	0.081	1.500	0.276	1.487	2.389	0.278	9.472

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	11	11	10	11	9
N.S.	1	1.00	1.00	0.77	0.85	0.85	0.77	0.85	0.69
time (sec)	N/A	0.139	0.003	1.488	0.193	0.267	0.030	0.273	0.032

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	108	31	149	191	51	109	117
N.S.	1	1.00	0.95	0.27	1.31	1.68	0.45	0.96	1.03
time (sec)	N/A	0.273	0.034	1.465	0.280	0.271	0.161	0.283	0.288

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	37	65	28	113	27	53	93	25
N.S.	1	1.03	1.81	0.78	3.14	0.75	1.47	2.58	0.69
time (sec)	N/A	0.195	0.036	1.467	0.285	0.274	0.185	0.281	0.056

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	128	34	171	17085	199	125	307
N.S.	1	1.00	0.94	0.25	1.26	125.62	1.46	0.92	2.26
time (sec)	N/A	0.290	0.063	1.490	0.286	1.108	0.844	0.281	9.482

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	108	35	152	239	70	109	117
N.S.	1	1.00	0.95	0.31	1.33	2.10	0.61	0.96	1.03
time (sec)	N/A	0.309	0.028	1.492	0.285	0.285	0.145	0.285	0.388

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	148	36	195	513	148	137	286
N.S.	1	1.00	0.96	0.23	1.27	3.33	0.96	0.89	1.86
time (sec)	N/A	0.299	0.099	1.482	0.301	0.294	0.672	0.284	10.292

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	125	38	174	18086	189	124	300
N.S.	1	1.00	0.92	0.28	1.28	132.99	1.39	0.91	2.21
time (sec)	N/A	0.338	0.065	1.485	0.289	1.151	0.833	0.289	10.470

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	164	39	207	54479	580	149	1168
N.S.	1	1.00	0.93	0.22	1.18	309.54	3.30	0.85	6.64
time (sec)	N/A	0.339	0.122	1.500	0.276	1.793	4.670	0.286	10.149

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.146	0.001	1.475	0.197	0.258	0.022	0.267	0.023

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	31	76	151	73	70	156
N.S.	1	1.00	0.94	0.58	1.43	2.85	1.38	1.32	2.94
time (sec)	N/A	0.211	0.039	1.515	0.282	0.427	0.170	0.265	0.444

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	203	38	160	91748	187	290	312
N.S.	1	1.00	1.64	0.31	1.29	739.90	1.51	2.34	2.52
time (sec)	N/A	0.274	0.048	1.483	0.292	2.281	0.968	0.271	9.462

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	283	36	296	96349	187	270	305
N.S.	1	1.00	1.02	0.13	1.07	347.83	0.68	0.97	1.10
time (sec)	N/A	0.411	0.192	1.480	0.283	2.070	0.927	0.269	9.512

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	249	65	202	592528	0	299	5082
N.S.	1	1.00	1.68	0.44	1.36	4003.57	0.00	2.02	34.34
time (sec)	N/A	0.375	0.079	1.508	0.281	42.648	0.000	0.273	9.890

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	180	221	108	224	334837	0	339	1393
N.S.	1	1.05	1.28	0.63	1.30	1946.73	0.00	1.97	8.10
time (sec)	N/A	0.387	0.314	1.524	0.311	30.976	0.000	0.276	9.841

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	235	263	147	284	343626	0	387	1002
N.S.	1	1.06	1.19	0.67	1.29	1554.87	0.00	1.75	4.53
time (sec)	N/A	0.544	0.520	1.512	0.372	42.834	0.000	0.435	9.665

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	285	313	188	345	343822	0	435	1056
N.S.	1	1.07	1.18	0.71	1.30	1292.56	0.00	1.64	3.97
time (sec)	N/A	0.652	0.322	1.536	0.295	59.773	0.000	0.360	9.823

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	311	58	328	622377	0	338	5042
N.S.	1	1.00	0.97	0.18	1.03	1951.03	0.00	1.06	15.81
time (sec)	N/A	0.518	0.338	1.501	0.295	43.661	0.000	0.476	9.946

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	349	319	105	350	352423	0	360	1383
N.S.	1	1.02	0.94	0.31	1.03	1033.50	0.00	1.06	4.06
time (sec)	N/A	0.527	0.195	1.525	0.295	26.609	0.000	0.273	9.884

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	408	366	144	412	358509	0	410	1001
N.S.	1	1.04	0.93	0.37	1.05	909.92	0.00	1.04	2.54
time (sec)	N/A	0.669	0.287	1.530	0.310	58.341	0.000	0.276	9.754

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	456	411	185	472	358702	0	459	1053
N.S.	1	1.04	0.94	0.42	1.08	820.83	0.00	1.05	2.41
time (sec)	N/A	0.783	0.370	1.514	0.296	82.838	0.000	0.286	9.843

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	15	15	15	15	15
N.S.	1	1.00	0.82	0.73	1.36	1.36	1.36	1.36	1.36
time (sec)	N/A	0.156	0.002	1.455	0.206	0.267	0.031	0.277	0.032

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	14	8	12	12	8	12	11
N.S.	1	1.00	1.27	0.73	1.09	1.09	0.73	1.09	1.00
time (sec)	N/A	0.156	0.001	1.465	0.195	0.264	0.027	0.259	0.024

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	11	9	7	7	7	5	7	6
N.S.	1	1.22	1.00	0.78	0.78	0.78	0.56	0.78	0.67
time (sec)	N/A	0.145	0.001	0.020	0.335	0.270	0.020	0.262	0.018

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.149	0.001	1.476	0.203	0.256	0.025	0.270	0.002

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	5	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.00
time (sec)	N/A	0.157	0.001	1.640	0.230	0.267	0.038	0.264	0.032

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	12	12	10	7	7
N.S.	1	1.00	0.82	0.73	1.09	1.09	0.91	0.64	0.64
time (sec)	N/A	0.156	0.002	1.633	0.207	0.275	0.050	0.264	9.102

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	17	17	17	7	7
N.S.	1	1.00	0.82	0.73	1.55	1.55	1.55	0.64	0.64
time (sec)	N/A	0.156	0.001	1.597	0.206	0.249	0.057	0.259	9.124

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	256	74	222	0	0	338	2478
N.S.	1	1.00	1.55	0.45	1.35	0.00	0.00	2.05	15.02
time (sec)	N/A	0.432	0.237	1.580	0.304	0.000	0.000	0.272	9.917

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	301	95	240	0	0	430	3810
N.S.	1	1.00	1.60	0.51	1.28	0.00	0.00	2.29	20.27
time (sec)	N/A	0.489	0.320	1.564	0.340	0.000	0.000	0.288	9.425

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	318	102	257	0	0	445	5673
N.S.	1	1.00	1.55	0.50	1.25	0.00	0.00	2.17	27.67
time (sec)	N/A	0.500	0.306	1.573	0.290	0.000	0.000	0.283	9.556

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	342	73	351	0	0	373	2469
N.S.	1	1.00	1.01	0.22	1.04	0.00	0.00	1.11	7.33
time (sec)	N/A	0.584	0.330	1.650	0.290	0.000	0.000	0.280	9.940

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	427	88	399	0	0	443	3798
N.S.	1	1.00	1.11	0.23	1.04	0.00	0.00	1.15	9.89
time (sec)	N/A	0.686	0.255	1.603	0.286	0.000	0.000	0.273	9.422

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	445	103	429	0	0	459	5664
N.S.	1	1.00	1.11	0.26	1.07	0.00	0.00	1.14	14.09
time (sec)	N/A	0.699	0.280	1.524	0.291	0.000	0.000	0.281	9.525

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	192	257	127	243	710521	0	375	1626
N.S.	1	1.04	1.40	0.69	1.32	3861.53	0.00	2.04	8.84
time (sec)	N/A	0.429	0.151	1.531	0.283	212.801	0.000	0.273	9.941

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	211	302	137	260	0	0	460	2611
N.S.	1	1.04	1.49	0.67	1.28	0.00	0.00	2.27	12.86
time (sec)	N/A	0.490	0.227	1.535	0.292	0.000	0.000	0.280	9.964

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	228	338	157	299	0	0	487	3943
N.S.	1	1.01	1.50	0.70	1.33	0.00	0.00	2.16	17.52
time (sec)	N/A	0.552	0.219	1.539	0.299	0.000	0.000	0.277	10.223

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	361	359	124	374	0	0	393	1623
N.S.	1	1.02	1.02	0.35	1.06	0.00	0.00	1.11	4.60
time (sec)	N/A	0.576	0.272	1.532	0.289	0.000	0.000	0.279	9.939

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	403	415	133	416	0	0	460	2605
N.S.	1	1.02	1.05	0.34	1.05	0.00	0.00	1.16	6.59
time (sec)	N/A	0.665	0.357	1.523	0.300	0.000	0.000	0.276	10.167

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	420	460	154	458	0	0	486	3939
N.S.	1	1.01	1.10	0.37	1.10	0.00	0.00	1.17	9.45
time (sec)	N/A	0.737	0.302	1.550	0.302	0.000	0.000	0.275	10.393

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	262	309	173	316	0	0	434	1687
N.S.	1	1.09	1.28	0.72	1.31	0.00	0.00	1.80	7.00
time (sec)	N/A	0.623	0.273	1.545	0.303	0.000	0.000	0.281	9.885

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	288	359	190	343	0	0	529	2680
N.S.	1	1.07	1.34	0.71	1.28	0.00	0.00	1.97	10.00
time (sec)	N/A	0.711	0.339	1.538	0.290	0.000	0.000	0.278	10.073

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	299	380	206	377	0	0	561	2696
N.S.	1	1.05	1.33	0.72	1.32	0.00	0.00	1.97	9.46
time (sec)	N/A	0.678	0.282	1.622	0.309	0.000	0.000	0.287	10.023

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	434	411	170	446	0	0	453	1686
N.S.	1	1.05	1.00	0.41	1.08	0.00	0.00	1.10	4.08
time (sec)	N/A	0.748	0.362	1.536	0.293	0.000	0.000	0.274	10.017

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	483	473	186	497	0	0	530	2680
N.S.	1	1.04	1.02	0.40	1.07	0.00	0.00	1.14	5.79
time (sec)	N/A	0.876	0.517	1.560	0.319	0.000	0.000	0.284	10.476

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	494	500	201	535	0	0	562	2695
N.S.	1	1.03	1.04	0.42	1.11	0.00	0.00	1.17	5.61
time (sec)	N/A	0.860	0.405	1.625	0.290	0.000	0.000	0.276	10.188

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	323	360	219	389	0	0	494	1747
N.S.	1	1.10	1.23	0.75	1.33	0.00	0.00	1.69	5.96
time (sec)	N/A	0.797	0.381	1.547	0.290	0.000	0.000	0.283	10.136

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	356	422	240	429	0	0	601	2747
N.S.	1	1.08	1.27	0.73	1.30	0.00	0.00	1.82	8.30
time (sec)	N/A	0.875	0.335	1.570	0.303	0.000	0.000	0.289	10.321

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	368	439	256	463	0	0	633	2764
N.S.	1	1.05	1.26	0.73	1.33	0.00	0.00	1.81	7.92
time (sec)	N/A	0.860	0.337	1.603	0.299	0.000	0.000	0.282	10.572

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	492	461	216	517	0	0	514	1743
N.S.	1	1.06	1.00	0.47	1.12	0.00	0.00	1.11	3.77
time (sec)	N/A	0.944	0.463	1.544	0.301	0.000	0.000	0.274	10.098

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	516	544	530	237	579	0	0	603	2741
N.S.	1	1.05	1.03	0.46	1.12	0.00	0.00	1.17	5.31
time (sec)	N/A	1.043	0.715	1.552	0.307	0.000	0.000	0.273	10.162

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	556	555	253	613	0	0	635	2757
N.S.	1	1.04	1.04	0.47	1.15	0.00	0.00	1.19	5.16
time (sec)	N/A	1.035	0.529	1.579	0.323	0.000	0.000	0.284	10.448

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	79	96	0	72	61	0	0
N.S.	1	1.00	0.65	0.79	0.00	0.60	0.50	0.00	0.00
time (sec)	N/A	0.254	10.065	1.565	0.000	0.143	1.035	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	90	0	79	95	0	0
N.S.	1	1.00	0.93	1.03	0.00	0.91	1.09	0.00	0.00
time (sec)	N/A	0.248	10.059	1.564	0.000	0.108	1.138	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	83	95	0	73	90	0	0
N.S.	1	1.00	0.93	1.07	0.00	0.82	1.01	0.00	0.00
time (sec)	N/A	0.246	10.049	1.548	0.000	0.122	1.134	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	85	101	0	82	66	0	0
N.S.	1	1.00	0.67	0.80	0.00	0.65	0.52	0.00	0.00
time (sec)	N/A	0.262	10.050	1.561	0.000	0.112	1.113	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	131	193	0	128	102	0	0
N.S.	1	1.00	0.51	0.75	0.00	0.50	0.40	0.00	0.00
time (sec)	N/A	0.357	10.117	1.577	0.000	0.113	1.260	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	80	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	5.71	0.86	0.86
time (sec)	N/A	0.138	1.775	1.529	0.233	0.306	2.844	0.282	9.034

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	24	25	34	104	22	23
N.S.	1	1.00	0.93	0.83	0.86	1.17	3.59	0.76	0.79
time (sec)	N/A	0.178	10.042	1.605	0.234	0.289	3.843	0.280	8.955

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	27	24	23	33	109	22	20
N.S.	1	1.00	1.08	0.96	0.92	1.32	4.36	0.88	0.80
time (sec)	N/A	0.183	10.046	1.609	0.237	0.291	4.165	0.277	8.953

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	44	44	133	30	29
N.S.	1	1.00	1.00	0.92	1.16	1.16	3.50	0.79	0.76
time (sec)	N/A	0.190	10.051	1.642	0.235	0.281	5.197	0.275	9.022

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	58	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	4.83	0.83	0.83
time (sec)	N/A	0.135	0.229	1.502	0.314	0.273	1.633	0.263	8.932

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	281	287	0	204	260	0	0
N.S.	1	1.00	0.73	0.75	0.00	0.53	0.68	0.00	0.00
time (sec)	N/A	0.688	10.225	2.011	0.000	0.174	3.294	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	145	51	45	0	835	1287	101	64
N.S.	1	1.33	0.47	0.41	0.00	7.66	11.81	0.93	0.59
time (sec)	N/A	0.440	0.013	1.492	0.000	0.989	0.577	0.284	9.139

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	143	47	41	0	799	1287	101	65
N.S.	1	1.31	0.43	0.38	0.00	7.33	11.81	0.93	0.60
time (sec)	N/A	0.359	0.012	1.483	0.000	1.024	0.554	0.280	0.201

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	207	187	195	209	210	216	240	237
N.S.	1	1.00	0.90	0.94	1.00	1.01	1.04	1.15	1.14
time (sec)	N/A	0.500	0.101	1.505	0.203	0.276	0.591	0.261	9.073

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	169	154	159	169	170	172	192	189
N.S.	1	0.99	0.91	0.94	0.99	1.00	1.01	1.13	1.11
time (sec)	N/A	0.444	0.072	1.500	0.197	0.295	0.546	0.274	9.081

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	131	119	123	129	130	128	144	141
N.S.	1	0.99	0.90	0.93	0.98	0.98	0.97	1.09	1.07
time (sec)	N/A	0.369	0.061	1.490	0.199	0.279	0.519	0.266	9.050

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	94	88	89	91	92	88	98	96
N.S.	1	0.98	0.92	0.93	0.95	0.96	0.92	1.02	1.00
time (sec)	N/A	0.331	0.044	1.535	0.201	0.266	0.479	0.259	8.967

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	81	75	75	77	80	70	77	76
N.S.	1	1.01	0.94	0.94	0.96	1.00	0.88	0.96	0.95
time (sec)	N/A	0.309	0.035	1.540	0.197	0.317	2.392	0.271	9.101

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	79	77	75	77	85	70	94	74
N.S.	1	0.98	0.95	0.93	0.95	1.05	0.86	1.16	0.91
time (sec)	N/A	0.327	0.048	1.510	0.208	0.312	49.265	0.269	9.139

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	96	88	90	93	101	0	123	92
N.S.	1	1.01	0.93	0.95	0.98	1.06	0.00	1.29	0.97
time (sec)	N/A	0.338	0.073	1.514	0.191	0.331	0.000	0.259	9.129

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	129	128	120	125	127	0	180	123
N.S.	1	1.01	1.00	0.94	0.98	0.99	0.00	1.41	0.96
time (sec)	N/A	0.375	0.067	1.507	0.207	0.320	0.000	0.267	9.229

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	165	164	156	166	168	0	230	161
N.S.	1	1.01	1.00	0.95	1.01	1.02	0.00	1.40	0.98
time (sec)	N/A	0.410	0.069	1.515	0.197	0.325	0.000	0.276	9.807

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	206	194	193	208	210	0	281	200
N.S.	1	1.00	0.95	0.94	1.01	1.02	0.00	1.37	0.98
time (sec)	N/A	0.454	0.152	1.527	0.198	0.339	0.000	0.263	0.262

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	353	351	242	351	342	469	446	358
N.S.	1	1.01	1.01	0.70	1.01	0.98	1.35	1.28	1.03
time (sec)	N/A	0.561	0.092	1.537	0.281	0.305	0.983	0.276	0.323

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	321	311	204	313	321	513	434	313
N.S.	1	1.02	0.98	0.65	0.99	1.02	1.62	1.37	0.99
time (sec)	N/A	0.521	0.110	1.540	0.277	0.304	0.847	0.277	9.855

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	317	306	194	311	304	423	394	311
N.S.	1	1.02	0.98	0.62	1.00	0.97	1.36	1.26	1.00
time (sec)	N/A	0.502	0.106	1.518	0.288	0.316	0.945	0.282	9.921

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	284	266	156	269	281	469	380	267
N.S.	1	1.02	0.95	0.56	0.96	1.01	1.68	1.36	0.96
time (sec)	N/A	0.473	0.109	1.532	0.293	0.298	0.770	0.287	10.583

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	275	264	146	267	249	376	340	264
N.S.	1	1.00	0.96	0.53	0.97	0.91	1.37	1.24	0.96
time (sec)	N/A	0.480	0.106	1.531	0.304	0.310	0.916	0.269	10.119

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	250	231	112	225	568	427	286	225
N.S.	1	1.02	0.94	0.46	0.92	2.32	1.74	1.17	0.92
time (sec)	N/A	0.406	0.148	1.525	0.304	0.307	0.775	0.275	9.760

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	229	104	223	600	342	248	222
N.S.	1	1.00	0.95	0.43	0.93	2.50	1.42	1.03	0.92
time (sec)	N/A	0.370	0.148	1.537	0.309	0.305	0.869	0.284	9.753

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	224	159	217	560	408	265	204
N.S.	1	1.00	0.99	0.70	0.96	2.47	1.80	1.17	0.90
time (sec)	N/A	0.396	0.114	1.542	0.288	0.307	1.222	0.266	9.502

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	218	155	214	565	326	228	201
N.S.	1	1.00	0.97	0.69	0.96	2.52	1.46	1.02	0.90
time (sec)	N/A	0.382	0.108	1.547	0.284	0.312	1.464	0.272	0.288

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	220	159	217	556	411	258	209
N.S.	1	1.00	0.97	0.70	0.96	2.45	1.81	1.14	0.92
time (sec)	N/A	0.401	0.109	1.553	0.284	0.318	3.954	0.258	9.299

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	220	155	214	584	328	217	207
N.S.	1	1.00	0.98	0.69	0.95	2.60	1.46	0.96	0.92
time (sec)	N/A	0.389	0.087	1.552	0.287	0.318	7.487	0.270	9.216

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	231	170	234	610	432	271	219
N.S.	1	1.00	0.95	0.70	0.97	2.52	1.79	1.12	0.90
time (sec)	N/A	0.418	0.123	1.534	0.279	0.309	43.346	0.281	9.259

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	231	170	234	595	0	293	220
N.S.	1	1.00	0.95	0.70	0.96	2.44	0.00	1.20	0.90
time (sec)	N/A	0.388	0.128	1.551	0.291	0.307	0.000	0.270	9.161

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	266	205	260	262	0	371	253
N.S.	1	1.00	0.96	0.74	0.94	0.95	0.00	1.34	0.91
time (sec)	N/A	0.453	0.140	1.528	0.290	0.289	0.000	0.275	9.250

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	266	205	260	295	0	333	253
N.S.	1	1.00	0.95	0.73	0.93	1.05	0.00	1.19	0.90
time (sec)	N/A	0.439	0.138	1.531	0.297	0.301	0.000	0.276	9.203

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	308	239	307	317	0	413	286
N.S.	1	1.00	0.98	0.76	0.98	1.01	0.00	1.32	0.91
time (sec)	N/A	0.482	0.110	1.563	0.290	0.295	0.000	0.278	9.570

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	311	240	307	335	0	387	287
N.S.	1	1.00	0.99	0.76	0.97	1.06	0.00	1.23	0.91
time (sec)	N/A	0.474	0.118	1.545	0.289	0.295	0.000	0.267	0.295

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	346	277	353	355	0	467	323
N.S.	1	1.00	0.99	0.79	1.01	1.01	0.00	1.33	0.92
time (sec)	N/A	0.517	0.124	1.657	0.292	0.304	0.000	0.275	10.004

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	216	205	217	222	303	236	293	356
N.S.	1	0.98	0.93	0.99	1.01	1.38	1.07	1.33	1.62
time (sec)	N/A	0.542	0.140	1.674	0.207	0.281	13.529	0.266	9.653

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	177	167	179	180	257	189	242	233
N.S.	1	0.98	0.93	0.99	1.00	1.43	1.05	1.34	1.29
time (sec)	N/A	0.463	0.120	1.520	0.199	0.293	12.664	0.271	9.730

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	135	129	138	138	202	141	212	155
N.S.	1	0.96	0.92	0.99	0.99	1.44	1.01	1.51	1.11
time (sec)	N/A	0.400	0.101	1.521	0.195	0.314	11.380	0.280	9.828

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	99	93	100	98	143	100	205	103
N.S.	1	0.96	0.90	0.97	0.95	1.39	0.97	1.99	1.00
time (sec)	N/A	0.343	0.065	1.542	0.200	0.259	5.434	0.273	0.090

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	98	95	97	100	145	0	123	100
N.S.	1	0.98	0.95	0.97	1.00	1.45	0.00	1.23	1.00
time (sec)	N/A	0.340	0.120	1.500	0.199	0.305	0.000	0.269	9.616

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	108	97	101	116	172	0	130	109
N.S.	1	0.99	0.89	0.93	1.06	1.58	0.00	1.19	1.00
time (sec)	N/A	0.353	0.079	1.495	0.201	0.305	0.000	0.269	9.794

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	128	118	123	138	208	0	196	130
N.S.	1	0.98	0.91	0.95	1.06	1.60	0.00	1.51	1.00
time (sec)	N/A	0.396	0.124	1.497	0.206	0.295	0.000	0.275	9.745

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	174	160	169	181	261	0	269	175
N.S.	1	0.99	0.91	0.97	1.03	1.49	0.00	1.54	1.00
time (sec)	N/A	0.444	0.112	1.501	0.210	0.311	0.000	0.278	9.651

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	212	198	209	226	310	0	324	216
N.S.	1	0.99	0.93	0.98	1.06	1.45	0.00	1.51	1.01
time (sec)	N/A	0.486	0.215	1.518	0.202	0.321	0.000	0.283	10.334

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	379	364	244	369	488	500	443	481
N.S.	1	1.03	0.99	0.66	1.00	1.32	1.36	1.20	1.30
time (sec)	N/A	0.953	0.428	1.528	0.337	0.295	85.665	0.269	0.354

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	358	319	204	325	455	0	435	362
N.S.	1	1.07	0.95	0.61	0.97	1.36	0.00	1.30	1.08
time (sec)	N/A	1.160	0.186	1.535	0.289	0.322	0.000	0.282	10.370

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	337	315	193	321	423	449	387	358
N.S.	1	1.03	0.96	0.59	0.98	1.29	1.37	1.18	1.09
time (sec)	N/A	0.700	0.279	1.543	0.291	0.321	76.438	0.272	10.180

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	315	282	157	277	920	0	338	287
N.S.	1	1.06	0.95	0.53	0.93	3.09	0.00	1.13	0.96
time (sec)	N/A	0.795	0.183	1.540	0.282	0.373	0.000	0.272	10.322

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	296	277	147	270	946	401	289	280
N.S.	1	1.03	0.96	0.51	0.94	3.28	1.39	1.00	0.97
time (sec)	N/A	0.613	0.182	1.557	0.287	0.323	67.315	0.261	0.344

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	280	255	130	259	874	0	313	246
N.S.	1	1.03	0.94	0.48	0.96	3.23	0.00	1.15	0.91
time (sec)	N/A	0.541	0.170	1.526	0.290	0.364	0.000	0.277	10.059

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	217	251	123	254	861	377	268	241
N.S.	1	0.82	0.95	0.47	0.96	3.26	1.43	1.02	0.91
time (sec)	N/A	0.504	0.169	1.526	0.293	0.322	3.097	0.269	9.768

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	275	255	187	258	860	457	301	244
N.S.	1	1.04	0.96	0.71	0.97	3.25	1.72	1.14	0.92
time (sec)	N/A	0.506	0.176	1.581	0.297	0.335	63.973	0.279	9.511

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	271	250	183	258	902	0	257	245
N.S.	1	1.04	0.96	0.70	0.99	3.47	0.00	0.99	0.94
time (sec)	N/A	0.493	0.173	1.539	0.300	0.300	0.000	0.274	9.472

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	285	255	195	267	902	0	306	247
N.S.	1	1.06	0.95	0.72	0.99	3.35	0.00	1.14	0.92
time (sec)	N/A	0.519	0.181	1.545	0.298	0.394	0.000	0.277	9.592

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	285	253	193	268	897	0	260	248
N.S.	1	1.06	0.94	0.71	0.99	3.32	0.00	0.96	0.92
time (sec)	N/A	0.515	0.181	1.544	0.296	0.421	0.000	0.273	9.439

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	314	281	215	292	982	0	328	274
N.S.	1	1.06	0.95	0.72	0.98	3.31	0.00	1.10	0.92
time (sec)	N/A	0.690	0.192	1.562	0.310	0.422	0.000	0.265	9.421

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	314	280	214	292	959	0	342	274
N.S.	1	1.06	0.94	0.72	0.98	3.23	0.00	1.15	0.92
time (sec)	N/A	0.689	0.191	1.579	0.304	0.399	0.000	0.268	9.519

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	356	319	251	323	442	0	431	310
N.S.	1	1.07	0.96	0.75	0.97	1.32	0.00	1.29	0.93
time (sec)	N/A	0.937	0.198	1.555	0.307	0.673	0.000	0.278	9.798

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	356	317	250	323	475	0	385	310
N.S.	1	1.06	0.95	0.75	0.96	1.42	0.00	1.15	0.93
time (sec)	N/A	0.913	0.208	1.554	0.294	0.270	0.000	0.267	9.523

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	397	370	287	374	507	0	475	348
N.S.	1	1.06	0.99	0.77	1.00	1.35	0.00	1.27	0.93
time (sec)	N/A	1.057	0.390	1.565	0.297	0.283	0.000	0.273	9.716

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	262	246	260	275	396	0	341	449
N.S.	1	0.98	0.92	0.98	1.03	1.49	0.00	1.28	1.69
time (sec)	N/A	0.625	0.152	1.526	0.204	0.250	0.000	0.282	9.236

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	223	208	218	233	353	0	291	293
N.S.	1	0.99	0.92	0.96	1.03	1.56	0.00	1.29	1.30
time (sec)	N/A	0.534	0.125	1.504	0.201	0.255	0.000	0.277	9.362

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	181	170	176	191	295	0	230	204
N.S.	1	0.97	0.91	0.95	1.03	1.59	0.00	1.24	1.10
time (sec)	N/A	0.484	0.103	1.505	0.199	0.259	0.000	0.278	9.567

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	142	145	137	147	225	0	142	152
N.S.	1	0.97	0.99	0.94	1.01	1.54	0.00	0.97	1.04
time (sec)	N/A	0.409	0.076	1.558	0.202	0.272	0.000	0.269	0.112

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	105	105	99	109	158	0	96	112
N.S.	1	0.96	0.96	0.91	1.00	1.45	0.00	0.88	1.03
time (sec)	N/A	0.358	0.061	1.520	0.201	0.252	0.000	0.273	9.161

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	115	104	113	129	187	0	126	123
N.S.	1	1.01	0.91	0.99	1.13	1.64	0.00	1.11	1.08
time (sec)	N/A	0.374	0.109	1.530	0.203	0.272	0.000	0.273	0.188

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	133	121	115	144	250	0	172	135
N.S.	1	0.99	0.90	0.86	1.07	1.87	0.00	1.28	1.01
time (sec)	N/A	0.395	0.093	1.533	0.207	0.263	0.000	0.273	9.154

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	161	149	153	182	316	0	185	167
N.S.	1	0.99	0.91	0.94	1.12	1.94	0.00	1.13	1.02
time (sec)	N/A	0.430	0.112	1.518	0.211	0.278	0.000	0.276	9.352

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	217	200	212	232	396	0	317	222
N.S.	1	1.00	0.92	0.97	1.06	1.82	0.00	1.45	1.02
time (sec)	N/A	0.507	0.149	1.528	0.211	0.285	0.000	0.275	9.496

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	256	238	253	280	448	0	372	265
N.S.	1	0.99	0.92	0.98	1.09	1.74	0.00	1.44	1.03
time (sec)	N/A	0.557	0.199	1.525	0.213	0.312	0.000	0.279	0.323

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	447	411	284	424	667	0	491	575
N.S.	1	1.07	0.99	0.68	1.02	1.60	0.00	1.18	1.38
time (sec)	N/A	1.513	0.514	1.623	0.322	0.275	0.000	0.277	9.483

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	426	380	244	380	634	0	483	425
N.S.	1	1.11	0.99	0.64	0.99	1.65	0.00	1.26	1.11
time (sec)	N/A	1.944	0.434	1.543	0.286	0.283	0.000	0.277	9.353

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	407	362	233	376	602	0	435	420
N.S.	1	1.09	0.97	0.62	1.00	1.61	0.00	1.16	1.12
time (sec)	N/A	1.326	0.384	1.543	0.291	0.281	0.000	0.279	9.627

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	378	329	195	330	1278	0	384	338
N.S.	1	1.10	0.95	0.57	0.96	3.70	0.00	1.11	0.98
time (sec)	N/A	1.240	0.247	1.544	0.284	0.294	0.000	0.280	9.661

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	364	323	186	326	1318	0	338	335
N.S.	1	1.08	0.96	0.55	0.97	3.92	0.00	1.01	1.00
time (sec)	N/A	0.901	0.352	1.543	0.296	0.287	0.000	0.272	9.328

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	337	300	166	311	1224	0	359	295
N.S.	1	1.07	0.95	0.53	0.98	3.87	0.00	1.14	0.93
time (sec)	N/A	0.880	0.224	1.527	0.313	0.294	0.000	0.267	9.263

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	275	294	159	305	1213	0	313	290
N.S.	1	0.90	0.96	0.52	0.99	3.95	0.00	1.02	0.94
time (sec)	N/A	0.760	0.220	1.531	0.291	0.311	0.000	0.277	9.303

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	263	284	151	296	1158	0	334	280
N.S.	1	0.87	0.94	0.50	0.98	3.85	0.00	1.11	0.93
time (sec)	N/A	0.638	0.221	1.565	0.299	0.285	0.000	0.272	9.207

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	253	279	146	291	1184	0	290	275
N.S.	1	0.87	0.96	0.50	1.00	4.05	0.00	0.99	0.94
time (sec)	N/A	0.535	0.212	1.555	0.286	0.279	0.000	0.271	9.150

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	269	286	218	300	1206	0	336	276
N.S.	1	0.89	0.94	0.72	0.99	3.98	0.00	1.11	0.91
time (sec)	N/A	0.604	0.233	1.556	0.288	0.308	0.000	0.271	11.933

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	262	283	216	302	1217	0	307	279
N.S.	1	0.87	0.94	0.72	1.00	4.04	0.00	1.02	0.93
time (sec)	N/A	0.602	0.224	1.583	0.286	0.294	0.000	0.266	10.052

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	339	303	230	317	1254	0	352	293
N.S.	1	1.07	0.96	0.73	1.00	3.96	0.00	1.11	0.92
time (sec)	N/A	0.642	0.233	1.657	0.292	0.295	0.000	0.278	9.328

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	337	299	228	318	1247	0	305	293
N.S.	1	1.07	0.95	0.72	1.01	3.95	0.00	0.97	0.93
time (sec)	N/A	0.630	0.240	1.556	0.291	0.303	0.000	0.279	0.325

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	374	328	253	343	1340	0	374	321
N.S.	1	1.09	0.96	0.74	1.00	3.91	0.00	1.09	0.94
time (sec)	N/A	0.998	0.245	1.545	0.287	0.296	0.000	0.271	9.339

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	372	324	252	343	1317	0	388	321
N.S.	1	1.09	0.95	0.74	1.01	3.86	0.00	1.14	0.94
time (sec)	N/A	0.973	0.248	1.554	0.294	0.285	0.000	0.271	9.286

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	416	366	289	376	621	0	479	359
N.S.	1	1.09	0.96	0.76	0.99	1.63	0.00	1.26	0.94
time (sec)	N/A	1.420	0.438	1.554	0.294	0.267	0.000	0.273	9.395

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	414	376	288	376	654	0	433	359
N.S.	1	1.09	0.99	0.76	0.99	1.72	0.00	1.14	0.94
time (sec)	N/A	1.397	0.483	1.577	0.295	0.277	0.000	0.279	9.345

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	457	419	325	427	686	0	523	397
N.S.	1	1.08	0.99	0.77	1.01	1.62	0.00	1.23	0.94
time (sec)	N/A	1.658	0.507	1.575	0.298	0.272	0.000	0.272	9.449

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	45	44	44	53	45	56
N.S.	1	1.00	1.09	0.83	0.81	0.81	0.98	0.83	1.04
time (sec)	N/A	0.251	0.016	1.499	0.292	0.251	0.062	0.261	0.099

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	25	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.83	0.80
time (sec)	N/A	0.206	0.005	1.505	0.288	0.264	0.038	0.269	0.034

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	53	36	37	37	44	38	49
N.S.	1	1.00	1.20	0.82	0.84	0.84	1.00	0.86	1.11
time (sec)	N/A	0.235	0.010	1.502	0.280	0.338	0.062	0.272	8.967

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	45	50	33	34	34	42	35	63
N.S.	1	1.10	1.22	0.80	0.83	0.83	1.02	0.85	1.54
time (sec)	N/A	0.228	0.009	1.512	0.274	0.329	0.060	0.261	0.081

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	53	35	36	36	46	38	48
N.S.	1	1.00	1.26	0.83	0.86	0.86	1.10	0.90	1.14
time (sec)	N/A	0.224	0.010	1.526	0.295	0.387	0.088	0.258	8.956

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	60	42	43	48	49	45	55
N.S.	1	1.00	1.22	0.86	0.88	0.98	1.00	0.92	1.12
time (sec)	N/A	0.228	0.018	1.518	0.283	0.341	0.101	0.255	0.079

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	28	33	27	29	25
N.S.	1	1.00	1.00	0.81	0.88	1.03	0.84	0.91	0.78
time (sec)	N/A	0.209	0.008	1.598	0.269	0.328	0.046	0.262	8.981

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	45	47	35	34	34	42	35	63
N.S.	1	1.10	1.15	0.85	0.83	0.83	1.02	0.85	1.54
time (sec)	N/A	0.230	0.011	1.679	0.285	0.248	0.062	0.263	9.024

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	53	33	32	32	41	33	63
N.S.	1	1.00	1.36	0.85	0.82	0.82	1.05	0.85	1.62
time (sec)	N/A	0.226	0.012	1.514	0.285	0.264	0.061	0.265	0.091

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	43	43	49	43	43
N.S.	1	1.00	1.00	0.80	0.78	0.78	0.89	0.78	0.78
time (sec)	N/A	0.234	0.005	0.143	0.195	0.242	0.016	0.260	0.028

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	43	43	49	43	43
N.S.	1	1.00	1.00	0.80	0.78	0.78	0.89	0.78	0.78
time (sec)	N/A	0.210	0.004	0.132	0.197	0.274	0.019	0.260	0.024

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	41	40	40	46	40	40
N.S.	1	1.00	1.00	0.82	0.80	0.80	0.92	0.80	0.80
time (sec)	N/A	0.202	0.003	0.137	0.206	0.283	0.017	0.255	0.024

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	44	39	38
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.96	0.85	0.83
time (sec)	N/A	0.205	0.005	0.026	0.199	0.261	0.045	0.265	0.029

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	38	45	41	39	38
N.S.	1	1.00	1.00	0.89	0.86	1.02	0.93	0.89	0.86
time (sec)	N/A	0.208	0.007	0.029	0.211	0.268	0.056	0.270	0.029

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	38	45	44	39	38
N.S.	1	1.00	1.00	0.89	0.86	1.02	1.00	0.89	0.86
time (sec)	N/A	0.210	0.006	0.032	0.204	0.263	0.100	0.307	0.030

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	79	79	92	79	79
N.S.	1	1.00	1.18	0.98	0.96	0.96	1.12	0.96	0.96
time (sec)	N/A	0.281	0.005	1.528	0.203	0.257	0.020	0.255	0.040

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	79	79	94	79	79
N.S.	1	1.00	1.18	0.98	0.96	0.96	1.15	0.96	0.96
time (sec)	N/A	0.263	0.004	1.554	0.208	0.265	0.019	0.264	0.038

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	76	76	88	76	76
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.14	0.99	0.99
time (sec)	N/A	0.276	0.004	1.539	0.199	0.260	0.021	0.263	0.037

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	74	74	88	75	74
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.00	0.85	0.84
time (sec)	N/A	0.245	0.010	1.487	0.208	0.256	0.070	0.265	0.041

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	74	73	81	82	74	73
N.S.	1	1.00	1.00	0.89	0.88	0.98	0.99	0.89	0.88
time (sec)	N/A	0.267	0.010	1.509	0.216	0.268	0.069	0.266	0.042

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	75	74	81	87	75	74
N.S.	1	1.00	1.00	0.89	0.88	0.96	1.04	0.89	0.88
time (sec)	N/A	0.259	0.011	1.554	0.215	0.273	0.115	0.290	0.038

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	139	116	115	115	138	115	115
N.S.	1	1.00	1.26	1.05	1.05	1.05	1.25	1.05	1.05
time (sec)	N/A	0.300	0.005	1.529	0.200	0.267	0.022	0.267	0.095

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	139	116	115	115	138	115	115
N.S.	1	1.00	1.26	1.05	1.05	1.05	1.25	1.05	1.05
time (sec)	N/A	0.291	0.005	1.582	0.210	0.272	0.022	0.268	0.095

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	134	113	112	112	134	112	112
N.S.	1	1.00	1.28	1.08	1.07	1.07	1.28	1.07	1.07
time (sec)	N/A	0.319	0.005	1.532	0.194	0.284	0.025	0.261	0.092

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	110	109	109	131	110	109
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.03	0.87	0.86
time (sec)	N/A	0.297	0.011	1.494	0.206	0.271	0.073	0.275	0.103

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	125	110	109	117	128	110	109
N.S.	1	1.00	1.00	0.88	0.87	0.94	1.02	0.88	0.87
time (sec)	N/A	0.303	0.012	1.514	0.211	0.285	0.085	0.267	0.106

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	126	111	110	117	131	111	110
N.S.	1	1.00	1.00	0.88	0.87	0.93	1.04	0.88	0.87
time (sec)	N/A	0.309	0.011	1.544	0.208	0.264	0.127	0.277	8.985

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	181	152	151	151	184	151	151
N.S.	1	1.00	1.31	1.10	1.09	1.09	1.33	1.09	1.09
time (sec)	N/A	0.343	0.006	1.615	0.220	0.253	0.024	0.264	9.157

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	181	152	151	151	185	151	151
N.S.	1	1.00	1.31	1.10	1.09	1.09	1.34	1.09	1.09
time (sec)	N/A	0.327	0.005	1.569	0.217	0.268	0.024	0.267	0.163

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	173	148	147	147	178	147	147
N.S.	1	1.00	1.33	1.14	1.13	1.13	1.37	1.13	1.13
time (sec)	N/A	0.369	0.005	1.532	0.208	0.278	0.025	0.273	0.157

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	145	144	144	175	145	144
N.S.	1	1.00	1.00	0.87	0.87	0.87	1.05	0.87	0.87
time (sec)	N/A	0.333	0.011	1.494	0.213	0.266	0.088	0.268	0.162

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	162	145	144	153	168	145	144
N.S.	1	1.00	1.00	0.90	0.89	0.94	1.04	0.90	0.89
time (sec)	N/A	0.364	0.011	1.504	0.193	0.279	0.106	0.272	9.067

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	147	146	153	175	147	146
N.S.	1	1.00	1.00	0.89	0.88	0.92	1.05	0.89	0.88
time (sec)	N/A	0.365	0.013	1.608	0.209	0.263	0.171	0.270	9.048

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	191	67	190	4798	178	206	319
N.S.	1	1.00	0.93	0.33	0.93	23.40	0.87	1.00	1.56
time (sec)	N/A	0.445	0.099	1.539	0.289	0.995	0.743	0.277	8.974

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	184	59	181	4261	150	191	340
N.S.	1	1.00	0.95	0.31	0.94	22.08	0.78	0.99	1.76
time (sec)	N/A	0.436	0.084	1.541	0.305	0.963	0.732	0.277	9.079

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	200	49	173	4628	160	174	266
N.S.	1	1.00	1.09	0.27	0.95	25.29	0.87	0.95	1.45
time (sec)	N/A	0.405	0.063	1.543	0.278	1.216	0.752	0.277	9.114

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	172	176	37	159	4671	160	162	274
N.S.	1	0.97	0.99	0.21	0.90	26.39	0.90	0.92	1.55
time (sec)	N/A	0.406	0.089	1.521	0.282	1.212	0.709	0.280	0.254

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	176	190	176	4588	0	176	716
N.S.	1	1.00	0.96	1.03	0.96	24.93	0.00	0.96	3.89
time (sec)	N/A	0.400	0.085	1.530	0.294	1.269	0.000	0.276	9.125

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	184	207	186	4524	0	198	723
N.S.	1	1.00	0.96	1.08	0.97	23.56	0.00	1.03	3.77
time (sec)	N/A	0.402	0.226	1.536	0.279	1.167	0.000	0.277	8.929

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	192	209	177	4279	0	202	701
N.S.	1	1.00	0.95	1.03	0.87	21.08	0.00	1.00	3.45
time (sec)	N/A	0.412	0.164	1.677	0.317	1.070	0.000	0.282	8.993

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	191	174	67	163	2077	110	176	180
N.S.	1	1.01	0.92	0.35	0.86	10.93	0.58	0.93	0.95
time (sec)	N/A	0.435	0.138	1.583	0.288	0.932	1.084	0.283	0.211

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	205	186	73	185	2358	124	186	194
N.S.	1	1.02	0.93	0.36	0.92	11.79	0.62	0.93	0.97
time (sec)	N/A	0.448	0.183	1.582	0.303	0.947	0.845	0.286	8.977

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	201	189	71	179	2118	116	183	175
N.S.	1	1.01	0.95	0.36	0.90	10.64	0.58	0.92	0.88
time (sec)	N/A	0.416	0.183	1.563	0.290	0.933	0.672	0.283	8.999

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	232	199	235	203	5018	0	213	490
N.S.	1	1.05	0.90	1.06	0.91	22.60	0.00	0.96	2.21
time (sec)	N/A	0.538	0.152	1.562	0.292	1.081	0.000	0.280	0.374

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	242	213	245	222	4976	0	233	488
N.S.	1	1.05	0.92	1.06	0.96	21.54	0.00	1.01	2.11
time (sec)	N/A	0.587	0.266	1.555	0.278	1.130	0.000	0.276	9.279

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	253	221	249	220	4774	0	245	733
N.S.	1	1.05	0.91	1.03	0.91	19.73	0.00	1.01	3.03
time (sec)	N/A	0.608	0.171	1.614	0.276	1.087	0.000	0.282	9.200

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	273	225	271	236	5373	0	264	537
N.S.	1	1.04	0.86	1.03	0.90	20.51	0.00	1.01	2.05
time (sec)	N/A	0.665	0.174	1.560	0.328	1.187	0.000	0.279	9.289

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	221	198	87	203	2163	148	203	216
N.S.	1	1.03	0.92	0.40	0.94	10.06	0.69	0.94	1.00
time (sec)	N/A	0.507	0.189	1.535	0.300	1.084	4.671	0.285	0.235

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	249	214	96	223	2519	170	210	232
N.S.	1	1.04	0.90	0.40	0.93	10.54	0.71	0.88	0.97
time (sec)	N/A	0.547	0.274	1.525	0.306	1.055	1.710	0.282	9.028

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	235	213	92	219	2251	163	209	212
N.S.	1	1.04	0.95	0.41	0.97	10.00	0.72	0.93	0.94
time (sec)	N/A	0.505	0.286	1.519	0.290	0.924	1.066	0.280	0.264

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	286	229	269	246	5229	0	248	540
N.S.	1	1.11	0.89	1.05	0.96	20.35	0.00	0.96	2.10
time (sec)	N/A	0.702	0.201	1.551	0.293	1.107	0.000	0.297	9.238

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	299	248	277	266	5112	0	268	793
N.S.	1	1.12	0.93	1.04	1.00	19.15	0.00	1.00	2.97
time (sec)	N/A	0.771	0.222	1.553	0.307	1.143	0.000	0.292	9.352

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	313	253	283	265	4911	0	278	778
N.S.	1	1.13	0.92	1.03	0.96	17.79	0.00	1.01	2.82
time (sec)	N/A	0.823	0.224	1.515	0.288	1.211	0.000	0.289	9.297

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	334	255	301	283	5550	0	299	870
N.S.	1	1.12	0.86	1.01	0.95	18.62	0.00	1.00	2.92
time (sec)	N/A	0.902	0.386	1.536	0.289	1.768	0.000	0.282	9.794

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	260	230	110	248	2364	201	236	253
N.S.	1	1.05	0.93	0.44	1.00	9.53	0.81	0.95	1.02
time (sec)	N/A	0.588	0.250	1.523	0.281	1.984	156.077	0.280	0.289

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	285	241	119	260	2646	214	238	265
N.S.	1	1.06	0.89	0.44	0.96	9.80	0.79	0.88	0.98
time (sec)	N/A	0.626	0.363	1.549	0.294	1.127	6.081	0.285	0.267

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	268	239	116	254	2344	202	233	247
N.S.	1	1.07	0.96	0.46	1.02	9.38	0.81	0.93	0.99
time (sec)	N/A	0.585	0.242	1.513	0.283	0.905	1.803	0.282	0.305

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	339	259	305	293	5370	0	284	871
N.S.	1	1.16	0.89	1.05	1.01	18.45	0.00	0.98	2.99
time (sec)	N/A	0.871	0.256	1.543	0.300	1.063	0.000	0.279	12.945

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	352	279	313	313	5250	0	304	840
N.S.	1	1.17	0.93	1.04	1.04	17.44	0.00	1.01	2.79
time (sec)	N/A	0.967	0.292	1.553	0.322	1.102	0.000	0.279	11.846

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	364	284	319	312	5049	0	315	825
N.S.	1	1.17	0.92	1.03	1.01	16.29	0.00	1.02	2.66
time (sec)	N/A	1.033	0.283	1.562	0.332	1.070	0.000	0.285	12.130

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	388	284	336	330	5670	0	326	918
N.S.	1	1.14	0.84	0.99	0.97	16.68	0.00	0.96	2.70
time (sec)	N/A	1.149	0.501	1.561	0.313	1.440	0.000	0.278	12.233

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	57	29	26	26	54	27	26
N.S.	1	1.00	1.97	1.00	0.90	0.90	1.86	0.93	0.90
time (sec)	N/A	0.232	0.021	1.528	0.271	0.262	0.071	0.269	11.520

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	57	29	26	26	54	27	26
N.S.	1	1.00	1.97	1.00	0.90	0.90	1.86	0.93	0.90
time (sec)	N/A	0.196	0.009	1.508	0.269	0.252	0.081	0.262	0.029

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	29	28	28	54	29	27
N.S.	1	1.00	1.87	0.94	0.90	0.90	1.74	0.94	0.87
time (sec)	N/A	0.231	0.020	1.525	0.271	0.258	0.069	0.274	0.069

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	29	28	28	54	29	27
N.S.	1	1.00	1.87	0.94	0.90	0.90	1.74	0.94	0.87
time (sec)	N/A	0.200	0.010	1.615	0.375	0.267	0.078	0.286	0.031

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	146	116	51	52	100	149	154
N.S.	1	1.00	2.92	2.32	1.02	1.04	2.00	2.98	3.08
time (sec)	N/A	0.252	0.048	1.674	0.382	0.270	0.141	0.278	11.157

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	149	119	166	53	110	114	156
N.S.	1	1.00	2.81	2.25	3.13	1.00	2.08	2.15	2.94
time (sec)	N/A	0.258	0.079	1.498	0.349	0.265	0.160	0.284	11.037

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	148	117	167	56	109	97	155
N.S.	1	1.00	2.74	2.17	3.09	1.04	2.02	1.80	2.87
time (sec)	N/A	0.241	0.048	1.516	0.352	0.285	0.133	0.282	11.056

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	147	118	52	53	102	90	155
N.S.	1	1.00	2.77	2.23	0.98	1.00	1.92	1.70	2.92
time (sec)	N/A	0.246	0.059	1.525	0.320	0.279	0.149	0.268	10.956

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	79	79	90	85	82
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.93	0.88	0.85
time (sec)	N/A	0.306	0.030	1.185	0.231	0.271	0.019	0.264	0.051

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	79	79	90	85	82
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.93	0.88	0.85
time (sec)	N/A	0.292	0.029	1.161	0.240	0.281	0.019	0.273	0.043

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	79	79	90	85	82
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.93	0.88	0.85
time (sec)	N/A	0.289	0.027	1.136	0.239	0.273	0.019	0.266	0.042

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	80	79	79	90	85	82
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.93	0.88	0.85
time (sec)	N/A	0.274	0.019	1.191	0.209	0.283	0.019	0.268	0.043

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	76	76	87	82	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.95	0.89	0.86
time (sec)	N/A	0.268	0.015	0.101	0.205	0.289	0.021	0.292	0.042

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	78	74	74	85	81	77
N.S.	1	1.00	1.00	0.89	0.84	0.84	0.97	0.92	0.88
time (sec)	N/A	0.256	0.030	0.030	0.194	0.272	0.076	0.266	0.047

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	81	74	81	82	81	77
N.S.	1	1.00	1.00	0.94	0.86	0.94	0.95	0.94	0.90
time (sec)	N/A	0.269	0.041	0.033	0.211	0.276	0.083	0.260	0.047

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	78	78	74	81	83	78	76
N.S.	1	1.00	0.91	0.91	0.86	0.94	0.97	0.91	0.88
time (sec)	N/A	0.270	0.061	0.034	0.225	0.278	0.141	0.263	0.043

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	76	76	75	81	83	77	75
N.S.	1	1.00	0.88	0.88	0.87	0.94	0.97	0.90	0.87
time (sec)	N/A	0.267	0.060	0.034	0.206	0.272	0.324	0.264	0.043

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	77	74	75	81	83	75	74
N.S.	1	1.00	0.90	0.86	0.87	0.94	0.97	0.87	0.86
time (sec)	N/A	0.266	0.060	0.033	0.273	0.277	1.234	0.274	10.461

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	163	152	151	151	167	157	151
N.S.	1	1.00	1.00	0.93	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.423	0.046	2.062	0.213	0.343	0.028	0.266	0.118

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	163	152	151	151	167	157	151
N.S.	1	1.00	1.00	0.93	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.394	0.031	2.092	0.236	0.325	0.031	0.270	10.531

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	150	152	151	151	167	157	151
N.S.	1	1.00	0.95	0.96	0.96	0.96	1.06	0.99	0.96
time (sec)	N/A	0.386	0.076	2.076	0.196	0.379	0.041	0.271	0.112

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	163	152	151	151	167	157	151
N.S.	1	1.00	1.03	0.96	0.96	0.96	1.06	0.99	0.96
time (sec)	N/A	0.370	0.030	2.061	0.195	0.360	0.027	0.271	0.110

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	125	149	148	148	163	154	148
N.S.	1	1.00	0.82	0.97	0.97	0.97	1.07	1.01	0.97
time (sec)	N/A	0.386	0.093	2.006	0.199	0.436	0.030	0.264	0.108

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	154	147	146	146	162	153	146
N.S.	1	1.00	1.03	0.99	0.98	0.98	1.09	1.03	0.98
time (sec)	N/A	0.359	0.053	1.503	0.198	0.275	0.126	0.266	10.494

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	152	150	146	153	156	152	145
N.S.	1	1.00	1.03	1.02	0.99	1.04	1.06	1.03	0.99
time (sec)	N/A	0.385	0.073	1.518	0.193	0.285	0.137	0.281	0.118

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	127	148	146	153	158	150	145
N.S.	1	1.00	0.86	1.01	0.99	1.04	1.07	1.02	0.99
time (sec)	N/A	0.385	0.087	1.492	0.197	0.297	0.193	0.279	9.831

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	147	147	153	158	150	145
N.S.	1	1.00	0.81	0.97	0.97	1.01	1.04	0.99	0.95
time (sec)	N/A	0.360	0.077	1.483	0.209	0.281	0.401	0.272	0.096

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	125	144	147	153	156	149	145
N.S.	1	1.00	0.82	0.95	0.97	1.01	1.03	0.98	0.95
time (sec)	N/A	0.357	0.084	1.498	0.215	0.278	1.410	0.283	0.069

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	223	221	217	217	246	229	205
N.S.	1	1.00	1.00	0.99	0.97	0.97	1.10	1.03	0.92
time (sec)	N/A	0.522	0.058	2.058	0.193	0.272	0.035	0.267	0.198

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	223	221	217	217	246	229	205
N.S.	1	1.00	1.00	0.99	0.97	0.97	1.10	1.03	0.92
time (sec)	N/A	0.474	0.052	2.051	0.202	0.274	0.032	0.271	9.008

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	223	221	217	217	246	229	205
N.S.	1	1.00	1.05	1.04	1.02	1.02	1.16	1.08	0.97
time (sec)	N/A	0.453	0.055	2.029	0.196	0.277	0.033	0.267	0.181

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	223	221	217	217	246	229	205
N.S.	1	1.00	1.05	1.04	1.02	1.02	1.16	1.08	0.97
time (sec)	N/A	0.435	0.039	2.048	0.207	0.266	0.032	0.272	0.181

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	170	218	214	214	243	226	202
N.S.	1	1.00	0.82	1.05	1.03	1.03	1.17	1.09	0.98
time (sec)	N/A	0.459	0.094	2.037	0.204	0.391	0.037	0.268	0.180

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	214	215	212	212	240	224	199
N.S.	1	1.00	1.07	1.08	1.06	1.06	1.20	1.12	1.00
time (sec)	N/A	0.420	0.090	1.517	0.196	0.372	0.184	0.274	9.129

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	172	219	212	219	236	224	199
N.S.	1	1.00	0.87	1.11	1.07	1.11	1.19	1.13	1.01
time (sec)	N/A	0.457	0.159	1.503	0.212	0.376	0.192	0.264	9.554

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	174	217	212	219	238	222	199
N.S.	1	1.00	0.88	1.10	1.07	1.11	1.20	1.12	1.01
time (sec)	N/A	0.455	0.136	1.475	0.195	0.393	0.244	0.260	0.154

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	172	216	212	219	236	221	199
N.S.	1	1.00	0.82	1.03	1.01	1.05	1.13	1.06	0.95
time (sec)	N/A	0.444	0.122	1.491	0.209	0.354	0.459	0.275	0.147

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	170	215	212	219	235	220	199
N.S.	1	1.00	0.81	1.03	1.01	1.05	1.12	1.05	0.95
time (sec)	N/A	0.429	0.129	1.492	0.207	0.290	1.495	0.267	9.393

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	329	334	165	378	15635	0	375	1271
N.S.	1	0.99	1.01	0.50	1.14	47.24	0.00	1.13	3.84
time (sec)	N/A	1.117	0.470	1.545	0.303	1.775	0.000	0.289	9.232

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	314	299	138	332	15451	0	351	1236
N.S.	1	1.00	0.96	0.44	1.06	49.36	0.00	1.12	3.95
time (sec)	N/A	1.052	0.218	1.551	0.293	2.360	0.000	0.278	0.146

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	298	290	123	313	14746	0	329	1170
N.S.	1	1.01	0.99	0.42	1.06	50.16	0.00	1.12	3.98
time (sec)	N/A	1.036	0.186	1.553	0.303	2.301	0.000	0.271	9.253

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	281	272	104	300	14875	0	291	1161
N.S.	1	1.02	0.99	0.38	1.09	54.09	0.00	1.06	4.22
time (sec)	N/A	0.956	0.342	1.551	0.313	1.462	0.000	0.280	9.119

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	257	254	82	266	15235	0	271	1150
N.S.	1	0.99	0.98	0.32	1.03	58.82	0.00	1.05	4.44
time (sec)	N/A	0.549	0.268	1.562	0.295	1.782	0.000	0.275	9.344

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	256	258	259	290	15327	0	278	1731
N.S.	1	0.99	1.00	1.00	1.12	59.41	0.00	1.08	6.71
time (sec)	N/A	0.614	0.181	1.548	0.289	84.140	0.000	0.272	9.186

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	257	260	290	15238	0	274	1802
N.S.	1	1.00	1.02	1.03	1.15	60.23	0.00	1.08	7.12
time (sec)	N/A	0.599	0.270	1.546	0.290	85.937	0.000	0.279	9.355

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	258	257	251	271	15424	0	267	6948
N.S.	1	0.99	0.99	0.97	1.04	59.32	0.00	1.03	26.72
time (sec)	N/A	0.568	0.318	1.567	0.283	45.126	0.000	0.281	9.843

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	274	264	276	302	15204	0	287	1842
N.S.	1	0.99	0.96	1.00	1.09	55.09	0.00	1.04	6.67
time (sec)	N/A	0.620	0.443	1.566	0.291	74.149	0.000	0.274	10.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	335	334	162	364	16147	0	351	1241
N.S.	1	0.99	0.99	0.48	1.08	47.91	0.00	1.04	3.68
time (sec)	N/A	0.924	0.429	1.556	0.300	1.957	0.000	0.270	9.313

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	310	294	136	329	16285	0	329	1229
N.S.	1	1.00	0.95	0.44	1.06	52.36	0.00	1.06	3.95
time (sec)	N/A	0.842	0.224	1.546	0.288	1.991	0.000	0.281	0.158

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	289	280	115	283	12153	0	301	816
N.S.	1	1.00	0.97	0.40	0.98	41.91	0.00	1.04	2.81
time (sec)	N/A	0.651	0.187	1.532	0.280	1.604	0.000	0.288	0.138

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	292	285	113	311	12617	0	311	827
N.S.	1	1.01	0.99	0.39	1.08	43.66	0.00	1.08	2.86
time (sec)	N/A	0.721	0.185	1.543	0.285	1.660	0.000	0.290	10.613

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	274	268	120	292	12636	0	302	835
N.S.	1	0.99	0.97	0.43	1.06	45.78	0.00	1.09	3.03
time (sec)	N/A	0.730	0.164	1.525	0.288	1.521	0.000	0.274	9.775

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	301	269	293	302	12541	0	314	1660
N.S.	1	1.04	0.93	1.01	1.04	43.39	0.00	1.09	5.74
time (sec)	N/A	0.748	0.212	1.559	0.287	21.328	0.000	0.276	9.775

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	317	285	298	329	12556	0	324	1684
N.S.	1	1.05	0.95	0.99	1.09	41.71	0.00	1.08	5.59
time (sec)	N/A	0.795	0.312	1.604	0.290	22.128	0.000	0.276	10.014

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	323	292	293	316	12231	0	333	1632
N.S.	1	1.06	0.95	0.96	1.03	39.97	0.00	1.09	5.33
time (sec)	N/A	0.809	0.496	1.588	0.321	17.921	0.000	0.283	9.832

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	354	303	332	365	16568	0	359	1924
N.S.	1	1.05	0.90	0.98	1.08	49.02	0.00	1.06	5.69
time (sec)	N/A	0.927	0.504	1.560	0.311	71.983	0.000	0.284	9.839

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	364	342	168	391	12967	0	381	916
N.S.	1	1.06	0.99	0.49	1.13	37.59	0.00	1.10	2.66
time (sec)	N/A	1.085	0.305	1.544	0.298	2.149	0.000	0.287	0.593

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	339	315	173	366	12939	0	359	908
N.S.	1	1.04	0.97	0.53	1.13	39.81	0.00	1.10	2.79
time (sec)	N/A	1.045	0.259	1.553	0.298	1.961	0.000	0.278	9.593

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	306	287	151	308	6926	0	315	627
N.S.	1	1.03	0.97	0.51	1.04	23.32	0.00	1.06	2.11
time (sec)	N/A	0.729	0.249	1.552	0.318	1.467	0.000	0.270	9.371

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	333	297	158	344	7190	0	335	640
N.S.	1	1.03	0.92	0.49	1.07	22.26	0.00	1.04	1.98
time (sec)	N/A	0.838	0.331	1.667	0.285	1.740	0.000	0.280	9.433

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	320	295	149	327	6984	0	329	630
N.S.	1	1.02	0.94	0.48	1.04	22.31	0.00	1.05	2.01
time (sec)	N/A	0.736	0.250	1.710	0.281	1.524	0.000	0.287	9.293

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	371	311	339	368	12815	0	371	1716
N.S.	1	1.07	0.90	0.98	1.06	36.93	0.00	1.07	4.95
time (sec)	N/A	0.998	0.283	1.582	0.302	21.986	0.000	0.280	9.747

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	391	336	345	400	12951	0	386	1747
N.S.	1	1.08	0.93	0.95	1.10	35.78	0.00	1.07	4.83
time (sec)	N/A	1.054	0.543	1.563	0.285	22.720	0.000	0.291	9.869

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	398	337	340	390	12435	0	395	1697
N.S.	1	1.11	0.94	0.94	1.08	34.54	0.00	1.10	4.71
time (sec)	N/A	1.109	0.558	1.608	0.287	17.282	0.000	0.285	9.904

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	432	352	394	444	16697	0	427	1994
N.S.	1	1.09	0.89	1.00	1.12	42.27	0.00	1.08	5.05
time (sec)	N/A	1.307	0.572	1.589	0.288	101.305	0.000	0.281	10.349

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	583	601	132	765	0	81	129	0	0
N.S.	1	1.03	0.23	1.31	0.00	0.14	0.22	0.00	0.00
time (sec)	N/A	1.113	10.144	1.670	0.000	0.079	1.499	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	572	121	755	0	74	107	0	0
N.S.	1	1.02	0.22	1.35	0.00	0.13	0.19	0.00	0.00
time (sec)	N/A	0.855	10.088	1.631	0.000	0.082	1.410	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	547	114	746	0	66	107	0	0
N.S.	1	1.02	0.21	1.39	0.00	0.12	0.20	0.00	0.00
time (sec)	N/A	0.690	10.070	1.661	0.000	0.079	1.394	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	514	107	735	0	55	105	0	0
N.S.	1	1.01	0.21	1.44	0.00	0.11	0.21	0.00	0.00
time (sec)	N/A	0.567	10.060	1.601	0.000	0.088	1.094	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	518	523	128	740	0	190	105	0	0
N.S.	1	1.01	0.25	1.43	0.00	0.37	0.20	0.00	0.00
time (sec)	N/A	0.592	10.172	1.538	0.000	0.212	1.697	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	547	557	126	758	0	235	107	0	121
N.S.	1	1.02	0.23	1.39	0.00	0.43	0.20	0.00	0.22
time (sec)	N/A	0.746	10.120	1.661	0.000	0.114	1.432	0.000	9.724

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	569	579	131	763	0	246	112	0	0
N.S.	1	1.02	0.23	1.34	0.00	0.43	0.20	0.00	0.00
time (sec)	N/A	0.866	10.196	1.659	0.000	0.131	1.577	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	594	625	134	813	0	138	129	0	0
N.S.	1	1.05	0.23	1.37	0.00	0.23	0.22	0.00	0.00
time (sec)	N/A	1.034	10.107	2.049	0.000	0.096	6.884	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	574	592	127	793	0	130	129	0	0
N.S.	1	1.03	0.22	1.38	0.00	0.23	0.22	0.00	0.00
time (sec)	N/A	0.860	10.100	2.009	0.000	0.107	5.070	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	554	118	775	0	107	129	0	0
N.S.	1	1.02	0.22	1.43	0.00	0.20	0.24	0.00	0.00
time (sec)	N/A	0.721	10.098	1.910	0.000	0.095	4.003	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	532	107	759	0	99	109	0	0
N.S.	1	1.02	0.20	1.45	0.00	0.19	0.21	0.00	0.00
time (sec)	N/A	0.616	10.078	1.602	0.000	0.162	3.542	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	561	565	108	765	0	112	109	0	0
N.S.	1	1.01	0.19	1.36	0.00	0.20	0.19	0.00	0.00
time (sec)	N/A	0.698	10.067	1.557	0.000	0.212	3.309	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	537	109	771	0	98	107	0	0
N.S.	1	1.01	0.20	1.45	0.00	0.18	0.20	0.00	0.00
time (sec)	N/A	0.566	10.053	1.539	0.000	0.095	3.270	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	579	586	119	794	0	345	265	0	0
N.S.	1	1.01	0.21	1.37	0.00	0.60	0.46	0.00	0.00
time (sec)	N/A	0.866	10.104	1.542	0.000	0.123	5.697	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	607	620	121	806	0	377	267	0	136
N.S.	1	1.02	0.20	1.33	0.00	0.62	0.44	0.00	0.22
time (sec)	N/A	1.028	10.096	1.962	0.000	0.139	5.906	0.000	10.038

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	733	740	172	956	0	202	238	0	0
N.S.	1	1.01	0.23	1.30	0.00	0.28	0.32	0.00	0.00
time (sec)	N/A	2.083	9.910	1.758	0.000	0.093	2.328	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	681	691	158	920	0	177	223	0	0
N.S.	1	1.01	0.23	1.35	0.00	0.26	0.33	0.00	0.00
time (sec)	N/A	1.707	10.272	1.744	0.000	0.171	2.118	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	667	666	143	829	0	147	223	0	0
N.S.	1	1.00	0.21	1.24	0.00	0.22	0.33	0.00	0.00
time (sec)	N/A	1.297	10.144	1.685	0.000	0.126	2.018	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	639	634	135	863	0	141	194	0	0
N.S.	1	0.99	0.21	1.35	0.00	0.22	0.30	0.00	0.00
time (sec)	N/A	1.010	9.288	1.693	0.000	0.091	2.074	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	620	621	185	848	0	340	235	0	0
N.S.	1	1.00	0.30	1.37	0.00	0.55	0.38	0.00	0.00
time (sec)	N/A	1.005	9.188	1.565	0.000	0.232	4.294	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	646	213	829	0	307	236	0	0
N.S.	1	1.01	0.33	1.30	0.00	0.48	0.37	0.00	0.00
time (sec)	N/A	1.079	9.140	1.983	0.000	0.235	2.907	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	640	657	218	826	0	320	255	0	0
N.S.	1	1.03	0.34	1.29	0.00	0.50	0.40	0.00	0.00
time (sec)	N/A	1.149	9.784	1.929	0.000	0.244	2.976	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	637	675	238	822	0	348	265	0	0
N.S.	1	1.06	0.37	1.29	0.00	0.55	0.42	0.00	0.00
time (sec)	N/A	1.309	10.148	1.808	0.000	0.558	3.587	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	719	238	845	0	330	274	0	0
N.S.	1	1.04	0.34	1.22	0.00	0.48	0.39	0.00	0.00
time (sec)	N/A	1.499	10.448	1.776	0.000	0.642	3.607	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	652	649	180	874	0	346	240	0	0
N.S.	1	1.00	0.28	1.34	0.00	0.53	0.37	0.00	0.00
time (sec)	N/A	1.204	10.279	1.730	0.000	0.400	3.258	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	659	671	211	883	0	404	304	0	0
N.S.	1	1.02	0.32	1.34	0.00	0.61	0.46	0.00	0.00
time (sec)	N/A	1.294	10.431	1.816	0.000	0.167	4.756	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	711	719	213	901	0	439	308	0	0
N.S.	1	1.01	0.30	1.27	0.00	0.62	0.43	0.00	0.00
time (sec)	N/A	1.537	10.485	1.822	0.000	0.152	4.837	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	743	754	192	931	0	482	304	0	0
N.S.	1	1.01	0.26	1.25	0.00	0.65	0.41	0.00	0.00
time (sec)	N/A	1.774	10.195	1.872	0.000	0.139	4.713	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	791	789	179	1161	0	262	512	0	0
N.S.	1	1.00	0.23	1.47	0.00	0.33	0.65	0.00	0.00
time (sec)	N/A	2.327	10.563	1.778	0.000	0.091	4.225	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	742	743	162	1103	0	237	525	0	0
N.S.	1	1.00	0.22	1.49	0.00	0.32	0.71	0.00	0.00
time (sec)	N/A	1.945	10.361	1.739	0.000	0.134	3.752	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	723	717	148	889	0	207	525	0	0
N.S.	1	0.99	0.20	1.23	0.00	0.29	0.73	0.00	0.00
time (sec)	N/A	1.520	10.309	1.677	0.000	0.149	3.574	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	684	139	1024	0	201	444	0	0
N.S.	1	0.99	0.20	1.48	0.00	0.29	0.64	0.00	0.00
time (sec)	N/A	1.189	10.194	1.699	0.000	0.088	3.622	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	676	676	215	987	0	457	473	0	0
N.S.	1	1.00	0.32	1.46	0.00	0.68	0.70	0.00	0.00
time (sec)	N/A	1.181	10.448	1.583	0.000	0.249	8.305	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	698	224	946	0	424	474	0	0
N.S.	1	1.01	0.32	1.37	0.00	0.61	0.68	0.00	0.00
time (sec)	N/A	1.277	10.372	2.409	0.000	0.242	5.027	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	708	232	941	0	433	462	0	0
N.S.	1	1.02	0.33	1.36	0.00	0.62	0.67	0.00	0.00
time (sec)	N/A	1.321	10.345	2.332	0.000	0.263	5.169	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	725	243	920	0	434	484	0	0
N.S.	1	1.05	0.35	1.33	0.00	0.63	0.70	0.00	0.00
time (sec)	N/A	1.461	10.602	2.124	0.000	0.670	6.195	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	741	772	246	900	0	384	495	0	0
N.S.	1	1.04	0.33	1.21	0.00	0.52	0.67	0.00	0.00
time (sec)	N/A	1.703	10.605	1.993	0.000	0.820	6.221	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	707	191	920	0	382	476	0	0
N.S.	1	1.03	0.28	1.34	0.00	0.55	0.69	0.00	0.00
time (sec)	N/A	1.348	10.242	1.960	0.000	0.261	5.907	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	723	240	903	0	430	524	0	0
N.S.	1	1.04	0.35	1.30	0.00	0.62	0.76	0.00	0.00
time (sec)	N/A	1.481	10.560	1.855	0.000	0.394	7.999	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	767	240	916	0	446	536	0	0
N.S.	1	1.03	0.32	1.23	0.00	0.60	0.72	0.00	0.00
time (sec)	N/A	1.699	10.880	1.846	0.000	0.376	8.323	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	705	705	202	949	0	470	527	0	0
N.S.	1	1.00	0.29	1.35	0.00	0.67	0.75	0.00	0.00
time (sec)	N/A	1.385	10.444	1.834	0.000	0.262	7.320	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	714	726	226	958	0	525	573	0	0
N.S.	1	1.02	0.32	1.34	0.00	0.74	0.80	0.00	0.00
time (sec)	N/A	1.579	10.702	1.828	0.000	0.265	13.495	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	764	767	227	976	0	559	576	0	0
N.S.	1	1.00	0.30	1.28	0.00	0.73	0.75	0.00	0.00
time (sec)	N/A	1.787	10.543	1.913	0.000	0.227	14.172	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	796	803	194	1006	0	606	541	0	0
N.S.	1	1.01	0.24	1.26	0.00	0.76	0.68	0.00	0.00
time (sec)	N/A	2.013	10.429	2.226	0.000	0.170	12.155	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	120	114	0	0	0	112	0	0
N.S.	1	1.18	1.12	0.00	0.00	0.00	1.10	0.00	0.00
time (sec)	N/A	0.300	0.633	0.000	0.000	0.000	28.290	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	125	116	0	0	0	114	0	0
N.S.	1	1.17	1.08	0.00	0.00	0.00	1.07	0.00	0.00
time (sec)	N/A	0.288	0.631	0.000	0.000	0.000	43.564	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	125	116	0	0	0	114	0	0
N.S.	1	1.17	1.08	0.00	0.00	0.00	1.07	0.00	0.00
time (sec)	N/A	0.301	0.644	0.000	0.000	0.000	62.265	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	54	54	63	54	54
N.S.	1	1.00	1.00	0.81	0.79	0.79	0.93	0.79	0.79
time (sec)	N/A	0.238	0.011	0.092	0.196	0.306	0.018	0.263	0.036

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	57	57	66	57	57
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.90	0.78	0.78
time (sec)	N/A	0.245	0.004	0.096	0.198	0.282	0.017	0.272	0.034

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	102	102	121	102	102
N.S.	1	1.00	1.14	0.94	0.94	0.94	1.11	0.94	0.94
time (sec)	N/A	0.302	0.004	1.655	0.197	0.280	0.026	0.288	0.093

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	129	106	105	105	124	105	105
N.S.	1	1.00	1.13	0.93	0.92	0.92	1.09	0.92	0.92
time (sec)	N/A	0.307	0.006	1.512	0.203	0.285	0.020	0.262	0.088

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	150	150	180	150	150
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.19	0.99	0.99
time (sec)	N/A	0.356	0.006	1.462	0.213	0.278	0.027	0.271	0.175

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	185	154	153	153	184	153	153
N.S.	1	1.00	1.19	0.99	0.98	0.98	1.18	0.98	0.98
time (sec)	N/A	0.352	0.006	1.521	0.227	0.261	0.023	0.277	0.176

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	236	199	198	198	241	198	198
N.S.	1	1.00	1.22	1.03	1.03	1.03	1.25	1.03	1.03
time (sec)	N/A	0.421	0.007	1.546	0.231	0.288	0.027	0.271	9.304

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	241	202	201	201	245	201	201
N.S.	1	1.00	1.22	1.02	1.02	1.02	1.24	1.02	1.02
time (sec)	N/A	0.402	0.007	1.579	0.210	0.362	0.026	0.272	0.417

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	214	44	174	241149	0	276	1970
N.S.	1	1.00	1.61	0.33	1.31	1813.15	0.00	2.08	14.81
time (sec)	N/A	0.313	0.063	1.552	0.307	5.082	0.000	0.285	0.991

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	221	79	208	220680	0	325	846
N.S.	1	1.00	1.36	0.49	1.28	1362.22	0.00	2.01	5.22
time (sec)	N/A	0.399	0.082	1.550	0.288	6.173	0.000	0.276	9.018

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	296	42	277	254687	0	286	1952
N.S.	1	1.00	1.01	0.14	0.95	869.24	0.00	0.98	6.66
time (sec)	N/A	0.434	0.216	1.563	0.310	6.415	0.000	0.284	9.811

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	311	75	305	219615	0	305	838
N.S.	1	1.00	0.97	0.23	0.95	684.16	0.00	0.95	2.61
time (sec)	N/A	0.560	0.178	1.552	0.321	5.099	0.000	0.293	8.991

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	323	315	86	305	124301	517	311	478
N.S.	1	1.02	0.99	0.27	0.96	390.88	1.63	0.98	1.50
time (sec)	N/A	0.483	0.345	1.550	0.319	5.024	43.913	0.280	0.366

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	315	294	82	294	122993	0	300	559
N.S.	1	1.02	0.95	0.26	0.95	396.75	0.00	0.97	1.80
time (sec)	N/A	0.501	0.276	1.552	0.296	3.579	0.000	0.274	9.239

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	365	347	117	355	124838	0	348	832
N.S.	1	1.04	0.99	0.33	1.01	355.66	0.00	0.99	2.37
time (sec)	N/A	0.570	0.339	1.711	0.323	9.938	0.000	0.300	9.388

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	351	329	114	343	124542	0	334	521
N.S.	1	1.03	0.97	0.34	1.01	366.30	0.00	0.98	1.53
time (sec)	N/A	0.560	0.368	1.630	0.334	8.281	0.000	0.283	0.403

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	404	379	153	402	125011	0	384	879
N.S.	1	1.06	0.99	0.40	1.05	327.25	0.00	1.01	2.30
time (sec)	N/A	0.657	0.410	1.643	0.313	19.777	0.000	0.280	9.445

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	396	366	144	396	125996	0	375	888
N.S.	1	1.04	0.96	0.38	1.04	331.57	0.00	0.99	2.34
time (sec)	N/A	0.661	0.438	1.555	0.307	21.923	0.000	0.274	0.497

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	430	202	290	0	214	252	0	0
N.S.	1	1.03	0.48	0.69	0.00	0.51	0.60	0.00	0.00
time (sec)	N/A	0.661	10.662	2.043	0.000	0.223	3.383	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	407	215	269	0	205	212	0	0
N.S.	1	1.03	0.55	0.68	0.00	0.52	0.54	0.00	0.00
time (sec)	N/A	0.612	10.595	2.032	0.000	0.282	3.346	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	382	182	262	0	193	212	0	0
N.S.	1	1.04	0.49	0.71	0.00	0.52	0.57	0.00	0.00
time (sec)	N/A	0.581	10.685	2.080	0.000	0.125	3.233	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	211	254	0	170	158	0	0
N.S.	1	1.00	0.60	0.72	0.00	0.48	0.45	0.00	0.00
time (sec)	N/A	0.537	10.189	2.050	0.000	0.130	2.245	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	171	243	0	163	156	0	0
N.S.	1	1.00	0.52	0.73	0.00	0.49	0.47	0.00	0.00
time (sec)	N/A	0.438	10.133	1.894	0.000	0.125	2.100	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	208	274	0	0	204	0	0
N.S.	1	1.00	0.60	0.79	0.00	0.00	0.59	0.00	0.00
time (sec)	N/A	0.510	10.340	1.650	0.000	0.000	4.059	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	208	274	0	0	206	0	0
N.S.	1	1.00	0.61	0.80	0.00	0.00	0.60	0.00	0.00
time (sec)	N/A	0.525	10.361	2.404	0.000	0.000	2.919	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	204	269	0	0	230	0	0
N.S.	1	1.00	0.60	0.79	0.00	0.00	0.67	0.00	0.00
time (sec)	N/A	0.520	10.216	2.239	0.000	0.000	2.830	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	205	264	0	0	235	0	0
N.S.	1	1.00	0.57	0.74	0.00	0.00	0.66	0.00	0.00
time (sec)	N/A	0.553	10.281	2.075	0.000	0.000	2.901	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	330	175	261	0	0	211	0	0
N.S.	1	1.00	0.53	0.79	0.00	0.00	0.64	0.00	0.00
time (sec)	N/A	0.628	10.220	2.045	0.000	0.000	3.005	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	354	179	280	0	0	216	0	0
N.S.	1	0.98	0.50	0.78	0.00	0.00	0.60	0.00	0.00
time (sec)	N/A	0.631	10.238	2.140	0.000	0.000	3.048	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	345	145	262	0	166	189	0	0
N.S.	1	0.98	0.41	0.74	0.00	0.47	0.54	0.00	0.00
time (sec)	N/A	0.620	10.236	2.110	0.000	0.128	2.716	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	365	145	269	0	173	192	0	0
N.S.	1	0.97	0.39	0.72	0.00	0.46	0.51	0.00	0.00
time (sec)	N/A	0.658	10.243	2.145	0.000	0.266	2.854	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	388	146	277	0	196	246	0	0
N.S.	1	0.97	0.36	0.69	0.00	0.49	0.62	0.00	0.00
time (sec)	N/A	0.706	10.177	2.329	0.000	0.267	4.045	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	419	148	299	0	208	246	0	0
N.S.	1	0.99	0.35	0.70	0.00	0.49	0.58	0.00	0.00
time (sec)	N/A	0.739	10.180	2.619	0.000	0.173	4.198	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	488	225	338	0	264	462	0	0
N.S.	1	1.03	0.47	0.71	0.00	0.55	0.97	0.00	0.00
time (sec)	N/A	0.769	10.880	2.042	0.000	0.140	9.496	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	465	238	317	0	255	398	0	0
N.S.	1	1.03	0.53	0.70	0.00	0.56	0.88	0.00	0.00
time (sec)	N/A	0.733	10.748	1.988	0.000	0.162	9.142	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	440	205	310	0	247	398	0	0
N.S.	1	1.03	0.48	0.73	0.00	0.58	0.93	0.00	0.00
time (sec)	N/A	0.680	11.117	2.089	0.000	0.150	9.121	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	196	300	0	224	396	0	0
N.S.	1	1.00	0.48	0.73	0.00	0.55	0.97	0.00	0.00
time (sec)	N/A	0.614	10.705	2.063	0.000	0.131	5.195	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	175	287	0	214	394	0	0
N.S.	1	1.00	0.46	0.75	0.00	0.56	1.03	0.00	0.00
time (sec)	N/A	0.520	10.507	1.922	0.000	0.132	4.980	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	224	346	0	0	405	0	0
N.S.	1	1.00	0.56	0.86	0.00	0.00	1.00	0.00	0.00
time (sec)	N/A	0.606	10.517	1.653	0.000	0.000	11.830	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	222	346	0	0	406	0	0
N.S.	1	1.00	0.55	0.86	0.00	0.00	1.00	0.00	0.00
time (sec)	N/A	0.607	10.461	3.396	0.000	0.000	6.135	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	194	344	0	0	377	0	0
N.S.	1	1.00	0.48	0.85	0.00	0.00	0.93	0.00	0.00
time (sec)	N/A	0.618	10.340	3.311	0.000	0.000	4.896	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	194	343	0	0	381	0	0
N.S.	1	1.00	0.48	0.84	0.00	0.00	0.93	0.00	0.00
time (sec)	N/A	0.630	10.336	2.960	0.000	0.000	4.843	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	389	163	344	0	0	379	0	0
N.S.	1	1.01	0.42	0.89	0.00	0.00	0.98	0.00	0.00
time (sec)	N/A	0.652	10.274	2.845	0.000	0.000	5.484	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	386	165	344	0	0	386	0	0
N.S.	1	1.00	0.43	0.89	0.00	0.00	1.00	0.00	0.00
time (sec)	N/A	0.665	10.222	2.505	0.000	0.000	5.647	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	389	163	343	0	0	406	0	0
N.S.	1	0.99	0.42	0.88	0.00	0.00	1.04	0.00	0.00
time (sec)	N/A	0.662	10.219	2.385	0.000	0.000	5.253	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	402	164	299	0	0	415	0	0
N.S.	1	0.98	0.40	0.73	0.00	0.00	1.01	0.00	0.00
time (sec)	N/A	0.712	10.246	2.278	0.000	0.000	5.319	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	174	295	0	0	444	0	0
N.S.	1	1.00	0.46	0.78	0.00	0.00	1.18	0.00	0.00
time (sec)	N/A	0.810	10.309	2.253	0.000	0.000	6.502	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	400	174	324	0	0	449	0	0
N.S.	1	0.99	0.43	0.80	0.00	0.00	1.11	0.00	0.00
time (sec)	N/A	0.775	10.318	2.402	0.000	0.000	6.794	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	391	171	308	0	217	398	0	0
N.S.	1	0.98	0.43	0.77	0.00	0.54	1.00	0.00	0.00
time (sec)	N/A	0.792	10.338	2.833	0.000	0.210	6.667	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	411	172	317	0	227	401	0	0
N.S.	1	0.97	0.41	0.75	0.00	0.54	0.95	0.00	0.00
time (sec)	N/A	0.838	10.364	3.202	0.000	0.349	6.986	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	434	149	325	0	250	403	0	0
N.S.	1	0.97	0.33	0.72	0.00	0.56	0.90	0.00	0.00
time (sec)	N/A	0.870	10.217	3.374	0.000	0.133	10.769	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	455	151	347	0	258	403	0	0
N.S.	1	0.96	0.32	0.73	0.00	0.54	0.85	0.00	0.00
time (sec)	N/A	0.928	10.210	3.653	0.000	0.128	11.565	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	373	212	246	0	163	177	0	0
N.S.	1	1.03	0.59	0.68	0.00	0.45	0.49	0.00	0.00
time (sec)	N/A	0.581	10.152	2.182	0.000	0.124	2.825	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	350	212	235	0	156	156	0	0
N.S.	1	1.04	0.63	0.70	0.00	0.46	0.46	0.00	0.00
time (sec)	N/A	0.537	10.159	2.093	0.000	0.118	2.696	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	322	193	230	0	147	156	0	0
N.S.	1	1.05	0.63	0.75	0.00	0.48	0.51	0.00	0.00
time (sec)	N/A	0.507	10.189	2.271	0.000	0.122	2.618	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	160	222	0	133	129	0	0
N.S.	1	1.00	0.54	0.74	0.00	0.44	0.43	0.00	0.00
time (sec)	N/A	0.458	10.112	2.066	0.000	0.122	1.936	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	150	208	0	135	128	0	0
N.S.	1	1.00	0.54	0.75	0.00	0.49	0.46	0.00	0.00
time (sec)	N/A	0.387	10.102	1.895	0.000	0.114	1.507	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	159	216	0	0	126	0	0
N.S.	1	1.00	0.56	0.76	0.00	0.00	0.44	0.00	0.00
time (sec)	N/A	0.467	10.222	1.627	0.000	0.000	2.274	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	157	234	0	0	128	0	0
N.S.	1	1.00	0.51	0.76	0.00	0.00	0.41	0.00	0.00
time (sec)	N/A	0.494	10.227	2.029	0.000	0.000	1.810	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	148	225	0	139	126	0	118
N.S.	1	1.00	0.49	0.75	0.00	0.46	0.42	0.00	0.39
time (sec)	N/A	0.484	10.143	1.928	0.000	0.111	1.715	0.000	9.789

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	149	237	0	136	131	0	0
N.S.	1	1.00	0.46	0.73	0.00	0.42	0.41	0.00	0.00
time (sec)	N/A	0.524	10.165	2.051	0.000	0.114	1.884	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	147	243	0	150	158	0	0
N.S.	1	1.00	0.42	0.70	0.00	0.43	0.46	0.00	0.00
time (sec)	N/A	0.553	10.161	2.124	0.000	0.126	2.410	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	134	255	0	159	163	0	0
N.S.	1	1.00	0.36	0.68	0.00	0.42	0.43	0.00	0.00
time (sec)	N/A	0.605	10.206	2.009	0.000	0.122	2.641	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	373	220	302	0	242	202	0	0
N.S.	1	1.02	0.60	0.83	0.00	0.66	0.55	0.00	0.00
time (sec)	N/A	0.836	10.199	3.068	0.000	0.121	10.696	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	347	176	282	0	211	172	0	0
N.S.	1	1.01	0.51	0.82	0.00	0.62	0.50	0.00	0.00
time (sec)	N/A	0.667	10.169	2.807	0.000	0.116	8.390	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	319	166	264	0	223	172	0	0
N.S.	1	1.02	0.53	0.84	0.00	0.71	0.55	0.00	0.00
time (sec)	N/A	0.556	10.161	2.807	0.000	0.118	7.198	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	298	181	248	0	215	156	0	0
N.S.	1	0.99	0.60	0.82	0.00	0.71	0.52	0.00	0.00
time (sec)	N/A	0.494	10.135	1.799	0.000	0.126	6.388	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	327	165	251	0	201	156	0	0
N.S.	1	0.98	0.50	0.75	0.00	0.60	0.47	0.00	0.00
time (sec)	N/A	0.557	10.218	1.736	0.000	0.123	5.898	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	298	116	227	0	147	133	0	0
N.S.	1	0.98	0.38	0.75	0.00	0.49	0.44	0.00	0.00
time (sec)	N/A	0.482	10.083	1.876	0.000	0.092	5.279	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	116	230	0	129	131	0	0
N.S.	1	1.00	0.42	0.84	0.00	0.47	0.48	0.00	0.00
time (sec)	N/A	0.387	10.078	1.730	0.000	0.093	4.950	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	125	253	0	191	289	0	0
N.S.	1	1.00	0.39	0.78	0.00	0.59	0.89	0.00	0.00
time (sec)	N/A	0.654	10.127	1.732	0.000	0.123	7.639	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	349	123	268	0	215	291	0	133
N.S.	1	1.01	0.36	0.78	0.00	0.62	0.85	0.00	0.39
time (sec)	N/A	0.733	10.120	2.851	0.000	0.122	8.080	0.000	9.968

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	371	140	283	0	231	316	0	147
N.S.	1	1.01	0.38	0.77	0.00	0.63	0.86	0.00	0.40
time (sec)	N/A	0.828	10.110	2.836	0.000	0.123	7.472	0.000	10.027

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	392	136	299	0	216	321	0	0
N.S.	1	1.01	0.35	0.77	0.00	0.56	0.83	0.00	0.00
time (sec)	N/A	1.025	10.127	2.797	0.000	0.126	9.872	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	269	174	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.476	0.405	0.000	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	170	147	0	0	0	141	0	0
N.S.	1	1.19	1.03	0.00	0.00	0.00	0.99	0.00	0.00
time (sec)	N/A	0.351	0.735	0.000	0.000	0.000	20.537	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	175	175	145	0	0	0	143	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.391	0.757	0.000	0.000	0.000	47.592	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75
time (sec)	N/A	0.152	0.007	1.524	0.209	0.261	0.019	0.279	0.024

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	7	9	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.70	0.90	0.60
time (sec)	N/A	0.158	0.001	1.531	0.210	0.272	0.023	0.268	0.056

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	8	9	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.80	0.90	0.60
time (sec)	N/A	0.157	0.002	1.545	0.202	0.249	0.023	0.277	8.893

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	14	17	17	15	15	6
N.S.	1	1.00	2.10	1.40	1.70	1.70	1.50	1.50	0.60
time (sec)	N/A	0.147	0.022	1.528	0.199	0.259	0.041	0.276	0.106

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	16	16	24	16	16
N.S.	1	1.00	1.00	0.71	0.67	0.67	1.00	0.67	0.67
time (sec)	N/A	0.171	0.013	1.529	0.274	0.280	0.052	0.273	0.031

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	38	38	46	39	49
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.92	0.78	0.98
time (sec)	N/A	0.236	0.021	1.552	0.286	0.288	0.097	0.265	0.141

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	39	38	38	46	39	48
N.S.	1	1.00	0.92	0.78	0.76	0.76	0.92	0.78	0.96
time (sec)	N/A	0.232	0.016	1.593	0.287	0.265	0.089	0.272	8.885

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	56	47	46	46	56	48	52
N.S.	1	1.00	0.93	0.78	0.77	0.77	0.93	0.80	0.87
time (sec)	N/A	0.235	0.016	1.608	0.275	0.276	0.102	0.268	9.061

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	47	46	46	56	48	52
N.S.	1	1.00	0.87	0.78	0.77	0.77	0.93	0.80	0.87
time (sec)	N/A	0.237	0.017	1.527	0.292	0.275	0.103	0.261	9.024

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	55	50	39	38	38	48	35	46
N.S.	1	1.10	1.00	0.78	0.76	0.76	0.96	0.70	0.92
time (sec)	N/A	0.218	0.008	1.522	0.284	0.438	0.061	0.280	0.099

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	38	38	48	39	46
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.96	0.78	0.92
time (sec)	N/A	0.226	0.013	1.518	0.325	0.447	0.073	0.275	9.065

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	100	83	84	115	105	86	100
N.S.	1	1.00	0.91	0.75	0.76	1.05	0.95	0.78	0.91
time (sec)	N/A	0.314	0.094	1.663	0.283	0.414	0.202	0.276	9.366

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	83	84	115	105	86	100
N.S.	1	1.00	0.88	0.75	0.76	1.05	0.95	0.78	0.91
time (sec)	N/A	0.320	0.084	1.613	0.281	0.436	0.200	0.279	0.198

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	122	61	61	91	70	63	52
N.S.	1	1.00	1.51	0.75	0.75	1.12	0.86	0.78	0.64
time (sec)	N/A	0.252	0.530	1.588	0.289	0.276	0.097	0.277	0.060

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	73	74	126	82	76	77
N.S.	1	1.00	0.91	0.79	0.80	1.37	0.89	0.83	0.84
time (sec)	N/A	0.311	0.034	1.568	0.308	0.280	0.153	0.280	0.128

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	119	104	105	256	124	111	120
N.S.	1	1.00	0.80	0.70	0.71	1.73	0.84	0.75	0.81
time (sec)	N/A	0.369	0.063	1.577	0.292	0.283	0.314	0.265	9.484

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	121	104	105	257	124	111	121
N.S.	1	1.00	0.83	0.71	0.72	1.76	0.85	0.76	0.83
time (sec)	N/A	0.379	0.081	1.563	0.276	0.270	0.330	0.289	9.929

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	111	94	95	187	116	106	110
N.S.	1	1.00	0.78	0.66	0.67	1.32	0.82	0.75	0.77
time (sec)	N/A	0.353	0.065	1.553	0.325	0.323	0.308	0.291	11.763

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	111	94	95	187	116	106	111
N.S.	1	1.00	0.78	0.66	0.67	1.32	0.82	0.75	0.78
time (sec)	N/A	0.352	0.080	1.543	0.289	0.389	0.294	0.295	0.195

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	138	103	86	87	131	110	89	102
N.S.	1	1.22	0.91	0.76	0.77	1.16	0.97	0.79	0.90
time (sec)	N/A	0.329	0.056	1.585	0.287	0.481	0.237	0.273	9.749

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	111	94	95	187	119	99	111
N.S.	1	1.00	0.85	0.72	0.73	1.43	0.91	0.76	0.85
time (sec)	N/A	0.350	0.068	1.538	0.289	0.461	0.289	0.282	0.194

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	119	91	76	75	75	102	69	91
N.S.	1	1.20	0.92	0.77	0.76	0.76	1.03	0.70	0.92
time (sec)	N/A	0.293	0.019	1.759	0.284	0.300	0.198	0.291	9.678

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	130	0	0	0	860	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	5.31	0.00	0.00
time (sec)	N/A	0.406	0.633	0.000	0.000	0.000	25.498	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	83	108	118	118	305	1340	392	115
N.S.	1	0.99	1.29	1.40	1.40	3.63	15.95	4.67	1.37
time (sec)	N/A	0.270	0.281	1.760	0.224	0.286	0.783	0.289	9.422

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	120	80	78	160	598	196	76
N.S.	1	1.00	1.97	1.31	1.28	2.62	9.80	3.21	1.25
time (sec)	N/A	0.243	0.176	2.077	0.213	0.328	0.484	0.277	9.179

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	43	42	43	39	56	180	65	38
N.S.	1	1.05	1.02	1.05	0.95	1.37	4.39	1.59	0.93
time (sec)	N/A	0.208	0.131	0.068	0.206	0.528	0.303	0.282	9.746

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	17	12	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.42	1.00	1.00	1.00
time (sec)	N/A	0.144	0.002	0.026	0.201	0.817	0.018	0.274	11.319

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	73	0	43
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.74	0.00	1.02
time (sec)	N/A	0.206	0.106	0.000	0.000	0.000	1.221	0.000	11.197

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	388	0	49
N.S.	1	1.00	1.00	0.00	0.00	0.00	8.82	0.00	1.11
time (sec)	N/A	0.214	0.138	0.000	0.000	0.000	4.866	0.000	10.885

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	63	0	0	0	0	0	59
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	1.28
time (sec)	N/A	0.209	0.126	0.000	0.000	0.000	0.000	0.000	10.811

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	305	305	206	0	0	0	342	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	1.12	0.00	0.00
time (sec)	N/A	0.523	1.115	0.000	0.000	0.000	20.808	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	66	162	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.47	3.60	0.00	0.00
time (sec)	N/A	0.330	0.952	0.000	0.000	0.397	137.421	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	273	273	178	0	0	0	304	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	1.11	0.00	0.00
time (sec)	N/A	0.462	0.375	0.000	0.000	0.000	104.019	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	204	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	0.461	0.000	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	165	147	0	0	0	0	0	0
N.S.	1	1.02	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	0.774	0.000	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	20	0	0	20
N.S.	1	1.00	1.00	0.88	0.83	0.83	0.00	0.00	0.83
time (sec)	N/A	0.206	7.399	2.483	0.273	0.285	0.000	0.000	9.615

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	93	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	10.542	0.000	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	61	0	228	95
N.S.	1	1.00	1.00	0.00	0.00	2.18	0.00	8.14	3.39
time (sec)	N/A	0.271	0.528	0.000	0.000	0.269	0.000	0.330	9.356

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	45	45	46	138	77	119	0	237	124
N.S.	1	1.00	1.02	3.07	1.71	2.64	0.00	5.27	2.76
time (sec)	N/A	0.323	1.601	0.244	0.308	0.283	0.000	0.350	9.438

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	52	59	54	0	111	76
N.S.	1	1.00	1.00	1.68	1.90	1.74	0.00	3.58	2.45
time (sec)	N/A	0.362	0.931	0.056	0.296	0.286	0.000	0.350	10.776

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	45	45	41	103	92	88	0	155	106
N.S.	1	1.00	0.91	2.29	2.04	1.96	0.00	3.44	2.36
time (sec)	N/A	0.637	2.494	0.276	0.304	0.291	0.000	0.456	11.690

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [10] had the largest ratio of [.9333329999999999968]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	20	0.100
2	A	2	2	1.00	22	0.091
3	A	2	2	1.00	22	0.091
4	A	2	2	1.00	25	0.080
5	A	2	2	1.00	27	0.074
6	A	2	2	1.00	27	0.074
7	A	9	8	0.97	15	0.533
8	A	11	10	1.01	15	0.667
9	A	13	12	1.05	15	0.800
10	A	15	14	1.08	15	0.933
11	A	9	8	0.97	15	0.533
12	A	8	7	0.96	16	0.438
13	A	4	3	1.00	11	0.273
14	A	4	3	1.00	15	0.200
15	A	4	4	1.00	13	0.308
16	A	3	3	1.00	13	0.231
17	A	8	7	1.02	15	0.467
18	A	4	3	1.00	19	0.158
19	A	4	3	1.00	21	0.143
20	A	4	3	0.90	31	0.097
21	A	4	3	0.90	36	0.083
22	A	1	1	1.00	35	0.029

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	1	1	1.00	33	0.030
24	A	1	1	1.00	36	0.028
25	A	13	12	1.01	19	0.632
26	A	11	10	0.97	18	0.556
27	A	5	4	1.00	27	0.148
28	A	5	4	1.00	28	0.143
29	A	5	4	1.00	24	0.167
30	A	5	4	0.91	24	0.167
31	A	5	4	1.00	26	0.154
32	A	5	4	1.00	26	0.154
33	A	5	4	1.00	28	0.143
34	A	5	4	1.00	30	0.133
35	A	5	4	1.00	29	0.138
36	A	5	4	1.00	29	0.138
37	A	5	4	0.93	29	0.138
38	A	5	4	0.94	32	0.125
39	A	8	7	1.15	13	0.538
40	A	5	4	1.09	49	0.082
41	A	5	4	1.00	57	0.070
42	A	2	2	1.00	31	0.065
43	A	2	2	1.00	42	0.048
44	A	5	4	1.00	42	0.095
45	A	5	4	1.00	45	0.089
46	A	5	4	1.00	45	0.089
47	A	5	4	1.00	44	0.091
48	A	3	3	1.00	20	0.150
49	A	7	6	1.09	20	0.300
50	A	2	2	1.00	16	0.125
51	A	6	5	1.00	20	0.250
52	A	4	4	1.00	18	0.222
53	A	2	2	1.00	30	0.067
54	A	2	2	1.00	30	0.067
55	A	2	2	1.00	28	0.071
56	A	2	2	1.17	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	10	9	0.97	30	0.300
58	A	12	11	1.01	30	0.367
59	A	10	10	1.02	32	0.312
60	A	8	8	1.01	32	0.250
61	A	6	6	1.01	32	0.188
62	A	4	4	1.01	32	0.125
63	A	6	6	1.01	32	0.188
64	A	8	8	1.02	32	0.250
65	A	10	10	1.03	32	0.312
66	A	9	9	1.01	32	0.281
67	A	7	7	0.99	32	0.219
68	A	7	7	1.01	32	0.219
69	A	9	9	1.01	32	0.281
70	A	12	11	0.96	17	0.647
71	A	2	2	1.00	17	0.118
72	A	2	2	0.99	17	0.118
73	A	2	2	0.99	22	0.091
74	A	2	2	1.00	22	0.091
75	A	2	2	1.00	22	0.091
76	A	3	3	1.00	17	0.176
77	A	3	3	1.00	19	0.158
78	A	2	2	1.00	18	0.111
79	A	3	3	1.00	18	0.167
80	A	3	3	1.00	22	0.136
81	A	1	1	1.00	20	0.050
82	A	1	1	1.00	20	0.050
83	A	3	3	1.00	33	0.091
84	A	3	3	1.00	35	0.086
85	A	1	1	1.00	36	0.028
86	A	1	1	1.00	36	0.028
87	A	3	3	0.99	30	0.100
88	A	3	3	0.99	32	0.094
89	A	1	1	1.00	33	0.030
90	A	1	1	1.00	33	0.030

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	1	1	1.00	20	0.050
92	A	1	1	1.00	24	0.042
93	A	3	3	1.00	22	0.136
94	A	3	3	1.00	22	0.136
95	A	1	1	1.00	20	0.050
96	A	1	1	1.00	20	0.050
97	A	3	3	1.00	18	0.167
98	A	3	3	1.00	22	0.136
99	A	1	1	1.00	35	0.029
100	A	1	1	1.00	37	0.027
101	A	3	3	1.00	38	0.079
102	A	3	3	1.00	38	0.079
103	A	1	1	1.00	32	0.031
104	A	1	1	1.00	34	0.029
105	A	3	3	0.99	35	0.086
106	A	3	3	0.99	35	0.086
107	A	3	3	1.01	17	0.176
108	A	3	3	1.01	18	0.167
109	A	3	3	1.01	19	0.158
110	A	3	3	1.01	20	0.150
111	A	3	3	1.00	15	0.200
112	A	3	3	1.00	17	0.176
113	A	3	3	1.01	15	0.200
114	A	3	3	1.00	17	0.176
115	A	2	2	1.00	16	0.125
116	A	2	2	1.00	15	0.133
117	A	4	4	1.05	16	0.250
118	A	4	4	1.02	15	0.267
119	A	6	6	1.10	16	0.375
120	A	6	6	1.05	15	0.400
121	A	8	8	1.14	16	0.500
122	A	8	8	1.08	15	0.533
123	A	2	2	1.00	15	0.133
124	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	2	2	1.00	21	0.095
126	A	2	2	1.00	20	0.100
127	A	4	4	1.03	21	0.190
128	A	4	4	1.02	20	0.200
129	A	6	6	1.08	21	0.286
130	A	6	6	1.04	20	0.300
131	A	8	8	1.10	21	0.381
132	A	8	8	1.06	20	0.400
133	A	3	3	0.96	11	0.273
134	A	3	3	0.97	12	0.250
135	A	2	2	1.00	15	0.133
136	A	3	3	0.97	14	0.214
137	A	2	2	1.00	17	0.118
138	A	3	3	1.00	19	0.158
139	A	2	2	1.00	20	0.100
140	A	2	2	1.00	14	0.143
141	A	4	4	1.00	17	0.235
142	A	4	4	1.00	19	0.211
143	A	3	3	1.00	20	0.150
144	A	4	4	1.00	21	0.190
145	A	3	3	1.00	22	0.136
146	A	4	4	1.00	24	0.167
147	A	3	3	1.00	25	0.120
148	A	3	3	1.00	25	0.120
149	A	4	4	1.03	26	0.154
150	A	6	6	1.07	26	0.231
151	A	8	8	1.10	26	0.308
152	A	10	9	1.33	11	0.818
153	A	4	3	1.00	12	0.250
154	A	2	2	1.00	15	0.133
155	A	10	9	1.33	14	0.643
156	A	9	8	1.02	17	0.471
157	A	3	3	1.00	19	0.158
158	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
159	A	2	2	1.00	14	0.143
160	A	2	2	1.00	17	0.118
161	A	6	5	1.03	19	0.263
162	A	2	2	1.00	20	0.100
163	A	3	3	1.00	21	0.143
164	A	2	2	1.00	22	0.091
165	A	3	3	1.00	24	0.125
166	A	2	2	1.00	25	0.080
167	A	2	2	1.00	19	0.105
168	A	2	2	1.00	17	0.118
169	A	2	2	1.00	20	0.100
170	A	2	2	1.00	19	0.105
171	A	2	2	1.00	31	0.065
172	A	3	3	1.05	31	0.097
173	A	5	5	1.06	31	0.161
174	A	7	7	1.07	31	0.226
175	A	2	2	1.00	30	0.067
176	A	4	4	1.02	30	0.133
177	A	6	6	1.04	30	0.200
178	A	8	8	1.04	30	0.267
179	A	2	2	1.00	21	0.095
180	A	2	2	1.00	21	0.095
181	A	2	2	1.22	19	0.105
182	A	2	2	1.00	19	0.105
183	A	2	2	1.00	21	0.095
184	A	2	2	1.00	21	0.095
185	A	2	2	1.00	21	0.095
186	A	2	2	1.00	36	0.056
187	A	2	2	1.00	41	0.049
188	A	2	2	1.00	46	0.043
189	A	2	2	1.00	35	0.057
190	A	2	2	1.00	40	0.050
191	A	2	2	1.00	45	0.044
192	A	4	4	1.04	36	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	4	4	1.04	41	0.098
194	A	4	4	1.01	46	0.087
195	A	4	4	1.02	35	0.114
196	A	4	4	1.02	40	0.100
197	A	4	4	1.01	45	0.089
198	A	6	6	1.09	36	0.167
199	A	6	6	1.07	41	0.146
200	A	6	6	1.05	46	0.130
201	A	6	6	1.05	35	0.171
202	A	6	6	1.04	40	0.150
203	A	6	6	1.03	45	0.133
204	A	8	8	1.10	36	0.222
205	A	8	8	1.08	41	0.195
206	A	8	8	1.05	46	0.174
207	A	8	8	1.06	35	0.229
208	A	8	8	1.05	40	0.200
209	A	8	8	1.04	45	0.178
210	A	2	2	1.00	17	0.118
211	A	2	2	1.00	18	0.111
212	A	2	2	1.00	19	0.105
213	A	2	2	1.00	20	0.100
214	A	2	2	1.00	22	0.091
215	A	1	1	1.00	23	0.043
216	A	1	1	1.00	26	0.038
217	A	1	1	1.00	28	0.036
218	A	1	1	1.00	31	0.032
219	A	1	1	1.00	15	0.067
220	A	2	2	1.00	42	0.048
221	A	3	3	1.33	11	0.273
222	A	3	3	1.31	15	0.200
223	A	4	3	1.00	30	0.100
224	A	4	3	0.99	30	0.100
225	A	4	3	0.99	30	0.100
226	A	4	3	0.98	30	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	4	3	1.01	30	0.100
228	A	4	3	0.98	30	0.100
229	A	4	3	1.01	30	0.100
230	A	4	3	1.01	30	0.100
231	A	4	3	1.01	30	0.100
232	A	4	3	1.00	30	0.100
233	A	4	4	1.01	30	0.133
234	A	4	4	1.02	30	0.133
235	A	4	4	1.02	30	0.133
236	A	4	4	1.02	30	0.133
237	A	4	4	1.00	30	0.133
238	A	4	4	1.02	28	0.143
239	A	2	2	1.00	27	0.074
240	A	2	2	1.00	30	0.067
241	A	2	2	1.00	30	0.067
242	A	2	2	1.00	30	0.067
243	A	2	2	1.00	30	0.067
244	A	2	2	1.00	30	0.067
245	A	2	2	1.00	30	0.067
246	A	2	2	1.00	30	0.067
247	A	2	2	1.00	30	0.067
248	A	2	2	1.00	30	0.067
249	A	2	2	1.00	30	0.067
250	A	2	2	1.00	30	0.067
251	A	4	3	0.98	30	0.100
252	A	4	3	0.98	30	0.100
253	A	4	3	0.96	30	0.100
254	A	4	3	0.96	30	0.100
255	A	4	3	0.98	30	0.100
256	A	4	3	0.99	30	0.100
257	A	4	3	0.98	30	0.100
258	A	4	3	0.99	30	0.100
259	A	4	3	0.99	30	0.100
260	A	4	4	1.03	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	8	8	1.07	30	0.267
262	A	3	3	1.03	30	0.100
263	A	7	7	1.06	30	0.233
264	A	4	4	1.03	30	0.133
265	A	5	5	1.03	28	0.179
266	A	14	13	0.82	27	0.481
267	A	4	4	1.04	30	0.133
268	A	4	4	1.04	30	0.133
269	A	4	4	1.06	30	0.133
270	A	4	4	1.06	30	0.133
271	A	4	4	1.06	30	0.133
272	A	4	4	1.06	30	0.133
273	A	4	4	1.07	30	0.133
274	A	4	4	1.06	30	0.133
275	A	4	4	1.06	30	0.133
276	A	4	3	0.98	30	0.100
277	A	4	3	0.99	30	0.100
278	A	4	3	0.97	30	0.100
279	A	4	3	0.97	30	0.100
280	A	4	3	0.96	30	0.100
281	A	4	3	1.01	30	0.100
282	A	4	3	0.99	30	0.100
283	A	4	3	0.99	30	0.100
284	A	4	3	1.00	30	0.100
285	A	4	3	0.99	30	0.100
286	A	5	5	1.07	30	0.167
287	A	11	11	1.11	30	0.367
288	A	6	6	1.09	30	0.200
289	A	9	9	1.10	30	0.300
290	A	5	5	1.08	30	0.167
291	A	8	8	1.07	30	0.267
292	A	16	15	0.90	30	0.500
293	A	16	15	0.87	28	0.536
294	A	14	13	0.87	27	0.481

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	15	14	0.89	30	0.467
296	A	14	13	0.87	30	0.433
297	A	5	5	1.07	30	0.167
298	A	6	6	1.07	30	0.200
299	A	6	6	1.09	30	0.200
300	A	6	6	1.09	30	0.200
301	A	6	6	1.09	30	0.200
302	A	6	6	1.09	30	0.200
303	A	6	6	1.08	30	0.200
304	A	2	2	1.00	16	0.125
305	A	2	2	1.00	16	0.125
306	A	2	2	1.00	16	0.125
307	A	8	7	1.10	14	0.500
308	A	2	2	1.00	16	0.125
309	A	2	2	1.00	16	0.125
310	A	2	2	1.00	16	0.125
311	A	9	8	1.10	14	0.571
312	A	8	7	1.00	16	0.438
313	A	2	2	1.00	21	0.095
314	A	2	2	1.00	19	0.105
315	A	2	2	1.00	18	0.111
316	A	2	2	1.00	21	0.095
317	A	2	2	1.00	21	0.095
318	A	2	2	1.00	21	0.095
319	A	3	3	1.00	23	0.130
320	A	3	3	1.00	21	0.143
321	A	3	3	1.00	20	0.150
322	A	2	2	1.00	23	0.087
323	A	2	2	1.00	23	0.087
324	A	2	2	1.00	23	0.087
325	A	3	3	1.00	23	0.130
326	A	3	3	1.00	21	0.143
327	A	3	3	1.00	20	0.150
328	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	2	2	1.00	23	0.087
330	A	2	2	1.00	23	0.087
331	A	3	3	1.00	23	0.130
332	A	3	3	1.00	21	0.143
333	A	3	3	1.00	20	0.150
334	A	2	2	1.00	23	0.087
335	A	2	2	1.00	23	0.087
336	A	2	2	1.00	23	0.087
337	A	2	2	1.00	23	0.087
338	A	2	2	1.00	23	0.087
339	A	2	2	1.00	21	0.095
340	A	11	10	0.97	20	0.500
341	A	2	2	1.00	23	0.087
342	A	2	2	1.00	23	0.087
343	A	2	2	1.00	23	0.087
344	A	10	9	1.01	23	0.391
345	A	12	11	1.02	21	0.524
346	A	11	10	1.01	20	0.500
347	A	4	4	1.05	23	0.174
348	A	4	4	1.05	23	0.174
349	A	4	4	1.05	23	0.174
350	A	4	4	1.04	23	0.174
351	A	12	11	1.03	23	0.478
352	A	14	13	1.04	21	0.619
353	A	13	12	1.04	20	0.600
354	A	6	6	1.11	23	0.261
355	A	6	6	1.12	23	0.261
356	A	6	6	1.13	23	0.261
357	A	6	6	1.12	23	0.261
358	A	14	13	1.05	23	0.565
359	A	16	15	1.06	21	0.714
360	A	15	14	1.07	20	0.700
361	A	8	8	1.16	23	0.348
362	A	8	8	1.17	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	8	8	1.17	23	0.348
364	A	8	8	1.14	23	0.348
365	A	6	5	1.00	20	0.250
366	A	5	4	1.00	18	0.222
367	A	6	5	1.00	20	0.250
368	A	5	4	1.00	18	0.222
369	A	5	4	1.00	27	0.148
370	A	5	4	1.00	29	0.138
371	A	5	4	1.00	28	0.143
372	A	5	4	1.00	28	0.143
373	A	2	2	1.00	36	0.056
374	A	2	2	1.00	36	0.056
375	A	2	2	1.00	36	0.056
376	A	2	2	1.00	34	0.059
377	A	2	2	1.00	33	0.061
378	A	2	2	1.00	36	0.056
379	A	2	2	1.00	36	0.056
380	A	2	2	1.00	36	0.056
381	A	2	2	1.00	36	0.056
382	A	2	2	1.00	36	0.056
383	A	2	2	1.00	38	0.053
384	A	2	2	1.00	38	0.053
385	A	3	3	1.00	38	0.079
386	A	3	3	1.00	36	0.083
387	A	3	3	1.00	35	0.086
388	A	3	3	1.00	38	0.079
389	A	3	3	1.00	38	0.079
390	A	3	3	1.00	38	0.079
391	A	2	2	1.00	38	0.053
392	A	2	2	1.00	38	0.053
393	A	2	2	1.00	38	0.053
394	A	2	2	1.00	38	0.053
395	A	3	3	1.00	38	0.079
396	A	3	3	1.00	36	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
397	A	3	3	1.00	35	0.086
398	A	3	3	1.00	38	0.079
399	A	3	3	1.00	38	0.079
400	A	3	3	1.00	38	0.079
401	A	2	2	1.00	38	0.053
402	A	2	2	1.00	38	0.053
403	A	8	8	0.99	38	0.211
404	A	8	8	1.00	38	0.211
405	A	8	8	1.01	38	0.211
406	A	8	8	1.02	36	0.222
407	A	2	2	0.99	35	0.057
408	A	2	2	0.99	38	0.053
409	A	2	2	1.00	38	0.053
410	A	2	2	0.99	38	0.053
411	A	2	2	0.99	38	0.053
412	A	3	3	0.99	38	0.079
413	A	4	4	1.00	38	0.105
414	A	3	3	1.00	38	0.079
415	A	4	4	1.01	36	0.111
416	A	14	13	0.99	35	0.371
417	A	4	4	1.04	38	0.105
418	A	4	4	1.05	38	0.105
419	A	4	4	1.06	38	0.105
420	A	4	4	1.05	38	0.105
421	A	5	5	1.06	38	0.132
422	A	16	15	1.04	38	0.395
423	A	12	11	1.03	38	0.289
424	A	14	13	1.03	36	0.361
425	A	13	12	1.02	35	0.343
426	A	6	6	1.07	38	0.158
427	A	6	6	1.08	38	0.158
428	A	6	6	1.11	38	0.158
429	A	6	6	1.09	38	0.158
430	A	12	12	1.03	25	0.480

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
431	A	10	10	1.02	25	0.400
432	A	7	7	1.02	23	0.304
433	A	5	5	1.01	22	0.227
434	A	8	7	1.01	25	0.280
435	A	10	9	1.02	25	0.360
436	A	12	11	1.02	25	0.440
437	A	10	10	1.05	25	0.400
438	A	8	8	1.03	25	0.320
439	A	7	7	1.02	25	0.280
440	A	4	4	1.02	25	0.160
441	A	7	7	1.01	23	0.304
442	A	5	5	1.01	22	0.227
443	A	12	11	1.01	25	0.440
444	A	14	13	1.02	25	0.520
445	A	19	19	1.01	35	0.543
446	A	16	16	1.01	35	0.457
447	A	13	13	1.00	33	0.394
448	A	11	11	0.99	32	0.344
449	A	14	13	1.00	35	0.371
450	A	14	13	1.01	35	0.371
451	A	13	12	1.03	35	0.343
452	A	16	15	1.06	35	0.429
453	A	18	17	1.04	35	0.486
454	A	14	13	1.00	35	0.371
455	A	15	14	1.02	35	0.400
456	A	18	17	1.01	35	0.486
457	A	20	19	1.01	35	0.543
458	A	21	21	1.00	35	0.600
459	A	18	18	1.00	35	0.514
460	A	15	15	0.99	33	0.455
461	A	13	13	0.99	32	0.406
462	A	16	15	1.00	35	0.429
463	A	16	15	1.01	35	0.429
464	A	15	14	1.02	35	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
465	A	18	17	1.05	35	0.486
466	A	20	19	1.04	35	0.543
467	A	16	15	1.03	35	0.429
468	A	18	17	1.04	35	0.486
469	A	20	19	1.03	35	0.543
470	A	16	15	1.00	35	0.429
471	A	18	17	1.02	35	0.486
472	A	20	19	1.00	35	0.543
473	A	22	21	1.01	35	0.600
474	A	4	4	1.18	20	0.200
475	A	2	2	1.17	21	0.095
476	A	2	2	1.17	23	0.087
477	A	2	2	1.00	23	0.087
478	A	2	2	1.00	26	0.077
479	A	3	3	1.00	25	0.120
480	A	3	3	1.00	28	0.107
481	A	3	3	1.00	25	0.120
482	A	3	3	1.00	28	0.107
483	A	3	3	1.00	25	0.120
484	A	3	3	1.00	28	0.107
485	A	2	2	1.00	26	0.077
486	A	2	2	1.00	29	0.069
487	A	2	2	1.00	25	0.080
488	A	2	2	1.00	28	0.071
489	A	4	4	1.02	25	0.160
490	A	3	3	1.02	28	0.107
491	A	6	6	1.04	25	0.240
492	A	5	5	1.03	28	0.179
493	A	8	8	1.06	25	0.320
494	A	7	7	1.04	28	0.250
495	A	2	2	1.03	30	0.067
496	A	2	2	1.03	30	0.067
497	A	2	2	1.04	30	0.067
498	A	2	2	1.00	28	0.071

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
499	A	2	2	1.00	27	0.074
500	A	2	2	1.00	30	0.067
501	A	2	2	1.00	30	0.067
502	A	2	2	1.00	30	0.067
503	A	2	2	1.00	30	0.067
504	A	9	8	1.00	30	0.267
505	A	4	4	0.98	30	0.133
506	A	4	4	0.98	30	0.133
507	A	4	4	0.97	30	0.133
508	A	4	4	0.97	30	0.133
509	A	4	4	0.99	30	0.133
510	A	2	2	1.03	30	0.067
511	A	2	2	1.03	30	0.067
512	A	2	2	1.03	30	0.067
513	A	2	2	1.00	28	0.071
514	A	2	2	1.00	27	0.074
515	A	2	2	1.00	30	0.067
516	A	2	2	1.00	30	0.067
517	A	2	2	1.00	30	0.067
518	A	2	2	1.00	30	0.067
519	A	4	4	1.01	30	0.133
520	A	4	4	1.00	30	0.133
521	A	4	4	0.99	30	0.133
522	A	4	4	0.98	30	0.133
523	A	11	10	1.00	30	0.333
524	A	6	6	0.99	30	0.200
525	A	6	6	0.98	30	0.200
526	A	6	6	0.97	30	0.200
527	A	6	6	0.97	30	0.200
528	A	6	6	0.96	30	0.200
529	A	2	2	1.03	30	0.067
530	A	2	2	1.04	30	0.067
531	A	2	2	1.05	30	0.067
532	A	2	2	1.00	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
533	A	2	2	1.00	27	0.074
534	A	7	6	1.00	30	0.200
535	A	2	2	1.00	30	0.067
536	A	2	2	1.00	30	0.067
537	A	2	2	1.00	30	0.067
538	A	2	2	1.00	30	0.067
539	A	2	2	1.00	30	0.067
540	A	3	3	1.02	30	0.100
541	A	3	3	1.01	30	0.100
542	A	4	4	1.02	30	0.133
543	A	3	3	0.99	30	0.100
544	A	4	4	0.98	30	0.133
545	A	4	4	0.98	28	0.143
546	A	6	6	1.00	27	0.222
547	A	9	8	1.00	30	0.267
548	A	4	4	1.01	30	0.133
549	A	4	4	1.01	30	0.133
550	A	4	4	1.01	30	0.133
551	A	2	2	1.00	30	0.067
552	A	2	2	1.19	25	0.080
553	A	2	2	1.00	28	0.071
554	A	2	2	1.00	22	0.091
555	A	2	2	1.00	35	0.057
556	A	2	2	1.00	35	0.057
557	A	2	2	1.00	22	0.091
558	A	4	3	1.00	25	0.120
559	A	3	3	1.00	15	0.200
560	A	3	3	1.00	15	0.200
561	A	3	3	1.00	20	0.150
562	A	3	3	1.00	20	0.150
563	A	10	9	1.10	17	0.529
564	A	3	3	1.00	25	0.120
565	A	3	3	1.00	35	0.086
566	A	3	3	1.00	35	0.086

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
567	A	3	3	1.00	22	0.136
568	A	3	3	1.00	25	0.120
569	A	3	3	1.00	15	0.200
570	A	3	3	1.00	15	0.200
571	A	3	3	1.00	20	0.150
572	A	3	3	1.00	20	0.150
573	A	13	12	1.22	17	0.706
574	A	3	3	1.00	25	0.120
575	A	10	9	1.20	18	0.500
576	A	2	2	1.00	36	0.056
577	A	4	4	0.99	19	0.211
578	A	4	4	1.00	19	0.211
579	A	3	3	1.05	17	0.176
580	A	1	1	1.00	9	0.111
581	A	3	3	1.00	19	0.158
582	A	3	3	1.00	19	0.158
583	A	3	3	1.00	19	0.158
584	A	2	2	1.00	38	0.053
585	A	2	2	1.00	58	0.034
586	A	2	2	1.00	30	0.067
587	A	2	2	1.00	36	0.056
588	A	4	4	1.02	35	0.114
589	A	1	1	1.00	46	0.022
590	A	11	10	1.00	24	0.417
591	A	1	1	1.00	48	0.021
592	A	1	1	1.00	45	0.022
593	A	1	1	1.00	69	0.014
594	A	1	1	1.00	86	0.012

CHAPTER 3

Listing of integrals

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3.29 $\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx$ . . . 411
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3.54 $\int \left(a+bx^3\right)^2\left(ac+adx+bcx^3+bdx^4\right)\,dx$ . . . 560
3.55 $\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx$ . . . 565
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3.62 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx$ 621
3.63 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx$ 629
3.64 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx$ 638
3.65 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx$ 646
3.66 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$ 655
3.67 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx$ 665
3.68 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx$ 673
3.69 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx$ 682
3.70 $\int \frac{(a+bx)^2}{c+dx^3}\,dx$ 692
3.71 $\int \frac{(a+bx)^3}{c+dx^3}\,dx$ 701
3.72 $\int \frac{(a+bx)^4}{c+dx^3}\,dx$ 708
3.73 $\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx$ 715
3.74 $\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx$ 722
3.75 $\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx$ 729
3.76 $\int \frac{2x^2+x^4}{1+x^3}\,dx$ 737
3.77 $\int \frac{2x^2+x^4}{1-x^3}\,dx$ 742
3.78 $\int \frac{1-x+4x^3}{1+x^3}\,dx$ 747
3.79 $\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$ 752
3.80 $\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$ 758
3.81 $\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$ 765
3.82 $\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$ 771
3.83 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$ 777
3.84 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$ 784
3.85 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$ 791
3.86 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$ 797
3.87 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$ 803

3.88 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$ 811

3.89 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$ 819

3.90 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$ 825

3.91 $\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$ 831

3.92 $\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$ 837

3.93 $\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$ 843

3.94 $\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$ 850

3.95 $\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx$ 857

3.96 $\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx$ 863

3.97 $\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx$ 869

3.98 $\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx$ 876

3.99 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$ 883

3.100 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$ 889

3.101 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$ 895

3.102 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$ 902

3.103 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$ 909

3.104 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$ 915

3.105 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$ 921

3.106 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$ 929

3.107 $\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$ 937

3.108 $\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx$ 945

3.109 $\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx$ 953

3.110 $\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx$ 961

3.111 $\int \frac{c+dx}{\sqrt{1+x^3}}\,dx$ 969

3.112 $\int \frac{c+dx}{\sqrt{1-x^3}}\,dx$ 975

3.113 $\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx$ 981

3.114 $\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx$ 987

3.115 $\int \frac{c+dx}{a-bx^4}\,dx$ 993

3.116 $\int \frac{c+dx}{a+bx^4}\,dx$ 999
3.117 $\int \frac{c+dx}{(a-bx^4)^2}\,dx$ 1005
3.118 $\int \frac{c+dx}{(a+bx^4)^2}\,dx$ 1011
3.119 $\int \frac{c+dx}{(a-bx^4)^3}\,dx$ 1018
3.120 $\int \frac{c+dx}{(a+bx^4)^3}\,dx$ 1025
3.121 $\int \frac{c+dx}{(a-bx^4)^4}\,dx$ 1033
3.122 $\int \frac{c+dx}{(a+bx^4)^4}\,dx$ 1040
3.123 $\int \frac{c+dx}{1-x^4}\,dx$ 1048
3.124 $\int \frac{c+dx}{1+x^4}\,dx$ 1053
3.125 $\int \frac{c+dx+ex^2}{a-bx^4}\,dx$ 1058
3.126 $\int \frac{c+dx+ex^2}{a+bx^4}\,dx$ 1064
3.127 $\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$ 1071
3.128 $\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$ 1078
3.129 $\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx$ 1085
3.130 $\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx$ 1093
3.131 $\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$ 1101
3.132 $\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$ 1109
3.133 $\int a(e+fx^4)^2\,dx$ 1117
3.134 $\int bx(e+fx^4)^2\,dx$ 1121
3.135 $\int (a+bx)\,(e+fx^4)^2\,dx$ 1125
3.136 $\int cx^2(e+fx^4)^2\,dx$ 1129
3.137 $\int (a+cx^2)\,(e+fx^4)^2\,dx$ 1133
3.138 $\int (bx+cx^2)\,(e+fx^4)^2\,dx$ 1137
3.139 $\int (a+bx+cx^2)\,(e+fx^4)^2\,dx$ 1141
3.140 $\int dx^3(e+fx^4)^2\,dx$ 1146
3.141 $\int (a+dx^3)\,(e+fx^4)^2\,dx$ 1150
3.142 $\int (bx+dx^3)\,(e+fx^4)^2\,dx$ 1155
3.143 $\int (a+bx+dx^3)\,(e+fx^4)^2\,dx$ 1160
3.144 $\int (cx^2+dx^3)\,(e+fx^4)^2\,dx$ 1165
3.145 $\int (a+cx^2+dx^3)\,(e+fx^4)^2\,dx$ 1170
3.146 $\int (bx+cx^2+dx^3)\,(e+fx^4)^2\,dx$ 1175
3.147 $\int (c+dx+ex^2+fx^3)\,(a+bx^4)^2\,dx$ 1180
3.148 $\int (c+dx+ex^2+fx^3)\,(a+bx^4)^3\,dx$ 1186
3.149 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$ 1192
3.150 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$ 1199
3.151 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$ 1207
3.152 $\int \frac{a}{2+3x^4}\,dx$ 1215

3.153 $\int \frac{bx}{2+3x^4}\,dx$ . . . 1222
3.154 $\int \frac{a+bx}{2+3x^4}\,dx$ . . . 1227
3.155 $\int \frac{cx^2}{2+3x^4}\,dx$ . . . 1233
3.156 $\int \frac{a+cx^2}{2+3x^4}\,dx$ . . . 1240
3.157 $\int \frac{bx+cx^2}{2+3x^4}\,dx$ . . . 1249
3.158 $\int \frac{a+bx+cx^2}{2+3x^4}\,dx$ . . . 1255
3.159 $\int \frac{dx^3}{2+3x^4}\,dx$ . . . 1262
3.160 $\int \frac{a+dx^3}{2+3x^4}\,dx$ . . . 1267
3.161 $\int \frac{bx+dx^3}{2+3x^4}\,dx$ . . . 1273
3.162 $\int \frac{a+bx+dx^3}{2+3x^4}\,dx$ . . . 1278
3.163 $\int \frac{cx^2+dx^3}{2+3x^4}\,dx$ . . . 1284
3.164 $\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx$ . . . 1290
3.165 $\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx$ . . . 1297
3.166 $\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx$ . . . 1303
3.167 $\int \frac{1+x+x^2+x^3}{1-x^4}\,dx$ . . . 1311
3.168 $\int \frac{1+x+x^2+x^3}{1+x^4}\,dx$ . . . 1315
3.169 $\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx$ . . . 1320
3.170 $\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx$ . . . 1326
3.171 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$ . . . 1333
3.172 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$ . . . 1339
3.173 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$ . . . 1347
3.174 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$ . . . 1355
3.175 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$ . . . 1364
3.176 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$ . . . 1371
3.177 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$ . . . 1379
3.178 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$ . . . 1388
3.179 $\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx$ . . . 1398
3.180 $\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx$ . . . 1403
3.181 $\int \frac{1-x^4}{1+x+x^2+x^3}\,dx$ . . . 1407
3.182 $\int \frac{1+x+x^2+x^3}{1-x^4}\,dx$ . . . 1411
3.183 $\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx$ . . . 1415
3.184 $\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx$ . . . 1420
3.185 $\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx$ . . . 1425
3.186 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx$ . . . 1430

3.187 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx$ . . . 1436
3.188 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx$ . . . 1443
3.189 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx$ . . . 1450
3.190 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx$ . . . 1457
3.191 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx$ . . . 1465
3.192 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx$ . . . 1473
3.193 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx$ . . . 1480
3.194 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx$ . . . 1487
3.195 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx$ . . . 1495
3.196 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx$ . . . 1503
3.197 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx$ . . . 1511
3.198 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx$ . . . 1520
3.199 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx$ . . . 1528
3.200 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx$ . . . 1536
3.201 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx$ . . . 1544
3.202 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx$ . . . 1553
3.203 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx$ . . . 1562
3.204 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx$ . . . 1571
3.205 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx$ . . . 1580
3.206 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx$ . . . 1590
3.207 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx$ . . . 1600
3.208 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$ . . . 1610
3.209 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$ . . . 1620
3.210 $\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx$ . . . 1630
3.211 $\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx$ . . . 1635
3.212 $\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx$ . . . 1640
3.213 $\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx$ . . . 1645
3.214 $\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx$ . . . 1650
3.215 $\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx$ . . . 1656
3.216 $\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx$ . . . 1660
3.217 $\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx$ . . . 1664
3.218 $\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx$ . . . 1668
3.219 $\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx$ . . . 1672

3.220 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$ . . . 1677
3.221 $\int \frac{1+x}{1+x^5}\,dx$ . . . 1683
3.222 $\int \frac{1-x}{1-x^5}\,dx$ . . . 1689
3.223 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$ . . . 1695
3.224 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$ . . . 1701
3.225 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$ . . . 1707
3.226 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$ . . . 1713
3.227 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)}\,dx$ . . . 1718
3.228 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx$ . . . 1723
3.229 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx$ . . . 1728
3.230 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)}\,dx$ . . . 1733
3.231 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx$ . . . 1739
3.232 $\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx$ . . . 1745
3.233 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$ . . . 1751
3.234 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$ . . . 1760
3.235 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$ . . . 1769
3.236 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$ . . . 1777
3.237 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$ . . . 1785
3.238 $\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$ . . . 1793
3.239 $\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx$ . . . 1801
3.240 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx$ . . . 1809
3.241 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx$ . . . 1816
3.242 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx$ . . . 1824
3.243 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx$ . . . 1832
3.244 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$ . . . 1839
3.245 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$ . . . 1847
3.246 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$ . . . 1855
3.247 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$ . . . 1862
3.248 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx$ . . . 1869
3.249 $\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx$ . . . 1876
3.250 $\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx$ . . . 1883
3.251 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$ . . . 1890
3.252 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$ . . . 1898

3.253 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$ . . . 1904
3.254 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$ . . . 1910
3.255 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2}\,dx$ . . . 1915
3.256 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^2}\,dx$ . . . 1920
3.257 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^2}\,dx$ . . . 1925
3.258 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2}\,dx$ . . . 1931
3.259 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^2}\,dx$ . . . 1937
3.260 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$ . . . 1944
3.261 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$ . . . 1955
3.262 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$ . . . 1964
3.263 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$ . . . 1973
3.264 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$ . . . 1983
3.265 $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$ . . . 1992
3.266 $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx$ . . . 2001
3.267 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx$ . . . 2013
3.268 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2}\,dx$ . . . 2022
3.269 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2}\,dx$ . . . 2031
3.270 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2}\,dx$ . . . 2040
3.271 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx$ . . . 2049
3.272 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^2}\,dx$ . . . 2057
3.273 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^2}\,dx$ . . . 2065
3.274 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^2}\,dx$ . . . 2073
3.275 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^2}\,dx$ . . . 2082
3.276 $\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$ . . . 2091
3.277 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$ . . . 2099
3.278 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$ . . . 2106
3.279 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$ . . . 2113
3.280 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$ . . . 2119
3.281 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^3}\,dx$ . . . 2124
3.282 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3}\,dx$ . . . 2129
3.283 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3}\,dx$ . . . 2134

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3.591 $\int (a+bx^n)^{\frac{-1-n}{n}}\,(c+dx^n)^{\frac{-1-n}{n}}\,(ac-bdx^{2n})\,dx$ 4463

3.592 $\int (hx)^{-1-n-np}\,(a+bx^n)^p\,(c+dx^n)^p\,(ac-bdx^{2n})\,dx$ 4467

3.593 $\int (a+bx^n)^p\,(c+dx^n)^p\,\left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right)\,dx$ 4472

3.594 $\int (hx)^m\,(a+bx^n)^p\,(c+dx^n)^p\,\left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)}+\frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right)\,dx$ 4476

3.1 $\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx$

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3.1.1 Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx=\frac{2(b^2c-abd+a^2e)\sqrt{a+bx}}{b^3}+\frac{2(bd-2ae)(a+bx)^{3/2}}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}$$

output
```
2/3*(-2*a*e+b*d)*(b*x+a)^(3/2)/b^3+2/5*e*(b*x+a)^(5/2)/b^3+2*(a^2*e-a*b*d+
b^2*c)*(b*x+a)^(1/2)/b^3
```

3.1.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx=\frac{2\sqrt{a+bx}(8a^2e-2ab(5d+2ex)+b^2(15c+x(5d+3ex)))}{15b^3}$$

input
```
Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(8*a^2*e - 2*a*b*(5*d + 2*e*x) + b^2*(15*c + x*(5*d + 3*e
*x))))/(15*b^3)
```

3.1.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a^2e-abd+b^2c}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}(bd-2ae)}{b^2}+\frac{e(a+bx)^{3/2}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{a+bx}(a^2e-abd+b^2c)}{b^3}+\frac{2(a+bx)^{3/2}(bd-2ae)}{3b^3}+\frac{2e(a+bx)^{5/2}}{5b^3}$$

input
```
Int[(c + d*x + e*x^2)/Sqrt[a + b*x],x]
```

output
```
(2*(b^2*c - a*b*d + a^2*e)*Sqrt[a + b*x])/b^3 + (2*(b*d - 2*a*e)*(a + b*x)
^(3/2))/(3*b^3) + (2*e*(a + b*x)^(5/2))/(5*b^3)
```

3.1.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1.4 Maple [A] (verified)

Time = 5.73 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{16\left(\frac{\left(3e\,x^2+5dx+15c\right)b^2}{8}-\frac{5a\left(\frac{2ex}{5}+d\right)b}{4}+a^2e\right)\sqrt{bx+a}}{15b^3}$	48
gosper	$\frac{2\sqrt{bx+a}\,\left(3b^2e\,x^2-4abex+5b^2dx+8a^2e-10abd+15b^2c\right)}{15b^3}$	53
trager	$\frac{2\sqrt{bx+a}\,\left(3b^2e\,x^2-4abex+5b^2dx+8a^2e-10abd+15b^2c\right)}{15b^3}$	53
risch	$\frac{2\sqrt{bx+a}\,\left(3b^2e\,x^2-4abex+5b^2dx+8a^2e-10abd+15b^2c\right)}{15b^3}$	53
derivativedivides	$\frac{\frac{2e(bx+a)^{\frac{5}{2}}}{5}-\frac{4ae(bx+a)^{\frac{3}{2}}}{3}+\frac{2bd(bx+a)^{\frac{3}{2}}}{3}+2a^2e\sqrt{bx+a}-2abd\sqrt{bx+a}+2b^2c\sqrt{bx+a}}{b^3}$	75
default	$\frac{\frac{2e(bx+a)^{\frac{5}{2}}}{5}-\frac{4ae(bx+a)^{\frac{3}{2}}}{3}+\frac{2bd(bx+a)^{\frac{3}{2}}}{3}+2a^2e\sqrt{bx+a}-2abd\sqrt{bx+a}+2b^2c\sqrt{bx+a}}{b^3}$	75

input
```
int((e*x^2+d*x+c)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
16/15*(1/8*(3*e*x^2+5*d*x+15*c)*b^2-5/4*a*(2/5*e*x+d)*b+a^2*e)*(b*x+a)^(1/
2)/b^3
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx=\frac{2\,(3\,b^2ex^2+15\,b^2c-10\,abd+8\,a^2e+(5\,b^2d-4\,abe)x)\sqrt{bx+a}}{15\,b^3}$$

input
```
integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*e*x^2 + 15*b^2*c - 10*a*b*d + 8*a^2*e + (5*b^2*d - 4*a*b*e)*x)
*sqrt(b*x + a)/b^3
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.42

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx = \begin{cases} \frac{2c\sqrt{a+bx}+\frac{2d\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2e\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} & \text{for } b\neq 0 \\ \frac{cx+\frac{dx^2}{2}+\frac{ex^3}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

output
```
Piecewise(((2*c*sqrt(a + b*x) + 2*d*(-a*sqrt(a + b*x) + (a + b*x)**(3/2)/3
)/b + 2*e*(a**2*sqrt(a + b*x) - 2*a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/
5)/b**2)/b, Ne(b, 0)), ((c*x + d*x**2/2 + e*x**3/3)/sqrt(a), True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx = \frac{2\left(15\sqrt{bx+a}c+\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}\right)}{15b}$$

input
```
integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + (
3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2)/b
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx = \frac{2\left(15\sqrt{bx+a}c+\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}\right)}{15b}$$

input
```
integrate((e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(b*x + a)*c + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b + (
3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^2)/b
```

3.1.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+b\,x}\,\left(3\,e\,{(a+b\,x)}^2+15\,b^2\,c+15\,a^2\,e-10\,a\,e\,(a+b\,x)+5\,b\,d\,(a+b\,x)-15\,a\,b\,d\right)}{15\,b^3}$$

input
```
int((c + d*x + e*x^2)/(a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(1/2)*(3*e*(a + b*x)^2 + 15*b^2*c + 15*a^2*e - 10*a*e*(a + b*
x) + 5*b*d*(a + b*x) - 15*a*b*d))/(15*b^3)
```

3.2 $\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx$

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3.2.1 Optimal result

Integrand size = 22, antiderivative size = 161

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx = \frac{2(b^2c-abd+a^2e)^2\sqrt{a+bx}}{b^5} + \frac{4(bd-2ae)\left(b^2c-abd+a^2e\right)(a+bx)^{3/2}}{3b^5} - \frac{2(6abde-6a^2e^2-b^2(d^2+2ce))\,(a+bx)^{5/2}}{5b^5} + \frac{4e(bd-2ae)(a+bx)^{7/2}}{7b^5} + \frac{2e^2(a+bx)^{9/2}}{9b^5}$$

output
```
4/3*(-2*a*e+b*d)*(a^2*e-a*b*d+b^2*c)*(b*x+a)^(3/2)/b^5-2/5*(6*a*b*d*e-6*a^
2*e^2-b^2*(2*c*e+d^2))*(b*x+a)^(5/2)/b^5+4/7*e*(-2*a*e+b*d)*(b*x+a)^(7/2)/
b^5+2/9*e^2*(b*x+a)^(9/2)/b^5+2*(a^2*e-a*b*d+b^2*c)^2*(b*x+a)^(1/2)/b^5
```

3.2.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(128a^4e^2-32a^3be(9d+2ex)+24a^2b^2(7d^2+6dex+2e(7c+ex^2))-4ab^3(21c(5d+2ex)+x(2}{315b^5}$$

input
```
Integrate[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(128*a^4*e^2 - 32*a^3*b*e*(9*d + 2*e*x) + 24*a^2*b^2*(7*d
^2 + 6*d*e*x + 2*e*(7*c + e*x^2)) - 4*a*b^3*(21*c*(5*d + 2*e*x) + x*(21*d^
2 + 27*d*e*x + 10*e^2*x^2)) + b^4*(315*c^2 + 42*c*x*(5*d + 3*e*x) + x^2*(6
3*d^2 + 90*d*e*x + 35*e^2*x^2))))/(315*b^5)
```

3.2.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx+ex^2\right)^2}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(a+bx)^{3/2}\left(6a^2e^2-6abde+b^2\left(2ce+d^2\right)\right)}{b^4}+\frac{2\sqrt{a+bx}(bd-2ae)\left(a^2e-abd+b^2c\right)}{b^4}+\frac{\left(a^2e-abd+b^2c\right)^2}{b^4\sqrt{a+bx}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(a+bx)^{5/2}\left(-6a^2e^2+6abde-\left(b^2\left(2ce+d^2\right)\right)\right)}{5b^5}+\frac{4(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)}{3b^5}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^2}{b^5}+\frac{4e(a+bx)^{7/2}(bd-2ae)}{7b^5}+\frac{2e^2(a+bx)^{9/2}}{9b^5}$$

input
```
Int[(c + d*x + e*x^2)^2/Sqrt[a + b*x],x]
```

output
```
(2*(b^2*c - a*b*d + a^2*e)^2*Sqrt[a + b*x])/b^5 + (4*(b*d - 2*a*e)*(b^2*c
- a*b*d + a^2*e)*(a + b*x)^(3/2))/(3*b^5) - (2*(6*a*b*d*e - 6*a^2*e^2 - b^
2*(d^2 + 2*c*e))*(a + b*x)^(5/2))/(5*b^5) + (4*e*(b*d - 2*a*e)*(a + b*x)^(
7/2))/(7*b^5) + (2*e^2*(a + b*x)^(9/2))/(9*b^5)
```

3.2.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 5.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{2e^2(bx+a)^{\frac{9}{2}}}{9}+\frac{4(-2ae+bd)e(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left(2\left(a^2e-abd+b^2c\right)e+(-2ae+bd)^2\right)(bx+a)^{\frac{5}{2}}}{5}+\frac{4\left(a^2e-abd+b^2c\right)(-2ae+bd)(bx+a)^{\frac{3}{2}}}{3}+2(a}{b^5}$
default	$\frac{\frac{2e^2(bx+a)^{\frac{9}{2}}}{9}+\frac{4(-2ae+bd)e(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left(2\left(a^2e-abd+b^2c\right)e+(-2ae+bd)^2\right)(bx+a)^{\frac{5}{2}}}{5}+\frac{4\left(a^2e-abd+b^2c\right)(-2ae+bd)(bx+a)^{\frac{3}{2}}}{3}+2(a}{b^5}$
pseudoelliptic	$\frac{256\sqrt{bx+a}\left(\frac{7\left(\frac{5e^2x^4}{2}+9\left(\frac{5dx}{7}+c\right)x^2e+\frac{9d^2x^2}{2}+15cdx+\frac{45c^2}{2}\right)b^4}{64}-\frac{105\left(\frac{2e^2x^3}{21}+\frac{2\left(\frac{9dx}{14}+c\right)xe}{5}+d\left(\frac{dx}{5}+c\right)\right)a\,b^3}{32}+\frac{21a^2\left(\frac{x^2e^2}{7}+\right.}{}}{315b^5}$
gosper	$\frac{2\sqrt{bx+a}\,(35e^2x^4b^4-40a\,b^3e^2x^3+90b^4de\,x^3+48a^2b^2e^2x^2-108a\,b^3de\,x^2+126b^4ce\,x^2+63b^4d^2x^2-64a^3b\,e^2x+144a^2b^2de}{315b^5}$
trager	$\frac{2\sqrt{bx+a}\,(35e^2x^4b^4-40a\,b^3e^2x^3+90b^4de\,x^3+48a^2b^2e^2x^2-108a\,b^3de\,x^2+126b^4ce\,x^2+63b^4d^2x^2-64a^3b\,e^2x+144a^2b^2de}{315b^5}$
risch	$\frac{2\sqrt{bx+a}\,(35e^2x^4b^4-40a\,b^3e^2x^3+90b^4de\,x^3+48a^2b^2e^2x^2-108a\,b^3de\,x^2+126b^4ce\,x^2+63b^4d^2x^2-64a^3b\,e^2x+144a^2b^2de}{315b^5}$

input
```
int((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^5*(1/9*e^2*(b*x+a)^(9/2)+2/7*(-2*a*e+b*d)*e*(b*x+a)^(7/2)+1/5*(2*(a^2*
e-a*b*d+b^2*c)*e+(-2*a*e+b*d)^2)*(b*x+a)^(5/2)+2/3*(a^2*e-a*b*d+b^2*c)*(-2
*a*e+b*d)*(b*x+a)^(3/2)+(a^2*e-a*b*d+b^2*c)^2*(b*x+a)^(1/2))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\left(35\,b^4e^2x^4+315\,b^4c^2-420\,ab^3cd+168\,a^2b^2d^2+128\,a^4e^2+10\,(9\,b^4de-4\,ab^3e^2)x^3+3\,(21\,b^4d^2+16\,a^2b\right.}{}$$

input
```
integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*e^2*x^4 + 315*b^4*c^2 - 420*a*b^3*c*d + 168*a^2*b^2*d^2 + 12
8*a^4*e^2 + 10*(9*b^4*d*e - 4*a*b^3*e^2)*x^3 + 3*(21*b^4*d^2 + 16*a^2*b^2*
e^2 + 6*(7*b^4*c - 6*a*b^3*d)*e)*x^2 + 48*(7*a^2*b^2*c - 6*a^3*b*d)*e + 2*
(105*b^4*c*d - 42*a*b^3*d^2 - 32*a^3*b*e^2 - 12*(7*a*b^3*c - 6*a^2*b^2*d)*
e)*x)*sqrt(b*x + a)/b^5
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{e^2(a+bx)^{\frac{9}{2}}}{9b^4}+\frac{(a+bx)^{\frac{7}{2}}\left(-4ae^2+2bde\right)}{7b^4}+\frac{(a+bx)^{\frac{5}{2}}\cdot\left(6a^2e^2-6abde+2b^2ce+b^2d^2\right)}{5b^4}+\frac{(a+bx)^{\frac{3}{2}}\left(-4a^3e^2+6a^2bde-4ab^2ce-2ab^2d^2+2b^3cd\right)}{3b^4}+\frac{\sqrt{a+bx}\left(a^4e^2-\right.}{}\right.}{b} \\ \frac{c^2x+cdx^2+\frac{dex^4}{2}+\frac{e^2x^5}{5}+\frac{x^3\cdot\left(2ce+d^2\right)}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate((e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

output
```
Piecewise((2*(e**2*(a + b*x)**(9/2)/(9*b**4) + (a + b*x)**(7/2)*(-4*a*e**2
 + 2*b*d*e)/(7*b**4) + (a + b*x)**(5/2)*(6*a**2*e**2 - 6*a*b*d*e + 2*b**2*
c*e + b**2*d**2)/(5*b**4) + (a + b*x)**(3/2)*(-4*a**3*e**2 + 6*a**2*b*d*e
- 4*a*b**2*c*e - 2*a*b**2*d**2 + 2*b**3*c*d)/(3*b**4) + sqrt(a + b*x)*(a**
4*e**2 - 2*a**3*b*d*e + 2*a**2*b**2*c*e + a**2*b**2*d**2 - 2*a*b**3*c*d +
b**4*c**2)/b**4)/b, Ne(b, 0)), ((c**2*x + c*d*x**2 + d*e*x**4/2 + e**2*x**
5/5 + x**3*(2*c*e + d**2)/3)/sqrt(a), True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx = \frac{2\left(315\sqrt{bx+a}c^2+42\,c\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}\right)+\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)\right.}{b^2}\right.}{}$$

input
```
integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(315*sqrt(b*x + a)*c^2 + 42*c*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*
a)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*e/b^2) + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*
a^2)*d^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)
^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(b*
x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*s
qrt(b*x + a)*a^4)*e^2/b^4)/b
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx = \frac{2\left(315\sqrt{bx+a}c^2+\frac{210\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)cd}{b}+\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)d^2}{b^2}+\frac{42\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}\right.}{b^2}\right.}{}$$

input
```
integrate((e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(b*x + a)*c^2 + 210*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*c
*d/b + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2
)*d^2/b^2 + 42*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a
)*a^2)*c*e/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x +
a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^(9/2) - 180*(
b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315
*sqrt(b*x + a)*a^4)*e^2/b^4)/b
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx+ex^2)^2}{\sqrt{a+bx}}\,dx = \frac{2\,e^2\,(a+b\,x)^{9/2}}{9\,b^5} + \frac{(a+b\,x)^{5/2}\,(12\,a^2\,e^2-12\,a\,b\,d\,e+2\,b^2\,d^2+4\,c\,b^2\,e)}{5\,b^5} + \frac{2\,\sqrt{a+b\,x}\,(e\,a^2-d\,a\,b+c\,b^2)^2}{b^5} - \frac{(8\,a\,e^2-4\,b\,d\,e)\,(a+b\,x)^{7/2}}{7\,b^5} - \frac{4\,(2\,a\,e-b\,d)\,(a+b\,x)^{3/2}\,(e\,a^2-d\,a\,b+c\,b^2)}{3\,b^5}$$

input
```
int((c + d*x + e*x^2)^2/(a + b*x)^(1/2),x)
```

output
```
(2*e^2*(a + b*x)^(9/2))/(9*b^5) + ((a + b*x)^(5/2)*(12*a^2*e^2 + 2*b^2*d^2
 + 4*b^2*c*e - 12*a*b*d*e))/(5*b^5) + (2*(a + b*x)^(1/2)*(b^2*c + a^2*e -
a*b*d)^2)/b^5 - ((8*a*e^2 - 4*b*d*e)*(a + b*x)^(7/2))/(7*b^5) - (4*(2*a*e
- b*d)*(a + b*x)^(3/2)*(b^2*c + a^2*e - a*b*d))/(3*b^5)
```

3.3 $\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx$

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3.3.1 Optimal result

Integrand size = 22, antiderivative size = 274

$$\begin{aligned}\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx &= \frac{2(b^2c-abd+a^2e)^3\sqrt{a+bx}}{b^7} \\ &+ \frac{2(bd-2ae)\left(b^2c-abd+a^2e\right)^2(a+bx)^{3/2}}{b^7} \\ &- \frac{6(b^2c-abd+a^2e)\left(5abde-5a^2e^2-b^2(d^2+ce)\right)(a+bx)^{5/2}}{5b^7} \\ &- \frac{2(bd-2ae)\left(10abde-10a^2e^2-b^2(d^2+6ce)\right)(a+bx)^{7/2}}{7b^7} \\ &- \frac{2e(5abde-5a^2e^2-b^2(d^2+ce))(a+bx)^{9/2}}{3b^7} \\ &+ \frac{6e^2(bd-2ae)(a+bx)^{11/2}}{11b^7} + \frac{2e^3(a+bx)^{13/2}}{13b^7}\end{aligned}$$

output
```
2*(-2*a*e+b*d)*(a^2*e-a*b*d+b^2*c)^2*(b*x+a)^(3/2)/b^7-6/5*(a^2*e-a*b*d+b^
2*c)*(5*a*b*d*e-5*a^2*e^2-b^2*(c*e+d^2))*(b*x+a)^(5/2)/b^7-2/7*(-2*a*e+b*d
)*(10*a*b*d*e-10*a^2*e^2-b^2*(6*c*e+d^2))*(b*x+a)^(7/2)/b^7-2/3*e*(5*a*b*d
*e-5*a^2*e^2-b^2*(c*e+d^2))*(b*x+a)^(9/2)/b^7+6/11*e^2*(-2*a*e+b*d)*(b*x+a
)^(11/2)/b^7+2/13*e^3*(b*x+a)^(13/2)/b^7+2*(a^2*e-a*b*d+b^2*c)^3*(b*x+a)^(
1/2)/b^7
```

3.3.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\sqrt{a+bx}(5120a^6e^3-1280a^5be^2(13d+2ex)+128a^4b^2e(143d^2+65dex+e(143c+15ex^2))-16a^3b^3(429}{}$$

input
```
Integrate[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(5120*a^6*e^3 - 1280*a^5*b*e^2*(13*d + 2*e*x) + 128*a^4*b
^2*e*(143*d^2 + 65*d*e*x + e*(143*c + 15*e*x^2)) - 16*a^3*b^3*(429*d^3 + 5
72*d^2*e*x + 78*d*e*(33*c + 5*e*x^2) + 4*e^2*x*(143*c + 25*e*x^2)) + 8*a^2
*b^4*(3003*c^2*e + 429*c*(7*d^2 + 6*d*e*x + 2*e^2*x^2) + x*(429*d^3 + 858*
d^2*e*x + 650*d*e^2*x^2 + 175*e^3*x^3)) + b^6*(15015*c^3 + 3003*c^2*x*(5*d
 + 3*e*x) + 143*c*x^2*(63*d^2 + 90*d*e*x + 35*e^2*x^2) + 5*x^3*(429*d^3 +
1001*d^2*e*x + 819*d*e^2*x^2 + 231*e^3*x^3)) - 2*a*b^5*(3003*c^2*(5*d + 2*
e*x) + 286*c*x*(21*d^2 + 27*d*e*x + 10*e^2*x^2) + x^2*(1287*d^3 + 2860*d^2
*e*x + 2275*d*e^2*x^2 + 630*e^3*x^3))))/(15015*b^7)
```

3.3.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx$$
$$\Big\downarrow \; 1140$$
$$\int \left(\frac{3e(a+bx)^{7/2}\left(5a^2e^2-5abde+b^2\left(ce+d^2\right)\right)}{b^6}+\frac{(a+bx)^{5/2}(bd-2ae)\left(10a^2e^2-10abde+b^2\left(6ce+d^2\right)\right)}{b^6}+\frac{3}{}\right.$$
$$\Big\downarrow \; 2009$$

$$
-\frac{2e(a+bx)^{9/2}\left(-5a^2e^2+5abde-\left(b^2\left(ce+d^2\right)\right)\right)}{3b^7}-\frac{2(a+bx)^{7/2}(bd-2ae)\left(-10a^2e^2+10abde-\left(b^2\left(6ce+d^2\right)\right)\right)}{7b^7}-\frac{6(a+bx)^{5/2}\left(a^2e-abd+b^2c\right)\left(-5a^2e^2+5abde-\left(b^2\left(ce+d^2\right)\right)\right)}{5b^7}+\frac{2(a+bx)^{3/2}(bd-2ae)\left(a^2e-abd+b^2c\right)^2}{b^7}+\frac{2\sqrt{a+bx}\left(a^2e-abd+b^2c\right)^3}{b^7}+\frac{6e^2(a+bx)^{11/2}(bd-2ae)}{11b^7}+\frac{2e^3(a+bx)^{13/2}}{13b^7}
$$

input
```
Int[(c + d*x + e*x^2)^3/Sqrt[a + b*x],x]
```

output
```
(2*(b^2*c - a*b*d + a^2*e)^3*Sqrt[a + b*x])/b^7 + (2*(b*d - 2*a*e)*(b^2*c
- a*b*d + a^2*e)^2*(a + b*x)^(3/2))/b^7 - (6*(b^2*c - a*b*d + a^2*e)*(5*a*
b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(5/2))/(5*b^7) - (2*(b*d -
2*a*e)*(10*a*b*d*e - 10*a^2*e^2 - b^2*(d^2 + 6*c*e))*(a + b*x)^(7/2))/(7*b
^7) - (2*e*(5*a*b*d*e - 5*a^2*e^2 - b^2*(d^2 + c*e))*(a + b*x)^(9/2))/(3*b
^7) + (6*e^2*(b*d - 2*a*e)*(a + b*x)^(11/2))/(11*b^7) + (2*e^3*(a + b*x)^(
13/2))/(13*b^7)
```

3.3.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$\frac{2048\left(\left(\frac{231e^{3}x^{6}}{1024}+\frac{1001\left(\frac{9dx}{11}+c\right)x^{4}e^{2}}{1024}+\frac{9009x^{2}\left(\frac{5}{9}d^{2}x^{2}+\frac{10}{7}cdx+c^{2}\right)e}{5120}+\frac{3003c^{3}}{1024}+\frac{429d^{3}x^{3}}{1024}+\frac{3003c^{2}dx}{1024}+\frac{9009c\,d^{2}x^{2}}{5120}\right)b^{6}-\frac{3003(}{}}{}$
derivativedivides	$\frac{2e^{3}(bx+a)^{\frac{13}{2}}}{13}+\frac{6(-2ae+bd)e^{2}(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(\left(a^{2}e-abd+b^{2}c\right)e^{2}+2(-2ae+bd)^{2}e+e\left(2\left(a^{2}e-abd+b^{2}c\right)e+(-2ae+bd)^{2}\right)\right)(bx+a)^{\frac{9}{2}}}{9}+$
default	$\frac{2e^{3}(bx+a)^{\frac{13}{2}}}{13}+\frac{6(-2ae+bd)e^{2}(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(\left(a^{2}e-abd+b^{2}c\right)e^{2}+2(-2ae+bd)^{2}e+e\left(2\left(a^{2}e-abd+b^{2}c\right)e+(-2ae+bd)^{2}\right)\right)(bx+a)^{\frac{9}{2}}}{9}+$
gosper	$2\sqrt{bx+a}\,(1155e^{3}x^{6}b^{6}-1260a\,b^{5}e^{3}x^{5}+4095b^{6}d\,e^{2}x^{5}+1400a^{2}b^{4}e^{3}x^{4}-4550a\,b^{5}d\,e^{2}x^{4}+5005b^{6}c\,e^{2}x^{4}+5005b^{6}d^{2}e\,x^{4}-160$
trager	$2\sqrt{bx+a}\,(1155e^{3}x^{6}b^{6}-1260a\,b^{5}e^{3}x^{5}+4095b^{6}d\,e^{2}x^{5}+1400a^{2}b^{4}e^{3}x^{4}-4550a\,b^{5}d\,e^{2}x^{4}+5005b^{6}c\,e^{2}x^{4}+5005b^{6}d^{2}e\,x^{4}-160$
risch	$2\sqrt{bx+a}\,(1155e^{3}x^{6}b^{6}-1260a\,b^{5}e^{3}x^{5}+4095b^{6}d\,e^{2}x^{5}+1400a^{2}b^{4}e^{3}x^{4}-4550a\,b^{5}d\,e^{2}x^{4}+5005b^{6}c\,e^{2}x^{4}+5005b^{6}d^{2}e\,x^{4}-160$

input
```
int((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2048/3003*((231/1024*e^3*x^6+1001/1024*(9/11*d*x+c)*x^4*e^2+9009/5120*x^2*
(5/9*d^2*x^2+10/7*c*d*x+c^2)*e+3003/1024*c^3+429/1024*d^3*x^3+3003/1024*c^
2*d*x+9009/5120*c*d^2*x^2)*b^6-3003/512*(6/143*e^3*x^5+(5/33*d*x^4+4/21*c*
x^3)*e^2+(4/21*d^2*x^3+18/35*c*d*x^2+2/5*c^2*x)*e+d*(3/35*d^2*x^2+2/5*c*d*
x+c^2))*a*b^5+3003/640*(25/429*e^3*x^4+2/7*x^2*(25/33*d*x+c)*e^2+(2/7*d^2*
x^2+6/7*c*d*x+c^2)*e+d^2*(1/7*d*x+c))*a^2*b^4-1287/160*(50/1287*e^3*x^3+(5
/33*d*x^2+2/9*c*x)*e^2+d*(2/9*d*x+c)*e+1/6*d^3)*a^3*b^3+143/40*e*a^4*(15/1
43*x^2*e^2+(5/11*d*x+c)*e+d^2)*b^2-13/4*e^2*(2/13*e*x+d)*a^5*b+a^6*e^3)*(b
*x+a)^(1/2)/b^7
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx$$

$$=\frac{2\left(1155\,b^{6}e^{3}x^{6}+15015\,b^{6}c^{3}-30030\,ab^{5}c^{2}d+24024\,a^{2}b^{4}cd^{2}-6864\,a^{3}b^{3}d^{3}+5120\,a^{6}e^{3}+315\left(13\,b^{6}de^{2}-\right.\right.}{}$$

input
```
integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/15015*(1155*b^6*e^3*x^6 + 15015*b^6*c^3 - 30030*a*b^5*c^2*d + 24024*a^2*
b^4*c*d^2 - 6864*a^3*b^3*d^3 + 5120*a^6*e^3 + 315*(13*b^6*d*e^2 - 4*a*b^5*
e^3)*x^5 + 35*(143*b^6*d^2*e + 40*a^2*b^4*e^3 + 13*(11*b^6*c - 10*a*b^5*d)
*e^2)*x^4 + 5*(429*b^6*d^3 - 320*a^3*b^3*e^3 - 104*(11*a*b^5*c - 10*a^2*b^
4*d)*e^2 + 286*(9*b^6*c*d - 4*a*b^5*d^2)*e)*x^3 + 1664*(11*a^4*b^2*c - 10*
a^5*b*d)*e^2 + 3*(3003*b^6*c*d^2 - 858*a*b^5*d^3 + 640*a^4*b^2*e^3 + 208*(
11*a^2*b^4*c - 10*a^3*b^3*d)*e^2 + 143*(21*b^6*c^2 - 36*a*b^5*c*d + 16*a^2
*b^4*d^2)*e)*x^2 + 1144*(21*a^2*b^4*c^2 - 36*a^3*b^3*c*d + 16*a^4*b^2*d^2)
*e + (15015*b^6*c^2*d - 12012*a*b^5*c*d^2 + 3432*a^2*b^4*d^3 - 2560*a^5*b*
e^3 - 832*(11*a^3*b^3*c - 10*a^4*b^2*d)*e^2 - 572*(21*a*b^5*c^2 - 36*a^2*b
^4*c*d + 16*a^3*b^3*d^2)*e)*x)*sqrt(b*x + a)/b^7
```

3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(277) = 554$.

Time = 1.10 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.30

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx = \begin{cases} 2\Bigg(\frac{e^3(a+bx)^{\frac{13}{2}}}{13b^6}+\frac{(a+bx)^{\frac{11}{2}}(-6ae^3+3bde^2)}{11b^6}+\frac{(a+bx)^{\frac{9}{2}}\cdot(15a^2e^3-15abde^2+3b^2ce^2+3b^2d^2e)}{9b^6}+\frac{(a+bx)^{\frac{7}{2}}(-20a^3e^3+30a^2bde^2-12ab^2ce^2-12ab^2d^2e+6b^3c\cdots}{7b^6} \\ \frac{c^3x+\frac{3c^2dx^2}{2}+\frac{de^2x^6}{2}+\frac{e^3x^7}{7}+\frac{x^5\cdot(3ce^2+3d^2e)}{5}+\frac{x^4\cdot(6cde+d^3)}{4}+\frac{x^3\cdot(3c^2e+3cd^2)}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate((e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

output
```
Piecewise((2*(e**3*(a + b*x)**(13/2)/(13*b**6) + (a + b*x)**(11/2)*(-6*a*e
**3 + 3*b*d*e**2)/(11*b**6) + (a + b*x)**(9/2)*(15*a**2*e**3 - 15*a*b*d*e*
*2 + 3*b**2*c*e**2 + 3*b**2*d**2*e)/(9*b**6) + (a + b*x)**(7/2)*(-20*a**3*
e**3 + 30*a**2*b*d*e**2 - 12*a*b**2*c*e**2 - 12*a*b**2*d**2*e + 6*b**3*c*d
*e + b**3*d**3)/(7*b**6) + (a + b*x)**(5/2)*(15*a**4*e**3 - 30*a**3*b*d*e*
*2 + 18*a**2*b**2*c*e**2 + 18*a**2*b**2*d**2*e - 18*a*b**3*c*d*e - 3*a*b**
3*d**3 + 3*b**4*c**2*e + 3*b**4*c*d**2)/(5*b**6) + (a + b*x)**(3/2)*(-6*a*
*5*e**3 + 15*a**4*b*d*e**2 - 12*a**3*b**2*c*e**2 - 12*a**3*b**2*d**2*e + 1
8*a**2*b**3*c*d*e + 3*a**2*b**3*d**3 - 6*a*b**4*c**2*e - 6*a*b**4*c*d**2 +
 3*b**5*c**2*d)/(3*b**6) + sqrt(a + b*x)*(a**6*e**3 - 3*a**5*b*d*e**2 + 3*
a**4*b**2*c*e**2 + 3*a**4*b**2*d**2*e - 6*a**3*b**3*c*d*e - a**3*b**3*d**3
 + 3*a**2*b**4*c**2*e + 3*a**2*b**4*c*d**2 - 3*a*b**5*c**2*d + b**6*c**3)/
b**6)/b, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + d*e**2*x**6/2 + e**3*x**7
/7 + x**5*(3*c*e**2 + 3*d**2*e)/5 + x**4*(6*c*d*e + d**3)/4 + x**3*(3*c**2
*e + 3*c*d**2)/3)/sqrt(a), True))
```

3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(250) = 500$.

Time = 0.20 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.92

$$\int \frac{(c + dx + ex^2)^3}{\sqrt{a + bx}}\,dx$$
$$= \frac{2\left(15015\sqrt{bx+a}c^3 + 3003\,c^2\left(\frac{5\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}a\right)d}{b} + \frac{\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+a}a^2\right)e}{b^2}\right) + 143\,c\left(\frac{21\left(3\,(bx\right.}{}\right.\right.}{}$$

input
```
integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/15015*(15015*sqrt(b*x + a)*c^3 + 3003*c^2*(5*((b*x + a)^(3/2) - 3*sqrt(b
*x + a)*a)*d/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x +
 a)*a^2)*e/b^2) + 143*c*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15
*sqrt(b*x + a)*a^2)*d^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
 + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + (35*(b*x + a)^
(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3
/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4) + 429*(5*(b*x + a)^(7/2) - 21*(b
*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 +
 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b^4 + 65*(63*(b*
x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*
x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2
/b^5 + 5*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^
(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*
x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6)/b
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(250) = 500$.

Time = 0.27 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.92

$$\int \frac{(c + dx + ex^2)^3}{\sqrt{a + bx}}\, dx$$

$$= \frac{2\left(15015\sqrt{bx+a}c^3 + \frac{15015\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}a\right)c^2d}{b} + \frac{3003\left(3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+a}a^2\right)cd^2}{b^2} + \frac{3003\left(3(bx+a)^{\frac{5}{2}} - \right.}{}}{}$$

input
```
integrate((e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/15015*(15015*sqrt(b*x + a)*c^3 + 15015*((b*x + a)^(3/2) - 3*sqrt(b*x + a
)*a)*c^2*d/b + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*
x + a)*a^2)*c*d^2/b^2 + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 1
5*sqrt(b*x + a)*a^2)*c^2*e/b^2 + 429*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/
2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 2574*(5*(b
*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*
x + a)*a^3)*c*d*e/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a +
378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)
*d^2*e/b^4 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x +
a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*e^2/b^4
+ 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^
2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a
)*a^5)*d*e^2/b^5 + 5*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 500
5*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^
4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*e^3/b^6)/b
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx = \frac{2\,e^3\,(a+b\,x)^{13/2}}{13\,b^7} - \frac{(12\,a\,e^3-6\,b\,d\,e^2)\,(a+b\,x)^{11/2}}{11\,b^7} + \frac{(a+b\,x)^{9/2}\,(30\,a^2\,e^3-30\,a\,b\,d\,e^2+6\,b^2\,d^2\,e+6\,c\,b^2\,e^2)}{9\,b^7} + \frac{2\,\sqrt{a+b\,x}\,(e\,a^2-d\,a\,b+c\,b^2)^3}{b^7} + \frac{(a+b\,x)^{5/2}\,(30\,a^4\,e^3-60\,a^3\,b\,d\,e^2+36\,a^2\,b^2\,c\,e^2+36\,a^2\,b^2\,d^2\,e-36\,a\,b^3\,c\,d\,e-6\,a\,b^3\,d^3+6\,b^4\,c^2\,e+6\,b^4}{5\,b^7} - \frac{2\,(2\,a\,e-b\,d)\,(a+b\,x)^{7/2}\,(10\,a^2\,e^2-10\,a\,b\,d\,e+b^2\,d^2+6\,c\,b^2\,e)}{7\,b^7} - \frac{2\,(2\,a\,e-b\,d)\,(a+b\,x)^{3/2}\,(e\,a^2-d\,a\,b+c\,b^2)^2}{b^7}$$

input
```
int((c + d*x + e*x^2)^3/(a + b*x)^(1/2),x)
```

output
```
(2*e^3*(a + b*x)^(13/2))/(13*b^7) - ((12*a*e^3 - 6*b*d*e^2)*(a + b*x)^(11/
2))/(11*b^7) + ((a + b*x)^(9/2)*(30*a^2*e^3 + 6*b^2*c*e^2 + 6*b^2*d^2*e -
30*a*b*d*e^2))/(9*b^7) + (2*(a + b*x)^(1/2)*(b^2*c + a^2*e - a*b*d)^3)/b^7
 + ((a + b*x)^(5/2)*(30*a^4*e^3 - 6*a*b^3*d^3 + 6*b^4*c*d^2 + 6*b^4*c^2*e
+ 36*a^2*b^2*c*e^2 + 36*a^2*b^2*d^2*e - 60*a^3*b*d*e^2 - 36*a*b^3*c*d*e))/
(5*b^7) - (2*(2*a*e - b*d)*(a + b*x)^(7/2)*(10*a^2*e^2 + b^2*d^2 + 6*b^2*c
*e - 10*a*b*d*e))/(7*b^7) - (2*(2*a*e - b*d)*(a + b*x)^(3/2)*(b^2*c + a^2*
e - a*b*d)^2)/b^7
```

3.3. $\int \frac{(c+dx+ex^2)^3}{\sqrt{a+bx}}\,dx$

3.4 $\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}} \, dx$

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3.4.1 Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}} \, dx = \frac{2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx}}{b^4} + \frac{2(b^2d-2abe+3a^2f)(a+bx)^{3/2}}{3b^4} + \frac{2(be-3af)(a+bx)^{5/2}}{5b^4} + \frac{2f(a+bx)^{7/2}}{7b^4}$$

output
```
2/3*(3*a^2*f-2*a*b*e+b^2*d)*(b*x+a)^(3/2)/b^4+2/5*(-3*a*f+b*e)*(b*x+a)^(5/
2)/b^4+2/7*f*(b*x+a)^(7/2)/b^4+2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x+a)^(1
/2)/b^4
```

3.4.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.72

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}} \, dx = \frac{2\sqrt{a+bx}(-48a^3f+8a^2b(7e+3fx)-2ab^2(35d+x(14e+9fx))+b^3(105c+x(35d+3x(7e+5fx))))}{105b^4}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x) - 2*a*b^2*(35*d + x*(1
4*e + 9*f*x)) + b^3*(105*c + x*(35*d + 3*x*(7*e + 5*f*x)))))/(105*b^4)
```

3.4.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{\sqrt{a+bx}(3a^2f-2abe+b^2d)}{b^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{b^3\sqrt{a+bx}}+\frac{(a+bx)^{3/2}(be-3af)}{b^3}+\frac{f(a+bx)^{5/2}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{2\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{2(a+bx)^{5/2}(be-3af)}{5b^4}+\frac{2f(a+bx)^{7/2}}{7b^4}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x],x]
```

output
```
(2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x])/b^4 + (2*(b^2*d - 2*
a*b*e + 3*a^2*f)*(a + b*x)^(3/2))/(3*b^4) + (2*(b*e - 3*a*f)*(a + b*x)^(5/
2))/(5*b^4) + (2*f*(a + b*x)^(7/2))/(7*b^4)
```

3.4.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.4.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{32\left(\frac{\left(-5f\,x^{3}-7e\,x^{2}-\frac{35}{3}dx-35c\right)b^{3}}{16}+\frac{35\left(\frac{9}{35}f\,x^{2}+\frac{2}{5}ex+d\right)a\,b^{2}}{24}-\frac{7\left(\frac{3fx}{7}+e\right)a^{2}b}{6}+f\,a^{3}\right)\sqrt{bx+a}}{35b^{4}}$
gosper	$-\frac{2\sqrt{bx+a}\,\left(-15b^{3}f\,x^{3}+18a\,b^{2}f\,x^{2}-21b^{3}e\,x^{2}-24xf\,a^{2}b+28a\,b^{2}ex-35b^{3}dx+48f\,a^{3}-56a^{2}be+70a\,b^{2}d-105b^{3}c\right)}{105b^{4}}$
trager	$-\frac{2\sqrt{bx+a}\,\left(-15b^{3}f\,x^{3}+18a\,b^{2}f\,x^{2}-21b^{3}e\,x^{2}-24xf\,a^{2}b+28a\,b^{2}ex-35b^{3}dx+48f\,a^{3}-56a^{2}be+70a\,b^{2}d-105b^{3}c\right)}{105b^{4}}$
risch	$-\frac{2\sqrt{bx+a}\,\left(-15b^{3}f\,x^{3}+18a\,b^{2}f\,x^{2}-21b^{3}e\,x^{2}-24xf\,a^{2}b+28a\,b^{2}ex-35b^{3}dx+48f\,a^{3}-56a^{2}be+70a\,b^{2}d-105b^{3}c\right)}{105b^{4}}$
derivativedivides	$\frac{\frac{2f(bx+a)^{\frac{7}{2}}}{7}-\frac{6af(bx+a)^{\frac{5}{2}}}{5}+\frac{2be(bx+a)^{\frac{5}{2}}}{5}+2a^{2}f(bx+a)^{\frac{3}{2}}-\frac{4abe(bx+a)^{\frac{3}{2}}}{3}+\frac{2b^{2}d(bx+a)^{\frac{3}{2}}}{3}-2a^{3}f\sqrt{bx+a}+2a^{2}be\sqrt{bx+a}-2a\,b}{b^{4}}$
default	$\frac{\frac{2f(bx+a)^{\frac{7}{2}}}{7}-\frac{6af(bx+a)^{\frac{5}{2}}}{5}+\frac{2be(bx+a)^{\frac{5}{2}}}{5}+2a^{2}f(bx+a)^{\frac{3}{2}}-\frac{4abe(bx+a)^{\frac{3}{2}}}{3}+\frac{2b^{2}d(bx+a)^{\frac{3}{2}}}{3}-2a^{3}f\sqrt{bx+a}+2a^{2}be\sqrt{bx+a}-2a\,b}{b^{4}}$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-32/35*(1/16*(-5*f*x^3-7*e*x^2-35/3*d*x-35*c)*b^3+35/24*(9/35*f*x^2+2/5*e*
x+d)*a*b^2-7/6*(3/7*f*x+e)*a^2*b+f*a^3)*(b*x+a)^(1/2)/b^4
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.79

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx = \frac{2\,(15\,b^3fx^3+105\,b^3c-70\,ab^2d+56\,a^2be-48\,a^3f+3\,(7\,b^3e-6\,ab^2f)x^2+(35\,b^3d-28\,ab^2e+24\,a^2bf)x)\sqrt{bx+a}}{105\,b^4}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^3*f*x^3 + 105*b^3*c - 70*a*b^2*d + 56*a^2*b*e - 48*a^3*f + 3*(
7*b^3*e - 6*a*b^2*f)*x^2 + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x)*sqrt(b*
x + a)/b^4
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.43

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx = \begin{cases} \frac{2c\sqrt{a+bx}+\frac{2d\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2e\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}+\frac{2f\left(-a^3\sqrt{a+bx}+a^2(a+bx)^{\frac{3}{2}}-\frac{3a(a+bx)^{\frac{5}{2}}}{5}+\frac{(a+bx)^{\frac{7}{2}}}{7}\right)}{b^3}}{b} & \text{for}\ b\neq 0 \\ \frac{cx+\frac{dx^2}{2}+\frac{ex^3}{3}+\frac{fx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x+a)**(1/2),x)
```

output
```
Piecewise(((2*c*sqrt(a + b*x) + 2*d*(-a*sqrt(a + b*x) + (a + b*x)**(3/2)/3
)/b + 2*e*(a**2*sqrt(a + b*x) - 2*a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/
5)/b**2 + 2*f*(-a**3*sqrt(a + b*x) + a**2*(a + b*x)**(3/2) - 3*a*(a + b*x)
**(5/2)/5 + (a + b*x)**(7/2)/7)/b**3)/b, Ne(b, 0)), ((c*x + d*x**2/2 + e*x
**3/3 + f*x**4/4)/sqrt(a), True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.12

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\left(105\sqrt{bx+a}c + \frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b} + \frac{7\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2} + \frac{3\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(b\right.}{b^3}}{105\,b}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b
+ 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^
2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 -
 35*sqrt(b*x + a)*a^3)*f/b^3)/b
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.12

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\left(105\sqrt{bx+a}c + \frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b} + \frac{7\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2} + \frac{3\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(b\right.}{b^3}}{105\,b}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(b*x + a)*c + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b
+ 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*e/b^
2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 -
 35*sqrt(b*x + a)*a^3)*f/b^3)/b
```

3.4.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx = \frac{(a+b\,x)^{3/2}\,(6\,f\,a^2-4\,e\,a\,b+2\,d\,b^2)}{3\,b^4} - \frac{(6\,a\,f-2\,b\,e)\,(a+b\,x)^{5/2}}{5\,b^4} + \frac{\sqrt{a+b\,x}\,(-2\,f\,a^3+2\,e\,a^2\,b-2\,d\,a\,b^2+2\,c\,b^3)}{b^4} + \frac{2\,f\,(a+b\,x)^{7/2}}{7\,b^4}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x)^(1/2),x)
```

output
```
((a + b*x)^(3/2)*(2*b^2*d + 6*a^2*f - 4*a*b*e))/(3*b^4) - ((6*a*f - 2*b*e)
*(a + b*x)^(5/2))/(5*b^4) + ((a + b*x)^(1/2)*(2*b^3*c - 2*a^3*f - 2*a*b^2*
d + 2*a^2*b*e))/b^4 + (2*f*(a + b*x)^(7/2))/(7*b^4)
```

3.5 $\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$

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3.5.1 Optimal result

Integrand size = 27, antiderivative size = 320

$$\begin{aligned}\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx &= \frac{2(b^3c-ab^2d+a^2be-a^3f)^2\sqrt{a+bx}}{b^7} \\ &+ \frac{4(b^2d-2abe+3a^2f)\,(b^3c-ab^2d+a^2be-a^3f)\,(a+bx)^{3/2}}{3b^7} \\ &+ \frac{2(b^4(d^2+2ce)-20a^3bef+15a^4f^2-6ab^3(de+cf)+6a^2b^2(e^2+2df))\,(a+bx)^{5/2}}{5b^7} \\ &+ \frac{4(10a^2bef-10a^3f^2+b^3(de+cf)-2ab^2(e^2+2df))\,(a+bx)^{7/2}}{7b^7} \\ &- \frac{2(10abef-15a^2f^2-b^2(e^2+2df))\,(a+bx)^{9/2}}{9b^7} \\ &+ \frac{4f(be-3af)(a+bx)^{11/2}}{11b^7}+\frac{2f^2(a+bx)^{13/2}}{13b^7}\end{aligned}$$

output
```
4/3*(3*a^2*f-2*a*b*e+b^2*d)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x+a)^(3/2)/b
^7+2/5*(b^4*(2*c*e+d^2)-20*a^3*b*e*f+15*a^4*f^2-6*a*b^3*(c*f+d*e)+6*a^2*b^
2*(2*d*f+e^2))*(b*x+a)^(5/2)/b^7+4/7*(10*a^2*b*e*f-10*a^3*f^2+b^3*(c*f+d*e
)-2*a*b^2*(2*d*f+e^2))*(b*x+a)^(7/2)/b^7-2/9*(10*a*b*e*f-15*a^2*f^2-b^2*(2
*d*f+e^2))*(b*x+a)^(9/2)/b^7+4/11*f*(-3*a*f+b*e)*(b*x+a)^(11/2)/b^7+2/13*f
^2*(b*x+a)^(13/2)/b^7+2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)^2*(b*x+a)^(1/2)/b^7
```

3.5.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(15360a^6f^2-2560a^5bf(13e+3fx)+128a^4b^2(143e^2+130efx+f(286d+45fx^2))-32a^3b^3(1}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(15360*a^6*f^2 - 2560*a^5*b*f*(13*e + 3*f*x) + 128*a^4*b^
2*(143*e^2 + 130*e*f*x + f*(286*d + 45*f*x^2)) - 32*a^3*b^3*(1287*c*f + 14
3*d*(9*e + 4*f*x) + 2*x*(143*e^2 + 195*e*f*x + 75*f^2*x^2)) + 8*a^2*b^4*(3
003*d^2 + 858*d*x*(3*e + 2*f*x) + 858*c*(7*e + 3*f*x) + x^2*(858*e^2 + 130
0*e*f*x + 525*f^2*x^2)) + b^6*(45045*c^2 + 858*c*x*(35*d + 3*x*(7*e + 5*f*
x)) + x^2*(9009*d^2 + 1430*d*x*(9*e + 7*f*x) + 35*x^2*(143*e^2 + 234*e*f*x
 + 99*f^2*x^2))) - 4*a*b^5*(429*c*(35*d + x*(14*e + 9*f*x)) + x*(3003*d^2
+ 143*d*x*(27*e + 20*f*x) + 5*x^2*(286*e^2 + 455*e*f*x + 189*f^2*x^2)))))/
(45045*b^7)
```

3.5.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{(a+bx)^{7/2}\left(15a^2f^2-10abef+b^2(2df+e^2)\right)}{b^6}+\frac{2(a+bx)^{5/2}\left(-10a^3f^2+10a^2bef-2ab^2(2df+e^2)+b^3(c\right.}{b^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(a+bx)^{9/2}\left(-15a^2f^2+10abef-\left(b^2\left(2df+e^2\right)\right)\right)}{9b^7}+\frac{4(a+bx)^{7/2}\left(-10a^3f^2+10a^2bef-2ab^2\left(2df+e^2\right)+b^3(cf+de)\right)}{7b^7}+\frac{4(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^7}+\frac{2\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^2}{b^7}+\frac{2(a+bx)^{5/2}\left(15a^4f^2-20a^3bef+6a^2b^2\left(2df+e^2\right)-6ab^3(cf+de)+b^4\left(2ce+d^2\right)\right)}{5b^7}+\frac{4f(a+bx)^{11/2}(be-3af)}{11b^7}+\frac{2f^2(a+bx)^{13/2}}{13b^7}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)^2/Sqrt[a + b*x],x]
```

output
```
(2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*Sqrt[a + b*x])/b^7 + (4*(b^2*d -
2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a + b*x)^(3/2))/(3
*b^7) + (2*(b^4*(d^2 + 2*c*e) - 20*a^3*b*e*f + 15*a^4*f^2 - 6*a*b^3*(d*e +
 c*f) + 6*a^2*b^2*(e^2 + 2*d*f))*(a + b*x)^(5/2))/(5*b^7) + (4*(10*a^2*b*e
*f - 10*a^3*f^2 + b^3*(d*e + c*f) - 2*a*b^2*(e^2 + 2*d*f))*(a + b*x)^(7/2)
)/(7*b^7) - (2*(10*a*b*e*f - 15*a^2*f^2 - b^2*(e^2 + 2*d*f))*(a + b*x)^(9/
2))/(9*b^7) + (4*f*(b*e - 3*a*f)*(a + b*x)^(11/2))/(11*b^7) + (2*f^2*(a +
b*x)^(13/2))/(13*b^7)
```

3.5.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.5.4 Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{2f^2(bx+a)^{\frac{13}{2}}}{13}+\frac{4(-3af+be)f(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(2\left(3a^2f-2aeb+b^2d\right)f+(-3af+be)^2\right)(bx+a)^{\frac{9}{2}}}{9}+\frac{2\left(2\left(-f\,a^3+a^2be-a\,b^2d+b^3c\right)f+2\left(3a\right.\right.}{7}$
pseudoelliptic	$2048\left(\frac{\left(\frac{231f^2x^6}{2}+273ef\,x^5+\frac{1001\left(df+\frac{e^2}{2}\right)x^4}{3}+429(cf+de)x^3+\frac{3003\left(ce+\frac{d^2}{2}\right)x^2}{5}+1001cdx+\frac{3003c^2}{2}\right)b^6}{512}-\frac{1001\left(\frac{9f^2x^5}{143}+\frac{5ef\,x^4}{33}\right.}{}\right.$
default	$\frac{2f^2(bx+a)^{\frac{13}{2}}}{13}-\frac{4(3af-be)f(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(-2\left(-3a^2f+2aeb-b^2d\right)f+(3af-be)^2\right)(bx+a)^{\frac{9}{2}}}{9}+\frac{2\left(-2\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)f+2\left(-\right.\right.}{7}$
gosper	$2\sqrt{bx+a}\,(3465f^2x^6b^6-3780a\,b^5f^2x^5+8190b^6ef\,x^5+4200a^2b^4f^2x^4-9100a\,b^5ef\,x^4+10010b^6df\,x^4+5005b^6e^2x^4-4800a$
trager	$2\sqrt{bx+a}\,(3465f^2x^6b^6-3780a\,b^5f^2x^5+8190b^6ef\,x^5+4200a^2b^4f^2x^4-9100a\,b^5ef\,x^4+10010b^6df\,x^4+5005b^6e^2x^4-4800a$
risch	$2\sqrt{bx+a}\,(3465f^2x^6b^6-3780a\,b^5f^2x^5+8190b^6ef\,x^5+4200a^2b^4f^2x^4-9100a\,b^5ef\,x^4+10010b^6df\,x^4+5005b^6e^2x^4-4800a$

input
```
int((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^7*(1/13*f^2*(b*x+a)^(13/2)+2/11*(-3*a*f+b*e)*f*(b*x+a)^(11/2)+1/9*(2*(
3*a^2*f-2*a*b*e+b^2*d)*f+(-3*a*f+b*e)^2)*(b*x+a)^(9/2)+1/7*(2*(-a^3*f+a^2*
b*e-a*b^2*d+b^3*c)*f+2*(3*a^2*f-2*a*b*e+b^2*d)*(-3*a*f+b*e))*(b*x+a)^(7/2)
+1/5*(2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(-3*a*f+b*e)+(3*a^2*f-2*a*b*e+b^2*d
)^2)*(b*x+a)^(5/2)+2/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(3*a^2*f-2*a*b*e+b^2
*d)*(b*x+a)^(3/2)+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)^2*(b*x+a)^(1/2))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.30

$$\int\frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$$
$$=\frac{2\,(3465\,b^6f^2x^6+45045\,b^6c^2-60060\,ab^5cd+24024\,a^2b^4d^2+18304\,a^4b^2e^2+15360\,a^6f^2+630\,(13\,b^6ef-}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45045*(3465*b^6*f^2*x^6 + 45045*b^6*c^2 - 60060*a*b^5*c*d + 24024*a^2*b^
4*d^2 + 18304*a^4*b^2*e^2 + 15360*a^6*f^2 + 630*(13*b^6*e*f - 6*a*b^5*f^2)
*x^5 + 35*(143*b^6*e^2 + 120*a^2*b^4*f^2 + 26*(11*b^6*d - 10*a*b^5*e)*f)*x
^4 + 10*(1287*b^6*d*e - 572*a*b^5*e^2 - 480*a^3*b^3*f^2 + 13*(99*b^6*c - 8
8*a*b^5*d + 80*a^2*b^4*e)*f)*x^3 + 3*(3003*b^6*d^2 + 2288*a^2*b^4*e^2 + 19
20*a^4*b^2*f^2 + 858*(7*b^6*c - 6*a*b^5*d)*e - 52*(99*a*b^5*c - 88*a^2*b^4
*d + 80*a^3*b^3*e)*f)*x^2 + 6864*(7*a^2*b^4*c - 6*a^3*b^3*d)*e - 416*(99*a
^3*b^3*c - 88*a^4*b^2*d + 80*a^5*b*e)*f + 2*(15015*b^6*c*d - 6006*a*b^5*d^
2 - 4576*a^3*b^3*e^2 - 3840*a^5*b*f^2 - 1716*(7*a*b^5*c - 6*a^2*b^4*d)*e +
 104*(99*a^2*b^4*c - 88*a^3*b^3*d + 80*a^4*b^2*e)*f)*x)*sqrt(b*x + a)/b^7
```

3.5.6 Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.76

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$$

$$= \begin{cases} \frac{2\left(\frac{f^2(a+bx)^{\frac{13}{2}}}{13b^6}+\frac{(a+bx)^{\frac{11}{2}}\left(-6af^2+2bef\right)}{11b^6}+\frac{(a+bx)^{\frac{9}{2}}\cdot\left(15a^2f^2-10abef+2b^2df+b^2e^2\right)}{9b^6}+\frac{(a+bx)^{\frac{7}{2}}\left(-20a^3f^2+20a^2bef-8ab^2df-4ab^2e^2+2b^3cf+2b^3de\right)}{7b^6}+ \cdots\right.}{\cdots} \\ \frac{c^2x+cdx^2+\frac{efx^6}{3}+\frac{f^2x^7}{7}+\frac{x^5\cdot\left(2df+e^2\right)}{5}+\frac{x^4\cdot(2cf+2de)}{4}+\frac{x^3\cdot\left(2ce+d^2\right)}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)**2/(b*x+a)**(1/2),x)
```

output
```
Piecewise((2*(f**2*(a + b*x)**(13/2)/(13*b**6) + (a + b*x)**(11/2)*(-6*a*f
**2 + 2*b*e*f)/(11*b**6) + (a + b*x)**(9/2)*(15*a**2*f**2 - 10*a*b*e*f + 2
*b**2*d*f + b**2*e**2)/(9*b**6) + (a + b*x)**(7/2)*(-20*a**3*f**2 + 20*a**
2*b*e*f - 8*a*b**2*d*f - 4*a*b**2*e**2 + 2*b**3*c*f + 2*b**3*d*e)/(7*b**6)
 + (a + b*x)**(5/2)*(15*a**4*f**2 - 20*a**3*b*e*f + 12*a**2*b**2*d*f + 6*a
**2*b**2*e**2 - 6*a*b**3*c*f - 6*a*b**3*d*e + 2*b**4*c*e + b**4*d**2)/(5*b
**6) + (a + b*x)**(3/2)*(-6*a**5*f**2 + 10*a**4*b*e*f - 8*a**3*b**2*d*f -
4*a**3*b**2*e**2 + 6*a**2*b**3*c*f + 6*a**2*b**3*d*e - 4*a*b**4*c*e - 2*a*
b**4*d**2 + 2*b**5*c*d)/(3*b**6) + sqrt(a + b*x)*(a**6*f**2 - 2*a**5*b*e*f
 + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*f - 2*a**3*b**3*d*e +
2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**6*c**2)/b**6)/b, Ne(b
, 0)), ((c**2*x + c*d*x**2 + e*f*x**6/3 + f**2*x**7/7 + x**5*(2*d*f + e**2
)/5 + x**4*(2*c*f + 2*d*e)/4 + x**3*(2*c*e + d**2)/3)/sqrt(a), True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\left(45045\sqrt{bx+a}c^2+858\,c\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}+\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)e}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx\right.}{}\right.\right.}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/45045*(45045*sqrt(b*x + a)*c^2 + 858*c*(35*((b*x + a)^(3/2) - 3*sqrt(b*x
 + a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x +
 a)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a
)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3) + 3003*(3*(b*x + a)^(5/2) - 10*
(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 143*(35*(b*x + a)^(9/2
) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*
a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 + 286*(35*(b*x + a)^(9/2)*f + 45*(b*e
 - 4*a*f)*(b*x + a)^(7/2) - 189*(a*b*e - 2*a^2*f)*(b*x + a)^(5/2) + 105*(3
*a^2*b*e - 4*a^3*f)*(b*x + a)^(3/2) - 315*(a^3*b*e - a^4*f)*sqrt(b*x + a))
*d/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^
(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt
(b*x + a)*a^5)*e*f/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*
a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(
5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6)/b
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\left(45045\sqrt{bx+a}c^2+\frac{30030\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)cd}{b}+\frac{3003\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)d^2}{b^2}+\frac{6006\left(3\,(bx+a)^{\frac{5}{2}}-10\right.}{}\right.}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)^2/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(b*x + a)*c^2 + 30030*((b*x + a)^(3/2) - 3*sqrt(b*x + a
)*a)*c*d/b + 3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x
+ a)*a^2)*d^2/b^2 + 6006*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sq
rt(b*x + a)*a^2)*c*e/b^2 + 2574*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
+ 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*e/b^3 + 2574*(5*(b*x +
a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a
)*a^3)*c*f/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*
x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^
4 + 286*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*
a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d*f/b^4 + 130*(63*(
b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(
b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*e*f
/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)
^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b
*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*f^2/b^6)/b
```

3.5.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+b\,x}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)^2}{b^7}+\frac{2\,f^2\,(a+b\,x)^{13/2}}{13\,b^7}$$
$$-\frac{(a+b\,x)^{7/2}\,\left(40\,a^3\,f^2-40\,a^2\,b\,e\,f+8\,a\,b^2\,e^2+16\,d\,a\,b^2\,f-4\,d\,b^3\,e-4\,c\,b^3\,f\right)}{7\,b^7}$$
$$+\frac{(a+b\,x)^{9/2}\,\left(30\,a^2\,f^2-20\,a\,b\,e\,f+2\,b^2\,e^2+4\,d\,b^2\,f\right)}{9\,b^7}$$
$$+\frac{(a+b\,x)^{5/2}\,\left(30\,a^4\,f^2-40\,a^3\,b\,e\,f+24\,a^2\,b^2\,d\,f+12\,a^2\,b^2\,e^2-12\,a\,b^3\,d\,e-12\,c\,a\,b^3\,f+2\,b^4\,d^2+4\,c\,b^4\,e}{5\,b^7}$$
$$-\frac{\left(12\,a\,f^2-4\,b\,e\,f\right)\,(a+b\,x)^{11/2}}{11\,b^7}$$
$$+\frac{4\,(a+b\,x)^{3/2}\,\left(3\,f\,a^2-2\,e\,a\,b+d\,b^2\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^7}$$

input
```
int((c + d*x + e*x^2 + f*x^3)^2/(a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e)^2)/b^7 + (2*f^2*(a
+ b*x)^(13/2))/(13*b^7) - ((a + b*x)^(7/2)*(40*a^3*f^2 + 8*a*b^2*e^2 - 4*b
^3*c*f - 4*b^3*d*e + 16*a*b^2*d*f - 40*a^2*b*e*f))/(7*b^7) + ((a + b*x)^(9
/2)*(30*a^2*f^2 + 2*b^2*e^2 + 4*b^2*d*f - 20*a*b*e*f))/(9*b^7) + ((a + b*x
)^(5/2)*(2*b^4*d^2 + 30*a^4*f^2 + 12*a^2*b^2*e^2 + 4*b^4*c*e - 12*a*b^3*c*
f - 12*a*b^3*d*e - 40*a^3*b*e*f + 24*a^2*b^2*d*f))/(5*b^7) - ((12*a*f^2 -
4*b*e*f)*(a + b*x)^(11/2))/(11*b^7) + (4*(a + b*x)^(3/2)*(b^2*d + 3*a^2*f
- 2*a*b*e)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^7)
```

3.5. $\int \frac{(c+dx+ex^2+fx^3)^2}{\sqrt{a+bx}}\,dx$

3.6 $\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx$

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3.6.1 Optimal result

Integrand size = 27, antiderivative size = 708

$$\begin{aligned}
\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx &= \frac{2(b^3c-ab^2d+a^2be-a^3f)^3\sqrt{a+bx}}{b^{10}} \\
&+ \frac{2(b^2d-2abe+3a^2f)(b^3c-ab^2d+a^2be-a^3f)^2(a+bx)^{3/2}}{b^{10}} \\
&+ \frac{6(b^3c-ab^2d+a^2be-a^3f)(b^4(d^2+ce)-16a^3bef+12a^4f^2-ab^3(5de+3cf)+a^2b^2(5e^2+9df))(a+}{5b^{10}} \\
&- \frac{2(168a^5bef^2-84a^6f^3-b^6(d^3+6cde+3c^2f)-105a^4b^2f(e^2+df)+12ab^5(d^2e+ce^2+2cdf)-30a^2b}{7b^{10}} \\
&+ \frac{2(70a^4bef^2-42a^5f^3-35a^3b^2f(e^2+df)+b^5(d^2e+ce^2+2cdf)-5ab^4(de^2+d^2f+2cef)+5a^2b^3(e^3}{3b^{10}} \\
&- \frac{6(56a^3bef^2-42a^4f^3-21a^2b^2f(e^2+df)-b^4(de^2+d^2f+2cef)+2ab^3(e^3+6def+3cf^2))(a+bx)^{11}}{11b^{10}} \\
&+ \frac{2(84a^2bef^2-84a^3f^3-21ab^2f(e^2+df)+b^3(e^3+6def+3cf^2))(a+bx)^{13/2}}{13b^{10}} \\
&- \frac{2f(8abef-12a^2f^2-b^2(e^2+df))(a+bx)^{15/2}}{5b^{10}} \\
&+ \frac{6f^2(be-3af)(a+bx)^{17/2}}{17b^{10}} + \frac{2f^3(a+bx)^{19/2}}{19b^{10}}
\end{aligned}$$

output
```
2*(3*a^2*f-2*a*b*e+b^2*d)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)^2*(b*x+a)^(3/2)/b
^10+6/5*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b^4*(c*e+d^2)-16*a^3*b*e*f+12*a^4*
f^2-a*b^3*(3*c*f+5*d*e)+a^2*b^2*(9*d*f+5*e^2))*(b*x+a)^(5/2)/b^10-2/7*(168
*a^5*b*e*f^2-84*a^6*f^3-b^6*(3*c^2*f+6*c*d*e+d^3)-105*a^4*b^2*f*(d*f+e^2)+
12*a*b^5*(2*c*d*f+c*e^2+d^2*e)-30*a^2*b^4*(2*c*e*f+d^2*f+d*e^2)+20*a^3*b^3
*(3*c*f^2+6*d*e*f+e^3))*(b*x+a)^(7/2)/b^10+2/3*(70*a^4*b*e*f^2-42*a^5*f^3-
35*a^3*b^2*f*(d*f+e^2)+b^5*(2*c*d*f+c*e^2+d^2*e)-5*a*b^4*(2*c*e*f+d^2*f+d*
e^2)+5*a^2*b^3*(3*c*f^2+6*d*e*f+e^3))*(b*x+a)^(9/2)/b^10-6/11*(56*a^3*b*e*
f^2-42*a^4*f^3-21*a^2*b^2*f*(d*f+e^2)-b^4*(2*c*e*f+d^2*f+d*e^2)+2*a*b^3*(3
*c*f^2+6*d*e*f+e^3))*(b*x+a)^(11/2)/b^10+2/13*(84*a^2*b*e*f^2-84*a^3*f^3-2
1*a*b^2*f*(d*f+e^2)+b^3*(3*c*f^2+6*d*e*f+e^3))*(b*x+a)^(13/2)/b^10-2/5*f*(
8*a*b*e*f-12*a^2*f^2-b^2*(d*f+e^2))*(b*x+a)^(15/2)/b^10+6/17*f^2*(-3*a*f+b
*e)*(b*x+a)^(17/2)/b^10+2/19*f^3*(b*x+a)^(19/2)/b^10+2*(-a^3*f+a^2*b*e-a*b
^2*d+b^3*c)^3*(b*x+a)^(1/2)/b^10
```

3.6.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 913, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\sqrt{a+bx}(-1376256a^9f^3+229376a^8bf^2(19e+3fx)-14336a^7b^2f(323e^2+152efx+f(323d+36fx^2))}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(-1376256*a^9*f^3 + 229376*a^8*b*f^2*(19*e + 3*f*x) - 143
36*a^7*b^2*f*(323*e^2 + 152*e*f*x + f*(323*d + 36*f*x^2)) + 1024*a^6*b^3*(
1615*e^3 + 2261*e^2*f*x + 114*e*f*(85*d + 14*f*x^2) + f^2*(4845*c + 2261*d
*x + 420*f*x^3)) - 256*a^5*b^4*(20995*d^2*f + 3230*c*f*(13*e + 3*f*x) + 32
3*d*(65*e^2 + 60*e*f*x + 21*f^2*x^2) + x*(3230*e^3 + 6783*e^2*f*x + 5320*e
*f^2*x^2 + 1470*f^3*x^3)) + 128*a^4*b^5*(4199*d^2*(11*e + 5*f*x) + 323*c*(
143*e^2 + 130*e*f*x + 45*f^2*x^2) + x^2*(4845*e^3 + 11305*e^2*f*x + 9310*e
*f^2*x^2 + 2646*f^3*x^3) + 323*d*(286*c*f + 5*x*(13*e^2 + 18*e*f*x + 7*f^2
*x^2))) - 16*a^3*b^6*(138567*d^3 + 415701*c^2*f + 8398*d^2*x*(22*e + 15*f*
x) + 1292*c*x*(143*e^2 + 195*e*f*x + 75*f^2*x^2) + x^3*(32300*e^3 + 79135*
e^2*f*x + 67032*e*f^2*x^2 + 19404*f^3*x^3) + 323*d*(286*c*(9*e + 4*f*x) +
5*x^2*(78*e^2 + 120*e*f*x + 49*f^2*x^2))) + b^9*(4849845*c^3 + 138567*c^2*
x*(35*d + 3*x*(7*e + 5*f*x)) + 323*c*x^2*(9009*d^2 + 1430*d*x*(9*e + 7*f*x
) + 35*x^2*(143*e^2 + 234*e*f*x + 99*f^2*x^2)) + x^3*(692835*d^3 + 146965*
d^2*x*(11*e + 9*f*x) + 6783*d*x^2*(195*e^2 + 330*e*f*x + 143*f^2*x^2) + 23
1*x^3*(1615*e^3 + 4199*e^2*f*x + 3705*e*f^2*x^2 + 1105*f^3*x^3))) + 8*a^2*
b^7*(138567*c^2*(7*e + 3*f*x) + 323*c*(3003*d^2 + 858*d*x*(3*e + 2*f*x) +
x^2*(858*e^2 + 1300*e*f*x + 525*f^2*x^2)) + x*(138567*d^3 + 8398*d^2*x*(33
*e + 25*f*x) + 323*d*x^2*(650*e^2 + 1050*e*f*x + 441*f^2*x^2) + 7*x^3*(807
5*e^3 + 20349*e^2*f*x + 17556*e*f^2*x^2 + 5148*f^3*x^3))) - 2*a*b^8*(13...
```

3.6.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{3f(a+bx)^{13/2}\left(12a^2f^2-8abef+b^2\left(df+e^2\right)\right)}{b^9}+\frac{(a+bx)^{11/2}\left(-84a^3f^3+84a^2bef^2-21ab^2f\left(df+e^2\right)+b^3\right.}{b^9}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2f(a+bx)^{15/2}\left(-12a^2f^2+8abef-\left(b^2\left(df+e^2\right)\right)\right)}{5b^{10}}+$$
$$\frac{2(a+bx)^{13/2}\left(-84a^3f^3+84a^2bef^2-21ab^2f\left(df+e^2\right)+b^3\left(3cf^2+6def+e^3\right)\right)}{13b^{10}}+$$
$$\frac{2(a+bx)^{3/2}\left(3a^2f-2abe+b^2d\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^2}{b^{10}}+$$
$$\frac{2\sqrt{a+bx}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)^3}{b^{10}}+$$
$$\frac{6(a+bx)^{5/2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)\left(12a^4f^2-16a^3bef+a^2b^2\left(9df+5e^2\right)-ab^3(3cf+5de)+b^4\left(ce+d^2\right)\right.}{5b^{10}}$$
$$\frac{6(a+bx)^{11/2}\left(-42a^4f^3+56a^3bef^2-21a^2b^2f\left(df+e^2\right)+2ab^3\left(3cf^2+6def+e^3\right)-b^4\left(2cef+d^2f+de^2\right)\right)}{11b^{10}}+$$
$$\frac{2(a+bx)^{9/2}\left(-42a^5f^3+70a^4bef^2-35a^3b^2f\left(df+e^2\right)+5a^2b^3\left(3cf^2+6def+e^3\right)-5ab^4\left(2cef+d^2f+de^2\right)+b\right.}{3b^{10}}$$
$$\frac{2(a+bx)^{7/2}\left(-84a^6f^3+168a^5bef^2-105a^4b^2f\left(df+e^2\right)+20a^3b^3\left(3cf^2+6def+e^3\right)-30a^2b^4\left(2cef+d^2f+de\right.\right.}{7b^{10}}$$
$$\frac{6f^2(a+bx)^{17/2}(be-3af)}{17b^{10}}+\frac{2f^3(a+bx)^{19/2}}{19b^{10}}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)^3/Sqrt[a + b*x],x]
```

output
```
(2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^3*Sqrt[a + b*x])/b^10 + (2*(b^2*d -
 2*a*b*e + 3*a^2*f)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)^2*(a + b*x)^(3/2))
/b^10 + (6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(b^4*(d^2 + c*e) - 16*a^3*b
*e*f + 12*a^4*f^2 - a*b^3*(5*d*e + 3*c*f) + a^2*b^2*(5*e^2 + 9*d*f))*(a +
b*x)^(5/2))/(5*b^10) - (2*(168*a^5*b*e*f^2 - 84*a^6*f^3 - b^6*(d^3 + 6*c*d
*e + 3*c^2*f) - 105*a^4*b^2*f*(e^2 + d*f) + 12*a*b^5*(d^2*e + c*e^2 + 2*c*
d*f) - 30*a^2*b^4*(d*e^2 + d^2*f + 2*c*e*f) + 20*a^3*b^3*(e^3 + 6*d*e*f +
3*c*f^2))*(a + b*x)^(7/2))/(7*b^10) + (2*(70*a^4*b*e*f^2 - 42*a^5*f^3 - 35
*a^3*b^2*f*(e^2 + d*f) + b^5*(d^2*e + c*e^2 + 2*c*d*f) - 5*a*b^4*(d*e^2 +
d^2*f + 2*c*e*f) + 5*a^2*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(9/2))/(
3*b^10) - (6*(56*a^3*b*e*f^2 - 42*a^4*f^3 - 21*a^2*b^2*f*(e^2 + d*f) - b^4
*(d*e^2 + d^2*f + 2*c*e*f) + 2*a*b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^
(11/2))/(11*b^10) + (2*(84*a^2*b*e*f^2 - 84*a^3*f^3 - 21*a*b^2*f*(e^2 + d*
f) + b^3*(e^3 + 6*d*e*f + 3*c*f^2))*(a + b*x)^(13/2))/(13*b^10) - (2*f*(8*
a*b*e*f - 12*a^2*f^2 - b^2*(e^2 + d*f))*(a + b*x)^(15/2))/(5*b^10) + (6*f^
2*(b*e - 3*a*f)*(a + b*x)^(17/2))/(17*b^10) + (2*f^3*(a + b*x)^(19/2))/(19
*b^10)
```

3.6. $\int \frac{\left(c+dx+ex^2+fx^3\right)^3}{\sqrt{a+bx}}\,dx$

3.6.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.6.4 Maple [A] (verified)

Time = 5.42 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{131072\left(\frac{\left(-\frac{12155 f^{3} x^{9}}{2}-\frac{40755 e\, f^{2} x^{8}}{2}-\frac{46189 f\left(d f+e^{2}\right) x^{7}}{2}+17765\left(-\frac{1}{2} e^{3}-\frac{3}{2} c\, f^{2}-3 d e f\right) x^{6}+62985\left(\left(-\frac{d^{2}}{2}-c e\right) f-\frac{d\, e^{2}}{2}\right) x^{5}+ \cdots\right.}{3276\cdots}\right.}{\cdots}$
derivativedivides	Expression too large to display
default	Expression too large to display
gosper	Expression too large to display
trager	Expression too large to display
risch	Expression too large to display

input
```
int((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-131072/230945*(1/32768*(-12155/2*f^3*x^9-40755/2*e*f^2*x^8-46189/2*f*(d*f
+e^2)*x^7+17765*(-1/2*e^3-3/2*c*f^2-3*d*e*f)*x^6+62985*((-1/2*d^2-c*e)*f-1
/2*d*e^2)*x^5+230945/3*(-c*f*d-1/2*e*(c*e+d^2))*x^4+692835/7*(-1/2*c^2*f-(
c*e+1/6*d^2)*d)*x^3-138567/2*c*(c*e+d^2)*x^2-230945/2*c^2*d*x-230945/2*c^3
)*b^9+230945/32768*(9/323*f^3*x^8+8/85*e*f^2*x^7+7/65*f*(d*f+e^2)*x^6+6/14
3*(3*c*f^2+6*d*e*f+e^3)*x^5+5/33*(f*(2*c*e+d^2)+d*e^2)*x^4+4/21*(2*c*f*d+e
*(c*e+d^2))*x^3+3/35*(3*c^2*f+6*c*d*e+d^3)*x^2+2/5*c*(c*e+d^2)*x+c^2*d)*a*
b^8-46189/8192*(12/323*f^3*x^7+28/221*e*f^2*x^6+21/143*f*(d*f+e^2)*x^5+25/
143*(1/3*e^3+c*f^2+2*d*e*f)*x^4+50/231*(f*(2*c*e+d^2)+d*e^2)*x^3+2/7*(2*c*
f*d+e*(c*e+d^2))*x^2+1/7*(3*c^2*f+6*c*d*e+d^3)*x+c^2*e+c*d^2)*a^2*b^7+1385
67/28672*(196/4199*f^3*x^6+392/2431*e*f^2*x^5+245/1287*f*(d*f+e^2)*x^4+100
/429*(1/3*e^3+c*f^2+2*d*e*f)*x^3+10/33*(f*(2*c*e+d^2)+d*e^2)*x^2+4/9*(2*c*
f*d+e*(c*e+d^2))*x+1/3*d^3+c^2*f+2*c*d*e)*a^3*b^6-46189/5376*a^4*(1323/461
89*f^3*x^5+245/2431*e*f^2*x^4+35/286*f*(d*f+e^2)*x^3+15/143*(3/2*c*f^2+3*d
*e*f+1/2*e^3)*x^2+5/11*((c*e+1/2*d^2)*f+1/2*d*e^2)*x+c*f*d+1/2*e*(c*e+d^2)
)*b^5+20995/2688*(147/4199*f^3*x^4+28/221*e*f^2*x^3+21/130*f*(d*f+e^2)*x^2
+1/13*(3*c*f^2+6*d*e*f+e^3)*x+(c*e+1/2*d^2)*f+1/2*d*e^2)*a^5*b^4-1615/448*
(28/323*f^3*x^3+28/85*e*f^2*x^2+7/15*f*(d*f+e^2)*x+1/3*e^3+c*f^2+2*d*e*f)*
a^6*b^3+323/96*f*(36/323*f^2*x^2+8/17*e*f*x+d*f+e^2)*a^7*b^2-19/6*(3/19*f*
x+e)*f^2*a^8*b+a^9*f^3)*(b*x+a)^(1/2)/b^10
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 1221, normalized size of antiderivative = 1.72

$$\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/4849845*(255255*b^9*f^3*x^9 + 4849845*b^9*c^3 - 9699690*a*b^8*c^2*d + 77
59752*a^2*b^7*c*d^2 - 2217072*a^3*b^6*d^3 + 1653760*a^6*b^3*e^3 - 1376256*
a^9*f^3 + 45045*(19*b^9*e*f^2 - 6*a*b^8*f^3)*x^8 + 3003*(323*b^9*e^2*f + 9
6*a^2*b^7*f^3 + 19*(17*b^9*d - 16*a*b^8*e)*f^2)*x^7 + 231*(1615*b^9*e^3 -
1344*a^3*b^6*f^3 + 19*(255*b^9*c - 238*a*b^8*d + 224*a^2*b^7*e)*f^2 + 646*
(15*b^9*d*e - 7*a*b^8*e^2)*f)*x^6 + 63*(20995*b^9*d*e^2 - 6460*a*b^8*e^3 +
 5376*a^4*b^5*f^3 - 76*(255*a*b^8*c - 238*a^2*b^7*d + 224*a^3*b^6*e)*f^2 +
 323*(65*b^9*d^2 + 56*a^2*b^7*e^2 + 10*(13*b^9*c - 12*a*b^8*d)*e)*f)*x^5 +
 35*(46189*b^9*d^2*e + 12920*a^2*b^7*e^3 - 10752*a^5*b^4*f^3 + 4199*(11*b^
9*c - 10*a*b^8*d)*e^2 + 152*(255*a^2*b^7*c - 238*a^3*b^6*d + 224*a^4*b^5*e
)*f^2 + 646*(143*b^9*c*d - 65*a*b^8*d^2 - 56*a^3*b^6*e^2 - 10*(13*a*b^8*c
- 12*a^2*b^7*d)*e)*f)*x^4 + 5*(138567*b^9*d^3 - 103360*a^3*b^6*e^3 + 86016
*a^6*b^3*f^3 - 33592*(11*a*b^8*c - 10*a^2*b^7*d)*e^2 - 1216*(255*a^3*b^6*c
 - 238*a^4*b^5*d + 224*a^5*b^4*e)*f^2 + 92378*(9*b^9*c*d - 4*a*b^8*d^2)*e
+ 323*(1287*b^9*c^2 - 2288*a*b^8*c*d + 1040*a^2*b^7*d^2 + 896*a^4*b^5*e^2
+ 160*(13*a^2*b^7*c - 12*a^3*b^6*d)*e)*f)*x^3 + 537472*(11*a^4*b^5*c - 10*
a^5*b^4*d)*e^2 + 19456*(255*a^6*b^3*c - 238*a^7*b^2*d + 224*a^8*b*e)*f^2 +
 3*(969969*b^9*c*d^2 - 277134*a*b^8*d^3 + 206720*a^4*b^5*e^3 - 172032*a^7*
b^2*f^3 + 67184*(11*a^2*b^7*c - 10*a^3*b^6*d)*e^2 + 2432*(255*a^4*b^5*c -
238*a^5*b^4*d + 224*a^6*b^3*e)*f^2 + 46189*(21*b^9*c^2 - 36*a*b^8*c*d +...
```

3.6.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1622 vs. 2(758) = 1516.

Time = 2.07 (sec) , antiderivative size = 1622, normalized size of antiderivative = 2.29

$$\int \frac{(c + dx + ex^2 + fx^3)^3}{\sqrt{a + bx}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)**3/(b*x+a)**(1/2),x)
```

output
```
Piecewise((2*(f**3*(a + b*x)**(19/2)/(19*b**9) + (a + b*x)**(17/2)*(-9*a*f
**3 + 3*b*e*f**2)/(17*b**9) + (a + b*x)**(15/2)*(36*a**2*f**3 - 24*a*b*e*f
**2 + 3*b**2*d*f**2 + 3*b**2*e**2*f)/(15*b**9) + (a + b*x)**(13/2)*(-84*a*
*3*f**3 + 84*a**2*b*e*f**2 - 21*a*b**2*d*f**2 - 21*a*b**2*e**2*f + 3*b**3*
c*f**2 + 6*b**3*d*e*f + b**3*e**3)/(13*b**9) + (a + b*x)**(11/2)*(126*a**4
*f**3 - 168*a**3*b*e*f**2 + 63*a**2*b**2*d*f**2 + 63*a**2*b**2*e**2*f - 18
*a*b**3*c*f**2 - 36*a*b**3*d*e*f - 6*a*b**3*e**3 + 6*b**4*c*e*f + 3*b**4*d
**2*f + 3*b**4*d*e**2)/(11*b**9) + (a + b*x)**(9/2)*(-126*a**5*f**3 + 210*
a**4*b*e*f**2 - 105*a**3*b**2*d*f**2 - 105*a**3*b**2*e**2*f + 45*a**2*b**3
*c*f**2 + 90*a**2*b**3*d*e*f + 15*a**2*b**3*e**3 - 30*a*b**4*c*e*f - 15*a*
b**4*d**2*f - 15*a*b**4*d*e**2 + 6*b**5*c*d*f + 3*b**5*c*e**2 + 3*b**5*d**
2*e)/(9*b**9) + (a + b*x)**(7/2)*(84*a**6*f**3 - 168*a**5*b*e*f**2 + 105*a
**4*b**2*d*f**2 + 105*a**4*b**2*e**2*f - 60*a**3*b**3*c*f**2 - 120*a**3*b*
*3*d*e*f - 20*a**3*b**3*e**3 + 60*a**2*b**4*c*e*f + 30*a**2*b**4*d**2*f +
30*a**2*b**4*d*e**2 - 24*a*b**5*c*d*f - 12*a*b**5*c*e**2 - 12*a*b**5*d**2*
e + 3*b**6*c**2*f + 6*b**6*c*d*e + b**6*d**3)/(7*b**9) + (a + b*x)**(5/2)*
(-36*a**7*f**3 + 84*a**6*b*e*f**2 - 63*a**5*b**2*d*f**2 - 63*a**5*b**2*e**
2*f + 45*a**4*b**3*c*f**2 + 90*a**4*b**3*d*e*f + 15*a**4*b**3*e**3 - 60*a*
*3*b**4*c*e*f - 30*a**3*b**4*d**2*f - 30*a**3*b**4*d*e**2 + 36*a**2*b**5*c
*d*f + 18*a**2*b**5*c*e**2 + 18*a**2*b**5*d**2*e - 9*a*b**6*c**2*f - 18...
```

3.6.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. 2(673) = 1346.

Time = 0.22 (sec) , antiderivative size = 1360, normalized size of antiderivative = 1.92

$$\int \frac{(c + dx + ex^2 + fx^3)^3}{\sqrt{a + bx}}\, dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/4849845*(4849845*sqrt(b*x + a)*c^3 + 138567*c^2*(35*((b*x + a)^(3/2) - 3
*sqrt(b*x + a)*a)*d/b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*s
qrt(b*x + a)*a^2)*e/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35
*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*f/b^3) + 323*c*(3003*(3*(b*x
+ a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d^2/b^2 + 143*(3
5*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*
(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*e^2/b^4 + 286*(35*(b*x + a)^(
9/2)*f + 45*(b*e - 4*a*f)*(b*x + a)^(7/2) - 189*(a*b*e - 2*a^2*f)*(b*x + a
)^(5/2) + 105*(3*a^2*b*e - 4*a^3*f)*(b*x + a)^(3/2) - 315*(a^3*b*e - a^4*f
)*sqrt(b*x + a))*d/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a
+ 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2
)*a^4 - 693*sqrt(b*x + a)*a^5)*e*f/b^5 + 15*(231*(b*x + a)^(13/2) - 1638*(
b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 +
9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a
^6)*f^2/b^6) + 138567*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x
+ a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3 + 4199*(315*(b*x + a)^(11/2
)*f + 385*(b*e - 5*a*f)*(b*x + a)^(9/2) - 990*(2*a*b*e - 5*a^2*f)*(b*x + a
)^(7/2) + 1386*(3*a^2*b*e - 5*a^3*f)*(b*x + a)^(5/2) - 1155*(4*a^3*b*e - 5
*a^4*f)*(b*x + a)^(3/2) + 3465*(a^4*b*e - a^5*f)*sqrt(b*x + a))*d^2/b^5 +
1615*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(...
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1412 vs. $2(673) = 1346$.

Time = 0.29 (sec) , antiderivative size = 1412, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)^3/(b*x+a)^(1/2),x, algorithm="giac")
```

output

```
2/4849845*(4849845*sqrt(b*x + a)*c^3 + 4849845*((b*x + a)^(3/2) - 3*sqrt(b
*x + a)*a)*c^2*d/b + 969969*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15
*sqrt(b*x + a)*a^2)*c*d^2/b^2 + 969969*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(
3/2)*a + 15*sqrt(b*x + a)*a^2)*c^2*e/b^2 + 138567*(5*(b*x + a)^(7/2) - 21*
(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d^3/b^3
 + 831402*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a
^2 - 35*sqrt(b*x + a)*a^3)*c*d*e/b^3 + 415701*(5*(b*x + a)^(7/2) - 21*(b*x
 + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c^2*f/b^3 +
 46189*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a
^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d^2*e/b^4 + 46189*(3
5*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*
(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*e^2/b^4 + 92378*(35*(b*x +
a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)
^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*c*d*f/b^4 + 20995*(63*(b*x + a)^(11/2)
 - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*
a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*e^2/b^5 + 20995*
(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1
386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5
)*d^2*f/b^5 + 41990*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*
x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 ...
```

3.6. $\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx$

3.6.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 896, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx$$

$$= \frac{(a+b\,x)^{11/2}\,(252\,a^4\,f^3-336\,a^3\,b\,e\,f^2+126\,a^2\,b^2\,d\,f^2+126\,a^2\,b^2\,e^2\,f-72\,a\,b^3\,d\,e\,f-12\,a\,b^3\,e^3-36\,c\,a\,b^3}{11\,b^{10}}$$

$$+\frac{2\sqrt{a+b\,x}\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)^3}{b^{10}}$$

$$+\frac{(a+b\,x)^{9/2}\,(-252\,a^5\,f^3+420\,a^4\,b\,e\,f^2-210\,a^3\,b^2\,d\,f^2-210\,a^3\,b^2\,e^2\,f+180\,a^2\,b^3\,d\,e\,f+30\,a^2\,b^3\,e^3+9}{9\,b^{10}}$$

$$+\frac{2\,f^3\,(a+b\,x)^{19/2}}{19\,b^{10}}$$

$$+\frac{(a+b\,x)^{13/2}\,(-168\,a^3\,f^3+168\,a^2\,b\,e\,f^2-42\,a\,b^2\,e^2\,f-42\,d\,a\,b^2\,f^2+2\,b^3\,e^3+12\,d\,b^3\,e\,f+6\,c\,b^3\,f^2)}{13\,b^{10}}$$

$$-\frac{(18\,a\,f^3-6\,b\,e\,f^2)\,(a+b\,x)^{17/2}}{17\,b^{10}}$$

$$+\frac{(a+b\,x)^{15/2}\,(72\,a^2\,f^3-48\,a\,b\,e\,f^2+6\,b^2\,e^2\,f+6\,d\,b^2\,f^2)}{15\,b^{10}}$$

$$-\frac{(a+b\,x)^{5/2}\,(72\,a^7\,f^3-168\,a^6\,b\,e\,f^2+126\,a^5\,b^2\,d\,f^2+126\,a^5\,b^2\,e^2\,f-90\,a^4\,b^3\,c\,f^2-180\,a^4\,b^3\,d\,e\,f-30}{}$$

$$+\frac{(a+b\,x)^{7/2}\,(168\,a^6\,f^3-336\,a^5\,b\,e\,f^2+210\,a^4\,b^2\,d\,f^2+210\,a^4\,b^2\,e^2\,f-120\,a^3\,b^3\,c\,f^2-240\,a^3\,b^3\,d\,e\,f-}{}$$

$$+\frac{2\,(a+b\,x)^{3/2}\,(3\,f\,a^2-2\,e\,a\,b+d\,b^2)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)^2}{b^{10}}$$

input
```
int((c + d*x + e*x^2 + f*x^3)^3/(a + b*x)^(1/2),x)
```

output
```
((a + b*x)^(11/2)*(252*a^4*f^3 - 12*a*b^3*e^3 + 6*b^4*d*e^2 + 6*b^4*d^2*f
+ 126*a^2*b^2*d*f^2 + 126*a^2*b^2*e^2*f + 12*b^4*c*e*f - 36*a*b^3*c*f^2 -
336*a^3*b*e*f^2 - 72*a*b^3*d*e*f))/(11*b^10) + (2*(a + b*x)^(1/2)*(b^3*c -
 a^3*f - a*b^2*d + a^2*b*e)^3)/b^10 + ((a + b*x)^(9/2)*(6*b^5*c*e^2 - 252*
a^5*f^3 + 6*b^5*d^2*e + 30*a^2*b^3*e^3 + 90*a^2*b^3*c*f^2 - 210*a^3*b^2*d*
f^2 - 210*a^3*b^2*e^2*f + 12*b^5*c*d*f - 30*a*b^4*d*e^2 - 30*a*b^4*d^2*f +
 420*a^4*b*e*f^2 + 180*a^2*b^3*d*e*f - 60*a*b^4*c*e*f))/(9*b^10) + (2*f^3*
(a + b*x)^(19/2))/(19*b^10) + ((a + b*x)^(13/2)*(2*b^3*e^3 - 168*a^3*f^3 +
 6*b^3*c*f^2 + 12*b^3*d*e*f - 42*a*b^2*d*f^2 - 42*a*b^2*e^2*f + 168*a^2*b*
e*f^2))/(13*b^10) - ((18*a*f^3 - 6*b*e*f^2)*(a + b*x)^(17/2))/(17*b^10) +
((a + b*x)^(15/2)*(72*a^2*f^3 + 6*b^2*d*f^2 + 6*b^2*e^2*f - 48*a*b*e*f^2))
/(15*b^10) - ((a + b*x)^(5/2)*(72*a^7*f^3 + 6*a*b^6*d^3 - 6*b^7*c*d^2 - 6*
b^7*c^2*e - 30*a^4*b^3*e^3 - 36*a^2*b^5*c*e^2 - 36*a^2*b^5*d^2*e + 60*a^3*
b^4*d*e^2 - 90*a^4*b^3*c*f^2 + 60*a^3*b^4*d^2*f + 126*a^5*b^2*d*f^2 + 126*
a^5*b^2*e^2*f + 18*a*b^6*c^2*f - 168*a^6*b*e*f^2 - 72*a^2*b^5*c*d*f + 120*
a^3*b^4*c*e*f - 180*a^4*b^3*d*e*f + 36*a*b^6*c*d*e))/(5*b^10) + ((a + b*x)
^(7/2)*(2*b^6*d^3 + 168*a^6*f^3 + 6*b^6*c^2*f - 40*a^3*b^3*e^3 + 60*a^2*b^
4*d*e^2 - 120*a^3*b^3*c*f^2 + 60*a^2*b^4*d^2*f + 210*a^4*b^2*d*f^2 + 210*a
^4*b^2*e^2*f + 12*b^6*c*d*e - 24*a*b^5*c*e^2 - 24*a*b^5*d^2*e - 336*a^5*b*
e*f^2 + 120*a^2*b^4*c*e*f - 240*a^3*b^3*d*e*f - 48*a*b^5*c*d*f))/(7*b^1...
```

3.6. $\int \frac{(c+dx+ex^2+fx^3)^3}{\sqrt{a+bx}}\,dx$

3.7 $\int \frac{c+dx}{a+bx^3} \, dx$

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3.7.1 Optimal result

Integrand size = 15, antiderivative size = 161

$$\int \frac{c+dx}{a+bx^3} \, dx = -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} - \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

output
```
1/3*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1/6*(c-a^(
1/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)-
1/3*(b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2)
)/a^(2/3)/b^(2/3)*3^(1/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int \frac{c+dx}{a+bx^3} \, dx$$

$$= \frac{-2\sqrt{3}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2}\right.\right.}{6a^{2/3}b^{2/3}}$$

input
```
Integrate[(c + d*x)/(a + b*x^3),x]
```

output
```
(-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

3.7.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+bx^3}\,dx$$

$$\Big\downarrow \text{2399}$$

$$\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{1}{2}\sqrt[3]{b}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}\sqrt[3]{b}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \; 1103$$

3.7. $\int \frac{c+dx}{a+bx^3}\,dx$

$$\frac{-\frac{1}{2}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

input
```
Int[(c + d*x)/(a + b*x^3),x]
```

output
```
((c - (a^(1/3)*d)/b^(1/3))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) +
 (-((Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(1/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)*b^(1/3))
```

3.7.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.7.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.75 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^2}}{3b}$
default	$c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\right.}{6b\left(\frac{a}{b}\right)}\right.$

input
```
int((d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*sum((_R*d+c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.7.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 1931, normalized size of antiderivative = 11.99

$$\int \frac{c+dx}{a+bx^3} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*
d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 +
a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)
*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(
1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2)
+ (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d - 1/2*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*
(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3
- a*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/1
2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3
)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d
^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^
2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*log(
-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*
d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 +
a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)...
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int \frac{c+dx}{a+bx^3} \, dx$$
$$= \operatorname{RootSum}\left(27t^3a^2b^2 + 9tabcd + ad^3 - bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd + 3tabc^2 + 2acd^2}{ad^3 + bc^3}\right)\right)\right)$$

input
```
integrate((d*x+c)/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t
*log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)
)))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\int \frac{c+dx}{a+bx^3}\,dx = \frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(d*(a/b)^(1/3) + c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(d*(a/b)^(1/3) - c)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(d*(a/b)^(1/3) - c)*log(x + (a/b)^(
1/3))/(b*(a/b)^(2/3))
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{c+dx}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(bc-\left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(bc+\left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}$$

input
```
integrate((d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(d*(-a/b)^(1/3) + c)*(
-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a
```

3.7.9 Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \frac{c+dx}{a+bx^3}\,dx = \sum_{k=1}^{3} \ln\left(b\left(c\,d+d^2\,x+\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)^2\,a\,b\,9 + \mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)\,b\,c\,x\,3\right)\right)\,\mathrm{root}\left(27\,a^2\,b^2\,z^3 + 9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)$$

input
```
int((c + d*x)/(a + b*x^3),x)
```

output
```
symsum(log(b*(c*d + d^2*x + 9*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 -
b*c^3, z, k)^2*a*b + 3*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3,
z, k)*b*c*x))*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z, k), k,
 1, 3)
```

3.8 $\int \frac{c+dx}{(a+bx^3)^2}\,dx$

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3.8.1 Optimal result

Integrand size = 15, antiderivative size = 189

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}$$

output
```
1/3*x*(d*x+c)/a/(b*x^3+a)+1/9*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x
)/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/9*(2*b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(
1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx$$
$$= \frac{\frac{6ax(c+dx)}{a+bx^3} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{\left(-2\sqrt[3]{a}\sqrt[3]{b}c+a^{2/3}d\right)\log}{b}}{18a^2}$$

input
```
Integrate[(c + d*x)/(a + b*x^3)^2,x]
```

output
```
((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)
*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b
^(1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(
1/3)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3
))/(18*a^2)
```

3.8.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2394, 25, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx$$
$$\Big\downarrow \text{ 2394}$$
$$\frac{x(c+dx)}{3a(a+bx^3)} - \frac{\int -\frac{2c+dx}{bx^3+a}dx}{3a}$$
$$\Big\downarrow \text{ 25}$$
$$\frac{\int \frac{2c+dx}{bx^3+a}dx}{3a} + \frac{x(c+dx)}{3a(a+bx^3)}$$
$$\Big\downarrow \text{ 2399}$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 1082}$$

3.8. $\int \frac{c+dx}{\left(a+bx^3\right)^2}\,dx$

$$\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{-\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

input
```
Int[(c + d*x)/(a + b*x^3)^2,x]
```

output
```
(x*(c + d*x))/(3*a*(a + b*x^3)) + (((2*c - (a^(1/3)*d)/b^(1/3))*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*(2*b^(1/3)*c + a^(1/3)*d
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - ((2*c - (a^(1/3)
*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)
*b^(1/3)))/(3*a)
```

3.8. $\int \frac{c+dx}{(a+bx^3)^2}\,dx$

3.8.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.8.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.78 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\frac{d x^2}{3a}+\frac{cx}{3a}}{b x^3+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_R d+2c\right)\ln\left(x-\_R\right)}{\_R^2}}{9ba}$
default	$c\left(\frac{x}{3a(b x^3+a)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\right)+d\left(\frac{x^2}{3a(b x^3+a)}+\frac{-\frac{\ln\left(x+\left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right.}\right.$

input
```
int((d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3*d/a*x^2+1/3*c/a*x)/(b*x^3+a)+1/9/b/a*sum((_R*d+2*c)/_R^2*ln(x-_R),_R=
RootOf(_Z^3*b+a))
```

3.8.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 2088, normalized size of antiderivative = 11.05

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
+ a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/
(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)
/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqr
t(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2
))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(
a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(
3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))
^(1/3)))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3
 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c
^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((
8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) + 3*sqrt
(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^
3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*s
qrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b
^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4
*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*
b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3)...
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.56

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx$$
$$= \operatorname{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right) + \frac{cx + dx^2}{3a^2 + 3abx^3}$$

input
```
integrate((d*x+c)/(b*x**3+a)**2,x)
```

output
```
RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda
(_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d*
*3 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = \frac{dx^2+cx}{3\,(abx^3+a^2)} + \frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(d*x^2 + c*x)/(a*b*x^3 + a^2) + 1/9*sqrt(3)*(d*(a/b)^(1/3) + 2*c)*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) + 1/18*(
d*(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/
3)) - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(2\,bc-\left(-ab^2\right)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}\,a} - \frac{\left(2\,bc+\left(-ab^2\right)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}\,a} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{dx^2+cx}{3\,(bx^3+a)a}$$

input
```
integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)
^(1/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(d*x^2 + c
*x)/((b*x^3 + a)*a)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{b\left(2\,c\,d+d^2\,x+\mathrm{root}(729\,a^5\,b^2\,z^3+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k)^2\,a^3\,b\,81+\mathrm{root}(729\,a^5\,b^2\,z^3\right.}{a^2\,9}\right.\right.$$
$$\left.+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k)\right)+\frac{\frac{d\,x^2}{3\,a}+\frac{c\,x}{3\,a}}{b\,x^3+a}$$

input
```
int((c + d*x)/(a + b*x^3)^2,x)
```

output
```
symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z
- 8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*
b*c*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) + (c*x)/(3*a))
/(a + b*x^3)
```

3.8. $\int \frac{c+dx}{(a+bx^3)^2}\, dx$

3.9 $\int \frac{c+dx}{(a+bx^3)^3}\,dx$

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3.9.1 Optimal result

Integrand size = 15, antiderivative size = 215

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = \frac{x(c+dx)}{6a\,(a+bx^3)^2}+\frac{x(5c+4dx)}{18a^2\,(a+bx^3)}-\frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}$$

output
```
1/6*x*(d*x+c)/a/(b*x^3+a)^2+1/18*x*(4*d*x+5*c)/a^2/(b*x^3+a)+1/27*(5*b^(1/
3)*c-2*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(2/3)-1/54*(5*b^(1/3)*c-
2*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(2/3)-1/2
7*(5*b^(1/3)*c+2*a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/
2))/a^(8/3)/b^(2/3)*3^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx$$

$$= \frac{\frac{9a^2x(c+dx)}{(a+bx^3)^2} + \frac{3ax(5c+4dx)}{a+bx^3} - \frac{2\sqrt{3}\sqrt[3]{a}\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{2\left(5\sqrt[3]{a}\sqrt[3]{b}c-2a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{(-5\ \cdots}{\cdots}}{54a^3}$$

input
```
Integrate[(c + d*x)/(a + b*x^3)^3,x]
```

output
```
((9*a^2*x*(c + d*x))/(a + b*x^3)^2 + (3*a*x*(5*c + 4*d*x))/(a + b*x^3) - (
2*Sqrt[3]*a^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]])/b^(2/3) + (2*(5*a^(1/3)*b^(1/3)*c - 2*a^(2/3)*d)*Log[a^(1
/3) + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*c + 2*a^(2/3)*d)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(54*a^3)
```

3.9.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {2394, 25, 2394, 27, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \ 2394$$

$$\frac{x(c+dx)}{6a(a+bx^3)^2} - \frac{\int -\frac{5c+4dx}{(bx^3+a)^2}dx}{6a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{5c+4dx}{(bx^3+a)^2}dx}{6a} + \frac{x(c+dx)}{6a(a+bx^3)^2}$$

$$\Big\downarrow \ 2394$$

$$\frac{\frac{x(5c+4dx)}{3a(a+bx^3)}-\frac{\int -\frac{2(5c+2dx)}{bx^3+a}dx}{3a}}{6a}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2\int \frac{5c+2dx}{bx^3+a}dx}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 2399$$

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}\left(5\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}\left(5\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}$$
$$\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 25$$

3.9. $\int \frac{c+dx}{\left(a+bx^3\right)^3}\,dx$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5}{3a}$$
$$\overline{\qquad 6a \qquad}$$
$$\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+$$
$$\overline{\qquad 6a \qquad}$$
$$\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$\downarrow$ 1082

$$\frac{2\left(\frac{\frac{1}{2}\sqrt[3]{b}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}$$
$$\overline{\qquad 6a \qquad}$$
$$\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$\downarrow$ 217

3.9. $\int \frac{c+dx}{(a+bx^3)^3}\,dx$

$$\frac{\frac{2\left(\frac{\frac{1}{2}\sqrt[3]{b}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}$$

$$\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{2\left(\frac{-\frac{1}{2}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}$$

$$\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

input
```
Int[(c + d*x)/(a + b*x^3)^3,x]
```

output
```
(x*(c + d*x))/(6*a*(a + b*x^3)^2) + ((x*(5*c + 4*d*x))/(3*a*(a + b*x^3)) +
 (2*(((5*c - (2*a^(1/3)*d)/b^(1/3))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b
^(1/3)) + (-((Sqrt[3]*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]])/b^(1/3)) - ((5*c - (2*a^(1/3)*d)/b^(1/3))*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)*b^(1/3))))/(3*a))/(6*a)
```

3.9. $\int \frac{c+dx}{(a+bx^3)^3}\,dx$

3.9.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.9.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.80 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{2bd\,x^5}{9a^2}+\frac{5bc\,x^4}{18a^2}+\frac{7d\,x^2}{18a}+\frac{4cx}{9a}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(2\_Rd+5c\right)\ln\left(x-\_R\right)}{\_R^2}}{27a^2b}$
default	$c\left(\frac{x}{6a(b\,x^3+a)^2}+\frac{\frac{5x}{18a\left(b\,x^3+a\right)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6a}}{a}\right)+d\left(\frac{x}{6a(b\,x^3\right.$

input
```
int((d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(2/9*b*d/a^2*x^5+5/18*b*c/a^2*x^4+7/18*d/a*x^2+4/9*c/a*x)/(b*x^3+a)^2+1/27
/a^2/b*sum((2*_R*d+5*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.9.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 2215, normalized size of antiderivative = 10.30

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/108*(24*b*d*x^5 + 30*b*c*x^4 + 42*a*d*x^2 + 48*a*c*x - 2*(a^2*b^2*x^6 +
2*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^
8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*s
qrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3
)/(a^8*b^2))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*
a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)
*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d - 25/2*((1/2)^(1/3)*(I*sqrt(3) +
1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/
3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^
8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 + 40*a*c*d^2 +
 (125*b*c^3 + 8*a*d^3)*x) + ((a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)
/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3
 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3))) + 3*sqrt(
1/3)*(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)
*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)
 - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*
b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b)
))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^...
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.68

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx$$

$$= \operatorname{RootSum}\left(19683t^3a^8b^2 + 810ta^3bcd + 8ad^3 - 125bc^3, \left(t \mapsto t\log\left(x + \frac{1458t^2a^6bd + 675ta^3bc^2 + 40acd^2}{8ad^3 + 125bc^3}\right.\right.\right.$$
$$+ \frac{8acx + 7adx^2 + 5bcx^4 + 4bdx^5}{18a^4 + 36a^3bx^3 + 18a^2b^2x^6}$$

input
```
integrate((d*x+c)/(b*x**3+a)**3,x)
```

output
```
RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3,
 Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*
d**2)/(8*a*d**3 + 125*b*c**3)))) + (8*a*c*x + 7*a*d*x**2 + 5*b*c*x**4 + 4*
b*d*x**5)/(18*a**4 + 36*a**3*b*x**3 + 18*a**2*b**2*x**6)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = \frac{4\,bdx^5+5\,bcx^4+7\,adx^2+8\,acx}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)} + \frac{\sqrt{3}\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(4*b*d*x^5 + 5*b*c*x^4 + 7*a*d*x^2 + 8*a*c*x)/(a^2*b^2*x^6 + 2*a^3*b*
x^3 + a^4) + 1/27*sqrt(3)*(2*d*(a/b)^(1/3) + 5*c)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) + 1/54*(2*d*(a/b)^(1/3) -
5*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/27*(2*
d*(a/b)^(1/3) - 5*c)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.90

$$\int \frac{c+dx}{\left(a+bx^3\right)^3}\,dx = -\frac{\sqrt{3}\left(5\,bc-2\,\left(-ab^2\right)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}\,a^2} - \frac{\left(5\,bc+2\,\left(-ab^2\right)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{2}{3}}\,a^2} - \frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3} + \frac{4\,bdx^5+5\,bcx^4+7\,adx^2+8\,acx}{18\,\left(bx^3+a\right)^2a^2}$$

input
```
integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(5*b*c - 2*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*c + 2*(-a*b^2)^(1/
3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27
*(2*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/
18*(4*b*d*x^5 + 5*b*c*x^4 + 7*a*d*x^2 + 8*a*c*x)/((b*x^3 + a)^2*a^2)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.96

$$\int \frac{c+dx}{\left(a+bx^3\right)^3}\,dx = \frac{\frac{7\,d\,x^2}{18\,a}+\frac{4\,c\,x}{9\,a}+\frac{5\,b\,c\,x^4}{18\,a^2}+\frac{2\,b\,d\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \left(\sum_{k=1}^{3}\ln\left(\frac{b\left(10\,c\,d+4\,d^2\,x+\mathrm{root}(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)^2\,a^5\,b\,729+\mathrm{roo}\right.}{a^4\,81}\right.\right.$$

$$\left.\left.+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)\right)\right.$$

input
```
int((c + d*x)/(a + b*x^3)^3,x)
```

output
```
((7*d*x^2)/(18*a) + (4*c*x)/(9*a) + (5*b*c*x^4)/(18*a^2) + (2*b*d*x^5)/(9*
a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((b*(10*c*d + 4*d^2*x + 729*
root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3, z, k)^2*a^
5*b + 135*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3,
z, k)*a^2*b*c*x))/(81*a^4))*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125
*b*c^3 + 8*a*d^3, z, k), k, 1, 3)
```

3.9. $\int \frac{c+dx}{(a+bx^3)^3}\,dx$

3.10 $\int \frac{c+dx}{(a+bx^3)^4}\,dx$

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3.10.1 Optimal result

Integrand size = 15, antiderivative size = 240

$$\begin{aligned}\int \frac{c+dx}{(a+bx^3)^4}\,dx = {} & \frac{x(c+dx)}{9a(a+bx^3)^3}+\frac{x(8c+7dx)}{54a^2(a+bx^3)^2}+\frac{2x(10c+7dx)}{81a^3(a+bx^3)} \\ & -\frac{2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}} \\ & +\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} \\ & -\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}\end{aligned}$$

output
```
1/9*x*(d*x+c)/a/(b*x^3+a)^3+1/54*x*(7*d*x+8*c)/a^2/(b*x^3+a)^2+2/81*x*(7*d
*x+10*c)/a^3/(b*x^3+a)+2/243*(20*b^(1/3)*c-7*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)
*x)/a^(11/3)/b^(2/3)-1/243*(20*b^(1/3)*c-7*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(2/3)-2/243*(20*b^(1/3)*c+7*a^(1/3)*d)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*3^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx$$

$$= \frac{\frac{54a^3x(c+dx)}{(a+bx^3)^3} + \frac{9a^2x(8c+7dx)}{(a+bx^3)^2} + \frac{12ax(10c+7dx)}{a+bx^3} - \frac{4\sqrt{3}\sqrt[3]{a}\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log\left(\cdots\right.}{b^{2/3}}\cdots}{486a^4}$$

input
```
Integrate[(c + d*x)/(a + b*x^3)^4,x]
```

output
```
((54*a^3*x*(c + d*x))/(a + b*x^3)^3 + (9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^
2 + (12*a*x*(10*c + 7*d*x))/(a + b*x^3) - (4*Sqrt[3]*a^(1/3)*(20*b^(1/3)*c
 + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*
(20*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (
2*(-20*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/b^(2/3))/(486*a^4)
```

3.10.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.933$, Rules used = {2394, 25, 2394, 27, 2394, 25, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx$$

$$\Big\downarrow \text{ 2394}$$

$$\frac{x(c+dx)}{9a\,(a+bx^3)^3} - \frac{\int -\frac{8c+7dx}{(bx^3+a)^3}dx}{9a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{8c+7dx}{(bx^3+a)^3}dx}{9a} + \frac{x(c+dx)}{9a\,(a+bx^3)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(8c+7dx)}{6a(a+bx^3)^2}-\frac{\int -\frac{4(10c+7dx)}{\left(bx^3+a\right)^2}dx}{6a}}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{10c+7dx}{\left(bx^3+a\right)^2}dx}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{2\left(\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}-\frac{\int -\frac{20c+7dx}{bx^3+a}dx}{3a}\right)}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2\left(\frac{\int \frac{20c+7dx}{bx^3+a}dx}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{2399}$$

$$\frac{\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{a}\left(40\sqrt[3]{b}c+7\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{16}$$

3.10. $\int \frac{c+dx}{(a+bx^3)^4}\,dx$

$$\frac{\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{a}\left(40\sqrt[3]{b}c+7\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x(8c+7dx)}{6a\left(a+bx^3\right)^2}}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$\downarrow$ 1142

$$\frac{\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(\ldots}{3a\ldots}\ldots}{3a}\ldots}{9a}$$

$$\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$\downarrow$ 25

$$\frac{\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(10\ldots}{3a(\ldots}\ldots}{3a}\ldots}{9a}$$

$$\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$\downarrow$ 27

3.10. $\int \frac{c+dx}{\left(a+bx^3\right)^4}\,dx$

$$\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\right.}{3a}}{9a}$$

$$\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}\right.}{3a}}{9a}$$

$$\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\left(\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\right)}{3a}}{9a}$$

$$\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{1103}$$

3.10. $\int \frac{c+dx}{\left(a+bx^3\right)^4}\,dx$

$$\frac{2\left(\frac{\frac{-\frac{1}{2}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(10c+7dx)}{3a(a+bx^3)}\right)}{3a}}{9a}$$

$$\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

input
```
Int[(c + d*x)/(a + b*x^3)^4,x]
```

output
```
(x*(c + d*x))/(9*a*(a + b*x^3)^3) + ((x*(8*c + 7*d*x))/(6*a*(a + b*x^3)^2)
 + (2*((x*(10*c + 7*d*x))/(3*a*(a + b*x^3)) + (((20*c - (7*a^(1/3)*d)/b^(1
/3))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*(20*b^(1/
3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3))
- ((20*c - (7*a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/2)/(3*a^(2/3)*b^(1/3)))/(3*a)))/(3*a))/(9*a)
```

3.10.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.10.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 9.84 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.46

method	result
risch	$\frac{\frac{14 d b^2 x^8}{81 a^3}+\frac{20 c b^2 x^7}{81 a^3}+\frac{77 b d x^5}{162 a^2}+\frac{52 b c x^4}{81 a^2}+\frac{67 d x^2}{162 a}+\frac{41 c x}{81 a}}{(b x^3+a)^3}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(7\_R d+20c)\ln(x-\_R)}{\_R^2}\right)}{243 a^3 b}$
default	$c\left(\frac{x}{9a(b x^3+a)^3}+\frac{\frac{4x}{27a(b x^3+a)^2}+\frac{8\left(\frac{5x}{18a(b x^3+a)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6a}\right)}{9a}}{a}\right.$

input
```
int((d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

3.10. $\int \frac{c+dx}{(a+bx^3)^4}\,dx$

output
```
(14/81*d/a^3*b^2*x^8+20/81*c/a^3*b^2*x^7+77/162*b*d/a^2*x^5+52/81*b*c/a^2*
x^4+67/162*d/a*x^2+41/81*c/a*x)/(b*x^3+a)^3+2/243/a^3/b*sum((7*_R*d+20*c)/
_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.10.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 2308, normalized size of antiderivative = 9.62

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

output
```
1/972*(168*b^2*d*x^8 + 240*b^2*c*x^7 + 462*a*b*d*x^5 + 624*a*b*c*x^4 + 402
*a^2*d*x^2 + 492*a^2*c*x - 2*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 +
a^6)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000
*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(
a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^1
1*b^2))^(1/3)))*log(7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)
/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d
*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d - 400*(4^(1/3)*(I*sqrt(3) + 1
)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^
2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d
^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 +
7840*a*c*d^2 + 4*(8000*b*c^3 + 343*a*d^3)*x) + ((a^3*b^3*x^9 + 3*a^4*b^2*x
^6 + 3*a^5*b*x^3 + a^6)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)
/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d
*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3))) + 3*sqrt(1/3)*(a^3*b^3*x^9 + 3*a^4*b^2*
x^6 + 3*a^5*b*x^3 + a^6)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 34
3*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(
2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +...
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.77

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx$$
$$= \operatorname{RootSum}\left(14348907t^3a^{11}b^2 + 408240ta^4bcd + 2744ad^3 - 64000bc^3, \left(t \mapsto t\log\left(x + \frac{413343t^2a^8bd + 194}{1372ad^3}\right.\right.\right.$$
$$+ \frac{82a^2cx + 67a^2dx^2 + 104abcx^4 + 77abdx^5 + 40b^2cx^7 + 28b^2dx^8}{162a^6 + 486a^5bx^3 + 486a^4b^2x^6 + 162a^3b^3x^9}$$

input
```
integrate((d*x+c)/(b*x**3+a)**4,x)
```

output
```
RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 6
4000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4
*b*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (82*a**2*c*x +
67*a**2*d*x**2 + 104*a*b*c*x**4 + 77*a*b*d*x**5 + 40*b**2*c*x**7 + 28*b**2
*d*x**8)/(162*a**6 + 486*a**5*b*x**3 + 486*a**4*b**2*x**6 + 162*a**3*b**3*
x**9)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = \frac{28\,b^2dx^8 + 40\,b^2cx^7 + 77\,abdx^5 + 104\,abcx^4 + 67\,a^2dx^2 + 82\,a^2cx}{162\,(a^3b^3x^9 + 3\,a^4b^2x^6 + 3\,a^5bx^3 + a^6)}$$
$$+ \frac{2\sqrt{3}\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}} + 20\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+ \frac{\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}} - 20\,c\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{2\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}} - 20\,c\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

output
```
1/162*(28*b^2*d*x^8 + 40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a*b*c*x^4 + 67*a^2
*d*x^2 + 82*a^2*c*x)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6) + 2
/243*sqrt(3)*(7*d*(a/b)^(1/3) + 20*c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3
))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/243*(7*d*(a/b)^(1/3) - 20*c)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) - 2/243*(7*d*(a/b)^(
1/3) - 20*c)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.91

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = -\frac{2\sqrt{3}\left(20\,bc-7\left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\left(-ab^2\right)^{\frac{2}{3}}a^3} - \frac{\left(20\,bc+7\left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(-ab^2\right)^{\frac{2}{3}}a^3} - \frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4} + \frac{28\,b^2dx^8+40\,b^2cx^7+77\,abdx^5+104\,abcx^4+67\,a^2dx^2+82\,a^2cx}{162\left(bx^3+a\right)^3a^3}$$

input
```
integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

output
```
-2/243*sqrt(3)*(20*b*c - 7*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*c + 7*(-a*b^2)
^(1/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) -
2/243*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^
4 + 1/162*(28*b^2*d*x^8 + 40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a*b*c*x^4 + 67
*a^2*d*x^2 + 82*a^2*c*x)/((b*x^3 + a)^3*a^3)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\frac{b\left(560\,c\,d + 196\,d^2\,x + \mathrm{root}(14348907\,a^{11}\,b^2\,z^3 + 408240\,a^4\,b\,c\,d\,z - 64000\,b\,c^3 + 2744\,a\,d^3, z, k)\right.}{a}\right.\right.$$
$$\left.\left. + 408240\,a^4\,b\,c\,d\,z - 64000\,b\,c^3 + 2744\,a\,d^3, z, k)\right)\right)$$
$$+ \frac{\frac{67\,d\,x^2}{162\,a} + \frac{41\,c\,x}{81\,a} + \frac{20\,b^2\,c\,x^7}{81\,a^3} + \frac{14\,b^2\,d\,x^8}{81\,a^3} + \frac{52\,b\,c\,x^4}{81\,a^2} + \frac{77\,b\,d\,x^5}{162\,a^2}}{a^3 + 3\,a^2\,b\,x^3 + 3\,a\,b^2\,x^6 + b^3\,x^9}$$

input
```
int((c + d*x)/(a + b*x^3)^4,x)
```

output
```
symsum(log((b*(560*c*d + 196*d^2*x + 59049*root(14348907*a^11*b^2*z^3 + 40
8240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)^2*a^7*b + 9720*root(143
48907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)*
a^3*b*c*x))/(6561*a^6))*root(14348907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z -
64000*b*c^3 + 2744*a*d^3, z, k), k, 1, 3) + ((67*d*x^2)/(162*a) + (41*c*x)
/(81*a) + (20*b^2*c*x^7)/(81*a^3) + (14*b^2*d*x^8)/(81*a^3) + (52*b*c*x^4)
/(81*a^2) + (77*b*d*x^5)/(162*a^2))/(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2
*x^6)
```

3.11 $\int \frac{a+bx}{d+ex^3}\,dx$

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3.11.1 Optimal result

Integrand size = 15, antiderivative size = 161

$$\int \frac{a+bx}{d+ex^3}\,dx = -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}-\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}$$

output
```
-1/3*(b*d^(1/3)-a*e^(1/3))*ln(d^(1/3)+e^(1/3)*x)/d^(2/3)/e^(2/3)-1/6*(a-b*
d^(1/3)/e^(1/3))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(1/3)
-1/3*(b*d^(1/3)+a*e^(1/3))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2
))/d^(2/3)/e^(2/3)*3^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int \frac{a+bx}{d+ex^3}\,dx$$
$$=\frac{-2\sqrt{3}\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)-\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\left(2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e\right.\right.}{6d^{2/3}e^{2/3}}$$

input
```
Integrate[(a + b*x)/(d + e*x^3),x]
```

output
```
(-2*Sqrt[3]*(b*d^(1/3) + a*e^(1/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqr
t[3]] - (b*d^(1/3) - a*e^(1/3))*(2*Log[d^(1/3) + e^(1/3)*x] - Log[d^(2/3)
- d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))
```

3.11.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{d+ex^3}\,dx$$

$$\Big\downarrow \; 2399$$

$$\frac{\int \frac{\sqrt[3]{d}\left(2\sqrt[3]{e}a+b\sqrt[3]{d}\right)+\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{1}{\sqrt[3]{e}x+\sqrt[3]{d}}dx}{3d^{2/3}}$$

$$\Big\downarrow \; 16$$

$$\frac{\int \frac{\sqrt[3]{d}\left(2\sqrt[3]{e}a+b\sqrt[3]{d}\right)+\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{3}{2}\sqrt[3]{d}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{1}{2}\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int -\frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3}{2}\sqrt[3]{d}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\int\frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{1}{2}\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int\frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3}{2}\sqrt[3]{d}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\int\frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{1}{2}\sqrt[3]{e}\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\frac{1}{2}\sqrt[3]{e}\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{3\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\sqrt[3]{e}}}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{1}{2}\sqrt[3]{e}\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{\sqrt{3}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \text{ 1103}$$

3.11. $\int \frac{a+bx}{d+ex^3}\,dx$

$$\frac{-\frac{\sqrt{3}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}-\frac{1}{2}\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

input
```
Int[(a + b*x)/(d + e*x^3),x]
```

output
```
((a - (b*d^(1/3))/e^(1/3))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(1/3)) +
 (-((Sqrt[3]*(b*d^(1/3) + a*e^(1/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sq
rt[3]])/e^(1/3)) - ((a - (b*d^(1/3))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3
)*x + e^(2/3)*x^2])/2)/(3*d^(2/3)*e^(1/3))
```

3.11.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.11.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(e\_Z^3+d\right)}\frac{\left(\_Rb+a\right)\ln\left(x-\_R\right)}{\_R^2}}{3e}$
default	$a\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)+b\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{d}{e}\right.\right.}{6e\left(\right.}\right.$

input
```
int((b*x+a)/(e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
1/3/e*sum((_R*b+a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

3.11.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 1961, normalized size of antiderivative = 12.18

$$\int \frac{a+bx}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)/(e*x^3+d),x, algorithm="fricas")
```

output
```
-1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^
3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d +
a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/4*((1/2)^(1/3)
*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(
1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2)
- (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/2*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*
(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d
- a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d + (b^3*d + a^3*e)*x) + 1/1
2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e
)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3
*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^
2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(
d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*e)))*log(
-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^
3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d +
a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2
*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)...
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int \frac{a+bx}{d+ex^3}\,dx$$
$$=\operatorname{RootSum}\left(27t^3d^2e^2+9tabde-a^3e+b^3d,\left(t\mapsto t\log\left(x+\frac{9t^2bd^2e+3ta^2de+2ab^2d}{a^3e+b^3d}\right)\right)\right)$$

input
```
integrate((b*x+a)/(e*x**3+d),x)
```

output
```
RootSum(27*_t**3*d**2*e**2 + 9*_t*a*b*d*e - a**3*e + b**3*d, Lambda(_t, _t
*log(x + (9*_t**2*b*d**2*e + 3*_t*a**2*d*e + 2*a*b**2*d)/(a**3*e + b**3*d)
)))
```

3.11.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)/(e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{a+bx}{d+ex^3}\,dx = -\frac{\sqrt{3}\left(ae-\left(-de^2\right)^{\frac{1}{3}}b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(ae+\left(-de^2\right)^{\frac{1}{3}}b\right)\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(b\left(-\frac{d}{e}\right)^{\frac{1}{3}}+a\right)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,d}$$

input
```
integrate((b*x+a)/(e*x^3+d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(a*e - (-d*e^2)^(1/3)*b)*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/
3))/(-d/e)^(1/3))/(-d*e^2)^(2/3) - 1/6*(a*e + (-d*e^2)^(1/3)*b)*log(x^2 +
x*(-d/e)^(1/3) + (-d/e)^(2/3))/(-d*e^2)^(2/3) - 1/3*(b*(-d/e)^(1/3) + a)*(
-d/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/d
```

3.11.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \frac{a+bx}{d+ex^3}\,dx = \sum_{k=1}^{3} \ln\left(e\left(a\,b+b^2\,x+\mathrm{root}\left(27\,d^2\,e^2\,z^3+9\,a\,b\,d\,e\,z+b^3\,d-a^3\,e,z,k\right)^2 d\,e\,9 + \mathrm{root}\left(27\,d^2\,e^2\,z^3+9\,a\,b\,d\,e\,z+b^3\,d-a^3\,e,z,k\right)\,a\,e\,x\,3\right)\right)\mathrm{root}\left(27\,d^2\,e^2\,z^3 + 9\,a\,b\,d\,e\,z+b^3\,d-a^3\,e,z,k\right)$$

input
```
int((a + b*x)/(d + e*x^3),x)
```

output
```
symsum(log(e*(a*b + b^2*x + 9*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d -
a^3*e, z, k)^2*d*e + 3*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a^3*e,
z, k)*a*e*x))*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a^3*e, z, k), k,
 1, 3)
```

3.12 $\int \frac{a+bx}{d-ex^3}\,dx$

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3.12.1 Optimal result

Integrand size = 16, antiderivative size = 161

$$\int \frac{a+bx}{d-ex^3}\,dx=-\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}}-\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}}+\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}$$

output
```
-1/3*(b*d^(1/3)+a*e^(1/3))*ln(d^(1/3)-e^(1/3)*x)/d^(2/3)/e^(2/3)+1/6*(b*d^
(1/3)+a*e^(1/3))*ln(d^(2/3)+d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(2/3)
-1/3*(b*d^(1/3)-a*e^(1/3))*arctan(1/3*(d^(1/3)+2*e^(1/3)*x)/d^(1/3)*3^(1/2
))/d^(2/3)/e^(2/3)*3^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int \frac{a+bx}{d-ex^3}\,dx$$

$$=\frac{-2\sqrt{3}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)-\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\left(2\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)-\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e\right.\right.}{6d^{2/3}e^{2/3}}$$

input
```
Integrate[(a + b*x)/(d - e*x^3),x]
```

output
```
(-2*Sqrt[3]*(b*d^(1/3) - a*e^(1/3))*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqr
t[3]] - (b*d^(1/3) + a*e^(1/3))*(2*Log[d^(1/3) - e^(1/3)*x] - Log[d^(2/3)
+ d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))
```

3.12.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2400, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{d-ex^3}dx$$

$$\Big\downarrow \text{2400}$$

$$\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e}x}dx}{3d^{2/3}}-\frac{\int \frac{\sqrt[3]{d}\left(b\sqrt[3]{d}-2a\sqrt[3]{e}\right)-\left(\sqrt[3]{e}a+b\sqrt[3]{d}\right)\sqrt[3]{e}x}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\int \frac{\sqrt[3]{d}\left(b\sqrt[3]{d}-2a\sqrt[3]{e}\right)-\left(\sqrt[3]{e}a+b\sqrt[3]{d}\right)\sqrt[3]{e}x}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}-\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\frac{3}{2}\sqrt[3]{d}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\int \frac{1}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{1}{2}\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{\sqrt[3]{e}\left(2\sqrt[3]{e}x+\sqrt[3]{d}\right)}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}-\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3}{2}\sqrt[3]{d}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\int\frac{1}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{1}{2}\sqrt[3]{e}\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int\frac{2\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}-\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \; 1082$$

$$-\frac{-\frac{1}{2}\sqrt[3]{e}\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int\frac{2\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{3\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\int\frac{1}{-\left(\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)}{\sqrt[3]{e}}}{3d^{2/3}\sqrt[3]{e}}-\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{\sqrt{3}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\arctan\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{\sqrt[3]{e}}-\frac{1}{2}\sqrt[3]{e}\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int\frac{2\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}-\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{\frac{\sqrt{3}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\arctan\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{\sqrt[3]{e}}-\frac{1}{2}\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{3d^{2/3}\sqrt[3]{e}}-\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

input
```
Int[(a + b*x)/(d - e*x^3),x]
```

3.12. $\int \frac{a+bx}{d-ex^3}\,dx$

output
```
-1/3*((a + (b*d^(1/3))/e^(1/3))*Log[d^(1/3) - e^(1/3)*x])/(d^(2/3)*e^(1/3)
) - ((Sqrt[3]*(b*d^(1/3) - a*e^(1/3))*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/S
qrt[3]])/e^(1/3) - ((a + (b*d^(1/3))/e^(1/3))*Log[d^(2/3) + d^(1/3)*e^(1/3
)*x + e^(2/3)*x^2])/2)/(3*d^(2/3)*e^(1/3))
```

3.12.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2400
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Simp[r*((B*r + A*s)/(3*a*
s))   Int[1/(r - s*x), x], x] - Simp[r/(3*a*s)   Int[(r*(B*r - 2*A*s) - s*(
B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
 NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

3.12.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.68 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.21

method	result
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(e\_Z^3-d\right)}\frac{(\_Rb+a)\ln(x-\_R)}{\_R^2}}{3e}$
default	$a\left(-\frac{\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^2+\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)+b\left(-\frac{\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2+\left(\right.\right.}{6\ldots}\right.$

input
```
int((b*x+a)/(-e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
-1/3/e*sum((_R*b+a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e-d))
```

3.12.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 1905, normalized size of antiderivative = 11.83

$$\int \frac{a+bx}{d-ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)/(-e*x^3+d),x, algorithm="fricas")
```

output
```
-1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d -
a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)
/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/36*(9*(I*sqrt(3
) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^
(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*
(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/6*(9*(I*sqrt(3) + 1)*(-1/
54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b
*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^
3*e)/(d^2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d - (b^3*d - a^3*e)*x) + 1/36*(9
*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(
d^2*e^2))^(1/3) + 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^
3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) +
 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2)
)^(1/3)))^2*d*e - 144*a*b)/(d*e)) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*
d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(-1/36*(
9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/
(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e
^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/6*(9*(I*sqrt(3
) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^
(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/...
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.48

$$\int \frac{a+bx}{d-ex^3}\,dx = -\operatorname{RootSum}\left(27t^3d^2e^2 - 9tabde - a^3e - b^3d, \left(t \mapsto t\log\left(x + \frac{9t^2bd^2e - 3ta^2de - 2ab^2d}{a^3e - b^3d}\right)\right)\right)$$

input
```
integrate((b*x+a)/(-e*x**3+d),x)
```

output
```
-RootSum(27*_t**3*d**2*e**2 - 9*_t*a*b*d*e - a**3*e - b**3*d, Lambda(_t, _
t*log(x + (9*_t**2*b*d**2*e - 3*_t*a**2*d*e - 2*a*b**2*d)/(a**3*e - b**3*d
))))
```

3.12.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx}{d-ex^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)/(-e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \frac{a+bx}{d-ex^3}\,dx = \frac{\left(ae+(de^2)^{\frac{1}{3}}b\right)\log\left(x^2+x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,(de^2)^{\frac{2}{3}}} - \frac{\left(b\left(\frac{d}{e}\right)^{\frac{1}{3}}+a\right)\left(\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,d} + \frac{\sqrt{3}\left((de^2)^{\frac{1}{3}}ae-(de^2)^{\frac{2}{3}}b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,de^2}$$

input
```
integrate((b*x+a)/(-e*x^3+d),x, algorithm="giac")
```

output
```
1/6*(a*e + (d*e^2)^(1/3)*b)*log(x^2 + x*(d/e)^(1/3) + (d/e)^(2/3))/(d*e^2)
^(2/3) - 1/3*(b*(d/e)^(1/3) + a)*(d/e)^(1/3)*log(abs(x - (d/e)^(1/3)))/d +
 1/3*sqrt(3)*((d*e^2)^(1/3)*a*e - (d*e^2)^(2/3)*b)*arctan(1/3*sqrt(3)*(2*x
 + (d/e)^(1/3))/(d/e)^(1/3))/(d*e^2)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int \frac{a+bx}{d-ex^3}\,dx = \sum_{k=1}^{3} \ln\left(e\left(a\,b+b^2\,x-\mathrm{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)^2\,d\,e\,9 - \mathrm{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)\,a\,e\,x\,3\right)\right)\,\mathrm{root}\left(27\,d^2\,e^2\,z^3 - 9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)$$

input
```
int((a + b*x)/(d - e*x^3),x)
```

output
```
symsum(log(e*(a*b + b^2*x - 9*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d +
a^3*e, z, k)^2*d*e - 3*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a^3*e,
z, k)*a*e*x))*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a^3*e, z, k), k,
 1, 3)
```

3.13 $\int \frac{1+x}{1+x^3} dx$

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3.13.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1+x}{1+x^3} dx = -\frac{2\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-2/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{1+x^3} dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 + x)/(1 + x^3),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

3.13.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2019, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{x^3+1} \, dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 1083}$$

$$-2 \int \frac{1}{-(2x-1)^2-3}d(2x-1)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2 \arctan \left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(1 + x)/(1 + x^3),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

3.13.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.13.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$
meijerg	$\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}+\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}$ -

input
```
int((1+x)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1+x}{1+x^3}\,dx=\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)$$

input
```
integrate((1+x)/(x^3+1),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1+x}{1+x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((1+x)/(x**3+1),x)
```

output
```
2*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1+x}{1+x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

input
```
integrate((1+x)/(x^3+1),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1+x}{1+x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

input
```
integrate((1+x)/(x^3+1),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

3.13.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1+x}{1+x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,x-1)}{3}\right)}{3}$$

input
```
int((x + 1)/(x^3 + 1),x)
```

output
```
(2*3^(1/2)*atan((3^(1/2)*(2*x - 1))/3))/3
```

3.14 $\int \frac{1-x}{1-x^3} \, dx$

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3.14.8 Giac [A] (verification not implemented) 326
3.14.9 Mupad [B] (verification not implemented) 327

3.14.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1-x}{1-x^3} \, dx = \frac{2 \arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{1-x^3} \, dx = \frac{2 \arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 - x)/(1 - x^3),x]
```

output
```
(2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

3.14.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2019, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{1-x^3} \, dx$$

$$\Big\downarrow \ 2019$$

$$\int \frac{1}{x^2+x+1}dx$$

$$\Big\downarrow \ 1083$$

$$-2\int \frac{1}{-(2x+1)^2-3}d(2x+1)$$

$$\Big\downarrow \ 217$$

$$\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(1 - x)/(1 - x^3),x]
```

output
```
(2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

3.14.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.14.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result
default	$\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\text{arct}\right.}{3(x^3)^{\frac{2}{3}}}$

input
```
int((1-x)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate((1-x)/(-x^3+1),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((1-x)/(-x**3+1),x)
```

output
```
2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate((1-x)/(-x^3+1),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate((1-x)/(-x^3+1),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,x+1)}{3}\right)}{3}$$

input
```
int((x - 1)/(x^3 - 1),x)
```

output
```
(2*3^(1/2)*atan((3^(1/2)*(2*x + 1))/3))/3
```

3.15 $\int \frac{1+x}{1-x^3} \, dx$

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3.15.9 Mupad [B] (verification not implemented) 332

3.15.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1+x}{1-x^3} \, dx = -\frac{2}{3} \log(1-x) + \frac{1}{3} \log \left(1+x+x^2\right)$$

output
```
-2/3*ln(1-x)+1/3*ln(x^2+x+1)
```

3.15.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{1-x^3} \, dx = -\frac{2}{3} \log(1-x) + \frac{1}{3} \log \left(1+x+x^2\right)$$

input
```
Integrate[(1 + x)/(1 - x^3),x]
```

output
```
(-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

3.15.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2400, 16, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{1-x^3} \, dx$$

$$\Big\downarrow \ 2400$$

$$\frac{2}{3} \int \frac{1}{1-x} dx - \frac{1}{3} \int -\frac{2x+1}{x^2+x+1} dx$$

$$\Big\downarrow \ 16$$

$$-\frac{1}{3} \int -\frac{2x+1}{x^2+x+1} dx - \frac{2}{3} \log(1-x)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{3} \int \frac{2x+1}{x^2+x+1} dx - \frac{2}{3} \log(1-x)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{3} \log\left(x^2+x+1\right) - \frac{2}{3} \log(1-x)$$

input
```
Int[(1 + x)/(1 - x^3),x]
```

output
```
(-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

3.15.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2400
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Simp[r*((B*r + A*s)/(3*a*
s))   Int[1/(r - s*x), x], x] - Simp[r/(3*a*s)   Int[(r*(B*r - 2*A*s) - s*(
B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
 NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

3.15.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result
default	$-\frac{2\ln(-1+x)}{3}+\frac{\ln(x^2+x+1)}{3}$
norman	$-\frac{2\ln(-1+x)}{3}+\frac{\ln(x^2+x+1)}{3}$
risch	$-\frac{2\ln(-1+x)}{3}+\frac{\ln(x^2+x+1)}{3}$
parallelrisch	$-\frac{2\ln(-1+x)}{3}+\frac{\ln(x^2+x+1)}{3}$
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{}\right.}{3(x^3)^{\frac{2}{3}}}$

input
```
int((1+x)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-2/3*ln(-1+x)+1/3*ln(x^2+x+1)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{1-x^3}\,dx = \frac{1}{3}\,\log\left(x^2+x+1\right) - \frac{2}{3}\,\log\left(x-1\right)$$

input
```
integrate((1+x)/(-x^3+1),x, algorithm="fricas")
```

output
```
1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1+x}{1-x^3}\,dx = -\frac{2\log\left(x-1\right)}{3} + \frac{\log\left(x^2+x+1\right)}{3}$$

input
```
integrate((1+x)/(-x**3+1),x)
```

output
```
-2*log(x - 1)/3 + log(x**2 + x + 1)/3
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{1-x^3}\,dx = \frac{1}{3}\,\log\left(x^2+x+1\right) - \frac{2}{3}\,\log\left(x-1\right)$$

input
```
integrate((1+x)/(-x^3+1),x, algorithm="maxima")
```

output
```
1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1+x}{1-x^3} \, dx = \frac{1}{3} \log \left(x^2+x+1\right) - \frac{2}{3} \log \left(|x-1|\right)$$

input
```
integrate((1+x)/(-x^3+1),x, algorithm="giac")
```

output
```
1/3*log(x^2 + x + 1) - 2/3*log(abs(x - 1))
```

3.15.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{1-x^3} \, dx = \frac{\ln \left(x^2+x+1\right)}{3} - \frac{2 \ln \left(x-1\right)}{3}$$

input
```
int(-(x + 1)/(x^3 - 1),x)
```

output
```
log(x + x^2 + 1)/3 - (2*log(x - 1))/3
```

3.16 $\int \frac{1-x}{1+x^3} \, dx$

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3.16.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1-x}{1+x^3} \, dx = \frac{2}{3}\log(1+x) - \frac{1}{3}\log\left(1-x+x^2\right)$$

output
```
2/3*ln(1+x)-1/3*ln(x^2-x+1)
```

3.16.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{1+x^3} \, dx = \frac{2}{3}\log(1+x) - \frac{1}{3}\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 - x)/(1 + x^3),x]
```

output
```
(2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

3.16.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2399, 16, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{x^3+1} \, dx$$

$$\Big\downarrow \ 2399$$

$$\frac{1}{3}\int \frac{1-2x}{x^2-x+1}dx+\frac{2}{3}\int \frac{1}{x+1}dx$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\int \frac{1-2x}{x^2-x+1}dx+\frac{2}{3}\log(x+1)$$

$$\Big\downarrow \ 1103$$

$$\frac{2}{3}\log(x+1)-\frac{1}{3}\log\left(x^2-x+1\right)$$

input
```
Int[(1 - x)/(1 + x^3),x]
```

output
```
(2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

3.16.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.16.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result
default	$\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{3}$
norman	$\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{3}$
risch	$\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{3}$
parallelrisch	$\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{3}$
meijerg	$\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}+\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}-\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3\right.}{6(x^3)^{\frac{2}{3}}}$

input
```
int((1-x)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
2/3*ln(1+x)-1/3*ln(x^2-x+1)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-x}{1+x^3}\,dx=-\frac{1}{3}\,\log\left(x^2-x+1\right)+\frac{2}{3}\,\log\left(x+1\right)$$

input
```
integrate((1-x)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1-x}{1+x^3}\,dx = \frac{2\log{(x+1)}}{3} - \frac{\log{(x^2-x+1)}}{3}$$

input
```
integrate((1-x)/(x**3+1),x)
```

output
```
2*log(x + 1)/3 - log(x**2 - x + 1)/3
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-x}{1+x^3}\,dx = -\frac{1}{3}\log\left(x^2-x+1\right) + \frac{2}{3}\log\left(x+1\right)$$

input
```
integrate((1-x)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1-x}{1+x^3}\,dx = -\frac{1}{3}\log\left(x^2-x+1\right) + \frac{2}{3}\log\left(|x+1|\right)$$

input
```
integrate((1-x)/(x^3+1),x, algorithm="giac")
```

output
```
-1/3*log(x^2 - x + 1) + 2/3*log(abs(x + 1))
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-x}{1+x^3}\,dx = \frac{2\ln(x+1)}{3} - \frac{\ln(x^2-x+1)}{3}$$

input
```
int(-(x - 1)/(x^3 + 1),x)
```

output
```
(2*log(x + 1))/3 - log(x^2 - x + 1)/3
```

3.17 $\int \frac{3-x}{1-x^3} \, dx$

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3.17.1 Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{3-x}{1-x^3} \, dx = \frac{4 \arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log(1-x) + \frac{1}{3}\log\left(1+x+x^2\right)$$

output
```
-2/3*ln(1-x)+1/3*ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.17.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{3-x}{1-x^3} \, dx = \frac{4 \arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log(1-x) + \frac{1}{3}\log\left(1+x+x^2\right)$$

input
```
Integrate[(3 - x)/(1 - x^3),x]
```

output
```
(4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2
]/3
```

3.17.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2400, 16, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-x}{1-x^3} \, dx$$

$$\Big\downarrow \text{2400}$$

$$\frac{2}{3}\int \frac{1}{1-x}dx-\frac{1}{3}\int -\frac{2x+7}{x^2+x+1}dx$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{3}\int -\frac{2x+7}{x^2+x+1}dx-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{2x+7}{x^2+x+1}dx-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(6\int \frac{1}{x^2+x+1}dx+\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\int \frac{2x+1}{x^2+x+1}dx-12\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\int \frac{2x+1}{x^2+x+1}dx+4\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(4\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\log\left(x^2+x+1\right)\right)-\frac{2}{3}\log(1-x)$$

input
```
Int[(3 - x)/(1 - x^3),x]
```

output
```
(-2*Log[1 - x])/3 + (4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 + x + x^2
])/3
```

3.17.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2400
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Simp[r*((B*r + A*s)/(3*a*
s))   Int[1/(r - s*x), x], x] - Simp[r/(3*a*s)   Int[(r*(B*r - 2*A*s) - s*(
B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
 NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

3.17.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result
default	$-\frac{2\ln(-1+x)}{3}+\frac{\ln(x^2+x+1)}{3}+\frac{4\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$-\frac{2\ln(-1+x)}{3}+\frac{\ln(16x^2+16x+16)}{3}+\frac{4\sqrt{3}\arctan\left(\frac{(2+4x)\sqrt{3}}{6}\right)}{3}$
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{(x^3)^{\frac{1}{3}}}+\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\ \mathrm{arct}\right.}{3(x^3)^{\frac{2}{3}}}$

input
```
int((3-x)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-2/3*ln(-1+x)+1/3*ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{3-x}{1-x^3}\,dx=\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

input
```
integrate((3-x)/(-x^3+1),x, algorithm="fricas")
```

output
```
4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log
(x - 1)
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{3-x}{1-x^3}\,dx=-\frac{2\log(x-1)}{3}+\frac{\log(x^2+x+1)}{3}+\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((3-x)/(-x**3+1),x)
```

output
```
-2*log(x - 1)/3 + log(x**2 + x + 1)/3 + 4*sqrt(3)*atan(2*sqrt(3)*x/3 + sqr
t(3)/3)/3
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{3-x}{1-x^3}\,dx = \frac{4}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{3}\,\log\left(x^2+x+1\right) - \frac{2}{3}\,\log\left(x-1\right)$$

input
```
integrate((3-x)/(-x^3+1),x, algorithm="maxima")
```

output
```
4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log
(x - 1)
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{3-x}{1-x^3}\,dx = \frac{4}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{3}\,\log\left(x^2+x+1\right) - \frac{2}{3}\,\log\left(|x-1|\right)$$

input
```
integrate((3-x)/(-x^3+1),x, algorithm="giac")
```

output
```
4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log
(abs(x - 1))
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{3-x}{1-x^3}\,dx = -\frac{2\ln(x-1)}{3} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)$$

input
```
int((x - 3)/(x^3 - 1),x)
```

output
```
log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*2i)/3 + 1/3) - log(x - (3^(1/2)*1i
)/2 + 1/2)*((3^(1/2)*2i)/3 - 1/3) - (2*log(x - 1))/3
```

3.18 $\int \frac{c+dx}{c^3+d^3x^3} dx$

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3.18.1 Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{c+dx}{c^3+d^3x^3} dx = -\frac{2\arctan\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

output
```
-2/3*arctan(1/3*(-2*d*x+c)/c*3^(1/2))/c/d*3^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{c+dx}{c^3+d^3x^3} dx = \frac{2\arctan\left(\frac{-c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

input
```
Integrate[(c + d*x)/(c^3 + d^3*x^3),x]
```

output
```
(2*ArcTan[(-c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

3.18.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2019, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{c^3+d^3x^3} \, dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{c^2-cdx+d^2x^2}dx$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2\int \frac{1}{-\left(1-\frac{2dx}{c}\right)^2-3}d\left(1-\frac{2dx}{c}\right)}{cd}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{2\arctan\left(\frac{1-\frac{2dx}{c}}{\sqrt{3}}\right)}{\sqrt{3}cd}$$

input
```
Int[(c + d*x)/(c^3 + d^3*x^3),x]
```

output
```
(-2*ArcTan[(1 - (2*d*x)/c)/Sqrt[3]])/(Sqrt[3]*c*d)
```

3.18.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.18.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{2d\sqrt{3}\,x}{3c}-\frac{\sqrt{3}}{3}\right)}{3dc}$	29
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x\,d^{2}-cd\right)\sqrt{3}}{3cd}\right)}{3cd}$	35

input
```
int((d*x+c)/(d^3*x^3+c^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)/d/c*arctan(2/3*d*3^(1/2)/c*x-1/3*3^(1/2))
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx=\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx-c)}{3\,c}\right)}{3\,cd}$$

input
```
integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

3.18.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \frac{-\frac{\sqrt{3}i\log\left(x+\frac{-c-\sqrt{3}ic}{2d}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

input
```
integrate((d*x+c)/(d**3*x**3+c**3),x)
```

output
```
(-sqrt(3)*I*log(x + (-c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (-c +
sqrt(3)*I*c)/(2*d))/3)/(c*d)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,d^2x-cd)}{3\,cd}\right)}{3\,cd}$$

input
```
integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x - c*d)/(c*d))/(c*d)
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx-c)}{3\,c}\right)}{3\,cd}$$

input
```
integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = -\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,d\,x}{3\,c}\right)}{3\,c\,d}$$

input
```
int((c + d*x)/(c^3 + d^3*x^3),x)
```

output
```
-(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*d*x)/(3*c)))/(3*c*d)
```

3.19 $\int \frac{c-dx}{c^3-d^3x^3} \, dx$

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3.19.7 Maxima [A] (verification not implemented) 352
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3.19.9 Mupad [B] (verification not implemented) 353

3.19.1 Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{c-dx}{c^3-d^3x^3} \, dx = \frac{2\arctan\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

output
```
2/3*arctan(1/3*(2*d*x+c)/c*3^(1/2))/c/d*3^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{c-dx}{c^3-d^3x^3} \, dx = \frac{2\arctan\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

input
```
Integrate[(c - d*x)/(c^3 - d^3*x^3),x]
```

output
```
(2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

3.19.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2019, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-dx}{c^3-d^3x^3}dx$$

$$\Big\downarrow \; 2019$$

$$\int \frac{1}{c^2+cdx+d^2x^2}dx$$

$$\Big\downarrow \; 1082$$

$$-\frac{2\int \frac{1}{-\left(\frac{2dx}{c}+1\right)^2-3}d\left(\frac{2dx}{c}+1\right)}{cd}$$

$$\Big\downarrow \; 217$$

$$\frac{2\arctan\left(\frac{\frac{2dx}{c}+1}{\sqrt{3}}\right)}{\sqrt{3}cd}$$

input
```
Int[(c - d*x)/(c^3 - d^3*x^3),x]
```

output
```
(2*ArcTan[(1 + (2*d*x)/c)/Sqrt[3]])/(Sqrt[3]*c*d)
```

3.19.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.19.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{2d\sqrt{3}\,x}{3c}+\frac{\sqrt{3}}{3}\right)}{3dc}$	29
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x\,d^{2}+cd\right)\sqrt{3}}{3cd}\right)}{3cd}$	34

input
```
int((-d*x+c)/(-d^3*x^3+c^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)/d/c*arctan(2/3*d*3^(1/2)/c*x+1/3*3^(1/2))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx+c)}{3\,c}\right)}{3\,cd}$$

input
```
integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

3.19.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx = \frac{-\frac{\sqrt{3}i\log\left(x+\frac{c-\sqrt{3}ic}{2d}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

input
```
integrate((-d*x+c)/(-d**3*x**3+c**3),x)
```

output
```
(-sqrt(3)*I*log(x + (c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (c + sq
rt(3)*I*c)/(2*d))/3)/(c*d)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,d^2x+cd)}{3\,cd}\right)}{3\,cd}$$

input
```
integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x + c*d)/(c*d))/(c*d)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx+c)}{3\,c}\right)}{3\,cd}$$

input
```
integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx=\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,d\,x}{3\,c}\right)}{3\,c\,d}$$

input
```
int((c - d*x)/(c^3 - d^3*x^3),x)
```

output
```
(2*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*d*x)/(3*c)))/(3*c*d)
```

3.20 $\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx$

3.20.1 Optimal result . . . 354
3.20.2 Mathematica [A] (verified) . . . 354
3.20.3 Rubi [A] (verified) . . . 355
3.20.4 Maple [B] (verified) . . . 356
3.20.5 Fricas [A] (verification not implemented) . . . 357
3.20.6 Sympy [C] (verification not implemented) . . . 357
3.20.7 Maxima [B] (verification not implemented) . . . 358
3.20.8 Giac [A] (verification not implemented) . . . 358
3.20.9 Mupad [B] (verification not implemented) . . . 359

3.20.1 Optimal result

Integrand size = 31, antiderivative size = 39

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx=-\frac{2B\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

output
```
-2/3*B*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)*3^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}\,dx=-\frac{2B\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

input
```
Integrate[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
```

output
```
(-2*B*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/3))
```

3.20.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2019, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{\frac{a^{2/3}}{\sqrt[3]{b}B}-\frac{\sqrt[3]{a}x}{B}+\frac{\sqrt[3]{b}x^2}{B}}dx$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2B\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{2B\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

input
```
Int[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
```

output
```
(-2*B*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/3))
```

3.20.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(28) = 56$.

Time = 1.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 5.00

method	result
default	$B\,b^{\frac{1}{3}}\left(a^{\frac{1}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+b^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln}{}\right.\right.$

input
```
int((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
B*b^(1/3)*(a^(1/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1)))+b^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1
/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/
3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a+bx^3}\,dx = \left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3 - 3\,a^{\frac{2}{3}}b^{\frac{1}{3}}x + 3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{2}{3}}b^{\frac{2}{3}}x^2 + ab^{\frac{1}{3}}x - a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}} - a}{bx^3 + a}\right)\right.$$

input
```
integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sqrt(
1/3)*(2*a^(2/3)*b^(2/3)*x^2 + a*b^(1/3)*x - a^(4/3))*sqrt(-1/a^(2/3)) - a)
/(b*x^3 + a)), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*b^(1/3)*x - a^(1/3))/a^(1
/3))/a^(1/3)]
```

3.20.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.26

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a+bx^3}\,dx = \frac{B\left(-\frac{\sqrt{3}i\log\left(x+\frac{-B\sqrt[3]{a}-\sqrt{3}iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3} + \frac{\sqrt{3}i\log\left(x+\frac{-B\sqrt[3]{a}+\sqrt{3}iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3}\right)}{\sqrt[3]{a}}$$

input
```
integrate((a**(1/3)*b**(1/3)*B+b**(2/3)*B*x)/(b*x**3+a),x)
```

output
```
B*(-sqrt(3)*I*log(x + (-B*a**(1/3) - sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3)))
/3 + sqrt(3)*I*log(x + (-B*a**(1/3) + sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3))
)/3)/a**(1/3)
```

3.20.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.18

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a + bx^3}\,dx = \frac{\sqrt{3}\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(B*b^(2/3)*(a/b)^(1/3) + B*a^(1/3)*b^(1/3))*arctan(1/3*sqrt(3)
*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(B*b^(2/3)*(a/b)^(
1/3) - B*a^(1/3)*b^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^
(2/3)) - 1/3*(B*b^(2/3)*(a/b)^(1/3) - B*a^(1/3)*b^(1/3))*log(x + (a/b)^(1/
3))/(b*(a/b)^(2/3))
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a + bx^3}\,dx = \frac{2\sqrt{3}Bb^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,b^{\frac{2}{3}}x - a^{\frac{1}{3}}b^{\frac{1}{3}}\right)}{3\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}\right)}{3\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}$$

input
```
integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*B*b^(1/3)*arctan(1/3*sqrt(3)*(2*b^(2/3)*x - a^(1/3)*b^(1/3))/s
qrt(a^(2/3)*b^(2/3)))/sqrt(a^(2/3)*b^(2/3))
```

3.20.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a + bx^3}\,dx = \frac{2\sqrt{3}\,B\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{3}\sqrt{b}}{3\sqrt{-b}} - \frac{2\sqrt{3}\,b^{5/6}\,x}{3\,a^{1/3}\sqrt{-b}}\right)}{3\,a^{1/3}\sqrt{-b}}$$

input
```
int((B*a^(1/3)*b^(1/3) + B*b^(2/3)*x)/(a + b*x^3),x)
```

output
```
(2*3^(1/2)*B*b^(1/2)*atanh((3^(1/2)*b^(1/2))/(3*(-b)^(1/2)) - (2*3^(1/2)*b
^(5/6)*x)/(3*a^(1/3)*(-b)^(1/2))))/(3*a^(1/3)*(-b)^(1/2))
```

3.21 $\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx$

3.21.1 Optimal result . . . 360
3.21.2 Mathematica [B] (verified) . . . 360
3.21.3 Rubi [A] (verified) . . . 361
3.21.4 Maple [B] (verified) . . . 362
3.21.5 Fricas [A] (verification not implemented) . . . 363
3.21.6 Sympy [C] (verification not implemented) . . . 363
3.21.7 Maxima [B] (verification not implemented) . . . 364
3.21.8 Giac [A] (verification not implemented) . . . 364
3.21.9 Mupad [B] (verification not implemented) . . . 365

3.21.1 Optimal result

Integrand size = 36, antiderivative size = 41

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx=\frac{2B\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

output
```
2/3*B*arctan(1/3*(a^(1/3)+2*(-b)^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)*3^(1/2)
```

3.21.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 129 vs. 2(41) = 82.

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.15

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx=\frac{\sqrt[3]{-b}B\left(2\sqrt{3}\left(\sqrt[3]{-b}-\sqrt[3]{b}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(\sqrt[3]{-b}+\sqrt[3]{b}\right)\left(2\log\left(\sqrt[3]{a}\right.\right.\right.}{6\sqrt[3]{a}b^{2/3}}$$

input
```
Integrate[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

output
```
((-b)^(1/3)*B*(2*Sqrt[3]*((-b)^(1/3) - b^(1/3))*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]] + ((-b)^(1/3) + b^(1/3))*(2*Log[a^(1/3) + b^(1/3)*x] - L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])))/(6*a^(1/3)*b^(2/3))
```

3.21.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2019, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{-\frac{a^{2/3}(-b)^{2/3}}{bB}+\frac{\sqrt[3]{a}x}{B}+\frac{\sqrt[3]{-b}x^2}{B}}dx$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2B\int \frac{1}{-\left(\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}$$

$$\Big\downarrow \text{217}$$

$$\frac{2B\arctan\left(\frac{\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

input
```
Int[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

output
```
(2*B*ArcTan[(1 + (2*(-b)^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/3))
```

3.21.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.21.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(30) = 60$.

Time = 1.52 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.93

method	result
default	$-B\,b^{\frac{1}{3}}\left(-a^{\frac{1}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\left(-b\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.\right.$

input
```
int((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x,method=_RETURNVERBOS
E)
```

output
```
-B*b^(1/3)*(-a^(1/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1)))+(-b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))*(-1)^(1/3)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.78

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx = \left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\,a^{\frac{2}{3}}(-b)^{\frac{1}{3}}\,x-3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{2}{3}}(-b)^{\frac{2}{3}}\,x^2-a(-b)^{\frac{1}{3}}\,x\right.}{bx^3+a}\right.\right.$$

input
```
integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="fr
icas")
```

output
```
[sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*a^(2/3)*(-b)^(1/3)*x - 3*sq
rt(1/3)*(2*a^(2/3)*(-b)^(2/3)*x^2 - a*(-b)^(1/3)*x - a^(4/3))*sqrt(-1/a^(2
/3)) - a)/(b*x^3 + a)), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*(-b)^(1/3)*x + a
^(1/3))/a^(1/3))/a^(1/3)]
```

3.21.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx = -\frac{B\left(-\frac{\sqrt{3}i\log\left(-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}-\frac{\sqrt{3}i\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}+\frac{\sqrt{3}i\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}+x\right)}{3}\right)}{\sqrt[3]{a}}$$

input
```
integrate((a**(1/3)*(-b)**(1/3)*B-(-b)**(2/3)*B*x)/(b*x**3+a),x)
```

output
```
-B*(-sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) - sqrt(3)*I*a**(1/3)*(-b)**
(2/3)/(2*b) + x)/3 + sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) + sqrt(3)*I
*a**(1/3)*(-b)**(2/3)/(2*b) + x)/3)/a**(1/3)
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 4.24

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}-Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="ma
xima")
```

output
```
-1/3*sqrt(3)*(B*(-b)^(2/3)*(a/b)^(1/3) - B*a^(1/3)*(-b)^(1/3))*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2/3)) - 1/6*(B*(-b)^(2/
3)*(a/b)^(1/3) + B*a^(1/3)*(-b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/
3))/(b*(a/b)^(2/3)) + 1/3*(B*(-b)^(2/3)*(a/b)^(1/3) + B*a^(1/3)*(-b)^(1/3)
)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx = -\frac{2\sqrt{3}B(-b)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\left(2(-b)^{\frac{2}{3}}x+a^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}}}\right)}{3\sqrt{a^{\frac{2}{3}}(-b)^{\frac{2}{3}}}}$$

input
```
integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="gi
ac")
```

output
```
-2/3*sqrt(3)*B*(-b)^(1/3)*arctan(-1/3*sqrt(3)*(2*(-b)^(2/3)*x + a^(1/3)*(-
b)^(1/3))/sqrt(a^(2/3)*(-b)^(2/3)))/sqrt(a^(2/3)*(-b)^(2/3))
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx=-\frac{2\,\sqrt{3}\,B\,\sqrt{-b}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sqrt{-b}}{3\,\sqrt{b}}-\frac{2\,\sqrt{3}\,\sqrt{b}\,x}{3\,a^{1/3}\,{\left(-b\right)}^{1/6}}\right)}{3\,a^{1/3}\,\sqrt{b}}$$

input
```
int(-(B*(-b)^(2/3)*x - B*a^(1/3)*(-b)^(1/3))/(a + b*x^3),x)
```

output
```
-(2*3^(1/2)*B*(-b)^(1/2)*atanh((3^(1/2)*(-b)^(1/2))/(3*b^(1/2)) - (2*3^(1/
2)*b^(1/2)*x)/(3*a^(1/3)*(-b)^(1/6))))/(3*a^(1/3)*b^(1/2))
```

3.22 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx$

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3.22.1 Optimal result

Integrand size = 35, antiderivative size = 118

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx = -\frac{B\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

output
```
-1/3*B*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(2/3)+1/6*B*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(2/3)-1/3*B*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/
a^(1/3)*3^(1/2))/a^(1/3)/b^(2/3)*3^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx = \frac{B\left(-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6\sqrt[3]{a}b^{2/3}}$$

input
```
Integrate[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

output
```
(B*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
 + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(1/3
)*b^(2/3))
```

3.22.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{Bx+Cx^2}{a+bx^3}-\frac{Cx^2}{a+bx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}-\frac{B\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

input
```
Int[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

output
```
-((B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b
^(2/3))) - (B*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)) + (B*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3))
```

3.22.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.22.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.57 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

method	result	size
risch	$-\frac{C\ln\left(b\,x^{3}+a\right)}{3b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(C\_R+B\right)\ln\left(x-\_R\right)}{\_R}}{3b}$	47
default	$-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{B\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	94

input
```
int(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*C/b*ln(b*x^3+a)+1/3/b*sum(1/_R*(C*_R+B)*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.63

$$\int\left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)dx$$
$$=\left[\frac{3\sqrt{\frac{1}{3}}Bab\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+\left(-ab^2\right)^{\frac{2}{3}}B\log\left(b\right.}{6\,ab^2}\right.$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*B*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sq
rt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1
/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*B*log(b^2*x^2 +
 (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*B*log(b*x - (-a*b
^2)^(1/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*B*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan
(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^
(2/3)*B*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2
/3)*B*log(b*x - (-a*b^2)^(1/3)))/(a*b^2)]
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.22

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx = B\,\mathrm{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

input
```
integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x)/(b*x**3+a),x)
```

output
```
B*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx \\ = -\frac{C\log\left(bx^3+a\right)}{3\,b}+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}+B\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ +\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}-B\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ -\frac{\sqrt{3}\left(2\,Ca-\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}\end{aligned}$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*C*log(b*x^3 + a)/b + 1/6*(2*C*(a/b)^(1/3) + B)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(b*(a/b)^(1/3)) + 1/3*(C*(a/b)^(1/3) - B)*log(x + (a/b)^(
1/3))/(b*(a/b)^(1/3)) - 1/9*sqrt(3)*(2*C*a - (3*B*(a/b)^(2/3) + 2*C*a/b)*b
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx=\frac{\sqrt{3}B\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{1}{3}}}-\frac{B\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{1}{3}}}-\frac{B\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*B*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(-a*b^
2)^(1/3) - 1/6*B*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(1/3) -
 1/3*B*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a
```

3.22.9 Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx=-\frac{B\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(B-\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(B+\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}$$

input
```
int((B*x + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

output
```
(log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*a^(1/3))*(B - 3^(1/2)*B*1i))/(6*
a^(1/3)*b^(2/3)) - (B*log(b^(1/3)*x + a^(1/3)))/(3*a^(1/3)*b^(2/3)) + (log
(3^(1/2)*a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*(B + 3^(1/2)*B*1i))/(6*a^(1
/3)*b^(2/3))
```

3.22. $\int \left(-\frac{Cx^2}{a+bx^3} + \frac{Bx+Cx^2}{a+bx^3}\right) dx$

3.23 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx$

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3.23.1 Optimal result

Integrand size = 33, antiderivative size = 118

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx = -\frac{A\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

output
```
1/3*A*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(1/3)-1/6*A*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)-1/3*A*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a
^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)*3^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx = -\frac{A\left(2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6a^{2/3}\sqrt[3]{b}}$$

input
```
Integrate[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

output
```
-1/6*(A*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(
1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(2
/3)*b^(1/3))
```

3.23.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{A+Cx^2}{a+bx^3}-\frac{Cx^2}{a+bx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{A\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

input
```
Int[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

output
```
-((A*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b
^(1/3))) + (A*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) - (A*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

3.23.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.23.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{C\ln\left(b\,x^{3}+a\right)}{3b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(C\_R^{2}+A\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3b}$	49
default	$\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{A\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	94

input
```
int(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*C/b*ln(b*x^3+a)+1/3/b*sum((C*_R^2+A)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a
))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.58

$$\int\left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)dx$$
$$=\left[\frac{3\sqrt{\frac{1}{3}}Aab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-(a^2b)^{\frac{2}{3}}A\log\left(abx^2\right.}{6\,a^2b}\right.$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*A*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^
(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)
*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*A*log(a*b*x^2 - (
a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*A*log(a*b*x + (a^2*b)^
(2/3)))/(a^2*b), 1/6*(6*sqrt(1/3)*A*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(
1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a
^2*b)^(2/3)*A*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)
^(2/3)*A*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b)]
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.19

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)\,dx = A\,\mathrm{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

input
```
integrate(-C*x**2/(b*x**3+a)+(C*x**2+A)/(b*x**3+a),x)
```

output
```
A*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)\,dx$$
$$= -\frac{C\log\left(bx^3+a\right)}{3\,b} - \frac{\sqrt{3}\left(2\,Ca-\left(3\,A\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} + \frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}-A\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+A\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*C*log(b*x^3 + a)/b - 1/9*sqrt(3)*(2*C*a - (3*A*(a/b)^(1/3) + 2*C*a/b)
*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*(
a/b)^(2/3) - A)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1
/3*(C*(a/b)^(2/3) + A)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx = -\frac{A\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a} + \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\,A\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab} + \frac{(-ab^2)^{\frac{1}{3}}\,A\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*A*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*(-a*b^2)^(1
/3)*A*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/6*(-
a*b^2)^(1/3)*A*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx = \frac{A\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,b^{1/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(A-\sqrt{3}\,A\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(A+\sqrt{3}\,A\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}}$$

input
```
int((A + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

output

```
(A*log(b^(1/3)*x + a^(1/3)))/(3*a^(2/3)*b^(1/3)) - (log(a^(1/3) - 2*b^(1/3
)*x - 3^(1/2)*a^(1/3)*1i)*(A - 3^(1/2)*A*1i))/(6*a^(2/3)*b^(1/3)) - (log(2
*b^(1/3)*x - 3^(1/2)*a^(1/3)*1i - a^(1/3))*(A + 3^(1/2)*A*1i))/(6*a^(2/3)*
b^(1/3))
```

3.23. $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)\,dx$

3.24 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right) dx$

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3.24.1 Optimal result

Integrand size = 36, antiderivative size = 161

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right) dx = -\frac{\left(A\sqrt[3]{b}+\sqrt[3]{a}B\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} - \frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

output
```
1/3*(A*b^(1/3)-a^(1/3)*B)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1/6*(A-a^(
1/3)*B/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)-
1/3*(A*b^(1/3)+a^(1/3)*B)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2)
)/a^(2/3)/b^(2/3)*3^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right) dx$$
$$= \frac{-2\sqrt{3}\left(A\sqrt[3]{b}+\sqrt[3]{a}B\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.\right.}{6a^{2/3}b^{2/3}}$$

input
```
Integrate[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

output
```
(-2*Sqrt[3]*(A*b^(1/3) + a^(1/3)*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] + (A*b^(1/3) - a^(1/3)*B)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

3.24.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{A+Bx+Cx^2}{a+bx^3}-\frac{Cx^2}{a+bx^3}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\left(\sqrt[3]{a}B+A\sqrt[3]{b}\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}-\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}$$

input
```
Int[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

output
```
-(((A*b^(1/3) + a^(1/3)*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((A*b^(1/3) - a^(1/3)*B)*Log[a^(1/3) + b^
(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((A - (a^(1/3)*B)/b^(1/3))*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

3.24.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.24.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.72 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.32

method	result
risch	$-\frac{C\ln\left(b\,x^{3}+a\right)}{3b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\left(C\,\_R^{2}+B\,\_R+A\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3b}$
default	$\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{A\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\right.\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$

input
```
int(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*C/b*ln(b*x^3+a)+1/3/b*sum((C*_R^2+B*_R+A)/_R^2*ln(x-_R),_R=RootOf(_Z^
3*b+a))
```

3.24.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 1961, normalized size of antiderivative = 12.18

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx=\text{Too large to display}$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^
3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a +
A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)
*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(
1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2)
- (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b - 1/2*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*
(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a
- A^3*b)/(a^2*b^2))^(1/3)))*A^2*a*b + 2*A*B^2*a + (B^3*a + A^3*b)*x) + 1/1
2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b
)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3
*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1
/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^
2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(
a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*b)))*log(
-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^
3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a +
A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b + 1/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)...
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx$$
$$=\text{RootSum}\left(27t^3a^2b^2+9tABab-A^3b+B^3a,\left(t\mapsto t\log\left(x+\frac{9t^2Ba^2b+3tA^2ab+2AB^2a}{A^3b+B^3a}\right)\right)\right)$$

input
```
integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x+A)/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**2*b**2 + 9*_t*A*B*a*b - A**3*b + B**3*a, Lambda(_t, _t
*log(x + (9*_t**2*B*a**2*b + 3*_t*A**2*a*b + 2*A*B**2*a)/(A**3*b + B**3*a)
)))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.17

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx$$
$$= -\frac{C\log\left(bx^3+a\right)}{3\,b} - \frac{\sqrt{3}\left(2\,Ca-\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\,A\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} + \frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}-A\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}+A\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*C*log(b*x^3 + a)/b - 1/9*sqrt(3)*(2*C*a - (3*B*(a/b)^(2/3) + 3*A*(a/b
)^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/
(a*b) + 1/6*(2*C*(a/b)^(2/3) + B*(a/b)^(1/3) - A)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(C*(a/b)^(2/3) - B*(a/b)^(1/3) + A)*l
og(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx=-\frac{\sqrt{3}\left(Ab-\left(-ab^2\right)^{\frac{1}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}}-\frac{\left(Ab+\left(-ab^2\right)^{\frac{1}{3}}B\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}}-\frac{\left(Bb\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(A*b - (-a*b^2)^(1/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(A*b + (-a*b^2)^(1/3)*B)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(B*b*(-a/b)^(1/3) + A*
b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx=\sum_{k=1}^{3}\ln\left(b\left(B^2\,x+A\,B+\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,A\,B\,a\,b\,z+B^3\,a-A^3\,b,z,k\right)^2\,a\,b\,9+A\,\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,A\,B\,a\,b\,z+B^3\,a-A^3\,b,z,k\right)\,b\,x\,3\right)\right)\,\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,A\,B\,a\,b\,z+B^3\,a-A^3\,b,z,k\right)$$

input
```
int((A + B*x + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

output
```
symsum(log(b*(B^2*x + A*B + 9*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a -
A^3*b, z, k)^2*a*b + 3*A*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A^3*b
, z, k)*b*x))*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A^3*b, z, k), k,
 1, 3)
```

3.25 $\int \frac{bx+cx^2}{d+ex^3} \, dx$

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3.25.1 Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{bx+cx^2}{d+ex^3} \, dx = -\frac{b \arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}} - \frac{b \log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{b \log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6\sqrt[3]{d}e^{2/3}} + \frac{c \log\left(d+ex^3\right)}{3e}$$

output
```
-1/3*b*ln(d^(1/3)+e^(1/3)*x)/d^(1/3)/e^(2/3)+1/6*b*ln(d^(2/3)-d^(1/3)*e^(1
/3)*x+e^(2/3)*x^2)/d^(1/3)/e^(2/3)+1/3*c*ln(e*x^3+d)/e-1/3*b*arctan(1/3*(d
^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(1/3)/e^(2/3)*3^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int \frac{bx+cx^2}{d+ex^3} \, dx = \frac{-2\sqrt{3}b\sqrt[3]{e} \arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right) - 2b\sqrt[3]{e}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right) + b\sqrt[3]{e}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right) + 2c\sqrt[3]{d}\log}{6\sqrt[3]{d}e}$$

input
```
Integrate[(b*x + c*x^2)/(d + e*x^3),x]
```

output
```
(-2*Sqrt[3]*b*e^(1/3)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*b*e^
(1/3)*Log[d^(1/3) + e^(1/3)*x] + b*e^(1/3)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x
 + e^(2/3)*x^2] + 2*c*d^(1/3)*Log[d + e*x^3])/(6*d^(1/3)*e)
```

3.25.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2027, 2410, 27, 792, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{d+ex^3}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x(b+cx)}{d+ex^3}dx$$

$$\Big\downarrow \text{2410}$$

$$\int \frac{bx}{ex^3+d}dx+c\int \frac{x^2}{ex^3+d}dx$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{x}{ex^3+d}dx+c\int \frac{x^2}{ex^3+d}dx$$

$$\Big\downarrow \text{792}$$

$$b\int \frac{x}{ex^3+d}dx+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \text{821}$$

$$b\left(\frac{\int \frac{\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\int \frac{1}{\sqrt[3]{e}x+\sqrt[3]{d}}dx}{3\sqrt[3]{d}\sqrt[3]{e}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \text{16}$$

$$b\left(\frac{\int \frac{\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \text{1142}$$

$$b\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{2\sqrt[3]{e}}}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \text{25}$$

$$b\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{\int \frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{2\sqrt[3]{e}}}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \text{27}$$

$$b\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \text{1082}$$

$$b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\sqrt[3]{e}}-\frac{1}{2}\int \frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \text{217}$$

3.25. $\int \frac{bx+cx^2}{d+ex^3}\,dx$

$$b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$\big\downarrow$ 1103

$$b\left(\frac{\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{2\sqrt[3]{e}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

input
```
Int[(b*x + c*x^2)/(d + e*x^3),x]
```

output
```
b*(-1/3*Log[d^(1/3) + e^(1/3)*x]/(d^(1/3)*e^(2/3)) + (-((Sqrt[3]*ArcTan[(1
 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/e^(1/3)) + Log[d^(2/3) - d^(1/3)*e^(1/
3)*x + e^(2/3)*x^2]/(2*e^(1/3)))/(3*d^(1/3)*e^(1/3))) + (c*Log[d + e*x^3])
/(3*e)
```

3.25.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.25. $\int \frac{bx+cx^2}{d+ex^3}\,dx$

3.25.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3}e+d\right)}\frac{\left(\_R^{2}c+\_Rb\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3e}$	36
default	$b\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)+\frac{c\ln\left(e\,x^{3}+d\right)}{3e}$	108

input
```
int((c*x^2+b*x)/(e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
1/3/e*sum((_R^2*c+_R*b)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

3.25.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 1344, normalized size of antiderivative = 10.03

$$\int \frac{bx+cx^2}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="fricas")
```

output
```
-1/12*(2*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*
d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*e*log(1/4*(3*(I*sqrt(3) + 1)*(-1/54*c^3
/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*d
*e^2 + (3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d
- b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e + b^2*e*x + c^2*d) - ((3*(I*sqrt(3)
 + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1
/3) - 2*c/e)*e + 3*sqrt(1/3)*e*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 +
1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*e^2 + 4*
(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e
)/(d*e^3))^(1/3) - 2*c/e)*c*e + 4*c^2)/e^2) + 6*c)*log(-1/4*(3*(I*sqrt(3)
+ 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/
3) - 2*c/e)^2*d*e^2 - (3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2)
 + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e + 2*b^2*e*x - c^2*d
+ 3/4*sqrt(1/3)*((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/
54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*d*e^2 + 2*c*d*e)*sqrt(-((3*(I*s
qrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^
3))^(1/3) - 2*c/e)^2*e^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/
(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*e + 4*c^2)/e^2))
- ((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^
3*e)/(d*e^3))^(1/3) - 2*c/e)*e - 3*sqrt(1/3)*e*sqrt(-((3*(I*sqrt(3) + 1...
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.56

$$\int \frac{bx + cx^2}{d + ex^3}\,dx$$
$$= \text{RootSum}\left(27t^3de^3 - 27t^2cde^2 + 9tc^2de + b^3e - c^3d, \left(t \mapsto t\log\left(x + \frac{9t^2de^2 - 6tcde + c^2d}{b^2e}\right)\right)\right)$$

input
```
integrate((c*x**2+b*x)/(e*x**3+d),x)
```

output
```
RootSum(27*_t**3*d*e**3 - 27*_t**2*c*d*e**2 + 9*_t*c**2*d*e + b**3*e - c**
3*d, Lambda(_t, _t*log(x + (9*_t**2*d*e**2 - 6*_t*c*d*e + c**2*d)/(b**2*e)
)))
```

3.25.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{bx + cx^2}{d + ex^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

$$\int \frac{bx + cx^2}{d + ex^3}\,dx = \frac{\sqrt{3}b \arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,(-de^2)^{\frac{1}{3}}} - \frac{b\log\left(x^2 + x\left(-\frac{d}{e}\right)^{\frac{1}{3}} + \left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,(-de^2)^{\frac{1}{3}}} - \frac{b\left(-\frac{d}{e}\right)^{\frac{2}{3}}\log\left(\left|x - \left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,d} + \frac{c\log\left(|ex^3 + d|\right)}{3\,e}$$

input
```
integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/(-d*e^
2)^(1/3) - 1/6*b*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/(-d*e^2)^(1/3) -
 1/3*b*(-d/e)^(2/3)*log(abs(x - (-d/e)^(1/3)))/d + 1/3*c*log(abs(e*x^3 + d
))/e
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.18

$$\int \frac{bx + cx^2}{d + ex^3} dx = \sum_{k=1}^{3} \ln \left(-\mathrm{root}\left(27\,d\,e^3\,z^3 - 27\,c\,d\,e^2\,z^2 + 9\,c^2\,d\,e\,z + b^3\,e - c^3\,d, z, k\right)\,\left(6\,c\,d\,e - \mathrm{root}\left(27\,d\,e^3\,z^3 - 27\,c\,d\,e^2\,z^2 + 9\,c^2\,d\,e\,z + b^3\,e - c^3\,d, z, k\right)\,d\,e^2\,9\right) + c^2\,d + b^2\,e\,x\right)\,\mathrm{root}\left(27\,d\,e^3\,z^3 - 27\,c\,d\,e^2\,z^2 + 9\,c^2\,d\,e\,z + b^3\,e - c^3\,d, z, k\right)$$

input
```
int((b*x + c*x^2)/(d + e*x^3),x)
```

output
```
symsum(log(c^2*d - root(27*d*e^3*z^3 - 27*c*d*e^2*z^2 + 9*c^2*d*e*z + b^3*
e - c^3*d, z, k)*(6*c*d*e - 9*root(27*d*e^3*z^3 - 27*c*d*e^2*z^2 + 9*c^2*d
*e*z + b^3*e - c^3*d, z, k)*d*e^2) + b^2*e*x)*root(27*d*e^3*z^3 - 27*c*d*e
^2*z^2 + 9*c^2*d*e*z + b^3*e - c^3*d, z, k), k, 1, 3)
```

3.26 $\int \frac{a+cx^2}{d-ex^3}\,dx$

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3.26.1 Optimal result

Integrand size = 18, antiderivative size = 134

$$\int \frac{a+cx^2}{d-ex^3}\,dx=\frac{a\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}}-\frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{c\log\left(d-ex^3\right)}{3e}$$

output
```
-1/3*a*ln(d^(1/3)-e^(1/3)*x)/d^(2/3)/e^(1/3)+1/6*a*ln(d^(2/3)+d^(1/3)*e^(1
/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(1/3)-1/3*c*ln(-e*x^3+d)/e+1/3*a*arctan(1/3*(
d^(1/3)+2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(2/3)/e^(1/3)*3^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^2}{d-ex^3}\,dx=\frac{2\sqrt{3}ae^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)-2ae^{2/3}\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)+ae^{2/3}\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)-2cd^{2/3}\text{lo}}{6d^{2/3}e}$$

input
```
Integrate[(a + c*x^2)/(d - e*x^3),x]
```

output
```
(2*Sqrt[3]*a*e^(2/3)*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*a*e^(
2/3)*Log[d^(1/3) - e^(1/3)*x] + a*e^(2/3)*Log[d^(2/3) + d^(1/3)*e^(1/3)*x
+ e^(2/3)*x^2] - 2*c*d^(2/3)*Log[d - e*x^3])/(6*d^(2/3)*e)
```

3.26.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {2410, 27, 750, 16, 792, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{d-ex^3}dx$$

$$\Big\downarrow \text{2410}$$

$$\int \frac{a}{d-ex^3}dx+c\int \frac{x^2}{d-ex^3}dx$$

$$\Big\downarrow \text{27}$$

$$a\int \frac{1}{d-ex^3}dx+c\int \frac{x^2}{d-ex^3}dx$$

$$\Big\downarrow \text{750}$$

$$a\left(\frac{\int \frac{\sqrt[3]{e}x+2\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e}x}dx}{3d^{2/3}}\right)+c\int \frac{x^2}{d-ex^3}dx$$

$$\Big\downarrow \text{16}$$

$$a\left(\frac{\int \frac{\sqrt[3]{e}x+2\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}-\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)+c\int \frac{x^2}{d-ex^3}dx$$

$$\Big\downarrow \text{792}$$

$$a\left(\frac{\int \frac{\sqrt[3]{e}x+2\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}-\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-\frac{c\log\left(d-ex^3\right)}{3e}$$

$\Big\downarrow$ 1142

$$a\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{\int \frac{\sqrt[3]{e}\left(2\sqrt[3]{e}x+\sqrt[3]{d}\right)}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{2\sqrt[3]{e}}}{3d^{2/3}}-\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-\frac{c\log\left(d-ex^3\right)}{3e}$$

$\Big\downarrow$ 27

$$a\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{1}{2}\int \frac{2\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}-\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-\frac{c\log\left(d-ex^3\right)}{3e}$$

$\Big\downarrow$ 1082

$$a\left(\frac{\frac{1}{2}\int \frac{2\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)}{\sqrt[3]{e}}}{3d^{2/3}}-\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-\frac{c\log\left(d-ex^3\right)}{3e}$$

$\Big\downarrow$ 217

$$a\left(\frac{\frac{1}{2}\int \frac{2\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{\sqrt[3]{e}}}{3d^{2/3}}-\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-\frac{c\log\left(d-ex^3\right)}{3e}$$

$\Big\downarrow$ 1103

3.26. $\int \frac{a+cx^2}{d-ex^3}\,dx$

$$a\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{\sqrt[3]{e}}+\frac{\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{2\sqrt[3]{e}}}{3d^{2/3}}-\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-\frac{c\log\left(d-ex^3\right)}{3e}$$

input
```
Int[(a + c*x^2)/(d - e*x^3),x]
```

output
```
a*(-1/3*Log[d^(1/3) - e^(1/3)*x]/(d^(2/3)*e^(1/3)) + ((Sqrt[3]*ArcTan[(1 +
 (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/e^(1/3) + Log[d^(2/3) + d^(1/3)*e^(1/3)*
x + e^(2/3)*x^2]/(2*e^(1/3)))/(3*d^(2/3))) - (c*Log[d - e*x^3])/(3*e)
```

3.26.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.26.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

method	result	size
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3} e-d\right)}\frac{\left(\_R^{2} c+a\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3e}$	36
default	$a\left(-\frac{\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)-\frac{c\ln\left(-e\,x^{3}+d\right)}{3e}$	110

input
```
int((c*x^2+a)/(-e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
-1/3/e*sum((_R^2*c+a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e-d))
```

3.26.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 1040, normalized size of antiderivative = 7.76

$$\int \frac{a+cx^2}{d-ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="fricas")
```

output
```
-1/12*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 +
a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)
*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*d*
e + a*e*x + c*d) - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) -
(c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e + 3*sqrt(1/3)*e*sqrt(-(((1
/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^
2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a
^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*c*e + 4*c^2)/e^
2) - 6*c)*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c
^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*d*e + 2*a*e*x + 3/2*sqrt(1/3)*
d*e*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2
+ a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1
)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*c
*e + 4*c^2)/e^2) - c*d) - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^
2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e - 3*sqrt(1/3)*e*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e
^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/
e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*c*e + 4*
c^2)/e^2) - 6*c)*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*
e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*d*e + 2*a*e*x - 3/2*...
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.52

$$\int \frac{a+cx^2}{d-ex^3}\,dx = -\operatorname{RootSum}\left(27t^3d^2e^3 - 27t^2cd^2e^2 + 9tc^2d^2e - a^3e^2 - c^3d^2, \left(t \mapsto t\log\left(x + \frac{-3tde + cd}{ae}\right)\right)\right)$$

input
```
integrate((c*x**2+a)/(-e*x**3+d),x)
```

output
```
-RootSum(27*_t**3*d**2*e**3 - 27*_t**2*c*d**2*e**2 + 9*_t*c**2*d**2*e - a*
*3*e**2 - c**3*d**2, Lambda(_t, _t*log(x + (-3*_t*d*e + c*d)/(a*e))))
```

3.26.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{d-ex^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \frac{a+cx^2}{d-ex^3}\,dx = -\frac{a\left(\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,d} - \frac{c\log\left(\left|ex^3-d\right|\right)}{3\,e} + \frac{\sqrt{3}(de^2)^{\frac{1}{3}}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,de} + \frac{(de^2)^{\frac{1}{3}}\,a\log\left(x^2+x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,de}$$

input
```
integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="giac")
```

output
```
-1/3*a*(d/e)^(1/3)*log(abs(x - (d/e)^(1/3)))/d - 1/3*c*log(abs(e*x^3 - d))
/e + 1/3*sqrt(3)*(d*e^2)^(1/3)*a*arctan(1/3*sqrt(3)*(2*x + (d/e)^(1/3))/(d
/e)^(1/3))/(d*e) + 1/6*(d*e^2)^(1/3)*a*log(x^2 + x*(d/e)^(1/3) + (d/e)^(2/
3))/(d*e)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.33

$$\int \frac{a+cx^2}{d-ex^3}\,dx = \sum_{k=1}^{3} \ln\left(-\left(c + \mathrm{root}\left(27\,d^2\,e^3\,z^3+27\,c\,d^2\,e^2\,z^2+9\,c^2\,d^2\,e\,z+c^3\,d^2+a^3\,e^2,z,k\right)\,e\,3\right)\,\left(c\,d + \mathrm{root}\left(27\,d^2\,e^3\,z^3+27\,c\,d^2\,e^2\,z^2+9\,c^2\,d^2\,e\,z+c^3\,d^2+a^3\,e^2,z,k\right)\,d\,e\,3 + a\,e\,x\right)\right)\,\mathrm{root}\left(27\,d^2\,e^3\,z^3+27\,c\,d^2\,e^2\,z^2+9\,c^2\,d^2\,e\,z+c^3\,d^2+a^3\,e^2,z,k\right)$$

input
```
int((a + c*x^2)/(d - e*x^3),x)
```

output
```
symsum(log(-(c + 3*root(27*d^2*e^3*z^3 + 27*c*d^2*e^2*z^2 + 9*c^2*d^2*e*z
+ c^3*d^2 + a^3*e^2, z, k)*e)*(c*d + 3*root(27*d^2*e^3*z^3 + 27*c*d^2*e^2*
z^2 + 9*c^2*d^2*e*z + c^3*d^2 + a^3*e^2, z, k)*d*e + a*e*x))*root(27*d^2*e
^3*z^3 + 27*c*d^2*e^2*z^2 + 9*c^2*d^2*e*z + c^3*d^2 + a^3*e^2, z, k), k, 1
, 3)
```

3.27 $\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx$

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3.27.1 Optimal result

Integrand size = 27, antiderivative size = 37

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx=-\frac{2\arctan\left(\frac{a-2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}+\frac{\log(a+bx)}{b}$$

output
```
ln(b*x+a)/b-2/3*arctan(1/3*(-2*b*x+a)/a*3^(1/2))/b*3^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.95

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx =\frac{2\sqrt{3}\arctan\left(\frac{-a+2bx}{\sqrt{3}a}\right)+2\log(a+bx)-\log\left(a^2-abx+b^2x^2\right)+\log\left(a^3+b^3x^3\right)}{3b}$$

input
```
Integrate[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-a + 2*b*x)/(Sqrt[3]*a)] + 2*Log[a + b*x] - Log[a^2 - a
*b*x + b^2*x^2] + Log[a^3 + b^3*x^3])/(3*b)
```

3.27.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2407, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}dx$$

$$\Big\downarrow \text{2407}$$

$$\frac{a\int \frac{1}{\frac{a^2}{b^2}-\frac{xa}{b}+x^2}dx}{b^2}+\frac{\int \frac{1}{\frac{a}{b}+x}dx}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\int \frac{1}{\frac{a^2}{b^2}-\frac{xa}{b}+x^2}dx}{b^2}+\frac{\log(a+bx)}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\int \frac{1}{-\left(1-\frac{2bx}{a}\right)^2-3}d\left(1-\frac{2bx}{a}\right)}{b}+\frac{\log(a+bx)}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{\log(a+bx)}{b}-\frac{2\arctan\left(\frac{1-\frac{2bx}{a}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

input
```
Int[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

output
```
(-2*ArcTan[(1 - (2*b*x)/a)/Sqrt[3]])/(Sqrt[3]*b) + Log[a + b*x]/b
```

3.27.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2407
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Simp[C/b
Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]]
 /; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && Pol
yQ[P2, x, 2]
```

3.27.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

method	result	size
risch	$\frac{\ln(bx+a)}{b}+\left(\sum_{\_R=\operatorname{RootOf}\left(3b^2\_Z^2+1\right)}\_R\ln\left(3ab\_R+2bx-a\right)\right)$	42
default	$\frac{\ln(bx+a)}{b}+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2b^2x-ab\right)\sqrt{3}}{3ab}\right)}{3b}$	43

input
```
int((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x,method=_RETURNVERBOSE)
```

output
```
ln(b*x+a)/b+sum(_R*ln(3*_R*a*b+2*b*x-a),_R=RootOf(3*_Z^2*b^2+1))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{2a^2 + b^2x^2}{a^3 + b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx-a)}{3\,a}\right) + 3\,\log\left(bx+a\right)}{3\,b}$$

input
```
integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a) + 3*log(b*x + a))/b
```

3.27.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{2a^2 + b^2x^2}{a^3 + b^3x^3}\,dx = \frac{-\frac{\sqrt{3}i\log\left(x+\frac{-a-\sqrt{3}ia}{2b}\right)}{3} + \frac{\sqrt{3}i\log\left(x+\frac{-a+\sqrt{3}ia}{2b}\right)}{3} + \log\left(\frac{a}{b}+x\right)}{b}$$

input
```
integrate((b**2*x**2+2*a**2)/(b**3*x**3+a**3),x)
```

output
```
(-sqrt(3)*I*log(x + (-a - sqrt(3)*I*a)/(2*b))/3 + sqrt(3)*I*log(x + (-a +
sqrt(3)*I*a)/(2*b))/3 + log(a/b + x))/b
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{2a^2 + b^2x^2}{a^3 + b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,b^2x-ab)}{3\,ab}\right)}{3\,b} + \frac{\log\left(bx+a\right)}{b}$$

input
```
integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x - a*b)/(a*b))/b + log(b*x + a)/b
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx-a)}{3\,a}\right)}{3\,b} + \frac{\log\left(|bx+a|\right)}{b}$$

input
```
integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a)/b + log(abs(b*x + a))/b
```

3.27.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.27

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx = \frac{\ln(a+b\,x)}{b} - \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}\,a^3\,b^4}{4\,a^3\,b^4+4\,x\,a^2\,b^5} - \frac{4\sqrt{3}\,a^2\,b^5\,x}{4\,a^3\,b^4+4\,x\,a^2\,b^5}\right)}{3\,b}$$

input
```
int((2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x)
```

output
```
log(a + b*x)/b - (2*3^(1/2)*atan((4*3^(1/2)*a^3*b^4)/(4*a^3*b^4 + 4*a^2*b^
5*x) - (4*3^(1/2)*a^2*b^5*x)/(4*a^3*b^4 + 4*a^2*b^5*x)))/(3*b)
```

3.28 $\int \frac{2a^2+b^2x^2}{a^3-b^3x^3} \, dx$

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3.28.1 Optimal result

Integrand size = 28, antiderivative size = 39

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3} \, dx = \frac{2\arctan\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b} - \frac{\log(a-bx)}{b}$$

output
```
-ln(-b*x+a)/b+2/3*arctan(1/3*(2*b*x+a)/a*3^(1/2))/b*3^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.82

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3} \, dx = \frac{2\sqrt{3}\arctan\left(\frac{a+2bx}{\sqrt{3}a}\right) - 2\log(a-bx) + \log\left(a^2+abx+b^2x^2\right) - \log\left(a^3-b^3x^3\right)}{3b}$$

input
```
Integrate[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)] - 2*Log[a - b*x] + Log[a^2 + a*
b*x + b^2*x^2] - Log[a^3 - b^3*x^3])/(3*b)
```

3.28.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2407, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}dx$$

$$\Big\downarrow \ 2407$$

$$\frac{a\int \frac{1}{\frac{a^2}{b^2}+\frac{xa}{b}+x^2}dx}{b^2}-\frac{\int \frac{1}{x-\frac{a}{b}}dx}{b}$$

$$\Big\downarrow \ 16$$

$$\frac{a\int \frac{1}{\frac{a^2}{b^2}+\frac{xa}{b}+x^2}dx}{b^2}-\frac{\log(a-bx)}{b}$$

$$\Big\downarrow \ 1082$$

$$-\frac{2\int \frac{1}{-\left(\frac{2bx}{a}+1\right)^2-3}d\left(\frac{2bx}{a}+1\right)}{b}-\frac{\log(a-bx)}{b}$$

$$\Big\downarrow \ 217$$

$$\frac{2\arctan\left(\frac{\frac{2bx}{a}+1}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{\log(a-bx)}{b}$$

input
```
Int[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
```

output
```
(2*ArcTan[(1 + (2*b*x)/a)/Sqrt[3]])/(Sqrt[3]*b) - Log[a - b*x]/b
```

3.28.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2407
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Simp[C/b
Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]]
 /; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && Pol
yQ[P2, x, 2]
```

3.28.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{2\arctan\left(\frac{(2bx+a)\sqrt{3}}{3a}\right)\sqrt{3}}{3b}-\frac{\ln(bx-a)}{b}$	38
default	$-\frac{\ln(-bx+a)}{b}+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2b^{2}x+ab\right)\sqrt{3}}{3ab}\right)}{3b}$	44

input
```
int((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctan(1/3*(2*b*x+a)/a*3^(1/2))/b*3^(1/2)-1/b*ln(b*x-a)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx+a)}{3\,a}\right)-3\,\log\left(bx-a\right)}{3\,b}$$

input
```
integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a) - 3*log(b*x - a))/b
```

3.28.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx=-\frac{\frac{\sqrt{3}i\log\left(x+\frac{a-\sqrt{3}ia}{2b}\right)}{3}-\frac{\sqrt{3}i\log\left(x+\frac{a+\sqrt{3}ia}{2b}\right)}{3}+\log\left(-\frac{a}{b}+x\right)}{b}$$

input
```
integrate((b**2*x**2+2*a**2)/(-b**3*x**3+a**3),x)
```

output
```
-(sqrt(3)*I*log(x + (a - sqrt(3)*I*a)/(2*b))/3 - sqrt(3)*I*log(x + (a + sq
rt(3)*I*a)/(2*b))/3 + log(-a/b + x))/b
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,b^2x+ab)}{3\,ab}\right)}{3\,b}-\frac{\log\left(bx-a\right)}{b}$$

input
```
integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x + a*b)/(a*b))/b - log(b*x - a)/b
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx+a)}{3\,a}\right)}{3\,b} - \frac{\log\left(|bx-a|\right)}{b}$$

input
```
integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a)/b - log(abs(b*x - a))/b
```

3.28.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.21

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}\,a^3\,b^4}{4\,a^3\,b^4-4\,a^2\,b^5\,x}+\frac{4\sqrt{3}\,a^2\,b^5\,x}{4\,a^3\,b^4-4\,a^2\,b^5\,x}\right)}{3\,b} - \frac{\ln\left(a-b\,x\right)}{b}$$

input
```
int((2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x)
```

output
```
(2*3^(1/2)*atan((4*3^(1/2)*a^3*b^4)/(4*a^3*b^4 - 4*a^2*b^5*x) + (4*3^(1/2)
*a^2*b^5*x)/(4*a^3*b^4 - 4*a^2*b^5*x)))/(3*b) - log(a - b*x)/b
```

3.29 $\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx$

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3.29.1 Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx=-\frac{2C\arctan\left(\frac{1-\sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{C\log\left(2+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

output
```
C*ln(2+b^(1/3)*x)/b^(1/3)-2/3*C*arctan(1/3*(1-b^(1/3)*x)*3^(1/2))/b^(1/3)*
3^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx=\frac{C\left(2\sqrt{3}\arctan\left(\frac{-1+\sqrt[3]{b}x}{\sqrt{3}}\right)+2\log\left(2+\sqrt[3]{b}x\right)-\log\left(4-2\sqrt[3]{b}x+b^{2/3}x^2\right)+\log(8+\right.}{3\sqrt[3]{b}}$$

input
```
Integrate[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

output
```
(C*(2*Sqrt[3]*ArcTan[(-1 + b^(1/3)*x)/Sqrt[3]] + 2*Log[2 + b^(1/3)*x] - Lo
g[4 - 2*b^(1/3)*x + b^(2/3)*x^2] + Log[8 + b*x^3]))/(3*b^(1/3))
```

3.29.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2402, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b^{2/3}Cx^2+8C}{bx^3+8}dx$$

$$\Big\downarrow \text{ 2402}$$

$$\frac{2C\int \frac{1}{x^2-\frac{2x}{\sqrt[3]{b}}+\frac{4}{b^{2/3}}}dx}{b^{2/3}}+\frac{C\int \frac{1}{x+\frac{2}{\sqrt[3]{b}}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{2C\int \frac{1}{x^2-\frac{2x}{\sqrt[3]{b}}+\frac{4}{b^{2/3}}}dx}{b^{2/3}}+\frac{C\log\left(\sqrt[3]{b}x+2\right)}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2C\int \frac{1}{-\left(1-\sqrt[3]{b}x\right)^2-3}d\left(1-\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{C\log\left(\sqrt[3]{b}x+2\right)}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{C\log\left(\sqrt[3]{b}x+2\right)}{\sqrt[3]{b}}-\frac{2C\arctan\left(\frac{1-\sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

input
```
Int[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

output
```
(-2*C*ArcTan[(1 - b^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (C*Log[2 + b^(1
/3)*x])/b^(1/3)
```

3.29.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2402
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Simp[C
/b   Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x]
, x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a,
b}, x] && PolyQ[P2, x, 2]
```

3.29.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(37) = 74$.

Time = 1.52 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.40

method	result	size
default	$C\left(\frac{8^{\frac{1}{3}}\ln\left(x+8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}-\frac{8^{\frac{1}{3}}\ln\left(x^{2}-8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}x+8^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{8^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{8^{\frac{2}{3}}x}{4\left(\frac{1}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{\ln(b\,x^{3}+8)}{3b^{\frac{1}{3}}}\right)$	115
meijerg	$\frac{2C\left(\frac{b^{\frac{1}{3}}x\ln\left(1+\frac{(b\,x^{3})^{\frac{1}{3}}}{2}\right)}{(b\,x^{3})^{\frac{1}{3}}}-\frac{b^{\frac{1}{3}}x\ln\left(1-\frac{(b\,x^{3})^{\frac{1}{3}}}{2}+\frac{(b\,x^{3})^{\frac{2}{3}}}{4}\right)}{2(b\,x^{3})^{\frac{1}{3}}}+\frac{b^{\frac{1}{3}}x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(b\,x^{3})^{\frac{1}{3}}}{4-(b\,x^{3})^{\frac{1}{3}}}\right)}{(b\,x^{3})^{\frac{1}{3}}}\right)}{3b^{\frac{1}{3}}}+\frac{C\ln\left(1+\frac{b\,x^{3}}{8}\right)}{3b^{\frac{1}{3}}}$	123

input
```
int((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x,method=_RETURNVERBOSE)
```

output
```
C*(1/3/b*8^(1/3)/(1/b)^(2/3)*ln(x+8^(1/3)*(1/b)^(1/3))-1/6/b*8^(1/3)/(1/b)
^(2/3)*ln(x^2-8^(1/3)*(1/b)^(1/3)*x+8^(2/3)*(1/b)^(2/3))+1/3/b*8^(1/3)/(1/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(2/3)/(1/b)^(1/3)*x-1))+1/3/b^(
1/3)*ln(b*x^3+8))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.79

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx = \left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{bx^3+6\sqrt{\frac{1}{3}}\left(bx^2+b^{\frac{2}{3}}x-2\,b^{\frac{1}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-6\,b^{\frac{1}{3}}x-4}{bx^3+8}\right)+Cb^{\frac{2}{3}}\log\left(bx+2\,b^{\frac{2}{3}}\right)}{b}, -\right.$$

input
```
integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="fricas")
```

output
```
[(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((b*x^3 + 6*sqrt(1/3)*(b*x^2 + b^(2/3)
*x - 2*b^(1/3))*sqrt(-1/b^(2/3)) - 6*b^(1/3)*x - 4)/(b*x^3 + 8)) + C*b^(2/
3)*log(b*x + 2*b^(2/3)))/b, (2*sqrt(1/3)*C*b^(2/3)*arctan(sqrt(1/3)*(b^(2/
3)*x - b^(1/3))/b^(1/3)) + C*b^(2/3)*log(b*x + 2*b^(2/3)))/b]
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx = \operatorname{RootSum}\left(3t^3b^{\frac{5}{3}}-3t^2Cb^{\frac{4}{3}}+tC^2b-C^3b^{\frac{2}{3}},\left(t\mapsto t\log\left(x+\frac{3t\sqrt[3]{b}-C}{C\sqrt[3]{b}}\right)\right)\right)$$

input
```
integrate((8*C+b**(2/3)*C*x**2)/(b*x**3+8),x)
```

output
```
RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
 Lambda(_t, _t*log(x + (3*_t*b**(1/3) - C)/(C*b**(1/3)))))
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx = \frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(b^{\frac{2}{3}}x-b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{3\,b^{\frac{1}{3}}}+\frac{C\log\left(\frac{b^{\frac{1}{3}}x+2}{b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}$$

input
```
integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(b^(2/3)*x - b^(1/3))/b^(1/3))/b^(1/3) +
C*log((b^(1/3)*x + 2)/b^(1/3))/b^(1/3)
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(36) = 72.

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.40

$$\begin{aligned}\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx &= \frac{2}{3}\sqrt{3}C\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(x+\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{b}\right)^{\frac{1}{3}}}\right)\\ &-\frac{1}{3}\left(Cb^{\frac{2}{3}}\left(-\frac{1}{b}\right)^{\frac{2}{3}}+2\,C\right)\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(\left|x-2\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right|\right)\\ &+\frac{1}{3}\left(C\left(-\frac{1}{b}\right)^{\frac{1}{3}}+\frac{C}{b^{\frac{1}{3}}}\right)\log\left(x^2+2\,x\left(-\frac{1}{b}\right)^{\frac{1}{3}}+4\left(-\frac{1}{b}\right)^{\frac{2}{3}}\right)\end{aligned}$$

input
```
integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*C*(-1/b)^(1/3)*arctan(1/3*sqrt(3)*(x + (-1/b)^(1/3))/(-1/b)^(1
/3)) - 1/3*(C*b^(2/3)*(-1/b)^(2/3) + 2*C)*(-1/b)^(1/3)*log(abs(x - 2*(-1/b
)^(1/3))) + 1/3*(C*(-1/b)^(1/3) + C/b^(1/3))*log(x^2 + 2*x*(-1/b)^(1/3) +
4*(-1/b)^(2/3))
```

3.29.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.06

$$\int \frac{8C + b^{2/3}Cx^2}{8 + bx^3}\,dx = \sum_{k=1}^{3} \ln\left(-\frac{\left(C - \mathrm{root}\left(27\,b^3\,z^3 - 27\,C\,b^{8/3}\,z^2 + 9\,C^2\,b^{7/3}\,z - 9\,C^3\,b^2, z, k\right)\,b^{1/3}\,3\right)\,\left(-C\right.}{b}\right.$$

input
```
int((8*C + C*b^(2/3)*x^2)/(b*x^3 + 8),x)
```

output
```
symsum(log(-(8*(C - 3*root(27*b^3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b^(7/3)*z
 - 9*C^3*b^2, z, k)*b^(1/3))*(3*root(27*b^3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2
*b^(7/3)*z - 9*C^3*b^2, z, k)*b^(1/3) - C + C*b^(1/3)*x))/b^(5/3))*root(27
*b^3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b^(7/3)*z - 9*C^3*b^2, z, k), k, 1, 3)
```

3.30 $\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx$

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3.30.1 Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx=-\frac{C\arctan\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}}+\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)$$

output
```
1/4*C*ln(a^(1/3)+2*x)-1/6*C*arctan(1/3*(a^(1/3)-4*x)/a^(1/3)*3^(1/2))*3^(1
/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx=\frac{1}{12}C\left(-2\sqrt{3}\arctan\left(\frac{1-\frac{4x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}+2x\right)-\log\left(a^{2/3}-2\sqrt[3]{a}x+4x^2\right)+\log\left(a+8x^3\right)\right)$$

input
```
Integrate[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

output
```
(C*(-2*Sqrt[3]*ArcTan[(1 - (4*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3) + 2*x]
- Log[a^(2/3) - 2*a^(1/3)*x + 4*x^2] + Log[a + 8*x^3]))/12
```

3.30.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2402, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}dx$$

$$\Big\downarrow \text{ 2402}$$

$$\frac{1}{8}\sqrt[3]{a}C\int \frac{1}{x^2-\frac{\sqrt[3]{a}x}{2}+\frac{a^{2/3}}{4}}dx+\frac{1}{4}C\int \frac{1}{x+\frac{\sqrt[3]{a}}{2}}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{8}\sqrt[3]{a}C\int \frac{1}{x^2-\frac{\sqrt[3]{a}x}{2}+\frac{a^{2/3}}{4}}dx+\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{2}C\int \frac{1}{-\left(1-\frac{4x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{4x}{\sqrt[3]{a}}\right)+\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{4}C\log\left(\sqrt[3]{a}+2x\right)-\frac{C\arctan\left(\frac{1-\frac{4x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

output
```
-1/2*(C*ArcTan[(1 - (4*x)/a^(1/3))/Sqrt[3]])/Sqrt[3] + (C*Log[a^(1/3) + 2*
x])/4
```

3.30.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2402
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Simp[C
/b   Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x]
, x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a,
b}, x] && PolyQ[P2, x, 2]
```

3.30.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(34) = 68$.

Time = 1.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.04

method	result	size
default	$C\left(a^{\frac{2}{3}}\left(\frac{8^{\frac{2}{3}}\ln\left(x+\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}}{8}\right)}{24a^{\frac{2}{3}}}-\frac{8^{\frac{2}{3}}\ln\left(x^{2}-\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}x}{8}+\frac{8^{\frac{1}{3}}a^{\frac{2}{3}}}{8}\right)}{48a^{\frac{2}{3}}}+\frac{8^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,8^{\frac{1}{3}}x}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{24a^{\frac{2}{3}}}\right)+\frac{\ln\left(8x^{3}+a\right)}{12}\right)$	96

input
```
int((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*(a^(2/3)*(1/24*8^(2/3)/a^(2/3)*ln(x+1/8*8^(2/3)*a^(1/3))-1/48*8^(2/3)/a^
(2/3)*ln(x^2-1/8*8^(2/3)*a^(1/3)*x+1/8*8^(1/3)*a^(2/3))+1/24*8^(2/3)/a^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*8^(1/3)/a^(1/3)*x-1)))+1/12*ln(8*x^3+a))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx = \frac{1}{6}\sqrt{3}C\arctan\left(\frac{4\sqrt{3}a^{\frac{2}{3}}x-\sqrt{3}a}{3\,a}\right)+\frac{1}{4}C\log\left(2\,x+a^{\frac{1}{3}}\right)$$

input
```
integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*C*arctan(1/3*(4*sqrt(3)*a^(2/3)*x - sqrt(3)*a)/a) + 1/4*C*log(
2*x + a^(1/3))
```

3.30.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx = C\left(\frac{\log\left(\frac{\sqrt[3]{a}}{2}+x\right)}{4} - \frac{\sqrt{3}i\log\left(x+\frac{-C\sqrt[3]{a}-\sqrt{3}iC\sqrt[3]{a}}{4C}\right)}{12}+\frac{\sqrt{3}i\log\left(x+\frac{-C\sqrt[3]{a}+\sqrt{3}iC\sqrt[3]{a}}{4C}\right)}{12}\right)$$

input
```
integrate((a**(2/3)*C+2*C*x**2)/(8*x**3+a),x)
```

output
```
C*(log(a**(1/3)/2 + x)/4 - sqrt(3)*I*log(x + (-C*a**(1/3) - sqrt(3)*I*C*a*
*(1/3))/(4*C))/12 + sqrt(3)*I*log(x + (-C*a**(1/3) + sqrt(3)*I*C*a**(1/3))
/(4*C))/12)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3}\,dx = \frac{1}{6}\,\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(4\,x - a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) + \frac{1}{4}\,C\log\left(x + \frac{1}{2}\,a^{\frac{1}{3}}\right)$$

input
```
integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*C*arctan(1/3*sqrt(3)*(4*x - a^(1/3))/a^(1/3)) + 1/4*C*log(x +
1/2*a^(1/3))
```

3.30.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3}\,dx = -\frac{\sqrt{3}\left(-i\,\sqrt{3}|a| - a\right)C\arctan\left(\frac{\sqrt{3}\left(4\,x+(-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{12\,a} - \frac{\left(C(-a)^{\frac{2}{3}} + 2\,Ca^{\frac{2}{3}}\right)(-a)^{\frac{1}{3}}\log\left(\left|x - \frac{1}{2}\,(-a)^{\frac{1}{3}}\right|\right)}{12\,a}$$

input
```
integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*(-I*sqrt(3)*abs(a) - a)*C*arctan(1/3*sqrt(3)*(4*x + (-a)^(1/
3))/(-a)^(1/3))/a - 1/12*(C*(-a)^(2/3) + 2*C*a^(2/3))*(-a)^(1/3)*log(abs(x
 - 1/2*(-a)^(1/3)))/a
```

3.30.9 Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.09

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx = \sum_{k=1}^{3} \ln\left(-\frac{a^{2/3}\left(C-12\,\mathrm{root}(1728\,a^2\,z^3-432\,C\,a^2\,z^2+36\,C^2\,a^2\,z-9\,C^3\,a^2,z,k)\right)\,(4$$

input
```
int((C*a^(2/3) + 2*C*x^2)/(a + 8*x^3),x)
```

output
```
symsum(log(-(a^(2/3)*(C - 12*root(1728*a^2*z^3 - 432*C*a^2*z^2 + 36*C^2*a^
2*z - 9*C^3*a^2, z, k))*(4*C*x - C*a^(1/3) + 12*root(1728*a^2*z^3 - 432*C*
a^2*z^2 + 36*C^2*a^2*z - 9*C^3*a^2, z, k)*a^(1/3)))/128)*root(1728*a^2*z^3
 - 432*C*a^2*z^2 + 36*C^2*a^2*z - 9*C^3*a^2, z, k), k, 1, 3)
```

3.31 $\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx$

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3.31.1 Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx=\frac{2C\arctan\left(\frac{1-\sqrt[3]{-b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}}-\frac{C\log\left(2+\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

output
```
-C*ln(2+(-b)^(1/3)*x)/(-b)^(1/3)+2/3*C*arctan(1/3*(1-(-b)^(1/3)*x)*3^(1/2)
)/(-b)^(1/3)*3^(1/2)
```

3.31.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx=\frac{C\left(-2\sqrt{3}b^{2/3}\arctan\left(\frac{1+\sqrt[3]{b}x}{\sqrt{3}}\right)+2b^{2/3}\log\left(2-\sqrt[3]{b}x\right)-b^{2/3}\log\left(4+2\sqrt[3]{b}x+b^2\right.\right.}{3b}$$

input
```
Integrate[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

output
```
(C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 + b^(1/3)*x)/Sqrt[3]] + 2*b^(2/3)*Log[2 -
 b^(1/3)*x] - b^(2/3)*Log[4 + 2*b^(1/3)*x + b^(2/3)*x^2] + (-b)^(2/3)*Log[
8 - b*x^3]))/(3*b)
```

3.31.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2403, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-b)^{2/3}Cx^2+8C}{bx^3-8}dx$$

$$\Big\downarrow \text{2403}$$

$$-\frac{2C\int \frac{1}{x^2-\frac{2x}{\sqrt[3]{-b}}+\frac{4}{(-b)^{2/3}}}dx}{(-b)^{2/3}}-\frac{C\int \frac{1}{x+\frac{2}{\sqrt[3]{-b}}}dx}{\sqrt[3]{-b}}$$

$$\Big\downarrow \text{16}$$

$$-\frac{2C\int \frac{1}{x^2-\frac{2x}{\sqrt[3]{-b}}+\frac{4}{(-b)^{2/3}}}dx}{(-b)^{2/3}}-\frac{C\log\left(\sqrt[3]{-b}x+2\right)}{\sqrt[3]{-b}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2C\int \frac{1}{-\left(1-\sqrt[3]{-b}x\right)^2-3}d\left(1-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}-\frac{C\log\left(\sqrt[3]{-b}x+2\right)}{\sqrt[3]{-b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{2C\arctan\left(\frac{1-\sqrt[3]{-b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}}-\frac{C\log\left(\sqrt[3]{-b}x+2\right)}{\sqrt[3]{-b}}$$

input
```
Int[(8*C + (-b)^(2/3)*C*x^2)/(-8 + b*x^3),x]
```

output
```
(2*C*ArcTan[(1 - (-b)^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*(-b)^(1/3)) - (C*Log[2 +
 (-b)^(1/3)*x])/(-b)^(1/3)
```

3.31.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2403
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a)^(1/3)/(-b)^(1/3)},
Simp[C/b   Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^
2), x], x]] /; EqQ[A*(-b)^(2/3) - (-a)^(1/3)*(-b)^(1/3)*B - 2*(-a)^(2/3)*C,
 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(46) = 92$.

Time = 1.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.70

method	result
meijerg	$\frac{2Cx\left(\ln\left(1-\frac{(bx^3)^{\frac{1}{3}}}{2}\right)-\frac{\ln\left(1+\frac{(bx^3)^{\frac{1}{3}}}{2}+\frac{(bx^3)^{\frac{2}{3}}}{4}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(bx^3)^{\frac{1}{3}}}{4+(bx^3)^{\frac{1}{3}}}\right)\right)}{3(bx^3)^{\frac{1}{3}}}-\frac{C\ln\left(1-\frac{bx^3}{8}\right)}{3(-b)^{\frac{1}{3}}}$
default	$C\left(\frac{8^{\frac{1}{3}}\ln\left(x-8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}-\frac{8^{\frac{1}{3}}\ln\left(x^2+8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}x+8^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{1}{b}\right)^{\frac{2}{3}}}-\frac{8^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{8^{\frac{2}{3}}x}{4\left(\frac{1}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{(-b)^{\frac{2}{3}}\ln(bx^3-8)}{3b}\right)$

input
```
int((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x,method=_RETURNVERBOSE)
```

output
```
2/3*C*x/(b*x^3)^(1/3)*(ln(1-1/2*(b*x^3)^(1/3))-1/2*ln(1+1/2*(b*x^3)^(1/3)+
1/4*(b*x^3)^(2/3))-3^(1/2)*arctan(1/4*3^(1/2)*(b*x^3)^(1/3)/(1+1/4*(b*x^3)
^(1/3))))-1/3*C/(-b)^(1/3)*ln(1-1/8*b*x^3)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.19

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx=\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{bx^3-6\sqrt{\frac{1}{3}}\left(bx^2-(-b)^{\frac{2}{3}}x+2(-b)^{\frac{1}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+6(-b)^{\frac{1}{3}}x+4}{bx^3-8}\right)+C(-b)^{\frac{2}{3}}\log\right.$$

$$\left.-\frac{2\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(\sqrt{\frac{1}{3}}\left((-b)^{\frac{2}{3}}x-(-b)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\right)-C(-b)^{\frac{2}{3}}\log\left(bx-2(-b)^{\frac{2}{3}}\right)}{b}\right]$$

input
```
integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="fricas")
```

output
```
[(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((b*x^3 - 6*sqrt(1/3)*(b*x^2 - (-b)^
(2/3)*x + 2*(-b)^(1/3))*sqrt((-b)^(1/3)/b) + 6*(-b)^(1/3)*x + 4)/(b*x^3 -
8)) + C*(-b)^(2/3)*log(b*x - 2*(-b)^(2/3)))/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b
)^(1/3)/b)*arctan(sqrt(1/3)*((-b)^(2/3)*x - (-b)^(1/3))*sqrt(-(-b)^(1/3)/b
)) - C*(-b)^(2/3)*log(b*x - 2*(-b)^(2/3)))/b]
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx = \text{RootSum}\left(3t^3b^2-3t^2Cb(-b)^{\frac{2}{3}}+tC^2(-b)^{\frac{4}{3}}-C^3b,\left(t\mapsto t\log\left(-\frac{3t}{C}+x+\frac{(-b}{b}\right.\right.\right.$$

input
```
integrate((8*C+(-b)**(2/3)*C*x**2)/(b*x**3-8),x)
```

output
```
RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(4/3) - C**
3*b, Lambda(_t, _t*log(-3*_t/C + x + (-b)**(2/3)/b)))
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(45) = 90$.

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

$$\begin{aligned}\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx &= \frac{\left(C(-b)^{\frac{2}{3}}-Cb^{\frac{2}{3}}\right)\log\left(b^{\frac{2}{3}}x^2+2\,b^{\frac{1}{3}}x+4\right)}{3\,b}\\&+\frac{\left(C(-b)^{\frac{2}{3}}+2\,Cb^{\frac{2}{3}}\right)\log\left(\frac{b^{\frac{1}{3}}x-2}{b^{\frac{1}{3}}}\right)}{3\,b}\\&+\frac{2\sqrt{3}\left(C(-b)^{\frac{2}{3}}\,b^{\frac{4}{3}}-\left(C(-b)^{\frac{2}{3}}\,b^{\frac{1}{3}}+3\,Cb\right)b\right)\arctan\left(\frac{\sqrt{3}\left(b^{\frac{2}{3}}x+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{9\,b^{\frac{7}{3}}}\end{aligned}$$

input
```
integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="maxima")
```

output
```
1/3*(C*(-b)^(2/3) - C*b^(2/3))*log(b^(2/3)*x^2 + 2*b^(1/3)*x + 4)/b + 1/3*
(C*(-b)^(2/3) + 2*C*b^(2/3))*log((b^(1/3)*x - 2)/b^(1/3))/b + 2/9*sqrt(3)*
(C*(-b)^(2/3)*b^(4/3) - (C*(-b)^(2/3)*b^(1/3) + 3*C*b)*b)*arctan(1/3*sqrt(
3)*(b^(2/3)*x + b^(1/3))/b^(1/3))/b^(7/3)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx = -\frac{2\sqrt{3}C|b|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}b^{\frac{1}{3}}\left(x+\frac{1}{b^{\frac{1}{3}}}\right)\right)}{3\,b} + \frac{\left(2\,C+\frac{C(-b)^{\frac{2}{3}}}{b^{\frac{2}{3}}}\right)\log\left(\left|x-\frac{2}{b^{\frac{1}{3}}}\right|\right)}{3\,b^{\frac{1}{3}}}$$

input
```
integrate((8*C+(-b)^(2/3)*C*x^2)/(b*x^3-8),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*C*abs(b)^(2/3)*arctan(1/3*sqrt(3)*b^(1/3)*(x + 1/b^(1/3)))/b
+ 1/3*(2*C + C*(-b)^(2/3)/b^(2/3))*log(abs(x - 2/b^(1/3)))/b^(1/3)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.09

$$\int \frac{8C+(-b)^{2/3}Cx^2}{-8+bx^3}\,dx = \sum_{k=1}^{3}\ln\left(\frac{8\,C^2}{(-b)^{5/3}} + \mathrm{root}\left(27\,b^3\,z^3-27\,C\,(-b)^{8/3}\,z^2-9\,C^2\,(-b)^{7/3}\,z-9\,C^3\,b^2,z,k\right)\left(-\frac{\mathrm{root}\left(27\,b^3\,z^3-27\,C\,(-b)^{8/3}\,z^2-9\,C^2\,(\right.}{b}\right.\right.$$

input
```
int((8*C + C*(-b)^(2/3)*x^2)/(b*x^3 - 8),x)
```

output
```
symsum(log((8*C^2)/(-b)^(5/3) + root(27*b^3*z^3 - 27*C*(-b)^(8/3)*z^2 - 9*
C^2*(-b)^(7/3)*z - 9*C^3*b^2, z, k)*((48*C)/(-b)^(4/3) - (72*root(27*b^3*z
^3 - 27*C*(-b)^(8/3)*z^2 - 9*C^2*(-b)^(7/3)*z - 9*C^3*b^2, z, k))/b + (24*
C*x)/b) - (8*C^2*x)/(-b)^(4/3))*root(27*b^3*z^3 - 27*C*(-b)^(8/3)*z^2 - 9*
C^2*(-b)^(7/3)*z - 9*C^3*b^2, z, k), k, 1, 3)
```

3.32 $\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx$

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3.32.1 Optimal result

Integrand size = 26, antiderivative size = 47

$$\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx=\frac{C\arctan\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

output
```
-1/4*C*ln((-a)^(1/3)+2*x)+1/6*C*arctan(1/3*(1-4*x/(-a)^(1/3))*3^(1/2))*3^(
1/2)
```

3.32.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(47) = 94.

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.26

$$\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx=\frac{C\left(2\sqrt{3}(-a)^{2/3}\arctan\left(\frac{1+\frac{4x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2(-a)^{2/3}\log\left(\sqrt[3]{a}-2x\right)+(-a)^{2/3}\log\left(a^{2/3}\right.\right.}{12a^{2/3}}$$

input
```
Integrate[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
```

output
```
(C*(2*Sqrt[3]*(-a)^(2/3)*ArcTan[(1 + (4*x)/a^(1/3))/Sqrt[3]] - 2*(-a)^(2/3
)*Log[a^(1/3) - 2*x] + (-a)^(2/3)*Log[a^(2/3) + 2*a^(1/3)*x + 4*x^2] - a^(
2/3)*Log[-a + 8*x^3]))/(12*a^(2/3))
```

3.32.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2403, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-a)^{2/3}C+2Cx^2}{a-8x^3}\,dx$$

$$\Big\downarrow \; 2403$$

$$-\frac{1}{8}\sqrt[3]{-a}C\int \frac{1}{x^2-\frac{1}{2}\sqrt[3]{-a}x+\frac{1}{4}(-a)^{2/3}}dx-\frac{1}{4}C\int \frac{1}{x+\frac{\sqrt[3]{-a}}{2}}dx$$

$$\Big\downarrow \; 16$$

$$-\frac{1}{8}\sqrt[3]{-a}C\int \frac{1}{x^2-\frac{1}{2}\sqrt[3]{-a}x+\frac{1}{4}(-a)^{2/3}}dx-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

$$\Big\downarrow \; 1082$$

$$-\frac{1}{2}C\int \frac{1}{-\left(1-\frac{4x}{\sqrt[3]{-a}}\right)^2-3}d\left(1-\frac{4x}{\sqrt[3]{-a}}\right)-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

$$\Big\downarrow \; 217$$

$$\frac{C\arctan\left(\frac{1-\frac{4x}{\sqrt[3]{-a}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}C\log\left(\sqrt[3]{-a}+2x\right)$$

input
```
Int[((-a)^(2/3)*C + 2*C*x^2)/(a - 8*x^3),x]
```

output
```
(C*ArcTan[(1 - (4*x)/(-a)^(1/3))/Sqrt[3]])/(2*Sqrt[3]) - (C*Log[(-a)^(1/3)
 + 2*x])/4
```

3.32.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2403
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a)^(1/3)/(-b)^(1/3)},
Simp[C/b   Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^
2), x], x]] /; EqQ[A*(-b)^(2/3) - (-a)^(1/3)*(-b)^(1/3)*B - 2*(-a)^(2/3)*C,
 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(36) = 72$.

Time = 1.52 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

method	result
default	$C\left((-a)^{\frac{2}{3}}\left(-\frac{8^{\frac{2}{3}}\ln\left(x-\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}}{8}\right)}{24a^{\frac{2}{3}}}+\frac{8^{\frac{2}{3}}\ln\left(x^2+\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}x}{8}+\frac{8^{\frac{1}{3}}a^{\frac{2}{3}}}{8}\right)}{48a^{\frac{2}{3}}}+\frac{8^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,8^{\frac{1}{3}}x}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{24a^{\frac{2}{3}}}\right)-\frac{\ln(-8x^3+a)}{12}\right.$

input
```
int(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*((-a)^(2/3)*(-1/24*8^(2/3)/a^(2/3)*ln(x-1/8*8^(2/3)*a^(1/3))+1/48*8^(2/3
)/a^(2/3)*ln(x^2+1/8*8^(2/3)*a^(1/3)*x+1/8*8^(1/3)*a^(2/3))+1/24*8^(2/3)/a
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*8^(1/3)/a^(1/3)*x+1)))-1/12*ln(-8*x^3
+a))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{(-a)^{2/3}C + 2Cx^2}{a - 8x^3}\,dx = \frac{1}{6}\sqrt{3}C\arctan\left(\frac{4\sqrt{3}(-a)^{\frac{2}{3}}x + \sqrt{3}a}{3a}\right) - \frac{1}{4}C\log\left(2x + (-a)^{\frac{1}{3}}\right)$$

input
```
integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*C*arctan(1/3*(4*sqrt(3)*(-a)^(2/3)*x + sqrt(3)*a)/a) - 1/4*C*l
og(2*x + (-a)^(1/3))
```

3.32.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.02

$$\int \frac{(-a)^{2/3}C + 2Cx^2}{a - 8x^3}\,dx = -C\left(\frac{\log\left(-\frac{a}{2(-a)^{\frac{2}{3}}} + x\right)}{4} + \frac{\sqrt{3}i\log\left(\frac{a}{4(-a)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{4(-a)^{\frac{2}{3}}} + x\right)}{12} - \frac{\sqrt{3}i\log\left(\frac{a}{4(-a)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{4(-a)^{\frac{2}{3}}} + x\right)}{12}\right)$$

input
```
integrate(((-a)**(2/3)*C+2*C*x**2)/(-8*x**3+a),x)
```

output
```
-C*(log(-a/(2*(-a)**(2/3)) + x)/4 + sqrt(3)*I*log(a/(4*(-a)**(2/3)) - sqrt
(3)*I*a/(4*(-a)**(2/3)) + x)/12 - sqrt(3)*I*log(a/(4*(-a)**(2/3)) + sqrt(3
)*I*a/(4*(-a)**(2/3)) + x)/12)
```

3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(36) = 72$.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.98

$$\int \frac{(-a)^{2/3}C + 2Cx^2}{a - 8x^3}\,dx = \frac{\sqrt{3}C(-a)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(4\,x+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{6\,a^{\frac{2}{3}}} + \frac{\left(C(-a)^{\frac{2}{3}} - Ca^{\frac{2}{3}}\right)\log\left(4\,x^2 + 2\,a^{\frac{1}{3}}x + a^{\frac{2}{3}}\right)}{12\,a^{\frac{2}{3}}} - \frac{\left(2\,C(-a)^{\frac{2}{3}} + Ca^{\frac{2}{3}}\right)\log\left(x - \frac{1}{2}\,a^{\frac{1}{3}}\right)}{12\,a^{\frac{2}{3}}}$$

input
```
integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*C*(-a)^(2/3)*arctan(1/3*sqrt(3)*(4*x + a^(1/3))/a^(1/3))/a^(2/
3) + 1/12*(C*(-a)^(2/3) - C*a^(2/3))*log(4*x^2 + 2*a^(1/3)*x + a^(2/3))/a^
(2/3) - 1/12*(2*C*(-a)^(2/3) + C*a^(2/3))*log(x - 1/2*a^(1/3))/a^(2/3)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{(-a)^{2/3}C + 2Cx^2}{a - 8x^3}\,dx = \frac{1}{6}\,\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(4\,x + a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{\left(2\,C(-a)^{\frac{2}{3}} + Ca^{\frac{2}{3}}\right)\log\left(\left|x - \frac{1}{2}\,a^{\frac{1}{3}}\right|\right)}{12\,a^{\frac{2}{3}}}$$

input
```
integrate(((-a)^(2/3)*C+2*C*x^2)/(-8*x^3+a),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*C*arctan(1/3*sqrt(3)*(4*x + a^(1/3))/a^(1/3)) - 1/12*(2*C*(-a)
^(2/3) + C*a^(2/3))*log(abs(x - 1/2*a^(1/3)))/a^(2/3)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.02

$$\int \frac{(-a)^{2/3}C + 2Cx^2}{a - 8x^3}\,dx = \sum_{k=1}^{3} \ln\left(-\frac{(C + 12\operatorname{root}(1728\,a^2\,z^3 + 432\,C\,a^2\,z^2 + 36\,C^2\,a^2\,z + 9\,C^3\,a^2, z, k))\,\left(C\right.}{}\right.$$

input
```
int((2*C*x^2 + C*(-a)^(2/3))/(a - 8*x^3),x)
```

output
```
symsum(log(-((C + 12*root(1728*a^2*z^3 + 432*C*a^2*z^2 + 36*C^2*a^2*z + 9*
C^3*a^2, z, k))*(C*a + 12*root(1728*a^2*z^3 + 432*C*a^2*z^2 + 36*C^2*a^2*z
 + 9*C^3*a^2, z, k)*a + 4*C*(-a)^(2/3)*x))/128)*root(1728*a^2*z^3 + 432*C*
a^2*z^2 + 36*C^2*a^2*z + 9*C^3*a^2, z, k), k, 1, 3)
```

3.33 $\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$

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3.33.1 Optimal result

Integrand size = 28, antiderivative size = 50

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=-\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

output
```
C*ln((a/b)^(1/3)+x)/b-2/3*C*arctan(1/3*(1-2*x/(a/b)^(1/3))*3^(1/2))/b*3^(1
/2)
```

3.33.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(50) = 100.

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.92

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=\frac{C\left(-2\sqrt{3}\left(\frac{a}{b}\right)^{2/3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(\frac{a}{b}\right)^{2/3}b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\left(\frac{a}{b}\right)^{2/3}\right.}{3a^{2/3}b}$$

input
```
Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

output
```
(C*(-2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - (a/b)^(2/3)*b^(2/3
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*Log[a + b*x^3])
)/(3*a^(2/3)*b)
```

3.33.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2406, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2C\left(\frac{a}{b}\right)^{2/3}+Cx^2}{a+bx^3}\,dx$$

$$\Big\downarrow \text{2406}$$

$$\frac{C\sqrt[3]{\frac{a}{b}}\int \frac{1}{x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}}dx}{b}+\frac{C\int \frac{1}{x+\sqrt[3]{\frac{a}{b}}}dx}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{C\sqrt[3]{\frac{a}{b}}\int \frac{1}{x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}}dx}{b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2C\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

input
```
Int[(2*(a/b)^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

output
```
(-2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^
(1/3) + x])/b
```

3.33.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2406
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Simp[C/b
 Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]
] /; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Po
lyQ[P2, x, 2]
```

3.33.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(43) = 86.

Time = 1.51 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.32

method	result	size
default	$C\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\frac{\ln\left(b\,x^{3}+a\right)}{3b}\right)$	116

input
```
int((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*(2*(a/b)^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1)))+1/3*ln(b*x^3+a)/b)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^{2}}{a+bx^{3}}\,dx=\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)+3\,C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

input
```
integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 3
*C*log(x + (a/b)^(1/3)))/b
```

3.33.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.00

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=\frac{C\left(\log\left(\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

input
```
integrate((2*(a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

output
```
C*(log(a/(b*(a/b)**(2/3)) + x) - sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) - sqr
t(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) +
 sqrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}+\frac{C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

input
```
integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/b + C*lo
g(x + (a/b)^(1/3))/b
```

3.33.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.86

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx = -\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b} - \frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2} + \frac{\left(3ab^2+i\sqrt{3}\sqrt{a^2b^4}\right)C\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^3}$$

input
```
integrate((2*(a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1
/3*(C*b^2*(-a/b)^(2/3) + 2*(a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b
)^(1/3)))/(a*b^2) + 1/6*(3*a*b^2 + I*sqrt(3)*sqrt(a^2*b^4))*C*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.44

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx = \sum_{k=1}^{3}\ln\left(-\frac{\left(C-\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z-9\,C^3\,a^2,z,k\right)\,b\,3\right)\,\left(\cdot\right.}{}\right.$$

input
```
int((C*x^2 + 2*C*(a/b)^(2/3))/(a + b*x^3),x)
```

output
```
symsum(log(-((C - 3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
 - 9*C^3*a^2, z, k)*b)*(3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a
^2*b*z - 9*C^3*a^2, z, k)*a*b - C*a + 2*C*b*x*(a/b)^(2/3)))/b^3)*root(27*a
^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z - 9*C^3*a^2, z, k), k, 1, 3)
```

3.34 $\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx$

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3.34.1 Optimal result

Integrand size = 30, antiderivative size = 53

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

output
```
-C*ln((-a/b)^(1/3)+x)/b+2/3*C*arctan(1/3*(1-2*x/(-a/b)^(1/3))*3^(1/2))/b*3
^(1/2)
```

3.34.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 150 vs. 2(53) = 106.

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.83

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=\frac{C\left(2\sqrt{3}\left(-\frac{a}{b}\right)^{2/3}b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\left(-\frac{a}{b}\right)^{2/3}b^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+(-\right.}{3a^{2/3}b}$$

input
```
Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

output
```
(C*(2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] - 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (-(a/b))^(2/3
)*b^(2/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*Log[a -
 b*x^3]))/(3*a^(2/3)*b)
```

3.34.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2406, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2C\left(-\frac{a}{b}\right)^{2/3}+Cx^2}{a-bx^3}dx$$

$$\Big\downarrow \text{2406}$$

$$-\frac{C\sqrt[3]{-\frac{a}{b}}\int \frac{1}{x^2-\sqrt[3]{-\frac{a}{b}}x+\left(-\frac{a}{b}\right)^{2/3}}dx}{b}-\frac{C\int \frac{1}{x+\sqrt[3]{-\frac{a}{b}}}dx}{b}$$

$$\Big\downarrow \text{16}$$

$$-\frac{C\sqrt[3]{-\frac{a}{b}}\int \frac{1}{x^2-\sqrt[3]{-\frac{a}{b}}x+\left(-\frac{a}{b}\right)^{2/3}}dx}{b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2C\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

input
```
Int[(2*(-(a/b))^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

output
```
(2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(-(a
/b))^(1/3) + x])/b
```

3.34.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2406
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Simp[C/b
 Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]
] /; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Po
lyQ[P2, x, 2]
```

3.34.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(46) = 92$.

Time = 1.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

method	result	siz
default	$C\left(2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-\frac{\ln\left(-b\,x^{3}+a\right)}{3b}\right)$	119

input
```
int((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*(2*(-a/b)^(2/3)*(-1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(2/3)*
ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*(1+
2/(a/b)^(1/3)*x)*3^(1/2)))-1/3*ln(-b*x^3+a)/b)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)-3\,C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

input
```
integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(3)*a)/a) -
3*C*log(x + (-a/b)^(1/3)))/b
```

3.34.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.08

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C + Cx^2}{a - bx^3}\,dx = -\frac{C\left(\log\left(-\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x\right) + \frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x\right)}{3} - \frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x\right)}{3}\right)}{b}$$

input
```
integrate((2*(-a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

output
```
-C*(log(-a/(b*(-a/b)**(2/3)) + x) + sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3)) -
sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3
)) + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.15

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C + Cx^2}{a - bx^3}\,dx = -\frac{2\sqrt{3}\left(Ca - \left(3C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}} + \frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab} - \frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}} - C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*(C*a - (3*C*(a/b)^(1/3)*(-a/b)^(2/3) + C*a/b)*b)*arctan(1/3*s
qrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b) - 1/3*(C*(a/b)^(2/3) - C*(-a
/b)^(2/3))*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(C
*(a/b)^(2/3) + 2*C*(-a/b)^(2/3))*log(x - (a/b)^(1/3))/(b*(a/b)^(2/3))
```

3.34.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.06

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx = -\frac{\sqrt{3}\left(ab^2-i\sqrt{3}\sqrt{a^2b^4}\right)C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^3} - \frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

input
```
integrate((2*(-a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(a*b^2 - I*sqrt(3)*sqrt(a^2*b^4))*C*arctan(1/3*sqrt(3)*(2*x +
 (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/3*(C*b^2*(a/b)^(2/3) + 2*(-a*b^2)^(
2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/(a*b^2)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.25

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx = \sum_{k=1}^{3}\ln\left(-\frac{\left(C+\mathrm{root}(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z+9\,C^3\,a^2,z,k)\,b\,3\right)}{}\right.$$

input
```
int((C*x^2 + 2*C*(-a/b)^(2/3))/(a - b*x^3),x)
```

output
```
symsum(log(-((C + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
 + 9*C^3*a^2, z, k)*b)*(C*a + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9
*C^2*a^2*b*z + 9*C^3*a^2, z, k)*a*b + 2*C*b*x*(-a/b)^(2/3)))/b^3)*root(27*
a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z + 9*C^3*a^2, z, k), k, 1, 3
)
```

3.35 $\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx$

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3.35.1 Optimal result

Integrand size = 29, antiderivative size = 54

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=-\frac{2C\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

output
```
C*ln((-a/b)^(1/3)-x)/b-2/3*C*arctan(1/3*(1+2*x/(-a/b)^(1/3))*3^(1/2))/b*3^
(1/2)
```

3.35.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 149 vs. 2(54) = 108.

Time = 0.05 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.76

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=\frac{C\left(-2\sqrt{3}\left(-\frac{a}{b}\right)^{2/3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(-\frac{a}{b}\right)^{2/3}b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-(\right.}{3a^{2/3}b}$$

input
```
Integrate[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

output
```
(C*(-2*Sqrt[3]*(-(a/b))^(2/3)*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]] + 2*(-(a/b))^(2/3)*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - (-(a/b))^(2/
3)*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*Log[a
+ b*x^3]))/(3*a^(2/3)*b)
```

3.35.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2408, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2C\left(-\frac{a}{b}\right)^{2/3}+Cx^2}{a+bx^3}dx$$

$$\Big\downarrow \text{2408}$$

$$-\frac{C\sqrt[3]{-\frac{a}{b}}\int \frac{1}{x^2+\sqrt[3]{-\frac{a}{b}}x+\left(-\frac{a}{b}\right)^{2/3}}dx}{b}-\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}dx}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{C\log \left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{C\sqrt[3]{-\frac{a}{b}}\int \frac{1}{x^2+\sqrt[3]{-\frac{a}{b}}x+\left(-\frac{a}{b}\right)^{2/3}}dx}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2C\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1\right)}{b}+\frac{C\log \left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{2C\arctan\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}$$

input
```
Int[(2*(-(a/b))^(2/3)*C + C*x^2)/(a + b*x^3),x]
```

output
```
(-2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(-(
a/b))^(1/3) - x])/b
```

3.35.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2408
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Simp[-C/b
   Int[1/(q - x), x], x] + Simp[(B - C*q)/b   Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &
& PolyQ[P2, x, 2]
```

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(47) = 94.

Time = 1.48 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.17

method	result	size
default	$C\left(2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\frac{\ln\left(b\,x^{3}+a\right)}{3b}\right)$	117

input
```
int((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*(2*(-a/b)^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1)))+1/3*ln(b*x^3+a)/b)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx=\frac{2\,\sqrt{3}C\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)+3\,C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

input
```
integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) +
3*C*log(x - (-a/b)^(1/3)))/b
```

3.35.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.02

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx = \frac{C\left(\log\left(\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

input
```
integrate((2*(-a/b)**(2/3)*C+C*x**2)/(b*x**3+a),x)
```

output
```
C*(log(a/(b*(-a/b)**(2/3)) + x) - sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3)) - s
qrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3
)) + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(47) = 94.

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.11

$$\begin{aligned}\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx &= \\ &-\frac{2\sqrt{3}\left(Ca-\left(3\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} \\ &+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

input
```
integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*(C*a - (3*C*(a/b)^(1/3)*(-a/b)^(2/3) + C*a/b)*b)*arctan(1/3*s
qrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/3*(C*(a/b)^(2/3) - C*(-a
/b)^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(C
*(a/b)^(2/3) + 2*C*(-a/b)^(2/3))*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.69

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx = -\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b} - \frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

input
```
integrate((2*(-a/b)^(2/3)*C+C*x^2)/(b*x^3+a),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1
/3*(C*b^2*(-a/b)^(2/3) + 2*(-a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/(a*b^2)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.20

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3}C+Cx^2}{a+bx^3}\,dx = \sum_{k=1}^{3}\ln\left(-\frac{\left(C-\mathrm{root}(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z-9\,C^3\,a^2,z,k)\;b\,3\right)}{}\right.$$

input
```
int((C*x^2 + 2*C*(-a/b)^(2/3))/(a + b*x^3),x)
```

output
```
symsum(log(-((C - 3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
 - 9*C^3*a^2, z, k)*b)*(3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a
^2*b*z - 9*C^3*a^2, z, k)*a*b - C*a + 2*C*b*x*(-a/b)^(2/3)))/b^3)*root(27*
a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z - 9*C^3*a^2, z, k), k, 1, 3
)
```

3.36 $\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx$

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3.36.1 Optimal result

Integrand size = 29, antiderivative size = 53

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=\frac{2C\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

output
```
-C*ln((a/b)^(1/3)-x)/b+2/3*C*arctan(1/3*(1+2*x/(a/b)^(1/3))*3^(1/2))/b*3^(
1/2)
```

3.36.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 147 vs. 2(53) = 106.

Time = 0.05 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.77

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=\frac{C\left(2\sqrt{3}\left(\frac{a}{b}\right)^{2/3}b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\left(\frac{a}{b}\right)^{2/3}b^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+\left(\frac{a}{b}\right)^{2/3}b^{2}\right.}{3a^{2/3}b}$$

input
```
Integrate[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

output
```
(C*(2*Sqrt[3]*(a/b)^(2/3)*b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] - 2*(a/b)^(2/3)*b^(2/3)*Log[a^(1/3) - b^(1/3)*x] + (a/b)^(2/3)*b^(2/3)
*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*Log[a - b*x^3]))
/(3*a^(2/3)*b)
```

3.36.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2408, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2C\left(\frac{a}{b}\right)^{2/3}+Cx^2}{a-bx^3}dx$$

$$\Big\downarrow \text{2408}$$

$$\frac{C\sqrt[3]{\frac{a}{b}}\int \frac{1}{x^2+\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}}dx}{b}+\frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}dx}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{C\sqrt[3]{\frac{a}{b}}\int \frac{1}{x^2+\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}}dx}{b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2C\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1\right)}{b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{2C\arctan\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

input
```
Int[(2*(a/b)^(2/3)*C + C*x^2)/(a - b*x^3),x]
```

output
```
(2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^(
1/3) - x])/b
```

3.36.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2408
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Simp[-C/b
   Int[1/(q - x), x], x] + Simp[(B - C*q)/b   Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &
& PolyQ[P2, x, 2]
```

3.36.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(46) = 92.

Time = 1.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.23

method	result	size
default	$C\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-\frac{\ln\left(-b\,x^{3}+a\right)}{3b}\right)$	118

input
```
int((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*(2*(a/b)^(2/3)*(-1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(2/3)*l
n(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*(1+2
/(a/b)^(1/3)*x)*3^(1/2)))-1/3*ln(-b*x^3+a)/b)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx=\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right)-3\,C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

input
```
integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) + sqrt(3)*a)/a) - 3
*C*log(x - (a/b)^(1/3)))/b
```

3.36.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx = -\frac{C\left(\log\left(-\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

input
```
integrate((2*(a/b)**(2/3)*C+C*x**2)/(-b*x**3+a),x)
```

output
```
-C*(log(-a/(b*(a/b)**(2/3)) + x) + sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) - sq
rt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) +
 sqrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx = \frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

input
```
integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - C*lo
g(x - (a/b)^(1/3))/b
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.60

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx = \frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b} - \frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

input
```
integrate((2*(a/b)^(2/3)*C+C*x^2)/(-b*x^3+a),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - 1/3*
(C*b^2*(a/b)^(2/3) + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3
)))/(a*b^2)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.23

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3}C+Cx^2}{a-bx^3}\,dx = \sum_{k=1}^{3}\ln\left(-\frac{\left(C+\mathrm{root}(27\,a^2\,b^3\,z^3+27\,C\,a^2\,b^2\,z^2+9\,C^2\,a^2\,b\,z+9\,C^3\,a^2,z,k)\,b\,3\right)\left(C\right.}{}\right.$$

input
```
int((C*x^2 + 2*C*(a/b)^(2/3))/(a - b*x^3),x)
```

output
```
symsum(log(-((C + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z
 + 9*C^3*a^2, z, k)*b)*(C*a + 3*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9
*C^2*a^2*b*z + 9*C^3*a^2, z, k)*a*b + 2*C*b*x*(a/b)^(2/3)))/b^3)*root(27*a
^2*b^3*z^3 + 27*C*a^2*b^2*z^2 + 9*C^2*a^2*b*z + 9*C^3*a^2, z, k), k, 1, 3)
```

3.37 $\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx$

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3.37.1 Optimal result

Integrand size = 29, antiderivative size = 61

$$\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx=-\frac{2C\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

output
```
C*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-2/3*C*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/b^(1/3)*3^(1/2)
```

3.37.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56

$$\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx=\frac{C\left(-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right.\right.}{3\sqrt[3]{b}}$$

input
```
Integrate[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

output
```
(C*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3)
 + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + Log[a + b
*x^3]))/(3*b^(1/3))
```

3.37.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2402, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx$$

$$\Big\downarrow \text{2402}$$

$$\frac{\sqrt[3]{a}C\int \frac{1}{x^2-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+\frac{a^{2/3}}{b^{2/3}}}dx}{b^{2/3}}+\frac{C\int \frac{1}{x+\frac{\sqrt[3]{a}}{\sqrt[3]{b}}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\sqrt[3]{a}C\int \frac{1}{x^2-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+\frac{a^{2/3}}{b^{2/3}}}dx}{b^{2/3}}+\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2C\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2C\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

input
```
Int[(2*a^(2/3)*C + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

output
```
(-2*C*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (C*
Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

3.37.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2402
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Simp[C
/b   Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x]
, x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a,
b}, x] && PolyQ[P2, x, 2]
```

3.37.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(44) = 88$.

Time = 1.56 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.84

method	result	size
default	$C\left(2a^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\frac{\ln\left(b\,x^{3}+a\right)}{3b^{\frac{1}{3}}}\right)$	112

input
```
int((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*(2*a^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1)))+1/3/b^(1/3)*ln(b*x^3+a))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.62

$$\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx = \left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3-3\,a^{\frac{2}{3}}b^{\frac{1}{3}}x+3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{1}{3}}bx^2+a^{\frac{2}{3}}b^{\frac{2}{3}}x-ab^{\frac{1}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-a}{bx^3+a}\right)+Cb^{\frac{2}{3}}\log\left(b\right.}{b}\right.$$

input
```
integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sq
rt(1/3)*(2*a^(1/3)*b*x^2 + a^(2/3)*b^(2/3)*x - a*b^(1/3))*sqrt(-1/b^(2/3))
 - a)/(b*x^3 + a)) + C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b, (2*sqrt(1/3)
*C*b^(2/3)*arctan(sqrt(1/3)*(2*a^(2/3)*b^(2/3)*x - a*b^(1/3))/(a*b^(1/3)))
 + C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b]
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{2a^{2/3}C+b^{2/3}Cx^2}{a+bx^3}\,dx = \operatorname{RootSum}\left(3t^3b^{\frac{5}{3}}-3t^2Cb^{\frac{4}{3}}+tC^2b-C^3b^{\frac{2}{3}},\left(t\mapsto t\log\left(x+\frac{3t\sqrt[3]{a}\sqrt[3]{b}-C\sqrt[3]{a}}{2C\sqrt[3]{b}}\right)\right.\right.$$

input
```
integrate((2*a**(2/3)*C+b**(2/3)*C*x**2)/(b*x**3+a),x)
```

output
```
RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
 Lambda(_t, _t*log(x + (3*_t*a**(1/3)*b**(1/3) - C*a**(1/3))/(2*C*b**(1/3)
))))
```

3.37.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(46) = 92$.

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.66

$$\int \frac{2a^{2/3}C + b^{2/3}Cx^2}{a + bx^3}\,dx = -\frac{2\sqrt{3}\left(Cab^{\frac{2}{3}} - \left(3\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \frac{Ca}{b^{\frac{1}{3}}}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} + \frac{\left(Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} - Ca^{\frac{2}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2\,Ca^{\frac{2}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*(C*a*b^(2/3) - (3*C*a^(2/3)*(a/b)^(1/3) + C*a/b^(1/3))*b)*arc
tan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/3*(C*b^(2/3)*(a
/b)^(2/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/
3)) + 1/3*(C*b^(2/3)*(a/b)^(2/3) + 2*C*a^(2/3))*log(x + (a/b)^(1/3))/(b*(a
/b)^(2/3))
```

3.37.8 Giac [F(-1)]

Timed out.

$$\int \frac{2a^{2/3}C + b^{2/3}Cx^2}{a + bx^3}\,dx = \text{Timed out}$$

input
```
integrate((2*a^(2/3)*C+b^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

output
```
Timed out
```

3.37.9 Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.16

$$\int \frac{2a^{2/3}C + b^{2/3}Cx^2}{a + bx^3}\,dx = \sum_{k=1}^{3} \ln\left(-\frac{a^{2/3}\left(C - \operatorname{root}\left(27\,a^2\,b^3\,z^3 - 27\,C\,a^2\,b^{8/3}\,z^2 + 9\,C^2\,a^2\,b^{7/3}\,z - 9\,C^3\,a^2\,b^2,\right.\right.}{}\right.$$

input
```
int((2*C*a^(2/3) + C*b^(2/3)*x^2)/(a + b*x^3),x)
```

output
```
symsum(log(-(a^(2/3)*(C - 3*root(27*a^2*b^3*z^3 - 27*C*a^2*b^(8/3)*z^2 + 9
*C^2*a^2*b^(7/3)*z - 9*C^3*a^2*b^2, z, k)*b^(1/3))*(3*root(27*a^2*b^3*z^3
- 27*C*a^2*b^(8/3)*z^2 + 9*C^2*a^2*b^(7/3)*z - 9*C^3*a^2*b^2, z, k)*a^(1/3
)*b^(1/3) - C*a^(1/3) + 2*C*b^(1/3)*x))/b^(5/3))*root(27*a^2*b^3*z^3 - 27*
C*a^2*b^(8/3)*z^2 + 9*C^2*a^2*b^(7/3)*z - 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

3.38 $\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

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3.38.2 Mathematica [A] (verified) 465
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3.38.1 Optimal result

Integrand size = 32, antiderivative size = 70

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = -\frac{2C\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{-b}}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

output
```
C*ln(a^(1/3)-(-b)^(1/3)*x)/(-b)^(1/3)-2/3*C*arctan(1/3*(a^(1/3)+2*(-b)^(1/
3)*x)/a^(1/3)*3^(1/2))/(-b)^(1/3)*3^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.66

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx =$$

$$-\frac{C\left(-2\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+(-b)^{2/3}}{3b}$$

input
```
Integrate[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

output
```
-1/3*(C*(-2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] +
2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2] + (-b)^(2/3)*Log[a + b*x^3]))/b
```

3.38.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2405, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}dx$$

$$\Big\downarrow \text{2405}$$

$$-\frac{\sqrt[3]{a}C\int \frac{1}{x^2+\frac{\sqrt[3]{a}x}{\sqrt[3]{-b}}+\frac{a^{2/3}}{(-b)^{2/3}}}dx}{(-b)^{2/3}}-\frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{-b}}-x}dx}{\sqrt[3]{-b}}$$

$$\Big\downarrow \text{16}$$

$$\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}-\frac{\sqrt[3]{a}C\int \frac{1}{x^2+\frac{\sqrt[3]{a}x}{\sqrt[3]{-b}}+\frac{a^{2/3}}{(-b)^{2/3}}}dx}{(-b)^{2/3}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2C\int \frac{1}{-\left(\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{-b}}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}-\frac{2C\arctan\left(\frac{\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-b}}$$

input
```
Int[(-2*a^(2/3)*C - (-b)^(2/3)*C*x^2)/(a + b*x^3),x]
```

output
```
(-2*C*ArcTan[(1 + (2*(-b)^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*(-b)^(1/3))
 + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1/3)
```

3.38.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2405
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/(-b)^(1/3)}, Sim
p[-C/b   Int[1/(q - x), x], x] + Simp[(B - C*q)/b   Int[1/(q^2 + q*x + x^2)
, x], x]] /; EqQ[A*(-b)^(2/3) + a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C, 0]] /;
FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.38.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(53) = 106$.

Time = 1.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.67

method	result	size
default	$C\left(-2a^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-\frac{(-b)^{\frac{2}{3}}\ln\left(b\,x^{3}+a\right)}{3b}\right)$	117

input
```
int((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*(-2*a^(2/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1)))-1/3*(-b)^(2/3)*ln(b*x^3+a)/b)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.93

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = \left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,bx^3+3\,a^{\frac{2}{3}}(-b)^{\frac{1}{3}}x-3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{1}{3}}bx^2+a^{\frac{2}{3}}(-b)^{\frac{2}{3}}x+a(-b)^{\frac{1}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}-a}{bx^3+a}\right.}{b}\right.$$

input
```
integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[(sqrt(1/3)*C*b*sqrt((-b)^(1/3)/b)*log((2*b*x^3 + 3*a^(2/3)*(-b)^(1/3)*x -
 3*sqrt(1/3)*(2*a^(1/3)*b*x^2 + a^(2/3)*(-b)^(2/3)*x + a*(-b)^(1/3))*sqrt(
(-b)^(1/3)/b) - a)/(b*x^3 + a)) - C*(-b)^(2/3)*log(b*x + a^(1/3)*(-b)^(2/3
)))/b, -(2*sqrt(1/3)*C*b*sqrt(-(-b)^(1/3)/b)*arctan(sqrt(1/3)*(2*a^(2/3)*(
-b)^(2/3)*x + a*(-b)^(1/3))*sqrt(-(-b)^(1/3)/b)/a) + C*(-b)^(2/3)*log(b*x
+ a^(1/3)*(-b)^(2/3)))/b]
```

3.38.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = -\operatorname{RootSum}\left(3t^3b^2-3t^2Cb(-b)^{\frac{2}{3}}+tC^2(-b)^{\frac{4}{3}}-C^3b,\left(t\mapsto t\log\left(\frac{3t\sqrt[3]{a}}{2C}-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b}+x\right)\right)\right)$$

input
```
integrate((-2*a**(2/3)*C-(-b)**(2/3)*C*x**2)/(b*x**3+a),x)
```

output
```
-RootSum(3*_t**3*b**2 - 3*_t**2*C*b*(-b)**(2/3) + _t*C**2*(-b)**(4/3) - C*
*3*b, Lambda(_t, _t*log(3*_t*a**(1/3)/(2*C) - a**(1/3)*(-b)**(2/3)/(2*b) +
 x)))
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(53) = 106.

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.47

$$\int \frac{-2a^{2/3}C-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = \frac{2\sqrt{3}\left(Ca(-b)^{\frac{2}{3}}-\left(3\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{Ca(-b)^{\frac{2}{3}}}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} - \frac{\left(C(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-Ca^{\frac{2}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(C(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,Ca^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*(C*a*(-b)^(2/3) - (3*C*a^(2/3)*(a/b)^(1/3) + C*a*(-b)^(2/3)/b)
*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) - 1/3*(C*(-b
)^(2/3)*(a/b)^(2/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b
*(a/b)^(2/3)) - 1/3*(C*(-b)^(2/3)*(a/b)^(2/3) + 2*C*a^(2/3))*log(x + (a/b)
^(1/3))/(b*(a/b)^(2/3))
```

3.38.8 Giac [F(-1)]

Timed out.

$$\int \frac{-2a^{2/3}C - (-b)^{2/3}Cx^2}{a + bx^3}\,dx = \text{Timed out}$$

input
```
integrate((-2*a^(2/3)*C-(-b)^(2/3)*C*x^2)/(b*x^3+a),x, algorithm="giac")
```

output
```
Timed out
```

3.38.9 Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.16

$$\int \frac{-2a^{2/3}C - (-b)^{2/3}Cx^2}{a + bx^3}\,dx = \sum_{k=1}^{3} \ln \left(\text{root}\left(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,(-b)^{8/3}\,z^2 - 9\,C^2\,a^2\,(-b)^{7/3}\,z + 9\,C^3\,a^2\,b^2, z\right.\right.$$

input
```
int(-(2*C*a^(2/3) + C*(-b)^(2/3)*x^2)/(a + b*x^3),x)
```

output
```
symsum(log(root(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 - 9*C^2*a^2*(-b)^
(7/3)*z + 9*C^3*a^2*b^2, z, k)*((6*C*a)/(-b)^(4/3) + (9*root(27*a^2*b^3*z^
3 + 27*C*a^2*(-b)^(8/3)*z^2 - 9*C^2*a^2*(-b)^(7/3)*z + 9*C^3*a^2*b^2, z, k
)*a)/b - (6*C*a^(2/3)*x)/b) - (C^2*a)/(-b)^(5/3) - (2*C^2*a^(2/3)*x)/(-b)^
(4/3))*root(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 - 9*C^2*a^2*(-b)^(7/3
)*z + 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

3.39 $\int \frac{-3+x^2}{-1+x^3} \, dx$

3.39.1 Optimal result 471
3.39.2 Mathematica [A] (verified) 471
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3.39.1 Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{-3+x^2}{-1+x^3} \, dx = \sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - \frac{2}{3}\log(1-x) + \frac{5}{6}\log\left(1+x+x^2\right)$$

output
```
-2/3*ln(1-x)+5/6*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \frac{-3+x^2}{-1+x^3} \, dx = \sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - \log(1-x) + \frac{1}{2}\log\left(1+x+x^2\right) + \frac{1}{3}\log\left(1-x^3\right)$$

input
```
Integrate[(-3 + x^2)/(-1 + x^3),x]
```

output
```
Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[1 - x] + Log[1 + x + x^2]/2 + Log[
1 - x^3]/3
```

3.39.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2414, 16, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-3}{x^3-1}\,dx$$

$$\Big\downarrow \text{2414}$$

$$\frac{2}{3}\int \frac{1}{1-x}dx-\frac{1}{3}\int -\frac{5x+7}{x^2+x+1}dx$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{3}\int -\frac{5x+7}{x^2+x+1}dx-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{5x+7}{x^2+x+1}dx-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{9}{2}\int \frac{1}{x^2+x+1}dx+\frac{5}{2}\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{5}{2}\int \frac{2x+1}{x^2+x+1}dx-9\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{5}{2}\int \frac{2x+1}{x^2+x+1}dx+3\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(3\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{5}{2}\log\left(x^2+x+1\right)\right)-\frac{2}{3}\log(1-x)$$

input
```
Int[(-3 + x^2)/(-1 + x^3),x]
```

output
```
(-2*Log[1 - x])/3 + (3*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + (5*Log[1 + x +
x^2])/2)/3
```

3.39.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2414
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Simp[q*((A + B*q
 + C*q^2)/(3*a))   Int[1/(q - x), x], x] + Simp[q/(3*a)   Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

3.39.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{2\ln(-1+x)}{3}+\frac{5\ln(x^2+x+1)}{6}+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}$	32
risch	$-\frac{2\ln(-1+x)}{3}+\frac{5\ln(9x^2+9x+9)}{6}+\sqrt{3}\,\arctan\left(\frac{2(\frac{3}{2}+3x)\sqrt{3}}{9}\right)$	36
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{(x^3)^{\frac{1}{3}}}+\frac{\ln(-x^3+1)}{3}$	73

input
```
int((x^2-3)/(x^3-1),x,method=_RETURNVERBOSE)
```

output
```
-2/3*ln(-1+x)+5/6*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{-3+x^2}{-1+x^3}\,dx=\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

input
```
integrate((x^2-3)/(x^3-1),x, algorithm="fricas")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(x -
 1)
```

3.39.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{-3+x^2}{-1+x^3}\,dx=-\frac{2\log\left(x-1\right)}{3}+\frac{5\log\left(x^2+x+1\right)}{6}+\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)$$

input
```
integrate((x**2-3)/(x**3-1),x)
```

output
```
-2*log(x - 1)/3 + 5*log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqr
t(3)/3)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{-3+x^2}{-1+x^3}\,dx = \sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{6}\log\left(x^2+x+1\right) - \frac{2}{3}\log\left(x-1\right)$$

input
```
integrate((x^2-3)/(x^3-1),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(x -
 1)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{-3+x^2}{-1+x^3}\,dx = \sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{6}\log\left(x^2+x+1\right) - \frac{2}{3}\log\left(|x-1|\right)$$

input
```
integrate((x^2-3)/(x^3-1),x, algorithm="giac")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(abs
(x - 1))
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{-3+x^2}{-1+x^3}\,dx = -\frac{2\,\ln\left(x-1\right)}{3} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{5}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{5}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)$$

input
```
int((x^2 - 3)/(x^3 - 1),x)
```

output
```
log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/2 + 5/6) - log(x - (3^(1/2)*1i
)/2 + 1/2)*((3^(1/2)*1i)/2 - 5/6) - (2*log(x - 1))/3
```

3.39. $\int \frac{-3+x^2}{-1+x^3}\,dx$

3.40 $\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx$

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3.40.1 Optimal result

Integrand size = 49, antiderivative size = 70

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx = -\frac{2\left(\frac{B}{\sqrt[3]{a}}+\frac{C}{\sqrt[3]{b}}\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}+\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

output
```
C*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-2/3*(B/a^(1/3)+C/b^(1/3))*arctan(1/3*(a^(1
/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx = \frac{-2\sqrt{3}\left(\sqrt[3]{b}B+\sqrt[3]{a}C\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\sqrt[3]{a}C\left(2\log\left(\sqrt[3]{\cdots}\right.\right.}{3\sqrt[3]{\cdots}}$$

input
```
Integrate[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/
(a + b*x^3),x]
```

output
```
(-2*Sqrt[3]*(b^(1/3)*B + a^(1/3)*C)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] + a^(1/3)*C*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2] + Log[a + b*x^3]))/(3*a^(1/3)*b^(1/3))
```

3.40.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used = {2402, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2a^{2/3}C+\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}dx$$

$$\Big\downarrow \text{2402}$$

$$\frac{\left(\sqrt[3]{a}C+\sqrt[3]{b}B\right)\int \frac{1}{x^2-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+\frac{a^{2/3}}{b^{2/3}}}dx}{b^{2/3}}+\frac{C\int \frac{1}{x+\frac{\sqrt[3]{a}}{\sqrt[3]{b}}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\left(\sqrt[3]{a}C+\sqrt[3]{b}B\right)\int \frac{1}{x^2-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+\frac{a^{2/3}}{b^{2/3}}}dx}{b^{2/3}}+\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(\sqrt[3]{a}C+\sqrt[3]{b}B\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}\sqrt[3]{b}}+\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}C+\sqrt[3]{b}B\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

input
```
Int[(a^(1/3)*b^(1/3)*B + 2*a^(2/3)*C + b^(2/3)*B*x + b^(2/3)*C*x^2)/(a + b
*x^3),x]
```

output
```
(-2*(b^(1/3)*B + a^(1/3)*C)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(
Sqrt[3]*a^(1/3)*b^(1/3)) + (C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

3.40.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2402
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Simp[C
/b   Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x]
, x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a,
b}, x] && PolyQ[P2, x, 2]
```

3.40.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(51) = 102$.

Time = 1.59 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.10

method	result
default	$\left(a^{\frac{1}{3}} b^{\frac{1}{3}} B+2 a^{\frac{2}{3}} C\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+b^{\frac{2}{3}} B\left(-\frac{\ln\left(x+\right.}{3b}\right.$

input
```
int((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3+a),x,
method=_RETURNVERBOSE)
```

output
```
(a^(1/3)*b^(1/3)*B+2*a^(2/3)*C)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b
/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+b^(2/3)*B*(-1/3/b/(a/b)^(1/3)*ln(x
+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1
/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*C/b^(1/3)*l
n(b*x^3+a)
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(53) = 106.

Time = 1.99 (sec) , antiderivative size = 430, normalized size of antiderivative = 6.14

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx=\left[\frac{\sqrt{\frac{1}{3}}b\sqrt{-\frac{C^2ab^{\frac{1}{3}}+2\,BCa^{\frac{2}{3}}b^{\frac{2}{3}}+B^2a^{\frac{1}{3}}b}{ab}}\log\left(-\frac{C^3a^2+B^3ab-2\left(C^3ab+B^3\right.}{}\right.}{}\right.$$

input
```
integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3
+a),x, algorithm="fricas")
```

output
```
[(sqrt(1/3)*b*sqrt(-(C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b
)/(a*b))*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 + 3*(C^3*a +
B^3*b)*a^(2/3)*b^(1/3)*x - 3*sqrt(1/3)*((2*B^2*b*x^2 + C^2*a*x + B*C*a)*a^
(2/3)*b^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*a^(1/3) - (2*B*C*a*b
*x^2 - B^2*a*b*x + C^2*a^2)*b^(1/3))*sqrt(-(C^2*a*b^(1/3) + 2*B*C*a^(2/3)*
b^(2/3) + B^2*a^(1/3)*b)/(a*b)))/(b*x^3 + a)) + C*b^(2/3)*log(b*x + a^(1/3
)*b^(2/3)))/b, (2*sqrt(1/3)*b*sqrt((C^2*a*b^(1/3) + 2*B*C*a^(2/3)*b^(2/3)
+ B^2*a^(1/3)*b)/(a*b))*arctan(sqrt(1/3)*((2*C^2*x + B*C)*a^(2/3)*b^(2/3)
- (2*B*C*b*x + B^2*b)*a^(1/3) + (2*B^2*b*x - C^2*a)*b^(1/3))*sqrt((C^2*a*b
^(1/3) + 2*B*C*a^(2/3)*b^(2/3) + B^2*a^(1/3)*b)/(a*b))/(C^3*a + B^3*b)) +
C*b^(2/3)*log(b*x + a^(1/3)*b^(2/3)))/b]
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx=\text{Timed out}$$

input
```
integrate((a**(1/3)*b**(1/3)*B+2*a**(2/3)*C+b**(2/3)*B*x+b**(2/3)*C*x**2)/
(b*x**3+a),x)
```

output
```
Timed out
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(53) = 106$.

Time = 0.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.37

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx=$$

$$-\frac{\sqrt{3}\left(2\,Cab^{\frac{2}{3}}-\left(6\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+3\,Ba^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+\frac{2\,Ca}{b}\right)b^{\frac{2}{3}}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$
$$-\frac{\left(2\,Ca^{\frac{2}{3}}+Ba^{\frac{1}{3}}b^{\frac{1}{3}}-\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{\frac{2}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(2\,Ca^{\frac{2}{3}}+Ba^{\frac{1}{3}}b^{\frac{1}{3}}+\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{\frac{2}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3
+a),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*(2*C*a*b^(2/3) - (6*C*a^(2/3)*(a/b)^(1/3) + 3*B*a^(1/3)*b^(1/
3)*(a/b)^(1/3) + (3*B*(a/b)^(2/3) + 2*C*a/b)*b^(2/3))*b)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) - 1/6*(2*C*a^(2/3) + B*a^(1/3)*b^
(1/3) - (2*C*(a/b)^(2/3) + B*(a/b)^(1/3))*b^(2/3))*log(x^2 - x*(a/b)^(1/3)
 + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(2*C*a^(2/3) + B*a^(1/3)*b^(1/3) + (
C*(a/b)^(2/3) - B*(a/b)^(1/3))*b^(2/3))*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3
))
```

3.40.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + 2a^{2/3}C + b^{2/3}Bx + b^{2/3}Cx^2}{a + bx^3}\,dx = \text{Timed out}$$

input
```
integrate((a^(1/3)*b^(1/3)*B+2*a^(2/3)*C+b^(2/3)*B*x+b^(2/3)*C*x^2)/(b*x^3
+a),x, algorithm="giac")
```

output
```
Timed out
```

3.40.9 Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 386, normalized size of antiderivative = 5.51

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + 2a^{2/3}C + b^{2/3}Bx + b^{2/3}Cx^2}{a + bx^3}\,dx = \sum_{k=1}^{3} \ln\left(-\frac{x\left(2\,C^2\,a^{2/3}\,b^{2/3} - B^2\,b^{4/3} + B\,C\,a^{1/3}\,b\right)}{b^2} + \frac{a^{1/3}\,(B$$

input
```
int((2*C*a^(2/3) + B*a^(1/3)*b^(1/3) + C*b^(2/3)*x^2 + B*b^(2/3)*x)/(a + b
*x^3),x)
```

output
```
symsum(log((a^(1/3)*(B*b^(1/3) + C*a^(1/3))^2)/b^(5/3) - (x*(2*C^2*a^(2/3)
*b^(2/3) - B^2*b^(4/3) + B*C*a^(1/3)*b))/b^2 + (root(27*a^2*b^3*z^3 - 27*C
*a^2*b^(8/3)*z^2 + 18*B*C*a^(5/3)*b^(8/3)*z + 9*C^2*a^2*b^(7/3)*z + 9*B^2*
a^(4/3)*b^3*z - 18*B*C^2*a^(5/3)*b^(7/3) - 9*B^2*C*a^(4/3)*b^(8/3) - 9*C^3
*a^2*b^2, z, k)*(9*root(27*a^2*b^3*z^3 - 27*C*a^2*b^(8/3)*z^2 + 18*B*C*a^(
5/3)*b^(8/3)*z + 9*C^2*a^2*b^(7/3)*z + 9*B^2*a^(4/3)*b^3*z - 18*B*C^2*a^(5
/3)*b^(7/3) - 9*B^2*C*a^(4/3)*b^(8/3) - 9*C^3*a^2*b^2, z, k)*a*b^(1/3) - 6
*C*a + 3*B*a^(1/3)*b^(2/3)*x + 6*C*a^(2/3)*b^(1/3)*x))/b^(4/3))*root(27*a^
2*b^3*z^3 - 27*C*a^2*b^(8/3)*z^2 + 18*B*C*a^(5/3)*b^(8/3)*z + 9*C^2*a^2*b^
(7/3)*z + 9*B^2*a^(4/3)*b^3*z - 18*B*C^2*a^(5/3)*b^(7/3) - 9*B^2*C*a^(4/3)
*b^(8/3) - 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

3.40. $\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+2a^{2/3}C+b^{2/3}Bx+b^{2/3}Cx^2}{a+bx^3}\,dx$

3.41 $\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

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3.41.1 Optimal result

Integrand size = 57, antiderivative size = 88

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx=\frac{2\left(bB+\sqrt[3]{a}(-b)^{2/3}C\right)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

output
```
C*ln(a^(1/3)-(-b)^(1/3)*x)/(-b)^(1/3)+2/3*(b*B+a^(1/3)*(-b)^(2/3)*C)*arcta
n(1/3*(a^(1/3)+2*(-b)^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b*3^(1/2)
```

3.41.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(88) = 176.

Time = 0.57 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.70

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx=\frac{2\sqrt{3}\sqrt[3]{b}\left(\left((-b)^{2/3}-\sqrt[3]{-b^2}\right)B+2\sqrt[3]{a}\sqrt[3]{b}C\right)\arctan}{}$$

input
```
Integrate[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-b)^(2/3
)*C*x^2)/(a + b*x^3),x]
```

output
```
(2*Sqrt[3]*b^(1/3)*(((-b)^(2/3) - (-b^2)^(1/3))*B + 2*a^(1/3)*b^(1/3)*C)*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (-2*b*((-(-b)^(2/3) + b^(2/3)
)*B + 2*a^(1/3)*(-b)^(1/3)*C)*Log[a^(1/3) + b^(1/3)*x] + ((-b)^(5/3)*B + b
^(5/3)*B + 2*a^(1/3)*(-b)^(1/3)*b*C)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2] - 2*a^(1/3)*(-b)^(2/3)*(-b^2)^(1/3)*C*Log[a + b*x^3])/(-b^2)^(1/
3))/(6*a^(1/3)*b)
```

3.41.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2405, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2a^{2/3}C+\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}dx$$

$$\Big\downarrow \text{2405}$$

$$\frac{\left(\sqrt[3]{-b}B-\sqrt[3]{a}C\right)\int \frac{1}{x^2+\frac{\sqrt[3]{a}x}{\sqrt[3]{-b}}+\frac{a^{2/3}}{(-b)^{2/3}}}dx}{(-b)^{2/3}}-\frac{C\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{-b}}-x}dx}{\sqrt[3]{-b}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\left(\sqrt[3]{-b}B-\sqrt[3]{a}C\right)\int \frac{1}{x^2+\frac{\sqrt[3]{a}x}{\sqrt[3]{-b}}+\frac{a^{2/3}}{(-b)^{2/3}}}dx}{(-b)^{2/3}}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}-\frac{2\left(\sqrt[3]{-b}B-\sqrt[3]{a}C\right)\int \frac{1}{-\left(\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}\sqrt[3]{-b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\left(\sqrt[3]{-b}B-\sqrt[3]{a}C\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{-b}}+\frac{C\log\left(\sqrt[3]{a}-\sqrt[3]{-b}x\right)}{\sqrt[3]{-b}}$$

input
```
Int[(a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C - (-b)^(2/3)*B*x - (-b)^(2/3)*C*x^
2)/(a + b*x^3),x]
```

output
```
(2*((-b)^(1/3)*B - a^(1/3)*C)*ArcTan[(1 + (2*(-b)^(1/3)*x)/a^(1/3))/Sqrt[3
]])/(Sqrt[3]*a^(1/3)*(-b)^(1/3)) + (C*Log[a^(1/3) - (-b)^(1/3)*x])/(-b)^(1
/3)
```

3.41.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2405
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/(-b)^(1/3)}, Sim
p[-C/b   Int[1/(q - x), x], x] + Simp[(B - C*q)/b   Int[1/(q^2 + q*x + x^2)
, x], x]] /; EqQ[A*(-b)^(2/3) + a^(1/3)*(-b)^(1/3)*B - 2*a^(2/3)*C, 0]] /;
FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.41.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(67) = 134.

Time = 1.59 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.58

method	result
default	$\left(a^{\frac{1}{3}}(-b)^{\frac{1}{3}} B-2 a^{\frac{2}{3}} C\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-(-b)^{\frac{2}{3}} B$

input
```
int((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^2)/(b*
x^3+a),x,method=_RETURNVERBOSE)
```

output
```
(a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/
6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-(-b)^(2/3)*B*(-1/3/b/(a/b)^(1/3
)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/
3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-1/3*C*(-b
)^(2/3)*ln(b*x^3+a)/b
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(67) = 134.

Time = 1.66 (sec) , antiderivative size = 470, normalized size of antiderivative = 5.34

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx = \left[\frac{\sqrt{\frac{1}{3}}b\sqrt{\frac{C^2a(-b)^{\frac{1}{3}}-2BCa^{\frac{2}{3}}(-b)^{\frac{2}{3}}-B^2a^{\frac{1}{3}}b}{ab}}\log\left(-\frac{C^3a^2+B\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^
2)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[(sqrt(1/3)*b*sqrt((C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1
/3)*b)/(a*b))*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3 - 3*(C^3
*a + B^3*b)*a^(2/3)*(-b)^(1/3)*x + 3*sqrt(1/3)*((2*B^2*b*x^2 + C^2*a*x + B
*C*a)*a^(2/3)*(-b)^(2/3) + (2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*a^(1/3) +
 (2*B*C*a*b*x^2 - B^2*a*b*x + C^2*a^2)*(-b)^(1/3))*sqrt((C^2*a*(-b)^(1/3)
- 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b)))/(b*x^3 + a)) - C*(-b)^
(2/3)*log(b*x + a^(1/3)*(-b)^(2/3)))/b, -(2*sqrt(1/3)*b*sqrt(-(C^2*a*(-b)^
(1/3) - 2*B*C*a^(2/3)*(-b)^(2/3) - B^2*a^(1/3)*b)/(a*b))*arctan(sqrt(1/3)*
((2*C^2*x + B*C)*a^(2/3)*(-b)^(2/3) - (2*B*C*b*x + B^2*b)*a^(1/3) - (2*B^2
*b*x - C^2*a)*(-b)^(1/3))*sqrt(-(C^2*a*(-b)^(1/3) - 2*B*C*a^(2/3)*(-b)^(2/
3) - B^2*a^(1/3)*b)/(a*b))/(C^3*a + B^3*b)) + C*(-b)^(2/3)*log(b*x + a^(1/
3)*(-b)^(2/3)))/b]
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - 2a^{2/3}C - (-b)^{2/3}Bx - (-b)^{2/3}Cx^2}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate((a**(1/3)*(-b)**(1/3)*B-2*a**(2/3)*C-(-b)**(2/3)*B*x-(-b)**(2/3)
*C*x**2)/(b*x**3+a),x)
```

output
```
Timed out
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(67) = 134$.

Time = 0.31 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - 2a^{2/3}C - (-b)^{2/3}Bx - (-b)^{2/3}Cx^2}{a+bx^3}\,dx = \frac{\sqrt{3}\left(2\,Ca(-b)^{\frac{2}{3}} - \left(6\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - 3\,Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.\right.}{}$$
$$+ \frac{\left(2\,Ca^{\frac{2}{3}} - Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}} - \left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}} + B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)(-b)^{\frac{2}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{\left(2\,Ca^{\frac{2}{3}} - Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}} + \left(C\left(\frac{a}{b}\right)^{\frac{2}{3}} - B\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)(-b)^{\frac{2}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^
2)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*(2*C*a*(-b)^(2/3) - (6*C*a^(2/3)*(a/b)^(1/3) - 3*B*a^(1/3)*(-b
)^(1/3)*(a/b)^(1/3) + (3*B*(a/b)^(2/3) + 2*C*a/b)*(-b)^(2/3))*b)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*a^(2/3) - B*a^
(1/3)*(-b)^(1/3) - (2*C*(a/b)^(2/3) + B*(a/b)^(1/3))*(-b)^(2/3))*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(2*C*a^(2/3) - B*a^(1/
3)*(-b)^(1/3) + (C*(a/b)^(2/3) - B*(a/b)^(1/3))*(-b)^(2/3))*log(x + (a/b)^
(1/3))/(b*(a/b)^(2/3))
```

3.41.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - 2a^{2/3}C - (-b)^{2/3}Bx - (-b)^{2/3}Cx^2}{a + bx^3}\,dx = \text{Timed out}$$

input
```
integrate((a^(1/3)*(-b)^(1/3)*B-2*a^(2/3)*C-(-b)^(2/3)*B*x-(-b)^(2/3)*C*x^
2)/(b*x^3+a),x, algorithm="giac")
```

output
```
Timed out
```

3.41.9 Mupad [B] (verification not implemented)

Time = 11.27 (sec) , antiderivative size = 444, normalized size of antiderivative = 5.05

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - 2a^{2/3}C - (-b)^{2/3}Bx - (-b)^{2/3}Cx^2}{a + bx^3}\,dx = \sum_{k=1}^{3} \ln\left(\text{root}\left(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,(-b)^{8/3}\,z^2 + 18\,B\right.\right.$$

input
```
int(-(2*C*a^(2/3) + B*(-b)^(2/3)*x - B*a^(1/3)*(-b)^(1/3) + C*(-b)^(2/3)*x
^2)/(a + b*x^3),x)
```

output
```
symsum(log(root(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 + 18*B*C*a^(5/3)*
(-b)^(8/3)*z + 9*B^2*a^(4/3)*b^3*z - 9*C^2*a^2*(-b)^(7/3)*z - 18*B*C^2*a^(
5/3)*(-b)^(7/3) + 9*B^2*C*a^(4/3)*(-b)^(8/3) + 9*C^3*a^2*b^2, z, k)*((6*C*
a)/(-b)^(4/3) - (x*(3*B*a^(1/3)*(-b)^(4/3) + 6*C*a^(2/3)*b))/b^2 + (9*root
(27*a^2*b^3*z^3 + 27*C*a^2*(-b)^(8/3)*z^2 + 18*B*C*a^(5/3)*(-b)^(8/3)*z +
9*B^2*a^(4/3)*b^3*z - 9*C^2*a^2*(-b)^(7/3)*z - 18*B*C^2*a^(5/3)*(-b)^(7/3)
 + 9*B^2*C*a^(4/3)*(-b)^(8/3) + 9*C^3*a^2*b^2, z, k)*a)/b) + (B^2*a^(1/3)*
b^2 + C^2*a*(-b)^(4/3) - 2*B*C*a^(2/3)*(-b)^(5/3))/b^3 - (x*(2*C^2*a^(2/3)
*(-b)^(2/3) - B^2*(-b)^(4/3) + B*C*a^(1/3)*b))/b^2)*root(27*a^2*b^3*z^3 +
27*C*a^2*(-b)^(8/3)*z^2 + 18*B*C*a^(5/3)*(-b)^(8/3)*z + 9*B^2*a^(4/3)*b^3*
z - 9*C^2*a^2*(-b)^(7/3)*z - 18*B*C^2*a^(5/3)*(-b)^(7/3) + 9*B^2*C*a^(4/3)
*(-b)^(8/3) + 9*C^3*a^2*b^2, z, k), k, 1, 3)
```

3.41. $\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-2a^{2/3}C-(-b)^{2/3}Bx-(-b)^{2/3}Cx^2}{a+bx^3}\,dx$

3.42 $\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3}\,dx$

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3.42.1 Optimal result

Integrand size = 31, antiderivative size = 11

$$\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3}\,dx=\frac{\log(B-Cx)}{C}$$

output
```
ln(-C*x+B)/C
```

3.42.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3}\,dx=\frac{\log(-B+Cx)}{C}$$

input
```
Integrate[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
```

output
```
Log[-B + C*x]/C
```

3.42.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B^2+BCx+C^2x^2}{C^3x^3-B^3}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{Cx-B}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log(B-Cx)}{C}$$

input
```
Int[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
```

output
```
Log[B - C*x]/C
```

3.42.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.42.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\ln(-Cx+B)}{C}$	12
norman	$\frac{\ln(-Cx+B)}{C}$	12
risch	$\frac{\ln(-Cx+B)}{C}$	12
parallelrisch	$\frac{\ln(-Cx+B)}{C}$	12

input
```
int((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x,method=_RETURNVERBOSE)
```

output
```
ln(-C*x+B)/C
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3} \, dx = \frac{\log(Cx - B)}{C}$$

input
```
integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="fricas")
```

output
```
log(C*x - B)/C
```

3.42.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3} \, dx = \frac{\log(-B + Cx)}{C}$$

input
```
integrate((C**2*x**2+B*C*x+B**2)/(C**3*x**3-B**3),x)
```

output
```
log(-B + C*x)/C
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3}\,dx = \frac{\log(Cx - B)}{C}$$

input
```
integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="maxima")
```

output
```
log(C*x - B)/C
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3}\,dx = \frac{\log(|Cx - B|)}{C}$$

input
```
integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="giac")
```

output
```
log(abs(C*x - B))/C
```

3.42.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3}\,dx = \frac{\ln(C\,x - B)}{C}$$

input
```
int(-(B^2 + C^2*x^2 + B*C*x)/(B^3 - C^3*x^3),x)
```

output
```
log(C*x - B)/C
```

3.43 $\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx$

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3.43.9 Mupad [B] (verification not implemented) 499

3.43.1 Optimal result

Integrand size = 42, antiderivative size = 21

$$\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx=\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

output
```
C*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)
```

3.43.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx=\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

input
```
Integrate[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

output
```
(C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

3.43.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{\frac{\sqrt[3]{a}}{C}+\frac{\sqrt[3]{b}x}{C}}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

input
```
Int[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

output
```
(C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

3.43.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.43.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result
risch	$\frac{C\ln\left(a^{\frac{1}{3}}b^{\frac{2}{3}}+bx\right)}{b^{\frac{1}{3}}}$
default	$C\left(a^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-b^{\frac{1}{3}}a^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln}{}\right.\right.$

input
```
int((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x,method=_RETU
RNVERBOSE)
```

output
```
C/b^(1/3)*ln(a^(1/3)*b^(2/3)+b*x)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx=\frac{C\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}$$

input
```
integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algor
ithm="fricas")
```

output
```
C*log(b*x + a^(1/3)*b^(2/3))/b^(1/3)
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a+bx^3}\,dx = \frac{C\log\left(\sqrt[3]{a}b^{\frac{2}{3}} + bx\right)}{\sqrt[3]{b}}$$

input
```
integrate((a**(2/3)*C-a**(1/3)*b**(1/3)*C*x+b**(2/3)*C*x**2)/(b*x**3+a),x)
```

output
```
C*log(a**(1/3)*b**(2/3) + b*x)/b**(1/3)
```

3.43.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(15) = 30.

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 10.00

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(2\,Cab^{\frac{2}{3}} + \left(3\,Ca^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} - 3\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - \frac{2\,Ca}{b^{\frac{1}{3}}}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} + \frac{\left(2\,Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} - Ca^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - Ca^{\frac{2}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} + Ca^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + Ca^{\frac{2}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algor
ithm="maxima")
```

output
```
-1/9*sqrt(3)*(2*C*a*b^(2/3) + (3*C*a^(1/3)*b^(1/3)*(a/b)^(2/3) - 3*C*a^(2/
3)*(a/b)^(1/3) - 2*C*a/b^(1/3))*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a*b) + 1/6*(2*C*b^(2/3)*(a/b)^(2/3) - C*a^(1/3)*b^(1/3)*(a/b
)^(1/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)
) + 1/3*(C*b^(2/3)*(a/b)^(2/3) + C*a^(1/3)*b^(1/3)*(a/b)^(1/3) + C*a^(2/3)
)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx = \frac{C\log\left(\left|b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right|\right)}{b^{\frac{1}{3}}}$$

input
```
integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algor
ithm="giac")
```

output
```
C*log(abs(b^(1/3)*x + a^(1/3)))/b^(1/3)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx = \frac{C\,\ln\left(x + \frac{a^{1/3}}{b^{1/3}}\right)}{b^{1/3}}$$

input
```
int((C*a^(2/3) + C*b^(2/3)*x^2 - C*a^(1/3)*b^(1/3)*x)/(a + b*x^3),x)
```

output
```
(C*log(x + a^(1/3)/b^(1/3)))/b^(1/3)
```

3.44 $\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

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3.44.8 Giac [C] (verification not implemented) 505
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3.44.1 Optimal result

Integrand size = 42, antiderivative size = 71

$$\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx = -\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B+\sqrt[3]{\frac{a}{b}}C\right)\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}a}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

output
```
C*ln((a/b)^(1/3)+x)/b-2/3*(a/b)^(2/3)*(B+(a/b)^(1/3)*C)*arctan(1/3*(1-2*x/
(a/b)^(1/3))*3^(1/2))/a*3^(1/2)
```

3.44.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 247 vs. 2(71) = 142.

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.48

$$\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx = \frac{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}B+\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\left(B+2\sqrt[3]{\frac{a}{b}}C\right)\right)\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\cdots\right.}{\cdots}$$

input
```
Integrate[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

output
```
(2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(
1/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 2*b^(1/3)*(-
(a^(2/3)*B) + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*(a/b)^(1/3)*C))*Log[a^(1/
3) + b^(1/3)*x] + b^(1/3)*(a^(2/3)*B - a^(1/3)*(a/b)^(1/3)*b^(1/3)*(B + 2*
(a/b)^(1/3)*C))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*C*Log
[a + b*x^3])/(6*a*b)
```

3.44.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2406, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\sqrt[3]{\frac{a}{b}}+2C\left(\frac{a}{b}\right)^{2/3}+Bx+Cx^2}{a+bx^3}dx$$

$$\Big\downarrow \text{2406}$$

$$\frac{\left(C\sqrt[3]{\frac{a}{b}}+B\right)\int \frac{1}{x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}}dx}{b}+\frac{C\int \frac{1}{x+\sqrt[3]{\frac{a}{b}}}dx}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{\left(C\sqrt[3]{\frac{a}{b}}+B\right)\int \frac{1}{x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}}dx}{b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(C\sqrt[3]{\frac{a}{b}}+B\right)\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b\sqrt[3]{\frac{a}{b}}}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)\left(C\sqrt[3]{\frac{a}{b}}+B\right)}{\sqrt{3}b\sqrt[3]{\frac{a}{b}}}$$

input
```
Int[((a/b)^(1/3)*B + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x]
```

output
```
(-2*(B + (a/b)^(1/3)*C)*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*
(a/b)^(1/3)*b) + (C*Log[(a/b)^(1/3) + x])/b
```

3.44.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.44. $\int \frac{\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

rule 2406
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Simp[C/b
 Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]
] /; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Po
lyQ[P2, x, 2]
```

3.44.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(60) = 120$.

Time = 1.50 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.08

method	result
default	$\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}} C+\left(\frac{a}{b}\right)^{\frac{1}{3}} B\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+B\left(-\frac{\ln(\ldots}{3\ldots}\right.$

input
```
int(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x,method=_RETURNVE
RBOSE)
```

output
```
(2*(a/b)^(2/3)*C+(a/b)^(1/3)*B)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b
/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+B*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a
/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*C/b*ln(b*x^3+a)
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(60) = 120$.

Time = 1.98 (sec) , antiderivative size = 429, normalized size of antiderivative = 6.04

$$\int \frac{\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3}\,dx = \left[\frac{C\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + \sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}} + B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}} + C^2a}{a}}\log\left(-\frac{C^3a^2 + \cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm
="fricas")
```

output
```
[(C*log(x + (a/b)^(1/3)) + sqrt(1/3)*sqrt(-(2*B*C*b*(a/b)^(2/3) + B^2*b*(a
/b)^(1/3) + C^2*a)/a)*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x^3
+ 3*(C^3*a*b + B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a
*b*x + C^2*a^2 - (2*B^2*b^2*x^2 + C^2*a*b*x + B*C*a*b)*(a/b)^(2/3) - (2*C^
2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt(-(2*B*C*b*(a/b)^(2/3) +
 B^2*b*(a/b)^(1/3) + C^2*a)/a))/(b*x^3 + a)))/b, (2*sqrt(1/3)*sqrt((2*B*C*
b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)*arctan(sqrt(1/3)*(2*B^2*b*x
- C^2*a + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) - (2*B*C*b*x + B^2*b)*(a/b)^(1/3
))*sqrt((2*B*C*b*(a/b)^(2/3) + B^2*b*(a/b)^(1/3) + C^2*a)/a)/(C^3*a + B^3*
b)) + C*log(x + (a/b)^(1/3)))/b]
```

3.44.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3}\,dx = \text{Timed out}$$

input
```
integrate(((a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

output
```
Timed out
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3}\,dx = \frac{C\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b} - \frac{2\sqrt{3}\left(Ca - \left(3B\left(\frac{a}{b}\right)^{\frac{2}{3}} + \frac{4Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab}$$

input
```
integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm
="maxima")
```

output
```
C*log(x + (a/b)^(1/3))/b - 2/9*sqrt(3)*(C*a - (3*B*(a/b)^(2/3) + 4*C*a/b)*
b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

3.44.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3}\,dx = \frac{\left(2Cab + \left(-a^2b^4\right)^{\frac{1}{3}}B\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2 - i\sqrt{3}\sqrt{a^2b^4}} - \frac{2\sqrt{3}\left(Cab + \left(ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} - \frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}} + Bb^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(ab^2\right)^{\frac{1}{3}}Bb + 2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^2}$$

input
```
integrate(((a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algorithm
="giac")
```

output
```
(2*C*a*b + (-a^2*b^4)^(1/3)*B)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(3
*a*b^2 - I*sqrt(3)*sqrt(a^2*b^4)) - 2/3*sqrt(3)*(C*a*b + (a*b^2)^(2/3)*B)*
arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2
*(-a/b)^(2/3) + B*b^2*(-a/b)^(1/3) + (a*b^2)^(1/3)*B*b + 2*(a*b^2)^(2/3)*C
)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 436, normalized size of antiderivative = 6.14

$$\int \frac{\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3} C + Bx + Cx^2}{a + bx^3}\, dx = \sum_{k=1}^{3} \ln\left(\frac{C^2\, a + B^2\, b\, \left(\frac{a}{b}\right)^{1/3} + 2\, B\, C\, b\, \left(\frac{a}{b}\right)^{2/3}}{b^3} + \frac{\mathrm{root}\left(27\, a^2\, b^3\, z^3 - 27\, C\, a^2\, b^2\, z^2 + 18\, B\, C\, a\, b^2\, z\, \left(\frac{a}{b}\right)^{2/3} + 9\, B^2\, a\, b^2\, z\, \left(\frac{a}{b}\right)^{1/3} + 9\, C^2\, a^2\, b\, z - 18\, B\, C^2\, a\, b\, \left(\frac{a}{b}\right)^{2/3}\right.}{} - \frac{x\left(2\, C^2\, \left(\frac{a}{b}\right)^{2/3} - B^2 + B\, C\, \left(\frac{a}{b}\right)^{1/3}\right)}{b^2}\right) \mathrm{root}\left(27\, a^2\, b^3\, z^3 - 27\, C\, a^2\, b^2\, z^2 + 18\, B\, C\, a\, b^2\, z\, \left(\frac{a}{b}\right)^{2/3} + 9\, B^2\, a\, b^2\, z\, \left(\right.\right.$$

input
```
int((B*x + C*x^2 + B*(a/b)^(1/3) + 2*C*(a/b)^(2/3))/(a + b*x^3),x)
```

output
```
symsum(log((C^2*a + B^2*b*(a/b)^(1/3) + 2*B*C*b*(a/b)^(2/3))/b^3 + (root(2
7*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^
2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(a/b)^(2/3) - 9*B^2*C*a*b*(
a/b)^(1/3) - 9*C^3*a^2, z, k)*(9*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 +
18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 1
8*B*C^2*a*b*(a/b)^(2/3) - 9*B^2*C*a*b*(a/b)^(1/3) - 9*C^3*a^2, z, k)*a*b -
 6*C*a + 3*B*b*x*(a/b)^(1/3) + 6*C*b*x*(a/b)^(2/3)))/b^2 - (x*(2*C^2*(a/b)
^(2/3) - B^2 + B*C*(a/b)^(1/3)))/b^2)*root(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z
^2 + 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*
z - 18*B*C^2*a*b*(a/b)^(2/3) - 9*B^2*C*a*b*(a/b)^(1/3) - 9*C^3*a^2, z, k),
 k, 1, 3)
```

3.45 $\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

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3.45.1 Optimal result

Integrand size = 45, antiderivative size = 76

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = \frac{2\left(B+\sqrt[3]{-\frac{a}{b}}C\right)\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-\frac{a}{b}}b} - \frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

output
```
-C*ln((-a/b)^(1/3)+x)/b+2/3*(B+(-a/b)^(1/3)*C)*arctan(1/3*(1-2*x/(-a/b)^(1
/3))*3^(1/2))/(-a/b)^(1/3)/b*3^(1/2)
```

3.45.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 288 vs. 2(76) = 152.

Time = 0.19 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.79

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = -\frac{\left(a^{2/3}B-\sqrt[3]{a}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}B-2\sqrt[3]{a}\left(-\frac{a}{b}\right)^{2/3}\sqrt[3]{b}C\right)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}ab^{2/3}} - \frac{\left(a^{2/3}B+\sqrt[3]{a}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}B+2\sqrt[3]{a}\left(-\frac{a}{b}\right)^{2/3}\sqrt[3]{b}C\right)\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3ab^{2/3}} - \frac{\left(-a^{2/3}B-\sqrt[3]{a}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}B-2\sqrt[3]{a}\left(-\frac{a}{b}\right)^{2/3}\sqrt[3]{b}C\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6ab^{2/3}} - \frac{C\log\left(a-bx^3\right)}{3b}$$

input
```
Integrate[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3
),x]
```

output
```
-(((a^(2/3)*B - a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-(a/b))^(2/3
)*b^(1/3)*C)*ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a
*b^(2/3))) - ((a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B + 2*a^(1/3)*(-
(a/b))^(2/3)*b^(1/3)*C)*Log[a^(1/3) - b^(1/3)*x])/(3*a*b^(2/3)) - ((-(a^(2
/3)*B) - a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*B - 2*a^(1/3)*(-(a/b))^(2/3)*b^(1/
3)*C)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a*b^(2/3)) - (C*L
og[a - b*x^3])/(3*b)
```

3.45.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2406, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.45. $\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

$$\int \frac{B\sqrt[3]{-\frac{a}{b}}+2C\left(-\frac{a}{b}\right)^{2/3}+Bx+Cx^2}{a-bx^3}\,dx$$
$$\Big\downarrow \; 2406$$
$$-\frac{\left(C\sqrt[3]{-\frac{a}{b}}+B\right)\int \frac{1}{x^2-\sqrt[3]{-\frac{a}{b}}x+\left(-\frac{a}{b}\right)^{2/3}}dx}{b}-\frac{C\int \frac{1}{x+\sqrt[3]{-\frac{a}{b}}}dx}{b}$$
$$\Big\downarrow \; 16$$
$$-\frac{\left(C\sqrt[3]{-\frac{a}{b}}+B\right)\int \frac{1}{x^2-\sqrt[3]{-\frac{a}{b}}x+\left(-\frac{a}{b}\right)^{2/3}}dx}{b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$
$$\Big\downarrow \; 1082$$
$$-\frac{2\left(C\sqrt[3]{-\frac{a}{b}}+B\right)\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b\sqrt[3]{-\frac{a}{b}}}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$
$$\Big\downarrow \; 217$$
$$\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)\left(C\sqrt[3]{-\frac{a}{b}}+B\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

input
```
Int[((-(a/b))^(1/3)*B + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

output
```
(2*(B + (-(a/b))^(1/3)*C)*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqr
t[3]*(-(a/b))^(1/3)*b) - (C*Log[(-(a/b))^(1/3) + x])/b
```

3.45. $\int \frac{\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

3.45.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2406
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Simp[C/b
 Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]
] /; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Po
lyQ[P2, x, 2]
```

3.45.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(65) = 130$.

Time = 1.52 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.95

method	result
default	$\left(2\left(-\frac{a}{b}\right)^{\frac{2}{3}} C+\left(-\frac{a}{b}\right)^{\frac{1}{3}} B\right)\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+B\left(\right.$

input
```
int(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x,method=_RETUR
NVERBOSE)
```

output
```
(2*(-a/b)^(2/3)*C+(-a/b)^(1/3)*B)*(-1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/
6/b/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2
)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))+B*(-1/3/b/(a/b)^(1/3)*ln(x-(a/b
)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/b
/(a/b)^(1/3)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))-1/3*C*ln(-b*x^3+a)/b
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(65) = 130$.

Time = 1.02 (sec) , antiderivative size = 459, normalized size of antiderivative = 6.04

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3}\,dx = \left[-\frac{C\log\left(x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) - \sqrt{\frac{1}{3}}\sqrt{\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}} + B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}} - C^2a}{a}}\log\Bigg(\right.}{}\right.$$

$$-\frac{2\sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}} + B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}} - C^2a}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,B^2bx + C^2a + \left(2\,C^2bx + BCb\right)\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \left(2\,BCbx + B^2b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{2\,BCb(}{}}}{C^3a - B^3b}\right.}{b}$$

input
```
integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algori
thm="fricas")
```

output
```
[-(C*log(x + (-a/b)^(1/3)) - sqrt(1/3)*sqrt((2*B*C*b*(-a/b)^(2/3) + B^2*b*
(-a/b)^(1/3) - C^2*a)/a)*log(-(C^3*a^2 - B^3*a*b + 2*(C^3*a*b - B^3*b^2)*x
^3 - 3*(C^3*a*b - B^3*b^2)*x*(-a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B
^2*a*b*x - C^2*a^2 + (2*B^2*b^2*x^2 - C^2*a*b*x - B*C*a*b)*(-a/b)^(2/3) -
(2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(-a/b)^(1/3))*sqrt((2*B*C*b*(-a/b)^(
2/3) + B^2*b*(-a/b)^(1/3) - C^2*a)/a))/(b*x^3 - a)))/b, -(2*sqrt(1/3)*sqrt
(-(2*B*C*b*(-a/b)^(2/3) + B^2*b*(-a/b)^(1/3) - C^2*a)/a)*arctan(-sqrt(1/3)
*(2*B^2*b*x + C^2*a + (2*C^2*b*x + B*C*b)*(-a/b)^(2/3) - (2*B*C*b*x + B^2*
b)*(-a/b)^(1/3))*sqrt(-(2*B*C*b*(-a/b)^(2/3) + B^2*b*(-a/b)^(1/3) - C^2*a)
/a)/(C^3*a - B^3*b)) + C*log(x + (-a/b)^(1/3)))/b]
```

3.45.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3}\,dx = \text{Timed out}$$

input
```
integrate(((-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
```

output
```
Timed out
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(65) = 130.

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.13

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3}\,dx =$$

$$-\frac{\sqrt{3}\left(2\,Ca - \left(6\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}} - 3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}} + 3\,B\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$
$$-\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}} - B\left(\frac{a}{b}\right)^{\frac{1}{3}} - B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}} + B\left(\frac{a}{b}\right)^{\frac{1}{3}} + B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algori
thm="maxima")
```

output
```
-1/9*sqrt(3)*(2*C*a - (6*C*(a/b)^(1/3)*(-a/b)^(2/3) - 3*B*(a/b)^(2/3) + 3*
B*(a/b)^(1/3)*(-a/b)^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(
1/3))/(a/b)^(1/3))/(a*b) - 1/6*(2*C*(a/b)^(2/3) - 2*C*(-a/b)^(2/3) - B*(a/
b)^(1/3) - B*(-a/b)^(1/3))*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)
^(2/3)) - 1/3*(C*(a/b)^(2/3) + 2*C*(-a/b)^(2/3) + B*(a/b)^(1/3) + B*(-a/b)
^(1/3))*log(x - (a/b)^(1/3))/(b*(a/b)^(2/3))
```

3.45.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.09

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3}\,dx = -\frac{\left(2\,Cab - \left(-a^2b^4\right)^{\frac{1}{3}}B\right)\log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab^2 + i\,\sqrt{3}\sqrt{a^2b^4}} - \frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + Bb^2\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(-ab^2\right)^{\frac{1}{3}}Bb + 2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2} + \frac{\sqrt{3}\left(\left(9\left(-a^2b^4\right)^{\frac{1}{3}}ab^2 + 27^{\frac{5}{6}}\left(-a^2b^4\right)^{\frac{5}{6}}\right)B - 18\left(a^2b^3 + i\,\sqrt{3}\sqrt{a^4b^6}\right)C\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{54\,a^2b^4}$$

input
```
integrate(((-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algori
thm="giac")
```

output
```
-(2*C*a*b - (-a^2*b^4)^(1/3)*B)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(3*
a*b^2 + I*sqrt(3)*sqrt(a^2*b^4)) - 1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1
/3) + (-a*b^2)^(1/3)*B*b + 2*(-a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/
b)^(1/3)))/(a*b^2) + 1/54*sqrt(3)*((9*(-a^2*b^4)^(1/3)*a*b^2 + 27^(5/6)*(-
a^2*b^4)^(5/6))*B - 18*(a^2*b^3 + I*sqrt(3)*sqrt(a^4*b^6))*C)*arctan(1/3*s
qrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 11.33 (sec) , antiderivative size = 456, normalized size of antiderivative = 6.00

$$\int \frac{\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3} C + Bx + Cx^2}{a - bx^3}\,dx = \sum_{k=1}^{3} \ln\left(\frac{B^2\,b\,\left(-\frac{a}{b}\right)^{1/3} - C^2\,a + 2\,B\,C\,b\,\left(-\frac{a}{b}\right)^{2/3}}{b^3}\right.$$
$$-\frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,b^2\,z^2 - 18\,B\,C\,a\,b^2\,z\,\left(-\frac{a}{b}\right)^{2/3} - 9\,B^2\,a\,b^2\,z\,\left(-\frac{a}{b}\right)^{1/3} + 9\,C^2\,a^2\,b\,z - 18\,B\,C^2\,a\,b\,\left(-\right.\right.}{}$$
$$\left.-\frac{x\left(2\,C^2\,\left(-\frac{a}{b}\right)^{2/3} - B^2 + B\,C\,\left(-\frac{a}{b}\right)^{1/3}\right)}{b^2}\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,b^2\,z^2 - 18\,B\,C\,a\,b^2\,z\,\left(-\frac{a}{b}\right)^{2/3} - 9\,B^2\,a\right.$$

input
```
int((B*x + C*x^2 + B*(-a/b)^(1/3) + 2*C*(-a/b)^(2/3))/(a - b*x^3),x)
```

output
```
symsum(log((B^2*b*(-a/b)^(1/3) - C^2*a + 2*B*C*b*(-a/b)^(2/3))/b^3 - (root
(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 - 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a
*b^2*z*(-a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) - 9*B^2*C*
a*b*(-a/b)^(1/3) + 9*C^3*a^2, z, k)*(6*C*a + 9*root(27*a^2*b^3*z^3 + 27*C*
a^2*b^2*z^2 - 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9
*C^2*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) - 9*B^2*C*a*b*(-a/b)^(1/3) + 9*C^
3*a^2, z, k)*a*b + 3*B*b*x*(-a/b)^(1/3) + 6*C*b*x*(-a/b)^(2/3)))/b^2 - (x*
(2*C^2*(-a/b)^(2/3) - B^2 + B*C*(-a/b)^(1/3)))/b^2)*root(27*a^2*b^3*z^3 +
27*C*a^2*b^2*z^2 - 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3
) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) - 9*B^2*C*a*b*(-a/b)^(1/3) +
 9*C^3*a^2, z, k), k, 1, 3)
```

3.46 $\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx$

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3.46.1 Optimal result

Integrand size = 45, antiderivative size = 78

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx=\frac{2\left(B-\sqrt[3]{-\frac{a}{b}}C\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-\frac{a}{b}}b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

output
```
C*ln((-a/b)^(1/3)-x)/b+2/3*(B-(-a/b)^(1/3)*C)*arctan(1/3*(1+2*x/(-a/b)^(1/
3))*3^(1/2))/(-a/b)^(1/3)/b*3^(1/2)
```

3.46.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 253 vs. $2(78) = 156$.

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.24

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a+bx^3}\,dx = \frac{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}B + \sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\left(-B + 2\sqrt[3]{-\frac{a}{b}}C\right)\right)\arctan\left(\right.}{}$$

input
```
Integrate[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*
x^3),x]
```

output
```
(2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (-(a/b))^(1/3)*b^(1/3)*(-B + 2*(-(
a/b))^(1/3)*C))*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] - 2*b^(
1/3)*(a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^(1/3)*(B - 2*(-(a/b))^(1/3)*C))
*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)*(-(a/b))^(1/3)*b^
(1/3)*(B - 2*(-(a/b))^(1/3)*C))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2] + 2*a*C*Log[a + b*x^3])/(6*a*b)
```

3.46.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {2408, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-B\sqrt[3]{-\frac{a}{b}} + 2C\left(-\frac{a}{b}\right)^{2/3} + Bx + Cx^2}{a+bx^3}\,dx$$

$$\Big\downarrow \text{2408}$$

$$\frac{\left(B - C\sqrt[3]{-\frac{a}{b}}\right)\int \frac{1}{x^2+\sqrt[3]{-\frac{a}{b}}x+\left(-\frac{a}{b}\right)^{2/3}}dx}{b} - \frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}dx}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{\left(B-C\sqrt[3]{-\frac{a}{b}}\right)\int \frac{1}{x^2+\sqrt[3]{-\frac{a}{b}}x+\left(-\frac{a}{b}\right)^{2/3}}dx}{b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

$\Big\downarrow$ 1082

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{2\left(B-C\sqrt[3]{-\frac{a}{b}}\right)\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1\right)}{b\sqrt[3]{-\frac{a}{b}}}$$

$\Big\downarrow$ 217

$$\frac{2\arctan\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)\left(B-C\sqrt[3]{-\frac{a}{b}}\right)}{\sqrt{3}b\sqrt[3]{-\frac{a}{b}}}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

input
```
Int[(-((-(a/b))^(1/3)*B) + 2*(-(a/b))^(2/3)*C + B*x + C*x^2)/(a + b*x^3),x
]
```

output
```
(2*(B - (-(a/b))^(1/3)*C)*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqr
t[3]*(-(a/b))^(1/3)*b) + (C*Log[(-(a/b))^(1/3) - x])/b
```

3.46.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2408
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Simp[-C/b
   Int[1/(q - x), x], x] + Simp[(B - C*q)/b   Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &
& PolyQ[P2, x, 2]
```

3.46.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(67) = 134$.

Time = 1.52 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.85

method	result
default	$\left(2\left(-\frac{a}{b}\right)^{\frac{2}{3}} C-\left(-\frac{a}{b}\right)^{\frac{1}{3}} B\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+B\Bigg(-$

input
```
int((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x,method=_RETUR
NVERBOSE)
```

output
```
(2*(-a/b)^(2/3)*C-(-a/b)^(1/3)*B)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6
/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+B*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)
^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*C/b*ln(b*x^3+a)
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(68) = 136$.

Time = 1.03 (sec) , antiderivative size = 450, normalized size of antiderivative = 5.77

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3}\,dx = \left[\frac{C\log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) + \sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(-\frac{a}{b}\right)^{\frac{2}{3}} - B^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}} + C^2a}{a}}\log}{}\right.$$

input
```
integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algori
thm="fricas")
```

output
```
[(C*log(x - (-a/b)^(1/3)) + sqrt(1/3)*sqrt(-(2*B*C*b*(-a/b)^(2/3) - B^2*b*
(-a/b)^(1/3) + C^2*a)/a)*log(-(C^3*a^2 + B^3*a*b - 2*(C^3*a*b + B^3*b^2)*x
^3 + 3*(C^3*a*b + B^3*b^2)*x*(-a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B
^2*a*b*x + C^2*a^2 - (2*B^2*b^2*x^2 + C^2*a*b*x + B*C*a*b)*(-a/b)^(2/3) +
(2*C^2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(-a/b)^(1/3))*sqrt(-(2*B*C*b*(-a/b)^
(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a))/(b*x^3 + a)))/b, (2*sqrt(1/3)*sqrt
((2*B*C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)*arctan(sqrt(1/3)*(
2*B^2*b*x - C^2*a + (2*C^2*b*x + B*C*b)*(-a/b)^(2/3) + (2*B*C*b*x + B^2*b)
*(-a/b)^(1/3))*sqrt((2*B*C*b*(-a/b)^(2/3) - B^2*b*(-a/b)^(1/3) + C^2*a)/a)
/(C^3*a + B^3*b)) + C*log(x - (-a/b)^(1/3)))/b]
```

3.46.6 Sympy [F(-1)]

Timed out.

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B + 2\left(-\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a + bx^3}\,dx = \text{Timed out}$$

input
```
integrate((-(-a/b)**(1/3)*B+2*(-a/b)**(2/3)*C+B*x+C*x**2)/(b*x**3+a),x)
```

output
```
Timed out
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(68) = 136.

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.06

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx =$$
$$-\frac{\sqrt{3}\left(2\,Ca-\left(6\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}+3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,B\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$
$$+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}+B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,C\left(-\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}-B\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algori
thm="maxima")
```

output
```
-1/9*sqrt(3)*(2*C*a - (6*C*(a/b)^(1/3)*(-a/b)^(2/3) + 3*B*(a/b)^(2/3) - 3*
B*(a/b)^(1/3)*(-a/b)^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(
1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*(a/b)^(2/3) - 2*C*(-a/b)^(2/3) + B*(a/
b)^(1/3) + B*(-a/b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)
^(2/3)) + 1/3*(C*(a/b)^(2/3) + 2*C*(-a/b)^(2/3) - B*(a/b)^(1/3) - B*(-a/b)
^(1/3))*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.71

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx =$$
$$-\frac{2\sqrt{3}\left(Cab+\left(-ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}$$
$$-\frac{\left(Cb^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}Bb+2\left(-ab^2\right)^{\frac{2}{3}}C\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

input
```
integrate((-(-a/b)^(1/3)*B+2*(-a/b)^(2/3)*C+B*x+C*x^2)/(b*x^3+a),x, algori
thm="giac")
```

output
```
-2/3*sqrt(3)*(C*a*b + (-a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(-a/b)^(2/3) + B*b^2*(-a/b)^(1/3)
 - (-a*b^2)^(1/3)*B*b + 2*(-a*b^2)^(2/3)*C)*(-a/b)^(1/3)*log(abs(x - (-a/b
)^(1/3)))/(a*b^2)
```

3.46.9 Mupad [B] (verification not implemented)

Time = 11.20 (sec) , antiderivative size = 453, normalized size of antiderivative = 5.81

$$\int \frac{-\sqrt[3]{-\frac{a}{b}}B+2\left(-\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a+bx^3}\,dx=\sum_{k=1}^{3}\ln\left(\frac{C^2\,a-B^2\,b\,\left(-\frac{a}{b}\right)^{1/3}+2\,B\,C\,b\,\left(-\frac{a}{b}\right)^{2/3}}{b^3}\right.$$
$$-\frac{\mathrm{root}\Big(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+18\,B\,C\,a\,b^2\,z\,\left(-\frac{a}{b}\right)^{2/3}-9\,B^2\,a\,b^2\,z\,\left(-\frac{a}{b}\right)^{1/3}+9\,C^2\,a^2\,b\,z-18\,B\,C^2\,a\,b\,(-}{}$$
$$\left.+\frac{x\left(B^2-2\,C^2\left(-\frac{a}{b}\right)^{2/3}+B\,C\left(-\frac{a}{b}\right)^{1/3}\right)}{b^2}\right)\,\mathrm{root}\Big(27\,a^2\,b^3\,z^3-27\,C\,a^2\,b^2\,z^2+18\,B\,C\,a\,b^2\,z\left(-\frac{a}{b}\right)^{2/3}-9\,B^2\,a\,b$$

input
```
int((B*x + C*x^2 - B*(-a/b)^(1/3) + 2*C*(-a/b)^(2/3))/(a + b*x^3),x)
```

output
```
symsum(log((C^2*a - B^2*b*(-a/b)^(1/3) + 2*B*C*b*(-a/b)^(2/3))/b^3 - (root
(27*a^2*b^3*z^3 - 27*C*a^2*b^2*z^2 + 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a
*b^2*z*(-a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) + 9*B^2*C*
a*b*(-a/b)^(1/3) - 9*C^3*a^2, z, k)*(6*C*a - 9*root(27*a^2*b^3*z^3 - 27*C*
a^2*b^2*z^2 + 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3) + 9
*C^2*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) + 9*B^2*C*a*b*(-a/b)^(1/3) - 9*C^
3*a^2, z, k)*a*b + 3*B*b*x*(-a/b)^(1/3) - 6*C*b*x*(-a/b)^(2/3)))/b^2 + (x*
(B^2 - 2*C^2*(-a/b)^(2/3) + B*C*(-a/b)^(1/3)))/b^2)*root(27*a^2*b^3*z^3 -
27*C*a^2*b^2*z^2 + 18*B*C*a*b^2*z*(-a/b)^(2/3) - 9*B^2*a*b^2*z*(-a/b)^(1/3
) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(-a/b)^(2/3) + 9*B^2*C*a*b*(-a/b)^(1/3) -
 9*C^3*a^2, z, k), k, 1, 3)
```

3.47 $\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

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3.47.1 Optimal result

Integrand size = 44, antiderivative size = 75

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = -\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B-\sqrt[3]{\frac{a}{b}}C\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}a}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

output
```
-C*ln((a/b)^(1/3)-x)/b-2/3*(a/b)^(2/3)*(B-(a/b)^(1/3)*C)*arctan(1/3*(1+2*x
/(a/b)^(1/3))*3^(1/2))/a*3^(1/2)
```

3.47.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(75) = 150.

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.25

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx = \frac{-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}B+\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\left(B-2\sqrt[3]{\frac{a}{b}}C\right)\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{}$$

input
```
Integrate[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x
]
```

output
```
(-2*Sqrt[3]*a^(1/3)*b^(1/3)*(a^(1/3)*B + (a/b)^(1/3)*b^(1/3)*(B - 2*(a/b)^
(1/3)*C))*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*(a^(2/3)
*B + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(1/3)*C))*Log[a^(1/3) - b^(
1/3)*x] + b^(1/3)*(a^(2/3)*B + a^(1/3)*(a/b)^(1/3)*b^(1/3)*(-B + 2*(a/b)^(
1/3)*C))*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*C*Log[a - b*
x^3])/(6*a*b)
```

3.47.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2408, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-B\sqrt[3]{\frac{a}{b}}+2C\left(\frac{a}{b}\right)^{2/3}+Bx+Cx^2}{a-bx^3}dx$$

$$\Big\downarrow \text{ 2408}$$

$$\frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}dx}{b}-\frac{\left(B-C\sqrt[3]{\frac{a}{b}}\right)\int \frac{1}{x^2+\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}}dx}{b}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{\left(B-C\sqrt[3]{\frac{a}{b}}\right)\int \frac{1}{x^2+\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}}dx}{b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2\left(B-C\sqrt[3]{\frac{a}{b}}\right)\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1\right)}{b\sqrt[3]{\frac{a}{b}}}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2\arctan\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)\left(B-C\sqrt[3]{\frac{a}{b}}\right)}{\sqrt{3}b\sqrt[3]{\frac{a}{b}}}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

input
```
Int[(-((a/b)^(1/3)*B) + 2*(a/b)^(2/3)*C + B*x + C*x^2)/(a - b*x^3),x]
```

output
```
(-2*(B - (a/b)^(1/3)*C)*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*
(a/b)^(1/3)*b) - (C*Log[(a/b)^(1/3) - x])/b
```

3.47.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.47. $\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx$

rule 2408
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Simp[-C/b
   Int[1/(q - x), x], x] + Simp[(B - C*q)/b   Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &
& PolyQ[P2, x, 2]
```

3.47.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(64) = 128.

Time = 1.52 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.97

method	result
default	$\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}} C-\left(\frac{a}{b}\right)^{\frac{1}{3}} B\right)\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+B\left(-\frac{\ln}{}\right.$

input
```
int((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x,method=_RETURN
VERBOSE)
```

output
```
(2*(a/b)^(2/3)*C-(a/b)^(1/3)*B)*(-1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6/
b/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))+B*(-1/3/b/(a/b)^(1/3)*ln(x-(a/b)^
(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/b/(
a/b)^(1/3)*arctan(1/3*(1+2/(a/b)^(1/3)*x)*3^(1/2)))-1/3*C*ln(-b*x^3+a)/b
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(65) = 130$.

Time = 1.78 (sec) , antiderivative size = 450, normalized size of antiderivative = 6.00

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3}\,dx = \left[-\frac{C\log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - \sqrt{\frac{1}{3}}\sqrt{\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}} - B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}} - C^2a}{a}}\log\left(-\frac{C^3a^2 \cdots}{\cdots}\right.}{\cdots} \right.$$

$$-\frac{2\sqrt{\frac{1}{3}}\sqrt{-\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}} - B^2b\left(\frac{a}{b}\right)^{\frac{1}{3}} - C^2a}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,B^2bx + C^2a + \left(2\,C^2bx + BCb\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} + \left(2\,BCbx + B^2b\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{2\,BCb\left(\frac{a}{b}\right)^{\frac{2}{3}} - \cdots}{\cdots}}}{C^3a - B^3b}\right.}{b}$$

input
```
integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="fricas")
```

output
```
[-(C*log(x - (a/b)^(1/3)) - sqrt(1/3)*sqrt((2*B*C*b*(a/b)^(2/3) - B^2*b*(a
/b)^(1/3) - C^2*a)/a)*log(-(C^3*a^2 - B^3*a*b + 2*(C^3*a*b - B^3*b^2)*x^3
- 3*(C^3*a*b - B^3*b^2)*x*(a/b)^(2/3) + 3*sqrt(1/3)*(2*B*C*a*b*x^2 - B^2*a
*b*x - C^2*a^2 + (2*B^2*b^2*x^2 - C^2*a*b*x - B*C*a*b)*(a/b)^(2/3) + (2*C^
2*a*b*x^2 - B*C*a*b*x - B^2*a*b)*(a/b)^(1/3))*sqrt((2*B*C*b*(a/b)^(2/3) -
B^2*b*(a/b)^(1/3) - C^2*a)/a))/(b*x^3 - a)))/b, -(2*sqrt(1/3)*sqrt(-(2*B*C
*b*(a/b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)*arctan(-sqrt(1/3)*(2*B^2*b*
x + C^2*a + (2*C^2*b*x + B*C*b)*(a/b)^(2/3) + (2*B*C*b*x + B^2*b)*(a/b)^(1
/3))*sqrt(-(2*B*C*b*(a/b)^(2/3) - B^2*b*(a/b)^(1/3) - C^2*a)/a)/(C^3*a - B
^3*b)) + C*log(x - (a/b)^(1/3)))/b]
```

3.47.6 Sympy [F(-1)]

Timed out.

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx=\text{Timed out}$$

input
```
integrate((-(a/b)**(1/3)*B+2*(a/b)**(2/3)*C+B*x+C*x**2)/(-b*x**3+a),x)
```

output
```
Timed out
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx=-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}-\frac{2\sqrt{3}\left(Ca+\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}-\frac{4\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

input
```
integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="maxima")
```

output
```
-C*log(x - (a/b)^(1/3))/b - 2/9*sqrt(3)*(C*a + (3*B*(a/b)^(2/3) - 4*C*a/b)
*b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.67

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B+2\left(\frac{a}{b}\right)^{2/3}C+Bx+Cx^2}{a-bx^3}\,dx=\frac{2\sqrt{3}\left(Cab-\left(ab^2\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}-\frac{\left(Cb^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+Bb^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-\left(ab^2\right)^{\frac{1}{3}}Bb+2\left(ab^2\right)^{\frac{2}{3}}C\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

input
```
integrate((-(a/b)^(1/3)*B+2*(a/b)^(2/3)*C+B*x+C*x^2)/(-b*x^3+a),x, algorit
hm="giac")
```

output
```
2/3*sqrt(3)*(C*a*b - (a*b^2)^(2/3)*B)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3
))/(a/b)^(1/3))/(a*b^2) - 1/3*(C*b^2*(a/b)^(2/3) + B*b^2*(a/b)^(1/3) - (a*
b^2)^(1/3)*B*b + 2*(a*b^2)^(2/3)*C)*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/
(a*b^2)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 11.39 (sec) , antiderivative size = 435, normalized size of antiderivative = 5.80

$$\int \frac{-\sqrt[3]{\frac{a}{b}}B + 2\left(\frac{a}{b}\right)^{2/3}C + Bx + Cx^2}{a - bx^3}\,dx = \sum_{k=1}^{3} \ln\left(-\frac{C^2\,a + B^2\,b\,\left(\frac{a}{b}\right)^{1/3} - 2\,B\,C\,b\,\left(\frac{a}{b}\right)^{2/3}}{b^3}\right.$$
$$-\frac{\mathrm{root}\left(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,b^2\,z^2 - 18\,B\,C\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{2/3} + 9\,B^2\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{1/3} + 9\,C^2\,a^2\,b\,z - 18\,B\,C^2\,a\,b\,\left(\frac{a}{b}\right)^{2/3}\right.}{}$$
$$\left. + \frac{x\left(B^2 - 2\,C^2\,\left(\frac{a}{b}\right)^{2/3} + B\,C\,\left(\frac{a}{b}\right)^{1/3}\right)}{b^2}\right)\mathrm{root}\left(27\,a^2\,b^3\,z^3 + 27\,C\,a^2\,b^2\,z^2 - 18\,B\,C\,a\,b^2\,z\,\left(\frac{a}{b}\right)^{2/3} + 9\,B^2\,a\,b^2\,z\,\left(\frac{a}{b}\right)\right.$$

input
```
int((B*x + C*x^2 - B*(a/b)^(1/3) + 2*C*(a/b)^(2/3))/(a - b*x^3),x)
```

output
```
symsum(log((x*(B^2 - 2*C^2*(a/b)^(2/3) + B*C*(a/b)^(1/3)))/b^2 - (root(27*
a^2*b^3*z^3 + 27*C*a^2*b^2*z^2 - 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*
z*(a/b)^(1/3) + 9*C^2*a^2*b*z - 18*B*C^2*a*b*(a/b)^(2/3) + 9*B^2*C*a*b*(a/
b)^(1/3) + 9*C^3*a^2, z, k)*(6*C*a + 9*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*
z^2 - 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b
*z - 18*B*C^2*a*b*(a/b)^(2/3) + 9*B^2*C*a*b*(a/b)^(1/3) + 9*C^3*a^2, z, k)
*a*b - 3*B*b*x*(a/b)^(1/3) + 6*C*b*x*(a/b)^(2/3)))/b^2 - (C^2*a + B^2*b*(a
/b)^(1/3) - 2*B*C*b*(a/b)^(2/3))/b^3)*root(27*a^2*b^3*z^3 + 27*C*a^2*b^2*z
^2 - 18*B*C*a*b^2*z*(a/b)^(2/3) + 9*B^2*a*b^2*z*(a/b)^(1/3) + 9*C^2*a^2*b*
z - 18*B*C^2*a*b*(a/b)^(2/3) + 9*B^2*C*a*b*(a/b)^(1/3) + 9*C^3*a^2, z, k),
 k, 1, 3)
```

3.48 $\int \frac{a+ax+cx^2}{1-x^3} \, dx$

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3.48.9 Mupad [B] (verification not implemented) 533

3.48.1 Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \frac{a+ax+cx^2}{1-x^3} \, dx = -\frac{1}{3}(2a+c)\log(1-x)+\frac{1}{3}(a-c)\log\left(1+x+x^2\right)$$

output
```
-1/3*(2*a+c)*ln(1-x)+1/3*(a-c)*ln(x^2+x+1)
```

3.48.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{a+ax+cx^2}{1-x^3} \, dx = \frac{1}{3}\left(-((2a+c)\log(1-x))+(a-c)\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[(a + a*x + c*x^2)/(1 - x^3),x]
```

output
```
(-((2*a + c)*Log[1 - x]) + (a - c)*Log[1 + x + x^2])/3
```

3.48.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2414, 16, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+a+cx^2}{1-x^3}dx$$

$$\Big\downarrow \text{ 2414}$$

$$\frac{1}{3}\int \frac{a-c+2(a-c)x}{x^2+x+1}dx+\frac{1}{3}(2a+c)\int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\int \frac{a-c+2(a-c)x}{x^2+x+1}dx-\frac{1}{3}(2a+c)\log(1-x)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{3}(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}(2a+c)\log(1-x)$$

input
```
Int[(a + a*x + c*x^2)/(1 - x^3),x]
```

output
```
-1/3*((2*a + c)*Log[1 - x]) + ((a - c)*Log[1 + x + x^2])/3
```

3.48.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, 
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2414
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Simp[q*((A + B*q
 + C*q^2)/(3*a))   Int[1/(q - x), x], x] + Simp[q/(3*a)   Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

3.48.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

method	result
default	$\left(-\frac{2a}{3}-\frac{c}{3}\right)\ln\left(-1+x\right)+\frac{(a-c)\ln(x^2+x+1)}{3}$
norman	$\left(-\frac{2a}{3}-\frac{c}{3}\right)\ln\left(-1+x\right)+\left(\frac{a}{3}-\frac{c}{3}\right)\ln\left(x^2+x+1\right)$
parallelrisch	$-\frac{2\ln(-1+x)a}{3}-\frac{\ln(-1+x)c}{3}+\frac{\ln(x^2+x+1)a}{3}-\frac{\ln(x^2+x+1)c}{3}$
risch	$-\frac{2\ln(-1+x)a}{3}-\frac{\ln(-1+x)c}{3}+\frac{\ln(-x^2-x-1)a}{3}-\frac{\ln(-x^2-x-1)c}{3}$
meijerg	$-\frac{c\ln(-x^3+1)}{3}-\frac{a\,x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-\frac{ax\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\right.}{}\right.}{}$

input
```
int((c*x^2+a*x+a)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
(-2/3*a-1/3*c)*ln(-1+x)+1/3*(a-c)*ln(x^2+x+1)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx=\frac{1}{3}\,(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}\,(2\,a+c)\log\left(x-1\right)$$

input
```
integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="fricas")
```

output
```
1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx = \frac{(a-c)\log(x^2+x+1)}{3} - \frac{(2a+c)\log(x-1)}{3}$$

input
```
integrate((c*x**2+a*x+a)/(-x**3+1),x)
```

output
```
(a - c)*log(x**2 + x + 1)/3 - (2*a + c)*log(x - 1)/3
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx = \frac{1}{3}(a-c)\log\left(x^2+x+1\right) - \frac{1}{3}(2a+c)\log(x-1)$$

input
```
integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="maxima")
```

output
```
1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx = \frac{1}{3}(a-c)\log\left(x^2+x+1\right) - \frac{1}{3}(2a+c)\log(|x-1|)$$

input
```
integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="giac")
```

output
```
1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(abs(x - 1))
```

3.48.9 Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx = \frac{a\,\ln\left(x^2+x+1\right)}{3} - \frac{c\,\ln\left(x-1\right)}{3} - \frac{2\,a\,\ln\left(x-1\right)}{3} - \frac{c\,\ln\left(x^2+x+1\right)}{3}$$

input
```
int(-(a + a*x + c*x^2)/(x^3 - 1),x)
```

output
```
(a*log(x + x^2 + 1))/3 - (c*log(x - 1))/3 - (2*a*log(x - 1))/3 - (c*log(x
+ x^2 + 1))/3
```

3.49 $\int \frac{a+bx+cx^2}{1-x^3} dx$

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3.49.8 Giac [A] (verification not implemented) 539
3.49.9 Mupad [B] (verification not implemented) 539

3.49.1 Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \frac{(a-b)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}(a+b+c)\log(1-x) + \frac{1}{6}(a+b-2c)\log\left(1+x+x^2\right)$$

output
```
-1/3*(a+b+c)*ln(1-x)+1/6*(a+b-2*c)*ln(x^2+x+1)+1/3*(a-b)*arctan(1/3*(1+2*x
)*3^(1/2))*3^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \frac{1}{6}\left(2\sqrt{3}(a-b)\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2(a+b)\log(1-x) + (a+b)\log\left(1+x+x^2\right) - 2c\log\left(1-x^3\right)\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(1 - x^3),x]
```

output
```
(2*Sqrt[3]*(a - b)*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*(a + b)*Log[1 - x] + (a +
 b)*Log[1 + x + x^2] - 2*c*Log[1 - x^3])/6
```

3.49.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2414, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{1-x^3} \, dx$$

$$\Big\downarrow \text{2414}$$

$$\frac{1}{3}\int \frac{2a-b-c+(a+b-2c)x}{x^2+x+1}dx+\frac{1}{3}(a+b+c)\int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\int \frac{2a-b-c+(a+b-2c)x}{x^2+x+1}dx-\frac{1}{3}\log(1-x)(a+b+c)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{1}{2}(a+b-2c)\int \frac{2x+1}{x^2+x+1}dx+\frac{3}{2}(a-b)\int \frac{1}{x^2+x+1}dx\right)-\frac{1}{3}\log(1-x)(a+b+c)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{1}{2}(a+b-2c)\int \frac{2x+1}{x^2+x+1}dx-3(a-b)\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{1}{3}\log(1-x)(a+b+c)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}(a+b-2c)\int \frac{2x+1}{x^2+x+1}dx+\sqrt{3}(a-b)\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(1-x)(a+b+c)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\sqrt{3}(a-b)\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2+x+1\right)(a+b-2c)\right)-\frac{1}{3}\log(1-x)(a+b+c)$$

input
```
Int[(a + b*x + c*x^2)/(1 - x^3),x]
```

output
```
-1/3*((a + b + c)*Log[1 - x]) + (Sqrt[3]*(a - b)*ArcTan[(1 + 2*x)/Sqrt[3]]
 + ((a + b - 2*c)*Log[1 + x + x^2])/2)/3
```

3.49.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2414
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Simp[q*((A + B*q
 + C*q^2)/(3*a))   Int[1/(q - x), x], x] + Simp[q/(3*a)   Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

3.49.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

method	result
default	$\left(-\frac{c}{3}-\frac{b}{3}-\frac{a}{3}\right)\ln\left(-1+x\right)+\frac{(a+b-2c)\ln(x^2+x+1)}{6}+\frac{2\left(\frac{3a}{2}-\frac{3b}{2}\right)\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$
risch	$-\frac{\ln(-1+x)c}{3}-\frac{\ln(-1+x)b}{3}-\frac{\ln(-1+x)a}{3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^2+(-a-b+2c)\_Z+a^2-ab-ac+b^2-bc+c^2\right)}\_R\ln\left(\left(-\_Ra-ac+b\right.\right.\right.}{3}$
meijerg	$-\frac{c\ln(-x^3+1)}{3}-\frac{bx^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-\frac{ax\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\right.}{2}\right.}{3}$

input
```
int((c*x^2+b*x+a)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*c-1/3*b-1/3*a)*ln(-1+x)+1/6*(a+b-2*c)*ln(x^2+x+1)+2/9*(3/2*a-3/2*b)*
arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \frac{1}{3}\sqrt{3}(a-b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right)-\frac{1}{3}(a+b+c)\log\left(x-1\right)$$

input
```
integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(
x^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

3.49.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 323, normalized size of antiderivative = 5.87

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx$$

$$= -\frac{(a+b+c)\log\left(x+\frac{a^2c-a^2(a+b+c)-2ab^2+bc^2-2bc(a+b+c)+b(a+b+c)^2}{a^3-b^3}\right)}{3} - \left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}\right.$$

$$\left.-\frac{\sqrt{3}i(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}i(a-b)}{6}\right)-2ab^2+bc^2-6bc\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}i(a-b)}{6}\right)}{a^3-b^3}\right.$$

$$-\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}\right.$$

$$\left.+\frac{\sqrt{3}i(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}i(a-b)}{6}\right)-2ab^2+bc^2-6bc\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}i(a-b)}{6}\right)}{a^3-b^3}\right.$$

input
```
integrate((c*x**2+b*x+a)/(-x**3+1),x)
```

output
```
-(a + b + c)*log(x + (a**2*c - a**2*(a + b + c) - 2*a*b**2 + b*c**2 - 2*b*
c*(a + b + c) + b*(a + b + c)**2)/(a**3 - b**3))/3 - (-a/6 - b/6 + c/3 - s
qrt(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 - b/6 + c/3 - sqrt(3)*I
*(a - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a -
 b)/6) + 9*b*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/6)**2)/(a**3 - b**3)) -
 (-a/6 - b/6 + c/3 + sqrt(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 -
 b/6 + c/3 + sqrt(3)*I*(a - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6
+ c/3 + sqrt(3)*I*(a - b)/6) + 9*b*(-a/6 - b/6 + c/3 + sqrt(3)*I*(a - b)/6
)**2)/(a**3 - b**3))
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \frac{1}{3}\,\sqrt{3}(a-b)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,(a+b-2\,c)\log\left(x^2+x+1\right) - \frac{1}{3}\,(a+b+c)\log\left(x-1\right)$$

input
```
integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(
x^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \frac{1}{3}\left(\sqrt{3}a-\sqrt{3}b\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,(a+b-2\,c)\log\left(x^2+x+1\right) - \frac{1}{3}\,(a+b+c)\log\left(|x-1|\right)$$

input
```
integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="giac")
```

output
```
1/3*(sqrt(3)*a - sqrt(3)*b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2
*c)*log(x^2 + x + 1) - 1/3*(a + b + c)*log(abs(x - 1))
```

3.49.9 Mupad [B] (verification not implemented)

Time = 10.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.58

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{a}{6}+\frac{b}{6}-\frac{c}{3}-\frac{\sqrt{3}\,a\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,b\,1\mathrm{i}}{6}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{a}{6}+\frac{b}{6}-\frac{c}{3}+\frac{\sqrt{3}\,a\,1\mathrm{i}}{6}-\frac{\sqrt{3}\,b\,1\mathrm{i}}{6}\right) - \ln\left(x-1\right)\left(\frac{a}{3}+\frac{b}{3}+\frac{c}{3}\right)$$

input
```
int(-(a + b*x + c*x^2)/(x^3 - 1),x)
```

output
```
log(x - (3^(1/2)*1i)/2 + 1/2)*(a/6 + b/6 - c/3 - (3^(1/2)*a*1i)/6 + (3^(1/
2)*b*1i)/6) + log(x + (3^(1/2)*1i)/2 + 1/2)*(a/6 + b/6 - c/3 + (3^(1/2)*a*
1i)/6 - (3^(1/2)*b*1i)/6) - log(x - 1)*(a/3 + b/3 + c/3)
```

3.49. $\int \frac{a+bx+cx^2}{1-x^3} \, dx$

3.50 $\int \frac{1+x+x^2}{1-x^3} \, dx$

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3.50.1 Optimal result

Integrand size = 16, antiderivative size = 8

$$\int \frac{1+x+x^2}{1-x^3} \, dx = -\log(1-x)$$

output
```
-ln(1-x)
```

3.50.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{1-x^3} \, dx = -\log(1-x)$$

input
```
Integrate[(1 + x + x^2)/(1 - x^3),x]
```

output
```
-Log[1 - x]
```

3.50.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{1-x^3}dx$$
$$\Big\downarrow \text{ 2019}$$
$$\int \frac{1}{1-x}dx$$
$$\Big\downarrow \text{ 16}$$
$$-\log(1-x)$$

input
```
Int[(1 + x + x^2)/(1 - x^3),x]
```

output
```
-Log[1 - x]
```

3.50.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.50.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result
default	$-\ln\left(-1+x\right)$
norman	$-\ln\left(-1+x\right)$
risch	$-\ln\left(-1+x\right)$
parallelrisch	$-\ln\left(-1+x\right)$
meijerg	$-\frac{\ln\left(-x^{3}+1\right)}{3}-\frac{x^{2}\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{3}\right)^{\frac{1}{3}}}{2+\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^{3}\right)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right.\right.}{}\right.}{}$

input
```
int((x^2+x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(-1+x)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2}{1-x^3}\,dx=-\log\left(x-1\right)$$

input
```
integrate((x^2+x+1)/(-x^3+1),x, algorithm="fricas")
```

output
```
-log(x - 1)
```

3.50.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1+x+x^2}{1-x^3}\,dx=-\log\left(x-1\right)$$

input
```
integrate((x**2+x+1)/(-x**3+1),x)
```

output
```
-log(x - 1)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\log{(x-1)}$$

input
```
integrate((x^2+x+1)/(-x^3+1),x, algorithm="maxima")
```

output
```
-log(x - 1)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\log{(|x-1|)}$$

input
```
integrate((x^2+x+1)/(-x^3+1),x, algorithm="giac")
```

output
```
-log(abs(x - 1))
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\ln{(x-1)}$$

input
```
int(-(x + x^2 + 1)/(x^3 - 1),x)
```

output
```
-log(x - 1)
```

3.51 $\int \frac{1-x+3x^2}{1-x^3} dx$

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3.51.1 Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

output
```
-ln(-x^3+1)+2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

input
```
Integrate[(1 - x + 3*x^2)/(1 - x^3),x]
```

output
```
(2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

3.51.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2410, 792, 2019, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2-x+1}{1-x^3}dx$$

$$\Big\downarrow \text{ 2410}$$

$$\int \frac{1-x}{1-x^3}dx+3\int \frac{x^2}{1-x^3}dx$$

$$\Big\downarrow \text{ 792}$$

$$\int \frac{1-x}{1-x^3}dx-\log \left(1-x^3\right)$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{x^2+x+1}dx-\log \left(1-x^3\right)$$

$$\Big\downarrow \text{ 1083}$$

$$-2\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\log \left(1-x^3\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2\arctan \left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\log \left(1-x^3\right)$$

input
```
Int[(1 - x + 3*x^2)/(1 - x^3),x]
```

output
```
(2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

3.51.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.51.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result
default	$-\ln\left(-1+x\right)-\ln\left(x^{2}+x+1\right)+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\ln\left(4x^{2}+4x+4\right)-\ln\left(-1+x\right)$
meijerg	$-\ln\left(-x^{3}+1\right)+\frac{x^{2}\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{2+\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^{3}\right)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)\right.}{}\right.}{}$

input
```
int((3*x^2-x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(-1+x)-ln(x^2+x+1)+2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \log\left(x^2+x+1\right) - \log\left(x-1\right)$$

input
```
integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(x - 1)
```

3.51.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = -\log\left(x-1\right)$$

input
```
integrate((3*x**2-x+1)/(-x**3+1),x)
```

output
```
-log(x - 1)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \log\left(x^2+x+1\right) - \log\left(x-1\right)$$

input
```
integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(x - 1)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \log\left(x^2+x+1\right) - \log\left(|x-1|\right)$$

input
```
integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(abs(x -
 1))
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.10

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = -\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right) - \ln\left(x-1\right) - \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\,1i}{3} + \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,1i}{3}$$

input
```
int(-(3*x^2 - x + 1)/(x^3 - 1),x)
```

output
```
(3^(1/2)*log(x + (3^(1/2)*1i)/2 + 1/2)*1i)/3 - log(x + (3^(1/2)*1i)/2 + 1/
2) - log(x - 1) - (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/3 - log(x - (
3^(1/2)*1i)/2 + 1/2)
```

3.52 $\int \frac{1+x+4x^2}{1-x^3} dx$

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3.52.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1+x+4x^2}{1-x^3} dx = -2\log(1-x) - \log\left(1+x+x^2\right)$$

output
```
-2*ln(1-x)-ln(x^2+x+1)
```

3.52.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1+x+4x^2}{1-x^3} dx = -2\log(1-x) - \log\left(1+x+x^2\right)$$

input
```
Integrate[(1 + x + 4*x^2)/(1 - x^3),x]
```

output
```
-2*Log[1 - x] - Log[1 + x + x^2]
```

3.52.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2414, 16, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+x+1}{1-x^3}dx$$

$$\Big\downarrow \text{ 2414}$$

$$\frac{1}{3}\int -\frac{3(2x+1)}{x^2+x+1}dx+2\int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\int -\frac{3(2x+1)}{x^2+x+1}dx-2\log(1-x)$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{2x+1}{x^2+x+1}dx-2\log(1-x)$$

$$\Big\downarrow \text{ 1103}$$

$$-\log\left(x^2+x+1\right)-2\log(1-x)$$

input
```
Int[(1 + x + 4*x^2)/(1 - x^3),x]
```

output
```
-2*Log[1 - x] - Log[1 + x + x^2]
```

3.52.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2414
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Simp[q*((A + B*q
 + C*q^2)/(3*a))   Int[1/(q - x), x], x] + Simp[q/(3*a)   Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

3.52.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result
default	$-2\ln(-1+x)-\ln(x^2+x+1)$
norman	$-2\ln(-1+x)-\ln(x^2+x+1)$
risch	$-2\ln(-1+x)-\ln(x^2+x+1)$
parallelrisch	$-2\ln(-1+x)-\ln(x^2+x+1)$
meijerg	$-\frac{4\ln(-x^3+1)}{3}-\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln(1+(x}{}\right.}{}$

input
```
int((4*x^2+x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(-1+x)-ln(x^2+x+1)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\log\left(x^2+x+1\right) - 2\,\log\left(x-1\right)$$

input
```
integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="fricas")
```

output
```
-log(x^2 + x + 1) - 2*log(x - 1)
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -2\log\left(x-1\right) - \log\left(x^2+x+1\right)$$

input
```
integrate((4*x**2+x+1)/(-x**3+1),x)
```

output
```
-2*log(x - 1) - log(x**2 + x + 1)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\log\left(x^2+x+1\right) - 2\,\log\left(x-1\right)$$

input
```
integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="maxima")
```

output
```
-log(x^2 + x + 1) - 2*log(x - 1)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\log\left(x^2+x+1\right) - 2\,\log\left(|x-1|\right)$$

input
```
integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="giac")
```

output
```
-log(x^2 + x + 1) - 2*log(abs(x - 1))
```

3.52.9 Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\ln\left(x^2+x+1\right) - 2\,\ln\left(x-1\right)$$

input
```
int(-(x + 4*x^2 + 1)/(x^3 - 1),x)
```

output
```
- log(x + x^2 + 1) - 2*log(x - 1)
```

3.53 $\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\, dx$

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3.53.1 Optimal result

Integrand size = 30, antiderivative size = 113

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\, dx = a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

output
```
a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x^7+3/4*a^
2*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*c*x^13+1/14*b^4*d*
x^14
```

3.53.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\, dx = a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

input
```
Integrate[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x
^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (
b^4*c*x^13)/13 + (b^4*d*x^14)/14
```

3.53.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^3 \left(ac+adx+bcx^3+bdx^4\right) \, dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(a^4c+a^4dx+4a^3bcx^3+4a^3bdx^4+6a^2b^2cx^6+6a^2b^2dx^7+4ab^3cx^9+4ab^3dx^{10}+b^4cx^{12}+b^4dx^{13}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

input
```
Int[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x
^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (
b^4*c*x^13)/13 + (b^4*d*x^14)/14
```

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.53.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

method	result
default	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}x^{11}d\,b^3a+\frac{1}{13}$
norman	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}x^{11}d\,b^3a+\frac{1}{13}$
risch	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}x^{11}d\,b^3a+\frac{1}{13}$
parallelrisch	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}x^{11}d\,b^3a+\frac{1}{13}$
gosper	$\frac{x(1430b^4d\,x^{13}+1540b^4c\,x^{12}+7280a\,b^3d\,x^{10}+8008a\,b^3c\,x^9+15015a^2b^2d\,x^7+17160a^2b^2c\,x^6+16016a^3bd\,x^4+20020a^3bc\,x^3+100}{20020}$

input
```
int((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*d*x^5*b*a^3+6/7*a^2*b^2*c*x^7+3/4*x^
8*b^2*d*a^2+2/5*a*b^3*c*x^10+4/11*x^11*d*b^3*a+1/13*b^4*c*x^13+1/14*b^4*d*
x^14
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int (a+bx^3)^3\left(ac+adx+bcx^3+bdx^4\right)dx=\frac{1}{14}b^4dx^{14}+\frac{1}{13}b^4cx^{13}+\frac{4}{11}ab^3dx^{11}+\frac{2}{5}ab^3cx^{10}+\frac{3}{4}a^2b^2dx^8+\frac{6}{7}a^2b^2cx^7+\frac{4}{5}a^3bdx^5+a^3bcx^4+\frac{1}{2}a^4dx^2+a^4cx$$

input
```
integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

output
```
1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1
/2*a^4*d*x^2 + a^4*c*x
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4) \, dx = a^4cx + \frac{a^4dx^2}{2} + a^3bcx^4 + \frac{4a^3bdx^5}{5} + \frac{6a^2b^2cx^7}{7} + \frac{3a^2b^2dx^8}{4} + \frac{2ab^3cx^{10}}{5} + \frac{4ab^3dx^{11}}{11} + \frac{b^4cx^{13}}{13} + \frac{b^4dx^{14}}{14}$$

input
```
integrate((b*x**3+a)**3*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

output
```
a**4*c*x + a**4*d*x**2/2 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/5 + 6*a**2*b**2
*c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/1
1 + b**4*c*x**13/13 + b**4*d*x**14/14
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4) \, dx = \frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{2}a^4dx^2 + a^4cx$$

input
```
integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

output
```
1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1
/2*a^4*d*x^2 + a^4*c*x
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\,dx = \frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{2}a^4dx^2 + a^4cx$$

input
```
integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

output
```
1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1
/2*a^4*d*x^2 + a^4*c*x
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\,dx = \frac{d\,a^4\,x^2}{2} + c\,a^4\,x + \frac{4\,d\,a^3\,b\,x^5}{5} + c\,a^3\,b\,x^4 + \frac{3\,d\,a^2\,b^2\,x^8}{4} + \frac{6\,c\,a^2\,b^2\,x^7}{7} + \frac{4\,d\,a\,b^3\,x^{11}}{11} + \frac{2\,c\,a\,b^3\,x^{10}}{5} + \frac{d\,b^4\,x^{14}}{14} + \frac{c\,b^4\,x^{13}}{13}$$

input
```
int((a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

output
```
(a^4*d*x^2)/2 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14 + a^4*c*x + (6*a^2*b^2*c
*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + a^3*b*c*x^4 + (2*a*b^3*c*x^10)/5 + (4*a^3*
b*d*x^5)/5 + (4*a*b^3*d*x^11)/11
```

3.54 $\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4) \, dx$

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3.54.1 Optimal result

Integrand size = 30, antiderivative size = 88

$$\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4) \, dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5 +\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

output
```
a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c*x^7+3/8*
a*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11
```

3.54.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4) \, dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5 +\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

input
```
Integrate[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2
*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

3.54.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right) \, dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(a^3c+a^3dx+3a^2bcx^3+3a^2bdx^4+3ab^2cx^6+3ab^2dx^7+b^3cx^9+b^3dx^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

input
```
Int[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2
*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

3.54.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.54.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

method	result	size
default	$a^{3}cx+\frac{1}{2}a^{3}d\,x^{2}+\frac{3}{4}a^{2}bc\,x^{4}+\frac{3}{5}x^{5}bd\,a^{2}+\frac{3}{7}a\,b^{2}c\,x^{7}+\frac{3}{8}x^{8}b^{2}da+\frac{1}{10}b^{3}c\,x^{10}+\frac{1}{11}b^{3}d\,x^{11}$	75
norman	$a^{3}cx+\frac{1}{2}a^{3}d\,x^{2}+\frac{3}{4}a^{2}bc\,x^{4}+\frac{3}{5}x^{5}bd\,a^{2}+\frac{3}{7}a\,b^{2}c\,x^{7}+\frac{3}{8}x^{8}b^{2}da+\frac{1}{10}b^{3}c\,x^{10}+\frac{1}{11}b^{3}d\,x^{11}$	75
risch	$a^{3}cx+\frac{1}{2}a^{3}d\,x^{2}+\frac{3}{4}a^{2}bc\,x^{4}+\frac{3}{5}x^{5}bd\,a^{2}+\frac{3}{7}a\,b^{2}c\,x^{7}+\frac{3}{8}x^{8}b^{2}da+\frac{1}{10}b^{3}c\,x^{10}+\frac{1}{11}b^{3}d\,x^{11}$	75
parallelrisch	$a^{3}cx+\frac{1}{2}a^{3}d\,x^{2}+\frac{3}{4}a^{2}bc\,x^{4}+\frac{3}{5}x^{5}bd\,a^{2}+\frac{3}{7}a\,b^{2}c\,x^{7}+\frac{3}{8}x^{8}b^{2}da+\frac{1}{10}b^{3}c\,x^{10}+\frac{1}{11}b^{3}d\,x^{11}$	75
gosper	$\frac{x\left(280b^{3}d\,x^{10}+308b^{3}c\,x^{9}+1155a\,b^{2}d\,x^{7}+1320a\,b^{2}c\,x^{6}+1848a^{2}bd\,x^{4}+2310a^{2}x^{3}bc+1540a^{3}dx+3080c\,a^{3}\right)}{3080}$	76

input
```
int((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*x^5*b*d*a^2+3/7*a*b^2*c*x^7+3/8*
x^8*b^2*d*a+1/10*b^3*c*x^10+1/11*b^3*d*x^11
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int (a+bx^3)^2\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

input
```
integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

output
```
1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/
5*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right)\,dx = a^3cx + \frac{a^3dx^2}{2} + \frac{3a^2bcx^4}{4} + \frac{3a^2bdx^5}{5} + \frac{3ab^2cx^7}{7} + \frac{3ab^2dx^8}{8} + \frac{b^3cx^{10}}{10} + \frac{b^3dx^{11}}{11}$$

input
```
integrate((b*x**3+a)**2*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

output
```
a**3*c*x + a**3*d*x**2/2 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x**5/5 + 3*a*b**
2*c*x**7/7 + 3*a*b**2*d*x**8/8 + b**3*c*x**10/10 + b**3*d*x**11/11
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right)\,dx = \frac{1}{11}\,b^3dx^{11} + \frac{1}{10}\,b^3cx^{10} + \frac{3}{8}\,ab^2dx^8 + \frac{3}{7}\,ab^2cx^7 + \frac{3}{5}\,a^2bdx^5 + \frac{3}{4}\,a^2bcx^4 + \frac{1}{2}\,a^3dx^2 + a^3cx$$

input
```
integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

output
```
1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/
5*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right)\,dx = \frac{1}{11}\,b^3dx^{11} + \frac{1}{10}\,b^3cx^{10} + \frac{3}{8}\,ab^2dx^8 + \frac{3}{7}\,ab^2cx^7 + \frac{3}{5}\,a^2bdx^5 + \frac{3}{4}\,a^2bcx^4 + \frac{1}{2}\,a^3dx^2 + a^3cx$$

input
```
integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

output
```
1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/
5*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.54.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right)\, dx = \frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{3\,d\,a^2\,b\,x^5}{5}+\frac{3\,c\,a^2\,b\,x^4}{4}+\frac{3\,d\,a\,b^2\,x^8}{8}+\frac{3\,c\,a\,b^2\,x^7}{7}+\frac{d\,b^3\,x^{11}}{11}+\frac{c\,b^3\,x^{10}}{10}$$

input
```
int((a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

output
```
(a^3*d*x^2)/2 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + a^3*c*x + (3*a^2*b*c*x
^4)/4 + (3*a*b^2*c*x^7)/7 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*d*x^8)/8
```

3.55 $\int (a+bx^3)(ac+adx+bcx^3+bdx^4)\,dx$

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3.55.9 Mupad [B] (verification not implemented) 569

3.55.1 Optimal result

Integrand size = 28, antiderivative size = 60

$$\int (a+bx^3)(ac+adx+bcx^3+bdx^4)\,dx=a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

output
```
a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*a*b*d*x^5+1/7*b^2*c*x^7+1/8*b^2*d*
x^8
```

3.55.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+bx^3)(ac+adx+bcx^3+bdx^4)\,dx=a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

input
```
Integrate[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7
+ (b^2*d*x^8)/8
```

3.55.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)\,(ac+adx+bcx^3+bdx^4)\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int (a^2c+a^2dx+2abcx^3+2abdx^4+b^2cx^6+b^2dx^7)\,dx$$

$$\Big\downarrow \text{2009}$$

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

input
```
Int[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7
+ (b^2*d*x^8)/8
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.55.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
default	$a^2cx+\frac{1}{2}a^2d\,x^2+\frac{1}{2}abc\,x^4+\frac{2}{5}x^5dba+\frac{1}{7}b^2c\,x^7+\frac{1}{8}b^2d\,x^8$	51
norman	$a^2cx+\frac{1}{2}a^2d\,x^2+\frac{1}{2}abc\,x^4+\frac{2}{5}x^5dba+\frac{1}{7}b^2c\,x^7+\frac{1}{8}b^2d\,x^8$	51
risch	$a^2cx+\frac{1}{2}a^2d\,x^2+\frac{1}{2}abc\,x^4+\frac{2}{5}x^5dba+\frac{1}{7}b^2c\,x^7+\frac{1}{8}b^2d\,x^8$	51
parallelrisch	$a^2cx+\frac{1}{2}a^2d\,x^2+\frac{1}{2}abc\,x^4+\frac{2}{5}x^5dba+\frac{1}{7}b^2c\,x^7+\frac{1}{8}b^2d\,x^8$	51
gosper	$\frac{x(35b^2d\,x^7+40b^2c\,x^6+112abd\,x^4+140abc\,x^3+140a^2dx+280a^2c)}{280}$	52

input
```
int((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*x^5*d*b*a+1/7*b^2*c*x^7+1/8*b^2*d*
x^8
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx^3)\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{2}\,a^2dx^2+a^2cx$$

input
```
integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

output
```
1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*
x^2 + a^2*c*x
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int (a+bx^3)(ac+adx+bcx^3+bdx^4)\,dx = a^2cx+\frac{a^2dx^2}{2}+\frac{abcx^4}{2}+\frac{2abdx^5}{5}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}$$

input
```
integrate((b*x**3+a)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

output
```
a**2*c*x + a**2*d*x**2/2 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 + b**2*c*x**7/7 +
 b**2*d*x**8/8
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx^3)(ac+adx+bcx^3+bdx^4)\,dx = \frac{1}{8}b^2dx^8+\frac{1}{7}b^2cx^7+\frac{2}{5}abdx^5+\frac{1}{2}abcx^4+\frac{1}{2}a^2dx^2+a^2cx$$

input
```
integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

output
```
1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*
x^2 + a^2*c*x
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx^3)(ac+adx+bcx^3+bdx^4)\,dx = \frac{1}{8}b^2dx^8+\frac{1}{7}b^2cx^7+\frac{2}{5}abdx^5+\frac{1}{2}abcx^4+\frac{1}{2}a^2dx^2+a^2cx$$

input
```
integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

output
```
1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*
x^2 + a^2*c*x
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx^3)\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{2\,d\,a\,b\,x^5}{5}+\frac{c\,a\,b\,x^4}{2}+\frac{d\,b^2\,x^8}{8}+\frac{c\,b^2\,x^7}{7}$$

input
```
int((a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

output
```
(a^2*d*x^2)/2 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + a^2*c*x + (a*b*c*x^4)/2 +
(2*a*b*d*x^5)/5
```

3.56 $\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx$

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3.56.8 Giac [A] (verification not implemented) 573
3.56.9 Mupad [B] (verification not implemented) 573

3.56.1 Optimal result

Integrand size = 30, antiderivative size = 12

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx = cx+\frac{dx^2}{2}$$

output
```
c*x+1/2*d*x^2
```

3.56.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx = cx+\frac{dx^2}{2}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
```

output
```
c*x + (d*x^2)/2
```

3.56.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int (c+dx)dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(c+dx)^2}{2d}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
```

output
```
(c + d*x)^2/(2*d)
```

3.56.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.56.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{x(dx+2c)}{2}$	11
default	$cx+\frac{1}{2}d\,x^2$	11
norman	$cx+\frac{1}{2}d\,x^2$	11
risch	$cx+\frac{1}{2}d\,x^2$	11
parallelrisch	$cx+\frac{1}{2}d\,x^2$	11
parts	$cx+\frac{1}{2}d\,x^2$	11

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(d*x+2*c)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx=\frac{1}{2}\,dx^2+cx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/2*d*x^2 + c*x
```

3.56.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx=cx+\frac{dx^2}{2}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a),x)
```

output
```
c*x + d*x**2/2
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx=\frac{1}{2}\,dx^2+cx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/2*d*x^2 + c*x
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx=\frac{1}{2}\,dx^2+cx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/2*d*x^2 + c*x
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx=\frac{d\,x^2}{2}+c\,x$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x)
```

output
```
c*x + (d*x^2)/2
```

3.57 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx$

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3.57.1 Optimal result

Integrand size = 30, antiderivative size = 161

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx = -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} - \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

output
```
1/3*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1/6*(c-a^(
1/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)-
1/3*(b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2)
)/a^(2/3)/b^(2/3)*3^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx$$
$$= \frac{-2\sqrt{3}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2}\right.\right.}{6a^{2/3}b^{2/3}}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

output
```
(-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

3.57.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2019, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{c+dx}{a+bx^3}dx$$

$$\Big\downarrow \text{ 2399}$$

$$\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{1}{2}\sqrt[3]{b}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{217}$$

3.57. $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx$

$$\frac{\frac{1}{2}\sqrt[3]{b}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\frac{1}{2}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

output
```
((c - (a^(1/3)*d)/b^(1/3))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) +
 (-((Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(1/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)*b^(1/3))
```

3.57.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.57. $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx$

3.57.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^2}}{3b}$
default	$c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\right.}{6b\left(\frac{a}{b}\right)}\right.$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/b*sum((_R*d+c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.57.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 1931, normalized size of antiderivative = 11.99

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*
d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 +
a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)
*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(
1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2)
+ (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d - 1/2*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*
(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3
- a*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/1
2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3
)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d
^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^
2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*log(
-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*
d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 +
a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)...
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^2}\, dx$$
$$= \text{RootSum}\left(27t^3a^2b^2 + 9tabcd + ad^3 - bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd + 3tabc^2 + 2acd^2}{ad^3 + bc^3}\right)\right)\right)$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**2,x)
```

output
```
RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t
*log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)
)))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx = \frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(d*(a/b)^(1/3) + c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(d*(a/b)^(1/3) - c)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(d*(a/b)^(1/3) - c)*log(x + (a/b)^(
1/3))/(b*(a/b)^(2/3))
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(bc-\left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(bc+\left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(d*(-a/b)^(1/3) + c)*(
-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a
```

3.57.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx = \sum_{k=1}^{3}\ln\left(b\left(c\,d+d^2\,x+\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)^2\,a\,b\,9+\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)\,b\,c\,x\,3\right)\right)\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,a\,b\,c\,d\,z+a\,d^3-b\,c^3,z,k\right)$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x)
```

output
```
symsum(log(b*(c*d + d^2*x + 9*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 -
b*c^3, z, k)^2*a*b + 3*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3,
z, k)*b*c*x))*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z, k), k,
 1, 3)
```

3.58 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3} dx$

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3.58.1 Optimal result

Integrand size = 30, antiderivative size = 189

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3} dx = \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}$$

output
```
1/3*x*(d*x+c)/a/(b*x^3+a)+1/9*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x
)/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/9*(2*b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(
1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx$$
$$=\frac{\frac{6ax(c+dx)}{a+bx^3}-\frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{\left(-2\sqrt[3]{a}\sqrt[3]{b}c+a^{2/3}d\right)\log}{b}}{18a^2}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

output
```
((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)
*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b
^(1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(
1/3)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3
))/(18*a^2)
```

3.58.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {2019, 2394, 25, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx$$
$$\Big\downarrow \text{ 2019}$$
$$\int \frac{c+dx}{(a+bx^3)^2}dx$$
$$\Big\downarrow \text{ 2394}$$
$$\frac{x(c+dx)}{3a(a+bx^3)}-\frac{\int -\frac{2c+dx}{bx^3+a}dx}{3a}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2c+dx}{bx^3+a}dx}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$\downarrow$ 2399

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$\downarrow$ 16

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$\downarrow$ 1142

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$\downarrow$ 25

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$\downarrow$ 1082

3.58. $\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{-\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

output
```
(x*(c + d*x))/(3*a*(a + b*x^3)) + (((2*c - (a^(1/3)*d)/b^(1/3))*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*(2*b^(1/3)*c + a^(1/3)*d
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - ((2*c - (a^(1/3)
*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)
*b^(1/3)))/(3*a)
```

3.58. $\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^3}\,dx$

3.58.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.58.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\frac{d x^2}{3a}+\frac{cx}{3a}}{b x^3+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_R d+2c\right)\ln\left(x-\_R\right)}{\_R^2}}{9ba}$
default	$c\left(\frac{x}{3a(b x^3+a)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\right)+d\left(\frac{x^2}{3a(b x^3+a)}+\frac{-\frac{\ln\left(x+\left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right.}\right.$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/3*d/a*x^2+1/3*c/a*x)/(b*x^3+a)+1/9/b/a*sum((_R*d+2*c)/_R^2*ln(x-_R),_R=
RootOf(_Z^3*b+a))
```

3.58.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 2088, normalized size of antiderivative = 11.05

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
+ a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/
(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)
/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqr
t(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2
))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(
a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(
3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))
^(1/3)))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3
 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c
^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((
8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) + 3*sqrt
(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^
3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*s
qrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b
^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4
*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*
b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3)...
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.56

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx$$
$$= \text{RootSum}\left(729t^3a^5b^2+54ta^2bcd+ad^3-8bc^3,\left(t\mapsto t\log\left(x+\frac{81t^2a^4bd+36ta^2bc^2+4acd^2}{ad^3+8bc^3}\right)\right)\right) + \frac{cx+dx^2}{3a^2+3abx^3}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**3,x)
```

output
```
RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda
(_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d*
*3 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx = \frac{dx^2+cx}{3\,(abx^3+a^2)} + \frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/3*(d*x^2 + c*x)/(a*b*x^3 + a^2) + 1/9*sqrt(3)*(d*(a/b)^(1/3) + 2*c)*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) + 1/18*(
d*(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/
3)) - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}\left(2\,bc-(-ab^2)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{\left(2\,bc+(-ab^2)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{dx^2+cx}{3\,(bx^3+a)a}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)
^(1/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(d*x^2 + c
*x)/((b*x^3 + a)*a)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{b\left(2\,c\,d+d^2\,x+\mathrm{root}(729\,a^5\,b^2\,z^3+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k)^2\,a^3\,b\,81+\mathrm{root}(729\,a^5\,b^2\,z^3}{a^2\,9} + 54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k)\right)\right) + \frac{\frac{d\,x^2}{3\,a}+\frac{c\,x}{3\,a}}{b\,x^3+a}$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x)
```

output
```
symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z
- 8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*
b*c*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) + (c*x)/(3*a))
/(a + b*x^3)
```

3.58. $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx$

3.59 $\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right) \, dx$

3.59.1 Optimal result 593
3.59.2 Mathematica [C] (verified) 594
3.59.3 Rubi [A] (verified) 595
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3.59.6 Sympy [A] (verification not implemented) 600
3.59.7 Maxima [F] 601
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3.59.1 Optimal result

Integrand size = 32, antiderivative size = 585

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right) \, dx = \frac{810a^3d\sqrt{a+bx^3}}{1729b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{54a^2(1729cx+935dx^2)\sqrt{a+bx^3}}{323323}+\frac{30a(247cx+187dx^2)\left(a+bx^3\right)^{3/2}}{46189}$$

$$+\frac{2}{323}\left(19cx+17dx^2\right)\left(a+bx^3\right)^{5/2}-\frac{405\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{(1\cdots}{(1\cdots}\right.\right.}{1729b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
30/46189*a*(187*d*x^2+247*c*x)*(b*x^3+a)^(3/2)+2/323*(17*d*x^2+19*c*x)*(b*
x^3+a)^(5/2)+54/323323*a^2*(935*d*x^2+1729*c*x)*(b*x^3+a)^(1/2)+810/1729*a
^3*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-405/1729*3^(1
/4)*a^(10/3)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2)
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)+54/323323*3^(3/4)*a^3*(a^(1/3)+b^(1/3)*x)*El
lipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*(1729*b^(1/3)*c-935*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.59.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.13

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{a^2x\sqrt{a+bx^3}\left(2c\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2}{3},\frac{5}{3},-\frac{b}{}\right.\right.}{2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
(a^2*x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-5/2, 1/3, 4/3, -((b*x^3)/a)
] + d*x*Hypergeometric2F1[-5/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x
^3)/a])
```

3.59.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2391, 2392, 27, 2392, 27, 2392, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right) \, dx$$

$$\Big\downarrow \text{2391}$$

$$\int \left(a+bx^3\right)^{5/2}(c+dx)dx$$

$$\Big\downarrow \text{2392}$$

$$\frac{15}{2}a\int \frac{2}{323}(19c+17dx)\left(bx^3+a\right)^{3/2}dx+\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{15}{323}a\int (19c+17dx)\left(bx^3+a\right)^{3/2}dx+\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$$\Big\downarrow \text{2392}$$

$$\frac{15}{323}a\left(\frac{9}{2}a\int \frac{2}{143}(247c+187dx)\sqrt{bx^3+a}dx+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(247cx+187dx^2\right)\right)+\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{15}{323}a\left(\frac{9}{143}a\int (247c+187dx)\sqrt{bx^3+a}dx+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(247cx+187dx^2\right)\right)+\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$$\Big\downarrow \text{2392}$$

$$\frac{15}{323}a\left(\frac{9}{143}a\left(\frac{3}{2}a\int \frac{2(1729c+935dx)}{35\sqrt{bx^3+a}}dx+\frac{2}{35}\sqrt{a+bx^3}\left(1729cx+935dx^2\right)\right)+\frac{2}{143}\left(a+bx^3\right)^{3/2}(247cx+187dx$$
$$\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{15}{323}a\left(\frac{9}{143}a\left(\frac{3}{35}a\int\frac{1729c+935dx}{\sqrt{bx^3+a}}dx+\frac{2}{35}\sqrt{a+bx^3}\left(1729cx+935dx^2\right)\right)+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(247cx+187dx^2\right)$$
$$\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$\downarrow$ 2417

$$\frac{15}{323}a\left(\frac{9}{143}a\left(\frac{3}{35}a\left(\left(1729c-\frac{935\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{bx^3+a}}dx+\frac{935d\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{35}\sqrt{a+bx}$$
$$\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$\downarrow$ 759

$$\frac{15}{323}a\left(\frac{9}{143}a\left(\frac{3}{35}a\left(\frac{935d\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(1729c-\frac{935(1}{}\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}$$
$$\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$\downarrow$ 2416

$$\frac{15}{323}a\left(\frac{9}{143}a\left(\frac{3}{35}a\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(1729c-\frac{935\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.\right.$$
$$\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

input
```
Int[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

3.59. $\int\left(a+bx^3\right)^{3/2}\left(ac+adx+bcx^3+bdx^4\right)\,dx$

output
```
(2*(19*c*x + 17*d*x^2)*(a + b*x^3)^(5/2))/323 + (15*a*((2*(247*c*x + 187*d
*x^2)*(a + b*x^3)^(3/2))/143 + (9*a*((2*(1729*c*x + 935*d*x^2)*Sqrt[a + b*
x^3])/35 + (3*a*((935*d*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])))/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(1729*c - (935*(1 - Sqrt[3])*a^(1/3)*
d)/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/35))/143))/323
```

3.59.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2391
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[PolynomialQuoti
ent[Pq, a + b*x^n, x]*(a + b*x^n)^(p + 1), x] /; FreeQ[{a, b, p}, x] && Pol
yQ[Pq, x] && IGtQ[n, 0] && GeQ[Expon[Pq, x], n] && EqQ[PolynomialRemainder[
Pq, a + b*x^n, x], 0]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.59.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.34

method	result	size
risch	Expression too large to display	786
elliptic	Expression too large to display	830
default	Expression too large to display	1618

input
```
int((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
2/323323*x*(17017*b^2*d*x^7+19019*b^2*c*x^6+53669*a*b*d*x^4+63973*a*b*c*x^
3+61897*a^2*d*x+91637*a^2*c)*(b*x^3+a)^(1/2)+81/323323*a^3*(-3458/3*I*c*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-1870/3
*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

3.59.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.20

$$\int (a+bx^3)^{3/2} (ac+adx+bcx^3+bdx^4)\ dx = \frac{2\left(140049\,a^3\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-75735\,a^3\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weiers}\right.\right.}{}$$

input
```
integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas
")
```

output
```
2/323323*(140049*a^3*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 75735*a
^3*sqrt(b)*d*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
 + (17017*b^3*d*x^8 + 19019*b^3*c*x^7 + 53669*a*b^2*d*x^5 + 63973*a*b^2*c*
x^4 + 61897*a^2*b*d*x^2 + 91637*a^2*b*c*x)*sqrt(b*x^3 + a))/b
```

3.59.6 Sympy [A] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.45

$$\int (a+bx^3)^{3/2}\left(ac+adx+bcx^3+bdx^4\right)\,dx = \frac{a^{\frac{5}{2}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{5}{2}}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{2a^{\frac{3}{2}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{3}{2}}bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}b^2cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}b^2dx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

output
```
a**(5/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(5/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a**(3/2)*b*c*x**4*gamma(4/3)*
hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a*
*(3/2)*b*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(8/3)) + sqrt(a)*b**2*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (
10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b**2*d*x**8*ga
mma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11
/3))
```

3.59.7 Maxima [F]

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \left(bdx^4+bcx^3+adx+ac\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)
```

3.59.8 Giac [F]

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \left(bdx^4+bcx^3+adx+ac\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \left(b\,x^3+a\right)^{3/2}\left(b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c\right)\,dx$$

input
```
int((a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

output
```
int((a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x)
```

3.60 $\int \sqrt{a+bx^3}(ac+adx+bcx^3+bdx^4)\ dx$

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3.60.1 Optimal result

Integrand size = 32, antiderivative size = 556

$$\int \sqrt{a+bx^3}(ac+adx+bcx^3+bdx^4)\ dx = \frac{54a^2d\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{18a(91cx+55dx^2)\sqrt{a+bx^3}}{5005}+\frac{2}{143}(13cx+11dx^2)\left(a+bx^3\right)^{3/2}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(91\sqrt[3]{b}c-55\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\right.}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/143*(11*d*x^2+13*c*x)*(b*x^3+a)^(3/2)+18/5005*a*(55*d*x^2+91*c*x)*(b*x^3
+a)^(1/2)+54/91*a^2*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))-27/91*3^(1/4)*a^(7/3)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2
)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+18/5005*3^(3/4)*a^2*(a^(1/3)+b^
(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2))),I*3^(1/2)+2*I)*(91*b^(1/3)*c-55*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/
2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.60.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.60 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.14

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx = \frac{ax\sqrt{a+bx^3}\left(2c\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
(a*x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3
)/a])
```

3.60.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2391, 2392, 27, 2392, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx$$

$$\Big\downarrow \; 2391$$

$$\int \left(a+bx^3\right)^{3/2}(c+dx)dx$$

$$\Big\downarrow \; 2392$$

$$\frac{9}{2}a\int \frac{2}{143}(13c+11dx)\sqrt{bx^3+a}dx+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

$$\Big\downarrow \; 27$$

$$\frac{9}{143}a\int (13c+11dx)\sqrt{bx^3+a}dx+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

$$\Big\downarrow \; 2392$$

$$\frac{9}{143}a\left(\frac{3}{2}a\int \frac{2(91c+55dx)}{35\sqrt{bx^3+a}}dx+\frac{2}{35}\sqrt{a+bx^3}\left(91cx+55dx^2\right)\right)+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

$$\Big\downarrow \; 27$$

$$\frac{9}{143}a\left(\frac{3}{35}a\int \frac{91c+55dx}{\sqrt{bx^3+a}}dx+\frac{2}{35}\sqrt{a+bx^3}\left(91cx+55dx^2\right)\right)+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

$$\Big\downarrow \; 2417$$

$$\frac{9}{143}a\left(\frac{3}{35}a\left(\left(91c-\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{55d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{35}\sqrt{a+bx^3}(91cx+55$$

$$\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

$$\Big\downarrow \; 759$$

$$\frac{9}{143}a\left(\frac{3}{35}a\left(\frac{55d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(91c-\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}$$

$$\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

$$\Big\downarrow \; 2416$$

3.60. $\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx$

$$\frac{9}{143}a\left(\frac{3}{35}a\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(91c-\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\right.}{\sqrt[3]{b}x+\left(1+\right.}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.$$

$$\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

input
```
Int[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
(2*(13*c*x + 11*d*x^2)*(a + b*x^3)^(3/2))/143 + (9*a*((2*(91*c*x + 55*d*x^
2)*Sqrt[a + b*x^3])/35 + (3*a*((55*d*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])))/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(91*c - (55*(1 - Sqrt[3]
)*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/35))/143
```

3.60. $\int \sqrt{a+bx^3}(ac+adx+bcx^3+bdx^4)\,dx$

3.60.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2391
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[PolynomialQuoti
ent[Pq, a + b*x^n, x]*(a + b*x^n)^(p + 1), x] /; FreeQ[{a, b, p}, x] && Pol
yQ[Pq, x] && IGtQ[n, 0] && GeQ[Expon[Pq, x], n] && EqQ[PolynomialRemainder[
Pq, a + b*x^n, x], 0]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.60.4 Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.37

method	result
risch	$\frac{2x\left(385bd\,x^{4}+455bc\,x^{3}+880adx+1274ac\right)\sqrt{b\,x^{3}+a}}{5005}+\frac{27a^{2}\left(-\frac{182ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\cdots}{-\frac{3\left(-a\,b^{2}\right)\cdots}{2b}\cdots}}\cdots}{\cdots}\cdots\right)}{\cdots}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
2/5005*x*(385*b*d*x^4+455*b*c*x^3+880*a*d*x+1274*a*c)*(b*x^3+a)^(1/2)+27/5
005*a^2*(-182/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*
b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2))-110/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))))
```

3.60.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.17

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx$$
$$=\frac{2\left(2457\,a^2\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-1485\,a^2\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\right.\right.}{5005\,b}$$

input
```
integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas
")
```

output
```
2/5005*(2457*a^2*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 1485*a^2*sq
rt(b)*d*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (3
85*b^2*d*x^5 + 455*b^2*c*x^4 + 880*a*b*d*x^2 + 1274*a*b*c*x)*sqrt(b*x^3 +
a))/b
```

3.60.6 Sympy [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.31

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx = \frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

output
```
a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x**4*gamma(4/3)*hyp
er((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)
*b*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(8/3))
```

3.60.7 Maxima [F]

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx = \int \left(bdx^4+bcx^3+adx+ac\right)\sqrt{bx^3+a}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

3.60.8 Giac [F]

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \left(bdx^4+bcx^3+adx+ac\right)\sqrt{bx^3+a}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \sqrt{b\,x^3+a}\,\left(b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c\right)\,dx$$

input
```
int((a + b*x^3)^(1/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

output
```
int((a + b*x^3)^(1/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x)
```

3.61 $\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx$

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3.61.1 Optimal result

Integrand size = 32, antiderivative size = 525

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx = \frac{6ad\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2}{35}\left(7cx+5dx^2\right)\sqrt{a+bx^3}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(7\sqrt[3]{b}c-5\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{(1-}{(1+}\right.\right.}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/35*(5*d*x^2+7*c*x)*(b*x^3+a)^(1/2)+6/7*a*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))-3/7*3^(1/4)*a^(4/3)*d*(a^(1/3)+b^(1/3)*x)*Ellip
ticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/35*3^(3/
4)*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*c-5*a^(1/3)*d*(1-3^(1/2
)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.61.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.14

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx = \frac{x\sqrt{a+bx^3}\left(2c\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)] +
d*x*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3)/
a])
```

3.61.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2019, 2392, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \; 2019$$

$$\int \sqrt{a+bx^3}(c+dx)dx$$

$$\Big\downarrow \; 2392$$

$$\frac{3}{2}a\int \frac{2(7c+5dx)}{35\sqrt{bx^3+a}}dx+\frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

$$\Big\downarrow \; 27$$

$$\frac{3}{35}a\int \frac{7c+5dx}{\sqrt{bx^3+a}}dx+\frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

$$\Big\downarrow \; 2417$$

$$\frac{3}{35}a\left(\left(7c-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{5d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

$$\Big\downarrow \; 759$$

$$\frac{3}{35}a\left(\frac{5d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7c-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\Big(}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.$$

$$\frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

$$\Big\downarrow \; 2416$$

3.61. $\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx$

$$\frac{3}{35}a\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7c-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.$$

$$\frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]
```

output
```
(2*(7*c*x + 5*d*x^2)*Sqrt[a + b*x^3])/35 + (3*a*((5*d*((2*Sqrt[a + b*x^3])
/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]
]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[
3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(7*c -
(5*(1 - Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) +
 b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/35
```

3.61. $\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx$

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.61. $\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx$

3.61.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 744, normalized size of antiderivative = 1.42

method	result
risch	$\frac{2x(5dx+7c)\sqrt{b\,x^3+a}}{35}+\frac{3a\left(-\frac{14ic\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\right.}{2}\right.}{}}}{}\right.}{}$
elliptic	$\frac{2d\,x^2\sqrt{b\,x^3+a}}{7}+\frac{2cx\sqrt{b\,x^3+a}}{5}-\frac{2iac\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x\right.}{}}}{}$
default	Expression too large to display

3.61. $\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*x*(5*d*x+7*c)*(b*x^3+a)^(1/2)+3/35*a*(-14/3*I*c*3^(1/2)/b*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-10/3*I*d*3^(1/2)/b*(-a*b^
2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.61.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.13

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}}\,dx$$
$$= \frac{2\left(21\,a\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right) - 15\,a\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b}\right.\right.\right.}{35\,b}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="fricas
")
```

output
```
2/35*(21*a*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 15*a*sqrt(b)*d*we
ierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*b*d*x^2 +
 7*b*c*x)*sqrt(b*x^3 + a))/b
```

3.61.6 Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.31

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((1/2, 4/3
), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + b*d*x**5*gam
ma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gam
ma(8/3))
```

3.61.7 Maxima [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx = \int \frac{bdx^4+bcx^3+adx+ac}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

3.61.8 Giac [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}} dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{\sqrt{bx^3 + a}} dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}} dx = \int \frac{b\,d\,x^4 + b\,c\,x^3 + a\,d\,x + a\,c}{\sqrt{b\,x^3 + a}} dx$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(1/2),x)
```

output
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(1/2), x)
```

3.62 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx$

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3.62.1 Optimal result

Integrand size = 32, antiderivative size = 490

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx=\frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3
)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b
^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c-a^(1/
3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

3.62.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.15

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F}\right.}{2\sqrt{a+bx^3}}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
 + d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[a + b*x^3]
)
```

3.62.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2019, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{c+dx}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2417}$$

$$\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{d\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x]
```

3.62. $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx$

output
```
(d*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3
^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (2*Sq
rt[2 + Sqrt[3]]*(c - ((1 - Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

3.62.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.62.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.47

method	result
elliptic	$-\frac{2ic\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\,F}{3b\sqrt{b\,x^3+a}}$
default	Expression too large to display

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.62.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.09

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(\sqrt{b}c\,\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}d\,\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weier}\right.\right.}{b}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="fricas
")
```

output
```
2*(sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - sqrt(b)*d*weierstrassZeta
(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

3.62.6 Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.16

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(3/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

3.62.7 Maxima [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

3.62.8 Giac [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \int \frac{b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x)
```

output
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2), x)
```

3.63 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx$

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3.63.1 Optimal result

Integrand size = 32, antiderivative size = 522

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \frac{2x(c+dx)}{3a\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c+\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*x*(d*x+c)/a/(b*x^3+a)^(1/2)-2/3*d*(b*x^3+a)^(1/2)/a/b^(2/3)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))+1/3*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2
)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/9*(a^(1/3)+b^
(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c+a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2
*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.63.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.18

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \frac{x\left(4c+2c\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+3dx\sqrt{1+\frac{bx^3}{a}}\,\text{Hy}\right.}{6a\sqrt{a+bx^3}}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x]
```

output
```
(x*(4*c + 2*c*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^
3)/a)] + 3*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x
^3)/a)]))/(6*a*Sqrt[a + b*x^3])
```

3.63.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2019, 2394, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{c+dx}{(a+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{ 2394}$$

$$\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}-\frac{2\int -\frac{c-dx}{2\sqrt{bx^3+a}}dx}{3a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{c-dx}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 2417}$$

$$\frac{\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}+\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{\ldots}\right.}{\sqrt{bx^3\ldots}}}{\sqrt[3]{b}}}{3a}$$

$$\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 2416}$$

3.63. $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx$

$$\frac{\dfrac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - d\left(\frac{2\sqrt{}}{\sqrt[3]{b}\left(\left(1+\sqrt{}\right.\right.}\right.}{3a}$$

$$\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x]
```

output
```
(2*x*(c + d*x))/(3*a*Sqrt[a + b*x^3]) + (-((d*((2*Sqrt[a + b*x^3])/(b^(1/3
)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b
^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3)) + (2*Sqrt[2 + Sqrt[3]]*(c + ((1 - Sqr
t[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a)
```

3.63.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.63. $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx$

3.63.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.47

method	result
elliptic	$-\frac{2b\left(-\frac{d\,x^2}{3ab}-\frac{cx}{3ba}\right)}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2ic\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\cdots\right.}{\left(-\cdots\right.}}}{9ab\sqrt{b\,x\cdots}}$
default	Expression too large to display

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/3/a/b*d*x^2-1/3/b/a*c*x)/((x^3+a/b)*b)^(1/2)-2/9*I*c/a*3^(1/2)/b*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/9*I*d/a*3^(1
/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.63.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.18

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \frac{2\left((bcx^3+ac)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+(bdx^3+ad)\sqrt{b}\text{weierstra}\right.}{3\,(ab^2x^3}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="fricas
")
```

output
```
2/3*((b*c*x^3 + a*c)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (b*d*x^3
+ a*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x
)) + (b*d*x^2 + b*c*x)*sqrt(b*x^3 + a))/(a*b^2*x^3 + a^2*b)
```

3.63.6 Sympy [A] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.31

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(5/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 5
/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(7/3)) + b*d*x**5*
gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)
*gamma(8/3))
```

3.63.7 Maxima [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

3.63.8 Giac [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{5/2}} \, dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{5}{2}}} \, dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{5/2}} \, dx = \int \frac{b\,d\,x^4 + b\,c\,x^3 + a\,d\,x + a\,c}{(b\,x^3 + a)^{5/2}} \, dx$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x)
```

output
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2), x)
```

3.64 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx$

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3.64.1 Optimal result

Integrand size = 32, antiderivative size = 554

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx = \frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}$$
$$+\frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}}-\frac{10d\sqrt{a+bx^3}}{27a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{5\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}c+5\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{27\sqrt[4]{3}a^2b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*x*(d*x+c)/a/(b*x^3+a)^(3/2)+2/27*x*(5*d*x+7*c)/a^2/(b*x^3+a)^(1/2)-10/
27*d*(b*x^3+a)^(1/2)/a^2/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+5/27*d*(a
^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)
/a^(5/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/81*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^
(1/3)*c+5*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)
/a^2/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.22

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx = \frac{4cx(10a+7bx^3)+14cx(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\right.}{54a^2\,(a+bx}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x]
```

output
```
(4*c*x*(10*a + 7*b*x^3) + 14*c*x*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeom
etric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 27*d*x^2*(a + b*x^3)*Sqrt[1 + (b*x
^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(54*a^2*(a + b*x^3)
^(3/2))
```

3.64.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2019, 2394, 27, 2394, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx$$

$\Big\downarrow$ 2019

$$\int \frac{c+dx}{(a+bx^3)^{5/2}}dx$$

$\Big\downarrow$ 2394

$$\frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}-\frac{2\int -\frac{7c+5dx}{2(bx^3+a)^{3/2}}dx}{9a}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{7c+5dx}{(bx^3+a)^{3/2}}dx}{9a}+\frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}$$

$\Big\downarrow$ 2394

$$\frac{\frac{2x(7c+5dx)}{3a\sqrt{a+bx^3}}-\frac{2\int -\frac{7c-5dx}{2\sqrt{bx^3+a}}dx}{3a}}{9a}+\frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{7c-5dx}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x(7c+5dx)}{3a\sqrt{a+bx^3}}}{9a}+\frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}$$

$\Big\downarrow$ 2417

$$\frac{\frac{\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+7c\right)\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{5d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}+\frac{2x(7c+5dx)}{3a\sqrt{a+bx^3}}}{9a}+\frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}$$

$\Big\downarrow$ 759

$$\frac{\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+7c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{5d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}}{9a}$$

$$\frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}$$

3.64. $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx$

$$\Big\downarrow \text{2416}$$

$$\frac{\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+7c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-5d\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}\right.}{3a}}{9a}$$

$$\frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x]
```

output
```
(2*x*(c + d*x))/(9*a*(a + b*x^3)^(3/2)) + ((2*x*(7*c + 5*d*x))/(3*a*Sqrt[a
 + b*x^3]) + ((-5*d*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))
)/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(7*c + (5*(1 - Sqrt[3])*a^(1/3)*d)/b^(1/3
))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a))/(9*a)
```

3.64. $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx$

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.64. $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx$

3.64.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.46

method	result	size
elliptic	Expression too large to display	809
default	Expression too large to display	1782

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(2/9*d/a/b^2*x^2+2/9*c/a/b^2*x)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2*b*(-5/27/a^2
/b*d*x^2-7/27/a^2/b*c*x)/((x^3+a/b)*b)^(1/2)-14/81*I/a^2*c*3^(1/2)/b*(-a*b
^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+10/81*I*d/a^2*3^(1/
2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.64.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.28

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx = \frac{2\left(7\left(b^2cx^6+2\,abcx^3+a^2c\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+5\left(b^2dx^6+\right.\right.}{}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="fricas
")
```

output
```
2/27*(7*(b^2*c*x^6 + 2*a*b*c*x^3 + a^2*c)*sqrt(b)*weierstrassPInverse(0, -
4*a/b, x) + 5*(b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d)*sqrt(b)*weierstrassZeta(0,
 -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*b^2*d*x^5 + 7*b^2*c*x^4 +
 8*a*b*d*x^2 + 10*a*b*c*x)*sqrt(b*x^3 + a))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 +
 a^4*b)
```

3.64.6 Sympy [A] (verification not implemented)

Time = 11.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.29

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{7}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{7}{2}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{7}{2}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{3},\frac{7}{2}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(7/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 7/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 7/2), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(5/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 7
/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)*gamma(7/3)) + b*d*x**5*
gamma(5/3)*hyper((5/3, 7/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)
*gamma(8/3))
```

3.64.7 Maxima [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{7/2}}\, dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{7}{2}}}\, dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)
```

3.64.8 Giac [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{7/2}}\, dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{7}{2}}}\, dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{7/2}}\, dx = \int \frac{b\,d\,x^4 + b\,c\,x^3 + a\,d\,x + a\,c}{(b\,x^3 + a)^{7/2}}\, dx$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x)
```

output
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2), x)
```

3.65 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx$

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3.65.1 Optimal result

Integrand size = 32, antiderivative size = 581

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx=\frac{2x(c+dx)}{15a(a+bx^3)^{5/2}}+\frac{2x(13c+11dx)}{135a^2(a+bx^3)^{3/2}}$$

$$+\frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}}-\frac{22d\sqrt{a+bx^3}}{81a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{11\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{27\,3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(91\sqrt[3]{b}c+55\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots}\right.\right.}{405\sqrt[4]{3}a^3b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/15*x*(d*x+c)/a/(b*x^3+a)^(5/2)+2/135*x*(11*d*x+13*c)/a^2/(b*x^3+a)^(3/2)
+2/405*x*(55*d*x+91*c)/a^3/(b*x^3+a)^(1/2)-22/81*d*(b*x^3+a)^(1/2)/a^3/b^(
2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+11/81*d*(a^(1/3)+b^(1/3)*x)*EllipticE
((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/b^(2/3)/(b*x^3+a)^
(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)+2/1215*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(91*b^(1/3)*c+55*a^(1/3)*d*(1-
3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^3/b^(2/3)/(b*x^3+a)^
(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)
```

3.65.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.24

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx = \frac{4cx(157a^2+221abx^3+91b^2x^6)+182cx(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeomet}}{81}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x]
```

output
```
(4*c*x*(157*a^2 + 221*a*b*x^3 + 91*b^2*x^6) + 182*c*x*(a + b*x^3)^2*Sqrt[1
 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 405*d*x^2*(
a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 7/2, 5/3, -((b*x^3
)/a)])/(810*a^3*(a + b*x^3)^(5/2))
```

3.65.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2019, 2394, 27, 2394, 27, 2394, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{c+dx}{(a+bx^3)^{7/2}}dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}-\frac{2\int -\frac{13c+11dx}{2(bx^3+a)^{5/2}}dx}{15a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{13c+11dx}{(bx^3+a)^{5/2}}dx}{15a}+\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{2x(13c+11dx)}{9a(a+bx^3)^{3/2}}-\frac{2\int -\frac{91c+55dx}{2\left(bx^3+a\right)^{3/2}}dx}{9a}}{15a}+\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{91c+55dx}{\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2x(13c+11dx)}{9a(a+bx^3)^{3/2}}}{15a}+\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{2x(91c+55dx)}{3a\sqrt{a+bx^3}}-\frac{2\int -\frac{91c-55dx}{2\sqrt{bx^3+a}}dx}{3a}}{9a}+\frac{2x(13c+11dx)}{9a(a+bx^3)^{3/2}}}{15a}+\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{91c-55dx}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x(91c+55dx)}{3a\sqrt{a+bx^3}}}{9a}+\frac{2x(13c+11dx)}{9a(a+bx^3)^{3/2}}}{15a}+\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

$$\Big\downarrow \text{2417}$$

3.65. $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}dx$

$$\frac{\frac{\frac{\left(\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+91c\right)\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{55d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}+\frac{2x(91c+55dx)}{3a\sqrt{a+bx^3}}}{9a}+\frac{2x(13c+11dx)}{9a(a+bx^3)^{3/2}}}{15a}+\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

$\downarrow$ 759

$$\frac{\frac{\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+91c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{55d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt{bx^3+a}}}{\sqrt[3]{b}}}{3a}}{9a}}{15a}$$

$$\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

$\downarrow$ 2416

$$\frac{\frac{\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+91c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-55d\left(\frac{2\sqrt{a+bx}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}\right.}{3a}}{9a}}{1}$$

$$\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x]
```

3.65. $\int \frac{ac+adx+bcx^3+bdx^4}{\left(a+bx^3\right)^{9/2}}\,dx$

output
```
(2*x*(c + d*x))/(15*a*(a + b*x^3)^(5/2)) + ((2*x*(13*c + 11*d*x))/(9*a*(a
+ b*x^3)^(3/2)) + ((2*x*(91*c + 55*d*x))/(3*a*Sqrt[a + b*x^3]) + ((-55*d*(
(2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (2*Sqrt[2
 + Sqrt[3]]*(91*c + (55*(1 - Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3]))/(3*a))/(9*a))/(15*a)
```

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.65.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 853, normalized size of antiderivative = 1.47

method	result	size
elliptic	Expression too large to display	853
default	Expression too large to display	1902

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
(2/15*d/a/b^3*x^2+2/15*c/a/b^3*x)*(b*x^3+a)^(1/2)/(x^3+a/b)^3+(22/135/b^2/
a^2*d*x^2+26/135/b^2/a^2*c*x)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2*b*(-11/81/a^3/
b*d*x^2-91/405/a^3/b*c*x)/((x^3+a/b)*b)^(1/2)-182/1215*I*c/a^3*3^(1/2)/b*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+22/243*I*d/a^3*
3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b...
```

3.65.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.37

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx = \frac{2\left(91\,(b^3cx^9+3\,ab^2cx^6+3\,a^2bcx^3+a^3c)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right.}{}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="fricas
")
```

output
```
2/405*(91*(b^3*c*x^9 + 3*a*b^2*c*x^6 + 3*a^2*b*c*x^3 + a^3*c)*sqrt(b)*weie
rstrassPInverse(0, -4*a/b, x) + 55*(b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*b*d*
x^3 + a^3*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*
a/b, x)) + (55*b^3*d*x^8 + 91*b^3*c*x^7 + 143*a*b^2*d*x^5 + 221*a*b^2*c*x^
4 + 115*a^2*b*d*x^2 + 157*a^2*b*c*x)*sqrt(b*x^3 + a))/(a^3*b^4*x^9 + 3*a^4
*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)
```

3.65.6 Sympy [A] (verification not implemented)

Time = 36.62 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.28

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{9}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{9}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{9}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{9}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{9}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{9}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(9/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 9/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
7/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 9/2), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(7/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 9
/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(9/2)*gamma(7/3)) + b*d*x**5*
gamma(5/3)*hyper((5/3, 9/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(9/2)
*gamma(8/3))
```

3.65.7 Maxima [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx = \int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

3.65.8 Giac [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{9/2}} dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{9}{2}}} dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{9/2}} dx = \int \frac{b\,d\,x^4 + b\,c\,x^3 + a\,d\,x + a\,c}{(b\,x^3 + a)^{9/2}} dx$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x)
```

output
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2), x)
```

3.66 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$

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3.66.1 Optimal result

Integrand size = 32, antiderivative size = 590

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2e\sqrt{a+bx^3}}{3b} + \frac{2fx\sqrt{a+bx^3}}{5b} + \frac{2gx^2\sqrt{a+bx^3}}{7b} + \frac{2(7bd-4ag)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Elliptic}}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*e*(b*x^3+a)^(1/2)/b+2/5*f*x*(b*x^3+a)^(1/2)/b+2/7*g*x^2*(b*x^3+a)^(1/2
)/b+2/7*(-4*a*g+7*b*d)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))-1/7*3^(1/4)*a^(1/3)*(-4*a*g+7*b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(
1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/105*(a^(1/3)+b^(1
/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*(-2*a*f+5*b*c)-5*a^(1/3)*(-4*a*g+7*b*d)*(
1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1
/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.66.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.23

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{\sqrt{a + bx^3}} \, dx$$
$$= \frac{4(a + bx^3)(35e + 3x(7f + 5gx)) + 42(5bc - 2af)x\sqrt{1 + \frac{bx^3}{a}} \text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, -\frac{bx^3}{a}\right) + 15(7b}{210b\sqrt{a + bx^3}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3],x]
```

output
```
(4*(a + b*x^3)*(35*e + 3*x*(7*f + 5*g*x)) + 42*(5*b*c - 2*a*f)*x*Sqrt[1 +
(b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 15*(7*b*d - 4*
a*g)*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)
])/(210*b*Sqrt[a + b*x^3])
```

3.66.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2427, 27, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2427}$$

$$\frac{2\int \frac{7bfx^3+7bex^2+(7bd-4ag)x+7bc}{2\sqrt{bx^3+a}}dx}{7b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7bfx^3+7bex^2+(7bd-4ag)x+7bc}{\sqrt{bx^3+a}}dx}{7b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{2427}$$

$$\frac{\frac{2\int \frac{35b^2ex^2+5b(7bd-4ag)x+7b(5bc-2af)}{2\sqrt{bx^3+a}}dx}{5b}+\frac{14}{5}fx\sqrt{a+bx^3}}{7b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{35b^2ex^2+5b(7bd-4ag)x+7b(5bc-2af)}{\sqrt{bx^3+a}}dx}{5b}+\frac{14}{5}fx\sqrt{a+bx^3}}{7b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{2425}$$

$$\frac{\frac{35b^2e\int \frac{x^2}{\sqrt{bx^3+a}}dx+\int \frac{7b(5bc-2af)+5b(7bd-4ag)x}{\sqrt{bx^3+a}}dx}{5b}+\frac{14}{5}fx\sqrt{a+bx^3}}{7b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{793}$$

$$\frac{\frac{\int \frac{7b(5bc-2af)+5b(7bd-4ag)x}{\sqrt{bx^3+a}}dx+\frac{70}{3}be\sqrt{a+bx^3}}{5b}+\frac{14}{5}fx\sqrt{a+bx^3}}{7b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{2417}$$

$$\frac{\frac{b^{2/3}\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd-4ag)\right)\int \frac{1}{\sqrt{bx^3+a}}dx+5b^{2/3}(7bd-4ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{70}{3}be\sqrt{a+bx^3}}{5b}+\frac{14}{5}fx\sqrt{a+bx^3}}{7b}-\cdots$$

$$\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \; 759$$

$$\frac{\frac{5b^{2/3}(7bd-4ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\cdots\right.}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots}{5b}\cdots}{7b}$$

$$\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \; 2416$$

$$\frac{\frac{\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd-4ag)\right)}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\cdots}{\cdots}}{\cdots}$$

$$\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

input

```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3],x]
```

3.66. $\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$

output
```
(2*g*x^2*Sqrt[a + b*x^3])/(7*b) + ((14*f*x*Sqrt[a + b*x^3])/5 + ((70*b*e*S
qrt[a + b*x^3])/3 + 5*b^(2/3)*(7*b*d - 4*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3
)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b
^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])) + (2*Sqrt[2 + Sqrt[3]]*b^(1/3)*(7*b^(1/3)*(5*b*
c - 2*a*f) - 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3]))/(5*b))/(7*b)
```

3.66.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.66.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{2g\,x^{2}\sqrt{b\,x^{3}+a}}{7b}+\frac{2fx\sqrt{b\,x^{3}+a}}{5b}+\frac{2e\sqrt{b\,x^{3}+a}}{3b}-\frac{2i\left(c-\frac{2af}{5b}\right)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-\cdots}{\cdots}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\cdots}}\cdots}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*g*x^2*(b*x^3+a)^(1/2)/b+2/5*f*x*(b*x^3+a)^(1/2)/b+2/3*e*(b*x^3+a)^(1/2
)/b-2/3*I*(c-2/5*a/b*f)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*
(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))-2/3*I*(d-4/7*a/b*g)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)))
```

3.66. $\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$

3.66.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.15

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = \frac{2\left(21\,(5\,bc-2\,af)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-15\,(7\,bd-4\,ag)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weiers}\right.\right.}{105\,b^2}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(21*(5*b*c - 2*a*f)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - 15*(
7*b*d - 4*a*g)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -
4*a/b, x)) + (15*b*g*x^2 + 21*b*f*x + 35*b*e)*sqrt(b*x^3 + a))/b^2
```

3.66.6 Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.32

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = e\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

output
```
e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True)
) + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((1/2, 4
/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + g*x**5*gam
ma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gam
ma(8/3))
```

3.66.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

3.66.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx=\int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(1/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(1/2), x)
```

3.67 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx$

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3.67.1 Optimal result

Integrand size = 32, antiderivative size = 594

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx=\frac{2x(bc-af+(bd-ag)x+bex^2)}{3ab\sqrt{a+bx^3}}$$

$$-\frac{2e\sqrt{a+bx^3}}{3ab}-\frac{2(bd-4ag)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt{2-\sqrt{3}}(bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}(bc+2af)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\operatorname{arcs}\right.}{3\sqrt[4]{3}ab^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*x*(b*c-a*f+(-a*g+b*d)*x+b*e*x^2)/a/b/(b*x^3+a)^(1/2)-2/3*e*(b*x^3+a)^(
1/2)/a/b-2/3*(-4*a*g+b*d)*(b*x^3+a)^(1/2)/a/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))+1/3*(-4*a*g+b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)
-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/9*(a^(1/3)+b^(
1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*(2*a*f+b*c)+a^(1/3)*(-4*a*g+b*d)*(1-3^(1/2
)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(5/3)/(b*x^3+a)^(1/2)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.67.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.22

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \frac{4bcx-4a(e+x(f-3gx))+2(bc+2af)x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}}{6ab}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x]
```

output
```
(4*b*c*x - 4*a*(e + x*(f - 3*g*x)) + 2*(b*c + 2*a*f)*x*Sqrt[1 + (b*x^3)/a]
*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*(b*d - 4*a*g)*x^2*Sqrt
[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqr
t[a + b*x^3])
```

3.67.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 589, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2397, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}-\frac{2\int -\frac{-3b^2ex^2-b(bd-4ag)x+b(bc+2af)}{2\sqrt{bx^3+a}}dx}{3ab^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-3b^2ex^2-b(bd-4ag)x+b(bc+2af)}{\sqrt{bx^3+a}}dx}{3ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2425}$$

$$\frac{\int \frac{b(bc+2af)-b(bd-4ag)x}{\sqrt{bx^3+a}}dx-3b^2e\int \frac{x^2}{\sqrt{bx^3+a}}dx}{3ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{793}$$

$$\frac{\int \frac{b(bc+2af)-b(bd-4ag)x}{\sqrt{bx^3+a}}dx-2be\sqrt{a+bx^3}}{3ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2417}$$

$$\frac{b^{2/3}\left(\sqrt[3]{b}(2af+bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\int \frac{1}{\sqrt{bx^3+a}}dx-b^{2/3}(bd-4ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-2be\sqrt{a+bx^3}}{3ab^2}$$
$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

$$\frac{-b^{2/3}(bd-4ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+b}}}{3ab^2}$$
$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2416}$$

3.67. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx$

$$\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{b}(2af+bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4\right.}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x]
```

output
```
(2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(3*a*b*Sqrt[a + b*x^3]) + (-2*
b*e*Sqrt[a + b*x^3] - b^(2/3)*(b*d - 4*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])) + (2*Sqrt[2 + Sqrt[3]]*b^(1/3)*(b^(1/3)*(b*c + 2*
a*f) + (1 - Sqrt[3])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a*
b^2)
```

3.67.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

3.67. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx$

3.67.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	821
default	Expression too large to display	1547

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/3*(a*g-b*d)/a/b^2*x^2+1/3*(a*f-b*c)/b^2/a*x+1/3*e/b^2)/((x^3+a/b)*
b)^(1/2)-2/3*I*(f/b-1/3*(a*f-b*c)/a/b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(g/b+1/3*(a*g-b*d)/a/b)*3^(1/2)/b
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.67.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.26

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(\left((b^2c+2\,abf)x^3+abc+2\,a^2f\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(\right.}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(((b^2*c + 2*a*b*f)*x^3 + a*b*c + 2*a^2*f)*sqrt(b)*weierstrassPInverse
(0, -4*a/b, x) + ((b^2*d - 4*a*b*g)*x^3 + a*b*d - 4*a^2*g)*sqrt(b)*weierst
rassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - sqrt(b*x^3 + a)*(
a*b*e - (b^2*d - a*b*g)*x^2 - (b^2*c - a*b*f)*x))/(a*b^3*x^3 + a^2*b^2)
```

3.67.6 Sympy [A] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.32

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = e\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

output
```
e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), Tru
e)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**
3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((4/
3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + g*x**
5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/
2)*gamma(8/3))
```

3.67.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

3.67.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2), x)
```

3.68 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx$

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3.68.1 Optimal result

Integrand size = 32, antiderivative size = 628

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx=\frac{2x(bc-af+(bd-ag)x+bex^2)}{9ab\,(a+bx^3)^{3/2}}$$
$$-\frac{2(5bd+4ag)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2(3ae-x(7bc+2af+(5bd+4ag)x))}{27a^2b\sqrt{a+bx^3}}$$
$$+\frac{\sqrt{2-\sqrt{3}}(5bd+4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}(7bc+2af)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd+4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\text{a}\right.}{27\sqrt[4]{3}a^2b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*x*(b*c-a*f+(-a*g+b*d)*x+b*e*x^2)/a/b/(b*x^3+a)^(3/2)-2/27*(3*a*e-x*(7*
b*c+2*a*f+(4*a*g+5*b*d)*x))/a^2/b/(b*x^3+a)^(1/2)-2/27*(4*a*g+5*b*d)*(b*x^
3+a)^(1/2)/a^2/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/27*(4*a*g+5*b*d)*
(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/
4)/a^(5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/81*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)
*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^
(1/3)*(2*a*f+7*b*c)+a^(1/3)*(4*a*g+5*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(3/4)/a^2/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.68.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.27

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = \frac{140b^2cx^4+40abx(5c+fx^3)-4a^2(15e+x(5f+27gx))+10(7bc+2af}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]
```

output
```
(140*b^2*c*x^4 + 40*a*b*x*(5*c + f*x^3) - 4*a^2*(15*e + x*(5*f + 27*g*x))
+ 10*(7*b*c + 2*a*f)*x*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1
/3, 1/2, 4/3, -((b*x^3)/a)] + 27*(5*b*d + 4*a*g)*x^2*(a + b*x^3)*Sqrt[1 +
(b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(270*a^2*b*(a +
 b*x^3)^(3/2))
```

3.68.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2397, 27, 2393, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{ 2397}$$

$$\frac{2x(x(bd-ag)-af+bc+bex^2)}{9ab(a+bx^3)^{3/2}}-\frac{2\int -\frac{3b^2ex^2+b(5bd+4ag)x+b(7bc+2af)}{2(bx^3+a)^{3/2}}dx}{9ab^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3b^2ex^2+b(5bd+4ag)x+b(7bc+2af)}{(bx^3+a)^{3/2}}dx}{9ab^2}+\frac{2x(x(bd-ag)-af+bc+bex^2)}{9ab(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 2393}$$

$$\frac{-\frac{2\int -\frac{b(7bc+2af-(5bd+4ag)x)}{2\sqrt{bx^3+a}}dx}{3a}-\frac{2(3abe-bx(x(4ag+5bd)+2af+7bc))}{3a\sqrt{a+bx^3}}}{9ab^2}+\frac{2x(x(bd-ag)-af+bc+bex^2)}{9ab(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{b\int \frac{7bc+2af-(5bd+4ag)x}{\sqrt{bx^3+a}}dx}{3a}-\frac{2(3abe-bx(x(4ag+5bd)+2af+7bc))}{3a\sqrt{a+bx^3}}}{9ab^2}+\frac{2x(x(bd-ag)-af+bc+bex^2)}{9ab(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 2417}$$

$$\frac{\frac{b\left(\left(\frac{(1-\sqrt{3})\sqrt[3]{a}(4ag+5bd)}{\sqrt[3]{b}}+2af+7bc\right)\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{(4ag+5bd)\int \frac{\sqrt[3]{b}x+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{3a}-\frac{2(3abe-bx(x(4ag+5bd)+2af+7bc))}{3a\sqrt{a+bx^3}}}{9ab^2}+\frac{2x(x(bd-ag)-af+bc+bex^2)}{9ab(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\frac{b\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+5bd)}{\sqrt[3]{b}}+2af+7bc\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{(4ag+5b}{}\right.}{3a}}{9ab^2}$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{9ab\left(a+bx^3\right)^{3/2}}$$

$\downarrow$ 2416

$$\frac{b\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+5bd)}{\sqrt[3]{b}}+2af+7bc\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{(4ag+5b}{}\right.}{3a}$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{9ab\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]
```

3.68. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx$

output
```
(2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(9*a*b*(a + b*x^3)^(3/2)) + ((
-2*(3*a*b*e - b*x*(7*b*c + 2*a*f + (5*b*d + 4*a*g)*x)))/(3*a*Sqrt[a + b*x^
3]) + (b*(-(((5*b*d + 4*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])))/b^(1/3)) + (2*Sqrt[2 + Sqrt[3]]*(7*b*c + 2*a*f + ((1 - Sqrt[3])
*a^(1/3)*(5*b*d + 4*a*g))/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(3*a)
)/(9*a*b^2)
```

3.68.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

3.68. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}} \, dx$

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.68.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	861
default	Expression too large to display	1673

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/9/a/b^3*(a*g-b*d)*x^2-2/9/a/b^3*(a*f-b*c)*x-2/9/b^3*e)*(b*x^3+a)^(1/2)
/(x^3+a/b)^2-2*b*(-1/27/a^2/b^2*(4*a*g+5*b*d)*x^2-1/27/a^2/b^2*(2*a*f+7*b*
c)*x)/((x^3+a/b)*b)^(1/2)-2/81*I/a^2/b^2*(2*a*f+7*b*c)*3^(1/2)*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/81*I/a^2/b^2*(4*a*g+5*b
*d)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2...
```

3.68.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(((7b^3c+2ab^2f)x^6+7a^2bc+2a^3f+2(7ab^2c+2a^2bf)x^3)\sqrt{b}\text{weiers}\right.}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/27*(((7*b^3*c + 2*a*b^2*f)*x^6 + 7*a^2*b*c + 2*a^3*f + 2*(7*a*b^2*c + 2*
a^2*b*f)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + ((5*b^3*d + 4*a*
b^2*g)*x^6 + 5*a^2*b*d + 4*a^3*g + 2*(5*a*b^2*d + 4*a^2*b*g)*x^3)*sqrt(b)*
weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + ((5*b^3*d
+ 4*a*b^2*g)*x^5 + (7*b^3*c + 2*a*b^2*f)*x^4 - 3*a^2*b*e + (8*a*b^2*d + a^
2*b*g)*x^2 + (10*a*b^2*c - a^2*b*f)*x)*sqrt(b*x^3 + a))/(a^2*b^4*x^6 + 2*a
^3*b^3*x^3 + a^4*b^2)
```

3.68.6 Sympy [A] (verification not implemented)

Time = 47.42 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = e\left(\begin{cases} -\frac{2}{9ab\sqrt{a+bx^3}+9b^2x^3\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^3}{3a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(5/2),x)
```

output
```
e*Piecewise((-2/(9*a*b*sqrt(a + b*x**3) + 9*b**2*x**3*sqrt(a + b*x**3)), N
e(b, 0)), (x**3/(3*a**(5/2)), True)) + c*x*gamma(1/3)*hyper((1/3, 5/2), (4
/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(4/3)) + d*x**2*gamma(2/3
)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(5/
3)) + f*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_polar(I*pi)/a
)/(3*a**(5/2)*gamma(7/3)) + g*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3))
```

3.68.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2), x)
```

3.68.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx=\int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx=\int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2), x)
```

3.69 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx$

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3.69.1 Optimal result

Integrand size = 32, antiderivative size = 676

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \frac{2x(bc-af+(bd-ag)x+bex^2)}{15ab(a+bx^3)^{5/2}}$$
$$+\frac{2x(7(13bc+2af)+5(11bd+4ag)x)}{405a^3b\sqrt{a+bx^3}}-\frac{2(11bd+4ag)\sqrt{a+bx^3}}{81a^3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{2(9ae-x(13bc+2af+(11bd+4ag)x))}{135a^2b(a+bx^3)^{3/2}}$$
$$+\frac{\sqrt{2-\sqrt{3}}(11bd+4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}a^{8/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(13bc+2af)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}(11bd+4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Ellipticl}}{405\sqrt[4]{3}a^3b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/15*x*(b*c-a*f+(-a*g+b*d)*x+b*e*x^2)/a/b/(b*x^3+a)^(5/2)-2/135*(9*a*e-x*(
13*b*c+2*a*f+(4*a*g+11*b*d)*x))/a^2/b/(b*x^3+a)^(3/2)+2/405*x*(14*a*f+91*b
*c+5*(4*a*g+11*b*d)*x)/a^3/b/(b*x^3+a)^(1/2)-2/81*(4*a*g+11*b*d)*(b*x^3+a)
^(1/2)/a^3/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/81*(4*a*g+11*b*d)*(a^
(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/
a^(8/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)+2/1215*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b
^(1/3)*(2*a*f+13*b*c)+5*a^(1/3)*(4*a*g+11*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)*3^(3/4)/a^3/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.69.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.25 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.29

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \frac{4004b^3cx^7+44ab^2x^4(221c+14fx^3)+44a^2bx(157c+34fx^3)-4a^3(297e}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x]
```

output
```
(4004*b^3*c*x^7 + 44*a*b^2*x^4*(221*c + 14*f*x^3) + 44*a^2*b*x*(157*c + 34
*f*x^3) - 4*a^3*(297*e + x*(77*f + 405*g*x)) + 154*(13*b*c + 2*a*f)*x*(a +
 b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a
)] + 405*(11*b*d + 4*a*g)*x^2*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[2/3, 7/2, 5/3, -((b*x^3)/a)])/(8910*a^3*b*(a + b*x^3)^(5/2))
```

3.69.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2397, 27, 2393, 27, 2394, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\,(a+bx^3)^{5/2}}-\frac{2\int -\frac{9b^2ex^2+b(11bd+4ag)x+b(13bc+2af)}{2(bx^3+a)^{5/2}}dx}{15ab^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{9b^2ex^2+b(11bd+4ag)x+b(13bc+2af)}{(bx^3+a)^{5/2}}dx}{15ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\,(a+bx^3)^{5/2}}$$

$$\Big\downarrow \text{2393}$$

$$\frac{-\frac{2\int -\frac{b(7(13bc+2af)+5(11bd+4ag)x)}{2\left(bx^3+a\right)^{3/2}}dx}{9a}-\frac{2(9abe-bx(x(4ag+11bd)+2af+13bc))}{9a(a+bx^3)^{3/2}}}{15ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\,(a+bx^3)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{b\int \frac{7(13bc+2af)+5(11bd+4ag)x}{\left(bx^3+a\right)^{3/2}}dx}{9a}-\frac{2(9abe-bx(x(4ag+11bd)+2af+13bc))}{9a(a+bx^3)^{3/2}}}{15ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\,(a+bx^3)^{5/2}}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{b\left(\frac{2x(7(2af+13bc)+5x(4ag+11bd))}{3a\sqrt{a+bx^3}}-\frac{2\int -\frac{7(13bc+2af)-5(11bd+4ag)x}{2\sqrt{bx^3+a}}dx}{3a}\right)}{9a}-\frac{2(9abe-bx(x(4ag+11bd)+2af+13bc))}{9a(a+bx^3)^{3/2}}}{15ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\,(a+bx^3)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{b\left(\frac{\int \frac{7(13bc+2af)-5(11bd+4ag)x}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x(7(2af+13bc)+5x(4ag+11bd))}{3a\sqrt{a+bx^3}}\right)}{9a}-\frac{2(9abe-bx(x(4ag+11bd)+2af+13bc))}{9a(a+bx^3)^{3/2}}}{15ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}}$$

$\big\downarrow$ 2417

$$\frac{\frac{b\left(\frac{\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+11bd)}{\sqrt[3]{b}}+14af+91bc\right)\int\frac{1}{\sqrt{bx^3+a}}dx-\frac{5(4ag+11bd)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}+\frac{2x(7(2af+13bc)+5x(4ag+11bd))}{3a\sqrt{a+bx^3}}\right)}{9a}-\frac{2(9abe-bx(x(4\cdots}{9a\cdots}}{15ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}}$$

$\big\downarrow$ 759

$$\frac{\frac{b\left(\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+11bd)}{\sqrt[3]{b}}+14af+91bc\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{5(4\cdots}{\cdots}}{3a}\cdots\right)}{9a}\cdots}{15ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}}$$

$\big\downarrow$ 2416

3.69. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx$

$$b\left(\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+11bd)}{\sqrt[3]{b}}+14af+91bc\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{5(4a\cdots}{\cdots}}{3a\cdots}\cdots\right.$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x]
```

output
```
(2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(15*a*b*(a + b*x^3)^(5/2)) + (
(-2*(9*a*b*e - b*x*(13*b*c + 2*a*f + (11*b*d + 4*a*g)*x)))/(9*a*(a + b*x^3
)^(3/2)) + (b*((2*x*(7*(13*b*c + 2*a*f) + 5*(11*b*d + 4*a*g)*x))/(3*a*Sqrt
[a + b*x^3]) + ((-5*(11*b*d + 4*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])))/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(91*b*c + 14*a*f + (5*(1
 - Sqrt[3])*a^(1/3)*(11*b*d + 4*a*g))/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3]))/(3*a)))/(9*a))/(15*a*b^2)
```

3.69 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^3\right)^{7/2}}\,dx$

3.69.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

3.69. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.69.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 921, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	921
default	Expression too large to display	1793

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/15/a/b^4*(a*g-b*d)*x^2-2/15/a/b^4*(a*f-b*c)*x-2/15/b^4*e)*(b*x^3+a)^(1
/2)/(x^3+a/b)^3+(2/135/a^2/b^3*(4*a*g+11*b*d)*x^2+2/135/a^2/b^3*(2*a*f+13*
b*c)*x)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2*b*(-1/81*(4*a*g+11*b*d)/a^3/b^2*x^2-
7/405*(2*a*f+13*b*c)/a^3/b^2*x)/((x^3+a/b)*b)^(1/2)-14/1215*I*(2*a*f+13*b*
c)/a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+2/243*I*(4*a*g+11*b*d)/a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^...
```

3.69.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.55

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \frac{2\left(7\left((13\,b^4c+2\,ab^3f)x^9+3\,(13\,ab^3c+2\,a^2b^2f)x^6+13\,a^3bc+2\,a^4f+3\right.\right.}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="fricas")
```

output
```
2/405*(7*((13*b^4*c + 2*a*b^3*f)*x^9 + 3*(13*a*b^3*c + 2*a^2*b^2*f)*x^6 +
13*a^3*b*c + 2*a^4*f + 3*(13*a^2*b^2*c + 2*a^3*b*f)*x^3)*sqrt(b)*weierstra
ssPInverse(0, -4*a/b, x) + 5*((11*b^4*d + 4*a*b^3*g)*x^9 + 3*(11*a*b^3*d +
 4*a^2*b^2*g)*x^6 + 11*a^3*b*d + 4*a^4*g + 3*(11*a^2*b^2*d + 4*a^3*b*g)*x^
3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) +
 (5*(11*b^4*d + 4*a*b^3*g)*x^8 + 7*(13*b^4*c + 2*a*b^3*f)*x^7 + 13*(11*a*b
^3*d + 4*a^2*b^2*g)*x^5 - 27*a^3*b*e + 17*(13*a*b^3*c + 2*a^2*b^2*f)*x^4 +
 5*(23*a^2*b^2*d + a^3*b*g)*x^2 + (157*a^2*b^2*c - 7*a^3*b*f)*x)*sqrt(b*x^
3 + a))/(a^3*b^5*x^9 + 3*a^4*b^4*x^6 + 3*a^5*b^3*x^3 + a^6*b^2)
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(7/2),x)
```

output
```
Timed out
```

3.69.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2), x)
```

3.69.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{(b\,x^3+a)^{7/2}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2), x)
```

3.70 $\int \frac{(a+bx)^2}{c+dx^3} \, dx$

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3.70.1 Optimal result

Integrand size = 17, antiderivative size = 186

$$\int \frac{(a+bx)^2}{c+dx^3} \, dx = -\frac{a\left(2b\sqrt[3]{c}+a\sqrt[3]{d}\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}} - \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} + \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d}$$

output
```
-1/3*a*(2*b*c^(1/3)-a*d^(1/3))*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(2/3)+1/6*a
*(2*b*c^(1/3)-a*d^(1/3))*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)
/d^(2/3)+1/3*b^2*ln(d*x^3+c)/d-1/3*a*(2*b*c^(1/3)+a*d^(1/3))*arctan(1/3*(c
^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/d^(2/3)*3^(1/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = \frac{\left(2abc^{2/3}+a^2\sqrt[3]{c}\sqrt[3]{d}\right)\arctan\left(\frac{-\sqrt[3]{c}+2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}cd^{2/3}} + \frac{\left(-2abc^{2/3}+a^2\sqrt[3]{c}\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3cd^{2/3}} - \frac{\left(-2abc^{2/3}+a^2\sqrt[3]{c}\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6cd^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d}$$

input
```
Integrate[(a + b*x)^2/(c + d*x^3),x]
```

output
```
((2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1/3))*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sq
rt[3]*c^(1/3))])/(Sqrt[3]*c*d^(2/3)) + ((-2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1
/3))*Log[c^(1/3) + d^(1/3)*x])/(3*c*d^(2/3)) - ((-2*a*b*c^(2/3) + a^2*c^(1
/3)*d^(1/3))*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c*d^(2/3))
 + (b^2*Log[c + d*x^3])/(3*d)
```

3.70.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2410, 792, 2399, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx$$

$$\Big\downarrow \text{ 2410}$$

$$\int \frac{a^2+2bxa}{dx^3+c}dx+b^2\int \frac{x^2}{dx^3+c}dx$$

$$\Big\downarrow \text{ 792}$$

$$\int \frac{a^2+2bxa}{dx^3+c}dx+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{2399}$$

$$\frac{\int \frac{a\left(2\sqrt[3]{c}\left(\sqrt[3]{d}a+b\sqrt[3]{c}\right)+\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}\sqrt[3]{d}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{a\left(2\sqrt[3]{c}\left(\sqrt[3]{d}a+b\sqrt[3]{c}\right)+\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{2\sqrt[3]{c}\left(\sqrt[3]{d}a+b\sqrt[3]{c}\right)+\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{1142}$$

$$\frac{a\left(\frac{3}{2}\sqrt[3]{c}\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{1}{2}\left(a-\frac{2b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\left(\frac{3}{2}\sqrt[3]{c}\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\left(a-\frac{2b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\left(\frac{3}{2}\sqrt[3]{c}\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\sqrt[3]{d}\left(a-\frac{2b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{1082}$$

3.70. $\int \frac{(a+bx)^2}{c+dx^3}\,dx$

$$\frac{a\left(\frac{1}{2}\sqrt[3]{d}\left(a-\frac{2b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$\Big\downarrow$ 217

$$\frac{a\left(\frac{1}{2}\sqrt[3]{d}\left(a-\frac{2b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$\Big\downarrow$ 1103

$$\frac{a\left(-\frac{\sqrt{3}\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{1}{2}\left(a-\frac{2b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

input
```
Int[(a + b*x)^2/(c + d*x^3),x]
```

output
```
-1/3*(a*(2*b*c^(1/3) - a*d^(1/3))*Log[c^(1/3) + d^(1/3)*x])/(c^(2/3)*d^(2/
3)) + (a*(-((Sqrt[3]*(2*b*c^(1/3) + a*d^(1/3))*ArcTan[(1 - (2*d^(1/3)*x)/c
^(1/3))/Sqrt[3]])/d^(1/3)) - ((a - (2*b*c^(1/3))/d^(1/3))*Log[c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2])/2))/(3*c^(2/3)*d^(1/3)) + (b^2*Log[c + d*x
^3])/(3*d)
```

3.70. $\int\frac{(a+bx)^2}{c+dx^3}\,dx$

3.70.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.70.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.65 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^3 d+c)} \frac{(\_R^2 b^2+2\_R ab+a^2)\ln(x-\_R)}{\_R^2}}{3d}$
default	$a^2\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)+2ab\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right.\right.}{6d}\right.$

input
```
int((b*x+a)^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*sum((_R^2*b^2+2*_R*a*b+a^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

3.70.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 5014, normalized size of antiderivative = 26.96

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2/(d*x^3+c),x, algorithm="fricas")
```

output
```
Too large to include
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx$$
$$= \text{RootSum}\left(27t^3c^2d^3 - 27t^2b^2c^2d^2 + t\left(18a^3bcd^2 + 9b^4c^2d\right) - a^6d^2 + 2a^3b^3cd - b^6c^2, \left(t \mapsto t\log\left(x + \frac{18t^2b\right.\right.\right.$$

input
```
integrate((b*x+a)**2/(d*x**3+c),x)
```

output
```
RootSum(27*_t**3*c**2*d**3 - 27*_t**2*b**2*c**2*d**2 + _t*(18*a**3*b*c*d**
2 + 9*b**4*c**2*d) - a**6*d**2 + 2*a**3*b**3*c*d - b**6*c**2, Lambda(_t, _
t*log(x + (18*_t**2*b*c**2*d**2 + 3*_t*a**3*c*d**2 - 12*_t*b**3*c**2*d + 7
*a**3*b**2*c*d + 2*b**5*c**2)/(a**5*d**2 + 8*a**2*b**3*c*d)))
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = -\frac{\sqrt{3}\left(2\,b^2c - \left(6\,ab\left(\frac{c}{d}\right)^{\frac{2}{3}} + 3\,a^2\left(\frac{c}{d}\right)^{\frac{1}{3}} + \frac{2\,b^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd} + \frac{\left(2\,b^2\left(\frac{c}{d}\right)^{\frac{2}{3}} + 2\,ab\left(\frac{c}{d}\right)^{\frac{1}{3}} - a^2\right)\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{\left(b^2\left(\frac{c}{d}\right)^{\frac{2}{3}} - 2\,ab\left(\frac{c}{d}\right)^{\frac{1}{3}} + a^2\right)\log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)^2/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*(2*b^2*c - (6*a*b*(c/d)^(2/3) + 3*a^2*(c/d)^(1/3) + 2*b^2*c/d
)*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d) + 1/6*(2*b^
2*(c/d)^(2/3) + 2*a*b*(c/d)^(1/3) - a^2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(
2/3))/(d*(c/d)^(2/3)) + 1/3*(b^2*(c/d)^(2/3) - 2*a*b*(c/d)^(1/3) + a^2)*lo
g(x + (c/d)^(1/3))/(d*(c/d)^(2/3))
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = \frac{b^2\log\left(|dx^3+c|\right)}{3\,d} - \frac{\sqrt{3}\left(a^2d-2\left(-cd^2\right)^{\frac{1}{3}}ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}} - \frac{\left(a^2d+2\left(-cd^2\right)^{\frac{1}{3}}ab\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}} - \frac{\left(2\,abd\left(-\frac{c}{d}\right)^{\frac{1}{3}}+a^2d\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

input
```
integrate((b*x+a)^2/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^2*log(abs(d*x^3 + c))/d - 1/3*sqrt(3)*(a^2*d - 2*(-c*d^2)^(1/3)*a*b)
*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(-c*d^2)^(2/3) - 1/
6*(a^2*d + 2*(-c*d^2)^(1/3)*a*b)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/
(-c*d^2)^(2/3) - 1/3*(2*a*b*d*(-c/d)^(1/3) + a^2*d)*(-c/d)^(1/3)*log(abs(x
 - (-c/d)^(1/3)))/(c*d)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = \sum_{k=1}^{3} \ln\Big(b^4 c + \mathrm{root}\big(27\,c^2\,d^3\,z^3 - 27\,b^2\,c^2\,d^2\,z^2 + 18\,a^3\,b\,c\,d^2\,z + 9\,b^4\,c^2\,d\,z + 2\,a^3\,b^3\,c\,d - b^6\,c^2 - a^6\,d^2, z, k\big)^2\,c\,d^2\,9 + 2\,a^3\,b\,d - \mathrm{root}\big(27\,c^2\,d^3\,z^3 - 27\,b^2\,c^2\,d^2\,z^2 + 18\,a^3\,b\,c\,d^2\,z + 9\,b^4\,c^2\,d\,z + 2\,a^3\,b^3\,c\,d - b^6\,c^2 - a^6\,d^2, z, k\big)\,b^2\,c\,d\,6 + \mathrm{root}\big(27\,c^2\,d^3\,z^3 - 27\,b^2\,c^2\,d^2\,z^2 + 18\,a^3\,b\,c\,d^2\,z + 9\,b^4\,c^2\,d\,z + 2\,a^3\,b^3\,c\,d - b^6\,c^2 - a^6\,d^2, z, k\big)\,a^2\,d^2\,x\,3 + 3\,a^2\,b^2\,d\,x\Big)\,\mathrm{root}\big(27\,c^2\,d^3\,z^3 - 27\,b^2\,c^2\,d^2\,z^2 + 18\,a^3\,b\,c\,d^2\,z + 9\,b^4\,c^2\,d\,z + 2\,a^3\,b^3\,c\,d - b^6\,c^2 - a^6\,d^2, z, k\big)$$

input
```
int((a + b*x)^2/(c + d*x^3),x)
```

output
```
symsum(log(b^4*c + 9*root(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c
*d^2*z + 9*b^4*c^2*d*z + 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)^2*c*d^2
+ 2*a^3*b*d - 6*root(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*
z + 9*b^4*c^2*d*z + 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)*b^2*c*d + 3*r
oot(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z + 9*b^4*c^2*d*z
 + 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)*a^2*d^2*x + 3*a^2*b^2*d*x)*roo
t(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z + 9*b^4*c^2*d*z +
 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k), k, 1, 3)
```

3.71 $\int \frac{(a+bx)^3}{c+dx^3}\,dx$

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3.71.1 Optimal result

Integrand size = 17, antiderivative size = 222

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \frac{b^3x}{d}+\frac{\left(b^3c-3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} - \frac{\left(b^3c+3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{\left(b^3c+3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} + \frac{ab^2\log\left(c+dx^3\right)}{d}$$

output
```
b^3*x/d-1/3*(b^3*c+3*a^2*b*c^(1/3)*d^(2/3)-a^3*d)*ln(c^(1/3)+d^(1/3)*x)/c^
(2/3)/d^(4/3)+1/6*(b^3*c+3*a^2*b*c^(1/3)*d^(2/3)-a^3*d)*ln(c^(2/3)-c^(1/3)
*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/d^(4/3)+a*b^2*ln(d*x^3+c)/d+1/3*(b^3*c-3*a
^2*b*c^(1/3)*d^(2/3)-a^3*d)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/
2))/c^(2/3)/d^(4/3)*3^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx$$
$$= \frac{6b^3c^{2/3}\sqrt[3]{d}x + 2\sqrt{3}\left(b^3c - 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) - 2\left(b^3c + 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\log\left(\sqrt[3]{c}+\right.}{6c^{2/3}d^{4/3}}$$

input
```
Integrate[(a + b*x)^3/(c + d*x^3),x]
```

output
```
(6*b^3*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^
3*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] - 2*(b^3*c + 3*a^2*b*c^(1
/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x] + (b^3*c + 3*a^2*b*c^(1/3)*d
^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2] + 6*a*b^2*c
^(2/3)*d^(1/3)*Log[c + d*x^3])/(6*c^(2/3)*d^(4/3))
```

3.71.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx$$
$$\Big\downarrow \text{2426}$$
$$\int \left(\frac{b^3}{d} - \frac{a^3(-d) - 3a^2bdx - 3ab^2dx^2 + b^3c}{d\,(c+dx^3)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\left(a^3(-d)-3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}}+\frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{\left(a^3(-d)+3a^2b\sqrt[3]{c}d^{2/3}+b^3c\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{ab^2\log\left(c+dx^3\right)}{d}+\frac{b^3x}{d}$$

input
```
Int[(a + b*x)^3/(c + d*x^3),x]
```

output
```
(b^3*x)/d + ((b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*ArcTan[(c^(1/3) - 2
*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(4/3)) - ((b^3*c + 3*a^
2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(4/3))
 + ((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (a*b^2*Log[c + d*x^3])/d
```

3.71.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.71.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.30

method	result
risch	$\frac{b^3x}{d}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^3d+c)}\frac{(3\_R^2ab^2d+3a^2bd\_R+a^3d-b^3c)\ln(x-\_R)}{\_R^2}}{3d^2}$
default	$\frac{b^3x}{d}+\frac{(a^3d-b^3c)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)+3d\,a^2b\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right.\right.}{6d(}\cdots\right.}{d}$

input
```
int((b*x+a)^3/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
b^3*x/d+1/3/d^2*sum((3*_R^2*a*b^2*d+3*_R*a^2*b*d+a^3*d-b^3*c)/_R^2*ln(x-_R
),_R=RootOf(_Z^3*d+c))
```

3.71.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 7245, normalized size of antiderivative = 32.64

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3/(d*x^3+c),x, algorithm="fricas")
```

output
```
Too large to include
```

3.71.6 Sympy [A] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \frac{b^3x}{d} + \text{RootSum}\left(27t^3c^2d^4 - 81t^2ab^2c^2d^3 + t\left(27a^5bcd^3 + 54a^2b^4c^2d^2\right) - a^9d^3 + 3a^6b^3cd^2 - 3a^3b^6c^2d + b^9c^3, \left(\right.\right.$$

input
```
integrate((b*x+a)**3/(d*x**3+c),x)
```

output
```
b**3*x/d + RootSum(27*_t**3*c**2*d**4 - 81*_t**2*a*b**2*c**2*d**3 + _t*(27
*a**5*b*c*d**3 + 54*a**2*b**4*c**2*d**2) - a**9*d**3 + 3*a**6*b**3*c*d**2
- 3*a**3*b**6*c**2*d + b**9*c**3, Lambda(_t, _t*log(x + (27*_t**2*a**2*b*c
**2*d**3 + 3*_t*a**6*c*d**3 - 60*_t*a**3*b**3*c**2*d**2 + 3*_t*b**6*c**3*d
 + 15*a**7*b**2*c*d**2 + 15*a**4*b**5*c**2*d - 3*a*b**8*c**3)/(a**9*d**3 +
 24*a**6*b**3*c*d**2 + 3*a**3*b**6*c**2*d - b**9*c**3))))
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \frac{b^3x}{d} - \frac{\sqrt{3}\left(\left(b^3\left(\frac{c}{d}\right)^{\frac{1}{3}} + 2\,ab^2\right)c - \left(3\,a^2b\left(\frac{c}{d}\right)^{\frac{2}{3}} + a^3\left(\frac{c}{d}\right)^{\frac{1}{3}} + \frac{2\,ab^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd} + \frac{\left(b^3c + \left(6\,ab^2\left(\frac{c}{d}\right)^{\frac{2}{3}} + 3\,a^2b\left(\frac{c}{d}\right)^{\frac{1}{3}} - a^3\right)d\right)\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{\left(b^3c - \left(3\,ab^2\left(\frac{c}{d}\right)^{\frac{2}{3}} - 3\,a^2b\left(\frac{c}{d}\right)^{\frac{1}{3}} + a^3\right)d\right)\log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)^3/(d*x^3+c),x, algorithm="maxima")
```

output
```
b^3*x/d - 1/3*sqrt(3)*((b^3*(c/d)^(1/3) + 2*a*b^2)*c - (3*a^2*b*(c/d)^(2/3
) + a^3*(c/d)^(1/3) + 2*a*b^2*c/d)*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3
))/(c/d)^(1/3))/(c*d) + 1/6*(b^3*c + (6*a*b^2*(c/d)^(2/3) + 3*a^2*b*(c/d)^
(1/3) - a^3)*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(d^2*(c/d)^(2/3)) -
 1/3*(b^3*c - (3*a*b^2*(c/d)^(2/3) - 3*a^2*b*(c/d)^(1/3) + a^3)*d)*log(x +
 (c/d)^(1/3))/(d^2*(c/d)^(2/3))
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \frac{b^3x}{d} + \frac{ab^2\log\left(|dx^3+c|\right)}{d} + \frac{\sqrt{3}\left(b^3c-a^3d+3\left(-cd^2\right)^{\frac{1}{3}}a^2b\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}} + \frac{\left(b^3c-a^3d-3\left(-cd^2\right)^{\frac{1}{3}}a^2b\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}} - \frac{\left(3a^2bd^3\left(-\frac{c}{d}\right)^{\frac{1}{3}}-b^3cd^2+a^3d^3\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^3}$$

input
```
integrate((b*x+a)^3/(d*x^3+c),x, algorithm="giac")
```

output
```
b^3*x/d + a*b^2*log(abs(d*x^3 + c))/d + 1/3*sqrt(3)*(b^3*c - a^3*d + 3*(-c
*d^2)^(1/3)*a^2*b)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(
-c*d^2)^(2/3) + 1/6*(b^3*c - a^3*d - 3*(-c*d^2)^(1/3)*a^2*b)*log(x^2 + x*(
-c/d)^(1/3) + (-c/d)^(2/3))/(-c*d^2)^(2/3) - 1/3*(3*a^2*b*d^3*(-c/d)^(1/3)
 - b^3*c*d^2 + a^3*d^3)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^3)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 10.21 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,c^2\,d^4\,z^3-81\,a\,b^2\,c^2\,d^3\,z^2+54\,a^2\,b^4\,c^2\,d^2\,z+27\,a^5\,b\,c\,d^3\,z+3\,a^6\,b^3\,c\,d^2-3\,a^3\,b^6\,c^2\,d+b^9\,c^3-a^9\right.\right.\right.$$
$$+\,x\left(6\,d\,a^4\,b^2+3\,c\,a\,b^5\right)+6\,a^2\,b^4\,c+3\,a^5\,b\,d\Big)\,\mathrm{root}\big(27\,c^2\,d^4\,z^3-81\,a\,b^2\,c^2\,d^3\,z^2$$
$$\left.+\,54\,a^2\,b^4\,c^2\,d^2\,z+27\,a^5\,b\,c\,d^3\,z+3\,a^6\,b^3\,c\,d^2-3\,a^3\,b^6\,c^2\,d+b^9\,c^3-a^9\,d^3,z,k\big)\right)+\frac{b^3\,x}{d}$$

input
```
int((a + b*x)^3/(c + d*x^3),x)
```

output
```
symsum(log(root(27*c^2*d^4*z^3 - 81*a*b^2*c^2*d^3*z^2 + 54*a^2*b^4*c^2*d^2
*z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3*a^3*b^6*c^2*d + b^9*c^3 - a^9*
d^3, z, k)*(x*(3*a^3*d^2 - 3*b^3*c*d) + 9*root(27*c^2*d^4*z^3 - 81*a*b^2*c
^2*d^3*z^2 + 54*a^2*b^4*c^2*d^2*z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3
*a^3*b^6*c^2*d + b^9*c^3 - a^9*d^3, z, k)*c*d^2 - 18*a*b^2*c*d) + x*(6*a^4
*b^2*d + 3*a*b^5*c) + 6*a^2*b^4*c + 3*a^5*b*d)*root(27*c^2*d^4*z^3 - 81*a*
b^2*c^2*d^3*z^2 + 54*a^2*b^4*c^2*d^2*z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^
2 - 3*a^3*b^6*c^2*d + b^9*c^3 - a^9*d^3, z, k), k, 1, 3) + (b^3*x)/d
```

3.72 $\int \frac{(a+bx)^4}{c+dx^3}\,dx$

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3.72.1 Optimal result

Integrand size = 17, antiderivative size = 282

$$\begin{aligned}\int \frac{(a+bx)^4}{c+dx^3}\,dx &= \frac{4ab^3x}{d}+\frac{b^4x^2}{2d}\\ &\quad+\frac{\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}d-a^4d^{4/3}\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{5/3}}\\ &\quad+\frac{\left(b\sqrt[3]{c}(b^3c-4a^3d)-\sqrt[3]{d}(4ab^3c-a^4d)\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{5/3}}\\ &\quad-\frac{\left(b\sqrt[3]{c}(b^3c-4a^3d)-\sqrt[3]{d}(4ab^3c-a^4d)\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{5/3}}\\ &\quad+\frac{2a^2b^2\log\left(c+dx^3\right)}{d}\end{aligned}$$

output
```
4*a*b^3*x/d+1/2*b^4*x^2/d+1/3*(b*c^(1/3)*(-4*a^3*d+b^3*c)-d^(1/3)*(-a^4*d+
4*a*b^3*c))*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(5/3)-1/6*(b*c^(1/3)*(-4*a^3*d
+b^3*c)-d^(1/3)*(-a^4*d+4*a*b^3*c))*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/c^(2/3)/d^(5/3)+2*a^2*b^2*ln(d*x^3+c)/d+1/3*(b^4*c^(4/3)+4*a*b^3*c*d^(
1/3)-4*a^3*b*c^(1/3)*d-a^4*d^(4/3))*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/
3)*3^(1/2))/c^(2/3)/d^(5/3)*3^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx$$

$$= \frac{24ab^3d^{2/3}x + 3b^4d^{2/3}x^2 + \frac{2\sqrt{3}\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}d-a^4d^{4/3}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}} + \frac{2\left(b^4c^{4/3}-4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}d+\right.}{c^{2/3}}}{6d}$$

input
```
Integrate[(a + b*x)^4/(c + d*x^3),x]
```

output
```
(24*a*b^3*d^(2/3)*x + 3*b^4*d^(2/3)*x^2 + (2*Sqrt[3]*(b^4*c^(4/3) + 4*a*b^
3*c*d^(1/3) - 4*a^3*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(1 - (2*d^(1/3)*x)/c
^(1/3))/Sqrt[3]])/c^(2/3) + (2*(b^4*c^(4/3) - 4*a*b^3*c*d^(1/3) - 4*a^3*b*
c^(1/3)*d + a^4*d^(4/3))*Log[c^(1/3) + d^(1/3)*x])/c^(2/3) - ((b^4*c^(4/3)
 - 4*a*b^3*c*d^(1/3) - 4*a^3*b*c^(1/3)*d + a^4*d^(4/3))*Log[c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3) + 12*a^2*b^2*d^(2/3)*Log[c + d*x^3])
/(6*d^(5/3))
```

3.72.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx$$

$$\Big\downarrow \text{2426}$$

$$\int \left(-\frac{a^4(-d)+bx\left(b^3c-4a^3d\right)-6a^2b^2dx^2+4ab^3c}{d\left(c+dx^3\right)}+\frac{4ab^3}{d}+\frac{b^4x}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2b^2\log\left(c+dx^3\right)}{d}+\frac{\left(a^4\left(-d^{4/3}\right)-4a^3b\sqrt[3]{c}d+4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{5/3}}+\frac{\left(a^4(-d)-\frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}+4ab^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}+\frac{\left(b\sqrt[3]{c}(b^3c-4a^3d)-\sqrt[3]{d}(4ab^3c-a^4d)\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{5/3}}+\frac{4ab^3x}{d}+\frac{b^4x^2}{2d}$$

input
```
Int[(a + b*x)^4/(c + d*x^3),x]
```

output
```
(4*a*b^3*x)/d + (b^4*x^2)/(2*d) + ((b^4*c^(4/3) + 4*a*b^3*c*d^(1/3) - 4*a^
3*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/
3))])/(Sqrt[3]*c^(2/3)*d^(5/3)) + ((b*c^(1/3)*(b^3*c - 4*a^3*d) - d^(1/3)*
(4*a*b^3*c - a^4*d))*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(5/3)) + ((4*a
*b^3*c - a^4*d - (b*c^(1/3)*(b^3*c - 4*a^3*d))/d^(1/3))*Log[c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (2*a^2*b^2*Log[c + d*x
^3])/d
```

3.72.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
 + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.72.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.32

method	result
risch	$\frac{b^4x^2}{2d}+\frac{4a\,b^3x}{d}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^3d+c)}\frac{\left(6a^2b^2d\,\_R^2+b\left(4a^3d-b^3c\right)\_R+a^4d-4a\,b^3c\right)\ln\left(x-\_R\right)}{\_R^2}}{3d^2}$
default	$\frac{b^3\left(\frac{1}{2}b\,x^2+4ax\right)}{d}+\frac{\left(a^4d-4a\,b^3c\right)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)+\left(4d\,a^3b-b^4c\right)\left(-\frac{\ln\left(x \ldots\right.}{3\ldots}\ldots\right.}{d}$

input
```
int((b*x+a)^4/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*b^4*x^2/d+4*a*b^3*x/d+1/3/d^2*sum((6*a^2*b^2*d*_R^2+b*(4*a^3*d-b^3*c)*
_R+a^4*d-4*a*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

3.72.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.54 (sec) , antiderivative size = 8787, normalized size of antiderivative = 31.16

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^4/(d*x^3+c),x, algorithm="fricas")
```

output
```
Too large to include
```

3.72.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**4/(d*x**3+c),x)
```

output
```
Timed out
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = -\frac{\sqrt{3}\left(\left(b^4\left(\frac{c}{d}\right)^{\frac{2}{3}}+4\,ab^3\left(\frac{c}{d}\right)^{\frac{1}{3}}+4\,a^2b^2\right)c-\left(4\,a^3b\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^4\left(\frac{c}{d}\right)^{\frac{1}{3}}+\frac{4\,a^2b^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd} + \frac{b^4x^2+8\,ab^3x}{2\,d} - \frac{\left(\left(b^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-4\,ab^3\right)c-\left(12\,a^2b^2\left(\frac{c}{d}\right)^{\frac{2}{3}}+4\,a^3b\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^4\right)d\right)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{\left(\left(b^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-4\,ab^3\right)c+\left(6\,a^2b^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-4\,a^3b\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^4\right)d\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)^4/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*((b^4*(c/d)^(2/3) + 4*a*b^3*(c/d)^(1/3) + 4*a^2*b^2)*c - (4*a
^3*b*(c/d)^(2/3) + a^4*(c/d)^(1/3) + 4*a^2*b^2*c/d)*d)*arctan(1/3*sqrt(3)*
(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d) + 1/2*(b^4*x^2 + 8*a*b^3*x)/d - 1/6
*((b^4*(c/d)^(1/3) - 4*a*b^3)*c - (12*a^2*b^2*(c/d)^(2/3) + 4*a^3*b*(c/d)^
(1/3) - a^4)*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(d^2*(c/d)^(2/3)) +
 1/3*((b^4*(c/d)^(1/3) - 4*a*b^3)*c + (6*a^2*b^2*(c/d)^(2/3) - 4*a^3*b*(c/
d)^(1/3) + a^4)*d)*log(x + (c/d)^(1/3))/(d^2*(c/d)^(2/3))
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx$$
$$= \frac{2\,a^2b^2\log\left(|dx^3+c|\right)}{d} + \frac{\sqrt{3}\left(4\,ab^3cd - a^4d^2 - (-cd^2)^{\frac{1}{3}}\,b^4c + 4\,(-cd^2)^{\frac{1}{3}}\,a^3bd\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,(-cd^2)^{\frac{2}{3}}\,d} + \frac{\left(4\,ab^3cd - a^4d^2 + (-cd^2)^{\frac{1}{3}}\,b^4c - 4\,(-cd^2)^{\frac{1}{3}}\,a^3bd\right)\log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(-cd^2)^{\frac{2}{3}}\,d} + \frac{b^4dx^2 + 8\,ab^3dx}{2\,d^2} + \frac{\left(b^4cd^4\left(-\frac{c}{d}\right)^{\frac{1}{3}} - 4\,a^3bd^5\left(-\frac{c}{d}\right)^{\frac{1}{3}} + 4\,ab^3cd^4 - a^4d^5\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^5}$$

input
```
integrate((b*x+a)^4/(d*x^3+c),x, algorithm="giac")
```

output
```
2*a^2*b^2*log(abs(d*x^3 + c))/d + 1/3*sqrt(3)*(4*a*b^3*c*d - a^4*d^2 - (-c
*d^2)^(1/3)*b^4*c + 4*(-c*d^2)^(1/3)*a^3*b*d)*arctan(1/3*sqrt(3)*(2*x + (-
c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*d) + 1/6*(4*a*b^3*c*d - a^4*d^2
+ (-c*d^2)^(1/3)*b^4*c - 4*(-c*d^2)^(1/3)*a^3*b*d)*log(x^2 + x*(-c/d)^(1/3
) + (-c/d)^(2/3))/((-c*d^2)^(2/3)*d) + 1/2*(b^4*d*x^2 + 8*a*b^3*d*x)/d^2 +
 1/3*(b^4*c*d^4*(-c/d)^(1/3) - 4*a^3*b*d^5*(-c/d)^(1/3) + 4*a*b^3*c*d^4 -
a^4*d^5)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^5)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\left(27\,c^2\,d^5\,z^3-162\,a^2\,b^2\,c^2\,d^4\,z^2+171\,a^4\,b^4\,c^2\,d^3\,z+36\,a\,b^7\,c^3\,d^2\,z+36\,a^7\,b\,c\,d^4\,z-6\,a^6\,b^6\,c^2\,d^2+4 \ldots\right.\right.\right.$$
$$+\frac{4\,a^7\,b\,d^2+19\,a^4\,b^4\,c\,d+4\,a\,b^7\,c^2}{d}+\frac{x\,(10\,a^6\,b^2\,d^2+16\,a^3\,b^5\,c\,d+b^8\,c^2)}{d}\Bigg)\,\mathrm{root}\big(27\,c^2\,d^5\,z^3$$
$$-162\,a^2\,b^2\,c^2\,d^4\,z^2+171\,a^4\,b^4\,c^2\,d^3\,z+36\,a\,b^7\,c^3\,d^2\,z+36\,a^7\,b\,c\,d^4\,z-6\,a^6\,b^6\,c^2\,d^2$$
$$\left.+4\,a^9\,b^3\,c\,d^3+4\,a^3\,b^9\,c^3\,d-b^{12}\,c^4-a^{12}\,d^4,z,k\big)\right)+\frac{b^4\,x^2}{2\,d}+\frac{4\,a\,b^3\,x}{d}$$

input
```
int((a + b*x)^4/(c + d*x^3),x)
```

output
```
symsum(log(root(27*c^2*d^5*z^3 - 162*a^2*b^2*c^2*d^4*z^2 + 171*a^4*b^4*c^2
*d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*z - 6*a^6*b^6*c^2*d^2 + 4*a^9
*b^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12*d^4, z, k)*((x*(3*a^4*d^3 -
 12*a*b^3*c*d^2))/d + 9*root(27*c^2*d^5*z^3 - 162*a^2*b^2*c^2*d^4*z^2 + 17
1*a^4*b^4*c^2*d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*z - 6*a^6*b^6*c^
2*d^2 + 4*a^9*b^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12*d^4, z, k)*c*d
^2 - 36*a^2*b^2*c*d) + (4*a*b^7*c^2 + 4*a^7*b*d^2 + 19*a^4*b^4*c*d)/d + (x
*(b^8*c^2 + 10*a^6*b^2*d^2 + 16*a^3*b^5*c*d))/d)*root(27*c^2*d^5*z^3 - 162
*a^2*b^2*c^2*d^4*z^2 + 171*a^4*b^4*c^2*d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7
*b*c*d^4*z - 6*a^6*b^6*c^2*d^2 + 4*a^9*b^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*
c^4 - a^12*d^4, z, k), k, 1, 3) + (b^4*x^2)/(2*d) + (4*a*b^3*x)/d
```

3.73 $\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx$

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3.73.1 Optimal result

Integrand size = 22, antiderivative size = 272

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \frac{2bcx}{e}+\frac{c^2x^2}{2e}+\frac{\left(c^2d^{4/3}+2bcd\sqrt[3]{e}-a\left(2b\sqrt[3]{d}+a\sqrt[3]{e}\right)e\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{5/3}}$$
$$-\frac{\left(\sqrt[3]{e}(2bcd-a^2e)-\sqrt[3]{d}(c^2d-2abe)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{5/3}}$$
$$+\frac{\left(\sqrt[3]{e}(2bcd-a^2e)-\sqrt[3]{d}(c^2d-2abe)\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{5/3}}$$
$$+\frac{(b^2+2ac)\log(d+ex^3)}{3e}$$

output
```
2*b*c*x/e+1/2*c^2*x^2/e-1/3*(e^(1/3)*(-a^2*e+2*b*c*d)-d^(1/3)*(-2*a*b*e+c^
2*d))*ln(d^(1/3)+e^(1/3)*x)/d^(2/3)/e^(5/3)+1/6*(e^(1/3)*(-a^2*e+2*b*c*d)-
d^(1/3)*(-2*a*b*e+c^2*d))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3
)/e^(5/3)+1/3*(2*a*c+b^2)*ln(e*x^3+d)/e+1/3*(c^2*d^(4/3)+2*b*c*d*e^(1/3)-a
*(2*b*d^(1/3)+a*e^(1/3))*e)*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/
2))/d^(2/3)/e^(5/3)*3^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx$$

$$= \frac{12bce^{2/3}x+3c^2e^{2/3}x^2+\frac{2\sqrt{3}(cd^{2/3}-ae^{2/3})\left(cd^{2/3}+2b\sqrt[3]{d}\sqrt[3]{e}+ae^{2/3}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{2/3}}+\frac{2\left(c^2d^{4/3}-2bcd\sqrt[3]{e}+a\left(-2b\sqrt[3]{d}+\right.\right.}{d^{2/3}}}{}$$

input
```
Integrate[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

output
```
(12*b*c*e^(2/3)*x + 3*c^2*e^(2/3)*x^2 + (2*Sqrt[3]*(c*d^(2/3) - a*e^(2/3))
*(c*d^(2/3) + 2*b*d^(1/3)*e^(1/3) + a*e^(2/3))*ArcTan[(1 - (2*e^(1/3)*x)/d
^(1/3))/Sqrt[3]])/d^(2/3) + (2*(c^2*d^(4/3) - 2*b*c*d*e^(1/3) + a*(-2*b*d^
(1/3) + a*e^(1/3))*e)*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - ((c^2*d^(4/3) -
2*b*c*d*e^(1/3) + a*(-2*b*d^(1/3) + a*e^(1/3))*e)*Log[d^(2/3) - d^(1/3)*e^
(1/3)*x + e^(2/3)*x^2])/d^(2/3) + 2*(b^2 + 2*a*c)*e^(2/3)*Log[d + e*x^3])/
(6*e^(5/3))
```

3.73.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx$$

$$\Big\downarrow \; 2426$$

$$\int \left(-\frac{a^2(-e)-ex^2(2ac+b^2)+x(c^2d-2abe)+2bcd}{e(d+ex^3)}+\frac{2bc}{e}+\frac{c^2x}{e}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(a^2(-e)-\frac{\sqrt[3]{d}(c^2d-2abe)}{\sqrt[3]{e}}+2bcd\right)}{6d^{2/3}e^{4/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}(2bcd-a^2e)-\sqrt[3]{d}(c^2d-2abe)\right)}{3d^{2/3}e^{5/3}}+\frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(-ae\left(a\sqrt[3]{e}+2b\sqrt[3]{d}\right)+2bcd\sqrt[3]{e}+c^2d^{4/3}\right)}{\sqrt{3}d^{2/3}e^{5/3}}+\frac{(2ac+b^2)\log(d+ex^3)}{3e}+\frac{2bcx}{e}+\frac{c^2x^2}{2e}$$

input
```
Int[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

output
```
(2*b*c*x)/e + (c^2*x^2)/(2*e) + ((c^2*d^(4/3) + 2*b*c*d*e^(1/3) - a*(2*b*d
^(1/3) + a*e^(1/3))*e)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/
(Sqrt[3]*d^(2/3)*e^(5/3)) - ((e^(1/3)*(2*b*c*d - a^2*e) - d^(1/3)*(c^2*d -
 2*a*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(5/3)) + ((2*b*c*d - a^2
*e - (d^(1/3)*(c^2*d - 2*a*b*e))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x
+ e^(2/3)*x^2])/(6*d^(2/3)*e^(4/3)) + ((b^2 + 2*a*c)*Log[d + e*x^3])/(3*e)
```

3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.73.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.74 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.31

method	result
risch	$\frac{c^2x^2}{2e}+\frac{2bcx}{e}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3e+d\right)}\frac{\left(e\left(2ac+b^2\right)\_R^2+\left(2aeb-c^2d\right)\_R+a^2e-2bcd\right)\ln\left(x-\_R\right)}{\_R^2}}{3e^2}$
default	$\frac{c\left(\frac{1}{2}cx^2+2bx\right)}{e}+\frac{(a^2e-2bcd)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)+(2aeb-c^2d)\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right.\right.}{3e\left(\frac{d}{e}\right)}\right.}{e}$

input
```
int((c*x^2+b*x+a)^2/(e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*c^2*x^2/e+2*b*c*x/e+1/3/e^2*sum((e*(2*a*c+b^2)*_R^2+(2*a*b*e-c^2*d)*_R
+a^2*e-2*b*c*d)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

3.73.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 12827, normalized size of antiderivative = 47.16

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.73.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**2/(e*x**3+d),x)
```

output
```
Timed out
```

3.73.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \frac{(b^2+2\,ac)\log\left(|ex^3+d|\right)}{3\,e} + \frac{\sqrt{3}\left(2\,bcde - a^2e^2 - (-de^2)^{\frac{1}{3}}\,c^2d + 2\,(-de^2)^{\frac{1}{3}}\,abe\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,(-de^2)^{\frac{2}{3}}\,e} + \frac{\left(2\,bcde - a^2e^2 + (-de^2)^{\frac{1}{3}}\,c^2d - 2\,(-de^2)^{\frac{1}{3}}\,abe\right)\log\left(x^2 + x\left(-\frac{d}{e}\right)^{\frac{1}{3}} + \left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,(-de^2)^{\frac{2}{3}}\,e} + \frac{c^2ex^2+4\,bcex}{2\,e^2} + \frac{\left(c^2de^4\left(-\frac{d}{e}\right)^{\frac{1}{3}} - 2\,abe^5\left(-\frac{d}{e}\right)^{\frac{1}{3}} + 2\,bcde^4 - a^2e^5\right)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,de^5}$$

input
```
integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="giac")
```

output
```
1/3*(b^2 + 2*a*c)*log(abs(e*x^3 + d))/e + 1/3*sqrt(3)*(2*b*c*d*e - a^2*e^2
 - (-d*e^2)^(1/3)*c^2*d + 2*(-d*e^2)^(1/3)*a*b*e)*arctan(1/3*sqrt(3)*(2*x
+ (-d/e)^(1/3))/(-d/e)^(1/3))/((-d*e^2)^(2/3)*e) + 1/6*(2*b*c*d*e - a^2*e^
2 + (-d*e^2)^(1/3)*c^2*d - 2*(-d*e^2)^(1/3)*a*b*e)*log(x^2 + x*(-d/e)^(1/3
) + (-d/e)^(2/3))/((-d*e^2)^(2/3)*e) + 1/2*(c^2*e*x^2 + 4*b*c*e*x)/e^2 + 1
/3*(c^2*d*e^4*(-d/e)^(1/3) - 2*a*b*e^5*(-d/e)^(1/3) + 2*b*c*d*e^4 - a^2*e^
5)*(-d/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/(d*e^5)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 769, normalized size of antiderivative = 2.83

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{2\,a^3\,b\,e^2+3\,a^2\,c^2\,d\,e+b^4\,d\,e+2\,b\,c^3\,d^2}{e}\right.\right.$$

$$+\frac{x\,(-2\,a^3\,c\,e^2+3\,a^2\,b^2\,e^2+2\,b^3\,c\,d\,e+c^4\,d^2)}{e}$$

$$-\mathrm{root}\big(27\,d^2\,e^5\,z^3-54\,a\,c\,d^2\,e^4\,z^2-27\,b^2\,d^2\,e^4\,z^2+27\,a^2\,c^2\,d^2\,e^3\,z+18\,b\,c^3\,d^3\,e^2\,z+18\,a^3\,b\,d\,e^4\,z+9\,b^4\,d^2\,e^3\,z$$

$$-54\,a\,c\,d^2\,e^4\,z^2-27\,b^2\,d^2\,e^4\,z^2+27\,a^2\,c^2\,d^2\,e^3\,z+18\,b\,c^3\,d^3\,e^2\,z+18\,a^3\,b\,d\,e^4\,z$$

$$+9\,b^4\,d^2\,e^3\,z+6\,a\,b^4\,c\,d^2\,e^2-9\,a^2\,b^2\,c^2\,d^2\,e^2-6\,a^4\,b\,c\,d\,e^3-6\,a\,b\,c^4\,d^3\,e-2\,a^3\,c^3\,d^2\,e^2$$

$$\left.+2\,b^3\,c^3\,d^3\,e+2\,a^3\,b^3\,d\,e^3-b^6\,d^2\,e^2-c^6\,d^4-a^6\,e^4,z,k\big)\right)+\frac{c^2\,x^2}{2\,e}+\frac{2\,b\,c\,x}{e}$$

input
```
int((a + b*x + c*x^2)^2/(d + e*x^3),x)
```

output
```
symsum(log((2*a^3*b*e^2 + 2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e)/e + (x*(c
^4*d^2 - 2*a^3*c*e^2 + 3*a^2*b^2*e^2 + 2*b^3*c*d*e))/e - 3*root(27*d^2*e^5
*z^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4*z^2 + 27*a^2*c^2*d^2*e^3*z + 18
*b*c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4*d^2*e^3*z + 6*a*b^4*c*d^2*e^2
- 9*a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 - 6*a*b*c^4*d^3*e - 2*a^3*c^3*d^
2*e^2 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b^6*d^2*e^2 - c^6*d^4 - a^6*e^
4, z, k)*e*(2*b^2*d - 3*root(27*d^2*e^5*z^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*
d^2*e^4*z^2 + 27*a^2*c^2*d^2*e^3*z + 18*b*c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z
 + 9*b^4*d^2*e^3*z + 6*a*b^4*c*d^2*e^2 - 9*a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c
*d*e^3 - 6*a*b*c^4*d^3*e - 2*a^3*c^3*d^2*e^2 + 2*b^3*c^3*d^3*e + 2*a^3*b^3
*d*e^3 - b^6*d^2*e^2 - c^6*d^4 - a^6*e^4, z, k)*d*e + 4*a*c*d - a^2*e*x +
2*b*c*d*x))*root(27*d^2*e^5*z^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4*z^2
+ 27*a^2*c^2*d^2*e^3*z + 18*b*c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4*d^2
*e^3*z + 6*a*b^4*c*d^2*e^2 - 9*a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 - 6*a
*b*c^4*d^3*e - 2*a^3*c^3*d^2*e^2 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b^6
*d^2*e^2 - c^6*d^4 - a^6*e^4, z, k), k, 1, 3) + (c^2*x^2)/(2*e) + (2*b*c*x
)/e
```

3.74 $\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx$

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3.74.1 Optimal result

Integrand size = 22, antiderivative size = 416

$$\begin{aligned}\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx =& -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}\\ &-\frac{\left(c^3d^2-3b^2cd^{4/3}e^{2/3}-3ac^2d^{4/3}e^{2/3}-b^3de-6abcde+3a^2b\sqrt[3]{d}e^{5/3}+a^3e^2\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{7/3}}\\ &+\frac{\left(c^3d^2-6abcde-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(b^2cd+ac^2d-a^2be)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}}\\ &-\frac{\left(c^3d^2-6abcde-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(b^2cd+ac^2d-a^2be)\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{7/3}}\\ &-\frac{(bc^2d-ab^2e-a^2ce)\log\left(d+ex^3\right)}{e^2}\end{aligned}$$

output
```
-(-6*a*b*c*e-b^3*e+c^3*d)*x/e^2+3/2*c*(a*c+b^2)*x^2/e+b*c^2*x^3/e+1/4*c^3*
x^4/e+1/3*(c^3*d^2-6*a*b*c*d*e-e*(-a^3*e+b^3*d)+3*d^(1/3)*e^(2/3)*(-a^2*b*
e+a*c^2*d+b^2*c*d))*ln(d^(1/3)+e^(1/3)*x)/d^(2/3)/e^(7/3)-1/6*(c^3*d^2-6*a
*b*c*d*e-e*(-a^3*e+b^3*d)+3*d^(1/3)*e^(2/3)*(-a^2*b*e+a*c^2*d+b^2*c*d))*ln
(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(7/3)-(-a^2*c*e-a*b^2*e+
b*c^2*d)*ln(e*x^3+d)/e^2-1/3*(c^3*d^2-3*b^2*c*d^(4/3)*e^(2/3)-3*a*c^2*d^(4
/3)*e^(2/3)-b^3*d*e-6*a*b*c*d*e+3*a^2*b*d^(1/3)*e^(5/3)+a^3*e^2)*arctan(1/
3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(2/3)/e^(7/3)*3^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx$$

$$= \frac{12\sqrt[3]{e}(-c^3d+b^3e+6abce)\,x+18c(b^2+ac)\,e^{4/3}x^2+12bc^2e^{4/3}x^3+3c^3e^{4/3}x^4-\frac{4\sqrt{3}\left(c^3d^2-3ac^2d^{4/3}e^{2/3}+e\left(-b^3\right.\right.}{}}{}$$

input
```
Integrate[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

output
```
(12*e^(1/3)*(-(c^3*d) + b^3*e + 6*a*b*c*e)*x + 18*c*(b^2 + a*c)*e^(4/3)*x^
2 + 12*b*c^2*e^(4/3)*x^3 + 3*c^3*e^(4/3)*x^4 - (4*Sqrt[3]*(c^3*d^2 - 3*a*c
^2*d^(4/3)*e^(2/3) + e*(-(b^3*d) + 3*a^2*b*d^(1/3)*e^(2/3) + a^3*e) - 3*c*
(b^2*d^(4/3)*e^(2/3) + 2*a*b*d*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt
[3]])/d^(2/3) + (4*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c^2*d^(4/3)*e^
(2/3) - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*Log[d^(
1/3) + e^(1/3)*x])/d^(2/3) - (2*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c
^2*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3
*e^2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3) + 12*e^(1/3)
*(-(b*c^2*d) + a*b^2*e + a^2*c*e)*Log[d + e*x^3])/(12*e^(7/3))
```

3.74.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx$$
$$\Big\downarrow \text{ 2426}$$

$$\int \left(\frac{-e(b^3d-a^3e)-3ex^2(a^2(-c)e-ab^2e+bc^2d)-3ex(a^2(-b)e+ac^2d+b^2cd)-6abcde+c^3d^2}{e^2(d+ex^3)}-\frac{-6abce+b}{}\right.$$

$\big\downarrow$ 2009

$$-\frac{\log(d+ex^3)(a^2(-c)e-ab^2e+bc^2d)}{e^2}-\frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(a^3e^2+3a^2b\sqrt[3]{d}e^{5/3}-6abcde-3ac^2d^{4/3}e^{2/3}-b^3de-3b^2cd^{4/3}e^{2/3}+c^3d^2\right)}{\sqrt{3}d^{2/3}e^{7/3}}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(a^2(-b)e+ac^2d+b^2cd)-6abcde+c^3d^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(a^2(-b)e+ac^2d+b^2cd)-6abcde+c^3d^2\right)}{3d^{2/3}e^{7/3}}-\frac{x(-6abce+b^3(-e)+c^3d)}{e^2}+\frac{3cx^2(ac+b^2)}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}$$

input
```
Int[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

output
```
-(((c^3*d - b^3*e - 6*a*b*c*e)*x)/e^2) + (3*c*(b^2 + a*c)*x^2)/(2*e) + (b*
c^2*x^3)/e + (c^3*x^4)/(4*e) - ((c^3*d^2 - 3*b^2*c*d^(4/3)*e^(2/3) - 3*a*c
^2*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e + 3*a^2*b*d^(1/3)*e^(5/3) + a^3
*e^2)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*
e^(7/3)) + ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)
*(b^2*c*d + a*c^2*d - a^2*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/
3)) - ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2
*c*d + a*c^2*d - a^2*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])
/(6*d^(2/3)*e^(7/3)) - ((b*c^2*d - a*b^2*e - a^2*c*e)*Log[d + e*x^3])/e^2
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.74.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.69 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.44

method	result
risch	$\frac{c^3x^4}{4e}+\frac{b\,c^2x^3}{e}+\frac{3a\,c^2x^2}{2e}+\frac{3b^2c\,x^2}{2e}+\frac{6abcx}{e}+\frac{b^3x}{e}-\frac{c^3dx}{e^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3e+d\right)}\left(3e\left(a^2ce+a\,b^2e-b\,c^2d\right)\_R^2+3e\left(a^2be\right.\right.}{3e^3}$
default	$\frac{\frac{1}{4}c^3x^4e+b\,c^2x^3e+\frac{3}{2}a\,c^2e\,x^2+\frac{3}{2}b^2ce\,x^2+6abcex+b^3ex-c^3dx}{e^2}+\frac{\left(a^3e^2-6abcde-b^3de+c^3d^2\right)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right.}{}$

input
```
int((c*x^2+b*x+a)^3/(e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*c^3*x^4/e+b*c^2*x^3/e+3/2/e*a*c^2*x^2+3/2/e*b^2*c*x^2+6/e*a*b*c*x+1/e*
b^3*x-1/e^2*c^3*d*x+1/3/e^3*sum((3*e*(a^2*c*e+a*b^2*e-b*c^2*d)*_R^2+3*e*(a
^2*b*e-a*c^2*d-b^2*c*d)*_R+a^3*e^2-6*a*b*c*d*e-b^3*d*e+c^3*d^2)/_R^2*ln(x-
_R),_R=RootOf(_Z^3*e+d))
```

3.74.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.41 (sec) , antiderivative size = 29479, normalized size of antiderivative = 70.86

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.74.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + bx + cx^2)^3}{d + ex^3} dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**3/(e*x**3+d),x)
```

output
```
Timed out
```

3.74.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + bx + cx^2)^3}{d + ex^3} dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx =$$

$$-\frac{\sqrt{3}\left(c^3d^2-b^3de-6\,abcde+a^3e^2+3\,(-de^2)^{\frac{1}{3}}\,b^2cd+3\,(-de^2)^{\frac{1}{3}}\,ac^2d-3\,(-de^2)^{\frac{1}{3}}\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2x\right.}{3\left(\right.}\right.}{3\,(-de^2)^{\frac{2}{3}}\,e}$$
$$-\frac{\left(c^3d^2-b^3de-6\,abcde+a^3e^2-3\,(-de^2)^{\frac{1}{3}}\,b^2cd-3\,(-de^2)^{\frac{1}{3}}\,ac^2d+3\,(-de^2)^{\frac{1}{3}}\,a^2be\right)\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right.}{6\,(-de^2)^{\frac{2}{3}}\,e}$$
$$-\frac{(bc^2d-ab^2e-a^2ce)\log(|ex^3+d|)}{e^2}$$
$$+\frac{c^3e^3x^4+4\,bc^2e^3x^3+6\,b^2ce^3x^2+6\,ac^2e^3x^2-4\,c^3de^2x+4\,b^3e^3x+24\,abce^3x}{4\,e^4}$$
$$+\frac{\left(3\,b^2cde^8\left(-\frac{d}{e}\right)^{\frac{1}{3}}+3\,ac^2de^8\left(-\frac{d}{e}\right)^{\frac{1}{3}}-3\,a^2be^9\left(-\frac{d}{e}\right)^{\frac{1}{3}}-c^3d^2e^7+b^3de^8+6\,abcde^8-a^3e^9\right)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\right.\right.}{3\,de^9}$$

input
```
integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(c^3*d^2 - b^3*d*e - 6*a*b*c*d*e + a^3*e^2 + 3*(-d*e^2)^(1/3)
*b^2*c*d + 3*(-d*e^2)^(1/3)*a*c^2*d - 3*(-d*e^2)^(1/3)*a^2*b*e)*arctan(1/3
*sqrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/((-d*e^2)^(2/3)*e) - 1/6*(c^3*
d^2 - b^3*d*e - 6*a*b*c*d*e + a^3*e^2 - 3*(-d*e^2)^(1/3)*b^2*c*d - 3*(-d*e
^2)^(1/3)*a*c^2*d + 3*(-d*e^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-d/e)^(1/3) + (
-d/e)^(2/3))/((-d*e^2)^(2/3)*e) - (b*c^2*d - a*b^2*e - a^2*c*e)*log(abs(e*
x^3 + d))/e^2 + 1/4*(c^3*e^3*x^4 + 4*b*c^2*e^3*x^3 + 6*b^2*c*e^3*x^2 + 6*a
*c^2*e^3*x^2 - 4*c^3*d*e^2*x + 4*b^3*e^3*x + 24*a*b*c*e^3*x)/e^4 + 1/3*(3*
b^2*c*d*e^8*(-d/e)^(1/3) + 3*a*c^2*d*e^8*(-d/e)^(1/3) - 3*a^2*b*e^9*(-d/e)
^(1/3) - c^3*d^2*e^7 + b^3*d*e^8 + 6*a*b*c*d*e^8 - a^3*e^9)*(-d/e)^(1/3)*l
og(abs(x - (-d/e)^(1/3)))/(d*e^9)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 1700, normalized size of antiderivative = 4.09

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2)^3/(d + e*x^3),x)
```

output
```
x*((b^3 + 6*a*b*c)/e - (c^3*d)/e^2) + symsum(log(root(27*d^2*e^7*z^3 + 81*
b*c^2*d^3*e^5*z^2 - 81*a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b
^2*c*d^2*e^5*z + 27*a^2*b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*
c^4*d^4*e^3*z + 54*a^4*c^2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3
*e^4*z - 27*a*c^5*d^4*e^3*z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18
*a^4*b*c^4*d^3*e^3 + 18*a*b^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2
*c^2*d^2*e^4 + 27*a^2*b^5*c^2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^
3*c^3*d^3*e^3 - 9*a^7*b*c*d*e^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*
a^6*c^3*d^2*e^4 - 3*a^3*c^6*d^4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e
+ 3*a^6*b^3*d*e^5 + b^9*d^3*e^3 - c^9*d^6 - a^9*e^6, z, k)*((3*x*(a^3*e^4
- b^3*d*e^3 + c^3*d^2*e^2 - 6*a*b*c*d*e^3))/e^2 - (3*(6*a*b^2*d*e^3 - 6*b*
c^2*d^2*e^2 + 6*a^2*c*d*e^3))/e^2 + 9*root(27*d^2*e^7*z^3 + 81*b*c^2*d^3*e
^5*z^2 - 81*a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b^2*c*d^2*e^
5*z + 27*a^2*b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*c^4*d^4*e^3
*z + 54*a^4*c^2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3*e^4*z - 27
*a*c^5*d^4*e^3*z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18*a^4*b*c^4*
d^3*e^3 + 18*a*b^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2*c^2*d^2*e^
4 + 27*a^2*b^5*c^2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^3*c^3*d^3*e
^3 - 9*a^7*b*c*d*e^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*a^6*c^3*d^2
*e^4 - 3*a^3*c^6*d^4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e + 3*a^6*...
```

3.75 $\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx$

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3.75.1 Optimal result

Integrand size = 22, antiderivative size = 645

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx$$
$$= -\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2}$$
$$- \frac{c(c^3d-4b^3e-12abce)\,x^3}{3e^2} + \frac{c^2(3b^2+2ac)\,x^4}{2e} + \frac{4bc^3x^5}{5e} + \frac{c^4x^6}{6e}$$
$$- \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\left(4c^3d^2+6c^2\left(bd^{5/3}\sqrt[3]{e}-ad^{4/3}e^{2/3}\right)-12abcde-e\left(b^3d+3ab^2d^{2/3}\sqrt[3]{e}-3a^2b\sqrt[3]{d}e^{2/3}-a^3\cdots\right.\right.}{\sqrt{3}d^{2/3}e^{8/3}}$$
$$+ \frac{\left(\sqrt[3]{e}(6b^2c^2d^2+4ac^3d^2-4ab^3de-12a^2bcde+a^4e^2)+\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2\cdots\right.}{3d^{2/3}e^{8/3}}$$
$$- \frac{\left(\sqrt[3]{e}(6b^2c^2d^2+4ac^3d^2-4ab^3de-12a^2bcde+a^4e^2)+\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2\cdots\right.}{6d^{2/3}e^{8/3}}$$
$$+ \frac{(c^4d^2-12abc^2de+6a^2b^2e^2-4ce(b^3d-a^3e))\log(d+ex^3)}{3e^3}$$

output
```
-2*(-6*a^2*b*c*e-2*a*b^3*e+2*a*c^3*d+3*b^2*c^2*d)*x/e^2-1/2*(-6*a^2*c^2*e-
12*a*b^2*c*e-b^4*e+4*b*c^3*d)*x^2/e^2-1/3*c*(-12*a*b*c*e-4*b^3*e+c^3*d)*x^
3/e^2+1/2*c^2*(2*a*c+3*b^2)*x^4/e+4/5*b*c^3*x^5/e+1/6*c^4*x^6/e+1/3*(e^(1/
3)*(a^4*e^2-12*a^2*b*c*d*e-4*a*b^3*d*e+4*a*c^3*d^2+6*b^2*c^2*d^2)+d^(1/3)*
(b^4*d*e+12*a*b^2*c*d*e+6*a^2*c^2*d*e-4*b*(a^3*e^2+c^3*d^2)))*ln(d^(1/3)+e
^(1/3)*x)/d^(2/3)/e^(8/3)-1/6*(e^(1/3)*(a^4*e^2-12*a^2*b*c*d*e-4*a*b^3*d*e
+4*a*c^3*d^2+6*b^2*c^2*d^2)+d^(1/3)*(b^4*d*e+12*a*b^2*c*d*e+6*a^2*c^2*d*e-
4*b*(a^3*e^2+c^3*d^2)))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/
e^(8/3)+1/3*(c^4*d^2-12*a*b*c^2*d*e+6*a^2*b^2*e^2-4*c*e*(-a^3*e+b^3*d))*ln
(e*x^3+d)/e^3-1/3*(b*d^(1/3)+a*e^(1/3))*(4*c^3*d^2+6*c^2*(b*d^(5/3)*e^(1/3
)-a*d^(4/3)*e^(2/3))-12*a*b*c*d*e-e*(b^3*d+3*a*b^2*d^(2/3)*e^(1/3)-3*a^2*b
*d^(1/3)*e^(2/3)-a^3*e))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))
/d^(2/3)/e^(8/3)*3^(1/2)
```

3.75.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx$$

$$= \frac{60e^{2/3}(-3b^2c^2d-2ac^3d+2ab^3e+6a^2bce)\,x+15e^{2/3}(-4bc^3d+b^4e+12ab^2ce+6a^2c^2e)\,x^2+10ce^{2/3}(-c}{}$$

input
```
Integrate[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

output
```
(60*e^(2/3)*(-3*b^2*c^2*d - 2*a*c^3*d + 2*a*b^3*e + 6*a^2*b*c*e)*x + 15*e^
(2/3)*(-4*b*c^3*d + b^4*e + 12*a*b^2*c*e + 6*a^2*c^2*e)*x^2 + 10*c*e^(2/3)
*(-(c^3*d) + 4*b^3*e + 12*a*b*c*e)*x^3 + 15*c^2*(3*b^2 + 2*a*c)*e^(5/3)*x^
4 + 24*b*c^3*e^(5/3)*x^5 + 5*c^4*e^(5/3)*x^6 + (10*Sqrt[3]*(b*d^(1/3) + a*
e^(1/3))*(-4*c^3*d^2 + c^2*(-6*b*d^(5/3)*e^(1/3) + 6*a*d^(4/3)*e^(2/3)) +
12*a*b*c*d*e + e*(b^3*d + 3*a*b^2*d^(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3
) - a^3*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (10*(4*
a*c^3*d^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a*b^3*d*e^(4/3
) + a^4*e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) - 4*b*(c^3*d^(
7/3) + 3*a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(1/3) + e^(1/3)*x])/d^(
2/3) - (5*(4*a*c^3*d^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a
*b^3*d*e^(4/3) + a^4*e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) -
 4*b*(c^3*d^(7/3) + 3*a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(2/3) - d^
(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3) + (10*(c^4*d^2 - 12*a*b*c^2*d*e +
6*a^2*b^2*e^2 + 4*c*e*(-(b^3*d) + a^3*e))*Log[d + e*x^3])/e^(1/3))/(30*e^(
8/3))
```

3.75.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx+cx^2\right)^4}{d+ex^3}\,dx$$

$$\Big\downarrow \text{2426}$$

$$\int \left(-\frac{x\left(-6a^2c^2e-12ab^2ce+b^4(-e)+4bc^3d\right)}{e^2}-\frac{2\left(-6a^2bce-2ab^3e+2ac^3d+3b^2c^2d\right)}{e^2}+\frac{a^4e^2-12a^2bcde-x(\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x^2(-6a^2c^2e-12ab^2ce+b^4(-e)+4bc^3d)}{2e^2}-\frac{2x(-6a^2bce-2ab^3e+2ac^3d+3b^2c^2d)}{e^2}-$$

$$\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(-e\left(a^3(-e)-3a^2b\sqrt[3]{d}e^{2/3}+3ab^2d^{2/3}\sqrt[3]{e}+b^3d\right)+6c^2\left(bd^{5/3}\sqrt[3]{e}-ad^{4/3}e^{2/3}\right.\right.}{\sqrt{3}d^{2/3}e^{8/3}}$$

$$\frac{\log(d+ex^3)\left(-4ce(b^3d-a^3e)+6a^2b^2e^2-12abc^2de+c^4d^2\right)}{3e^3}-$$

$$\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(a^4e^2-12a^2bcde+\frac{\sqrt[3]{d}(-4b(a^3e^2+c^3d^2)+6a^2c^2de+12ab^2cde+b^4de)}{\sqrt[3]{e}}-4ab^3de+4ac^3d^2+\right.}{6d^{2/3}e^{7/3}}$$

$$\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}(a^4e^2-12a^2bcde-4ab^3de+4ac^3d^2+6b^2c^2d^2)+\sqrt[3]{d}(-4b(a^3e^2+c^3d^2)+6a^2c^2de+12ab\right.}{3d^{2/3}e^{8/3}}$$

$$\frac{cx^3(-12abce-4b^3e+c^3d)}{3e^2}+\frac{c^2x^4(2ac+3b^2)}{2e}+\frac{4bc^3x^5}{5e}+\frac{c^4x^6}{6e}$$

input
```
Int[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

output
```
(-2*(3*b^2*c^2*d + 2*a*c^3*d - 2*a*b^3*e - 6*a^2*b*c*e)*x)/e^2 - ((4*b*c^3
*d - b^4*e - 12*a*b^2*c*e - 6*a^2*c^2*e)*x^2)/(2*e^2) - (c*(c^3*d - 4*b^3*
e - 12*a*b*c*e)*x^3)/(3*e^2) + (c^2*(3*b^2 + 2*a*c)*x^4)/(2*e) + (4*b*c^3*
x^5)/(5*e) + (c^4*x^6)/(6*e) - ((b*d^(1/3) + a*e^(1/3))*(4*c^3*d^2 + 6*c^2
*(b*d^(5/3)*e^(1/3) - a*d^(4/3)*e^(2/3)) - 12*a*b*c*d*e - e*(b^3*d + 3*a*b
^2*d^(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^3*e))*ArcTan[(d^(1/3) - 2
*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(8/3)) + ((e^(1/3)*(6*b
^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^(1/
3)*(b^4*d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))*L
og[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(8/3)) - ((6*b^2*c^2*d^2 + 4*a*c^3*d
^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2 + (d^(1/3)*(b^4*d*e + 12*a*b^2
*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))/e^(1/3))*Log[d^(2/3) -
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(7/3)) + ((c^4*d^2 - 12*a*b
*c^2*d*e + 6*a^2*b^2*e^2 - 4*c*e*(b^3*d - a^3*e))*Log[d + e*x^3])/(3*e^3)
```

3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.75.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.69 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.54

method	result
risch	$\frac{c^4x^6}{6e}+\frac{4b\,c^3x^5}{5e}+\frac{a\,c^3x^4}{e}+\frac{3b^2c^2x^4}{2e}+\frac{4ab\,c^2x^3}{e}+\frac{4b^3c\,x^3}{3e}-\frac{c^4d\,x^3}{3e^2}+\frac{3a^2c^2x^2}{e}+\frac{6a\,b^2c\,x^2}{e}+\frac{b^4x^2}{2e}-\frac{2b\,c^3d\,x^2}{e^2}+1$
default	$\frac{\frac{1}{6}c^4x^6e+\frac{4}{5}b\,c^3x^5e+a\,c^3e\,x^4+\frac{3}{2}b^2c^2e\,x^4+4ab\,c^2e\,x^3+\frac{4}{3}b^3ce\,x^3-\frac{1}{3}c^4d\,x^3+3a^2c^2e\,x^2+6a\,b^2ce\,x^2+\frac{1}{2}b^4e\,x^2-2b\,c^3d\,x^2+12a^2bcex+4a\,b}{e^2}$

input
```
int((c*x^2+b*x+a)^4/(e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
1/6*c^4*x^6/e+4/5*b*c^3*x^5/e+1/e*a*c^3*x^4+3/2/e*b^2*c^2*x^4+4/e*a*b*c^2*
x^3+4/3/e*b^3*c*x^3-1/3/e^2*c^4*d*x^3+3/e*a^2*c^2*x^2+6/e*a*b^2*c*x^2+1/2/
e*b^4*x^2-2/e^2*b*c^3*d*x^2+12/e*a^2*b*c*x+4/e*a*b^3*x-4/e^2*a*c^3*d*x-6/e
^2*x*b^2*c^2*d+1/3/e^3*sum(((4*a^3*c*e^2+6*a^2*b^2*e^2-12*a*b*c^2*d*e-4*b^
3*c*d*e+c^4*d^2)*_R^2+_R*(4*a^3*b*e^2-6*a^2*c^2*d*e-12*a*b^2*c*d*e-b^4*d*e
+4*b*c^3*d^2)+a^4*e^2-12*a^2*b*c*d*e-4*a*b^3*d*e+4*a*c^3*d^2+6*b^2*c^2*d^2
)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

3.75.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 91.41 (sec) , antiderivative size = 47284, normalized size of antiderivative = 73.31

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**4/(e*x**3+d),x)
```

output
```
Timed out
```

3.75.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx =$$

$$-\frac{\sqrt{3}\Big(6\,b^2c^2d^2e+4\,ac^3d^2e-4\,ab^3de^2-12\,a^2bcde^2+a^4e^3-4\,(-de^2)^{\frac{1}{3}}\,bc^3d^2+(-de^2)^{\frac{1}{3}}\,b^4de+12\,(-de^2)}{3\,(-de^2)^{\frac{2}{3}}\,e^2}$$

$$-\frac{\Big(6\,b^2c^2d^2e+4\,ac^3d^2e-4\,ab^3de^2-12\,a^2bcde^2+a^4e^3+4\,(-de^2)^{\frac{1}{3}}\,bc^3d^2-(-de^2)^{\frac{1}{3}}\,b^4de-12\,(-de^2)^{\frac{1}{3}}\,a}{6\,(-de^2)^{\frac{2}{3}}\,e^2}$$

$$+\frac{(c^4d^2-4\,b^3cde-12\,abc^2de+6\,a^2b^2e^2+4\,a^3ce^2)\log\left(|ex^3+d|\right)}{3\,e^3}$$

$$+\frac{5\,c^4e^5x^6+24\,bc^3e^5x^5+45\,b^2c^2e^5x^4+30\,ac^3e^5x^4-10\,c^4de^4x^3+40\,b^3ce^5x^3+120\,abc^2e^5x^3-60\,bc^3de^4x}{30\,e}$$

$$-\frac{\Big(4\,bc^3d^2e^{11}\left(-\frac{d}{e}\right)^{\frac{1}{3}}-b^4de^{12}\left(-\frac{d}{e}\right)^{\frac{1}{3}}-12\,ab^2cde^{12}\left(-\frac{d}{e}\right)^{\frac{1}{3}}-6\,a^2c^2de^{12}\left(-\frac{d}{e}\right)^{\frac{1}{3}}+4\,a^3be^{13}\left(-\frac{d}{e}\right)^{\frac{1}{3}}+6\,b^2c^2d^2e}{3\,de^{13}}$$

input
```
integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(6*b^2*c^2*d^2*e + 4*a*c^3*d^2*e - 4*a*b^3*d*e^2 - 12*a^2*b*c
*d*e^2 + a^4*e^3 - 4*(-d*e^2)^(1/3)*b*c^3*d^2 + (-d*e^2)^(1/3)*b^4*d*e + 1
2*(-d*e^2)^(1/3)*a*b^2*c*d*e + 6*(-d*e^2)^(1/3)*a^2*c^2*d*e - 4*(-d*e^2)^(
1/3)*a^3*b*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/((-d
*e^2)^(2/3)*e^2) - 1/6*(6*b^2*c^2*d^2*e + 4*a*c^3*d^2*e - 4*a*b^3*d*e^2 -
12*a^2*b*c*d*e^2 + a^4*e^3 + 4*(-d*e^2)^(1/3)*b*c^3*d^2 - (-d*e^2)^(1/3)*b
^4*d*e - 12*(-d*e^2)^(1/3)*a*b^2*c*d*e - 6*(-d*e^2)^(1/3)*a^2*c^2*d*e + 4*
(-d*e^2)^(1/3)*a^3*b*e^2)*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/((-d*e^
2)^(2/3)*e^2) + 1/3*(c^4*d^2 - 4*b^3*c*d*e - 12*a*b*c^2*d*e + 6*a^2*b^2*e^
2 + 4*a^3*c*e^2)*log(abs(e*x^3 + d))/e^3 + 1/30*(5*c^4*e^5*x^6 + 24*b*c^3*
e^5*x^5 + 45*b^2*c^2*e^5*x^4 + 30*a*c^3*e^5*x^4 - 10*c^4*d*e^4*x^3 + 40*b^
3*c*e^5*x^3 + 120*a*b*c^2*e^5*x^3 - 60*b*c^3*d*e^4*x^2 + 15*b^4*e^5*x^2 +
180*a*b^2*c*e^5*x^2 + 90*a^2*c^2*e^5*x^2 - 180*b^2*c^2*d*e^4*x - 120*a*c^3
*d*e^4*x + 120*a*b^3*e^5*x + 360*a^2*b*c*e^5*x)/e^6 - 1/3*(4*b*c^3*d^2*e^1
1*(-d/e)^(1/3) - b^4*d*e^12*(-d/e)^(1/3) - 12*a*b^2*c*d*e^12*(-d/e)^(1/3)
- 6*a^2*c^2*d*e^12*(-d/e)^(1/3) + 4*a^3*b*e^13*(-d/e)^(1/3) + 6*b^2*c^2*d^
2*e^11 + 4*a*c^3*d^2*e^11 - 4*a*b^3*d*e^12 - 12*a^2*b*c*d*e^12 + a^4*e^13)
*(-d/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/(d*e^13)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 2971, normalized size of antiderivative = 4.61

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2)^4/(d + e*x^3),x)
```

output
```
x^2*((b^4 + 6*a^2*c^2 + 12*a*b^2*c)/(2*e) - (2*b*c^3*d)/e^2) - x^3*((c^4*d
)/(3*e^2) - (4*b*c*(3*a*c + b^2))/(3*e)) + symsum(log(root(27*d^2*e^9*z^3
+ 324*a*b*c^2*d^3*e^7*z^2 + 108*b^3*c*d^3*e^7*z^2 - 108*a^3*c*d^2*e^8*z^2
- 162*a^2*b^2*d^2*e^8*z^2 - 27*c^4*d^4*e^6*z^2 - 72*a*b*c^6*d^5*e^4*z + 21
6*a^2*b^2*c^4*d^4*e^5*z + 144*a^3*b^3*c^2*d^3*e^6*z - 108*a^5*b^2*c*d^2*e^
7*z + 108*a^2*b^5*c*d^3*e^6*z - 36*a^4*b*c^3*d^3*e^6*z + 36*a*b^4*c^3*d^4*
e^5*z + 144*b^3*c^5*d^5*e^4*z + 90*b^6*c^2*d^4*e^5*z - 144*a^3*c^5*d^4*e^5
*z + 90*a^6*c^2*d^2*e^7*z + 171*a^4*b^4*d^2*e^7*z + 36*a*b^7*d^3*e^6*z + 3
6*a^7*b*d*e^8*z + 9*c^8*d^6*e^3*z + 36*a^7*b^4*c*d^2*e^6 - 36*a^7*b*c^4*d^
3*e^5 - 36*a^4*b^7*c*d^3*e^5 - 36*a^4*b*c^7*d^5*e^3 - 36*a*b^7*c^4*d^5*e^3
 + 36*a*b^4*c^7*d^6*e^2 + 12*a*b^10*c*d^4*e^4 + 108*a^5*b^5*c^2*d^3*e^5 -
108*a^5*b^2*c^5*d^4*e^4 + 108*a^2*b^5*c^5*d^5*e^3 - 96*a^6*b^3*c^3*d^3*e^5
 + 96*a^3*b^6*c^3*d^4*e^4 - 96*a^3*b^3*c^6*d^5*e^3 - 54*a^8*b^2*c^2*d^2*e^
6 - 54*a^2*b^8*c^2*d^4*e^4 - 54*a^2*b^2*c^8*d^6*e^2 - 9*a^4*b^4*c^4*d^4*e^
4 - 12*a^10*b*c*d*e^7 - 12*a*b*c^10*d^7*e - 6*b^6*c^6*d^6*e^2 + 4*b^9*c^3*
d^5*e^3 - 6*a^6*c^6*d^4*e^4 - 4*a^9*c^3*d^2*e^6 - 4*a^3*c^9*d^6*e^2 - 6*a^
6*b^6*d^2*e^6 + 4*a^3*b^9*d^3*e^5 + 4*b^3*c^9*d^7*e + 4*a^9*b^3*d*e^7 - b^
12*d^4*e^4 - c^12*d^8 - a^12*e^8, z, k)*((x*(3*a^4*e^5 + 12*a*c^3*d^2*e^3
+ 18*b^2*c^2*d^2*e^3 - 12*a*b^3*d*e^4 - 36*a^2*b*c*d*e^4))/e^3 - (6*c^4*d^
3*e^3 + 36*a^2*b^2*d*e^5 - 24*b^3*c*d^2*e^4 + 24*a^3*c*d*e^5 - 72*a*b*c...
```

3.76 $\int \frac{2x^2+x^4}{1+x^3}\,dx$

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3.76.1 Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{2x^2+x^4}{1+x^3}\,dx=\frac{x^2}{2}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\frac{1}{2}\log\left(1-x+x^2\right)$$

output
```
1/2*x^2+ln(1+x)+1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{2x^2+x^4}{1+x^3}\,dx=\frac{1}{6}\left(3x^2-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+2\log(1+x)-\log\left(1-x+x^2\right)+4\log\left(1+x^3\right)\right)$$

input
```
Integrate[(2*x^2 + x^4)/(1 + x^3),x]
```

output
```
(3*x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log[1 - x +
 x^2] + 4*Log[1 + x^3])/6
```

3.76.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2027, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+2x^2}{x^3+1} \, dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^2\left(x^2+2\right)}{x^3+1}dx$$

$$\Big\downarrow \text{2426}$$

$$\int \left(\frac{(2x-1)x}{x^3+1}+x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^2}{2}+\frac{1}{2}\log\left(x^2-x+1\right)+\log(x+1)$$

input
```
Int[(2*x^2 + x^4)/(1 + x^3),x]
```

output
```
x^2/2 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x] + Log[1 - x + x^2]/
2
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.76.4 Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^2}{2}+\ln(1+x)+\frac{\ln(x^2-x+1)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	38
risch	$\frac{x^2}{2}+\frac{\ln(4x^2-4x+4)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}+\ln(1+x)$	40
meijerg	$\frac{x^2}{2}-\frac{x^2\left(-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{\left(x^3\right)^{\frac{2}{3}}}+\frac{\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2\left(x^3\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2-\left(x^3\right)^{\frac{1}{3}}}\right)}{\left(x^3\right)^{\frac{2}{3}}}\right)}{3}+\frac{2\ln(x^3+1)}{3}$	89

input
```
int((x^4+2*x^2)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+ln(1+x)+1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{2x^2+x^4}{1+x^3}\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)+\log\left(x+1\right)$$

input
```
integrate((x^4+2*x^2)/(x^3+1),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
 + log(x + 1)
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{2x^2+x^4}{1+x^3}\,dx = \frac{x^2}{2} + \log(x+1) + \frac{\log(x^2-x+1)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**4+2*x**2)/(x**3+1),x)
```

output
```
x**2/2 + log(x + 1) + log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - s
qrt(3)/3)/3
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{2x^2+x^4}{1+x^3}\,dx = \frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\log\left(x^2-x+1\right) + \log(x+1)$$

input
```
integrate((x^4+2*x^2)/(x^3+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
 + log(x + 1)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{2x^2+x^4}{1+x^3}\,dx = \frac{1}{2}x^2 - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\log\left(x^2-x+1\right) + \log(|x+1|)$$

input
```
integrate((x^4+2*x^2)/(x^3+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
 + log(abs(x + 1))
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{2x^2+x^4}{1+x^3}\,dx = \ln(x+1) + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{x^2}{2}$$

input
```
int((2*x^2 + x^4)/(x^3 + 1),x)
```

output
```
log(x + 1) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/2) - log(x
+ (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/2) + x^2/2
```

3.77 $\int \frac{2x^2+x^4}{1-x^3}\,dx$

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3.77.1 Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{2x^2+x^4}{1-x^3}\,dx=-\frac{x^2}{2}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(1-x)-\frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
-1/2*x^2-ln(1-x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{2x^2+x^4}{1-x^3}\,dx=\frac{1}{6}\left(-3x^2-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-2\log(1-x)+\log\left(1+x+x^2\right)-4\log\left(1-x^3\right)\right)$$

input
```
Integrate[(2*x^2 + x^4)/(1 - x^3),x]
```

output
```
(-3*x^2 - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] + Log[1 + x +
 x^2] - 4*Log[1 - x^3])/6
```

3.77.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2027, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+2x^2}{1-x^3}\,dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x^2(x^2+2)}{1-x^3}dx$$

$$\Big\downarrow \text{ 2426}$$

$$\int \left(\frac{x(2x+1)}{1-x^3}-x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)$$

input
```
Int[(2*x^2 + x^4)/(1 - x^3),x]
```

output
```
-1/2*x^2 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x] - Log[1 + x + x^
2]/2
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.77.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{x^{2}}{2}-\ln\left(-1+x\right)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln\left(x^{2}+x+1\right)}{2}$	36
default	$-\frac{x^{2}}{2}-\ln\left(-1+x\right)-\frac{\ln\left(x^{2}+x+1\right)}{2}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	38
meijerg	$\frac{(-1)^{\frac{1}{3}}\left(\frac{3x^{2}(-1)^{\frac{2}{3}}}{2}+\frac{x^{2}(-1)^{\frac{2}{3}}\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{2+\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{\left(x^{3}\right)^{\frac{2}{3}}}\right)}{3}-\frac{2\ln\left(-x^{3}+1\right)}{3}$	90

input
```
int((x^4+2*x^2)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2-ln(-1+x)-1/3*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2))-1/2*ln(x^2+x+1)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{2x^2+x^4}{1-x^3}\,dx=-\frac{1}{2}\,x^{2}-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{2}\,\log\left(x^{2}+x+1\right)-\log\left(x-1\right)$$

input
```
integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="fricas")
```

output
```
-1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1
) - log(x - 1)
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{2x^2+x^4}{1-x^3}\,dx = -\frac{x^2}{2} - \log(x-1) - \frac{\log(x^2+x+1)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**4+2*x**2)/(-x**3+1),x)
```

output
```
-x**2/2 - log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 +
sqrt(3)/3)/3
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{2x^2+x^4}{1-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{2}\,\log\left(x^2+x+1\right) - \log(x-1)$$

input
```
integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="maxima")
```

output
```
-1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1
) - log(x - 1)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{2x^2+x^4}{1-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{2}\,\log\left(x^2+x+1\right) - \log(|x-1|)$$

input
```
integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="giac")
```

output
```
-1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1
) - log(abs(x - 1))
```

3.77.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{2x^2+x^4}{1-x^3}\,dx = -\ln(x-1)+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\frac{x^2}{2}$$

input

```
int(-(2*x^2 + x^4)/(x^3 - 1),x)
```

output

```
log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/2) - log(x - 1) - log(x
+ (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2) - x^2/2
```

3.78 $\int \frac{1-x+4x^3}{1+x^3}\,dx$

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3.78.7 Maxima [A] (verification not implemented) 750
3.78.8 Giac [A] (verification not implemented) 750
3.78.9 Mupad [B] (verification not implemented) 751

3.78.1 Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = 4x+\frac{4\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

output
```
4*x-2/3*ln(1+x)+1/3*ln(x^2-x+1)+4/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = 4x-\frac{4\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 - x + 4*x^3)/(1 + x^3),x]
```

output
```
4*x - (4*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - 
x + x^2]/3
```

3.78.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^3-x+1}{x^3+1}\,dx$$

$$\Big\downarrow \text{ 2426}$$

$$\int \left(4-\frac{x+3}{x^3+1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(x^2-x+1\right)+4x-\frac{2}{3}\log(x+1)$$

input
```
Int[(1 - x + 4*x^3)/(1 + x^3),x]
```

output
```
4*x + (4*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x
 + x^2]/3
```

3.78.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.78.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result
default	$4x-\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{3}-\frac{4\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$
risch	$4x-\frac{2\ln(1+x)}{3}+\frac{\ln(16x^2-16x+16)}{3}-\frac{4\sqrt{3}\arctan\left(\frac{(-2+4x)\sqrt{3}}{6}\right)}{3}$
meijerg	$4x-\frac{4x\left(\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{(x^3)^{\frac{1}{3}}}-\frac{\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2(x^3)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{(x^3)^{\frac{1}{3}}}\right)}{3}+\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}-\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}$

input
```
int((4*x^3-x+1)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
4*x-2/3*ln(1+x)+1/3*ln(x^2-x+1)-4/3*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{1-x+4x^3}{1+x^3}\,dx=-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+4x+\frac{1}{3}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

input
```
integrate((4*x^3-x+1)/(x^3+1),x, algorithm="fricas")
```

output
```
-4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) -
2/3*log(x + 1)
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = 4x - \frac{2\log(x+1)}{3} + \frac{\log(x^2-x+1)}{3} - \frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((4*x**3-x+1)/(x**3+1),x)
```

output
```
4*x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3 - 4*sqrt(3)*atan(2*sqrt(3)*x/3
- sqrt(3)/3)/3
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = -\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 4x + \frac{1}{3}\log\left(x^2-x+1\right) - \frac{2}{3}\log(x+1)$$

input
```
integrate((4*x^3-x+1)/(x^3+1),x, algorithm="maxima")
```

output
```
-4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) -
2/3*log(x + 1)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = -\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 4x + \frac{1}{3}\log\left(x^2-x+1\right) - \frac{2}{3}\log(|x+1|)$$

input
```
integrate((4*x^3-x+1)/(x^3+1),x, algorithm="giac")
```

output
```
-4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) -
2/3*log(abs(x + 1))
```

3.78.9 Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = 4\,x-\frac{2\,\ln\left(x+1\right)}{3}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)$$

input
```
int((4*x^3 - x + 1)/(x^3 + 1),x)
```

output
```
4*x - (2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*2i)/3 + 1
/3) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*2i)/3 - 1/3)
```

3.79 $\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$

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3.79.1 Optimal result

Integrand size = 18, antiderivative size = 230

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3
^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^
2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1+x)*Ellip
ticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*
((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1
/2)
```

3.79.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.20

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \left(1+\sqrt{3}\right) x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right) + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

input
```
Integrate[(1 + Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

output
```
(1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (x^2*Hypergeometr
ic2F1[1/2, 2/3, 5/3, -x^3])/2
```

3.79.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2417}$$

$$2\sqrt{3}\int \frac{1}{\sqrt{x^3+1}}dx+\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

input
```
Int[(1 + Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sq
rt[1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -
7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

3.79.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.79.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.79 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.20

method	result
meijerg	$x {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{x^2 {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2}+\sqrt{3}\, x {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$
elliptic	$\frac{2\left(1+\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-}{-}}}{}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{}$

input
```
int((1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/3,1/2],[4/3],-x^3)+1/2*x^2*hypergeom([1/2,2/3],[5/3],-x^3)+
3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

3.79.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.09

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = 2\left(\sqrt{3}+1\right)\text{weierstrassPInverse}(0,-4,x) - 2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(3) + 1)*weierstrassPInverse(0, -4, x) - 2*weierstrassZeta(0, -4, w
eierstrassPInverse(0, -4, x))
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.40

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5
/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gam
ma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I
*pi))/(3*gamma(4/3))
```

3.79.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

3.79.8 Giac [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.36

$$\begin{aligned}&\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx\\&=\sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)\\&-\frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,E\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\\&+\frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,F\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\end{aligned}$$

input
```
int((x + 3^(1/2) + 1)/(x^3 + 1)^(1/2),x)
```

output
```
3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, -x^3) - (6*((x + (3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((
3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
 ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) + (6*((x + (3^(1/2)*
1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*elliptic
F(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((
3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.80 $\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$

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3.80.1 Optimal result

Integrand size = 22, antiderivative size = 257

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2*(-x^3+1)^(1/2)/(1-x+3^(1/2))+3^(1/4)*(1-x)*EllipticE((1-x-3^(1/2))/(1-x
+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2)
)^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-4*3^(1/4)*(1-x)*El
lipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2
))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2
)^(1/2)
```

3.80.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.17

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \left(1+\sqrt{3}\right) x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right) - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)$$

input
```
Integrate[(1 + Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

output
```
(1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] - (x^2*Hypergeometri
c2F1[1/2, 2/3, 5/3, x^3])/2
```

3.80.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2417}$$

$$2\sqrt{3}\int \frac{1}{\sqrt{1-x^3}}dx+\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{759}$$

$$\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{2416}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

input
```
Int[(1 + Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

output
```
(-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)
/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*S
qrt[1 - x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)],
-7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

3.80.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.80.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.61 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.16

method	result
meijerg	$x_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)-\frac{x^2{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2}+\sqrt{3}\,x_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)$
elliptic	$-\frac{2i\left(1+\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+$

input
```
int((1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/3,1/2],[4/3],x^3)-1/2*x^2*hypergeom([1/2,2/3],[5/3],x^3)+3^
(1/2)*x*hypergeom([1/3,1/2],[4/3],x^3)
```

3.80.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.09

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -2\left(i\sqrt{3}+i\right)\text{weierstrassPInverse}(0,4,x) - 2i\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(3) + I)*weierstrassPInverse(0, 4, x) - 2*I*weierstrassZeta(0, 4
, weierstrassPInverse(0, 4, x))
```

3.80.6 Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.38

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

output
```
-x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamm
a(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(
3*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_po
lar(2*I*pi))/(3*gamma(4/3))
```

3.80.7 Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.80.8 Giac [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.33

$$\begin{aligned}
&\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx \\
&= \sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right) \\
&+ \frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\
&- \frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}
\end{aligned}$$

input
```
int((3^(1/2) - x + 1)/(1 - x^3)^(1/2),x)
```

output
```
3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, x^3) + (6*(x^3 - 1)^(1/2)*(-(x - (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*el
lipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1
) + x^3)^(1/2)) - (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2
)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.80. $\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$

3.81 $\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}} dx$

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3.81.1 Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}} dx = \frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
2*(x^3-1)^(1/2)/(1-x-3^(1/2))-3^(1/4)*(1-x)*EllipticE((1-x+3^(1/2))/(1-x-3
^(1/2)),2*I-I*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*
2^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

3.81.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}} dx = -\frac{x\sqrt{1-x^3}\left(-2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{2\sqrt{-1+x^3}}$$

input
```
Integrate[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - x^3]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x
^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/Sqrt[-1 + x^3]
```

3.81.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{2418\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \text{2418}$$

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

output
```
(2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)
/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)
]*Sqrt[-1 + x^3])
```

3.81.3.1 Defintions of rubi rules used

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.81.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.72 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.66

method	result
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x\, {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}} - \frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x^2\, {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2\sqrt{\operatorname{signum}(x^3-1)}} + \frac{\sqrt{3}\,\sqrt{-\operatorname{signum}(x^3-1)}\, x\, {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}$
elliptic	$\frac{2\left(1+\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$

input
```
int((1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],x
^3)-1/2/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeom([1/2,2/3]
,[5/3],x^3)+3^(1/2)/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom
([1/3,1/2],[4/3],x^3)
```

3.81.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.15

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = 2\left(\sqrt{3}+1\right)\text{weierstrassPInverse}(0,4,x) + 2\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(3) + 1)*weierstrassPInverse(0, 4, x) + 2*weierstrassZeta(0, 4, wei
erstrassPInverse(0, 4, x))
```

3.81.6 Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \frac{ix^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

output
```
I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - sqrt(3)
*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - I*x*gamma
(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

3.81.7 Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

3.81.8 Giac [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.26

$$\begin{aligned}
&\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx \\
&= \frac{\sqrt{3}\,x\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)}{\sqrt{x^3-1}} \\
&\quad + \frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\
&\quad - \frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}
\end{aligned}$$

input
```
int((3^(1/2) - x + 1)/(x^3 - 1)^(1/2),x)
```

output
```
(3^(1/2)*x*(1 - x^3)^(1/2)*hypergeom([1/3, 1/2], 4/3, x^3))/(x^3 - 1)^(1/2
) + (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3
^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -
((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
1) + x^3)^(1/2) - (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^
(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2)
```

3.81. $\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

3.82 $\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$

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3.82.1 Optimal result

Integrand size = 20, antiderivative size = 135

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-2*(-x^3-1)^(1/2)/(1+x-3^(1/2))+3^(1/4)*(1+x)*EllipticE((1+x+3^(1/2))/(1+x
-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/
2*2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

3.82.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \frac{x\sqrt{1+x^3}\left(2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

input
```
Integrate[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

output
```
(x*Sqrt[1 + x^3]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] +
 x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

3.82.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2418}$$

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

input
```
Int[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

output
```
(-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)
*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x
)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2
)]*Sqrt[-1 - x^3])
```

3.82.3.1 Defintions of rubi rules used

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.82.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

method	result
meijerg	$-ix\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{ix^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2}-i\sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$
elliptic	$-\frac{2i\left(1+\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}\ldots}{\ldots}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ldots}{\ldots}$

input
```
int((1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*x*hypergeom([1/3,1/2],[4/3],-x^3)-1/2*I*x^2*hypergeom([1/2,2/3],[5/3],-
x^3)-I*3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

3.82.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.17

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -2\left(i\sqrt{3}+i\right)\text{weierstrassPInverse}(0,-4,x) + 2i\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(3) + I)*weierstrassPInverse(0, -4, x) + 2*I*weierstrassZeta(0,
-4, weierstrassPInverse(0, -4, x))
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.73

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

output
```
-I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamm
a(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(
I*pi))/(3*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_
polar(I*pi))/(3*gamma(4/3))
```

3.82.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

3.82.8 Giac [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.67

$$\begin{aligned}
&\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx \\
&= \frac{\sqrt{3}\,x\,\sqrt{x^3+1}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)}{\sqrt{-x^3-1}} \\
&\quad - \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\
&\quad + \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}
\end{aligned}$$

input
```
int((x + 3^(1/2) + 1)/(- x^3 - 1)^(1/2),x)
```

output
```
(3^(1/2)*x*(x^3 + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -x^3))/(- x^3 - 1)^(
1/2) - (6*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 -
1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) + (6*(x^3 + 1)^(1/2)*(
(x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2))^(1/2))
```

3.82. $\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}} \, dx$

3.83 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$

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3.83.1 Optimal result

Integrand size = 33, antiderivative size = 468

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*(b*x^3+a)^(1/2)/b^(1/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)*
(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)+4*3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6
^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.83.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.19

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2
, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3
)/a)]))/(2*Sqrt[a + b*x^3])
```

3.83.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{ 2417}$$

$$2\sqrt{3}\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx+\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx$$

$$\Big\downarrow \text{ 759}$$

$$\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

3.83. $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}dx$

output
```
(2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.83.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.83.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(346) = 692$.

Time = 1.75 (sec) , antiderivative size = 1003, normalized size of antiderivative = 2.14

method	result	size
default	Expression too large to display	1003

input
```
int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
))-2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)...
```

3.83.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.10

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{2\left(a^{\frac{1}{3}}\sqrt{b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-b^{\frac{5}{6}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2*(a^(1/3)*sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - b^(5/
6)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

3.83.6 Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.26

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3+a)**(1/2),x)
```

output
```
b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1
/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

3.83.7 Maxima [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

3.83.8 Giac [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(a + b*x^3)^(1/2),x)
```

output
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(a + b*x^3)^(1/2), x)
```

3.84 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$

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3.84.7 Maxima [F] . . . 790
3.84.8 Giac [F] . . . 790
3.84.9 Mupad [F(-1)] . . . 790

3.84.1 Optimal result

Integrand size = 35, antiderivative size = 481

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2*(-b*x^3+a)^(1/2)/b^(1/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+3^(1/4)*a^(1/
3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3
)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
/b^(1/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)-4*3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b
^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.84.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.19

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = \frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)-\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
(x*Sqrt[1 - (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2
, 4/3, (b*x^3)/a] - b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/(2*Sqrt[a - b*x^3])
```

3.84.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$$

$$\Big\downarrow \text{2417}$$

$$2\sqrt{3}\sqrt[3]{a}\int \frac{1}{\sqrt{a-bx^3}}dx+\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{759}$$

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{2416}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

3.84. $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$

output
```
(-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (4*3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

3.84.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.84. $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$

3.84.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 948 vs. 2(359) = 718.

Time = 1.80 (sec) , antiderivative size = 949, normalized size of antiderivative = 1.97

method	result	size
default	Expression too large to display	949

input
```
int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3)
)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2
)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b
^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I
*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2
)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^
(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3
/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3)*3^
(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/
3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)
*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/...
```

3.84.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\right.\right.}{b}$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="
fricas")
```

output
```
-2*(a^(1/3)*sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + sqrt
(-b)*b^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/
b
```

3.84.6 Sympy [A] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.27

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

output
```
-b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x*
*3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyp
er((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

3.84.7 Maxima [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="
maxima")
```

output
```
-integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x)
```

3.84.8 Giac [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="
giac")
```

output
```
integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=-\int \frac{b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{a-b\,x^3}}\,dx$$

input
```
int(-(b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(a - b*x^3)^(1/2),x)
```

output
```
-int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(a - b*x^3)^(1/2), x)
```

3.85 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$

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3.85.1 Optimal result

Integrand size = 36, antiderivative size = 271

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
2*(b*x^3-a)^(1/2)/b^(1/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-3^(1/4)*a^(1/3)
*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*
x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b
^(1/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))^2)^(1/2)
```

3.85.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.34

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)-\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
(x*Sqrt[1 - (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2
, 4/3, (b*x^3)/a] - b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/(2*Sqrt[-a + b*x^3])
```

3.85.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}\,dx$$

$$\Big\downarrow \ 2418$$

$$\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
(2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/
((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

3.85.3.1 Defintions of rubi rules used

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.85.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(206) = 412$.

Time = 1.76 (sec) , antiderivative size = 952, normalized size of antiderivative = 3.51

method	result	size
default	Expression too large to display	952

input
```
int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/3*I/b^(2/3)*3^(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))
/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)
/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3
))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*a^(1/3)*3^(1/2)/b*(a*b
^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2
)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(
1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1
/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*
(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1
/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b...
```

3.85.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.18

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$$
$$=\frac{2\left(a^{\frac{1}{3}}\sqrt{b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+b^{\frac{5}{6}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="f
ricas")
```

output
```
2*(a^(1/3)*sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + b^(5/6
)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

3.85.6 Sympy [A] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.41

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3-a)**(1/2),x)
```

output
```
I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*
gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(
3*a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a
)/(3*a**(1/6)*gamma(4/3))
```

3.85.7 Maxima [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="m
axima")
```

output
```
-integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

3.85.8 Giac [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = -\int \frac{b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{b\,x^3-a}}\,dx$$

input
```
int(-(b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(b*x^3 - a)^(1/2),x)
```

output
```
-int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(b*x^3 - a)^(1/2), x)
```

3.86 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$

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3.86.9 Mupad [F(-1)] . . . 802

3.86.1 Optimal result

Integrand size = 36, antiderivative size = 266

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2*(-b*x^3-a)^(1/2)/b^(1/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+3^(1/4)*a^(1/3
)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(
1/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))^2)^(1/2)
```

3.86.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.35

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2
, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3
)/a)]))/(2*Sqrt[-a - b*x^3])
```

3.86.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$$

$$\Big\downarrow \text{2418}$$

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(
1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

3.86.3.1 Defintions of rubi rules used

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.86.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1011 vs. 2(201) = 402.

Time = 1.72 (sec) , antiderivative size = 1012, normalized size of antiderivative = 3.80

method	result	size
default	Expression too large to display	1012

input
```
int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*
(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))-2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/...
```

3.86.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx =$$
$$-\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInve}\right.\right.}{b}$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="f
ricas")
```

output
```
-2*(a^(1/3)*sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - sqr
t(-b)*b^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)
))/b
```

3.86.6 Sympy [A] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.48

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

output
```
-I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4
/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*h
yper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

3.86.7 Maxima [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)
```

3.86.8 Giac [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{-b\,x^3-a}}\,dx$$

input
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(- a - b*x^3)^(1/2),x)
```

output
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(- a - b*x^3)^(1/2), x)
```

3.87 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$

3.87.1 Optimal result . . . 803
3.87.2 Mathematica [C] (verified) . . . 804
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3.87.1 Optimal result

Integrand size = 30, antiderivative size = 520

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\Bigg(\arcsin}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*(b/a)^(1/3)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1
/4)*a^(1/3)*(b/a)^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/
2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((
b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(-a^(1/3)*(b/a)^(1/3)*(1-3^(1/2))+b^(1/3)*(1+3^(1/2)))*(1/2*6^(1/2)+1/
2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.87.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.17

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

input
```
Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -
((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)
]))/(2*Sqrt[a + b*x^3])
```

3.87.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 516, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2417}$$

$$\left(-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}+\sqrt{3}+1\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}+\sqrt{3}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}+\sqrt{3}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{\sqrt[3]{b}}$$

input
```
Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
((b/a)^(1/3)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/
3) + (2*Sqrt[2 + Sqrt[3]]*(1 + Sqrt[3] - ((1 - Sqrt[3])*a^(1/3)*(b/a)^(1/3
))/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.87.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

3.87. $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.87.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(386) = 772.

Time = 1.74 (sec) , antiderivative size = 1004, normalized size of antiderivative = 1.93

method	result	size
default	Expression too large to display	1004

input
```
int((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))-2*I/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I
*(b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)...
```

3.87.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{2\left(\sqrt{b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0\right.\right.\right.}{b}$$

input
```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - sqrt(b)*(b/a)
^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

3.87.6 Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.24

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a)**(1/2),x)
```

output
```
x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*
pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x
**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper
((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

3.87.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

3.87.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/(a + b*x^3)^(1/2),x)
```

output
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/(a + b*x^3)^(1/2), x)
```

3.88 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$

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3.88.1 Optimal result

Integrand size = 32, antiderivative size = 533

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2*(b/a)^(1/3)*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+3
^(1/4)*a^(1/3)*(b/a)^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/
2)-1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3
)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/3*(a^(1/3)-b^(1/3)*x)*Ell
ipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*(-a^(1/3)*(b/a)^(1/3)*(1-3^(1/2))+b^(1/3)*(1+3^(1/2)))*(1/2*
6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.88.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.17

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(-2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

input
```
Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - (b*x^3)/a]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2,
4/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a
]))/Sqrt[a - b*x^3]
```

3.88.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1}{\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2417}$$

$$\left(-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}+\sqrt{3}+1\right)\int \frac{1}{\sqrt{a-bx^3}}dx+\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-\frac{2\sqrt{2+\sqrt{3}}\left(-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}+\sqrt{3}+1\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\left(\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}\right.}{\sqrt[3]{b}}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}+\sqrt{3}+1\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

input
```
Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
((b/a)^(1/3)*((-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/
3)*x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^
(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) -
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])))/b^(1
/3) - (2*Sqrt[2 + Sqrt[3]]*(1 + Sqrt[3] - ((1 - Sqrt[3])*a^(1/3)*(b/a)^(1/
3))/b^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

3.88.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

3.88. $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.88.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(399) = 798$.

Time = 1.78 (sec) , antiderivative size = 950, normalized size of antiderivative = 1.78

method	result	size
default	Expression too large to display	950

input
```
int((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^
3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3
)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I/b*(a*b^
2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1
/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(a*b
^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2
)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+
1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1...
```

3.88.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.11

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(\sqrt{-b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\right.\right.}{b}$$

input
```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

output
```
-2*(sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + sqrt(-b)*(b/
a)^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

3.88.6 Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.24

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+a)**(1/2),x)
```

output
```
-x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2
*I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,),
b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*
hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3
))
```

3.88.7 Maxima [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

output
```
-integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

3.88.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \int \frac{\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{a-b\,x^3}}\,dx$$

input
```
int((3^(1/2) - x*(b/a)^(1/3) + 1)/(a - b*x^3)^(1/2),x)
```

output
```
int((3^(1/2) - x*(b/a)^(1/3) + 1)/(a - b*x^3)^(1/2), x)
```

3.89 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$

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3.89.1 Optimal result

Integrand size = 33, antiderivative size = 256

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a+bx^3}}{b\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
2*(b/a)^(2/3)*(b*x^3-a)^(1/2)/b/(1-(b/a)^(1/3)*x-3^(1/2))-3^(1/4)*(1-(b/a)
^(1/3)*x)*EllipticE((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2)),2*
I-I*3^(1/2))*((1+(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(1-(b/a)^(1/3)*x-3^(1/2))^
2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(b/a)^(1/3)/(b*x^3-a)^(1/2)/((-1+(b/a)^
(1/3)*x)/(1-(b/a)^(1/3)*x-3^(1/2))^2)^(1/2)
```

3.89.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.35

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(-2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

input
```
Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - (b*x^3)/a]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2,
4/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a
]))/Sqrt[-a + b*x^3]
```

3.89.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1}{\sqrt{bx^3-a}}\,dx$$

$$\Big\downarrow \; 2418$$

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{bx^3-a}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}\sqrt{bx^3-a}}$$

input
```
Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
(2*(b/a)^(2/3)*Sqrt[-a + b*x^3])/(b*(1 - Sqrt[3] - (b/a)^(1/3)*x)) - (3^(1
/4)*Sqrt[2 + Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^
(2/3)*x^2)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3]
- (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(
1/3)*Sqrt[-((1 - (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2)]*Sqrt[-a
+ b*x^3])
```

3.89.3.1 Defintions of rubi rules used

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.89.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(211) = 422$.

Time = 1.67 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.72

method	result	size
default	Expression too large to display	953

input
```
int((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3
-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)
/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I/b*(a*b^2
)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*
b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/
3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2
)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(a*b^2
)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*
b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2
/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3)...
```

3.89.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.20

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(\sqrt{b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},\right.\right.\right.}{b}$$

input
```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + sqrt(b)*(b/a)^
(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

3.89.6 Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a)**(1/2),x)
```

output
```
I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt
(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/
a)/(3*sqrt(a)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**
3/a)/(3*sqrt(a)*gamma(4/3))
```

3.89.7 Maxima [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx=\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

3.89.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx=\int \frac{\sqrt{3}-x\,{\left(\frac{b}{a}\right)}^{1/3}+1}{\sqrt{b\,x^3-a}}\,dx$$

input
```
int((3^(1/2) - x*(b/a)^(1/3) + 1)/(b*x^3 - a)^(1/2),x)
```

output
```
int((3^(1/2) - x*(b/a)^(1/3) + 1)/(b*x^3 - a)^(1/2), x)
```

3.90 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$

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3.90.1 Optimal result

Integrand size = 33, antiderivative size = 251

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2*(b/a)^(2/3)*(-b*x^3-a)^(1/2)/b/(1+(b/a)^(1/3)*x-3^(1/2))+3^(1/4)*(1+(b/
a)^(1/3)*x)*EllipticE((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2)),
2*I-I*3^(1/2))*((1-(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(1+(b/a)^(1/3)*x-3^(1/2)
)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(b/a)^(1/3)/(-b*x^3-a)^(1/2)/((-1-(b/
a)^(1/3)*x)/(1+(b/a)^(1/3)*x-3^(1/2))^2)^(1/2)
```

3.90.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.37

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2(1+\sqrt{3})\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

input
```
Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -
((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)
]))/(2*Sqrt[-a - b*x^3])
```

3.90.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1}{\sqrt{-a-bx^3}}\,dx$$

$$\Big\downarrow \text{2418}$$

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}\sqrt{-a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}$$

input
```
Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(-2*(b/a)^(2/3)*Sqrt[-a - b*x^3])/(b*(1 - Sqrt[3] + (b/a)^(1/3)*x)) + (3^(
1/4)*Sqrt[2 + Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)
^(2/3)*x^2)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3]
 + (b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^
(1/3)*Sqrt[-((1 + (b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2)]*Sqrt[-a
 - b*x^3])
```

3.90.3.1 Defintions of rubi rules used

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.90.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(208) = 416.

Time = 1.73 (sec) , antiderivative size = 1013, normalized size of antiderivative = 4.04

method	result	size
default	Expression too large to display	1013

input
```
int((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))-2*I/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-
b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3
*I*(b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1...
```

3.90.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(\sqrt{-b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInv}\right.\right.}{b}$$

input
```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="fricas"
)
```

output
```
-2*(sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - sqrt(-b)*(b
/a)^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

3.90.6 Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.52

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-a)**(1/2),x)
```

output
```
-I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar
(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2)
, (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) - I*x*gamma(1/3
)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3
))
```

3.90.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

3.90.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{-b\,x^3-a}}\,dx$$

input
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/(- a - b*x^3)^(1/2),x)
```

output
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/(- a - b*x^3)^(1/2), x)
```

3.91 $\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$

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3.91.1 Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3
^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^
2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.91.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.39

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \left(1-\sqrt{3}\right)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right) + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

input
```
Integrate[(1 - Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

output
```
(1 - Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (x^2*Hypergeometr
ic2F1[1/2, 2/3, 5/3, -x^3])/2
```

3.91.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2416}$$

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[(1 - Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sq
rt[1 + x^3])
```

3.91.3.1 Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.91.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.72 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.38

method	result
meijerg	$x {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -x^3\right) + \frac{x^2 {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2} - \sqrt{3}\, x {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -x^3\right)$
elliptic	$\frac{2\left(1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} + 2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} + 2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\cdots$

input
```
int((1+x-3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/3,1/2],[4/3],-x^3)+1/2*x^2*hypergeom([1/2,2/3],[5/3],-x^3)-
3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

3.91.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.17

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = -2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,-4,x) - 2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(3) - 1)*weierstrassPInverse(0, -4, x) - 2*weierstrassZeta(0, -4,
weierstrassPInverse(0, -4, x))
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5
/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi)
)/(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I
*pi))/(3*gamma(4/3))
```

3.91.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

3.91.8 Giac [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.46

$$\begin{aligned}
&\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx \\
&= -\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right) \\
&\quad - \frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\
&\quad + \frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}
\end{aligned}$$

input
```
int((x - 3^(1/2) + 1)/(x^3 + 1)^(1/2),x)
```

output
```
(6*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2
))^(1/2) - (6*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((
x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2))^(1/2) - 3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, -x^3)
```

3.91. $\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$

3.92 $\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$

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3.92.1 Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx=-\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2*(-x^3+1)^(1/2)/(1-x+3^(1/2))+3^(1/4)*(1-x)*EllipticE((1-x-3^(1/2))/(1-x
+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2)
)^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.92.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx=\left(1-\sqrt{3}\right)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)$$

input
```
Integrate[(1 - Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

output
```
(1 - Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] - (x^2*Hypergeometri
c2F1[1/2, 2/3, 5/3, x^3])/2
```

3.92.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

input
```
Int[(1 - Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

output
```
(-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)
/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*S
qrt[1 - x^3])
```

3.92.3.1 Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.92.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.61 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.30

method	result
meijerg	$x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)-\frac{x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2}-\sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)$
elliptic	$-\frac{2i\left(1-\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}\right.}}{}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$

input
```
int((1-x-3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/3,1/2],[4/3],x^3)-1/2*x^2*hypergeom([1/2,2/3],[5/3],x^3)-3^
(1/2)*x*hypergeom([1/3,1/2],[4/3],x^3)
```

3.92.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.16

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -2\left(-i\sqrt{3}+i\right)\text{weierstrassPInverse}(0,4,x) - 2i\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(3) + I)*weierstrassPInverse(0, 4, x) - 2*I*weierstrassZeta(0,
4, weierstrassPInverse(0, 4, x))
```

3.92.6 Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.68

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -\frac{x^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

output
```
-x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamm
a(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*
I*pi))/(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_po
lar(2*I*pi))/(3*gamma(4/3))
```

3.92.7 Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx=\int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.92.8 Giac [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx=\int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.42

$$\begin{aligned}\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx &= -\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)\\
&+\frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\\
&-\frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\end{aligned}$$

input
```
int(-(x + 3^(1/2) - 1)/(1 - x^3)^(1/2),x)
```

output
```
(6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/
2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - 3^(1/2)*x*hypergeom([1/3,
1/2], 4/3, x^3) - (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2
)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.92. $\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$

3.93 $\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

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3.93.1 Optimal result

Integrand size = 22, antiderivative size = 264

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
2*(x^3-1)^(1/2)/(1-x-3^(1/2))+4*3^(1/4)*(1-x)*EllipticF((1-x+3^(1/2))/(1-x
-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2)
)^2)^(1/2)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-3^(1/4)*(1-x)*Elli
pticE((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^
2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^
(1/2)
```

3.93.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.24

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = -\frac{x\sqrt{1-x^3}\left(2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{2\sqrt{-1+x^3}}$$

input
```
Integrate[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - x^3]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x
^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/Sqrt[-1 + x^3]
```

3.93.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 2419}$$

$$\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx-2\sqrt{3}\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{ 2418}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} + \frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}$$

input
```
Int[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

output
```
(2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)
/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)
]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2
)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x
)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

3.93.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.93.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.62 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

method	result
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x\, {}_2F_1(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3)}{\sqrt{\operatorname{signum}(x^3-1)}} - \frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x^2\, {}_2F_1(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3)}{2\sqrt{\operatorname{signum}(x^3-1)}} - \frac{\sqrt{3}\,\sqrt{-\operatorname{signum}(x^3-1)}\, x\, {}_2F_1(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3)}{\sqrt{\operatorname{signum}(x^3-1)}}$
elliptic	$\frac{2\left(1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}\cdots}{\frac{3}{2}+\cdots}}\cdots}{}$

input
```
int((1-x-3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],x
^3)-1/2/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeom([1/2,2/3]
,[5/3],x^3)-3^(1/2)/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom
([1/3,1/2],[4/3],x^3)
```

3.93.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.08

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = -2\left(\sqrt{3}-1\right)\mathrm{weierstrassPInverse}(0,4,x) + 2\,\mathrm{weierstrassZeta}(0,4,\mathrm{weierstrassPInverse}(0,4,x))$$

input
```
integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(3) - 1)*weierstrassPInverse(0, 4, x) + 2*weierstrassZeta(0, 4, we
ierstrassPInverse(0, 4, x))
```

3.93.6 Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

output
```
I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - I*x*gam
ma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + sqrt(3)*I*x*gamma
(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

3.93.7 Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

3.93.8 Giac [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.24

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = -\frac{\sqrt{3}\,x\,\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)}{\sqrt{x^3-1}} + \frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} - \frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int(-(x + 3^(1/2) - 1)/(x^3 - 1)^(1/2),x)
```

output
```
(6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/
2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
 x^3)^(1/2) - (3^(1/2)*x*(1 - x^3)^(1/2)*hypergeom([1/3, 1/2], 4/3, x^3))/
(x^3 - 1)^(1/2) - (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^
(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2)
```

3.93. $\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

3.94 $\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$

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3.94.1 Optimal result

Integrand size = 22, antiderivative size = 247

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-2*(-x^3-1)^(1/2)/(1+x-3^(1/2))-4*3^(1/4)*(1+x)*EllipticF((1+x+3^(1/2))/(1
+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/
2))^2)^(1/2)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+3^(1/4)*(1+x)*E
llipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2
))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))
^2)^(1/2)
```

3.94.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.27

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \frac{x\sqrt{1+x^3}\left(-2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

input
```
Integrate[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

output
```
(x*Sqrt[1 + x^3]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3]
 + x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

3.94.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 2419}$$

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-2\sqrt{3}\int \frac{1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{ 2418}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

input
```
Int[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

output
```
(-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)
*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x
)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2
)]*Sqrt[-1 - x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^
2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] +
x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]
)
```

3.94.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.94.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.64 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.21

method	result
meijerg	$-ix\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{ix^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2}+i\sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$
elliptic	$-\frac{2i\left(1-\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i}{}\right.}}{}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$

input
```
int((1+x-3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*x*hypergeom([1/3,1/2],[4/3],-x^3)-1/2*I*x^2*hypergeom([1/2,2/3],[5/3],-
x^3)+I*3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

3.94.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.09

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -2\left(-i\sqrt{3}+i\right)\mathrm{weierstrassPInverse}(0,-4,x) + 2i\,\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(3) + I)*weierstrassPInverse(0, -4, x) + 2*I*weierstrassZeta(0,
 -4, weierstrassPInverse(0, -4, x))
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{ix^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+x-3**(1/2))/(-x**3-1)**(1/2),x)
```

output
```
-I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamm
a(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(
3*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_
polar(I*pi))/(3*gamma(4/3))
```

3.94.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

3.94.8 Giac [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.46

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{\sqrt{3}\,x\,\sqrt{x^3+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)}{\sqrt{-x^3-1}} - \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} + \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int((x - 3^(1/2) + 1)/(- x^3 - 1)^(1/2),x)
```

output
```
(6*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2
)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (6*(x^3 + 1)^(1/2)*((x + (3
^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*e
llipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2))^(1/2)) - (3^(1/2)*x*(x^3 + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3,
-x^3))/(- x^3 - 1)^(1/2)
```

3.94. $\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$

3.95 $\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx$

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3.95.1 Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = -\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-2*(x^3+1)^(1/2)/(1+x+3^(1/2))+3^(1/4)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+
3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))
^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.95.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.37

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = \left(-1+\sqrt{3}\right)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right) - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

input
```
Integrate[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3],x]
```

output
```
(-1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] - (x^2*Hypergeomet
ric2F1[1/2, 2/3, 5/3, -x^3])/2
```

3.95.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}-1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \; 2416$$

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

input
```
Int[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3],x]
```

output
```
(-2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)
/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 + x^3])
```

3.95.3.1 Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.95.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.70 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.38

method	result
meijerg	$-x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2}+\sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$
elliptic	$\frac{2\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\cdots}}{\cdots}$
default	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\cdots}{-\frac{3}{2}+\cdots}}}{\cdots}$

input
```
int((-1-x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x*hypergeom([1/3,1/2],[4/3],-x^3)-1/2*x^2*hypergeom([1/2,2/3],[5/3],-x^3)
+3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

3.95.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.17

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = 2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,-4,x) + 2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(3) - 1)*weierstrassPInverse(0, -4, x) + 2*weierstrassZeta(0, -4, w
eierstrassPInverse(0, -4, x))
```

3.95.6 Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.73

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = -\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-1-x+3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
-x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(
5/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*ga
mma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(
I*pi))/(3*gamma(4/3))
```

3.95.7 Maxima [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = \int -\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

3.95.8 Giac [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = \int -\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((-1-x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

3.95.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.48

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx$$
$$= \sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)$$
$$+ \frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$
$$- \frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int(-(x - 3^(1/2) + 1)/(x^3 + 1)^(1/2),x)
```

output
```
3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, -x^3) + (6*((x + (3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((
3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
 ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (6*((x + (3^(1/2)*
1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*elliptic
F(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((
3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.95. $\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx$

3.96 $\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx$

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3.96.1 Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx=\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
2*(-x^3+1)^(1/2)/(1-x+3^(1/2))-3^(1/4)*(1-x)*EllipticE((1-x-3^(1/2))/(1-x+
3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))
^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.96.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.30

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx=\frac{1}{2}x\left(2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)$$

input
```
Integrate[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3],x]
```

output
```
(x*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + x*Hypergeomet
ric2F1[1/2, 2/3, 5/3, x^3]))/2
```

3.96.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{2416}$$

$$\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

input
```
Int[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3],x]
```

output
```
(2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sq
rt[1 - x^3])
```

3.96.3.1 Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.96.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.65 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result
meijerg	$-x\,{}_{2}F_{1}\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^{3}\right)+\frac{x^{2}\,{}_{2}F_{1}\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^{3}\right)}{2}+\sqrt{3}\,x\,{}_{2}F_{1}\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^{3}\right)$
elliptic	$-\frac{2i\left(\sqrt{3}-1\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\cdots$
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\cdots$

input
```
int((-1+x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x*hypergeom([1/3,1/2],[4/3],x^3)+1/2*x^2*hypergeom([1/2,2/3],[5/3],x^3)+3
^(1/2)*x*hypergeom([1/3,1/2],[4/3],x^3)
```

3.96.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.16

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = -2\left(i\sqrt{3}-i\right)\text{weierstrassPInverse}(0,4,x) + 2i\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(3) - I)*weierstrassPInverse(0, 4, x) + 2*I*weierstrassZeta(0, 4
, weierstrassPInverse(0, 4, x))
```

3.96.6 Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.68

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-1+x+3**(1/2))/(-x**3+1)**(1/2),x)
```

output
```
x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3
*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_pol
ar(2*I*pi))/(3*gamma(4/3))
```

3.96.7 Maxima [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.96.8 Giac [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((-1+x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.39

$$\begin{aligned}
&\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx \\
&= \sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right) \\
&\quad - \frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\
&\quad + \frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}
\end{aligned}$$

input
```
int((x + 3^(1/2) - 1)/(1 - x^3)^(1/2),x)
```

output
```
3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, x^3) - (6*(x^3 - 1)^(1/2)*(-(x - (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*el
lipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1
) + x^3)^(1/2)) + (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2
)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.96. $\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx$

3.97 $\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx$

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3.97.1 Optimal result

Integrand size = 18, antiderivative size = 263

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = -\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2*(x^3-1)^(1/2)/(1-x-3^(1/2))-4*3^(1/4)*(1-x)*EllipticF((1-x+3^(1/2))/(1-
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2
))^2)^(1/2)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)+3^(1/4)*(1-x)*Ell
ipticE((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))
^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)
^(1/2)
```

3.97.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.24

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = \frac{x\sqrt{1-x^3}\left(2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{2\sqrt{-1+x^3}}$$

input
```
Integrate[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]
```

output
```
(x*Sqrt[1 - x^3]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] +
 x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])
```

3.97.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \ 2419$$

$$2\sqrt{3}\int \frac{1}{\sqrt{x^3-1}}dx-\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \ 760$$

$$-\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \ 2418$$

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}$$

input
```
Int[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]
```

output
```
(-2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)
*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x
)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2
)]*Sqrt[-1 + x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^
2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] -
x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]
)
```

3.97.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.97.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.68 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.37

method	result
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x\, {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}+\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x^2\, {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2\sqrt{\operatorname{signum}(x^3-1)}}+\frac{\sqrt{3}\,\sqrt{-\operatorname{signum}(x^3-1)}\, x\, {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}$
elliptic	$\frac{2\left(\sqrt{3}-1\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$

input
```
int((-1+x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],
x^3)+1/2/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeom([1/2,2/3
],[5/3],x^3)+3^(1/2)/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeo
m([1/3,1/2],[4/3],x^3)
```

3.97.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.08

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = 2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,4,x) - 2\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(3) - 1)*weierstrassPInverse(0, 4, x) - 2*weierstrassZeta(0, 4, wei
erstrassPInverse(0, 4, x))
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = -\frac{ix^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-1+x+3**(1/2))/(x**3-1)**(1/2),x)
```

output
```
-I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - sqrt(3
)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + I*x*gamm
a(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

3.97.7 Maxima [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

3.97.8 Giac [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx=\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((-1+x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.24

$$\begin{aligned}
&\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx \\
&=\frac{\sqrt{3}\,x\,\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)}{\sqrt{x^3-1}} \\
&-\frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} \\
&+\frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}
\end{aligned}$$

input
```
int((x + 3^(1/2) - 1)/(x^3 - 1)^(1/2),x)
```

output
```
(3^(1/2)*x*(1 - x^3)^(1/2)*hypergeom([1/3, 1/2], 4/3, x^3))/(x^3 - 1)^(1/2
) - (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3
^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -
((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
1) + x^3)^(1/2) + (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^
(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2)
```

3.97. $\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx$

3.98 $\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx$

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3.98.1 Optimal result

Integrand size = 22, antiderivative size = 248

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2*(-x^3-1)^(1/2)/(1+x-3^(1/2))+4*3^(1/4)*(1+x)*EllipticF((1+x+3^(1/2))/(1+
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2
))^2)^(1/2)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)-3^(1/4)*(1+x)*El
lipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2)
)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^
2)^(1/2)
```

3.98.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.27

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = -\frac{x\sqrt{1+x^3}\left(-2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

input
```
Integrate[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3],x]
```

output
```
-1/2*(x*Sqrt[1 + x^3]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
-x^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/Sqrt[-1 - x^3]
```

3.98.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}-1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2419}$$

$$2\sqrt{3}\int \frac{1}{\sqrt{-x^3-1}}dx-\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2418}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}} + \frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

input
```
Int[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3],x]
```

output
```
(2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)
/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)
]*Sqrt[-1 - x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2
)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x
)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

3.98.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.98.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.62 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.21

method	result
meijerg	$ix\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{ix^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2}-i\sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$
elliptic	$-\frac{2i\left(\sqrt{3}-1\right)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\cdots\right.}}{\cdots}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\cdots}}{\cdots}$

input
```
int((-1-x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*x*hypergeom([1/3,1/2],[4/3],-x^3)+1/2*I*x^2*hypergeom([1/2,2/3],[5/3],-x
^3)-I*3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

3.98.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.09

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = -2\left(i\sqrt{3}-i\right)\text{weierstrassPInverse}(0,-4,x) - 2i\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(3) - I)*weierstrassPInverse(0, -4, x) - 2*I*weierstrassZeta(0,
-4, weierstrassPInverse(0, -4, x))
```

3.98.6 Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \frac{ix^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-1-x+3**(1/2))/(-x**3-1)**(1/2),x)
```

output
```
I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma
(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I
*pi))/(3*gamma(4/3)) + I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_p
olar(I*pi))/(3*gamma(4/3))
```

3.98.7 Maxima [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \int -\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

3.98.8 Giac [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \int -\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((-1-x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

3.98.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx &= \frac{\sqrt{3}\,x\,\sqrt{x^3+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)}{\sqrt{-x^3-1}} \\ &+ \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} \\ &- \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\end{aligned}$$

input
```
int(-(x - 3^(1/2) + 1)/(- x^3 - 1)^(1/2),x)
```

output
```
(3^(1/2)*x*(x^3 + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -x^3))/(- x^3 - 1)^(
1/2) + (6*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 -
1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (6*(x^3 + 1)^(1/2)*(
(x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2))^(1/2))
```

3.98. $\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx$

3.99 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$

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3.99.1 Optimal result

Integrand size = 35, antiderivative size = 256

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*(b*x^3+a)^(1/2)/b^(1/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)*
(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

3.99.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.35

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2(-1+\sqrt{3})\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\right.\right.}{2\sqrt{a+bx^3}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1
/2, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x
^3)/a)]))/(2*Sqrt[a + b*x^3])
```

3.99.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$$
$$\Big\downarrow \text{2416}$$
$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{b}x+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.99.3.1 Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.99.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. 2(189) = 378.

Time = 1.76 (sec) , antiderivative size = 1003, normalized size of antiderivative = 3.92

method	result	size
default	Expression too large to display	1003

input
```
int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
))+2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)...
```

3.99.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.19

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx =$$
$$-\frac{2\left(a^{\frac{1}{3}}\sqrt{b}(\sqrt{3}-1)\,\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+b^{\frac{5}{6}}\,\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,\right.\right.\right.}{b}$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fr
icas")
```

output
```
-2*(a^(1/3)*sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + b^(5
/6)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

3.99.6 Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.48

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

output
```
b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + x*gamma(1/3)*hyper((1
/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

3.99.7 Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

3.99.8 Giac [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}-1\right)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(a + b*x^3)^(1/2),x)
```

output
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(a + b*x^3)^(1/2), x)
```

3.100 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$

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3.100.1 Optimal result

Integrand size = 37, antiderivative size = 263

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2*(-b*x^3+a)^(1/2)/b^(1/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+3^(1/4)*a^(1/
3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3
)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
/b^(1/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)
```

3.100.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.34

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2(-1+\sqrt{3})\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right.\right.}{2\sqrt{a-bx^3}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/
3, 1/2, 4/3, (b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^
3)/a]))/Sqrt[a - b*x^3]
```

3.100.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
(-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

3.100.3.1 Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.100.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 948 vs. $2(196) = 392$.

Time = 1.74 (sec) , antiderivative size = 949, normalized size of antiderivative = 3.61

method	result	size
default	Expression too large to display	949

input
```
int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-2/3*I/b^(2/3)*3^(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))
/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)
/(-b*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/
3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*Ellipt
icF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*
3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*a^(1/3)*3^(1/2)/b*(a*
b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/
2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*Elliptic
F(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^
(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-
I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^
(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)...
```

3.100.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$$
$$= \frac{2\left(a^{\frac{1}{3}}\sqrt{-b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\right.\right.\right.}{b}$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="
fricas")
```

output
```
2*(a^(1/3)*sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - sqrt(
-b)*b^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

3.100.6 Sympy [A] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.49

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

output
```
-b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3
,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + x*gamma(1/3)*hyp
er((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

3.100.7 Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="
maxima")
```

output
```
-integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x)
```

3.100.8 Giac [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="
giac")
```

output
```
integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}-1\right)}{\sqrt{a-b\,x^3}}\,dx$$

input
```
int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(a - b*x^3)^(1/2),x)
```

output
```
int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(a - b*x^3)^(1/2), x)
```

3.101 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$

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3.101.1 Optimal result

Integrand size = 38, antiderivative size = 497

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
2*(b*x^3-a)^(1/2)/b^(1/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+4*3^(1/4)*a^(1/
3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3
)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))
/b^(1/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2)
)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(1/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^
(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.101.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.18

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right.\right.}{2\sqrt{-a+bx^3}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/
3, 1/2, 4/3, (b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^
3)/a]))/Sqrt[-a + b*x^3]
```

3.101.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2419}$$

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx-2\sqrt{3}\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{760}$$

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{2418}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

3.101. $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}dx$

output
```
(2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/
((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) + (4*3^(1/4)*Sqr
t[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)]
, -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - S
qrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

3.101.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.101. $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$

3.101.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 951 vs. 2(375) = 750.

Time = 1.73 (sec) , antiderivative size = 952, normalized size of antiderivative = 1.92

method	result	size
default	Expression too large to display	952

input
```
int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/3*I/b^(2/3)*3^(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))
/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)
/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3
))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*a^(1/3)*3^(1/2)/b*(a*b
^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2
)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(
1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1
/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*
(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1
/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b...
```

3.101.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.10

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = -\frac{2\left(a^{\frac{1}{3}}\sqrt{b}\left(\sqrt{3}-1\right)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-b^{\frac{5}{6}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="f
ricas")
```

output
```
-2*(a^(1/3)*sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - b^(5/
6)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

3.101.6 Sympy [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.23

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

output
```
I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*
gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*a**(1/
6)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a
)/(3*a**(1/6)*gamma(4/3))
```

3.101.7 Maxima [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="m
axima")
```

output
```
-integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)
```

3.101.8 Giac [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{b^{1/3}\,x+a^{1/3}\,(\sqrt{3}-1)}{\sqrt{b\,x^3-a}}\,dx$$

input
```
int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(b*x^3 - a)^(1/2),x)
```

output
```
int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(b*x^3 - a)^(1/2), x)
```

3.102 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$

3.102.1 Optimal result 902
3.102.2 Mathematica [C] (verified) 903
3.102.3 Rubi [A] (verified) 903
3.102.4 Maple [B] (verified) 906
3.102.5 Fricas [C] (verification not implemented) 907
3.102.6 Sympy [A] (verification not implemented) 907
3.102.7 Maxima [F] 908
3.102.8 Giac [F] 908
3.102.9 Mupad [F(-1)] 908

3.102.1 Optimal result

Integrand size = 38, antiderivative size = 488

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right.}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2*(-b*x^3-a)^(1/2)/b^(1/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-4*3^(1/4)*a^(1
/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)
*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b
^(1/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(1/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.102.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.19

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2(-1+\sqrt{3})\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\right.\right.}{2\sqrt{-a-bx^3}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1
/2, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x
^3)/a)]))/(2*Sqrt[-a - b*x^3])
```

3.102.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2419}$$

$$\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx-2\sqrt{3}\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^3-a}}dx$$

$$\Big\downarrow \text{760}$$

$$\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{2418}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

3.102. $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}dx$

output
```
(-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(
1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) - (4*3^(1/4)*Sq
rt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

3.102.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.102.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1011 vs. $2(366) = 732$.

Time = 1.74 (sec) , antiderivative size = 1012, normalized size of antiderivative = 2.07

method	result	size
default	Expression too large to display	1012

input
```
int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-...
```

3.102.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = \frac{2\left(a^{\frac{1}{3}}\sqrt{-b}\left(\sqrt{3}-1\right)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\right.\right.}{b}$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="f
ricas")
```

output
```
2*(a^(1/3)*sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + sqrt
(-b)*b^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
)/b
```

3.102.6 Sympy [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.26

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

output
```
-I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*h
yper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

3.102.7 Maxima [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)
```

3.102.8 Giac [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{b^{1/3}\,x - a^{1/3}\,(\sqrt{3}-1)}{\sqrt{-b\,x^3-a}}\,dx$$

input
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(- a - b*x^3)^(1/2),x)
```

output
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(- a - b*x^3)^(1/2), x)
```

3.103 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$

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3.103.1 Optimal result

Integrand size = 32, antiderivative size = 241

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\,E\left(\arcsin\left(\frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*(b/a)^(2/3)*(b*x^3+a)^(1/2)/b/(1+(b/a)^(1/3)*x+3^(1/2))-3^(1/4)*(1+(b/a)
^(1/3)*x)*EllipticE((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2)),I*
3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((1-(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/
(1+(b/a)^(1/3)*x+3^(1/2))^2)^(1/2)/(b/a)^(1/3)/(b*x^3+a)^(1/2)/((1+(b/a)^(
1/3)*x)/(1+(b/a)^(1/3)*x+3^(1/2))^2)^(1/2)
```

3.103.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.37

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

input
```
Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
 -((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2*Sqrt[a + b*x^3])
```

3.103.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{2416}$$

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}\sqrt{a+bx^3}}$$

input
```
Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(2*(b/a)^(2/3)*Sqrt[a + b*x^3])/(b*(1 + Sqrt[3] + (b/a)^(1/3)*x)) - (3^(1/
4)*Sqrt[2 - Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)^(
2/3)*x^2)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] +
 (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1
/3)*Sqrt[(1 + (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

3.103.3.1 Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.103. $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$

3.103.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(196) = 392.

Time = 1.72 (sec) , antiderivative size = 1004, normalized size of antiderivative = 4.17

method	result	size
default	Expression too large to display	1004

input
```
int((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))+2*I/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I
*(b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)...
```

3.103.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.22

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(\sqrt{b}(\sqrt{3}-1)\,\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\right.\right.}{b}$$

input
```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + sqrt(b)*(b/a
)^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

3.103.6 Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.51

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a)**(1/2),x)
```

output
```
x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*
pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/
3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + x*gamma(1/3)*hyper
((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

3.103.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

3.103.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx=\int \frac{x\,{\left(\frac{b}{a}\right)}^{1/3}-\sqrt{3}+1}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(a + b*x^3)^(1/2),x)
```

output
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(a + b*x^3)^(1/2), x)
```

3.104 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$

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3.104.1 Optimal result

Integrand size = 34, antiderivative size = 248

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\,E\left(\arcsin\left(\frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2*(b/a)^(2/3)*(-b*x^3+a)^(1/2)/b/(1-(b/a)^(1/3)*x+3^(1/2))+3^(1/4)*(1-(b/
a)^(1/3)*x)*EllipticE((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2)),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(b/a)^(1/3)*x+(b/a)^(2/3)*x^2
)/(1-(b/a)^(1/3)*x+3^(1/2))^2)^(1/2)/(b/a)^(1/3)/(-b*x^3+a)^(1/2)/((1-(b/a
)^(1/3)*x)/(1-(b/a)^(1/3)*x+3^(1/2))^2)^(1/2)
```

3.104.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.36

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2(-1+\sqrt{3})\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

input
```
Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2,
4/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a
]))/Sqrt[a - b*x^3]
```

3.104.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1}{\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)}$$

input
```
Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
(-2*(b/a)^(2/3)*Sqrt[a - b*x^3])/(b*(1 + Sqrt[3] - (b/a)^(1/3)*x)) + (3^(1
/4)*Sqrt[2 - Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^
(2/3)*x^2)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 - Sqrt[3]
- (b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(
1/3)*Sqrt[(1 - (b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*Sqrt[a - b*
x^3])
```

3.104.3.1 Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.104.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(203) = 406$.

Time = 1.74 (sec) , antiderivative size = 950, normalized size of antiderivative = 3.83

method	result	size
default	Expression too large to display	950

input
```
int((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^
3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3
)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2*I/b*(a*b^
2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1
/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(a*b
^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2
)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+
1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1...
```

3.104.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \frac{2\left(\sqrt{-b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0\right.\right.\right.}{b}$$

input
```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

output
```
2*(sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - sqrt(-b)*(b/a
)^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

3.104.6 Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.52

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+a)**(1/2),x)
```

output
```
-x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2
*I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2),
(4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + x*gamma(1/3)*
hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3
))
```

3.104.7 Maxima [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

output
```
-integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

3.104.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{a-b\,x^3}}\,dx$$

input
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(a - b*x^3)^(1/2),x)
```

output
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(a - b*x^3)^(1/2), x)
```

3.105 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$

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3.105.1 Optimal result

Integrand size = 35, antiderivative size = 549

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
2*(b/a)^(1/3)*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-2/3
*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*
x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*(1-3^(1/2))-a^(1/3)*(b/a)^(
1/3)*(1+3^(1/2)))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(
1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(b*x^
3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^
2)^(1/2)-3^(1/4)*a^(1/3)*(b/a)^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*
((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^
2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1
/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.105.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.16

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2(-1+\sqrt{3})\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

input
```
Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2,
4/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a
]))/Sqrt[-a + b*x^3]
```

3.105.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1}{\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{ 2419}$$

$$\left(-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}-\sqrt{3}+1\right)\int \frac{1}{\sqrt{bx^3-a}}dx+\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}\left(-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}-\sqrt{3}+1\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}}\cdots\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{ 2418}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\left(\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}\right)}{\sqrt[3]{b}}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}-\sqrt{3}+1\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

input
```
Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
((b/a)^(1/3)*((2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^
(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*
a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3)
- b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])))/
b^(1/3) - (2*Sqrt[2 - Sqrt[3]]*(1 - Sqrt[3] - ((1 + Sqrt[3])*a^(1/3)*(b/a)
^(1/3))/b^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 +
 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

3.105.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

3.105. $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.105.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(415) = 830$.

Time = 1.69 (sec) , antiderivative size = 953, normalized size of antiderivative = 1.74

method	result	size
default	Expression too large to display	953

input
```
int((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3
-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)
/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2*I/b*(a*b^2
)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*
b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/
3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2
)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(a*b^2
)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*
b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2
/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3)...
```

3.105.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = -\frac{2\left(\sqrt{b}(\sqrt{3}-1)\,\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\right.\right.\right.}{b}$$

input
```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - sqrt(b)*(b/a)
^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

3.105.6 Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.21

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a)**(1/2),x)
```

output
```
I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt
(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sq
rt(a)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**
3/a)/(3*sqrt(a)*gamma(4/3))
```

3.105.7 Maxima [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

3.105.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{b\,x^3-a}}\,dx$$

input
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(b*x^3 - a)^(1/2),x)
```

output
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(b*x^3 - a)^(1/2), x)
```

3.106 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$

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3.106.1 Optimal result

Integrand size = 35, antiderivative size = 540

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$+\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2*(b/a)^(1/3)*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+2/
3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*(1-3^(1/2))-a^(1/3)*(b/a)^(1
/3)*(1+3^(1/2)))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(-b*x^3
-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)+3^(1/4)*a^(1/3)*(b/a)^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.106.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.17

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$$
$$= \frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2(-1+\sqrt{3})\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

input
```
Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
 -((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2*Sqrt[-a - b*x^3])
```

3.106.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2419}$$

$$\left(-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}-\sqrt{3}+1\right)\int \frac{1}{\sqrt{-bx^3-a}}dx+\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{760}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2-\sqrt{3}}\left(-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}-\sqrt{3}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{2418}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}-\sqrt{3}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\left(\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right.}{\sqrt[3]{b}}$$

input
```
Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
((b/a)^(1/3)*((-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])))
/b^(1/3) + (2*Sqrt[2 - Sqrt[3]]*(1 - Sqrt[3] - ((1 + Sqrt[3])*a^(1/3)*(b/a
)^(1/3))/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
+ 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

3.106.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

3.106. $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.106.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(406) = 812$.

Time = 1.75 (sec) , antiderivative size = 1013, normalized size of antiderivative = 1.88

method	result	size
default	Expression too large to display	1013

input
```
int((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+2*I/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-
b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3
*I*(b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1...
```

3.106.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.10

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \frac{2\left(\sqrt{-b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInvers}\right.\right.}{b}$$

input
```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="fricas"
)
```

output
```
2*(sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + sqrt(-b)*(b/
a)^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

3.106.6 Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.24

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-a)**(1/2),x)
```

output
```
-I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar
(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3
)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3
))
```

3.106.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

3.106.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{x\,\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{-b\,x^3-a}}\,dx$$

input
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(- a - b*x^3)^(1/2),x)
```

output
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(- a - b*x^3)^(1/2), x)
```

3.107 $\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$

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3.107.1 Optimal result

Integrand size = 17, antiderivative size = 490

$$
\begin{aligned}
\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx &= \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} \\
&- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\
&+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

output
```
2*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3
)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b
^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c-a^(1/
3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

3.107.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.15

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x)/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
 + d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[a + b*x^3]
)
```

3.107.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{ 2417}$$

$$\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{d\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

input
```
Int[(c + d*x)/Sqrt[a + b*x^3],x]
```

3.107. $\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$

output
```
(d*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3
^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (2*Sq
rt[2 + Sqrt[3]]*(c - ((1 - Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

3.107.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.107.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.47

method	result
default	$-\frac{2 i c \sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}} \sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}} \sqrt{-\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\; F}{3b\sqrt{b\,x^{3}+a}}$
elliptic	$-\frac{2 i c \sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}} \sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}} \sqrt{-\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\; F}{3b\sqrt{b\,x^{3}+a}}$

input
```
int((d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.107.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.09

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

input
```
integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - sqrt(b)*d*weierstrassZeta
(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

3.107.6 Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.16

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(b*x**3+a)**(1/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

3.107.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(b*x^3 + a), x)
```

3.107.8 Giac [F]

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(b*x^3 + a), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx=\int \frac{c+d\,x}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c + d*x)/(a + b*x^3)^(1/2),x)
```

output
```
int((c + d*x)/(a + b*x^3)^(1/2), x)
```

3.108 $\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx$

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3.108.1 Optimal result

Integrand size = 18, antiderivative size = 503

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \frac{2d\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c+\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
2*d*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1
/3)*d*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(
1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(
2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)-2/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^
(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3
)*c+a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2
/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

3.108.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.15

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \frac{x\sqrt{1-\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

input
```
Integrate[(c + d*x)/Sqrt[a - b*x^3],x]
```

output
```
(x*Sqrt[1 - (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] +
d*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[a - b*x^3])
```

3.108.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx$$

$$\Big\downarrow \text{2417}$$

$$\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\int \frac{1}{\sqrt{a-bx^3}}dx-\frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{2416}$$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\frac{d\left(\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}\right)}{\sqrt[3]{b}}$$

input
```
Int[(c + d*x)/Sqrt[a - b*x^3],x]
```

3.108. $\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx$

output
```
-((d*((-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) +
 (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) +
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
- b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])))/b^(1/3)) - (
2*Sqrt[2 + Sqrt[3]]*(c + ((1 - Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*S
qrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]
*Sqrt[a - b*x^3])
```

3.108.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.108.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.35

method	result
default	$\frac{2ic\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,F\left(\sqrt{3}\,\sqrt{-\frac{i\left(x\cdots\right.}{}}\right.\cdots}{3b\sqrt{-b\,x^{3}+a}}$
elliptic	$\frac{2ic\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,F\left(\sqrt{3}\,\sqrt{-\frac{i\left(x\cdots\right.}{}}\right.\cdots}{3b\sqrt{-b\,x^{3}+a}}$

input
```
int((d*x+c)/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*c*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/
2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1
/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2/3*I*d*3
^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^
(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1
/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^
(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/
(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b
^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

3.108.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.09

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(\sqrt{-b}c\,\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}d\,\mathrm{weierstrassZeta}\left(0,\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

input
```
integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(-b)*c*weierstrassPInverse(0, 4*a/b, x) - sqrt(-b)*d*weierstrassZe
ta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

3.108.6 Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.16

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(-b*x**3+a)**(1/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sq
rt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp
_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

3.108.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \int \frac{dx+c}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```

3.108.8 Giac [F]

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \int \frac{dx+c}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \int \frac{c+d\,x}{\sqrt{a-b\,x^3}}\,dx$$

input
```
int((c + d*x)/(a - b*x^3)^(1/2),x)
```

output
```
int((c + d*x)/(a - b*x^3)^(1/2), x)
```

3.109 $\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx$

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3.109.1 Optimal result

Integrand size = 19, antiderivative size = 515

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = -\frac{2d\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$
$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$
$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}c+\left(1+\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
-2*d*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-2/3*(a^(1/3)
-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)
*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*c+a^(1/3)*d*(1+3^(1/2)))*((a^(2/3)+a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1
/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)
-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*d*(a
^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a
^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2
/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)
```

3.109.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.15

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = \frac{x\sqrt{1-\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

input
```
Integrate[(c + d*x)/Sqrt[-a + b*x^3],x]
```

output
```
(x*Sqrt[1 - (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] +
d*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[-a + b*x^3])
```

3.109.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{bx^3-a}}\,dx$$

$$\Big\downarrow \text{2419}$$

$$\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\int \frac{1}{\sqrt{bx^3-a}}dx-\frac{d\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{760}$$

$$-\frac{d\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{2418}$$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\frac{d\left(\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}\right)}{\sqrt[3]{b}}$$

input
```
Int[(c + d*x)/Sqrt[-a + b*x^3],x]
```

3.109. $\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx$

output
```
-((d*((2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) -
 (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) +
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3)
- b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3
)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])))/b^(1/3))
 - (2*Sqrt[2 - Sqrt[3]]*(c + ((1 + Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) -
 b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3]
)*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3
)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(1/
3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)
*x)^2)]*Sqrt[-a + b*x^3])
```

3.109.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.109.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.33

method	result
default	$\frac{2ic\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,F\left(\sqrt{3}\,\sqrt{-\frac{i\left(x \cdots\right.}{\cdots}}\cdots\right)\cdots}{3b\sqrt{b\,x^{3}-a}}$
elliptic	$\frac{2ic\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,F\left(\sqrt{3}\,\sqrt{-\frac{i\left(x \cdots\right.}{\cdots}}\cdots\right)\cdots}{3b\sqrt{b\,x^{3}-a}}$

input
```
int((d*x+c)/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*c*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/
2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x
^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/
3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2/3*I*d*3^
(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(
1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2
)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1
/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a
*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

3.109.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(\sqrt{b}c\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{b}d\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

input
```
integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(b)*c*weierstrassPInverse(0, 4*a/b, x) - sqrt(b)*d*weierstrassZeta(
0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

3.109.6 Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.14

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = -\frac{icx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(b*x**3-a)**(1/2),x)
```

output
```
-I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*gamma(4/3
)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*ga
mma(5/3))
```

3.109.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

3.109.8 Giac [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = \int \frac{c+d\,x}{\sqrt{b\,x^3-a}}\,dx$$

input
```
int((c + d*x)/(b*x^3 - a)^(1/2),x)
```

output
```
int((c + d*x)/(b*x^3 - a)^(1/2), x)
```

3.110 $\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx$

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3.110.1 Optimal result

Integrand size = 20, antiderivative size = 508

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = -\frac{2d\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}c-\left(1+\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2*d*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+2/3*(a^(1/3)
+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(
1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*c-a^(1/3)*d*(1+3^(1/2)))*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*
6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b
^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*d*(a^(1
/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3
)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-
b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))^2)^(1/2)
```

3.110.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.15

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

input
```
Integrate[(c + d*x)/Sqrt[-a - b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
 + d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[-a - b*x^3
])
```

3.110.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx$$

$$\Big\downarrow \; 2419$$

$$\left(c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-bx^3-a}}dx+\frac{d\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \; 760$$

$$\frac{d\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \; 2418$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\frac{d\left(\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)}{\sqrt[3]{b}}$$

input
```
Int[(c + d*x)/Sqrt[-a - b*x^3],x]
```

3.110. $\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx$

output
```
(d*((-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) +
(3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])))/b^(1/3) +
 (2*Sqrt[2 - Sqrt[3]]*(c - ((1 + Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(1/3)
*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2)]*Sqrt[-a - b*x^3])
```

3.110.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.110.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.43

method	result
default	$-\frac{2ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{-b\,x^{3}-a}}$
elliptic	$-\frac{2ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{-b\,x^{3}-a}}$

input
```
int((d*x+c)/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a
*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.110.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.09

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(\sqrt{-b}c\,\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}d\,\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

input
```
integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(-b)*c*weierstrassPInverse(0, -4*a/b, x) - sqrt(-b)*d*weierstrassZ
eta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

3.110.6 Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.16

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = -\frac{icx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(-b*x**3-a)**(1/2),x)
```

output
```
-I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*s
qrt(a)*gamma(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*
exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

3.110.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \int \frac{dx+c}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

3.110.8 Giac [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \int \frac{dx+c}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \int \frac{c+d\,x}{\sqrt{-b\,x^3-a}}\,dx$$

input
```
int((c + d*x)/(- a - b*x^3)^(1/2),x)
```

output
```
int((c + d*x)/(- a - b*x^3)^(1/2), x)
```

3.111 $\int \frac{c+dx}{\sqrt{1+x^3}}\,dx$

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3.111.1 Optimal result

Integrand size = 15, antiderivative size = 246

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \frac{2d\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2*d*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*d*(1+x)*EllipticE((1+x-3^(1/2))/(1
+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+2/3*(1+x)*Ellipti
cF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(c-d*(1-3^(1/2)))*(1/2*6^(1/
2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((
1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.111.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.17

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = cx\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right) + \frac{1}{2}dx^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

input
```
Integrate[(c + d*x)/Sqrt[1 + x^3],x]
```

output
```
c*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (d*x^2*Hypergeometric2F1[1/2,
 2/3, 5/3, -x^3])/2
```

3.111.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \; 2417$$

$$\left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{x^3+1}}dx+d\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \; 759$$

$$d\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \; 2416$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+d\left(\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)$$

input
```
Int[(c + d*x)/Sqrt[1 + x^3],x]
```

output
```
d*((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x
)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] +
x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]
*Sqrt[1 + x^3])) + (2*Sqrt[2 + Sqrt[3]]*(c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt
[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1
+ Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^
2]*Sqrt[1 + x^3])
```

3.111.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.111.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.64 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.13

method	result
meijerg	$\frac{d\,x^{2}{}_{2}F_{1}\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^{3}\right)}{2}+c x\,{}_{2}F_{1}\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^{3}\right)$
default	$\frac{2c\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}+\frac{2d\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}$
elliptic	$\frac{2c\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}+\frac{2d\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}$

input
```
int((d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*x^2*hypergeom([1/2,2/3],[5/3],-x^3)+c*x*hypergeom([1/3,1/2],[4/3],-x
^3)
```

3.111.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = 2\,c\mathrm{weierstrassPInverse}(0,-4,x) - 2\,d\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2*c*weierstrassPInverse(0, -4, x) - 2*d*weierstrassZeta(0, -4, weierstrass
PInverse(0, -4, x))
```

3.111.6 Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.25

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(x**3+1)**(1/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/
3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3
*gamma(5/3))
```

3.111.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \int \frac{dx+c}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(x^3 + 1), x)
```

3.111.8 Giac [F]

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \int \frac{dx+c}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(x^3 + 1), x)
```

3.111.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.52

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx =$$

$$-\frac{2d\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}\right.}}$$

$$+\frac{2c\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int((c + d*x)/(x^3 + 1)^(1/2),x)
```

output
```
(2*c*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x +
1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*d*(((3^(1/2)*1i)/2 - 1/2)*ellipticF
(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3
^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
))*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2
)
```

3.112 $\int \frac{c+dx}{\sqrt{1-x^3}}\,dx$

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3.112.1 Optimal result

Integrand size = 17, antiderivative size = 271

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \frac{2d\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
2*d*(-x^3+1)^(1/2)/(1-x+3^(1/2))-3^(1/4)*d*(1-x)*EllipticE((1-x-3^(1/2))/(
1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1
/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-2/3*(1-x)*Ellip
ticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(c+d-d*3^(1/2))*(1/2*6^(1/
2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/(
(1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.14

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = cx\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right) + \frac{1}{2}dx^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)$$

input
```
Integrate[(c + d*x)/Sqrt[1 - x^3],x]
```

output
```
c*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + (d*x^2*Hypergeometric2F1[1/2,
2/3, 5/3, x^3])/2
```

3.112.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \ 2417$$

$$\left(c-\sqrt{3}d+d\right)\int \frac{1}{\sqrt{1-x^3}}dx - d\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \ 759$$

$$-d\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \ 2416$$

$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-d\left(\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}\right)$$

input
```
Int[(c + d*x)/Sqrt[1 - x^3],x]
```

output
```
-(d*((-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1
- x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3]
 - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)
^2]*Sqrt[1 - x^3]))) - (2*Sqrt[2 + Sqrt[3]]*(c + d - Sqrt[3]*d)*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x
)^2]*Sqrt[1 - x^3])
```

3.112.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.112.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.11

method	result
meijerg	$\frac{d x^2 {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2}+c x {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)$
default	$-\frac{2ic\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2id\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$
elliptic	$-\frac{2ic\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2id\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$

input
```
int((d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*x^2*hypergeom([1/2,2/3],[5/3],x^3)+c*x*hypergeom([1/3,1/2],[4/3],x^3
)
```

3.112.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = -2i\,c\text{weierstrassPInverse}(0,4,x) + 2i\,d\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*I*c*weierstrassPInverse(0, 4, x) + 2*I*d*weierstrassZeta(0, 4, weierstr
assPInverse(0, 4, x))
```

3.112.6 Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.24

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(-x**3+1)**(1/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(
4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi)
)/(3*gamma(5/3))
```

3.112.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \int \frac{dx+c}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(-x^3 + 1), x)
```

3.112.8 Giac [F]

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \int \frac{dx+c}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(-x^3 + 1), x)
```

3.112.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.50

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx =$$

$$-\frac{2c\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

$$-\frac{2d\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)\cdots}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\cdots\right.}}$$

input
```
int((c + d*x)/(1 - x^3)^(1/2),x)
```

output
```
- (2*c*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)
/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x
 - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) -
 (2*d*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1
i)/2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 -
1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x +
(3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1
/2))
```

3.113 $\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx$

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3.113.6 Sympy [A] (verification not implemented) . . . 985
3.113.7 Maxima [F] . . . 985
3.113.8 Giac [F] . . . 985
3.113.9 Mupad [B] (verification not implemented) . . . 986

3.113.1 Optimal result

Integrand size = 15, antiderivative size = 275

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = -\frac{2d\sqrt{-1+x^3}}{1-\sqrt{3}-x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2-\sqrt{3}}\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2*d*(x^3-1)^(1/2)/(1-x-3^(1/2))-2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^
(1/2)),2*I-I*3^(1/2))*(c+d+d*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)
/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/
2)+3^(1/4)*d*(1-x)*EllipticE((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*((
x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(x^3-1)^(1/2)/((
-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

3.113.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.21

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = \frac{x\sqrt{1-x^3}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{2\sqrt{-1+x^3}}$$

input
```
Integrate[(c + d*x)/Sqrt[-1 + x^3],x]
```

output
```
(x*Sqrt[1 - x^3]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + d*x*Hypergeo
metric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])
```

3.113.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \text{2419}$$

$$\left(c+\sqrt{3}d+d\right)\int \frac{1}{\sqrt{x^3-1}}dx-d\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$-d\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2418}$$

$$-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-d\left(\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}\right)$$

input
```
Int[(c + d*x)/Sqrt[-1 + x^3],x]
```

output
```
-(d*((2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
- x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3]
 - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3]))) - (2*Sqrt[2 - Sqrt[3]]*(c + d + Sqrt[3]*d)*(1 - x
)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[
3] - x)^2)]*Sqrt[-1 + x^3])
```

3.113.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.113.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.62 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.24

method	result
meijerg	$\frac{d\sqrt{-\operatorname{signum}(x^3-1)}\,x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2\sqrt{\operatorname{signum}(x^3-1)}}+\frac{c\sqrt{-\operatorname{signum}(x^3-1)}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}$
default	$\frac{2c\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2d\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{x-}$
elliptic	$\frac{2c\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2d\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{x-}$

input
```
int((d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeom([1/2,2/3],[
5/3],x^3)+c/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/
2],[4/3],x^3)
```

3.113.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = 2\,c\mathrm{weierstrassPInverse}(0,4,x) - 2\,d\mathrm{weierstrassZeta}(0,4,\mathrm{weierstrassPInverse}(0,4,x))$$

input
```
integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2*c*weierstrassPInverse(0, 4, x) - 2*d*weierstrassZeta(0, 4, weierstrassPI
nverse(0, 4, x))
```

3.113.6 Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.20

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx=-\frac{icx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{idx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(x**3-1)**(1/2),x)
```

output
```
-I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - I*d*x**
2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3))
```

3.113.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx=\int \frac{dx+c}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(x^3 - 1), x)
```

3.113.8 Giac [F]

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx=\int \frac{dx+c}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(x^3 - 1), x)
```

3.113.9 Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.36

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = -\frac{2c\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$
$$-\frac{2d\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{F}\left(\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{E}\left(\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}}{}}\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right.}}$$

input
```
int((c + d*x)/(x^3 - 1)^(1/2),x)
```

output
```
- (2*c*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*d*(((3^(1/2)*1i)/2 - 1/2)*elli
pticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/
2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin((-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
x^3)^(1/2)
```

3.114 $\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx$

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3.114.1 Optimal result

Integrand size = 17, antiderivative size = 261

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = -\frac{2d\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2\sqrt{2-\sqrt{3}}\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-2*d*(-x^3-1)^(1/2)/(1+x-3^(1/2))+2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3
^(1/2)),2*I-I*3^(1/2))*(c-d*(1+3^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x
+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)
^(1/2)+3^(1/4)*d*(1+x)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2)
)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(-x^3-1)^(1/
2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

3.114.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.24

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = \frac{x\sqrt{1+x^3}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

input
```
Integrate[(c + d*x)/Sqrt[-1 - x^3],x]
```

output
```
(x*Sqrt[1 + x^3]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + d*x*Hyperge
ometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

3.114.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2419}$$

$$\left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{-x^3-1}}dx+d\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$d\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2418}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+$$
$$d\left(\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}\right)$$

input
```
Int[(c + d*x)/Sqrt[-1 - x^3],x]
```

output
```
d*((-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 +
 x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3]
+ x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x
)^2)]*Sqrt[-1 - x^3])) + (2*Sqrt[2 - Sqrt[3]]*(c - (1 + Sqrt[3])*d)*(1 + x
)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] +
x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[
3] + x)^2)]*Sqrt[-1 - x^3])
```

3.114.3.1 Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.114.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.58 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.14

method	result
meijerg	$-\frac{i d x^2 {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2} - icx\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$
default	$-\frac{2ic\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}} - \frac{2id\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$
elliptic	$-\frac{2ic\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}} - \frac{2id\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$

input
```
int((d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*d*x^2*hypergeom([1/2,2/3],[5/3],-x^3)-I*c*x*hypergeom([1/3,1/2],[4/
3],-x^3)
```

3.114.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = -2i\,c\,\mathrm{weierstrassPInverse}(0,-4,x) + 2i\,d\,\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2*I*c*weierstrassPInverse(0, -4, x) + 2*I*d*weierstrassZeta(0, -4, weiers
trassPInverse(0, -4, x))
```

3.114.6 Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.25

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = -\frac{icx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(-x**3-1)**(1/2),x)
```

output
```
-I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma
(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi
))/(3*gamma(5/3))
```

3.114.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = \int \frac{dx+c}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(-x^3 - 1), x)
```

3.114.8 Giac [F]

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = \int \frac{dx+c}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(-x^3 - 1), x)
```

3.114.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.55

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\,c\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

$$-\frac{2\,d\,\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\right.}}$$

input
```
int((c + d*x)/(- x^3 - 1)^(1/2),x)
```

output
```
(2*c*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((
3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1
/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2
*d*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2
 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1
/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3
/2))^(1/2))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.115 $\int \frac{c+dx}{a-bx^4} dx$

3.115.1 Optimal result 993
3.115.2 Mathematica [A] (verified) 993
3.115.3 Rubi [A] (verified) 994
3.115.4 Maple [C] (verified) 995
3.115.5 Fricas [C] (verification not implemented) 995
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3.115.8 Giac [B] (verification not implemented) 996
3.115.9 Mupad [B] (verification not implemented) 997

3.115.1 Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{c+dx}{a-bx^4} dx = \frac{c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{c\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

output
```
1/2*c*arctan(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(1/4)+1/2*c*arctanh(b^(1/4)*x/a^
(1/4))/a^(3/4)/b^(1/4)+1/2*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

3.115.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.54

$$\int \frac{c+dx}{a-bx^4} dx = \frac{2\sqrt[4]{b}c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \left(\sqrt[4]{b}c+\sqrt[4]{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right) + \sqrt[4]{b}c\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right) - \sqrt[4]{a}d\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{4a^{3/4}\sqrt{b}}$$

input
```
Integrate[(c + d*x)/(a - b*x^4),x]
```

output
```
(2*b^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(1/4)*c + a^(1/4)*d)*Log[a^(
1/4) - b^(1/4)*x] + b^(1/4)*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*d*Log[a^(
1/4) + b^(1/4)*x] + a^(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*a^(3/4)*Sqrt[
b])
```

3.115.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a-bx^4}\,dx$$
$$\Big\downarrow \text{ 2415}$$
$$\int \left(\frac{c}{a-bx^4}+\frac{dx}{a-bx^4}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{c\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

input
```
Int[(c + d*x)/(a - b*x^4),x]
```

output
```
(c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (c*ArcTanh[(b^(1/4)*
x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*S
qrt[a]*Sqrt[b])
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.115.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.39

method	result	size
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	34
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}$	87

input
```
int((d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*sum((_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.115.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 39057, normalized size of antiderivative = 448.93

$$\int \frac{c+dx}{a-bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.115.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{c+dx}{a-bx^4}\,dx =$$
$$-\operatorname{RootSum}\left(256t^4a^3b^2-32t^2a^2bd^2-16tabc^2d+ad^4-bc^4,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3bd^2+16t^2a^2bc^2d\ldots}{4acd\ldots}\right.\right.\right.$$

input
```
integrate((d*x+c)/(-b*x**4+a),x)
```

output
```
-RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a
*d**4 - b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 + 16*_t**2*a
**2*b*c**2*d + 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4
 + b*c**5))))
```

3.115.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(57) = 114$.

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{c+dx}{a-bx^4}\,dx = \frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}} - \frac{d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}} - \frac{c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}$$

input
```
integrate((d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + 1/4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*d*log(sqrt(
b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*c*log((sqrt(b)*x - sqrt(sqrt(a)*
sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b)))
```

3.115.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.61

$$\int \frac{c+dx}{a-bx^4}\,dx = \frac{\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}bd-(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}bd-(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2}$$

input
```
integrate((d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/(a*b) - 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) +
sqrt(-a/b))/(a*b) - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d - (-a*b^3)^(1/4)*b
*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2)
- 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d - (-a*b^3)^(1/4)*b*c)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2)
```

3.115.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.09

$$\int \frac{c+dx}{a-bx^4}\,dx = \begin{cases} \frac{2\,c+3\,d\,x}{6\,b\,x^3} \\ \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,(-b)^{1/4}\,x}{a^{1/4}}-1\right)\left(2\,a^{1/4}\,d+\sqrt{2}\,(-b)^{1/4}\,c\right)}{4\,a^{3/4}\,\sqrt{-b}} - \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,(-b)^{1/4}\,x}{a^{1/4}}+1\right)\left(4\,a^{1/4}\,d-2\,\sqrt{2}\,(-b)^{1/4}\,c\right)}{8\,a^{3/4}\,\sqrt{-b}} + \frac{\sqrt{2}\,c\,\ln\left(\frac{\sqrt{-b}\,x^2+\sqrt{a}+\sqrt{2}\,a^{1/4}\,(\cdots}{\sqrt{-b}\,x^2+\sqrt{a}-\sqrt{2}\,a^{1/4}\,(\cdots}\right.}{8\,a^{3/4}\,(-b)^{1/4}} \end{cases}$$

input
```
int((c + d*x)/(a - b*x^4),x)
```

output

```
piecewise(a == 0, (2*c + 3*d*x)/(6*b*x^3), a ~= 0, (atan((2^(1/2)*(-b)^(1/
4)*x)/a^(1/4) - 1)*(2*a^(1/4)*d + 2^(1/2)*(-b)^(1/4)*c))/(4*a^(3/4)*(-b)^(
1/2)) - (atan((2^(1/2)*(-b)^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*d - 2*2^(1/2)
*(-b)^(1/4)*c))/(8*a^(3/4)*(-b)^(1/2)) + (2^(1/2)*c*log(((-b)^(1/2)*x^2 +
a^(1/2) + 2^(1/2)*a^(1/4)*(-b)^(1/4)*x)/((-b)^(1/2)*x^2 + a^(1/2) - 2^(1/2
)*a^(1/4)*(-b)^(1/4)*x)))/(8*a^(3/4)*(-b)^(1/4)))
```

3.115. $\int \frac{c+dx}{a-bx^4}\,dx$

3.116 $\int \frac{c+dx}{a+bx^4}\,dx$

3.116.1 Optimal result 999
3.116.2 Mathematica [A] (verified) 999
3.116.3 Rubi [A] (verified) 1000
3.116.4 Maple [C] (verified) 1001
3.116.5 Fricas [C] (verification not implemented) 1001
3.116.6 Sympy [A] (verification not implemented) 1002
3.116.7 Maxima [A] (verification not implemented) 1002
3.116.8 Giac [A] (verification not implemented) 1003
3.116.9 Mupad [B] (verification not implemented) 1003

3.116.1 Optimal result

Integrand size = 15, antiderivative size = 219

$$\int \frac{c+dx}{a+bx^4}\,dx=\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

output
```
1/4*c*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)*2^(1/2)+1/4*c*a
rctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)*2^(1/2)-1/8*c*ln(-a^(1/
4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)+1/8*c*ln
(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)+1/
2*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.84

$$\int \frac{c+dx}{a+bx^4}\,dx = \frac{-2\left(\sqrt{2}\sqrt[4]{b}c+2\sqrt[4]{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{b}c-2\sqrt[4]{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\sqrt{2}\sqrt[4]{b}c\left(-\log\right.}{8a^{3/4}\sqrt{b}}$$

input
```
Integrate[(c + d*x)/(a + b*x^4),x]
```

output
```
(-2*(Sqrt[2]*b^(1/4)*c + 2*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)] + 2*(Sqrt[2]*b^(1/4)*c - 2*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)] + Sqrt[2]*b^(1/4)*c*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a
^(3/4)*Sqrt[b])
```

3.116.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+bx^4}\,dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{c}{a+bx^4}+\frac{dx}{a+bx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

input
```
Int[(c + d*x)/(a + b*x^4),x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (c*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)
*b^(1/4))
```

3.116.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.116.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.15

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(\_R d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	32
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}$	124

input
```
int((d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum((_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.116.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 41851, normalized size of antiderivative = 191.10

$$\int \frac{c+dx}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.116.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$\int \frac{c+dx}{a+bx^4}\,dx$$
$$=\mathrm{RootSum}\left(256t^4a^3b^2+32t^2a^2bd^2-16tabc^2d+ad^4+bc^4,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3bd^2-16t^2a^2bc^2d-\right.\right.\right.$$
$$4acd^4-$$

input
```
integrate((d*x+c)/(b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a*
d**4 + b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 - 16*_t**2*a*
*2*b*c**2*d - 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4
- b*c**5))))
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{a+bx^4}\,dx=\frac{\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{1}{4}}}$$
$$+\frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-2\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}$$
$$+\frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c+2\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}$$

input
```
integrate((d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3
/4)*b^(1/4)) - 1/8*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x +
 sqrt(a))/(a^(3/4)*b^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(1/4)*c - 2*sqrt(a)*d
)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*b
^(1/4)*c + 2*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*
b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4))
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{a+bx^4}\,dx = \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}bd-(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}bd-(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2}$$

input
```
integrate((d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(
a*b) - 1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(
a/b))/(a*b) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt
(2)*(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2)
```

3.116.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int \frac{c+dx}{a+bx^4}\,dx = \begin{cases} -\frac{2\,c+3\,d\,x}{6\,b\,x^3} & \text{if } \\ \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{a^{1/4}}-1\right)\left(2\,a^{1/4}\,d+\sqrt{2}\,b^{1/4}\,c\right)}{4\,a^{3/4}\,\sqrt{b}} - \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{a^{1/4}}+1\right)\left(4\,a^{1/4}\,d-2\,\sqrt{2}\,b^{1/4}\,c\right)}{8\,a^{3/4}\,\sqrt{b}} + \frac{\sqrt{2}\,c\,\ln\left(\frac{\sqrt{a}+\sqrt{b}\,x^2+\sqrt{2}\,a^{1/4}\,b^{1/4}\,x}{\sqrt{a}+\sqrt{b}\,x^2-\sqrt{2}\,a^{1/4}\,b^{1/4}\,x}\right)}{8\,a^{3/4}\,b^{1/4}} & \text{if } \end{cases}$$

input
```
int((c + d*x)/(a + b*x^4),x)
```

output
```
piecewise(a == 0, -(2*c + 3*d*x)/(6*b*x^3), a ~= 0, (atan((2^(1/2)*b^(1/4)
*x)/a^(1/4) - 1)*(2*a^(1/4)*d + 2^(1/2)*b^(1/4)*c))/(4*a^(3/4)*b^(1/2)) -
(atan((2^(1/2)*b^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*d - 2*2^(1/2)*b^(1/4)*c)
)/(8*a^(3/4)*b^(1/2)) + (2^(1/2)*c*log((a^(1/2) + b^(1/2)*x^2 + 2^(1/2)*a^
(1/4)*b^(1/4)*x)/(a^(1/2) + b^(1/2)*x^2 - 2^(1/2)*a^(1/4)*b^(1/4)*x)))/(8*
a^(3/4)*b^(1/4)))
```

3.116. $\int \frac{c+dx}{a+bx^4}\,dx$

3.117 $\int \frac{c+dx}{(a-bx^4)^2}\,dx$

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3.117.1 Optimal result

Integrand size = 16, antiderivative size = 110

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx=\frac{x(c+dx)}{4a(a-bx^4)}+\frac{3c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{3c\,\mathrm{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}}+\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

output
```
1/4*x*(d*x+c)/a/(-b*x^4+a)+3/8*c*arctan(b^(1/4)*x/a^(1/4))/a^(7/4)/b^(1/4)
+3/8*c*arctanh(b^(1/4)*x/a^(1/4))/a^(7/4)/b^(1/4)+1/4*d*arctanh(x^2*b^(1/2
)/a^(1/2))/a^(3/2)/b^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.53

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx$$

$$=\frac{\frac{4ax(c+dx)}{a-bx^4}+\frac{6\sqrt[4]{a}c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}-\frac{\left(3\sqrt[4]{a}\sqrt[4]{b}c+2\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{\left(3\sqrt[4]{a}\sqrt[4]{b}c-2\sqrt{a}d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt{b}}+\frac{2\sqrt{a}d\,\mathrm{l}}{}}{16a^2}$$

input
```
Integrate[(c + d*x)/(a - b*x^4)^2,x]
```

output
```
((4*a*x*(c + d*x))/(a - b*x^4) + (6*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])
/b^(1/4) - ((3*a^(1/4)*b^(1/4)*c + 2*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/
Sqrt[b] + ((3*a^(1/4)*b^(1/4)*c - 2*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/S
qrt[b] + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

3.117.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a-bx^4)^2} \, dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(c+dx)}{4a(a-bx^4)}-\frac{\int -\frac{3c+2dx}{a-bx^4}dx}{4a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3c+2dx}{a-bx^4}dx}{4a}+\frac{x(c+dx)}{4a(a-bx^4)}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\int \left(\frac{3c}{a-bx^4}+\frac{2dx}{a-bx^4}\right)dx}{4a}+\frac{x(c+dx)}{4a(a-bx^4)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{3c\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{4a}+\frac{x(c+dx)}{4a(a-bx^4)}$$

input
```
Int[(c + d*x)/(a - b*x^4)^2,x]
```

output
```
(x*(c + d*x))/(4*a*(a - b*x^4)) + ((3*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^
(3/4)*b^(1/4)) + (3*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) +
(d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(4*a)
```

3.117.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.117.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{\frac{d\,x^2}{4a}+\frac{cx}{4a}}{-b\,x^4+a}-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(2\_R d+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$	69
default	$c\left(\frac{x}{4a\left(-b\,x^4+a\right)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{16a^2}\right)+d\left(\frac{x^2}{4a\left(-b\,x^4+a\right)}+\frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)$	128

input
```
int((d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*d/a*x^2+1/4*c/a*x)/(-b*x^4+a)-1/16/b/a*sum((2*_R*d+3*c)/_R^3*ln(x-_R)
,_R=RootOf(_Z^4*b-a))
```

3.117.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 40560, normalized size of antiderivative = 368.73

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.117.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.42

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx$$
$$= \text{RootSum}\left(65536t^4a^7b^2 - 2048t^2a^4bd^2 + 1152ta^2bc^2d + 16ad^4 - 81bc^4, \left(t \mapsto t\log\left(x + \frac{32768t^3a^6bd^2 + \cdots}{\cdots}\right.\right.\right.$$
$$+ \frac{-cx-dx^2}{-4a^2+4abx^4}$$

input
```
integrate((d*x+c)/(-b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*b**2 - 2048*_t**2*a**4*b*d**2 + 1152*_t*a**2*b*c*
*2*d + 16*a*d**4 - 81*b*c**4, Lambda(_t, _t*log(x + (32768*_t**3*a**6*b*d*
*2 + 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 + 1296*_t*a**2*b*c**4 + 3
60*a*c**2*d**3)/(192*a*c*d**4 + 243*b*c**5)))) + (-c*x - d*x**2)/(-4*a**2
+ 4*a*b*x**4)
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx = -\frac{dx^2+cx}{4\,(abx^4-a^2)} + \frac{\frac{6\,c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{3\,c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{16\,a}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(d*x^2 + c*x)/(a*b*x^4 - a^2) + 1/16*(6*c*arctan(sqrt(b)*x/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*d*log(sqrt(b)*x^2 + sqrt(
a))/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) -
 3*c*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqr
t(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))))/a
```

3.117.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(79) = 158$.

Time = 0.27 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.31

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx = \frac{3\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b} - \frac{3\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b} - \frac{dx^2+cx}{4\,(bx^4-a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}bd+3\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}bd+3\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

output
```
3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b
))/(a^2*b) - 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4
) + sqrt(-a/b))/(a^2*b) - 1/4*(d*x^2 + c*x)/((b*x^4 - a)*a) + 1/16*sqrt(2)
*(2*sqrt(2)*sqrt(-a*b)*b*d + 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(2*sqrt(2)
*sqrt(-a*b)*b*d + 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^2)
```

3.117.9 Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.57

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(-\frac{b^2\left(3\,c\,d^2+2\,d^3\,x+\mathrm{root}(65536\,a^7\,b^2\,z^4-2048\,a^4\,b\,d^2\,z^2+1152\,a^2\,b\,c^2\,d\,z-81\,b\,c^4+16\,a\,d^4,\cdots}{\cdots}\right.\right.$$

$$\left.\left.-2048\,a^4\,b\,d^2\,z^2+1152\,a^2\,b\,c^2\,d\,z-81\,b\,c^4+16\,a\,d^4,z,k)\right)\right)+\frac{\frac{d\,x^2}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^4}$$

input
```
int((c + d*x)/(a - b*x^4)^2,x)
```

output
```
symsum(log(-(b^2*(3*c*d^2 + 2*d^3*x + 192*root(65536*a^7*b^2*z^4 - 2048*a^
4*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c^4 + 16*a*d^4, z, k)^2*a^3*b*c -
128*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*
b*c^4 + 16*a*d^4, z, k)^2*a^3*b*d*x + 36*root(65536*a^7*b^2*z^4 - 2048*a^4
*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c^4 + 16*a*d^4, z, k)*a*b*c^2*x))/(
16*a^3))*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d^2*z^2 + 1152*a^2*b*c^2*d*z
- 81*b*c^4 + 16*a*d^4, z, k), k, 1, 4) + ((d*x^2)/(4*a) + (c*x)/(4*a))/(a
- b*x^4)
```

3.118 $\int \frac{c+dx}{(a+bx^4)^2}\,dx$

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3.118.1 Optimal result

Integrand size = 15, antiderivative size = 241

$$\begin{aligned}\int \frac{c+dx}{(a+bx^4)^2}\,dx &= \frac{x(c+dx)}{4a(a+bx^4)}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}-\frac{3c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}\\
&\quad+\frac{3c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}\\
&\quad+\frac{3c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{b}}\end{aligned}$$

output
```
1/4*x*(d*x+c)/a/(b*x^4+a)+3/16*c*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7
/4)/b^(1/4)*2^(1/2)+3/16*c*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(
1/4)*2^(1/2)-3/32*c*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(
7/4)/b^(1/4)*2^(1/2)+3/32*c*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(7/4)/b^(1/4)*2^(1/2)+1/4*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(1
/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.93

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx$$
$$=\frac{\frac{8a^{3/4}x(c+dx)}{a+bx^4}-\frac{2\left(3\sqrt{2}\sqrt[4]{b}c+4\sqrt[4]{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}}+\frac{2\left(3\sqrt{2}\sqrt[4]{b}c-4\sqrt[4]{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}}-\frac{3\sqrt{2}c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{\sqrt[4]{b}}}{32a^{7/4}}$$

input
```
Integrate[(c + d*x)/(a + b*x^4)^2,x]
```

output
```
((8*a^(3/4)*x*(c + d*x))/(a + b*x^4) - (2*(3*Sqrt[2]*b^(1/4)*c + 4*a^(1/4)
*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] + (2*(3*Sqrt[2]*b^(1/
4)*c - 4*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (3*
Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4)
+ (3*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(
1/4))/(32*a^(7/4))
```

3.118.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx$$
$$\Big\downarrow \text{2394}$$
$$\frac{x(c+dx)}{4a(a+bx^4)}-\frac{\int -\frac{3c+2dx}{bx^4+a}dx}{4a}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{3c+2dx}{bx^4+a}dx}{4a}+\frac{x(c+dx)}{4a(a+bx^4)}$$
$$\Big\downarrow \text{2415}$$

$$\frac{\int \left(\frac{3c}{bx^4+a}+\frac{2dx}{bx^4+a}\right)dx}{4a}+\frac{x(c+dx)}{4a\left(a+bx^4\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{3c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{3c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{3c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{3c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{d\,\text{arct}}{}}{4a}$$

$$\frac{x(c+dx)}{4a\left(a+bx^4\right)}$$

input
```
Int[(c + d*x)/(a + b*x^4)^2,x]
```

output
```
(x*(c + d*x))/(4*a*(a + b*x^4)) + ((d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt
[a]*Sqrt[b]) - (3*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(3/4)*b^(1/4)) + (3*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*b^(1/4)) - (3*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (3*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)))/(4*a)
```

3.118.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.118.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.27

method	result
risch	$\frac{\frac{d x^2}{4a}+\frac{cx}{4a}}{b x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b+a)}\frac{(2\_Rd+3c)\ln(x-\_R)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(b x^4+a)}+\frac{3(\frac{a}{b})^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+(\frac{a}{b})^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-(\frac{a}{b})^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(\frac{a}{b})^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(\frac{a}{b})^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+d\left(\frac{x^2}{4a(b x^4+a)}\right.$

input
```
int((d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*d/a*x^2+1/4*c/a*x)/(b*x^4+a)+1/16/b/a*sum((2*_R*d+3*c)/_R^3*ln(x-_R),
_R=RootOf(_Z^4*b+a))
```

3.118.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 43065, normalized size of antiderivative = 178.69

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.118.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.64

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx$$
$$= \text{RootSum}\left(65536t^4a^7b^2 + 2048t^2a^4bd^2 - 1152ta^2bc^2d + 16ad^4 + 81bc^4, \left(t \mapsto t\log\left(x + \frac{-32768t^3a^6bd^2 -\right.\right.\right.$$
$$+ \frac{cx+dx^2}{4a^2+4abx^4}$$

input
```
integrate((d*x+c)/(b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*b**2 + 2048*_t**2*a**4*b*d**2 - 1152*_t*a**2*b*c*
*2*d + 16*a*d**4 + 81*b*c**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*b*d
**2 - 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 - 1296*_t*a**2*b*c**4 +
360*a*c**2*d**3)/(192*a*c*d**4 - 243*b*c**5)))) + (c*x + d*x**2)/(4*a**2 +
 4*a*b*x**4)
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx = \frac{dx^2+cx}{4(abx^4+a^2)}$$
$$+ \frac{\frac{3\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{3\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-4\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}} + \frac{2}{}}{32a}$$

input
```
integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(d*x^2 + c*x)/(a*b*x^4 + a^2) + 1/32*(3*sqrt(2)*c*log(sqrt(b)*x^2 + sq
rt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 3*sqrt(2)*c*log(sqr
t(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(3*s
qrt(2)*a^(1/4)*b^(1/4)*c - 4*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x +
sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt
(b))*b^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(1/4)*c + 4*sqrt(a)*d)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)))/a
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx = \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b} - \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b} + \frac{dx^2+cx}{4\,(bx^4+a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}bd+3\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}bd+3\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2}$$

input
```
integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
(a^2*b) - 3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(a^2*b) + 1/4*(d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sq
rt(2)*sqrt(a*b)*b*d + 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)
*b*d + 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))
/(a/b)^(1/4))/(a^2*b^2)
```

3.118.9 Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.17

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx$$

$$= \left(\sum_{k=1}^{4}\ln\left(\frac{b^2\left(3\,c\,d^2+2\,d^3\,x-\mathrm{root}(65536\,a^7\,b^2\,z^4+2048\,a^4\,b\,d^2\,z^2-1152\,a^2\,b\,c^2\,d\,z+81\,b\,c^4+16\,a\,d^4,z,\right.}{}\right.\right.$$

$$\left.\left.+2048\,a^4\,b\,d^2\,z^2-1152\,a^2\,b\,c^2\,d\,z+81\,b\,c^4+16\,a\,d^4,z,k)\right)\right) + \frac{\frac{d\,x^2}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a}$$

input
```
int((c + d*x)/(a + b*x^4)^2,x)
```

output
```
symsum(log((b^2*(3*c*d^2 + 2*d^3*x - 192*root(65536*a^7*b^2*z^4 + 2048*a^4
*b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^4 + 16*a*d^4, z, k)^2*a^3*b*c + 1
28*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b
*c^4 + 16*a*d^4, z, k)^2*a^3*b*d*x - 36*root(65536*a^7*b^2*z^4 + 2048*a^4*
b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^4 + 16*a*d^4, z, k)*a*b*c^2*x))/(1
6*a^3))*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^2*z^2 - 1152*a^2*b*c^2*d*z +
 81*b*c^4 + 16*a*d^4, z, k), k, 1, 4) + ((d*x^2)/(4*a) + (c*x)/(4*a))/(a +
 b*x^4)
```

3.118. $\int \frac{c+dx}{(a+bx^4)^2} \, dx$

3.119 $\int \frac{c+dx}{(a-bx^4)^3}\,dx$

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3.119.8 Giac [B] (verification not implemented) 1023
3.119.9 Mupad [B] (verification not implemented) 1024

3.119.1 Optimal result

Integrand size = 16, antiderivative size = 136

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = \frac{x(c+dx)}{8a\,(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2\,(a-bx^4)} + \frac{21c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{21c\,\mathrm{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{3d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

output
```
1/8*x*(d*x+c)/a/(-b*x^4+a)^2+1/32*x*(6*d*x+7*c)/a^2/(-b*x^4+a)+21/64*c*arc
tan(b^(1/4)*x/a^(1/4))/a^(11/4)/b^(1/4)+21/64*c*arctanh(b^(1/4)*x/a^(1/4))
/a^(11/4)/b^(1/4)+3/16*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.42

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx$$
$$= \frac{\frac{16a^2x(c+dx)}{(a-bx^4)^2} + \frac{4ax(7c+6dx)}{a-bx^4} + \frac{42\sqrt[4]{a}c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} - \frac{3\left(7\sqrt[4]{a}\sqrt[4]{b}c+4\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{3\left(7\sqrt[4]{a}\sqrt[4]{b}c-4\sqrt{a}d\right)\log\left(\sqrt[4]{\cdots}\right.}{\sqrt{b}}}{128a^3}$$

input
```
Integrate[(c + d*x)/(a - b*x^4)^3,x]
```

output
```
((16*a^2*x*(c + d*x))/(a - b*x^4)^2 + (4*a*x*(7*c + 6*d*x))/(a - b*x^4) +
(42*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(1/4) - (3*(7*a^(1/4)*b^(1/4)
*c + 4*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sqrt[b] + (3*(7*a^(1/4)*b^(1/4
)*c - 4*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt[b] + (12*Sqrt[a]*d*Log[S
qrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

3.119.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}-\frac{\int -\frac{7c+6dx}{(a-bx^4)^2}dx}{8a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7c+6dx}{(a-bx^4)^2}dx}{8a}+\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(7c+6dx)}{4a(a-bx^4)}-\frac{\int -\frac{3(7c+4dx)}{a-bx^4}dx}{4a}}{8a}+\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{7c+4dx}{a-bx^4}dx}{4a}+\frac{x(7c+6dx)}{4a(a-bx^4)}}{8a}+\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{3\int\left(\frac{7c}{a-bx^4}+\frac{4dx}{a-bx^4}\right)dx}{4a}+\frac{x(7c+6dx)}{4a(a-bx^4)}}{8a}+\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3\left(\frac{7c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{7c\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{2d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\right)}{4a}+\frac{x(7c+6dx)}{4a(a-bx^4)}}{8a}+\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}$$

input
```
Int[(c + d*x)/(a - b*x^4)^3,x]
```

output
```
(x*(c + d*x))/(8*a*(a - b*x^4)^2) + ((x*(7*c + 6*d*x))/(4*a*(a - b*x^4)) +
 (3*((7*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (7*c*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (2*d*ArcTanh[(Sqrt[b]*x^2)/Sqr
t[a]])/(Sqrt[a]*Sqrt[b])))/(4*a))/(8*a)
```

3.119.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.119.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

method	result
risch	$\frac{-\frac{3bd\,x^{6}}{16a^{2}}-\frac{7bc\,x^{5}}{32a^{2}}+\frac{5d\,x^{2}}{16a}+\frac{11cx}{32a}}{(-b\,x^{4}+a)^{2}}-\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(4\_Rd+7c\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{128a^{2}b}$
default	$c\left(\frac{x}{8a(-b\,x^{4}+a)^{2}}+\frac{\frac{7x}{32a\left(-b\,x^{4}+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{128a^{2}}}{a}\right)+d\left(\frac{x^{2}}{8a(-b\,x^{4}+a)^{2}}+\frac{\frac{3x^{2}}{16a\left(-b\,x^{4}+a\right)}+\cdots}{a}\right.$

input
```
int((d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/16*b*d/a^2*x^6-7/32*b*c/a^2*x^5+5/16*d/a*x^2+11/32*c/a*x)/(-b*x^4+a)^2
-3/128/a^2/b*sum((4*_R*d+7*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.119.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 40637, normalized size of antiderivative = 298.80

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.119.6 Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx =$$
$$-\operatorname{RootSum}\left(268435456t^4a^{11}b^2 - 4718592t^2a^6bd^2 - 2709504ta^3bc^2d + 20736ad^4 - 194481bc^4, \left(t \mapsto t\log\right.\right.$$
$$-\frac{-11acx - 10adx^2 + 7bcx^5 + 6bdx^6}{32a^4 - 64a^3bx^4 + 32a^2b^2x^8}$$

input
```
integrate((d*x+c)/(-b*x**4+a)**3,x)
```

output
```
-RootSum(268435456*_t**4*a**11*b**2 - 4718592*_t**2*a**6*b*d**2 - 2709504*
_t*a**3*b*c**2*d + 20736*a*d**4 - 194481*b*c**4, Lambda(_t, _t*log(x + (-6
7108864*_t**3*a**9*b*d**2 + 9633792*_t**2*a**6*b*c**2*d + 589824*_t*a**4*d
**4 - 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 + 4537
89*b*c**5)))) - (-11*a*c*x - 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a*
*4 - 64*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.37

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = -\frac{6\,bdx^6 + 7\,bcx^5 - 10\,adx^2 - 11\,acx}{32\,(a^2b^2x^8 - 2\,a^3bx^4 + a^4)} + \frac{3\left(\frac{14\,c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{4\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{4\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{7\,c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\right)}{128\,a^2}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/(a^2*b^2*x^8 - 2*a^3
*b*x^4 + a^4) + 3/128*(14*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))) + 4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b
)) - 4*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 7*c*log((sqrt(b)*x
 - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sq
rt(sqrt(a)*sqrt(b))))/a^2
```

3.119.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(104) = 208$.

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.00

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = \frac{21\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^3b} - \frac{21\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^3b} - \frac{3\,\sqrt{2}\left(4\,\sqrt{2}\sqrt{-ab}bd-7\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} - \frac{3\,\sqrt{2}\left(4\,\sqrt{2}\sqrt{-ab}bd-7\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} - \frac{6\,bdx^6+7\,bcx^5-10\,adx^2-11\,acx}{32\,(bx^4-a)^2a^2}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

output
```
21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a
/b))/(a^3*b) - 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^
(1/4) + sqrt(-a/b))/(a^3*b) - 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d - 7*
(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)
^(1/4))/(a^3*b^2) - 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d - 7*(-a*b^3)^(
1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a
^3*b^2) - 1/32*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a
)^2*a^2)
```

3.119.9 Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.32

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = \frac{\frac{5\,d\,x^2}{16\,a}+\frac{11\,c\,x}{32\,a}-\frac{7\,b\,c\,x^5}{32\,a^2}-\frac{3\,b\,d\,x^6}{16\,a^2}}{a^2-2\,a\,b\,x^4+b^2\,x^8}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\frac{b^2\left(63\,c\,d^2+36\,d^3\,x+\mathrm{root}(268435456\,a^{11}\,b^2\,z^4-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-\right.}{}\right.\right.$$
$$\left.-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-194481\,b\,c^4+20736\,a\,d^4,z,k)\right)$$

input
```
int((c + d*x)/(a - b*x^4)^3,x)
```

output
```
((5*d*x^2)/(16*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a^2) - (3*b*d*x^6)/(
16*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(-(3*b^2*(63*c*d^2 + 36*d
^3*x + 7168*root(268435456*a^11*b^2*z^4 - 4718592*a^6*b*d^2*z^2 + 2709504*
a^3*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4, z, k)^2*a^5*b*c + 1176*root(26
8435456*a^11*b^2*z^4 - 4718592*a^6*b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194
481*b*c^4 + 20736*a*d^4, z, k)*a^2*b*c^2*x - 4096*root(268435456*a^11*b^2*
z^4 - 4718592*a^6*b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b*c^4 + 20736
*a*d^4, z, k)^2*a^5*b*d*x))/(2048*a^6))*root(268435456*a^11*b^2*z^4 - 4718
592*a^6*b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4, z,
 k), k, 1, 4)
```

3.120 $\int \frac{c+dx}{(a+bx^4)^3} dx$

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3.120.1 Optimal result

Integrand size = 15, antiderivative size = 266

$$\begin{aligned}\int \frac{c+dx}{(a+bx^4)^3} dx = {} & \frac{x(c+dx)}{8a(a+bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a+bx^4)} + \frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} \\ & - \frac{21c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} \\ & - \frac{21c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}} \\ & + \frac{21c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{b}}\end{aligned}$$

output
```
1/8*x*(d*x+c)/a/(b*x^4+a)^2+1/32*x*(6*d*x+7*c)/a^2/(b*x^4+a)+21/128*c*arct
an(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/b^(1/4)*2^(1/2)+21/128*c*arctan(
1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/b^(1/4)*2^(1/2)-21/256*c*ln(-a^(1/4)
*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)+21/256*c*
ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)
+3/16*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.94

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx$$
$$=\frac{\frac{32a^{7/4}x(c+dx)}{(a+bx^4)^2}+\frac{8a^{3/4}x(7c+6dx)}{a+bx^4}-\frac{6\left(7\sqrt{2}\sqrt[4]{b}c+8\sqrt[4]{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}}+\frac{6\left(7\sqrt{2}\sqrt[4]{b}c-8\sqrt[4]{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}}-\,2}{256a^{11/4}}$$

input
```
Integrate[(c + d*x)/(a + b*x^4)^3,x]
```

output
```
((32*a^(7/4)*x*(c + d*x))/(a + b*x^4)^2 + (8*a^(3/4)*x*(7*c + 6*d*x))/(a +
 b*x^4) - (6*(7*Sqrt[2]*b^(1/4)*c + 8*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/Sqrt[b] + (6*(7*Sqrt[2]*b^(1/4)*c - 8*a^(1/4)*d)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (21*Sqrt[2]*c*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (21*Sqrt[2]*c*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(256*a^(11/4))
```

3.120.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx$$
$$\Big\downarrow \; 2394$$
$$\frac{x(c+dx)}{8a(a+bx^4)^2}-\frac{\int -\frac{7c+6dx}{(bx^4+a)^2}dx}{8a}$$
$$\Big\downarrow \; 25$$
$$\frac{\int \frac{7c+6dx}{(bx^4+a)^2}dx}{8a}+\frac{x(c+dx)}{8a(a+bx^4)^2}$$
$$\Big\downarrow \; 2394$$

$$\frac{\frac{x(7c+6dx)}{4a(a+bx^4)}-\frac{\int -\frac{3(7c+4dx)}{bx^4+a}dx}{4a}}{8a}+\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{7c+4dx}{bx^4+a}dx}{4a}+\frac{x(7c+6dx)}{4a(a+bx^4)}}{8a}+\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{3\int \left(\frac{7c}{bx^4+a}+\frac{4dx}{bx^4+a}\right)dx}{4a}+\frac{x(7c+6dx)}{4a(a+bx^4)}}{8a}+\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{3\left(-\frac{7c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{7c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{7c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{7c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{2d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\right)}{4a}+\cdots}{8a}$$

$$\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

input
```
Int[(c + d*x)/(a + b*x^4)^3,x]
```

output
```
(x*(c + d*x))/(8*a*(a + b*x^4)^2) + ((x*(7*c + 6*d*x))/(4*a*(a + b*x^4)) +
 (3*((2*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - (7*c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (7*c*ArcTan
[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (7*c*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(
1/4)) + (7*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(1/4))))/(4*a))/(8*a)
```

3.120. $\int \frac{c+dx}{(a+bx^4)^3}\,dx$

3.120.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.120.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\frac{3bd\,x^{6}}{16a^{2}}+\frac{7bc\,x^{5}}{32a^{2}}+\frac{5d\,x^{2}}{16a}+\frac{11cx}{32a}}{(b\,x^{4}+a)^{2}}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^{4}b+a)}\frac{(4\_Rd+7c)\ln(x-\_R)}{\_R^{3}}\right)}{128a^{2}b}$
default	$c\left(\frac{x}{8a(b\,x^{4}+a)^{2}}+\frac{\frac{7x}{32a(b\,x^{4}+a)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^{2}}}{a}\right)+d\left(\frac{\cdots}{8a\cdots}\right.$

input
```
int((d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/16*b*d/a^2*x^6+7/32*b*c/a^2*x^5+5/16*d/a*x^2+11/32*c/a*x)/(b*x^4+a)^2+3
/128/a^2/b*sum((4*_R*d+7*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.120.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 43180, normalized size of antiderivative = 162.33

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.120.6 Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.72

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx$$
$$= \operatorname{RootSum}\left(268435456t^4a^{11}b^2 + 4718592t^2a^6bd^2 - 2709504ta^3bc^2d + 20736ad^4 + 194481bc^4, \left(t \mapsto t\log\left(\right.\right.\right.$$
$$+ \frac{11acx + 10adx^2 + 7bcx^5 + 6bdx^6}{32a^4 + 64a^3bx^4 + 32a^2b^2x^8}$$

input
```
integrate((d*x+c)/(b*x**4+a)**3,x)
```

output
```
RootSum(268435456*_t**4*a**11*b**2 + 4718592*_t**2*a**6*b*d**2 - 2709504*_
t*a**3*b*c**2*d + 20736*a*d**4 + 194481*b*c**4, Lambda(_t, _t*log(x + (-67
108864*_t**3*a**9*b*d**2 - 9633792*_t**2*a**6*b*c**2*d - 589824*_t*a**4*d*
*4 - 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 - 45378
9*b*c**5)))) + (11*a*c*x + 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4
 + 64*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \frac{6\,bdx^6+7\,bcx^5+10\,adx^2+11\,acx}{32\,(a^2b^2x^8+2\,a^3bx^4+a^4)} + \frac{3\left(\frac{7\,\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{7\,\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{2\left(7\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-8\,\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}\right.}{256\,a^2}$$

input
```
integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(6*b*d*x^6 + 7*b*c*x^5 + 10*a*d*x^2 + 11*a*c*x)/(a^2*b^2*x^8 + 2*a^3*
b*x^4 + a^4) + 3/256*(7*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4
)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 7*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(7*sqrt(2)*a^(1/4)*b^(1/
4)*c - 8*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 2*(7
*sqrt(2)*a^(1/4)*b^(1/4)*c + 8*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x
- sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sq
rt(b))*b^(1/4)))/a^2
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.96

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \frac{21\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b} - \frac{21\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ab}bd+7(ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ab}bd+7(ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} + \frac{6\,bdx^6+7\,bcx^5+10\,adx^2+11\,acx}{32\,(bx^4+a)^2a^2}$$

input
```
integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

output
```
21/256*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(a^3*b) - 21/256*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4)
 + sqrt(a/b))/(a^3*b) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b*d + 7*(a*b^3)
^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^3*b^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b*d + 7*(a*b^3)^(1/4)*b*c)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^2) + 1/32
*(6*b*d*x^6 + 7*b*c*x^5 + 10*a*d*x^2 + 11*a*c*x)/((b*x^4 + a)^2*a^2)
```

3.120.9 Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.18

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \frac{\frac{5\,d\,x^2}{16\,a}+\frac{11\,c\,x}{32\,a}+\frac{7\,b\,c\,x^5}{32\,a^2}+\frac{3\,b\,d\,x^6}{16\,a^2}}{a^2+2\,a\,b\,x^4+b^2\,x^8} + \left(\sum_{k=1}^{4}\ln\left(\frac{b^2\left(63\,c\,d^2+36\,d^3\,x-\mathrm{root}(268435456\,a^{11}\,b^2\,z^4+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+1\right.}{}\right.\right.$$

$$\left.+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+194481\,b\,c^4+20736\,a\,d^4,z,k)\right)$$

input
```
int((c + d*x)/(a + b*x^4)^3,x)
```

output
```
((5*d*x^2)/(16*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a^2) + (3*b*d*x^6)/(
16*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log((3*b^2*(63*c*d^2 + 36*d^
3*x - 7168*root(268435456*a^11*b^2*z^4 + 4718592*a^6*b*d^2*z^2 - 2709504*a
^3*b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4, z, k)^2*a^5*b*c - 1176*root(268
435456*a^11*b^2*z^4 + 4718592*a^6*b*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 1944
81*b*c^4 + 20736*a*d^4, z, k)*a^2*b*c^2*x + 4096*root(268435456*a^11*b^2*z
^4 + 4718592*a^6*b*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*c^4 + 20736*
a*d^4, z, k)^2*a^5*b*d*x))/(2048*a^6))*root(268435456*a^11*b^2*z^4 + 47185
92*a^6*b*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4, z,
k), k, 1, 4)
```

3.120. $\int \frac{c+dx}{(a+bx^4)^3}\,dx$

3.121 $\int \frac{c+dx}{(a-bx^4)^4} dx$

3.121.1 Optimal result 1033
3.121.2 Mathematica [A] (verified) 1033
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3.121.1 Optimal result

Integrand size = 16, antiderivative size = 162

$$\int \frac{c+dx}{(a-bx^4)^4} dx = \frac{x(c+dx)}{12a(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a-bx^4)} + \frac{77c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{77c\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{5d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

output
```
1/12*x*(d*x+c)/a/(-b*x^4+a)^3+1/96*x*(10*d*x+11*c)/a^2/(-b*x^4+a)^2+1/384*
x*(60*d*x+77*c)/a^3/(-b*x^4+a)+77/256*c*arctan(b^(1/4)*x/a^(1/4))/a^(15/4)
/b^(1/4)+77/256*c*arctanh(b^(1/4)*x/a^(1/4))/a^(15/4)/b^(1/4)+5/32*d*arcta
nh(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.34

$$\int \frac{c+dx}{(a-bx^4)^4} dx$$

$$= \frac{\frac{128a^3x(c+dx)}{(a-bx^4)^3} + \frac{16a^2x(11c+10dx)}{(a-bx^4)^2} + \frac{4ax(77c+60dx)}{a-bx^4} + \frac{462\sqrt[4]{a}c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} - \frac{3\left(77\sqrt[4]{a}\sqrt[4]{b}c+40\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{3}{}}{1536a^4}$$

input
```
Integrate[(c + d*x)/(a - b*x^4)^4,x]
```

output
```
((128*a^3*x*(c + d*x))/(a - b*x^4)^3 + (16*a^2*x*(11*c + 10*d*x))/(a - b*x
^4)^2 + (4*a*x*(77*c + 60*d*x))/(a - b*x^4) + (462*a^(1/4)*c*ArcTan[(b^(1/
4)*x)/a^(1/4)])/b^(1/4) - (3*(77*a^(1/4)*b^(1/4)*c + 40*Sqrt[a]*d)*Log[a^(
1/4) - b^(1/4)*x])/Sqrt[b] + (3*(77*a^(1/4)*b^(1/4)*c - 40*Sqrt[a]*d)*Log[
a^(1/4) + b^(1/4)*x])/Sqrt[b] + (120*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])
/Sqrt[b])/(1536*a^4)
```

3.121.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2394, 25, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a-bx^4)^4} \, dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(c+dx)}{12a(a-bx^4)^3}-\frac{\int -\frac{11c+10dx}{(a-bx^4)^3}dx}{12a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{11c+10dx}{(a-bx^4)^3}dx}{12a}+\frac{x(c+dx)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(11c+10dx)}{8a(a-bx^4)^2}-\frac{\int -\frac{77c+60dx}{(a-bx^4)^2}dx}{8a}}{12a}+\frac{x(c+dx)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{77c+60dx}{(a-bx^4)^2}dx}{8a}+\frac{x(11c+10dx)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x(77c+60dx)}{4a(a-bx^4)}-\frac{\int -\frac{3(77c+40dx)}{a-bx^4}dx}{4a}}{8a}+\frac{x(11c+10dx)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\int \frac{77c+40dx}{a-bx^4}dx}{4a}+\frac{x(77c+60dx)}{4a(a-bx^4)}}{8a}+\frac{x(11c+10dx)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{\frac{3\int \left(\frac{77c}{a-bx^4}+\frac{40dx}{a-bx^4}\right)dx}{4a}+\frac{x(77c+60dx)}{4a(a-bx^4)}}{8a}+\frac{x(11c+10dx)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{3\left(\frac{77c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{77c\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{20d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\right)}{4a}+\frac{x(77c+60dx)}{4a(a-bx^4)}}{8a}+\frac{x(11c+10dx)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx)}{12a(a-bx^4)^3}$$

input
```
Int[(c + d*x)/(a - b*x^4)^4,x]
```

output
```
(x*(c + d*x))/(12*a*(a - b*x^4)^3) + ((x*(11*c + 10*d*x))/(8*a*(a - b*x^4)
^2) + ((x*(77*c + 60*d*x))/(4*a*(a - b*x^4)) + (3*((77*c*ArcTan[(b^(1/4)*x
)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (77*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a
^(3/4)*b^(1/4)) + (20*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))
)/(4*a))/(8*a))/(12*a)
```

3.121.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.121.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{\frac{5 d b^{2} x^{10}}{32 a^{3}}+\frac{77 c b^{2} x^{9}}{384 a^{3}}-\frac{5 b d x^{6}}{12 a^{2}}-\frac{33 b c x^{5}}{64 a^{2}}+\frac{11 d x^{2}}{32 a}+\frac{51 c x}{128 a}}{\left(-b x^{4}+a\right)^{3}}-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4} b-a\right)} \frac{\left(40 \_R d+77 c\right) \ln\left(x-\_R\right)}{\_R^{3}}}{512 a^{3} b}$	113
default	$\frac{\frac{5 d b^{2} x^{10}}{32 a^{3}}+\frac{77 c b^{2} x^{9}}{384 a^{3}}-\frac{5 b d x^{6}}{12 a^{2}}-\frac{33 b c x^{5}}{64 a^{2}}+\frac{11 d x^{2}}{32 a}+\frac{51 c x}{128 a}}{\left(-b x^{4}+a\right)^{3}}+\frac{\frac{77 c \left(\frac{a}{b}\right)^{\frac{1}{4}} \left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2 \arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4 a}+\frac{10 d \ln\left(\frac{a+x^{2} \sqrt{a b}}{a-x^{2} \sqrt{a b}}\right)}{\sqrt{a b}}}{128 a^{3}}$	165

input
```
int((d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9-5/12*b*d/a^2*x^6-33/64*b*c/a^2*x
^5+11/32*d/a*x^2+51/128*c/a*x)/(-b*x^4+a)^3-1/512/a^3/b*sum((40*_R*d+77*c)
/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.121.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 40780, normalized size of antiderivative = 251.73

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.121.6 Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx$$
$$= \text{RootSum}\left(68719476736t^4a^{15}b^2 - 838860800t^2a^8bd^2 + 485703680ta^4bc^2d + 2560000ad^4 - 35153041bc^4, \left(\right.\right.$$
$$+ \frac{-153a^2cx - 132a^2dx^2 + 198abcx^5 + 160abdx^6 - 77b^2cx^9 - 60b^2dx^{10}}{-384a^6 + 1152a^5bx^4 - 1152a^4b^2x^8 + 384a^3b^3x^{12}}$$

input
```
integrate((d*x+c)/(-b*x**4+a)**4,x)
```

output
```
RootSum(68719476736*_t**4*a**15*b**2 - 838860800*_t**2*a**8*b*d**2 + 48570
3680*_t*a**4*b*c**2*d + 2560000*a*d**4 - 35153041*b*c**4, Lambda(_t, _t*lo
g(x + (429496729600*_t**3*a**12*b*d**2 + 62170071040*_t**2*a**8*b*c**2*d -
 2621440000*_t*a**5*d**4 + 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2*
d**3)/(788480000*a*c*d**4 + 2706784157*b*c**5)))) + (-153*a**2*c*x - 132*a
**2*d*x**2 + 198*a*b*c*x**5 + 160*a*b*d*x**6 - 77*b**2*c*x**9 - 60*b**2*d*
x**10)/(-384*a**6 + 1152*a**5*b*x**4 - 1152*a**4*b**2*x**8 + 384*a**3*b**3
*x**12)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.38

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx$$
$$= -\frac{60\,b^2dx^{10}+77\,b^2cx^9-160\,abdx^6-198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\,(a^3b^3x^{12}-3\,a^4b^2x^8+3\,a^5bx^4-a^6)}$$
$$+\frac{\frac{154\,c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{40\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{77\,c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{512\,a^3}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

output
```
-1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*a*b*c*x^5 + 132
*a^2*d*x^2 + 153*a^2*c*x)/(a^3*b^3*x^12 - 3*a^4*b^2*x^8 + 3*a^5*b*x^4 - a^
6) + 1/512*(154*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(b))) + 40*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d
*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 77*c*log((sqrt(b)*x - sqrt
(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(b))))/a^3
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(129) = 258.

Time = 0.28 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.83

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx = \frac{77\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^4b}$$
$$-\frac{77\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^4b}$$
$$+\frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{-ab}bd+77\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2}$$
$$+\frac{\sqrt{2}\left(40\,\sqrt{2}\sqrt{-ab}bd+77\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2}$$
$$-\frac{60\,b^2dx^{10}+77\,b^2cx^9-160\,abdx^6-198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\,(bx^4-a)^3a^3}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

output
```
77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-
a/b))/(a^4*b) - 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b
)^(1/4) + sqrt(-a/b))/(a^4*b) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d +
 77*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-
a/b)^(1/4))/(a^4*b^2) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d + 77*(-a*
b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/(a^4*b^2) - 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*
a*b*c*x^5 + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

3.121.9 Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.17

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(-\frac{b^2\left(1925\,c\,d^2+1000\,d^3\,x+\mathrm{root}(68719476736\,a^{15}\,b^2\,z^4-838860800\,a^8\,b\,d^2\,z^2+485703680\,a^4\right.}{}\right.\right.$$

$$\left.\left. -838860800\,a^8\,b\,d^2\,z^2+485703680\,a^4\,b\,c^2\,d\,z-35153041\,b\,c^4+2560000\,a\,d^4,z,k\right)\right)$$

$$+\frac{\frac{11\,d\,x^2}{32\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}-\frac{33\,b\,c\,x^5}{64\,a^2}-\frac{5\,b\,d\,x^6}{12\,a^2}}{a^3-3\,a^2\,b\,x^4+3\,a\,b^2\,x^8-b^3\,x^{12}}$$

input
```
int((c + d*x)/(a - b*x^4)^4,x)
```

output
```
symsum(log(-(b^2*(1925*c*d^2 + 1000*d^3*x + 315392*root(68719476736*a^15*b
^2*z^4 - 838860800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^
4 + 2560000*a*d^4, z, k)^2*a^7*b*c + 47432*root(68719476736*a^15*b^2*z^4 -
 838860800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560
000*a*d^4, z, k)*a^3*b*c^2*x - 163840*root(68719476736*a^15*b^2*z^4 - 8388
60800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*a
*d^4, z, k)^2*a^7*b*d*x))/(32768*a^9))*root(68719476736*a^15*b^2*z^4 - 838
860800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*
a*d^4, z, k), k, 1, 4) + ((11*d*x^2)/(32*a) + (51*c*x)/(128*a) + (77*b^2*c
*x^9)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) - (33*b*c*x^5)/(64*a^2) - (5*b*d
*x^6)/(12*a^2))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

3.122 $\int \frac{c+dx}{(a+bx^4)^4} \, dx$

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3.122.1 Optimal result

Integrand size = 15, antiderivative size = 291

$$\int \frac{c+dx}{(a+bx^4)^4} \, dx = \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a+bx^4)} + \frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{77c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{77c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}} - \frac{77c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{77c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

output
```
1/12*x*(d*x+c)/a/(b*x^4+a)^3+1/96*x*(10*d*x+11*c)/a^2/(b*x^4+a)^2+1/384*x*
(60*d*x+77*c)/a^3/(b*x^4+a)+77/512*c*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/
a^(15/4)/b^(1/4)*2^(1/2)+77/512*c*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1
5/4)/b^(1/4)*2^(1/2)-77/1024*c*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b
^(1/2))/a^(15/4)/b^(1/4)*2^(1/2)+77/1024*c*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*b^(1/2))/a^(15/4)/b^(1/4)*2^(1/2)+5/32*d*arctan(x^2*b^(1/2)/a^(1
/2))/a^(7/2)/b^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.94

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{256a^{11/4}x(c+dx)}{(a+bx^4)^3} + \frac{32a^{7/4}x(11c+10dx)}{(a+bx^4)^2} + \frac{8a^{3/4}x(77c+60dx)}{a+bx^4} - \frac{6\left(77\sqrt{2}\sqrt[4]{b}c+80\sqrt[4]{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{6\left(77\sqrt{2}\sqrt[4]{b}c-80\sqrt[4]{\ldots}\right.}{\ldots}}{3072a^{15/4}}$$

input
```
Integrate[(c + d*x)/(a + b*x^4)^4,x]
```

output
```
((256*a^(11/4)*x*(c + d*x))/(a + b*x^4)^3 + (32*a^(7/4)*x*(11*c + 10*d*x))
/(a + b*x^4)^2 + (8*a^(3/4)*x*(77*c + 60*d*x))/(a + b*x^4) - (6*(77*Sqrt[2
]*b^(1/4)*c + 80*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[
b] + (6*(77*Sqrt[2]*b^(1/4)*c - 80*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/Sqrt[b] - (231*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/b^(1/4) + (231*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(3072*a^(15/4))
```

3.122.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2394, 25, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx$$

$$\Big\downarrow \text{ 2394}$$

$$\frac{x(c+dx)}{12a(a+bx^4)^3} - \frac{\int -\frac{11c+10dx}{(bx^4+a)^3}dx}{12a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{11c+10dx}{(bx^4+a)^3}dx}{12a} + \frac{x(c+dx)}{12a(a+bx^4)^3}$$

$$\Big\downarrow \ 2394$$

$$\frac{\frac{x(11c+10dx)}{8a(a+bx^4)^2}-\frac{\int -\frac{77c+60dx}{(bx^4+a)^2}dx}{8a}}{12a}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{77c+60dx}{(bx^4+a)^2}dx}{8a}+\frac{x(11c+10dx)}{8a(a+bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \ 2394$$

$$\frac{\frac{\frac{x(77c+60dx)}{4a\left(a+bx^4\right)}-\frac{\int -\frac{3(77c+40dx)}{bx^4+a}dx}{4a}}{8a}+\frac{x(11c+10dx)}{8a(a+bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3\int \frac{77c+40dx}{bx^4+a}dx}{4a}+\frac{x(77c+60dx)}{4a\left(a+bx^4\right)}}{8a}+\frac{x(11c+10dx)}{8a(a+bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \ 2415$$

$$\frac{\frac{\frac{3\int \left(\frac{77c}{bx^4+a}+\frac{40dx}{bx^4+a}\right)dx}{4a}+\frac{x(77c+60dx)}{4a\left(a+bx^4\right)}}{8a}+\frac{x(11c+10dx)}{8a(a+bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{3\left(-\frac{77c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{77c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{77c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{77c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{20d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\right)}{4a}}{8a}}{12a}$$

$$\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

input

```
Int[(c + d*x)/(a + b*x^4)^4,x]
```

3.122. $\int \frac{c+dx}{(a+bx^4)^4}\,dx$

output
```
(x*(c + d*x))/(12*a*(a + b*x^4)^3) + ((x*(11*c + 10*d*x))/(8*a*(a + b*x^4)
^2) + ((x*(77*c + 60*d*x))/(4*a*(a + b*x^4)) + (3*((20*d*ArcTan[(Sqrt[b]*x
^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - (77*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (77*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (77*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (77*c*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)))
)/(4*a))/(8*a))/(12*a)
```

3.122.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.122.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\frac{5d\,b^{2}x^{10}}{32a^{3}}+\frac{77c\,b^{2}x^{9}}{384a^{3}}+\frac{5bd\,x^{6}}{12a^{2}}+\frac{33bc\,x^{5}}{64a^{2}}+\frac{11d\,x^{2}}{32a}+\frac{51cx}{128a}}{\left(b\,x^{4}+a\right)^{3}}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}b+a\right)}\frac{\left(40\_Rd+77c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{512a^{3}b}$
default	$\frac{\frac{5d\,b^{2}x^{10}}{32a^{3}}+\frac{77c\,b^{2}x^{9}}{384a^{3}}+\frac{5bd\,x^{6}}{12a^{2}}+\frac{33bc\,x^{5}}{64a^{2}}+\frac{11d\,x^{2}}{32a}+\frac{51cx}{128a}}{\left(b\,x^{4}+a\right)^{3}}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{8a}}{128a^{3}}$

input
```
int((d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+5/12*b*d/a^2*x^6+33/64*b*c/a^2*x
^5+11/32*d/a*x^2+51/128*c/a*x)/(b*x^4+a)^3+1/512/a^3/b*sum((40*_R*d+77*c)/
_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.122.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.06 (sec) , antiderivative size = 43302, normalized size of antiderivative = 148.80

$$\int \frac{c+dx}{\left(a+bx^4\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.122.6 Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.79

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx$$
$$=\operatorname{RootSum}\left(68719476736t^4a^{15}b^2+838860800t^2a^8bd^2-485703680ta^4bc^2d+2560000ad^4+35153041bc^4,\ \left(\right.\right.$$
$$+\frac{153a^2cx+132a^2dx^2+198abcx^5+160abdx^6+77b^2cx^9+60b^2dx^{10}}{384a^6+1152a^5bx^4+1152a^4b^2x^8+384a^3b^3x^{12}}$$

input
```
integrate((d*x+c)/(b*x**4+a)**4,x)
```

output
```
RootSum(68719476736*_t**4*a**15*b**2 + 838860800*_t**2*a**8*b*d**2 - 48570
3680*_t*a**4*b*c**2*d + 2560000*a*d**4 + 35153041*b*c**4, Lambda(_t, _t*lo
g(x + (-429496729600*_t**3*a**12*b*d**2 - 62170071040*_t**2*a**8*b*c**2*d
- 2621440000*_t*a**5*d**4 - 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2
*d**3)/(788480000*a*c*d**4 - 2706784157*b*c**5)))) + (153*a**2*c*x + 132*a
**2*d*x**2 + 198*a*b*c*x**5 + 160*a*b*d*x**6 + 77*b**2*c*x**9 + 60*b**2*d*
x**10)/(384*a**6 + 1152*a**5*b*x**4 + 1152*a**4*b**2*x**8 + 384*a**3*b**3*
x**12)
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.04

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx=\frac{60\,b^2dx^{10}+77\,b^2cx^9+160\,abdx^6+198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\,(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6)}$$
$$+\frac{\frac{77\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{77\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\left(77\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-80\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}}{1024\,a^3}$$

input
```
integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 132*
a^2*d*x^2 + 153*a^2*c*x)/(a^3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6
) + 1/1024*(77*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqr
t(a))/(a^(3/4)*b^(1/4)) - 77*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b
^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(1/4)*c -
80*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/s
qrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 2*(77*sqrt
(2)*a^(1/4)*b^(1/4)*c + 80*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sq
rt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b
))*b^(1/4)))/a^3
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{c+dx}{(a+bx^4)^4}\,dx &= \frac{77\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b} \\ &\quad - \frac{77\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b} \\ &\quad + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}bd+77\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2} \\ &\quad + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}bd+77\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2} \\ &\quad + \frac{60\,b^2dx^{10}+77\,b^2cx^9+160\,abdx^6+198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\,(bx^4+a)^3a^3}\end{aligned}$$

input
```
integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

output
```
77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b
))/(a^4*b) - 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(a^4*b) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*
b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a^4*b^2) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3)^(1/4)*b
*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^2)
+ 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 13
2*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

3.122.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.20

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(\frac{b^2\left(1925\,c\,d^2+1000\,d^3\,x-\mathrm{root}(68719476736\,a^{15}\,b^2\,z^4+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\right.}{\ldots}\right.\right.$$
$$\left.+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\,c^2\,d\,z+35153041\,b\,c^4+2560000\,a\,d^4,z,k)\right)$$
$$+\frac{\frac{11\,d\,x^2}{32\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{33\,b\,c\,x^5}{64\,a^2}+\frac{5\,b\,d\,x^6}{12\,a^2}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$

input
```
int((c + d*x)/(a + b*x^4)^4,x)
```

output
```
symsum(log((b^2*(1925*c*d^2 + 1000*d^3*x - 315392*root(68719476736*a^15*b^
2*z^4 + 838860800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4
 + 2560000*a*d^4, z, k)^2*a^7*b*c - 47432*root(68719476736*a^15*b^2*z^4 +
838860800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 25600
00*a*d^4, z, k)*a^3*b*c^2*x + 163840*root(68719476736*a^15*b^2*z^4 + 83886
0800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a*
d^4, z, k)^2*a^7*b*d*x))/(32768*a^9))*root(68719476736*a^15*b^2*z^4 + 8388
60800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a
*d^4, z, k), k, 1, 4) + ((11*d*x^2)/(32*a) + (51*c*x)/(128*a) + (77*b^2*c*
x^9)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) + (33*b*c*x^5)/(64*a^2) + (5*b*d*
x^6)/(12*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

3.123 $\int \frac{c+dx}{1-x^4} \, dx$

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3.123.1 Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{c+dx}{1-x^4} \, dx = \frac{1}{2} c \arctan(x) + \frac{1}{2} c \operatorname{arctanh}(x) + \frac{1}{2} d \operatorname{arctanh}\left(x^2\right)$$

output
```
1/2*c*arctan(x)+1/2*c*arctanh(x)+1/2*d*arctanh(x^2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{c+dx}{1-x^4} \, dx = \frac{1}{4}\left(2c\arctan(x)-(c+d)\log(1-x)+c\log(1+x)-d\log(1+x)+d\log\left(1+x^2\right)\right)$$

input
```
Integrate[(c + d*x)/(1 - x^4),x]
```

output
```
(2*c*ArcTan[x] - (c + d)*Log[1 - x] + c*Log[1 + x] - d*Log[1 + x] + d*Log[
1 + x^2])/4
```

3.123.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{1-x^4} \, dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{c}{1-x^4}+\frac{dx}{1-x^4}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c\arctan(x)+\frac{1}{2}c\text{arctanh}(x)+\frac{1}{2}d\text{arctanh}\left(x^2\right)$$

input
```
Int[(c + d*x)/(1 - x^4),x]
```

output
```
(c*ArcTan[x])/2 + (c*ArcTanh[x])/2 + (d*ArcTanh[x^2])/2
```

3.123.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.123.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(18) = 36$.

Time = 1.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

method	result
default	$\frac{(-c-d)\ln(-1+x)}{4}-\frac{(-c+d)\ln(1+x)}{4}+\frac{d\ln(x^2+1)}{4}+\frac{c\arctan(x)}{2}$
meijerg	$\frac{d\,\mathrm{arctanh}(x^2)}{2}-\frac{cx\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}$
parallelrisch	$\frac{\ln(1+x)c}{4}-\frac{\ln(1+x)d}{4}-\frac{\ln(-1+x)c}{4}-\frac{\ln(-1+x)d}{4}+\frac{\ln(x-i)d}{4}-\frac{i\ln(x-i)c}{4}+\frac{\ln(x+i)d}{4}+\frac{i\ln(x+i)c}{4}$
risch	$\frac{d\ln(c^4x^2+4d^4x^2+c^4+4d^4)}{4}+\frac{c\arctan\left(\frac{c^4x}{c^4+4d^4}+\frac{4d^4x}{c^4+4d^4}\right)}{2}+\frac{c\arctan\left(\frac{2cd}{c^2-2d^2}\right)}{2}+\frac{\ln(-x-1)c}{4}-\frac{\ln(-x-1)d}{4}-\frac{\ln(-1}{4}$

input
```
int((d*x+c)/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c-d)*ln(-1+x)-1/4*(-c+d)*ln(1+x)+1/4*d*ln(x^2+1)+1/2*c*arctan(x)
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{c+dx}{1-x^4}\,dx = \frac{1}{2}\,c\arctan\left(x\right)+\frac{1}{4}\,d\log\left(x^2+1\right)+\frac{1}{4}\,(c-d)\log\left(x+1\right)-\frac{1}{4}\,(c+d)\log\left(x-1\right)$$

input
```
integrate((d*x+c)/(-x^4+1),x, algorithm="fricas")
```

output
```
1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1/4*(c + d
)*log(x - 1)
```

3.123.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 13.04

$$\int \frac{c+dx}{1-x^4}\,dx = \frac{(c-d)\log\left(x+\frac{c^4(c-d)+5c^2d^3+c^2d(c-d)^2-2d^4(c-d)+2d^2(c-d)^3}{c^5+4cd^4}\right)}{4} - \frac{(c+d)\log\left(x+\frac{-c^4(c+d)+5c^2d^3+c^2d(c+d)^2+2d^4(c+d)-2d^2(c+d)^3}{c^5+4cd^4}\right)}{4} - \left(-\frac{ic}{4} - \frac{d}{4}\right)\log\left(x+\frac{-4c^4\left(-\frac{ic}{4}-\frac{d}{4}\right)+5c^2d^3+16c^2d\left(-\frac{ic}{4}-\frac{d}{4}\right)^2+8d^4\left(-\frac{ic}{4}-\frac{d}{4}\right)-128d^2\left(-\frac{ic}{4}-\frac{d}{4}\right)^3}{c^5+4cd^4}\right) - \left(\frac{ic}{4} - \frac{d}{4}\right)\log\left(x+\frac{-4c^4\left(\frac{ic}{4}-\frac{d}{4}\right)+5c^2d^3+16c^2d\left(\frac{ic}{4}-\frac{d}{4}\right)^2+8d^4\left(\frac{ic}{4}-\frac{d}{4}\right)-128d^2\left(\frac{ic}{4}-\frac{d}{4}\right)^3}{c^5+4cd^4}\right)$$

input
```
integrate((d*x+c)/(-x**4+1),x)
```

output
```
(c - d)*log(x + (c**4*(c - d) + 5*c**2*d**3 + c**2*d*(c - d)**2 - 2*d**4*(
c - d) + 2*d**2*(c - d)**3)/(c**5 + 4*c*d**4))/4 - (c + d)*log(x + (-c**4*
(c + d) + 5*c**2*d**3 + c**2*d*(c + d)**2 + 2*d**4*(c + d) - 2*d**2*(c + d
)**3)/(c**5 + 4*c*d**4))/4 - (-I*c/4 - d/4)*log(x + (-4*c**4*(-I*c/4 - d/4
) + 5*c**2*d**3 + 16*c**2*d*(-I*c/4 - d/4)**2 + 8*d**4*(-I*c/4 - d/4) - 12
8*d**2*(-I*c/4 - d/4)**3)/(c**5 + 4*c*d**4)) - (I*c/4 - d/4)*log(x + (-4*c
**4*(I*c/4 - d/4) + 5*c**2*d**3 + 16*c**2*d*(I*c/4 - d/4)**2 + 8*d**4*(I*c
/4 - d/4) - 128*d**2*(I*c/4 - d/4)**3)/(c**5 + 4*c*d**4))
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{c+dx}{1-x^4}\,dx = \frac{1}{2}c\arctan(x) + \frac{1}{4}d\log\left(x^2+1\right) + \frac{1}{4}(c-d)\log(x+1) - \frac{1}{4}(c+d)\log(x-1)$$

input
```
integrate((d*x+c)/(-x^4+1),x, algorithm="maxima")
```

output
```
1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1/4*(c + d
)*log(x - 1)
```

3.123.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{c+dx}{1-x^4}\,dx = \frac{1}{2}c\arctan(x)+\frac{1}{4}d\log\left(x^2+1\right)+\frac{1}{4}(c-d)\log(|x+1|)-\frac{1}{4}(c+d)\log(|x-1|)$$

input
```
integrate((d*x+c)/(-x^4+1),x, algorithm="giac")
```

output
```
1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(abs(x + 1)) - 1/4*(
c + d)*log(abs(x - 1))
```

3.123.9 Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.17

$$\begin{aligned}\int \frac{c+dx}{1-x^4}\,dx = &-\frac{(-1)^{1/4}\operatorname{atan}\left((-1)^{3/4}\sqrt{2}\,x+1\right)\left(\sqrt{2}\,c+2\,(-1)^{1/4}\,d\right)}{4}\\ &-\frac{(-1)^{1/4}\operatorname{atan}\left((-1)^{3/4}\sqrt{2}\,x-1\right)\left(2\sqrt{2}\,c-4\,(-1)^{1/4}\,d\right)}{8}\\ &+\frac{(-1)^{1/4}\sqrt{2}\,c\,\ln\left(\frac{x^2+(-1)^{1/4}\sqrt{2}\,x+1\mathrm{i}}{x^2-(-1)^{1/4}\sqrt{2}\,x+1\mathrm{i}}\right)}{8}\end{aligned}$$

input
```
int(-(c + d*x)/(x^4 - 1),x)
```

output
```
((-1)^(1/4)*2^(1/2)*c*log((x^2 + (-1)^(1/4)*2^(1/2)*x + 1i)/(x^2 - (-1)^(1
/4)*2^(1/2)*x + 1i)))/8 - ((-1)^(1/4)*atan((-1)^(3/4)*2^(1/2)*x - 1)*(2*2^
(1/2)*c - 4*(-1)^(1/4)*d))/8 - ((-1)^(1/4)*atan((-1)^(3/4)*2^(1/2)*x + 1)*
(2^(1/2)*c + 2*(-1)^(1/4)*d))/4
```

3.124 $\int \frac{c+dx}{1+x^4} dx$

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3.124.1 Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{c+dx}{1+x^4} dx = \frac{1}{2} d \arctan \left(x^2\right) - \frac{c \arctan \left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{c \arctan \left(1+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{c \log \left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{c \log \left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

output
```
1/2*d*arctan(x^2)+1/4*c*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*c*arctan(1+x*2^(1
/2))*2^(1/2)-1/8*c*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/8*c*ln(1+x^2+x*2^(1/2))*2
^(1/2)
```

3.124.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{1+x^4} dx = \frac{1}{4}\left(-\left(\left(\sqrt[4]{-1}c+id\right) \log \left(\sqrt[4]{-1}-x\right)\right) + \left(-(-1)^{3/4}c + id\right) \log \left((-1)^{3/4}-x\right) + \left(\sqrt[4]{-1}c-id\right) \log \left(\sqrt[4]{-1}+x\right) + \left((-1)^{3/4}c+id\right) \log \left((-1)^{3/4}+x\right)\right)$$

input
```
Integrate[(c + d*x)/(1 + x^4),x]
```

output
```
(-(((-1)^(1/4)*c + I*d)*Log[(-1)^(1/4) - x]) + (-((-1)^(3/4)*c) + I*d)*Log
[(-1)^(3/4) - x] + ((-1)^(1/4)*c - I*d)*Log[(-1)^(1/4) + x] + ((-1)^(3/4)*
c + I*d)*Log[(-1)^(3/4) + x])/4
```

3.124.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^4+1}\,dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{c}{x^4+1}+\frac{dx}{x^4+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{c\arctan\left(\sqrt{2}x+1\right)}{2\sqrt{2}}+\frac{1}{2}d\arctan\left(x^2\right)-\frac{c\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{c\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}$$

input
```
Int[(c + d*x)/(1 + x^4),x]
```

output
```
(d*ArcTan[x^2])/2 - (c*ArcTan[1 - Sqrt[2]*x])/(2*Sqrt[2]) + (c*ArcTan[1 +
Sqrt[2]*x])/(2*Sqrt[2]) - (c*Log[1 - Sqrt[2]*x + x^2])/(4*Sqrt[2]) + (c*Lo
g[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

3.124.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.124.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.28

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^{4}+1\right)}\frac{\left(\_R d+c\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{4}$
default	$\frac{c\sqrt{2}\,\left(\ln\left(\frac{1+x^{2}+\sqrt{2}\,x}{1+x^{2}-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{8}+\frac{d\arctan\left(x^{2}\right)}{2}$
meijerg	$\frac{d\arctan\left(x^{2}\right)}{2}+\frac{c\left(-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}+\sqrt{x^{4}}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}{2-\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{2}\,\left(x^{4}\right)^{\frac{1}{4}}+\sqrt{x^{4}}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(x^{4}\right.}{2+\sqrt{2}\,\left(x^{4}\right.}\right.}{\left(x^{4}\right)^{\frac{1}{4}}}}{4}$

input
```
int((d*x+c)/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.124.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 34376, normalized size of antiderivative = 350.78

$$\int \frac{c+dx}{1+x^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(x^4+1),x, algorithm="fricas")
```

output
```
Too large to include
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{c+dx}{1+x^4}\,dx = \text{RootSum}\left(256t^4+32t^2d^2-16tc^2d+c^4+d^4,\left(t\mapsto t\log\left(x+\frac{128t^3d^2+16t^2c^2d+4tc^4+8td^4-5c^2d^3}{c^5-4cd^4}\right.\right.\right.$$

input
```
integrate((d*x+c)/(x**4+1),x)
```

output
```
RootSum(256*_t**4 + 32*_t**2*d**2 - 16*_t*c**2*d + c**4 + d**4, Lambda(_t,
 _t*log(x + (128*_t**3*d**2 + 16*_t**2*c**2*d + 4*_t*c**4 + 8*_t*d**4 - 5*
c**2*d**3)/(c**5 - 4*c*d**4))))
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{c+dx}{1+x^4}\,dx &= \frac{1}{8}\sqrt{2}c\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}c\log\left(x^2-\sqrt{2}x+1\right)\\ &\quad+\frac{1}{4}\left(\sqrt{2}c-2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)\\ &\quad+\frac{1}{4}\left(\sqrt{2}c+2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)\end{aligned}$$

input
```
integrate((d*x+c)/(x^4+1),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x
 + 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(s
qrt(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \frac{c+dx}{1+x^4}\,dx = \frac{1}{8}\sqrt{2}c\log\left(x^2+\sqrt{2}x+1\right) - \frac{1}{8}\sqrt{2}c\log\left(x^2-\sqrt{2}x+1\right) + \frac{1}{4}\left(\sqrt{2}c-2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{4}\left(\sqrt{2}c+2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)$$

input
```
integrate((d*x+c)/(x^4+1),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x
 + 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(s
qrt(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

3.124.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{c+dx}{1+x^4}\,dx = \mathrm{atan}\left(\sqrt{2}\,x-1\right)\left(\frac{d}{2}+\frac{\sqrt{2}\,c}{4}\right) - \mathrm{atan}\left(\sqrt{2}\,x+1\right)\left(\frac{d}{2}-\frac{\sqrt{2}\,c}{4}\right) + \frac{\sqrt{2}\,c\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{8}$$

input
```
int((c + d*x)/(x^4 + 1),x)
```

output
```
atan(2^(1/2)*x - 1)*(d/2 + (2^(1/2)*c)/4) - atan(2^(1/2)*x + 1)*(d/2 - (2^
(1/2)*c)/4) + (2^(1/2)*c*log((2^(1/2)*x + x^2 + 1)/(x^2 - 2^(1/2)*x + 1)))
/8
```

3.125 $\int \frac{c+dx+ex^2}{a-bx^4} dx$

3.125.1 Optimal result 1058
3.125.2 Mathematica [A] (verified) 1058
3.125.3 Rubi [A] (verified) 1059
3.125.4 Maple [C] (verified) 1060
3.125.5 Fricas [C] (verification not implemented) 1060
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3.125.8 Giac [B] (verification not implemented) 1062
3.125.9 Mupad [B] (verification not implemented) 1063

3.125.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{c+dx+ex^2}{a-bx^4} dx = \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

output
```
1/2*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)+1/2*arctan(b^(1/4)*x/a^
(1/4))*(-e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*arctanh(b^(1/4)*x/a^(1/4
))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

3.125.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.61

$$\int \frac{c+dx+ex^2}{a-bx^4} dx = \frac{2\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \left(\sqrt{b}c+\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{a}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right) + \sqrt{b}c\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right) - \sqrt[4]{}}{4a^{3/4}b^{3/4}}$$

input
```
Integrate[(c + d*x + e*x^2)/(a - b*x^4),x]
```

output
```
(2*(Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + a^(1
/4)*b^(1/4)*d + Sqrt[a]*e)*Log[a^(1/4) - b^(1/4)*x] + Sqrt[b]*c*Log[a^(1/4
) + b^(1/4)*x] - a^(1/4)*b^(1/4)*d*Log[a^(1/4) + b^(1/4)*x] + Sqrt[a]*e*Lo
g[a^(1/4) + b^(1/4)*x] + a^(1/4)*b^(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*
a^(3/4)*b^(3/4))
```

3.125.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx$$

$$\Big\downarrow \text{ 2415}$$

$$\int \left(\frac{c+ex^2}{a-bx^4}+\frac{dx}{a-bx^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}c-\sqrt{a}e\right)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2a^{3/4}b^{3/4}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

input
```
Int[(c + d*x + e*x^2)/(a - b*x^4),x]
```

output
```
((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4
)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b])
```

3.125.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.125.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.34

method	result	size
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(\_R^2 e+\_R d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	39
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	139

input
```
int((e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*sum((_R^2*e+_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.125.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.58 (sec) , antiderivative size = 120560, normalized size of antiderivative = 1039.31

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.125.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(105) = 210.

Time = 5.33 (sec) , antiderivative size = 471, normalized size of antiderivative = 4.06

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx =$$
$$-\operatorname{RootSum}\Big(256t^4a^3b^3+t^2\left(-64a^2b^2ce-32a^2b^2d^2\right)+t\left(-16a^2bde^2-16ab^2c^2d\right)-a^2e^4+2abc^2e^2-4ab$$

input
```
integrate((e*x**2+d*x+c)/(-b*x**4+a),x)
```

output
```
-RootSum(256*_t**4*a**3*b**3 + _t**2*(-64*a**2*b**2*c*e - 32*a**2*b**2*d**
2) + _t*(-16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) - a**2*e**4 + 2*a*b*c**2*e*
*2 - 4*a*b*c*d**2*e + a*b*d**4 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t*
*3*a**4*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2
 + 48*_t**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e - 16*_t**2*a**2
*b**3*c**3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 + 16*_t*a**2*b
**2*c**3*e**2 - 36*_t*a**2*b**2*c**2*d**2*e - 8*_t*a**2*b**2*c*d**4 + 4*_t
*a*b**3*c**5 + 3*a**3*d*e**5 - 5*a**2*b*c*d**3*e**2 + 2*a**2*b*d**5*e + 5*
a*b**2*c**4*d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 + a**2*b*c**2*e**4 - 8*a*
*2*b*c*d**2*e**3 + 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d
**2*e - 4*a*b**2*c**2*d**4 - b**3*c**6))))
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.32

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx = \frac{d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}} - \frac{d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
1/4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*d*log(sqrt(b)*x^2
 - sqrt(a))/(sqrt(a)*sqrt(b)) + 1/2*(sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)
*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(s
qrt(b)*c + sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x +
 sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.23

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx = -\frac{\sqrt{2}\left(b^2c-\sqrt{2}(-ab^3)^{\frac{1}{4}}bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c+\sqrt{2}(-ab^3)^{\frac{1}{4}}bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c-\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}} + \frac{\sqrt{2}\left(b^2c-\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(b^2*c - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/
4*sqrt(2)*(b^2*c + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*s
qrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a
/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 - sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 725, normalized size of antiderivative = 6.25

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx = \sum_{k=1}^{4} \ln\Big(-b^2\,c\,d^2+b^2\,c^2\,e-b^2\,d^3\,x-a\,b\,e^3$$
$$-\mathrm{root}\big(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2$$
$$-\mathrm{root}\big(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z$$
$$-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+a\,b\,d^4-a^2\,e^4-b^2\,c^4,z,k\big)\;b^3\,c^2\,x\,4$$
$$+\mathrm{root}\big(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2$$
$$-\mathrm{root}\big(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z$$
$$-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+a\,b\,d^4-a^2\,e^4-b^2\,c^4,z,k\big)\;a\,b^2\,e^2\,x\,4$$
$$+\mathrm{root}\big(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z$$
$$-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+a\,b\,d^4-a^2\,e^4-b^2\,c^4,z,k\big)\;a\,b^2\,d\,e\,8$$
$$+2\,b^2\,c\,d\,e\,x\Big)\;\mathrm{root}\big(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z$$
$$+16\,a\,b^2\,c^2\,d\,z-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+a\,b\,d^4-a^2\,e^4-b^2\,c^4,z,k\big)$$

input
```
int((c + d*x + e*x^2)/(a - b*x^4),x)
```

output
```
symsum(log(b^2*c^2*e - b^2*c*d^2 - b^2*d^3*x - a*b*e^3 - 16*root(256*a^3*b
^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a
*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4
, z, k)^2*a*b^3*c - 4*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b
^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c
^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*b^3*c^2*x + 16*root(256*a^3*b^
3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*
b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4,
 z, k)^2*a*b^3*d*x - 4*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*
b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*
c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*a*b^2*e^2*x + 8*root(256*a^3*
b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*
a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^
4, z, k)*a*b^2*d*e + 2*b^2*c*d*e*x)*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*
z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d
^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k), k, 1, 4)
```

3.126 $\int \frac{c+dx+ex^2}{a+bx^4} dx$

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3.126.1 Optimal result

Integrand size = 20, antiderivative size = 277

$$\int \frac{c+dx+ex^2}{a+bx^4} dx = \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

output
```
1/2*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*
x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1
/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+c*b^
(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e
*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)
/a^(1/4))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.83

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = \frac{-2\left(\sqrt{2}\sqrt{b}c+2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt{b}c-2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1+\right.}{8a^{3/4}b^{3}}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^4),x]
```

output
```
(-2*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)
*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*
(Sqrt[b]*c - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a^(3/4
)*b^(3/4))
```

3.126.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx$$

$$\Big\downarrow \ 2415$$

$$\int \left(\frac{c+ex^2}{a+bx^4}+\frac{dx}{a+bx^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^4),x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt
[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)
) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c
 - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*S
qrt[2]*a^(3/4)*b^(3/4))
```

3.126.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.126.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4} b+a\right)}\frac{\left(\_R^{2} e+\_R d+c\right) \ln\left(x-\_R\right)}{\_R^{3}}}{4 b}$
default	$\frac{c \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8 a}+\frac{d \arctan\left(x^{2} \sqrt{\frac{b}{a}}\right)}{2 \sqrt{a b}}+\frac{e \sqrt{2}\, \left(\ln\left(\frac{x^{2}-\left(\right.}{x^{2}+\left(\right.}\right.\right.}{}$

input
```
int((e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum((_R^2*e+_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.126.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 121386, normalized size of antiderivative = 438.22

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.126.6 Sympy [A] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.68

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx$$
$$= \mathrm{RootSum}\Big(256t^4a^3b^3 + t^2\cdot\left(64a^2b^2ce + 32a^2b^2d^2\right) + t\left(16a^2bde^2 - 16ab^2c^2d\right) + a^2e^4 + 2abc^2e^2 - 4abcd^2e$$

input
```
integrate((e*x**2+d*x+c)/(b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*b**3 + _t**2*(64*a**2*b**2*c*e + 32*a**2*b**2*d**2)
 + _t*(16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) + a**2*e**4 + 2*a*b*c**2*e**2
- 4*a*b*c*d**2*e + a*b*d**4 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a
**4*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2 + 4
8*_t**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e + 16*_t**2*a**2*b**
3*c**3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 - 16*_t*a**2*b**2*
c**3*e**2 + 36*_t*a**2*b**2*c**2*d**2*e + 8*_t*a**2*b**2*c*d**4 + 4*_t*a*b
**3*c**5 + 3*a**3*d*e**5 + 5*a**2*b*c*d**3*e**2 - 2*a**2*b*d**5*e + 5*a*b*
*2*c**4*d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 - a**2*b*c**2*e**4 + 8*a**2*b
*c*d**2*e**3 - 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d**2*
e - 4*a*b**2*c**2*d**4 + b**3*c**6))))
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{c+dx+ex^2}{a+bx^4}\,dx &= \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}} \\ &\quad-\frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}} \\ &\quad+\frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-2\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{3}{4}}} \\ &\quad+\frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e+2\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{3}{4}}}\end{aligned}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1
/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*e)*l
og(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) +
1/4*(sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e - 2*sqrt(a)*sqr
t(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sq
rt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 1/4*(sqrt(2)*a^(
1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e + 2*sqrt(a)*sqrt(b)*d)*arctan(1
/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4))
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4
)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) -
 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4
)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) +
 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^
3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.126.9 Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.57

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = \sum_{k=1}^{4} \ln\Big(b^2\,c\,d^2 - b^2\,c^2\,e + b^2\,d^3\,x - a\,b\,e^3$$
$$-\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z - 4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2$$
$$-\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z$$
$$-4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + a\,b\,d^4 + a^2\,e^4 + b^2\,c^4, z, k\big)\;b^3\,c^2\,x\,4$$
$$+\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z - 4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2$$
$$+\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z$$
$$-4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + a\,b\,d^4 + a^2\,e^4 + b^2\,c^4, z, k\big)\;a\,b^2\,e^2\,x\,4$$
$$-\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z$$
$$-4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + a\,b\,d^4 + a^2\,e^4 + b^2\,c^4, z, k\big)\;a\,b^2\,d\,e\,8$$
$$-2\,b^2\,c\,d\,e\,x\Big)\;\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z$$
$$-16\,a\,b^2\,c^2\,d\,z - 4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + a\,b\,d^4 + a^2\,e^4 + b^2\,c^4, z, k\big)$$

input
```
int((c + d*x + e*x^2)/(a + b*x^4),x)
```

output
```
symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - a*b*e^3 - 16*root(256*a^3*b
^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a
*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4
, z, k)^2*a*b^3*c - 4*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b
^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c
^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*b^3*c^2*x + 16*root(256*a^3*b^
3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*
b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4,
 z, k)^2*a*b^3*d*x + 4*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*
b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*
c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*a*b^2*e^2*x - 8*root(256*a^3*
b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*
a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^
4, z, k)*a*b^2*d*e - 2*b^2*c*d*e*x)*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*
z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d
^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k), k, 1, 4)
```

3.127 $\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$

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3.127.1 Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = \frac{x(c+dx+ex^2)}{4a(a-bx^4)} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

output
```
1/4*x*(e*x^2+d*x+c)/a/(-b*x^4+a)+1/4*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(3/2
)/b^(1/2)+1/8*arctan(b^(1/4)*x/a^(1/4))*(-e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b
^(3/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3
/4)
```

3.127.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.45

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$$
$$= \frac{\frac{4ax(c+x(d+ex))}{a-bx^4} - \frac{2\sqrt[4]{a}\left(-3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} - \frac{\left(3\sqrt[4]{a}\sqrt{b}c+2\sqrt{a}\sqrt[4]{b}d+a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{b^{3/4}} + \frac{\left(3\sqrt[4]{a}\sqrt{b}c-2\sqrt{a}\sqrt[4]{b}\cdots\right.}{\cdots}}{16a^2}$$

input
```
Integrate[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

output
```
((4*a*x*(c + x*(d + e*x)))/(a - b*x^4) - (2*a^(1/4)*(-3*Sqrt[b]*c + Sqrt[a
]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((3*a^(1/4)*Sqrt[b]*c + 2*Sqrt
[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((3*a^(1/4)
*Sqrt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^
(3/4) + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

3.127.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$$

$$\Big\downarrow \text{ 2394}$$

$$\frac{x(c+dx+ex^2)}{4a(a-bx^4)}-\frac{\int -\frac{ex^2+2dx+3c}{a-bx^4}dx}{4a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{ex^2+2dx+3c}{a-bx^4}dx}{4a}+\frac{x(c+dx+ex^2)}{4a(a-bx^4)}$$

$$\Big\downarrow \text{ 2415}$$

$$\frac{\int \left(\frac{2dx}{a-bx^4}+\frac{ex^2+3c}{a-bx^4}\right)dx}{4a}+\frac{x(c+dx+ex^2)}{4a(a-bx^4)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}c-\sqrt{a}e\right)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{2a^{3/4}b^{3/4}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{4a}+\frac{x(c+dx+ex^2)}{4a(a-bx^4)}$$

input
```
Int[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

output
```
(x*(c + d*x + e*x^2))/(4*a*(a - b*x^4)) + (((3*Sqrt[b]*c - Sqrt[a]*e)*ArcT
an[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*
ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^
2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(4*a)
```

3.127.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.127.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{e x^3}{4a}+\frac{d x^2}{4a}+\frac{cx}{4a}}{-b x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(\_R^2 e+2\_R d+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(-b x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{16a^2}\right)+d\left(\frac{x^2}{4a(-b x^4+a)}+\frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)+e\left(\frac{x^3}{4a(-b x}\right.$

input
```
int((e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/a*e*x^3+1/4*d/a*x^2+1/4*c/a*x)/(-b*x^4+a)-1/16/b/a*sum((_R^2*e+2*_R*d
+3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.127.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 116982, normalized size of antiderivative = 801.25

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.127.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(131) = 262.

Time = 40.49 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.48

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$$
$$= \text{RootSum}\Big(65536t^4a^7b^3 + t^2\left(-3072a^4b^2ce - 2048a^4b^2d^2\right) + t\left(128a^3bde^2 + 1152a^2b^2c^2d\right) - a^2e^4 + 18abc^2$$
$$+ \frac{-cx-dx^2-ex^3}{-4a^2+4abx^4}$$

input
```
integrate((e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**
2*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*
a*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _
t*log(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 9830
4*_t**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5
*b**2*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_
t*a**4*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*
d**2*e + 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5
 - 120*a**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*
a*b**2*c**3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3
+ 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576
*a*b**2*c**2*d**4 - 729*b**3*c**6)))) + (-c*x - d*x**2 - e*x**3)/(-4*a**2
+ 4*a*b*x**4)
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.31

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = -\frac{ex^3+dx^2+cx}{4\,(abx^4-a^2)} + \frac{\frac{2\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(3\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,a}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(e*x^3 + d*x^2 + c*x)/(a*b*x^4 - a^2) + 1/16*(2*d*log(sqrt(b)*x^2 + s
qrt(a))/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b
)) + 2*(3*sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*sqrt(b)*c + sqrt(a)*e)*log((sq
rt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a
```

3.127.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(107) = 214$.

Time = 0.28 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.10

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = -\frac{\sqrt{2}\left(3\,b^2c-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}\left(3\,b^2c+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}\left(3\,b^2c-\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} + \frac{\sqrt{2}\left(3\,b^2c-\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{ex^3+dx^2+cx}{4\,(bx^4-a)a}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(3*b^2*c - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*ar
ctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4
)*a) - 1/16*sqrt(2)*(3*b^2*c + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c - sqr
t(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4
)*a) - 1/4*(e*x^3 + d*x^2 + c*x)/((b*x^4 - a)*a)
```

3.127.9 Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.27

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = \frac{\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^4} + \left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\right.\right.\right.$$
$$-\frac{-9\,b^2\,c^2\,e+12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}-\frac{x\,(2\,b^2\,d^3-3\,b^2\,c\,d\,e)}{16\,a^3}\Big)\,\mathrm{root}\big(65536\,a^7\,b^3\,z^4$$
$$-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e$$
$$\left.+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4-81\,b^2\,c^4-a^2\,e^4,z,k\big)\right)$$

input
```
int((c + d*x + e*x^2)/(a - b*x^4)^2,x)
```

output
```
((d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a - b*x^4) + symsum(log(- r
oot(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152
*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 1
6*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k)*(root(65536*a^7*b^3*z^4 - 3072*a^4
*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e
^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4
, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 + 4*a^2*b^2*e^2))/(16*a^
3) - (b^2*d*e)/a) - (12*b^2*c*d^2 - 9*b^2*c^2*e + a*b*e^3)/(64*a^3) - (x*(
2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3))*root(65536*a^7*b^3*z^4 - 3072*a^4*b^2*
c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z
- 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z,
k), k, 1, 4)
```

3.128 $\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$

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3.128.1 Optimal result

Integrand size = 20, antiderivative size = 308

$$\begin{aligned}\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx &= \frac{x(c+dx+ex^2)}{4a(a+bx^4)}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} \\ &\quad -\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} \\ &\quad +\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} \\ &\quad -\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} \\ &\quad +\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}\end{aligned}$$

output
```
1/4*x*(e*x^2+d*x+c)/a/(b*x^4+a)+1/4*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/
b^(1/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2
)+3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^(1/2)+1/16*
arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/
4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2)
)/a^(7/4)/b^(3/4)*2^(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$$

$$= \frac{\frac{8ax(c+x(d+ex))}{a+bx^4} - \frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{b}c+4\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{b}c-4\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1+\cdots\right.}{b^{3/4}}}{32a^2}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

output
```
((8*a*x*(c + x*(d + e*x)))/(a + b*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c +
 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/b^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(1/4)*b^(1/4)*d +
 Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sq
rt[2]*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[b]*c - a^(3/4)*e
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(32*a^2
)
```

3.128.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$$

$$\Big\downarrow \; 2394$$

$$\frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)} - \frac{\int -\frac{ex^2+2dx+3c}{bx^4+a}dx}{4a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{ex^2+2dx+3c}{bx^4+a}dx}{4a} + \frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)}$$

$$\Big\downarrow \; 2415$$

$$\frac{\int \left(\frac{2dx}{bx^4+a}+\frac{ex^2+3c}{bx^4+a}\right)dx}{4a}+\frac{x(c+dx+ex^2)}{4a(a+bx^4)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(3\sqrt{b}c-\sqrt{}\right.}{}}{4a}$$

$$\frac{x(c+dx+ex^2)}{4a(a+bx^4)}$$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

output
```
(x*(c + d*x + e*x^2))/(4*a*(a + b*x^4)) + ((d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]
])/(Sqrt[a]*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((3
*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)))/
(4*a)
```

3.128.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.128.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.26

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{cx}{4a}}{b\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{\left(\_R^2e+2\_Rd+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+d\left(\frac{x^2}{4a(b\,x^4+a)}\right.$

input
```
int((e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/a*e*x^3+1/4*d/a*x^2+1/4*c/a*x)/(b*x^4+a)+1/16/b/a*sum((_R^2*e+2*_R*d+
3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.128.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 124258, normalized size of antiderivative = 403.44

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.128.6 Sympy [A] (verification not implemented)

Time = 40.16 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.64

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$$
$$=\operatorname{RootSum}\left(65536t^4a^7b^3+t^2\cdot\left(3072a^4b^2ce+2048a^4b^2d^2\right)+t\left(128a^3bde^2-1152a^2b^2c^2d\right)+a^2e^4+18abc^2\right.$$
$$+\frac{cx+dx^2+ex^3}{4a^2+4abx^4}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a
*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t
*log(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304
*_t**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*
b**2*d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t
*a**4*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d
**2*e + 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5
+ 120*a**2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a
*b**2*c**3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 -
 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*
a*b**2*c**2*d**4 + 729*b**3*c**6))) + (c*x + d*x**2 + e*x**3)/(4*a**2 + 4
*a*b*x**4)
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx=\frac{ex^3+dx^2+cx}{4\,(abx^4+a^2)}$$
$$+\frac{\frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-4\sqrt{a}\sqrt{b}d\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{}}}}{32\,a}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(e*x^3 + d*x^2 + c*x)/(a*b*x^4 + a^2) + 1/32*(sqrt(2)*(3*sqrt(b)*c - s
qrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*
b^(3/4)) - sqrt(2)*(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(
1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)
*c + sqrt(2)*a^(3/4)*b^(1/4)*e - 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqr
t(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(
3/4)*b^(1/4)*e + 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sq
rt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b
))*b^(3/4)))/a
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$$

$$= \frac{ex^3+dx^2+cx}{4\,(bx^4+a)a} + \frac{\sqrt{2}\Big(2\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\Big(2\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$- \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(e*x^3 + d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a
*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)
*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*
(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*
e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

3.128.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.53

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^2}\,dx = \frac{\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a} + \left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a \right.\right.\right.$$

$$- \frac{9\,b^2\,c^2\,e-12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3} + \frac{x\,(2\,b^2\,d^3-3\,b^2\,c\,d\,e)}{16\,a^3}\Bigg)\,\mathrm{root}\big(65536\,a^7\,b^3\,z^4$$

$$+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e$$

$$\left.+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\big)\right)$$

input
```
int((c + d*x + e*x^2)/(a + b*x^4)^2,x)
```

output
```
((d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a + b*x^4) + symsum(log((x*
(2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3) - (9*b^2*c^2*e - 12*b^2*c*d^2 + a*b*e^
3)/(64*a^3) - root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2
*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*
a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)*(root(65536*a^7*b^3
*z^4 + 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z
+ 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^
2*c^4 + a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 - 4*a^2*b
^2*e^2))/(16*a^3) + (b^2*d*e)/a))*root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*
e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z -
48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)
, k, 1, 4)
```

3.129 $\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx$

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3.129.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx = \frac{x(c+dx+ex^2)}{8a(a-bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)} + \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

output
```
1/8*x*(e*x^2+d*x+c)/a/(-b*x^4+a)^2+1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(-b*x^4+
a)+3/16*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)+1/64*arctan(b^(1/4)
*x/a^(1/4))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)+1/64*arctanh(b^(1
/4)*x/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)
```

3.129.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.36

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx$$
$$= \frac{\frac{16a^2x(c+x(d+ex))}{(a-bx^4)^2}+\frac{4ax(7c+x(6d+5ex))}{a-bx^4}+\frac{2\sqrt[4]{a}(21\sqrt{b}c-5\sqrt{a}e)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{\left(21\sqrt[4]{a}\sqrt{b}c+12\sqrt{a}\sqrt[4]{b}d+5a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{b^{3/4}}}{128a^3}$$

input
```
Integrate[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

output
```
((16*a^2*x*(c + x*(d + e*x)))/(a - b*x^4)^2 + (4*a*x*(7*c + x*(6*d + 5*e*x
)))/(a - b*x^4) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*
x)/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c + 12*Sqrt[a]*b^(1/4)*d + 5*a
^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*Sqrt[b]*c - 12*
Sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (12*S
qrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

3.129.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2394, 25, 2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx$$
$$\Big\downarrow \text{2394}$$
$$\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}-\frac{\int -\frac{5ex^2+6dx+7c}{(a-bx^4)^2}dx}{8a}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{5ex^2+6dx+7c}{(a-bx^4)^2}dx}{8a}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}$$
$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}-\frac{\int -\frac{5ex^2+12dx+21c}{a-bx^4}dx}{4a}}{8a}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{5ex^2+12dx+21c}{a-bx^4}dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}}{8a}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{\int \left(\frac{12dx}{a-bx^4}+\frac{5ex^2+21c}{a-bx^4}\right)dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}}{8a}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(21\sqrt{b}c-5\sqrt{a}e)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(5\sqrt{a}e+21\sqrt{b}c)}{2a^{3/4}b^{3/4}}+\frac{6d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}}{8a}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}$$

input
```
Int[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

output
```
(x*(c + d*x + e*x^2))/(8*a*(a - b*x^4)^2) + ((x*(7*c + 6*d*x + 5*e*x^2))/(
4*a*(a - b*x^4)) + (((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/
4)])/(2*a^(3/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*
x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + (6*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(S
qrt[a]*Sqrt[b]))/(4*a))/(8*a)
```

3.129. $\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx$

3.129.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.129.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64

method	result
risch	$\frac{-\frac{5be\,x^7}{32a^2}-\frac{3bd\,x^6}{16a^2}-\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}}{\left(-b\,x^4+a\right)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(5\_R^2 e+12\_R d+21c\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2 b}$
default	$c\left(\frac{x}{8a\left(-b\,x^4+a\right)^2}+\frac{\frac{7x}{32a\left(-b\,x^4+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{128a^2}}{a}\right)+d\left(\frac{x^2}{8a\left(-b\,x^4+a\right)^2}+\frac{\frac{3x^2}{16a\left(-b\,x^4+a\right)}+\cdots}{a}\right.$

input
```
int((e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-5/32*b*e/a^2*x^7-3/16*b*d/a^2*x^6-7/32*b*c/a^2*x^5+9/32/a*e*x^3+5/16*d/a
*x^2+11/32*c/a*x)/(-b*x^4+a)^2-1/128/a^2/b*sum((5*_R^2*e+12*_R*d+21*c)/_R^
3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.129.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.14 (sec) , antiderivative size = 118710, normalized size of antiderivative = 663.18

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

output
```
Timed out
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^3}\,dx=-\frac{5\,bex^7+6\,bdx^6+7\,bcx^5-9\,aex^3-10\,adx^2-11\,acx}{32\,\left(a^2b^2x^8-2\,a^3bx^4+a^4\right)}+\frac{\frac{12\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{12\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,\sqrt{b}c+5\,\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^2}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 - 9*a*e*x^3 - 10*a*d*x^2 - 11*a*c
*x)/(a^2*b^2*x^8 - 2*a^3*b*x^4 + a^4) + 1/128*(12*d*log(sqrt(b)*x^2 + sqrt
(a))/(sqrt(a)*sqrt(b)) - 12*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b))
 + 2*(21*sqrt(b)*c - 5*sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*sqrt(b)*c + 5*sqrt(a)*e)*log
((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^2
```

3.129.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(139) = 278$.

Time = 0.29 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.87

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx$$

$$= -\frac{\sqrt{2}\left(21\,b^2c-12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{\sqrt{2}\left(21\,b^2c+12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{\sqrt{2}(21\,b^2c-5\,\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$+\frac{\sqrt{2}(21\,b^2c-5\,\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{5\,bex^7+6\,bdx^6+7\,bcx^5-9\,aex^3-10\,adx^2-11\,acx}{32\,(bx^4-a)^2a^2}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

output
```
-1/128*sqrt(2)*(21*b^2*c - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)
^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 5*
sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log
(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) + 1/256*s
qrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sq
rt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 -
 9*a*e*x^3 - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a)^2*a^2)
```

3.129.9 Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 826, normalized size of antiderivative = 4.61

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx = \frac{\frac{5\,d\,x^2}{16\,a}+\frac{9\,e\,x^3}{32\,a}+\frac{11\,c\,x}{32\,a}-\frac{7\,b\,c\,x^5}{32\,a^2}-\frac{3\,b\,d\,x^6}{16\,a^2}-\frac{5\,b\,e\,x^7}{32\,a^2}}{a^2-2\,a\,b\,x^4+b^2\,x^8}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(125\,a\,e^3+3024\,b\,c\,d^2-2205\,b\,c^2\,e+1728\,b\,d^3\,x+\mathrm{root}(268435456\,a^{11}\,b^3\,z^4-6881280\,a^6\right.}{}\right.\right.$$
$$-6881280\,a^6\,b^2\,c\,e\,z^2-4718592\,a^6\,b^2\,d^2\,z^2+2709504\,a^3\,b^2\,c^2\,d\,z+153600\,a^4\,b\,d\,e^2\,z$$
$$\left.-60480\,a\,b\,c\,d^2\,e+22050\,a\,b\,c^2\,e^2+20736\,a\,b\,d^4-625\,a^2\,e^4-194481\,b^2\,c^4,z,k)\right)$$

input
```
int((c + d*x + e*x^2)/(a - b*x^4)^3,x)
```

output
```
((5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a
^2) - (3*b*d*x^6)/(16*a^2) - (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 - 2*a*b*
x^4) + symsum(log(-(b*(125*a*e^3 + 3024*b*c*d^2 - 2205*b*c^2*e + 1728*b*d^
3*x + 344064*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 47185
92*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 6048
0*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b
^2*c^4, z, k)^2*a^5*b^2*c + 3200*root(268435456*a^11*b^3*z^4 - 6881280*a^6
*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*
a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 62
5*a^2*e^4 - 194481*b^2*c^4, z, k)*a^3*b*e^2*x - 2520*b*c*d*e*x + 56448*roo
t(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z
^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e +
22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^
2*b^2*c^2*x - 196608*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2
 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*
z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 -
194481*b^2*c^4, z, k)^2*a^5*b^2*d*x - 15360*root(268435456*a^11*b^3*z^4 -
6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*
z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a
*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*r...
```

3.129. $\int \frac{c+dx+ex^2}{(a-bx^4)^3} \, dx$

3.130 $\int \frac{c+dx+ex^2}{(a+bx^4)^3} dx$

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3.130.1 Optimal result

Integrand size = 20, antiderivative size = 341

$$\begin{aligned}\int \frac{c+dx+ex^2}{(a+bx^4)^3} dx &= \frac{x(c+dx+ex^2)}{8a(a+bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} + \frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} \\ &- \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}} \\ &+ \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}} \\ &- \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}} \\ &+ \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}\end{aligned}$$

output
```
1/8*x*(e*x^2+d*x+c)/a/(b*x^4+a)^2+1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(b*x^4+a)
+3/16*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)-1/256*ln(-a^(1/4)*b^(1
/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^
(3/4)*2^(1/2)+1/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*
e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x
*2^(1/2)/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)*2^(1/2)+1/12
8*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/
b^(3/4)*2^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx$$
$$= \frac{\frac{32a^2x(c+x(d+ex))}{(a+bx^4)^2}+\frac{8ax(7c+x(6d+5ex))}{a+bx^4}-\frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}c+24\sqrt[4]{a}\sqrt[4]{b}d+5\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}c-24\sqrt[4]{}\right.}{}}{}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```

output
```
((32*a^2*x*(c + x*(d + e*x)))/(a + b*x^4)^2 + (8*a*x*(7*c + x*(6*d + 5*e*x
)))/(a + b*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1/4)*b^(1/4)*d
+ 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) +
(2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a
]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*(-21*a^(1
/4)*Sqrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

3.130.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2394, 25, 2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx$$
$$\Big\downarrow \; 2394$$
$$\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}-\frac{\int -\frac{5ex^2+6dx+7c}{(bx^4+a)^2}dx}{8a}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{5ex^2+6dx+7c}{(bx^4+a)^2}dx}{8a}+\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}$$

$\Big\downarrow$ 2394

$$\frac{\frac{x(7c+6dx+5ex^2)}{4a(a+bx^4)}-\frac{\int -\frac{5ex^2+12dx+21c}{bx^4+a}dx}{4a}}{8a}+\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{5ex^2+12dx+21c}{bx^4+a}dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a+bx^4)}}{8a}+\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}$$

$\Big\downarrow$ 2415

$$\frac{\frac{\int \left(\frac{12dx}{bx^4+a}+\frac{5ex^2+21c}{bx^4+a}\right)dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a+bx^4)}}{8a}+\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(5\sqrt{a}e+21\sqrt{b}c)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(5\sqrt{a}e+21\sqrt{b}c)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{(21\sqrt{b}c-5\sqrt{a}e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{(21\sqrt{b}c-5\sqrt{a}e)\log\left(\sqrt{2}\ldots\right.}{4\sqrt{2}a^{3\ldots}}}{4a}}{8a}$$

$$\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}$$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```

output
```
(x*(c + d*x + e*x^2))/(8*a*(a + b*x^4)^2) + ((x*(7*c + 6*d*x + 5*e*x^2))/(
4*a*(a + b*x^4)) + ((6*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
- ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((21*Sqrt[b]*c - 5
*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqr
t[2]*a^(3/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)))/(4*a))/(8
*a)
```

3.130. $\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx$

3.130.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.130.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.33

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\operatorname{RootOf}(\_Z^4b+a)}\frac{(5\_R^2e+12\_Rd+21c)\ln(x-\_R)}{\_R^3}}{128a^2b}$
default	$c\left(\frac{x}{8a(b\,x^4+a)^2}+\frac{\frac{7x}{32a(b\,x^4+a)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+d\left(\frac{}{8a}\right.$

input
```
int((e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/32*b*e/a^2*x^7+3/16*b*d/a^2*x^6+7/32*b*c/a^2*x^5+9/32/a*e*x^3+5/16*d/a*
x^2+11/32*c/a*x)/(b*x^4+a)^2+1/128/a^2/b*sum((5*_R^2*e+12*_R*d+21*c)/_R^3*
ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.130.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.02 (sec) , antiderivative size = 124787, normalized size of antiderivative = 365.94

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

output
```
Timed out
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^3}\,dx = \frac{5\,bex^7+6\,bdx^6+7\,bcx^5+9\,aex^3+10\,adx^2+11\,acx}{32\,\left(a^2b^2x^8+2\,a^3bx^4+a^4\right)}$$

$$+\frac{\frac{\sqrt{2}\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-24}{a^{\frac{3}{4}}}}{256\,a^2}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*e*x^3 + 10*a*d*x^2 + 11*a*c*
x)/(a^2*b^2*x^8 + 2*a^3*b*x^4 + a^4) + 1/256*(sqrt(2)*(21*sqrt(b)*c - 5*sq
rt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b
^(3/4)) - sqrt(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(3
/4)*c + 5*sqrt(2)*a^(3/4)*b^(1/4)*e - 24*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqr
t(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/
4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(3/4)*c + 5*sq
rt(2)*a^(3/4)*b^(1/4)*e + 24*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt
(a)*sqrt(b))*b^(3/4)))/a^2
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.97

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = \frac{5\,bex^7+6\,bdx^6+7\,bcx^5+9\,aex^3+10\,adx^2+11\,acx}{32\,(bx^4+a)^2a^2} + \frac{\sqrt{2}\Big(12\,\sqrt{2}\sqrt{ab}b^2d+21\,(ab^3)^{\frac14}\,b^2c+5\,(ab^3)^{\frac34}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\,\left(\frac{a}{b}\right)^{\frac14}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\Big(12\,\sqrt{2}\sqrt{ab}b^2d+21\,(ab^3)^{\frac14}\,b^2c+5\,(ab^3)^{\frac34}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\,\left(\frac{a}{b}\right)^{\frac14}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\Big(21\,(ab^3)^{\frac14}\,b^2c-5\,(ab^3)^{\frac34}\,e\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3} - \frac{\sqrt{2}\Big(21\,(ab^3)^{\frac14}\,b^2c-5\,(ab^3)^{\frac34}\,e\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*e*x^3 + 10*a*d*x^2 + 11*a*c*
x)/((b*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a
*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b
^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)
^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e
)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.42

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = \frac{\frac{5\,d\,x^2}{16\,a}+\frac{9\,e\,x^3}{32\,a}+\frac{11\,c\,x}{32\,a}+\frac{7\,b\,c\,x^5}{32\,a^2}+\frac{3\,b\,d\,x^6}{16\,a^2}+\frac{5\,b\,e\,x^7}{32\,a^2}}{a^2+2\,a\,b\,x^4+b^2\,x^8}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(125\,a\,e^3-3024\,b\,c\,d^2+2205\,b\,c^2\,e-1728\,b\,d^3\,x+\mathrm{root}(268435456\,a^{11}\,b^3\,z^4+6881280\,a^6 \cdots\right.}{\cdots}\right.\right.$$
$$+6881280\,a^6\,b^2\,c\,e\,z^2+4718592\,a^6\,b^2\,d^2\,z^2-2709504\,a^3\,b^2\,c^2\,d\,z+153600\,a^4\,b\,d\,e^2\,z$$
$$\left.\left.-60480\,a\,b\,c\,d^2\,e+22050\,a\,b\,c^2\,e^2+20736\,a\,b\,d^4+625\,a^2\,e^4+194481\,b^2\,c^4,z,k)\right)\right)$$

input
```
int((c + d*x + e*x^2)/(a + b*x^4)^3,x)
```

output

```
((5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a
^2) + (3*b*d*x^6)/(16*a^2) + (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 + 2*a*b*
x^4) + symsum(log(-(b*(125*a*e^3 - 3024*b*c*d^2 + 2205*b*c^2*e - 1728*b*d^
3*x + 344064*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 47185
92*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 6048
0*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b
^2*c^4, z, k)^2*a^5*b^2*c - 3200*root(268435456*a^11*b^3*z^4 + 6881280*a^6
*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*
a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 62
5*a^2*e^4 + 194481*b^2*c^4, z, k)*a^3*b*e^2*x + 2520*b*c*d*e*x + 56448*roo
t(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z
^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e +
22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^
2*b^2*c^2*x - 196608*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2
 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*
z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 +
194481*b^2*c^4, z, k)^2*a^5*b^2*d*x + 15360*root(268435456*a^11*b^3*z^4 +
6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*
z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a
*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*r...
```

3.130. $\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^3}\,dx$

3.131 $\int \frac{c+dx+ex^2}{(a-bx^4)^4} dx$

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3.131.1 Optimal result

Integrand size = 21, antiderivative size = 211

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4} dx = \frac{x(c+dx+ex^2)}{12a(a-bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a-bx^4)} + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

output
```
1/12*x*(e*x^2+d*x+c)/a/(-b*x^4+a)^3+1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(-b*x
^4+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c)/a^3/(-b*x^4+a)+5/32*d*arctanh(x^2*b
^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)+1/256*arctan(b^(1/4)*x/a^(1/4))*(-15*e*a^(
1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)+1/256*arctanh(b^(1/4)*x/a^(1/4))*(15*e
*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)
```

3.131.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.31

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$$
$$= \frac{\frac{128a^3x(c+x(d+ex))}{(a-bx^4)^3}+\frac{4ax(77c+15x(4d+3ex))}{a-bx^4}+\frac{16a^2x(11c+x(10d+9ex))}{(a-bx^4)^2}+\frac{6\sqrt[4]{a}\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{3\left(77\sqrt[4]{a}\sqrt{b}c+\right.}{}}{1536a^4}$$

input
```
Integrate[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

output
```
((128*a^3*x*(c + x*(d + e*x)))/(a - b*x^4)^3 + (4*a*x*(77*c + 15*x*(4*d +
3*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a - b*x^4)^2
+ (6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^
(3/4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Lo
g[a^(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c - 40*Sqrt[a]*b^
(1/4)*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (120*Sqrt[a]*d
*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

3.131.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2394, 25, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$$
$$\Big\downarrow \text{2394}$$
$$\frac{x(c+dx+ex^2)}{12a(a-bx^4)^3}-\frac{\int -\frac{9ex^2+10dx+11c}{(a-bx^4)^3}dx}{12a}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{9ex^2+10dx+11c}{(a-bx^4)^3}dx}{12a}+\frac{x(c+dx+ex^2)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}-\frac{\int -\frac{45ex^2+60dx+77c}{(a-bx^4)^2}dx}{8a}}{12a}+\frac{x(c+dx+ex^2)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{45ex^2+60dx+77c}{(a-bx^4)^2}dx}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx+ex^2)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x(77c+60dx+45ex^2)}{4a(a-bx^4)}-\frac{\int -\frac{3(15ex^2+40dx+77c)}{a-bx^4}dx}{4a}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx+ex^2)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{15ex^2+40dx+77c}{a-bx^4}dx}{4a}+\frac{x(77c+60dx+45ex^2)}{4a(a-bx^4)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx+ex^2)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{\frac{3\int \left(\frac{40dx}{a-bx^4}+\frac{15ex^2+77c}{a-bx^4}\right)dx}{4a}+\frac{x(77c+60dx+45ex^2)}{4a(a-bx^4)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx+ex^2)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(77\sqrt{b}c-15\sqrt{a}e)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(15\sqrt{a}e+77\sqrt{b}c)}{2a^{3/4}b^{3/4}}+\frac{20d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\right)}{4a}+\frac{x(77c+60dx+45ex^2)}{4a(a-bx^4)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx+ex^2)}{12a(a-bx^4)^3}$$

input

```
Int[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

3.131. $\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$

output
```
(x*(c + d*x + e*x^2))/(12*a*(a - b*x^4)^3) + ((x*(11*c + 10*d*x + 9*e*x^2)
)/(8*a*(a - b*x^4)^2) + ((x*(77*c + 60*d*x + 45*e*x^2))/(4*a*(a - b*x^4))
+ (3*(((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/
4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*b^(3/4)) + (20*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt
[b])))/(4*a))/(8*a))/(12*a)
```

3.131.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.131.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(-b\,x^4+a)^3}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{\left(15\_R^2e+40\_Rd+77c\right.}{\_R^3}}{512a^3b}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(-b\,x^4+a)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+}{}$

input
```
int((e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9-21/64*b*e/
a^2*x^7-5/12*b*d/a^2*x^6-33/64*b*c/a^2*x^5+113/384/a*e*x^3+11/32*d/a*x^2+5
1/128*c/a*x)/(-b*x^4+a)^3-1/512/a^3/b*sum((15*_R^2*e+40*_R*d+77*c)/_R^3*ln
(x-_R),_R=RootOf(_Z^4*b-a))
```

3.131.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.18 (sec) , antiderivative size = 118903, normalized size of antiderivative = 563.52

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

output
```
Timed out
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.32

$$\int \frac{c+dx+ex^2}{\left(a-bx^4\right)^4}\,dx = -\frac{45\,b^2ex^{11}+60\,b^2dx^{10}+77\,b^2cx^9-126\,abex^7-160\,abdx^6-198\,abcx^5+113\,a^2ex^3+132\,a^2dx^2+153\,a}{384\,(a^3b^3x^{12}-3\,a^4b^2x^8+3\,a^5bx^4-a^6)} + \frac{\frac{40\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,\sqrt{b}c+15\,\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512\,a^3}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

output
```
-1/384*(45*b^2*e*x^11 + 60*b^2*d*x^10 + 77*b^2*c*x^9 - 126*a*b*e*x^7 - 160
*a*b*d*x^6 - 198*a*b*c*x^5 + 113*a^2*e*x^3 + 132*a^2*d*x^2 + 153*a^2*c*x)/
(a^3*b^3*x^12 - 3*a^4*b^2*x^8 + 3*a^5*b*x^4 - a^6) + 1/512*(40*d*log(sqrt(
b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log(sqrt(b)*x^2 - sqrt(a))/(sqr
t(a)*sqrt(b)) + 2*(77*sqrt(b)*c - 15*sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*sqrt(b)*c + 15
*sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt
(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^3
```

3.131.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(170) = 340$.

Time = 0.29 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.75

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$$
$$= -\frac{\sqrt{2}\left(77\,b^2c-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{\sqrt{2}\left(77\,b^2c+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{\sqrt{2}(77\,b^2c-15\,\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$+\frac{\sqrt{2}(77\,b^2c-15\,\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{45\,b^2ex^{11}+60\,b^2dx^{10}+77\,b^2cx^9-126\,abex^7-160\,abdx^6-198\,abcx^5+113\,a^2ex^3+132\,a^2dx^2+153\,a}{384\,(bx^4-a)^3a^3}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

output
```
-1/512*sqrt(2)*(77*b^2*c - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqrt(-a*b)*b
*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 1
5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(
1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)
*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) + 1/1
024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4
) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^2*e*x^11 + 60*b^2*d*x^1
0 + 77*b^2*c*x^9 - 126*a*b*e*x^7 - 160*a*b*d*x^6 - 198*a*b*c*x^5 + 113*a^2
*e*x^3 + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

3.131. $\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$

3.131.9 Mupad [B] (verification not implemented)

Time = 9.95 (sec) , antiderivative size = 874, normalized size of antiderivative = 4.14

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$$
$$= \frac{\frac{11\,d\,x^2}{32\,a}+\frac{113\,e\,x^3}{384\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{15\,b^2\,e\,x^{11}}{128\,a^3}-\frac{33\,b\,c\,x^5}{64\,a^2}-\frac{5\,b\,d\,x^6}{12\,a^2}-\frac{21\,b\,e\,x^7}{64\,a^2}}{a^3-3\,a^2\,b\,x^4+3\,a\,b^2\,x^8-b^3\,x^{12}}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(3375\,a\,e^3+123200\,b\,c\,d^2-88935\,b\,c^2\,e+64000\,b\,d^3\,x+\mathrm{root}(68719476736\,a^{15}\,b^3\,z^4-12\right.}{}\right.\right.$$
$$-1211105280\,a^8\,b^2\,c\,e\,z^2-838860800\,a^8\,b^2\,d^2\,z^2+485703680\,a^4\,b^2\,c^2\,d\,z$$
$$+18432000\,a^5\,b\,d\,e^2\,z-7392000\,a\,b\,c\,d^2\,e+2668050\,a\,b\,c^2\,e^2+2560000\,a\,b\,d^4$$
$$\left.\left.-35153041\,b^2\,c^4-50625\,a^2\,e^4,z,k)\right)\right)$$

input
```
int((c + d*x + e*x^2)/(a - b*x^4)^4,x)
```

output
```
((11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^
9)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) - (33*b
*c*x^5)/(64*a^2) - (5*b*d*x^6)/(12*a^2) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^
3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log(-(b*(3375*a*e^3 + 123200*
b*c*d^2 - 88935*b*c^2*e + 64000*b*d^3*x + 20185088*root(68719476736*a^15*b
^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 48570368
0*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050
*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)^2
*a^7*b^2*c + 115200*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e
*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^
5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4
- 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^4*b*e^2*x - 92400*b*c*d*e*x +
3035648*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 83886
0800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z
- 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b
^2*c^4 - 50625*a^2*e^4, z, k)*a^3*b^2*c^2*x - 10485760*root(68719476736*a^
15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 4857
03680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 266
8050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z,
k)^2*a^7*b^2*d*x - 614400*root(68719476736*a^15*b^3*z^4 - 1211105280*a^...
```

3.132 $\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$

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3.132.1 Optimal result

Integrand size = 20, antiderivative size = 372

$$\begin{aligned}\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx &= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3}+\frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)}\\
&+\frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}}\\
&+\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}}\\
&-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}\\
&+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}\end{aligned}$$

output

```
1/12*x*(e*x^2+d*x+c)/a/(b*x^4+a)^3+1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(b*x^4
+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c)/a^3/(b*x^4+a)+5/32*d*arctan(x^2*b^(1/
2)/a^(1/2))/a^(7/2)/b^(1/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x
^2*b^(1/2))*(-15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1/1024*l
n(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-15*e*a^(1/2)+77*c*b^(1/
2))/a^(15/4)/b^(3/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(1
5*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1/512*arctan(1+b^(1/4)*
x*2^(1/2)/a^(1/4))*(15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{256a^3x(c+x(d+ex))}{(a+bx^4)^3}+\frac{8ax(77c+15x(4d+3ex))}{a+bx^4}+\frac{32a^2x(11c+x(10d+9ex))}{(a+bx^4)^2}-\frac{6\sqrt[4]{a}\left(77\sqrt{2}\sqrt{b}c+80\sqrt[4]{a}\sqrt[4]{b}d+15\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\ldots}{\ldots}\right)}{b^{3/4}}\ldots}{\ldots}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

output
```
((256*a^3*x*(c + x*(d + e*x)))/(a + b*x^4)^3 + (8*a*x*(77*c + 15*x*(4*d +
3*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a + b*x^4)^2
- (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqr
t[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (6*a^(1/4)*(77*
Sqrt[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*Sqrt[b]*
c + 15*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/
b^(3/4) + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^4)
```

3.132.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2394, 25, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$

$$\Big\downarrow \; 2394$$

$$\frac{x(c+dx+ex^2)}{12a(a+bx^4)^3}-\frac{\int -\frac{9ex^2+10dx+11c}{(bx^4+a)^3}dx}{12a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{9ex^2+10dx+11c}{(bx^4+a)^3}dx}{12a}+\frac{x(c+dx+ex^2)}{12a(a+bx^4)^3}$$

$\Big\downarrow$ 2394

$$\frac{\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}-\frac{\int -\frac{45ex^2+60dx+77c}{(bx^4+a)^2}dx}{8a}}{12a}+\frac{x(c+dx+ex^2)}{12a(a+bx^4)^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{45ex^2+60dx+77c}{(bx^4+a)^2}dx}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}+\frac{x(c+dx+ex^2)}{12a(a+bx^4)^3}$$

$\Big\downarrow$ 2394

$$\frac{\frac{\frac{x(77c+60dx+45ex^2)}{4a(a+bx^4)}-\frac{\int -\frac{3(15ex^2+40dx+77c)}{bx^4+a}dx}{4a}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}+\frac{x(c+dx+ex^2)}{12a(a+bx^4)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{15ex^2+40dx+77c}{bx^4+a}dx}{4a}+\frac{x(77c+60dx+45ex^2)}{4a(a+bx^4)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}+\frac{x(c+dx+ex^2)}{12a(a+bx^4)^3}$$

$\Big\downarrow$ 2415

$$\frac{\frac{\frac{3\int \left(\frac{40dx}{bx^4+a}+\frac{15ex^2+77c}{bx^4+a}\right)dx}{4a}+\frac{x(77c+60dx+45ex^2)}{4a(a+bx^4)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}+\frac{x(c+dx+ex^2)}{12a(a+bx^4)^3}$$

$\Big\downarrow$ 2009

$$\frac{\frac{3\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(15\sqrt{a}e+77\sqrt{b}c)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(15\sqrt{a}e+77\sqrt{b}c)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{(77\sqrt{b}c-15\sqrt{a}e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{(77\sqrt{b}c-15\sqrt{a}e)\log\ldots}{4\ldots}\ldots}{4a}\ldots}{8a}\ldots}{12a}$$

$$\frac{x(c+dx+ex^2)}{12a(a+bx^4)^3}$$

3.132. $\int \frac{c+dx+ex^2}{(a+bx^4)^4}dx$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

output
```
(x*(c + d*x + e*x^2))/(12*a*(a + b*x^4)^3) + ((x*(11*c + 10*d*x + 9*e*x^2)
)/(8*a*(a + b*x^4)^2) + ((x*(77*c + 60*d*x + 45*e*x^2))/(4*a*(a + b*x^4))
+ (3*((20*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - ((77*Sqrt[b
]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(3/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a]*e
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/
4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))))/(4*a))/(8*a))/(12
*a)
```

3.132.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.132.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{(15\_R^2e+40\_Rd+77c}{\_R^3}}{512a^3b}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(b\,x^4+a)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\right.}{8a}}{}$

input
```
int((e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+21/64*b*e/
a^2*x^7+5/12*b*d/a^2*x^6+33/64*b*c/a^2*x^5+113/384/a*e*x^3+11/32*d/a*x^2+5
1/128*c/a*x)/(b*x^4+a)^3+1/512/a^3/b*sum((15*_R^2*e+40*_R*d+77*c)/_R^3*ln(
x-_R),_R=RootOf(_Z^4*b+a))
```

3.132.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.97 (sec) , antiderivative size = 124960, normalized size of antiderivative = 335.91

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

output
```
Timed out
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$
$$= \frac{45\,b^2ex^{11}+60\,b^2dx^{10}+77\,b^2cx^9+126\,abex^7+160\,abdx^6+198\,abcx^5+113\,a^2ex^3+132\,a^2dx^2+153\,a^2cx}{384\,(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6)}$$
$$+\frac{\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+15\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e\cdots\right.}{\cdots}}{1024\,a^3}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/384*(45*b^2*e*x^11 + 60*b^2*d*x^10 + 77*b^2*c*x^9 + 126*a*b*e*x^7 + 160*
a*b*d*x^6 + 198*a*b*c*x^5 + 113*a^2*e*x^3 + 132*a^2*d*x^2 + 153*a^2*c*x)/(
a^3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6) + 1/1024*(sqrt(2)*(77*sq
rt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt
(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*sqrt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)
*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt
(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e - 80*sqrt(a)*sqrt(b)*
d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)
*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)
*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e + 80*sqrt(a)*sqrt(b)*d)*arctan(1
/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^3
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$

$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$

$$+ \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$

$$+ \frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$

$$- \frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$

$$+ \frac{45\,b^2ex^{11}+60\,b^2dx^{10}+77\,b^2cx^9+126\,abex^7+160\,abdx^6+198\,abcx^5+113\,a^2ex^3+132\,a^2dx^2+153\,a}{384\,(bx^4+a)^3a^3}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a
*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*
b^2*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))
/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b
^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/10
24*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^2*e*x^11 + 60*b^2*d*x^
10 + 77*b^2*c*x^9 + 126*a*b*e*x^7 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 113*a^
2*e*x^3 + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

3.132.9 Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 873, normalized size of antiderivative = 2.35

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$
$$= \frac{\frac{11\,d\,x^2}{32\,a}+\frac{113\,e\,x^3}{384\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{15\,b^2\,e\,x^{11}}{128\,a^3}+\frac{33\,b\,c\,x^5}{64\,a^2}+\frac{5\,b\,d\,x^6}{12\,a^2}+\frac{21\,b\,e\,x^7}{64\,a^2}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(3375\,a\,e^3-123200\,b\,c\,d^2+88935\,b\,c^2\,e-64000\,b\,d^3\,x+\mathrm{root}(68719476736\,a^{15}\,b^3\,z^4+12\right.}{}\right.\right.$$
$$+1211105280\,a^8\,b^2\,c\,e\,z^2+838860800\,a^8\,b^2\,d^2\,z^2-485703680\,a^4\,b^2\,c^2\,d\,z$$
$$+18432000\,a^5\,b\,d\,e^2\,z-7392000\,a\,b\,c\,d^2\,e+2668050\,a\,b\,c^2\,e^2+2560000\,a\,b\,d^4$$
$$\left.+35153041\,b^2\,c^4+50625\,a^2\,e^4,z,k)\right)$$

input
```
int((c + d*x + e*x^2)/(a + b*x^4)^4,x)
```

output
```
((11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^
9)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (33*b
*c*x^5)/(64*a^2) + (5*b*d*x^6)/(12*a^2) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^
3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log(-(b*(3375*a*e^3 - 123200*
b*c*d^2 + 88935*b*c^2*e - 64000*b*d^3*x + 20185088*root(68719476736*a^15*b
^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 48570368
0*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050
*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)^2
*a^7*b^2*c - 115200*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e
*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^
5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4
+ 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^4*b*e^2*x + 92400*b*c*d*e*x +
3035648*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 83886
0800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z
- 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b
^2*c^4 + 50625*a^2*e^4, z, k)*a^3*b^2*c^2*x - 10485760*root(68719476736*a^
15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 4857
03680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 266
8050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z,
k)^2*a^7*b^2*d*x + 614400*root(68719476736*a^15*b^3*z^4 + 1211105280*a^...
```

3.133 $\int a(e+fx^4)^2\,dx$

3.133.1 Optimal result 1117
3.133.2 Mathematica [A] (verified) 1117
3.133.3 Rubi [A] (verified) 1118
3.133.4 Maple [A] (verified) 1119
3.133.5 Fricas [A] (verification not implemented) 1119
3.133.6 Sympy [A] (verification not implemented) 1119
3.133.7 Maxima [A] (verification not implemented) 1120
3.133.8 Giac [A] (verification not implemented) 1120
3.133.9 Mupad [B] (verification not implemented) 1120

3.133.1 Optimal result

Integrand size = 11, antiderivative size = 28

$$\int a(e+fx^4)^2\,dx = ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9$$

output
```
a*e^2*x+2/5*a*e*f*x^5+1/9*a*f^2*x^9
```

3.133.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int a(e+fx^4)^2\,dx = a\left(e^2x+\frac{2}{5}efx^5+\frac{f^2x^9}{9}\right)$$

input
```
Integrate[a*(e + f*x^4)^2,x]
```

output
```
a*(e^2*x + (2*e*f*x^5)/5 + (f^2*x^9)/9)
```

3.133.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {27, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \; 27$$

$$a\int \left(fx^4+e\right)^2 dx$$

$$\Big\downarrow \; 747$$

$$a\int \left(f^2x^8+2efx^4+e^2\right) dx$$

$$\Big\downarrow \; 2009$$

$$a\left(e^2x+\frac{2}{5}efx^5+\frac{f^2x^9}{9}\right)$$

input
```
Int[a*(e + f*x^4)^2,x]
```

output
```
a*(e^2*x + (2*e*f*x^5)/5 + (f^2*x^9)/9)
```

3.133.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.133.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
default	$\left(\frac{1}{9}f^{2}x^{9}+\frac{2}{5}ef\,x^{5}+e^{2}x\right)a$	24
parallelrisch	$\left(\frac{1}{9}f^{2}x^{9}+\frac{2}{5}ef\,x^{5}+e^{2}x\right)a$	24
norman	$a\,e^{2}x+\frac{2}{5}aef\,x^{5}+\frac{1}{9}a\,f^{2}x^{9}$	25
risch	$a\,e^{2}x+\frac{2}{5}aef\,x^{5}+\frac{1}{9}a\,f^{2}x^{9}$	25
gosper	$\frac{x\left(5f^{2}x^{8}+18ef\,x^{4}+45e^{2}\right)a}{45}$	26

input
```
int(a*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/9*f^2*x^9+2/5*e*f*x^5+e^2*x)*a
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int a\left(e+fx^{4}\right)^{2}\,dx=\frac{1}{9}\,af^{2}x^{9}+\frac{2}{5}\,aefx^{5}+ae^{2}x$$

input
```
integrate(a*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/9*a*f^2*x^9 + 2/5*a*e*f*x^5 + a*e^2*x
```

3.133.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int a\left(e+fx^{4}\right)^{2}\,dx=ae^{2}x+\frac{2aefx^{5}}{5}+\frac{af^{2}x^{9}}{9}$$

input
```
integrate(a*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int a(e+fx^4)^2\,dx = \frac{1}{45}\left(5\,f^2x^9+18\,efx^5+45\,e^2x\right)a$$

input
```
integrate(a*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/45*(5*f^2*x^9 + 18*e*f*x^5 + 45*e^2*x)*a
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int a(e+fx^4)^2\,dx = \frac{1}{45}\left(5\,f^2x^9+18\,efx^5+45\,e^2x\right)a$$

input
```
integrate(a*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/45*(5*f^2*x^9 + 18*e*f*x^5 + 45*e^2*x)*a
```

3.133.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int a(e+fx^4)^2\,dx = \frac{a\,x\,(45\,e^2+18\,e\,f\,x^4+5\,f^2\,x^8)}{45}$$

input
```
int(a*(e + f*x^4)^2,x)
```

output
```
(a*x*(45*e^2 + 5*f^2*x^8 + 18*e*f*x^4))/45
```

3.134 $\int bx(e+fx^4)^2\,dx$

3.134.1 Optimal result 1121
3.134.2 Mathematica [A] (verified) 1121
3.134.3 Rubi [A] (verified) 1122
3.134.4 Maple [A] (verified) 1123
3.134.5 Fricas [A] (verification not implemented) 1123
3.134.6 Sympy [A] (verification not implemented) 1123
3.134.7 Maxima [A] (verification not implemented) 1124
3.134.8 Giac [A] (verification not implemented) 1124
3.134.9 Mupad [B] (verification not implemented) 1124

3.134.1 Optimal result

Integrand size = 12, antiderivative size = 33

$$\int bx(e+fx^4)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

output
```
1/2*b*e^2*x^2+1/3*b*e*f*x^6+1/10*b*f^2*x^10
```

3.134.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int bx(e+fx^4)^2\,dx=b\left(\frac{e^2x^2}{2}+\frac{1}{3}efx^6+\frac{f^2x^{10}}{10}\right)$$

input
```
Integrate[b*x*(e + f*x^4)^2,x]
```

output
```
b*((e^2*x^2)/2 + (e*f*x^6)/3 + (f^2*x^10)/10)
```

3.134.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int bx(e+fx^4)^2\,dx$$

$$\Big\downarrow \; 27$$

$$b\int x(fx^4+e)^2\,dx$$

$$\Big\downarrow \; 802$$

$$b\int (f^2x^9+2efx^5+e^2x)\,dx$$

$$\Big\downarrow \; 2009$$

$$b\left(\frac{e^2x^2}{2}+\frac{1}{3}efx^6+\frac{f^2x^{10}}{10}\right)$$

input
```
Int[b*x*(e + f*x^4)^2,x]
```

output
```
b*((e^2*x^2)/2 + (e*f*x^6)/3 + (f^2*x^10)/10)
```

3.134.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.134.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
default	$\left(\frac{1}{10} f^{2} x^{10}+\frac{1}{3} e f \,x^{6}+\frac{1}{2} x^{2} e^{2}\right) b$	27
parallelrisch	$\left(\frac{1}{10} f^{2} x^{10}+\frac{1}{3} e f \,x^{6}+\frac{1}{2} x^{2} e^{2}\right) b$	27
gosper	$\frac{x^{2} \left(3 f^{2} x^{8}+10 e f \,x^{4}+15 e^{2}\right) b}{30}$	28
norman	$\frac{1}{2} b \,e^{2} x^{2}+\frac{1}{3} b e f \,x^{6}+\frac{1}{10} b \,f^{2} x^{10}$	28
risch	$\frac{1}{2} b \,e^{2} x^{2}+\frac{1}{3} b e f \,x^{6}+\frac{1}{10} b \,f^{2} x^{10}$	28

input
```
int(b*x*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/10*f^2*x^10+1/3*e*f*x^6+1/2*x^2*e^2)*b
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx\left(e+fx^4\right)^2\,dx=\frac{1}{10}\,bf^2x^{10}+\frac{1}{3}\,befx^6+\frac{1}{2}\,be^2x^2$$

input
```
integrate(b*x*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/10*b*f^2*x^10 + 1/3*b*e*f*x^6 + 1/2*b*e^2*x^2
```

3.134.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int bx\left(e+fx^4\right)^2\,dx=\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}$$

input
```
integrate(b*x*(f*x**4+e)**2,x)
```

output
```
b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx(e+fx^4)^2\,dx = \frac{1}{30}\left(3\,f^2x^{10}+10\,efx^6+15\,e^2x^2\right)b$$

input
```
integrate(b*x*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/30*(3*f^2*x^10 + 10*e*f*x^6 + 15*e^2*x^2)*b
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx(e+fx^4)^2\,dx = \frac{1}{30}\left(3\,f^2x^{10}+10\,efx^6+15\,e^2x^2\right)b$$

input
```
integrate(b*x*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/30*(3*f^2*x^10 + 10*e*f*x^6 + 15*e^2*x^2)*b
```

3.134.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx(e+fx^4)^2\,dx = \frac{b\,x^2\,\left(15\,e^2+10\,e\,f\,x^4+3\,f^2\,x^8\right)}{30}$$

input
```
int(b*x*(e + f*x^4)^2,x)
```

output
```
(b*x^2*(15*e^2 + 3*f^2*x^8 + 10*e*f*x^4))/30
```

3.135 $\int (a+bx)\left(e+fx^4\right)^2\,dx$

3.135.1 Optimal result 1125
3.135.2 Mathematica [A] (verified) 1125
3.135.3 Rubi [A] (verified) 1126
3.135.4 Maple [A] (verified) 1127
3.135.5 Fricas [A] (verification not implemented) 1127
3.135.6 Sympy [A] (verification not implemented) 1127
3.135.7 Maxima [A] (verification not implemented) 1128
3.135.8 Giac [A] (verification not implemented) 1128
3.135.9 Mupad [B] (verification not implemented) 1128

3.135.1 Optimal result

Integrand size = 15, antiderivative size = 60

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}$$

output
```
a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2
*x^10
```

3.135.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}$$

input
```
Integrate[(a + b*x)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9
+ (b*f^2*x^10)/10
```

3.135.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(ae^2+2aefx^4+af^2x^8+be^2x+2befx^5+bf^2x^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

input
```
Int[(a + b*x)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9
+ (b*f^2*x^10)/10
```

3.135.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.135.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
gosper	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}$	51
default	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}$	51
norman	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}$	51
risch	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}$	51
parallelrisch	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}$	51

input
```
int((b*x+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2
*x^10
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{2}\,be^2x^2+ae^2x$$

input
```
integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^
2*x^2 + a*e^2*x
```

3.135.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}$$

input
```
integrate((b*x+a)*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
 b*f**2*x**10/10
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{2}\,be^2x^2+ae^2x$$

input
```
integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^
2*x^2 + a*e^2*x
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{2}\,be^2x^2+ae^2x$$

input
```
integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^
2*x^2 + a*e^2*x
```

3.135.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=\frac{b\,e^2\,x^2}{2}+a\,e^2\,x+\frac{b\,e\,f\,x^6}{3}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{b\,f^2\,x^{10}}{10}+\frac{a\,f^2\,x^9}{9}$$

input
```
int((e + f*x^4)^2*(a + b*x),x)
```

output
```
(b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + a*e^2*x + (2*a*e*f*x^5)/
5 + (b*e*f*x^6)/3
```

3.136 $\int cx^2(e+fx^4)^2\,dx$

3.136.1 Optimal result 1129
3.136.2 Mathematica [A] (verified) 1129
3.136.3 Rubi [A] (verified) 1130
3.136.4 Maple [A] (verified) 1131
3.136.5 Fricas [A] (verification not implemented) 1131
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3.136.7 Maxima [A] (verification not implemented) 1132
3.136.8 Giac [A] (verification not implemented) 1132
3.136.9 Mupad [B] (verification not implemented) 1132

3.136.1 Optimal result

Integrand size = 14, antiderivative size = 33

$$\int cx^2(e+fx^4)^2\,dx=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

output
```
1/3*c*e^2*x^3+2/7*c*e*f*x^7+1/11*c*f^2*x^11
```

3.136.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int cx^2(e+fx^4)^2\,dx=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

input
```
Integrate[c*x^2*(e + f*x^4)^2,x]
```

output
```
(c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

3.136.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int cx^2\left(e+fx^4\right)^2\,dx$$
$$\Big\downarrow \; 27$$
$$c\int x^2\left(fx^4+e\right)^2\,dx$$
$$\Big\downarrow \; 802$$
$$c\int \left(f^2x^{10}+2efx^6+e^2x^2\right)dx$$
$$\Big\downarrow \; 2009$$
$$c\left(\frac{e^2x^3}{3}+\frac{2}{7}efx^7+\frac{f^2x^{11}}{11}\right)$$

input
```
Int[c*x^2*(e + f*x^4)^2,x]
```

output
```
c*((e^2*x^3)/3 + (2*e*f*x^7)/7 + (f^2*x^11)/11)
```

3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.136.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
default	$\left(\frac{1}{11}f^2x^{11}+\frac{2}{7}ef\,x^7+\frac{1}{3}e^2x^3\right)c$	27
parallelrisch	$\left(\frac{1}{11}f^2x^{11}+\frac{2}{7}ef\,x^7+\frac{1}{3}e^2x^3\right)c$	27
gosper	$\frac{x^3(21f^2x^8+66ef\,x^4+77e^2)c}{231}$	28
norman	$\frac{1}{3}c\,e^2x^3+\frac{2}{7}cef\,x^7+\frac{1}{11}c\,f^2x^{11}$	28
risch	$\frac{1}{3}c\,e^2x^3+\frac{2}{7}cef\,x^7+\frac{1}{11}c\,f^2x^{11}$	28

input
```
int(c*x^2*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/11*f^2*x^11+2/7*e*f*x^7+1/3*e^2*x^3)*c
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2(e+fx^4)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{2}{7}\,cefx^7+\frac{1}{3}\,ce^2x^3$$

input
```
integrate(c*x^2*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/11*c*f^2*x^11 + 2/7*c*e*f*x^7 + 1/3*c*e^2*x^3
```

3.136.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int cx^2(e+fx^4)^2\,dx=\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

input
```
integrate(c*x**2*(f*x**4+e)**2,x)
```

output
```
c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2(e+fx^4)^2\,dx = \frac{1}{231}\left(21\,f^2x^{11}+66\,efx^7+77\,e^2x^3\right)c$$

input
```
integrate(c*x^2*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/231*(21*f^2*x^11 + 66*e*f*x^7 + 77*e^2*x^3)*c
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2(e+fx^4)^2\,dx = \frac{1}{231}\left(21\,f^2x^{11}+66\,efx^7+77\,e^2x^3\right)c$$

input
```
integrate(c*x^2*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/231*(21*f^2*x^11 + 66*e*f*x^7 + 77*e^2*x^3)*c
```

3.136.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2(e+fx^4)^2\,dx = \frac{c\,x^3\,(77\,e^2+66\,e\,f\,x^4+21\,f^2\,x^8)}{231}$$

input
```
int(c*x^2*(e + f*x^4)^2,x)
```

output
```
(c*x^3*(77*e^2 + 21*f^2*x^8 + 66*e*f*x^4))/231
```

3.137 $\int (a+cx^2) (e+fx^4)^2 \, dx$

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3.137.2 Mathematica [A] (verified) . . . 1133
3.137.3 Rubi [A] (verified) . . . 1134
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3.137.5 Fricas [A] (verification not implemented) . . . 1135
3.137.6 Sympy [A] (verification not implemented) . . . 1135
3.137.7 Maxima [A] (verification not implemented) . . . 1136
3.137.8 Giac [A] (verification not implemented) . . . 1136
3.137.9 Mupad [B] (verification not implemented) . . . 1136

3.137.1 Optimal result

Integrand size = 17, antiderivative size = 60

$$\int (a+cx^2) (e+fx^4)^2 \, dx = ae^2x + \frac{1}{3}ce^2x^3 + \frac{2}{5}aefx^5 + \frac{2}{7}cefx^7 + \frac{1}{9}af^2x^9 + \frac{1}{11}cf^2x^{11}$$

output
```
a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2
*x^11
```

3.137.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+cx^2) (e+fx^4)^2 \, dx = ae^2x + \frac{1}{3}ce^2x^3 + \frac{2}{5}aefx^5 + \frac{2}{7}cefx^7 + \frac{1}{9}af^2x^9 + \frac{1}{11}cf^2x^{11}$$

input
```
Integrate[(a + c*x^2)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/
9 + (c*f^2*x^11)/11
```

3.137.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(ae^2+2aefx^4+af^2x^8+ce^2x^2+2cefx^6+cf^2x^{10}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

input
```
Int[(a + c*x^2)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/
9 + (c*f^2*x^11)/11
```

3.137.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.137.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
gosper	$a\,e^{2} x+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51
default	$a\,e^{2} x+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51
norman	$a\,e^{2} x+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51
risch	$a\,e^{2} x+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51
parallelrisch	$a\,e^{2} x+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51

input
```
int((c*x^2+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2
*x^11
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+cx^2)\left(e+fx^4\right)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cefx^7+\frac{2}{5}\,aefx^5+\frac{1}{3}\,ce^2x^3+ae^2x$$

input
```
integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^
2*x^3 + a*e^2*x
```

3.137.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+cx^2)\left(e+fx^4\right)^2\,dx=ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

input
```
integrate((c*x**2+a)*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7
 + c*f**2*x**11/11
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cefx^7+\frac{2}{5}\,aefx^5+\frac{1}{3}\,ce^2x^3+ae^2x$$

input
```
integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^
2*x^3 + a*e^2*x
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cefx^7+\frac{2}{5}\,aefx^5+\frac{1}{3}\,ce^2x^3+ae^2x$$

input
```
integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^
2*x^3 + a*e^2*x
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{c\,e^2\,x^3}{3}+a\,e^2\,x+\frac{2\,c\,e\,f\,x^7}{7}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{c\,f^2\,x^{11}}{11}+\frac{a\,f^2\,x^9}{9}$$

input
```
int((a + c*x^2)*(e + f*x^4)^2,x)
```

output
```
(a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (c*f^2*x^11)/11 + a*e^2*x + (2*a*e*f*x^5)/
5 + (2*c*e*f*x^7)/7
```

3.138 $\int (bx + cx^2) (e + fx^4)^2 \, dx$

3.138.1 Optimal result 1137
3.138.2 Mathematica [A] (verified) 1137
3.138.3 Rubi [A] (verified) 1138
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3.138.5 Fricas [A] (verification not implemented) 1139
3.138.6 Sympy [A] (verification not implemented) 1139
3.138.7 Maxima [A] (verification not implemented) 1140
3.138.8 Giac [A] (verification not implemented) 1140
3.138.9 Mupad [B] (verification not implemented) 1140

3.138.1 Optimal result

Integrand size = 19, antiderivative size = 65

$$\int (bx + cx^2) (e + fx^4)^2 \, dx = \frac{1}{2} be^2 x^2 + \frac{1}{3} ce^2 x^3 + \frac{1}{3} befx^6 + \frac{2}{7} cefx^7 + \frac{1}{10} bf^2 x^{10} + \frac{1}{11} cf^2 x^{11}$$

output
```
1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^2*x^10+1/
11*c*f^2*x^11
```

3.138.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int (bx + cx^2) (e + fx^4)^2 \, dx = \frac{1}{2} be^2 x^2 + \frac{1}{3} ce^2 x^3 + \frac{1}{3} befx^6 + \frac{2}{7} cefx^7 + \frac{1}{10} bf^2 x^{10} + \frac{1}{11} cf^2 x^{11}$$

input
```
Integrate[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

output
```
(b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x
^10)/10 + (c*f^2*x^11)/11
```

3.138.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2027, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{2027}$$

$$\int x(b+cx)\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(be^2x+2befx^5+bf^2x^9+ce^2x^2+2cefx^6+cf^2x^{10}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

input
```
Int[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

output
```
(b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x
^10)/10 + (c*f^2*x^11)/11
```

3.138.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.138.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^2(210c f^2 x^9+231b f^2 x^8+660cef x^5+770bef x^4+770c e^2 x+1155e^2 b)}{2310}$	54
default	$\frac{1}{2}b e^2 x^2+\frac{1}{3}c e^2 x^3+\frac{1}{3}bef x^6+\frac{2}{7}cef x^7+\frac{1}{10}b f^2 x^{10}+\frac{1}{11}c f^2 x^{11}$	54
norman	$\frac{1}{2}b e^2 x^2+\frac{1}{3}c e^2 x^3+\frac{1}{3}bef x^6+\frac{2}{7}cef x^7+\frac{1}{10}b f^2 x^{10}+\frac{1}{11}c f^2 x^{11}$	54
risch	$\frac{1}{2}b e^2 x^2+\frac{1}{3}c e^2 x^3+\frac{1}{3}bef x^6+\frac{2}{7}cef x^7+\frac{1}{10}b f^2 x^{10}+\frac{1}{11}c f^2 x^{11}$	54
parallelrisch	$\frac{1}{2}b e^2 x^2+\frac{1}{3}c e^2 x^3+\frac{1}{3}bef x^6+\frac{2}{7}cef x^7+\frac{1}{10}b f^2 x^{10}+\frac{1}{11}c f^2 x^{11}$	54

input
```
int((c*x^2+b*x)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2310*x^2*(210*c*f^2*x^9+231*b*f^2*x^8+660*c*e*f*x^5+770*b*e*f*x^4+770*c*
e^2*x+1155*b*e^2)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (bx+cx^2)\left(e+fx^4\right)^2\,dx=\frac{1}{11}cf^2x^{11}+\frac{1}{10}bf^2x^{10}+\frac{2}{7}cefx^7+\frac{1}{3}befx^6+\frac{1}{3}ce^2x^3+\frac{1}{2}be^2x^2$$

input
```
integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/3*c*
e^2*x^3 + 1/2*b*e^2*x^2
```

3.138.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (bx+cx^2)\left(e+fx^4\right)^2\,dx=\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

input
```
integrate((c*x**2+b*x)*(f*x**4+e)**2,x)
```

output
```
b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x
**7/7 + c*f**2*x**11/11
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (bx+cx^2)\left(e+fx^4\right)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2$$

input
```
integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/3*c*
e^2*x^3 + 1/2*b*e^2*x^2
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (bx+cx^2)\left(e+fx^4\right)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2$$

input
```
integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/3*c*
e^2*x^3 + 1/2*b*e^2*x^2
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (bx+cx^2)\left(e+fx^4\right)^2\,dx=\frac{c\,e^2\,x^3}{3}+\frac{b\,e^2\,x^2}{2}+\frac{2\,c\,e\,f\,x^7}{7}+\frac{b\,e\,f\,x^6}{3}+\frac{c\,f^2\,x^{11}}{11}+\frac{b\,f^2\,x^{10}}{10}$$

input
```
int((b*x + c*x^2)*(e + f*x^4)^2,x)
```

output
```
(b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (b*e*f
*x^6)/3 + (2*c*e*f*x^7)/7
```

3.139 $\int (a+bx+cx^2)(e+fx^4)^2\,dx$

3.139.1 Optimal result 1141
3.139.2 Mathematica [A] (verified) 1141
3.139.3 Rubi [A] (verified) 1142
3.139.4 Maple [A] (verified) 1143
3.139.5 Fricas [A] (verification not implemented) 1143
3.139.6 Sympy [A] (verification not implemented) 1144
3.139.7 Maxima [A] (verification not implemented) 1144
3.139.8 Giac [A] (verification not implemented) 1144
3.139.9 Mupad [B] (verification not implemented) 1145

3.139.1 Optimal result

Integrand size = 20, antiderivative size = 92

$$\int (a+bx+cx^2)(e+fx^4)^2\,dx = ae^2x+\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

output
```
a*e^2*x+1/2*b*e^2*x^2+1/3*c*e^2*x^3+2/5*a*e*f*x^5+1/3*b*e*f*x^6+2/7*c*e*f*
x^7+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/11*c*f^2*x^11
```

3.139.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2)(e+fx^4)^2\,dx = ae^2x+\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

input
```
Integrate[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3
+ (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

3.139.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e+fx^4\right)^2 \left(a+bx+cx^2\right) dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(ae^2+2aefx^4+af^2x^8+be^2x+2befx^5+bf^2x^9+ce^2x^2+2cefx^6+cf^2x^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

input
```
Int[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3
+ (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

3.139.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.139.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result
gosper	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$
default	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$
norman	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$
risch	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$
parallelrisch	$a\,e^2x+\frac{1}{2}b\,e^2x^2+\frac{1}{3}c\,e^2x^3+\frac{2}{5}aef\,x^5+\frac{1}{3}bef\,x^6+\frac{2}{7}cef\,x^7+\frac{1}{9}a\,f^2x^9+\frac{1}{10}b\,f^2x^{10}+\frac{1}{11}c\,f^2x^{11}$

input
```
int((c*x^2+b*x+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a*e^2*x+1/2*b*e^2*x^2+1/3*c*e^2*x^3+2/5*a*e*f*x^5+1/3*b*e*f*x^6+2/7*c*e*f*
x^7+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/11*c*f^2*x^11
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int (a+bx+cx^2)\left(e+fx^4\right)^2\,dx = \frac{1}{11}cf^2x^{11}+\frac{1}{10}bf^2x^{10}+\frac{1}{9}af^2x^9+\frac{2}{7}cefx^7+\frac{1}{3}befx^6+\frac{2}{5}aefx^5+\frac{1}{3}ce^2x^3+\frac{1}{2}be^2x^2+ae^2x$$

input
```
integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*
e*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x
```

3.139.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$\int (a+bx+cx^2)(e+fx^4)^2\,dx = ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

input
```
integrate((c*x**2+b*x+a)*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
 b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int (a+bx+cx^2)(e+fx^4)^2\,dx = \frac{1}{11}cf^2x^{11} + \frac{1}{10}bf^2x^{10} + \frac{1}{9}af^2x^9 + \frac{2}{7}cefx^7 + \frac{1}{3}befx^6 + \frac{2}{5}aefx^5 + \frac{1}{3}ce^2x^3 + \frac{1}{2}be^2x^2 + ae^2x$$

input
```
integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*
e*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int (a+bx+cx^2)(e+fx^4)^2\,dx = \frac{1}{11}cf^2x^{11} + \frac{1}{10}bf^2x^{10} + \frac{1}{9}af^2x^9 + \frac{2}{7}cefx^7 + \frac{1}{3}befx^6 + \frac{2}{5}aefx^5 + \frac{1}{3}ce^2x^3 + \frac{1}{2}be^2x^2 + ae^2x$$

input
```
integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*
e*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x
```

3.139.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int (a+bx+cx^2)(e+fx^4)^2\,dx = \frac{c\,e^2\,x^3}{3}+\frac{b\,e^2\,x^2}{2}+a\,e^2\,x+\frac{2\,c\,e\,f\,x^7}{7}+\frac{b\,e\,f\,x^6}{3}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{c\,f^2\,x^{11}}{11}+\frac{b\,f^2\,x^{10}}{10}+\frac{a\,f^2\,x^9}{9}$$

input
```
int((e + f*x^4)^2*(a + b*x + c*x^2),x)
```

output
```
(b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (c*f^2*x
^11)/11 + a*e^2*x + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7
```

3.140 $\int dx^3(e+fx^4)^2\,dx$

3.140.1 Optimal result 1146
3.140.2 Mathematica [A] (verified) 1146
3.140.3 Rubi [A] (verified) 1147
3.140.4 Maple [A] (verified) 1148
3.140.5 Fricas [A] (verification not implemented) 1148
3.140.6 Sympy [B] (verification not implemented) 1148
3.140.7 Maxima [A] (verification not implemented) 1149
3.140.8 Giac [A] (verification not implemented) 1149
3.140.9 Mupad [B] (verification not implemented) 1149

3.140.1 Optimal result

Integrand size = 14, antiderivative size = 17

$$\int dx^3\left(e+fx^4\right)^2\,dx=\frac{d(e+fx^4)^3}{12f}$$

output
```
1/12*d*(f*x^4+e)^3/f
```

3.140.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int dx^3\left(e+fx^4\right)^2\,dx=\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

input
```
Integrate[d*x^3*(e + f*x^4)^2,x]
```

output
```
(d*e^2*x^4)/4 + (d*e*f*x^8)/4 + (d*f^2*x^12)/12
```

3.140.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int dx^3 \left(e+fx^4\right)^2 \, dx$$

$$\Big\downarrow \text{ 27}$$

$$d\int x^3 \left(fx^4+e\right)^2 dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{d\left(e+fx^4\right)^3}{12f}$$

input
```
Int[d*x^3*(e + f*x^4)^2,x]
```

output
```
(d*(e + f*x^4)^3)/(12*f)
```

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.140.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{d\left(f\,x^{4}+e\right)^{3}}{12f}$	16
gosper	$\frac{x^{4}\left(f^{2}x^{8}+3ef\,x^{4}+3e^{2}\right)d}{12}$	27
parallelrisch	$\left(\frac{1}{12}f^{2}x^{12}+\frac{1}{4}ef\,x^{8}+\frac{1}{4}e^{2}x^{4}\right)d$	27
norman	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{4}def\,x^{8}$	28
risch	$\frac{d\,f^{2}x^{12}}{12}+\frac{def\,x^{8}}{4}+\frac{d\,x^{4}e^{2}}{4}+\frac{d\,e^{3}}{12f}$	37

input
```
int(d*x^3*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*d*(f*x^4+e)^3/f
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int dx^{3}\left(e+fx^{4}\right)^{2}\,dx=\frac{1}{12}\,df^{2}x^{12}+\frac{1}{4}\,defx^{8}+\frac{1}{4}\,de^{2}x^{4}$$

input
```
integrate(d*x^3*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/4*d*e*f*x^8 + 1/4*d*e^2*x^4
```

3.140.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int dx^{3}\left(e+fx^{4}\right)^{2}\,dx=\frac{de^{2}x^{4}}{4}+\frac{defx^{8}}{4}+\frac{df^{2}x^{12}}{12}$$

input
```
integrate(d*x**3*(f*x**4+e)**2,x)
```

output
```
d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int dx^3(e+fx^4)^2\,dx=\frac{(fx^4+e)^3d}{12\,f}$$

input
```
integrate(d*x^3*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*(f*x^4 + e)^3*d/f
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int dx^3(e+fx^4)^2\,dx=\frac{(fx^4+e)^3d}{12\,f}$$

input
```
integrate(d*x^3*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*(f*x^4 + e)^3*d/f
```

3.140.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int dx^3(e+fx^4)^2\,dx=\frac{d\,x^4\,(3\,e^2+3\,e\,f\,x^4+f^2\,x^8)}{12}$$

input
```
int(d*x^3*(e + f*x^4)^2,x)
```

output
```
(d*x^4*(3*e^2 + f^2*x^8 + 3*e*f*x^4))/12
```

3.141 $\int (a+dx^3) (e+fx^4)^2 \, dx$

3.141.1 Optimal result . . . 1150
3.141.2 Mathematica [A] (verified) . . . 1150
3.141.3 Rubi [A] (verified) . . . 1151
3.141.4 Maple [A] (verified) . . . 1152
3.141.5 Fricas [A] (verification not implemented) . . . 1152
3.141.6 Sympy [A] (verification not implemented) . . . 1153
3.141.7 Maxima [A] (verification not implemented) . . . 1153
3.141.8 Giac [A] (verification not implemented) . . . 1153
3.141.9 Mupad [B] (verification not implemented) . . . 1154

3.141.1 Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \left(a+dx^3\right) \left(e+fx^4\right)^2 \, dx = ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{d(e+fx^4)^3}{12f}$$

output
```
a*e^2*x+2/5*a*e*f*x^5+1/9*a*f^2*x^9+1/12*d*(f*x^4+e)^3/f
```

3.141.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \left(a+dx^3\right) \left(e+fx^4\right)^2 \, dx = ae^2x+\frac{1}{4}de^2x^4+\frac{2}{5}aefx^5+\frac{1}{4}defx^8+\frac{1}{9}af^2x^9+\frac{1}{12}df^2x^{12}$$

input
```
Integrate[(a + d*x^3)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9
+ (d*f^2*x^12)/12
```

3.141.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2017, 27, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+dx^3)\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{2017}$$

$$\int a\left(fx^4+e\right)^2 dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{27}$$

$$a\int \left(fx^4+e\right)^2 dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{747}$$

$$a\int \left(f^2x^8+2efx^4+e^2\right) dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{2009}$$

$$a\left(e^2x+\frac{2}{5}efx^5+\frac{f^2x^9}{9}\right)+\frac{d\left(e+fx^4\right)^3}{12f}$$

input
```
Int[(a + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(d*(e + f*x^4)^3)/(12*f) + a*(e^2*x + (2*e*f*x^5)/5 + (f^2*x^9)/9)
```

3.141.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

3.141.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

method	result	size
gosper	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+a\,e^2x$	51
default	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+a\,e^2x$	51
norman	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+a\,e^2x$	51
risch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+a\,e^2x$	51
parallelrisch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+a\,e^2x$	51

input
```
int((d*x^3+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*d*f^2*x^12+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/5*a*e*f*x^5+1/4*d*x^4*e^2+a*
e^2*x
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (a+dx^3)\,(e+fx^4)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+ae^2x$$

input
```
integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^
2*x^4 + a*e^2*x
```

3.141.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \left(a+dx^3\right)\left(e+fx^4\right)^2\,dx=ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

input
```
integrate((d*x**3+a)*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + d*e**2*x**4/4 + d*e*f*x**8/4 +
 d*f**2*x**12/12
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \left(a+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+ae^2x$$

input
```
integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^
2*x^4 + a*e^2*x
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \left(a+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+ae^2x$$

input
```
integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^
2*x^4 + a*e^2*x
```

3.141.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (a+dx^3)(e+fx^4)^2\,dx=\frac{d\,e^2\,x^4}{4}+a\,e^2\,x+\frac{d\,e\,f\,x^8}{4}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{d\,f^2\,x^{12}}{12}+\frac{a\,f^2\,x^9}{9}$$

input
```
int((a + d*x^3)*(e + f*x^4)^2,x)
```

output
```
(a*f^2*x^9)/9 + (d*e^2*x^4)/4 + (d*f^2*x^12)/12 + a*e^2*x + (2*a*e*f*x^5)/
5 + (d*e*f*x^8)/4
```

3.142 $\int (bx+dx^3)(e+fx^4)^2\,dx$

3.142.1 Optimal result . . . 1155
3.142.2 Mathematica [A] (verified) . . . 1155
3.142.3 Rubi [A] (verified) . . . 1156
3.142.4 Maple [A] (verified) . . . 1157
3.142.5 Fricas [A] (verification not implemented) . . . 1157
3.142.6 Sympy [A] (verification not implemented) . . . 1158
3.142.7 Maxima [A] (verification not implemented) . . . 1158
3.142.8 Giac [A] (verification not implemented) . . . 1158
3.142.9 Mupad [B] (verification not implemented) . . . 1159

3.142.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int (bx+dx^3)(e+fx^4)^2\,dx = \frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

output
```
1/2*b*e^2*x^2+1/3*b*e*f*x^6+1/10*b*f^2*x^10+1/12*d*(f*x^4+e)^3/f
```

3.142.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int (bx+dx^3)(e+fx^4)^2\,dx = \frac{1}{2}be^2x^2+\frac{1}{4}de^2x^4+\frac{1}{3}befx^6+\frac{1}{4}defx^8+\frac{1}{10}bf^2x^{10}+\frac{1}{12}df^2x^{12}$$

input
```
Integrate[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4 + (b*f^2*x^1
0)/10 + (d*f^2*x^12)/12
```

3.142.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2017, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+dx^3)\left(e+fx^4\right)^2\,dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int bx\left(fx^4+e\right)^2dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{ 27}$$

$$b\int x\left(fx^4+e\right)^2dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{ 802}$$

$$b\int \left(f^2x^9+2efx^5+e^2x\right)dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{ 2009}$$

$$b\left(\frac{e^2x^2}{2}+\frac{1}{3}efx^6+\frac{f^2x^{10}}{10}\right)+\frac{d\left(e+fx^4\right)^3}{12f}$$

input
```
Int[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(d*(e + f*x^4)^3)/(12*f) + b*((e^2*x^2)/2 + (e*f*x^6)/3 + (f^2*x^10)/10)
```

3.142.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

3.142.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2$	54
norman	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2$	54
risch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2$	54
parallelrisch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2$	54
gosper	$\frac{x^2(5d\,f^2x^{10}+6b\,f^2x^8+15def\,x^6+20bef\,x^4+15d\,e^2x^2+30e^2b)}{60}$	56

input
```
int((d*x^3+b*x)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/4*d*e*f*x^8+1/3*b*e*f*x^6+1/4*d*x^4*e^2+
1/2*b*e^2*x^2
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int(bx+dx^3)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2$$

input
```
integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*
e^2*x^4 + 1/2*b*e^2*x^2
```

3.142.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int (bx + dx^3) (e + fx^4)^2 dx = \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

input
```
integrate((d*x**3+b*x)*(f*x**4+e)**2,x)
```

output
```
b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**
8/4 + d*f**2*x**12/12
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (bx + dx^3) (e + fx^4)^2 dx = \frac{1}{12} df^2x^{12} + \frac{1}{10} bf^2x^{10} + \frac{1}{4} defx^8 + \frac{1}{3} befx^6 + \frac{1}{4} de^2x^4 + \frac{1}{2} be^2x^2$$

input
```
integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*
e^2*x^4 + 1/2*b*e^2*x^2
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (bx + dx^3) (e + fx^4)^2 dx = \frac{1}{12} df^2x^{12} + \frac{1}{10} bf^2x^{10} + \frac{1}{4} defx^8 + \frac{1}{3} befx^6 + \frac{1}{4} de^2x^4 + \frac{1}{2} be^2x^2$$

input
```
integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*
e^2*x^4 + 1/2*b*e^2*x^2
```

3.142.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (bx + dx^3) (e + fx^4)^2 \, dx = \frac{d e^2 x^4}{4} + \frac{b e^2 x^2}{2} + \frac{d e f x^8}{4} + \frac{b e f x^6}{3} + \frac{d f^2 x^{12}}{12} + \frac{b f^2 x^{10}}{10}$$

input
```
int((b*x + d*x^3)*(e + f*x^4)^2,x)
```

output
```
(b*e^2*x^2)/2 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (d*f^2*x^12)/12 + (b*e*f
*x^6)/3 + (d*e*f*x^8)/4
```

3.143 $\int (a+bx+dx^3)(e+fx^4)^2\,dx$

3.143.1 Optimal result 1160
3.143.2 Mathematica [A] (verified) 1160
3.143.3 Rubi [A] (verified) 1161
3.143.4 Maple [A] (verified) 1162
3.143.5 Fricas [A] (verification not implemented) 1162
3.143.6 Sympy [A] (verification not implemented) 1163
3.143.7 Maxima [A] (verification not implemented) 1163
3.143.8 Giac [A] (verification not implemented) 1163
3.143.9 Mupad [B] (verification not implemented) 1164

3.143.1 Optimal result

Integrand size = 20, antiderivative size = 77

$$\int (a+bx+dx^3)(e+fx^4)^2\,dx = ae^2x+\frac{1}{2}be^2x^2+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{d(e+fx^4)^3}{12f}$$

output
```
a*e^2*x+1/2*b*e^2*x^2+2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2
*x^10+1/12*d*(f*x^4+e)^3/f
```

3.143.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int (a+bx+dx^3)(e+fx^4)^2\,dx = ae^2x+\frac{1}{2}be^2x^2+\frac{1}{4}de^2x^4+\frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{4}defx^8+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{1}{12}df^2x^{12}$$

input
```
Integrate[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3
+ (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*f^2*x^12)/12
```

3.143.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e+fx^4\right)^2 \left(a+bx+dx^3\right) \, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int (a+bx) \left(fx^4+e\right)^2dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{ 2389}$$

$$\int \left(bf^2x^9+af^2x^8+2befx^5+2aefx^4+be^2x+ae^2\right)dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{ 2009}$$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d\left(e+fx^4\right)^3}{12f}$$

input
```
Int[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9
+ (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

3.143.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.143.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
gosper	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2+a\,e^2x$
default	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2+a\,e^2x$
norman	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2+a\,e^2x$
risch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2+a\,e^2x$
parallelrisch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{2}b\,e^2x^2+a\,e^2x$

input
```
int((d*x^3+b*x+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/9*a*f^2*x^9+1/4*d*e*f*x^8+1/3*b*e*f*x^6+
2/5*a*e*f*x^5+1/4*d*x^4*e^2+1/2*b*e^2*x^2+a*e^2*x
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \left(a+bx+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2+ae^2x$$

input
```
integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 1/3*b*
e*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2 + a*e^2*x
```

3.143.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int (a+bx+dx^3)\left(e+fx^4\right)^2\,dx = ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

input
```
integrate((d*x**3+b*x+a)*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
 b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+bx+dx^3)\left(e+fx^4\right)^2\,dx = \frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2+ae^2x$$

input
```
integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 1/3*b*
e*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2 + a*e^2*x
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+bx+dx^3)\left(e+fx^4\right)^2\,dx = \frac{1}{12}\,df^2x^{12}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{2}\,be^2x^2+ae^2x$$

input
```
integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 1/3*b*
e*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2 + a*e^2*x
```

3.143.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+bx+dx^3)\left(e+fx^4\right)^2\,dx=\frac{d\,e^2\,x^4}{4}+\frac{b\,e^2\,x^2}{2}+a\,e^2\,x+\frac{d\,e\,f\,x^8}{4}+\frac{b\,e\,f\,x^6}{3}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{d\,f^2\,x^{12}}{12}+\frac{b\,f^2\,x^{10}}{10}+\frac{a\,f^2\,x^9}{9}$$

input
```
int((e + f*x^4)^2*(a + b*x + d*x^3),x)
```

output
```
(b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (d*f^2*x
^12)/12 + a*e^2*x + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4
```

3.144 $\int (cx^2 + dx^3) (e + fx^4)^2 \, dx$

3.144.1 Optimal result 1165
3.144.2 Mathematica [A] (verified) 1165
3.144.3 Rubi [A] (verified) 1166
3.144.4 Maple [A] (verified) 1167
3.144.5 Fricas [A] (verification not implemented) 1167
3.144.6 Sympy [A] (verification not implemented) 1168
3.144.7 Maxima [A] (verification not implemented) 1168
3.144.8 Giac [A] (verification not implemented) 1168
3.144.9 Mupad [B] (verification not implemented) 1169

3.144.1 Optimal result

Integrand size = 21, antiderivative size = 50

$$\int (cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{1}{3}ce^2x^3 + \frac{2}{7}cefx^7 + \frac{1}{11}cf^2x^{11} + \frac{d(e + fx^4)^3}{12f}$$

output
```
1/3*c*e^2*x^3+2/7*c*e*f*x^7+1/11*c*f^2*x^11+1/12*d*(f*x^4+e)^3/f
```

3.144.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int (cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{1}{3}ce^2x^3 + \frac{1}{4}de^2x^4 + \frac{2}{7}cefx^7 + \frac{1}{4}defx^8 + \frac{1}{11}cf^2x^{11} + \frac{1}{12}df^2x^{12}$$

input
```
Integrate[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 + (c*f^2*x
^11)/11 + (d*f^2*x^12)/12
```

3.144.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2017, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2+dx^3\right)\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{2017}$$

$$\int cx^2\left(fx^4+e\right)^2 dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{27}$$

$$c\int x^2\left(fx^4+e\right)^2 dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{802}$$

$$c\int \left(f^2x^{10}+2efx^6+e^2x^2\right) dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{e^2x^3}{3}+\frac{2}{7}efx^7+\frac{f^2x^{11}}{11}\right)+\frac{d\left(e+fx^4\right)^3}{12f}$$

input
```
Int[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(d*(e + f*x^4)^3)/(12*f) + c*((e^2*x^3)/3 + (2*e*f*x^7)/7 + (f^2*x^11)/11)
```

3.144.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

3.144.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{x^{3}\left(77d\,f^{2}x^{9}+84c\,f^{2}x^{8}+231def\,x^{5}+264cef\,x^{4}+231d\,e^{2}x+308c\,e^{2}\right)}{924}$	54
default	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}$	54
norman	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}$	54
risch	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}$	54
parallelrisch	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{11}c\,f^{2}x^{11}+\frac{1}{4}def\,x^{8}+\frac{2}{7}cef\,x^{7}+\frac{1}{4}d\,x^{4}e^{2}+\frac{1}{3}c\,e^{2}x^{3}$	54

input
```
int((d*x^3+c*x^2)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/924*x^3*(77*d*f^2*x^9+84*c*f^2*x^8+231*d*e*f*x^5+264*c*e*f*x^4+231*d*e^2
*x+308*c*e^2)
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3$$

input
```
integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7 + 1/4*d*
e^2*x^4 + 1/3*c*e^2*x^3
```

3.144.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int (cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

input
```
integrate((d*x**3+c*x**2)*(f*x**4+e)**2,x)
```

output
```
c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x
**8/4 + d*f**2*x**12/12
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (cx^2+dx^3) (e+fx^4)^2 \, dx = \frac{1}{12} df^2x^{12} + \frac{1}{11} cf^2x^{11} + \frac{1}{4} defx^8 + \frac{2}{7} cefx^7 + \frac{1}{4} de^2x^4 + \frac{1}{3} ce^2x^3$$

input
```
integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7 + 1/4*d*
e^2*x^4 + 1/3*c*e^2*x^3
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (cx^2+dx^3) (e+fx^4)^2 \, dx = \frac{1}{12} df^2x^{12} + \frac{1}{11} cf^2x^{11} + \frac{1}{4} defx^8 + \frac{2}{7} cefx^7 + \frac{1}{4} de^2x^4 + \frac{1}{3} ce^2x^3$$

input
```
integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7 + 1/4*d*
e^2*x^4 + 1/3*c*e^2*x^3
```

3.144.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (cx^2 + dx^3)\,(e + fx^4)^2\,dx = \frac{d\,e^2\,x^4}{4} + \frac{c\,e^2\,x^3}{3} + \frac{d\,e\,f\,x^8}{4} + \frac{2\,c\,e\,f\,x^7}{7} + \frac{d\,f^2\,x^{12}}{12} + \frac{c\,f^2\,x^{11}}{11}$$

input
```
int((e + f*x^4)^2*(c*x^2 + d*x^3),x)
```

output
```
(c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12 + (2*c*e
*f*x^7)/7 + (d*e*f*x^8)/4
```

3.145 $\int (a+cx^2+dx^3)(e+fx^4)^2 \, dx$

3.145.1 Optimal result 1170
3.145.2 Mathematica [A] (verified) 1170
3.145.3 Rubi [A] (verified) 1171
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3.145.7 Maxima [A] (verification not implemented) 1173
3.145.8 Giac [A] (verification not implemented) 1173
3.145.9 Mupad [B] (verification not implemented) 1174

3.145.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int (a+cx^2+dx^3)(e+fx^4)^2 \, dx = ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

output
```
a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2
*x^11+1/12*d*(f*x^4+e)^3/f
```

3.145.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int (a+cx^2+dx^3)(e+fx^4)^2 \, dx = ae^2x+\frac{1}{3}ce^2x^3+\frac{1}{4}de^2x^4+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{4}defx^8+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}+\frac{1}{12}df^2x^{12}$$

input
```
Integrate[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/
7 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

3.145.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2017, 1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx^4)^2 (a+cx^2+dx^3) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int (cx^2+a)(fx^4+e)^2 dx + \frac{d(e+fx^4)^3}{12f}$$

$$\Big\downarrow \text{1468}$$

$$\int (cf^2x^{10}+af^2x^8+2cefx^6+2aefx^4+ce^2x^2+ae^2) dx + \frac{d(e+fx^4)^3}{12f}$$

$$\Big\downarrow \text{2009}$$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

input
```
Int[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/
9 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

3.145.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

3.145.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
gosper	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{3}c\,e^2x^3+a\,e^2x$
default	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{3}c\,e^2x^3+a\,e^2x$
norman	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{3}c\,e^2x^3+a\,e^2x$
risch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{3}c\,e^2x^3+a\,e^2x$
parallelrisch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{11}c\,f^2x^{11}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{2}{7}cef\,x^7+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,x^4e^2+\frac{1}{3}c\,e^2x^3+a\,e^2x$

input
```
int((d*x^3+c*x^2+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/7*c*e*f*x^7+
2/5*a*e*f*x^5+1/4*d*x^4*e^2+1/3*c*e^2*x^3+a*e^2*x
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+cx^2+dx^3)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3+ae^2x$$

input
```
integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/7*c*
e*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + a*e^2*x
```

3.145.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int \left(a+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx = ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

input
```
integrate((d*x**3+c*x**2+a)*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7
 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \left(a+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx = \frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3+ae^2x$$

input
```
integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/7*c*
e*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + a*e^2*x
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \left(a+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx = \frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{2}{5}\,aefx^5+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3+ae^2x$$

input
```
integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/7*c*
e*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + a*e^2*x
```

3.145.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+cx^2+dx^3)(e+fx^4)^2 \, dx = \frac{d\,e^2\,x^4}{4} + \frac{c\,e^2\,x^3}{3} + a\,e^2\,x + \frac{d\,e\,f\,x^8}{4} + \frac{2\,c\,e\,f\,x^7}{7} + \frac{2\,a\,e\,f\,x^5}{5} + \frac{d\,f^2\,x^{12}}{12} + \frac{c\,f^2\,x^{11}}{11} + \frac{a\,f^2\,x^9}{9}$$

input
```
int((e + f*x^4)^2*(a + c*x^2 + d*x^3),x)
```

output
```
(a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (c*f^2*x^11)/11 + (d*f^2*x
^12)/12 + a*e^2*x + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4
```

3.146 $\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx$

3.146.1 Optimal result . . . 1175
3.146.2 Mathematica [A] (verified) . . . 1175
3.146.3 Rubi [A] (verified) . . . 1176
3.146.4 Maple [A] (verified) . . . 1177
3.146.5 Fricas [A] (verification not implemented) . . . 1178
3.146.6 Sympy [A] (verification not implemented) . . . 1178
3.146.7 Maxima [A] (verification not implemented) . . . 1178
3.146.8 Giac [A] (verification not implemented) . . . 1179
3.146.9 Mupad [B] (verification not implemented) . . . 1179

3.146.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

output
```
1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^2*x^10+1/
11*c*f^2*x^11+1/12*d*(f*x^4+e)^3/f
```

3.146.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{4}de^2x^4+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{4}defx^8+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}+\frac{1}{12}df^2x^{12}$$

input
```
Integrate[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (2*c*e*f*x
^7)/7 + (d*e*f*x^8)/4 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (d*f^2*x^12)/1
2
```

3.146.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2017, 2027, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx^4)^2 (bx+cx^2+dx^3) \, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int (cx^2+bx) (fx^4+e)^2 dx+\frac{d(e+fx^4)^3}{12f}$$

$$\Big\downarrow \text{ 2027}$$

$$\int x(b+cx) (fx^4+e)^2 dx+\frac{d(e+fx^4)^3}{12f}$$

$$\Big\downarrow \text{ 2123}$$

$$\int (cf^2x^{10}+bf^2x^9+2cefx^6+2befx^5+ce^2x^2+be^2x) dx+\frac{d(e+fx^4)^3}{12f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

input
```
Int[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x
^10)/10 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

3.146.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.146.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result
gosper	$\frac{x^2(385d f^2x^{10}+420c f^2x^9+462b f^2x^8+1155def x^6+1320cef x^5+1540bef x^4+1155d e^2x^2+1540c e^2x+2310e^2b)}{4620}$
default	$\frac{1}{12}d f^2x^{12}+\frac{1}{11}c f^2x^{11}+\frac{1}{10}b f^2x^{10}+\frac{1}{4}def x^8+\frac{2}{7}cef x^7+\frac{1}{3}bef x^6+\frac{1}{4}d x^4e^2+\frac{1}{3}c e^2x^3+\frac{1}{2}b$
norman	$\frac{1}{12}d f^2x^{12}+\frac{1}{11}c f^2x^{11}+\frac{1}{10}b f^2x^{10}+\frac{1}{4}def x^8+\frac{2}{7}cef x^7+\frac{1}{3}bef x^6+\frac{1}{4}d x^4e^2+\frac{1}{3}c e^2x^3+\frac{1}{2}b$
risch	$\frac{1}{12}d f^2x^{12}+\frac{1}{11}c f^2x^{11}+\frac{1}{10}b f^2x^{10}+\frac{1}{4}def x^8+\frac{2}{7}cef x^7+\frac{1}{3}bef x^6+\frac{1}{4}d x^4e^2+\frac{1}{3}c e^2x^3+\frac{1}{2}b$
parallelrisch	$\frac{1}{12}d f^2x^{12}+\frac{1}{11}c f^2x^{11}+\frac{1}{10}b f^2x^{10}+\frac{1}{4}def x^8+\frac{2}{7}cef x^7+\frac{1}{3}bef x^6+\frac{1}{4}d x^4e^2+\frac{1}{3}c e^2x^3+\frac{1}{2}b$

input
```
int((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4620*x^2*(385*d*f^2*x^10+420*c*f^2*x^9+462*b*f^2*x^8+1155*d*e*f*x^6+1320
*c*e*f*x^5+1540*b*e*f*x^4+1155*d*e^2*x^2+1540*c*e^2*x+2310*b*e^2)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (bx + cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{1}{12} df^2x^{12} + \frac{1}{11} cf^2x^{11} + \frac{1}{10} bf^2x^{10} + \frac{1}{4} defx^8 + \frac{2}{7} cefx^7 + \frac{1}{3} befx^6 + \frac{1}{4} de^2x^4 + \frac{1}{3} ce^2x^3 + \frac{1}{2} be^2x^2$$

input
```
integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 2/7*
c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

3.146.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int (bx + cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

input
```
integrate((d*x**3+c*x**2+b*x)*(f*x**4+e)**2,x)
```

output
```
b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x
**7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (bx + cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{1}{12} df^2x^{12} + \frac{1}{11} cf^2x^{11} + \frac{1}{10} bf^2x^{10} + \frac{1}{4} defx^8 + \frac{2}{7} cefx^7 + \frac{1}{3} befx^6 + \frac{1}{4} de^2x^4 + \frac{1}{3} ce^2x^3 + \frac{1}{2} be^2x^2$$

input
```
integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 2/7*
c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{10}bf^2x^{10}+\frac{1}{4}defx^8+\frac{2}{7}cefx^7+\frac{1}{3}befx^6+\frac{1}{4}de^2x^4+\frac{1}{3}ce^2x^3+\frac{1}{2}be^2x^2$$

input
```
integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 2/7*
c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

3.146.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx = \frac{d\,e^2\,x^4}{4}+\frac{c\,e^2\,x^3}{3}+\frac{b\,e^2\,x^2}{2}+\frac{d\,e\,f\,x^8}{4}+\frac{2\,c\,e\,f\,x^7}{7}+\frac{b\,e\,f\,x^6}{3}+\frac{d\,f^2\,x^{12}}{12}+\frac{c\,f^2\,x^{11}}{11}+\frac{b\,f^2\,x^{10}}{10}$$

input
```
int((e + f*x^4)^2*(b*x + c*x^2 + d*x^3),x)
```

output
```
(b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (c*f^2*x
^11)/11 + (d*f^2*x^12)/12 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/
4
```

3.147 $\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx$

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3.147.1 Optimal result

Integrand size = 25, antiderivative size = 109

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7 +\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{f(a+bx^4)^3}{12b}$$

output
```
a^2*c*x+1/2*a^2*d*x^2+1/3*a^2*e*x^3+2/5*a*b*c*x^5+1/3*a*b*d*x^6+2/7*a*b*e*
x^7+1/9*b^2*c*x^9+1/10*b^2*d*x^10+1/11*b^2*e*x^11+1/12*f*(b*x^4+a)^3/b
```

3.147.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5 +\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9 +\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx^{12}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5
+ (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d
*x^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

3.147.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^2 (c+dx+ex^2+fx^3) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int (ex^2+dx+c) (bx^4+a)^2 dx + \frac{f(a+bx^4)^3}{12b}$$

$$\Big\downarrow \text{2188}$$

$$\int (b^2ex^{10}+b^2dx^9+b^2cx^8+2abex^6+2abdx^5+2abcx^4+a^2ex^2+a^2dx+a^2c) dx + \frac{f(a+bx^4)^3}{12b}$$

$$\Big\downarrow \text{2009}$$

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f(a+bx^4)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3
+ (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f
*(a + b*x^4)^3)/(12*b)
```

3.147.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.147.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2$
default	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2$
norman	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2$
risch	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2$
parallelrisch	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*a*b*f*x^
8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*a^2*f*x^4+1/3*a^2*e*x^3+1/
2*a^2*d*x^2+a^2*c*x
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx = \frac{1}{12}b^2fx^{12}+\frac{1}{11}b^2ex^{11}+\frac{1}{10}b^2dx^{10}+\frac{1}{9}b^2cx^9+\frac{1}{4}abfx^8+\frac{2}{7}abex^7+\frac{1}{3}abdx^6+\frac{2}{5}abcx^5+\frac{1}{4}a^2fx^4+\frac{1}{3}a^2ex^3+\frac{1}{2}a^2dx^2+a^2cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*
a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4
+ 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

3.147.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx = a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{a^2fx^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{abfx^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}+\frac{b^2fx^{12}}{12}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

output
```
a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5
+ a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x*
*10/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{1}{12}b^2fx^{12}+\frac{1}{11}b^2ex^{11}+\frac{1}{10}b^2dx^{10}+\frac{1}{9}b^2cx^9+\frac{1}{4}abfx^8+\frac{2}{7}abex^7+\frac{1}{3}abdx^6+\frac{2}{5}abcx^5+\frac{1}{4}a^2fx^4+\frac{1}{3}a^2ex^3+\frac{1}{2}a^2dx^2+a^2cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*
a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4
+ 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{1}{12}b^2fx^{12}+\frac{1}{11}b^2ex^{11}+\frac{1}{10}b^2dx^{10}+\frac{1}{9}b^2cx^9+\frac{1}{4}abfx^8+\frac{2}{7}abex^7+\frac{1}{3}abdx^6+\frac{2}{5}abcx^5+\frac{1}{4}a^2fx^4+\frac{1}{3}a^2ex^3+\frac{1}{2}a^2dx^2+a^2cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*
a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4
+ 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

3.147.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{f\,a^2\,x^4}{4}+\frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{f\,a\,b\,x^8}{4}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{f\,b^2\,x^{12}}{12}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

input
```
int((a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^2*d*x^2)/2 + (b^2*c*x^9)/9 + (a^2*e*x^3)/3 + (b^2*d*x^10)/10 + (a^2*f*x
^4)/4 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12 + a^2*c*x + (2*a*b*c*x^5)/5 + (a
*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4
```

3.148 $\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx$

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3.148.8 Giac [A] (verification not implemented) . . . 1190
3.148.9 Mupad [B] (verification not implemented) . . . 1191

3.148.1 Optimal result

Integrand size = 25, antiderivative size = 151

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{f(a+bx^4)^4}{16b}$$

output
```
a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+3/5*a^2*b*c*x^5+1/2*a^2*b*d*x^6+3/7*a^
2*b*e*x^7+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^11+1/13*b^3*c*x
^13+1/14*b^3*d*x^14+1/15*b^3*e*x^15+1/16*f*(b*x^4+a)^4/b
```

3.148.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{4}ab^2fx^{12}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{16}b^3fx^{16}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/
5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9
)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*
c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

3.148.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2017, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^3 \left(c+dx+ex^2+fx^3\right) \, dx$$

$$\Big\downarrow \; 2017$$

$$\int \left(ex^2+dx+c\right) \left(bx^4+a\right)^3dx+\frac{f\left(a+bx^4\right)^4}{16b}$$

$$\Big\downarrow \; 2188$$

$$\int \left(b^3ex^{14}+b^3dx^{13}+b^3cx^{12}+3ab^2ex^{10}+3ab^2dx^9+3ab^2cx^8+3a^2bex^6+3a^2bdx^5+3a^2bcx^4+a^3ex^2+a^3dx+\right.$$
$$\frac{f\left(a+bx^4\right)^4}{16b}$$

$$\Big\downarrow \; 2009$$

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6
)/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2
*e*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a
+ b*x^4)^4)/(16*b)
```

3.148.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.148.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

method	result
gosper	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$
default	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$
norman	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$
risch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$
parallelrisch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c\,x$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+1/4*f*a^3*x^4+3/5*a^2*b*c*x^5+1/2*a^2*
b*d*x^6+3/7*a^2*b*e*x^7+3/8*f*a^2*b*x^8+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+
3/11*a*b^2*e*x^11+1/4*a*b^2*f*x^12+1/13*b^3*c*x^13+1/14*b^3*d*x^14+1/15*b^
3*e*x^15+1/16*b^3*f*x^16
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = \frac{1}{16}b^3fx^{16}+\frac{1}{15}b^3ex^{15}+\frac{1}{14}b^3dx^{14}+\frac{1}{13}b^3cx^{13}+\frac{1}{4}ab^2fx^{12}+\frac{3}{11}ab^2ex^{11}+\frac{3}{10}ab^2dx^{10}+\frac{1}{3}ab^2cx^9+\frac{3}{8}a^2bfx^8+\frac{3}{7}a^2bex^7+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3fx^4+\frac{1}{3}a^3ex^3+\frac{1}{2}a^3dx^2+a^3cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 +
 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1
/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.148.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{a^3fx^4}{4}+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3a^2bfx^8}{8}+\frac{ab^2cx^9}{3}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{ab^2fx^{12}}{4}+\frac{b^3cx^{13}}{13}+\frac{b^3dx^{14}}{14}+\frac{b^3ex^{15}}{15}+\frac{b^3fx^{16}}{16}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

output
```
a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5
/5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x**
*9/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3
*c*x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c + dx + ex^2 + fx^3) (a + bx^4)^3 \, dx = \frac{1}{16} b^3 f x^{16} + \frac{1}{15} b^3 e x^{15} + \frac{1}{14} b^3 d x^{14} + \frac{1}{13} b^3 c x^{13} + \frac{1}{4} a b^2 f x^{12} + \frac{3}{11} a b^2 e x^{11} + \frac{3}{10} a b^2 d x^{10} + \frac{1}{3} a b^2 c x^9 + \frac{3}{8} a^2 b f x^8 + \frac{3}{7} a^2 b e x^7 + \frac{1}{2} a^2 b d x^6 + \frac{3}{5} a^2 b c x^5 + \frac{1}{4} a^3 f x^4 + \frac{1}{3} a^3 e x^3 + \frac{1}{2} a^3 d x^2 + a^3 c x$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 +
 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1
/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c + dx + ex^2 + fx^3) (a + bx^4)^3 \, dx = \frac{1}{16} b^3 f x^{16} + \frac{1}{15} b^3 e x^{15} + \frac{1}{14} b^3 d x^{14} + \frac{1}{13} b^3 c x^{13} + \frac{1}{4} a b^2 f x^{12} + \frac{3}{11} a b^2 e x^{11} + \frac{3}{10} a b^2 d x^{10} + \frac{1}{3} a b^2 c x^9 + \frac{3}{8} a^2 b f x^8 + \frac{3}{7} a^2 b e x^7 + \frac{1}{2} a^2 b d x^6 + \frac{3}{5} a^2 b c x^5 + \frac{1}{4} a^3 f x^4 + \frac{1}{3} a^3 e x^3 + \frac{1}{2} a^3 d x^2 + a^3 c x$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 +
 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1
/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.148.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{f\,a^3\,x^4}{4}+\frac{e\,a^3\,x^3}{3}+\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{3\,f\,a^2\,b\,x^8}{8}+\frac{3\,e\,a^2\,b\,x^7}{7}+\frac{d\,a^2\,b\,x^6}{2}+\frac{3\,c\,a^2\,b\,x^5}{5}+\frac{f\,a\,b^2\,x^{12}}{4}+\frac{3\,e\,a\,b^2\,x^{11}}{11}+\frac{3\,d\,a\,b^2\,x^{10}}{10}+\frac{c\,a\,b^2\,x^9}{3}+\frac{f\,b^3\,x^{16}}{16}+\frac{e\,b^3\,x^{15}}{15}+\frac{d\,b^3\,x^{14}}{14}+\frac{c\,b^3\,x^{13}}{13}$$

input
```
int((a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^3*d*x^2)/2 + (b^3*c*x^13)/13 + (a^3*e*x^3)/3 + (b^3*d*x^14)/14 + (a^3*f
*x^4)/4 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16 + a^3*c*x + (3*a^2*b*c*x^5)/5
+ (a*b^2*c*x^9)/3 + (a^2*b*d*x^6)/2 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^7
)/7 + (3*a*b^2*e*x^11)/11 + (3*a^2*b*f*x^8)/8 + (a*b^2*f*x^12)/4
```

3.149 $\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^2}\,dx$

3.149.1 Optimal result 1192
3.149.2 Mathematica [A] (verified) 1192
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3.149.9 Mupad [B] (verification not implemented) 1198

3.149.1 Optimal result

Integrand size = 26, antiderivative size = 155

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^2}\,dx=\frac{af+bx(c+dx+ex^2)}{4ab\left(a-bx^4\right)}+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

output
```
1/4*(a*f+b*x*(e*x^2+d*x+c))/a/b/(-b*x^4+a)+1/4*d*arctanh(x^2*b^(1/2)/a^(1/
2))/a^(3/2)/b^(1/2)+1/8*arctan(b^(1/4)*x/a^(1/4))*(-e*a^(1/2)+3*c*b^(1/2))
/a^(7/4)/b^(3/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^
(7/4)/b^(3/4)
```

3.149.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.42

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^2}\,dx$$
$$=\frac{\frac{4a(af+bx(c+x(d+ex)))}{a-bx^4}-2\sqrt[4]{a}\sqrt[4]{b}\left(-3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\sqrt[4]{b}\left(3\sqrt[4]{a}\sqrt{b}c+2\sqrt{a}\sqrt[4]{b}d+a^{3/4}e\right)\log\left(\right.}{16a^2b}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

output
```
((4*a*(a*f + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2*a^(1/4)*b^(1/4)*(-3*S
qrt[b]*c + Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - b^(1/4)*(3*a^(1/4)*Sqr
t[b]*c + 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x] + b^(1/
4)*(3*a^(1/4)*Sqrt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b
^(1/4)*x] + 2*Sqrt[a]*Sqrt[b]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^2*b)
```

3.149.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$$

$$\Big\downarrow \ 2393$$

$$\frac{af+bx(c+dx+ex^2)}{4ab\,(a-bx^4)}-\frac{\int -\frac{ex^2+2dx+3c}{a-bx^4}dx}{4a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{ex^2+2dx+3c}{a-bx^4}dx}{4a}+\frac{af+bx(c+dx+ex^2)}{4ab\,(a-bx^4)}$$

$$\Big\downarrow \ 2415$$

$$\frac{\int \left(\frac{2dx}{a-bx^4}+\frac{ex^2+3c}{a-bx^4}\right)dx}{4a}+\frac{af+bx(c+dx+ex^2)}{4ab\,(a-bx^4)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}c-\sqrt{a}e\right)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{2a^{3/4}b^{3/4}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{4a}+\frac{af+bx(c+dx+ex^2)}{4ab\,(a-bx^4)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

output
```
(a*f + b*x*(c + d*x + e*x^2))/(4*a*b*(a - b*x^4)) + (((3*Sqrt[b]*c - Sqrt[
a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ((3*Sqrt[b]*c + S
qrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + (d*ArcTanh[(
Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(4*a)
```

3.149.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.149.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{cx}{4a}+\frac{f}{4b}}{-b\,x^4+a}-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(\_R^2 e+2\_R d+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(-b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{16a^2}\right)+d\left(\frac{x^2}{4a(-b\,x^4+a)}+\frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)+e\left(\frac{x^3}{4a(-b\,x\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/a*e*x^3+1/4*d/a*x^2+1/4*c/a*x+1/4*f/b)/(-b*x^4+a)-1/16/b/a*sum((_R^2*
e+2*_R*d+3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.149.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 117016, normalized size of antiderivative = 754.94

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.149.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(139) = 278.

Time = 43.72 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.35

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$$
$$= \text{RootSum}\left(65536t^4a^7b^3 + t^2\left(-3072a^4b^2ce - 2048a^4b^2d^2\right) + t\left(128a^3bde^2 + 1152a^2b^2c^2d\right) - a^2e^4 + 18abc^2\right.$$
$$+ \frac{-af - bcx - bdx^2 - bex^3}{-4a^2b + 4ab^2x^4}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**
2*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*
a*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _
t*log(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 9830
4*_t**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5
*b**2*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_
t*a**4*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*
d**2*e + 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5
 - 120*a**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*
a*b**2*c**3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3
+ 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576
*a*b**2*c**2*d**4 - 729*b**3*c**6))) + (-a*f - b*c*x - b*d*x**2 - b*e*x**
3)/(-4*a**2*b + 4*a*b**2*x**4)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.29

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx = -\frac{bex^3+bdx^2+bcx+af}{4\,(ab^2x^4-a^2b)} + \frac{\frac{2\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(3\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,a}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(b*e*x^3 + b*d*x^2 + b*c*x + a*f)/(a*b^2*x^4 - a^2*b) + 1/16*(2*d*log
(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))
/(sqrt(a)*sqrt(b)) + 2*(3*sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*sqrt(b)*c + sq
rt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)
*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a
```

3.149.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(116) = 232$.

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.03

$$\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^2}\,dx$$
$$= -\frac{\sqrt{2}\left(3\,b^2c - 2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd + \sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$- \frac{\sqrt{2}\left(3\,b^2c + 2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd - \sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$- \frac{\sqrt{2}(3\,b^2c - \sqrt{-ab}be)\log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$+ \frac{\sqrt{2}(3\,b^2c - \sqrt{-ab}be)\log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{bex^3 + bdx^2 + bcx + af}{4\,(bx^4 - a)ab}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(3*b^2*c - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*ar
ctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4
)*a) - 1/16*sqrt(2)*(3*b^2*c + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c - sqr
t(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4
)*a) - 1/4*(b*e*x^3 + b*d*x^2 + b*c*x + a*f)/((b*x^4 - a)*a*b)
```

3.149.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.12

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\right.\right.\right.$$
$$\left. -\frac{-9\,b^2\,c^2\,e+12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}-\frac{x\,(2\,b^2\,d^3-3\,b^2\,c\,d\,e)}{16\,a^3}\right)\mathrm{root}\Big(65536\,a^7\,b^3\,z^4$$
$$-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e$$
$$\left.\left.+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4-81\,b^2\,c^4-a^2\,e^4,z,k\Big)\right)\right)+\frac{\frac{f}{4\,b}+\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^4}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x)
```

output
```
symsum(log(- root(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*
d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a
*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k)*(root(65536*a^7*b^3*
z^4 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z +
 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2
*c^4 - a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 + 4*a^2*b^
2*e^2))/(16*a^3) - (b^2*d*e)/a) - (12*b^2*c*d^2 - 9*b^2*c^2*e + a*b*e^3)/(
64*a^3) - (x*(2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3))*root(65536*a^7*b^3*z^4 -
 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*
a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4
- a^2*e^4, z, k), k, 1, 4) + (f/(4*b) + (d*x^2)/(4*a) + (e*x^3)/(4*a) + (c
*x)/(4*a))/(a - b*x^4)
```

3.150 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$

3.150.1 Optimal result 1199
3.150.2 Mathematica [A] (verified) 1200
3.150.3 Rubi [A] (verified) 1200
3.150.4 Maple [C] (verified) 1202
3.150.5 Fricas [C] (verification not implemented) 1203
3.150.6 Sympy [F(-1)] 1204
3.150.7 Maxima [A] (verification not implemented) 1204
3.150.8 Giac [B] (verification not implemented) 1205
3.150.9 Mupad [B] (verification not implemented) 1206

3.150.1 Optimal result

Integrand size = 26, antiderivative size = 188

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx = \frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{8ab(a-bx^4)^2} + \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

output
```
1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(-b*x^4+a)+1/8*(a*f+b*x*(e*x^2+d*x+c))/a/b/
(-b*x^4+a)^2+3/16*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)+1/64*arct
an(b^(1/4)*x/a^(1/4))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)+1/64*ar
ctanh(b^(1/4)*x/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)
```

3.150.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.35

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$$
$$= \frac{\frac{4ax(7c+x(6d+5ex))}{a-bx^4}+\frac{16a^2(af+bx(c+x(d+ex)))}{b(a-bx^4)^2}+\frac{2\sqrt[4]{a}\left(21\sqrt{b}c-5\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{\left(21\sqrt[4]{a}\sqrt{b}c+12\sqrt{a}\sqrt[4]{b}d+5a^{3/4}e\right)\log\left(\sqrt[4]{\cdots}\right.}{b^{3/4}}}{128a^3}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

output
```
((4*a*x*(7*c + x*(6*d + 5*e*x)))/(a - b*x^4) + (16*a^2*(a*f + b*x*(c + x*(
d + e*x))))/(b*(a - b*x^4)^2) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*Ar
cTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c + 12*Sqrt[a]*b
^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*S
qrt[b]*c - 12*Sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b
^(3/4) + (12*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

3.150.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2393, 25, 2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$$
$$\Big\downarrow \text{2393}$$
$$\frac{af+bx(c+dx+ex^2)}{8ab(a-bx^4)^2}-\frac{\int -\frac{5ex^2+6dx+7c}{(a-bx^4)^2}dx}{8a}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{5ex^2+6dx+7c}{(a-bx^4)^2}dx}{8a}+\frac{af+bx(c+dx+ex^2)}{8ab(a-bx^4)^2}$$
$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}-\frac{\int -\frac{5ex^2+12dx+21c}{a-bx^4}dx}{4a}}{8a}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{5ex^2+12dx+21c}{a-bx^4}dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}}{8a}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{\int \left(\frac{12dx}{a-bx^4}+\frac{5ex^2+21c}{a-bx^4}\right)dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}}{8a}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(21\sqrt{b}c-5\sqrt{a}e)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(5\sqrt{a}e+21\sqrt{b}c)}{2a^{3/4}b^{3/4}}+\frac{6d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}}{8a}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

output
```
(a*f + b*x*(c + d*x + e*x^2))/(8*a*b*(a - b*x^4)^2) + ((x*(7*c + 6*d*x + 5
*e*x^2))/(4*a*(a - b*x^4)) + (((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4
)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh
[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + (6*d*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a]])/(Sqrt[a]*Sqrt[b]))/(4*a))/(8*a)
```

3.150. $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$

3.150.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.150.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.64

method	result
risch	$\frac{-\frac{5be\,x^7}{32a^2}-\frac{3bd\,x^6}{16a^2}-\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}+\frac{f}{8b}}{\left(-b\,x^4+a\right)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4b-a\right)}\frac{\left(5\_R^2e+12\_Rd+21c\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2b}$
default	$c\left(\frac{x}{8a\left(-b\,x^4+a\right)^2}+\frac{\frac{7x}{32a\left(-b\,x^4+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{128a^2}}{a}\right)+d\left(\frac{x^2}{8a\left(-b\,x^4+a\right)^2}+\frac{\frac{3x^2}{16a\left(-b\,x^4+a\right)}+\cdots}{a}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-5/32*b*e/a^2*x^7-3/16*b*d/a^2*x^6-7/32*b*c/a^2*x^5+9/32/a*e*x^3+5/16*d/a
*x^2+11/32*c/a*x+1/8*f/b)/(-b*x^4+a)^2-1/128/a^2/b*sum((5*_R^2*e+12*_R*d+2
1*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.150.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.66 (sec) , antiderivative size = 118761, normalized size of antiderivative = 631.71

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.150.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

output
```
Timed out
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.32

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx = -\frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5-9\,abex^3-10\,abdx^2-11\,abcx-4\,a^2f}{32\,(a^2b^3x^8-2\,a^3b^2x^4+a^4b)} + \frac{\frac{12\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{12\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(21\sqrt{b}c-5\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^2}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 - 9*a*b*e*x^3 - 10*a*b*d*x^
2 - 11*a*b*c*x - 4*a^2*f)/(a^2*b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(1
2*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 12*d*log(sqrt(b)*x^2 -
sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(21*sqrt(b)*c - 5*sqrt(a)*e)*arctan(sqrt(b)
*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*sq
rt(b)*c + 5*sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x
+ sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^2
```

3.150.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(148) = 296$.

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.87

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$$
$$= -\frac{\sqrt{2}\left(21\,b^2c-12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{\sqrt{2}\left(21\,b^2c+12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{\sqrt{2}(21\,b^2c-5\,\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$+\frac{\sqrt{2}(21\,b^2c-5\,\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5-9\,abex^3-10\,abdx^2-11\,abcx-4\,a^2f}{32\,(bx^4-a)^2a^2b}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

output
```
-1/128*sqrt(2)*(21*b^2*c - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)
^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 5*
sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log
(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) + 1/256*s
qrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sq
rt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c
*x^5 - 9*a*b*e*x^3 - 10*a*b*d*x^2 - 11*a*b*c*x - 4*a^2*f)/((b*x^4 - a)^2*a
^2*b)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 832, normalized size of antiderivative = 4.43

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(125\,a\,e^3+3024\,b\,c\,d^2-2205\,b\,c^2\,e+1728\,b\,d^3\,x+\mathrm{root}(268435456\,a^{11}\,b^3\,z^4-6881280\,a^6\,b^2\right.}{}\right.\right.$$
$$-6881280\,a^6\,b^2\,c\,e\,z^2-4718592\,a^6\,b^2\,d^2\,z^2+2709504\,a^3\,b^2\,c^2\,d\,z+153600\,a^4\,b\,d\,e^2\,z$$
$$\left.-60480\,a\,b\,c\,d^2\,e+22050\,a\,b\,c^2\,e^2+20736\,a\,b\,d^4-625\,a^2\,e^4-194481\,b^2\,c^4,z,k)\right)$$
$$+\frac{\frac{f}{8b}+\frac{5\,d\,x^2}{16\,a}+\frac{9\,e\,x^3}{32\,a}+\frac{11\,c\,x}{32\,a}-\frac{7\,b\,c\,x^5}{32\,a^2}-\frac{3\,b\,d\,x^6}{16\,a^2}-\frac{5\,b\,e\,x^7}{32\,a^2}}{a^2-2\,a\,b\,x^4+b^2\,x^8}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x)
```

output
```
symsum(log(-(b*(125*a*e^3 + 3024*b*c*d^2 - 2205*b*c^2*e + 1728*b*d^3*x + 3
44064*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*
b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c
*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4,
 z, k)^2*a^5*b^2*c + 3200*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*
e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d
*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e
^4 - 194481*b^2*c^4, z, k)*a^3*b*e^2*x - 2520*b*c*d*e*x + 56448*root(26843
5456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 27
09504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a
*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^2*b^2*c
^2*x - 196608*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718
592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 604
80*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*
b^2*c^4, z, k)^2*a^5*b^2*d*x - 15360*root(268435456*a^11*b^3*z^4 - 6881280
*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153
600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4
- 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435
456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 270
9504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050...
```

3.151 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4} dx$

3.151.1 Optimal result 1207
3.151.2 Mathematica [A] (verified) 1208
3.151.3 Rubi [A] (verified) 1208
3.151.4 Maple [C] (verified) 1211
3.151.5 Fricas [C] (verification not implemented) 1211
3.151.6 Sympy [F(-1)] 1212
3.151.7 Maxima [A] (verification not implemented) 1212
3.151.8 Giac [B] (verification not implemented) 1213
3.151.9 Mupad [B] (verification not implemented) 1214

3.151.1 Optimal result

Integrand size = 26, antiderivative size = 220

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4} dx = \frac{x(11c+10dx+9ex^2)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a-bx^4)} + \frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3} + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

output
```
1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(-b*x^4+a)^2+1/384*x*(45*e*x^2+60*d*x+77*
c)/a^3/(-b*x^4+a)+1/12*(a*f+b*x*(e*x^2+d*x+c))/a/b/(-b*x^4+a)^3+5/32*d*arc
tanh(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)+1/256*arctan(b^(1/4)*x/a^(1/4))*
(-15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)+1/256*arctanh(b^(1/4)*x/a^(1
/4))*(15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)
```

3.151.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.30

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$$

$$= \frac{\frac{4ax(77c+15x(4d+3ex))}{a-bx^4}+\frac{16a^2x(11c+x(10d+9ex))}{(a-bx^4)^2}-\frac{128a^3(af+bx(c+x(d+ex)))}{b(-a+bx^4)^3}+\frac{6\sqrt[4]{a}(77\sqrt{b}c-15\sqrt{a}e)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{3(77\sqrt[4]{\cdots}}{\cdots}}{1536\cdots}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

output
```
((4*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10
*d + 9*e*x)))/(a - b*x^4)^2 - (128*a^3*(a*f + b*x*(c + x*(d + e*x))))/(b*(
-a + b*x^4)^3) + (6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*
x)/a^(1/4)])/b^(3/4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 1
5*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c
- 40*Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) +
 (120*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

3.151.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2393, 25, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$$

$$\Big\downarrow \text{2393}$$

$$\frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3}-\frac{\int -\frac{9ex^2+10dx+11c}{(a-bx^4)^3}dx}{12a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{9ex^2+10dx+11c}{(a-bx^4)^3}dx}{12a}+\frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3}$$

$$\Big\downarrow \; 2394$$

$$\frac{\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}-\frac{\int -\frac{45ex^2+60dx+77c}{(a-bx^4)^2}dx}{8a}}{12a}+\frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{45ex^2+60dx+77c}{(a-bx^4)^2}dx}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3}$$

$$\Big\downarrow \; 2394$$

$$\frac{\frac{\frac{x(77c+60dx+45ex^2)}{4a(a-bx^4)}-\frac{\int -\frac{3(15ex^2+40dx+77c)}{a-bx^4}dx}{4a}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\int \frac{15ex^2+40dx+77c}{a-bx^4}dx}{4a}+\frac{x(77c+60dx+45ex^2)}{4a(a-bx^4)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{\frac{3\int \left(\frac{40dx}{a-bx^4}+\frac{15ex^2+77c}{a-bx^4}\right)dx}{4a}+\frac{x(77c+60dx+45ex^2)}{4a(a-bx^4)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(77\sqrt{b}c-15\sqrt{a}e)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(15\sqrt{a}e+77\sqrt{b}c)}{2a^{3/4}b^{3/4}}+\frac{20d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\right)}{4a}+\frac{x(77c+60dx+45ex^2)}{4a(a-bx^4)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

3.151. $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$

output
```
(a*f + b*x*(c + d*x + e*x^2))/(12*a*b*(a - b*x^4)^3) + ((x*(11*c + 10*d*x
+ 9*e*x^2))/(8*a*(a - b*x^4)^2) + ((x*(77*c + 60*d*x + 45*e*x^2))/(4*a*(a
- b*x^4)) + (3*(((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)]
)/(2*a^(3/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)
/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + (20*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sq
rt[a]*Sqrt[b])))/(4*a))/(8*a))/(12*a)
```

3.151.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.151. $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$

3.151.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}+\frac{f}{12b}}{(-b\,x^4+a)^3}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{(15\_R^2e+40\_R}{}}{512a^3b}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}+\frac{f}{12b}}{(-b\,x^4+a)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{4a}}{}$

input
```
int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9-21/64*b*e/
a^2*x^7-5/12*b*d/a^2*x^6-33/64*b*c/a^2*x^5+113/384/a*e*x^3+11/32*d/a*x^2+5
1/128*c/a*x+1/12*f/b)/(-b*x^4+a)^3-1/512/a^3/b*sum((15*_R^2*e+40*_R*d+77*c
)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.151.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.98 (sec) , antiderivative size = 118945, normalized size of antiderivative = 540.66

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.151.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx = \text{Timed out}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

output
```
Timed out
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.35

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx = -\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9-126\,ab^2ex^7-160\,ab^2dx^6-198\,ab^2cx^5+113\,a^2bex^3+132\,a^2bdx^2+}{384\,(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b)} + \frac{\frac{40\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512\,a^3}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

output
```
-1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 126*a*b^2*e*x^7 - 1
60*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153
*a^2*b*c*x + 32*a^3*f)/(a^3*b^4*x^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6
*b) + 1/512*(40*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log(
sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(77*sqrt(b)*c - 15*sqrt(a)*e)
*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sq
rt(b)) - (77*sqrt(b)*c + 15*sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(
b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)))/a^3
```

3.151.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(179) = 358$.

Time = 0.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.76

$$
\begin{aligned}
\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx
&= -\frac{\sqrt{2}\left(77\,b^2c-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3} \\
&\quad -\frac{\sqrt{2}\left(77\,b^2c+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3} \\
&\quad -\frac{\sqrt{2}(77\,b^2c-15\,\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3} \\
&\quad +\frac{\sqrt{2}(77\,b^2c-15\,\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3} \\
&\quad -\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9-126\,ab^2ex^7-160\,ab^2dx^6-198\,ab^2cx^5+113\,a^2bex^3+132\,a^2bdx^2+}{384\,(bx^4-a)^3a^3b}
\end{aligned}
$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

output
```
-1/512*sqrt(2)*(77*b^2*c - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqrt(-a*b)*b
*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 1
5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(
1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)
*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) + 1/1
024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4
) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^1
0 + 77*b^3*c*x^9 - 126*a*b^2*e*x^7 - 160*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 1
13*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 32*a^3*f)/((b*x^4 - a)^
3*a^3*b)
```

3.151.9 Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 880, normalized size of antiderivative = 4.00

$$\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^4} \, dx$$
$$= \left(\sum_{k=1}^{4} \ln \left(-\frac{b \left(3375\, a\, e^3 + 123200\, b\, c\, d^2 - 88935\, b\, c^2\, e + 64000\, b\, d^3\, x + \mathrm{root}(68719476736\, a^{15}\, b^3\, z^4 - 1211105280\, a^8\, b^2\, c\, e\, z^2 - 838860800\, a^8\, b^2\, d^2\, z^2 + 485703680\, a^4\, b^2\, c^2\, d\, z + 18432000\, a^5\, b\, d\, e^2\, z - 7392000\, a\, b\, c\, d^2\, e + 2668050\, a\, b\, c^2\, e^2 + 2560000\, a\, b\, d^4 - 35153041\, b^2\, c^4 - 50625\, a^2\, e^4, z, k\right)}{\cdots} \right) \right.$$
$$+ \frac{\frac{f}{12\, b} + \frac{11\, d\, x^2}{32\, a} + \frac{113\, e\, x^3}{384\, a} + \frac{51\, c\, x}{128\, a} + \frac{77\, b^2\, c\, x^9}{384\, a^3} + \frac{5\, b^2\, d\, x^{10}}{32\, a^3} + \frac{15\, b^2\, e\, x^{11}}{128\, a^3} - \frac{33\, b\, c\, x^5}{64\, a^2} - \frac{5\, b\, d\, x^6}{12\, a^2} - \frac{21\, b\, e\, x^7}{64\, a^2}}{a^3 - 3\, a^2\, b\, x^4 + 3\, a\, b^2\, x^8 - b^3\, x^{12}}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x)
```

output
```
symsum(log(-(b*(3375*a*e^3 + 123200*b*c*d^2 - 88935*b*c^2*e + 64000*b*d^3*
x + 20185088*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 -
838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e
^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153
041*b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2*c + 115200*root(68719476736*a
^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485
703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 26
68050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z,
 k)*a^4*b*e^2*x - 92400*b*c*d*e*x + 3035648*root(68719476736*a^15*b^3*z^4
 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b
^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^3*b^2*
c^2*x - 10485760*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^
2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b
*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 3
5153041*b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x - 614400*root(6871947
6736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2
 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*
e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e
^4, z, k)*a^4*b*d*e))/(2097152*a^9))*root(68719476736*a^15*b^3*z^4 - 12...
```

3.152 $\int \frac{a}{2+3x^4} dx$

3.152.1 Optimal result . . . 1215
3.152.2 Mathematica [A] (verified) . . . 1215
3.152.3 Rubi [A] (verified) . . . 1216
3.152.4 Maple [C] (verified) . . . 1219
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3.152.9 Mupad [B] (verification not implemented) . . . 1221

3.152.1 Optimal result

Integrand size = 11, antiderivative size = 101

$$\int \frac{a}{2+3x^4} dx = -\frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\arctan\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}$$

output
```
1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)-1/4
8*a*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*a*ln(6^(3/4)*x+3*x^2+6^(1/2)
)*6^(3/4)
```

3.152.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

$$\int \frac{a}{2+3x^4} dx = \frac{a\left(-2\arctan\left(1-\sqrt[4]{6}x\right)+2\arctan\left(1+\sqrt[4]{6}x\right)-\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)+\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)\right)}{8\sqrt[4]{6}}$$

input
```
Integrate[a/(2 + 3*x^4),x]
```

output
```
(a*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] - Log[2 - 2*6^(1/4)
*x + Sqrt[6]*x^2] + Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(1/4))
```

3.152.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {27, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a}{3x^4+2}dx$$

$$\Big\downarrow \text{27}$$

$$a\int \frac{1}{3x^4+2}dx$$

$$\Big\downarrow \text{755}$$

$$a\left(\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{3}x^2+\sqrt{2}}{3x^4+2}dx}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1476}$$

$$a\left(\frac{\frac{\int \frac{1}{x^2-\frac{2^{3/4}x}{\sqrt[4]{3}}+\sqrt{\frac{2}{3}}}dx}{2\sqrt{3}}+\frac{\int \frac{1}{x^2+\frac{2^{3/4}x}{\sqrt[4]{3}}+\sqrt{\frac{2}{3}}}dx}{2\sqrt{3}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1082}$$

$$a\left(\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{2}}+\frac{\frac{\int \frac{1}{-\left(1-\sqrt[4]{6}x\right)^2-1}d\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\int \frac{1}{-\left(\sqrt[4]{6}x+1\right)^2-1}d\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{217}$$

$$a\left(\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{2}}+\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1479}$$

$$a\left(\frac{-\frac{\int -\frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}-\frac{\int -\frac{6^{3/4}\left(\sqrt[4]{6}x+1\right)}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}+\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{25}$$

$$a\left(\frac{\frac{\int \frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}+\frac{\int \frac{6^{3/4}\left(\sqrt[4]{6}x+1\right)}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}+\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{27}$$

$$a\left(\frac{\frac{\int \frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt[4]{6}x+1}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\sqrt{2}}+\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1103}$$

$$a\left(\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}+\frac{\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{2\ 2^{3/4}\sqrt[4]{3}}-\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{2\ 2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}\right)$$

input
```
Int[a/(2 + 3*x^4),x]
```

output
```
a*((-(ArcTan[1 - 6^(1/4)*x]/(2^(3/4)*3^(1/4))) + ArcTan[1 + 6^(1/4)*x]/(2^
(3/4)*3^(1/4)))/(2*Sqrt[2]) + (-1/2*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2]/(2^(3
/4)*3^(1/4)) + Log[Sqrt[6] + 6^(3/4)*x + 3*x^2]/(2*2^(3/4)*3^(1/4)))/(2*Sq
rt[2]))
```

3.152. $\int \frac{a}{2+3x^4}\,dx$

3.152.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.152.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.25

method	result
risch	$\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}$
meijerg	$\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}}{8+3^{\frac{1}{4}}8}\right.}{\left(x^4\right)^{\frac{1}{4}}}\right.}{96}$

input
```
int(a/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*a*sum(1/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

3.152.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{a}{2+3x^4}\,dx = {} & \frac{1}{96}\cdot 24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\log\left(12\,ax+24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\right)\\ & +\frac{1}{96}i\cdot 24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\log\left(12\,ax+i\cdot 24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\right)\\ & -\frac{1}{96}i\cdot 24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\log\left(12\,ax-i\cdot 24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\right)\\ & -\frac{1}{96}\cdot 24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\log\left(12\,ax-24^{\frac{3}{4}}\left(-a^4\right)^{\frac{1}{4}}\right)\end{aligned}$$

input
```
integrate(a/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/96*24^(3/4)*(-a^4)^(1/4)*log(12*a*x + 24^(3/4)*(-a^4)^(1/4)) + 1/96*I*24
^(3/4)*(-a^4)^(1/4)*log(12*a*x + I*24^(3/4)*(-a^4)^(1/4)) - 1/96*I*24^(3/4
)*(-a^4)^(1/4)*log(12*a*x - I*24^(3/4)*(-a^4)^(1/4)) - 1/96*24^(3/4)*(-a^4
)^(1/4)*log(12*a*x - 24^(3/4)*(-a^4)^(1/4))
```

3.152.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{a}{2+3x^4}\,dx = a\left(-\frac{6^{\frac{3}{4}}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48} + \frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{24}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}x+1\right)}{24}\right)$$

input
```
integrate(a/(3*x**4+2),x)
```

output
```
a*(-6**(3/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*log(x**2 +
 6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*atan(6**(1/4)*x - 1)/24 + 6**(3/4
)*atan(6**(1/4)*x + 1)/24)
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.22

$$\int \frac{a}{2+3x^4}\,dx$$
$$= \frac{1}{48}\left(2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+3^{\frac{3}{4}}2^{\frac{3}{4}}\,\mathrm{l}\right.$$

input
```
integrate(a/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/48*(2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*
2^(3/4))) + 2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^
(1/4)*2^(3/4))) + 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sq
rt(2)) - 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*a
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{a}{2+3x^4}\,dx$$
$$= \frac{1}{48}\left(2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(a/(3*x^4+2),x, algorithm="giac")
```

output
```
1/48*(2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))
) + 2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)))
+ 6^(3/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 6^(3/4)*log(x^2 -
 sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*a
```

3.152.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.36

$$\int \frac{a}{2+3x^4}\,dx = -\frac{(-1)^{1/4}\,6144^{3/4}\,a\left(\mathrm{atan}\left(\frac{(-1)^{1/4}\,6144^{1/4}\,x}{8}\right)\,1\mathrm{i}+\mathrm{atanh}\left(\frac{(-1)^{1/4}\,6144^{1/4}\,x}{8}\right)\,1\mathrm{i}\right)}{3072}$$

input
```
int(a/(3*x^4 + 2),x)
```

output
```
-((-1)^(1/4)*6144^(3/4)*a*(atan(((-1)^(1/4)*6144^(1/4)*x)/8)*1i + atanh(((
-1)^(1/4)*6144^(1/4)*x)/8)*1i))/3072
```

3.153 $\int \frac{bx}{2+3x^4} dx$

3.153.1 Optimal result 1222
3.153.2 Mathematica [A] (verified) 1222
3.153.3 Rubi [A] (verified) 1223
3.153.4 Maple [A] (verified) 1224
3.153.5 Fricas [A] (verification not implemented) 1224
3.153.6 Sympy [A] (verification not implemented) 1225
3.153.7 Maxima [A] (verification not implemented) 1225
3.153.8 Giac [A] (verification not implemented) 1225
3.153.9 Mupad [B] (verification not implemented) 1226

3.153.1 Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{bx}{2+3x^4} dx = \frac{b \arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

output
```
1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

3.153.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{bx}{2+3x^4} dx = \frac{b \arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

input
```
Integrate[(b*x)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

3.153.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {27, 807, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx}{3x^4+2}dx$$
$$\Big\downarrow \text{27}$$
$$b\int \frac{x}{3x^4+2}dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}b\int \frac{1}{3x^4+2}dx^2$$
$$\Big\downarrow \text{216}$$
$$\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

input
```
Int[(b*x)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

3.153.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.153.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$	16
risch	$\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$	16
meijerg	$\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,x^2}{2}\right)}{12}$	19

input
```
int(b*x/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{bx}{2+3x^4}\,dx = \frac{1}{12}\sqrt{6}b\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

input
```
integrate(b*x/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

3.153.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{bx}{2+3x^4}\,dx = \frac{\sqrt{6}b\,\text{atan}\left(\frac{\sqrt{6}x^2}{2}\right)}{12}$$

input
```
integrate(b*x/(3*x**4+2),x)
```

output
```
sqrt(6)*b*atan(sqrt(6)*x**2/2)/12
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{bx}{2+3x^4}\,dx = \frac{1}{12}\sqrt{6}b\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

input
```
integrate(b*x/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{bx}{2+3x^4}\,dx = \frac{1}{12}\sqrt{6}b\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

input
```
integrate(b*x/(3*x^4+2),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{bx}{2+3x^4}\,dx = \frac{\sqrt{6}\,b\,\mathrm{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}$$

input
```
int((b*x)/(3*x^4 + 2),x)
```

output
```
(6^(1/2)*b*atan((6^(1/2)*x^2)/2))/12
```

3.154 $\int \frac{a+bx}{2+3x^4} \, dx$

3.154.1 Optimal result 1227
3.154.2 Mathematica [A] (verified) 1227
3.154.3 Rubi [A] (verified) 1228
3.154.4 Maple [C] (verified) 1229
3.154.5 Fricas [C] (verification not implemented) 1230
3.154.6 Sympy [A] (verification not implemented) 1230
3.154.7 Maxima [A] (verification not implemented) 1230
3.154.8 Giac [A] (verification not implemented) 1231
3.154.9 Mupad [B] (verification not implemented) 1232

3.154.1 Optimal result

Integrand size = 15, antiderivative size = 123

$$\int \frac{a+bx}{2+3x^4} \, dx = \frac{b \arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a \arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a \arctan\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a \log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a \log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}$$

output
```
1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)-1/4
8*a*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*a*ln(6^(3/4)*x+3*x^2+6^(1/2)
)*6^(3/4)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

3.154.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87

$$\int \frac{a+bx}{2+3x^4} \, dx = \frac{-2\left(\sqrt[4]{6}a+2b\right) \arctan\left(1-\sqrt[4]{6}x\right) + 2\left(\sqrt[4]{6}a-2b\right) \arctan\left(1+\sqrt[4]{6}x\right) + \sqrt[4]{6}a\left(-\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)\right.}{8\sqrt{6}}$$

input
```
Integrate[(a + b*x)/(2 + 3*x^4),x]
```

output
```
(-2*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*(6^(1/4)*a - 2*b)*ArcTan[1
 + 6^(1/4)*x] + 6^(1/4)*a*(-Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + Log[2 + 2
*6^(1/4)*x + Sqrt[6]*x^2]))/(8*Sqrt[6])
```

3.154.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{3x^4+2}\,dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{a}{3x^4+2}+\frac{bx}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\arctan\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

input
```
Int[(a + b*x)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/
4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x +
 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

3.154.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.154.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.24

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_Rb+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$
meijerg	$\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{2}\sqrt{3}\,x^2}{2}\right)}{12}+\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\cdots}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}\cdots\right)}{96}$

input
```
int((b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R*b+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

3.154.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 12348, normalized size of antiderivative = 100.39

$$\int \frac{a+bx}{2+3x^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.154.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.72

$$\int \frac{a+bx}{2+3x^4}\,dx$$
$$= \text{RootSum}\left(18432t^4+384t^2b^2-96ta^2b+3a^4+2b^4, \left(t\mapsto t\log\left(x+\frac{3072t^3b^2+192t^2a^2b+24ta^4+32tb}{3a^5-8ab^4}\right.\right.\right.$$

input
```
integrate((b*x+a)/(3*x**4+2),x)
```

output
```
RootSum(18432*_t**4 + 384*_t**2*b**2 - 96*_t*a**2*b + 3*a**4 + 2*b**4, Lam
bda(_t, _t*log(x + (3072*_t**3*b**2 + 192*_t**2*a**2*b + 24*_t*a**4 + 32*_
t*b**4 - 10*a**2*b**3)/(3*a**5 - 8*a*b**4))))
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{a+bx}{2+3x^4}\,dx &= \frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)-\frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\\ &+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a-2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\ &+\frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\end{aligned}$$

input
```
integrate((b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/48*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) - 1/
48*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24
*sqrt(3)*(3^(1/4)*2^(3/4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*s
qrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)
*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{a+bx}{2+3x^4}\,dx =& \frac{1}{48}\cdot 6^{\frac{3}{4}}a\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)\\ &-\frac{1}{48}\cdot 6^{\frac{3}{4}}a\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)\\ &+\frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\\ &+\frac{1}{24}\left(6^{\frac{3}{4}}a+2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\end{aligned}$$

input
```
integrate((b*x+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/48*6^(3/4)*a*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*6^(3/4)
*a*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(3/4)*a - 2*sqrt
(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*
(6^(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2
/3)^(1/4)))
```

3.154.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.97

$$\int \frac{a+bx}{2+3x^4}\,dx = \frac{2^{3/4}\,3^{3/4}\,a\,\ln\left(x^2+\frac{6^{3/4}\,x}{3}+\frac{\sqrt{6}}{3}\right)}{48} - \frac{2^{3/4}\,3^{3/4}\,a\,\ln\left(x^2-\frac{6^{3/4}\,x}{3}+\frac{\sqrt{6}}{3}\right)}{48} + \frac{2^{3/4}\,3^{3/4}\,a\,\mathrm{atan}\left(6^{1/4}\,x-1\right)}{24} + \frac{2^{3/4}\,3^{3/4}\,a\,\mathrm{atan}\left(6^{1/4}\,x+1\right)}{24} + \frac{\sqrt{2}\,\sqrt{3}\,b\,\mathrm{atan}\left(6^{1/4}\,x-1\right)}{12} - \frac{\sqrt{2}\,\sqrt{3}\,b\,\mathrm{atan}\left(6^{1/4}\,x+1\right)}{12}$$

input
```
int((a + b*x)/(3*x^4 + 2),x)
```

output
```
(2^(3/4)*3^(3/4)*a*log((6^(3/4)*x)/3 + 6^(1/2)/3 + x^2))/48 - (2^(3/4)*3^(
3/4)*a*log(6^(1/2)/3 - (6^(3/4)*x)/3 + x^2))/48 + (2^(3/4)*3^(3/4)*a*atan(
6^(1/4)*x - 1))/24 + (2^(3/4)*3^(3/4)*a*atan(6^(1/4)*x + 1))/24 + (2^(1/2)
*3^(1/2)*b*atan(6^(1/4)*x - 1))/12 - (2^(1/2)*3^(1/2)*b*atan(6^(1/4)*x + 1
))/12
```

3.155 $\int \frac{cx^2}{2+3x^4}\,dx$

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3.155.1 Optimal result

Integrand size = 14, antiderivative size = 101

$$\int \frac{cx^2}{2+3x^4}\,dx=-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}-\frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}$$

output
```
1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)+1/2
4*c*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*c*ln(6^(3/4)*x+3*x^2+6^(1/2)
)*6^(1/4)
```

3.155.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

$$\int \frac{cx^2}{2+3x^4}\,dx=\frac{c\left(-2\arctan\left(1-\sqrt[4]{6}x\right)+2\arctan\left(1+\sqrt[4]{6}x\right)+\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)-\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)\right)}{4\ 6^{3/4}}$$

input
```
Integrate[(c*x^2)/(2 + 3*x^4),x]
```

output
```
(c*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] + Log[2 - 2*6^(1/4)
*x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(4*6^(3/4))
```

3.155.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cx^2}{3x^4+2}dx$$

$$\Big\downarrow \text{27}$$

$$c\int \frac{x^2}{3x^4+2}dx$$

$$\Big\downarrow \text{826}$$

$$c\left(\frac{\int \frac{\sqrt{3}x^2+\sqrt{2}}{3x^4+2}dx}{2\sqrt{3}}-\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1476}$$

$$c\left(\frac{\frac{\int \frac{1}{x^2-\frac{2^{3/4}x}{\sqrt[4]{3}}+\sqrt{\frac{2}{3}}}dx}{2\sqrt{3}}+\frac{\int \frac{1}{x^2+\frac{2^{3/4}x}{\sqrt[4]{3}}+\sqrt{\frac{2}{3}}}dx}{2\sqrt{3}}}{2\sqrt{3}}-\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1082}$$

$$c\left(\frac{\frac{\int \frac{1}{-\left(1-\sqrt[4]{6}x\right)^2-1}d\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\int \frac{1}{-\left(\sqrt[4]{6}x+1\right)^2-1}d\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}-\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{217}$$

$$c\left(\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}-\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1479}$$

$$c\left(\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}-\frac{-\frac{\int -\frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}-\frac{\int -\frac{6^{3/4}\left(\sqrt[4]{6}x+1\right)}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}\right)$$

$$\Big\downarrow \ 25$$

$$c\left(\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}-\frac{\frac{\int \frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}+\frac{\int \frac{6^{3/4}\left(\sqrt[4]{6}x+1\right)}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}\right)$$

$$\Big\downarrow \ 27$$

$$c\left(\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}-\frac{\frac{\int \frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt[4]{6}x+1}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \ 1103$$

$$c\left(\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}-\frac{\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{2\ 2^{3/4}\sqrt[4]{3}}-\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{2\ 2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}\right)$$

input
```
Int[(c*x^2)/(2 + 3*x^4),x]
```

output
```
c*((-(ArcTan[1 - 6^(1/4)*x]/(2^(3/4)*3^(1/4))) + ArcTan[1 + 6^(1/4)*x]/(2^
(3/4)*3^(1/4)))/(2*Sqrt[3]) - (-1/2*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2]/(2^(3
/4)*3^(1/4)) + Log[Sqrt[6] + 6^(3/4)*x + 3*x^2]/(2*2^(3/4)*3^(1/4)))/(2*Sq
rt[3]))
```

3.155. $\int \frac{cx^2}{2+3x^4}\,dx$

3.155.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.155.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.25

method	result
risch	$\frac{c\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^{4}+2\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{12}$
default	$\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^{3}\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}\cdots}{8+3\cdots}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}\cdots\right)}{216}$

input
```
int(c*x^2/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*c*sum(1/_R*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

3.155.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \frac{cx^2}{2+3x^4}\,dx = {} & \frac{1}{216}\cdot 54^{\frac{3}{4}}\left(-c^4\right)^{\frac{1}{4}}\log\left(3\,c^3x+54^{\frac{1}{4}}\left(-c^4\right)^{\frac{3}{4}}\right)\\ & -\frac{1}{216}i\cdot 54^{\frac{3}{4}}\left(-c^4\right)^{\frac{1}{4}}\log\left(3\,c^3x+i\cdot 54^{\frac{1}{4}}\left(-c^4\right)^{\frac{3}{4}}\right)\\ & +\frac{1}{216}i\cdot 54^{\frac{3}{4}}\left(-c^4\right)^{\frac{1}{4}}\log\left(3\,c^3x-i\cdot 54^{\frac{1}{4}}\left(-c^4\right)^{\frac{3}{4}}\right)\\ & -\frac{1}{216}\cdot 54^{\frac{3}{4}}\left(-c^4\right)^{\frac{1}{4}}\log\left(3\,c^3x-54^{\frac{1}{4}}\left(-c^4\right)^{\frac{3}{4}}\right)\end{aligned}$$

input
```
integrate(c*x^2/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/216*54^(3/4)*(-c^4)^(1/4)*log(3*c^3*x + 54^(1/4)*(-c^4)^(3/4)) - 1/216*I
*54^(3/4)*(-c^4)^(1/4)*log(3*c^3*x + I*54^(1/4)*(-c^4)^(3/4)) + 1/216*I*54
^(3/4)*(-c^4)^(1/4)*log(3*c^3*x - I*54^(1/4)*(-c^4)^(3/4)) - 1/216*54^(3/4
)*(-c^4)^(1/4)*log(3*c^3*x - 54^(1/4)*(-c^4)^(3/4))
```

3.155.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{cx^2}{2+3x^4}\,dx = c\left(\frac{\sqrt[4]{6}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}-\frac{\sqrt[4]{6}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}+\frac{\sqrt[4]{6}\,\mathrm{atan}\left(\sqrt[4]{6}x-1\right)}{12}+\frac{\sqrt[4]{6}\,\mathrm{atan}\left(\sqrt[4]{6}x+1\right)}{12}\right)$$

input
```
integrate(c*x**2/(3*x**4+2),x)
```

output
```
c*(6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/24 - 6**(1/4)*log(x**2 +
6**(3/4)*x/3 + sqrt(6)/3)/24 + 6**(1/4)*atan(6**(1/4)*x - 1)/12 + 6**(1/4)
*atan(6**(1/4)*x + 1)/12)
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.22

$$\int \frac{cx^2}{2+3x^4}\,dx = \frac{1}{24}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-3^{\frac{1}{4}}2^{\frac{1}{4}}\,\mathrm{l}$$

input
```
integrate(c*x^2/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/24*(2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*
2^(3/4))) + 2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^
(1/4)*2^(3/4))) - 3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sq
rt(2)) + 3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*c
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{cx^2}{2+3x^4}\,dx$$
$$=\frac{1}{24}\left(2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(c*x^2/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*(2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))
) + 2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)))
- 6^(1/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 6^(1/4)*log(x^2 -
 sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*c
```

3.155.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.32

$$\int \frac{cx^2}{2+3x^4}\,dx=\frac{(-1)^{1/4}\,24^{1/4}\,c\left(\mathrm{atan}\left(\frac{(-1)^{1/4}\,24^{1/4}\,x}{2}\right)-\mathrm{atanh}\left(\frac{(-1)^{1/4}\,24^{1/4}\,x}{2}\right)\right)}{12}$$

input
```
int((c*x^2)/(3*x^4 + 2),x)
```

output
```
((-1)^(1/4)*24^(1/4)*c*(atan(((-1)^(1/4)*24^(1/4)*x)/2) - atanh(((-1)^(1/4
)*24^(1/4)*x)/2)))/12
```

3.156 $\int \frac{a+cx^2}{2+3x^4}\,dx$

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3.156.1 Optimal result

Integrand size = 17, antiderivative size = 141

$$\int \frac{a+cx^2}{2+3x^4}\,dx = -\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}$$

output
```
-1/48*ln(-6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/48*ln(6^(3/4
)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(-1+6^(1/4)*x)*(2*c
+a*6^(1/2))*6^(1/4)+1/24*arctan(1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)
```

3.156.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int \frac{a+cx^2}{2+3x^4}\,dx = \frac{-2\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)-\left(\sqrt{6}a-2c\right)\left(\log\left(2-2\sqrt[4]{6}x+\sqrt{\right.\right.}{8\ 6^{3/4}}$$

input
```
Integrate[(a + c*x^2)/(2 + 3*x^4),x]
```

output
```
(-2*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a + 2*c)*ArcTan[1
 + 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - Lo
g[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

3.156.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{3x^4+2}dx$$

$$\Big\downarrow \text{1482}$$

$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{3x^4+2}dx+\frac{1}{12}\left(\sqrt{6}a+2c\right)\int \frac{3x^2+\sqrt{6}}{3x^4+2}dx$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{1}{2}\int \frac{1}{x^2-\frac{2^{3/4}x}{\sqrt[4]{3}}+\sqrt{\frac{2}{3}}}dx+\frac{1}{2}\int \frac{1}{x^2+\frac{2^{3/4}x}{\sqrt[4]{3}}+\sqrt{\frac{2}{3}}}dx\right)+\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{3x^4+2}dx$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{3x^4+2}dx+\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{\sqrt[4]{3}\int \frac{1}{-\left(1-\sqrt[4]{6}x\right)^2-1}d\left(1-\sqrt[4]{6}x\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\int \frac{1}{-\left(\sqrt[4]{6}x+1\right)^2-1}d\left(\sqrt[4]{6}x+1\right)}{2^{3/4}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{3x^4+2}dx+$$
$$\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{\sqrt[4]{3}\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}}\right)$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\left(-\frac{\sqrt[4]{3}\int -\frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}}-\frac{\sqrt[4]{3}\int -\frac{6^{3/4}\left(\sqrt[4]{6}x+1\right)}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}}\right)+$$
$$\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{\sqrt[4]{3}\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\left(\frac{\sqrt[4]{3}\int \frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}}+\frac{\sqrt[4]{3}\int \frac{6^{3/4}\left(\sqrt[4]{6}x+1\right)}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}}\right)+$$
$$\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{\sqrt[4]{3}\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\left(\frac{\sqrt[4]{3}\int \frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}}+\frac{3}{2}\int \frac{\sqrt[4]{6}x+1}{3x^2+6^{3/4}x+\sqrt{6}}dx\right)+$$
$$\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{\sqrt[4]{3}\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{\sqrt[4]{3}\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}}\right)+$$
$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\left(\frac{\sqrt[4]{3}\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{2\ 2^{3/4}}-\frac{\sqrt[4]{3}\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{2\ 2^{3/4}}\right)$$

input
```
Int[(a + c*x^2)/(2 + 3*x^4),x]
```

3.156. $\int \frac{a+cx^2}{2+3x^4}\,dx$

output
```
((Sqrt[6]*a + 2*c)*(-((3^(1/4)*ArcTan[1 - 6^(1/4)*x])/2^(3/4)) + (3^(1/4)*
ArcTan[1 + 6^(1/4)*x])/2^(3/4)))/12 + ((Sqrt[6]*a - 2*c)*(-1/2*(3^(1/4)*Lo
g[Sqrt[6] - 6^(3/4)*x + 3*x^2])/2^(3/4) + (3^(1/4)*Log[Sqrt[6] + 6^(3/4)*x
 + 3*x^2])/(2*2^(3/4))))/12
```

3.156.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

3.156.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)} \frac{\left(\_R^2 c+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}\cdots}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}\cdots}\right)\cdots\right.}{\cdots}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}\cdots}{8+3\cdots}\right)}{\left(x^4\right)^{\cdots}}\cdots\right.}{216}$

input
```
int((c*x^2+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^2*c+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(104) = 208$.

Time = 0.28 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.50

$$
\begin{aligned}
&\int \frac{a+cx^2}{2+3x^4}\,dx \\
&= -\frac{1}{24}\sqrt{-12\,ac+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\log\Bigg(-3\left(9\,a^4-4\,c^4\right)x \\
&\quad+\left(9\,a^3-6\,ac^2-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}c\right)\sqrt{-12\,ac+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\Bigg) \\
&\quad+\frac{1}{24}\sqrt{-12\,ac+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\log\Bigg(-3\left(9\,a^4-4\,c^4\right)x \\
&\quad-\left(9\,a^3-6\,ac^2-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}c\right)\sqrt{-12\,ac+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\Bigg) \\
&\quad-\frac{1}{24}\sqrt{-12\,ac-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\log\Bigg(-3\left(9\,a^4-4\,c^4\right)x \\
&\quad+\left(9\,a^3-6\,ac^2+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}c\right)\sqrt{-12\,ac-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\Bigg) \\
&\quad+\frac{1}{24}\sqrt{-12\,ac-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\log\Bigg(-3\left(9\,a^4-4\,c^4\right)x \\
&\quad-\left(9\,a^3-6\,ac^2+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}c\right)\sqrt{-12\,ac-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\Bigg)
\end{aligned}
$$

input
```
integrate((c*x^2+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
-1/24*sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))*log(-3*(9*
a^4 - 4*c^4)*x + (9*a^3 - 6*a*c^2 - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c
^4)*c)*sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))) + 1/24*s
qrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))*log(-3*(9*a^4 - 4
*c^4)*x - (9*a^3 - 6*a*c^2 - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)*c)*
sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))) - 1/24*sqrt(-12
*a*c - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))*log(-3*(9*a^4 - 4*c^4)*x
 + (9*a^3 - 6*a*c^2 + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)*c)*sqrt(-1
2*a*c - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))) + 1/24*sqrt(-12*a*c -
sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))*log(-3*(9*a^4 - 4*c^4)*x - (9*a
^3 - 6*a*c^2 + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)*c)*sqrt(-12*a*c -
 sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))
```

3.156.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.48

$$\int \frac{a+cx^2}{2+3x^4}\,dx$$
$$=\text{RootSum}\left(55296t^4+2304t^2ac+9a^4+12a^2c^2+4c^4,\left(t\mapsto t\log\left(x+\frac{-4608t^3c+72ta^3-144tac^2}{9a^4-4c^4}\right)\right)\right)$$

input
```
integrate((c*x**2+a)/(3*x**4+2),x)
```

output
```
RootSum(55296*_t**4 + 2304*_t**2*a*c + 9*a**4 + 12*a**2*c**2 + 4*c**4, Lam
bda(_t, _t*log(x + (-4608*_t**3*c + 72*_t*a**3 - 144*_t*a*c**2)/(9*a**4 -
4*c**4))))
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{a+cx^2}{2+3x^4}\,dx =& \frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a+\sqrt{2}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\ &+\frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a+\sqrt{2}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\ &+\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\\ &-\frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\end{aligned}$$

input
```
integrate((c*x^2+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2
*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)
*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/48*3^(
1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x +
 sqrt(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 -
 3^(1/4)*2^(3/4)*x + sqrt(2))
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int \frac{a+cx^2}{2+3x^4}\,dx = \frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((c*x^2+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(
2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)
^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*log(x
^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*l
og(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.156.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.23

$$\int \frac{a+cx^2}{2+3x^4}\,dx = -2\,\mathrm{atanh}\left(\frac{216\,a^2\,x\,\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3+18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2-12\,c^3} - \frac{144\,c^2\,x\,\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3+18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2-12\,c^3}\right)\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}} + 2\,\mathrm{atanh}\left(\frac{216\,a^2\,x\,\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3-18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2+12\,c^3} - \frac{144\,c^2\,x\,\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3-18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2+12\,c^3}\right)\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}$$

input
```
int((a + c*x^2)/(3*x^4 + 2),x)
```

output
```
2*atanh((216*a^2*x*((6^(1/2)*a^2*1i)/192 - (a*c)/48 - (6^(1/2)*c^2*1i)/288
)^(1/2))/(6^(1/2)*a^3*9i - 18*a^2*c + 12*c^3 - 6^(1/2)*a*c^2*6i) - (144*c^
2*x*((6^(1/2)*a^2*1i)/192 - (a*c)/48 - (6^(1/2)*c^2*1i)/288)^(1/2))/(6^(1/
2)*a^3*9i - 18*a^2*c + 12*c^3 - 6^(1/2)*a*c^2*6i))*((6^(1/2)*a^2*1i)/192 -
 (a*c)/48 - (6^(1/2)*c^2*1i)/288)^(1/2) - 2*atanh((216*a^2*x*((6^(1/2)*c^2
*1i)/288 - (6^(1/2)*a^2*1i)/192 - (a*c)/48)^(1/2))/(6^(1/2)*a^3*9i + 18*a^
2*c - 12*c^3 - 6^(1/2)*a*c^2*6i) - (144*c^2*x*((6^(1/2)*c^2*1i)/288 - (6^(
1/2)*a^2*1i)/192 - (a*c)/48)^(1/2))/(6^(1/2)*a^3*9i + 18*a^2*c - 12*c^3 -
6^(1/2)*a*c^2*6i))*((6^(1/2)*c^2*1i)/288 - (6^(1/2)*a^2*1i)/192 - (a*c)/48
)^(1/2)
```

3.156. $\int \frac{a+cx^2}{2+3x^4}\,dx$

3.157 $\int \frac{bx+cx^2}{2+3x^4}\,dx$

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3.157.1 Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\arctan\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}$$

output
```
1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)+1/2
4*c*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*c*ln(6^(3/4)*x+3*x^2+6^(1/2)
)*6^(1/4)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

3.157.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \frac{-2\left(\sqrt[4]{6}b+c\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\left(-\sqrt[4]{6}b+c\right)\arctan\left(1+\sqrt[4]{6}x\right)+c\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)-c\log}{4\ 6^{3/4}}$$

input
```
Integrate[(b*x + c*x^2)/(2 + 3*x^4),x]
```

output
```
(-2*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*(-(6^(1/4)*b) + c)*ArcTan[1
+ 6^(1/4)*x] + c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - c*Log[2 + 2*6^(1/4)*
x + Sqrt[6]*x^2])/(4*6^(3/4))
```

3.157.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2027, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{3x^4+2}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x(b+cx)}{3x^4+2}dx$$

$$\Big\downarrow \text{ 2370}$$

$$\int \left(\frac{bx}{3x^4+2}+\frac{cx^2}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}$$

input
```
Int[(b*x + c*x^2)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/
4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x +
 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4))
```

3.157.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

3.157.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.27

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^{4}+2\right)}\frac{\left(\_R^{2}c+\_Rb\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{12}$
default	$\frac{b\arctan\left(\frac{x^{2}\sqrt{6}}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^{3}\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\cdots\right)}{216}$

input
```
int((c*x^2+b*x)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^2*c+_R*b)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

3.157.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 12741, normalized size of antiderivative = 103.59

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.157.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\int \frac{bx+cx^2}{2+3x^4}\,dx$$
$$= \text{RootSum}\left(27648t^4+576t^2b^2+96tbc^2+3b^4+2c^4,\left(t\mapsto t\log\left(x+\frac{-1152t^3c^2+288t^2b^3-36tb^2c^2+3b}{6b^4c-c^5}\right.\right.\right.$$

input
```
integrate((c*x**2+b*x)/(3*x**4+2),x)
```

output
```
RootSum(27648*_t**4 + 576*_t**2*b**2 + 96*_t*b*c**2 + 3*b**4 + 2*c**4, Lam
bda(_t, _t*log(x + (-1152*_t**3*c**2 + 288*_t**2*b**3 - 36*_t*b**2*c**2 +
3*b**5 - 3*b*c**4)/(6*b**4*c - c**5))))
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.20

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}c-2\sqrt{3}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$
$$+\frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}c+2\sqrt{3}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-\frac{1}{24}$$
$$\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

input
```
integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c - 2*sqrt(3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*
(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c + 2*sqr
t(3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) - 1/24
*3^(1/4)*2^(1/4)*c*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*3
^(1/4)*2^(1/4)*c*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$
\begin{aligned}
\int \frac{bx+cx^2}{2+3x^4}\,dx = & -\frac{1}{24}\cdot 6^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) \\
& +\frac{1}{24}\cdot 6^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) \\
& -\frac{1}{12}\left(\sqrt{6}b-6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) \\
& +\frac{1}{12}\left(\sqrt{6}b+6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)
\end{aligned}
$$

input
```
integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")
```

output
```
-1/24*6^(1/4)*c*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*6^(1/4
)*c*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/12*(sqrt(6)*b - 6^(1/
4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/12*(
sqrt(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)
^(1/4)))
```

3.157.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.32

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \sum_{k=1}^{4}\ln\left(9\,b^3\,x-6\,c^3-\mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)\,b\,c\,144+\mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)^2\,b\,x\,864+\mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)\,c^2\,x\,72\right)\,\mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)$$

input
```
int((b*x + c*x^2)/(3*x^4 + 2),x)
```

output
```
symsum(log(9*b^3*x - 6*c^3 - 144*root(z^4 + (b^2*z^2)/48 + (b*c^2*z)/288 +
 c^4/13824 + b^4/9216, z, k)*b*c + 864*root(z^4 + (b^2*z^2)/48 + (b*c^2*z)
/288 + c^4/13824 + b^4/9216, z, k)^2*b*x + 72*root(z^4 + (b^2*z^2)/48 + (b
*c^2*z)/288 + c^4/13824 + b^4/9216, z, k)*c^2*x)*root(z^4 + (b^2*z^2)/48 +
 (b*c^2*z)/288 + c^4/13824 + b^4/9216, z, k), k, 1, 4)
```

3.158 $\int \frac{a+bx+cx^2}{2+3x^4}\,dx$

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3.158.1 Optimal result

Integrand size = 20, antiderivative size = 163

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}} - \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}}$$

output
```
1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)-1/48*ln(-6^(3/4)*x+3*x^2+6^(1/2))*(
-2*c+a*6^(1/2))*6^(1/4)+1/48*ln(6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*
6^(1/4)+1/24*arctan(-1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(1+6^
(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)
```

3.158.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.79

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx$$
$$= \frac{-2\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)-\left(\sqrt{6}a-2c\right)\left(\right.}{8\ 6^{3/4}}$$

input
```
Integrate[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

output
```
(-2*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a -
 2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6
^(1/4)*x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

3.158.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{3x^4+2}\,dx$$
$$\Big\downarrow \ 2415$$
$$\int \left(\frac{a+cx^2}{3x^4+2}+\frac{bx}{3x^4+2}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\arctan\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

input
```
Int[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1
/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)
) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((S
qrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4))
```

3.158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.158.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.21

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^2 c+\_R b+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}}{8+3}\right)}{\left(x^4\right)^{\frac{3}{4}}}\right.}{216}$

input
```
int((c*x^2+b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^2*c+_R*b+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

3.158.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 46651, normalized size of antiderivative = 286.20

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.158.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(144) = 288.

Time = 2.39 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.79

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx$$
$$= \text{RootSum}\left(55296t^4 + t^2\cdot\left(2304ac+1152b^2\right) + t\left(-288a^2b+192bc^2\right) + 9a^4 + 12a^2c^2 - 24ab^2c + 6b^4 + 4c\right.$$

input
```
integrate((c*x**2+b*x+a)/(3*x**4+2),x)
```

output
```
RootSum(55296*_t**4 + _t**2*(2304*a*c + 1152*b**2) + _t*(-288*a**2*b + 192
*b*c**2) + 9*a**4 + 12*a**2*c**2 - 24*a*b**2*c + 6*b**4 + 4*c**4, Lambda(_
t, _t*log(x + (-13824*_t**3*a**2*c + 27648*_t**3*a*b**2 + 9216*_t**3*c**3
+ 1728*_t**2*a**3*b + 3456*_t**2*a*b*c**2 - 2304*_t**2*b**3*c + 216*_t*a**
5 - 576*_t*a**3*c**2 + 1296*_t*a**2*b**2*c + 288*_t*a*b**4 + 288*_t*a*c**4
 + 288*_t*b**2*c**3 + 90*a**4*b*c - 90*a**3*b**3 + 60*a*b**3*c**2 - 24*b**
5*c + 24*b*c**5)/(27*a**6 - 18*a**4*c**2 + 144*a**3*b**2*c - 72*a**2*b**4
- 12*a**2*c**4 + 96*a*b**2*c**3 - 48*b**4*c**2 + 8*c**6))))
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.15

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = \frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) - \frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a-2\sqrt{3}\sqrt{2}b+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a+2\sqrt{3}\sqrt{2}b+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

input
```
integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(
3/4)*x + sqrt(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(
3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*(3^(3/4)*2^(3/4)*a - 2*sqrt(3
)*sqrt(2)*b + 2*3^(1/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x
 + 3^(1/4)*2^(3/4))) + 1/24*(3^(3/4)*2^(3/4)*a + 2*sqrt(3)*sqrt(2)*b + 2*3
^(1/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4
)))
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx$$
$$= \frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$
$$+\frac{1}{24}\left(6^{\frac{3}{4}}a+2\sqrt{6}b+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$
$$+\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$
$$-\frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4
)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b + 2*6^(1/4)
*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^
(3/4)*a - 2*6^(1/4)*c)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48
*(6^(3/4)*a - 2*6^(1/4)*c)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.158.9 Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.66

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = \sum_{k=1}^{4}\ln\Bigg(9\,a\,b^2-9\,a^2\,c$$
$$-\mathrm{root}\left(z^4+\frac{z^2\,(2304\,a\,c+1152\,b^2)}{55296}-\frac{z\,(288\,a^2\,b-192\,b\,c^2)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}\right.$$
$$\left.+\frac{a^4}{6144},z,k\right)\left(\mathrm{root}\left(z^4+\frac{z^2\,(2304\,a\,c+1152\,b^2)}{55296}-\frac{z\,(288\,a^2\,b-192\,b\,c^2)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}\right.\right.$$
$$\left.+144\,b\,c+x\,(108\,a^2-72\,c^2)\Big)-6\,c^3+x\,(9\,b^3-18\,a\,b\,c)\right)\mathrm{root}\left(z^4+\frac{z^2\,(2304\,a\,c+1152\,b^2)}{55296}\right.$$
$$\left.-\frac{z\,(288\,a^2\,b-192\,b\,c^2)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)\Bigg)$$

input
```
int((a + b*x + c*x^2)/(3*x^4 + 2),x)
```

output
```
symsum(log(9*a*b^2 - 9*a^2*c - root(z^4 + (z^2*(2304*a*c + 1152*b^2))/5529
6 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304 + (a^2*c^2)/4608 +
c^4/13824 + b^4/9216 + a^4/6144, z, k)*(root(z^4 + (z^2*(2304*a*c + 1152*b
^2))/55296 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304 + (a^2*c^2
)/4608 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*(864*a - 864*b*x) + 144*b*
c + x*(108*a^2 - 72*c^2)) - 6*c^3 + x*(9*b^3 - 18*a*b*c))*root(z^4 + (z^2*
(2304*a*c + 1152*b^2))/55296 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*
c)/2304 + (a^2*c^2)/4608 + c^4/13824 + b^4/9216 + a^4/6144, z, k), k, 1, 4
)
```

3.158. $\int \frac{a+bx+cx^2}{2+3x^4}\,dx$

3.159 $\int \frac{dx^3}{2+3x^4} dx$

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3.159.8 Giac [A] (verification not implemented) . . . 1265
3.159.9 Mupad [B] (verification not implemented) . . . 1266

3.159.1 Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{dx^3}{2+3x^4} dx = \frac{1}{12} d \log \left(2+3x^4\right)$$

output
```
1/12*d*ln(3*x^4+2)
```

3.159.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{dx^3}{2+3x^4} dx = \frac{1}{12} d \log \left(2+3x^4\right)$$

input
```
Integrate[(d*x^3)/(2 + 3*x^4),x]
```

output
```
(d*Log[2 + 3*x^4])/12
```

3.159.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{dx^3}{3x^4+2}\,dx$$

$$\Big\downarrow \; 27$$

$$d\int \frac{x^3}{3x^4+2}dx$$

$$\Big\downarrow \; 792$$

$$\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
Int[(d*x^3)/(2 + 3*x^4),x]
```

output
```
(d*Log[2 + 3*x^4])/12
```

3.159.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

3.159.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{d \ln\left(x^4+\frac{2}{3}\right)}{12}$	10
derivativedivides	$\frac{d \ln\left(3x^4+2\right)}{12}$	12
default	$\frac{d \ln\left(3x^4+2\right)}{12}$	12
norman	$\frac{d \ln\left(3x^4+2\right)}{12}$	12
meijerg	$\frac{d \ln\left(\frac{3x^4}{2}+1\right)}{12}$	12
risch	$\frac{d \ln\left(3x^4+2\right)}{12}$	12

input
```
int(d*x^3/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*d*ln(x^4+2/3)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

input
```
integrate(d*x^3/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/12*d*log(3*x^4 + 2)
```

3.159.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{d\log\left(3x^4+2\right)}{12}$$

input
```
integrate(d*x**3/(3*x**4+2),x)
```

output
```
d*log(3*x**4 + 2)/12
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

input
```
integrate(d*x^3/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/12*d*log(3*x^4 + 2)
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

input
```
integrate(d*x^3/(3*x^4+2),x, algorithm="giac")
```

output
```
1/12*d*log(3*x^4 + 2)
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{d\,\ln\left(x^4+\frac{2}{3}\right)}{12}$$

input
```
int((d*x^3)/(3*x^4 + 2),x)
```

output
```
(d*log(x^4 + 2/3))/12
```

3.160 $\int \frac{a+dx^3}{2+3x^4}\,dx$

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3.160.8 Giac [A] (verification not implemented) 1271
3.160.9 Mupad [B] (verification not implemented) 1272

3.160.1 Optimal result

Integrand size = 17, antiderivative size = 114

$$\int \frac{a+dx^3}{2+3x^4}\,dx=-\frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\arctan\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}-\frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}+\frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)+1/1
2*d*ln(3*x^4+2)-1/48*a*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*a*ln(6^(3
/4)*x+3*x^2+6^(1/2))*6^(3/4)
```

3.160.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.95

$$\int \frac{a+dx^3}{2+3x^4}\,dx=\frac{1}{48}\left(-2\,6^{3/4}a\arctan\left(1-\sqrt[4]{6}x\right)+2\,6^{3/4}a\arctan\left(1+\sqrt[4]{6}x\right)-6^{3/4}a\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)+6^{3/4}a\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)+4d\log\left(2+3x^4\right)\right)$$

input
```
Integrate[(a + d*x^3)/(2 + 3*x^4),x]
```

output
```
(-2*6^(3/4)*a*ArcTan[1 - 6^(1/4)*x] + 2*6^(3/4)*a*ArcTan[1 + 6^(1/4)*x] -
6^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(3/4)*a*Log[2 + 2*6^(1/4)
*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

3.160.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+dx^3}{3x^4+2}dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{a}{3x^4+2}+\frac{dx^3}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\arctan\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
Int[(a + d*x^3)/(2 + 3*x^4),x]
```

output
```
-1/4*(a*ArcTan[1 - 6^(1/4)*x])/6^(1/4) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1
/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] +
 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*Log[2 + 3*x^4])/12
```

3.160.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.160.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.27

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^{4}+2\right)}\frac{\left(\_R^{3}d+a\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{d\ln\left(3x^{4}+2\right)}{12}$
meijerg	$\frac{d\ln\left(\frac{3x^{4}}{2}+1\right)}{12}+\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\right.}{96}$

input
```
int((d*x^3+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^3*d+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

3.160.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(83) = 166$.

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.68

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(\sqrt{3}\sqrt{\sqrt{6}\sqrt{-a^4}}+2\,d\right)\log\left(3\,ax+\sqrt{3}\sqrt{\sqrt{6}\sqrt{-a^4}}\right) - \frac{1}{24}\left(\sqrt{3}\sqrt{\sqrt{6}\sqrt{-a^4}}-2\,d\right)\log\left(3\,ax-\sqrt{3}\sqrt{\sqrt{6}\sqrt{-a^4}}\right) + \frac{1}{24}\left(\sqrt{3}\sqrt{-\sqrt{6}\sqrt{-a^4}}+2\,d\right)\log\left(3\,ax+\sqrt{3}\sqrt{-\sqrt{6}\sqrt{-a^4}}\right) - \frac{1}{24}\left(\sqrt{3}\sqrt{-\sqrt{6}\sqrt{-a^4}}-2\,d\right)\log\left(3\,ax-\sqrt{3}\sqrt{-\sqrt{6}\sqrt{-a^4}}\right)$$

input
```
integrate((d*x^3+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/24*(sqrt(3)*sqrt(sqrt(6)*sqrt(-a^4)) + 2*d)*log(3*a*x + sqrt(3)*sqrt(sqr
t(6)*sqrt(-a^4))) - 1/24*(sqrt(3)*sqrt(sqrt(6)*sqrt(-a^4)) - 2*d)*log(3*a*
x - sqrt(3)*sqrt(sqrt(6)*sqrt(-a^4))) + 1/24*(sqrt(3)*sqrt(-sqrt(6)*sqrt(-
a^4)) + 2*d)*log(3*a*x + sqrt(3)*sqrt(-sqrt(6)*sqrt(-a^4))) - 1/24*(sqrt(3
)*sqrt(-sqrt(6)*sqrt(-a^4)) - 2*d)*log(3*a*x - sqrt(3)*sqrt(-sqrt(6)*sqrt(
-a^4)))
```

3.160.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.45

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \text{RootSum}\left(165888t^4-55296t^3d+6912t^2d^2-384td^3+27a^4+8d^4,\left(t\mapsto t\log\left(x+\frac{24t-2d}{3a}\right)\right)\right)$$

input
```
integrate((d*x**3+a)/(3*x**4+2),x)
```

output
```
RootSum(165888*_t**4 - 55296*_t**3*d + 6912*_t**2*d**2 - 384*_t*d**3 + 27*
a**4 + 8*d**4, Lambda(_t, _t*log(x + (24*_t - 2*d)/(3*a))))
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.31

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d+3a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

input
```
integrate((d*x^3+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x
- 3^(1/4)*2^(3/4))) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*lo
g(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^
(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{48}\left(6^{\frac{3}{4}}a+4d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-\frac{1}{48}\left(6^{\frac{3}{4}}a-4d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4
))) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
- 1/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.160.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \ln\left(x - \frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}-\frac{6^{1/4}\,\sqrt{\frac{3}{4}\mathrm{i}}\,a}{12}\right)+\ln\left(x+\frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}+\frac{6^{1/4}\,\sqrt{\frac{3}{4}\mathrm{i}}\,a}{12}\right)+\ln\left(x-\frac{(-1)\right.$$

input
```
int((a + d*x^3)/(3*x^4 + 2),x)
```

output
```
log(x - ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(3i/4)^(1/2)*a)/1
2) + log(x + ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(3i/4)^(1/2)
*a)/12) + log(x - ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(-3i/4)
^(1/2)*a)/12) + log(x + ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(
-3i/4)^(1/2)*a)/12)
```

3.161 $\int \frac{bx+dx^3}{2+3x^4} dx$

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3.161.2 Mathematica [C] (verified) 1273
3.161.3 Rubi [A] (verified) 1274
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3.161.8 Giac [B] (verification not implemented) 1277
3.161.9 Mupad [B] (verification not implemented) 1277

3.161.1 Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{bx+dx^3}{2+3x^4} dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} + \frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/12*d*ln(3*x^4+2)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

3.161.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

$$\int \frac{bx+dx^3}{2+3x^4} dx = \frac{1}{24}\left(i\sqrt{6}b+2d\right)\log\left(\sqrt{6}-3ix^2\right) + \frac{1}{24}\left(-i\sqrt{6}b+2d\right)\log\left(\sqrt{6}+3ix^2\right)$$

input
```
Integrate[(b*x + d*x^3)/(2 + 3*x^4),x]
```

output
```
((I*Sqrt[6]*b + 2*d)*Log[Sqrt[6] - (3*I)*x^2])/24 + (((-I)*Sqrt[6]*b + 2*d
)*Log[Sqrt[6] + (3*I)*x^2])/24
```

3.161.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2027, 1577, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+dx^3}{3x^4+2}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x\left(b+dx^2\right)}{3x^4+2}dx$$

$$\Big\downarrow \text{ 1577}$$

$$\frac{1}{2}\int \frac{dx^2+b}{3x^4+2}dx^2$$

$$\Big\downarrow \text{ 452}$$

$$\frac{1}{2}\left(b\int \frac{1}{3x^4+2}dx^2+d\int \frac{x^2}{3x^4+2}dx^2\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(d\int \frac{x^2}{3x^4+2}dx^2+\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{\sqrt{6}}\right)$$

$$\Big\downarrow \text{ 240}$$

$$\frac{1}{2}\left(\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{\sqrt{6}}+\frac{1}{6}d\log\left(3x^4+2\right)\right)$$

input
```
Int[(b*x + d*x^3)/(2 + 3*x^4),x]
```

output
```
((b*ArcTan[Sqrt[3/2]*x^2])/Sqrt[6] + (d*Log[2 + 3*x^4])/6)/2
```

3.161.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.161.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{d\ln(3x^4+2)}{12}+\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$	28
risch	$\frac{d\ln(9x^4+6)}{12}+\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}$	28
meijerg	$\frac{d\ln\left(\frac{3x^4}{2}+1\right)}{12}+\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,x^2}{2}\right)}{12}$	31

input
```
int((d*x^3+b*x)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*d*ln(3*x^4+2)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{bx+dx^3}{2+3x^4}\,dx = \frac{1}{12}\sqrt{6}b\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)+\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2) + 1/12*d*log(3*x^4 + 2)
```

3.161.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{bx+dx^3}{2+3x^4}\,dx = \left(-\frac{\sqrt{6}ib}{24}+\frac{d}{12}\right)\log\left(x^2-\frac{\sqrt{6}i}{3}\right)+\left(\frac{\sqrt{6}ib}{24}+\frac{d}{12}\right)\log\left(x^2+\frac{\sqrt{6}i}{3}\right)$$

input
```
integrate((d*x**3+b*x)/(3*x**4+2),x)
```

output
```
(-sqrt(6)*I*b/24 + d/12)*log(x**2 - sqrt(6)*I/3) + (sqrt(6)*I*b/24 + d/12)
*log(x**2 + sqrt(6)*I/3)
```

3.161.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(27) = 54.

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.14

$$\begin{aligned}\int \frac{bx+dx^3}{2+3x^4}\,dx &= -\frac{1}{12}\sqrt{3}\sqrt{2}b\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\ &\quad+\frac{1}{12}\sqrt{3}\sqrt{2}b\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\ &\quad+\frac{1}{12}d\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{12}d\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\end{aligned}$$

input
```
integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*
2^(3/4))) + 1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x
 - 3^(1/4)*2^(3/4))) + 1/12*d*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2
)) + 1/12*d*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

3.161.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.58

$$\int \frac{bx+dx^3}{2+3x^4}\,dx = -\frac{1}{12}\sqrt{6}b\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\sqrt{6}b\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}d\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) + \frac{1}{12}d\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="giac")
```

output
```
-1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))
) + 1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/
4))) + 1/12*d*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/12*d*log(x^
2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{bx+dx^3}{2+3x^4}\,dx = \frac{d\,\ln\left(x^4+\frac{2}{3}\right)}{12} + \frac{\sqrt{6}\,b\,\mathrm{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}$$

input
```
int((b*x + d*x^3)/(3*x^4 + 2),x)
```

output
```
(d*log(x^4 + 2/3))/12 + (6^(1/2)*b*atan((6^(1/2)*x^2)/2))/12
```

3.162 $\int \frac{a+bx+dx^3}{2+3x^4} dx$

3.162.1 Optimal result 1278
3.162.2 Mathematica [A] (verified) 1278
3.162.3 Rubi [A] (verified) 1279
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3.162.5 Fricas [C] (verification not implemented) 1281
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3.162.7 Maxima [A] (verification not implemented) 1282
3.162.8 Giac [A] (verification not implemented) 1282
3.162.9 Mupad [B] (verification not implemented) 1283

3.162.1 Optimal result

Integrand size = 20, antiderivative size = 136

$$\int \frac{a+bx+dx^3}{2+3x^4} dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\arctan\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} - \frac{a\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{a\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)+1/1
2*d*ln(3*x^4+2)-1/48*a*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*a*ln(6^(3
/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{a+bx+dx^3}{2+3x^4} dx = \frac{1}{48}\left(-2\sqrt{6}\left(\sqrt[4]{6}a+2b\right)\arctan\left(1-\sqrt[4]{6}x\right) + 2\sqrt{6}\left(\sqrt[4]{6}a-2b\right)\arctan\left(1+\sqrt[4]{6}x\right) - 6^{3/4}a\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right) + 6^{3/4}a\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right) + 4d\log\left(2+3x^4\right)\right)$$

input
```
Integrate[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

output
```
(-2*Sqrt[6]*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*Sqrt[6]*(6^(1/4)*a
 - 2*b)*ArcTan[1 + 6^(1/4)*x] - 6^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^
2] + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

3.162.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+dx^3}{3x^4+2}\,dx$$
$$\Big\downarrow \text{2415}$$
$$\int \left(\frac{a}{3x^4+2}+\frac{x\left(b+dx^2\right)}{3x^4+2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\arctan\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
Int[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/
4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x +
 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) +
(d*Log[2 + 3*x^4])/12
```

3.162.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.162.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.25

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+\_R b+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan\left(\frac{x^2\sqrt{6}}{2}\right)\sqrt{6}}{12}+\frac{d\ln(3x}{12}$
meijerg	$\frac{d\ln\left(\frac{3x^4}{2}+1\right)}{12}+\frac{\sqrt{6}\,b\arctan\left(\frac{\sqrt{2}\sqrt{3}\,x^2}{2}\right)}{12}+\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1\right.}{}\right.}{96}$

input
```
int((d*x^3+b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^3*d+_R*b+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

3.162.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 17085, normalized size of antiderivative = 125.62

$$\int \frac{a + bx + dx^3}{2 + 3x^4} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.162.6 Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.46

$$\int \frac{a + bx + dx^3}{2 + 3x^4} \, dx$$
$$= \text{RootSum}\left(165888t^4 - 55296t^3d + t^2 \cdot \left(3456b^2 + 6912d^2\right) + t\left(-864a^2b - 576b^2d - 384d^3\right) + 27a^4 + 72a^2\right.$$

input
```
integrate((d*x**3+b*x+a)/(3*x**4+2),x)
```

output
```
RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(3456*b**2 + 6912*d**2) + _t*
(-864*a**2*b - 576*b**2*d - 384*d**3) + 27*a**4 + 72*a**2*b*d + 18*b**4 +
24*b**2*d**2 + 8*d**4, Lambda(_t, _t*log(x + (27648*_t**3*b**2 + 1728*_t**
2*a**2*b - 6912*_t**2*b**2*d + 216*_t*a**4 - 288*_t*a**2*b*d + 288*_t*b**4
 + 576*_t*b**2*d**2 - 18*a**4*d - 90*a**2*b**3 + 12*a**2*b*d**2 - 24*b**4*
d - 16*b**2*d**3)/(27*a**5 - 72*a*b**4))))
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.26

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = \frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d+3\,a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a-2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

input
```
integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*log(sqrt(3)*x^2 + 3^(1/4
)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d - 3*a)
*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*sqrt(3)*(3^(1/4)*2^
(3/4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)*b)*arctan(1/6*3^(3/
4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a+2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a+4\,d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a-4\,d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*(6^(3/4)*a - 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(
2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)
^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqr
t(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)
*(2/3)^(1/4)*x + sqrt(2/3))
```

3.162.9 Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.26

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = \sum_{k=1}^{4} \ln\left(x\left(9\,a^2\,d+9\,b^3+6\,b\,d^2\right)+9\,a\,b^2-6\,a\,d^2-\mathrm{root}\left(z^4-\frac{d\,z^3}{3}\right.\right.$$
$$+\frac{z^2\left(3456\,b^2+6912\,d^2\right)}{165888}-\frac{z\left(864\,a^2\,b+576\,b^2\,d+384\,d^3\right)}{165888}+\frac{a^2\,b\,d}{2304}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{b^4}{9216}$$
$$+\frac{a^4}{6144},z,k\Bigg)\left(\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\left(3456\,b^2+6912\,d^2\right)}{165888}-\frac{z\left(864\,a^2\,b+576\,b^2\,d+384\,d^3\right)}{165888}+\frac{a^2\,b\,d}{2304}+\frac{b^2\,d^2}{6912}+\right.\right.$$
$$\left.\left.-144\,a\,d+x\left(108\,a^2+144\,b\,d\right)\right)\right)\mathrm{root}\left(z^4-\frac{d\,z^3}{3}+\frac{z^2\left(3456\,b^2+6912\,d^2\right)}{165888}\right.$$
$$\left.-\frac{z\left(864\,a^2\,b+576\,b^2\,d+384\,d^3\right)}{165888}+\frac{a^2\,b\,d}{2304}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\right)$$

input
```
int((a + b*x + d*x^3)/(3*x^4 + 2),x)
```

output
```
symsum(log(x*(9*a^2*d + 6*b*d^2 + 9*b^3) + 9*a*b^2 - 6*a*d^2 - root(z^4 -
(d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^2))/165888 - (z*(864*a^2*b + 576*b^2*d
 + 384*d^3))/165888 + (a^2*b*d)/2304 + (b^2*d^2)/6912 + d^4/20736 + b^4/92
16 + a^4/6144, z, k)*(root(z^4 - (d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^2))/1
65888 - (z*(864*a^2*b + 576*b^2*d + 384*d^3))/165888 + (a^2*b*d)/2304 + (b
^2*d^2)/6912 + d^4/20736 + b^4/9216 + a^4/6144, z, k)*(864*a - 864*b*x) -
144*a*d + x*(144*b*d + 108*a^2)))*root(z^4 - (d*z^3)/3 + (z^2*(3456*b^2 +
6912*d^2))/165888 - (z*(864*a^2*b + 576*b^2*d + 384*d^3))/165888 + (a^2*b*
d)/2304 + (b^2*d^2)/6912 + d^4/20736 + b^4/9216 + a^4/6144, z, k), k, 1, 4
)
```

3.163 $\int \frac{cx^2+dx^3}{2+3x^4} dx$

3.163.1 Optimal result 1284
3.163.2 Mathematica [A] (verified) 1284
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3.163.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{cx^2+dx^3}{2+3x^4} dx = -\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}}+\frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)+1/1
2*d*ln(3*x^4+2)+1/24*c*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*c*ln(6^(3
/4)*x+3*x^2+6^(1/2))*6^(1/4)
```

3.163.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.95

$$\int \frac{cx^2+dx^3}{2+3x^4} dx = \frac{1}{24}\left(-2\sqrt[4]{6}c\arctan\left(1-\sqrt[4]{6}x\right)+2\sqrt[4]{6}c\arctan\left(1+\sqrt[4]{6}x\right) + \sqrt[4]{6}c\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)-\sqrt[4]{6}c\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right) + 2d\log\left(2+3x^4\right)\right)$$

input
```
Integrate[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
(-2*6^(1/4)*c*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*c*ArcTan[1 + 6^(1/4)*x] +
6^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - 6^(1/4)*c*Log[2 + 2*6^(1/4)
*x + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

3.163.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2027, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cx^2+dx^3}{3x^4+2}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^2(c+dx)}{3x^4+2}dx$$

$$\Big\downarrow \text{2370}$$

$$\int \left(\frac{cx^2}{3x^4+2}+\frac{dx^3}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
Int[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
-1/2*(c*ArcTan[1 - 6^(1/4)*x])/6^(3/4) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3
/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] +
 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

3.163.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

3.163.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^{4}+2\right)}\frac{\left(\_R^{3} d+\_R^{2} c\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{12}$
default	$\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}+\frac{d\ln\left(3x^{4}+2\right)}{12}$
meijerg	$\frac{d\ln\left(\frac{3x^{4}}{2}+1\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^{3}\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+ \right.}{216}$

input
```
int((d*x^3+c*x^2)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^3*d+_R^2*c)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

3.163.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.10

$$\int \frac{cx^2 + dx^3}{2 + 3x^4}\,dx = \frac{1}{24}\left(\sqrt{2}\sqrt{\sqrt{6}\sqrt{-c^4}} + 2\,d\right)\log\left(6\,c^3x + \sqrt{6}\sqrt{2}\sqrt{-c^4}\sqrt{\sqrt{6}\sqrt{-c^4}}\right) - \frac{1}{24}\left(\sqrt{2}\sqrt{\sqrt{6}\sqrt{-c^4}} - 2\,d\right)\log\left(6\,c^3x - \sqrt{6}\sqrt{2}\sqrt{-c^4}\sqrt{\sqrt{6}\sqrt{-c^4}}\right) - \frac{1}{24}\left(\sqrt{2}\sqrt{-\sqrt{6}\sqrt{-c^4}} - 2\,d\right)\log\left(6\,c^3x + \sqrt{6}\sqrt{2}\sqrt{-c^4}\sqrt{-\sqrt{6}\sqrt{-c^4}}\right) + \frac{1}{24}\left(\sqrt{2}\sqrt{-\sqrt{6}\sqrt{-c^4}} + 2\,d\right)\log\left(6\,c^3x - \sqrt{6}\sqrt{2}\sqrt{-c^4}\sqrt{-\sqrt{6}\sqrt{-c^4}}\right)$$

input
```
integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/24*(sqrt(2)*sqrt(sqrt(6)*sqrt(-c^4)) + 2*d)*log(6*c^3*x + sqrt(6)*sqrt(2
)*sqrt(-c^4)*sqrt(sqrt(6)*sqrt(-c^4))) - 1/24*(sqrt(2)*sqrt(sqrt(6)*sqrt(-
c^4)) - 2*d)*log(6*c^3*x - sqrt(6)*sqrt(2)*sqrt(-c^4)*sqrt(sqrt(6)*sqrt(-c
^4))) - 1/24*(sqrt(2)*sqrt(-sqrt(6)*sqrt(-c^4)) - 2*d)*log(6*c^3*x + sqrt(
6)*sqrt(2)*sqrt(-c^4)*sqrt(-sqrt(6)*sqrt(-c^4))) + 1/24*(sqrt(2)*sqrt(-sqr
t(6)*sqrt(-c^4)) + 2*d)*log(6*c^3*x - sqrt(6)*sqrt(2)*sqrt(-c^4)*sqrt(-sqr
t(6)*sqrt(-c^4)))
```

3.163.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{cx^2 + dx^3}{2 + 3x^4}\,dx = \operatorname{RootSum}\left(41472t^4 - 13824t^3d + 1728t^2d^2 - 96td^3 + 3c^4 + 2d^4, \left(t \mapsto t\log\left(x + \frac{3456t^3 - 864t^2d + 72td}{3c^3}\right.\right.\right.$$

input
```
integrate((d*x**3+c*x**2)/(3*x**4+2),x)
```

output
```
RootSum(41472*_t**4 - 13824*_t**3*d + 1728*_t**2*d**2 - 96*_t*d**3 + 3*c**
4 + 2*d**4, Lambda(_t, _t*log(x + (3456*_t**3 - 864*_t**2*d + 72*_t*d**2 -
 2*d**3)/(3*c**3))))
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.33

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d-\sqrt{3}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d+\sqrt{3}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{12}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{12}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

input
```
integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(
1/4)*2^(3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt
(3)*c)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/12*3^(1/4)*2^(1/
4)*c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/12*3^
(1/4)*2^(1/4)*c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))
)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{12}\cdot 6^{\frac{1}{4}}c\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\cdot 6^{\frac{1}{4}}c\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}\left(6^{\frac{1}{4}}c-2\,d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) + \frac{1}{24}\left(6^{\frac{1}{4}}c+2\,d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4
))) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
+ 1/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.163.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx = \ln\left(x - \frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}+\frac{6^{1/4}\,\sqrt{-\frac{1}{2}\mathrm{i}}\,c}{12}\right)+\ln\left(x+\frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}-\frac{6^{1/4}\,\sqrt{-\frac{1}{2}\mathrm{i}}\,c}{12}\right)+\ln\left(x-\frac{(\right.$$

input
```
int((c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

output
```
log(x - ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(-1i/2)^(1/2)*c)/
12) + log(x + ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(-1i/2)^(1/
2)*c)/12) + log(x - ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(1i/2
)^(1/2)*c)/12) + log(x + ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*
(1i/2)^(1/2)*c)/12)
```

3.164 $\int \frac{a+cx^2+dx^3}{2+3x^4} dx$

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3.164.1 Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{a+cx^2+dx^3}{2+3x^4} dx = -\frac{(\sqrt{6}a+2c)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{(\sqrt{6}a+2c)\arctan\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}} - \frac{(\sqrt{6}a-2c)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{(\sqrt{6}a-2c)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/12*d*ln(3*x^4+2)-1/48*ln(-6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1
/4)+1/48*ln(6^(3/4)*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(
-1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(1+6^(1/4)*x)*(2*c+a*6^(1
/2))*6^(1/4)
```

3.164.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.96

$$\int \frac{a+cx^2+dx^3}{2+3x^4} dx = \frac{1}{48}\left(-2\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right) + 2\sqrt[4]{6}\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right) - \sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right) + \sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right) + 4d\log\left(2+3x^4\right)\right)$$

input
```
Integrate[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
(-2*6^(1/4)*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(Sqrt[6]*a
 + 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 - 2*6^(1/4
)*x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 + 2*6^(1/4)*x + Sqrt[
6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

3.164.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2+dx^3}{3x^4+2}dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{a+cx^2}{3x^4+2}+\frac{dx^3}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\arctan\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
Int[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
-1/4*((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/6^(3/4) + ((Sqrt[6]*a + 2*c
)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^
(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x
 + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

3.164.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.164.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4+2\right)} \frac{\left(\_R^3 d+\_R^2 c+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}}\right.\right.}{}$
meijerg	$\frac{d\ln\left(\frac{3x^4}{2}+1\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\right.}{216}$

input
```
int((d*x^3+c*x^2+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^3*d+_R^2*c+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

3.164.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(115) = 230$.

Time = 0.29 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.33

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(2\,d-\sqrt{-12\,ac+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\right)\log\left(-3\left(9\,a^4-4\,c^4\right)x + \left(9\,a^3-6\,ac^2-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}c\right)\sqrt{-12\,ac+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\right) + \frac{1}{24}\left(2\,d+\sqrt{-12\,ac+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\right)\log\left(-3\left(9\,a^4-4\,c^4\right)x - \left(9\,a^3-6\,ac^2-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}c\right)\sqrt{-12\,ac+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\right) + \frac{1}{24}\left(2\,d-\sqrt{-12\,ac-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\right)\log\left(-3\left(9\,a^4-4\,c^4\right)x + \left(9\,a^3-6\,ac^2+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}c\right)\sqrt{-12\,ac-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\right) + \frac{1}{24}\left(2\,d+\sqrt{-12\,ac-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\right)\log\left(-3\left(9\,a^4-4\,c^4\right)x - \left(9\,a^3-6\,ac^2+\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}c\right)\sqrt{-12\,ac-\sqrt{6}\sqrt{-9\,a^4+12\,a^2c^2-4\,c^4}}\right)$$

input
```
integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/24*(2*d - sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))*log
(-3*(9*a^4 - 4*c^4)*x + (9*a^3 - 6*a*c^2 - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^
2 - 4*c^4)*c)*sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))) +
 1/24*(2*d + sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))*lo
g(-3*(9*a^4 - 4*c^4)*x - (9*a^3 - 6*a*c^2 - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c
^2 - 4*c^4)*c)*sqrt(-12*a*c + sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))
+ 1/24*(2*d - sqrt(-12*a*c - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))*l
og(-3*(9*a^4 - 4*c^4)*x + (9*a^3 - 6*a*c^2 + sqrt(6)*sqrt(-9*a^4 + 12*a^2*
c^2 - 4*c^4)*c)*sqrt(-12*a*c - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))
 + 1/24*(2*d + sqrt(-12*a*c - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4)))*
log(-3*(9*a^4 - 4*c^4)*x - (9*a^3 - 6*a*c^2 + sqrt(6)*sqrt(-9*a^4 + 12*a^2
*c^2 - 4*c^4)*c)*sqrt(-12*a*c - sqrt(6)*sqrt(-9*a^4 + 12*a^2*c^2 - 4*c^4))
)
```

3.164.6 Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.96

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx$$
$$=\text{RootSum}\left(165888t^4-55296t^3d+t^2\cdot\left(6912ac+6912d^2\right)+t\left(-1152acd-384d^3\right)+27a^4+36a^2c^2+48\right.$$

input
```
integrate((d*x**3+c*x**2+a)/(3*x**4+2),x)
```

output
```
RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 6912*d**2) + _t*(
-1152*a*c*d - 384*d**3) + 27*a**4 + 36*a**2*c**2 + 48*a*c*d**2 + 12*c**4 +
 8*d**4, Lambda(_t, _t*log(x + (-13824*_t**3*c + 3456*_t**2*c*d + 216*_t*a
**3 - 432*_t*a*c**2 - 288*_t*c*d**2 - 18*a**3*d + 36*a*c**2*d + 8*c*d**3)/
(27*a**4 - 12*c**4))))
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx &= -\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c-2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\\
&\quad+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)\\
&\quad+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\\
&\quad+\frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\end{aligned}$$

input
```
integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
-1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d - 3*a)*log
(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(sqrt(
3)*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4
)*x + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*
arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)
*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2
*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.89

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c+4\,d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c-4\,d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(
2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)
^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c + 4*d)
*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4
)*c - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

3.164.9 Mupad [B] (verification not implemented)

Time = 10.29 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.86

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx = \ln\left(-2\,c+\sqrt{6}\,a\,1\mathrm{i}+x\,\sqrt{3\mathrm{i}\,\sqrt{6}\,a^2-12\,a\,c-2\mathrm{i}\,\sqrt{6}\,c^2}\right)\left(\frac{d}{12}+\frac{\sqrt{\frac{3\mathrm{i}\,\sqrt{6}\,a^2}{4}-3\,a\,c-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{2}}}{12}\right)+\ln\left(2\,c-\sqrt{6}\,a\,1\mathrm{i}+x\,\sqrt{3\mathrm{i}\,\sqrt{6}\,a^2-12\,a\,c-2\mathrm{i}\,\sqrt{6}\,c^2}\right)\left(\frac{d}{12}-\frac{\sqrt{\frac{3\mathrm{i}\,\sqrt{6}\,a^2}{4}-3\,a\,c-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{2}}}{12}\right)+\ln\left(2\,c+\sqrt{6}\,a\,1\mathrm{i}+x\,\sqrt{-3\mathrm{i}\,\sqrt{6}\,a^2-12\,a\,c+2\mathrm{i}\,\sqrt{6}\,c^2}\right)\left(\frac{d}{12}-\frac{\sqrt{-\frac{3\mathrm{i}\,\sqrt{6}\,a^2}{4}-3\,a\,c+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{2}}}{12}\right)+\ln\left(2\,c+\sqrt{6}\,a\,1\mathrm{i}-x\,\sqrt{-3\mathrm{i}\,\sqrt{6}\,a^2-12\,a\,c+2\mathrm{i}\,\sqrt{6}\,c^2}\right)\left(\frac{d}{12}+\frac{\sqrt{-\frac{3\mathrm{i}\,\sqrt{6}\,a^2}{4}-3\,a\,c+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{2}}}{12}\right)$$

input
```
int((a + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

output
```
log(6^(1/2)*a*1i - 2*c + x*(6^(1/2)*a^2*3i - 12*a*c - 6^(1/2)*c^2*2i)^(1/2
))*(d/12 + ((6^(1/2)*a^2*3i)/4 - 3*a*c - (6^(1/2)*c^2*1i)/2)^(1/2)/12) + l
og(2*c - 6^(1/2)*a*1i + x*(6^(1/2)*a^2*3i - 12*a*c - 6^(1/2)*c^2*2i)^(1/2)
)*(d/12 - ((6^(1/2)*a^2*3i)/4 - 3*a*c - (6^(1/2)*c^2*1i)/2)^(1/2)/12) + lo
g(2*c + 6^(1/2)*a*1i + x*(6^(1/2)*c^2*2i - 6^(1/2)*a^2*3i - 12*a*c)^(1/2))
*(d/12 - ((6^(1/2)*c^2*1i)/2 - (6^(1/2)*a^2*3i)/4 - 3*a*c)^(1/2)/12) + log
(2*c + 6^(1/2)*a*1i - x*(6^(1/2)*c^2*2i - 6^(1/2)*a^2*3i - 12*a*c)^(1/2))*
(d/12 + ((6^(1/2)*c^2*1i)/2 - (6^(1/2)*a^2*3i)/4 - 3*a*c)^(1/2)/12)
```

3.165 $\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx$

3.165.1 Optimal result 1297
3.165.2 Mathematica [A] (verified) 1297
3.165.3 Rubi [A] (verified) 1298
3.165.4 Maple [C] (verified) 1299
3.165.5 Fricas [C] (verification not implemented) 1300
3.165.6 Sympy [A] (verification not implemented) 1300
3.165.7 Maxima [A] (verification not implemented) 1301
3.165.8 Giac [A] (verification not implemented) 1301
3.165.9 Mupad [B] (verification not implemented) 1302

3.165.1 Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\arctan\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} - \frac{c\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{4\ 6^{3/4}} + \frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)+1/1
2*d*ln(3*x^4+2)+1/24*c*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*c*ln(6^(3
/4)*x+3*x^2+6^(1/2))*6^(1/4)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(-2\sqrt[4]{6}\left(\sqrt[4]{6}b+c\right)\arctan\left(1-\sqrt[4]{6}x\right) + 2\sqrt[4]{6}\left(-\sqrt[4]{6}b+c\right)\arctan\left(1+\sqrt[4]{6}x\right) + \sqrt[4]{6}c\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right) - \sqrt[4]{6}c\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right) + 2d\log\left(2+3x^4\right)\right)$$

input
```
Integrate[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
(-2*6^(1/4)*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(-(6^(1/4)*b
) + c)*ArcTan[1 + 6^(1/4)*x] + 6^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2
] - 6^(1/4)*c*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

3.165.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2028, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2+dx^3}{3x^4+2} \, dx$$

$$\Big\downarrow \text{2028}$$

$$\int \frac{x(b+cx+dx^2)}{3x^4+2}dx$$

$$\Big\downarrow \text{2370}$$

$$\int \left(\frac{x(b+dx^2)}{3x^4+2}+\frac{cx^2}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
Int[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/
4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x +
 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) +
(d*Log[2 + 3*x^4])/12
```

3.165.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

3.165.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.28

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^{4}+2\right)}\frac{\left(\_R^{3} d+\_R^{2} c+\_R b\right) \ln\left(x-\_R\right)}{\_R^{3}}\right)}{12}$
default	$\frac{b \arctan\left(\frac{x^{2}\sqrt{6}}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\, 6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}-\frac{\sqrt{3}\, 6^{\frac{1}{4}} x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}+\frac{\sqrt{3}\, 6^{\frac{1}{4}} x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\, 6^{\frac{3}{4}} x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\, 6^{\frac{3}{4}} x}{6}-1\right)\right)}{144}+\frac{d\ln(3x}{12}$
meijerg	$\frac{d\ln\left(\frac{3x^{4}}{2}+1\right)}{12}+\frac{54^{\frac{3}{4}} c\left(\frac{x^{3}\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}} 8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}} 8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\right.}{216}$

input
```
int((d*x^3+c*x^2+b*x)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^3*d+_R^2*c+_R*b)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

3.165.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 18086, normalized size of antiderivative = 132.99

$$\int \frac{bx + cx^2 + dx^3}{2 + 3x^4} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.165.6 Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.39

$$\int \frac{bx + cx^2 + dx^3}{2 + 3x^4} \, dx$$
$$= \text{RootSum}\left(82944t^4 - 27648t^3d + t^2 \cdot \left(1728b^2 + 3456d^2\right) + t\left(-288b^2d + 288bc^2 - 192d^3\right) + 9b^4 + 12b^2d^2\right.$$

input
```
integrate((d*x**3+c*x**2+b*x)/(3*x**4+2),x)
```

output
```
RootSum(82944*_t**4 - 27648*_t**3*d + _t**2*(1728*b**2 + 3456*d**2) + _t*(
-288*b**2*d + 288*b*c**2 - 192*d**3) + 9*b**4 + 12*b**2*d**2 - 24*b*c**2*d
 + 6*c**4 + 4*d**4, Lambda(_t, _t*log(x + (-3456*_t**3*c**2 + 864*_t**2*b*
*3 + 864*_t**2*c**2*d - 144*_t*b**3*d - 108*_t*b**2*c**2 - 72*_t*c**2*d**2
 + 9*b**5 + 6*b**3*d**2 + 9*b**2*c**2*d - 9*b*c**4 + 2*c**2*d**3)/(18*b**4
*c - 3*c**5))))
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.28

$$\int \frac{bx + cx^2 + dx^3}{2 + 3x^4}\,dx = \frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c - 6\,b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x + 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c + 6\,b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x - 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d - \sqrt{3}c\right)\log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right) + \frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d + \sqrt{3}c\right)\log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right)$$

input
```
integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c - 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*
(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c
 + 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/72
*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(1/4)
*2^(3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt(3)*
c)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{bx + cx^2 + dx^3}{2 + 3x^4}\,dx = -\frac{1}{12}\left(\sqrt{6}b - 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\left(\sqrt{6}b + 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}\left(6^{\frac{1}{4}}c - 2\,d\right)\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) + \frac{1}{24}\left(6^{\frac{1}{4}}c + 2\,d\right)\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")
```

output
```
-1/12*(sqrt(6)*b - 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/12*(sqrt(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3
/4)*(2*x - sqrt(2)*(2/3)^(1/4))) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2
)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2
/3)^(1/4)*x + sqrt(2/3))
```

3.165.9 Mupad [B] (verification not implemented)

Time = 10.47 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.21

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx$$
$$= \sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(z^4 - \frac{d\,z^3}{3} + \frac{z^2\,(1728\,b^2+3456\,d^2)}{82944} - \frac{z\,(-288\,b\,c^2+288\,b^2\,d+192\,d^3)}{82944} - \frac{b\,c^2\,d}{3456} + \frac{b^2\,d^2}{6912} + \frac{d^4}{20736} + \frac{c^4}{13824} + \frac{b^4}{9216}, z, k\right)\left(144\,b\,c + x\,(144\,b\,d - 72\,c^2) - \mathrm{root}\left(z^4 - \frac{d\,z^3}{3} + \frac{z^2\,(1728\,b^2+3456\,d^2)}{82944} - \frac{z\,(-288\,b\,c^2+288\,b^2\,d+192\,d^3)}{82944} - \frac{b\,c^2\,d}{3456} + \frac{b^2\,d^2}{6912} + \frac{d^4}{20736} + \frac{c}{138}\right.\right.\right.$$
$$\left.\left. + x\,(9\,b^3 + 6\,b\,d^2 - 6\,c^2\,d) - 6\,c^3 + 12\,b\,c\,d\right)\mathrm{root}\left(z^4 - \frac{d\,z^3}{3} + \frac{z^2\,(1728\,b^2+3456\,d^2)}{82944} - \frac{z\,(-288\,b\,c^2+288\,b^2\,d+192\,d^3)}{82944} - \frac{b\,c^2\,d}{3456} + \frac{b^2\,d^2}{6912} + \frac{d^4}{20736} + \frac{c^4}{13824} + \frac{b^4}{9216}, z, k\right)$$

input
```
int((b*x + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

output
```
symsum(log(x*(6*b*d^2 - 6*c^2*d + 9*b^3) - root(z^4 - (d*z^3)/3 + (z^2*(17
28*b^2 + 3456*d^2))/82944 - (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944
- (b*c^2*d)/3456 + (b^2*d^2)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k
)*(144*b*c + x*(144*b*d - 72*c^2) - 864*root(z^4 - (d*z^3)/3 + (z^2*(1728*
b^2 + 3456*d^2))/82944 - (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 - (
b*c^2*d)/3456 + (b^2*d^2)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k)*b
*x) - 6*c^3 + 12*b*c*d)*root(z^4 - (d*z^3)/3 + (z^2*(1728*b^2 + 3456*d^2))
/82944 - (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 - (b*c^2*d)/3456 +
(b^2*d^2)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k), k, 1, 4)
```

3.166 $\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx$

3.166.1 Optimal result 1303
3.166.2 Mathematica [A] (verified) 1304
3.166.3 Rubi [A] (verified) 1304
3.166.4 Maple [C] (verified) 1306
3.166.5 Fricas [C] (verification not implemented) 1306
3.166.6 Sympy [B] (verification not implemented) 1307
3.166.7 Maxima [A] (verification not implemented) 1308
3.166.8 Giac [A] (verification not implemented) 1309
3.166.9 Mupad [B] (verification not implemented) 1309

3.166.1 Optimal result

Integrand size = 25, antiderivative size = 176

$$\begin{aligned}\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx &= \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} \\ &\quad + \frac{\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}} \\ &\quad - \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} \\ &\quad + \frac{\left(\sqrt{6}a-2c\right)\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\ 6^{3/4}} + \frac{1}{12}d\log\left(2+3x^4\right)\end{aligned}$$

output
```
1/12*d*ln(3*x^4+2)+1/12*b*arctan(1/2*x^2*6^(1/2))*6^(1/2)-1/48*ln(-6^(3/4)
*x+3*x^2+6^(1/2))*(-2*c+a*6^(1/2))*6^(1/4)+1/48*ln(6^(3/4)*x+3*x^2+6^(1/2)
)*(-2*c+a*6^(1/2))*6^(1/4)+1/24*arctan(-1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/
4)+1/24*arctan(1+6^(1/4)*x)*(2*c+a*6^(1/2))*6^(1/4)
```

3.166.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.93

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{48}\left(-2\sqrt[4]{6}\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)\arctan\left(1-\sqrt[4]{6}x\right) + 2\sqrt[4]{6}\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)\arctan\left(1+\sqrt[4]{6}x\right) - \sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right) + \sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)+4d\log\left(2+3x^4\right)\right)$$

input
```
Integrate[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
(-2*6^(1/4)*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*6^(1
/4)*(Sqrt[6]*a - 2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[
6]*a - 2*c)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)
*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

3.166.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2+dx^3}{3x^4+2}\,dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{a+cx^2}{3x^4+2}+\frac{x\left(b+dx^2\right)}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(\sqrt{6}a+2c)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{(\sqrt{6}a+2c)\arctan\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}}-\frac{(\sqrt{6}a-2c)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{(\sqrt{6}a-2c)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
Int[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1
/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)
) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((S
qrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 +
 3*x^4])/12
```

3.166.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.166.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.22

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(3\_Z^{4}+2\right)}\frac{\left(\_R^{3}d+\_R^{2}c+\_Rb+a\right)\ln\left(x-\_R\right)}{\_R^{3}}}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan\left(\frac{x^{2}\sqrt{6}}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\cdots}{\cdots}$
meijerg	$\frac{d\ln\left(\frac{3x^{4}}{2}+1\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^{3}\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\cdots\right.}{216}$

input
```
int((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^3*d+_R^2*c+_R*b+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

3.166.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 54479, normalized size of antiderivative = 309.54

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.166.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(156) = 312$.

Time = 4.67 (sec) , antiderivative size = 580, normalized size of antiderivative = 3.30

$$\int \frac{a + bx + cx^2 + dx^3}{2 + 3x^4}\, dx$$
$$= \text{RootSum}\left(165888t^4 - 55296t^3d + t^2 \cdot \left(6912ac + 3456b^2 + 6912d^2\right) + t\left(-864a^2b - 1152acd - 576b^2d + \right.\right.$$

input
```
integrate((d*x**3+c*x**2+b*x+a)/(3*x**4+2),x)
```

output
```
RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 3456*b**2 + 6912*
d**2) + _t*(-864*a**2*b - 1152*a*c*d - 576*b**2*d + 576*b*c**2 - 384*d**3)
 + 27*a**4 + 72*a**2*b*d + 36*a**2*c**2 - 72*a*b**2*c + 48*a*c*d**2 + 18*b
**4 + 24*b**2*d**2 - 48*b*c**2*d + 12*c**4 + 8*d**4, Lambda(_t, _t*log(x +
 (-41472*_t**3*a**2*c + 82944*_t**3*a*b**2 + 27648*_t**3*c**3 + 5184*_t**2
*a**3*b + 10368*_t**2*a**2*c*d - 20736*_t**2*a*b**2*d + 10368*_t**2*a*b*c*
*2 - 6912*_t**2*b**3*c - 6912*_t**2*c**3*d + 648*_t*a**5 - 864*_t*a**3*b*d
 - 1728*_t*a**3*c**2 + 3888*_t*a**2*b**2*c - 864*_t*a**2*c*d**2 + 864*_t*a
*b**4 + 1728*_t*a*b**2*d**2 - 1728*_t*a*b*c**2*d + 864*_t*a*c**4 + 1152*_t
*b**3*c*d + 864*_t*b**2*c**3 + 576*_t*c**3*d**2 - 54*a**5*d + 270*a**4*b*c
 - 270*a**3*b**3 + 36*a**3*b*d**2 + 144*a**3*c**2*d - 324*a**2*b**2*c*d +
24*a**2*c*d**3 - 72*a*b**4*d + 180*a*b**3*c**2 - 48*a*b**2*d**3 + 72*a*b*c
**2*d**2 - 72*a*c**4*d - 72*b**5*c - 48*b**3*c*d**2 - 72*b**2*c**3*d + 72*
b*c**5 - 16*c**3*d**3)/(81*a**6 - 54*a**4*c**2 + 432*a**3*b**2*c - 216*a**
2*b**4 - 36*a**2*c**4 + 288*a*b**2*c**3 - 144*b**4*c**2 + 24*c**6)))
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.18

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = -\frac{1}{144} \cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c - 2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d - 3\,a\right)\log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right) + \frac{1}{144} \cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c + 2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d - 3\,a\right)\log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right) + \frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a + 2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c - 6\sqrt{2}b\right)\arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x + 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a + 2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c + 6\sqrt{2}b\right)\arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x - 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

input
```
integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
-1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d - 3*a)*log
(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(sqrt(
3)*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4
)*x + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c -
 6*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4)))
+ 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c + 6*sqrt(2)*b)*a
rctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.85

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx$$
$$= \frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$
$$+ \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$
$$+ \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c + 4d\right)\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$
$$- \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c - 4d\right)\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4
)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b + 2*6^(1/4)
*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^
(3/4)*a - 2*6^(1/4)*c + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
- 1/48*(6^(3/4)*a - 2*6^(1/4)*c - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + s
qrt(2/3))
```

3.166.9 Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 1168, normalized size of antiderivative = 6.64

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

output
```
symsum(log(9*a*b^2 - 864*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/2
4 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*
z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 -
(a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 +
 b^4/9216 + a^4/6144, z, k)^2*a - 9*a^2*c - 6*a*d^2 + 9*b^3*x - 6*c^3 + 14
4*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c
*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 -
(b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d
^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216 + a^4/6144, z
, k)*a*d - 144*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z
^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (
d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/
2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216
+ a^4/6144, z, k)*b*c + 12*b*c*d - 108*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24
 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)
/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^
2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736
 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*a^2*x + 864*root(z^4 - (d*z^3)/3
 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/
288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a...
```

3.166. $\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx$

3.167 $\int \frac{1+x+x^2+x^3}{1-x^4} \, dx$

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3.167.1 Optimal result

Integrand size = 19, antiderivative size = 8

$$\int \frac{1+x+x^2+x^3}{1-x^4} \, dx = -\log(1-x)$$

output
```
-ln(1-x)
```

3.167.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2+x^3}{1-x^4} \, dx = -\log(1-x)$$

input
```
Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

output
```
-Log[1 - x]
```

3.167.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+x+1}{1-x^4} \, dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{16}$$

$$-\log(1-x)$$

input
```
Int[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

output
```
-Log[1 - x]
```

3.167.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.167.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result
default	$-\ln\left(-1+x\right)$
norman	$-\ln\left(-1+x\right)$
risch	$-\ln\left(-1+x\right)$
parallelrisch	$-\ln\left(-1+x\right)$
meijerg	$-\frac{\ln\left(-x^{4}+1\right)}{4}-\frac{x^{3}\left(\ln\left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{4}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^{4}\right)^{\frac{1}{4}}\right)\right)}{4\left(x^{4}\right)^{\frac{3}{4}}}+\frac{\operatorname{arctanh}\left(x^{2}\right)}{2}-\frac{x\left(\ln\left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln\left(1+\right.\right.}{4(x}$

input
```
int((x^3+x^2+x+1)/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(-1+x)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx=-\log\left(x-1\right)$$

input
```
integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")
```

output
```
-log(x - 1)
```

3.167.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx=-\log\left(x-1\right)$$

input
```
integrate((x**3+x**2+x+1)/(-x**4+1),x)
```

output
```
-log(x - 1)
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log(x-1)$$

input
```
integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")
```

output
```
-log(x - 1)
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log(|x-1|)$$

input
```
integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")
```

output
```
-log(abs(x - 1))
```

3.167.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\ln(x-1)$$

input
```
int(-(x + x^2 + x^3 + 1)/(x^4 - 1),x)
```

output
```
-log(x - 1)
```

3.168 $\int \frac{1+x+x^2+x^3}{1+x^4} dx$

3.168.1 Optimal result 1315
3.168.2 Mathematica [A] (verified) 1315
3.168.3 Rubi [A] (verified) 1316
3.168.4 Maple [C] (verified) 1317
3.168.5 Fricas [B] (verification not implemented) 1317
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3.168.7 Maxima [A] (verification not implemented) 1318
3.168.8 Giac [A] (verification not implemented) 1319
3.168.9 Mupad [B] (verification not implemented) 1319

3.168.1 Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{1+x+x^2+x^3}{1+x^4} dx = \frac{\arctan(x^2)}{2} - \frac{\arctan(1-\sqrt{2}x)}{\sqrt{2}} + \frac{\arctan(1+\sqrt{2}x)}{\sqrt{2}} + \frac{1}{4}\log(1+x^4)$$

output
```
1/2*arctan(x^2)+1/4*ln(x^4+1)+1/2*arctan(-1+x*2^(1/2))*2^(1/2)+1/2*arctan(
1+x*2^(1/2))*2^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{1+x+x^2+x^3}{1+x^4} dx = \frac{1}{4}\left(-2\left(1+\sqrt{2}\right)\arctan\left(1-\sqrt{2}x\right) + 2\left(-1+\sqrt{2}\right)\arctan\left(1+\sqrt{2}x\right) + \log\left(1+x^4\right)\right)$$

input
```
Integrate[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

output
```
(-2*(1 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(-1 + Sqrt[2])*ArcTan[1 + Sqrt
[2]*x] + Log[1 + x^4])/4
```

3.168.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+x+1}{x^4+1} \, dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{x(x^2+1)}{x^4+1}+\frac{x^2+1}{x^4+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(x^2\right)}{2}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}+\frac{1}{4}\log\left(x^4+1\right)$$

input
```
Int[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

output
```
ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/Sqrt
[2] + Log[1 + x^4]/4
```

3.168.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.168.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\left(\_R^3+\_R^2+\_R+1\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+\sqrt{2}\,x}{1+x^2-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{8}+\frac{\arctan\left(x^2\right)}{2}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-\sqrt{2}\,x}{1+x^2+\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\,\mathrm{arc}\right.}{8}$
meijerg	$\frac{\ln\left(x^4+1\right)}{4}+\frac{x^3\sqrt{2}\ln\left(1-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}\right)}{4\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\mathrm{arcta}}{}$

input
```
int((x^3+x^2+x+1)/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^3+_R^2+_R+1)/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

3.168.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(41) = 82.

Time = 0.43 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.85

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = \frac{1}{4}\left(\sqrt{2\sqrt{2}-3}+1\right)\log\left(\sqrt{2\sqrt{2}-3}\left(\sqrt{2}+2\right)+2x+\sqrt{2}\right) - \frac{1}{4}\left(\sqrt{2\sqrt{2}-3}-1\right)\log\left(-\sqrt{2\sqrt{2}-3}\left(\sqrt{2}+2\right)+2x+\sqrt{2}\right) - \frac{1}{4}\left(\sqrt{-2\sqrt{2}-3}-1\right)\log\left(\left(\sqrt{2}-2\right)\sqrt{-2\sqrt{2}-3}+2x-\sqrt{2}\right) + \frac{1}{4}\left(\sqrt{-2\sqrt{2}-3}+1\right)\log\left(-\left(\sqrt{2}-2\right)\sqrt{-2\sqrt{2}-3}+2x-\sqrt{2}\right)$$

input
```
integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2*sqrt(2) - 3) + 1)*log(sqrt(2*sqrt(2) - 3)*(sqrt(2) + 2) + 2*x
+ sqrt(2)) - 1/4*(sqrt(2*sqrt(2) - 3) - 1)*log(-sqrt(2*sqrt(2) - 3)*(sqrt(
2) + 2) + 2*x + sqrt(2)) - 1/4*(sqrt(-2*sqrt(2) - 3) - 1)*log((sqrt(2) - 2
)*sqrt(-2*sqrt(2) - 3) + 2*x - sqrt(2)) + 1/4*(sqrt(-2*sqrt(2) - 3) + 1)*l
og(-(sqrt(2) - 2)*sqrt(-2*sqrt(2) - 3) + 2*x - sqrt(2))
```

3.168.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = \frac{\log\left(x^2-\sqrt{2}x+1\right)}{4} + \frac{\log\left(x^2+\sqrt{2}x+1\right)}{4} + 2 \cdot \left(\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x-1\right) + 2\left(-\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x+1\right)$$

input
```
integrate((x**3+x**2+x+1)/(x**4+1),x)
```

output
```
log(x**2 - sqrt(2)*x + 1)/4 + log(x**2 + sqrt(2)*x + 1)/4 + 2*(1/4 + sqrt(
2)/4)*atan(sqrt(2)*x - 1) + 2*(-1/4 + sqrt(2)/4)*atan(sqrt(2)*x + 1)
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = -\frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) + \frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(
2)*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(
2)*x + 1) + 1/4*log(x^2 - sqrt(2)*x + 1)
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = \frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="giac")
```

output
```
1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*(sqrt(2) + 1)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(2)*x + 1) + 1/4*l
og(x^2 - sqrt(2)*x + 1)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.94

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = \ln\left((16x-16)\left(\frac{\sqrt{-2\sqrt{2}-3}}{4}+\frac{1}{4}\right)-8x\right)\left(\frac{\sqrt{-2\sqrt{2}-3}}{4}+\frac{1}{4}\right) - \ln\left(8x+(16x-16)\left(\frac{\sqrt{-2\sqrt{2}-3}}{4}-\frac{1}{4}\right)\right)\left(\frac{\sqrt{-2\sqrt{2}-3}}{4}-\frac{1}{4}\right) - \ln\left(8x+(16x-16)\left(\frac{\sqrt{2\sqrt{2}-3}}{4}-\frac{1}{4}\right)\right)\left(\frac{\sqrt{2\sqrt{2}-3}}{4}-\frac{1}{4}\right) + \ln\left(8x-(16x-16)\left(\frac{\sqrt{2\sqrt{2}-3}}{4}+\frac{1}{4}\right)\right)\left(\frac{\sqrt{2\sqrt{2}-3}}{4}+\frac{1}{4}\right)$$

input
```
int((x + x^2 + x^3 + 1)/(x^4 + 1),x)
```

output
```
log((16*x - 16)*((- 2*2^(1/2) - 3)^(1/2)/4 + 1/4) - 8*x)*((- 2*2^(1/2) - 3
)^(1/2)/4 + 1/4) - log(8*x + (16*x - 16)*((- 2*2^(1/2) - 3)^(1/2)/4 - 1/4)
)*((- 2*2^(1/2) - 3)^(1/2)/4 - 1/4) - log(8*x + (16*x - 16)*((2*2^(1/2) -
3)^(1/2)/4 - 1/4))*((2*2^(1/2) - 3)^(1/2)/4 - 1/4) + log(8*x - (16*x - 16)
*((2*2^(1/2) - 3)^(1/2)/4 + 1/4))*((2*2^(1/2) - 3)^(1/2)/4 + 1/4)
```

3.169 $\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx$

3.169.1 Optimal result 1320
3.169.2 Mathematica [A] (verified) 1321
3.169.3 Rubi [A] (verified) 1321
3.169.4 Maple [C] (verified) 1322
3.169.5 Fricas [C] (verification not implemented) 1323
3.169.6 Sympy [A] (verification not implemented) 1323
3.169.7 Maxima [A] (verification not implemented) 1323
3.169.8 Giac [B] (verification not implemented) 1324
3.169.9 Mupad [B] (verification not implemented) 1325

3.169.1 Optimal result

Integrand size = 20, antiderivative size = 124

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx=-\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\log\left(a-bx^4\right)}{4b}$$

output
```
-1/4*ln(-b*x^4+a)/b-1/2*arctan(b^(1/4)*x/a^(1/4))*(a^(1/2)-b^(1/2))/a^(3/4
)/b^(3/4)+1/2*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)+1/2*arctanh(b^(
1/4)*x/a^(1/4))*(a^(1/2)+b^(1/2))/a^(3/4)/b^(3/4)
```

3.169.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.64

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = \frac{\left(-a^{3/4}+\sqrt[4]{a}\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2ab^{3/4}} - \frac{\left(a^{3/4}+\sqrt{a}\sqrt[4]{b}+\sqrt[4]{a}\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{4ab^{3/4}} - \frac{\left(-a^{3/4}+\sqrt{a}\sqrt[4]{b}-\sqrt[4]{a}\sqrt{b}\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{4ab^{3/4}} + \frac{\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{a}\sqrt{b}} - \frac{\log\left(a-bx^4\right)}{4b}$$

input
```
Integrate[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

output
```
((-a^(3/4) + a^(1/4)*Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b^(3/4)) -
 ((a^(3/4) + Sqrt[a]*b^(1/4) + a^(1/4)*Sqrt[b])*Log[a^(1/4) - b^(1/4)*x])/
(4*a*b^(3/4)) - ((-a^(3/4) + Sqrt[a]*b^(1/4) - a^(1/4)*Sqrt[b])*Log[a^(1/4
) + b^(1/4)*x])/(4*a*b^(3/4)) + Log[Sqrt[a] + Sqrt[b]*x^2]/(4*Sqrt[a]*Sqrt
[b]) - Log[a - b*x^4]/(4*b)
```

3.169.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+x+1}{a-bx^4}\,dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{x(x^2+1)}{a-bx^4}+\frac{x^2+1}{a-bx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\log\left(a-bx^4\right)}{4b}$$

input
```
Int[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

output
```
-1/2*((Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(a^(3/4)*b^(3/4)) +
 ((Sqrt[a] + Sqrt[b])*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - Log[a - b*x^4]/(4*b)
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.169.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.31

method	result	size
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(\_R^3+\_R^2+\_R+1\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	38
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\ln\left(-b\,x^4+a\right)}{4b}$	150

input
```
int((x^3+x^2+x+1)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*sum((_R^3+_R^2+_R+1)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.169.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 91748, normalized size of antiderivative = 739.90

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.169.6 Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.51

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx =$$
$$-\operatorname{RootSum}\left(256t^4a^3b^4-256t^3a^3b^3+t^2\cdot\left(96a^3b^2-96a^2b^3\right)+t\left(-16a^3b+32a^2b^2-16ab^3\right)+a^3-3a^2b\right.$$

input
```
integrate((x**3+x**2+x+1)/(-b*x**4+a),x)
```

output
```
-RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 -
 96*a**2*b**3) + _t*(-16*a**3*b + 32*a**2*b**2 - 16*a*b**3) + a**3 - 3*a**
2*b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**3 + 48*_t*
*2*a**3*b**2 + 16*_t**2*a**2*b**3 - 12*_t*a**3*b + 16*_t*a**2*b**2 - 4*_t*
a*b**3 + a**3 - 2*a**2*b + a*b**2)/(a**2*b - 2*a*b**2 + b**3))))
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.29

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = -\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}b} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}b} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(a) - sqrt(b))*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*
sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(a) - sqrt(b))*log(sqrt(b)*x^2 +
 sqrt(a))/(sqrt(a)*b) - 1/4*(sqrt(a) + sqrt(b))*log(sqrt(b)*x^2 - sqrt(a))
/(sqrt(a)*b) - 1/4*(sqrt(a) + sqrt(b))*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(
b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b))
```

3.169.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(84) = 168$.

Time = 0.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.34

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx$$

$$= -\frac{\log\left(|bx^4-a|\right)}{4\,b} + \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2 - \sqrt{2}\sqrt{-ab^3}b + \left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}$$
$$+ \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2 + \sqrt{2}\sqrt{-ab^3}b + \left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}$$
$$+ \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2 - \left(-ab^3\right)^{\frac{3}{4}}\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}$$
$$- \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2 - \left(-ab^3\right)^{\frac{3}{4}}\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}$$

input
```
integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*log(abs(b*x^4 - a))/b + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 - sqrt(2)*sqr
t(-a*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/
4))/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt
(-a*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4
))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4
))*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((
-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqr
t(-a/b))/(a*b^3)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.52

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = \sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2 + 16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+3\,a\,b^2-b^3 + a^3, z, k\right)\left(\mathrm{root}\left(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2+16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b- \right.\right.\right.$$
$$\left.\left.\left. - x\left(4\,a\,b^2-4\,b^3\right)\right)\right)\,\mathrm{root}\left(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2+16\,a^3\,b\,z + 16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+3\,a\,b^2-b^3+a^3, z, k\right)$$

input
```
int((x + x^2 + x^3 + 1)/(a - b*x^4),x)
```

output
```
symsum(log(-root(256*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 96*a
^2*b^3*z^2 + 16*a^3*b*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2 -
b^3 + a^3, z, k)*(root(256*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2
- 96*a^2*b^3*z^2 + 16*a^3*b*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*
b^2 - b^3 + a^3, z, k)*(16*a*b^3 - 16*a*b^3*x) - x*(4*a*b^2 - 4*b^3)))*roo
t(256*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 96*a^2*b^3*z^2 + 16
*a^3*b*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2 - b^3 + a^3, z, k
), k, 1, 4)
```

3.170 $\int \frac{1+x+x^2+x^3}{a+bx^4} dx$

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3.170.1 Optimal result

Integrand size = 19, antiderivative size = 277

$$\int \frac{1+x+x^2+x^3}{a+bx^4} dx = \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\log(a+bx^4)}{4b}$$

output
```
1/4*ln(b*x^4+a)/b+1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(
a^(1/2)-b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)-1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*b^(1/2))*(a^(1/2)-b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/2*arctan(
x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/
4))*(a^(1/2)+b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/
2)/a^(1/4))*(a^(1/2)+b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)
```

3.170.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.02

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx$$
$$= \frac{-2\sqrt[4]{a}\left(\sqrt{2}\sqrt{a}+2\sqrt[4]{a}\sqrt[4]{b}+\sqrt{2}\sqrt{b}\right)\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\left(\sqrt{2}\sqrt{a}-2\sqrt[4]{a}\sqrt[4]{b}+\sqrt{2}\sqrt{b}\right)\sqrt[4]{b}\,\mathrm{arct}}{}$$

input
```
Integrate[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

output
```
(-2*a^(1/4)*(Sqrt[2]*Sqrt[a] + 2*a^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*(Sqrt[2]*Sqrt[a] - 2
*a^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)] + Sqrt[2]*(a^(3/4) - a^(1/4)*Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(-a^(3/4) + a^(1/4)*Sqrt[b]
)*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a*Log
[a + b*x^4])/(8*a*b)
```

3.170.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+x+1}{a+bx^4}\,dx$$
$$\Big\downarrow \text{2415}$$
$$\int \left(\frac{x(x^2+1)}{a+bx^4}+\frac{x^2+1}{a+bx^4}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{(\sqrt{a}+\sqrt{b})\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{(\sqrt{a}+\sqrt{b})\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{(\sqrt{a}-\sqrt{b})\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{(\sqrt{a}-\sqrt{b})\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\log(a+bx^4)}{4b}$$

input
```
Int[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

output
```
ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sq
rt[a] + Sqrt[b])*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*b^(3/4)) + ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b])*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/
4)) + Log[a + b*x^4]/(4*b)
```

3.170.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.170.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.13

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(\_R^3+\_R^2+\_R+1\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{}$

input
```
int((x^3+x^2+x+1)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum((_R^3+_R^2+_R+1)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.170.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 96349, normalized size of antiderivative = 347.83

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.170.6 Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.68

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3b^4-256t^3a^3b^3+t^2\cdot\left(96a^3b^2+96a^2b^3\right)+t\left(-16a^3b-32a^2b^2-16ab^3\right)+a^3+3a^2b+\right.$$

input
```
integrate((x**3+x**2+x+1)/(b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 +
96*a**2*b**3) + _t*(-16*a**3*b - 32*a**2*b**2 - 16*a*b**3) + a**3 + 3*a**2
*b + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**3 - 48*_t**2
*a**3*b**2 + 16*_t**2*a**2*b**3 + 12*_t*a**3*b + 16*_t*a**2*b**2 + 4*_t*a*
b**3 - a**3 - 2*a**2*b - a*b**2)/(a**2*b + 2*a*b**2 + b**3))))
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.07

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}-\sqrt{a}\sqrt{b}+b\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}+\sqrt{a}\sqrt{b}-b\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\left(\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{a}\right)b+\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}+2a\right)\sqrt{b}-2a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{5}{4}}} + \frac{\left(\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{a}\right)b+\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}-2a\right)\sqrt{b}+2a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{5}{4}}}$$

input
```
integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(sqrt(2)*a^(3/4)*b^(1/4) - sqrt(a)*sqrt(b) + b)*log(sqrt(b)*x^
2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/8*sqrt(2)*(
sqrt(2)*a^(3/4)*b^(1/4) + sqrt(a)*sqrt(b) - b)*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/4*((sqrt(2)*a^(1/4)*b^(1
/4) - 2*sqrt(a))*b + (sqrt(2)*a^(3/4)*b^(1/4) + 2*a)*sqrt(b) - 2*a*sqrt(b)
)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 1/4*((sqrt(2)*a^(1/4)*
b^(1/4) + 2*sqrt(a))*b + (sqrt(2)*a^(3/4)*b^(1/4) - 2*a)*sqrt(b) + 2*a*sqr
t(b))*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt
(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4))
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.97

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \frac{\log\left(|bx^4+a|\right)}{4\,b} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2 - \sqrt{2}\sqrt{ab^3}b + (ab^3)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2 + \sqrt{2}\sqrt{ab^3}b + (ab^3)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2 - (ab^3)^{\frac{3}{4}}\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2 - (ab^3)^{\frac{3}{4}}\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

input
```
integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 - sqrt(2)*sqrt(
a*b^3)*b + (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt(a*b^3)
*b + (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*log(x^2 +
sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^
2 - (a*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.170.9 Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.10

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \sum_{k=1}^{4} \ln\Big(-\mathrm{root}\big(256\,a^3\,b^4\,z^4 - 256\,a^3\,b^3\,z^3 + 96\,a^3\,b^2\,z^2 + 96\,a^2\,b^3\,z^2 - 16\,a^3\,b\,z - 16\,a\,b^3\,z - 32\,a^2\,b^2\,z + 3\,a^2\,b + 3\,a\,b^2 + b^3 + a^3, z, k\big)\,\big(\mathrm{root}\big(256\,a^3\,b^4\,z^4 - 256\,a^3\,b^3\,z^3 + 96\,a^3\,b^2\,z^2 + 96\,a^2\,b^3\,z^2 - 16\,a^3\,b\,z - 16\,a\,b^3\,z - 32\,a^2\,b^2\,z + 3\,a^2\,b \ldots + x\left(4\,b^3 + 4\,a\,b^2\right)\big)\Big)\,\mathrm{root}\big(256\,a^3\,b^4\,z^4 - 256\,a^3\,b^3\,z^3 + 96\,a^3\,b^2\,z^2 + 96\,a^2\,b^3\,z^2 - 16\,a^3\,b\,z - 16\,a\,b^3\,z - 32\,a^2\,b^2\,z + 3\,a^2\,b + 3\,a\,b^2 + b^3 + a^3, z, k\big)$$

input
```
int((x + x^2 + x^3 + 1)/(a + b*x^4),x)
```

output
```
symsum(log(-root(256*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 96*a
^2*b^3*z^2 - 16*a^3*b*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2 +
b^3 + a^3, z, k)*(root(256*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2
+ 96*a^2*b^3*z^2 - 16*a^3*b*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*
b^2 + b^3 + a^3, z, k)*(16*a*b^3 - 16*a*b^3*x) + x*(4*a*b^2 + 4*b^3)))*roo
t(256*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 96*a^2*b^3*z^2 - 16
*a^3*b*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2 + b^3 + a^3, z, k
), k, 1, 4)
```

3.170. $\int \frac{1+x+x^2+x^3}{a+bx^4} \, dx$

3.171 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4} dx$

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3.171.1 Optimal result

Integrand size = 31, antiderivative size = 148

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4} dx = -\frac{gx}{b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}} +\frac{\left(bc+\sqrt{a}\sqrt{b}e+ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}} +\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

output
```
-g*x/b-1/4*f*ln(-b*x^4+a)/b+1/2*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(
1/2)+1/2*arctan(b^(1/4)*x/a^(1/4))*(b*c+a*g-e*a^(1/2)*b^(1/2))/a^(3/4)/b^(
5/4)+1/2*arctanh(b^(1/4)*x/a^(1/4))*(b*c+a*g+e*a^(1/2)*b^(1/2))/a^(3/4)/b^
(5/4)
```

3.171.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.68

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$$
$$= \frac{-4a^{3/4}\sqrt[4]{b}gx+2\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(bc+\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

output
```
(-4*a^(3/4)*b^(1/4)*g*x + 2*(b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4
)*x)/a^(1/4)] - (b*c + a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e + a*g)*Log[a^
(1/4) - b^(1/4)*x] + b*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*b^(3/4)*d*Log[
a^(1/4) + b^(1/4)*x] + Sqrt[a]*Sqrt[b]*e*Log[a^(1/4) + b^(1/4)*x] + a*g*Lo
g[a^(1/4) + b^(1/4)*x] + a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2] - a^
(3/4)*b^(1/4)*f*Log[a - b*x^4])/(4*a^(3/4)*b^(5/4))
```

3.171.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$$
$$\Big\downarrow \; 2424$$
$$\int \left(\frac{c+ex^2+gx^4}{a-bx^4}+\frac{x(d+fx^2)}{a-bx^4}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

output
```
-((g*x)/b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTanh[(b^(1/4)*x
)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sq
rt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

3.171.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.171.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{gx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(-\_R^{3}bf-\_R^{2}be-\_Rbd-ag-bc\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$-\frac{gx}{b}+\frac{\frac{(ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{bd\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{f\ln\left(-b\,x^{4}+\right.}{4}}{b}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-g*x/b+1/4/b^2*sum((-_R^3*b*f-_R^2*b*e-_R*b*d-a*g-b*c)/_R^3*ln(x-_R),_R=Ro
otOf(_Z^4*b-a))
```

3.171.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.65 (sec) , antiderivative size = 592528, normalized size of antiderivative = 4003.57

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{a - bx^4} dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{a - bx^4} dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

output
```
Timed out
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{a - bx^4} dx = -\frac{gx}{b} + \frac{\frac{2\left(b^{\frac{3}{2}}c - \sqrt{a}be + a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\left(b^{\frac{3}{2}}d - \sqrt{a}bf\right)\log\left(\sqrt{b}x^2 + \sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(b^{\frac{3}{2}}d + \sqrt{a}bf\right)\log\left(\sqrt{b}x^2 - \sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(b^{\frac{3}{2}}c + \sqrt{a}be + a\sqrt{b}g\right)\log}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{}}}}{4b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
-g*x/b + 1/4*(2*(b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*arctan(sqrt(b)*x/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d
 - sqrt(a)*b*f)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d + sqrt
(a)*b*f)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (b^(3/2)*c + sqrt(a)*b*e
 + a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(
sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

3.171.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(108) = 216$.

Time = 0.27 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.02

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$$

$$= -\frac{\sqrt{2}\left(b^2c+abg-\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c+abg+\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c+abg-\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}} + \frac{\sqrt{2}\left(b^2c+abg-\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}} - \frac{gx}{b} - \frac{f\log\left(|bx^4-a|\right)}{4\,b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(b^2*c + a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)
*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3
/4) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)
*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^
3)^(3/4) - 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*
x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + a*b*g -
 sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(
3/4) - g*x/b - 1/4*f*log(abs(b*x^4 - a))/b
```

3.171.9 Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 5082, normalized size of antiderivative = 34.34

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{a - bx^4} \, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x)
```

output
```
symsum(log(b^2*c^2*e - b^2*c*d^2 + a^2*e*g^2 - a^2*f^2*g - b^2*d^3*x - a*b
*e^3 - a*b*c*f^2 - a*b*d^2*g - 16*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3
 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b
^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z
+ 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*
c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2
*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4
*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b
^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b
^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k
)^2*a*b^3*c - 4*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*
z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^
3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2
*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b
^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a
^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4
*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b
^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b
*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*b^3*c^2*x - b^2*
c^2*f*x - a^2*f*g^2*x - 16*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 6...
```

3.172 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$

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3.172.1 Optimal result

Integrand size = 31, antiderivative size = 172

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx = \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab(a-bx^4)} + \frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}} + \frac{\left(3bc+\sqrt{a}\sqrt{b}e-ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

output
```
1/4*x*(b*f*x^3+b*e*x^2+b*d*x+a*g+b*c)/a/b/(-b*x^4+a)+1/4*d*arctanh(x^2*b^(
1/2)/a^(1/2))/a^(3/2)/b^(1/2)+1/8*arctan(b^(1/4)*x/a^(1/4))*(3*b*c-a*g-e*a
^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))*(3*b*c-a*g+
e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)
```

3.172.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$$
$$= \frac{\frac{4a^{3/4}\sqrt[4]{b}(a(f+gx)+bx(c+x(d+ex)))}{a-bx^4} - 2\left(-3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \left(3bc+2\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e-\right.}{16a^7}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

output
```
((4*a^(3/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2
*(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (3*b*c +
 2*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) - b^(1/4)*x] +
 (3*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) + b^(
1/4)*x] + 2*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(5
/4))
```

3.172.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2397, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{\int \frac{bex^2+2bdx+3bc-ag}{a-bx^4}dx}{4ab} + \frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\int \left(\frac{2bdx}{a-bx^4}+\frac{bex^2+3bc-ag}{a-bx^4}\right)dx}{4ab} + \frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{\sqrt{b}d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}}}{4ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

output
```
(x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + (((3*b*c
 - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4
)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a
^(3/4)*b^(1/4)) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a])/(4*a
*b)
```

3.172.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.172.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.63

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b-a)}\frac{\left(\_R^2 e+2\_R d-\frac{ag-3bc}{b}\right)\ln(x-\_R)}{\_R^3}}{16ba}$
default	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^4+a}+\frac{\frac{(-ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{bd\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{2\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+}{x-}\right.\right.}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{4ba}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/a*e*x^3+1/4*d/a*x^2+1/4*(a*g+b*c)/a/b*x+1/4*f/b)/(-b*x^4+a)-1/16/b/a*
sum((_R^2*e+2*_R*d-1/b*(a*g-3*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.172.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.98 (sec) , antiderivative size = 334837, normalized size of antiderivative = 1946.73

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.172.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

output
```
Timed out
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.30

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx=-\frac{bex^3+bdx^2+af+(bc+ag)x}{4\,(ab^2x^4-a^2b)} + \frac{\frac{2\sqrt{b}d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}}-\frac{2\sqrt{b}d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}}+\frac{2\left(3b^{\frac{3}{2}}c-\sqrt{a}be-a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(3b^{\frac{3}{2}}c+\sqrt{a}be-a\sqrt{b}g\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,ab}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(b*e*x^3 + b*d*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4 - a^2*b) + 1/16*
(2*sqrt(b)*d*log(sqrt(b)*x^2 + sqrt(a))/sqrt(a) - 2*sqrt(b)*d*log(sqrt(b)*
x^2 - sqrt(a))/sqrt(a) + 2*(3*b^(3/2)*c - sqrt(a)*b*e - a*sqrt(b)*g)*arcta
n(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
 - (3*b^(3/2)*c + sqrt(a)*b*e - a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)
*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(b))*sqrt(b)))/(a*b)
```

3.172.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(133) = 266$.

Time = 0.28 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$$
$$= -\frac{\sqrt{2}\left(3\,b^2c-abg-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}\left(3\,b^2c-abg+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}\left(3\,b^2c-abg-\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} + \frac{\sqrt{2}\left(3\,b^2c-abg-\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{bex^3+bdx^2+bcx+agx+af}{4\,(bx^4-a)ab}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)
*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b
^3)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*g + 2*sqrt(2)*(-a*b^3)^(1/4)*b*
d - sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)
^(1/4))/((-a*b^3)^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*
e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/3
2*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1
/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) - 1/4*(b*e*x^3 + b*d*x^2 + b*c*x + a*
g*x + a*f)/((b*x^4 - a)*a*b)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 1393, normalized size of antiderivative = 8.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(-\frac{-a^2\,e\,g^2+6\,a\,b\,c\,e\,g-4\,a\,b\,d^2\,g+a\,b\,e^3-9\,b^2\,c^2\,e+12\,b^2\,c\,d^2}{64\,a^3} - \frac{\mathrm{root}(65536\,a^7\,b^5\,z^4+1024\,a^5\,b^3\,e\,g\,z^2-3072\,a^4\,b^4\,c\,e\,z^2-2048\,a^4\,b^4\,d^2\,z^2-768\,a^3\,b^3\,c\,d\,g\,z+128\,a^4\,b^2}{} - \frac{b\,d\,x\,(2\,b\,d^2-3\,b\,c\,e+a\,e\,g)}{16\,a^3}\right) \mathrm{root}\big(65536\,a^7\,b^5\,z^4+1024\,a^5\,b^3\,e\,g\,z^2-3072\,a^4\,b^4\,c\,e\,z^2 - 2048\,a^4\,b^4\,d^2\,z^2-768\,a^3\,b^3\,c\,d\,g\,z+128\,a^4\,b^2\,d\,g^2\,z+128\,a^3\,b^3\,d\,e^2\,z+1152\,a^2\,b^4\,c^2\,d\,z + 16\,a^2\,b^2\,d^2\,e\,g-12\,a^2\,b^2\,c\,e^2\,g-48\,a\,b^3\,c\,d^2\,e+108\,a\,b^3\,c^3\,g+12\,a^3\,b\,c\,g^3-54\,a^2\,b^2\,c^2\,g^2 + 2\,a^3\,b\,e^2\,g^2+18\,a\,b^3\,c^2\,e^2+16\,a\,b^3\,d^4-81\,b^4\,c^4-a^2\,b^2\,e^4-a^4\,g^4, z, k\big)\right) + \frac{\frac{f}{4\,b}+\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{x\,(b\,c+a\,g)}{4\,a\,b}}{a-b\,x^4}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x)
```

output
```
symsum(log(- (12*b^2*c*d^2 - 9*b^2*c^2*e - a^2*e*g^2 + a*b*e^3 - 4*a*b*d^2
*g + 6*a*b*c*e*g)/(64*a^3) - (root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^
2 - 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 12
8*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^
2*d^2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a
^3*b*c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*
a*b^3*d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*b*(9*b^2*c^2*x + a^2
*g^2*x - 16*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c
*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z
+ 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^2*e*g - 12*a^2
*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 - 54*a^
2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 - 81*b^4
*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*a^3*b*g + a*b*e^2*x + 48*root(65536*a^
7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2
*z^2 - 768*a^3*b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1
152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c
*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e
^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*
g^4, z, k)*a^2*b^2*c - 4*a*b*d*e - 32*root(65536*a^7*b^5*z^4 + 1024*a^5*b^
3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c...
```

3.172. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$

3.173 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$

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3.173.1 Optimal result

Integrand size = 31, antiderivative size = 221

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab(a-bx^4)^2} + \frac{4af+x(7bc-ag+6bdx+5bex^2)}{32a^2b(a-bx^4)} + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e-3ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} + \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

output
```
1/8*x*(b*f*x^3+b*e*x^2+b*d*x+a*g+b*c)/a/b/(-b*x^4+a)^2+1/32*(4*a*f+x*(5*b*
e*x^2+6*b*d*x-a*g+7*b*c))/a^2/b/(-b*x^4+a)+3/16*d*arctanh(x^2*b^(1/2)/a^(1
/2))/a^(5/2)/b^(1/2)+1/64*arctan(b^(1/4)*x/a^(1/4))*(21*b*c-3*a*g-5*e*a^(1
/2)*b^(1/2))/a^(11/4)/b^(5/4)+1/64*arctanh(b^(1/4)*x/a^(1/4))*(21*b*c-3*a*
g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)
```

3.173.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.19

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$$
$$= \frac{\frac{4a^{3/4}\sqrt[4]{b}(a^2(4f+3gx)-b^2x^5(7c+x(6d+5ex))+abx(11c+x(10d+9ex+gx^3)))}{(a-bx^4)^2}+2\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

output
```
((4*a^(3/4)*b^(1/4)*(a^2*(4*f + 3*g*x) - b^2*x^5*(7*c + x*(6*d + 5*e*x)) +
 a*b*x*(11*c + x*(10*d + 9*e*x + g*x^3))))/(a - b*x^4)^2 + 2*(21*b*c - 5*S
qrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (21*b*c + 12*a^(1/
4)*b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) - b^(1/4)*x] + (21
*b*c - 12*a^(1/4)*b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) + b
^(1/4)*x] + 12*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)
*b^(5/4))
```

3.173.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2397, 2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{\int \frac{4bfx^3+5bex^2+6bdx+7bc-ag}{(a-bx^4)^2}dx}{8ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{2393}$$

$$\frac{\frac{x(-ag+7bc+6bdx+5bex^2)+4af}{4a(a-bx^4)}-\frac{\int -\frac{5bex^2+12bdx+3(7bc-ag)}{a-bx^4}dx}{4a}}{8ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{5bex^2+12bdx+3(7bc-ag)}{a-bx^4}dx}{4a}+\frac{x(-ag+7bc+6bdx+5bex^2)+4af}{4a(a-bx^4)}}{8ab}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{\int \left(\frac{12bdx}{a-bx^4}+\frac{5bex^2+3(7bc-ag)}{a-bx^4}\right)dx}{4a}+\frac{x(-ag+7bc+6bdx+5bex^2)+4af}{4a(a-bx^4)}}{8ab}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{6\sqrt{b}d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}}}{4a}+\frac{x(-ag+7bc+6bdx+5bex^2)+4af}{4a(a-bx^4)}}{8ab}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{8ab\left(a-bx^4\right)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

output
```
(x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + ((4*a*
f + x*(7*b*c - a*g + 6*b*d*x + 5*b*e*x^2))/(4*a*(a - b*x^4)) + (((21*b*c -
 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1
/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)
])/(2*a^(3/4)*b^(1/4)) + (6*Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt
[a])/(4*a))/(8*a*b)
```

3.173.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.173.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.67

method	result
risch	$\frac{-\frac{5bex^7}{32a^2}-\frac{3bdx^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{9ex^3}{32a}+\frac{5dx^2}{16a}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-bx^4+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{\left(5\_R^2e+12\_Rd-\frac{3(ag-7bc)}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2b}$
default	$\frac{-\frac{5bex^7}{32a^2}-\frac{3bdx^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{9ex^3}{32a}+\frac{5dx^2}{16a}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-bx^4+a)^2}+\frac{\frac{(-3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{3bd\ln\left(\frac{a-}{a-}\right.}{\sqrt{}}}{32a^2b}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-5/32*b*e/a^2*x^7-3/16*b*d/a^2*x^6+1/32*(a*g-7*b*c)/a^2*x^5+9/32/a*e*x^3+
5/16*d/a*x^2+1/32*(3*a*g+11*b*c)/a/b*x+1/8*f/b)/(-b*x^4+a)^2-1/128/a^2/b*s
um((5*_R^2*e+12*_R*d-3/b*(a*g-7*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.173.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.83 (sec) , antiderivative size = 343626, normalized size of antiderivative = 1554.87

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.173.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

output
```
Timed out
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.29

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$$
$$= -\frac{5\,b^2ex^7+6\,b^2dx^6-9\,abex^3+(7\,b^2c-abg)x^5-10\,abdx^2-4\,a^2f-(11\,abc+3\,a^2g)x}{32\,(a^2b^3x^8-2\,a^3b^2x^4+a^4b)}$$
$$+\frac{\frac{12\,\sqrt{b}d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}}-\frac{12\,\sqrt{b}d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}}+\frac{2\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be-3\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c+5\,\sqrt{a}be-3\,a\sqrt{b}g\right)\log\left(\cdots\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^2b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 - 9*a*b*e*x^3 + (7*b^2*c - a*b*g)*x^5 - 1
0*a*b*d*x^2 - 4*a^2*f - (11*a*b*c + 3*a^2*g)*x)/(a^2*b^3*x^8 - 2*a^3*b^2*x
^4 + a^4*b) + 1/128*(12*sqrt(b)*d*log(sqrt(b)*x^2 + sqrt(a))/sqrt(a) - 12*
sqrt(b)*d*log(sqrt(b)*x^2 - sqrt(a))/sqrt(a) + 2*(21*b^(3/2)*c - 5*sqrt(a)
*b*e - 3*a*sqrt(b)*g)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(b))*sqrt(b)) - (21*b^(3/2)*c + 5*sqrt(a)*b*e - 3*a*sqrt(b)*
g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(
b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

3.173.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(181) = 362$.

Time = 0.43 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.75

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$$
$$= -\frac{\sqrt{2}\left(21\,b^2c-3\,abg-12\,\sqrt{2}(-ab^3)^{\frac14}\,bd+5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}\right)}{2\left(-\frac{a}{b}\right)^{\frac14}}\right)}{128\,(-ab^3)^{\frac34}\,a^2}$$
$$-\frac{\sqrt{2}\left(21\,b^2c-3\,abg+12\,\sqrt{2}(-ab^3)^{\frac14}\,bd-5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}\right)}{2\left(-\frac{a}{b}\right)^{\frac14}}\right)}{128\,(-ab^3)^{\frac34}\,a^2}$$
$$-\frac{\sqrt{2}\left(21\,b^2c-3\,abg-5\,\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac34}\,a^2}$$
$$+\frac{\sqrt{2}\left(21\,b^2c-3\,abg-5\,\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac34}\,a^2}$$
$$-\frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5-abgx^5-9\,abex^3-10\,abdx^2-11\,abcx-3\,a^2gx-4\,a^2f}{32\,(bx^4-a)^2a^2b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

output
```
-1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqr
t(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))
/((-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c - 3*a*b*g + 12*sqrt(2)*(-a
*b^3)^(1/4)*b*d - 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/
b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3
*a*b*g - 5*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/
((-a*b^3)^(3/4)*a^2) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*
e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1
/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*e*x^3 - 1
0*a*b*d*x^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.53

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \frac{\frac{f}{8\,b}+\frac{5\,d\,x^2}{16\,a}+\frac{9\,e\,x^3}{32\,a}-\frac{x^5\,(7\,b\,c-a\,g)}{32\,a^2}+\frac{x\,(11\,b\,c+3\,a\,g)}{32\,a\,b}-\frac{3\,b\,d\,x^6}{16\,a^2}-\frac{5\,b\,e\,x^7}{32\,a^2}}{a^2-2\,a\,b\,x^4+b^2\,x^8} + \left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(268435456\,a^{11}\,b^5\,z^4+983040\,a^7\,b^3\,e\,g\,z^2-6881280\,a^6\,b^4\,c\,e\,z^2-4718592\,a^6\,b^4\,d^2\,z^2-774\right.\right.\right.$$

$$- \frac{-45\,a^2\,e\,g^2+630\,a\,b\,c\,e\,g-432\,a\,b\,d^2\,g+125\,a\,b\,e^3-2205\,b^2\,c^2\,e+3024\,b^2\,c\,d^2}{32768\,a^6} - \frac{x\,(216\,b^2\,d^3-315\,c\,e\,b^2\,d+45\,a\,e\,g\,b\,d)}{4096\,a^6}\Bigg)\,\mathrm{root}\big(268435456\,a^{11}\,b^5\,z^4+983040\,a^7\,b^3\,e\,g\,z^2 - 6881280\,a^6\,b^4\,c\,e\,z^2-4718592\,a^6\,b^4\,d^2\,z^2-774144\,a^4\,b^3\,c\,d\,g\,z+55296\,a^5\,b^2\,d\,g^2\,z + 153600\,a^4\,b^3\,d\,e^2\,z+2709504\,a^3\,b^4\,c^2\,d\,z+8640\,a^2\,b^2\,d^2\,e\,g-6300\,a^2\,b^2\,c\,e^2\,g - 60480\,a\,b^3\,c\,d^2\,e+111132\,a\,b^3\,c^3\,g+2268\,a^3\,b\,c\,g^3-23814\,a^2\,b^2\,c^2\,g^2+450\,a^3\,b\,e^2\,g^2 + 22050\,a\,b^3\,c^2\,e^2-625\,a^2\,b^2\,e^4+20736\,a\,b^3\,d^4-81\,a^4\,g^4-194481\,b^4\,c^4,z,k\big)\Bigg)$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x)
```

output
```
(f/(8*b) + (5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) - (x^5*(7*b*c - a*g))/(32*a
^2) + (x*(11*b*c + 3*a*g))/(32*a*b) - (3*b*d*x^6)/(16*a^2) - (5*b*e*x^7)/(
32*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(- root(268435456*a^11*b^
5*z^4 + 983040*a^7*b^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 4718592*a^6*b^4
*d^2*z^2 - 774144*a^4*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3
*d*e^2*z + 2709504*a^3*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*a^2*b^2*c
*e^2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 - 238
14*a^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 625*a^2*b^2
*e^4 + 20736*a*b^3*d^4 - 81*a^4*g^4 - 194481*b^4*c^4, z, k)*(root(26843545
6*a^11*b^5*z^4 + 983040*a^7*b^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 471859
2*a^6*b^4*d^2*z^2 - 774144*a^4*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 15360
0*a^4*b^3*d*e^2*z + 2709504*a^3*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*
a^2*b^2*c*e^2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*
g^3 - 23814*a^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 62
5*a^2*b^2*e^4 + 20736*a*b^3*d^4 - 81*a^4*g^4 - 194481*b^4*c^4, z, k)*((344
064*a^5*b^3*c - 49152*a^6*b^2*g)/(32768*a^6) - (6*b^3*d*x)/a) + (x*(144*a^
4*b*g^2 + 7056*a^2*b^3*c^2 + 400*a^3*b^2*e^2 - 2016*a^3*b^2*c*g))/(4096*a^
6) - (15*b^2*d*e)/(32*a^3)) - (3024*b^2*c*d^2 - 2205*b^2*c^2*e - 45*a^2*e*
g^2 + 125*a*b*e^3 - 432*a*b*d^2*g + 630*a*b*c*e*g)/(32768*a^6) - (x*(216*b
^2*d^3 - 315*b^2*c*d*e + 45*a*b*d*e*g))/(4096*a^6))*root(268435456*a^11...
```

3.173. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$

3.174 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$

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3.174.1 Optimal result

Integrand size = 31, antiderivative size = 266

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab(a-bx^4)^3} + \frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b(a-bx^4)} + \frac{8af+x(11bc-ag+10bdx+9bex^2)}{96a^2b(a-bx^4)^2} + \frac{\left(77bc-15\sqrt{a}\sqrt{b}e-7ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}} + \frac{\left(77bc+15\sqrt{a}\sqrt{b}e-7ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}} + \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

output
```
1/12*x*(b*f*x^3+b*e*x^2+b*d*x+a*g+b*c)/a/b/(-b*x^4+a)^3+1/384*x*(45*b*e*x^
2+60*b*d*x-7*a*g+77*b*c)/a^3/b/(-b*x^4+a)+1/96*(8*a*f+x*(9*b*e*x^2+10*b*d*
x-a*g+11*b*c))/a^2/b/(-b*x^4+a)^2+5/32*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(7
/2)/b^(1/2)+1/256*arctan(b^(1/4)*x/a^(1/4))*(77*b*c-7*a*g-15*e*a^(1/2)*b^(
1/2))/a^(15/4)/b^(5/4)+1/256*arctanh(b^(1/4)*x/a^(1/4))*(77*b*c-7*a*g+15*e
*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)
```

3.174.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.18

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$$
$$= \frac{\frac{4a^{3/4}\sqrt[4]{b}x(77bc-7ag+15bx(4d+3ex))}{a-bx^4}+\frac{16a^{7/4}\sqrt[4]{b}x(11bc-ag+bx(10d+9ex))}{(a-bx^4)^2}+\frac{128a^{11/4}\sqrt[4]{b}(a(f+gx)+bx(c+x(d+ex)))}{(a-bx^4)^3}+6\Big(77bc-}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

output
```
((4*a^(3/4)*b^(1/4)*x*(77*b*c - 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a - b*x^4)
 + (16*a^(7/4)*b^(1/4)*x*(11*b*c - a*g + b*x*(10*d + 9*e*x)))/(a - b*x^4)^
2 + (128*a^(11/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^
4)^3 + 6*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4
)] - 3*(77*b*c + 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*Log[
a^(1/4) - b^(1/4)*x] + 3*(77*b*c - 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[
b]*e - 7*a*g)*Log[a^(1/4) + b^(1/4)*x] + 120*a^(1/4)*b^(3/4)*d*Log[Sqrt[a]
 + Sqrt[b]*x^2])/(1536*a^(15/4)*b^(5/4))
```

3.174.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2397, 2393, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$$
$$\Big\downarrow \; 2397$$
$$\frac{\int \frac{8bfx^3+9bex^2+10bdx+11bc-ag}{(a-bx^4)^3}dx}{12ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$
$$\Big\downarrow \; 2393$$

$$\frac{\frac{x\left(-ag+11bc+10bdx+9bex^2\right)+8af}{8a(a-bx^4)^2}-\frac{\int -\frac{45bex^2+60bdx+7(11bc-ag)}{\left(a-bx^4\right)^2}dx}{8a}}{12ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{45bex^2+60bdx+7(11bc-ag)}{\left(a-bx^4\right)^2}dx}{8a}+\frac{x\left(-ag+11bc+10bdx+9bex^2\right)+8af}{8a(a-bx^4)^2}}{12ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 2394$$

$$\frac{\frac{\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{4a\left(a-bx^4\right)}-\frac{\int -\frac{3\left(15bex^2+40bdx+7(11bc-ag)\right)}{a-bx^4}dx}{4a}}{8a}+\frac{x\left(-ag+11bc+10bdx+9bex^2\right)+8af}{8a(a-bx^4)^2}}{12ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3\int \frac{15bex^2+40bdx+7(11bc-ag)}{a-bx^4}dx}{4a}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x\left(-ag+11bc+10bdx+9bex^2\right)+8af}{8a(a-bx^4)^2}}{12ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 2415$$

$$\frac{\frac{\frac{3\int \left(\frac{40bdx}{a-bx^4}+\frac{15bex^2+7(11bc-ag)}{a-bx^4}\right)dx}{4a}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x\left(-ag+11bc+10bdx+9bex^2\right)+8af}{8a(a-bx^4)^2}}{12ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{20\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{4a}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\ldots}{12ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

3.174. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

output
```
(x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + ((8*a
*f + x*(11*b*c - a*g + 10*b*d*x + 9*b*e*x^2))/(8*a*(a - b*x^4)^2) + ((x*(7
*(11*b*c - a*g) + 60*b*d*x + 45*b*e*x^2))/(4*a*(a - b*x^4)) + (3*(((77*b*c
 - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b
^(1/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/a^(
1/4)])/(2*a^(3/4)*b^(1/4)) + (20*Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])
/Sqrt[a]))/(4*a))/(8*a))/(12*a*b)
```

3.174.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.174.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\frac{15e\,b^{2}x^{11}}{128a^{3}}+\frac{5d\,b^{2}x^{10}}{32a^{3}}-\frac{7(ag-11bc)b\,x^{9}}{384a^{3}}-\frac{21be\,x^{7}}{64a^{2}}-\frac{5bd\,x^{6}}{12a^{2}}+\frac{3(ag-11bc)x^{5}}{64a^{2}}+\frac{113e\,x^{3}}{384a}+\frac{11d\,x^{2}}{32a}+\frac{(7ag+51bc)x}{128ab}+\frac{f}{12b}}{\left(-b\,x^{4}+a\right)^{3}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}b-a\right)}}{}$
default	$\frac{\frac{15e\,b^{2}x^{11}}{128a^{3}}+\frac{5d\,b^{2}x^{10}}{32a^{3}}-\frac{7(ag-11bc)b\,x^{9}}{384a^{3}}-\frac{21be\,x^{7}}{64a^{2}}-\frac{5bd\,x^{6}}{12a^{2}}+\frac{3(ag-11bc)x^{5}}{64a^{2}}+\frac{113e\,x^{3}}{384a}+\frac{11d\,x^{2}}{32a}+\frac{(7ag+51bc)x}{128ab}+\frac{f}{12b}}{\left(-b\,x^{4}+a\right)^{3}}+\frac{\left(-7ag+77bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\right.}{}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10-7/384*(a*g-11*b*c)/a^3*b*x^9-21
/64*b*e/a^2*x^7-5/12*b*d/a^2*x^6+3/64/a^2*(a*g-11*b*c)*x^5+113/384/a*e*x^3
+11/32*d/a*x^2+1/128*(7*a*g+51*b*c)/a/b*x+1/12*f/b)/(-b*x^4+a)^3-1/512/a^3
/b*sum((15*_R^2*e+40*_R*d-7*(a*g-11*b*c)/b)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b
-a))
```

3.174.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 59.77 (sec) , antiderivative size = 343822, normalized size of antiderivative = 1292.56

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.174.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

output
```
Timed out
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.30

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = -\frac{45\,b^3ex^{11}+60\,b^3dx^{10}-126\,ab^2ex^7-160\,ab^2dx^6+7\,(11\,b^3c-ab^2g)x^9+113\,a^2bex^3+132\,a^2bdx^2-18}{384\,(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b)} + \frac{\frac{40\,\sqrt{b}d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}}-\frac{40\,\sqrt{b}d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}}+\frac{2\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be-7\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c+15\,\sqrt{a}be-7\,a\sqrt{b}g\right)\log}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512\,a^3b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

output
```
-1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 - 126*a*b^2*e*x^7 - 160*a*b^2*d*x^6
+ 7*(11*b^3*c - a*b^2*g)*x^9 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 - 18*(11*
a*b^2*c - a^2*b*g)*x^5 + 32*a^3*f + 3*(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b^4*x
^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b) + 1/512*(40*sqrt(b)*d*log(sqr
t(b)*x^2 + sqrt(a))/sqrt(a) - 40*sqrt(b)*d*log(sqrt(b)*x^2 - sqrt(a))/sqrt
(a) + 2*(77*b^(3/2)*c - 15*sqrt(a)*b*e - 7*a*sqrt(b)*g)*arctan(sqrt(b)*x/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2
)*c + 15*sqrt(a)*b*e - 7*a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b
)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sq
rt(b)))/(a^3*b)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.64

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$$
$$= -\frac{\sqrt{2}\left(77\,b^2c-7\,abg-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{\sqrt{2}\left(77\,b^2c-7\,abg+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{\sqrt{2}(77\,b^2c-7\,abg-15\,\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$+\frac{\sqrt{2}(77\,b^2c-7\,abg-15\,\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x(-\frac{a}{b})^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9-7\,ab^2gx^9-126\,ab^2ex^7-160\,ab^2dx^6-198\,ab^2cx^5+18\,a^2bgx^5+113}{384\,(bx^4-a)^3a^3b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

output
```
-1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sq
rt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4)
)/((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c - 7*a*b*g + 40*sqrt(2)*(-
a*b^3)^(1/4)*b*d - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c
- 7*a*b*g - 15*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/
b))/((-a*b^3)^(3/4)*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a
*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^
3) - 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 -
 126*a*b^2*e*x^7 - 160*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 11
3*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 21*a^3*g*x + 32*a^3*f)/(
(b*x^4 - a)^3*a^3*b)
```

3.174.9 Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 1056, normalized size of antiderivative = 3.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(68719476736\,a^{15}\,b^5\,z^4 - 1211105280\,a^8\,b^4\,c\,e\,z^2 + 110100480\,a^9\,b^3\,e\,g\,z^2 - 838860800\,a^8\,b^4\,d\right.\right.\right.$$

$$- \frac{-735\,a^2\,e\,g^2 + 16170\,a\,b\,c\,e\,g - 11200\,a\,b\,d^2\,g + 3375\,a\,b\,e^3 - 88935\,b^2\,c^2\,e + 123200\,b^2\,c\,d^2}{2097152\,a^9}$$

$$- \frac{x\,(4000\,b^2\,d^3 - 5775\,c\,e\,b^2\,d + 525\,a\,e\,g\,b\,d)}{131072\,a^9}\Bigg)\,\mathrm{root}\big(68719476736\,a^{15}\,b^5\,z^4$$

$$- 1211105280\,a^8\,b^4\,c\,e\,z^2 + 110100480\,a^9\,b^3\,e\,g\,z^2 - 838860800\,a^8\,b^4\,d^2\,z^2$$

$$- 88309760\,a^5\,b^3\,c\,d\,g\,z + 485703680\,a^4\,b^4\,c^2\,d\,z + 4014080\,a^6\,b^2\,d\,g^2\,z$$

$$+ 18432000\,a^5\,b^3\,d\,e^2\,z + 672000\,a^2\,b^2\,d^2\,e\,g - 485100\,a^2\,b^2\,c\,e^2\,g - 7392000\,a\,b^3\,c\,d^2\,e$$

$$+ 12782924\,a\,b^3\,c^3\,g + 105644\,a^3\,b\,c\,g^3 - 1743126\,a^2\,b^2\,c^2\,g^2 + 22050\,a^3\,b\,e^2\,g^2$$

$$+ 2668050\,a\,b^3\,c^2\,e^2 - 50625\,a^2\,b^2\,e^4 + 2560000\,a\,b^3\,d^4 - 2401\,a^4\,g^4 - 35153041\,b^4\,c^4, z, k\big)\Bigg)$$

$$+ \frac{\frac{f}{12\,b} + \frac{11\,d\,x^2}{32\,a} + \frac{113\,e\,x^3}{384\,a} - \frac{3\,x^5\,(11\,b\,c-a\,g)}{64\,a^2} + \frac{7\,b\,x^9\,(11\,b\,c-a\,g)}{384\,a^3} + \frac{x\,(51\,b\,c+7\,a\,g)}{128\,a\,b} + \frac{5\,b^2\,d\,x^{10}}{32\,a^3} + \frac{15\,b^2\,e\,x^{11}}{128\,a^3} - \frac{5\,b\,d\,x^6}{12\,a^2} - \frac{21\,b\,e}{64\,a}}{a^3 - 3\,a^2\,b\,x^4 + 3\,a\,b^2\,x^8 - b^3\,x^{12}}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x)
```

output
```
symsum(log(- root(68719476736*a^15*b^5*z^4 - 1211105280*a^8*b^4*c*e*z^2 +
110100480*a^9*b^3*e*g*z^2 - 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c
*d*g*z + 485703680*a^4*b^4*c^2*d*z + 4014080*a^6*b^2*d*g^2*z + 18432000*a^
5*b^3*d*e^2*z + 672000*a^2*b^2*d^2*e*g - 485100*a^2*b^2*c*e^2*g - 7392000*
a*b^3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g^3 - 1743126*a^2*b^
2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2 - 50625*a^2*b^2*e^
4 + 2560000*a*b^3*d^4 - 2401*a^4*g^4 - 35153041*b^4*c^4, z, k)*(root(68719
476736*a^15*b^5*z^4 - 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z
^2 - 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d*g*z + 485703680*a^4*
b^4*c^2*d*z + 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b^3*d*e^2*z + 672000*
a^2*b^2*d^2*e*g - 485100*a^2*b^2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 1278292
4*a*b^3*c^3*g + 105644*a^3*b*c*g^3 - 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b
*e^2*g^2 + 2668050*a*b^3*c^2*e^2 - 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 -
 2401*a^4*g^4 - 35153041*b^4*c^4, z, k)*((20185088*a^7*b^3*c - 1835008*a^8
*b^2*g)/(2097152*a^9) - (5*b^3*d*x)/a^2) + (x*(1568*a^5*b*g^2 + 189728*a^3
*b^3*c^2 + 7200*a^4*b^2*e^2 - 34496*a^4*b^2*c*g))/(131072*a^9) - (75*b^2*d
*e)/(256*a^5)) - (123200*b^2*c*d^2 - 88935*b^2*c^2*e - 735*a^2*e*g^2 + 337
5*a*b*e^3 - 11200*a*b*d^2*g + 16170*a*b*c*e*g)/(2097152*a^9) - (x*(4000*b^
2*d^3 - 5775*b^2*c*d*e + 525*a*b*d*e*g))/(131072*a^9))*root(68719476736*a^
15*b^5*z^4 - 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 - 8...
```

3.174. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$

3.175 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$

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3.175.1 Optimal result

Integrand size = 30, antiderivative size = 319

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \frac{gx}{b}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} - \frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{f\log\left(a+bx^4\right)}{4b}$$

output

```
g*x/b+1/4*f*ln(b*x^4+a)/b+1/2*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2
)-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(b*c-a*g-e*a^(1/2
)*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*b^(1/2))*(b*c-a*g-e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/4*arc
tan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(b*c-a*g+e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5
/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(b*c-a*g+e*a^(1/2)*b^(
1/2))/a^(3/4)/b^(5/4)*2^(1/2)
```

3.175.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$$
$$= \frac{8a^{3/4}\sqrt[4]{b}gx-2\left(\sqrt{2}bc+2\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e-\sqrt{2}ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}bc-2\sqrt[4]{a}b^{3/4}d+\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

output
```
(8*a^(3/4)*b^(1/4)*g*x - 2*(Sqrt[2]*b*c + 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sq
rt[a]*Sqrt[b]*e - Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2
*(Sqrt[2]*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e - Sqrt[2]*
a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-(b*c) + Sqrt[a]*S
qrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + S
qrt[2]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*f*Log[a + b*x^4])/(8*a^(3/4)*b^(5/
4))
```

3.175.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$$
$$\Big\downarrow \text{2424}$$
$$\int \left(\frac{c+ex^2+gx^4}{a+bx^4}+\frac{x(d+fx^2)}{a+bx^4}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

output
```
(g*x)/b + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((b*c +
Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b
]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt
[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log
[a + b*x^4])/(4*b)
```

3.175.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.175.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.18

method	result
risch	$\frac{gx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(\_R^3 bf+\_R^2 be+\_R bd-ag+bc\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$\frac{gx}{b}+\frac{\frac{(-ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{bd\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-(}{x^2+(}\right.\right.}{}}{b}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
g*x/b+1/4/b^2*sum((_R^3*b*f+_R^2*b*e+_R*b*d-a*g+b*c)/_R^3*ln(x-_R),_R=Root
Of(_Z^4*b+a))
```

3.175.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 43.66 (sec) , antiderivative size = 622377, normalized size of antiderivative = 1951.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.175.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \frac{gx}{b}$$

$$+\frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f+b^2c-\sqrt{a}b^{\frac{3}{2}}e-abg\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-b^2c+\sqrt{a}b^{\frac{3}{2}}e+abg\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+ \cdots}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
g*x/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f + b^2*c - sqrt(a)*b^(3/2)*
e - a*b*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)
*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - b^2*c + sqrt(a)*b^(3/2)*e
 + a*b*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*
b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt
(2)*a^(5/4)*b^(5/4)*g - 2*sqrt(a)*b^2*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x +
 sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqr
t(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e
- sqrt(2)*a^(5/4)*b^(5/4)*g + 2*sqrt(a)*b^2*d)*arctan(1/2*sqrt(2)*(2*sqrt(
b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(
a)*sqrt(b))*b^(5/4)))/b
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.06

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$$
$$= \frac{gx}{b} + \frac{f\log\left(|bx^4+a|\right)}{4\,b} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-(ab^3)^{\frac14}b^2c+(ab^3)^{\frac14}abg-(ab^3)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4\,ab^3}$$
$$- \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-(ab^3)^{\frac14}b^2c+(ab^3)^{\frac14}abg-(ab^3)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4\,ab^3}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac14}b^2c-(ab^3)^{\frac14}abg-(ab^3)^{\frac34}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$
$$- \frac{\sqrt{2}\left((ab^3)^{\frac14}b^2c-(ab^3)^{\frac14}abg-(ab^3)^{\frac34}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
g*x/b + 1/4*f*log(abs(b*x^4 + a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d
 - (a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(s
qrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^
3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)
*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 9.95 (sec) , antiderivative size = 5042, normalized size of antiderivative = 15.81

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x)
```

output
```
symsum(log(b^2*c*d^2 - b^2*c^2*e - a^2*e*g^2 + a^2*f^2*g + b^2*d^3*x - a*b
*e^3 - a*b*c*f^2 - a*b*d^2*g - 16*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3
 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b
^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z
- 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*
c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2
*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4
*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b
^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b
^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k
)^2*a*b^3*c - 4*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*
z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^
3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2
*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b
^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a
^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4
*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b
^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b
^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*b^3*c^2*x + b^2*
c^2*f*x + a^2*f*g^2*x + 16*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 6...
```

3.176 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2} dx$

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3.176.1 Optimal result

Integrand size = 30, antiderivative size = 341

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2} dx = \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab(a+bx^4)} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} - \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$

output
```
1/4*x*(b*f*x^3+b*e*x^2+b*d*x-a*g+b*c)/a/b/(b*x^4+a)+1/4*d*arctan(x^2*b^(1/
2)/a^(1/2))/a^(3/2)/b^(1/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*b^(1/2))*(3*b*c+a*g-e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/32*ln(a^
(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(3*b*c+a*g-e*a^(1/2)*b^(1/2))
/a^(7/4)/b^(5/4)*2^(1/2)+1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(3*b*c+
a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(
1/2)/a^(1/4))*(3*b*c+a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)
```

3.176.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.94

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$$
$$= \frac{-\frac{8a^{3/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{a+bx^4}-2\left(3\sqrt{2}bc+4\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e+\sqrt{2}ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

output
```
((-8*a^(3/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4) -
2*(3*Sqrt[2]*b*c + 4*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[
2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b*c - 4*a^(
1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[2]*a*g)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c - Sqrt[
a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(32*a^(7/4)*b^(5/4))
```

3.176.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2397, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$$
$$\Big\downarrow \text{2397}$$
$$\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}-\frac{\int -\frac{bex^2+2bdx+3bc+ag}{bx^4+a}dx}{4ab}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{bex^2+2bdx+3bc+ag}{bx^4+a}dx}{4ab}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\int \left(\frac{2bdx}{bx^4+a}+\frac{bex^2+3bc+ag}{bx^4+a}\right)dx}{4ab}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+3\ldots\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}}{4ab}$$

$$\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

output
```
(x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + ((Sqrt[b
]*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - ((3*b*c + Sqrt[a]*Sqrt[b]*e +
 a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4))
 + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(
1/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)))/(4*a*b)
```

3.176.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.176.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(\_R^2 e+2\_R d+\frac{ag+3bc}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\frac{(ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{bd\arctan\left(x^2 \right.}{\sqrt{ab}}}{4ba}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/a*e*x^3+1/4*d/a*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+1/16/b/a*s
um((_R^2*e+2*_R*d+1/b*(a*g+3*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.176.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.61 (sec) , antiderivative size = 352423, normalized size of antiderivative = 1033.50

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.176.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

output
```
Timed out
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx = \frac{bex^3+bdx^2-af+(bc-ag)x}{4\,(ab^2x^4+a^2b)}$$

$$+\frac{\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}a^{\frac{3}{4}}\right.}{}}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b*e*x^3 + b*d*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 + a^2*b) + 1/32*(
sqrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 + sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*b^(3/2)*c -
sqrt(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + s
qrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(3/
4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g - 4*sqrt(a)*b^(3/2)*d)*arctan(1/2
*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + s
qrt(2)*a^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g + 4*sqrt(a)*b^(3/2)*d
)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a*b)
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.06

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$$
$$= \frac{bex^3+bdx^2+bcx-agx-af}{4\,(bx^4+a)ab} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$- \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(b*e*x^3 + b*d*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqr
t(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*
b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b
)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(
1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^
(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c +
(a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(a^2*b^3)
```

3.176.9 Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 1383, normalized size of antiderivative = 4.06

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\frac{a^2\,e\,g^2+6\,a\,b\,c\,e\,g-4\,a\,b\,d^2\,g+a\,b\,e^3+9\,b^2\,c^2\,e-12\,b^2\,c\,d^2}{64\,a^3}\right.\right.$$
$$-\frac{\mathrm{root}(65536\,a^7\,b^5\,z^4+1024\,a^5\,b^3\,e\,g\,z^2+3072\,a^4\,b^4\,c\,e\,z^2+2048\,a^4\,b^4\,d^2\,z^2-768\,a^3\,b^3\,c\,d\,g\,z-128\,a^4\,b^2}{}$$
$$\left.-\frac{b\,d\,x\,(-2\,b\,d^2+3\,b\,c\,e+a\,e\,g)}{16\,a^3}\right)\mathrm{root}\big(65536\,a^7\,b^5\,z^4+1024\,a^5\,b^3\,e\,g\,z^2+3072\,a^4\,b^4\,c\,e\,z^2$$
$$+2048\,a^4\,b^4\,d^2\,z^2-768\,a^3\,b^3\,c\,d\,g\,z-128\,a^4\,b^2\,d\,g^2\,z+128\,a^3\,b^3\,d\,e^2\,z-1152\,a^2\,b^4\,c^2\,d\,z$$
$$-16\,a^2\,b^2\,d^2\,e\,g+12\,a^2\,b^2\,c\,e^2\,g-48\,a\,b^3\,c\,d^2\,e+108\,a\,b^3\,c^3\,g+12\,a^3\,b\,c\,g^3+54\,a^2\,b^2\,c^2\,g^2$$
$$\left.+2\,a^3\,b\,e^2\,g^2+18\,a\,b^3\,c^2\,e^2+16\,a\,b^3\,d^4+81\,b^4\,c^4+a^2\,b^2\,e^4+a^4\,g^4,z,k\big)\right)$$
$$+\frac{\frac{d\,x^2}{4\,a}-\frac{f}{4\,b}+\frac{e\,x^3}{4\,a}+\frac{x\,(b\,c-a\,g)}{4\,a\,b}}{b\,x^4+a}$$

input

```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x)
```

output
```
symsum(log(- (9*b^2*c^2*e - 12*b^2*c*d^2 + a^2*e*g^2 + a*b*e^3 - 4*a*b*d^2
*g + 6*a*b*c*e*g)/(64*a^3) - (root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^
2 + 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 12
8*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^
2*d^2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a
^3*b*c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*
a*b^3*d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*b*(9*b^2*c^2*x + a^2
*g^2*x + 16*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c
*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z
+ 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^2*e*g + 12*a^2
*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 + 54*a^
2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 + 81*b^4
*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*a^3*b*g - a*b*e^2*x + 48*root(65536*a^
7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2
*z^2 - 768*a^3*b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1
152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c
*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e
^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*
g^4, z, k)*a^2*b^2*c + 4*a*b*d*e - 32*root(65536*a^7*b^5*z^4 + 1024*a^5*b^
3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c...
```

3.176. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$

3.177 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$

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3.177.1 Optimal result

Integrand size = 30, antiderivative size = 394

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx &= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2} - \frac{4af-x(7bc+ag+6bdx+5bex^2)}{32a^2b(a+bx^4)} \\
&+ \frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} \\
&+ \frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} \\
&- \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}} \\
&+ \frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}}
\end{aligned}$$

output
```
1/8*x*(b*f*x^3+b*e*x^2+b*d*x-a*g+b*c)/a/b/(b*x^4+a)^2+1/32*(-4*a*f+x*(5*b*
e*x^2+6*b*d*x+a*g+7*b*c))/a^2/b/(b*x^4+a)+3/16*d*arctan(x^2*b^(1/2)/a^(1/2
))/a^(5/2)/b^(1/2)-1/256*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2)
)*(21*b*c+3*a*g-5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+1/256*ln(a^(
1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(21*b*c+3*a*g-5*e*a^(1/2)*b^(1
/2))/a^(11/4)/b^(5/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(
21*b*c+3*a*g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+1/128*arctan(1+
b^(1/4)*x*2^(1/2)/a^(1/4))*(21*b*c+3*a*g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(
5/4)*2^(1/2)
```

3.177.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$$
$$= \frac{\frac{8a^{3/4}\sqrt[4]{b}x(7bc+ag+bx(6d+5ex))}{a+bx^4} - \frac{32a^{7/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{(a+bx^4)^2} - 2\left(21\sqrt{2}bc+24\sqrt[4]{a}b^{3/4}d+5\sqrt{2}\sqrt{a}\sqrt{b}e+3\sqrt{\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```

output
```
((8*a^(3/4)*b^(1/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x)))/(a + b*x^4) - (32
*a^(7/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4)^2 - 2*
(21*Sqrt[2]*b*c + 24*a^(1/4)*b^(3/4)*d + 5*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 3*S
qrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b*c -
24*a^(1/4)*b^(3/4)*d + 5*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 3*Sqrt[2]*a*g)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e
 - 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]
*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(5/4))
```

3.177.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2397, 25, 2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$$

$$\Big\downarrow \ 2397$$

$$\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2}-\frac{\int -\frac{4bfx^3+5bex^2+6bdx+7bc+ag}{(bx^4+a)^2}dx}{8ab}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{4bfx^3+5bex^2+6bdx+7bc+ag}{(bx^4+a)^2}dx}{8ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \ 2393$$

$$\frac{-\frac{\int -\frac{5bex^2+12bdx+3(7bc+ag)}{bx^4+a}dx}{4a}-\frac{4af-x(ag+7bc+6bdx+5bex^2)}{4a(a+bx^4)}}{8ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{5bex^2+12bdx+3(7bc+ag)}{bx^4+a}dx}{4a}-\frac{4af-x(ag+7bc+6bdx+5bex^2)}{4a(a+bx^4)}}{8ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \ 2415$$

$$\frac{\frac{\int \left(\frac{12bdx}{bx^4+a}+\frac{5bex^2+3(7bc+ag)}{bx^4+a}\right)dx}{4a}-\frac{4af-x(ag+7bc+6bdx+5bex^2)}{4a(a+bx^4)}}{8ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(5\sqrt{a}\sqrt{b}e+3ag+21bc)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(5\sqrt{a}\sqrt{b}e+3ag+21bc)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)(-5\sqrt{a}\sqrt{b}e+3ag+21bc)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\log(\ldots$$

$$4a \qquad 8ab$$

$$\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```

output
```
(x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) + (-1/4*
(4*a*f - x*(7*b*c + a*g + 6*b*d*x + 5*b*e*x^2))/(a*(a + b*x^4)) + ((6*Sqrt
[b]*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - ((21*b*c + 5*Sqrt[a]*Sqrt[b
]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b
^(1/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - ((21*b*c - 5*Sqrt[a]*Sqrt[b
]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(1/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4))
)/(4*a))/(8*a*b)
```

3.177.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.177.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.37

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{\left(5\_R^2e+12\_Rd+\frac{3ag+21bc}{b}\right)\ln(x-\_R)}{\_R^3}}{128a^2b}$
default	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\frac{(3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)\right.}{8a}}{}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/32*b*e/a^2*x^7+3/16*b*d/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/a*e*x^3+5
/16*d/a*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+a)^2+1/128/a^2/b*sum
((5*_R^2*e+12*_R*d+3/b*(a*g+7*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.177.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 58.34 (sec) , antiderivative size = 358509, normalized size of antiderivative = 909.92

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.177.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

output
```
Timed out
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx = \frac{5\,b^2ex^7+6\,b^2dx^6+9\,abex^3+(7\,b^2c+abg)x^5+10\,abdx^2-4\,a^2f+(11\,abc-3\,a^2g)x}{32\,(a^2b^3x^8+2\,a^3b^2x^4+a^4b)} + \frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 9*a*b*e*x^3 + (7*b^2*c + a*b*g)*x^5 + 10
*a*b*d*x^2 - 4*a^2*f + (11*a*b*c - 3*a^2*g)*x)/(a^2*b^3*x^8 + 2*a^3*b^2*x^
4 + a^4*b) + 1/256*(sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g)
*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4))
- sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g)*log(sqrt(b)*x^2 -
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^
(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*
g - 24*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4
)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4))
+ 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2
)*a^(5/4)*b^(3/4)*g + 24*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*
x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*
sqrt(b))*b^(3/4)))/(a^2*b)
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$$
$$= \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21(ab^3)^{\frac{1}{4}}b^2c+3(ab^3)^{\frac{1}{4}}abg+5(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3}$$
$$+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21(ab^3)^{\frac{1}{4}}b^2c+3(ab^3)^{\frac{1}{4}}abg+5(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128a^3b^3}$$
$$+\frac{\sqrt{2}\left(21(ab^3)^{\frac{1}{4}}b^2c+3(ab^3)^{\frac{1}{4}}abg-5(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256a^3b^3}$$
$$-\frac{\sqrt{2}\left(21(ab^3)^{\frac{1}{4}}b^2c+3(ab^3)^{\frac{1}{4}}abg-5(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256a^3b^3}$$
$$+\frac{5b^2ex^7+6b^2dx^6+7b^2cx^5+abgx^5+9abex^3+10abdx^2+11abcx-3a^2gx-4a^2f}{32(bx^4+a)^2a^2b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*
b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^
2*d + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/
256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3
/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt
(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*l
og(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*e*x^7
+ 6*b^2*d*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*e*x^3 + 10*a*b*d*x^2 + 11*
a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

3.177.9 Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 1001, normalized size of antiderivative = 2.54

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$$
$$= \frac{\frac{5\,d\,x^2}{16\,a}-\frac{f}{8\,b}+\frac{9\,e\,x^3}{32\,a}+\frac{x^5\,(7\,b\,c+a\,g)}{32\,a^2}+\frac{x\,(11\,b\,c-3\,a\,g)}{32\,a\,b}+\frac{3\,b\,d\,x^6}{16\,a^2}+\frac{5\,b\,e\,x^7}{32\,a^2}}{a^2+2\,a\,b\,x^4+b^2\,x^8}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\big(268435456\,a^{11}\,b^5\,z^4+983040\,a^7\,b^3\,e\,g\,z^2+6881280\,a^6\,b^4\,c\,e\,z^2+4718592\,a^6\,b^4\,d^2\,z^2-774\right.\right.$$
$$-\frac{45\,a^2\,e\,g^2+630\,a\,b\,c\,e\,g-432\,a\,b\,d^2\,g+125\,a\,b\,e^3+2205\,b^2\,c^2\,e-3024\,b^2\,c\,d^2}{32768\,a^6}$$
$$\left.-\frac{x\,(-216\,b^2\,d^3+315\,c\,e\,b^2\,d+45\,a\,e\,g\,b\,d)}{4096\,a^6}\right)\mathrm{root}\big(268435456\,a^{11}\,b^5\,z^4+983040\,a^7\,b^3\,e\,g\,z^2$$
$$+6881280\,a^6\,b^4\,c\,e\,z^2+4718592\,a^6\,b^4\,d^2\,z^2-774144\,a^4\,b^3\,c\,d\,g\,z-55296\,a^5\,b^2\,d\,g^2\,z$$
$$+153600\,a^4\,b^3\,d\,e^2\,z-2709504\,a^3\,b^4\,c^2\,d\,z-8640\,a^2\,b^2\,d^2\,e\,g+6300\,a^2\,b^2\,c\,e^2\,g$$
$$-60480\,a\,b^3\,c\,d^2\,e+111132\,a\,b^3\,c^3\,g+2268\,a^3\,b\,c\,g^3+23814\,a^2\,b^2\,c^2\,g^2+450\,a^3\,b\,e^2\,g^2$$
$$\left.+22050\,a\,b^3\,c^2\,e^2+625\,a^2\,b^2\,e^4+20736\,a\,b^3\,d^4+81\,a^4\,g^4+194481\,b^4\,c^4,z,k\big)\right)$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x)
```

output
```
((5*d*x^2)/(16*a) - f/(8*b) + (9*e*x^3)/(32*a) + (x^5*(7*b*c + a*g))/(32*a
^2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (3*b*d*x^6)/(16*a^2) + (5*b*e*x^7)/(
32*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log(- root(268435456*a^11*b^
5*z^4 + 983040*a^7*b^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 4718592*a^6*b^4
*d^2*z^2 - 774144*a^4*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3
*d*e^2*z - 2709504*a^3*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*a^2*b^2*c
*e^2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 + 238
14*a^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 625*a^2*b^2
*e^4 + 20736*a*b^3*d^4 + 81*a^4*g^4 + 194481*b^4*c^4, z, k)*(root(26843545
6*a^11*b^5*z^4 + 983040*a^7*b^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 471859
2*a^6*b^4*d^2*z^2 - 774144*a^4*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 15360
0*a^4*b^3*d*e^2*z - 2709504*a^3*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*
a^2*b^2*c*e^2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*
g^3 + 23814*a^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 62
5*a^2*b^2*e^4 + 20736*a*b^3*d^4 + 81*a^4*g^4 + 194481*b^4*c^4, z, k)*((344
064*a^5*b^3*c + 49152*a^6*b^2*g)/(32768*a^6) - (6*b^3*d*x)/a) + (x*(144*a^
4*b*g^2 + 7056*a^2*b^3*c^2 - 400*a^3*b^2*e^2 + 2016*a^3*b^2*c*g))/(4096*a^
6) + (15*b^2*d*e)/(32*a^3)) - (2205*b^2*c^2*e - 3024*b^2*c*d^2 + 45*a^2*e*
g^2 + 125*a*b*e^3 - 432*a*b*d^2*g + 630*a*b*c*e*g)/(32768*a^6) - (x*(315*b
^2*c*d*e - 216*b^2*d^3 + 45*a*b*d*e*g))/(4096*a^6))*root(268435456*a^11...
```

3.177. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$

3.178 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$

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3.178.1 Optimal result

Integrand size = 30, antiderivative size = 437

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx &= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab(a+bx^4)^3}+\frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b(a+bx^4)} \\ &\quad -\frac{8af-x(11bc+ag+10bdx+9bex^2)}{96a^2b(a+bx^4)^2}+\frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} \\ &\quad -\frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}} \\ &\quad +\frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}} \\ &\quad -\frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}} \\ &\quad +\frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}}\end{aligned}$$

output
```
1/12*x*(b*f*x^3+b*e*x^2+b*d*x-a*g+b*c)/a/b/(b*x^4+a)^3+1/384*x*(45*b*e*x^2
+60*b*d*x+7*a*g+77*b*c)/a^3/b/(b*x^4+a)+1/96*(-8*a*f+x*(9*b*e*x^2+10*b*d*x
+a*g+11*b*c))/a^2/b/(b*x^4+a)^2+5/32*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(7/2)
/b^(1/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(77*b*c
+7*a*g-15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)+1/1024*ln(a^(1/4)*b^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(77*b*c+7*a*g-15*e*a^(1/2)*b^(1/2))/a
^(15/4)/b^(5/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(77*b*c
+7*a*g+15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)+1/512*arctan(1+b^(1/
4)*x*2^(1/2)/a^(1/4))*(77*b*c+7*a*g+15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)
*2^(1/2)
```

3.178.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.94

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$$
$$= \frac{\frac{8a^{3/4}\sqrt[4]{b}x(77bc+7ag+15bx(4d+3ex))}{a+bx^4}+\frac{32a^{7/4}\sqrt[4]{b}x(11bc+ag+bx(10d+9ex))}{(a+bx^4)^2}-\frac{256a^{11/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{(a+bx^4)^3}-6\left(77\sqrt{2}\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```

output
```
((8*a^(3/4)*b^(1/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a + b*x^4)
 + (32*a^(7/4)*b^(1/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x)))/(a + b*x^4)^
2 - (256*a^(11/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^
4)^3 - 6*(77*Sqrt[2]*b*c + 80*a^(1/4)*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[
b]*e + 7*Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt
[2]*b*c - 80*a^(1/4)*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 7*Sqrt[2]*
a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(77*b*c - 15*Sqrt
[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2] + 3*Sqrt[2]*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^(5/4))
```

3.178.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2397, 25, 2393, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{12ab(a+bx^4)^3}-\frac{\int -\frac{8bfx^3+9bex^2+10bdx+11bc+ag}{(bx^4+a)^3}dx}{12ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{8bfx^3+9bex^2+10bdx+11bc+ag}{(bx^4+a)^3}dx}{12ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{2393}$$

$$\frac{-\frac{\int -\frac{45bex^2+60bdx+7(11bc+ag)}{(bx^4+a)^2}dx}{8a}-\frac{8af-x(ag+11bc+10bdx+9bex^2)}{8a(a+bx^4)^2}}{12ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{45bex^2+60bdx+7(11bc+ag)}{(bx^4+a)^2}dx}{8a}-\frac{8af-x(ag+11bc+10bdx+9bex^2)}{8a(a+bx^4)^2}}{12ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x\left(7(ag+11bc)+60bdx+45bex^2\right)}{4a\left(a+bx^4\right)}-\frac{\int -\frac{3\left(15bex^2+40bdx+7(11bc+ag)\right)}{bx^4+a}dx}{4a}}{8a}-\frac{8af-x(ag+11bc+10bdx+9bex^2)}{8a(a+bx^4)^2}}{12ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{15bex^2+40bdx+7(11bc+ag)}{bx^4+a}dx}{4a}+\frac{x\left(7(ag+11bc)+60bdx+45bex^2\right)}{4a\left(a+bx^4\right)}}{8a}-\frac{8af-x\left(ag+11bc+10bdx+9bex^2\right)}{8a(a+bx^4)^2}}{12ab}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

$\Big\downarrow$ 2415

$$\frac{\frac{\frac{3\int \left(\frac{40bdx}{bx^4+a}+\frac{15bex^2+7(11bc+ag)}{bx^4+a}\right)dx}{4a}+\frac{x\left(7(ag+11bc)+60bdx+45bex^2\right)}{4a\left(a+bx^4\right)}}{8a}-\frac{8af-x\left(ag+11bc+10bdx+9bex^2\right)}{8a(a+bx^4)^2}}{12ab}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\frac{3\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} \cdots\right.}{4a}}{8a}}{12\cdots}$$

$$\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```

output
```
(x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (-1/8
*(8*a*f - x*(11*b*c + a*g + 10*b*d*x + 9*b*e*x^2))/(a*(a + b*x^4)^2) + ((x
*(7*(11*b*c + a*g) + 60*b*d*x + 45*b*e*x^2))/(4*a*(a + b*x^4)) + (3*((20*S
qrt[b]*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - ((77*b*c + 15*Sqrt[a]*Sq
rt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*b^(1/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - ((77*b*c - 15*Sqrt[a]*
Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b
^(1/4))))/(4*a))/(8*a))/(12*a*b)
```

3.178. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$

3.178.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.178. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$

3.178.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.42

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}-\frac{(7ag-51bc)x}{128ab}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}}{}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}-\frac{(7ag-51bc)x}{128ab}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{(7ag+77bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\right.}{}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+7/384*(a*g+11*b*c)/a^3*b*x^9+21
/64*b*e/a^2*x^7+5/12*b*d/a^2*x^6+3/64/a^2*(a*g+11*b*c)*x^5+113/384/a*e*x^3
+11/32*d/a*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+1/512/a^3/
b*sum((15*_R^2*e+40*_R*d+7*(a*g+11*b*c)/b)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+
a))
```

3.178.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 82.84 (sec) , antiderivative size = 358702, normalized size of antiderivative = 820.83

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.178.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^4)^4} \, dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

output
```
Timed out
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.08

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^4)^4} \, dx = \frac{45\, b^3ex^{11} + 60\, b^3dx^{10} + 126\, ab^2ex^7 + 160\, ab^2dx^6 + 7\,(11\, b^3c + ab^2g)x^9 + 113\, a^2bex^3 + 132\, a^2bdx^2 + 18\,(11}{384\,(a^3b^4x^{12} + 3\, a^4b^3x^8 + 3\, a^5b^2x^4 + a^6b)}$$

$$+ \frac{\frac{\sqrt{2}\left(77\, b^{\frac{3}{2}}c - 15\, \sqrt{a}be + 7\, a\sqrt{b}g\right)\log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(77\, b^{\frac{3}{2}}c - 15\, \sqrt{a}be + 7\, a\sqrt{b}g\right)\log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\,\left(77\,\sqrt{2}a^{\frac{1}{4}}}{}}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 126*a*b^2*e*x^7 + 160*a*b^2*d*x^6 +
 7*(11*b^3*c + a*b^2*g)*x^9 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 18*(11*a
*b^2*c + a^2*b*g)*x^5 - 32*a^3*f + 3*(51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^4*x^
12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b) + 1/1024*(sqrt(2)*(77*b^(3/2)*
c - 15*sqrt(a)*b*e + 7*a*sqrt(b)*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1
/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*
e + 7*a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/
(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^(3/4)*b
^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g - 80*sqrt(a)*b^(3/2)*d)*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^
(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 1
5*sqrt(2)*a^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 80*sqrt(a)*b^(
3/2)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sq
rt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$$
$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77(ab^3)^{\frac14}b^2c+7(ab^3)^{\frac14}abg+15(ab^3)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{512a^4b^3}$$
$$+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77(ab^3)^{\frac14}b^2c+7(ab^3)^{\frac14}abg+15(ab^3)^{\frac34}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{512a^4b^3}$$
$$+\frac{\sqrt{2}\left(77(ab^3)^{\frac14}b^2c+7(ab^3)^{\frac14}abg-15(ab^3)^{\frac34}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{1024a^4b^3}$$
$$-\frac{\sqrt{2}\left(77(ab^3)^{\frac14}b^2c+7(ab^3)^{\frac14}abg-15(ab^3)^{\frac34}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{1024a^4b^3}$$
$$+\frac{45b^3ex^{11}+60b^3dx^{10}+77b^3cx^9+7ab^2gx^9+126ab^2ex^7+160ab^2dx^6+198ab^2cx^5+18a^2bgx^5+113}{384(bx^4+a)^3a^3b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*
b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b
^2*d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) +
1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3
)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024
*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4
)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^
3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*e*x^7
+ 160*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*e*x^3 + 1
32*a^2*b*d*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3
*b)
```

3.178.9 Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 1053, normalized size of antiderivative = 2.41

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$$

$$= \left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(68719476736\,a^{15}\,b^5\,z^4+1211105280\,a^8\,b^4\,c\,e\,z^2+110100480\,a^9\,b^3\,e\,g\,z^2+838860800\,a^8\,b^4\,d^2\right.\right.\right.$$

$$-\frac{735\,a^2\,e\,g^2+16170\,a\,b\,c\,e\,g-11200\,a\,b\,d^2\,g+3375\,a\,b\,e^3+88935\,b^2\,c^2\,e-123200\,b^2\,c\,d^2}{2097152\,a^9}$$

$$\left.-\frac{x\left(-4000\,b^2\,d^3+5775\,c\,e\,b^2\,d+525\,a\,e\,g\,b\,d\right)}{131072\,a^9}\right)\mathrm{root}\left(68719476736\,a^{15}\,b^5\,z^4\right.$$

$$+1211105280\,a^8\,b^4\,c\,e\,z^2+110100480\,a^9\,b^3\,e\,g\,z^2+838860800\,a^8\,b^4\,d^2\,z^2$$

$$-88309760\,a^5\,b^3\,c\,d\,g\,z-485703680\,a^4\,b^4\,c^2\,d\,z-4014080\,a^6\,b^2\,d\,g^2\,z$$

$$+18432000\,a^5\,b^3\,d\,e^2\,z-672000\,a^2\,b^2\,d^2\,e\,g+485100\,a^2\,b^2\,c\,e^2\,g-7392000\,a\,b^3\,c\,d^2\,e$$

$$+12782924\,a\,b^3\,c^3\,g+105644\,a^3\,b\,c\,g^3+1743126\,a^2\,b^2\,c^2\,g^2+22050\,a^3\,b\,e^2\,g^2$$

$$\left.\left.+2668050\,a\,b^3\,c^2\,e^2+50625\,a^2\,b^2\,e^4+2560000\,a\,b^3\,d^4+2401\,a^4\,g^4+35153041\,b^4\,c^4,z,k\right)\right)$$

$$+\frac{\frac{11\,d\,x^2}{32\,a}-\frac{f}{12\,b}+\frac{113\,e\,x^3}{384\,a}+\frac{3\,x^5\,(11\,b\,c+a\,g)}{64\,a^2}+\frac{7\,b\,x^9\,(11\,b\,c+a\,g)}{384\,a^3}+\frac{x\,(51\,b\,c-7\,a\,g)}{128\,a\,b}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{15\,b^2\,e\,x^{11}}{128\,a^3}+\frac{5\,b\,d\,x^6}{12\,a^2}+\frac{21\,b\,e}{64\,a}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x)
```

output
```
symsum(log(- root(68719476736*a^15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 +
110100480*a^9*b^3*e*g*z^2 + 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c
*d*g*z - 485703680*a^4*b^4*c^2*d*z - 4014080*a^6*b^2*d*g^2*z + 18432000*a^
5*b^3*d*e^2*z - 672000*a^2*b^2*d^2*e*g + 485100*a^2*b^2*c*e^2*g - 7392000*
a*b^3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g^3 + 1743126*a^2*b^
2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2 + 50625*a^2*b^2*e^
4 + 2560000*a*b^3*d^4 + 2401*a^4*g^4 + 35153041*b^4*c^4, z, k)*(root(68719
476736*a^15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z
^2 + 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d*g*z - 485703680*a^4*
b^4*c^2*d*z - 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b^3*d*e^2*z - 672000*
a^2*b^2*d^2*e*g + 485100*a^2*b^2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 1278292
4*a*b^3*c^3*g + 105644*a^3*b*c*g^3 + 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b
*e^2*g^2 + 2668050*a*b^3*c^2*e^2 + 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 +
 2401*a^4*g^4 + 35153041*b^4*c^4, z, k)*((20185088*a^7*b^3*c + 1835008*a^8
*b^2*g)/(2097152*a^9) - (5*b^3*d*x)/a^2) + (x*(1568*a^5*b*g^2 + 189728*a^3
*b^3*c^2 - 7200*a^4*b^2*e^2 + 34496*a^4*b^2*c*g))/(131072*a^9) + (75*b^2*d
*e)/(256*a^5)) - (88935*b^2*c^2*e - 123200*b^2*c*d^2 + 735*a^2*e*g^2 + 337
5*a*b*e^3 - 11200*a*b*d^2*g + 16170*a*b*c*e*g)/(2097152*a^9) - (x*(5775*b^
2*c*d*e - 4000*b^2*d^3 + 525*a*b*d*e*g))/(131072*a^9))*root(68719476736*a^
15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 + 8...
```

3.178. $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4} \, dx$

3.179 $\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx$

3.179.1 Optimal result 1398
3.179.2 Mathematica [A] (verified) 1398
3.179.3 Rubi [A] (verified) 1399
3.179.4 Maple [A] (verified) 1400
3.179.5 Fricas [B] (verification not implemented) 1400
3.179.6 Sympy [B] (verification not implemented) 1400
3.179.7 Maxima [B] (verification not implemented) 1401
3.179.8 Giac [B] (verification not implemented) 1401
3.179.9 Mupad [B] (verification not implemented) 1402

3.179.1 Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}(1-x)^4$$

output
```
-1/4*(1-x)^4
```

3.179.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}(-1+x)^4$$

input
```
Integrate[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

output
```
-1/4*(-1 + x)^4
```

3.179.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2006, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^4)^3}{(x^3+x^2+x+1)^3}dx$$

$$\Big\downarrow \text{ 2006}$$

$$\int (1-x)^3dx$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{1}{4}(1-x)^4$$

input
```
Int[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

output
```
-1/4*(1 - x)^4
```

3.179.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

3.179.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{(-1+x)^4}{4}$	8
parallelrisch	$-\frac{1}{4}x^4+x^3-\frac{3}{2}x^2+x$	16
gosper	$-\frac{x(x^3-4x^2+6x-4)}{4}$	17
risch	$-\frac{1}{4}x^4+x^3-\frac{3}{2}x^2+x-\frac{1}{4}$	17
norman	$\frac{-2x^5-2x^3-x^4-\frac{7}{4}x^2-\frac{1}{2}x-\frac{1}{4}x^8+\frac{1}{2}x^9-\frac{1}{4}x^{10}-\frac{3}{4}}{(x^3+x^2+x+1)^2}$	53

input
```
int((-x^4+1)^3/(x^3+x^2+x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-1+x)^4
```

3.179.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}\,x^4+x^3-\frac{3}{2}\,x^2+x$$

input
```
integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="fricas")
```

output
```
-1/4*x^4 + x^3 - 3/2*x^2 + x
```

3.179.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{x^4}{4}+x^3-\frac{3x^2}{2}+x$$

input
```
integrate((-x**4+1)**3/(x**3+x**2+x+1)**3,x)
```

output
```
-x**4/4 + x**3 - 3*x**2/2 + x
```

3.179.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}\,x^4 + x^3 - \frac{3}{2}\,x^2 + x$$

input
```
integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="maxima")
```

output
```
-1/4*x^4 + x^3 - 3/2*x^2 + x
```

3.179.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}\,x^4 + x^3 - \frac{3}{2}\,x^2 + x$$

input
```
integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="giac")
```

output
```
-1/4*x^4 + x^3 - 3/2*x^2 + x
```

3.179.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{x^4}{4} + x^3 - \frac{3\,x^2}{2} + x$$

input
```
int(-(x^4 - 1)^3/(x + x^2 + x^3 + 1)^3,x)
```

output
```
x - (3*x^2)/2 + x^3 - x^4/4
```

3.180 $\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} dx$

3.180.1 Optimal result 1403
3.180.2 Mathematica [A] (verified) 1403
3.180.3 Rubi [A] (verified) 1404
3.180.4 Maple [A] (verified) 1405
3.180.5 Fricas [A] (verification not implemented) 1405
3.180.6 Sympy [A] (verification not implemented) 1405
3.180.7 Maxima [A] (verification not implemented) 1406
3.180.8 Giac [A] (verification not implemented) 1406
3.180.9 Mupad [B] (verification not implemented) 1406

3.180.1 Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} dx = -\frac{1}{3}(1-x)^3$$

output
```
-1/3*(1-x)^3
```

3.180.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} dx = x - x^2 + \frac{x^3}{3}$$

input
```
Integrate[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]
```

output
```
x - x^2 + x^3/3
```

3.180.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^4)^2}{(x^3+x^2+x+1)^2}\,dx$$

$$\Big\downarrow \; 2019$$

$$\int (1-x)^2dx$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{3}(1-x)^3$$

input
```
Int[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]
```

output
```
-1/3*(1 - x)^3
```

3.180.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.180.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{(-1+x)^3}{3}$	8
gosper	$\frac{x(x^2-3x+3)}{3}$	12
parallelrisch	$\frac{1}{3}x^3-x^2+x$	13
risch	$\frac{1}{3}x^3-x^2+x-\frac{1}{3}$	14
norman	$\frac{-\frac{1}{3}x^2+\frac{2}{3}x+\frac{1}{3}x^4-\frac{2}{3}x^5+\frac{1}{3}x^6-\frac{1}{3}}{x^3+x^2+x+1}$	38

input
```
int((-x^4+1)^2/(x^3+x^2+x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-1+x)^3
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{1}{3}\,x^3-x^2+x$$

input
```
integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="fricas")
```

output
```
1/3*x^3 - x^2 + x
```

3.180.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{x^3}{3}-x^2+x$$

input
```
integrate((-x**4+1)**2/(x**3+x**2+x+1)**2,x)
```

output
```
x**3/3 - x**2 + x
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{1}{3}\,x^3 - x^2 + x$$

input
```
integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="maxima")
```

output
```
1/3*x^3 - x^2 + x
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{1}{3}\,x^3 - x^2 + x$$

input
```
integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="giac")
```

output
```
1/3*x^3 - x^2 + x
```

3.180.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{x\,(x^2-3\,x+3)}{3}$$

input
```
int((x^4 - 1)^2/(x + x^2 + x^3 + 1)^2,x)
```

output
```
(x*(x^2 - 3*x + 3))/3
```

3.181 $\int \frac{1-x^4}{1+x+x^2+x^3}\,dx$

3.181.1 Optimal result 1407
3.181.2 Mathematica [A] (verified) 1407
3.181.3 Rubi [A] (verified) 1408
3.181.4 Maple [A] (verified) 1409
3.181.5 Fricas [A] (verification not implemented) 1409
3.181.6 Sympy [A] (verification not implemented) 1409
3.181.7 Maxima [A] (verification not implemented) 1410
3.181.8 Giac [A] (verification not implemented) 1410
3.181.9 Mupad [B] (verification not implemented) 1410

3.181.1 Optimal result

Integrand size = 19, antiderivative size = 9

$$\int \frac{1-x^4}{1+x+x^2+x^3}\,dx = x-\frac{x^2}{2}$$

output
```
x-1/2*x^2
```

3.181.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^4}{1+x+x^2+x^3}\,dx = x-\frac{x^2}{2}$$

input
```
Integrate[(1 - x^4)/(1 + x + x^2 + x^3),x]
```

output
```
x - x^2/2
```

3.181.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^3+x^2+x+1} \, dx$$
$$\Big\downarrow \ 2019$$
$$\int (1-x)dx$$
$$\Big\downarrow \ 17$$
$$-\frac{1}{2}(1-x)^2$$

input
```
Int[(1 - x^4)/(1 + x + x^2 + x^3),x]
```

output
```
-1/2*(1 - x)^2
```

3.181.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.181.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{x(-2+x)}{2}$	7
default	$x-\frac{1}{2}x^2$	8
norman	$x-\frac{1}{2}x^2$	8
risch	$x-\frac{1}{2}x^2$	8
parallelrisch	$x-\frac{1}{2}x^2$	8
parts	$x-\frac{1}{2}x^2$	8

input
```
int((-x^4+1)/(x^3+x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(-2+x)
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1-x^4}{1+x+x^2+x^3}\,dx = -\frac{1}{2}\,x^2 + x$$

input
```
integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="fricas")
```

output
```
-1/2*x^2 + x
```

3.181.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{1-x^4}{1+x+x^2+x^3}\,dx = -\frac{x^2}{2} + x$$

input
```
integrate((-x**4+1)/(x**3+x**2+x+1),x)
```

output
```
-x**2/2 + x
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1-x^4}{1+x+x^2+x^3} \, dx = -\frac{1}{2} \, x^2 + x$$

input
```
integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="maxima")
```

output
```
-1/2*x^2 + x
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1-x^4}{1+x+x^2+x^3} \, dx = -\frac{1}{2} \, x^2 + x$$

input
```
integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="giac")
```

output
```
-1/2*x^2 + x
```

3.181.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{1-x^4}{1+x+x^2+x^3} \, dx = -\frac{x\,(x-2)}{2}$$

input
```
int(-(x^4 - 1)/(x + x^2 + x^3 + 1),x)
```

output
```
-(x*(x - 2))/2
```

3.182 $\int \frac{1+x+x^2+x^3}{1-x^4} dx$

3.182.1 Optimal result 1411
3.182.2 Mathematica [A] (verified) 1411
3.182.3 Rubi [A] (verified) 1412
3.182.4 Maple [A] (verified) 1413
3.182.5 Fricas [A] (verification not implemented) 1413
3.182.6 Sympy [A] (verification not implemented) 1413
3.182.7 Maxima [A] (verification not implemented) 1414
3.182.8 Giac [A] (verification not implemented) 1414
3.182.9 Mupad [B] (verification not implemented) 1414

3.182.1 Optimal result

Integrand size = 19, antiderivative size = 8

$$\int \frac{1+x+x^2+x^3}{1-x^4} \, dx = -\log(1-x)$$

output
```
-ln(1-x)
```

3.182.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2+x^3}{1-x^4} \, dx = -\log(1-x)$$

input
```
Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

output
```
-Log[1 - x]
```

3.182.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+x+1}{1-x^4}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{ 16}$$

$$-\log(1-x)$$

input
```
Int[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

output
```
-Log[1 - x]
```

3.182.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.182.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result
default	$-\ln\left(-1+x\right)$
norman	$-\ln\left(-1+x\right)$
risch	$-\ln\left(-1+x\right)$
parallelrisch	$-\ln\left(-1+x\right)$
meijerg	$-\frac{\ln\left(-x^{4}+1\right)}{4}-\frac{x^{3}\left(\ln\left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{4}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^{4}\right)^{\frac{1}{4}}\right)\right)}{4\left(x^{4}\right)^{\frac{3}{4}}}+\frac{\operatorname{arctanh}\left(x^{2}\right)}{2}-$ $x\left(\ln\left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(\right.\right.\right.$ / $4(x$

input
```
int((x^3+x^2+x+1)/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(-1+x)
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(x-1\right)$$

input
```
integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")
```

output
```
-log(x - 1)
```

3.182.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(x-1\right)$$

input
```
integrate((x**3+x**2+x+1)/(-x**4+1),x)
```

output
```
-log(x - 1)
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log(x-1)$$

input
```
integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")
```

output
```
-log(x - 1)
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log(|x-1|)$$

input
```
integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")
```

output
```
-log(abs(x - 1))
```

3.182.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\ln(x-1)$$

input
```
int(-(x + x^2 + x^3 + 1)/(x^4 - 1),x)
```

output
```
-log(x - 1)
```

3.183 $\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx$

3.183.1 Optimal result 1415
3.183.2 Mathematica [A] (verified) 1415
3.183.3 Rubi [A] (verified) 1416
3.183.4 Maple [A] (verified) 1417
3.183.5 Fricas [A] (verification not implemented) 1417
3.183.6 Sympy [A] (verification not implemented) 1418
3.183.7 Maxima [A] (verification not implemented) 1418
3.183.8 Giac [A] (verification not implemented) 1418
3.183.9 Mupad [B] (verification not implemented) 1419

3.183.1 Optimal result

Integrand size = 21, antiderivative size = 7

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx = \frac{1}{1-x}$$

output
```
1/(1-x)
```

3.183.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx = -\frac{1}{-1+x}$$

input
```
Integrate[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
```

output
```
-(-1 + x)^(-1)
```

3.183.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3+x^2+x+1)^2}{(1-x^4)^2} dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(1-x)^2} dx$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{1-x}$$

input
```
Int[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
```

output
```
(1 - x)^(-1)
```

3.183.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.183.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result
gosper	$-\frac{1}{-1+x}$
default	$-\frac{1}{-1+x}$
risch	$-\frac{1}{-1+x}$
parallelrisch	$-\frac{1}{-1+x}$
norman	$\frac{-x^3-x^2-x-1}{x^4-1}$
meijerg	$\frac{(-1)^{\frac{1}{4}}\left(-\frac{x^3(-1)^{\frac{3}{4}}}{-x^4+1}-\frac{3x^3(-1)^{\frac{3}{4}}\left(\ln\left(1-\left(x^4\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^4\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^4\right)^{\frac{1}{4}}\right)\right)}{4\left(x^4\right)^{\frac{3}{4}}}\right)}{4}+\frac{i\left(-\frac{ix^2}{-x^4+1}+i\,\mathrm{arctanh}\left(x^2\right)\right)}{2}+\frac{3(-1}{\ }$

input
```
int((x^3+x^2+x+1)^2/(-x^4+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(-1+x)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx=-\frac{1}{x-1}$$

input
```
integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="fricas")
```

output
```
-1/(x - 1)
```

3.183.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx = -\frac{1}{x-1}$$

input
```
integrate((x**3+x**2+x+1)**2/(-x**4+1)**2,x)
```

output
```
-1/(x - 1)
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx = -\frac{1}{x-1}$$

input
```
integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="maxima")
```

output
```
-1/(x - 1)
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx = -\frac{1}{x-1}$$

input
```
integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="giac")
```

output
```
-1/(x - 1)
```

3.183.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx = -\frac{1}{x-1}$$

input
```
int((x + x^2 + x^3 + 1)^2/(x^4 - 1)^2,x)
```

output
```
-1/(x - 1)
```

3.184 $\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx$

3.184.1 Optimal result 1420
3.184.2 Mathematica [A] (verified) 1420
3.184.3 Rubi [A] (verified) 1421
3.184.4 Maple [A] (verified) 1422
3.184.5 Fricas [A] (verification not implemented) 1422
3.184.6 Sympy [A] (verification not implemented) 1423
3.184.7 Maxima [A] (verification not implemented) 1423
3.184.8 Giac [A] (verification not implemented) 1423
3.184.9 Mupad [B] (verification not implemented) 1424

3.184.1 Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2(1-x)^2}$$

output
```
1/2/(1-x)^2
```

3.184.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2(-1+x)^2}$$

input
```
Integrate[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
```

output
```
1/(2*(-1 + x)^2)
```

3.184.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3+x^2+x+1)^3}{(1-x^4)^3} \, dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{(1-x)^3}dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{1}{2(1-x)^2}$$

input
```
Int[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
```

output
```
1/(2*(1 - x)^2)
```

3.184.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.184.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result
gosper	$\frac{1}{2(-1+x)^2}$
default	$\frac{1}{2(-1+x)^2}$
risch	$\frac{1}{2(-1+x)^2}$
parallelrisch	$\frac{1}{2(-1+x)^2}$
norman	$\frac{x+x^5+\frac{3}{2}x^4+\frac{3}{2}x^2+2x^3+\frac{1}{2}x^6+\frac{1}{2}}{(x^4-1)^2}$
meijerg	$-\frac{(-1)^{\frac{3}{4}}\left(\frac{(-1)^{\frac{1}{4}}x\left(-7x^4+11\right)}{4\left(-x^4+1\right)^2}-\frac{21x(-1)^{\frac{1}{4}}\left(\ln\left(1-\left(x^4\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^4\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^4\right)^{\frac{1}{4}}\right)\right)}{16\left(x^4\right)^{\frac{1}{4}}}\right)}{8}+\frac{5(-1)^{\frac{1}{4}}\left(-\frac{x^3(-1)^{\frac{3}{4}}\left(21x^4+\right.}{28\left(-x^4+1\right)^2}\right.}{}$

input
```
int((x^3+x^2+x+1)^3/(-x^4+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/(-1+x)^2
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2\,(x^2-2\,x+1)}$$

input
```
integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="fricas")
```

output
```
1/2/(x^2 - 2*x + 1)
```

3.184.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2x^2-4x+2}$$

input
```
integrate((x**3+x**2+x+1)**3/(-x**4+1)**3,x)
```

output
```
1/(2*x**2 - 4*x + 2)
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2\,(x^2-2\,x+1)}$$

input
```
integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="maxima")
```

output
```
1/2/(x^2 - 2*x + 1)
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2\,(x-1)^2}$$

input
```
integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="giac")
```

output
```
1/2/(x - 1)^2
```

3.184.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2(x-1)^2}$$

input
```
int(-(x + x^2 + x^3 + 1)^3/(x^4 - 1)^3,x)
```

output
```
1/(2*(x - 1)^2)
```

3.185 $\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx$

3.185.1 Optimal result 1425
3.185.2 Mathematica [A] (verified) 1425
3.185.3 Rubi [A] (verified) 1426
3.185.4 Maple [A] (verified) 1427
3.185.5 Fricas [B] (verification not implemented) 1427
3.185.6 Sympy [B] (verification not implemented) 1428
3.185.7 Maxima [B] (verification not implemented) 1428
3.185.8 Giac [A] (verification not implemented) 1428
3.185.9 Mupad [B] (verification not implemented) 1429

3.185.1 Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = \frac{1}{3(1-x)^3}$$

output
```
1/3/(1-x)^3
```

3.185.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3(-1+x)^3}$$

input
```
Integrate[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]
```

output
```
-1/3*1/(-1 + x)^3
```

3.185.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3+x^2+x+1)^4}{(1-x^4)^4}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(1-x)^4}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{3(1-x)^3}$$

input
```
Int[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]
```

output
```
1/(3*(1 - x)^3)
```

3.185.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.185.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{1}{3(-1+x)^{3}}$	8
default	$-\frac{1}{3(-1+x)^{3}}$	8
risch	$-\frac{1}{3(-1+x)^{3}}$	8
parallelrisch	$-\frac{1}{3(-1+x)^{3}}$	8
norman	$\frac{-4x^{4}-x^{8}-x-2x^{2}-\frac{10}{3}x^{3}-4x^{5}-\frac{10}{3}x^{6}-2x^{7}-\frac{1}{3}x^{9}-\frac{1}{3}}{(x^{4}-1)^{3}}$	54
meijerg	Expression too large to display	698

input
```
int((x^3+x^2+x+1)^4/(-x^4+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/(-1+x)^3
```

3.185.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3\,(x^3-3\,x^2+3\,x-1)}$$

input
```
integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="fricas")
```

output
```
-1/3/(x^3 - 3*x^2 + 3*x - 1)
```

3.185.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3x^3-9x^2+9x-3}$$

input
```
integrate((x**3+x**2+x+1)**4/(-x**4+1)**4,x)
```

output
```
-1/(3*x**3 - 9*x**2 + 9*x - 3)
```

3.185.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3\,(x^3-3\,x^2+3\,x-1)}$$

input
```
integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="maxima")
```

output
```
-1/3/(x^3 - 3*x^2 + 3*x - 1)
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3\,(x-1)^3}$$

input
```
integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="giac")
```

output
```
-1/3/(x - 1)^3
```

3.185.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{\left(1+x+x^2+x^3\right)^4}{\left(1-x^4\right)^4}\,dx = -\frac{1}{3\,(x-1)^3}$$

input
```
int((x + x^2 + x^3 + 1)^4/(x^4 - 1)^4,x)
```

output
```
-1/(3*(x - 1)^3)
```

3.186 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx$

3.186.1 Optimal result . . . 1430
3.186.2 Mathematica [A] (verified) . . . 1431
3.186.3 Rubi [A] (verified) . . . 1431
3.186.4 Maple [C] (verified) . . . 1432
3.186.5 Fricas [F(-1)] . . . 1433
3.186.6 Sympy [F(-1)] . . . 1433
3.186.7 Maxima [A] (verification not implemented) . . . 1433
3.186.8 Giac [B] (verification not implemented) . . . 1434
3.186.9 Mupad [B] (verification not implemented) . . . 1435

3.186.1 Optimal result

Integrand size = 36, antiderivative size = 165

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx = -\frac{gx}{b}-\frac{hx^2}{2b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(bc+\sqrt{a}\sqrt{b}e+ag\right)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{(bd+ah)\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

output
```
-g*x/b-1/2*h*x^2/b-1/4*f*ln(-b*x^4+a)/b+1/2*(a*h+b*d)*arctanh(x^2*b^(1/2)/
a^(1/2))/b^(3/2)/a^(1/2)+1/2*arctan(b^(1/4)*x/a^(1/4))*(b*c+a*g-e*a^(1/2)*
b^(1/2))/a^(3/4)/b^(5/4)+1/2*arctanh(b^(1/4)*x/a^(1/4))*(b*c+a*g+e*a^(1/2)
*b^(1/2))/a^(3/4)/b^(5/4)
```

3.186.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.55

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx$$
$$= \frac{-4a^{3/4}\sqrt{b}gx-2a^{3/4}\sqrt{b}hx^2+2\sqrt[4]{b}\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(b^{5/4}c+\sqrt[4]{a}bd+\sqrt{a}b^{3/4}e+a\sqrt[4]{}\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```

output
```
(-4*a^(3/4)*Sqrt[b]*g*x - 2*a^(3/4)*Sqrt[b]*h*x^2 + 2*b^(1/4)*(b*c - Sqrt[
a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(5/4)*c + a^(1/4)*b*d
 + Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g + a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] +
 (b^(5/4)*c - a^(1/4)*b*d + Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g - a^(5/4)*h)*L
og[a^(1/4) + b^(1/4)*x] + a^(1/4)*(b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2] -
 a^(3/4)*Sqrt[b]*f*Log[a - b*x^4])/(4*a^(3/4)*b^(3/2))
```

3.186.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx$$
$$\Big\downarrow \text{2424}$$
$$\int \left(\frac{c+ex^2+gx^4}{a-bx^4}+\frac{x\left(d+fx^2+hx^4\right)}{a-bx^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+bd)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}-\frac{hx^2}{2b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x]
```

output
```
-((g*x)/b) - (h*x^2)/(2*b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1
/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*Ar
cTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + ((b*d + a*h)*ArcTanh[(Sq
rt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - (f*Log[a - b*x^4])/(4*b)
```

3.186.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.186.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.58 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.45

method	result
risch	$-\frac{h\,x^{2}}{2b}-\frac{gx}{b}-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(bc+ag+\left(ah+bd\right)\_R+\_R^{2}be+\_R^{3}bf\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$-\frac{\frac{1}{2}h\,x^{2}+gx}{b}+\frac{\frac{\left(ag+bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{\left(ah+bd\right)\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*h*x^2/b-g*x/b-1/4/b^2*sum((b*c+a*g+(a*h+b*d)*_R+_R^2*b*e+_R^3*b*f)/_R
^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.186.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx=\text{Timed out}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas"
)
```

output
```
Timed out
```

3.186.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx=\text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

output
```
Timed out
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.35

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx=-\frac{hx^2+2\,gx}{2\,b} + \frac{\frac{2\left(b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b^{\frac{3}{2}}d-\sqrt{a}bf+a\sqrt{b}h\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b}-\frac{\left(b^{\frac{3}{2}}d+\sqrt{a}bf+a\sqrt{b}h\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b}-\frac{\left(b^{\frac{3}{2}}c+\sqrt{}\right.}{}}{4\,b}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima"
)
```

output
```
-1/2*(h*x^2 + 2*g*x)/b + 1/4*(2*(b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*ar
ctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(
b)) + (b^(3/2)*d - sqrt(a)*b*f + a*sqrt(b)*h)*log(sqrt(b)*x^2 + sqrt(a))/(
sqrt(a)*b) - (b^(3/2)*d + sqrt(a)*b*f + a*sqrt(b)*h)*log(sqrt(b)*x^2 - sqr
t(a))/(sqrt(a)*b) - (b^(3/2)*c + sqrt(a)*b*e + a*sqrt(b)*g)*log((sqrt(b)*x
 - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sq
rt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

3.186.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(123) = 246$.

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a-bx^4}\,dx = -\frac{\sqrt{2}\left(b^2c+abg-\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c+abg+\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(b^2c+abg-\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}} + \frac{\sqrt{2}\left(b^2c+abg-\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}} - \frac{f\log\left(|bx^4-a|\right)}{4\,b} - \frac{bhx^2+2\,bgx}{2\,b^2}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(b^2*c + a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(2)*(-a*b^3
)^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/
4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-
a*b^3)^(1/4)*b*d + sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*b)*b*e)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*s
qrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) +
 sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)
*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*f*log
(abs(b*x^4 - a))/b - 1/2*(b*h*x^2 + 2*b*g*x)/b^2
```

3.186.9 Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 2478, normalized size of antiderivative = 15.02

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{a - bx^4} \, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4),x)
```

output
```
symsum(log(- root(256*a^3*b^6*z^4 + 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*g*z^2
 - 64*a^3*b^4*d*h*z^2 - 64*a^2*b^5*c*e*z^2 - 32*a^4*b^3*h^2*z^2 + 96*a^3*b
^4*f^2*z^2 - 32*a^2*b^5*d^2*z^2 - 32*a^3*b^3*e*f*g*z - 32*a^3*b^3*d*f*h*z
+ 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*b^
2*g^2*h*z - 16*a^4*b^2*f*h^2*z + 16*a^3*b^3*e^2*h*z + 16*a^3*b^3*d*g^2*z +
 16*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z + 16*a*b^5*c
^2*d*z + 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h + 8*a^2*
b^3*c*d*f*g - 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g -
4*a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 + 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*
h^2 - 4*a^2*b^3*d^2*e*g + 4*a^2*b^3*d*e^2*f + 4*a^2*b^3*c*e^2*g - 4*a^2*b^
3*c*e*f^2 + 4*a^4*b*f*g^2*h - 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*
c*d^2*e + 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^
2*g^2 - 6*a^2*b^3*c^2*g^2 - 2*a^2*b^3*d^2*f^2 - 2*a^4*b*f^2*h^2 + 4*a^2*b^
3*d^3*h - 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a*b^4*d^4 + a^
5*h^4 - a^2*b^3*e^4 - a^4*b*g^4 - b^5*c^4, z, k)*((8*a*b^3*c*f - 8*a*b^3*d
*e - 8*a^2*b^2*e*h + 8*a^2*b^2*f*g)/b + root(256*a^3*b^6*z^4 + 256*a^3*b^5
*f*z^3 - 64*a^3*b^4*e*g*z^2 - 64*a^3*b^4*d*h*z^2 - 64*a^2*b^5*c*e*z^2 - 32
*a^4*b^3*h^2*z^2 + 96*a^3*b^4*f^2*z^2 - 32*a^2*b^5*d^2*z^2 - 32*a^3*b^3*e*
f*g*z - 32*a^3*b^3*d*f*h*z + 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*
a^2*b^4*c*d*g*z + 16*a^4*b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z + 16*a^3*b^3*...
```

3.187 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4} \, dx$

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3.187.1 Optimal result

Integrand size = 41, antiderivative size = 188

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4} \, dx = -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b} - \frac{\left(be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}+ai\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{\left(be+\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}+ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} + \frac{(bd+ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} - \frac{f\log\left(a-bx^4\right)}{4b}$$

output
```
-g*x/b-1/2*h*x^2/b-1/3*i*x^3/b-1/4*f*ln(-b*x^4+a)/b+1/2*(a*h+b*d)*arctanh(
x^2*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-1/2*arctan(b^(1/4)*x/a^(1/4))*(b*e+a*
i-(a*g+b*c)*b^(1/2)/a^(1/2))/a^(1/4)/b^(7/4)+1/2*arctanh(b^(1/4)*x/a^(1/4)
)*(b*e+a*i+(a*g+b*c)*b^(1/2)/a^(1/2))/a^(1/4)/b^(7/4)
```

3.187.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.60

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx$$
$$= \frac{-12b^{3/4}gx-6b^{3/4}hx^2-4b^{3/4}ix^3+\frac{6\left(b^{3/2}c-\sqrt{a}be+a\sqrt{b}g-a^{3/2}i\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}}-\frac{3\left(b^{3/2}c+\sqrt[4]{a}b^{5/4}d+\sqrt{a}be+a\sqrt{b}g+a^{5/4}\sqrt[4]{\ldots}\right.}{a^{3/4}}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x]
```

output
```
(-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 + (6*(b^(3/2)*c - Sqr
t[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(3/4) -
 (3*(b^(3/2)*c + a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g + a^(5/4)*b
^(1/4)*h + a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(3/4) + (3*(b^(3/2)*c -
a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g - a^(5/4)*b^(1/4)*h + a^(3/2
)*i)*Log[a^(1/4) + b^(1/4)*x])/a^(3/4) + (3*b^(1/4)*(b*d + a*h)*Log[Sqrt[a
] + Sqrt[b]*x^2])/Sqrt[a] - 3*b^(3/4)*f*Log[a - b*x^4])/(12*b^(7/4))
```

3.187.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx$$
$$\Big\downarrow \; 2424$$
$$\int \left(\frac{c+ex^2+gx^4+ix^6}{a-bx^4}+\frac{x\left(d+fx^2+hx^4\right)}{a-bx^4}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+bd)}{2\sqrt{a}b^{3/2}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x]
```

output
```
-((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - ((b*e - (Sqrt[b]*(b*c + a*g))
/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*e +
 (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(
1/4)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^
(3/2)) - (f*Log[a - b*x^4])/(4*b)
```

3.187.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.187.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51

method	result
risch	$-\frac{i x^3}{3b}-\frac{h x^2}{2b}-\frac{gx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b-a)}\frac{\left(-\_R^3 bf+(-ai-be)\_R^2+(-ah-bd)\_R-ag-bc\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$-\frac{\frac{1}{3}ix^3+\frac{1}{2}hx^2+gx}{b}+\frac{\frac{(ag+bc)\left(\frac{a}{b}\right)^{\frac14}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac14}}{x-\left(\frac{a}{b}\right)^{\frac14}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac14}}\right)\right)}{4a}+\frac{(ah+bd)\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{(ai+be)\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac14}}\right)-\ln\right)}{4b\left(\frac{a}{b}\right)^{\frac14}}}{b}$

input
```
int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBO
SE)
```

output
```
-1/3*i*x^3/b-1/2*h*x^2/b-g*x/b+1/4/b^2*sum((-_R^3*b*f+(-a*i-b*e)*_R^2+(-a*
h-b*d)*_R-a*g-b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.187.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx = \text{Timed out}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="f
ricas")
```

output
```
Timed out
```

3.187.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx = \text{Timed out}$$

input
```
integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

output
```
Timed out
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx = -\frac{2\,ix^3+3\,hx^2+6\,gx}{6\,b} + \frac{\frac{2\left(b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g-a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\left(b^{\frac{3}{2}}d-\sqrt{a}bf+a\sqrt{b}h\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(b^{\frac{3}{2}}d+\sqrt{a}bf+a\sqrt{b}h\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(b^{\frac{3}{2}}\right.}{}}{4\,b}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="m
axima")
```

output
```
-1/6*(2*i*x^3 + 3*h*x^2 + 6*g*x)/b + 1/4*(2*(b^(3/2)*c - sqrt(a)*b*e + a*s
qrt(b)*g - a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d - sqrt(a)*b*f + a*sqrt(b)*h)*log(
sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d + sqrt(a)*b*f + a*sqrt(b)*
h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (b^(3/2)*c + sqrt(a)*b*e + a*s
qrt(b)*g + a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x +
 sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

3.187.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(144) = 288$.

Time = 0.29 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.29

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{a - bx^4}\,dx = -\frac{f\log\left(\left|bx^4 - a\right|\right)}{4\,b}$$
$$-\frac{\sqrt{2}\left(b^3c + ab^2g - \sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d - \sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh - \sqrt{-ab}b^2e - \sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{4\left(-ab^3\right)^{\frac{3}{4}}b}$$
$$-\frac{\sqrt{2}\left(b^3c + ab^2g + \sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d + \sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh - \sqrt{-ab}b^2e + \sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{4\left(-ab^3\right)^{\frac{3}{4}}b}$$
$$-\frac{\sqrt{2}\left(b^3c + ab^2g - \sqrt{-ab}b^2e - \sqrt{-ab}abi\right)\log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}b}$$
$$+\frac{\sqrt{2}\left(b^3c + ab^2g - \sqrt{-ab}b^2e - \sqrt{-ab}abi\right)\log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}b}$$
$$-\frac{2\,b^2ix^3 + 3\,b^2hx^2 + 6\,b^2gx}{6\,b^3}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="g
iac")
```

output
```
-1/4*f*log(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(b^3*c + a*b^2*g - sqrt(2)*(-a*
b^3)^(1/4)*b^2*d - sqrt(2)*(-a*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e - sqrt(
-a*b)*a*b*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))
/((-a*b^3)^(3/4)*b) - 1/4*sqrt(2)*(b^3*c + a*b^2*g + sqrt(2)*(-a*b^3)^(1/4
)*b^2*d + sqrt(2)*(-a*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e + sqrt(-a*b)*a*b
*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*b) - 1/8*sqrt(2)*(b^3*c + a*b^2*g - sqrt(-a*b)*b^2*e - sqrt(-a*b)*
a*b*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*b) +
 1/8*sqrt(2)*(b^3*c + a*b^2*g - sqrt(-a*b)*b^2*e - sqrt(-a*b)*a*b*i)*log(x
^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*b) - 1/6*(2*b^2*
i*x^3 + 3*b^2*h*x^2 + 6*b^2*g*x)/b^3
```

3.187.9 Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 3810, normalized size of antiderivative = 20.27

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{a - bx^4}\, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4),x)
```

output
```
symsum(log(- (a^4*i^3 + a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^2*b^2*c*h^2
- a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i + a*b^3*c*f^2 + a*b^3*d^
2*g - a*b^3*c^2*i + 3*a^3*b*e*i^2 + a^3*b*g*h^2 - a^3*b*g^2*i - 2*a^2*b^2*
c*g*i - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h + 2*a*b^3*c*d*
h - 2*a*b^3*c*e*g - 2*a*b^3*d*e*f - 2*a^3*b*f*h*i)/b^2 - root(256*a^3*b^7*
z^4 + 256*a^3*b^6*f*z^3 - 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3
*b^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 - 64*a^2*b^6*c*e*z^2 - 32*a^4*b^4*h^2*z^
2 + 96*a^3*b^5*f^2*z^2 - 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*
b^3*e*h*i*z - 32*a^3*b^4*e*f*g*z - 32*a^3*b^4*d*f*h*z + 32*a^3*b^4*d*e*i*z
 + 32*a^3*b^4*c*g*h*z - 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z + 32*a^2*b
^5*c*d*g*z + 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^3*f*h^2*z
+ 16*a^4*b^3*d*i^2*z + 16*a^3*b^4*e^2*h*z + 16*a^3*b^4*d*g^2*z + 16*a^2*b^
5*c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z + 16*a*b^6*c^2*d*z + 1
6*a^3*b^4*f^3*z + 8*a^4*b^2*e*f*h*i - 8*a^4*b^2*d*g*h*i - 8*a^3*b^3*d*e*g*
h + 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^3*b^3*
c*d*h*i + 8*a^2*b^4*c*d*f*g - 8*a^2*b^4*c*d*e*h - 4*a^4*b^2*f^2*g*i + 4*a^
4*b^2*f*g^2*h + 4*a^4*b^2*e*g^2*i - 4*a^4*b^2*e*g*h^2 - 4*a^4*b^2*c*h^2*i
- 4*a^3*b^3*d^2*g*i + 4*a^4*b^2*d*f*i^2 + 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3*e^
2*f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^3*
b^3*d*f*g^2 + 4*a^2*b^4*c^2*f*h + 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2...
```

3.187. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a-bx^4}\,dx$

3.188 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx$

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3.188.1 Optimal result

Integrand size = 46, antiderivative size = 205

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx \\
&= -\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}-\frac{jx^4}{4b}-\frac{\left(be-\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}+ai\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} \\
&+\frac{\left(be+\frac{\sqrt{b}(bc+ag)}{\sqrt{a}}+ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{7/4}} \\
&+\frac{(bd+ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{(bf+aj)\log\left(a-bx^4\right)}{4b^2}
\end{aligned}$$

output
```
-g*x/b-1/2*h*x^2/b-1/3*i*x^3/b-1/4*j*x^4/b-1/4*(a*j+b*f)*ln(-b*x^4+a)/b^2+
1/2*(a*h+b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-1/2*arctan(b^(1
/4)*x/a^(1/4))*(b*e+a*i-(a*g+b*c)*b^(1/2)/a^(1/2))/a^(1/4)/b^(7/4)+1/2*arc
tanh(b^(1/4)*x/a^(1/4))*(b*e+a*i+(a*g+b*c)*b^(1/2)/a^(1/2))/a^(1/4)/b^(7/4
)
```

3.188.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.55

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx$$

$$= \frac{-12b^{3/4}gx-6b^{3/4}hx^2-4b^{3/4}ix^3-3b^{3/4}jx^4+\frac{6\left(b^{3/2}c-\sqrt{a}be+a\sqrt{b}g-a^{3/2}i\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}}-\frac{3\left(b^{3/2}c+\sqrt[4]{a}b^{5/4}d+\sqrt{a}be\right.}{}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b
*x^4),x]
```

output
```
(-12*b^(3/4)*g*x - 6*b^(3/4)*h*x^2 - 4*b^(3/4)*i*x^3 - 3*b^(3/4)*j*x^4 + (
6*(b^(3/2)*c - Sqrt[a]*b*e + a*Sqrt[b]*g - a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a
^(1/4)])/a^(3/4) - (3*(b^(3/2)*c + a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqr
t[b]*g + a^(5/4)*b^(1/4)*h + a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(3/4)
+ (3*(b^(3/2)*c - a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e + a*Sqrt[b]*g - a^(5/4)*
b^(1/4)*h + a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x])/a^(3/4) + (3*b^(1/4)*(b*d
 + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[a] - (3*(b*f + a*j)*Log[a - b*x^4
])/b^(1/4))/(12*b^(7/4))
```

3.188.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx$$

$$\Big\downarrow \; 2424$$

$$\int \left(\frac{c+ex^2+gx^4+ix^6}{a-bx^4}+\frac{x\left(d+fx^2+hx^4+jx^6\right)}{a-bx^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(ag+bc)}{\sqrt{a}}+ai+be\right)}{2\sqrt[4]{a}b^{7/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(ah+bd)}{2\sqrt{a}b^{3/2}}-\frac{(aj+bf)\log\left(a-bx^4\right)}{4b^2}-\frac{gx}{b}-\frac{hx^2}{2b}-\frac{ix^3}{3b}-\frac{jx^4}{4b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4),
x]
```

output
```
-((g*x)/b) - (h*x^2)/(2*b) - (i*x^3)/(3*b) - (j*x^4)/(4*b) - ((b*e - (Sqrt
[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^
(7/4)) + ((b*e + (Sqrt[b]*(b*c + a*g))/Sqrt[a] + a*i)*ArcTanh[(b^(1/4)*x)/
a^(1/4)])/(2*a^(1/4)*b^(7/4)) + ((b*d + a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]
])/(2*Sqrt[a]*b^(3/2)) - ((b*f + a*j)*Log[a - b*x^4])/(4*b^2)
```

3.188.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.188.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.50

method	result
risch	$-\frac{j x^4}{4b}-\frac{i x^3}{3b}-\frac{h x^2}{2b}-\frac{gx}{b}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(bc+ag+(ah+bd)\_R+(ai+be)\_R^2+(aj+bf)\_R^3\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$-\frac{\frac{1}{4}j x^4+\frac{1}{3}i x^3+\frac{1}{2}h x^2+gx}{b}+\frac{\frac{(ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(ah+bd)\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{(ai+be)\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right.\right.}{4b\left(\frac{a}{b}\right)\cdots}}{b}$

input
```
int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETUR
NVERBOSE)
```

output
```
-1/4*j*x^4/b-1/3*i*x^3/b-1/2*h*x^2/b-g*x/b-1/4/b^2*sum((b*c+a*g+(a*h+b*d)*
_R+(a*i+b*e)*_R^2+(a*j+b*f)*_R^3)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.188.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{a - bx^4} \, dx = \text{Timed out}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algori
thm="fricas")
```

output
```
Timed out
```

3.188.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{a - bx^4} \, dx = \text{Timed out}$$

input
```
integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

output
```
Timed out
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.25

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx = -\frac{3\,jx^4+4\,ix^3+6\,hx^2+12\,gx}{12\,b} + \frac{\frac{2\left(b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g-a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b^{\frac{3}{2}}d-\sqrt{a}bf+a\sqrt{b}h-a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b}-\frac{\left(b^{\frac{3}{2}}d+\sqrt{a}bf+a\sqrt{b}h+a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b}}{4\,b}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algori
thm="maxima")
```

output
```
-1/12*(3*j*x^4 + 4*i*x^3 + 6*h*x^2 + 12*g*x)/b + 1/4*(2*(b^(3/2)*c - sqrt(
a)*b*e + a*sqrt(b)*g - a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d - sqrt(a)*b*f + a*sqr
t(b)*h - a^(3/2)*j)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d +
sqrt(a)*b*f + a*sqrt(b)*h + a^(3/2)*j)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)
*b) - (b^(3/2)*c + sqrt(a)*b*e + a*sqrt(b)*g + a^(3/2)*i)*log((sqrt(b)*x -
 sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt
(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

3.188.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(159) = 318$.

Time = 0.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.17

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx =$$

$$-\frac{\sqrt{2}\left(b^3c+ab^2g-\sqrt{2}(-ab^3)^{\frac{1}{4}}b^2d-\sqrt{2}(-ab^3)^{\frac{1}{4}}abh-\sqrt{-ab}b^2e-\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2(-\frac{a}{b})^{\frac{1}{4}}}\right.}{4(-ab^3)^{\frac{3}{4}}b}$$
$$-\frac{\sqrt{2}\left(b^3c+ab^2g+\sqrt{2}(-ab^3)^{\frac{1}{4}}b^2d+\sqrt{2}(-ab^3)^{\frac{1}{4}}abh-\sqrt{-ab}b^2e+\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2(-\frac{a}{b})^{\frac{1}{4}}}\right.}{4(-ab^3)^{\frac{3}{4}}b}$$
$$-\frac{\sqrt{2}\left(b^3c+ab^2g-\sqrt{-ab}b^2e-\sqrt{-ab}abi\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8(-ab^3)^{\frac{3}{4}}b}$$
$$+\frac{\sqrt{2}\left(b^3c+ab^2g-\sqrt{-ab}b^2e-\sqrt{-ab}abi\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8(-ab^3)^{\frac{3}{4}}b}$$
$$-\frac{(bf+aj)\log\left(|bx^4-a|\right)}{4b^2}-\frac{3b^3jx^4+4b^3ix^3+6b^3hx^2+12b^3gx}{12b^4}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algori
thm="giac")
```

output
```
-1/4*sqrt(2)*(b^3*c + a*b^2*g - sqrt(2)*(-a*b^3)^(1/4)*b^2*d - sqrt(2)*(-a
*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e - sqrt(-a*b)*a*b*i)*arctan(1/2*sqrt(2
)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*b) - 1/4*sqrt
(2)*(b^3*c + a*b^2*g + sqrt(2)*(-a*b^3)^(1/4)*b^2*d + sqrt(2)*(-a*b^3)^(1/
4)*a*b*h - sqrt(-a*b)*b^2*e + sqrt(-a*b)*a*b*i)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*b) - 1/8*sqrt(2)*(b^3*
c + a*b^2*g - sqrt(-a*b)*b^2*e - sqrt(-a*b)*a*b*i)*log(x^2 + sqrt(2)*x*(-a
/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*b) + 1/8*sqrt(2)*(b^3*c + a*b^2*g
- sqrt(-a*b)*b^2*e - sqrt(-a*b)*a*b*i)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) +
sqrt(-a/b))/((-a*b^3)^(3/4)*b) - 1/4*(b*f + a*j)*log(abs(b*x^4 - a))/b^2 -
 1/12*(3*b^3*j*x^4 + 4*b^3*i*x^3 + 6*b^3*h*x^2 + 12*b^3*g*x)/b^4
```

3.188. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx$

3.188.9 Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 5673, normalized size of antiderivative = 27.67

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a-bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4),
x)
```

output
```
symsum(log(- (a^4*i^3 + a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^4*g*j^2 + a^
2*b^2*c*h^2 - a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i - 2*a^4*h*i*
j + a*b^3*c*f^2 + a*b^3*d^2*g - a*b^3*c^2*i + a^3*b*c*j^2 + 3*a^3*b*e*i^2
+ a^3*b*g*h^2 - a^3*b*g^2*i + 2*a^2*b^2*c*f*j - 2*a^2*b^2*c*g*i - 2*a^2*b^
2*d*e*j - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h + 2*a*b^3*c*
d*h - 2*a*b^3*c*e*g - 2*a*b^3*d*e*f - 2*a^3*b*d*i*j - 2*a^3*b*e*h*j + 2*a^
3*b*f*g*j - 2*a^3*b*f*h*i)/b^2 - root(256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3
+ 256*a^3*b^7*f*z^3 + 192*a^4*b^5*f*j*z^2 - 64*a^4*b^5*g*i*z^2 - 64*a^3*b^
6*e*g*z^2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*i*z^2 - 64*a^2*b^7*c*e*z^2 +
 96*a^5*b^4*j^2*z^2 - 32*a^4*b^5*h^2*z^2 + 96*a^3*b^6*f^2*z^2 - 32*a^2*b^7
*d^2*z^2 - 32*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i*z + 32*a^4*b^4*e*h*i*z -
32*a^4*b^4*e*g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*b^4*c*i*j*z - 32*a^3*b^5*
e*f*g*z - 32*a^3*b^5*d*f*h*z + 32*a^3*b^5*d*e*i*z + 32*a^3*b^5*c*g*h*z - 3
2*a^3*b^5*c*f*i*z - 32*a^3*b^5*c*e*j*z - 32*a^2*b^6*c*e*f*z + 32*a^2*b^6*c
*d*g*z - 16*a^5*b^3*h^2*j*z + 16*a^5*b^3*h*i^2*z + 48*a^5*b^3*f*j^2*z + 48
*a^4*b^4*f^2*j*z + 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*h^2*z - 16*a^3*b^5*d^
2*j*z + 16*a^4*b^4*d*i^2*z + 16*a^3*b^5*e^2*h*z + 16*a^3*b^5*d*g^2*z + 16*
a^2*b^6*c^2*h*z - 16*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2*z + 16*a*b^7*c^2*d
*z + 16*a^6*b^2*j^3*z + 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g*i*j + 8*a^5*b^2*e
*h*i*j + 8*a^4*b^3*e*f*h*i - 8*a^4*b^3*e*f*g*j - 8*a^4*b^3*d*g*h*i - 8*...
```

3.189 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx$

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3.189.8 Giac [A] (verification not implemented) . . . 1455
3.189.9 Mupad [B] (verification not implemented) . . . 1455

3.189.1 Optimal result

Integrand size = 35, antiderivative size = 337

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx &= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{(bd-ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}\\
&+\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}\\
&-\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}\\
&+\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{f\log\left(a+bx^4\right)}{4b}\end{aligned}$$

output
```
g*x/b+1/2*h*x^2/b+1/4*f*ln(b*x^4+a)/b+1/2*(-a*h+b*d)*arctan(x^2*b^(1/2)/a^
(1/2))/b^(3/2)/a^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))*(b*c-a*g-e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(
1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(b*c-a*g-e*a^(1/2)*b^(1/2))/a^(3/4)/b^
(5/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(b*c-a*g+e*a^(1/2)*
b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(
b*c-a*g+e*a^(1/2)*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)
```

3.189.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx$$
$$= \frac{-2\left(\sqrt{2}b^{5/4}c+2\sqrt[4]{a}bd+\sqrt{2}\sqrt{a}b^{3/4}e-\sqrt{2}a\sqrt[4]{b}g-2a^{5/4}h\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}b^{5/4}c-2\sqrt[4]{a}bd\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```

output
```
(-2*(Sqrt[2]*b^(5/4)*c + 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e - Sqrt[
2]*a*b^(1/4)*g - 2*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*
(Sqrt[2]*b^(5/4)*c - 2*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*e - Sqrt[2]*a
*b^(1/4)*g + 2*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + b^(1/4
)*(Sqrt[2]*(-(b*c) + Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*(2*x
*(2*g + h*x) + f*Log[a + b*x^4])))/(8*a^(3/4)*b^(3/2))
```

3.189.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx$$
$$\Big\downarrow \text{2424}$$
$$\int \left(\frac{c+ex^2+gx^4}{a+bx^4}+\frac{x\left(d+fx^2+hx^4\right)}{a+bx^4}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{2\sqrt{a}b^{3/2}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}+\frac{hx^2}{2b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x]
```

output
```
(g*x)/b + (h*x^2)/(2*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*S
qrt[a]*b^(3/2)) - ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e
- a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)
) - ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e
 - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(5/4)) + (f*Log[a + b*x^4])/(4*b)
```

3.189.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.189.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.65 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.22

method	result
risch	$\frac{h\,x^{2}}{2b}+\frac{gx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}b+a\right)}\frac{\left(bc-ag+\left(-ah+bd\right)\_R+\_R^{2}be+\_R^{3}bf\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$\frac{\frac{1}{2}h\,x^{2}+gx}{b}+\frac{\frac{\left(-ag+bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\left(-ah+bd\right)\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+}{b}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*h*x^2/b+g*x/b+1/4/b^2*sum((b*c-a*g+(-a*h+b*d)*_R+_R^2*b*e+_R^3*b*f)/_R
^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.189.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx=\text{Timed out}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.189.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx=\text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.04

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{a + bx^4} \, dx = \frac{hx^2 + 2\,gx}{2\,b}$$

$$+ \frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f + b^2c - \sqrt{a}b^{\frac{3}{2}}e - abg\right)\log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f - b^2c + \sqrt{a}b^{\frac{3}{2}}e + abg\right)\log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{2(\cdots}{\cdots}}{\cdots}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*(h*x^2 + 2*g*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f + b^2*c -
sqrt(a)*b^(3/2)*e - a*b*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + s
qrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - b^2*c + s
qrt(a)*b^(3/2)*e + a*b*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sq
rt(a))/(a^(3/4)*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*
b^(7/4)*e - sqrt(2)*a^(5/4)*b^(5/4)*g - 2*sqrt(a)*b^2*d + 2*a^(3/2)*b*h)*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqr
t(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4
)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*b^(5/4)*g + 2*sqrt(a)*b^
2*d - 2*a^(3/2)*b*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(
1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/b
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.11

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx = \frac{f\log\left(|bx^4+a|\right)}{4\,b} + \frac{bhx^2+2\,bgx}{2\,b^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d+\sqrt{2}\sqrt{ab}abh-(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d+\sqrt{2}\sqrt{ab}abh-(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*f*log(abs(b*x^4 + a))/b + 1/2*(b*h*x^2 + 2*b*g*x)/b^2 - 1/4*sqrt(2)*(s
qrt(2)*sqrt(a*b)*b^2*d + sqrt(2)*sqrt(a*b)*a*b*h - (a*b^3)^(1/4)*b^2*c + (
a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d + s
qrt(2)*sqrt(a*b)*a*b*h - (a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^
3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)
*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

3.189.9 Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 2469, normalized size of antiderivative = 7.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4),x)
```

output
```
symsum(log(root(256*a^3*b^6*z^4 - 256*a^3*b^5*f*z^3 - 64*a^3*b^4*e*g*z^2 -
 64*a^3*b^4*d*h*z^2 + 64*a^2*b^5*c*e*z^2 + 32*a^4*b^3*h^2*z^2 + 96*a^3*b^4
*f^2*z^2 + 32*a^2*b^5*d^2*z^2 + 32*a^3*b^3*e*f*g*z + 32*a^3*b^3*d*f*h*z -
32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^2*b^4*c*d*g*z + 16*a^4*b^2*
g^2*h*z - 16*a^4*b^2*f*h^2*z - 16*a^3*b^3*e^2*h*z - 16*a^3*b^3*d*g^2*z + 1
6*a^2*b^4*c^2*h*z - 16*a^2*b^4*d^2*f*z + 16*a^2*b^4*d*e^2*z - 16*a*b^5*c^2
*d*z - 16*a^3*b^3*f^3*z - 8*a^3*b^2*d*e*g*h + 8*a^3*b^2*c*f*g*h - 8*a^2*b^
3*c*d*f*g + 8*a^2*b^3*c*d*e*h + 4*a^3*b^2*e^2*f*h - 4*a^3*b^2*e*f^2*g - 4*
a^3*b^2*d*f^2*h + 4*a^3*b^2*d*f*g^2 - 4*a^2*b^3*c^2*f*h - 4*a^3*b^2*c*e*h^
2 + 4*a^2*b^3*d^2*e*g - 4*a^2*b^3*d*e^2*f - 4*a^2*b^3*c*e^2*g + 4*a^2*b^3*
c*e*f^2 - 4*a^4*b*f*g^2*h + 4*a^4*b*e*g*h^2 + 4*a*b^4*c^2*d*f - 4*a*b^4*c*
d^2*e - 4*a^4*b*d*h^3 - 4*a*b^4*c^3*g + 6*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*
g^2 + 6*a^2*b^3*c^2*g^2 + 2*a^2*b^3*d^2*f^2 + 2*a^4*b*f^2*h^2 - 4*a^2*b^3*
d^3*h - 4*a^3*b^2*c*g^3 + 2*a*b^4*c^2*e^2 + a^3*b^2*f^4 + a^2*b^3*e^4 + a^
4*b*g^4 + a*b^4*d^4 + a^5*h^4 + b^5*c^4, z, k)*((8*a*b^3*c*f - 8*a*b^3*d*e
 + 8*a^2*b^2*e*h - 8*a^2*b^2*f*g)/b + root(256*a^3*b^6*z^4 - 256*a^3*b^5*f
*z^3 - 64*a^3*b^4*e*g*z^2 - 64*a^3*b^4*d*h*z^2 + 64*a^2*b^5*c*e*z^2 + 32*a
^4*b^3*h^2*z^2 + 96*a^3*b^4*f^2*z^2 + 32*a^2*b^5*d^2*z^2 + 32*a^3*b^3*e*f*
g*z + 32*a^3*b^3*d*f*h*z - 32*a^3*b^3*c*g*h*z - 32*a^2*b^4*c*e*f*z + 32*a^
2*b^4*c*d*g*z + 16*a^4*b^2*g^2*h*z - 16*a^4*b^2*f*h^2*z - 16*a^3*b^3*e^...
```

3.189. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^4}\,dx$

3.190 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx$

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3.190.1 Optimal result

Integrand size = 40, antiderivative size = 384

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx &= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{(bd-ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} \\ &\quad -\frac{\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{7/4}} \\ &\quad +\frac{\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{7/4}} \\ &\quad -\frac{\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{7/4}} \\ &\quad +\frac{\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{f\log\left(a+bx^4\right)}{4b}\end{aligned}$$

output
```
g*x/b+1/2*h*x^2/b+1/3*i*x^3/b+1/4*f*ln(b*x^4+a)/b+1/2*(-a*h+b*d)*arctan(x^
2*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/
2)+x^2*b^(1/2))*(-(-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2
^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(-a*i+b*e)*
a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)*
x*2^(1/2)/a^(1/4))*((-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)
*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*((-a*i+b*e)*a^(1/2)+(-a*g
+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)
```

3.190.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.11

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{a + bx^4} \, dx$$

$$= \frac{24b^{3/4}gx + 12b^{3/4}hx^2 + 8b^{3/4}ix^3 + \frac{6\left(-\sqrt{2}b^{3/2}c - 2\sqrt[4]{a}b^{5/4}d - \sqrt{2}\sqrt{a}be + \sqrt{2}a\sqrt{b}g + 2a^{5/4}\sqrt[4]{b}h + \sqrt{2}a^{3/2}i\right)\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x]
```

output
```
(24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + (6*(-(Sqrt[2]*b^(3/
2)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g +
2*a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sq
rt[a]*b*e - Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - Sqrt[2]*a^(3/2)*i)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*Sqrt[2]*(b^(3/2)*c
- Sqrt[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (3*Sqrt[2]*(b^(3/2)*c - Sqrt[a]*b*e - a*S
qrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/a^(3/4) + 6*b^(3/4)*f*Log[a + b*x^4])/(24*b^(7/4))
```

3.190.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4} \, dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{c+ex^2+gx^4+ix^6}{a+bx^4}+\frac{x(d+fx^2+hx^4)}{a+bx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{2\sqrt{a}b^{3/2}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x]
```

output
```
(g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + ((b*d - a*h)*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a
*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4))
 + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c - a*g) - Sqrt[a
]*(b*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*
Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]*(b*e - a*i))*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b
^(7/4)) + (f*Log[a + b*x^4])/(4*b)
```

3.190.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.190.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.23

method	result
risch	$\frac{i\,x^3}{3b}+\frac{h\,x^2}{2b}+\frac{gx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(bc-ag+(-ah+bd)\_R+(-ai+be)\_R^2+\_R^3 bf\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$\frac{\frac{1}{3}i\,x^3+\frac{1}{2}h\,x^2+gx}{b}+\frac{\frac{(-ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{(-ah+bd)\arctan\left(x^2\right.}{2\sqrt{ab}}}{b}$

input
```
int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOS
E)
```

output
```
1/3*i*x^3/b+1/2*h*x^2/b+g*x/b+1/4/b^2*sum((b*c-a*g+(-a*h+b*d)*_R+(-a*i+b*e
)*_R^2+_R^3*b*f)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.190.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fr
icas")
```

output
```
Timed out
```

3.190.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx = \frac{2\,ix^3+3\,hx^2+6\,gx}{6\,b}$$

$$+ \frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f+b^2c-\sqrt{a}b^{\frac{3}{2}}e-abg+a^{\frac{3}{2}}\sqrt{b}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-b^2c+\sqrt{a}b^{\frac{3}{2}}e+abg-a^{\frac{3}{2}}\sqrt{b}i\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b\right.}{a^{\frac{3}{4}}b^{\frac{5}{4}}}}{}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="ma
xima")
```

output
```
1/6*(2*i*x^3 + 3*h*x^2 + 6*g*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*
f + b^2*c - sqrt(a)*b^(3/2)*e - a*b*g + a^(3/2)*sqrt(b)*i)*log(sqrt(b)*x^2
 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(
2)*a^(3/4)*b^(5/4)*f - b^2*c + sqrt(a)*b^(3/2)*e + a*b*g - a^(3/2)*sqrt(b)
*i)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4
)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^
(5/4)*b^(5/4)*g - sqrt(2)*a^(7/4)*b^(3/4)*i - 2*sqrt(a)*b^2*d + 2*a^(3/2)*
b*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(
a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*
b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*b^(5/4)*g - sqrt(2
)*a^(7/4)*b^(3/4)*i + 2*sqrt(a)*b^2*d - 2*a^(3/2)*b*h)*arctan(1/2*sqrt(2)*
(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sq
rt(sqrt(a)*sqrt(b))*b^(5/4)))/b
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.15

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx = \frac{f\log\left(|bx^4+a|\right)}{4\,b}+\frac{2\,b^2ix^3+3\,b^2hx^2+6\,b^2gx}{6\,b^3}$$
$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^3d+\sqrt{2}\sqrt{ab}ab^2h-(ab^3)^{\frac14}\,b^3c+(ab^3)^{\frac14}\,ab^2g-(ab^3)^{\frac34}\,be+(ab^3)^{\frac34}\,ai\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right.\right.}{2\left(\frac{a}{b}\right)^{\frac14}}\right.}{4\,ab^4}$$
$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^3d+\sqrt{2}\sqrt{ab}ab^2h-(ab^3)^{\frac14}\,b^3c+(ab^3)^{\frac14}\,ab^2g-(ab^3)^{\frac34}\,be+(ab^3)^{\frac34}\,ai\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right.\right.}{2\left(\frac{a}{b}\right)^{\frac14}}\right.}{4\,ab^4}$$
$$+\frac{\sqrt{2}\left((ab^3)^{\frac14}\,b^3c-(ab^3)^{\frac14}\,ab^2g-(ab^3)^{\frac34}\,be+(ab^3)^{\frac34}\,ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4}$$
$$-\frac{\sqrt{2}\left((ab^3)^{\frac14}\,b^3c-(ab^3)^{\frac14}\,ab^2g-(ab^3)^{\frac34}\,be+(ab^3)^{\frac34}\,ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="gi
ac")
```

output
```
1/4*f*log(abs(b*x^4 + a))/b + 1/6*(2*b^2*i*x^3 + 3*b^2*h*x^2 + 6*b^2*g*x)/
b^3 - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^3*d + sqrt(2)*sqrt(a*b)*a*b^2*h - (
a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e + (a*b^3)^(
3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b
^4) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^3*d + sqrt(2)*sqrt(a*b)*a*b^2*h - (
a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e + (a*b^3)^(
3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b
^4) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c - (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(
3/4)*b*e + (a*b^3)^(3/4)*a*i)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(a*b^4) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c - (a*b^3)^(1/4)*a*b^2*g - (a*b
^3)^(3/4)*b*e + (a*b^3)^(3/4)*a*i)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(
a/b))/(a*b^4)
```

3.190.9 Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 3798, normalized size of antiderivative = 9.89

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4),x)
```

output
```
symsum(log((a^4*i^3 - a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^2*b^2*c*h^2 -
a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i - a*b^3*c*f^2 - a*b^3*d^2*
g + a*b^3*c^2*i - 3*a^3*b*e*i^2 - a^3*b*g*h^2 + a^3*b*g^2*i - 2*a^2*b^2*c*
g*i - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h - 2*a*b^3*c*d*h
+ 2*a*b^3*c*e*g + 2*a*b^3*d*e*f + 2*a^3*b*f*h*i)/b^2 + root(256*a^3*b^7*z^
4 - 256*a^3*b^6*f*z^3 + 64*a^4*b^4*g*i*z^2 - 64*a^3*b^5*e*g*z^2 - 64*a^3*b
^5*d*h*z^2 - 64*a^3*b^5*c*i*z^2 + 64*a^2*b^6*c*e*z^2 + 32*a^4*b^4*h^2*z^2
+ 96*a^3*b^5*f^2*z^2 + 32*a^2*b^6*d^2*z^2 - 32*a^4*b^3*f*g*i*z + 32*a^4*b^
3*e*h*i*z + 32*a^3*b^4*e*f*g*z + 32*a^3*b^4*d*f*h*z - 32*a^3*b^4*d*e*i*z -
 32*a^3*b^4*c*g*h*z + 32*a^3*b^4*c*f*i*z - 32*a^2*b^5*c*e*f*z + 32*a^2*b^5
*c*d*g*z - 16*a^5*b^2*h*i^2*z + 16*a^4*b^3*g^2*h*z - 16*a^4*b^3*f*h^2*z +
16*a^4*b^3*d*i^2*z - 16*a^3*b^4*e^2*h*z - 16*a^3*b^4*d*g^2*z + 16*a^2*b^5*
c^2*h*z - 16*a^2*b^5*d^2*f*z + 16*a^2*b^5*d*e^2*z - 16*a*b^6*c^2*d*z - 16*
a^3*b^4*f^3*z - 8*a^4*b^2*e*f*h*i + 8*a^4*b^2*d*g*h*i - 8*a^3*b^3*d*e*g*h
+ 8*a^3*b^3*d*e*f*i + 8*a^3*b^3*c*f*g*h + 8*a^3*b^3*c*e*g*i - 8*a^3*b^3*c*
d*h*i - 8*a^2*b^4*c*d*f*g + 8*a^2*b^4*c*d*e*h + 4*a^4*b^2*f^2*g*i - 4*a^4*
b^2*f*g^2*h - 4*a^4*b^2*e*g^2*i + 4*a^4*b^2*e*g*h^2 + 4*a^4*b^2*c*h^2*i -
4*a^3*b^3*d^2*g*i - 4*a^4*b^2*d*f*i^2 - 4*a^4*b^2*c*g*i^2 + 4*a^3*b^3*e^2*
f*h - 4*a^3*b^3*e*f^2*g - 4*a^3*b^3*d*f^2*h - 4*a^3*b^3*c*f^2*i + 4*a^3*b^
3*d*f*g^2 - 4*a^2*b^4*c^2*f*h - 4*a^2*b^4*c^2*e*i - 4*a^3*b^3*c*e*h^2 +...
```

3.190. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{a+bx^4}\,dx$

3.191 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx$

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3.191.1 Optimal result

Integrand size = 45, antiderivative size = 402

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx &= \frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{jx^4}{4b}+\frac{(bd-ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}\\ &\quad-\frac{\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{7/4}}\\ &\quad+\frac{\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{7/4}}\\ &\quad-\frac{\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{7/4}}\\ &\quad+\frac{\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{7/4}}\\ &\quad+\frac{(bf-aj)\log\left(a+bx^4\right)}{4b^2}\end{aligned}$$

output
```
g*x/b+1/2*h*x^2/b+1/3*i*x^3/b+1/4*j*x^4/b+1/4*(-a*j+b*f)*ln(b*x^4+a)/b^2+1
/2*(-a*h+b*d)*arctan(x^2*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-1/8*ln(-a^(1/4)*
b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(
1/2))/a^(3/4)/b^(7/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*b^(1/2))*(-(-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)
+1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*((-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b
^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*((
-a*i+b*e)*a^(1/2)+(-a*g+b*c)*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)
```

3.191.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.11

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx$$

$$= \frac{24b^{3/4}gx+12b^{3/4}hx^2+8b^{3/4}ix^3+6b^{3/4}jx^4+\frac{6\left(-\sqrt{2}b^{3/2}c-2\sqrt[4]{a}b^{5/4}d-\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+2a^{5/4}\sqrt[4]{b}h+\sqrt{2}a^{3/2}i\right)\arctan\left(\right.}{a^{3/4}}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b
*x^4),x]
```

output
```
(24*b^(3/4)*g*x + 12*b^(3/4)*h*x^2 + 8*b^(3/4)*i*x^3 + 6*b^(3/4)*j*x^4 + (
6*(-(Sqrt[2]*b^(3/2)*c) - 2*a^(1/4)*b^(5/4)*d - Sqrt[2]*Sqrt[a]*b*e + Sqrt
[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (6*(Sqrt[2]*b^(3/2)*c - 2*a^(1/4)*b^(5
/4)*d + Sqrt[2]*Sqrt[a]*b*e - Sqrt[2]*a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h -
Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (3*S
qrt[2]*(b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (3*Sqrt[2]*(b^(3/2)*c -
 Sqrt[a]*b*e - a*Sqrt[b]*g + a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/a^(3/4) + (6*(b*f - a*j)*Log[a + b*x^4])/b^(1/4))/(2
4*b^(7/4))
```

3.191.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx$$

$$\Big\downarrow \ 2424$$

$$\int \left(\frac{c+ex^2+gx^4+ix^6}{a+bx^4}+\frac{x\left(d+fx^2+hx^4+jx^6\right)}{a+bx^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(bc-ag)+\sqrt{a}(be-ai)\right)}{2\sqrt{2}a^{3/4}b^{7/4}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(\sqrt{b}(bc-ag)-\sqrt{a}(be-ai)\right)}{4\sqrt{2}a^{3/4}b^{7/4}}+\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{2\sqrt{a}b^{3/2}}+\frac{(bf-aj)\log\left(a+bx^4\right)}{4b^2}+\frac{gx}{b}+\frac{hx^2}{2b}+\frac{ix^3}{3b}+\frac{jx^4}{4b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4),
x]
```

output
```
(g*x)/b + (h*x^2)/(2*b) + (i*x^3)/(3*b) + (j*x^4)/(4*b) + ((b*d - a*h)*Arc
Tan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)) - ((Sqrt[b]*(b*c - a*g) +
Sqrt[a]*(b*e - a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a
^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) + Sqrt[a]*(b*e - a*i))*ArcTan[1 +
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(7/4)) - ((Sqrt[b]*(b*c
 - a*g) - Sqrt[a]*(b*e - a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(7/4)) + ((Sqrt[b]*(b*c - a*g) - Sqrt[a]
*(b*e - a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*S
qrt[2]*a^(3/4)*b^(7/4)) + ((b*f - a*j)*Log[a + b*x^4])/(4*b^2)
```

3.191.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.191.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.26

method	result
risch	$\frac{j x^4}{4b}+\frac{i x^3}{3b}+\frac{h x^2}{2b}+\frac{g x}{b}+\frac{\sum_{\_R=\operatorname{RootOf}(\_Z^4 b+a)}\frac{\left(bc-ag+(-ah+bd)\_R+(-ai+be)\_R^2+(-aj+bf)\_R^3\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$\frac{\frac{1}{4}j x^4+\frac{1}{3}i x^3+\frac{1}{2}h x^2+g x}{b}+\frac{\frac{(-ag+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{(-ah+bd)\operatorname{arc}}{2\surd}}{}$

input
```
int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURN
VERBOSE)
```

output
```
1/4*j*x^4/b+1/3*i*x^3/b+1/2*h*x^2/b+g*x/b+1/4/b^2*sum((b*c-a*g+(-a*h+b*d)*
_R+(-a*i+b*e)*_R^2+(-a*j+b*f)*_R^3)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.191.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorit
hm="fricas")
```

output
```
Timed out
```

3.191.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.07

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx = \frac{3\,jx^4+4\,ix^3+6\,hx^2+12\,gx}{12\,b}$$

$$+\frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}j+b^2c-\sqrt{a}b^{\frac{3}{2}}e-abg+a^{\frac{3}{2}}\sqrt{b}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}j-b^2c+\sqrt{a}b^{\frac{3}{2}}e+abg-a^{\frac{3}{2}}\cdots}{a^{\frac{3}{4}}b^{\frac{5}{4}}}}{\cdots}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorit
hm="maxima")
```

output
```
1/12*(3*j*x^4 + 4*i*x^3 + 6*h*x^2 + 12*g*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3
/4)*b^(5/4)*f - sqrt(2)*a^(7/4)*b^(1/4)*j + b^2*c - sqrt(a)*b^(3/2)*e - a*
b*g + a^(3/2)*sqrt(b)*i)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqr
t(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - sqrt(2)*a^(
7/4)*b^(1/4)*j - b^2*c + sqrt(a)*b^(3/2)*e + a*b*g - a^(3/2)*sqrt(b)*i)*lo
g(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 2
*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*
b^(5/4)*g - sqrt(2)*a^(7/4)*b^(3/4)*i - 2*sqrt(a)*b^2*d + 2*a^(3/2)*b*h)*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqr
t(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4
)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt(2)*a^(5/4)*b^(5/4)*g - sqrt(2)*a^(7
/4)*b^(3/4)*i + 2*sqrt(a)*b^2*d - 2*a^(3/2)*b*h)*arctan(1/2*sqrt(2)*(2*sqr
t(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqr
t(a)*sqrt(b))*b^(5/4)))/b
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.14

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx = \frac{(bf-aj)\log\left(|bx^4+a|\right)}{4\,b^2}$$
$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^3d+\sqrt{2}\sqrt{ab}ab^2h-(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g-(ab^3)^{\frac{3}{4}}\,be+(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right.\right.}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{4\,ab^4}$$
$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^3d+\sqrt{2}\sqrt{ab}ab^2h-(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g-(ab^3)^{\frac{3}{4}}\,be+(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right.\right.}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{4\,ab^4}$$
$$+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^3c-(ab^3)^{\frac{1}{4}}\,ab^2g-(ab^3)^{\frac{3}{4}}\,be+(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4}$$
$$-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^3c-(ab^3)^{\frac{1}{4}}\,ab^2g-(ab^3)^{\frac{3}{4}}\,be+(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4}$$
$$+\frac{3\,b^3jx^4+4\,b^3ix^3+6\,b^3hx^2+12\,b^3gx}{12\,b^4}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorit
hm="giac")
```

output
```
1/4*(b*f - a*j)*log(abs(b*x^4 + a))/b^2 - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b
^3*d + sqrt(2)*sqrt(a*b)*a*b^2*h - (a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b
^2*g - (a*b^3)^(3/4)*b*e + (a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b
^3*d + sqrt(2)*sqrt(a*b)*a*b^2*h - (a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b
^2*g - (a*b^3)^(3/4)*b*e + (a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c
 - (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e + (a*b^3)^(3/4)*a*i)*log(x^2
+ sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) - 1/8*sqrt(2)*((a*b^3)^(1/4)*
b^3*c - (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e + (a*b^3)^(3/4)*a*i)*log
(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 1/12*(3*b^3*j*x^4 + 4*
b^3*i*x^3 + 6*b^3*h*x^2 + 12*b^3*g*x)/b^4
```

3.191.9 Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 5664, normalized size of antiderivative = 14.09

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{a + bx^4} \, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4),
x)
```

output
```
symsum(log((a^4*i^3 - a*b^3*e^3 + b^4*c*d^2 - b^4*c^2*e + a^4*g*j^2 + a^2*
b^2*c*h^2 - a^2*b^2*e*g^2 + a^2*b^2*f^2*g + 3*a^2*b^2*e^2*i - 2*a^4*h*i*j
- a*b^3*c*f^2 - a*b^3*d^2*g + a*b^3*c^2*i - a^3*b*c*j^2 - 3*a^3*b*e*i^2 -
a^3*b*g*h^2 + a^3*b*g^2*i + 2*a^2*b^2*c*f*j - 2*a^2*b^2*c*g*i - 2*a^2*b^2*
d*e*j - 2*a^2*b^2*d*f*i + 2*a^2*b^2*d*g*h - 2*a^2*b^2*e*f*h - 2*a*b^3*c*d*
h + 2*a*b^3*c*e*g + 2*a*b^3*d*e*f + 2*a^3*b*d*i*j + 2*a^3*b*e*h*j - 2*a^3*
b*f*g*j + 2*a^3*b*f*h*i)/b^2 + root(256*a^3*b^8*z^4 + 256*a^4*b^6*j*z^3 -
256*a^3*b^7*f*z^3 - 192*a^4*b^5*f*j*z^2 + 64*a^4*b^5*g*i*z^2 - 64*a^3*b^6*
e*g*z^2 - 64*a^3*b^6*d*h*z^2 - 64*a^3*b^6*c*i*z^2 + 64*a^2*b^7*c*e*z^2 + 9
6*a^5*b^4*j^2*z^2 + 32*a^4*b^5*h^2*z^2 + 96*a^3*b^6*f^2*z^2 + 32*a^2*b^7*d
^2*z^2 + 32*a^5*b^3*g*i*j*z - 32*a^4*b^4*f*g*i*z + 32*a^4*b^4*e*h*i*z - 32
*a^4*b^4*e*g*j*z - 32*a^4*b^4*d*h*j*z - 32*a^4*b^4*c*i*j*z + 32*a^3*b^5*e*
f*g*z + 32*a^3*b^5*d*f*h*z - 32*a^3*b^5*d*e*i*z - 32*a^3*b^5*c*g*h*z + 32*
a^3*b^5*c*f*i*z + 32*a^3*b^5*c*e*j*z - 32*a^2*b^6*c*e*f*z + 32*a^2*b^6*c*d
*g*z + 16*a^5*b^3*h^2*j*z - 16*a^5*b^3*h*i^2*z - 48*a^5*b^3*f*j^2*z + 48*a
^4*b^4*f^2*j*z + 16*a^4*b^4*g^2*h*z - 16*a^4*b^4*f*h^2*z + 16*a^3*b^5*d^2*
j*z + 16*a^4*b^4*d*i^2*z - 16*a^3*b^5*e^2*h*z - 16*a^3*b^5*d*g^2*z + 16*a^
2*b^6*c^2*h*z - 16*a^2*b^6*d^2*f*z + 16*a^2*b^6*d*e^2*z - 16*a*b^7*c^2*d*z
 + 16*a^6*b^2*j^3*z - 16*a^3*b^5*f^3*z - 8*a^5*b^2*f*g*i*j + 8*a^5*b^2*e*h
*i*j - 8*a^4*b^3*e*f*h*i + 8*a^4*b^3*e*f*g*j + 8*a^4*b^3*d*g*h*i + 8*a^...
```

3.191. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{a+bx^4}\,dx$

3.192 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2} \, dx$

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3.192.1 Optimal result

Integrand size = 36, antiderivative size = 184

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2} \, dx = \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{4ab(a-bx^4)} + \frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}} + \frac{\left(3bc+\sqrt{a}\sqrt{b}e-ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}} + \frac{(bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

output
```
1/4*x*(b*c+a*g+(a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(-b*x^4+a)+1/4*(-a*h+b*d)*
arctanh(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)+1/8*arctan(b^(1/4)*x/a^(1/4))
*(3*b*c-a*g-e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)+1/8*arctanh(b^(1/4)*x/a^(1/
4))*(3*b*c-a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)
```

3.192.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.40

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx$$
$$= \frac{\frac{4a^{3/4}\sqrt{b}(bx(c+x(d+ex))+a(f+x(g+hx)))}{a-bx^4} - 2\sqrt[4]{b}\left(-3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + \left(-3b^{5/4}c-2\sqrt[4]{a}bd-\sqrt{\ }\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]
```

output
```
((4*a^(3/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/(a - b*
x^4) - 2*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(
1/4)] + (-3*b^(5/4)*c - 2*a^(1/4)*b*d - Sqrt[a]*b^(3/4)*e + a*b^(1/4)*g +
2*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (3*b^(5/4)*c - 2*a^(1/4)*b*d + Sqr
t[a]*b^(3/4)*e - a*b^(1/4)*g + 2*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 2*a
^(1/4)*(-(b*d) + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(3/2))
```

3.192.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2397, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx$$
$$\Big\downarrow \ 2397$$
$$\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{4ab\,(a-bx^4)} - \frac{\int -\frac{b^2ex^2+2b(bd-ah)x+b(3bc-ag)}{a-bx^4}dx}{4ab^2}$$
$$\Big\downarrow \ 25$$
$$\frac{\int \frac{b^2ex^2+2b(bd-ah)x+b(3bc-ag)}{a-bx^4}dx}{4ab^2} + \frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{4ab\,(a-bx^4)}$$
$$\Big\downarrow \ 2415$$

$$\frac{\int \left(\frac{2b(bd-ah)x}{a-bx^4}+\frac{b^2ex^2+b(3bc-ag)}{a-bx^4}\right)dx}{4ab^2}+\frac{x(x(ah+bd)+ag+bc+bex^2+bfx^3)}{4ab(a-bx^4)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{2a^{3/4}}+\frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{2a^{3/4}}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{\sqrt{a}}}{4ab^2}+\frac{x(x(ah+bd)+ag+bc+bex^2+bfx^3)}{4ab(a-bx^4)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x]
```

output
```
(x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) +
((b^(3/4)*(3*b*c - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(
2*a^(3/4)) + (b^(3/4)*(3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x
)/a^(1/4)])/(2*a^(3/4)) + (Sqrt[b]*(b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[
a]])/Sqrt[a])/(4*a*b^2)
```

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.192.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

method	result
risch	$\frac{\frac{e\,x^{3}}{4a}+\frac{(ah+bd)x^{2}}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^{4}+a}-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(\_R^{2}e-\frac{2(ah-bd)\_R}{b}-\frac{ag-3bc}{b}\right)\ln\left(x-\_R\right)}{\_R^{3}}}{16ba}$
default	$\frac{\frac{e\,x^{3}}{4a}+\frac{(ah+bd)x^{2}}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^{4}+a}+\frac{\frac{(-ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(-2ah+2bd)\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\right.\right.}{}}{4ba}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/a*e*x^3+1/4*(a*h+b*d)/a/b*x^2+1/4*(a*g+b*c)/a/b*x+1/4*f/b)/(-b*x^4+a)
-1/16/b/a*sum((_R^2*e-2/b*(a*h-b*d)*_R-1/b*(a*g-3*b*c))/_R^3*ln(x-_R),_R=R
ootOf(_Z^4*b-a))
```

3.192.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 212.80 (sec) , antiderivative size = 710521, normalized size of antiderivative = 3861.53

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx=\text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="frica
s")
```

output
```
Too large to include
```

3.192.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

output
```
Timed out
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.32

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx = -\frac{bex^3+(bd+ah)x^2+af+(bc+ag)x}{4\,(ab^2x^4-a^2b)} + \frac{\frac{2\,(bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\,(bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be-a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(3\,b^{\frac{3}{2}}c+\sqrt{a}be-a\sqrt{b}g\right)\log\left(\frac{\sqrt{b}}{\sqrt{b}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,ab}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxim
a")
```

output
```
-1/4*(b*e*x^3 + (b*d + a*h)*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4 - a^2*b)
 + 1/16*(2*(b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 2*(b
*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(3*b^(3/2)*c -
sqrt(a)*b*e - a*sqrt(b)*g)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*b^(3/2)*c + sqrt(a)*b*e - a*sqrt(b)*
g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(
b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a*b)
```

3.192.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(145) = 290$.

Time = 0.27 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx =$$

$$-\frac{\sqrt{2}\left(3\,b^2c-abg-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$-\frac{\sqrt{2}\left(3\,b^2c-abg+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$-\frac{\sqrt{2}(3\,b^2c-abg-\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$+\frac{\sqrt{2}(3\,b^2c-abg-\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a}$$
$$-\frac{bex^3+bdx^2+ahx^2+bcx+agx+af}{4\,(bx^4-a)ab}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac"
)
```

output
```
-1/16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + 2*sqrt(2)*
(-a*b^3)^(1/4)*a*h + sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a
/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*
g + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - 2*sqrt(2)*(-a*b^3)^(1/4)*a*h - sqrt(-a*
b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a
*b^3)^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 +
 sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3
*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-
a/b))/((-a*b^3)^(3/4)*a) - 1/4*(b*e*x^3 + b*d*x^2 + a*h*x^2 + b*c*x + a*g*
x + a*f)/((b*x^4 - a)*a*b)
```

3.192.9 Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 1626, normalized size of antiderivative = 8.84

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^2,x)
```

output
```
symsum(log(- root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 + 1024*a^5*b^4*
e*g*z^2 - 3072*a^4*b^5*c*e*z^2 - 2048*a^6*b^3*h^2*z^2 - 2048*a^4*b^5*d^2*z
^2 + 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2*g^2*h*z - 128
*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z + 128*a^4*b^3*d*g^2*z + 128*a^3*b^
4*d*e^2*z + 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h + 96*a^2*b^3*c*d*e*h
 - 48*a^3*b^2*c*e*h^2 + 16*a^2*b^3*d^2*e*g - 12*a^2*b^3*c*e^2*g + 16*a^4*b
*e*g*h^2 - 48*a*b^4*c*d^2*e - 64*a^4*b*d*h^3 + 108*a*b^4*c^3*g + 96*a^3*b^
2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 - 54*a^2*b^3*c^2*g^2 - 64*a^2*b^3*d^3*h + 12
*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^4 - 81*b^5*c^4
 - a^2*b^3*e^4 - a^4*b*g^4, z, k)*(root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d
*h*z^2 + 1024*a^5*b^4*e*g*z^2 - 3072*a^4*b^5*c*e*z^2 - 2048*a^6*b^3*h^2*z^
2 - 2048*a^4*b^5*d^2*z^2 + 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128
*a^5*b^2*g^2*h*z - 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z + 128*a^4*b^
3*d*g^2*z + 128*a^3*b^4*d*e^2*z + 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*
h + 96*a^2*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 + 16*a^2*b^3*d^2*e*g - 12*a^2*
b^3*c*e^2*g + 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e - 64*a^4*b*d*h^3 + 108*a
*b^4*c^3*g + 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 - 54*a^2*b^3*c^2*g^2 -
 64*a^2*b^3*d^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 1
6*a^5*h^4 - 81*b^5*c^4 - a^2*b^3*e^4 - a^4*b*g^4, z, k)*((768*a^3*b^4*c -
256*a^4*b^3*g)/(64*a^3*b) - (x*(128*a^3*b^4*d - 128*a^4*b^3*h))/(16*a^3...
```

3.193 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2} dx$

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3.193.7 Maxima [A] (verification not implemented) . . . 1484
3.193.8 Giac [B] (verification not implemented) . . . 1485
3.193.9 Mupad [B] (verification not implemented) . . . 1486

3.193.1 Optimal result

Integrand size = 41, antiderivative size = 203

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2} dx = \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{4ab(a-bx^4)} - \frac{\left(be-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{\left(be+\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{(bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

output
```
1/4*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+b*f*x^3)/a/b/(-b*x^4+a)+1/4*(-a*h
+b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/8*arctan(b^(1/4)*x/a^
(1/4))*(b*e-3*a*i-(-a*g+3*b*c)*b^(1/2)/a^(1/2))/a^(5/4)/b^(7/4)+1/8*arctan
h(b^(1/4)*x/a^(1/4))*(b*e-3*a*i+(-a*g+3*b*c)*b^(1/2)/a^(1/2))/a^(5/4)/b^(7
/4)
```

3.193.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.49

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx$$

$$= \frac{\frac{4a^{3/4}b^{3/4}(bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{a-bx^4} + 2\left(3b^{3/2}c-\sqrt{a}be-a\sqrt{b}g+3a^{3/2}i\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + \left(-3b^{3/2}c-\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,
x]
```

output
```
((4*a^(3/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + x*(h + i*x)))))
/(a - b*x^4) + 2*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*A
rcTan[(b^(1/4)*x)/a^(1/4)] + (-3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d - Sqrt[a]
*b*e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1
/4)*x] + (3*b^(3/2)*c - 2*a^(1/4)*b^(5/4)*d + Sqrt[a]*b*e - a*Sqrt[b]*g +
2*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 2*a^(1/4)*b^
(1/4)*(-(b*d) + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(7/4))
```

3.193.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {2397, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx$$

$$\Big\downarrow \ 2397$$

$$\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{4ab\left(a-bx^4\right)} - \frac{\int -\frac{b(be-3ai)x^2+2b(bd-ah)x+b(3bc-ag)}{a-bx^4}dx}{4ab^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{b(be-3ai)x^2+2b(bd-ah)x+b(3bc-ag)}{a-bx^4}dx}{4ab^2} + \frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\int \left(\frac{2b(bd-ah)x}{a-bx^4}+\frac{b(be-3ai)x^2+b(3bc-ag)}{a-bx^4}\right)dx}{4ab^2}+\frac{x(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3)}{4ab(a-bx^4)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{2\sqrt[4]{a}}+\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{2\sqrt[4]{a}}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{\sqrt{a}}}{4ab^2}+\frac{x(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3)}{4ab(a-bx^4)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x]
```

output
```
(x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(4*a*b*(a - b*
x^4)) + (-1/2*(b^(1/4)*(b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*a*i)*Arc
Tan[(b^(1/4)*x)/a^(1/4)])/a^(1/4) + (b^(1/4)*(b*e + (Sqrt[b]*(3*b*c - a*g)
)/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)) + (Sqrt[b]*(b
*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a])/(4*a*b^2)
```

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.193.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.67

method	result
risch	$\frac{\frac{(ai+be)x^3}{4ab}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^4+a}-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(-(3ai-be)\_R^2-2(ah-bd)\_R-ag+3bc\right)\ln\left(x-\_R\right)}{\_R^3}}{16a\,b^2}$
default	$\frac{\frac{(ai+be)x^3}{4ab}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{f}{4b}}{-b\,x^4+a}+\frac{\frac{(-ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(-2ah+2bd)\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-}{4ba}$ (-3ai+

input
```
int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVER
BOSE)
```

output
```
(1/4*(a*i+b*e)/a/b*x^3+1/4*(a*h+b*d)/a/b*x^2+1/4*(a*g+b*c)/a/b*x+1/4*f/b)/
(-b*x^4+a)-1/16/a/b^2*sum((-(3*a*i-b*e)*_R^2-2*(a*h-b*d)*_R-a*g+3*b*c)/_R^
3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.193.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm=
"fricas")
```

output
```
Timed out
```

3.193.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

output
```
Timed out
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx$$
$$= -\frac{(be+ai)x^3+(bd+ah)x^2+af+(bc+ag)x}{4\,(ab^2x^4-a^2b)}$$
$$+\frac{\frac{2\,(bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{2\,(bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be-a\sqrt{b}g+3\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(3\,b^{\frac{3}{2}}c+\sqrt{a}be-a\sqrt{b}g-3\ldots}{\sqrt{a}\sqrt{\ldots}}}{16\,ab}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm=
"maxima")
```

output
```
-1/4*((b*e + a*i)*x^3 + (b*d + a*h)*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4
- a^2*b) + 1/16*(2*(b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)
) - 2*(b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(3*b^(3
/2)*c - sqrt(a)*b*e - a*sqrt(b)*g + 3*a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*b^(3/2)*c + sq
rt(a)*b*e - a*sqrt(b)*g - 3*a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(
b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)))/(a*b)
```

3.193.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(164) = 328$.

Time = 0.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.27

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx =$$

$$-\frac{\sqrt{2}\left(3\,b^3c-ab^2g-2\,\sqrt{2}(-ab^3)^{\frac14}\,b^2d+2\,\sqrt{2}(-ab^3)^{\frac14}\,abh-\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(\right.}{2\,(-\frac{a}{b})}\right.}{16\,(-ab^3)^{\frac34}\,ab}$$

$$-\frac{\sqrt{2}\left(3\,b^3c-ab^2g+2\,\sqrt{2}(-ab^3)^{\frac14}\,b^2d-2\,\sqrt{2}(-ab^3)^{\frac14}\,abh-\sqrt{-ab}b^2e-3\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(\right.}{2\,(-\frac{a}{b})}\right.}{16\,(-ab^3)^{\frac34}\,ab}$$

$$-\frac{\sqrt{2}(3\,b^3c-ab^2g-\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac34}\,ab}$$

$$+\frac{\sqrt{2}(3\,b^3c-ab^2g-\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac34}\,ab}$$

$$-\frac{bex^3+aix^3+bdx^2+ahx^2+bcx+agx+af}{4\,(bx^4-a)ab}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm=
"giac")
```

output
```
-1/16*sqrt(2)*(3*b^3*c - a*b^2*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b^2*d + 2*sqrt
(2)*(-a*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e + 3*sqrt(-a*b)*a*b*i)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a*b)
 - 1/16*sqrt(2)*(3*b^3*c - a*b^2*g + 2*sqrt(2)*(-a*b^3)^(1/4)*b^2*d - 2*sq
rt(2)*(-a*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e - 3*sqrt(-a*b)*a*b*i)*arctan
(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a*
b) - 1/32*sqrt(2)*(3*b^3*c - a*b^2*g - sqrt(-a*b)*b^2*e + 3*sqrt(-a*b)*a*b
*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a*b) +
1/32*sqrt(2)*(3*b^3*c - a*b^2*g - sqrt(-a*b)*b^2*e + 3*sqrt(-a*b)*a*b*i)*l
og(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a*b) - 1/4*(
b*e*x^3 + a*i*x^3 + b*d*x^2 + a*h*x^2 + b*c*x + a*g*x + a*f)/((b*x^4 - a)*
a*b)
```

3.193.9 Mupad [B] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 2611, normalized size of antiderivative = 12.86

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^2,x)
```

output
```
symsum(log((27*a^4*i^3 - a*b^3*e^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e - 12*a^2*b
^2*c*h^2 + a^2*b^2*e*g^2 + 9*a^2*b^2*e^2*i + 4*a*b^3*d^2*g - 27*a*b^3*c^2*
i - 27*a^3*b*e*i^2 + 4*a^3*b*g*h^2 - 3*a^3*b*g^2*i + 18*a^2*b^2*c*g*i - 8*
a^2*b^2*d*g*h + 24*a*b^3*c*d*h - 6*a*b^3*c*e*g)/(64*a^3*b^2) - root(65536*
a^7*b^7*z^4 - 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c*i*z^2 + 4096*a^5*b^5*d
*h*z^2 + 1024*a^5*b^5*e*g*z^2 - 3072*a^4*b^6*c*e*z^2 - 2048*a^6*b^4*h^2*z^
2 - 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z - 768*a^4*b^4*d*e*i*z + 768
*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z - 1152*a^6*b^2*h*i^2*z - 128*a^5*b^
3*g^2*h*z + 1152*a^5*b^3*d*i^2*z - 128*a^4*b^4*e^2*h*z - 1152*a^3*b^5*c^2*
h*z + 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z + 1152*a^2*b^6*c^2*d*z + 9
6*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b^3*c*e*g*i - 32*a^3*b^3*
d*e*g*h + 96*a^2*b^4*c*d*e*h - 12*a^4*b^2*e*g^2*i + 144*a^4*b^2*c*h^2*i -
48*a^3*b^3*d^2*g*i + 16*a^4*b^2*e*g*h^2 - 108*a^4*b^2*c*g*i^2 - 108*a^2*b^
4*c^2*e*i + 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h^2 + 16*a^2*b^4*d^2*e*g
- 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5*c*d^2*e + 108*a^5*b*e*i
^3 + 108*a*b^5*c^3*g - 54*a^4*b^2*e^2*i^2 + 162*a^3*b^3*c^2*i^2 + 96*a^3*b
^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 - 54*a^2*b^4*c^2*g^2 + 18*a^5*b*g^2*i^2 + 1
2*a^3*b^3*e^3*i - 64*a^4*b^2*d*h^3 - 64*a^2*b^4*d^3*h + 12*a^3*b^3*c*g^3 +
 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 - 81*a^6*i^4 - 81*b^6*c^4
- a^4*b^2*g^4 - a^2*b^4*e^4, z, l)*(root(65536*a^7*b^7*z^4 - 3072*a^6*b...
```

3.194 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2} dx$

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3.194.1 Optimal result

Integrand size = 46, antiderivative size = 225

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2} dx &= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{4ab(a-bx^4)} \\ &- \frac{\left(be-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} + \frac{\left(be+\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{7/4}} \\ &+ \frac{(bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} + \frac{j\log(a-bx^4)}{4b^2}\end{aligned}$$

output
```
1/4*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+(a*j+b*f)*x^3)/a/b/(-b*x^4+a)+1/4
*(-a*h+b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)+1/4*j*ln(-b*x^4+a
)/b^2-1/8*arctan(b^(1/4)*x/a^(1/4))*(b*e-3*a*i-(-a*g+3*b*c)*b^(1/2)/a^(1/2
))/a^(5/4)/b^(7/4)+1/8*arctanh(b^(1/4)*x/a^(1/4))*(b*e-3*a*i+(-a*g+3*b*c)*
b^(1/2)/a^(1/2))/a^(5/4)/b^(7/4)
```

3.194.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.50

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx$$

$$= \frac{\frac{4(a^2j+b^2x(c+x(d+ex))+ab(f+x(g+x(h+ix))))}{a(a-bx^4)} + \frac{2\sqrt[4]{b}\left(3b^{3/2}c-\sqrt{a}be-a\sqrt{b}g+3a^{3/2}i\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{\sqrt[4]{b}\left(-3b^{3/2}c-2\sqrt[4]{a}b^{5/4}d-\sqrt{}\right.}{}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b
*x^4)^2,x]
```

output
```
((4*(a^2*j + b^2*x*(c + x*(d + e*x)) + a*b*(f + x*(g + x*(h + i*x)))))/(a*
(a - b*x^4)) + (2*b^(1/4)*(3*b^(3/2)*c - Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(
3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(7/4) + (b^(1/4)*(-3*b^(3/2)*c - 2*
a^(1/4)*b^(5/4)*d - Sqrt[a]*b*e + a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h + 3*a^
(3/2)*i)*Log[a^(1/4) - b^(1/4)*x])/a^(7/4) + (b^(1/4)*(3*b^(3/2)*c - 2*a^(
1/4)*b^(5/4)*d + Sqrt[a]*b*e - a*Sqrt[b]*g + 2*a^(5/4)*b^(1/4)*h - 3*a^(3/
2)*i)*Log[a^(1/4) + b^(1/4)*x])/a^(7/4) + (2*Sqrt[b]*(b*d - a*h)*Log[Sqrt[
a] + Sqrt[b]*x^2])/a^(3/2) + 4*j*Log[a - b*x^4])/(16*b^2)
```

3.194.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2397, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{4ab\left(a-bx^4\right)}-\frac{\int -\frac{-4abjx^3+b(be-3ai)x^2+2b(bd-ah)x+b(3bc-ag)}{a-bx^4}dx}{4ab^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-4abjx^3+b(be-3ai)x^2+2b(bd-ah)x+b(3bc-ag)}{a-bx^4}dx}{4ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{4ab\left(a-bx^4\right)}$$

$$\Big\downarrow \text{ 2415}$$

$$\frac{\int \left(\frac{b(be-3ai)x^2+b(3bc-ag)}{a-bx^4}+\frac{x\left(2b(bd-ah)-4abjx^2\right)}{a-bx^4}\right)dx}{4ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{4ab\left(a-bx^4\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{2\sqrt[4]{a}}+\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}(3bc-ag)}{\sqrt{a}}-3ai+be\right)}{2\sqrt[4]{a}}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(bd-ah)}{\sqrt{a}}+aj\log}{4ab^2}$$
$$\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{4ab\left(a-bx^4\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^
2,x]
```

output
```
(x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(4*a*b
*(a - b*x^4)) + (-1/2*(b^(1/4)*(b*e - (Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 3*
a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(1/4) + (b^(1/4)*(b*e + (Sqrt[b]*(3*b*
c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)) + (Sq
rt[b]*(b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] + a*j*Log[a - b*
x^4])/(4*a*b^2)
```

3.194. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx$

3.194.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.194.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\frac{(ai+be)x^3}{4ab}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{aj+bf}{4b^2}}{-b\,x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{\left(-4j\_R^3-\frac{(3ai-be)\_R^2}{a}-\frac{2(ah-bd)\_R}{a}-\frac{ag-3bc}{a}\right)\ln(x-\_}{\_R^3}}{16b^2}$
default	$\frac{\frac{(ai+be)x^3}{4ab}+\frac{(ah+bd)x^2}{4ab}+\frac{(ag+bc)x}{4ab}+\frac{aj+bf}{4b^2}}{-b\,x^4+a}+\frac{\frac{(-ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{(-2ah+2bd)\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{(-3}{}}{4ba}$

input
```
int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RET
URNVERBOSE)
```

output
```
(1/4*(a*i+b*e)/a/b*x^3+1/4*(a*h+b*d)/a/b*x^2+1/4*(a*g+b*c)/a/b*x+1/4*(a*j+
b*f)/b^2)/(-b*x^4+a)-1/16/b^2*sum((-4*j*_R^3-1/a*(3*a*i-b*e)*_R^2-2/a*(a*h
-b*d)*_R-1/a*(a*g-3*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.194.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algo
rithm="fricas")
```

output
```
Timed out
```

3.194.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2
,x)
```

output
```
Timed out
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx$$
$$= -\frac{(b^2e+abi)x^3+abf+a^2j+(b^2d+abh)x^2+(b^2c+abg)x}{4\,(ab^3x^4-a^2b^2)}$$
$$+\frac{\frac{2\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be-a\sqrt{b}g+3\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\left(b^{\frac{3}{2}}d-a\sqrt{b}h+2\,a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b}-\frac{2\left(b^{\frac{3}{2}}d-a\sqrt{b}h-2\,a^{\frac{3}{2}}j\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b}}{16\,ab}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algo
rithm="maxima")
```

output
```
-1/4*((b^2*e + a*b*i)*x^3 + a*b*f + a^2*j + (b^2*d + a*b*h)*x^2 + (b^2*c +
 a*b*g)*x)/(a*b^3*x^4 - a^2*b^2) + 1/16*(2*(3*b^(3/2)*c - sqrt(a)*b*e - a*
sqrt(b)*g + 3*a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*
sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*(b^(3/2)*d - a*sqrt(b)*h + 2*a^(3/2)*j)
*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - 2*(b^(3/2)*d - a*sqrt(b)*h - 2*a
^(3/2)*j)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (3*b^(3/2)*c + sqrt(a)*
b*e - a*sqrt(b)*g - 3*a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(
sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)
))/(a*b)
```

3.194.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(184) = 368$.

Time = 0.28 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.16

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx = \frac{j\log(|bx^4-a|)}{4\,b^2} - \frac{\sqrt{2}\Big(3\,b^3c-ab^2g-2\,\sqrt{2}(-ab^3)^{\frac14}\,b^2d+2\,\sqrt{2}(-ab^3)^{\frac14}\,abh-\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi\Big)\arctan\left(\frac{\sqrt{2}\Big(2\,x+\sqrt{2}(}{2\,(-\frac{a}{b})}\right.}{16\,(-ab^3)^{\frac34}\,ab} - \frac{\sqrt{2}\Big(3\,b^3c-ab^2g+2\,\sqrt{2}(-ab^3)^{\frac14}\,b^2d-2\,\sqrt{2}(-ab^3)^{\frac14}\,abh-\sqrt{-ab}b^2e-3\,\sqrt{-ab}abi\Big)\arctan\left(\frac{\sqrt{2}\Big(2\,x-\sqrt{2}(}{2\,(-\frac{a}{b})}\right.}{16\,(-ab^3)^{\frac34}\,ab} - \frac{\sqrt{2}(3\,b^3c-ab^2g-\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi)\log\left(x^2+\sqrt{2}x(-\frac{a}{b})^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac34}\,ab} + \frac{\sqrt{2}(3\,b^3c-ab^2g-\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi)\log\left(x^2-\sqrt{2}x(-\frac{a}{b})^{\frac14}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac34}\,ab} - \frac{(be+ai)x^3+(bd+ah)x^2+(bc+ag)x+\frac{abf+a^2j}{b}}{4\,(bx^4-a)ab}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algo
rithm="giac")
```

output
```
1/4*j*log(abs(b*x^4 - a))/b^2 - 1/16*sqrt(2)*(3*b^3*c - a*b^2*g - 2*sqrt(2
)*(-a*b^3)^(1/4)*b^2*d + 2*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2*e
 + 3*sqrt(-a*b)*a*b*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a
/b)^(1/4))/((-a*b^3)^(3/4)*a*b) - 1/16*sqrt(2)*(3*b^3*c - a*b^2*g + 2*sqrt
(2)*(-a*b^3)^(1/4)*b^2*d - 2*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - sqrt(-a*b)*b^2
*e - 3*sqrt(-a*b)*a*b*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(
-a/b)^(1/4))/((-a*b^3)^(3/4)*a*b) - 1/32*sqrt(2)*(3*b^3*c - a*b^2*g - sqrt
(-a*b)*b^2*e + 3*sqrt(-a*b)*a*b*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt
(-a/b))/((-a*b^3)^(3/4)*a*b) + 1/32*sqrt(2)*(3*b^3*c - a*b^2*g - sqrt(-a*b
)*b^2*e + 3*sqrt(-a*b)*a*b*i)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b
))/((-a*b^3)^(3/4)*a*b) - 1/4*((b*e + a*i)*x^3 + (b*d + a*h)*x^2 + (b*c +
a*g)*x + (a*b*f + a^2*j)/b)/((b*x^4 - a)*a*b)
```

3.194.9 Mupad [B] (verification not implemented)

Time = 10.22 (sec) , antiderivative size = 3943, normalized size of antiderivative = 17.52

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^
2,x)
```

output
```
((b*f + a*j)/(4*b^2) + (x*(b*c + a*g))/(4*a*b) + (x^2*(b*d + a*h))/(4*a*b)
 + (x^3*(b*e + a*i))/(4*a*b))/(a - b*x^4) + symsum(log((27*a^4*i^3 - a*b^3
*e^3 - 12*b^4*c*d^2 + 9*b^4*c^2*e + 16*a^4*g*j^2 - 12*a^2*b^2*c*h^2 + a^2*
b^2*e*g^2 + 9*a^2*b^2*e^2*i - 48*a^4*h*i*j + 4*a*b^3*d^2*g - 27*a*b^3*c^2*
i - 48*a^3*b*c*j^2 - 27*a^3*b*e*i^2 + 4*a^3*b*g*h^2 - 3*a^3*b*g^2*i + 18*a
^2*b^2*c*g*i - 16*a^2*b^2*d*e*j - 8*a^2*b^2*d*g*h + 24*a*b^3*c*d*h - 6*a*b
^3*c*e*g + 48*a^3*b*d*i*j + 16*a^3*b*e*h*j)/(64*a^3*b^2) - root(65536*a^7*
b^8*z^4 - 65536*a^7*b^6*j*z^3 - 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^
2 + 4096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 - 3072*a^4*b^7*c*e*z^2 + 2
4576*a^7*b^4*j^2*z^2 - 2048*a^6*b^5*h^2*z^2 - 2048*a^4*b^7*d^2*z^2 + 1536*
a^6*b^3*g*i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^
4*e*h*i*z - 512*a^5*b^4*e*g*j*z + 1536*a^4*b^5*c*e*j*z - 768*a^4*b^5*d*e*i
*z + 768*a^4*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z + 1024*a^6*b^3*h^2*j*z - 11
52*a^6*b^3*h*i^2*z - 128*a^5*b^4*g^2*h*z + 1024*a^4*b^5*d^2*j*z + 1152*a^5
*b^4*d*i^2*z - 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z + 128*a^4*b^5*d*
g^2*z + 128*a^3*b^6*d*e^2*z + 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z -
192*a^5*b^2*e*h*i*j + 192*a^4*b^3*d*e*i*j - 192*a^4*b^3*c*g*h*j + 96*a^4*b
^3*d*g*h*i - 288*a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*
i - 32*a^3*b^4*d*e*g*h + 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*
b^2*g*h^2*i - 288*a^5*b^2*d*i^2*j + 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*...
```

3.194. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^2}\,dx$

3.195 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2} dx$

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3.195.1 Optimal result

Integrand size = 35, antiderivative size = 353

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2} dx \\
&= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{4ab(a+bx^4)} + \frac{(bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} \\
&\quad - \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} \\
&\quad + \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} \\
&\quad - \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}} \\
&\quad + \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}
\end{aligned}$$

output
```
1/4*x*(b*c-a*g+(-a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(b*x^4+a)+1/4*(a*h+b*d)*a
rctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1
/2)+a^(1/2)+x^2*b^(1/2))*(3*b*c+a*g-e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(
1/2)+1/32*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(3*b*c+a*g-e*a
^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a
^(1/4))*(3*b*c+a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*arctan(
1+b^(1/4)*x*2^(1/2)/a^(1/4))*(3*b*c+a*g+e*a^(1/2)*b^(1/2))/a^(7/4)/b^(5/4)
*2^(1/2)
```

3.195.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.02

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx$$

$$= \frac{-\frac{8a^{3/4}\sqrt{b}(-bx(c+x(d+ex))+a(f+x(g+hx)))}{a+bx^4} - 2\left(3\sqrt{2}b^{5/4}c + 4\sqrt[4]{a}bd + \sqrt{2}\sqrt{a}b^{3/4}e + \sqrt{2}a\sqrt[4]{b}g + 4a^{5/4}h\right)\arctan\left(\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]
```

output
```
((-8*a^(3/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(a
+ b*x^4) - 2*(3*Sqrt[2]*b^(5/4)*c + 4*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4
)*e + Sqrt[2]*a*b^(1/4)*g + 4*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^
(1/4)] + 2*(3*Sqrt[2]*b^(5/4)*c - 4*a^(1/4)*b*d + Sqrt[2]*Sqrt[a]*b^(3/4)*
e + Sqrt[2]*a*b^(1/4)*g - 4*a^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1
/4)] + Sqrt[2]*b^(1/4)*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*b*c - Sqrt[a]*
Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(
32*a^(7/4)*b^(3/2))
```

3.195.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2397, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{4ab(a+bx^4)}-\frac{\int -\frac{b^2ex^2+2b(bd+ah)x+b(3bc+ag)}{bx^4+a}dx}{4ab^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2ex^2+2b(bd+ah)x+b(3bc+ag)}{bx^4+a}dx}{4ab^2}+\frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{4ab(a+bx^4)}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\int \left(\frac{2b(bd+ah)x}{bx^4+a}+\frac{b^2ex^2+b(3bc+ag)}{bx^4+a}\right)dx}{4ab^2}+\frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{4ab(a+bx^4)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{2\sqrt{2}a^{3/4}}+\frac{b^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{2\sqrt{2}a^{3/4}}-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{}\right.}{4\sqrt{2}a^{3/4}}}{4ab^2}$$
$$\frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{4ab(a+bx^4)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x]
```

output
```
(x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) +
((Sqrt[b]*(b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (b^(3/4)*(3
*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)) + (b^(3/4)*(3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 +
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)) - (b^(3/4)*(3*b*c - Sqr
t[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/(4*Sqrt[2]*a^(3/4)) + (b^(3/4)*(3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)))/(4
*a*b^2)
```

3.195.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.195.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\frac{e x^3}{4a}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b+a)}\frac{\left(\_R^2 e+\frac{2(ah+bd)\_R}{b}+\frac{ag+3bc}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$\frac{\frac{e x^3}{4a}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b x^4+a}+\frac{\frac{(ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{(2ah+}{}}{4}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/a*e*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+
1/16/b/a*sum((_R^2*e+2/b*(a*h+b*d)*_R+1/b*(a*g+3*b*c))/_R^3*ln(x-_R),_R=Ro
otOf(_Z^4*b+a))
```

3.195.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

3.195.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

output
```
Timed out
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.06

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx = \frac{bex^3+(bd-ah)x^2-af+(bc-ag)x}{4\,(ab^2x^4+a^2b)}$$

$$+\frac{\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}a^{\frac{3}{4}}\right.}{}}{}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima
")
```

output
```
1/4*(b*e*x^3 + (b*d - a*h)*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 + a^2*b)
+ 1/32*(sqrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2
+ sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*b^(3
/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(
2)*a^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g - 4*sqrt(a)*b^(3/2)*d - 4
*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3
*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b
^(3/4)*g + 4*sqrt(a)*b^(3/2)*d + 4*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqr
t(sqrt(a)*sqrt(b))*b^(3/4)))/(a*b)
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.11

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx = \frac{bex^3+bdx^2-ahx^2+bcx-agx-af}{4\,(bx^4+a)ab} + \frac{\sqrt{2}\Big(2\sqrt{2}\sqrt{ab}b^2d+2\sqrt{2}\sqrt{ab}abh+3\,(ab^3)^{\frac14}\,b^2c+(ab^3)^{\frac14}\,abg+(ab^3)^{\frac34}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\Big(2\sqrt{2}\sqrt{ab}b^2d+2\sqrt{2}\sqrt{ab}abh+3\,(ab^3)^{\frac14}\,b^2c+(ab^3)^{\frac14}\,abg+(ab^3)^{\frac34}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac14}\,b^2c+(ab^3)^{\frac14}\,abg-(ab^3)^{\frac34}\,e\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3} - \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac14}\,b^2c+(ab^3)^{\frac14}\,abg-(ab^3)^{\frac34}\,e\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(b*e*x^3 + b*d*x^2 - a*h*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b)
+ 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*
(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sq
rt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2
*sqrt(2)*sqrt(a*b)*b^2*d + 2*sqrt(2)*sqrt(a*b)*a*b*h + 3*(a*b^3)^(1/4)*b^2
*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^
2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^
(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(a^2*b^3)
```

3.195.9 Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 1623, normalized size of antiderivative = 4.60

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^2,x)
```

output
```
symsum(log((12*b^3*c*d^2 - a*b^2*e^3 - 9*b^3*c^2*e + 4*a^3*g*h^2 + 4*a*b^2
*d^2*g + 12*a^2*b*c*h^2 - a^2*b*e*g^2 + 24*a*b^2*c*d*h - 6*a*b^2*c*e*g + 8
*a^2*b*d*g*h)/(64*a^3*b) - root(65536*a^7*b^6*z^4 + 4096*a^5*b^4*d*h*z^2 +
 1024*a^5*b^4*e*g*z^2 + 3072*a^4*b^5*c*e*z^2 + 2048*a^6*b^3*h^2*z^2 + 2048
*a^4*b^5*d^2*z^2 - 768*a^4*b^3*c*g*h*z - 768*a^3*b^4*c*d*g*z - 128*a^5*b^2
*g^2*h*z + 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*z - 128*a^4*b^3*d*g^2*
z + 128*a^3*b^4*d*e^2*z - 1152*a^2*b^5*c^2*d*z - 32*a^3*b^2*d*e*g*h - 96*a
^2*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 - 16*a^2*b^3*d^2*e*g + 12*a^2*b^3*c*e^
2*g - 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e + 64*a^4*b*d*h^3 + 108*a*b^4*c^3
*g + 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 54*a^2*b^3*c^2*g^2 + 64*a^2*
b^3*d^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 + 16*a*b^4*d^4 + 16*a^5*h^
4 + 81*b^5*c^4 + a^2*b^3*e^4 + a^4*b*g^4, z, k)*(root(65536*a^7*b^6*z^4 +
4096*a^5*b^4*d*h*z^2 + 1024*a^5*b^4*e*g*z^2 + 3072*a^4*b^5*c*e*z^2 + 2048*
a^6*b^3*h^2*z^2 + 2048*a^4*b^5*d^2*z^2 - 768*a^4*b^3*c*g*h*z - 768*a^3*b^4
*c*d*g*z - 128*a^5*b^2*g^2*h*z + 128*a^4*b^3*e^2*h*z - 1152*a^3*b^4*c^2*h*
z - 128*a^4*b^3*d*g^2*z + 128*a^3*b^4*d*e^2*z - 1152*a^2*b^5*c^2*d*z - 32*
a^3*b^2*d*e*g*h - 96*a^2*b^3*c*d*e*h - 48*a^3*b^2*c*e*h^2 - 16*a^2*b^3*d^2
*e*g + 12*a^2*b^3*c*e^2*g - 16*a^4*b*e*g*h^2 - 48*a*b^4*c*d^2*e + 64*a^4*b
*d*h^3 + 108*a*b^4*c^3*g + 96*a^3*b^2*d^2*h^2 + 2*a^3*b^2*e^2*g^2 + 54*a^2
*b^3*c^2*g^2 + 64*a^2*b^3*d^3*h + 12*a^3*b^2*c*g^3 + 18*a*b^4*c^2*e^2 +...
```

3.196 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx$

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3.196.1 Optimal result

Integrand size = 40, antiderivative size = 395

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx = \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{4ab(a+bx^4)}+\frac{(bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}-\frac{\left(\sqrt{b}(3bc+ag)+\sqrt{a}(be+3ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{7/4}}+\frac{\left(\sqrt{b}(3bc+ag)+\sqrt{a}(be+3ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{7/4}}-\frac{\left(\sqrt{b}(3bc+ag)-\sqrt{a}(be+3ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{7/4}}+\frac{\left(\sqrt{b}(3bc+ag)-\sqrt{a}(be+3ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{7/4}}$$

output
```
1/4*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+b*f*x^3)/a/b/(b*x^4+a)+1/4*(a*h
+b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/32*ln(-a^(1/4)*b^(1/4)
*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))
/a^(7/4)/b^(7/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(
1/2))*(-(3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+1
/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*
b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*
((3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)
```

3.196.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx$$
$$= \frac{-\frac{8a^{3/4}b^{3/4}(-bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{a+bx^4}-2\Big(3\sqrt{2}b^{3/2}c+4\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+4a^{5/4}\sqrt[4]{b}h+}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,
x]
```

output
```
((-8*a^(3/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + x*(h + i*x)
))))/(a + b*x^4) - 2*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*
Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2
)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b^(3/2)*c - 4*
a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g - 4*a^(5/4)*
b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] +
 Sqrt[2]*(-3*b^(3/2)*c + Sqrt[a]*b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b^(3/2)*c - Sq
rt[a]*b*e + a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/(32*a^(7/4)*b^(7/4))
```

3.196.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2397, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx$$

$$\Big\downarrow \; 2397$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{4ab\,(a+bx^4)}-\frac{\int -\frac{b(be+3ai)x^2+2b(bd+ah)x+b(3bc+ag)}{bx^4+a}dx}{4ab^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b(be+3ai)x^2+2b(bd+ah)x+b(3bc+ag)}{bx^4+a}dx}{4ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{4ab\,(a+bx^4)}$$

$$\Big\downarrow \; 2415$$

$$\frac{\int \left(\frac{2b(bd+ah)x}{bx^4+a}+\frac{b(be+3ai)x^2+b(3bc+ag)}{bx^4+a}\right)dx}{4ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{4ab\,(a+bx^4)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{2\sqrt{2}a^{3/4}}+\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{2\sqrt{2}a^{3/4}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{}}{4a}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{4ab\,(a+bx^4)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x]
```

output
```
(x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(4*a*b*(a + b*
x^4)) + ((Sqrt[b]*(b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (b^
(1/4)*(Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)) + (b^(1/4)*(Sqrt[b]*(3*b*c + a*g)
 + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)) - (b^(1/4)*(Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4))
+ (b^(1/4)*(Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)))/(4*a*b^2)
```

3.196.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.196.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.34

method	result
risch	$\frac{-\frac{(ai-be)x^3}{4ab}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b+a)}\frac{\left((3ai+be)\_R^2+2(ah+bd)\_R+ag+3bc\right)\ln(x-\_R)}{\_R^3}}{16a\,b^2}$
default	$\frac{-\frac{(ai-be)x^3}{4ab}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\frac{(ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}}{}$

input
```
int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERB
OSE)
```

output
```
(-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)
/(b*x^4+a)+1/16/a/b^2*sum(((3*a*i+b*e)*_R^2+2*(a*h+b*d)*_R+a*g+3*b*c)/_R^3
*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.196.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="
fricas")
```

output
```
Timed out
```

3.196.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

output
```
Timed out
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx = \frac{(be-ai)x^3+(bd-ah)x^2-af+(bc-ag)x}{4\,(ab^2x^4+a^2b)}$$

$$+ \frac{\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(3\sqrt{2}\right.}{}}{}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="
maxima")
```

output
```
1/4*((b*e - a*i)*x^3 + (b*d - a*h)*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 +
 a^2*b) + 1/32*(sqrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g - 3*a^(3/
2)*i)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3
/4)) - sqrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g - 3*a^(3/2)*i)*log
(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*
(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)
*b^(3/4)*g + 3*sqrt(2)*a^(7/4)*b^(1/4)*i - 4*sqrt(a)*b^(3/2)*d - 4*a^(3/2)
*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqr
t(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)
*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g
 + 3*sqrt(2)*a^(7/4)*b^(1/4)*i + 4*sqrt(a)*b^(3/2)*d + 4*a^(3/2)*sqrt(b)*h
)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a*b)
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.16

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^2}\,dx = \frac{bex^3-aix^3+bdx^2-ahx^2+bcx-agx-af}{4\,(bx^4+a)ab}$$
$$+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^3d+2\sqrt{2}\sqrt{ab}ab^2h+3\,(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g+(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\frac{\sqrt{2}\left(2x\right.}{2}\right.}{16\,a^2b^4}$$
$$+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^3d+2\sqrt{2}\sqrt{ab}ab^2h+3\,(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g+(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\frac{\sqrt{2}\left(2x\right.}{2}\right.}{16\,a^2b^4}$$
$$+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g-(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^4}$$
$$-\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^3c+(ab^3)^{\frac{1}{4}}\,ab^2g-(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^4}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="
giac")
```

output
```
1/4*(b*e*x^3 - a*i*x^3 + b*d*x^2 - a*h*x^2 + b*c*x - a*g*x - a*f)/((b*x^4
+ a)*a*b) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^3*d + 2*sqrt(2)*sqrt(a*b)*
a*b^2*h + 3*(a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g + (a*b^3)^(3/4)*b*
e + 3*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a
/b)^(1/4))/(a^2*b^4) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^3*d + 2*sqrt(2)
*sqrt(a*b)*a*b^2*h + 3*(a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g + (a*b^
3)^(3/4)*b*e + 3*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^3*c + (a
*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e - 3*(a*b^3)^(3/4)*a*i)*log(x^2 + s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4
)*b^3*c + (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3/4)*b*e - 3*(a*b^3)^(3/4)*a*i)
*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
```

3.196.9 Mupad [B] (verification not implemented)

Time = 10.17 (sec) , antiderivative size = 2605, normalized size of antiderivative = 6.59

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{(a + bx^4)^2} dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^2,x)
```

output
```
symsum(log(- root(65536*a^7*b^7*z^4 + 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*
c*i*z^2 + 4096*a^5*b^5*d*h*z^2 + 1024*a^5*b^5*e*g*z^2 + 3072*a^4*b^6*c*e*z
^2 + 2048*a^6*b^4*h^2*z^2 + 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z + 7
68*a^4*b^4*d*e*i*z - 768*a^4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z + 1152*a^6*
b^2*h*i^2*z - 128*a^5*b^3*g^2*h*z + 1152*a^5*b^3*d*i^2*z + 128*a^4*b^4*e^2
*h*z - 1152*a^3*b^5*c^2*h*z - 128*a^4*b^4*d*g^2*z + 128*a^3*b^5*d*e^2*z -
1152*a^2*b^6*c^2*d*z - 96*a^4*b^2*d*g*h*i - 288*a^3*b^3*c*d*h*i + 72*a^3*b
^3*c*e*g*i - 32*a^3*b^3*d*e*g*h - 96*a^2*b^4*c*d*e*h + 12*a^4*b^2*e*g^2*i
- 144*a^4*b^2*c*h^2*i - 48*a^3*b^3*d^2*g*i - 16*a^4*b^2*e*g*h^2 + 108*a^4*
b^2*c*g*i^2 + 108*a^2*b^4*c^2*e*i - 144*a^2*b^4*c*d^2*i - 48*a^3*b^3*c*e*h
^2 - 16*a^2*b^4*d^2*e*g + 12*a^2*b^4*c*e^2*g - 48*a^5*b*g*h^2*i - 48*a*b^5
*c*d^2*e + 108*a^5*b*e*i^3 + 108*a*b^5*c^3*g + 54*a^4*b^2*e^2*i^2 + 162*a^
3*b^3*c^2*i^2 + 96*a^3*b^3*d^2*h^2 + 2*a^3*b^3*e^2*g^2 + 54*a^2*b^4*c^2*g^
2 + 18*a^5*b*g^2*i^2 + 12*a^3*b^3*e^3*i + 64*a^4*b^2*d*h^3 + 64*a^2*b^4*d^
3*h + 12*a^3*b^3*c*g^3 + 18*a*b^5*c^2*e^2 + 16*a^5*b*h^4 + 16*a*b^5*d^4 +
81*a^6*i^4 + 81*b^6*c^4 + a^4*b^2*g^4 + a^2*b^4*e^4, z, l)*(root(65536*a^7
*b^7*z^4 + 3072*a^6*b^4*g*i*z^2 + 9216*a^5*b^5*c*i*z^2 + 4096*a^5*b^5*d*h*
z^2 + 1024*a^5*b^5*e*g*z^2 + 3072*a^4*b^6*c*e*z^2 + 2048*a^6*b^4*h^2*z^2 +
 2048*a^4*b^6*d^2*z^2 + 768*a^5*b^3*e*h*i*z + 768*a^4*b^4*d*e*i*z - 768*a^
4*b^4*c*g*h*z - 768*a^3*b^5*c*d*g*z + 1152*a^6*b^2*h*i^2*z - 128*a^5*b^...
```

3.197 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx$

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3.197.1 Optimal result

Integrand size = 45, antiderivative size = 417

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx \\
&= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{4ab\,(a+bx^4)} + \frac{(bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} \\
&\quad - \frac{\left(\sqrt{b}(3bc+ag)+\sqrt{a}(be+3ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{7/4}} \\
&\quad + \frac{\left(\sqrt{b}(3bc+ag)+\sqrt{a}(be+3ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{7/4}} \\
&\quad - \frac{\left(\sqrt{b}(3bc+ag)-\sqrt{a}(be+3ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{7/4}} \\
&\quad + \frac{\left(\sqrt{b}(3bc+ag)-\sqrt{a}(be+3ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{7/4}} + \frac{j\log(a+bx^4)}{4b^2}
\end{aligned}$$

output
```
1/4*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+(-a*j+b*f)*x^3)/a/b/(b*x^4+a)+1
/4*(a*h+b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)+1/4*j*ln(b*x^4+a)
/b^2-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+b*e)
*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1
/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/
2))/a^(7/4)/b^(7/4)*2^(1/2)+1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*
a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+1/16*arctan(
1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+b*e)*a^(1/2)+(a*g+3*b*c)*b^(1/2))/a^(
7/4)/b^(7/4)*2^(1/2)
```

3.197.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx$$

$$= \frac{\frac{8(a^2j+b^2x(c+x(d+ex))-ab(f+x(g+x(h+ix))))}{a(a+bx^4)} - \frac{2\sqrt[4]{b}\left(3\sqrt{2}b^{3/2}c+4\sqrt[4]{a}b^{5/4}d+\sqrt{2}\sqrt{a}be+\sqrt{2}a\sqrt{b}g+4a^{5/4}\sqrt[4]{b}h+3\sqrt{2}a^{3/2}i\right)\arctan\left(1-\cdots\right.}{a^{7/4}}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b
*x^4)^2,x]
```

output
```
((8*(a^2*j + b^2*x*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x)))))/(a*
(a + b*x^4)) - (2*b^(1/4)*(3*Sqrt[2]*b^(3/2)*c + 4*a^(1/4)*b^(5/4)*d + Sqr
t[2]*Sqrt[a]*b*e + Sqrt[2]*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a
^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*b^(1/4)*(3
*Sqrt[2]*b^(3/2)*c - 4*a^(1/4)*b^(5/4)*d + Sqrt[2]*Sqrt[a]*b*e + Sqrt[2]*a
*Sqrt[b]*g - 4*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*b^(1/4)*(-3*b^(3/2)*c + Sqrt[a]*
b*e - a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2])/a^(7/4) + (Sqrt[2]*b^(1/4)*(3*b^(3/2)*c - Sqrt[a]*b*e + a*S
qrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/a^(7/4) + 8*j*Log[a + b*x^4])/(32*b^2)
```

3.197.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2397, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}dx$$

$$\Big\downarrow \ 2397$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{4ab\,(a+bx^4)}-\frac{\int -\frac{4abjx^3+b(be+3ai)x^2+2b(bd+ah)x+b(3bc+ag)}{bx^4+a}dx}{4ab^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{4abjx^3+b(be+3ai)x^2+2b(bd+ah)x+b(3bc+ag)}{bx^4+a}dx}{4ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{4ab\,(a+bx^4)}$$

$$\Big\downarrow \ 2415$$

$$\frac{\int \left(\frac{b(be+3ai)x^2+b(3bc+ag)}{bx^4+a}+\frac{x\left(4abjx^2+2b(bd+ah)\right)}{bx^4+a}\right)dx}{4ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{4ab\,(a+bx^4)}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{2\sqrt{2}a^{3/4}}+\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}(ag+3bc)+\sqrt{a}(3ai+be)\right)}{2\sqrt{2}a^{3/4}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{4ab\,(a+bx^4)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^
2,x]
```

output
```
(x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(4*a*b
*(a + b*x^4)) + ((Sqrt[b]*(b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[
a] - (b^(1/4)*(Sqrt[b]*(3*b*c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)) + (b^(1/4)*(Sqrt[b]*(3*b*
c + a*g) + Sqrt[a]*(b*e + 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)) - (b^(1/4)*(Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*
a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(3/4)) + (b^(1/4)*(Sqrt[b]*(3*b*c + a*g) - Sqrt[a]*(b*e + 3*a*i))*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)) + a*j
*Log[a + b*x^4])/(4*a*b^2)
```

3.197.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.197. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx$

3.197.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.37

method	result
risch	$\frac{-\frac{(ai-be)x^3}{4ab}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}+\frac{aj-bf}{4b^2}}{b\,x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(4j\_R^3+\frac{(3ai+be)\_R^2}{a}+\frac{2(ah+bd)\_R}{a}+\frac{ag+3bc}{a}\right)\ln\left(x-\_\right.}{\_R^3}}{16b^2}$
default	$\frac{-\frac{(ai-be)x^3}{4ab}-\frac{(ah-bd)x^2}{4ab}-\frac{(ag-bc)x}{4ab}+\frac{aj-bf}{4b^2}}{b\,x^4+a}+\frac{\frac{(ag+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right.\right.}{8a}}{}$

input
```
int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETU
RNVERBOSE)
```

output
```
(-1/4*(a*i-b*e)/a/b*x^3-1/4*(a*h-b*d)/a/b*x^2-1/4*(a*g-b*c)/a/b*x+1/4*(a*j
-b*f)/b^2)/(b*x^4+a)+1/16/b^2*sum((4*j*_R^3+1/a*(3*a*i+b*e)*_R^2+2/a*(a*h+
b*d)*_R+(a*g+3*b*c)/a)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.197.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algor
ithm="fricas")
```

output
```
Timed out
```

3.197.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,
x)
```

output
```
Timed out
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx$$
$$= \frac{(b^2e-abi)x^3-abf+a^2j+(b^2d-abh)x^2+(b^2c-abg)x}{4\,(ab^3x^4+a^2b^2)}$$
$$+ \frac{\frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}j+3\,b^2c-\sqrt{a}b^{\frac{3}{2}}e+abg-3\,a^{\frac{3}{2}}\sqrt{b}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{7}{4}}b^{\frac{1}{4}}j-3\,b^2c+\sqrt{a}b^{\frac{3}{2}}e-abg+3\,a^{\frac{3}{2}}\sqrt{b}i\right)\log\left(\sqrt{b}x^2\right.}{a^{\frac{3}{4}}b^{\frac{5}{4}}}}{}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algor
ithm="maxima")
```

output
```
1/4*((b^2*e - a*b*i)*x^3 - a*b*f + a^2*j + (b^2*d - a*b*h)*x^2 + (b^2*c -
a*b*g)*x)/(a*b^3*x^4 + a^2*b^2) + 1/32*(sqrt(2)*(4*sqrt(2)*a^(7/4)*b^(1/4)
*j + 3*b^2*c - sqrt(a)*b^(3/2)*e + a*b*g - 3*a^(3/2)*sqrt(b)*i)*log(sqrt(b
)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(
4*sqrt(2)*a^(7/4)*b^(1/4)*j - 3*b^2*c + sqrt(a)*b^(3/2)*e - a*b*g + 3*a^(3
/2)*sqrt(b)*i)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(
3/4)*b^(5/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e
 + sqrt(2)*a^(5/4)*b^(5/4)*g + 3*sqrt(2)*a^(7/4)*b^(3/4)*i - 4*sqrt(a)*b^2
*d - 4*a^(3/2)*b*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 2*(3
*sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e + sqrt(2)*a^(5/4)*b
^(5/4)*g + 3*sqrt(2)*a^(7/4)*b^(3/4)*i + 4*sqrt(a)*b^2*d + 4*a^(3/2)*b*h)*
arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sq
rt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/(a*b)
```

3.197.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.17

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx$$
$$= \frac{j\log\left(|bx^4+a|\right)}{4\,b^2} + \frac{(be-ai)x^3+(bd-ah)x^2+(bc-ag)x-\frac{abf-a^2j}{b}}{4\,(bx^4+a)ab}$$
$$+ \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^3d+2\sqrt{2}\sqrt{ab}ab^2h+3\,(ab^3)^{\frac14}\,b^3c+(ab^3)^{\frac14}\,ab^2g+(ab^3)^{\frac34}\,be+3\,(ab^3)^{\frac34}\,ai\right)\arctan\left(\frac{\sqrt{2}\left(2x\right.}{2}\right.}{16\,a^2b^4}$$
$$+ \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^3d+2\sqrt{2}\sqrt{ab}ab^2h+3\,(ab^3)^{\frac14}\,b^3c+(ab^3)^{\frac14}\,ab^2g+(ab^3)^{\frac34}\,be+3\,(ab^3)^{\frac34}\,ai\right)\arctan\left(\frac{\sqrt{2}\left(2x\right.}{2}\right.}{16\,a^2b^4}$$
$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac14}\,b^3c+(ab^3)^{\frac14}\,ab^2g-(ab^3)^{\frac34}\,be-3\,(ab^3)^{\frac34}\,ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^4}$$
$$- \frac{\sqrt{2}\left(3\,(ab^3)^{\frac14}\,b^3c+(ab^3)^{\frac14}\,ab^2g-(ab^3)^{\frac34}\,be-3\,(ab^3)^{\frac34}\,ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^4}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algor
ithm="giac")
```

output
```
1/4*j*log(abs(b*x^4 + a))/b^2 + 1/4*((b*e - a*i)*x^3 + (b*d - a*h)*x^2 + (
b*c - a*g)*x - (a*b*f - a^2*j)/b)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*sqrt
(2)*sqrt(a*b)*b^3*d + 2*sqrt(2)*sqrt(a*b)*a*b^2*h + 3*(a*b^3)^(1/4)*b^3*c
+ (a*b^3)^(1/4)*a*b^2*g + (a*b^3)^(3/4)*b*e + 3*(a*b^3)^(3/4)*a*i)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 1/16*sqrt
(2)*(2*sqrt(2)*sqrt(a*b)*b^3*d + 2*sqrt(2)*sqrt(a*b)*a*b^2*h + 3*(a*b^3)^(
1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g + (a*b^3)^(3/4)*b*e + 3*(a*b^3)^(3/4)*a
*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4)
+ 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g - (a*b^3)^(3
/4)*b*e - 3*(a*b^3)^(3/4)*a*i)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(a^2*b^4) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^3*c + (a*b^3)^(1/4)*a*b^2*g
- (a*b^3)^(3/4)*b*e - 3*(a*b^3)^(3/4)*a*i)*log(x^2 - sqrt(2)*x*(a/b)^(1/4)
 + sqrt(a/b))/(a^2*b^4)
```

3.197.9 Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 3939, normalized size of antiderivative = 9.45

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^
2,x)
```

output
```
((x*(b*c - a*g))/(4*a*b) - (b*f - a*j)/(4*b^2) + (x^2*(b*d - a*h))/(4*a*b)
 + (x^3*(b*e - a*i))/(4*a*b))/(a + b*x^4) + symsum(log(- root(65536*a^7*b^
8*z^4 - 65536*a^7*b^6*j*z^3 + 3072*a^6*b^5*g*i*z^2 + 9216*a^5*b^6*c*i*z^2
+ 4096*a^5*b^6*d*h*z^2 + 1024*a^5*b^6*e*g*z^2 + 3072*a^4*b^7*c*e*z^2 + 245
76*a^7*b^4*j^2*z^2 + 2048*a^6*b^5*h^2*z^2 + 2048*a^4*b^7*d^2*z^2 - 1536*a^
6*b^3*g*i*j*z - 4608*a^5*b^4*c*i*j*z - 2048*a^5*b^4*d*h*j*z + 768*a^5*b^4*
e*h*i*z - 512*a^5*b^4*e*g*j*z - 1536*a^4*b^5*c*e*j*z + 768*a^4*b^5*d*e*i*z
 - 768*a^4*b^5*c*g*h*z - 768*a^3*b^6*c*d*g*z - 1024*a^6*b^3*h^2*j*z + 1152
*a^6*b^3*h*i^2*z - 128*a^5*b^4*g^2*h*z - 1024*a^4*b^5*d^2*j*z + 1152*a^5*b
^4*d*i^2*z + 128*a^4*b^5*e^2*h*z - 1152*a^3*b^6*c^2*h*z - 128*a^4*b^5*d*g^
2*z + 128*a^3*b^6*d*e^2*z - 1152*a^2*b^7*c^2*d*z - 4096*a^7*b^2*j^3*z - 19
2*a^5*b^2*e*h*i*j - 192*a^4*b^3*d*e*i*j + 192*a^4*b^3*c*g*h*j - 96*a^4*b^3
*d*g*h*i - 288*a^3*b^4*c*d*h*i + 192*a^3*b^4*c*d*g*j + 72*a^3*b^4*c*e*g*i
- 32*a^3*b^4*d*e*g*h - 96*a^2*b^5*c*d*e*h + 32*a^5*b^2*g^2*h*j - 48*a^5*b^
2*g*h^2*i - 288*a^5*b^2*d*i^2*j - 32*a^4*b^3*e^2*h*j + 576*a^5*b^2*c*i*j^2
 + 256*a^5*b^2*d*h*j^2 + 64*a^5*b^2*e*g*j^2 + 288*a^3*b^4*c^2*h*j + 32*a^4
*b^3*d*g^2*j + 12*a^4*b^3*e*g^2*i - 144*a^4*b^3*c*h^2*i - 48*a^3*b^4*d^2*g
*i - 16*a^4*b^3*e*g*h^2 + 108*a^4*b^3*c*g*i^2 - 32*a^3*b^4*d*e^2*j + 192*a
^4*b^3*c*e*j^2 + 288*a^2*b^5*c^2*d*j + 108*a^2*b^5*c^2*e*i - 144*a^2*b^5*c
*d^2*i - 48*a^3*b^4*c*e*h^2 - 16*a^2*b^5*d^2*e*g + 12*a^2*b^5*c*e^2*g -...
```

3.197. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^2}\,dx$

3.198 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3} dx$

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3.198.1 Optimal result

Integrand size = 36, antiderivative size = 241

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3} dx = \frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{8ab(a-bx^4)^2} + \frac{4af+x(7bc-ag+2(3bd-ah)x+5bex^2)}{32a^2b(a-bx^4)} + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e-3ag\right)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} + \frac{(3bd-ah)\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}$$

output
```
1/8*x*(b*c+a*g+(a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(-b*x^4+a)^2+1/32*(4*a*f+x
*(7*b*c-a*g+2*(-a*h+3*b*d)*x+5*b*e*x^2))/a^2/b/(-b*x^4+a)+1/16*(-a*h+3*b*d
)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)+1/64*arctan(b^(1/4)*x/a^(1/
4))*(21*b*c-3*a*g-5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)+1/64*arctanh(b^(1/
4)*x/a^(1/4))*(21*b*c-3*a*g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)
```

3.198.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx$$
$$= \frac{\frac{4a^{3/4}\sqrt{b}x(7bc+bx(6d+5ex)-a(g+2hx))}{a-bx^4}+\frac{16a^{7/4}\sqrt{b}(bx(c+x(d+ex))+a(f+x(g+hx)))}{(a-bx^4)^2}+2\sqrt[4]{b}\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\arctan}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]
```

output
```
((4*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) - a*(g + 2*h*x)))/(a - b*
x^4) + (16*a^(7/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/
(a - b*x^4)^2 + 2*b^(1/4)*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b
^(1/4)*x)/a^(1/4)] + (-21*b^(5/4)*c - 12*a^(1/4)*b*d - 5*Sqrt[a]*b^(3/4)*e
 + 3*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(5/4)*c -
 12*a^(1/4)*b*d + 5*Sqrt[a]*b^(3/4)*e - 3*a*b^(1/4)*g + 4*a^(5/4)*h)*Log[a
^(1/4) + b^(1/4)*x] - 4*a^(1/4)*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])
/(128*a^(11/4)*b^(3/2))
```

3.198.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2397, 25, 2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx$$
$$\Big\downarrow \text{2397}$$
$$\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}-\frac{\int -\frac{4b^2fx^3+5b^2ex^2+2b(3bd-ah)x+b(7bc-ag)}{(a-bx^4)^2}dx}{8ab^2}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{4b^2fx^3+5b^2ex^2+2b(3bd-ah)x+b(7bc-ag)}{(a-bx^4)^2}dx}{8ab^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \; 2393$$

$$\frac{\frac{x(b(7bc-ag)+2bx(3bd-ah)+5b^2ex^2)+4abf}{4a(a-bx^4)}-\frac{\int -\frac{5b^2ex^2+4b(3bd-ah)x+3b(7bc-ag)}{a-bx^4}dx}{4a}}{8ab^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{5b^2ex^2+4b(3bd-ah)x+3b(7bc-ag)}{a-bx^4}dx}{4a}+\frac{x(b(7bc-ag)+2bx(3bd-ah)+5b^2ex^2)+4abf}{4a(a-bx^4)}}{8ab^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{\int \left(\frac{4b(3bd-ah)x}{a-bx^4}+\frac{5b^2ex^2+3b(7bc-ag)}{a-bx^4}\right)dx}{4a}+\frac{x(b(7bc-ag)+2bx(3bd-ah)+5b^2ex^2)+4abf}{4a(a-bx^4)}}{8ab^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{2a^{3/4}}+\frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{2a^{3/4}}+\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(3bd-ah)}{\sqrt{a}}}{4a}+\frac{x(b(7bc-ag)+2bx(3b\cdots}{4a(\cdots}}{8ab^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x]
```

output
```
(x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2)
+ ((4*a*b*f + x*(b*(7*b*c - a*g) + 2*b*(3*b*d - a*h)*x + 5*b^2*e*x^2))/(4*
a*(a - b*x^4)) + ((b^(3/4)*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(
b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)) + (b^(3/4)*(21*b*c + 5*Sqrt[a]*Sqrt[b]*e
- 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)) + (2*Sqrt[b]*(3*b*d - a
*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a])/(4*a))/(8*a*b^2)
```

3.198. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx$

3.198.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.198. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx$

3.198.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.72

method	result
risch	$\frac{-\frac{5be\,x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{\left(5\_R^2e-\frac{4(ah-3bd)\_R}{b}-\frac{3(a}{}\right.}{\_R^3}}{128a^2b}$
default	$\frac{-\frac{5be\,x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}+\frac{\frac{(-3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}}{}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-5/32*b*e/a^2*x^7+1/16*(a*h-3*b*d)/a^2*x^6+1/32*(a*g-7*b*c)/a^2*x^5+9/32/
a*e*x^3+1/16*(a*h+5*b*d)/a/b*x^2+1/32*(3*a*g+11*b*c)/a/b*x+1/8*f/b)/(-b*x^
4+a)^2-1/128/a^2/b*sum((5*_R^2*e-4/b*(a*h-3*b*d)*_R-3/b*(a*g-7*b*c))/_R^3*
ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.198.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx=\text{Timed out}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="frica
s")
```

output
```
Timed out
```

3.198.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

output
```
Timed out
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.31

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx =$$
$$-\frac{5\,b^2ex^7+2\,(3\,b^2d-abh)x^6-9\,abex^3+(7\,b^2c-abg)x^5-4\,a^2f-2\,(5\,abd+a^2h)x^2-(11\,abc+3\,a^2g)x}{32\,(a^2b^3x^8-2\,a^3b^2x^4+a^4b)}$$
$$+\frac{\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be-3\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c+5\,\sqrt{a}be-3\,a\sqrt{\ldots}\right.}{\sqrt{a}\sqrt{\ldots}}}{128\,a^2b}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxim
a")
```

output
```
-1/32*(5*b^2*e*x^7 + 2*(3*b^2*d - a*b*h)*x^6 - 9*a*b*e*x^3 + (7*b^2*c - a*
b*g)*x^5 - 4*a^2*f - 2*(5*a*b*d + a^2*h)*x^2 - (11*a*b*c + 3*a^2*g)*x)/(a^
2*b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(4*(3*b*d - a*h)*log(sqrt(b)*x^
2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 4*(3*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a)
)/(sqrt(a)*sqrt(b)) + 2*(21*b^(3/2)*c - 5*sqrt(a)*b*e - 3*a*sqrt(b)*g)*arc
tan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b
)) - (21*b^(3/2)*c + 5*sqrt(a)*b*e - 3*a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(
sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

3.198.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(201) = 402.

Time = 0.28 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.80

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx =$$

$$-\frac{\sqrt{2}\left(21\,b^2c-3\,abg-12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+4\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah+5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{\sqrt{2}\left(21\,b^2c-3\,abg+12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-4\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah-5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{\sqrt{2}\left(21\,b^2c-3\,abg-5\,\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$+\frac{\sqrt{2}\left(21\,b^2c-3\,abg-5\,\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{5\,b^2ex^7+6\,b^2dx^6-2\,abhx^6+7\,b^2cx^5-abgx^5-9\,abex^3-10\,abdx^2-2\,a^2hx^2-11\,abcx-3\,a^2gx-4\,a}{32\,(bx^4-a)^2a^2b}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac"
)
```

output
```
-1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 4*sqr
t(2)*(-a*b^3)^(1/4)*a*h + 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(21*b
^2*c - 3*a*b*g + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 4*sqrt(2)*(-a*b^3)^(1/4)*
a*h - 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-
a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*s
qrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3
/4)*a^2) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*e)*log(x^2 -
 sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b^2*e
*x^7 + 6*b^2*d*x^6 - 2*a*b*h*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*e*x^3 -
 10*a*b*d*x^2 - 2*a^2*h*x^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 -
a)^2*a^2*b)
```

3.198.9 Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 1687, normalized size of antiderivative = 7.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^3}\,dx = \text{Too large to display}$$

input

```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^3,x)
```

output

```
(f/(8*b) + (9*e*x^3)/(32*a) - (x^5*(7*b*c - a*g))/(32*a^2) - (x^6*(3*b*d -
 a*h))/(16*a^2) + (x*(11*b*c + 3*a*g))/(32*a*b) + (x^2*(5*b*d + a*h))/(16*
a*b) - (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(- ro
ot(268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z
^2 - 6881280*a^6*b^5*c*e*z^2 - 524288*a^8*b^3*h^2*z^2 - 4718592*a^6*b^5*d^
2*z^2 + 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^
2*h*z - 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z + 55296*a^5*b^3*d*g
^2*z + 153600*a^4*b^4*d*e^2*z + 2709504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e
*g*h + 40320*a^2*b^3*c*d*e*h + 8640*a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2
 - 6300*a^2*b^3*c*e^2*g + 960*a^4*b*e*g*h^2 - 60480*a*b^4*c*d^2*e - 3072*a
^4*b*d*h^3 + 111132*a*b^4*c^3*g + 13824*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*
g^2 - 23814*a^2*b^3*c^2*g^2 - 27648*a^2*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 2
2050*a*b^4*c^2*e^2 - 625*a^2*b^3*e^4 - 81*a^4*b*g^4 + 20736*a*b^4*d^4 + 25
6*a^5*h^4 - 194481*b^5*c^4, z, k)*(root(268435456*a^11*b^6*z^4 + 3145728*a
^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 - 6881280*a^6*b^5*c*e*z^2 - 524288
*a^8*b^3*h^2*z^2 - 4718592*a^6*b^5*d^2*z^2 + 258048*a^5*b^3*c*g*h*z - 7741
44*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z - 51200*a^5*b^3*e^2*h*z - 90316
8*a^4*b^4*c^2*h*z + 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z + 27095
04*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h + 40320*a^2*b^3*c*d*e*h + 8640*a
^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 - 6300*a^2*b^3*c*e^2*g + 960*a^4*...
```

3.199 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx$

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3.199.1 Optimal result

Integrand size = 41, antiderivative size = 268

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx &= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{8ab\,(a-bx^4)^2} \\ &+ \frac{4af+x(7bc-ag+2(3bd-ah)x+(5be-3ai)x^2)}{32a^2b\,(a-bx^4)} \\ &- \frac{\left(5be-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{7/4}} \\ &+ \frac{\left(5be+\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{7/4}} + \frac{(3bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}\end{aligned}$$

output
```
1/8*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+b*f*x^3)/a/b/(-b*x^4+a)^2+1/32*(4
*a*f+x*(7*b*c-a*g+2*(-a*h+3*b*d)*x+(-3*a*i+5*b*e)*x^2))/a^2/b/(-b*x^4+a)+1
/16*(-a*h+3*b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)-1/64*arctan(
b^(1/4)*x/a^(1/4))*(5*b*e-3*a*i-3*(-a*g+7*b*c)*b^(1/2)/a^(1/2))/a^(9/4)/b^
(7/4)+1/64*arctanh(b^(1/4)*x/a^(1/4))*(5*b*e-3*a*i+3*(-a*g+7*b*c)*b^(1/2)/
a^(1/2))/a^(9/4)/b^(7/4)
```

3.199.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.34

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx$$
$$= \frac{-\frac{4a^{3/4}b^{3/4}x(-b(7c+x(6d+5ex))+a(g+x(2h+3ix)))}{a-bx^4}+\frac{16a^{7/4}b^{3/4}(bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{(a-bx^4)^2}+2\left(21b^{3/2}c-5\sqrt{a}be-\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,
x]
```

output
```
((-4*a^(3/4)*b^(3/4)*x*(-(b*(7*c + x*(6*d + 5*e*x))) + a*(g + x*(2*h + 3*i
*x))))/(a - b*x^4) + (16*a^(7/4)*b^(3/4)*(b*x*(c + x*(d + e*x)) + a*(f + x
*(g + x*(h + i*x)))))/(a - b*x^4)^2 + 2*(21*b^(3/2)*c - 5*Sqrt[a]*b*e - 3*
a*Sqrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] + (-21*b^(3/2)*c -
12*a^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h
 + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + (21*b^(3/2)*c - 12*a^(1/4)*b^(5
/4)*d + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)
*Log[a^(1/4) + b^(1/4)*x] - 4*a^(1/4)*b^(1/4)*(-3*b*d + a*h)*Log[Sqrt[a] +
 Sqrt[b]*x^2])/(128*a^(11/4)*b^(7/4))
```

3.199.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {2397, 25, 2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{8ab\left(a-bx^4\right)^2}-\frac{\int -\frac{4b^2fx^3+b(5be-3ai)x^2+2b(3bd-ah)x+b(7bc-ag)}{(a-bx^4)^2}dx}{8ab^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4b^2fx^3+b(5be-3ai)x^2+2b(3bd-ah)x+b(7bc-ag)}{(a-bx^4)^2}dx}{8ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{ 2393}$$

$$\frac{\frac{x\left(b(7bc-ag)+2bx(3bd-ah)+bx^2(5be-3ai)\right)+4abf}{4a(a-bx^4)}-\frac{\int -\frac{b(5be-3ai)x^2+4b(3bd-ah)x+3b(7bc-ag)}{a-bx^4}dx}{4a}}{8ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{b(5be-3ai)x^2+4b(3bd-ah)x+3b(7bc-ag)}{a-bx^4}dx}{4a}+\frac{x\left(b(7bc-ag)+2bx(3bd-ah)+bx^2(5be-3ai)\right)+4abf}{4a(a-bx^4)}}{8ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{ 2415}$$

$$\frac{\frac{\int \left(\frac{4b(3bd-ah)x}{a-bx^4}+\frac{b(5be-3ai)x^2+3b(7bc-ag)}{a-bx^4}\right)dx}{4a}+\frac{x\left(b(7bc-ag)+2bx(3bd-ah)+bx^2(5be-3ai)\right)+4abf}{4a(a-bx^4)}}{8ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{-\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{2\sqrt[4]{a}}+\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{2\sqrt[4]{a}}+\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(3bd-ah)}{\sqrt{a}}}{4a}+\frac{x\left(b(7bc-\right.}{}}{8ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x]
```

3.199. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx$

output
```
(x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(8*a*b*(a - b*
x^4)^2) + ((4*a*b*f + x*(b*(7*b*c - a*g) + 2*b*(3*b*d - a*h)*x + b*(5*b*e
- 3*a*i)*x^2))/(4*a*(a - b*x^4)) + (-1/2*(b^(1/4)*(5*b*e - (3*Sqrt[b]*(7*b
*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(1/4) + (b^(1/4
)*(5*b*e + (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTanh[(b^(1/4)*x)/
a^(1/4)])/(2*a^(1/4)) + (2*Sqrt[b]*(3*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqr
t[a]])/Sqrt[a])/(4*a))/(8*a*b^2)
```

3.199.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.199.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\frac{(3ai-5be)x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{(ai+9be)x^3}{32ab}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b-a)}\frac{(-(3ai-5be)\_R^2-4(\ldots}{\ldots}}{128a^2b^2\ldots}$
default	$\frac{\frac{(3ai-5be)x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{(ai+9be)x^3}{32ab}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}+\frac{\frac{(-3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\ldots\right.\right.}{4a}\ldots}{\ldots}$

input
```
int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVER
BOSE)
```

output
```
(1/32*(3*a*i-5*b*e)/a^2*x^7+1/16*(a*h-3*b*d)/a^2*x^6+1/32*(a*g-7*b*c)/a^2*
x^5+1/32*(a*i+9*b*e)/a/b*x^3+1/16*(a*h+5*b*d)/a/b*x^2+1/32*(3*a*g+11*b*c)/
a/b*x+1/8*f/b)/(-b*x^4+a)^2-1/128/a^2/b^2*sum((-(3*a*i-5*b*e)*_R^2-4*(a*h-
3*b*d)*_R-3*a*g+21*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.199.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx=\text{Timed out}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm=
"fricas")
```

output
```
Timed out
```

3.199.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{(a - bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

output
```
Timed out
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.28

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{(a - bx^4)^3}\,dx = -\frac{(5\,b^2e - 3\,abi)x^7 + 2\,(3\,b^2d - abh)x^6 + (7\,b^2c - abg)x^5 - (9\,abe + a^2i)x^3 - 4\,a^2f - 2\,(5\,abd + a^2h)x^2 -}{32\,(a^2b^3x^8 - 2\,a^3b^2x^4 + a^4b)} + \frac{\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be-3\,a\sqrt{b}g+3\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(21\,b^{\frac{3}{2}}c+5\,\sqrt{a}be}{}}{128\,a^2b}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm=
"maxima")
```

output
```
-1/32*((5*b^2*e - 3*a*b*i)*x^7 + 2*(3*b^2*d - a*b*h)*x^6 + (7*b^2*c - a*b*
g)*x^5 - (9*a*b*e + a^2*i)*x^3 - 4*a^2*f - 2*(5*a*b*d + a^2*h)*x^2 - (11*a
*b*c + 3*a^2*g)*x)/(a^2*b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(4*(3*b*d
 - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 4*(3*b*d - a*h)*log
(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(21*b^(3/2)*c - 5*sqrt(a)*b*
e - 3*a*sqrt(b)*g + 3*a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*b^(3/2)*c + 5*sqrt(a)*b*e - 3
*a*sqrt(b)*g - 3*a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(
b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a
^2*b)
```

3.199.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(228) = 456$.

Time = 0.28 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.97

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{(a - bx^4)^3}\, dx =$$

$$-\frac{\sqrt{2}\left(21\, b^3c - 3\, ab^2g - 12\, \sqrt{2}(-ab^3)^{\frac{1}{4}}\, b^2d + 4\, \sqrt{2}(-ab^3)^{\frac{1}{4}}\, abh - 5\, \sqrt{-ab}b^2e + 3\, \sqrt{-ab}abi\right) \arctan\left(\frac{\sqrt{2}\left(2\right.}{}\right.}{128\, (-ab^3)^{\frac{3}{4}}\, a^2b}$$

$$-\frac{\sqrt{2}\left(21\, b^3c - 3\, ab^2g + 12\, \sqrt{2}(-ab^3)^{\frac{1}{4}}\, b^2d - 4\, \sqrt{2}(-ab^3)^{\frac{1}{4}}\, abh - 5\, \sqrt{-ab}b^2e - 3\, \sqrt{-ab}abi\right) \arctan\left(\frac{\sqrt{2}\left(2\right.}{}\right.}{128\, (-ab^3)^{\frac{3}{4}}\, a^2b}$$

$$-\frac{\sqrt{2}(21\, b^3c - 3\, ab^2g - 5\, \sqrt{-ab}b^2e + 3\, \sqrt{-ab}abi) \log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256\, (-ab^3)^{\frac{3}{4}}\, a^2b}$$

$$+\frac{\sqrt{2}(21\, b^3c - 3\, ab^2g - 5\, \sqrt{-ab}b^2e + 3\, \sqrt{-ab}abi) \log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256\, (-ab^3)^{\frac{3}{4}}\, a^2b}$$

$$-\frac{5\, b^2ex^7 - 3\, abix^7 + 6\, b^2dx^6 - 2\, abhx^6 + 7\, b^2cx^5 - abgx^5 - 9\, abex^3 - a^2ix^3 - 10\, abdx^2 - 2\, a^2hx^2 - 11\, a}{32\, (bx^4 - a)^2a^2b}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm=
"giac")
```

output
```
-1/128*sqrt(2)*(21*b^3*c - 3*a*b^2*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b^2*d + 4
*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 5*sqrt(-a*b)*b^2*e + 3*sqrt(-a*b)*a*b*i)*a
rctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/
4)*a^2*b) - 1/128*sqrt(2)*(21*b^3*c - 3*a*b^2*g + 12*sqrt(2)*(-a*b^3)^(1/4
)*b^2*d - 4*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 5*sqrt(-a*b)*b^2*e - 3*sqrt(-a*
b)*a*b*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((
-a*b^3)^(3/4)*a^2*b) - 1/256*sqrt(2)*(21*b^3*c - 3*a*b^2*g - 5*sqrt(-a*b)*
b^2*e + 3*sqrt(-a*b)*a*b*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))
/((-a*b^3)^(3/4)*a^2*b) + 1/256*sqrt(2)*(21*b^3*c - 3*a*b^2*g - 5*sqrt(-a*
b)*b^2*e + 3*sqrt(-a*b)*a*b*i)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/
b))/((-a*b^3)^(3/4)*a^2*b) - 1/32*(5*b^2*e*x^7 - 3*a*b*i*x^7 + 6*b^2*d*x^6
 - 2*a*b*h*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*e*x^3 - a^2*i*x^3 - 10*a*
b*d*x^2 - 2*a^2*h*x^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a
^2*b)
```

3.199.9 Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 2680, normalized size of antiderivative = 10.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^3,x)
```

output
```
symsum(log((27*a^4*i^3 - 125*a*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e -
 336*a^2*b^2*c*h^2 + 45*a^2*b^2*e*g^2 + 225*a^2*b^2*e^2*i + 432*a*b^3*d^2*
g - 1323*a*b^3*c^2*i - 135*a^3*b*e*i^2 + 48*a^3*b*g*h^2 - 27*a^3*b*g^2*i +
 378*a^2*b^2*c*g*i - 288*a^2*b^2*d*g*h + 2016*a*b^3*c*d*h - 630*a*b^3*c*e*
g)/(32768*a^6*b^2) - root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2
+ 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z
^2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^
2*z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*
e*i*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2
*h*z + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*
h*z + 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2
*d*z + 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i
 - 5760*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 51
84*a^3*b^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4
*b^2*c*g*i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^
4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*
i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*
b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e
^2*g^2 - 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i -
27648*a^2*b^4*d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a...
```

3.200 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3} dx$

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3.200.1 Optimal result

Integrand size = 46, antiderivative size = 285

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3} dx = \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{8ab(a-bx^4)^2} + \frac{4a(bf-aj)+x(b(7bc-ag)+2b(3bd-ah)x+b(5be-3ai)x^2)}{32a^2b^2(a-bx^4)} - \frac{\left(5be-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{7/4}} + \frac{\left(5be+\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{9/4}b^{7/4}} + \frac{(3bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}$$

output
```
1/8*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+(a*j+b*f)*x^3)/a/b/(-b*x^4+a)^2+1
/32*(4*a*(-a*j+b*f)+x*(b*(-a*g+7*b*c)+2*b*(-a*h+3*b*d)*x+b*(-3*a*i+5*b*e)*
x^2))/a^2/b^2/(-b*x^4+a)+1/16*(-a*h+3*b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^
(5/2)/b^(3/2)-1/64*arctan(b^(1/4)*x/a^(1/4))*(5*b*e-3*a*i-3*(-a*g+7*b*c)*b
^(1/2)/a^(1/2))/a^(9/4)/b^(7/4)+1/64*arctanh(b^(1/4)*x/a^(1/4))*(5*b*e-3*a
*i+3*(-a*g+7*b*c)*b^(1/2)/a^(1/2))/a^(9/4)/b^(7/4)
```

3.200.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx$$
$$= \frac{-\frac{4a^{3/4}(8a^2j-b^2x(7c+x(6d+5ex))+abx(g+x(2h+3ix)))}{a-bx^4}+\frac{16a^{7/4}(a^2j+b^2x(c+x(d+ex))+ab(f+x(g+x(h+ix))))}{(a-bx^4)^2}+2\sqrt[4]{b}\Big(21b^{3/2}c-5}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b
*x^4)^3,x]
```

output
```
((-4*a^(3/4)*(8*a^2*j - b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g + x*(2*h
+ 3*i*x))))/(a - b*x^4) + (16*a^(7/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) + a
*b*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^2 + 2*b^(1/4)*(21*b^(3/2)*c - 5
*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^(1/4)] +
b^(1/4)*(-21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d - 5*Sqrt[a]*b*e + 3*a*Sqrt[b
]*g + 4*a^(5/4)*b^(1/4)*h + 3*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)*x] + b^(1/4
)*(21*b^(3/2)*c - 12*a^(1/4)*b^(5/4)*d + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 4
*a^(5/4)*b^(1/4)*h - 3*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x] - 4*a^(1/4)*Sqr
t[b]*(-3*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)*b^2)
```

3.200.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2397, 25, 2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx$$
$$\Big\downarrow \; 2397$$
$$\frac{x(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc)}{8ab(a-bx^4)^2}-\frac{\int -\frac{4b(bf-aj)x^3+b(5be-3ai)x^2+2b(3bd-ah)x+b(7bc-ag)}{(a-bx^4)^2}dx}{8ab^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4b(bf-aj)x^3+b(5be-3ai)x^2+2b(3bd-ah)x+b(7bc-ag)}{(a-bx^4)^2}dx}{8ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{2393}$$

$$\frac{\frac{x\left(b(7bc-ag)+2bx(3bd-ah)+bx^2(5be-3ai)\right)+4a(bf-aj)}{4a(a-bx^4)}-\frac{\int -\frac{b(5be-3ai)x^2+4b(3bd-ah)x+3b(7bc-ag)}{a-bx^4}dx}{4a}}{8ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b(5be-3ai)x^2+4b(3bd-ah)x+3b(7bc-ag)}{a-bx^4}dx}{4a}+\frac{x\left(b(7bc-ag)+2bx(3bd-ah)+bx^2(5be-3ai)\right)+4a(bf-aj)}{4a(a-bx^4)}}{8ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{\int \left(\frac{4b(3bd-ah)x}{a-bx^4}+\frac{b(5be-3ai)x^2+3b(7bc-ag)}{a-bx^4}\right)dx}{4a}+\frac{x\left(b(7bc-ag)+2bx(3bd-ah)+bx^2(5be-3ai)\right)+4a(bf-aj)}{4a(a-bx^4)}}{8ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{2\sqrt[4]{a}}+\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{3\sqrt{b}(7bc-ag)}{\sqrt{a}}-3ai+5be\right)}{2\sqrt[4]{a}}+\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(3bd-ah)}{\sqrt{a}}}{4a}+\frac{x(b(7bc-}{}}{8ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{8ab\left(a-bx^4\right)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^
3,x]
```

3.200. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx$

output
```
(x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(8*a*b
*(a - b*x^4)^2) + ((4*a*(b*f - a*j) + x*(b*(7*b*c - a*g) + 2*b*(3*b*d - a*
h)*x + b*(5*b*e - 3*a*i)*x^2))/(4*a*(a - b*x^4)) + (-1/2*(b^(1/4)*(5*b*e -
 (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcTan[(b^(1/4)*x)/a^(1/4)])/a
^(1/4) + (b^(1/4)*(5*b*e + (3*Sqrt[b]*(7*b*c - a*g))/Sqrt[a] - 3*a*i)*ArcT
anh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)) + (2*Sqrt[b]*(3*b*d - a*h)*ArcTanh[(
Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a])/(4*a))/(8*a*b^2)
```

3.200.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.200. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx$

3.200.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.72

method	result
risch	$\frac{\frac{(3ai-5be)x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{j\,x^4}{4b}+\frac{(ai+9be)x^3}{32ab}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}-\frac{aj-bf}{8b^2}}{\left(-b\,x^4+a\right)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4b-a\right)}\frac{\left(-(3ai-5be)\right.}{}}{}$
default	$\frac{\frac{(3ai-5be)x^7}{32a^2}+\frac{(ah-3bd)x^6}{16a^2}+\frac{(ag-7bc)x^5}{32a^2}+\frac{j\,x^4}{4b}+\frac{(ai+9be)x^3}{32ab}+\frac{(ah+5bd)x^2}{16ab}+\frac{(3ag+11bc)x}{32ab}-\frac{aj-bf}{8b^2}}{\left(-b\,x^4+a\right)^2}+\frac{\frac{(-3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right.}{4a}}{}$

input
```
int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RET
URNVERBOSE)
```

output
```
(1/32*(3*a*i-5*b*e)/a^2*x^7+1/16*(a*h-3*b*d)/a^2*x^6+1/32*(a*g-7*b*c)/a^2*
x^5+1/4*j*x^4/b+1/32*(a*i+9*b*e)/a/b*x^3+1/16*(a*h+5*b*d)/a/b*x^2+1/32*(3*
a*g+11*b*c)/a/b*x-1/8*(a*j-b*f)/b^2)/(-b*x^4+a)^2-1/128/a^2/b^2*sum((-(3*a
*i-5*b*e)*_R^2-4*(a*h-3*b*d)*_R-3*a*g+21*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4
*b-a))
```

3.200.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a-bx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algo
rithm="fricas")
```

output
```
Timed out
```

3.200.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3
,x)
```

output
```
Timed out
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.32

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx$$
$$= \frac{8\,a^2bjx^4-(5\,b^3e-3\,ab^2i)x^7-2\,(3\,b^3d-ab^2h)x^6-(7\,b^3c-ab^2g)x^5+4\,a^2bf-4\,a^3j+(9\,ab^2e+a^2bi)x^3}{32\,(a^2b^4x^8-2\,a^3b^3x^4+a^4b^2)}$$
$$+\frac{\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{4\,(3\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be-3\,a\sqrt{b}g+3\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c+5\,\sqrt{a}be\right.}{}}{128\,a^2b}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algo
rithm="maxima")
```

output
```
1/32*(8*a^2*b*j*x^4 - (5*b^3*e - 3*a*b^2*i)*x^7 - 2*(3*b^3*d - a*b^2*h)*x^
6 - (7*b^3*c - a*b^2*g)*x^5 + 4*a^2*b*f - 4*a^3*j + (9*a*b^2*e + a^2*b*i)*
x^3 + 2*(5*a*b^2*d + a^2*b*h)*x^2 + (11*a*b^2*c + 3*a^2*b*g)*x)/(a^2*b^4*x
^8 - 2*a^3*b^3*x^4 + a^4*b^2) + 1/128*(4*(3*b*d - a*h)*log(sqrt(b)*x^2 + s
qrt(a))/(sqrt(a)*sqrt(b)) - 4*(3*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sq
rt(a)*sqrt(b)) + 2*(21*b^(3/2)*c - 5*sqrt(a)*b*e - 3*a*sqrt(b)*g + 3*a^(3/
2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))*sqrt(b)) - (21*b^(3/2)*c + 5*sqrt(a)*b*e - 3*a*sqrt(b)*g - 3*a^(3/2)*i)
*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b)
)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

3.200.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(245) = 490$.

Time = 0.29 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^3}\,dx =$$

$$-\frac{\sqrt{2}\left(21\,b^3c-3\,ab^2g-12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d+4\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-5\,\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\right.}{}\right.}{128\,(-ab^3)^{\frac{3}{4}}\,a^2b}$$
$$-\frac{\sqrt{2}\left(21\,b^3c-3\,ab^2g+12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d-4\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-5\,\sqrt{-ab}b^2e-3\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(2\right.}{}\right.}{128\,(-ab^3)^{\frac{3}{4}}\,a^2b}$$
$$-\frac{\sqrt{2}\left(21\,b^3c-3\,ab^2g-5\,\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2b}$$
$$+\frac{\sqrt{2}\left(21\,b^3c-3\,ab^2g-5\,\sqrt{-ab}b^2e+3\,\sqrt{-ab}abi\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2b}$$
$$-\frac{5\,b^3ex^7-3\,ab^2ix^7+6\,b^3dx^6-2\,ab^2hx^6+7\,b^3cx^5-ab^2gx^5-8\,a^2bjx^4-9\,ab^2ex^3-a^2bix^3-10\,ab^2dx^2}{32\,(bx^4-a)^2a^2b^2}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algo
rithm="giac")
```

output
```
-1/128*sqrt(2)*(21*b^3*c - 3*a*b^2*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b^2*d + 4
*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 5*sqrt(-a*b)*b^2*e + 3*sqrt(-a*b)*a*b*i)*a
rctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/
4)*a^2*b) - 1/128*sqrt(2)*(21*b^3*c - 3*a*b^2*g + 12*sqrt(2)*(-a*b^3)^(1/4
)*b^2*d - 4*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 5*sqrt(-a*b)*b^2*e - 3*sqrt(-a*
b)*a*b*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((
-a*b^3)^(3/4)*a^2*b) - 1/256*sqrt(2)*(21*b^3*c - 3*a*b^2*g - 5*sqrt(-a*b)*
b^2*e + 3*sqrt(-a*b)*a*b*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))
/((-a*b^3)^(3/4)*a^2*b) + 1/256*sqrt(2)*(21*b^3*c - 3*a*b^2*g - 5*sqrt(-a*
b)*b^2*e + 3*sqrt(-a*b)*a*b*i)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/
b))/((-a*b^3)^(3/4)*a^2*b) - 1/32*(5*b^3*e*x^7 - 3*a*b^2*i*x^7 + 6*b^3*d*x
^6 - 2*a*b^2*h*x^6 + 7*b^3*c*x^5 - a*b^2*g*x^5 - 8*a^2*b*j*x^4 - 9*a*b^2*e
*x^3 - a^2*b*i*x^3 - 10*a*b^2*d*x^2 - 2*a^2*b*h*x^2 - 11*a*b^2*c*x - 3*a^2
*b*g*x - 4*a^2*b*f + 4*a^3*j)/((b*x^4 - a)^2*a^2*b^2)
```

3.200.9 Mupad [B] (verification not implemented)

Time = 10.02 (sec) , antiderivative size = 2696, normalized size of antiderivative = 9.46

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{(a - bx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^
3,x)
```

output
```
symsum(log((27*a^4*i^3 - 125*a*b^3*e^3 - 3024*b^4*c*d^2 + 2205*b^4*c^2*e -
 336*a^2*b^2*c*h^2 + 45*a^2*b^2*e*g^2 + 225*a^2*b^2*e^2*i + 432*a*b^3*d^2*
g - 1323*a*b^3*c^2*i - 135*a^3*b*e*i^2 + 48*a^3*b*g*h^2 - 27*a^3*b*g^2*i +
 378*a^2*b^2*c*g*i - 288*a^2*b^2*d*g*h + 2016*a*b^3*c*d*h - 630*a*b^3*c*e*
g)/(32768*a^6*b^2) - root(268435456*a^11*b^7*z^4 - 589824*a^8*b^4*g*i*z^2
+ 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z
^2 - 6881280*a^6*b^6*c*e*z^2 - 524288*a^8*b^4*h^2*z^2 - 4718592*a^6*b^6*d^
2*z^2 + 61440*a^6*b^3*e*h*i*z + 258048*a^5*b^4*c*g*h*z - 184320*a^5*b^4*d*
e*i*z - 774144*a^4*b^5*c*d*g*z - 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2
*h*z + 55296*a^6*b^3*d*i^2*z - 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*
h*z + 55296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z + 2709504*a^3*b^6*c^2
*d*z + 3456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i
 - 5760*a^3*b^3*d*e*g*h + 40320*a^2*b^4*c*d*e*h - 540*a^4*b^2*e*g^2*i - 51
84*a^3*b^3*d^2*g*i + 4032*a^4*b^2*c*h^2*i + 960*a^4*b^2*e*g*h^2 - 2268*a^4
*b^2*c*g*i^2 - 26460*a^2*b^4*c^2*e*i + 36288*a^2*b^4*c*d^2*i + 8640*a^2*b^
4*d^2*e*g - 6720*a^3*b^3*c*e*h^2 - 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*
i - 60480*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g - 1350*a^4*
b^2*e^2*i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e
^2*g^2 - 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i -
27648*a^2*b^4*d^3*h - 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a...
```

3.201 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx$

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3.201.1 Optimal result

Integrand size = 35, antiderivative size = 413

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx &= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{8ab(a+bx^4)^2}-\frac{4af-x(7bc+ag+2(3bd+ah)x+5bex^2)}{32a^2b(a+bx^4)} \\ &+\frac{(3bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}-\frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} \\ &+\frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} \\ &-\frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}} \\ &+\frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{5/4}}\end{aligned}$$

output
```
1/8*x*(b*c-a*g+(-a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(b*x^4+a)^2+1/32*(-4*a*f+
x*(7*b*c+a*g+2*(a*h+3*b*d)*x+5*b*e*x^2))/a^2/b/(b*x^4+a)+1/16*(a*h+3*b*d)*
arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)-1/256*ln(-a^(1/4)*b^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*b^(1/2))*(21*b*c+3*a*g-5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(
5/4)*2^(1/2)+1/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(21*b
*c+3*a*g-5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+1/128*arctan(-1+b^(
1/4)*x*2^(1/2)/a^(1/4))*(21*b*c+3*a*g+5*e*a^(1/2)*b^(1/2))/a^(11/4)/b^(5/4
)*2^(1/2)+1/128*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(21*b*c+3*a*g+5*e*a^(1
/2)*b^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)
```

3.201.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx$$
$$= \frac{\frac{8a^{3/4}\sqrt{b}x(7bc+bx(6d+5ex)+a(g+2hx))}{a+bx^4} - \frac{32a^{7/4}\sqrt{b}(-bx(c+x(d+ex))+a(f+x(g+hx)))}{(a+bx^4)^2} - 2\left(21\sqrt{2}b^{5/4}c + 24\sqrt[4]{a}bd + 5\sqrt{2}\sqrt{a}b\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```

output
```
((8*a^(3/4)*Sqrt[b]*x*(7*b*c + b*x*(6*d + 5*e*x) + a*(g + 2*h*x)))/(a + b*
x^4) - (32*a^(7/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x))) + a*(f + x*(g + h*x))
))/(a + b*x^4)^2 - 2*(21*Sqrt[2]*b^(5/4)*c + 24*a^(1/4)*b*d + 5*Sqrt[2]*Sq
rt[a]*b^(3/4)*e + 3*Sqrt[2]*a*b^(1/4)*g + 8*a^(5/4)*h)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b^(5/4)*c - 24*a^(1/4)*b*d + 5*Sqrt[2
]*Sqrt[a]*b^(3/4)*e + 3*Sqrt[2]*a*b^(1/4)*g - 8*a^(5/4)*h)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e
- 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*
b^(1/4)*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(3/2))
```

3.201.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2397, 25, 2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx$$

$$\Big\downarrow \ 2397$$

$$\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{8ab\,(a+bx^4)^2}-\frac{\int -\frac{4b^2fx^3+5b^2ex^2+2b(3bd+ah)x+b(7bc+ag)}{(bx^4+a)^2}dx}{8ab^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{4b^2fx^3+5b^2ex^2+2b(3bd+ah)x+b(7bc+ag)}{(bx^4+a)^2}dx}{8ab^2}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{8ab\,(a+bx^4)^2}$$

$$\Big\downarrow \ 2393$$

$$\frac{-\frac{\int -\frac{5b^2ex^2+4b(3bd+ah)x+3b(7bc+ag)}{bx^4+a}dx}{4a}-\frac{4abf-x\left(b(ag+7bc)+2bx(ah+3bd)+5b^2ex^2\right)}{4a(a+bx^4)}}{8ab^2}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{8ab\,(a+bx^4)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{5b^2ex^2+4b(3bd+ah)x+3b(7bc+ag)}{bx^4+a}dx}{4a}-\frac{4abf-x\left(b(ag+7bc)+2bx(ah+3bd)+5b^2ex^2\right)}{4a(a+bx^4)}}{8ab^2}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{8ab\,(a+bx^4)^2}$$

$$\Big\downarrow \ 2415$$

$$\frac{\frac{\int \left(\frac{4b(3bd+ah)x}{bx^4+a}+\frac{5b^2ex^2+3b(7bc+ag)}{bx^4+a}\right)dx}{4a}-\frac{4abf-x\left(b(ag+7bc)+2bx(ah+3bd)+5b^2ex^2\right)}{4a(a+bx^4)}}{8ab^2}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{8ab\,(a+bx^4)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{2\sqrt{2}a^{3/4}}+\frac{b^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{2\sqrt{2}a^{3/4}}-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3a\right.}{4\sqrt{2}a^{3/4}}}{4a}$$

$$8ab$$

$$\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x]
```

output
```
(x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2)
+ (-1/4*(4*a*b*f - x*(b*(7*b*c + a*g) + 2*b*(3*b*d + a*h)*x + 5*b^2*e*x^2)
)/(a*(a + b*x^4)) + ((2*Sqrt[b]*(3*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]
])/Sqrt[a] - (b^(3/4)*(21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 - (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)) + (b^(3/4)*(21*b*c + 5*Sqr
t[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2
]*a^(3/4)) - (b^(3/4)*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)) + (b^(3/4)*
(21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)))/(4*a))/(8*a*b^2)
```

3.201.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.201.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.41

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{\left(5\_R^2e+\frac{4(ah+3bd)\_R}{b}+\frac{3ag+}{b}\right.}{\_R^3}}{128a^2b}$
default	$\frac{\frac{5be\,x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}+\frac{9e\,x^3}{32a}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\frac{(3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\right.\right.}{8a}}{}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/32*b*e/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32/a
*e*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+
a)^2+1/128/a^2/b*sum((5*_R^2*e+4/b*(a*h+3*b*d)*_R+3/b*(a*g+7*b*c))/_R^3*ln
(x-_R),_R=RootOf(_Z^4*b+a))
```

3.201.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a + bx^4)^3}\, dx = \text{Timed out}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

3.201.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a + bx^4)^3}\, dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

output
```
Timed out
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.08

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a + bx^4)^3}\, dx$$
$$= \frac{5\, b^2 e x^7 + 2\,(3\, b^2 d + abh)x^6 + 9\, abex^3 + (7\, b^2 c + abg)x^5 - 4\, a^2 f + 2\,(5\, abd - a^2 h)x^2 + (11\, abc - 3\, a^2 g)x}{32\,(a^2 b^3 x^8 + 2\, a^3 b^2 x^4 + a^4 b)}$$
$$+ \frac{\frac{\sqrt{2}\left(21\, b^{\frac{3}{2}} c - 5\, \sqrt{a} be + 3\, a\sqrt{b} g\right) \log\left(\sqrt{b} x^2 + \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(21\, b^{\frac{3}{2}} c - 5\, \sqrt{a} be + 3\, a\sqrt{b} g\right) \log\left(\sqrt{b} x^2 - \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{3}{4}}} + \frac{2\left(21\, \sqrt{2} a^{\frac{1}{4}} b^{\frac{7}{4}} \cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima
")
```

output
```
1/32*(5*b^2*e*x^7 + 2*(3*b^2*d + a*b*h)*x^6 + 9*a*b*e*x^3 + (7*b^2*c + a*b
*g)*x^5 - 4*a^2*f + 2*(5*a*b*d - a^2*h)*x^2 + (11*a*b*c - 3*a^2*g)*x)/(a^2
*b^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sqrt(2)*(21*b^(3/2)*c - 5*sqrt(
a)*b*e + 3*a*sqrt(b)*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt
(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(
b)*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3
/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*s
qrt(2)*a^(5/4)*b^(3/4)*g - 24*sqrt(a)*b^(3/2)*d - 8*a^(3/2)*sqrt(b)*h)*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/
4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 24*sqrt
(a)*b^(3/2)*d + 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqr
t(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b)
)*b^(3/4)))/(a^2*b)
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx$$

$$= \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+4\sqrt{2}\sqrt{ab}abh+21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$

$$+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+4\sqrt{2}\sqrt{ab}abh+21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$

$$+\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$

$$-\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$

$$+\frac{5\,b^2ex^7+6\,b^2dx^6+2\,abhx^6+7\,b^2cx^5+abgx^5+9\,abex^3+10\,abdx^2-2\,a^2hx^2+11\,abcx-3\,a^2gx-4\,a}{32\left(bx^4+a\right)^2a^2b}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21
*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt
(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 4*sqrt(2)*sqrt(a*b)*a*b*h + 21*(a*b^3)^(
1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a
*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 + s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1
/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 2*
a*b*h*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*e*x^3 + 10*a*b*d*x^2 - 2*a^2*h
*x^2 + 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

3.201.9 Mupad [B] (verification not implemented)

Time = 10.02 (sec) , antiderivative size = 1686, normalized size of antiderivative = 4.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^3,x)
```

output
```
((9*e*x^3)/(32*a) - f/(8*b) + (x^5*(7*b*c + a*g))/(32*a^2) + (x^6*(3*b*d +
 a*h))/(16*a^2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (x^2*(5*b*d - a*h))/(16*
a*b) + (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log((302
4*b^3*c*d^2 - 125*a*b^2*e^3 - 2205*b^3*c^2*e + 48*a^3*g*h^2 + 432*a*b^2*d^
2*g + 336*a^2*b*c*h^2 - 45*a^2*b*e*g^2 + 2016*a*b^2*c*d*h - 630*a*b^2*c*e*
g + 288*a^2*b*d*g*h)/(32768*a^6*b) - root(268435456*a^11*b^6*z^4 + 3145728
*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g*z^2 + 6881280*a^6*b^5*c*e*z^2 + 5242
88*a^8*b^3*h^2*z^2 + 4718592*a^6*b^5*d^2*z^2 - 258048*a^5*b^3*c*g*h*z - 77
4144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*g^2*h*z + 51200*a^5*b^3*e^2*h*z - 903
168*a^4*b^4*c^2*h*z - 55296*a^5*b^3*d*g^2*z + 153600*a^4*b^4*d*e^2*z - 270
9504*a^3*b^5*c^2*d*z - 5760*a^3*b^2*d*e*g*h - 40320*a^2*b^3*c*d*e*h - 8640
*a^2*b^3*d^2*e*g - 6720*a^3*b^2*c*e*h^2 + 6300*a^2*b^3*c*e^2*g - 960*a^4*b
*e*g*h^2 - 60480*a*b^4*c*d^2*e + 3072*a^4*b*d*h^3 + 111132*a*b^4*c^3*g + 1
3824*a^3*b^2*d^2*h^2 + 450*a^3*b^2*e^2*g^2 + 23814*a^2*b^3*c^2*g^2 + 27648
*a^2*b^3*d^3*h + 2268*a^3*b^2*c*g^3 + 22050*a*b^4*c^2*e^2 + 625*a^2*b^3*e^
4 + 81*a^4*b*g^4 + 20736*a*b^4*d^4 + 256*a^5*h^4 + 194481*b^5*c^4, z, k)*(
root(268435456*a^11*b^6*z^4 + 3145728*a^7*b^4*d*h*z^2 + 983040*a^7*b^4*e*g
*z^2 + 6881280*a^6*b^5*c*e*z^2 + 524288*a^8*b^3*h^2*z^2 + 4718592*a^6*b^5*
d^2*z^2 - 258048*a^5*b^3*c*g*h*z - 774144*a^4*b^4*c*d*g*z - 18432*a^6*b^2*
g^2*h*z + 51200*a^5*b^3*e^2*h*z - 903168*a^4*b^4*c^2*h*z - 55296*a^5*b^...
```

3.201. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^3}\,dx$

3.202 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx$

3.202.1 Optimal result . . . 1553
3.202.2 Mathematica [A] (verified) . . . 1554
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3.202.1 Optimal result

Integrand size = 40, antiderivative size = 463

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx &= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{8ab(a+bx^4)^2}\\
&\quad-\frac{4af-x(7bc+ag+2(3bd+ah)x+(5be+3ai)x^2)}{32a^2b(a+bx^4)}+\frac{(3bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}\\
&\quad-\frac{\left(3\sqrt{b}(7bc+ag)+\sqrt{a}(5be+3ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{7/4}}\\
&\quad+\frac{\left(3\sqrt{b}(7bc+ag)+\sqrt{a}(5be+3ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{7/4}}\\
&\quad-\frac{\left(3\sqrt{b}(7bc+ag)-\sqrt{a}(5be+3ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{7/4}}\\
&\quad+\frac{\left(3\sqrt{b}(7bc+ag)-\sqrt{a}(5be+3ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{7/4}}\end{aligned}$$

output
```
1/8*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+b*f*x^3)/a/b/(b*x^4+a)^2+1/32*(
-4*a*f+x*(7*b*c+a*g+2*(a*h+3*b*d)*x+(3*a*i+5*b*e)*x^2))/a^2/b/(b*x^4+a)+1/
16*(a*h+3*b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)-1/256*ln(-a^(1/
4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+5*b*e)*a^(1/2)+3*(a*g+7
*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)
+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11
/4)/b^(7/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+5*b
*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/128*arctan(1
+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/
a^(11/4)/b^(7/4)*2^(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.02

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx$$

$$= \frac{\frac{8a^{3/4}b^{3/4}x(7bc+ag+bx(6d+5ex)+ax(2h+3ix))}{a+bx^4} - \frac{32a^{7/4}b^{3/4}(-bx(c+x(d+ex))+a(f+x(g+x(h+ix))))}{(a+bx^4)^2} - 2\left(21\sqrt{2}b^{3/2}c+24\sqrt[4]{a}b^{5/4}\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,
x]
```

output
```
((8*a^(3/4)*b^(3/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x) + a*x*(2*h + 3*i*x)
))/(a + b*x^4) - (32*a^(7/4)*b^(3/4)*(-(b*x*(c + x*(d + e*x))) + a*(f + x*
(g + x*(h + i*x)))))/(a + b*x^4)^2 - 2*(21*Sqrt[2]*b^(3/2)*c + 24*a^(1/4)*
b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g + 8*a^(5/4)*b^(1
/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(
21*Sqrt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sq
rt[2]*a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b^(3/2)*c + 5*Sqrt[a]*b*e - 3*
a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2] + Sqrt[2]*(21*b^(3/2)*c - 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g - 3*a^(3/2
)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(256*a^(11/4)
*b^(7/4))
```

3.202.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2397, 25, 2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{8ab\left(a+bx^4\right)^2}-\frac{\int -\frac{4b^2fx^3+b(5be+3ai)x^2+2b(3bd+ah)x+b(7bc+ag)}{(bx^4+a)^2}dx}{8ab^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4b^2fx^3+b(5be+3ai)x^2+2b(3bd+ah)x+b(7bc+ag)}{(bx^4+a)^2}dx}{8ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

$$\Big\downarrow \text{2393}$$

$$\frac{-\frac{\int -\frac{b(5be+3ai)x^2+4b(3bd+ah)x+3b(7bc+ag)}{bx^4+a}dx}{4a}-\frac{4abf-x\left(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be)\right)}{4a(a+bx^4)}}{8ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b(5be+3ai)x^2+4b(3bd+ah)x+3b(7bc+ag)}{bx^4+a}dx}{4a}-\frac{4abf-x\left(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be)\right)}{4a(a+bx^4)}}{8ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{\int \left(\frac{4b(3bd+ah)x}{bx^4+a}+\frac{b(5be+3ai)x^2+3b(7bc+ag)}{bx^4+a}\right)dx}{4a}-\frac{4abf-x\left(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be)\right)}{4a(a+bx^4)}}{8ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{2\sqrt{2}a^{3/4}}+\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{2\sqrt{2}a^{3/4}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right.}{4\sqrt{2}a^{3/}}}{4a}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x]
```

output
```
(x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(8*a*b*(a + b*
x^4)^2) + (-1/4*(4*a*b*f - x*(b*(7*b*c + a*g) + 2*b*(3*b*d + a*h)*x + b*(5
*b*e + 3*a*i)*x^2))/(a*(a + b*x^4)) + ((2*Sqrt[b]*(3*b*d + a*h)*ArcTan[(Sq
rt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (b^(1/4)*(3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]
*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)) + (b^(1/4)*(3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan
[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)) - (b^(1/4)*(3*Sqrt[
b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)) + (b^(1/4)*(3*Sqrt[b]*(7*b*c
 + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)))/(4*a))/(8*a*b^2)
```

3.202.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.202.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{(3ai+5be)x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}-\frac{(ai-9be)x^3}{32ab}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(bx^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{\left((3ai+5be)\_R^2+4(ah\right.}{}}{128a^2b^2}$
default	$\frac{\frac{(3ai+5be)x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}-\frac{(ai-9be)x^3}{32ab}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{f}{8b}}{(bx^4+a)^2}+\frac{\frac{(3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right.\right.}{8}}{}$

input
```
int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERB
OSE)
```

output
```
(1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*
x^5-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*a*g-11*b*c)/
a/b*x-1/8*f/b)/(b*x^4+a)^2+1/128/a^2/b^2*sum(((3*a*i+5*b*e)*_R^2+4*(a*h+3*
b*d)*_R+3*a*g+21*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.202.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="
fricas")
```

output
```
Timed out
```

3.202.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

output
```
Timed out
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.07

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx$$
$$= \frac{(5\,b^2e+3\,abi)x^7+2\,(3\,b^2d+abh)x^6+(7\,b^2c+abg)x^5+(9\,abe-a^2i)x^3-4\,a^2f+2\,(5\,abd-a^2h)x^2+(1}{32\,(a^2b^3x^8+2\,a^3b^2x^4+a^4b)}$$
$$+\frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g-3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+}{}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="
maxima")
```

output
```
1/32*((5*b^2*e + 3*a*b*i)*x^7 + 2*(3*b^2*d + a*b*h)*x^6 + (7*b^2*c + a*b*g
)*x^5 + (9*a*b*e - a^2*i)*x^3 - 4*a^2*f + 2*(5*a*b*d - a^2*h)*x^2 + (11*a*
b*c - 3*a^2*g)*x)/(a^2*b^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sqrt(2)*(
21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g - 3*a^(3/2)*i)*log(sqrt(b)*x^
2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*b
^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g - 3*a^(3/2)*i)*log(sqrt(b)*x^2 -
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(
1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g
 + 3*sqrt(2)*a^(7/4)*b^(1/4)*i - 24*sqrt(a)*b^(3/2)*d - 8*a^(3/2)*sqrt(b)*
h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)
*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)
*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 3
*sqrt(2)*a^(7/4)*b^(1/4)*i + 24*sqrt(a)*b^(3/2)*d + 8*a^(3/2)*sqrt(b)*h)*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqr
t(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^2*b)
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.14

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx$$

$$= \frac{5\,b^2ex^7+3\,abix^7+6\,b^2dx^6+2\,abhx^6+7\,b^2cx^5+abgx^5+9\,abex^3-a^2ix^3+10\,abdx^2-2\,a^2hx^2+11\,abc\cdots}{32\,(bx^4+a)^2a^2b}$$

$$+\frac{\sqrt{2}\Big(12\,\sqrt{2}\sqrt{ab}b^3d+4\,\sqrt{2}\sqrt{ab}ab^2h+21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g+5\,(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\Big)\arctan\Big(\cdots}{128\,a^3b^4}$$

$$+\frac{\sqrt{2}\Big(12\,\sqrt{2}\sqrt{ab}b^3d+4\,\sqrt{2}\sqrt{ab}ab^2h+21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g+5\,(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\Big)\arctan\Big(\cdots}{128\,a^3b^4}$$

$$+\frac{\sqrt{2}\Big(21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g-5\,(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\Big)\log\Big(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\Big)}{256\,a^3b^4}$$

$$-\frac{\sqrt{2}\Big(21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g-5\,(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\Big)\log\Big(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\Big)}{256\,a^3b^4}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="
giac")
```

output
```
1/32*(5*b^2*e*x^7 + 3*a*b*i*x^7 + 6*b^2*d*x^6 + 2*a*b*h*x^6 + 7*b^2*c*x^5
+ a*b*g*x^5 + 9*a*b*e*x^3 - a^2*i*x^3 + 10*a*b*d*x^2 - 2*a^2*h*x^2 + 11*a*
b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b) + 1/128*sqrt(2)*(12*sqr
t(2)*sqrt(a*b)*b^3*d + 4*sqrt(2)*sqrt(a*b)*a*b^2*h + 21*(a*b^3)^(1/4)*b^3*
c + 3*(a*b^3)^(1/4)*a*b^2*g + 5*(a*b^3)^(3/4)*b*e + 3*(a*b^3)^(3/4)*a*i)*a
rctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) + 1/1
28*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^3*d + 4*sqrt(2)*sqrt(a*b)*a*b^2*h + 21*
(a*b^3)^(1/4)*b^3*c + 3*(a*b^3)^(1/4)*a*b^2*g + 5*(a*b^3)^(3/4)*b*e + 3*(a
*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(a^3*b^4) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^3*c + 3*(a*b^3)^(1/4)*a*b
^2*g - 5*(a*b^3)^(3/4)*b*e - 3*(a*b^3)^(3/4)*a*i)*log(x^2 + sqrt(2)*x*(a/b
)^(1/4) + sqrt(a/b))/(a^3*b^4) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^3*c + 3
*(a*b^3)^(1/4)*a*b^2*g - 5*(a*b^3)^(3/4)*b*e - 3*(a*b^3)^(3/4)*a*i)*log(x^
2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4)
```

3.202.9 Mupad [B] (verification not implemented)

Time = 10.48 (sec) , antiderivative size = 2680, normalized size of antiderivative = 5.79

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^3,x)
```

output
```
symsum(log(- root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 412876
8*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 + 688
1280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2 +
61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z -
774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 5
5296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 55
296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 3
456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*
a^3*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b
^3*d^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g
*i^2 + 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*
g - 6720*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 6048
0*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*
i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 +
 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^
2*b^4*d^3*h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^
2 + 81*a^4*b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 8
1*a^6*i^4 + 194481*b^6*c^4, z, l)*(root(268435456*a^11*b^7*z^4 + 589824*a^
8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040
*a^7*b^5*e*g*z^2 + 6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4...
```

3.202. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^3}\,dx$

3.203 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3} dx$

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3.203.1 Optimal result

Integrand size = 45, antiderivative size = 480

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3} dx \\
&= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{8ab(a+bx^4)^2} \\
&- \frac{4a(bf+aj)-x(b(7bc+ag)+2b(3bd+ah)x+b(5be+3ai)x^2)}{32a^2b^2(a+bx^4)} \\
&+ \frac{(3bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}} - \frac{\left(3\sqrt{b}(7bc+ag)+\sqrt{a}(5be+3ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{7/4}} \\
&+ \frac{\left(3\sqrt{b}(7bc+ag)+\sqrt{a}(5be+3ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{7/4}} \\
&- \frac{\left(3\sqrt{b}(7bc+ag)-\sqrt{a}(5be+3ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{7/4}} \\
&+ \frac{\left(3\sqrt{b}(7bc+ag)-\sqrt{a}(5be+3ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{7/4}}
\end{aligned}$$

output
```
1/8*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+(-a*j+b*f)*x^3)/a/b/(b*x^4+a)^2
+1/32*(-4*a*(a*j+b*f)+x*(b*(a*g+7*b*c)+2*b*(a*h+3*b*d)*x+b*(3*a*i+5*b*e)*x
^2))/a^2/b^2/(b*x^4+a)+1/16*(a*h+3*b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2
)/b^(3/2)-1/256*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*
i+5*b*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/256*ln(
a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-(3*a*i+5*b*e)*a^(1/2)+3*(
a*g+7*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/128*arctan(-1+b^(1/4)*x*2^(
1/2)/a^(1/4))*((3*a*i+5*b*e)*a^(1/2)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/b^(7/
4)*2^(1/2)+1/128*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*((3*a*i+5*b*e)*a^(1/2
)+3*(a*g+7*b*c)*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx$$

$$= \frac{\frac{8a^{3/4}(-8a^2j+b^2x(7c+x(6d+5ex))+abx(g+x(2h+3ix)))}{a+bx^4}+\frac{32a^{7/4}(a^2j+b^2x(c+x(d+ex))-ab(f+x(g+x(h+ix))))}{(a+bx^4)^2}-2\sqrt[4]{b}\Big(21\sqrt{2}b^{3/2}c}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b
*x^4)^3,x]
```

output
```
((8*a^(3/4)*(-8*a^2*j + b^2*x*(7*c + x*(6*d + 5*e*x)) + a*b*x*(g + x*(2*h
+ 3*i*x))))/(a + b*x^4) + (32*a^(7/4)*(a^2*j + b^2*x*(c + x*(d + e*x)) - a
*b*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^2 - 2*b^(1/4)*(21*Sqrt[2]*b^(3/
2)*c + 24*a^(1/4)*b^(5/4)*d + 5*Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*
g + 8*a^(5/4)*b^(1/4)*h + 3*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)] + 2*b^(1/4)*(21*Sqrt[2]*b^(3/2)*c - 24*a^(1/4)*b^(5/4)*d + 5*
Sqrt[2]*Sqrt[a]*b*e + 3*Sqrt[2]*a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 3*Sqrt
[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*b^(1/4)*(
-21*b^(3/2)*c + 5*Sqrt[a]*b*e - 3*a*Sqrt[b]*g + 3*a^(3/2)*i)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(21*b^(3/2)*c
- 5*Sqrt[a]*b*e + 3*a*Sqrt[b]*g - 3*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^2)
```

3.203.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2397, 25, 2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{8ab\,(a+bx^4)^2}-\frac{\int -\frac{4b(bf+aj)x^3+b(5be+3ai)x^2+2b(3bd+ah)x+b(7bc+ag)}{(bx^4+a)^2}dx}{8ab^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4b(bf+aj)x^3+b(5be+3ai)x^2+2b(3bd+ah)x+b(7bc+ag)}{(bx^4+a)^2}dx}{8ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{8ab\,(a+bx^4)^2}$$

$$\Big\downarrow \text{2393}$$

$$\frac{-\frac{\int -\frac{b(5be+3ai)x^2+4b(3bd+ah)x+3b(7bc+ag)}{bx^4+a}dx}{4a}-\frac{4a(aj+bf)-x\left(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be)\right)}{4a(a+bx^4)}}{8ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{8ab\,(a+bx^4)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b(5be+3ai)x^2+4b(3bd+ah)x+3b(7bc+ag)}{bx^4+a}dx}{4a}-\frac{4a(aj+bf)-x\left(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be)\right)}{4a(a+bx^4)}}{8ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{8ab\,(a+bx^4)^2}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{\int \left(\frac{4b(3bd+ah)x}{bx^4+a}+\frac{b(5be+3ai)x^2+3b(7bc+ag)}{bx^4+a}\right)dx}{4a}-\frac{4a(aj+bf)-x\left(b(ag+7bc)+2bx(ah+3bd)+bx^2(3ai+5be)\right)}{4a(a+bx^4)}}{8ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{8ab\,(a+bx^4)^2}$$

$$\downarrow \textcolor{blue}{2009}$$

$$\frac{-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{2\sqrt{2}a^{3/4}}+\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{b}(ag+7bc)+\sqrt{a}(3ai+5be)\right)}{2\sqrt{2}a^{3/4}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right.}{4\sqrt{2}a^{3/}}}{4a}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{8ab\left(a+bx^4\right)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^
3,x]
```

output
```
(x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(8*a*b
*(a + b*x^4)^2) + (-1/4*(4*a*(b*f + a*j) - x*(b*(7*b*c + a*g) + 2*b*(3*b*d
 + a*h)*x + b*(5*b*e + 3*a*i)*x^2))/(a*(a + b*x^4)) + ((2*Sqrt[b]*(3*b*d +
 a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (b^(1/4)*(3*Sqrt[b]*(7*b*c
+ a*g) + Sqrt[a]*(5*b*e + 3*a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)) + (b^(1/4)*(3*Sqrt[b]*(7*b*c + a*g) + Sqrt[a]*(5*b*e
+ 3*a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)) - (
b^(1/4)*(3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)) + (b^(1/4)*(
3*Sqrt[b]*(7*b*c + a*g) - Sqrt[a]*(5*b*e + 3*a*i))*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)))/(4*a))/(8*a*b^2)
```

3.203.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.203.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.42

method	result
risch	$\frac{\frac{(3ai+5be)x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}-\frac{j\,x^4}{4b}-\frac{(ai-9be)x^3}{32ab}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{aj+bf}{8b^2}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{((3ai+5be)\_ \cdots}{\cdots}}{\cdots}$
default	$\frac{\frac{(3ai+5be)x^7}{32a^2}+\frac{(ah+3bd)x^6}{16a^2}+\frac{(ag+7bc)x^5}{32a^2}-\frac{j\,x^4}{4b}-\frac{(ai-9be)x^3}{32ab}-\frac{(ah-5bd)x^2}{16ab}-\frac{(3ag-11bc)x}{32ab}-\frac{aj+bf}{8b^2}}{(b\,x^4+a)^2}+\frac{(3ag+21bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}\right.\right.}{\cdots}$

input
```
int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETU
RNVERBOSE)
```

output
```
(1/32*(3*a*i+5*b*e)/a^2*x^7+1/16*(a*h+3*b*d)/a^2*x^6+1/32*(a*g+7*b*c)/a^2*
x^5-1/4*j*x^4/b-1/32*(a*i-9*b*e)/a/b*x^3-1/16*(a*h-5*b*d)/a/b*x^2-1/32*(3*
a*g-11*b*c)/a/b*x-1/8*(a*j+b*f)/b^2)/(b*x^4+a)^2+1/128/a^2/b^2*sum(((3*a*i
+5*b*e)*_R^2+4*(a*h+3*b*d)*_R+3*a*g+21*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b
+a))
```

3.203.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{(a + bx^4)^3} \, dx = \text{Timed out}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algor
ithm="fricas")
```

output
```
Timed out
```

3.203.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{(a + bx^4)^3} \, dx = \text{Timed out}$$

input
```
integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,
x)
```

output
```
Timed out
```

3.203.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.11

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{(a + bx^4)^3} \, dx =$$

$$-\frac{8\,a^2bjx^4 - (5\,b^3e + 3\,ab^2i)x^7 - 2\,(3\,b^3d + ab^2h)x^6 - (7\,b^3c + ab^2g)x^5 + 4\,a^2bf + 4\,a^3j - (9\,ab^2e - a^2bi)}{32\,(a^2b^4x^8 + 2\,a^3b^3x^4 + a^4b^2)}$$

$$+\frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c - 5\,\sqrt{a}be + 3\,a\sqrt{b}g - 3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c - 5\,\sqrt{a}be + 3\,a\sqrt{b}g - 3\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} +}{}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algor
ithm="maxima")
```

output
```
-1/32*(8*a^2*b*j*x^4 - (5*b^3*e + 3*a*b^2*i)*x^7 - 2*(3*b^3*d + a*b^2*h)*x
^6 - (7*b^3*c + a*b^2*g)*x^5 + 4*a^2*b*f + 4*a^3*j - (9*a*b^2*e - a^2*b*i)
*x^3 - 2*(5*a*b^2*d - a^2*b*h)*x^2 - (11*a*b^2*c - 3*a^2*b*g)*x)/(a^2*b^4*
x^8 + 2*a^3*b^3*x^4 + a^4*b^2) + 1/256*(sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*
b*e + 3*a*sqrt(b)*g - 3*a^(3/2)*i)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e
+ 3*a*sqrt(b)*g - 3*a^(3/2)*i)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2
)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 3*sqrt(2)*a^(7/4)*b^(1
/4)*i - 24*sqrt(a)*b^(3/2)*d - 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*
sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(
sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2)*a^
(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*g + 3*sqrt(2)*a^(7/4)*b^(1/4)*
i + 24*sqrt(a)*b^(3/2)*d + 8*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt
(a)*sqrt(b))*b^(3/4)))/(a^2*b)
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.17

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx$$

$$= \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^3d+4\sqrt{2}\sqrt{ab}ab^2h+21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g+5\,(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\frac{\sqrt{2}}{}\right.}{128\,a^3b^4}$$

$$+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^3d+4\sqrt{2}\sqrt{ab}ab^2h+21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g+5\,(ab^3)^{\frac{3}{4}}\,be+3\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\left(\right.}{128\,a^3b^4}$$

$$+\frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g-5\,(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^4}$$

$$-\frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^3c+3\,(ab^3)^{\frac{1}{4}}\,ab^2g-5\,(ab^3)^{\frac{3}{4}}\,be-3\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^4}$$

$$+\frac{5\,b^3ex^7+3\,ab^2ix^7+6\,b^3dx^6+2\,ab^2hx^6+7\,b^3cx^5+ab^2gx^5-8\,a^2bjx^4+9\,ab^2ex^3-a^2bix^3+10\,ab^2dx^2}{32\,(bx^4+a)^2a^2b^2}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algor
ithm="giac")
```

output
```
1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^3*d + 4*sqrt(2)*sqrt(a*b)*a*b^2*h +
21*(a*b^3)^(1/4)*b^3*c + 3*(a*b^3)^(1/4)*a*b^2*g + 5*(a*b^3)^(3/4)*b*e + 3
*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^3*b^4) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^3*d + 4*sqrt(2)*sq
rt(a*b)*a*b^2*h + 21*(a*b^3)^(1/4)*b^3*c + 3*(a*b^3)^(1/4)*a*b^2*g + 5*(a*
b^3)^(3/4)*b*e + 3*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a
/b)^(1/4))/(a/b)^(1/4))/(a^3*b^4) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^3*c
+ 3*(a*b^3)^(1/4)*a*b^2*g - 5*(a*b^3)^(3/4)*b*e - 3*(a*b^3)^(3/4)*a*i)*log
(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) - 1/256*sqrt(2)*(21*(a
*b^3)^(1/4)*b^3*c + 3*(a*b^3)^(1/4)*a*b^2*g - 5*(a*b^3)^(3/4)*b*e - 3*(a*b
^3)^(3/4)*a*i)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) + 1/
32*(5*b^3*e*x^7 + 3*a*b^2*i*x^7 + 6*b^3*d*x^6 + 2*a*b^2*h*x^6 + 7*b^3*c*x^
5 + a*b^2*g*x^5 - 8*a^2*b*j*x^4 + 9*a*b^2*e*x^3 - a^2*b*i*x^3 + 10*a*b^2*d
*x^2 - 2*a^2*b*h*x^2 + 11*a*b^2*c*x - 3*a^2*b*g*x - 4*a^2*b*f - 4*a^3*j)/(
(b*x^4 + a)^2*a^2*b^2)
```

3.203.9 Mupad [B] (verification not implemented)

Time = 10.19 (sec) , antiderivative size = 2695, normalized size of antiderivative = 5.61

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^
3,x)
```

output
```
symsum(log(- root(268435456*a^11*b^7*z^4 + 589824*a^8*b^4*g*i*z^2 + 412876
8*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040*a^7*b^5*e*g*z^2 + 688
1280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4718592*a^6*b^6*d^2*z^2 +
61440*a^6*b^3*e*h*i*z - 258048*a^5*b^4*c*g*h*z + 184320*a^5*b^4*d*e*i*z -
774144*a^4*b^5*c*d*g*z + 18432*a^7*b^2*h*i^2*z - 18432*a^6*b^3*g^2*h*z + 5
5296*a^6*b^3*d*i^2*z + 51200*a^5*b^4*e^2*h*z - 903168*a^4*b^5*c^2*h*z - 55
296*a^5*b^4*d*g^2*z + 153600*a^4*b^5*d*e^2*z - 2709504*a^3*b^6*c^2*d*z - 3
456*a^4*b^2*d*g*h*i - 24192*a^3*b^3*c*d*h*i + 7560*a^3*b^3*c*e*g*i - 5760*
a^3*b^3*d*e*g*h - 40320*a^2*b^4*c*d*e*h + 540*a^4*b^2*e*g^2*i - 5184*a^3*b
^3*d^2*g*i - 4032*a^4*b^2*c*h^2*i - 960*a^4*b^2*e*g*h^2 + 2268*a^4*b^2*c*g
*i^2 + 26460*a^2*b^4*c^2*e*i - 36288*a^2*b^4*c*d^2*i - 8640*a^2*b^4*d^2*e*
g - 6720*a^3*b^3*c*e*h^2 + 6300*a^2*b^4*c*e^2*g - 576*a^5*b*g*h^2*i - 6048
0*a*b^5*c*d^2*e + 540*a^5*b*e*i^3 + 111132*a*b^5*c^3*g + 1350*a^4*b^2*e^2*
i^2 + 13824*a^3*b^3*d^2*h^2 + 7938*a^3*b^3*c^2*i^2 + 450*a^3*b^3*e^2*g^2 +
 23814*a^2*b^4*c^2*g^2 + 162*a^5*b*g^2*i^2 + 1500*a^3*b^3*e^3*i + 27648*a^
2*b^4*d^3*h + 3072*a^4*b^2*d*h^3 + 2268*a^3*b^3*c*g^3 + 22050*a*b^5*c^2*e^
2 + 81*a^4*b^2*g^4 + 625*a^2*b^4*e^4 + 256*a^5*b*h^4 + 20736*a*b^5*d^4 + 8
1*a^6*i^4 + 194481*b^6*c^4, z, m)*(root(268435456*a^11*b^7*z^4 + 589824*a^
8*b^4*g*i*z^2 + 4128768*a^7*b^5*c*i*z^2 + 3145728*a^7*b^5*d*h*z^2 + 983040
*a^7*b^5*e*g*z^2 + 6881280*a^6*b^6*c*e*z^2 + 524288*a^8*b^4*h^2*z^2 + 4...
```

3.203. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^3}\,dx$

3.204 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx$

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3.204.1 Optimal result

Integrand size = 36, antiderivative size = 293

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx=\frac{x(bc+ag+(bd+ah)x+bex^2+bfx^3)}{12ab(a-bx^4)^3}+\frac{x(7(11bc-ag)+12(5bd-ah)x+45bex^2)}{384a^3b(a-bx^4)}+\frac{8af+x(11bc-ag+2(5bd-ah)x+9bex^2)}{96a^2b(a-bx^4)^2}+\frac{\left(77bc-15\sqrt{a}\sqrt{b}e-7ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}}+\frac{\left(77bc+15\sqrt{a}\sqrt{b}e-7ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}}+\frac{(5bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}$$

output
```
1/12*x*(b*c+a*g+(a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(-b*x^4+a)^3+1/384*x*(-7*
a*g+77*b*c+12*(-a*h+5*b*d)*x+45*b*e*x^2)/a^3/b/(-b*x^4+a)+1/96*(8*a*f+x*(1
1*b*c-a*g+2*(-a*h+5*b*d)*x+9*b*e*x^2))/a^2/b/(-b*x^4+a)^2+1/32*(-a*h+5*b*d
)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)+1/256*arctan(b^(1/4)*x/a^(1
/4))*(77*b*c-7*a*g-15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)+1/256*arctanh(b^
(1/4)*x/a^(1/4))*(77*b*c-7*a*g+15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)
```

3.204.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.23

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx$$
$$= \frac{\frac{4a^{3/4}\sqrt{b}x(77bc-7ag+60bdx-12ahx+45bex^2)}{a-bx^4}+\frac{16a^{7/4}\sqrt{b}x(11bc+bx(10d+9ex)-a(g+2hx))}{(a-bx^4)^2}+\frac{128a^{11/4}\sqrt{b}(bx(c+x(d+ex))+a(f+x(g+hx)}{(a-bx^4)^3}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x]
```

output
```
((4*a^(3/4)*Sqrt[b]*x*(77*b*c - 7*a*g + 60*b*d*x - 12*a*h*x + 45*b*e*x^2))
/(a - b*x^4) + (16*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d + 9*e*x) - a*(g +
 2*h*x)))/(a - b*x^4)^2 + (128*a^(11/4)*Sqrt[b]*(b*x*(c + x*(d + e*x)) + a
*(f + x*(g + h*x))))/(a - b*x^4)^3 + 6*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b
]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(77*b^(5/4)*c + 40*a^(1/4)*b*
d + 15*Sqrt[a]*b^(3/4)*e - 7*a*b^(1/4)*g - 8*a^(5/4)*h)*Log[a^(1/4) - b^(1
/4)*x] + 3*(77*b^(5/4)*c - 40*a^(1/4)*b*d + 15*Sqrt[a]*b^(3/4)*e - 7*a*b^(
1/4)*g + 8*a^(5/4)*h)*Log[a^(1/4) + b^(1/4)*x] - 24*a^(1/4)*(-5*b*d + a*h)
*Log[Sqrt[a] + Sqrt[b]*x^2])/(1536*a^(15/4)*b^(3/2))
```

3.204.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2397, 25, 2393, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx$$
$$\Big\downarrow \text{2397}$$
$$\frac{x(x(ah+bd)+ag+bc+bex^2+bfx^3)}{12ab(a-bx^4)^3}-\frac{\int -\frac{8b^2fx^3+9b^2ex^2+2b(5bd-ah)x+b(11bc-ag)}{(a-bx^4)^3}dx}{12ab^2}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{8b^2fx^3+9b^2ex^2+2b(5bd-ah)x+b(11bc-ag)}{(a-bx^4)^3}dx}{12ab^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \text{2393}$$

$$\frac{\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+9b^2ex^2\right)+8abf}{8a(a-bx^4)^2}-\frac{\int -\frac{45b^2ex^2+12b(5bd-ah)x+7b(11bc-ag)}{\left(a-bx^4\right)^2}dx}{8a}}{12ab^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{45b^2ex^2+12b(5bd-ah)x+7b(11bc-ag)}{\left(a-bx^4\right)^2}dx}{8a}+\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+9b^2ex^2\right)+8abf}{8a(a-bx^4)^2}}{12ab^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x\left(7b(11bc-ag)+12bx(5bd-ah)+45b^2ex^2\right)}{4a\left(a-bx^4\right)}-\frac{\int -\frac{3\left(15b^2ex^2+8b(5bd-ah)x+7b(11bc-ag)\right)}{a-bx^4}dx}{4a}}{8a}+\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+9b^2ex^2\right)+8abf}{8a(a-bx^4)^2}}{12ab^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{15b^2ex^2+8b(5bd-ah)x+7b(11bc-ag)}{a-bx^4}dx}{4a}+\frac{x\left(7b(11bc-ag)+12bx(5bd-ah)+45b^2ex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+9b^2ex^2\right)+8abf}{8a(a-bx^4)^2}}{12ab^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{\frac{3\int \left(\frac{8b(5bd-ah)x}{a-bx^4}+\frac{15b^2ex^2+7b(11bc-ag)}{a-bx^4}\right)dx}{4a}+\frac{x\left(7b(11bc-ag)+12bx(5bd-ah)+45b^2ex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+9b^2ex^2\right)+8abf}{8a(a-bx^4)^2}}{12ab^2}+\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

3.204. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3\left(\frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{2a^{3/4}}+\frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{2a^{3/4}}+\frac{4\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(5bd-ah)}{\sqrt{a}}\right)}{4a}+\frac{x\left(7b(11bc-ag)+1\right.}{4a\left(\right.}}{8a}$$

$$12ab^2$$

$$\frac{x\left(x(ah+bd)+ag+bc+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x]
```

output
```
(x*(b*c + a*g + (b*d + a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3)
 + ((8*a*b*f + x*(b*(11*b*c - a*g) + 2*b*(5*b*d - a*h)*x + 9*b^2*e*x^2))/(
8*a*(a - b*x^4)^2) + ((x*(7*b*(11*b*c - a*g) + 12*b*(5*b*d - a*h)*x + 45*b
^2*e*x^2))/(4*a*(a - b*x^4)) + (3*((b^(3/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e
 - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)) + (b^(3/4)*(77*b*c + 15
*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)) + (4
*Sqrt[b]*(5*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a]))/(4*a))/(8
*a))/(12*a*b^2)
```

3.204.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.204.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.75

method	result
risch	$\frac{\frac{15 e b^{2} x^{11}}{128 a^{3}}-\frac{(a h-5 b d) b x^{10}}{32 a^{3}}-\frac{7(a g-11 b c) b x^{9}}{384 a^{3}}-\frac{21 b e x^{7}}{64 a^{2}}+\frac{(a h-5 b d) x^{6}}{12 a^{2}}+\frac{3(a g-11 b c) x^{5}}{64 a^{2}}+\frac{113 e x^{3}}{384 a}+\frac{(a h+11 b d) x^{2}}{32 a b}+\frac{(7 a g+51 b c) x}{128 a b}+\frac{f}{12 b}}{\left(-b x^{4}+a\right)^{3}}-\frac{\_R=}{}$
default	$\frac{\frac{15 e b^{2} x^{11}}{128 a^{3}}-\frac{(a h-5 b d) b x^{10}}{32 a^{3}}-\frac{7(a g-11 b c) b x^{9}}{384 a^{3}}-\frac{21 b e x^{7}}{64 a^{2}}+\frac{(a h-5 b d) x^{6}}{12 a^{2}}+\frac{3(a g-11 b c) x^{5}}{64 a^{2}}+\frac{113 e x^{3}}{384 a}+\frac{(a h+11 b d) x^{2}}{32 a b}+\frac{(7 a g+51 b c) x}{128 a b}+\frac{f}{12 b}}{\left(-b x^{4}+a\right)^{3}}+\frac{(-7ag}{}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11-1/32*(a*h-5*b*d)/a^3*b*x^10-7/384*(a*g-11*b*c)/a^3*
b*x^9-21/64*b*e/a^2*x^7+1/12/a^2*(a*h-5*b*d)*x^6+3/64/a^2*(a*g-11*b*c)*x^5
+113/384/a*e*x^3+1/32*(a*h+11*b*d)/a/b*x^2+1/128*(7*a*g+51*b*c)/a/b*x+1/12
*f/b)/(-b*x^4+a)^3-1/512/a^3/b*sum((15*_R^2*e-8/b*(a*h-5*b*d)*_R-7*(a*g-11
*b*c)/b)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.204.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx = \text{Timed out}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="frica
s")
```

output
```
Timed out
```

3.204.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

output
```
Timed out
```

3.204.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx =$$

$$-\frac{45\,b^3ex^{11}-126\,ab^2ex^7+12\,(5\,b^3d-ab^2h)x^{10}+7\,(11\,b^3c-ab^2g)x^9+113\,a^2bex^3-32\,(5\,ab^2d-a^2bh)x}{384\,(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2}$$

$$+\frac{\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be-7\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c+15\,\sqrt{a}be-7\,a}{\sqrt{a}\sqrt{}}}{512\,a^3b}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxim
a")
```

output
```
-1/384*(45*b^3*e*x^11 - 126*a*b^2*e*x^7 + 12*(5*b^3*d - a*b^2*h)*x^10 + 7*
(11*b^3*c - a*b^2*g)*x^9 + 113*a^2*b*e*x^3 - 32*(5*a*b^2*d - a^2*b*h)*x^6
- 18*(11*a*b^2*c - a^2*b*g)*x^5 + 32*a^3*f + 12*(11*a^2*b*d + a^3*h)*x^2 +
 3*(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b^4*x^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4
 - a^6*b) + 1/512*(8*(5*b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqr
t(b)) - 8*(5*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(
77*b^(3/2)*c - 15*sqrt(a)*b*e - 7*a*sqrt(b)*g)*arctan(sqrt(b)*x/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2)*c + 15*
sqrt(a)*b*e - 7*a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt
(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(
a^3*b)
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.69

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx =$$

$$-\frac{\sqrt{2}\left(77\,b^2c-7\,abg-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+8\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah+15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$

$$-\frac{\sqrt{2}\left(77\,b^2c-7\,abg+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-8\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,ah-15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$

$$-\frac{\sqrt{2}\left(77\,b^2c-7\,abg-15\,\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$

$$+\frac{\sqrt{2}\left(77\,b^2c-7\,abg-15\,\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$

$$-\frac{45\,b^3ex^{11}+60\,b^3dx^{10}-12\,ab^2hx^{10}+77\,b^3cx^9-7\,ab^2gx^9-126\,ab^2ex^7-160\,ab^2dx^6+32\,a^2bhx^6-198}{384\,(bx^4-a)^3a}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac"
)
```

output
```
-1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 8*sqr
t(2)*(-a*b^3)^(1/4)*a*h + 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqr
t(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2)*(77*
b^2*c - 7*a*b*g + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 8*sqrt(2)*(-a*b^3)^(1/4)
*a*h - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/
(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g -
15*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3
)^(3/4)*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a*b)*b*e)*log
(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(
45*b^3*e*x^11 + 60*b^3*d*x^10 - 12*a*b^2*h*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g
*x^9 - 126*a*b^2*e*x^7 - 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 - 198*a*b^2*c*x^
5 + 18*a^2*b*g*x^5 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 12*a^3*h*x^2 + 15
3*a^2*b*c*x + 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b)
```

3.204.9 Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 1747, normalized size of antiderivative = 5.96

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a - bx^4)^4}\, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a - b*x^4)^4,x)
```

output
```
symsum(log(- root(68719476736*a^15*b^6*z^4 - 1211105280*a^8*b^5*c*e*z^2 +
335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 - 838860800*a^8*b^5*
d^2*z^2 - 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z + 17661952*
a^6*b^3*c*g*h*z + 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 8
02816*a^7*b^2*g^2*h*z - 3686400*a^6*b^3*e^2*h*z + 4014080*a^6*b^3*d*g^2*z
+ 18432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h + 2956800*a^2*b^3*c*d*
e*h + 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^2*c*e*h^2 - 485100*a^2*b^3*c*e
^2*g + 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c*d^2*e - 81920*a^4*b*d*h^3 + 1
2782924*a*b^4*c^3*g + 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 - 174
3126*a^2*b^3*c^2*g^2 - 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668
050*a*b^4*c^2*e^2 - 50625*a^2*b^3*e^4 - 2401*a^4*b*g^4 + 2560000*a*b^4*d^4
 + 4096*a^5*h^4 - 35153041*b^5*c^4, z, k)*(root(68719476736*a^15*b^6*z^4 -
 1211105280*a^8*b^5*c*e*z^2 + 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^
4*e*g*z^2 - 838860800*a^8*b^5*d^2*z^2 - 33554432*a^10*b^3*h^2*z^2 - 883097
60*a^5*b^4*c*d*g*z + 17661952*a^6*b^3*c*g*h*z + 485703680*a^4*b^5*c^2*d*z
- 97140736*a^5*b^4*c^2*h*z - 802816*a^7*b^2*g^2*h*z - 3686400*a^6*b^3*e^2*
h*z + 4014080*a^6*b^3*d*g^2*z + 18432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*
d*e*g*h + 2956800*a^2*b^3*c*d*e*h + 672000*a^2*b^3*d^2*e*g - 295680*a^3*b^
2*c*e*h^2 - 485100*a^2*b^3*c*e^2*g + 26880*a^4*b*e*g*h^2 - 7392000*a*b^4*c
*d^2*e - 81920*a^4*b*d*h^3 + 12782924*a*b^4*c^3*g + 614400*a^3*b^2*d^2*...
```

3.204. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a-bx^4)^4}\,dx$

3.205 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx$

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3.205.1 Optimal result

Integrand size = 41, antiderivative size = 331

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx &= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+bfx^3)}{12ab\,(a-bx^4)^3}\\ &\quad+\frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b\,(a-bx^4)}\\ &\quad+\frac{8af+x(11bc-ag+2(5bd-ah)x+3(3be-ai)x^2)}{96a^2b\,(a-bx^4)^2}\\ &\quad+\frac{\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{13/4}b^{7/4}}\\ &\quad+\frac{\left(15be+\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{13/4}b^{7/4}}+\frac{(5bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}\end{aligned}$$

output
```
1/12*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+b*f*x^3)/a/b/(-b*x^4+a)^3+1/384*
x*(-7*a*g+77*b*c+12*(-a*h+5*b*d)*x+15*(-a*i+3*b*e)*x^2)/a^3/b/(-b*x^4+a)+1
/96*(8*a*f+x*(11*b*c-a*g+2*(-a*h+5*b*d)*x+3*(-a*i+3*b*e)*x^2))/a^2/b/(-b*x
^4+a)^2+1/32*(-a*h+5*b*d)*arctanh(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)+1/2
56*arctanh(b^(1/4)*x/a^(1/4))*(15*b*e-5*a*i+7*(-a*g+11*b*c)*b^(1/2)/a^(1/2
))/a^(13/4)/b^(7/4)+1/256*arctan(b^(1/4)*x/a^(1/4))*(5*a*i-15*b*e+7*(-a*g+
11*b*c)*b^(1/2)/a^(1/2))/a^(13/4)/b^(7/4)
```

3.205.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.27

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx$$
$$= \frac{-\frac{4ab^{3/4}x(-77bc+7ag-15bx(4d+3ex)+3ax(4h+5ix))}{a-bx^4}-\frac{16a^2b^{3/4}x(-b(11c+x(10d+9ex))+a(g+x(2h+3ix)))}{(a-bx^4)^2}+\frac{128a^3b^{3/4}(bx(c+x(d+ex))}{(a-bx}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,
x]
```

output
```
((-4*a*b^(3/4)*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*
i*x)))/(a - b*x^4) - (16*a^2*b^(3/4)*x*(-(b*(11*c + x*(10*d + 9*e*x))) + a
*(g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a^3*b^(3/4)*(b*x*(c + x*(d +
 e*x)) + a*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^3 + 6*a^(1/4)*(77*b^(3/
2)*c - 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b^(1/4)*x)/a^
(1/4)] + 3*a^(1/4)*(-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e
+ 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) - b^(1/4)
*x] - 3*a^(1/4)*(-77*b^(3/2)*c + 40*a^(1/4)*b^(5/4)*d - 15*Sqrt[a]*b*e + 7
*a*Sqrt[b]*g - 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*Log[a^(1/4) + b^(1/4)*x]
 - 24*Sqrt[a]*b^(1/4)*(-5*b*d + a*h)*Log[Sqrt[a] + Sqrt[b]*x^2])/(1536*a^4
*b^(7/4))
```

3.205.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {2397, 25, 2393, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx$$
$$\Big\downarrow \text{2397}$$
$$\frac{x(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3)}{12ab\,(a-bx^4)^3}-\frac{\int -\frac{8b^2fx^3+3b(3be-ai)x^2+2b(5bd-ah)x+b(11bc-ag)}{(a-bx^4)^3}dx}{12ab^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{8b^2fx^3+3b(3be-ai)x^2+2b(5bd-ah)x+b(11bc-ag)}{(a-bx^4)^3}dx}{12ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 2393$$

$$\frac{\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+8abf}{8a(a-bx^4)^2}-\frac{\int -\frac{15b(3be-ai)x^2+12b(5bd-ah)x+7b(11bc-ag)}{\left(a-bx^4\right)^2}dx}{8a}}{12ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{15b(3be-ai)x^2+12b(5bd-ah)x+7b(11bc-ag)}{\left(a-bx^4\right)^2}dx}{8a}+\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+8abf}{8a(a-bx^4)^2}}{12ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 2394$$

$$\frac{\frac{\frac{x\left(7b(11bc-ag)+12bx(5bd-ah)+15bx^2(3be-ai)\right)}{4a\left(a-bx^4\right)}-\frac{\int -\frac{3\left(5b(3be-ai)x^2+8b(5bd-ah)x+7b(11bc-ag)\right)}{a-bx^4}dx}{4a}}{8a}+\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+}{8a(a-bx^4)^2}}{12ab^2}$$

$$\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3\int \frac{5b(3be-ai)x^2+8b(5bd-ah)x+7b(11bc-ag)}{a-bx^4}dx}{4a}+\frac{x\left(7b(11bc-ag)+12bx(5bd-ah)+15bx^2(3be-ai)\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+8ab}{8a(a-bx^4)^2}}{12ab^2}$$

$$\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 2415$$

3.205. $\displaystyle\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx$

$$\frac{\frac{\frac{3\int\left(\frac{8b(5bd-ah)x}{a-bx^4}+\frac{5b(3be-ai)x^2+7b(11bc-ag)}{a-bx^4}\right)dx}{4a}+\frac{x\left(7b(11bc-ag)+12bx(5bd-ah)+15bx^2(3be-ai)\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai))+}{8a(a-bx^4)^2}}{12ab^2}$$

$$\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

$\downarrow$ 2009

$$\frac{\frac{\frac{3\left(-\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{2\sqrt[4]{a}}+\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{2\sqrt[4]{a}}+\frac{4\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(5bd-ah)}{\sqrt{a}}\right)}{4a}+\frac{x(7l}{}}{8a}}{12ab^2}$$

$$\frac{x\left(x(ah+bd)+x^2(ai+be)+ag+bc+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x]
```

output
```
(x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + b*f*x^3))/(12*a*b*(a - b
*x^4)^3) + ((8*a*b*f + x*(b*(11*b*c - a*g) + 2*b*(5*b*d - a*h)*x + 3*b*(3*
b*e - a*i)*x^2))/(8*a*(a - b*x^4)^2) + ((x*(7*b*(11*b*c - a*g) + 12*b*(5*b
*d - a*h)*x + 15*b*(3*b*e - a*i)*x^2))/(4*a*(a - b*x^4)) + (3*(-1/2*(b^(1/
4)*(15*b*e - (7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*a*i)*ArcTan[(b^(1/4)*x
)/a^(1/4)])/a^(1/4) + (b^(1/4)*(15*b*e + (7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a
] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)) + (4*Sqrt[b]*(5*b*d -
 a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a]))/(4*a))/(8*a))/(12*a*b^2)
```

3.205.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.205.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.57 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.73

method	result
risch	$\frac{-\frac{5(ai-3be)b\,x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}+\frac{7(ai-3be)x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{(5ai+113be)x^3}{384ab}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7ag+51bc}{128ab}}{(-b\,x^4+a)^3}$
default	$\frac{-\frac{5(ai-3be)b\,x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}+\frac{7(ai-3be)x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{(5ai+113be)x^3}{384ab}+\frac{(ah+11bd)x^2}{32ab}+\frac{(7ag+51bc}{128ab}}{(-b\,x^4+a)^3}$

input
```
int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVER
BOSE)
```

output
```
(-5/128*(a*i-3*b*e)/a^3*b*x^11-1/32*(a*h-5*b*d)/a^3*b*x^10-7/384*(a*g-11*b
*c)/a^3*b*x^9+7/64*(a*i-3*b*e)/a^2*x^7+1/12/a^2*(a*h-5*b*d)*x^6+3/64/a^2*(
a*g-11*b*c)*x^5+1/384*(5*a*i+113*b*e)/a/b*x^3+1/32*(a*h+11*b*d)/a/b*x^2+1/
128*(7*a*g+51*b*c)/a/b*x+1/12*f/b)/(-b*x^4+a)^3-1/512/a^3/b^2*sum((-5*(a*i
-3*b*e)*_R^2-8*(a*h-5*b*d)*_R-7*a*g+77*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b
-a))
```

3.205.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx=\text{Timed out}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm=
"fricas")
```

output
```
Timed out
```

3.205.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{(a - bx^4)^4} \, dx = \text{Timed out}$$

input
```
integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

output
```
Timed out
```

3.205.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.30

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{(a - bx^4)^4} \, dx = -\frac{15\,(3\,b^3e - ab^2i)x^{11} + 12\,(5\,b^3d - ab^2h)x^{10} + 7\,(11\,b^3c - ab^2g)x^9 - 42\,(3\,ab^2e - a^2bi)x^7 - 32\,(5\,ab^2d - \cdots}{384\,(a^3b^4x^{12} - 3\,\cdots} + \frac{\frac{8\,(5\,bd - ah)\log\left(\sqrt{b}x^2 + \sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{8\,(5\,bd - ah)\log\left(\sqrt{b}x^2 - \sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(77\,b^{\frac{3}{2}}c - 15\,\sqrt{a}be - 7\,a\sqrt{b}g + 5\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(77\,b^{\frac{3}{2}}c + 15\,\sqrt{\cdots}\right.}{\cdots}}{512\,a^3b}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm=
"maxima")
```

output
```
-1/384*(15*(3*b^3*e - a*b^2*i)*x^11 + 12*(5*b^3*d - a*b^2*h)*x^10 + 7*(11*
b^3*c - a*b^2*g)*x^9 - 42*(3*a*b^2*e - a^2*b*i)*x^7 - 32*(5*a*b^2*d - a^2*
b*h)*x^6 - 18*(11*a*b^2*c - a^2*b*g)*x^5 + 32*a^3*f + (113*a^2*b*e + 5*a^3
*i)*x^3 + 12*(11*a^2*b*d + a^3*h)*x^2 + 3*(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b
^4*x^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b) + 1/512*(8*(5*b*d - a*h)*
log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 8*(5*b*d - a*h)*log(sqrt(b)
*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(77*b^(3/2)*c - 15*sqrt(a)*b*e - 7*a
*sqrt(b)*g + 5*a^(3/2)*i)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)
*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2)*c + 15*sqrt(a)*b*e - 7*a*sqr
t(b)*g - 5*a^(3/2)*i)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x +
 sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3*b)
```

3.205.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(284) = 568$.

Time = 0.29 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.82

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx =$$

$$-\frac{\sqrt{2}\left(77\,b^3c-7\,ab^2g-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d+8\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-15\,\sqrt{-ab}b^2e+5\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(\right.}{}\right.}{512\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$-\frac{\sqrt{2}\left(77\,b^3c-7\,ab^2g+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d-8\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-15\,\sqrt{-ab}b^2e-5\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}\left(\right.}{}\right.}{512\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$-\frac{\sqrt{2}(77\,b^3c-7\,ab^2g-15\,\sqrt{-ab}b^2e+5\,\sqrt{-ab}abi)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$+\frac{\sqrt{2}(77\,b^3c-7\,ab^2g-15\,\sqrt{-ab}b^2e+5\,\sqrt{-ab}abi)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$-\frac{45\,b^3ex^{11}-15\,ab^2ix^{11}+60\,b^3dx^{10}-12\,ab^2hx^{10}+77\,b^3cx^9-7\,ab^2gx^9-126\,ab^2ex^7+42\,a^2bix^7-160\,a}{}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm=
"giac")
```

output
```
-1/512*sqrt(2)*(77*b^3*c - 7*a*b^2*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b^2*d + 8
*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 15*sqrt(-a*b)*b^2*e + 5*sqrt(-a*b)*a*b*i)*
arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3
/4)*a^3*b) - 1/512*sqrt(2)*(77*b^3*c - 7*a*b^2*g + 40*sqrt(2)*(-a*b^3)^(1/
4)*b^2*d - 8*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 15*sqrt(-a*b)*b^2*e - 5*sqrt(-
a*b)*a*b*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/
((-a*b^3)^(3/4)*a^3*b) - 1/1024*sqrt(2)*(77*b^3*c - 7*a*b^2*g - 15*sqrt(-a
*b)*b^2*e + 5*sqrt(-a*b)*a*b*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a
/b))/((-a*b^3)^(3/4)*a^3*b) + 1/1024*sqrt(2)*(77*b^3*c - 7*a*b^2*g - 15*sq
rt(-a*b)*b^2*e + 5*sqrt(-a*b)*a*b*i)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sq
rt(-a/b))/((-a*b^3)^(3/4)*a^3*b) - 1/384*(45*b^3*e*x^11 - 15*a*b^2*i*x^11
+ 60*b^3*d*x^10 - 12*a*b^2*h*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 - 126*a*b
^2*e*x^7 + 42*a^2*b*i*x^7 - 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 - 198*a*b^2*c
*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*e*x^3 + 5*a^3*i*x^3 + 132*a^2*b*d*x^2 +
12*a^3*h*x^2 + 153*a^2*b*c*x + 21*a^3*g*x + 32*a^3*f)/((b*x^4 - a)^3*a^3*b
)
```

3.205.9 Mupad [B] (verification not implemented)

Time = 10.32 (sec) , antiderivative size = 2747, normalized size of antiderivative = 8.30

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{(a - bx^4)^4} \, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a - b*x^4)^4,x)
```

output
```
(f/(12*b) - (3*x^5*(11*b*c - a*g))/(64*a^2) - (x^6*(5*b*d - a*h))/(12*a^2)
 - (7*x^7*(3*b*e - a*i))/(64*a^2) + (7*b*x^9*(11*b*c - a*g))/(384*a^3) + (
x*(51*b*c + 7*a*g))/(128*a*b) + (b*x^10*(5*b*d - a*h))/(32*a^3) + (5*b*x^1
1*(3*b*e - a*i))/(128*a^3) + (x^2*(11*b*d + a*h))/(32*a*b) + (x^3*(113*b*e
 + 5*a*i))/(384*a*b))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsu
m(log((125*a^4*i^3 - 3375*a*b^3*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^2*e -
 4928*a^2*b^2*c*h^2 + 735*a^2*b^2*e*g^2 + 3375*a^2*b^2*e^2*i + 11200*a*b^3
*d^2*g - 29645*a*b^3*c^2*i - 1125*a^3*b*e*i^2 + 448*a^3*b*g*h^2 - 245*a^3*
b*g^2*i + 5390*a^2*b^2*c*g*i - 4480*a^2*b^2*d*g*h + 49280*a*b^3*c*d*h - 16
170*a*b^3*c*e*g)/(2097152*a^9*b^2) - root(68719476736*a^15*b^7*z^4 - 12111
05280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*
z^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i*z^2 - 838860800*a^
8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 8830
9760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 12288000*a^6*b^4*d*e*i*z
 + 485703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c
^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^
3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z + 89600*a^4
*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^
3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*i - 224000*a
^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2 - 53900*...
```

3.205. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a-bx^4)^4}\,dx$

3.206 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4} dx$

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3.206.1 Optimal result

Integrand size = 46, antiderivative size = 349

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4} dx &= \frac{x(bc+ag+(bd+ah)x+(be+ai)x^2+(bf+aj)x^3)}{12ab(a-bx^4)^3} \\ &+ \frac{x(7(11bc-ag)+12(5bd-ah)x+15(3be-ai)x^2)}{384a^3b(a-bx^4)} \\ &+ \frac{4a(2bf-aj)+x(b(11bc-ag)+2b(5bd-ah)x+3b(3be-ai)x^2)}{96a^2b^2(a-bx^4)^2} \\ &+ \frac{\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5(3be-ai)\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{13/4}b^{7/4}} \\ &+ \frac{\left(15be+\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{13/4}b^{7/4}} + \frac{(5bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}\end{aligned}$$

output
```
1/12*x*(b*c+a*g+(a*h+b*d)*x+(a*i+b*e)*x^2+(a*j+b*f)*x^3)/a/b/(-b*x^4+a)^3+
1/384*x*(-7*a*g+77*b*c+12*(-a*h+5*b*d)*x+15*(-a*i+3*b*e)*x^2)/a^3/b/(-b*x^
4+a)+1/96*(4*a*(-a*j+2*b*f)+x*(b*(-a*g+11*b*c)+2*b*(-a*h+5*b*d)*x+3*b*(-a*
i+3*b*e)*x^2))/a^2/b^2/(-b*x^4+a)^2+1/32*(-a*h+5*b*d)*arctanh(x^2*b^(1/2)/
a^(1/2))/a^(7/2)/b^(3/2)+1/256*arctanh(b^(1/4)*x/a^(1/4))*(15*b*e-5*a*i+7*
(-a*g+11*b*c)*b^(1/2)/a^(1/2))/a^(13/4)/b^(7/4)+1/256*arctan(b^(1/4)*x/a^(
1/4))*(5*a*i-15*b*e+7*(-a*g+11*b*c)*b^(1/2)/a^(1/2))/a^(13/4)/b^(7/4)
```

3.206.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.26

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx$$

$$= \frac{-\frac{4abx(-77bc+7ag-15bx(4d+3ex)+3ax(4h+5ix))}{a-bx^4}-\frac{16a^2(12a^2j-b^2x(11c+x(10d+9ex))+abx(g+x(2h+3ix)))}{(a-bx^4)^2}+\frac{128a^3(a^2j+b^2x(c+x(d+}{(a}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b
*x^4)^4,x]
```

output
```
((-4*a*b*x*(-77*b*c + 7*a*g - 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*x))
/(a - b*x^4) - (16*a^2*(12*a^2*j - b^2*x*(11*c + x*(10*d + 9*e*x)) + a*b*x
*(g + x*(2*h + 3*i*x))))/(a - b*x^4)^2 + (128*a^3*(a^2*j + b^2*x*(c + x*(d
 + e*x)) + a*b*(f + x*(g + x*(h + i*x)))))/(a - b*x^4)^3 + 6*a^(1/4)*b^(1/
4)*(77*b^(3/2)*c - 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*ArcTan[(b
^(1/4)*x)/a^(1/4)] + 3*a^(1/4)*b^(1/4)*(-77*b^(3/2)*c - 40*a^(1/4)*b^(5/4)
*d - 15*Sqrt[a]*b*e + 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h + 5*a^(3/2)*i)*L
og[a^(1/4) - b^(1/4)*x] + 3*a^(1/4)*b^(1/4)*(77*b^(3/2)*c - 40*a^(1/4)*b^(
5/4)*d + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 8*a^(5/4)*b^(1/4)*h - 5*a^(3/2)*
i)*Log[a^(1/4) + b^(1/4)*x] - 24*Sqrt[a]*Sqrt[b]*(-5*b*d + a*h)*Log[Sqrt[a
] + Sqrt[b]*x^2])/(1536*a^4*b^2)
```

3.206.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2397, 25, 2393, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{12ab\left(a-bx^4\right)^3}-\frac{\int -\frac{4b(2bf-aj)x^3+3b(3be-ai)x^2+2b(5bd-ah)x+b(11bc-ag)}{\left(a-bx^4\right)^3}dx}{12ab^2}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{4b(2bf-aj)x^3+3b(3be-ai)x^2+2b(5bd-ah)x+b(11bc-ag)}{\left(a-bx^4\right)^3}dx}{12ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{12ab\left(a-bx^4\right)^3}$$

$\Big\downarrow$ 2393

$$\frac{\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+4a(2bf-aj)}{8a\left(a-bx^4\right)^2}-\frac{\int -\frac{15b(3be-ai)x^2+12b(5bd-ah)x+7b(11bc-ag)}{\left(a-bx^4\right)^2}dx}{8a}}{12ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{12ab\left(a-bx^4\right)^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{15b(3be-ai)x^2+12b(5bd-ah)x+7b(11bc-ag)}{\left(a-bx^4\right)^2}dx}{8a}+\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+4a(2bf-aj)}{8a\left(a-bx^4\right)^2}}{12ab^2}+\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{12ab\left(a-bx^4\right)^3}$$

$\Big\downarrow$ 2394

$$\frac{\frac{\frac{x\left(7b(11bc-ag)+12bx(5bd-ah)+15bx^2(3be-ai)\right)}{4a\left(a-bx^4\right)}-\frac{\int -\frac{3\left(5b(3be-ai)x^2+8b(5bd-ah)x+7b(11bc-ag)\right)}{a-bx^4}dx}{4a}}{8a}+\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+}{8a\left(a-bx^4\right)^2}}{12ab^2}$$

$$\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{12ab\left(a-bx^4\right)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{5b(3be-ai)x^2+8b(5bd-ah)x+7b(11bc-ag)}{a-bx^4}dx}{4a}+\frac{x\left(7b(11bc-ag)+12bx(5bd-ah)+15bx^2(3be-ai)\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x\left(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai)\right)+4a(}{8a\left(a-bx^4\right)^2}}{12ab^2}$$

$$\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{12ab\left(a-bx^4\right)^3}$$

$\Big\downarrow$ 2415

3.206. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a-bx^4\right)^4}dx$

$$\frac{\frac{\frac{3\int\left(\frac{8b(5bd-ah)x}{a-bx^4}+\frac{5b(3be-ai)x^2+7b(11bc-ag)}{a-bx^4}\right)dx}{4a}+\frac{x\left(7b(11bc-ag)+12bx(5bd-ah)+15bx^2(3be-ai)\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x(b(11bc-ag)+2bx(5bd-ah)+3bx^2(3be-ai))+}{8a(a-bx^4)^2}}{12ab^2}$$

$$\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3\left(-\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{2\sqrt[4]{a}}+\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{7\sqrt{b}(11bc-ag)}{\sqrt{a}}-5ai+15be\right)}{2\sqrt[4]{a}}+\frac{4\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)(5bd-ah)}{\sqrt{a}}\right)}{4a}+\frac{x(7l}{}}{8a}}{12ab^2}$$

$$\frac{x\left(x(ah+bd)+x^2(ai+be)+x^3(aj+bf)+ag+bc\right)}{12ab\left(a-bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^
4,x]
```

output
```
(x*(b*c + a*g + (b*d + a*h)*x + (b*e + a*i)*x^2 + (b*f + a*j)*x^3))/(12*a*
b*(a - b*x^4)^3) + ((4*a*(2*b*f - a*j) + x*(b*(11*b*c - a*g) + 2*b*(5*b*d
- a*h)*x + 3*b*(3*b*e - a*i)*x^2))/(8*a*(a - b*x^4)^2) + ((x*(7*b*(11*b*c
- a*g) + 12*b*(5*b*d - a*h)*x + 15*b*(3*b*e - a*i)*x^2))/(4*a*(a - b*x^4))
 + (3*(-1/2*(b^(1/4)*(15*b*e - (7*Sqrt[b]*(11*b*c - a*g))/Sqrt[a] - 5*a*i)
*ArcTan[(b^(1/4)*x)/a^(1/4)])/a^(1/4) + (b^(1/4)*(15*b*e + (7*Sqrt[b]*(11*
b*c - a*g))/Sqrt[a] - 5*a*i)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)) + (
4*Sqrt[b]*(5*b*d - a*h)*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a]))/(4*a))/(
8*a))/(12*a*b^2)
```

3.206.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.206 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx$

3.206.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.73

method	result
risch	$\frac{-\frac{5(ai-3be)b\,x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}+\frac{7(ai-3be)x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{j\,x^4}{8b}+\frac{(5ai+113be)x^3}{384ab}+\frac{(ah+11bd)x^2}{32ab}+(7a}{\left(-b\,x^4+a\right)^3}$
default	$\frac{-\frac{5(ai-3be)b\,x^{11}}{128a^3}-\frac{(ah-5bd)b\,x^{10}}{32a^3}-\frac{7(ag-11bc)b\,x^9}{384a^3}+\frac{7(ai-3be)x^7}{64a^2}+\frac{(ah-5bd)x^6}{12a^2}+\frac{3(ag-11bc)x^5}{64a^2}+\frac{j\,x^4}{8b}+\frac{(5ai+113be)x^3}{384ab}+\frac{(ah+11bd)x^2}{32ab}+(7a}{\left(-b\,x^4+a\right)^3}$

input
```
int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RET
URNVERBOSE)
```

output
```
(-5/128*(a*i-3*b*e)/a^3*b*x^11-1/32*(a*h-5*b*d)/a^3*b*x^10-7/384*(a*g-11*b
*c)/a^3*b*x^9+7/64*(a*i-3*b*e)/a^2*x^7+1/12/a^2*(a*h-5*b*d)*x^6+3/64/a^2*(
a*g-11*b*c)*x^5+1/8*j*x^4/b+1/384*(5*a*i+113*b*e)/a/b*x^3+1/32*(a*h+11*b*d
)/a/b*x^2+1/128*(7*a*g+51*b*c)/a/b*x-1/24*(a*j-2*b*f)/b^2)/(-b*x^4+a)^3-1/
512/a^3/b^2*sum((-5*(a*i-3*b*e)*_R^2-8*(a*h-5*b*d)*_R-7*a*g+77*b*c)/_R^3*l
n(x-_R),_R=RootOf(_Z^4*b-a))
```

3.206.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{\left(a-bx^4\right)^4}\,dx=\text{Timed out}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algo
rithm="fricas")
```

output
```
Timed out
```

3.206.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx = \text{Timed out}$$

input
```
integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4
,x)
```

output
```
Timed out
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx =$$

$$-\frac{15\,(3\,b^4e-ab^3i)x^{11}+12\,(5\,b^4d-ab^3h)x^{10}+7\,(11\,b^4c-ab^3g)x^9+48\,a^3bjx^4-42\,(3\,ab^3e-a^2b^2i)x^7-\cdots}{38\cdots}$$

$$+\frac{\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{8\,(5\,bd-ah)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be-7\,a\sqrt{b}g+5\,a^{\frac{3}{2}}i\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c+15\,\sqrt{\cdots}\right.}{\cdots}}{512\,a^3b}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algo
rithm="maxima")
```

output
```
-1/384*(15*(3*b^4*e - a*b^3*i)*x^11 + 12*(5*b^4*d - a*b^3*h)*x^10 + 7*(11*
b^4*c - a*b^3*g)*x^9 + 48*a^3*b*j*x^4 - 42*(3*a*b^3*e - a^2*b^2*i)*x^7 - 3
2*(5*a*b^3*d - a^2*b^2*h)*x^6 - 18*(11*a*b^3*c - a^2*b^2*g)*x^5 + 32*a^3*b
*f - 16*a^4*j + (113*a^2*b^2*e + 5*a^3*b*i)*x^3 + 12*(11*a^2*b^2*d + a^3*b
*h)*x^2 + 3*(51*a^2*b^2*c + 7*a^3*b*g)*x)/(a^3*b^5*x^12 - 3*a^4*b^4*x^8 +
3*a^5*b^3*x^4 - a^6*b^2) + 1/512*(8*(5*b*d - a*h)*log(sqrt(b)*x^2 + sqrt(a
))/(sqrt(a)*sqrt(b)) - 8*(5*b*d - a*h)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)
*sqrt(b)) + 2*(77*b^(3/2)*c - 15*sqrt(a)*b*e - 7*a*sqrt(b)*g + 5*a^(3/2)*i
)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) - (77*b^(3/2)*c + 15*sqrt(a)*b*e - 7*a*sqrt(b)*g - 5*a^(3/2)*i)*lo
g((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))
/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3*b)
```

3.206.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(302) = 604$.

Time = 0.28 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.81

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx =$$

$$-\frac{\sqrt{2}\left(77\,b^3c-7\,ab^2g-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d+8\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-15\,\sqrt{-ab}b^2e+5\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}(\cdots}{\cdots}\right.}{512\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$-\frac{\sqrt{2}\left(77\,b^3c-7\,ab^2g+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,b^2d-8\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,abh-15\,\sqrt{-ab}b^2e-5\,\sqrt{-ab}abi\right)\arctan\left(\frac{\sqrt{2}(\cdots}{\cdots}\right.}{512\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$-\frac{\sqrt{2}(77\,b^3c-7\,ab^2g-15\,\sqrt{-ab}b^2e+5\,\sqrt{-ab}abi)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$+\frac{\sqrt{2}(77\,b^3c-7\,ab^2g-15\,\sqrt{-ab}b^2e+5\,\sqrt{-ab}abi)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3b}$$

$$-\frac{45\,b^4ex^{11}-15\,ab^3ix^{11}+60\,b^4dx^{10}-12\,ab^3hx^{10}+77\,b^4cx^9-7\,ab^3gx^9-126\,ab^3ex^7+42\,a^2b^2ix^7-160\cdots}{\cdots}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algo
rithm="giac")
```

output
```
-1/512*sqrt(2)*(77*b^3*c - 7*a*b^2*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b^2*d + 8
*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 15*sqrt(-a*b)*b^2*e + 5*sqrt(-a*b)*a*b*i)*
arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3
/4)*a^3*b) - 1/512*sqrt(2)*(77*b^3*c - 7*a*b^2*g + 40*sqrt(2)*(-a*b^3)^(1/
4)*b^2*d - 8*sqrt(2)*(-a*b^3)^(1/4)*a*b*h - 15*sqrt(-a*b)*b^2*e - 5*sqrt(-
a*b)*a*b*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/
((-a*b^3)^(3/4)*a^3*b) - 1/1024*sqrt(2)*(77*b^3*c - 7*a*b^2*g - 15*sqrt(-a
*b)*b^2*e + 5*sqrt(-a*b)*a*b*i)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a
/b))/((-a*b^3)^(3/4)*a^3*b) + 1/1024*sqrt(2)*(77*b^3*c - 7*a*b^2*g - 15*sq
rt(-a*b)*b^2*e + 5*sqrt(-a*b)*a*b*i)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sq
rt(-a/b))/((-a*b^3)^(3/4)*a^3*b) - 1/384*(45*b^4*e*x^11 - 15*a*b^3*i*x^11
+ 60*b^4*d*x^10 - 12*a*b^3*h*x^10 + 77*b^4*c*x^9 - 7*a*b^3*g*x^9 - 126*a*b
^3*e*x^7 + 42*a^2*b^2*i*x^7 - 160*a*b^3*d*x^6 + 32*a^2*b^2*h*x^6 - 198*a*b
^3*c*x^5 + 18*a^2*b^2*g*x^5 + 48*a^3*b*j*x^4 + 113*a^2*b^2*e*x^3 + 5*a^3*b
*i*x^3 + 132*a^2*b^2*d*x^2 + 12*a^3*b*h*x^2 + 153*a^2*b^2*c*x + 21*a^3*b*g
*x + 32*a^3*b*f - 16*a^4*j)/((b*x^4 - a)^3*a^3*b^2)
```

3.206.9 Mupad [B] (verification not implemented)

Time = 10.57 (sec) , antiderivative size = 2764, normalized size of antiderivative = 7.92

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a - b*x^4)^
4,x)
```

output
```
symsum(log((125*a^4*i^3 - 3375*a*b^3*e^3 - 123200*b^4*c*d^2 + 88935*b^4*c^
2*e - 4928*a^2*b^2*c*h^2 + 735*a^2*b^2*e*g^2 + 3375*a^2*b^2*e^2*i + 11200*
a*b^3*d^2*g - 29645*a*b^3*c^2*i - 1125*a^3*b*e*i^2 + 448*a^3*b*g*h^2 - 245
*a^3*b*g^2*i + 5390*a^2*b^2*c*g*i - 4480*a^2*b^2*d*g*h + 49280*a*b^3*c*d*h
 - 16170*a*b^3*c*e*g)/(2097152*a^9*b^2) - root(68719476736*a^15*b^7*z^4 -
1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5
*d*h*z^2 + 110100480*a^9*b^5*e*g*z^2 - 36700160*a^10*b^4*g*i*z^2 - 8388608
00*a^8*b^6*d^2*z^2 - 33554432*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z -
 88309760*a^5*b^5*c*d*g*z + 17661952*a^6*b^4*c*g*h*z - 12288000*a^6*b^4*d*
e*i*z + 485703680*a^4*b^6*c^2*d*z - 409600*a^8*b^2*h*i^2*z - 97140736*a^5*
b^5*c^2*h*z - 802816*a^7*b^3*g^2*h*z - 3686400*a^6*b^4*e^2*h*z + 2048000*a
^7*b^3*d*i^2*z + 4014080*a^6*b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z + 8960
0*a^4*b^2*d*g*h*i - 985600*a^3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 2688
00*a^3*b^3*d*e*g*h + 2956800*a^2*b^4*c*d*e*h - 14700*a^4*b^2*e*g^2*i - 224
000*a^3*b^3*d^2*g*i + 98560*a^4*b^2*c*h^2*i + 26880*a^4*b^2*e*g*h^2 - 5390
0*a^4*b^2*c*g*i^2 - 1778700*a^2*b^4*c^2*e*i + 2464000*a^2*b^4*c*d^2*i + 67
2000*a^2*b^4*d^2*e*g - 295680*a^3*b^3*c*e*h^2 - 485100*a^2*b^4*c*e^2*g - 8
960*a^5*b*g*h^2*i - 7392000*a*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*
b^5*c^3*g - 33750*a^4*b^2*e^2*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^
3*c^2*i^2 + 22050*a^3*b^3*e^2*g^2 - 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*...
```

3.206. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a-bx^4)^4}\,dx$

3.207 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx$

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3.207.1 Optimal result

Integrand size = 35, antiderivative size = 462

$$\begin{aligned}
\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx
&= \frac{x(bc-ag+(bd-ah)x+bex^2+bfx^3)}{12ab(a+bx^4)^3}+\frac{x(7(11bc+ag)+12(5bd+ah)x+45bex^2)}{384a^3b(a+bx^4)} \\
&\quad -\frac{8af-x(11bc+ag+2(5bd+ah)x+9bex^2)}{96a^2b(a+bx^4)^2}+\frac{(5bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} \\
&\quad -\frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}} \\
&\quad +\frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}} \\
&\quad -\frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}} \\
&\quad +\frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{5/4}}
\end{aligned}$$

output
```
1/12*x*(b*c-a*g+(-a*h+b*d)*x+b*e*x^2+b*f*x^3)/a/b/(b*x^4+a)^3+1/384*x*(7*a
*g+77*b*c+12*(a*h+5*b*d)*x+45*b*e*x^2)/a^3/b/(b*x^4+a)+1/96*(-8*a*f+x*(11*
b*c+a*g+2*(a*h+5*b*d)*x+9*b*e*x^2))/a^2/b/(b*x^4+a)^2+1/32*(a*h+5*b*d)*arc
tan(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1
/2)+a^(1/2)+x^2*b^(1/2))*(77*b*c+7*a*g-15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5
/4)*2^(1/2)+1/1024*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(77*b
*c+7*a*g-15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)+1/512*arctan(-1+b^
(1/4)*x*2^(1/2)/a^(1/4))*(77*b*c+7*a*g+15*e*a^(1/2)*b^(1/2))/a^(15/4)/b^(5
/4)*2^(1/2)+1/512*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(77*b*c+7*a*g+15*e*a
^(1/2)*b^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{8a^{3/4}\sqrt{b}x(77bc+7ag+60bdx+12ahx+45bex^2)}{a+bx^4}+\frac{32a^{7/4}\sqrt{b}x(11bc+bx(10d+9ex)+a(g+2hx))}{(a+bx^4)^2}-\frac{256a^{11/4}\sqrt{b}(-bx(c+x(d+ex))+a(f+x(g+h}{(a+bx^4)^3}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x]
```

output
```
((8*a^(3/4)*Sqrt[b]*x*(77*b*c + 7*a*g + 60*b*d*x + 12*a*h*x + 45*b*e*x^2))
/(a + b*x^4) + (32*a^(7/4)*Sqrt[b]*x*(11*b*c + b*x*(10*d + 9*e*x) + a*(g +
 2*h*x)))/(a + b*x^4)^2 - (256*a^(11/4)*Sqrt[b]*(-(b*x*(c + x*(d + e*x)))
+ a*(f + x*(g + h*x))))/(a + b*x^4)^3 - 6*(77*Sqrt[2]*b^(5/4)*c + 80*a^(1/
4)*b*d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g + 16*a^(5/4)
*h)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(5/4)*c - 80
*a^(1/4)*b*d + 15*Sqrt[2]*Sqrt[a]*b^(3/4)*e + 7*Sqrt[2]*a*b^(1/4)*g - 16*a
^(5/4)*h)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*b^(1/4)*(77*
b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2] + 3*Sqrt[2]*b^(1/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*
g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*
b^(3/2))
```

3.207.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2397, 25, 2393, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{12ab(a+bx^4)^3}-\frac{\int -\frac{8b^2fx^3+9b^2ex^2+2b(5bd+ah)x+b(11bc+ag)}{(bx^4+a)^3}dx}{12ab^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{8b^2fx^3+9b^2ex^2+2b(5bd+ah)x+b(11bc+ag)}{(bx^4+a)^3}dx}{12ab^2}+\frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{2393}$$

$$\frac{-\frac{\int -\frac{45b^2ex^2+12b(5bd+ah)x+7b(11bc+ag)}{(bx^4+a)^2}dx}{8a}-\frac{8abf-x(b(ag+11bc)+2bx(ah+5bd)+9b^2ex^2)}{8a(a+bx^4)^2}}{12ab^2}+\frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{45b^2ex^2+12b(5bd+ah)x+7b(11bc+ag)}{(bx^4+a)^2}dx}{8a}-\frac{8abf-x(b(ag+11bc)+2bx(ah+5bd)+9b^2ex^2)}{8a(a+bx^4)^2}}{12ab^2}+\frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x\left(7b(ag+11bc)+12bx(ah+5bd)+45b^2ex^2\right)}{4a\left(a+bx^4\right)}-\frac{\int -\frac{3\left(15b^2ex^2+8b(5bd+ah)x+7b(11bc+ag)\right)}{bx^4+a}dx}{4a}}{8a}-\frac{8abf-x(b(ag+11bc)+2bx(ah+5bd)+9b^2ex^2)}{8a(a+bx^4)^2}}{12ab^2}+\frac{x(x(bd-ah)-ag+bc+bex^2+bfx^3)}{12ab(a+bx^4)^3}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{15b^2ex^2+8b(5bd+ah)x+7b(11bc+ag)}{bx^4+a}dx}{4a}+\frac{x\left(7b(ag+11bc)+12bx(ah+5bd)+45b^2ex^2\right)}{4a\left(a+bx^4\right)}}{8a}-\frac{8abf-x\left(b(ag+11bc)+2bx(ah+5bd)+9b^2ex^2\right)}{8a(a+bx^4)^2}}{12ab^2}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

$\downarrow$ 2415

$$\frac{\frac{\frac{3\int \left(\frac{8b(5bd+ah)x}{bx^4+a}+\frac{15b^2ex^2+7b(11bc+ag)}{bx^4+a}\right)dx}{4a}+\frac{x\left(7b(ag+11bc)+12bx(ah+5bd)+45b^2ex^2\right)}{4a\left(a+bx^4\right)}}{8a}-\frac{8abf-x\left(b(ag+11bc)+2bx(ah+5bd)+9b^2ex^2\right)}{8a(a+bx^4)^2}}{12ab^2}+\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

$\downarrow$ 2009

$$\frac{\frac{3\left(-\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{2\sqrt{2}a^{3/4}}+\frac{b^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{2\sqrt{2}a^{3/4}}-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}\right.}{4\sqrt{2}a^{3/4}}}{4a}}{8a}$$

$$\frac{x\left(x(bd-ah)-ag+bc+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x]
```

3.207. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx$

output
```
(x*(b*c - a*g + (b*d - a*h)*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3)
 + (-1/8*(8*a*b*f - x*(b*(11*b*c + a*g) + 2*b*(5*b*d + a*h)*x + 9*b^2*e*x^
2))/(a*(a + b*x^4)^2) + ((x*(7*b*(11*b*c + a*g) + 12*b*(5*b*d + a*h)*x + 4
5*b^2*e*x^2))/(4*a*(a + b*x^4)) + (3*((4*Sqrt[b]*(5*b*d + a*h)*ArcTan[(Sqr
t[b]*x^2)/Sqrt[a]])/Sqrt[a] - (b^(3/4)*(77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*
a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)) + (b^(3/
4)*(77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)) - (b^(3/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e +
7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*
a^(3/4)) + (b^(3/4)*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4))))/(4*a))/(8*
a))/(12*a*b^2)
```

3.207.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

3.207. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx$

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.207.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.47

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}+\frac{113e\,x^3}{384a}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-51bc)x}{128ab}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{\_R=\text{I}}{}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}+\frac{113e\,x^3}{384a}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-51bc)x}{128ab}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{(7ag+}{}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*c)/a^3*
b*x^9+21/64*b*e/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a*g+11*b*c)*x^5
+113/384/a*e*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12
*f/b)/(b*x^4+a)^3+1/512/a^3/b*sum((15*_R^2*e+8/b*(a*h+5*b*d)*_R+7*(a*g+11*
b*c)/b)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.207.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx = \text{Timed out}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas
")
```

output
```
Timed out
```

3.207.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

output
```
Timed out
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.12

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx = \frac{45\,b^3ex^{11}+126\,ab^2ex^7+12\,(5\,b^3d+ab^2h)x^{10}+7\,(11\,b^3c+ab^2g)x^9+113\,a^2bex^3+32\,(5\,ab^2d+a^2bh)x^6}{384\,(a^3b^4x^{12}+3\,a^4b^3x^8+3\,a^5b^2x^4}$$
$$+\frac{\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be+7\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be+7\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}\right.}{}}{}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima
")
```

output
```
1/384*(45*b^3*e*x^11 + 126*a*b^2*e*x^7 + 12*(5*b^3*d + a*b^2*h)*x^10 + 7*(
11*b^3*c + a*b^2*g)*x^9 + 113*a^2*b*e*x^3 + 32*(5*a*b^2*d + a^2*b*h)*x^6 +
 18*(11*a*b^2*c + a^2*b*g)*x^5 - 32*a^3*f + 12*(11*a^2*b*d - a^3*h)*x^2 +
3*(51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^4*x^12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4
+ a^6*b) + 1/1024*(sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*a*sqrt(b)*g)
*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4))
- sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*a*sqrt(b)*g)*log(sqrt(b)*x^2
- sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a
^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4
)*g - 80*sqrt(a)*b^(3/2)*d - 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*s
qrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(s
qrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^
(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 80*sqrt(a)*b^(3/2)*d + 16*
a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b
)
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.11

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx$$

$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+8\sqrt{2}\sqrt{ab}abh+77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$
$$+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+8\sqrt{2}\sqrt{ab}abh+77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$
$$+\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$-\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c+7\,(ab^3)^{\frac{1}{4}}\,abg-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$+\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+12\,ab^2hx^{10}+77\,b^3cx^9+7\,ab^2gx^9+126\,ab^2ex^7+160\,ab^2dx^6+32\,a^2bhx^6+198}{384\,(bx^4+a)^3a}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77
*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqr
t(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 8*sqrt(2)*sqrt(a*b)*a*b*h + 77*(a*b^3)^
(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77
*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2
 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024*sqrt(2)*(77*(a*b^
3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqr
t(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*e*x^11 + 60*b^3*
d*x^10 + 12*a*b^2*h*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*e*x^7
+ 160*a*b^2*d*x^6 + 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 11
3*a^2*b*e*x^3 + 132*a^2*b*d*x^2 - 12*a^3*h*x^2 + 153*a^2*b*c*x - 21*a^3*g*
x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

3.207.9 Mupad [B] (verification not implemented)

Time = 10.10 (sec) , antiderivative size = 1743, normalized size of antiderivative = 3.77

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a + bx^4)^4} \, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^4)^4,x)
```

output
```
symsum(log((123200*b^3*c*d^2 - 3375*a*b^2*e^3 - 88935*b^3*c^2*e + 448*a^3*
g*h^2 + 11200*a*b^2*d^2*g + 4928*a^2*b*c*h^2 - 735*a^2*b*e*g^2 + 49280*a*b
^2*c*d*h - 16170*a*b^2*c*e*g + 4480*a^2*b*d*g*h)/(2097152*a^9*b) - root(68
719476736*a^15*b^6*z^4 + 1211105280*a^8*b^5*c*e*z^2 + 335544320*a^9*b^4*d*
h*z^2 + 110100480*a^9*b^4*e*g*z^2 + 838860800*a^8*b^5*d^2*z^2 + 33554432*a
^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z - 17661952*a^6*b^3*c*g*h*z - 48
5703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c^2*h*z - 802816*a^7*b^2*g^2*h*
z + 3686400*a^6*b^3*e^2*h*z - 4014080*a^6*b^3*d*g^2*z + 18432000*a^5*b^4*d
*e^2*z - 268800*a^3*b^2*d*e*g*h - 2956800*a^2*b^3*c*d*e*h - 672000*a^2*b^3
*d^2*e*g - 295680*a^3*b^2*c*e*h^2 + 485100*a^2*b^3*c*e^2*g - 26880*a^4*b*e
*g*h^2 - 7392000*a*b^4*c*d^2*e + 81920*a^4*b*d*h^3 + 12782924*a*b^4*c^3*g
+ 614400*a^3*b^2*d^2*h^2 + 22050*a^3*b^2*e^2*g^2 + 1743126*a^2*b^3*c^2*g^2
 + 2048000*a^2*b^3*d^3*h + 105644*a^3*b^2*c*g^3 + 2668050*a*b^4*c^2*e^2 +
50625*a^2*b^3*e^4 + 2401*a^4*b*g^4 + 2560000*a*b^4*d^4 + 4096*a^5*h^4 + 35
153041*b^5*c^4, z, k)*(root(68719476736*a^15*b^6*z^4 + 1211105280*a^8*b^5*
c*e*z^2 + 335544320*a^9*b^4*d*h*z^2 + 110100480*a^9*b^4*e*g*z^2 + 83886080
0*a^8*b^5*d^2*z^2 + 33554432*a^10*b^3*h^2*z^2 - 88309760*a^5*b^4*c*d*g*z -
 17661952*a^6*b^3*c*g*h*z - 485703680*a^4*b^5*c^2*d*z - 97140736*a^5*b^4*c
^2*h*z - 802816*a^7*b^2*g^2*h*z + 3686400*a^6*b^3*e^2*h*z - 4014080*a^6*b^
3*d*g^2*z + 18432000*a^5*b^4*d*e^2*z - 268800*a^3*b^2*d*e*g*h - 2956800...
```

3.207. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^4)^4}\,dx$

3.208 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$

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3.208.1 Optimal result

Integrand size = 40, antiderivative size = 516

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx \\
&= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+bfx^3)}{12ab\,(a+bx^4)^3} \\
&\quad+\frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)} \\
&\quad-\frac{8af-x(11bc+ag+2(5bd+ah)x+3(3be+ai)x^2)}{96a^2b\,(a+bx^4)^2}+\frac{(5bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}} \\
&\quad-\frac{\left(7\sqrt{b}(11bc+ag)+5\sqrt{a}(3be+ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{7/4}} \\
&\quad+\frac{\left(7\sqrt{b}(11bc+ag)+5\sqrt{a}(3be+ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{7/4}} \\
&\quad-\frac{\left(7\sqrt{b}(11bc+ag)-5\sqrt{a}(3be+ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{7/4}} \\
&\quad+\frac{\left(7\sqrt{b}(11bc+ag)-5\sqrt{a}(3be+ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{7/4}}
\end{aligned}$$

output
```
1/12*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+b*f*x^3)/a/b/(b*x^4+a)^3+1/384
*x*(7*a*g+77*b*c+12*(a*h+5*b*d)*x+15*(a*i+3*b*e)*x^2)/a^3/b/(b*x^4+a)+1/96
*(-8*a*f+x*(11*b*c+a*g+2*(a*h+5*b*d)*x+3*(a*i+3*b*e)*x^2))/a^2/b/(b*x^4+a)
^2+1/32*(a*h+5*b*d)*arctan(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)-1/1024*ln(
-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*(a*i+3*b*e)*a^(1/2)+7*
(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)+1/1024*ln(a^(1/4)*b^(1/4)*x
*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/
2))/a^(15/4)/b^(7/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5
*(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)+1/51
2*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c
)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{32a^{7/4}b^{3/4}x(11bc+ag+bx(10d+9ex)+ax(2h+3ix))}{(a+bx^4)^2}+\frac{8a^{3/4}b^{3/4}x(77bc+7ag+15bx(4d+3ex)+3ax(4h+5ix))}{a+bx^4}-\frac{256a^{11/4}b^{3/4}(-bx(c+x(d+ex}{(a+bx}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,
x]
```

output
```
((32*a^(7/4)*b^(3/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x) + a*x*(2*h + 3*i
*x)))/(a + b*x^4)^2 + (8*a^(3/4)*b^(3/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d +
 3*e*x) + 3*a*x*(4*h + 5*i*x)))/(a + b*x^4) - (256*a^(11/4)*b^(3/4)*(-(b*x
*(c + x*(d + e*x))) + a*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^3 - 6*(77*
Sqrt[2]*b^(3/2)*c + 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt
[2]*a*Sqrt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt[2]*b^(3/2)*c - 80*a^(1/4)*b^(5/4)
*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g - 16*a^(5/4)*b^(1/4)*h
 + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 3*Sqrt[2
]*(-77*b^(3/2)*c + 15*Sqrt[a]*b*e - 7*a*Sqrt[b]*g + 5*a^(3/2)*i)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]*(77*b^(3/2)*c -
15*Sqrt[a]*b*e + 7*a*Sqrt[b]*g - 5*a^(3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^(7/4))
```

3.208.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2397, 25, 2393, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{12ab\left(a+bx^4\right)^3}-\frac{\int -\frac{8b^2fx^3+3b(3be+ai)x^2+2b(5bd+ah)x+b(11bc+ag)}{(bx^4+a)^3}dx}{12ab^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{8b^2fx^3+3b(3be+ai)x^2+2b(5bd+ah)x+b(11bc+ag)}{(bx^4+a)^3}dx}{12ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{2393}$$

$$\frac{-\frac{\int -\frac{15b(3be+ai)x^2+12b(5bd+ah)x+7b(11bc+ag)}{\left(bx^4+a\right)^2}dx}{8a}-\frac{8abf-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3be)\right)}{8a(a+bx^4)^2}}{12ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{15b(3be+ai)x^2+12b(5bd+ah)x+7b(11bc+ag)}{\left(bx^4+a\right)^2}dx}{8a}-\frac{8abf-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3be)\right)}{8a(a+bx^4)^2}}{12ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x\left(7b(ag+11bc)+12bx(ah+5bd)+15bx^2(ai+3be)\right)}{4a\left(a+bx^4\right)}-\frac{\int -\frac{3\left(5b(3be+ai)x^2+8b(5bd+ah)x+7b(11bc+ag)\right)}{bx^4+a}dx}{4a}}{8a}-\frac{8abf-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3\right.}{8a(a+bx^4)^2}}{12ab^2}$$
$$\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$\downarrow 27$$

$$\frac{\frac{\frac{3\int \frac{5b(3be+ai)x^2+8b(5bd+ah)x+7b(11bc+ag)}{bx^4+a}dx}{4a}+\frac{x\left(7b(ag+11bc)+12bx(ah+5bd)+15bx^2(ai+3be)\right)}{4a\left(a+bx^4\right)}}{8a}-\frac{8abf-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3be)\right)}{8a(a+bx^4)^2}}{12ab^2}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$\downarrow 2415$$

$$\frac{\frac{\frac{3\int \left(\frac{8b(5bd+ah)x}{bx^4+a}+\frac{5b(3be+ai)x^2+7b(11bc+ag)}{bx^4+a}\right)dx}{4a}+\frac{x\left(7b(ag+11bc)+12bx(ah+5bd)+15bx^2(ai+3be)\right)}{4a\left(a+bx^4\right)}}{8a}-\frac{8abf-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3\right.}{8a(a+bx^4)^2}}{12ab^2}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$\downarrow 2009$$

$$\frac{3\left(-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{2\sqrt{2}a^{3/4}}+\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{2\sqrt{2}a^{3/4}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{\right.}}{4\sqrt{}}\right.}{4a}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)-ag+bc+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x]
```

3.208. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}dx$

output
```
(x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + b*f*x^3))/(12*a*b*(a + b
*x^4)^3) + (-1/8*(8*a*b*f - x*(b*(11*b*c + a*g) + 2*b*(5*b*d + a*h)*x + 3*
b*(3*b*e + a*i)*x^2))/(a*(a + b*x^4)^2) + ((x*(7*b*(11*b*c + a*g) + 12*b*(
5*b*d + a*h)*x + 15*b*(3*b*e + a*i)*x^2))/(4*a*(a + b*x^4)) + (3*((4*Sqrt[
b]*(5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (b^(1/4)*(7*Sqrt
[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)) + (b^(1/4)*(7*Sqrt[b]*(11*b*c + a*g) + 5*
Sqrt[a]*(3*b*e + a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)) - (b^(1/4)*(7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)
) + (b^(1/4)*(7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*e + a*i))*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4))))/(4*a
))/(8*a))/(12*a*b^2)
```

3.208.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

3.208. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.208.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.46

method	result
risch	$\frac{\frac{5(ai+3be)b\,x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{7(ai+3be)x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{(5ai-113be)x^3}{384ab}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-51bc)x}{128ab}}{(b\,x^4+a)^3}$
default	$\frac{\frac{5(ai+3be)b\,x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{7(ai+3be)x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{(5ai-113be)x^3}{384ab}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-51bc)x}{128ab}}{(b\,x^4+a)^3}$

input
```
int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERB
OSE)
```

output
```
(5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*
c)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a
*g+11*b*c)*x^5-1/384*(5*a*i-113*b*e)/a/b*x^3-1/32*(a*h-11*b*d)/a/b*x^2-1/1
28*(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+1/512/a^3/b^2*sum((5*(a*i+3*
b*e)*_R^2+8*(a*h+5*b*d)*_R+7*a*g+77*b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a)
)
```

3.208.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{(a + bx^4)^4} \, dx = \text{Timed out}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="
fricas")
```

output
```
Timed out
```

3.208.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{(a + bx^4)^4} \, dx = \text{Timed out}$$

input
```
integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

output
```
Timed out
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.12

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{(a + bx^4)^4} \, dx$$
$$= \frac{15\,(3\,b^3e + ab^2i)x^{11} + 12\,(5\,b^3d + ab^2h)x^{10} + 7\,(11\,b^3c + ab^2g)x^9 + 42\,(3\,ab^2e + a^2bi)x^7 + 32\,(5\,ab^2d + a^2b}{384\,(a^3b^4x^{12} + 3\,a^4b}$$
$$+ \frac{\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c - 15\,\sqrt{a}be + 7\,a\sqrt{b}g - 5\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c - 15\,\sqrt{a}be + 7\,a\sqrt{b}g - 5\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}}{}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="
maxima")
```

output
```
1/384*(15*(3*b^3*e + a*b^2*i)*x^11 + 12*(5*b^3*d + a*b^2*h)*x^10 + 7*(11*b
^3*c + a*b^2*g)*x^9 + 42*(3*a*b^2*e + a^2*b*i)*x^7 + 32*(5*a*b^2*d + a^2*b
*h)*x^6 + 18*(11*a*b^2*c + a^2*b*g)*x^5 - 32*a^3*f + (113*a^2*b*e - 5*a^3*
i)*x^3 + 12*(11*a^2*b*d - a^3*h)*x^2 + 3*(51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^
4*x^12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b) + 1/1024*(sqrt(2)*(77*b^(3
/2)*c - 15*sqrt(a)*b*e + 7*a*sqrt(b)*g - 5*a^(3/2)*i)*log(sqrt(b)*x^2 + sq
rt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)
*c - 15*sqrt(a)*b*e + 7*a*sqrt(b)*g - 5*a^(3/2)*i)*log(sqrt(b)*x^2 - sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*
b^(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 5
*sqrt(2)*a^(7/4)*b^(1/4)*i - 80*sqrt(a)*b^(3/2)*d - 16*a^(3/2)*sqrt(b)*h)*
arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sq
rt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^
(7/4)*c + 15*sqrt(2)*a^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*s
qrt(2)*a^(7/4)*b^(1/4)*i + 80*sqrt(a)*b^(3/2)*d + 16*a^(3/2)*sqrt(b)*h)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.17

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$$

$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^3d+8\sqrt{2}\sqrt{ab}ab^2h+77\,(ab^3)^{\frac{1}{4}}\,b^3c+7\,(ab^3)^{\frac{1}{4}}\,ab^2g+15\,(ab^3)^{\frac{3}{4}}\,be+5\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan\Bigg(}{512\,a^4b^4}$$

$$+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^3d+8\sqrt{2}\sqrt{ab}ab^2h+77\,(ab^3)^{\frac{1}{4}}\,b^3c+7\,(ab^3)^{\frac{1}{4}}\,ab^2g+15\,(ab^3)^{\frac{3}{4}}\,be+5\,(ab^3)^{\frac{3}{4}}\,ai\right)\arctan}{512\,a^4b^4}$$

$$+\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^3c+7\,(ab^3)^{\frac{1}{4}}\,ab^2g-15\,(ab^3)^{\frac{3}{4}}\,be-5\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^4}$$

$$-\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^3c+7\,(ab^3)^{\frac{1}{4}}\,ab^2g-15\,(ab^3)^{\frac{3}{4}}\,be-5\,(ab^3)^{\frac{3}{4}}\,ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^4}$$

$$+\frac{45\,b^3ex^{11}+15\,ab^2ix^{11}+60\,b^3dx^{10}+12\,ab^2hx^{10}+77\,b^3cx^9+7\,ab^2gx^9+126\,ab^2ex^7+42\,a^2bix^7+160\,a}{}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="
giac")
```

output
```
1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^3*d + 8*sqrt(2)*sqrt(a*b)*a*b^2*h +
77*(a*b^3)^(1/4)*b^3*c + 7*(a*b^3)^(1/4)*a*b^2*g + 15*(a*b^3)^(3/4)*b*e +
5*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^
(1/4))/(a^4*b^4) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^3*d + 8*sqrt(2)*s
qrt(a*b)*a*b^2*h + 77*(a*b^3)^(1/4)*b^3*c + 7*(a*b^3)^(1/4)*a*b^2*g + 15*(
a*b^3)^(3/4)*b*e + 5*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^4) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^3
*c + 7*(a*b^3)^(1/4)*a*b^2*g - 15*(a*b^3)^(3/4)*b*e - 5*(a*b^3)^(3/4)*a*i)
*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^4) - 1/1024*sqrt(2)*(
77*(a*b^3)^(1/4)*b^3*c + 7*(a*b^3)^(1/4)*a*b^2*g - 15*(a*b^3)^(3/4)*b*e -
5*(a*b^3)^(3/4)*a*i)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^4
) + 1/384*(45*b^3*e*x^11 + 15*a*b^2*i*x^11 + 60*b^3*d*x^10 + 12*a*b^2*h*x^
10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*e*x^7 + 42*a^2*b*i*x^7 + 160
*a*b^2*d*x^6 + 32*a^2*b*h*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2
*b*e*x^3 - 5*a^3*i*x^3 + 132*a^2*b*d*x^2 - 12*a^3*h*x^2 + 153*a^2*b*c*x -
21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3*b)
```

3.208.9 Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 2741, normalized size of antiderivative = 5.31

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6}{(a + bx^4)^4} \, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^4,x)
```

output
```
((3*x^5*(11*b*c + a*g))/(64*a^2) - f/(12*b) + (x^6*(5*b*d + a*h))/(12*a^2)
 + (7*x^7*(3*b*e + a*i))/(64*a^2) + (7*b*x^9*(11*b*c + a*g))/(384*a^3) + (
x*(51*b*c - 7*a*g))/(128*a*b) + (b*x^10*(5*b*d + a*h))/(32*a^3) + (5*b*x^1
1*(3*b*e + a*i))/(128*a^3) + (x^2*(11*b*d - a*h))/(32*a*b) + (x^3*(113*b*e
 - 5*a*i))/(384*a*b))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsu
m(log(- root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 40370
1760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*e*g*z
^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432*a^10
*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 176619
52*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2*d*z
+ 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*g^2*h
*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*b^4*d
*g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^3*b^3
*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*a^2*b
^4*c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a^4*b^
2*c*h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a^2*b^
4*c^2*e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680*a^3*
b^3*c*e*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a*b^5*
c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2*i^2
+ 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2*g...
```

3.208. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{(a+bx^4)^4}\,dx$

3.209 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$

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3.209.1 Optimal result

Integrand size = 45, antiderivative size = 534

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx \\
&= \frac{x(bc-ag+(bd-ah)x+(be-ai)x^2+(bf-aj)x^3)}{12ab\,(a+bx^4)^3} \\
&\quad+\frac{x(7(11bc+ag)+12(5bd+ah)x+15(3be+ai)x^2)}{384a^3b\,(a+bx^4)} \\
&\quad-\frac{4a(2bf+aj)-x(b(11bc+ag)+2b(5bd+ah)x+3b(3be+ai)x^2)}{96a^2b^2\,(a+bx^4)^2} \\
&\quad+\frac{(5bd+ah)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}b^{3/2}}-\frac{\left(7\sqrt{b}(11bc+ag)+5\sqrt{a}(3be+ai)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{7/4}} \\
&\quad+\frac{\left(7\sqrt{b}(11bc+ag)+5\sqrt{a}(3be+ai)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{7/4}} \\
&\quad-\frac{\left(7\sqrt{b}(11bc+ag)-5\sqrt{a}(3be+ai)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{7/4}} \\
&\quad+\frac{\left(7\sqrt{b}(11bc+ag)-5\sqrt{a}(3be+ai)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{7/4}}
\end{aligned}$$

output
```
1/12*x*(b*c-a*g+(-a*h+b*d)*x+(-a*i+b*e)*x^2+(-a*j+b*f)*x^3)/a/b/(b*x^4+a)^
3+1/384*x*(7*a*g+77*b*c+12*(a*h+5*b*d)*x+15*(a*i+3*b*e)*x^2)/a^3/b/(b*x^4+
a)+1/96*(-4*a*(a*j+2*b*f)+x*(b*(a*g+11*b*c)+2*b*(a*h+5*b*d)*x+3*b*(a*i+3*b
*e)*x^2))/a^2/b^2/(b*x^4+a)^2+1/32*(a*h+5*b*d)*arctan(x^2*b^(1/2)/a^(1/2))
/a^(7/2)/b^(3/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))
*(-5*(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)+
1/1024*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*(a*i+3*b*e)*a
^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)+1/512*arctan(-1+b^
(1/4)*x*2^(1/2)/a^(1/4))*(5*(a*i+3*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^
(15/4)/b^(7/4)*2^(1/2)+1/512*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*(a*i+3
*b*e)*a^(1/2)+7*(a*g+11*b*c)*b^(1/2))/a^(15/4)/b^(7/4)*2^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{8a^{3/4}bx(77bc+7ag+15bx(4d+3ex)+3ax(4h+5ix))}{a+bx^4} - \frac{32a^{7/4}\left(12a^2j-b^2x(11c+x(10d+9ex))-abx(g+x(2h+3ix))\right)}{(a+bx^4)^2} + \frac{256a^{11/4}(a^2j+b^2x(c+x}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b
*x^4)^4,x]
```

output
```
((8*a^(3/4)*b*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x) + 3*a*x*(4*h + 5*i*
x)))/(a + b*x^4) - (32*a^(7/4)*(12*a^2*j - b^2*x*(11*c + x*(10*d + 9*e*x))
 - a*b*x*(g + x*(2*h + 3*i*x))))/(a + b*x^4)^2 + (256*a^(11/4)*(a^2*j + b^
2*x*(c + x*(d + e*x)) - a*b*(f + x*(g + x*(h + i*x)))))/(a + b*x^4)^3 - 6*
b^(1/4)*(77*Sqrt[2]*b^(3/2)*c + 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*
b*e + 7*Sqrt[2]*a*Sqrt[b]*g + 16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*b^(1/4)*(77*Sqrt[2]*b^(3/2)*c
- 80*a^(1/4)*b^(5/4)*d + 15*Sqrt[2]*Sqrt[a]*b*e + 7*Sqrt[2]*a*Sqrt[b]*g -
16*a^(5/4)*b^(1/4)*h + 5*Sqrt[2]*a^(3/2)*i)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)] + 3*Sqrt[2]*b^(1/4)*(-77*b^(3/2)*c + 15*Sqrt[a]*b*e - 7*a*Sqrt[b
]*g + 5*a^(3/2)*i)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
+ 3*Sqrt[2]*b^(1/4)*(77*b^(3/2)*c - 15*Sqrt[a]*b*e + 7*a*Sqrt[b]*g - 5*a^(
3/2)*i)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(1
5/4)*b^2)
```

3.209.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {2397, 25, 2393, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{12ab\left(a+bx^4\right)^3}-\frac{\int -\frac{4b(2bf+aj)x^3+3b(3be+ai)x^2+2b(5bd+ah)x+b(11bc+ag)}{(bx^4+a)^3}dx}{12ab^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4b(2bf+aj)x^3+3b(3be+ai)x^2+2b(5bd+ah)x+b(11bc+ag)}{(bx^4+a)^3}dx}{12ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{12ab\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{2393}$$

$$\frac{-\frac{\int -\frac{15b(3be+ai)x^2+12b(5bd+ah)x+7b(11bc+ag)}{\left(bx^4+a\right)^2}dx}{8a}-\frac{4a(aj+2bf)-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3be)\right)}{8a(a+bx^4)^2}}{12ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{12ab\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{15b(3be+ai)x^2+12b(5bd+ah)x+7b(11bc+ag)}{\left(bx^4+a\right)^2}dx}{8a}-\frac{4a(aj+2bf)-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(ai+3be)\right)}{8a(a+bx^4)^2}}{12ab^2}+\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{12ab\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x\left(7b(ag+11bc)+12bx(ah+5bd)+15bx^2(ai+3be)\right)}{4a\left(a+bx^4\right)}-\frac{\int -\frac{3\left(5b(3be+ai)x^2+8b(5bd+ah)x+7b(11bc+ag)\right)}{bx^4+a}dx}{4a}}{8a}-\frac{4a(aj+2bf)-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx\right.}{8a(a+bx^4)^2}}{12ab^2}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{12ab\left(a+bx^4\right)^3}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{5b(3be+ai)x^2+8b(5bd+ah)x+7b(11bc+ag)}{bx^4+a}dx}{4a}+\frac{x\left(7b(ag+11bc)+12bx(ah+5bd)+15bx^2(ai+3be)\right)}{4a\left(a+bx^4\right)}}{8a}-\frac{4a(aj+2bf)-x\left(b(ag+11bc)+2bx(ah+5bd)+3bx^2(a\right.}{8a(a+bx^4)^2}}{12ab^2}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{12ab\left(a+bx^4\right)^3}$$

$\downarrow$ 2415

$$\frac{\frac{\frac{3\int \left(\frac{8b(5bd+ah)x}{bx^4+a}+\frac{5b(3be+ai)x^2+7b(11bc+ag)}{bx^4+a}\right)dx}{4a}+\frac{x\left(7b(ag+11bc)+12bx(ah+5bd)+15bx^2(ai+3be)\right)}{4a\left(a+bx^4\right)}}{8a}-\frac{4a(aj+2bf)-x\left(b(ag+11bc)+2bx(ah+5bd)+3ba\right.}{8a(a+bx^4)^2}}{12ab^2}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{12ab\left(a+bx^4\right)^3}$$

$\downarrow$ 2009

$$\frac{\frac{3\left(-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{2\sqrt{2}a^{3/4}}+\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(7\sqrt{b}(ag+11bc)+5\sqrt{a}(ai+3be)\right)}{2\sqrt{2}a^{3/4}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{\right.}}{4\sqrt{}}\right.}{4a}}{}$$

$$\frac{x\left(x(bd-ah)+x^2(be-ai)+x^3(bf-aj)-ag+bc\right)}{12ab\left(a+bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^
4,x]
```

3.209. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$

output
```
(x*(b*c - a*g + (b*d - a*h)*x + (b*e - a*i)*x^2 + (b*f - a*j)*x^3))/(12*a*
b*(a + b*x^4)^3) + (-1/8*(4*a*(2*b*f + a*j) - x*(b*(11*b*c + a*g) + 2*b*(5
*b*d + a*h)*x + 3*b*(3*b*e + a*i)*x^2))/(a*(a + b*x^4)^2) + ((x*(7*b*(11*b
*c + a*g) + 12*b*(5*b*d + a*h)*x + 15*b*(3*b*e + a*i)*x^2))/(4*a*(a + b*x^
4)) + (3*((4*Sqrt[b]*(5*b*d + a*h)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a]
- (b^(1/4)*(7*Sqrt[b]*(11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)) + (b^(1/4)*(7*Sqrt[b]*(
11*b*c + a*g) + 5*Sqrt[a]*(3*b*e + a*i))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/(2*Sqrt[2]*a^(3/4)) - (b^(1/4)*(7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[
a]*(3*b*e + a*i))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/
(4*Sqrt[2]*a^(3/4)) + (b^(1/4)*(7*Sqrt[b]*(11*b*c + a*g) - 5*Sqrt[a]*(3*b*
e + a*i))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[
2]*a^(3/4))))/(4*a))/(8*a))/(12*a*b^2)
```

3.209.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

3.209. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.209.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.58 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.47

method	result
risch	$\frac{\frac{5(ai+3be)b\,x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{7(ai+3be)x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{j\,x^4}{8b}-\frac{(5ai-113be)x^3}{384ab}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-}{12}}{(b\,x^4+a)^3}$
default	$\frac{\frac{5(ai+3be)b\,x^{11}}{128a^3}+\frac{(ah+5bd)b\,x^{10}}{32a^3}+\frac{7(ag+11bc)b\,x^9}{384a^3}+\frac{7(ai+3be)x^7}{64a^2}+\frac{(ah+5bd)x^6}{12a^2}+\frac{3(ag+11bc)x^5}{64a^2}-\frac{j\,x^4}{8b}-\frac{(5ai-113be)x^3}{384ab}-\frac{(ah-11bd)x^2}{32ab}-\frac{(7ag-}{12}}{(b\,x^4+a)^3}$

input
```
int((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETU
RNVERBOSE)
```

output
```
(5/128*(a*i+3*b*e)/a^3*b*x^11+1/32*(a*h+5*b*d)/a^3*b*x^10+7/384*(a*g+11*b*
c)/a^3*b*x^9+7/64*(a*i+3*b*e)/a^2*x^7+1/12/a^2*(a*h+5*b*d)*x^6+3/64/a^2*(a
*g+11*b*c)*x^5-1/8*j*x^4/b-1/384*(5*a*i-113*b*e)/a/b*x^3-1/32*(a*h-11*b*d)
/a/b*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/24*(a*j+2*b*f)/b^2)/(b*x^4+a)^3+1/51
2/a^3/b^2*sum((5*(a*i+3*b*e)*_R^2+8*(a*h+5*b*d)*_R+7*a*g+77*b*c)/_R^3*ln(x
-_R),_R=RootOf(_Z^4*b+a))
```

3.209.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{(a + bx^4)^4} dx = \text{Timed out}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algor
ithm="fricas")
```

output
```
Timed out
```

3.209.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{(a + bx^4)^4} dx = \text{Timed out}$$

input
```
integrate((j*x**7+i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,
x)
```

output
```
Timed out
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.15

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5 + ix^6 + jx^7}{(a + bx^4)^4} dx$$
$$= \frac{15\,(3\,b^4e + ab^3i)x^{11} + 12\,(5\,b^4d + ab^3h)x^{10} + 7\,(11\,b^4c + ab^3g)x^9 - 48\,a^3bjx^4 + 42\,(3\,ab^3e + a^2b^2i)x^7 + 32}{384\,(}$$
$$+ \frac{\frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c - 15\,\sqrt{a}be + 7\,a\sqrt{b}g - 5\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(77\,b^{\frac{3}{2}}c - 15\,\sqrt{a}be + 7\,a\sqrt{b}g - 5\,a^{\frac{3}{2}}i\right)\log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}}{}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algor
ithm="maxima")
```

output
```
1/384*(15*(3*b^4*e + a*b^3*i)*x^11 + 12*(5*b^4*d + a*b^3*h)*x^10 + 7*(11*b
^4*c + a*b^3*g)*x^9 - 48*a^3*b*j*x^4 + 42*(3*a*b^3*e + a^2*b^2*i)*x^7 + 32
*(5*a*b^3*d + a^2*b^2*h)*x^6 + 18*(11*a*b^3*c + a^2*b^2*g)*x^5 - 32*a^3*b*
f - 16*a^4*j + (113*a^2*b^2*e - 5*a^3*b*i)*x^3 + 12*(11*a^2*b^2*d - a^3*b*
h)*x^2 + 3*(51*a^2*b^2*c - 7*a^3*b*g)*x)/(a^3*b^5*x^12 + 3*a^4*b^4*x^8 + 3
*a^5*b^3*x^4 + a^6*b^2) + 1/1024*(sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e +
 7*a*sqrt(b)*g - 5*a^(3/2)*i)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
+ sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*e + 7*
a*sqrt(b)*g - 5*a^(3/2)*i)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + s
qrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a
^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(7/4)*b^(1/4)
*i - 80*sqrt(a)*b^(3/2)*d - 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sq
rt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sq
rt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^(
3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 5*sqrt(2)*a^(7/4)*b^(1/4)*i
 + 80*sqrt(a)*b^(3/2)*d + 16*a^(3/2)*sqrt(b)*h)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt
(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

3.209. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$

3.209.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.19

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx$$

$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^3d+8\sqrt{2}\sqrt{ab}ab^2h+77(ab^3)^{\frac{1}{4}}b^3c+7(ab^3)^{\frac{1}{4}}ab^2g+15(ab^3)^{\frac{3}{4}}be+5(ab^3)^{\frac{3}{4}}ai\right)\arctan\left(\cdots\right.}{512\,a^4b^4}$$

$$+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^3d+8\sqrt{2}\sqrt{ab}ab^2h+77(ab^3)^{\frac{1}{4}}b^3c+7(ab^3)^{\frac{1}{4}}ab^2g+15(ab^3)^{\frac{3}{4}}be+5(ab^3)^{\frac{3}{4}}ai\right)\arctan\cdots}{512\,a^4b^4}$$

$$+\frac{\sqrt{2}\left(77(ab^3)^{\frac{1}{4}}b^3c+7(ab^3)^{\frac{1}{4}}ab^2g-15(ab^3)^{\frac{3}{4}}be-5(ab^3)^{\frac{3}{4}}ai\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^4}$$

$$-\frac{\sqrt{2}\left(77(ab^3)^{\frac{1}{4}}b^3c+7(ab^3)^{\frac{1}{4}}ab^2g-15(ab^3)^{\frac{3}{4}}be-5(ab^3)^{\frac{3}{4}}ai\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^4}$$

$$+\frac{45\,b^4ex^{11}+15\,ab^3ix^{11}+60\,b^4dx^{10}+12\,ab^3hx^{10}+77\,b^4cx^9+7\,ab^3gx^9+126\,ab^3ex^7+42\,a^2b^2ix^7+160\cdots}{\cdots}$$

input
```
integrate((j*x^7+i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algor
ithm="giac")
```

output
```
1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^3*d + 8*sqrt(2)*sqrt(a*b)*a*b^2*h +
77*(a*b^3)^(1/4)*b^3*c + 7*(a*b^3)^(1/4)*a*b^2*g + 15*(a*b^3)^(3/4)*b*e +
5*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^
(1/4))/(a^4*b^4) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^3*d + 8*sqrt(2)*s
qrt(a*b)*a*b^2*h + 77*(a*b^3)^(1/4)*b^3*c + 7*(a*b^3)^(1/4)*a*b^2*g + 15*(
a*b^3)^(3/4)*b*e + 5*(a*b^3)^(3/4)*a*i)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^4) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^3
*c + 7*(a*b^3)^(1/4)*a*b^2*g - 15*(a*b^3)^(3/4)*b*e - 5*(a*b^3)^(3/4)*a*i)
*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^4) - 1/1024*sqrt(2)*(
77*(a*b^3)^(1/4)*b^3*c + 7*(a*b^3)^(1/4)*a*b^2*g - 15*(a*b^3)^(3/4)*b*e -
5*(a*b^3)^(3/4)*a*i)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^4
) + 1/384*(45*b^4*e*x^11 + 15*a*b^3*i*x^11 + 60*b^4*d*x^10 + 12*a*b^3*h*x^
10 + 77*b^4*c*x^9 + 7*a*b^3*g*x^9 + 126*a*b^3*e*x^7 + 42*a^2*b^2*i*x^7 + 1
60*a*b^3*d*x^6 + 32*a^2*b^2*h*x^6 + 198*a*b^3*c*x^5 + 18*a^2*b^2*g*x^5 - 4
8*a^3*b*j*x^4 + 113*a^2*b^2*e*x^3 - 5*a^3*b*i*x^3 + 132*a^2*b^2*d*x^2 - 12
*a^3*b*h*x^2 + 153*a^2*b^2*c*x - 21*a^3*b*g*x - 32*a^3*b*f - 16*a^4*j)/((b
*x^4 + a)^3*a^3*b^2)
```

3.209.9 Mupad [B] (verification not implemented)

Time = 10.45 (sec) , antiderivative size = 2757, normalized size of antiderivative = 5.16

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6+jx^7}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6 + j*x^7)/(a + b*x^4)^
4,x)
```

output
```
symsum(log(- root(68719476736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 +
403701760*a^9*b^5*c*i*z^2 + 335544320*a^9*b^5*d*h*z^2 + 110100480*a^9*b^5*
e*g*z^2 + 36700160*a^10*b^4*g*i*z^2 + 838860800*a^8*b^6*d^2*z^2 + 33554432
*a^10*b^4*h^2*z^2 + 2457600*a^7*b^3*e*h*i*z - 88309760*a^5*b^5*c*d*g*z - 1
7661952*a^6*b^4*c*g*h*z + 12288000*a^6*b^4*d*e*i*z - 485703680*a^4*b^6*c^2
*d*z + 409600*a^8*b^2*h*i^2*z - 97140736*a^5*b^5*c^2*h*z - 802816*a^7*b^3*
g^2*h*z + 3686400*a^6*b^4*e^2*h*z + 2048000*a^7*b^3*d*i^2*z - 4014080*a^6*
b^4*d*g^2*z + 18432000*a^5*b^5*d*e^2*z - 89600*a^4*b^2*d*g*h*i - 985600*a^
3*b^3*c*d*h*i + 323400*a^3*b^3*c*e*g*i - 268800*a^3*b^3*d*e*g*h - 2956800*
a^2*b^4*c*d*e*h + 14700*a^4*b^2*e*g^2*i - 224000*a^3*b^3*d^2*g*i - 98560*a
^4*b^2*c*h^2*i - 26880*a^4*b^2*e*g*h^2 + 53900*a^4*b^2*c*g*i^2 + 1778700*a
^2*b^4*c^2*e*i - 2464000*a^2*b^4*c*d^2*i - 672000*a^2*b^4*d^2*e*g - 295680
*a^3*b^3*c*e*h^2 + 485100*a^2*b^4*c*e^2*g - 8960*a^5*b*g*h^2*i - 7392000*a
*b^5*c*d^2*e + 7500*a^5*b*e*i^3 + 12782924*a*b^5*c^3*g + 33750*a^4*b^2*e^2
*i^2 + 614400*a^3*b^3*d^2*h^2 + 296450*a^3*b^3*c^2*i^2 + 22050*a^3*b^3*e^2
*g^2 + 1743126*a^2*b^4*c^2*g^2 + 2450*a^5*b*g^2*i^2 + 67500*a^3*b^3*e^3*i
+ 2048000*a^2*b^4*d^3*h + 81920*a^4*b^2*d*h^3 + 105644*a^3*b^3*c*g^3 + 266
8050*a*b^5*c^2*e^2 + 2401*a^4*b^2*g^4 + 50625*a^2*b^4*e^4 + 4096*a^5*b*h^4
 + 2560000*a*b^5*d^4 + 625*a^6*i^4 + 35153041*b^6*c^4, z, m)*(root(6871947
6736*a^15*b^7*z^4 + 1211105280*a^8*b^6*c*e*z^2 + 403701760*a^9*b^5*c*i*...
```

3.210 $\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx$

3.210.1 Optimal result . . . 1630
3.210.2 Mathematica [C] (verified) . . . 1630
3.210.3 Rubi [A] (verified) . . . 1631
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3.210.5 Fricas [A] (verification not implemented) . . . 1632
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3.210.8 Giac [F] . . . 1634
3.210.9 Mupad [F(-1)] . . . 1634

3.210.1 Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx=\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}$$

output
```
1/2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+1/2*c*(cos(2*arctan(b^(
1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2
*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/
(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

3.210.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx=\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{cx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x)/Sqrt[a + b*x^4],x]
```

output
```
(d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*
x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

3.210.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{c}{\sqrt{a+bx^4}}+\frac{dx}{\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

input
```
Int[(c + d*x)/Sqrt[a + b*x^4],x]
```

output
```
(d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])
```

3.210.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.210.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}$	96
elliptic	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}$	99

input
```
int((d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
+1/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.60

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \frac{4\,b^{\frac{3}{2}}c\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+a\sqrt{b}d\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)}{4\,ab}$$

input
```
integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(4*b^(3/2)*c*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + a*s
qrt(b)*d*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a))/(a*b)
```

3.210.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \frac{d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

3.210.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \int \frac{dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(b*x^4 + a), x)
```

3.210.8 Giac [F]

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx=\int \frac{dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(b*x^4 + a), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx=\int \frac{c+d\,x}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x)/(a + b*x^4)^(1/2),x)
```

output
```
int((c + d*x)/(a + b*x^4)^(1/2), x)
```

3.211 $\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx$

3.211.1 Optimal result 1635
3.211.2 Mathematica [C] (verified) 1635
3.211.3 Rubi [A] (verified) 1636
3.211.4 Maple [A] (verified) 1637
3.211.5 Fricas [A] (verification not implemented) 1637
3.211.6 Sympy [A] (verification not implemented) 1638
3.211.7 Maxima [F] 1638
3.211.8 Giac [F] 1638
3.211.9 Mupad [F(-1)] 1639

3.211.1 Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx=\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}+\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}$$

output
```
1/2*d*arctan(x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(1/2)+a^(1/4)*c*EllipticF(b^(
1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/(-b*x^4+a)^(1/2)
```

3.211.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx=\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}+\frac{cx\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)}{\sqrt{a-bx^4}}$$

input
```
Integrate[(c + d*x)/Sqrt[a - b*x^4],x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 - (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/Sqrt[a - b*x^4]
```

3.211.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{c}{\sqrt{a-bx^4}}+\frac{dx}{\sqrt{a-bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

input
```
Int[(c + d*x)/Sqrt[a - b*x^4],x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1
- (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a -
 b*x^4])
```

3.211.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

3.211.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{c\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}+\frac{d\arctan\left(\frac{x^2\sqrt{b}}{\sqrt{-b\,x^4+a}}\right)}{2\sqrt{b}}$	90
elliptic	$\frac{c\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}+\frac{d\ln\left(-\frac{2b\,x^2}{\sqrt{-b}}+2\sqrt{-b\,x^4+a}\right)}{2\sqrt{-b}}$	99

input
```
int((d*x+c)/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a
^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)+1/
2*d*arctan(x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(1/2)
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx$$
$$=\frac{4\sqrt{-b}bc\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-a\sqrt{-b}d\log\left(2\,bx^4-2\sqrt{-bx^4+a}\sqrt{-b}x^2-a\right)}{4\,ab}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(4*sqrt(-b)*b*c*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - a*
sqrt(-b)*d*log(2*b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(-b)*x^2 - a))/(a*b)
```

3.211.6 Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = d\left(\begin{cases} -\frac{i\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for } \left|\frac{bx^4}{a}\right|>1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise}\end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x+c)/(-b*x**4+a)**(1/2),x)
```

output
```
d*Piecewise((-I*acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4/a) > 1
), (asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) + c*x*gamma(1/4)*hyper(
(1/4, 1/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

3.211.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = \int \frac{dx+c}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(-b*x^4 + a), x)
```

3.211.8 Giac [F]

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = \int \frac{dx+c}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(-b*x^4 + a), x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = \int \frac{c+d\,x}{\sqrt{a-b\,x^4}}\,dx$$

input
```
int((c + d*x)/(a - b*x^4)^(1/2),x)
```

output
```
int((c + d*x)/(a - b*x^4)^(1/2), x)
```

3.212 $\int \frac{c+dx}{\sqrt{-a+bx^4}} dx$

3.212.1 Optimal result . . . 1640
3.212.2 Mathematica [C] (verified) . . . 1640
3.212.3 Rubi [A] (verified) . . . 1641
3.212.4 Maple [A] (verified) . . . 1642
3.212.5 Fricas [A] (verification not implemented) . . . 1642
3.212.6 Sympy [A] (verification not implemented) . . . 1643
3.212.7 Maxima [F] . . . 1643
3.212.8 Giac [F] . . . 1643
3.212.9 Mupad [F(-1)] . . . 1644

3.212.1 Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx=\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{-a+bx^4}}\right)}{2\sqrt{b}}+\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{-a+bx^4}}$$

output
```
1/2*d*arctanh(x^2*b^(1/2)/(b*x^4-a)^(1/2))/b^(1/2)+a^(1/4)*c*EllipticF(b^(
1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/(b*x^4-a)^(1/2)
```

3.212.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx=\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{-a+bx^4}}\right)}{2\sqrt{b}}+\frac{cx\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)}{\sqrt{-a+bx^4}}$$

input
```
Integrate[(c + d*x)/Sqrt[-a + b*x^4],x]
```

output
```
(d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 - (b
*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/Sqrt[-a + b*x^4]
```

3.212.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{bx^4-a}}dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{c}{\sqrt{bx^4-a}}+\frac{dx}{\sqrt{bx^4-a}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{bx^4-a}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}$$

input
```
Int[(c + d*x)/Sqrt[-a + b*x^4],x]
```

output
```
(d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[
1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[-
a + b*x^4])
```

3.212.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.212.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{c\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4-a}}+\frac{d\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4-a}\right)}{2\sqrt{b}}$	95
elliptic	$\frac{c\sqrt{1+\frac{x^2\sqrt{b}}{\sqrt{a}}}\,\sqrt{1-\frac{x^2\sqrt{b}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4-a}}+\frac{d\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4-a}\right)}{2\sqrt{b}}$	98

input
```
int((d*x+c)/(b*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/(-1/a^(1/2)*b^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)*(1-x^2*b^(1/2)/
a^(1/2))^(1/2)/(b*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*b^(1/2))^(1/2),I)+1
/2*d*ln(x^2*b^(1/2)+(b*x^4-a)^(1/2))/b^(1/2)
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = -\frac{4\,b^{\frac{3}{2}}c\left(\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)-a\sqrt{b}d\log\left(2\,bx^4+2\,\sqrt{bx^4-a}\sqrt{b}x^2-a\right)}{4\,ab}$$

input
```
integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(4*b^(3/2)*c*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - a*sq
rt(b)*d*log(2*b*x^4 + 2*sqrt(b*x^4 - a)*sqrt(b)*x^2 - a))/(a*b)
```

3.212.6 Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = d\left(\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for } \left|\frac{bx^4}{a}\right| > 1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise} \end{cases}\right) - \frac{icx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x+c)/(b*x**4-a)**(1/2),x)
```

output
```
d*Piecewise((acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4/a) > 1),
(-I*asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) - I*c*x*gamma(1/4)*hype
r((1/4, 1/2), (5/4,), b*x**4/a)/(4*sqrt(a)*gamma(5/4))
```

3.212.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = \int \frac{dx+c}{\sqrt{bx^4-a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(b*x^4 - a), x)
```

3.212.8 Giac [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = \int \frac{dx+c}{\sqrt{bx^4-a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(b*x^4 - a), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = \int \frac{c+d\,x}{\sqrt{b\,x^4-a}}\,dx$$

input
```
int((c + d*x)/(b*x^4 - a)^(1/2),x)
```

output
```
int((c + d*x)/(b*x^4 - a)^(1/2), x)
```

3.213 $\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx$

3.213.1 Optimal result . . . 1645
3.213.2 Mathematica [C] (verified) . . . 1645
3.213.3 Rubi [A] (verified) . . . 1646
3.213.4 Maple [C] (verified) . . . 1647
3.213.5 Fricas [A] (verification not implemented) . . . 1647
3.213.6 Sympy [C] (verification not implemented) . . . 1648
3.213.7 Maxima [F] . . . 1648
3.213.8 Giac [F] . . . 1649
3.213.9 Mupad [F(-1)] . . . 1649

3.213.1 Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}$$

output
```
1/2*d*arctan(x^2*b^(1/2)/(-b*x^4-a)^(1/2))/b^(1/2)+1/2*c*(cos(2*arctan(b^(
1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2
*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/
(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(-b*x^4-a)^(1/2)
```

3.213.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.67

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{-a-bx^4}}$$

input
```
Integrate[(c + d*x)/Sqrt[-a - b*x^4],x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*
x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[-a - b*x^4]
```

3.213.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{c}{\sqrt{-a-bx^4}}+\frac{dx}{\sqrt{-a-bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}$$

input
```
Int[(c + d*x)/Sqrt[-a - b*x^4],x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[-a - b*x^4])
```

3.213.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.213.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{c\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}-a}}+\frac{d\arctan\left(\frac{x^{2}\sqrt{b}}{\sqrt{-b\,x^{4}-a}}\right)}{2\sqrt{b}}$	101
elliptic	$\frac{c\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}-a}}+\frac{d\ln\left(-\frac{2b\,x^{2}}{\sqrt{-b}}+2\sqrt{-b\,x^{4}-a}\right)}{2\sqrt{-b}}$	110

input
```
int((d*x+c)/(-b*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/(-I/a^(1/2)*b^(1/2))^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(-b*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*b^(1/2))^(1/2)
,I)+1/2*d*arctan(x^2*b^(1/2)/(-b*x^4-a)^(1/2))/b^(1/2)
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = -\frac{4\sqrt{-b}bc\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+a\sqrt{-b}d\log\left(-2\,bx^4+2\,\sqrt{-bx^4-a}\sqrt{-b}x^2-a\right)}{4\,ab}$$

input
```
integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(4*sqrt(-b)*b*c*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) +
 a*sqrt(-b)*d*log(-2*b*x^4 + 2*sqrt(-b*x^4 - a)*sqrt(-b)*x^2 - a))/(a*b)
```

3.213.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = -\frac{id\,\text{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} - \frac{icx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x+c)/(-b*x**4-a)**(1/2),x)
```

output
```
-I*d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - I*c*x*gamma(1/4)*hyper((1/4
, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

3.213.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \int \frac{dx+c}{\sqrt{-bx^4-a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(-b*x^4 - a), x)
```

3.213.8 Giac [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \int \frac{dx+c}{\sqrt{-bx^4-a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(-b*x^4 - a), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \int \frac{c+d\,x}{\sqrt{-b\,x^4-a}}\,dx$$

input
```
int((c + d*x)/(- a - b*x^4)^(1/2),x)
```

output
```
int((c + d*x)/(- a - b*x^4)^(1/2), x)
```

3.214 $\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx$

3.214.1 Optimal result . . . 1650
3.214.2 Mathematica [C] (verified) . . . 1651
3.214.3 Rubi [A] (verified) . . . 1651
3.214.4 Maple [C] (verified) . . . 1652
3.214.5 Fricas [A] (verification not implemented) . . . 1653
3.214.6 Sympy [C] (verification not implemented) . . . 1654
3.214.7 Maxima [F] . . . 1654
3.214.8 Giac [F] . . . 1654
3.214.9 Mupad [F(-1)] . . . 1655

3.214.1 Optimal result

Integrand size = 22, antiderivative size = 257

$$\begin{aligned}
\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx &= \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} \\
&\quad-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} \\
&\quad+\frac{\sqrt[4]{a}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
1/2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+e*x*(b*x^4+a)^(1/2)/b^(
1/2)/(a^(1/2)+x^2*b^(1/2))-a^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2)
)^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^
(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(e+c*b^(1/2)/a^(1/2))
*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

3.214.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.51

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}} + \frac{ex^3\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^4],x]
```

output
```
(d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*
x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4] +
(e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])
/(3*Sqrt[a + b*x^4])
```

3.214.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \; 2424$$

$$\int \left(\frac{c+ex^2}{\sqrt{a+bx^4}}+\frac{dx}{\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[(c + d*x + e*x^2)/Sqrt[a + b*x^4],x]
```

output
```
(e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[
a] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2
]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4
])
```

3.214.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.214.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.75

method	result
default	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}+\frac{d\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}}\right.}{2\sqrt{b}}$
elliptic	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{d\ln\left(2x^{2}\sqrt{b}+2\sqrt{b\,x^{4}+a}\right)}{2\sqrt{b}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}$

input
```
int((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
+I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/
2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*d*ln(x^
2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.50

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{4\,a\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+a\sqrt{b}dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+4\,(bc-ae)\sqrt{b}x}{4\,abx}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(4*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) +
 a*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 4*(b*c
- a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 4*s
qrt(b*x^4 + a)*a*e)/(a*b*x)
```

3.214.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.40

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \frac{d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma
(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma
(7/4))
```

3.214.7 Maxima [F]

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

3.214.8 Giac [F]

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx=\int \frac{e\,x^2+d\,x+c}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2)/(a + b*x^4)^(1/2),x)
```

output
```
int((c + d*x + e*x^2)/(a + b*x^4)^(1/2), x)
```

3.215 $\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx$

3.215.1 Optimal result 1656
3.215.2 Mathematica [A] (verified) 1656
3.215.3 Rubi [A] (verified) 1657
3.215.4 Maple [A] (verified) 1657
3.215.5 Fricas [A] (verification not implemented) 1658
3.215.6 Sympy [C] (verification not implemented) 1658
3.215.7 Maxima [A] (verification not implemented) 1659
3.215.8 Giac [A] (verification not implemented) 1659
3.215.9 Mupad [B] (verification not implemented) 1659

3.215.1 Optimal result

Integrand size = 23, antiderivative size = 14

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{gx}{\sqrt{a+bx^4}}$$

output
```
g*x/(b*x^4+a)^(1/2)
```

3.215.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{gx}{\sqrt{a+bx^4}}$$

input
```
Integrate[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(g*x)/Sqrt[a + b*x^4]
```

3.215.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag - bgx^4}{(a + bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \; 908$$

$$\frac{gx}{\sqrt{a + bx^4}}$$

input
```
Int[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(g*x)/Sqrt[a + b*x^4]
```

3.215.3.1 Defintions of rubi rules used

rule 908
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

3.215.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
default	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
trager	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
elliptic	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
pseudoelliptic	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13

input
```
int((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
g*x/(b*x^4+a)^(1/2)
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ag - bgx^4}{(a + bx^4)^{3/2}}\, dx = \frac{gx}{\sqrt{bx^4 + a}}$$

input
```
integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
g*x/sqrt(b*x^4 + a)
```

3.215.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 80, normalized size of antiderivative = 5.71

$$\int \frac{ag - bgx^4}{(a + bx^4)^{3/2}}\, dx = \frac{gx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((-b*g*x**4+a*g)/(b*x**4+a)**(3/2),x)
```

output
```
g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp
_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ag - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{bx^4 + a}}$$

input
```
integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
g*x/sqrt(b*x^4 + a)
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ag - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{bx^4 + a}}$$

input
```
integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
g*x/sqrt(b*x^4 + a)
```

3.215.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ag - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{g\,x}{\sqrt{b\,x^4 + a}}$$

input
```
int((a*g - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

output
```
(g*x)/(a + b*x^4)^(1/2)
```

3.216 $\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx$

3.216.1 Optimal result 1660
3.216.2 Mathematica [A] (verified) 1660
3.216.3 Rubi [A] (verified) 1661
3.216.4 Maple [A] (verified) 1661
3.216.5 Fricas [A] (verification not implemented) 1662
3.216.6 Sympy [C] (verification not implemented) 1662
3.216.7 Maxima [A] (verification not implemented) 1663
3.216.8 Giac [A] (verification not implemented) 1663
3.216.9 Mupad [B] (verification not implemented) 1663

3.216.1 Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

output
```
1/2*(2*a*g*x+e*x^2)/a/(b*x^4+a)^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{x(2ag+ex)}{2a\sqrt{a+bx^4}}$$

input
```
Integrate[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(x*(2*a*g + e*x))/(2*a*Sqrt[a + b*x^4])
```

3.216.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2395}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag-bgx^4+ex}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 2395$$

$$\frac{2agx+ex^2}{2a\sqrt{a+bx^4}}$$

input
```
Int[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])
```

3.216.3.1 Defintions of rubi rules used

rule 2395
```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, Simp[-(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

3.216.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result
gosper	$\frac{x(2ag+ex)}{2\sqrt{b\,x^4+a}\,a}$
trager	$\frac{x(2ag+ex)}{2\sqrt{b\,x^4+a}\,a}$
elliptic	$\frac{e\,x^2}{2a\sqrt{b\,x^4+a}}+\frac{gx}{\sqrt{b\,x^4+a}}$
default	$ag\left(\frac{x}{2a\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{e\,x^2}{2a\sqrt{b\,x^4+a}}-gb\left(-\frac{x}{2b\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+}}{2b\sqrt{}}\right.$

input
```
int((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(2*a*g+e*x)/(b*x^4+a)^(1/2)/a
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt{bx^4+a}(2\,agx+ex^2)}{2\,(abx^4+a^2)}$$

input
```
integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(b*x^4 + a)*(2*a*g*x + e*x^2)/(a*b*x^4 + a^2)
```

3.216.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.84 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.59

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((-b*g*x**4+a*g+e*x)/(b*x**4+a)**(3/2),x)
```

output
```
g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp
_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*x*
*4/a))
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{2\,agx+ex^2}{2\,\sqrt{bx^4+a}a}$$

input
```
integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*a*g*x + e*x^2)/(sqrt(b*x^4 + a)*a)
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{\left(2\,g+\frac{ex}{a}\right)x}{2\,\sqrt{bx^4+a}}$$

input
```
integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(2*g + e*x/a)*x/sqrt(b*x^4 + a)
```

3.216.9 Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{g\,x+\frac{e\,x^2}{2\,a}}{\sqrt{b\,x^4+a}}$$

input
```
int((a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

output
```
(g*x + (e*x^2)/(2*a))/(a + b*x^4)^(1/2)
```

3.217 $\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx$

3.217.1 Optimal result 1664
3.217.2 Mathematica [A] (verified) 1664
3.217.3 Rubi [A] (verified) 1665
3.217.4 Maple [A] (verified) 1665
3.217.5 Fricas [A] (verification not implemented) 1666
3.217.6 Sympy [A] (verification not implemented) 1666
3.217.7 Maxima [A] (verification not implemented) 1667
3.217.8 Giac [A] (verification not implemented) 1667
3.217.9 Mupad [B] (verification not implemented) 1667

3.217.1 Optimal result

Integrand size = 28, antiderivative size = 25

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = -\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

output
```
1/2*(2*b*g*x-f)/b/(b*x^4+a)^(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{-f+2bgx}{2b\sqrt{a+bx^4}}$$

input
```
Integrate[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(-f + 2*b*g*x)/(2*b*Sqrt[a + b*x^4])
```

3.217.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{2395\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag-bgx^4+fx^3}{(a+bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \; 2395$$

$$-\frac{f-2bgx}{2b\sqrt{a+bx^4}}$$

input
```
Int[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(f - 2*b*g*x)/(b*Sqrt[a + b*x^4])
```

3.217.3.1 Defintions of rubi rules used

rule 2395
```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, Simp[-(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

3.217.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result
gosper	$\frac{2bgx-f}{2b\sqrt{b\,x^4+a}}$
trager	$\frac{2bgx-f}{2b\sqrt{b\,x^4+a}}$
elliptic	$-\frac{f}{2b\sqrt{b\,x^4+a}}+\frac{gx}{\sqrt{b\,x^4+a}}$
default	$-\frac{f}{2b\sqrt{b\,x^4+a}}+ag\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)-gb\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{}}{2b\sqrt{}}\right.$

input
```
int((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*b*g*x-f)/b/(b*x^4+a)^(1/2)
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{ag+fx^3-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx=\frac{\sqrt{bx^4+a}(2\,bgx-f)}{2\left(b^2x^4+ab\right)}$$

input
```
integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(b*x^4 + a)*(2*b*g*x - f)/(b^2*x^4 + a*b)
```

3.217.6 Sympy [A] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.36

$$\int \frac{ag+fx^3-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx=f\left(\begin{cases}-\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}-\frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((-b*g*x**4+f*x**3+a*g)/(b*x**4+a)**(3/2),x)
```

output
```
f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), Tru
e)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{ag + fx^3 - bgx^4}{(a + bx^4)^{3/2}}\, dx = \frac{2\, bgx - f}{2\sqrt{bx^4 + a}b}$$

input
```
integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*b*g*x - f)/(sqrt(b*x^4 + a)*b)
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{ag + fx^3 - bgx^4}{(a + bx^4)^{3/2}}\, dx = \frac{2\, gx - \frac{f}{b}}{2\sqrt{bx^4 + a}}$$

input
```
integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(2*g*x - f/b)/sqrt(b*x^4 + a)
```

3.217.9 Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{ag + fx^3 - bgx^4}{(a + bx^4)^{3/2}}\, dx = \frac{g\, x - \frac{f}{2\, b}}{\sqrt{b\, x^4 + a}}$$

input
```
int((a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

output
```
(g*x - f/(2*b))/(a + b*x^4)^(1/2)
```

3.218 $\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx$

3.218.1 Optimal result 1668
3.218.2 Mathematica [A] (verified) 1668
3.218.3 Rubi [A] (verified) 1669
3.218.4 Maple [A] (verified) 1669
3.218.5 Fricas [A] (verification not implemented) 1670
3.218.6 Sympy [A] (verification not implemented) 1670
3.218.7 Maxima [A] (verification not implemented) 1671
3.218.8 Giac [A] (verification not implemented) 1671
3.218.9 Mupad [B] (verification not implemented) 1671

3.218.1 Optimal result

Integrand size = 31, antiderivative size = 38

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx=-\frac{af-2abgx-bex^2}{2ab\sqrt{a+bx^4}}$$

output
```
1/2*(2*a*b*g*x+b*e*x^2-a*f)/a/b/(b*x^4+a)^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{-af+2abgx+bex^2}{2ab\sqrt{a+bx^4}}$$

input
```
Integrate[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(-(a*f) + 2*a*b*g*x + b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])
```

3.218.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2395}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag-bgx^4+ex+fx^3}{(a+bx^4)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 2395}$$
$$-\frac{-2abgx+af-bex^2}{2ab\sqrt{a+bx^4}}$$

input
```
Int[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(a*f - 2*a*b*g*x - b*e*x^2)/(a*b*Sqrt[a + b*x^4])
```

3.218.3.1 Defintions of rubi rules used

rule 2395
```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, Simp[-(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

3.218.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result
gosper	$\frac{2abgx+b e\,x^{2}-af}{2ab\sqrt{b\,x^{4}+a}}$
trager	$\frac{2abgx+b e\,x^{2}-af}{2ab\sqrt{b\,x^{4}+a}}$
elliptic	$-\frac{-b e\,x^{2}+af}{2\sqrt{b\,x^{4}+a}\,ab}+\frac{gx}{\sqrt{b\,x^{4}+a}}$
default	$-\frac{f}{2b\sqrt{b\,x^{4}+a}}+ag\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+\frac{e\,x^{2}}{2a\sqrt{b\,x^{4}+a}}-gb\left(-\frac{x}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\right.$

input
```
int((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a*b*g*x+b*e*x^2-a*f)/a/b/(b*x^4+a)^(1/2)
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{\sqrt{bx^4+a}(2\,abgx+bex^2-af)}{2\,(ab^2x^4+a^2b)}$$

input
```
integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*b)
```

3.218.6 Sympy [A] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.50

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx=f\left(\begin{cases}-\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}-\frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((-b*g*x**4+f*x**3+a*g+e*x)/(b*x**4+a)**(3/2),x)
```

output
```
f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), Tru
e)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1
+ b*x**4/a))
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt{bx^4+a}(2\,abgx+bex^2-af)}{2\,(ab^2x^4+a^2b)}$$

input
```
integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*b)
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{\left(2\,g+\frac{ex}{a}\right)x-\frac{f}{b}}{2\,\sqrt{bx^4+a}}$$

input
```
integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*((2*g + e*x/a)*x - f/b)/sqrt(b*x^4 + a)
```

3.218.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{g\,x-\frac{f}{2\,b}+\frac{e\,x^2}{2\,a}}{\sqrt{b\,x^4+a}}$$

input
```
int((a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

output
```
(g*x - f/(2*b) + (e*x^2)/(2*a))/(a + b*x^4)^(1/2)
```

3.219 $\int \frac{-1+x^4}{(1+x^4)^{3/2}} \, dx$

3.219.1 Optimal result 1672
3.219.2 Mathematica [A] (verified) 1672
3.219.3 Rubi [A] (verified) 1673
3.219.4 Maple [A] (verified) 1674
3.219.5 Fricas [A] (verification not implemented) 1674
3.219.6 Sympy [C] (verification not implemented) 1675
3.219.7 Maxima [A] (verification not implemented) 1675
3.219.8 Giac [A] (verification not implemented) 1675
3.219.9 Mupad [B] (verification not implemented) 1676

3.219.1 Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}} \, dx = -\frac{x}{\sqrt{1+x^4}}$$

output
```
-x/(x^4+1)^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}} \, dx = -\frac{x}{\sqrt{1+x^4}}$$

input
```
Integrate[(-1 + x^4)/(1 + x^4)^(3/2),x]
```

output
```
-(x/Sqrt[1 + x^4])
```

3.219.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-1}{(x^4+1)^{3/2}}\,dx$$

$$\Big\downarrow \; 908$$

$$-\frac{x}{\sqrt{x^4+1}}$$

input
```
Int[(-1 + x^4)/(1 + x^4)^(3/2),x]
```

output
```
-(x/Sqrt[1 + x^4])
```

3.219.3.1 Defintions of rubi rules used

rule 908
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

3.219.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{x}{\sqrt{x^4+1}}$	11
default	$-\frac{x}{\sqrt{x^4+1}}$	11
trager	$-\frac{x}{\sqrt{x^4+1}}$	11
risch	$-\frac{x}{\sqrt{x^4+1}}$	11
elliptic	$-\frac{x}{\sqrt{x^4+1}}$	11
pseudoelliptic	$-\frac{x}{\sqrt{x^4+1}}$	11
meijerg	$-x{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{4};-x^4\right)+\frac{x^5{}_2F_1\left(\frac{5}{4},\frac{3}{2};\frac{9}{4};-x^4\right)}{5}$	32

input
```
int((x^4-1)/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-x/(x^4+1)^(1/2)
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{x^4+1}}$$

input
```
integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-x/sqrt(x^4 + 1)
```

3.219.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((x**4-1)/(x**4+1)**(3/2),x)
```

output
```
x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi))/(4*gamma(9
/4)) - x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi))/(4*gam
ma(5/4))
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{x^4+1}}$$

input
```
integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-x/sqrt(x^4 + 1)
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{x^4+1}}$$

input
```
integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
-x/sqrt(x^4 + 1)
```

3.219.9 Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{x^4+1}}$$

input
```
int((x^4 - 1)/(x^4 + 1)^(3/2),x)
```

output
```
-x/(x^4 + 1)^(1/2)
```

3.220 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$

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3.220.1 Optimal result

Integrand size = 42, antiderivative size = 385

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx=\frac{f\sqrt{a+bx^4}}{2b}+\frac{gx\sqrt{a+bx^4}}{3b}+\frac{hx^2\sqrt{a+bx^4}}{4b}+\frac{ix^3\sqrt{a+bx^4}}{5b}+\frac{(5be-3ai)x\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{(2bd-ah)\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{\sqrt[4]{a}(5be-3ai)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\left(15be+\frac{5\sqrt{b}(3bc-ag)}{\sqrt{a}}-9ai\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}$$

output
```
1/4*(-a*h+2*b*d)*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/2*f*(b*x^4
+a)^(1/2)/b+1/3*g*x*(b*x^4+a)^(1/2)/b+1/4*h*x^2*(b*x^4+a)^(1/2)/b+1/5*i*x^
3*(b*x^4+a)^(1/2)/b+1/5*(-3*a*i+5*b*e)*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+
x^2*b^(1/2))-1/5*a^(1/4)*(-3*a*i+5*b*e)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1
/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)+1/30*a^(1/4)*(cos(2*arctan(b^(1/4)*x
/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arcta
n(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(15*b*e-9*a*i+5*(
-a*g+3*b*c)*b^(1/2)/a^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(
7/4)/(b*x^4+a)^(1/2)
```

3.220.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.73

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{30a\sqrt{b}f+20a\sqrt{b}gx+15a\sqrt{b}hx^2+12a\sqrt{b}ix^3+30b^{3/2}fx^4+20b^{3/2}gx^5+15b^{3/2}hx^6+12b^{3/2}ix^7+30bd\sqrt{}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4
],x]
```

output
```
(30*a*Sqrt[b]*f + 20*a*Sqrt[b]*g*x + 15*a*Sqrt[b]*h*x^2 + 12*a*Sqrt[b]*i*x
^3 + 30*b^(3/2)*f*x^4 + 20*b^(3/2)*g*x^5 + 15*b^(3/2)*h*x^6 + 12*b^(3/2)*i
*x^7 + 30*b*d*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 15*
a*h*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*Sqrt[b]*(-
3*b*c + a*g)*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x
^4)/a)] + 4*Sqrt[b]*(5*b*e - 3*a*i)*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric
2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^(3/2)*Sqrt[a + b*x^4])
```

3.220.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$$
$$\Big\downarrow \text{2424}$$
$$\int \left(\frac{c+ex^2+gx^4+ix^6}{\sqrt{a+bx^4}}+\frac{x\left(d+fx^2+hx^4\right)}{\sqrt{a+bx^4}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\frac{5\sqrt{b}(3bc-ag)}{\sqrt{a}}-9ai+15be\right)}{30b^{7/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(5be-3ai)E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)(2bd-ah)}{4b^{3/2}}+\frac{x\sqrt{a+bx^4}(5be-3ai)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\sqrt{a+bx^4}}{2b}+\frac{gx\sqrt{a+bx^4}}{3b}+\frac{hx^2\sqrt{a+bx^4}}{4b}+\frac{ix^3\sqrt{a+bx^4}}{5b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4],x]
```

output
```
(f*Sqrt[a + b*x^4])/(2*b) + (g*x*Sqrt[a + b*x^4])/(3*b) + (h*x^2*Sqrt[a +
b*x^4])/(4*b) + (i*x^3*Sqrt[a + b*x^4])/(5*b) + ((5*b*e - 3*a*i)*x*Sqrt[a
+ b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*b*d - a*h)*ArcTanh[(Sq
rt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*(5*b*e - 3*a*i)*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*
(15*b*e + (5*Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 9*a*i)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

3.220.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.220.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.75

method	result
elliptic	$\frac{i x^3\sqrt{b x^4+a}}{5b}+\frac{h x^2\sqrt{b x^4+a}}{4b}+\frac{gx\sqrt{b x^4+a}}{3b}+\frac{f\sqrt{b x^4+a}}{2b}+\frac{\left(c-\frac{ag}{3b}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+\frac{\left(d-\frac{ah}{2b}\right)\ln\left(2\right.}{}$
risch	$\frac{\left(12i x^3+15h x^2+20gx+30f\right)\sqrt{b x^4+a}}{60b}-\frac{\frac{10ag\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}-\frac{30bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+\frac{i(18a}{}}{}$
default	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+i\left(\frac{x^3\sqrt{b x^4+a}}{5b}-\frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}\right)$

input
```
int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
1/5*i*x^3*(b*x^4+a)^(1/2)/b+1/4*h*x^2*(b*x^4+a)^(1/2)/b+1/3*g*x*(b*x^4+a)^
(1/2)/b+1/2*f*(b*x^4+a)^(1/2)/b+(c-1/3*a/b*g)/(I/a^(1/2)*b^(1/2))^(1/2)*(1
-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*(d-1/2*a/b*h)*ln(2*x^2*b^
(1/2)+2*(b*x^4+a)^(1/2))/b^(1/2)+I*(e-3/5*a/b*i)*a^(1/2)/(I/a^(1/2)*b^(1/2
))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE
(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.53

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{24\,(5\,abe-3\,a^2i)\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+8\,(15\,b^2c-15\,abe-5\,abg+9\,a^2i)\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(a}{}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorit
hm="fricas")
```

output
```
1/120*(24*(5*a*b*e - 3*a^2*i)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a
/b)^(1/4)/x), -1) + 8*(15*b^2*c - 15*a*b*e - 5*a*b*g + 9*a^2*i)*sqrt(b)*x*
(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - 15*(2*a*b*d - a^2*h)
*sqrt(b)*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(12*a*b*i
*x^4 + 15*a*b*h*x^3 + 20*a*b*g*x^2 + 30*a*b*f*x + 60*a*b*e - 36*a^2*i)*sqr
t(b*x^4 + a))/(a*b^2*x)
```

3.220.6 Sympy [A] (verification not implemented)

Time = 3.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.68

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}hx^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{ah\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + f\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right) + \frac{d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{gx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{ix^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*h*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*h*asinh(sqrt(b)*x**2/sqrt(a))/
(4*b**(3/2)) + f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)
/(2*b), True)) + d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4
)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4
)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)
/(4*sqrt(a)*gamma(7/4)) + g*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + i*x**7*gamma(7/4)*hyper((1/
2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

3.220.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx=\int \frac{ix^6+hx^5+gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a
), x)
```

3.220.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx=\int \frac{ix^6+hx^5+gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a
), x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$$
$$=\int \frac{i\,x^6+h\,x^5+g\,x^4+f\,x^3+e\,x^2+d\,x+c}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^(1/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^(1/2), x
)
```

3.221 $\int \frac{1+x}{1+x^5} dx$

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3.221.1 Optimal result

Integrand size = 11, antiderivative size = 109

$$\int \frac{1+x}{1+x^5} dx = -\frac{1}{5}\sqrt[5]{-1}(1+\sqrt[5]{-1}) \log \left(\sqrt[5]{-1}-x\right) + \frac{1}{5}(-1)^{4/5} \left(1 -(-1)^{4/5}\right) \log \left(-(-1)^{4/5}-x\right)+\frac{1}{5}(-1)^{2/5} \left(1-(-1)^{2/5}\right) \log \left((-1)^{2/5}+x\right)-\frac{1}{5}(-1)^{3/5} \left(1+(-1)^{3/5}\right) \log \left(-(-\right.$$

output
```
-1/5*(-1)^(1/5)*(1+(-1)^(1/5))*ln((-1)^(1/5)-x)+1/5*(-1)^(4/5)*(1-(-1)^(4/
5))*ln(-(-1)^(4/5)-x)+1/5*(-1)^(2/5)*(1-(-1)^(2/5))*ln((-1)^(2/5)+x)-1/5*(
-1)^(3/5)*(1+(-1)^(3/5))*ln(-(-1)^(3/5)+x)
```

3.221.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int \frac{1+x}{1+x^5} dx = \text{RootSum}\left[1-\#1+\#1^2-\#1^3+\#1^4\&, \frac{\log(x-\#1)}{-1+2\#1-3\#1^2+4\#1^3}\&\right]$$

input
```
Integrate[(1 + x)/(1 + x^5),x]
```

output
```
RootSum[1 - #1 + #1^2 - #1^3 + #1^4 & , Log[x - #1]/(-1 + 2*#1 - 3*#1^2 +
4*#1^3) & ]
```

3.221.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2019, 2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{x^5+1} \, dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{x^4-x^3+x^2-x+1}dx$$
$$\Big\downarrow \text{2492}$$
$$\int \left(\frac{-2x+\sqrt{5}+1}{\sqrt{5}\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}-\frac{-2x-\sqrt{5}+1}{\sqrt{5}\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{5}\sqrt{5-2\sqrt{5}}\arctan\left(\frac{-4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{5}\sqrt{5+2\sqrt{5}}\arctan\left(\frac{-4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)+\frac{\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{2\sqrt{5}}-\frac{\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{2\sqrt{5}}$$

input
```
Int[(1 + x)/(1 + x^5),x]
```

output
```
-1/5*(Sqrt[5 - 2*Sqrt[5]]*ArcTan[(1 - Sqrt[5] - 4*x)/Sqrt[2*(5 + Sqrt[5])]
]) - (Sqrt[5 + 2*Sqrt[5]]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[2*(5 - Sqrt[5])]
])/5 + Log[2 - (1 - Sqrt[5])*x + 2*x^2]/(2*Sqrt[5]) - Log[2 - (1 + Sqrt[5]
)*x + 2*x^2]/(2*Sqrt[5])
```

3.221.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

3.221.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.41

method	result
risch	$\sum_{\_R=\operatorname{RootOf}(\_Z^4-\_Z^3+\_Z^2-\_Z+1)}\frac{\ln\left(x-\_R\right)}{4\_R^3-3\_R^2+2\_R-1}$
default	$\frac{\sqrt{5}\,\ln\left(x\sqrt{5}+2x^2-x+2\right)}{10}+\frac{2\left(-\frac{\sqrt{5}\,(\sqrt{5}-1)}{2}+5-\sqrt{5}\right)\arctan\left(\frac{\sqrt{5}+4x-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{\sqrt{5}\,\ln\left(-x\sqrt{5}+2x^2-x+2\right)}{10}-\frac{2\left(-\frac{\sqrt{5}\,(-\sqrt{5}-1)}{2}-\right.}{5}$
meijerg	$\frac{x\ln\left(1+(x^5)^{\frac{1}{5}}\right)}{5(x^5)^{\frac{1}{5}}}-\frac{x\cos\left(\frac{\pi}{5}\right)\ln\left(1-2\cos\left(\frac{\pi}{5}\right)(x^5)^{\frac{1}{5}}+(x^5)^{\frac{2}{5}}\right)}{5(x^5)^{\frac{1}{5}}}+\frac{2x\sin\left(\frac{\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{\pi}{5}\right)(x^5)^{\frac{1}{5}}}{1-\cos\left(\frac{\pi}{5}\right)(x^5)^{\frac{1}{5}}}\right)}{5(x^5)^{\frac{1}{5}}}+\frac{x\cos\left(\frac{2\pi}{5}\right)\ln\left(1+2\cos\right.}{5(x^5}$

input
```
int((1+x)/(x^5+1),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(4*_R^3-3*_R^2+2*_R-1)*ln(x-_R),_R=RootOf(_Z^4-_Z^3+_Z^2-_Z+1))
```

3.221.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(73) = 146$.

Time = 0.99 (sec) , antiderivative size = 835, normalized size of antiderivative = 7.66

$$\int \frac{1+x}{1+x^5} \, dx = \text{Too large to display}$$

input
```
integrate((1+x)/(x^5+1),x, algorithm="fricas")
```

output
```
-1/10*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))*log(3/8*(sqrt(5) + 5*sqrt(-2
/25*sqrt(5) - 1/5))^3 + 1/8*(3*sqrt(5) + 15*sqrt(-2/25*sqrt(5) - 1/5) + 8)
*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2 + 3/8*((sqrt(5) + 5*sqrt(-2/25*
sqrt(5) - 1/5))^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 11*x + 1
) - 1/10*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))*log(-3/8*(sqrt(5) + 5*sqr
t(-2/25*sqrt(5) - 1/5))^3 + (sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 + 11
*x - 9/2*sqrt(5) - 45/2*sqrt(-2/25*sqrt(5) - 1/5) - 14) + 1/10*(sqrt(5) +
5*sqrt(-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) +
5*sqrt(-2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/1
00*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2))*log(-1/8*(3*sqrt(5) + 15*sq
rt(-2/25*sqrt(5) - 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - (
sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 3/8*((sqrt(5) + 5*sqrt(-2/25*sq
rt(5) - 1/5))^2 - 12)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 5/4*sqrt(-
3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))^2 - 1/50*(sqrt(5) + 5*sqrt(-
2/25*sqrt(5) - 1/5))*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) - 3/100*(sqrt
(5) - 5*sqrt(-2/25*sqrt(5) - 1/5))^2)*((3*sqrt(5) + 15*sqrt(-2/25*sqrt(5)
- 1/5) + 8)*(sqrt(5) - 5*sqrt(-2/25*sqrt(5) - 1/5)) + 8*sqrt(5) + 40*sqrt(
-2/25*sqrt(5) - 1/5) + 36) + 22*x + 9/2*sqrt(5) + 45/2*sqrt(-2/25*sqrt(5)
- 1/5) + 2) + 1/10*(sqrt(5) - 5*sqrt(-3/100*(sqrt(5) + 5*sqrt(-2/25*sqrt(5
) - 1/5))^2 - 1/50*(sqrt(5) + 5*sqrt(-2/25*sqrt(5) - 1/5))*(sqrt(5) - 5...
```

3.221.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. $2(109) = 218$.

Time = 0.58 (sec) , antiderivative size = 1287, normalized size of antiderivative = 11.81

$$\int \frac{1+x}{1+x^5} \, dx = \text{Too large to display}$$

input
```
integrate((1+x)/(x**5+1),x)
```

output
```
sqrt(5)*log(x**2 + x*(-48/11 - 21*sqrt(5)/11 + 4*sqrt(10)*sqrt(sqrt(5) + 3
)/11 + 45*sqrt(2)*sqrt(sqrt(5) + 3)/22) - 1381*sqrt(10)*sqrt(sqrt(5) + 3)/
484 - 3045*sqrt(2)*sqrt(sqrt(5) + 3)/484 + 2213*sqrt(5)/242 + 5217/242)/10
 - sqrt(5)*log(x**2 + x*(-48/11 - 45*sqrt(2)*sqrt(3 - sqrt(5))/22 + 4*sqrt
(10)*sqrt(3 - sqrt(5))/11 + 21*sqrt(5)/11) - 2213*sqrt(5)/242 - 1381*sqrt(
10)*sqrt(3 - sqrt(5))/484 + 3045*sqrt(2)*sqrt(3 - sqrt(5))/484 + 5217/242)
/10 + 2*sqrt(-sqrt(10)*sqrt(3 - sqrt(5))/50 + 3/20)*atan(44*x/(-8*sqrt(5)*
sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt
(5)) + 15)) - 96/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*
sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*s
qrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) - 45*sqrt(2)*sqrt(3 - sqrt(5))/(-
8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - s
qrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqr
t(3 - sqrt(5)) + 15)) + 8*sqrt(10)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*sqrt(-2*s
qrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqr
t(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15
)) + 42*sqrt(5)/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*s
qrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15))) + 2*sqrt(-sqrt(10)*sqrt(sqrt(5...
```

3.221.7 Maxima [F]

$$\int \frac{1+x}{1+x^5}\,dx = \int \frac{x+1}{x^5+1}\,dx$$

input
```
integrate((1+x)/(x^5+1),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(x^5 + 1), x)
```

3.221.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int \frac{1+x}{1+x^5}\,dx = \frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4\,x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4\,x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right) - \frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{5}+1\right)+1\right) + \frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{5}-1\right)+1\right)$$

input
```
integrate((1+x)/(x^5+1),x, algorithm="giac")
```

output
```
1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))
+ 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)
) - 1/10*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/10*sqrt(5)*log(x^2
 + 1/2*x*(sqrt(5) - 1) + 1)
```

3.221.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \frac{1+x}{1+x^5}\,dx = \sum_{k=1}^{4} \ln\left(\operatorname{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)\left(-4\,x + \operatorname{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)\left(25\operatorname{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)+15\,x-15\right)+1\right)\right)\operatorname{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)$$

input
```
int((x + 1)/(x^5 + 1),x)
```

output
```
symsum(log(root(z^4 - z/25 + 1/125, z, k)*(root(z^4 - z/25 + 1/125, z, k)*
(25*root(z^4 - z/25 + 1/125, z, k) + 15*x - 15) - 4*x + 1))*root(z^4 - z/2
5 + 1/125, z, k), k, 1, 4)
```

3.222 $\int \frac{1-x}{1-x^5} dx$

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3.222.8 Giac [A] (verification not implemented) . . . 1694
3.222.9 Mupad [B] (verification not implemented) . . . 1694

3.222.1 Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \frac{1-x}{1-x^5} dx = -\frac{1}{5}(-1)^{2/5} \left(1 -(-1)^{2/5}\right) \log \left((-1)^{2/5}-x\right)+\frac{1}{5}(-1)^{3/5} \left(1+(-1)^{3/5}\right) \log \left(-(-1)^{3/5}-x\right)+\frac{1}{5}\sqrt[5]{-1}(1+\sqrt[5]{-1}) \log \left(\sqrt[5]{-1}+x\right.$$

output
```
-1/5*(-1)^(2/5)*(1-(-1)^(2/5))*ln((-1)^(2/5)-x)+1/5*(-1)^(3/5)*(1+(-1)^(3/
5))*ln(-(-1)^(3/5)-x)+1/5*(-1)^(1/5)*(1+(-1)^(1/5))*ln((-1)^(1/5)+x)-1/5*(
-1)^(4/5)*(1-(-1)^(4/5))*ln(-(-1)^(4/5)+x)
```

3.222.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.43

$$\int \frac{1-x}{1-x^5} dx = \text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{\log(x-\#1)}{1+2\#1+3\#1^2+4\#1^3}\&\right]$$

input
```
Integrate[(1 - x)/(1 - x^5),x]
```

output
```
RootSum[1 + #1 + #1^2 + #1^3 + #1^4 & , Log[x - #1]/(1 + 2*#1 + 3*#1^2 + 4
*#1^3) & ]
```

3.222.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2019, 2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{1-x^5}\,dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{x^4+x^3+x^2+x+1}dx$$

$$\Big\downarrow \text{ 2492}$$

$$\int \left(\frac{2x+\sqrt{5}+1}{\sqrt{5}\left(2x^2+\left(1+\sqrt{5}\right)x+2\right)}-\frac{2x-\sqrt{5}+1}{\sqrt{5}\left(2x^2+\left(1-\sqrt{5}\right)x+2\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}\sqrt{5-2\sqrt{5}}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)+\frac{1}{5}\sqrt{5+2\sqrt{5}}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)-\frac{\log\left(2x^2+\left(1-\sqrt{5}\right)x+2\right)}{2\sqrt{5}}+\frac{\log\left(2x^2+\left(1+\sqrt{5}\right)x+2\right)}{2\sqrt{5}}$$

input
```
Int[(1 - x)/(1 - x^5),x]
```

output
```
(Sqrt[5 - 2*Sqrt[5]]*ArcTan[(1 - Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]])/5
+ (Sqrt[5 + 2*Sqrt[5]]*ArcTan[(1 + Sqrt[5] + 4*x)/Sqrt[2*(5 - Sqrt[5])]])/
5 - Log[2 + (1 - Sqrt[5])*x + 2*x^2]/(2*Sqrt[5]) + Log[2 + (1 + Sqrt[5])*x
 + 2*x^2]/(2*Sqrt[5])
```

3.222.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

3.222.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.38

method	result
risch	$\sum_{\_R=\operatorname{RootOf}(\_Z^4+\_Z^3+\_Z^2+\_Z+1)}\frac{\ln(x-\_R)}{4\_R^3+3\_R^2+2\_R+1}$
default	$-\frac{\sqrt{5}\,\ln\left(-x\sqrt{5}+2x^2+x+2\right)}{10}-\frac{2\left(-\frac{\sqrt{5}\,(-\sqrt{5}+1)}{2}+\sqrt{5}-5\right)\arctan\left(\frac{-\sqrt{5}+4x+1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}+\frac{\sqrt{5}\,\ln\left(x\sqrt{5}+2x^2+x+2\right)}{10}+\frac{2\left(-\frac{\sqrt{5}\,(\sqrt{5}+1}{2}\right.}{}$
meijerg	$-\frac{x\left(\ln\left(1-(x^5)^{\frac{1}{5}}\right)+\cos\left(\frac{2\pi}{5}\right)\ln\left(1-2\cos\left(\frac{2\pi}{5}\right)(x^5)^{\frac{1}{5}}+(x^5)^{\frac{2}{5}}\right)-2\sin\left(\frac{2\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{2\pi}{5}\right)(x^5)^{\frac{1}{5}}}{1-\cos\left(\frac{2\pi}{5}\right)(x^5)^{\frac{1}{5}}}\right)-\cos\left(\frac{\pi}{5}\right)\ln\left(1+2\cos\left(\frac{\pi}{5}\right)(x\right.\right.}{5(x^5)^{\frac{1}{5}}}$

input
```
int((1-x)/(-x^5+1),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(4*_R^3+3*_R^2+2*_R+1)*ln(x-_R),_R=RootOf(_Z^4+_Z^3+_Z^2+_Z+1))
```

3.222.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(73) = 146$.

Time = 1.02 (sec) , antiderivative size = 799, normalized size of antiderivative = 7.33

$$\int \frac{1-x}{1-x^5}\,dx = \text{Too large to display}$$

input
```
integrate((1-x)/(-x^5+1),x, algorithm="fricas")
```

output
```
-1/10*(sqrt(5) - sqrt(2*sqrt(5) - 5))*log(3/8*(sqrt(5) + sqrt(2*sqrt(5) -
5))^3 + 1/8*(3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt
(5) - 5))^2 + 3/8*((sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 12)*(sqrt(5) - sqrt
(2*sqrt(5) - 5)) + 11*x - 1) - 1/10*(sqrt(5) + sqrt(2*sqrt(5) - 5))*log(-3
/8*(sqrt(5) + sqrt(2*sqrt(5) - 5))^3 - (sqrt(5) + sqrt(2*sqrt(5) - 5))^2 +
 11*x - 9/2*sqrt(5) - 9/2*sqrt(2*sqrt(5) - 5) + 14) + 1/10*(sqrt(5) + 5*sq
rt(-3/100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sqrt(5) + sqrt(2*sqrt(
5) - 5))*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(5) - sqrt(2*sqrt(5)
 - 5))^2))*log(-1/8*(3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqr
t(2*sqrt(5) - 5))^2 + (sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 3/8*((sqrt(5) +
sqrt(2*sqrt(5) - 5))^2 - 12)*(sqrt(5) - sqrt(2*sqrt(5) - 5)) + 5/4*sqrt(-3
/100*(sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - 1/50*(sqrt(5) + sqrt(2*sqrt(5) -
5))*(sqrt(5) - sqrt(2*sqrt(5) - 5)) - 3/100*(sqrt(5) - sqrt(2*sqrt(5) - 5)
)^2)*((3*sqrt(5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5) -
5)) - 8*sqrt(5) - 8*sqrt(2*sqrt(5) - 5) + 36) + 22*x + 9/2*sqrt(5) + 9/2*s
qrt(2*sqrt(5) - 5) - 2) + 1/10*(sqrt(5) - 5*sqrt(-3/100*(sqrt(5) + sqrt(2*
sqrt(5) - 5))^2 - 1/50*(sqrt(5) + sqrt(2*sqrt(5) - 5))*(sqrt(5) - sqrt(2*s
qrt(5) - 5)) - 3/100*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2))*log(-1/8*(3*sqrt(
5) + 3*sqrt(2*sqrt(5) - 5) - 8)*(sqrt(5) - sqrt(2*sqrt(5) - 5))^2 + (sqrt(
5) + sqrt(2*sqrt(5) - 5))^2 - 3/8*((sqrt(5) + sqrt(2*sqrt(5) - 5))^2 - ...
```

3.222.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. $2(109) = 218$.

Time = 0.55 (sec) , antiderivative size = 1287, normalized size of antiderivative = 11.81

$$\int \frac{1-x}{1-x^5}\,dx = \text{Too large to display}$$

input
```
integrate((1-x)/(-x**5+1),x)
```

output
```
sqrt(5)*log(x**2 + x*(-21*sqrt(5)/11 - 4*sqrt(10)*sqrt(3 - sqrt(5))/11 + 4
5*sqrt(2)*sqrt(3 - sqrt(5))/22 + 48/11) - 2213*sqrt(5)/242 - 1381*sqrt(10)
*sqrt(3 - sqrt(5))/484 + 3045*sqrt(2)*sqrt(3 - sqrt(5))/484 + 5217/242)/10
 - sqrt(5)*log(x**2 + x*(-45*sqrt(2)*sqrt(sqrt(5) + 3)/22 - 4*sqrt(10)*sqr
t(sqrt(5) + 3)/11 + 21*sqrt(5)/11 + 48/11) - 1381*sqrt(10)*sqrt(sqrt(5) +
3)/484 - 3045*sqrt(2)*sqrt(sqrt(5) + 3)/484 + 2213*sqrt(5)/242 + 5217/242)
/10 + 2*sqrt(-sqrt(10)*sqrt(3 - sqrt(5))/50 + 3/20)*atan(44*x/(-8*sqrt(5)*
sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt
(5)) + 15)) - 42*sqrt(5)/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) +
15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15
) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) - 8*sqrt(10)*sqrt(3 - sqr
t(5))/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sq
rt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt
(10)*sqrt(3 - sqrt(5)) + 15)) + 45*sqrt(2)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*s
qrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqr
t(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(
5)) + 15)) + 96/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*s
qrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15))) + 2*sqrt(-sqrt(10)*sqrt(sqrt(5...
```

3.222.7 Maxima [F]

$$\int \frac{1-x}{1-x^5}\,dx = \int \frac{x-1}{x^5-1}\,dx$$

input
```
integrate((1-x)/(-x^5+1),x, algorithm="maxima")
```

output
```
integrate((x - 1)/(x^5 - 1), x)
```

3.222.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int \frac{1-x}{1-x^5}\,dx = \frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) - \frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)$$

input
```
integrate((1-x)/(-x^5+1),x, algorithm="giac")
```

output
```
1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x - sqrt(5) + 1)/sqrt(2*sqrt(5) + 10))
+ 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x + sqrt(5) + 1)/sqrt(-2*sqrt(5) + 10)
) + 1/10*sqrt(5)*log(x^2 + 1/2*x*(sqrt(5) + 1) + 1) - 1/10*sqrt(5)*log(x^2
 - 1/2*x*(sqrt(5) - 1) + 1)
```

3.222.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

$$\int \frac{1-x}{1-x^5}\,dx = \sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)\left(4x + \mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)\left(25\,\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)+15x+15\right) + 1\right)\right)\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)$$

input
```
int((x - 1)/(x^5 - 1),x)
```

output
```
symsum(log(-root(z^4 + z/25 + 1/125, z, k)*(4*x + root(z^4 + z/25 + 1/125,
 z, k)*(25*root(z^4 + z/25 + 1/125, z, k) + 15*x + 15) + 1))*root(z^4 + z/
25 + 1/125, z, k), k, 1, 4)
```

3.223 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

3.223.1 Optimal result 1695
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3.223.1 Optimal result

Integrand size = 30, antiderivative size = 208

$$\begin{aligned}\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = {} & \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^6} \\ & -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^6}{6b^5} \\ & +\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^9}{9b^4} \\ & +\frac{(b^2d-abe+a^2f)\,x^{12}}{12b^3}+\frac{(be-af)x^{15}}{15b^2}+\frac{fx^{18}}{18b} \\ & -\frac{a^3(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3b^7}\end{aligned}$$

output
```
1/3*a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^3/b^6-1/6*a*(-a^3*f+a^2*b*e-a*b^2
*d+b^3*c)*x^6/b^5+1/9*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^9/b^4+1/12*(a^2*f-a
*b*e+b^2*d)*x^12/b^3+1/15*(-a*f+b*e)*x^15/b^2+1/18*f*x^18/b-1/3*a^3*(-a^3*
f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/b^7
```

3.223.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{bx^3(-60a^5f+30a^4b(2e+fx^3)-10a^3b^2(6d+3ex^3+2fx^6)+5a^2b^3(12c+6dx^3+4ex^6+3fx^9)+b^5x^6(2}{}$$

input
```
Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(b*x^3*(-60*a^5*f + 30*a^4*b*(2*e + f*x^3) - 10*a^3*b^2*(6*d + 3*e*x^3 + 2
*f*x^6) + 5*a^2*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) + b^5*x^6*(20*c +
 15*d*x^3 + 12*e*x^6 + 10*f*x^9) - a*b^4*x^3*(30*c + 20*d*x^3 + 15*e*x^6 +
 12*f*x^9)) + 60*a^3*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a + b*x^3]
)/(180*b^7)
```

3.223.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{ 2361}$$

$$\frac{1}{3}\int \frac{x^9\left(fx^9+ex^6+dx^3+c\right)}{bx^3+a}dx^3$$

$$\Big\downarrow \text{ 2123}$$

$$\frac{1}{3}\int \left(\frac{fx^{15}}{b}+\frac{(be-af)x^{12}}{b^2}+\frac{\left(fa^2-bea+b^2d\right)x^9}{b^3}+\frac{\left(-fa^3+bea^2-b^2da+b^3c\right)x^6}{b^4}+\frac{a\left(fa^3-bea^2+b^2da-\right.}{b^5}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{x^{12}(a^2f-abe+b^2d)}{4b^3}-\frac{a^3\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^7}+\frac{a^2x^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6}\right.$$

input
```
Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
((a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/b^6 - (a*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*x^6)/(2*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9)
/(3*b^4) + ((b^2*d - a*b*e + a^2*f)*x^12)/(4*b^3) + ((b*e - a*f)*x^15)/(5*
b^2) + (f*x^18)/(6*b) - (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b
*x^3])/b^7)/3
```

3.223.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.223.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.94

method	result
norman	$-\frac{(f a^3-a^2be+a b^2d-b^3c)x^9}{9b^4}-\frac{(af-be)x^{15}}{15b^2}+\frac{f x^{18}}{18b}+\frac{(a^2f-aeb+b^2d)x^{12}}{12b^3}+\frac{a(f a^3-a^2be+a b^2d-b^3c)x^6}{6b^5}-$
default	$-\frac{-\frac{1}{6}f x^{18}b^5+\frac{1}{5}a b^4 f x^{15}-\frac{1}{5}b^5 e x^{15}-\frac{1}{4}a^2b^3 f x^{12}+\frac{1}{4}a b^4 e x^{12}-\frac{1}{4}b^5 d x^{12}+\frac{1}{3}a^3b^2 f x^9-\frac{1}{3}a^2b^3 e x^9+\frac{1}{3}a b^4 d x^9-\frac{1}{3}b^5 c x^9-\frac{1}{2}f x^6 \cdots}{3b^6}$
parallelrisch	$10f x^{18}b^6-12x^{15}a b^5 f+12x^{15}b^6 e+15x^{12}a^2b^4 f-15x^{12}a b^5 e+15x^{12}b^6 d-20x^9a^3b^3 f+20x^9a^2b^4 e-20x^9a b^5 d+20x^9b^6 c+30x^6 a$
risch	$\frac{f x^{18}}{18b}-\frac{af x^{15}}{15b^2}+\frac{e x^{15}}{15b}+\frac{a^2f x^{12}}{12b^3}-\frac{ae x^{12}}{12b^2}+\frac{d x^{12}}{12b}-\frac{a^3f x^9}{9b^4}+\frac{a^2e x^9}{9b^3}-\frac{ad x^9}{9b^2}+\frac{c x^9}{9b}+\frac{f x^6a^4}{6b^5}-\frac{a^3e x^6}{6b^4}+$

input
```
int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/9/b^4*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)*x^9-1/15/b^2*(a*f-b*e)*x^15+1/18*f*
x^18/b+1/12*(a^2*f-a*b*e+b^2*d)*x^12/b^3+1/6*a/b^5*(a^3*f-a^2*b*e+a*b^2*d-
b^3*c)*x^6-1/3*a^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^6*x^3+1/3*a^3*(a^3*f-a^
2*b*e+a*b^2*d-b^3*c)/b^7*ln(b*x^3+a)
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.01

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{10\,b^6 f x^{18} + 12\,(b^6 e - a b^5 f) x^{15} + 15\,(b^6 d - a b^5 e + a^2 b^4 f) x^{12} + 20\,(b^6 c - a b^5 d + a^2 b^4 e - a^3 b^3 f) x^9 - 30\,(a}{}$$

input
```
integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/180*(10*b^6*f*x^18 + 12*(b^6*e - a*b^5*f)*x^15 + 15*(b^6*d - a*b^5*e + a
^2*b^4*f)*x^12 + 20*(b^6*c - a*b^5*d + a^2*b^4*e - a^3*b^3*f)*x^9 - 30*(a*
b^5*c - a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)*x^6 + 60*(a^2*b^4*c - a^3*b^3*d
 + a^4*b^2*e - a^5*b*f)*x^3 - 60*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)
*log(b*x^3 + a))/b^7
```

3.223.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.04

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{a^3(a^3f-a^2be+ab^2d-b^3c)\log(a+bx^3)}{3b^7} + x^{15}\left(-\frac{af}{15b^2}+\frac{e}{15b}\right) + x^{12}\left(\frac{a^2f}{12b^3}-\frac{ae}{12b^2}+\frac{d}{12b}\right) + x^9\left(-\frac{a^3f}{9b^4}+\frac{a^2e}{9b^3}-\frac{ad}{9b^2}+\frac{c}{9b}\right) + x^6\left(\frac{a^4f}{6b^5}-\frac{a^3e}{6b^4}+\frac{a^2d}{6b^3}-\frac{ac}{6b^2}\right) + x^3\left(-\frac{a^5f}{3b^6}+\frac{a^4e}{3b^5}-\frac{a^3d}{3b^4}+\frac{a^2c}{3b^3}\right) + \frac{fx^{18}}{18b}$$

input
```
integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

output
```
a**3*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**7) + x*
*15*(-a*f/(15*b**2) + e/(15*b)) + x**12*(a**2*f/(12*b**3) - a*e/(12*b**2)
+ d/(12*b)) + x**9*(-a**3*f/(9*b**4) + a**2*e/(9*b**3) - a*d/(9*b**2) + c/
(9*b)) + x**6*(a**4*f/(6*b**5) - a**3*e/(6*b**4) + a**2*d/(6*b**3) - a*c/(
6*b**2)) + x**3*(-a**5*f/(3*b**6) + a**4*e/(3*b**5) - a**3*d/(3*b**4) + a*
*2*c/(3*b**3)) + f*x**18/(18*b)
```

3.223.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{10\,b^5fx^{18}+12\,(b^5e-ab^4f)x^{15}+15\,(b^5d-ab^4e+a^2b^3f)x^{12}+20\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^9-30\,(a}{180\,b^6}$$
$$-\frac{(a^3b^3c-a^4b^2d+a^5be-a^6f)\log\left(bx^3+a\right)}{3\,b^7}$$

input
```
integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/180*(10*b^5*f*x^18 + 12*(b^5*e - a*b^4*f)*x^15 + 15*(b^5*d - a*b^4*e + a
^2*b^3*f)*x^12 + 20*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^9 - 30*(a*
b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^6 + 60*(a^2*b^3*c - a^3*b^2*d +
 a^4*b*e - a^5*f)*x^3)/b^6 - 1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)
*log(b*x^3 + a)/b^7
```

3.223.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.15

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{10\,b^5fx^{18}+12\,b^5ex^{15}-12\,ab^4fx^{15}+15\,b^5dx^{12}-15\,ab^4ex^{12}+15\,a^2b^3fx^{12}+20\,b^5cx^9-20\,ab^4dx^9+20\,a^2}{}$$
$$-\frac{(a^3b^3c-a^4b^2d+a^5be-a^6f)\log\left(\left|bx^3+a\right|\right)}{3\,b^7}$$

input
```
integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/180*(10*b^5*f*x^18 + 12*b^5*e*x^15 - 12*a*b^4*f*x^15 + 15*b^5*d*x^12 - 1
5*a*b^4*e*x^12 + 15*a^2*b^3*f*x^12 + 20*b^5*c*x^9 - 20*a*b^4*d*x^9 + 20*a^
2*b^3*e*x^9 - 20*a^3*b^2*f*x^9 - 30*a*b^4*c*x^6 + 30*a^2*b^3*d*x^6 - 30*a^
3*b^2*e*x^6 + 30*a^4*b*f*x^6 + 60*a^2*b^3*c*x^3 - 60*a^3*b^2*d*x^3 + 60*a^
4*b*e*x^3 - 60*a^5*f*x^3)/b^6 - 1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6
*f)*log(abs(b*x^3 + a))/b^7
```

3.223.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.14

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^{15}\left(\frac{e}{15\,b}-\frac{a\,f}{15\,b^2}\right)+x^{12}\left(\frac{d}{12\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{12\,b}\right)+x^9\left(\frac{c}{9\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{9\,b}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^6-e\,a^5\,b+d\,a^4\,b^2-c\,a^3\,b^3\right)}{3\,b^7}+\frac{f\,x^{18}}{18\,b}+\frac{a^2\,x^3\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{3\,b^2}-\frac{a\,x^6\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{6\,b}$$

input
```
int((x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

output
```
x^15*(e/(15*b) - (a*f)/(15*b^2)) + x^12*(d/(12*b) - (a*(e/b - (a*f)/b^2))/
(12*b)) + x^9*(c/(9*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(9*b)) + (log
(a + b*x^3)*(a^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e))/(3*b^7) + (f*x^18)/
(18*b) + (a^2*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(3*b^2) -
 (a*x^6*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(6*b)
```

3.224 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3} \, dx$

3.224.1 Optimal result 1701
3.224.2 Mathematica [A] (verified) 1701
3.224.3 Rubi [A] (verified) 1702
3.224.4 Maple [A] (verified) 1703
3.224.5 Fricas [A] (verification not implemented) 1704
3.224.6 Sympy [A] (verification not implemented) 1704
3.224.7 Maxima [A] (verification not implemented) 1705
3.224.8 Giac [A] (verification not implemented) 1705
3.224.9 Mupad [B] (verification not implemented) 1706

3.224.1 Optimal result

Integrand size = 30, antiderivative size = 170

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3} \, dx = -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^6}{6b^4} + \frac{(b^2d-abe+a^2f)\,x^9}{9b^3} + \frac{(be-af)x^{12}}{12b^2} + \frac{fx^{15}}{15b} + \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3b^6}$$

output
```
-1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^3/b^5+1/6*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)*x^6/b^4+1/9*(a^2*f-a*b*e+b^2*d)*x^9/b^3+1/12*(-a*f+b*e)*x^12/b^2+1/
15*f*x^15/b+1/3*a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/b^6
```

3.224.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3} \, dx = \frac{bx^3(60a^4f-30a^3b(2e+fx^3)+10a^2b^2(6d+3ex^3+2fx^6)-5ab^3(12c+6dx^3+4ex^6+3fx^9)+b^4x^3(30c}{180b^6}$$

input
```
Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(b*x^3*(60*a^4*f - 30*a^3*b*(2*e + f*x^3) + 10*a^2*b^2*(6*d + 3*e*x^3 + 2*
f*x^6) - 5*a*b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9) + b^4*x^3*(30*c + 20
*d*x^3 + 15*e*x^6 + 12*f*x^9)) - 60*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^
3*f)*Log[a + b*x^3])/(180*b^6)
```

3.224.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{ 2361}$$

$$\frac{1}{3}\int \frac{x^6\left(fx^9+ex^6+dx^3+c\right)}{bx^3+a}dx^3$$

$$\Big\downarrow \text{ 2123}$$

$$\frac{1}{3}\int \left(\frac{fx^{12}}{b}+\frac{(be-af)x^9}{b^2}+\frac{\left(fa^2-bea+b^2d\right)x^6}{b^3}+\frac{\left(-fa^3+bea^2-b^2da+b^3c\right)x^3}{b^4}+\frac{a\left(fa^3-bea^2+b^2da-b\right.}{b^5}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{x^9\left(a^2f-abe+b^2d\right)}{3b^3}+\frac{a^2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6}-\frac{ax^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^5}+\right.$$

input
```
Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(-((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/b^5) + ((b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x^6)/(2*b^4) + ((b^2*d - a*b*e + a^2*f)*x^9)/(3*b^3) + ((
b*e - a*f)*x^12)/(4*b^2) + (f*x^15)/(5*b) + (a^2*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a + b*x^3])/b^6)/3
```

3.224.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.224.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94

method	result
norman	$-\frac{(f a^3-a^2be+a b^2d-b^3c)x^6}{6b^4}-\frac{(af-be)x^{12}}{12b^2}+\frac{f x^{15}}{15b}+\frac{(a^2f-aeb+b^2d)x^9}{9b^3}+\frac{a(f a^3-a^2be+a b^2d-b^3c)x^3}{3b^5}-$ $a^2(f a^3-$
default	$\frac{\frac{1}{5}f x^{15}b^4-\frac{1}{4}a b^3 f x^{12}+\frac{1}{4}b^4 e x^{12}+\frac{1}{3}a^2b^2 f x^9-\frac{1}{3}a b^3 e x^9+\frac{1}{3}b^4 d x^9-\frac{1}{2}f x^6 a^3 b+\frac{1}{2}a^2b^2 e x^6-\frac{1}{2}a b^3 d x^6+\frac{1}{2}b^4 c x^6+a^4 f x^3-a^3 be}{3b^5}$
parallelrisch	$-\frac{-12f x^{15}b^5+15x^{12}a b^4 f-15x^{12}b^5 e-20x^9a^2b^3 f+20x^9 a b^4 e-20x^9 b^5 d+30x^6a^3b^2 f-30x^6a^2b^3 e+30x^6 a b^4 d-30x^6 b^5 c-60a^4}{18}$
risch	$\frac{f x^{15}}{15b}-\frac{af x^{12}}{12b^2}+\frac{e x^{12}}{12b}+\frac{a^2 f x^9}{9b^3}-\frac{ae x^9}{9b^2}+\frac{d x^9}{9b}-\frac{f x^6a^3}{6b^4}+\frac{a^2 e x^6}{6b^3}-\frac{ad x^6}{6b^2}+\frac{c x^6}{6b}+\frac{a^4 f x^3}{3b^5}-\frac{a^3 e x^3}{3b^4}+$ a

input
```
int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6/b^4*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)*x^6-1/12/b^2*(a*f-b*e)*x^12+1/15*f*
x^15/b+1/9*(a^2*f-a*b*e+b^2*d)*x^9/b^3+1/3*a/b^5*(a^3*f-a^2*b*e+a*b^2*d-b^
3*c)*x^3-1/3*a^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^6*ln(b*x^3+a)
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00

$$\int \frac{x^8(c + dx^3 + ex^6 + fx^9)}{a + bx^3}\, dx = \frac{12\, b^5 f x^{15} + 15\, (b^5 e - ab^4 f)x^{12} + 20\, (b^5 d - ab^4 e + a^2 b^3 f)x^9 + 30\, (b^5 c - ab^4 d + a^2 b^3 e - a^3 b^2 f)x^6 - 60\, (ab}{180\, b^6}$$

input
```
integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/180*(12*b^5*f*x^15 + 15*(b^5*e - a*b^4*f)*x^12 + 20*(b^5*d - a*b^4*e + a
^2*b^3*f)*x^9 + 30*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^6 - 60*(a*b
^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 + 60*(a^2*b^3*c - a^3*b^2*d +
a^4*b*e - a^5*f)*log(b*x^3 + a))/b^6
```

3.224.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\int \frac{x^8(c + dx^3 + ex^6 + fx^9)}{a + bx^3}\, dx = -\frac{a^2(a^3 f - a^2 be + ab^2 d - b^3 c)\log\left(a + bx^3\right)}{3b^6} + x^{12}\left(-\frac{af}{12b^2} + \frac{e}{12b}\right) + x^9\left(\frac{a^2 f}{9b^3} - \frac{ae}{9b^2} + \frac{d}{9b}\right) + x^6\left(-\frac{a^3 f}{6b^4} + \frac{a^2 e}{6b^3} - \frac{ad}{6b^2} + \frac{c}{6b}\right) + x^3\left(\frac{a^4 f}{3b^5} - \frac{a^3 e}{3b^4} + \frac{a^2 d}{3b^3} - \frac{ac}{3b^2}\right) + \frac{fx^{15}}{15b}$$

input
```
integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

output
```
-a**2*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**6) + x
**12*(-a*f/(12*b**2) + e/(12*b)) + x**9*(a**2*f/(9*b**3) - a*e/(9*b**2) +
d/(9*b)) + x**6*(-a**3*f/(6*b**4) + a**2*e/(6*b**3) - a*d/(6*b**2) + c/(6*
b)) + x**3*(a**4*f/(3*b**5) - a**3*e/(3*b**4) + a**2*d/(3*b**3) - a*c/(3*b
**2)) + f*x**15/(15*b)
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{12\,b^4fx^{15}+15\,(b^4e-ab^3f)x^{12}+20\,(b^4d-ab^3e+a^2b^2f)x^9+30\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^6-60\,(ab^3}{180\,b^5}$$
$$+\frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\log(bx^3+a)}{3\,b^6}$$

input
```
integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/180*(12*b^4*f*x^15 + 15*(b^4*e - a*b^3*f)*x^12 + 20*(b^4*d - a*b^3*e + a
^2*b^2*f)*x^9 + 30*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^6 - 60*(a*b^3
*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^3)/b^5 + 1/3*(a^2*b^3*c - a^3*b^2*d +
a^4*b*e - a^5*f)*log(b*x^3 + a)/b^6
```

3.224.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.13

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{12\,b^4fx^{15}+15\,b^4ex^{12}-15\,ab^3fx^{12}+20\,b^4dx^9-20\,ab^3ex^9+20\,a^2b^2fx^9+30\,b^4cx^6-30\,ab^3dx^6+30\,a^2b^2}{180\,b^5}$$
$$+\frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\log(|bx^3+a|)}{3\,b^6}$$

input
```
integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/180*(12*b^4*f*x^15 + 15*b^4*e*x^12 - 15*a*b^3*f*x^12 + 20*b^4*d*x^9 - 20
*a*b^3*e*x^9 + 20*a^2*b^2*f*x^9 + 30*b^4*c*x^6 - 30*a*b^3*d*x^6 + 30*a^2*b
^2*e*x^6 - 30*a^3*b*f*x^6 - 60*a*b^3*c*x^3 + 60*a^2*b^2*d*x^3 - 60*a^3*b*e
*x^3 + 60*a^4*f*x^3)/b^5 + 1/3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*l
og(abs(b*x^3 + a))/b^6
```

3.224.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^{12}\left(\frac{e}{12\,b}-\frac{a\,f}{12\,b^2}\right)+x^9\left(\frac{d}{9\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{9\,b}\right)+x^6\left(\frac{c}{6\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{6\,b}\right)-\frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^5-e\,a^4\,b+d\,a^3\,b^2-c\,a^2\,b^3\right)}{3\,b^6}+\frac{f\,x^{15}}{15\,b}-\frac{a\,x^3\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{3\,b}$$

input
```
int((x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

output
```
x^12*(e/(12*b) - (a*f)/(12*b^2)) + x^9*(d/(9*b) - (a*(e/b - (a*f)/b^2))/(9
*b)) + x^6*(c/(6*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(6*b)) - (log(a
+ b*x^3)*(a^5*f - a^2*b^3*c + a^3*b^2*d - a^4*b*e))/(3*b^6) + (f*x^15)/(15
*b) - (a*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(3*b)
```

3.225 $\int \frac{x^5\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$

3.225.1 Optimal result 1707
3.225.2 Mathematica [A] (verified) 1707
3.225.3 Rubi [A] (verified) 1708
3.225.4 Maple [A] (verified) 1709
3.225.5 Fricas [A] (verification not implemented) 1710
3.225.6 Sympy [A] (verification not implemented) 1710
3.225.7 Maxima [A] (verification not implemented) 1711
3.225.8 Giac [A] (verification not implemented) 1711
3.225.9 Mupad [B] (verification not implemented) 1712

3.225.1 Optimal result

Integrand size = 30, antiderivative size = 132

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^4} + \frac{(b^2d-abe+a^2f)\,x^6}{6b^3} + \frac{(be-af)x^9}{9b^2} + \frac{fx^{12}}{12b} - \frac{a(b^3c-ab^2d+a^2be-a^3f)\log\left(a+bx^3\right)}{3b^5}$$

output
```
1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^3/b^4+1/6*(a^2*f-a*b*e+b^2*d)*x^6/b^3
+1/9*(-a*f+b*e)*x^9/b^2+1/12*f*x^12/b-1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
*ln(b*x^3+a)/b^5
```

3.225.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.90

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{bx^3(-12a^3f+6a^2b(2e+fx^3)-2ab^2(6d+3ex^3+2fx^6)+b^3(12c+6dx^3+4ex^6+3fx^9))+12a(-b^3c+}{36b^5}$$

input
```
Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(b*x^3*(-12*a^3*f + 6*a^2*b*(2*e + f*x^3) - 2*a*b^2*(6*d + 3*e*x^3 + 2*f*x
^6) + b^3*(12*c + 6*d*x^3 + 4*e*x^6 + 3*f*x^9)) + 12*a*(-(b^3*c) + a*b^2*d
 - a^2*b*e + a^3*f)*Log[a + b*x^3])/(36*b^5)
```

3.225.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$$

$$\Big\downarrow \text{2361}$$

$$\frac{1}{3}\int \frac{x^3(fx^9+ex^6+dx^3+c)}{bx^3+a}dx^3$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{3}\int \left(\frac{fx^9}{b}+\frac{(be-af)x^6}{b^2}+\frac{(fa^2-bea+b^2d)\,x^3}{b^3}+\frac{-fa^3+bea^2-b^2da+b^3c}{b^4}+\frac{a(fa^3-bea^2+b^2da-b^3c)}{b^4\,(bx^3+a)}\right) d$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{x^6(a^2f-abe+b^2d)}{2b^3}-\frac{a\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^5}+\frac{x^3(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}+\frac{x^9}{}\right.$$

input
```
Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/b^4 + ((b^2*d - a*b*e + a^2*f)*
x^6)/(2*b^3) + ((b*e - a*f)*x^9)/(3*b^2) + (f*x^12)/(4*b) - (a*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/b^5)/3
```

3.225.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.225.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.93

method	result
norman	$-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)x^3}{3b^4}-\frac{(af-be)x^9}{9b^2}+\frac{f\,x^{12}}{12b}+\frac{(a^2f-aeb+b^2d)x^6}{6b^3}+\frac{a(f\,a^3-a^2be+a\,b^2d-b^3c)\ln(b\,x^3+a)}{3b^5}$
default	$-\frac{-\frac{1}{4}b^3f\,x^{12}+\frac{1}{3}a\,b^2f\,x^9-\frac{1}{3}b^3e\,x^9-\frac{1}{2}x^6f\,a^2b+\frac{1}{2}a\,b^2e\,x^6-\frac{1}{2}b^3d\,x^6+f\,a^3x^3-a^2be\,x^3+a\,b^2d\,x^3-b^3c\,x^3}{3b^4}+\frac{a(f\,a^3-a^2be+a\,b^2}{3b}$
parallelrisch	$\frac{3f\,x^{12}b^4-4x^9a\,b^3f+4x^9b^4e+6x^6a^2b^2f-6x^6a\,b^3e+6b^4d\,x^6-12a^3bf\,x^3+12a^2b^2e\,x^3-12a\,b^3d\,x^3+12b^4c\,x^3+12\ln(b\,x^3+a)a^4f}{36b^5}$
risch	$\frac{f\,x^{12}}{12b}-\frac{af\,x^9}{9b^2}+\frac{e\,x^9}{9b}+\frac{x^6f\,a^2}{6b^3}-\frac{ae\,x^6}{6b^2}+\frac{d\,x^6}{6b}-\frac{f\,a^3x^3}{3b^4}+\frac{a^2e\,x^3}{3b^3}-\frac{ad\,x^3}{3b^2}+\frac{c\,x^3}{3b}+\frac{a^4\ln(b\,x^3+a)f}{3b^5}-\frac{a^3\ln(}{}$

input
```
int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^4*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)*x^3-1/9/b^2*(a*f-b*e)*x^9+1/12*f*x^
12/b+1/6*(a^2*f-a*b*e+b^2*d)*x^6/b^3+1/3*a/b^5*(a^3*f-a^2*b*e+a*b^2*d-b^3*
c)*ln(b*x^3+a)
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$=\frac{3\,b^4fx^{12}+4\,(b^4e-ab^3f)x^9+6\,(b^4d-ab^3e+a^2b^2f)x^6+12\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^3-12\,(ab^3c-a}{36\,b^5}$$

input
```
integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/36*(3*b^4*f*x^12 + 4*(b^4*e - a*b^3*f)*x^9 + 6*(b^4*d - a*b^3*e + a^2*b^
2*f)*x^6 + 12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 12*(a*b^3*c -
a^2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a))/b^5
```

3.225.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx=\frac{a(a^3f-a^2be+ab^2d-b^3c)\log\left(a+bx^3\right)}{3b^5}+x^9\left(-\frac{af}{9b^2}+\frac{e}{9b}\right)$$
$$+x^6\left(\frac{a^2f}{6b^3}-\frac{ae}{6b^2}+\frac{d}{6b}\right)+x^3\left(-\frac{a^3f}{3b^4}+\frac{a^2e}{3b^3}-\frac{ad}{3b^2}+\frac{c}{3b}\right)+\frac{fx^{12}}{12b}$$

input
```
integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

output
```
a*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*x**3)/(3*b**5) + x**9*
(-a*f/(9*b**2) + e/(9*b)) + x**6*(a**2*f/(6*b**3) - a*e/(6*b**2) + d/(6*b)
) + x**3*(-a**3*f/(3*b**4) + a**2*e/(3*b**3) - a*d/(3*b**2) + c/(3*b)) + f
*x**12/(12*b)
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{3\,b^3fx^{12}+4\,(b^3e-ab^2f)x^9+6\,(b^3d-ab^2e+a^2bf)x^6+12\,(b^3c-ab^2d+a^2be-a^3f)x^3}{36\,b^4} - \frac{(ab^3c-a^2b^2d+a^3be-a^4f)\log\left(bx^3+a\right)}{3\,b^5}$$

input
```
integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/36*(3*b^3*f*x^12 + 4*(b^3*e - a*b^2*f)*x^9 + 6*(b^3*d - a*b^2*e + a^2*b*
f)*x^6 + 12*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/b^4 - 1/3*(a*b^3*c -
a^2*b^2*d + a^3*b*e - a^4*f)*log(b*x^3 + a)/b^5
```

3.225.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.09

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{3\,b^3fx^{12}+4\,b^3ex^9-4\,ab^2fx^9+6\,b^3dx^6-6\,ab^2ex^6+6\,a^2bfx^6+12\,b^3cx^3-12\,ab^2dx^3+12\,a^2bex^3-12\,a^3}{36\,b^4} - \frac{(ab^3c-a^2b^2d+a^3be-a^4f)\log\left(|bx^3+a|\right)}{3\,b^5}$$

input
```
integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/36*(3*b^3*f*x^12 + 4*b^3*e*x^9 - 4*a*b^2*f*x^9 + 6*b^3*d*x^6 - 6*a*b^2*e
*x^6 + 6*a^2*b*f*x^6 + 12*b^3*c*x^3 - 12*a*b^2*d*x^3 + 12*a^2*b*e*x^3 - 12
*a^3*f*x^3)/b^4 - 1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*log(abs(b*x^
3 + a))/b^5
```

3.225.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.07

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^9\left(\frac{e}{9\,b}-\frac{a\,f}{9\,b^2}\right)+x^6\left(\frac{d}{6\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{6\,b}\right)+x^3\left(\frac{c}{3\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^{12}}{12\,b}+\frac{\ln\left(b\,x^3+a\right)\,\left(f\,a^4-e\,a^3\,b+d\,a^2\,b^2-c\,a\,b^3\right)}{3\,b^5}$$

input
```
int((x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

output
```
x^9*(e/(9*b) - (a*f)/(9*b^2)) + x^6*(d/(6*b) - (a*(e/b - (a*f)/b^2))/(6*b)
) + x^3*(c/(3*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(3*b)) + (f*x^12)/(
12*b) + (log(a + b*x^3)*(a^4*f + a^2*b^2*d - a*b^3*c - a^3*b*e))/(3*b^5)
```

3.226 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$

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3.226.1 Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3} dx = \frac{(b^2d-abe+a^2f)\,x^3}{3b^3} + \frac{(be-af)x^6}{6b^2} + \frac{fx^9}{9b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3b^4}$$

output
```
1/3*(a^2*f-a*b*e+b^2*d)*x^3/b^3+1/6*(-a*f+b*e)*x^6/b^2+1/9*f*x^9/b+1/3*(-a
^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/b^4
```

3.226.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3} dx = \frac{bx^3(6a^2f-3ab(2e+fx^3)+b^2(6d+3ex^3+2fx^6))+6(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{18b^4}$$

input
```
Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(b*x^3*(6*a^2*f - 3*a*b*(2*e + f*x^3) + b^2*(6*d + 3*e*x^3 + 2*f*x^6)) + 6
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*b^4)
```

3.226.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2359, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}dx$$

$$\Big\downarrow \text{2359}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{bx^3+a}dx^3$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{3}\int \left(\frac{fx^6}{b}+\frac{(be-af)x^3}{b^2}+\frac{fa^2-bea+b^2d}{b^3}+\frac{-fa^3+bea^2-b^2da+b^3c}{b^3(bx^3+a)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{x^3(a^2f-abe+b^2d)}{b^3}+\frac{\log(a+bx^3)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{x^6(be-af)}{2b^2}+\frac{fx^9}{3b}\right)$$

input
```
Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(((b^2*d - a*b*e + a^2*f)*x^3)/b^3 + ((b*e - a*f)*x^6)/(2*b^2) + (f*x^9)/(
3*b) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/b^4)/3
```

3.226.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2359
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/(m
 + 1)   Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^
p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ
[Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.226.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

method	result
norman	$-\frac{(af-be)x^6}{6b^2}+\frac{f\,x^9}{9b}+\frac{(a^2f-aeb+b^2d)x^3}{3b^3}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)\ln(b\,x^3+a)}{3b^4}$
default	$\frac{\frac{1}{3}b^2f\,x^9-\frac{1}{2}abf\,x^6+\frac{1}{2}b^2e\,x^6+a^2f\,x^3-abe\,x^3+d\,x^3b^2}{3b^3}+\frac{(-f\,a^3+a^2be-a\,b^2d+b^3c)\ln(b\,x^3+a)}{3b^4}$
parallelrisch	$-\frac{-2b^3f\,x^9+3x^6a\,b^2f-3x^6b^3e-6a^2bf\,x^3+6a\,b^2e\,x^3-6b^3d\,x^3+6\ln(b\,x^3+a)a^3f-6\ln(b\,x^3+a)a^2be+6\ln(b\,x^3+a)a\,b^2d-6\ln(}{18b^4}$
risch	$\frac{f\,x^9}{9b}-\frac{af\,x^6}{6b^2}+\frac{e\,x^6}{6b}+\frac{a^2f\,x^3}{3b^3}-\frac{ae\,x^3}{3b^2}+\frac{d\,x^3}{3b}-\frac{\ln(b\,x^3+a)f\,a^3}{3b^4}+\frac{\ln(b\,x^3+a)a^2e}{3b^3}-\frac{\ln(b\,x^3+a)ad}{3b^2}+\frac{c\ln(b\,x^3+}{3b}$

input
```
int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6/b^2*(a*f-b*e)*x^6+1/9*f*x^9/b+1/3*(a^2*f-a*b*e+b^2*d)*x^3/b^3-1/3/b^4
*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)*ln(b*x^3+a)
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$=\frac{2\,b^3fx^9+3\,(b^3e-ab^2f)x^6+6\,(b^3d-ab^2e+a^2bf)x^3+6\,(b^3c-ab^2d+a^2be-a^3f)\log{(bx^3+a)}}{18\,b^4}$$

input
```
integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/18*(2*b^3*f*x^9 + 3*(b^3*e - a*b^2*f)*x^6 + 6*(b^3*d - a*b^2*e + a^2*b*f
)*x^3 + 6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a))/b^4
```

3.226.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^6\left(-\frac{af}{6b^2}+\frac{e}{6b}\right)+x^3\left(\frac{a^2f}{3b^3}-\frac{ae}{3b^2}+\frac{d}{3b}\right) + \frac{fx^9}{9b}-\frac{(a^3f-a^2be+ab^2d-b^3c)\log(a+bx^3)}{3b^4}$$

input
```
integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

output
```
x**6*(-a*f/(6*b**2) + e/(6*b)) + x**3*(a**2*f/(3*b**3) - a*e/(3*b**2) + d/
(3*b)) + f*x**9/(9*b) - (a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a + b*
x**3)/(3*b**4)
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{2\,b^2fx^9+3\,(b^2e-abf)x^6+6\,(b^2d-abe+a^2f)x^3}{18\,b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(bx^3+a)}{3\,b^4}$$

input
```
integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/18*(2*b^2*f*x^9 + 3*(b^2*e - a*b*f)*x^6 + 6*(b^2*d - a*b*e + a^2*f)*x^3)
/b^3 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/b^4
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{2\,b^2fx^9+3\,b^2ex^6-3\,abfx^6+6\,b^2dx^3-6\,abex^3+6\,a^2fx^3}{18\,b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(|bx^3+a|)}{3\,b^4}$$

input
```
integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/18*(2*b^2*f*x^9 + 3*b^2*e*x^6 - 3*a*b*f*x^6 + 6*b^2*d*x^3 - 6*a*b*e*x^3
+ 6*a^2*f*x^3)/b^3 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(abs(b*x^3
 + a))/b^4
```

3.226.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^6\left(\frac{e}{6\,b}-\frac{a\,f}{6\,b^2}\right)+x^3\left(\frac{d}{3\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{3\,b}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^4}+\frac{f\,x^9}{9\,b}$$

input
```
int((x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

output
```
x^6*(e/(6*b) - (a*f)/(6*b^2)) + x^3*(d/(3*b) - (a*(e/b - (a*f)/b^2))/(3*b)
) + (log(a + b*x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^4) + (f*x^9)
/(9*b)
```

3.227 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)}\,dx$

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3.227.1 Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{c+dx^3+ex^6+fx^9}{x\,(a+bx^3)}\,dx = \frac{(be-af)x^3}{3b^2}+\frac{fx^6}{6b}+\frac{c\log(x)}{a}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a+bx^3\right)}{3ab^3}$$

output
```
1/3*(-a*f+b*e)*x^3/b^2+1/6*f*x^6/b+c*ln(x)/a-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b
^3*c)*ln(b*x^3+a)/a/b^3
```

3.227.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x\,(a+bx^3)}\,dx = \frac{abx^3(2be-2af+bfx^3)+6b^3c\log(x)-2(b^3c-ab^2d+a^2be-a^3f)\log\left(a+bx^3\right)}{6ab^3}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
```

output
```
(a*b*x^3*(2*b*e - 2*a*f + b*f*x^3) + 6*b^3*c*Log[x] - 2*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*Log[a + b*x^3])/(6*a*b^3)
```

3.227.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)}dx$$

$$\Big\downarrow \ 2361$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^3(bx^3+a)}dx^3$$

$$\Big\downarrow \ 2123$$

$$\frac{1}{3}\int \left(\frac{fx^3}{b}+\frac{be-af}{b^2}+\frac{fa^3-bea^2+b^2da-b^3c}{ab^2(bx^3+a)}+\frac{c}{ax^3}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(-\frac{\log(a+bx^3)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{ab^3}+\frac{x^3(be-af)}{b^2}+\frac{c\log(x^3)}{a}+\frac{fx^6}{2b}\right)$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x]
```

output
```
(((b*e - a*f)*x^3)/b^2 + (f*x^6)/(2*b) + (c*Log[x^3])/a - ((b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(a*b^3))/3
```

3.227.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.227.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(-f\,x^{3} b+a f-b e\right)^{2}}{6 b^{3} f}+\frac{c\ln\left(x\right)}{a}+\frac{\left(f\,a^{3}-a^{2} b e+a\,b^{2} d-b^{3} c\right)\ln\left(b\,x^{3}+a\right)}{3 a\,b^{3}}$
norman	$-\frac{\left(a f-b e\right) x^{3}}{3 b^{2}}+\frac{f\,x^{6}}{6 b}+\frac{c\ln\left(x\right)}{a}+\frac{\left(f\,a^{3}-a^{2} b e+a\,b^{2} d-b^{3} c\right)\ln\left(b\,x^{3}+a\right)}{3 a\,b^{3}}$
parallelrisch	$\frac{x^{6} a\,b^{2} f-2 a^{2} b f\,x^{3}+2 a\,b^{2} e\,x^{3}+6 c\ln\left(x\right) b^{3}+2\ln\left(b\,x^{3}+a\right) a^{3} f-2\ln\left(b\,x^{3}+a\right) a^{2} b e+2\ln\left(b\,x^{3}+a\right) a\,b^{2} d-2\ln\left(b\,x^{3}+a\right) b^{3} c}{6 a\,b^{3}}$
risch	$\frac{f\,x^{6}}{6 b}-\frac{f a\,x^{3}}{3 b^{2}}+\frac{e\,x^{3}}{3 b}+\frac{f\,a^{2}}{6 b^{3}}-\frac{a e}{3 b^{2}}+\frac{e^{2}}{6 b f}+\frac{c\ln\left(x\right)}{a}+\frac{a^{2}\ln\left(-b\,x^{3}-a\right) f}{3 b^{3}}-\frac{a\ln\left(-b\,x^{3}-a\right) e}{3 b^{2}}+\frac{\ln\left(-b\,x^{3}-a\right) d}{3 b}-$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-b*f*x^3+a*f-b*e)^2/b^3/f+c*ln(x)/a+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)
/a/b^3*ln(b*x^3+a)
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)}\,dx = \frac{ab^2fx^6+6\,b^3c\log\left(x\right)+2\left(ab^2e-a^2bf\right)x^3-2\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{6\,ab^3}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(a*b^2*f*x^6 + 6*b^3*c*log(x) + 2*(a*b^2*e - a^2*b*f)*x^3 - 2*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a))/(a*b^3)
```

3.227.6 Sympy [A] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)}\,dx = x^3\left(-\frac{af}{3b^2}+\frac{e}{3b}\right)+\frac{fx^6}{6b}+\frac{c\log\left(x\right)}{a} + \frac{\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a}{b}+x^3\right)}{3ab^3}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a),x)
```

output
```
x**3*(-a*f/(3*b**2) + e/(3*b)) + f*x**6/(6*b) + c*log(x)/a + (a**3*f - a**
2*b*e + a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a*b**3)
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)}\,dx = \frac{c\log\left(x^3\right)}{3\,a}+\frac{bfx^6+2\,(be-af)x^3}{6\,b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{3\,ab^3}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*c*log(x^3)/a + 1/6*(b*f*x^6 + 2*(b*e - a*f)*x^3)/b^2 - 1/3*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/(a*b^3)
```

3.227.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)}\,dx = \frac{c\log\left(|x|\right)}{a}+\frac{bfx^6+2\,bex^3-2\,afx^3}{6\,b^2} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(|bx^3+a|\right)}{3\,ab^3}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a),x, algorithm="giac")
```

output
```
c*log(abs(x))/a + 1/6*(b*f*x^6 + 2*b*e*x^3 - 2*a*f*x^3)/b^2 - 1/3*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*log(abs(b*x^3 + a))/(a*b^3)
```

3.227.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x\,(a+bx^3)}\,dx = x^3\left(\frac{e}{3\,b}-\frac{a\,f}{3\,b^2}\right)+\frac{f\,x^6}{6\,b}+\frac{c\,\ln(x)}{a} - \frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a\,b^3}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)),x)
```

output
```
x^3*(e/(3*b) - (a*f)/(3*b^2)) + (f*x^6)/(6*b) + (c*log(x))/a - (log(a + b*
x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a*b^3)
```

3.228 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)} dx$

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3.228.9 Mupad [B] (verification not implemented) . . . 1727

3.228.1 Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)} dx = -\frac{c}{3ax^3}+\frac{fx^3}{3b}-\frac{(bc-ad)\log(x)}{a^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^2b^2}$$

output
```
-1/3*c/a/x^3+1/3*f*x^3/b-(-a*d+b*c)*ln(x)/a^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)*ln(b*x^3+a)/a^2/b^2
```

3.228.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)} dx = \frac{1}{3}\left(-\frac{c}{ax^3}+\frac{fx^3}{b}+\frac{3(-bc+ad)\log(x)}{a^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{a^2b^2}\right)$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
```

output
```
(-(c/(a*x^3)) + (f*x^3)/b + (3*(-(b*c) + a*d)*Log[x])/a^2 + ((b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(a^2*b^2))/3
```

3.228.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4 \left(a+bx^3\right)} \, dx$$

$$\Big\downarrow \text{ 2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^6 \left(bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{ 2123}$$

$$\frac{1}{3}\int \left(\frac{c}{ax^6}+\frac{f}{b}+\frac{-fa^3+bea^2-b^2da+b^3c}{a^2b\left(bx^3+a\right)}+\frac{ad-bc}{a^2x^3}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{\log\left(x^3\right)(bc-ad)}{a^2}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^2b^2}-\frac{c}{ax^3}+\frac{fx^3}{b}\right)$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x]
```

output
```
(-(c/(a*x^3)) + (f*x^3)/b - ((b*c - a*d)*Log[x^3])/a^2 + ((b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*Log[a + b*x^3])/(a^2*b^2))/3
```

3.228.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.228.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

method	result
default	$\frac{f\,x^{3}}{3b}-\frac{c}{3a\,x^{3}}+\frac{(ad-bc)\ln(x)}{a^{2}}-\frac{(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c)\ln(b\,x^{3}+a)}{3a^{2}b^{2}}$
norman	$\frac{-\frac{c}{3a}+\frac{f\,x^{6}}{3b}}{x^{3}}+\frac{(ad-bc)\ln(x)}{a^{2}}-\frac{(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c)\ln(b\,x^{3}+a)}{3a^{2}b^{2}}$
risch	$\frac{f\,x^{3}}{3b}-\frac{c}{3a\,x^{3}}+\frac{d\ln(x)}{a}-\frac{bc\ln(x)}{a^{2}}-\frac{a\ln(b\,x^{3}+a)f}{3b^{2}}+\frac{e\ln(b\,x^{3}+a)}{3b}-\frac{d\ln(b\,x^{3}+a)}{3a}+\frac{b\ln(b\,x^{3}+a)c}{3a^{2}}$
parallelrisch	$\frac{x^{6}f\,a^{2}b+3\ln(x)x^{3}a\,b^{2}d-3\ln(x)x^{3}b^{3}c-\ln(b\,x^{3}+a)x^{3}a^{3}f+\ln(b\,x^{3}+a)x^{3}a^{2}be-\ln(b\,x^{3}+a)x^{3}a\,b^{2}d+\ln(b\,x^{3}+a)x^{3}b^{3}c-a\,b^{2}c}{3a^{2}b^{2}x^{3}}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*f*x^3/b-1/3*c/a/x^3+(a*d-b*c)/a^2*ln(x)-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3
*c)/a^2/b^2*ln(b*x^3+a)
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx = \frac{a^2bfx^6+(b^3c-ab^2d+a^2be-a^3f)x^3\log(bx^3+a)-3\,(b^3c-ab^2d)x^3\log(x)-ab^2c}{3\,a^2b^2x^3}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*(a^2*b*f*x^6 + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3*log(b*x^3 + a)
- 3*(b^3*c - a*b^2*d)*x^3*log(x) - a*b^2*c)/(a^2*b^2*x^3)
```

3.228.6 Sympy [A] (verification not implemented)

Time = 49.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx = \frac{fx^3}{3b} - \frac{c}{3ax^3} + \frac{(ad-bc)\log(x)}{a^2} - \frac{(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{a}{b}+x^3\right)}{3a^2b^2}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a),x)
```

output
```
f*x**3/(3*b) - c/(3*a*x**3) + (a*d - b*c)*log(x)/a**2 - (a**3*f - a**2*b*e
 + a*b**2*d - b**3*c)*log(a/b + x**3)/(3*a**2*b**2)
```

3.228.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx = \frac{fx^3}{3\,b} - \frac{(bc-ad)\log(x^3)}{3\,a^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(bx^3+a)}{3\,a^2b^2} - \frac{c}{3\,ax^3}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*f*x^3/b - 1/3*(b*c - a*d)*log(x^3)/a^2 + 1/3*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*log(b*x^3 + a)/(a^2*b^2) - 1/3*c/(a*x^3)
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx = \frac{fx^3}{3\,b} - \frac{(bc-ad)\log(|x|)}{a^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(|bx^3+a|)}{3\,a^2b^2} + \frac{bcx^3-adx^3-ac}{3\,a^2x^3}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*f*x^3/b - (b*c - a*d)*log(abs(x))/a^2 + 1/3*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*log(abs(b*x^3 + a))/(a^2*b^2) + 1/3*(b*c*x^3 - a*d*x^3 - a*c)/(a
^2*x^3)
```

3.228.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)}\,dx = \frac{f\,x^3}{3\,b} - \frac{c}{3\,a\,x^3} + \frac{\ln(x)\,\left(a\,d-b\,c\right)}{a^2} + \frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^2\,b^2}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)),x)
```

output
```
(f*x^3)/(3*b) - c/(3*a*x^3) + (log(x)*(a*d - b*c))/a^2 + (log(a + b*x^3)*(
b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^2*b^2)
```

3.229 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx$

3.229.1 Optimal result . . . 1728
3.229.2 Mathematica [A] (verified) . . . 1728
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3.229.1 Optimal result

Integrand size = 30, antiderivative size = 95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx = -\frac{c}{6ax^6}+\frac{bc-ad}{3a^2x^3}+\frac{(b^2c-abd+a^2e)\log(x)}{a^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^3b}$$

output
```
-1/6*c/a/x^6+1/3*(-a*d+b*c)/a^2/x^3+(a^2*e-a*b*d+b^2*c)*ln(x)/a^3-1/3*(-a^
3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/a^3/b
```

3.229.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx = \frac{-\frac{a(ac-2bcx^3+2adx^3)}{x^6}+6(b^2c-abd+a^2e)\log(x)+\left(-2b^2c+2abd-2a^2e+\frac{2a^3f}{b}\right)\log(a+bx^3)}{6a^3}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
```

output
```
(-((a*(a*c - 2*b*c*x^3 + 2*a*d*x^3))/x^6) + 6*(b^2*c - a*b*d + a^2*e)*Log[
x] + (-2*b^2*c + 2*a*b*d - 2*a^2*e + (2*a^3*f)/b)*Log[a + b*x^3])/(6*a^3)
```

3.229.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}dx$$

$$\Big\downarrow \text{2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^9(bx^3+a)}dx^3$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{3}\int \left(\frac{c}{ax^9}+\frac{fa^3-bea^2+b^2da-b^3c}{a^3(bx^3+a)}+\frac{ea^2-bda+b^2c}{a^3x^3}+\frac{ad-bc}{a^2x^6}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{bc-ad}{a^2x^3}+\frac{\log(x^3)(a^2e-abd+b^2c)}{a^3}-\frac{\log(a+bx^3)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^3b}-\frac{c}{2ax^6}\right)$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x]
```

output
```
(-1/2*c/(a*x^6) + (b*c - a*d)/(a^2*x^3) + ((b^2*c - a*b*d + a^2*e)*Log[x^3
])/a^3 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(a^3*b))/3
```

3.229.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.229.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

method	result
default	$-\frac{c}{6a\,x^{6}}-\frac{ad-bc}{3a^{2}x^{3}}+\frac{\left(a^{2}e-abd+b^{2}c\right)\ln\left(x\right)}{a^{3}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{3a^{3}b}$
norman	$\frac{-\frac{c}{6a}-\frac{(ad-bc)x^{3}}{3a^{2}}}{x^{6}}+\frac{\left(a^{2}e-abd+b^{2}c\right)\ln\left(x\right)}{a^{3}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{3a^{3}b}$
risch	$\frac{-\frac{c}{6a}-\frac{(ad-bc)x^{3}}{3a^{2}}}{x^{6}}+\frac{e\ln\left(x\right)}{a}-\frac{\ln\left(x\right)bd}{a^{2}}+\frac{\ln\left(x\right)b^{2}c}{a^{3}}+\frac{\ln\left(-b\,x^{3}-a\right)f}{3b}-\frac{\ln\left(-b\,x^{3}-a\right)e}{3a}+\frac{b\ln\left(-b\,x^{3}-a\right)d}{3a^{2}}-\frac{b^{2}\ln\left(-b\,x\right.}{3a^{3}}$
parallelrisch	$\frac{6\ln\left(x\right)x^{6}a^{2}be-6\ln\left(x\right)x^{6}a\,b^{2}d+6\ln\left(x\right)x^{6}b^{3}c+2\ln\left(b\,x^{3}+a\right)x^{6}a^{3}f-2\ln\left(b\,x^{3}+a\right)x^{6}a^{2}be+2\ln\left(b\,x^{3}+a\right)x^{6}a\,b^{2}d-2\ln\left(b\,x^{3}+a\right)x^{6}b}{6a^{3}x^{6}b}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*c/a/x^6-1/3*(a*d-b*c)/a^2/x^3+(a^2*e-a*b*d+b^2*c)*ln(x)/a^3+1/3*(a^3*
f-a^2*b*e+a*b^2*d-b^3*c)/a^3/b*ln(b*x^3+a)
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)}\,dx = -\frac{2\,(b^3c-ab^2d+a^2be-a^3f)x^6\log\left(bx^3+a\right)-6\,(b^3c-ab^2d+a^2be)x^6\log\left(x\right)+a^2bc-2\,(ab^2c-a^2bd)x}{6\,a^3bx^6}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6*log(b*x^3 + a) - 6*(b^3*c
- a*b^2*d + a^2*b*e)*x^6*log(x) + a^2*b*c - 2*(a*b^2*c - a^2*b*d)*x^3)/(a^
3*b*x^6)
```

3.229.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a),x)
```

output
```
Timed out
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)}\,dx = \frac{\left(b^2c-abd+a^2e\right)\log\left(x^3\right)}{3\,a^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(bx^3+a\right)}{3\,a^3b} + \frac{2\,(bc-ad)x^3-ac}{6\,a^2x^6}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*(b^2*c - a*b*d + a^2*e)*log(x^3)/a^3 - 1/3*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*log(b*x^3 + a)/(a^3*b) + 1/6*(2*(b*c - a*d)*x^3 - a*c)/(a^2*x^6)
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.29

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)}\,dx = \frac{\left(b^2c-abd+a^2e\right)\log\left(|x|\right)}{a^3} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(|bx^3+a|\right)}{3\,a^3b} - \frac{3\,b^2cx^6-3\,abdx^6+3\,a^2ex^6-2\,abcx^3+2\,a^2dx^3+a^2c}{6\,a^3x^6}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a),x, algorithm="giac")
```

output
```
(b^2*c - a*b*d + a^2*e)*log(abs(x))/a^3 - 1/3*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*log(abs(b*x^3 + a))/(a^3*b) - 1/6*(3*b^2*c*x^6 - 3*a*b*d*x^6 + 3*a
^2*e*x^6 - 2*a*b*c*x^3 + 2*a^2*d*x^3 + a^2*c)/(a^3*x^6)
```

3.229.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\,(a+bx^3)}\,dx = \frac{\ln{(x)}\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{a^3} - \frac{\frac{c}{6\,a}+\frac{x^3\,(a\,d-b\,c)}{3\,a^2}}{x^6} - \frac{\ln{\left(b\,x^3+a\right)}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^3\,b}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)),x)
```

output
```
(log(x)*(b^2*c + a^2*e - a*b*d))/a^3 - (c/(6*a) + (x^3*(a*d - b*c))/(3*a^2
))/x^6 - (log(a + b*x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^3*b)
```

3.230 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)}\,dx$

3.230.1 Optimal result 1733
3.230.2 Mathematica [A] (verified) 1733
3.230.3 Rubi [A] (verified) 1734
3.230.4 Maple [A] (verified) 1735
3.230.5 Fricas [A] (verification not implemented) 1736
3.230.6 Sympy [F(-1)] 1736
3.230.7 Maxima [A] (verification not implemented) 1736
3.230.8 Giac [A] (verification not implemented) 1737
3.230.9 Mupad [B] (verification not implemented) 1737

3.230.1 Optimal result

Integrand size = 30, antiderivative size = 128

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)}\,dx = -\frac{c}{9ax^9}+\frac{bc-ad}{6a^2x^6}-\frac{b^2c-abd+a^2e}{3a^3x^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log(x)}{a^4} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^4}$$

output
```
-1/9*c/a/x^9+1/6*(-a*d+b*c)/a^2/x^6+1/3*(-a^2*e+a*b*d-b^2*c)/a^3/x^3-(-a^3
*f+a^2*b*e-a*b^2*d+b^3*c)*ln(x)/a^4+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(
b*x^3+a)/a^4
```

3.230.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)}\,dx = -\frac{c}{9ax^9}+\frac{bc-ad}{6a^2x^6}+\frac{-b^2c+abd-a^2e}{3a^3x^3} + \frac{(-b^3c+ab^2d-a^2be+a^3f)\log(x)}{a^4} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^4}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

output
```
-1/9*c/(a*x^9) + (b*c - a*d)/(6*a^2*x^6) + (-(b^2*c) + a*b*d - a^2*e)/(3*a
^3*x^3) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[x])/a^4 + ((b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/(3*a^4)
```

3.230.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^{12}\left(bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{3}\int \left(\frac{c}{ax^{12}}-\frac{b\left(fa^3-bea^2+b^2da-b^3c\right)}{a^4\left(bx^3+a\right)}+\frac{fa^3-bea^2+b^2da-b^3c}{a^4x^3}+\frac{ea^2-bda+b^2c}{a^3x^6}+\frac{ad-bc}{a^2x^9}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{bc-ad}{2a^2x^6}-\frac{a^2e-abd+b^2c}{a^3x^3}-\frac{\log\left(x^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^4}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2d+\right.}{a^4}\right.$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x]
```

output
```
(-1/3*c/(a*x^9) + (b*c - a*d)/(2*a^2*x^6) - (b^2*c - a*b*d + a^2*e)/(a^3*x
^3) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[x^3])/a^4 + ((b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*Log[a + b*x^3])/a^4)/3
```

3.230.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.230.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

method	result
default	$-\frac{c}{9a\,x^{9}}-\frac{ad-bc}{6a^{2}x^{6}}-\frac{a^{2}e-abd+b^{2}c}{3a^{3}x^{3}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(x\right)}{a^{4}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{3a^{4}}$
norman	$\frac{-\frac{c}{9a}-\frac{(ad-bc)x^{3}}{6a^{2}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{3a^{3}}}{x^{9}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(x\right)}{a^{4}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{3a^{4}}$
risch	$\frac{-\frac{c}{9a}-\frac{(ad-bc)x^{3}}{6a^{2}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{3a^{3}}}{x^{9}}+\frac{\ln\left(x\right)f}{a}-\frac{\ln\left(x\right)be}{a^{2}}+\frac{\ln\left(x\right)b^{2}d}{a^{3}}-\frac{\ln\left(x\right)b^{3}c}{a^{4}}-\frac{\ln\left(b\,x^{3}+a\right)f}{3a}+\frac{\ln\left(b\,x^{3}+a\right)be}{3a^{2}}-$
parallelrisch	$\frac{18\ln\left(x\right)x^{9}a^{3}f-18\ln\left(x\right)x^{9}a^{2}be+18\ln\left(x\right)x^{9}a\,b^{2}d-18\ln\left(x\right)x^{9}b^{3}c-6\ln\left(b\,x^{3}+a\right)x^{9}a^{3}f+6\ln\left(b\,x^{3}+a\right)x^{9}a^{2}be-6\ln\left(b\,x^{3}+a\right)x^{9}a\,b^{2}}{18a^{4}x^{9}}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/9*c/a/x^9-1/6*(a*d-b*c)/a^2/x^6-1/3*(a^2*e-a*b*d+b^2*c)/a^3/x^3+(a^3*f-
a^2*b*e+a*b^2*d-b^3*c)/a^4*ln(x)-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4*ln(
b*x^3+a)
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.99

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{10}(a + bx^3)}\,dx$$
$$= \frac{6\,(b^3c - ab^2d + a^2be - a^3f)x^9 \log\left(bx^3 + a\right) - 18\,(b^3c - ab^2d + a^2be - a^3f)x^9 \log\left(x\right) - 6\,(ab^2c - a^2bd + a}{18\,a^4x^9}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/18*(6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(b*x^3 + a) - 18*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^9*log(x) - 6*(a*b^2*c - a^2*b*d + a^3*e)*x
^6 - 2*a^3*c + 3*(a^2*b*c - a^3*d)*x^3)/(a^4*x^9)
```

3.230.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{10}(a + bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a),x)
```

output
```
Timed out
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{10}(a + bx^3)}\,dx = \frac{(b^3c - ab^2d + a^2be - a^3f)\log\left(bx^3 + a\right)}{3\,a^4} - \frac{(b^3c - ab^2d + a^2be - a^3f)\log\left(x^3\right)}{3\,a^4} - \frac{6\,(b^2c - abd + a^2e)x^6 - 3\,(abc - a^2d)x^3 + 2\,a^2c}{18\,a^3x^9}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(b*x^3 + a)/a^4 - 1/3*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*log(x^3)/a^4 - 1/18*(6*(b^2*c - a*b*d + a^2*e)*
x^6 - 3*(a*b*c - a^2*d)*x^3 + 2*a^2*c)/(a^3*x^9)
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.41

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)}\,dx$$
$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\log\left(|x|\right)}{a^4}+\frac{\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)\log\left(|bx^3+a|\right)}{3\,a^4b}$$
$$+\frac{11\,b^3cx^9-11\,ab^2dx^9+11\,a^2bex^9-11\,a^3fx^9-6\,ab^2cx^6+6\,a^2bdx^6-6\,a^3ex^6+3\,a^2bcx^3-3\,a^3dx^3-2}{18\,a^4x^9}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a),x, algorithm="giac")
```

output
```
-(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(abs(x))/a^4 + 1/3*(b^4*c - a*b^3*
d + a^2*b^2*e - a^3*b*f)*log(abs(b*x^3 + a))/(a^4*b) + 1/18*(11*b^3*c*x^9
- 11*a*b^2*d*x^9 + 11*a^2*b*e*x^9 - 11*a^3*f*x^9 - 6*a*b^2*c*x^6 + 6*a^2*b
*d*x^6 - 6*a^3*e*x^6 + 3*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^4*x^9)
```

3.230.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)}\,dx = \frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^4} - \frac{\frac{c}{9\,a}+\frac{x^3\,\left(a\,d-b\,c\right)}{6\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{3\,a^3}}{x^9} - \frac{\ln\left(x\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^4}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)),x)
```

output
```
(log(a + b*x^3)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^4) - (c/(9*a) +
(x^3*(a*d - b*c))/(6*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(3*a^3))/x^9 - (
log(x)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4
```

3.230. $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)}\,dx$

3.231 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx$

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3.231.1 Optimal result

Integrand size = 30, antiderivative size = 164

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx = -\frac{c}{12ax^{12}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{6a^3x^6}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\log(x)}{a^5}-\frac{b(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^5}$$

output
```
-1/12*c/a/x^12+1/9*(-a*d+b*c)/a^2/x^9+1/6*(-a^2*e+a*b*d-b^2*c)/a^3/x^6+1/3
*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^3+b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*l
n(x)/a^5-1/3*b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/a^5
```

3.231.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx = \frac{12ab^3cx^9-6a^2b^2x^6(c+2dx^3)+2a^3bx^3(2c+3dx^3+6ex^6)-a^4(3c+4dx^3+6ex^6+12fx^9)+36b(b^3c-}{36a^5x^{12}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```

output
```
(12*a*b^3*c*x^9 - 6*a^2*b^2*x^6*(c + 2*d*x^3) + 2*a^3*b*x^3*(2*c + 3*d*x^3
 + 6*e*x^6) - a^4*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + 36*b*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x^12*Log[x] - 12*b*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*x^12*Log[a + b*x^3])/(36*a^5*x^12)
```

3.231.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)}dx$$

$$\Big\downarrow \; 2361$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^{15}\left(bx^3+a\right)}dx^3$$

$$\Big\downarrow \; 2123$$

$$\frac{1}{3}\int \left(\frac{\left(fa^3-bea^2+b^2da-b^3c\right)b^2}{a^5\left(bx^3+a\right)}-\frac{\left(fa^3-bea^2+b^2da-b^3c\right)b}{a^5x^3}+\frac{fa^3-bea^2+b^2da-b^3c}{a^4x^6}+\frac{ea^2-bda+b^2c}{a^3x^9}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(\frac{bc-ad}{3a^2x^9}-\frac{a^2e-abd+b^2c}{2a^3x^6}+\frac{b\log\left(x^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^5}-\frac{b\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-ab^2\right.}{a^5}\right.$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x]
```

output
```
(-1/4*c/(a*x^12) + (b*c - a*d)/(3*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(2*a^
3*x^6) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x^3) + (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*Log[x^3])/a^5 - (b*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*Log[a + b*x^3])/a^5)/3
```

3.231.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.231.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.95

method	result
default	$-\frac{c}{12a\,x^{12}}-\frac{ad-bc}{9a^{2}x^{9}}-\frac{a^{2}e-abd+b^{2}c}{6a^{3}x^{6}}-\frac{f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c}{3a^{4}x^{3}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)b\ln\left(x\right)}{a^{5}}+\frac{b\left(f\,a^{3}-a^{2}be+a\,b^{2}\right.}{3a}$
norman	$\frac{-\frac{c}{12a}-\frac{(ad-bc)x^{3}}{9a^{2}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{6a^{3}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{3a^{4}}}{x^{12}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)b\ln\left(x\right)}{a^{5}}+\frac{b\left(f\,a^{3}-a^{2}be+a\,b^{2}d-\right.}{3a^{5}}$
risch	$\frac{-\frac{c}{12a}-\frac{(ad-bc)x^{3}}{9a^{2}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{6a^{3}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{3a^{4}}}{x^{12}}-\frac{b\ln\left(x\right)f}{a^{2}}+\frac{b^{2}\ln\left(x\right)e}{a^{3}}-\frac{b^{3}\ln\left(x\right)d}{a^{4}}+\frac{b^{4}\ln\left(x\right)c}{a^{5}}+\frac{b\ln}{}$
parallelrisch	$-\frac{36\ln\left(x\right)x^{12}a^{3}bf-36\ln\left(x\right)x^{12}a^{2}b^{2}e+36\ln\left(x\right)x^{12}a\,b^{3}d-36\ln\left(x\right)x^{12}b^{4}c-12\ln\left(b\,x^{3}+a\right)x^{12}a^{3}bf+12\ln\left(b\,x^{3}+a\right)x^{12}a^{2}b^{2}e-12}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/12*c/a/x^12-1/9*(a*d-b*c)/a^2/x^9-1/6*(a^2*e-a*b*d+b^2*c)/a^3/x^6-1/3*(
a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^3-(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b*l
n(x)+1/3*b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*ln(b*x^3+a)
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx =$$
$$-\frac{12\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}\log\left(bx^3+a\right)-36\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}\log\left(x\right)-12\,(ab^3c}{36\,a^5x^{12}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/36*(12*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12*log(b*x^3 + a) - 36
*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12*log(x) - 12*(a*b^3*c - a^2*b
^2*d + a^3*b*e - a^4*f)*x^9 + 6*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 3*a^4*
c - 4*(a^3*b*c - a^4*d)*x^3)/(a^5*x^12)
```

3.231.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a),x)
```

output
```
Timed out
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx$$
$$= -\frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\log\left(bx^3+a\right)}{3\,a^5}+\frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\log\left(x^3\right)}{3\,a^5}$$
$$+\frac{12\,(b^3c-ab^2d+a^2be-a^3f)x^9-6\,(ab^2c-a^2bd+a^3e)x^6-3\,a^3c+4\,(a^2bc-a^3d)x^3}{36\,a^4x^{12}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(b*x^3 + a)/a^5 + 1/3*(b^4
*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(x^3)/a^5 + 1/36*(12*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*x^9 - 6*(a*b^2*c - a^2*b*d + a^3*e)*x^6 - 3*a^3*c +
4*(a^2*b*c - a^3*d)*x^3)/(a^4*x^12)
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.40

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx$$
$$= \frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\log(|x|)}{a^5} - \frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\log(|bx^3+a|)}{3\,a^5b}$$
$$- \frac{25\,b^4cx^{12}-25\,ab^3dx^{12}+25\,a^2b^2ex^{12}-25\,a^3bfx^{12}-12\,ab^3cx^9+12\,a^2b^2dx^9-12\,a^3bex^9+12\,a^4fx^9+6}{36\,a^5x^{12}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a),x, algorithm="giac")
```

output
```
(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(abs(x))/a^5 - 1/3*(b^5*c - a*b
^4*d + a^2*b^3*e - a^3*b^2*f)*log(abs(b*x^3 + a))/(a^5*b) - 1/36*(25*b^4*c
*x^12 - 25*a*b^3*d*x^12 + 25*a^2*b^2*e*x^12 - 25*a^3*b*f*x^12 - 12*a*b^3*c
*x^9 + 12*a^2*b^2*d*x^9 - 12*a^3*b*e*x^9 + 12*a^4*f*x^9 + 6*a^2*b^2*c*x^6
- 6*a^3*b*d*x^6 + 6*a^4*e*x^6 - 4*a^3*b*c*x^3 + 4*a^4*d*x^3 + 3*a^4*c)/(a^
5*x^12)
```

3.231.9 Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx = \frac{\ln(x)\,\left(-f\,a^3\,b+e\,a^2\,b^2-d\,a\,b^3+c\,b^4\right)}{a^5} - \frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3\,b+e\,a^2\,b^2-d\,a\,b^3+c\,b^4\right)}{3\,a^5} - \frac{\frac{c}{12\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{9\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{6\,a^3}}{x^{12}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)),x)
```

output
```
(log(x)*(b^4*c + a^2*b^2*e - a*b^3*d - a^3*b*f))/a^5 - (log(a + b*x^3)*(b^
4*c + a^2*b^2*e - a*b^3*d - a^3*b*f))/(3*a^5) - (c/(12*a) - (x^9*(b^3*c -
a^3*f - a*b^2*d + a^2*b*e))/(3*a^4) + (x^3*(a*d - b*c))/(9*a^2) + (x^6*(b^
2*c + a^2*e - a*b*d))/(6*a^3))/x^12
```

3.231. $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)}\,dx$

3.232 $\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx$

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3.232.1 Optimal result

Integrand size = 30, antiderivative size = 205

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx &= -\frac{c}{15ax^{15}}+\frac{bc-ad}{12a^2x^{12}}-\frac{b^2c-abd+a^2e}{9a^3x^9} \\ &\quad+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^4x^6}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5x^3} \\ &\quad-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\log(x)}{a^6} \\ &\quad+\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\log(a+bx^3)}{3a^6}\end{aligned}$$

output
```
-1/15*c/a/x^15+1/12*(-a*d+b*c)/a^2/x^12+1/9*(-a^2*e+a*b*d-b^2*c)/a^3/x^9+1
/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^6-1/3*b*(-a^3*f+a^2*b*e-a*b^2*d+b^
3*c)/a^5/x^3-b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(x)/a^6+1/3*b^2*(-a^3*f+
a^2*b*e-a*b^2*d+b^3*c)*ln(b*x^3+a)/a^6
```

3.232.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx = -\frac{\frac{a(60b^4cx^{12}-30ab^3x^9(c+2dx^3)+10a^2b^2x^6(2c+3dx^3+6ex^6)-5a^3bx^3(3c+4dx^3+6ex^6+12fx^9)+a^4(12c+15dx^3+20ex^6+30fx^9))}{x^{15}}+180b^2}{180a^6}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

output
```
-1/180*((a*(60*b^4*c*x^12 - 30*a*b^3*x^9*(c + 2*d*x^3) + 10*a^2*b^2*x^6*(2
*c + 3*d*x^3 + 6*e*x^6) - 5*a^3*b*x^3*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9)
 + a^4*(12*c + 15*d*x^3 + 20*e*x^6 + 30*f*x^9)))/x^15 + 180*b^2*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*Log[x] - 60*b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*Log[a + b*x^3])/a^6
```

3.232.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^{18}\left(bx^3+a\right)}dx^3$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{3}\int \left(-\frac{\left(fa^3-bea^2+b^2da-b^3c\right)b^3}{a^6\left(bx^3+a\right)}+\frac{\left(fa^3-bea^2+b^2da-b^3c\right)b^2}{a^6x^3}-\frac{\left(fa^3-bea^2+b^2da-b^3c\right)b}{a^5x^6}+\frac{fa^3-bea\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{bc-ad}{4a^2x^{12}}-\frac{a^2e-abd+b^2c}{3a^3x^9}-\frac{b^2\log\left(x^3\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6}+\frac{b^2\log\left(a+bx^3\right)\left(a^3(-f)+a^2be-a\right.}{a^6}\right.$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x]
```

output
```
(-1/5*c/(a*x^15) + (b*c - a*d)/(4*a^2*x^12) - (b^2*c - a*b*d + a^2*e)/(3*a
^3*x^9) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^4*x^6) - (b*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f))/(a^5*x^3) - (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*Log[x^3])/a^6 + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a + b*x^
3])/a^6)/3
```

3.232.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.232.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.94

method	result
default	$-\frac{c}{15a\,x^{15}}-\frac{ad-bc}{12a^{2}x^{12}}-\frac{a^{2}e-abd+b^{2}c}{9a^{3}x^{9}}-\frac{f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c}{6a^{4}x^{6}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)b^{2}\ln\left(x\right)}{a^{6}}+\frac{\left(f\,a^{3}-a^{2}be+a\right.}{3a^{5}x^{3}}$
norman	$\frac{-\frac{c}{15a}-\frac{(ad-bc)x^{3}}{12a^{2}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{9a^{3}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{6a^{4}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)b\,x^{12}}{3a^{5}}}{x^{15}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)b^{2}}{a^{6}}$
risch	$\frac{-\frac{c}{15a}-\frac{(ad-bc)x^{3}}{12a^{2}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{9a^{3}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{6a^{4}}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)b\,x^{12}}{3a^{5}}}{x^{15}}+\frac{b^{2}\ln\left(x\right)f}{a^{3}}-\frac{b^{3}\ln\left(x\right)e}{a^{4}}+$
parallelrisch	$180\ln\left(x\right)x^{15}a^{3}b^{2}f-180\ln\left(x\right)x^{15}a^{2}b^{3}e+180\ln\left(x\right)x^{15}a\,b^{4}d-180\ln\left(x\right)x^{15}b^{5}c-60\ln\left(b\,x^{3}+a\right)x^{15}a^{3}b^{2}f+60\ln\left(b\,x^{3}+a\right)x^{15}a^{2}b^{3}e$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/15*c/a/x^15-1/12*(a*d-b*c)/a^2/x^12-1/9*(a^2*e-a*b*d+b^2*c)/a^3/x^9-1/6
*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^6+(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6*b
^2*ln(x)+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b/x^3-1/3*b^2*(a^3*f-a^2*b*
e+a*b^2*d-b^3*c)/a^6*ln(b*x^3+a)
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx = \frac{60\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{15}\log\left(bx^3+a\right)-180\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{15}\log\left(x\right)-60\,(ab^4c}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/180*(60*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15*log(b*x^3 + a) -
180*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^15*log(x) - 60*(a*b^4*c -
a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 30*(a^2*b^3*c - a^3*b^2*d + a^4*b*
e - a^5*f)*x^9 - 20*(a^3*b^2*c - a^4*b*d + a^5*e)*x^6 - 12*a^5*c + 15*(a^4
*b*c - a^5*d)*x^3)/(a^6*x^15)
```

3.232.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**16/(b*x**3+a),x)
```

output
```
Timed out
```

3.232.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx = \frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\log(bx^3+a)}{3\,a^6} - \frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\log(x^3)}{3\,a^6} - \frac{60\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}-30\,(ab^3c-a^2b^2d+a^3be-a^4f)x^9+20\,(a^2b^2c-a^3bd+a^4e)x^6+1}{180\,a^5x^{15}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(b*x^3 + a)/a^6 - 1/3*(b^
5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(x^3)/a^6 - 1/180*(60*(b^4*c - a
*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 30*(a*b^3*c - a^2*b^2*d + a^3*b*e - a
^4*f)*x^9 + 20*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 12*a^4*c - 15*(a^3*b*c
- a^4*d)*x^3)/(a^5*x^15)
```

3.232.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.37

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx = -\frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\log(|x|)}{a^6} + \frac{(b^6c-ab^5d+a^2b^4e-a^3b^3f)\log(|bx^3+a|)}{3\,a^6b} + \frac{137\,b^5cx^{15}-137\,ab^4dx^{15}+137\,a^2b^3ex^{15}-137\,a^3b^2fx^{15}-60\,ab^4cx^{12}+60\,a^2b^3dx^{12}-60\,a^3b^2ex^{12}+60\,a}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^16/(b*x^3+a),x, algorithm="giac")
```

output
```
-(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(abs(x))/a^6 + 1/3*(b^6*c -
a*b^5*d + a^2*b^4*e - a^3*b^3*f)*log(abs(b*x^3 + a))/(a^6*b) + 1/180*(137*
b^5*c*x^15 - 137*a*b^4*d*x^15 + 137*a^2*b^3*e*x^15 - 137*a^3*b^2*f*x^15 -
60*a*b^4*c*x^12 + 60*a^2*b^3*d*x^12 - 60*a^3*b^2*e*x^12 + 60*a^4*b*f*x^12
+ 30*a^2*b^3*c*x^9 - 30*a^3*b^2*d*x^9 + 30*a^4*b*e*x^9 - 30*a^5*f*x^9 - 20
*a^3*b^2*c*x^6 + 20*a^4*b*d*x^6 - 20*a^5*e*x^6 + 15*a^4*b*c*x^3 - 15*a^5*d
*x^3 - 12*a^5*c)/(a^6*x^15)
```

3.232.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{16}(a+bx^3)}\,dx$$
$$= \frac{\ln(b\,x^3+a)\,\left(-f\,a^3\,b^2+e\,a^2\,b^3-d\,a\,b^4+c\,b^5\right)}{3\,a^6} - \frac{\frac{c}{15\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{6\,a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{12\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{9\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^5}}{x^{15}} - \frac{\ln(x)\,\left(-f\,a^3\,b^2+e\,a^2\,b^3-d\,a\,b^4+c\,b^5\right)}{a^6}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^16*(a + b*x^3)),x)
```

output
```
(log(a + b*x^3)*(b^5*c + a^2*b^3*e - a^3*b^2*f - a*b^4*d))/(3*a^6) - (c/(1
5*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(6*a^4) + (x^3*(a*d - b*c
))/(12*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(9*a^3) + (b*x^12*(b^3*c - a^3
*f - a*b^2*d + a^2*b*e))/(3*a^5))/x^15 - (log(x)*(b^5*c + a^2*b^3*e - a^3*
b^2*f - a*b^4*d))/a^6
```

3.233 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

3.233.1 Optimal result 1751
3.233.2 Mathematica [A] (verified) 1752
3.233.3 Rubi [A] (verified) 1752
3.233.4 Maple [C] (verified) 1754
3.233.5 Fricas [A] (verification not implemented) 1755
3.233.6 Sympy [A] (verification not implemented) 1756
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3.233.8 Giac [A] (verification not implemented) 1758
3.233.9 Mupad [B] (verification not implemented) 1759

3.233.1 Optimal result

Integrand size = 30, antiderivative size = 348

$$\begin{aligned}\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx &= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{b^6}-\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^5}\\ &+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^7}{7b^4}+\frac{(b^2d-abe+a^2f)\,x^{10}}{10b^3}+\frac{(be-af)x^{13}}{13b^2}\\ &+\frac{fx^{16}}{16b}+\frac{a^{7/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{19/3}}\\ &-\frac{a^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{19/3}}\\ &+\frac{a^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{19/3}}\end{aligned}$$

output
```
a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^6-1/4*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3
*c)*x^4/b^5+1/7*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^7/b^4+1/10*(a^2*f-a*b*e+b
^2*d)*x^10/b^3+1/13*(-a*f+b*e)*x^13/b^2+1/16*f*x^16/b-1/3*a^(7/3)*(-a^3*f+
a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(19/3)+1/6*a^(7/3)*(-a^3*f+
a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(19/3)+
1/3*a^(7/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x
)/a^(1/3)*3^(1/2))/b^(19/3)*3^(1/2)
```

3.233.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.01

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= -\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{b^6}+\frac{a(-b^3c+ab^2d-a^2be+a^3f)\,x^4}{4b^5}$$
$$+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^7}{7b^4}+\frac{(b^2d-abe+a^2f)\,x^{10}}{10b^3}+\frac{(be-af)x^{13}}{13b^2}$$
$$+\frac{fx^{16}}{16b}+\frac{a^{7/3}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{19/3}}$$
$$+\frac{a^{7/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{19/3}}$$
$$-\frac{a^{7/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{19/3}}$$

input
```
Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
-((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f)*x^4)/(4*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f
)*x^7)/(7*b^4) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^3) + ((b*e - a*f)*x^
13)/(13*b^2) + (f*x^16)/(16*b) + (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(19/
3)) + (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3
)*x])/(3*b^(19/3)) - (a^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(19/3))
```

3.233.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2375, 27, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{2375}$$

$$\frac{\int \frac{16x^9((be-af)x^6+bdx^3+bc)}{bx^3+a}dx}{16b}+\frac{fx^{16}}{16b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^9((be-af)x^6+bdx^3+bc)}{bx^3+a}dx}{b}+\frac{fx^{16}}{16b}$$

$$\Big\downarrow \text{1812}$$

$$\frac{\int \left(\frac{(be-af)x^{12}}{b}+\frac{(fa^2-bea+b^2d)x^9}{b^2}+\frac{(-fa^3+bea^2-b^2da+b^3c)x^6}{b^3}-\frac{a(-fa^3+bea^2-b^2da+b^3c)x^3}{b^4}+\frac{a^2(-fa^3+bea^2-b^2da+b^3c)}{b^5}+\frac{fa^6}{}\right.}{b}$$

$$\frac{fx^{16}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{x^{10}(a^2f-abe+b^2d)}{10b^2}+\frac{x^7(a^3(-f)+a^2be-ab^2d+b^3c)}{7b^3}+\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5}-\frac{ax^4(a^3(-f)+a^2be-ab^2d+b^3c)}{4b^4}+\frac{a^{7/3}\arctan\left(\sqrt[3]{}\right.}{}}{}$$

$$\frac{fx^{16}}{16b}$$

input
```
Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(f*x^16)/(16*b) + ((a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5 - (a*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^4)/(4*b^4) + ((b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*x^7)/(7*b^3) + ((b^2*d - a*b*e + a^2*f)*x^10)/(10*b^2) + ((b*e
 - a*f)*x^13)/(13*b) + (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(16/3)) - (a^(7/3)
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(16/3)
) + (a^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(6*b^(16/3)))/b
```

3.233. $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

3.233.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.233.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.70

method	result
risch	$\frac{f\,x^{16}}{16b}-\frac{x^{13}af}{13b^2}+\frac{x^{13}e}{13b}+\frac{x^{10}a^2f}{10b^3}-\frac{x^{10}ae}{10b^2}+\frac{x^{10}d}{10b}-\frac{x^7a^3f}{7b^4}+\frac{x^7a^2e}{7b^3}-\frac{x^7ad}{7b^2}+\frac{x^7c}{7b}+\frac{a^4f\,x^4}{4b^5}-\frac{a^3e\,x^4}{4b^4}+\frac{a^2d\,x^4}{4b^3}$
default	$-\frac{-\frac{1}{16}f\,x^{16}b^5+\frac{1}{13}x^{13}a\,b^4f-\frac{1}{13}x^{13}b^5e-\frac{1}{10}x^{10}a^2b^3f+\frac{1}{10}x^{10}a\,b^4e-\frac{1}{10}x^{10}b^5d+\frac{1}{7}x^7a^3b^2f-\frac{1}{7}x^7a^2b^3e+\frac{1}{7}x^7a\,b^4d-\frac{1}{7}x^7b^5c-\frac{1}{4}a^4bf\,x^4}{b^6}$

3.233. $\int \frac{x^9\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$

input
```
int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/16*f*x^16/b-1/13/b^2*x^13*a*f+1/13/b*x^13*e+1/10/b^3*x^10*a^2*f-1/10/b^2
*x^10*a*e+1/10/b*x^10*d-1/7/b^4*x^7*a^3*f+1/7/b^3*x^7*a^2*e-1/7/b^2*x^7*a*
d+1/7/b*x^7*c+1/4/b^5*a^4*f*x^4-1/4/b^4*a^3*e*x^4+1/4/b^3*a^2*d*x^4-1/4/b^
2*a*c*x^4-1/b^6*a^5*f*x+1/b^5*a^4*e*x-1/b^4*a^3*d*x+1/b^3*a^2*c*x+1/3/b^7*
a^3*sum((a^3*f-a^2*b*e+a*b^2*d-b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.98

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$= \frac{1365\,b^5fx^{16}+1680\,(b^5e-ab^4f)x^{13}+2184\,(b^5d-ab^4e+a^2b^3f)x^{10}+3120\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x}{}$$

input
```
integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/21840*(1365*b^5*f*x^16 + 1680*(b^5*e - a*b^4*f)*x^13 + 2184*(b^5*d - a*b
^4*e + a^2*b^3*f)*x^10 + 3120*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^
7 - 5460*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4 - 7280*sqrt(3)*(a
^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*
b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 3640*(a^2*b^3*c - a^3*b^2*d + a^4*b*e -
a^5*f)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 7280*(a^2*b^3*
c - a^3*b^2*d + a^4*b*e - a^5*f)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 21840*
(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6
```

3.233.6 Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.35

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^{13}\left(-\frac{af}{13b^2}+\frac{e}{13b}\right)+x^{10}\left(\frac{a^2f}{10b^3}-\frac{ae}{10b^2}+\frac{d}{10b}\right)+x^7\left(-\frac{a^3f}{7b^4}+\frac{a^2e}{7b^3}-\frac{ad}{7b^2}+\frac{c}{7b}\right)$$
$$+x^4\left(\frac{a^4f}{4b^5}-\frac{a^3e}{4b^4}+\frac{a^2d}{4b^3}-\frac{ac}{4b^2}\right)+x\left(-\frac{a^5f}{b^6}+\frac{a^4e}{b^5}-\frac{a^3d}{b^4}+\frac{a^2c}{b^3}\right)$$
$$+\mathrm{RootSum}\left(27t^3b^{19}-a^{16}f^3+3a^{15}bef^2-3a^{14}b^2df^2-3a^{14}b^2e^2f+3a^{13}b^3cf^2+6a^{13}b^3def+a^{13}b^3e^3-6\right.$$
$$+\frac{fx^{16}}{16b}$$

input
```
integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

output
```
x**13*(-a*f/(13*b**2) + e/(13*b)) + x**10*(a**2*f/(10*b**3) - a*e/(10*b**2
) + d/(10*b)) + x**7*(-a**3*f/(7*b**4) + a**2*e/(7*b**3) - a*d/(7*b**2) +
c/(7*b)) + x**4*(a**4*f/(4*b**5) - a**3*e/(4*b**4) + a**2*d/(4*b**3) - a*c
/(4*b**2)) + x*(-a**5*f/b**6 + a**4*e/b**5 - a**3*d/b**4 + a**2*c/b**3) +
RootSum(27*_t**3*b**19 - a**16*f**3 + 3*a**15*b*e*f**2 - 3*a**14*b**2*d*f*
*2 - 3*a**14*b**2*e**2*f + 3*a**13*b**3*c*f**2 + 6*a**13*b**3*d*e*f + a**1
3*b**3*e**3 - 6*a**12*b**4*c*e*f - 3*a**12*b**4*d**2*f - 3*a**12*b**4*d*e*
*2 + 6*a**11*b**5*c*d*f + 3*a**11*b**5*c*e**2 + 3*a**11*b**5*d**2*e - 3*a*
*10*b**6*c**2*f - 6*a**10*b**6*c*d*e - a**10*b**6*d**3 + 3*a**9*b**7*c**2*
e + 3*a**9*b**7*c*d**2 - 3*a**8*b**8*c**2*d + a**7*b**9*c**3, Lambda(_t, _
t*log(3*_t*b**6/(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x))) + f
*x**16/(16*b)
```

3.233.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.01

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{455\,b^5fx^{16}+560\,(b^5e-ab^4f)x^{13}+728\,(b^5d-ab^4e+a^2b^3f)x^{10}+1040\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^7-}{7280\,b^6}$$
$$-\frac{\sqrt{3}(a^3b^3c-a^4b^2d+a^5be-a^6f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(a^3b^3c-a^4b^2d+a^5be-a^6f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(a^3b^3c-a^4b^2d+a^5be-a^6f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/7280*(455*b^5*f*x^16 + 560*(b^5*e - a*b^4*f)*x^13 + 728*(b^5*d - a*b^4*e
 + a^2*b^3*f)*x^10 + 1040*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^7 -
1820*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4 + 7280*(a^2*b^3*c - a
^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6 - 1/3*sqrt(3)*(a^3*b^3*c - a^4*b^2*d +
a^5*b*e - a^6*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*
(a/b)^(2/3)) + 1/6*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(b^7*(a/b)^(2/3)) - 1/3*(a^3*b^3*c - a^4*b^2*d +
 a^5*b*e - a^6*f)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(2/3))
```

3.233.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.28

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx =$$
$$-\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}\,a^2b^3c-(-ab^2)^{\frac{1}{3}}\,a^3b^2d+(-ab^2)^{\frac{1}{3}}\,a^4be-(-ab^2)^{\frac{1}{3}}\,a^5f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^7}$$
$$-\frac{\left((-ab^2)^{\frac{1}{3}}\,a^2b^3c-(-ab^2)^{\frac{1}{3}}\,a^3b^2d+(-ab^2)^{\frac{1}{3}}\,a^4be-(-ab^2)^{\frac{1}{3}}\,a^5f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^7}$$
$$+\frac{(a^3b^{13}c-a^4b^{12}d+a^5b^{11}e-a^6b^{10}f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{16}}$$
$$+\frac{455\,b^{15}fx^{16}+560\,b^{15}ex^{13}-560\,ab^{14}fx^{13}+728\,b^{15}dx^{10}-728\,ab^{14}ex^{10}+728\,a^2b^{13}fx^{10}+1040\,b^{15}cx^7-}{}$$

input
```
integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*((-a*b^2)^(1/3)*a^2*b^3*c - (-a*b^2)^(1/3)*a^3*b^2*d + (-a*b^
2)^(1/3)*a^4*b*e - (-a*b^2)^(1/3)*a^5*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))/b^7 - 1/6*((-a*b^2)^(1/3)*a^2*b^3*c - (-a*b^2)^(1/3)*
a^3*b^2*d + (-a*b^2)^(1/3)*a^4*b*e - (-a*b^2)^(1/3)*a^5*f)*log(x^2 + x*(-a
/b)^(1/3) + (-a/b)^(2/3))/b^7 + 1/3*(a^3*b^13*c - a^4*b^12*d + a^5*b^11*e
- a^6*b^10*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^16) + 1/7280*(4
55*b^15*f*x^16 + 560*b^15*e*x^13 - 560*a*b^14*f*x^13 + 728*b^15*d*x^10 - 7
28*a*b^14*e*x^10 + 728*a^2*b^13*f*x^10 + 1040*b^15*c*x^7 - 1040*a*b^14*d*x
^7 + 1040*a^2*b^13*e*x^7 - 1040*a^3*b^12*f*x^7 - 1820*a*b^14*c*x^4 + 1820*
a^2*b^13*d*x^4 - 1820*a^3*b^12*e*x^4 + 1820*a^4*b^11*f*x^4 + 7280*a^2*b^13
*c*x - 7280*a^3*b^12*d*x + 7280*a^4*b^11*e*x - 7280*a^5*b^10*f*x)/b^16
```

3.233.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.03

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^{13}\left(\frac{e}{13\,b}-\frac{a\,f}{13\,b^2}\right)+x^{10}\left(\frac{d}{10\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{10\,b}\right)+x^7\left(\frac{c}{7\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{7\,b}\right)$$
$$+\frac{f\,x^{16}}{16\,b}-\frac{a^{7/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{19/3}}$$
$$+\frac{a^2\,x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b^2}-\frac{a\,x^4\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{4\,b}$$
$$-\frac{a^{7/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{19/3}}$$
$$+\frac{a^{7/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{19/3}}$$

input
```
int((x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

output
```
x^13*(e/(13*b) - (a*f)/(13*b^2)) + x^10*(d/(10*b) - (a*(e/b - (a*f)/b^2))/
(10*b)) + x^7*(c/(7*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(7*b)) + (f*x
^16)/(16*b) - (a^(7/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d +
 a^2*b*e))/(3*b^(19/3)) + (a^2*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b)
)/b))/b^2 - (a*x^4*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(4*b) -
(a^(7/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 -
 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(19/3)) + (a^(7/3)*log(3^(
1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a
^3*f - a*b^2*d + a^2*b*e))/(3*b^(19/3))
```

3.234 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

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3.234.1 Optimal result

Integrand size = 30, antiderivative size = 316

$$\begin{aligned}\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx &= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4}+\frac{(b^2d-abe+a^2f)\,x^8}{8b^3}\\ &+\frac{(be-af)x^{11}}{11b^2}+\frac{fx^{14}}{14b}-\frac{a^{5/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{17/3}}\\ &-\frac{a^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{17/3}}\\ &+\frac{a^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{17/3}}\end{aligned}$$

output
```
-1/2*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/b^5+1/5*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)*x^5/b^4+1/8*(a^2*f-a*b*e+b^2*d)*x^8/b^3+1/11*(-a*f+b*e)*x^11/b^2+1/
14*f*x^14/b-1/3*a^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*
x)/b^(17/3)+1/6*a^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/b^(17/3)-1/3*a^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(17/3)*3^(1/2)
```

3.234.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.98

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{a(-b^3c+ab^2d-a^2be+a^3f)\,x^2}{2b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4}+\frac{(b^2d-abe+a^2f)\,x^8}{8b^3}+\frac{(be-af)x^{11}}{11b^2}+\frac{fx^{14}}{14b}+\frac{a^{5/3}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^{17/3}}+\frac{a^{5/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{17/3}}-\frac{a^{5/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{17/3}}$$

input
```
Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(2*b^5) + ((b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^8)/(8*b^3) +
 ((b*e - a*f)*x^11)/(11*b^2) + (f*x^14)/(14*b) + (a^(5/3)*(-(b^3*c) + a*b^
2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[
3]*b^(17/3)) + (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3)
 + b^(1/3)*x])/(3*b^(17/3)) - (a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3
*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(17/3))
```

3.234.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2375, 27, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

$$\Big\downarrow \; 2375$$

$$\frac{\int \frac{14x^7\left((be-af)x^6+bdx^3+bc\right)}{bx^3+a}dx}{14b}+\frac{fx^{14}}{14b}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{x^7\left((be-af)x^6+bdx^3+bc\right)}{bx^3+a}dx}{b}+\frac{fx^{14}}{14b}$$

$\Big\downarrow$ 1812

$$\frac{\int \left(\frac{(be-af)x^{10}}{b}+\frac{(fa^2-bea+b^2d)x^7}{b^2}+\frac{(-fa^3+bea^2-b^2da+b^3c)x^4}{b^3}-\frac{a(-fa^3+bea^2-b^2da+b^3c)x}{b^4}-\frac{(fa^5-bea^4+b^2da^3-b^3ca^2)x}{b^4(bx^3+a)}\right)dx}{b}+$$

$$\frac{fx^{14}}{14b}$$

$\Big\downarrow$ 2009

$$\frac{\frac{x^8(a^2f-abe+b^2d)}{8b^2}+\frac{x^5(a^3(-f)+a^2be-ab^2d+b^3c)}{5b^3}-\frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{2b^4}-\frac{a^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b}{\sqrt{3}b^{14/3}}}{b}$$

$$\frac{fx^{14}}{14b}$$

input
```
Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(f*x^14)/(14*b) + (-1/2*(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/b^4 +
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b^3) + ((b^2*d - a*b*e + a^2*
f)*x^8)/(8*b^2) + ((b*e - a*f)*x^11)/(11*b) - (a^(5/3)*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[
3]*b^(14/3)) - (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) +
b^(1/3)*x]])/(3*b^(14/3)) + (a^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(14/3)))/b
```

3.234. $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

3.234.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.234.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.65

method	result
risch	$\frac{f x^{14}}{14b} - \frac{x^{11} a f}{11b^2} + \frac{e x^{11}}{11b} + \frac{x^8 a^2 f}{8b^3} - \frac{ae x^8}{8b^2} + \frac{x^8 d}{8b} - \frac{x^5 a^3 f}{5b^4} + \frac{a^2 e x^5}{5b^3} - \frac{x^5 ad}{5b^2} + \frac{x^5 c}{5b} + \frac{x^2 a^4 f}{2b^5} - \frac{a^3 e x^2}{2b^4} + \frac{x^2 a^2 d}{2b^3} -$
default	$\frac{\frac{f x^{14} b^4}{14} + \frac{\left(-a b^3 f + b^4 e\right) x^{11}}{11} + \frac{\left(a^2 b^2 f - a b^3 e + b^4 d\right) x^8}{8} + \frac{\left(-a^3 b f + a^2 e b^2 - a b^3 d + b^4 c\right) x^5}{5} + \frac{\left(a^4 f - a^3 b e + a^2 b^2 d - a b^3 c\right) x^2}{2}}{b^5} - \frac{\left(-\frac{\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} \right.}{}$

input
```
int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/14*f*x^14/b-1/11/b^2*x^11*a*f+1/11/b*e*x^11+1/8/b^3*x^8*a^2*f-1/8/b^2*a*
e*x^8+1/8/b*x^8*d-1/5/b^4*x^5*a^3*f+1/5/b^3*a^2*e*x^5-1/5/b^2*x^5*a*d+1/5/
b*x^5*c+1/2/b^5*x^2*a^4*f-1/2/b^4*a^3*e*x^2+1/2/b^3*x^2*a^2*d-1/2/b^2*a*c*
x^2+1/3/b^6*a^2*sum((-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/_R*ln(x-_R),_R=RootOf(_
Z^3*b+a))
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.02

$$\int \frac{x^7(c + dx^3 + ex^6 + fx^9)}{a + bx^3}\, dx$$

$$= \frac{660\, b^4 f x^{14} + 840\, (b^4 e - ab^3 f)x^{11} + 1155\, (b^4 d - ab^3 e + a^2 b^2 f)x^8 + 1848\, (b^4 c - ab^3 d + a^2 b^2 e - a^3 b f)x^5 -}{}$$

input
```
integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/9240*(660*b^4*f*x^14 + 840*(b^4*e - a*b^3*f)*x^11 + 1155*(b^4*d - a*b^3*
e + a^2*b^2*f)*x^8 + 1848*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^5 - 46
20*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^2 + 3080*sqrt(3)*(a*b^3*c - a
^2*b^2*d + a^3*b*e - a^4*f)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2
/b^2)^(1/3) - sqrt(3)*a)/a) + 1540*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)
*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 30
80*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(a^2/b^2)^(1/3)*log(a*x + b*(a^
2/b^2)^(2/3)))/b^5
```

3.234.6 Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.62

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^{11}\left(-\frac{af}{11b^2}+\frac{e}{11b}\right)+x^8\left(\frac{a^2f}{8b^3}-\frac{ae}{8b^2}+\frac{d}{8b}\right)$$
$$+x^5\left(-\frac{a^3f}{5b^4}+\frac{a^2e}{5b^3}-\frac{ad}{5b^2}+\frac{c}{5b}\right)+x^2\left(\frac{a^4f}{2b^5}-\frac{a^3e}{2b^4}+\frac{a^2d}{2b^3}-\frac{ac}{2b^2}\right)$$
$$+\mathrm{RootSum}\left(27t^3b^{17}-a^{14}f^3+3a^{13}bef^2-3a^{12}b^2df^2-3a^{12}b^2e^2f+3a^{11}b^3cf^2+6a^{11}b^3def+a^{11}b^3e^3-6\right.$$
$$+\frac{fx^{14}}{14b}$$

input
```
integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

output
```
x**11*(-a*f/(11*b**2) + e/(11*b)) + x**8*(a**2*f/(8*b**3) - a*e/(8*b**2) +
 d/(8*b)) + x**5*(-a**3*f/(5*b**4) + a**2*e/(5*b**3) - a*d/(5*b**2) + c/(5
*b)) + x**2*(a**4*f/(2*b**5) - a**3*e/(2*b**4) + a**2*d/(2*b**3) - a*c/(2*
b**2)) + RootSum(27*_t**3*b**17 - a**14*f**3 + 3*a**13*b*e*f**2 - 3*a**12*
b**2*d*f**2 - 3*a**12*b**2*e**2*f + 3*a**11*b**3*c*f**2 + 6*a**11*b**3*d*e
*f + a**11*b**3*e**3 - 6*a**10*b**4*c*e*f - 3*a**10*b**4*d**2*f - 3*a**10*
b**4*d*e**2 + 6*a**9*b**5*c*d*f + 3*a**9*b**5*c*e**2 + 3*a**9*b**5*d**2*e
- 3*a**8*b**6*c**2*f - 6*a**8*b**6*c*d*e - a**8*b**6*d**3 + 3*a**7*b**7*c*
*2*e + 3*a**7*b**7*c*d**2 - 3*a**6*b**8*c**2*d + a**5*b**9*c**3, Lambda(_t
, _t*log(9*_t**2*b**11/(a**9*f**2 - 2*a**8*b*e*f + 2*a**7*b**2*d*f + a**7*
b**2*e**2 - 2*a**6*b**3*c*f - 2*a**6*b**3*d*e + 2*a**5*b**4*c*e + a**5*b**
4*d**2 - 2*a**4*b**5*c*d + a**3*b**6*c**2) + x))) + f*x**14/(14*b)
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.99

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{\sqrt{3}(a^2b^3c-a^3b^2d+a^4be-a^5f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{220\,b^4fx^{14}+280\,(b^4e-ab^3f)x^{11}+385\,(b^4d-ab^3e+a^2b^2f)x^8+616\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^5-}{3080\,b^5} + \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(1/3)) + 1/3080*(220*b^4*f*x^14
 + 280*(b^4*e - a*b^3*f)*x^11 + 385*(b^4*d - a*b^3*e + a^2*b^2*f)*x^8 + 61
6*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^5 - 1540*(a*b^3*c - a^2*b^2*d
+ a^3*b*e - a^4*f)*x^2)/b^5 + 1/6*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6*(a/b)^(1/3)) - 1/3*(a^2*b^3*
c - a^3*b^2*d + a^4*b*e - a^5*f)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(1/3))
```

3.234.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.37

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx =$$

$$-\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}ab^3c-(-ab^2)^{\frac{2}{3}}a^2b^2d+(-ab^2)^{\frac{2}{3}}a^3be-(-ab^2)^{\frac{2}{3}}a^4f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^7}$$
$$+\frac{\left((-ab^2)^{\frac{2}{3}}ab^3c-(-ab^2)^{\frac{2}{3}}a^2b^2d+(-ab^2)^{\frac{2}{3}}a^3be-(-ab^2)^{\frac{2}{3}}a^4f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^7}$$
$$-\frac{\left(a^2b^{12}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^{11}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^4b^{10}e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^5b^9f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{14}}$$
$$+\frac{220b^{13}fx^{14}+280b^{13}ex^{11}-280ab^{12}fx^{11}+385b^{13}dx^8-385ab^{12}ex^8+385a^2b^{11}fx^8+616b^{13}cx^5-616}{308}$$

input
```
integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*((-a*b^2)^(2/3)*a*b^3*c - (-a*b^2)^(2/3)*a^2*b^2*d + (-a*b^2)
^(2/3)*a^3*b*e - (-a*b^2)^(2/3)*a^4*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/b^7 + 1/6*((-a*b^2)^(2/3)*a*b^3*c - (-a*b^2)^(2/3)*a^2*
b^2*d + (-a*b^2)^(2/3)*a^3*b*e - (-a*b^2)^(2/3)*a^4*f)*log(x^2 + x*(-a/b)^
(1/3) + (-a/b)^(2/3))/b^7 - 1/3*(a^2*b^12*c*(-a/b)^(1/3) - a^3*b^11*d*(-a/
b)^(1/3) + a^4*b^10*e*(-a/b)^(1/3) - a^5*b^9*f*(-a/b)^(1/3))*(-a/b)^(1/3)*
log(abs(x - (-a/b)^(1/3)))/(a*b^14) + 1/3080*(220*b^13*f*x^14 + 280*b^13*e
*x^11 - 280*a*b^12*f*x^11 + 385*b^13*d*x^8 - 385*a*b^12*e*x^8 + 385*a^2*b^
11*f*x^8 + 616*b^13*c*x^5 - 616*a*b^12*d*x^5 + 616*a^2*b^11*e*x^5 - 616*a^
3*b^10*f*x^5 - 1540*a*b^12*c*x^2 + 1540*a^2*b^11*d*x^2 - 1540*a^3*b^10*e*x
^2 + 1540*a^4*b^9*f*x^2)/b^14
```

3.234.9 Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.99

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^{11}\left(\frac{e}{11\,b}-\frac{a\,f}{11\,b^2}\right)+x^8\left(\frac{d}{8\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{8\,b}\right)+x^5\left(\frac{c}{5\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{5\,b}\right)+\frac{f\,x^{14}}{14\,b}$$
$$-\frac{a^{5/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{17/3}}-\frac{a\,x^2\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{2\,b}$$
$$+\frac{a^{5/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{17/3}}$$
$$-\frac{a^{5/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{17/3}}$$

input
```
int((x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

output
```
x^11*(e/(11*b) - (a*f)/(11*b^2)) + x^8*(d/(8*b) - (a*(e/b - (a*f)/b^2))/(8
*b)) + x^5*(c/(5*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(5*b)) + (f*x^14
)/(14*b) - (a^(5/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^
2*b*e))/(3*b^(17/3)) - (a*x^2*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b
))/(2*b) + (a^(5/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1
/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(17/3)) - (a^(5
/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)
*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(17/3))
```

3.235 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

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3.235.1 Optimal result

Integrand size = 30, antiderivative size = 312

$$\begin{aligned}\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx &= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4}+\frac{(b^2d-abe+a^2f)\,x^7}{7b^3}\\&\quad+\frac{(be-af)x^{10}}{10b^2}+\frac{fx^{13}}{13b}-\frac{a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{16/3}}\\&\quad+\frac{a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{16/3}}\\&\quad-\frac{a^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{16/3}}\end{aligned}$$

output
```
-a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^5+1/4*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
*x^4/b^4+1/7*(a^2*f-a*b*e+b^2*d)*x^7/b^3+1/10*(-a*f+b*e)*x^10/b^2+1/13*f*x
^13/b+1/3*a^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(
16/3)-1/6*a^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/b^(16/3)-1/3*a^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(16/3)*3^(1/2)
```

3.235.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx &= \frac{a(-b^3c+ab^2d-a^2be+a^3f)\,x}{b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{4b^4}+\frac{(b^2d-abe+a^2f)\,x^7}{7b^3}\\ &+\frac{(be-af)x^{10}}{10b^2}+\frac{fx^{13}}{13b}+\frac{a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^{16/3}}\\ &-\frac{a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{16/3}}\\ &+\frac{a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{16/3}}\end{aligned}$$

input
```
Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^5 + ((b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^4)/(4*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e
 - a*f)*x^10)/(10*b^2) + (f*x^13)/(13*b) + (a^(4/3)*(-(b^3*c) + a*b^2*d -
a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(
16/3)) - (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(
1/3)*x])/(3*b^(16/3)) + (a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(16/3))
```

3.235.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2375, 27, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

$$\Big\downarrow \; 2375$$

$$\frac{\int \frac{13x^6((be-af)x^6+bdx^3+bc)}{bx^3+a}dx}{13b}+\frac{fx^{13}}{13b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^6((be-af)x^6+bdx^3+bc)}{bx^3+a}dx}{b}+\frac{fx^{13}}{13b}$$

$$\Big\downarrow \text{1812}$$

$$\frac{\int \left(\frac{(be-af)x^9}{b}+\frac{(fa^2-bea+b^2d)x^6}{b^2}+\frac{(-fa^3+bea^2-b^2da+b^3c)x^3}{b^3}-\frac{a(-fa^3+bea^2-b^2da+b^3c)}{b^4}+\frac{-fa^5+bea^4-b^2da^3+b^3ca^2}{b^4(bx^3+a)}\right)dx}{b}+\frac{fx^{13}}{13b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{x^7(a^2f-abe+b^2d)}{7b^2}+\frac{x^4(a^3(-f)+a^2be-ab^2d+b^3c)}{4b^3}-\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}-\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3}{\sqrt{3}b^{13/3}}}{b}$$

$$\frac{fx^{13}}{13b}$$

input
```
Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(f*x^13)/(13*b) + (-((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4) + ((b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*x^4)/(4*b^3) + ((b^2*d - a*b*e + a^2*f)*x
^7)/(7*b^2) + ((b*e - a*f)*x^10)/(10*b) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b
^(13/3)) + (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1
/3)*x]])/(3*b^(13/3)) - (a^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(13/3)))/b
```

3.235. $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

3.235.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.235.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.62

method	result
risch	$\frac{f\,x^{13}}{13b}-\frac{x^{10}af}{10b^2}+\frac{e\,x^{10}}{10b}+\frac{x^7a^2f}{7b^3}-\frac{ae\,x^7}{7b^2}+\frac{x^7d}{7b}-\frac{x^4a^3f}{4b^4}+\frac{a^2e\,x^4}{4b^3}-\frac{x^4ad}{4b^2}+\frac{x^4c}{4b}+\frac{x\,a^4f}{b^5}-\frac{a^3ex}{b^4}+\frac{x\,a^2d}{b^3}-$
default	$\frac{\frac{1}{13}f\,x^{13}b^4-\frac{1}{10}x^{10}a\,b^3f+\frac{1}{10}x^{10}b^4e+\frac{1}{7}x^7a^2b^2f-\frac{1}{7}x^7a\,b^3e+\frac{1}{7}b^4d\,x^7-\frac{1}{4}a^3bf\,x^4+\frac{1}{4}a^2b^2e\,x^4-\frac{1}{4}a\,b^3d\,x^4+\frac{1}{4}b^4c\,x^4+a^4fx-a^3bex+a^2b^2}{b^5}$

3.235. $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

input
```
int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/13*f*x^13/b-1/10/b^2*x^10*a*f+1/10/b*e*x^10+1/7/b^3*x^7*a^2*f-1/7/b^2*a*
e*x^7+1/7/b*x^7*d-1/4/b^4*x^4*a^3*f+1/4/b^3*a^2*e*x^4-1/4/b^2*x^4*a*d+1/4/
b*x^4*c+1/b^5*x*a^4*f-1/b^4*a^3*e*x+1/b^3*x*a^2*d-1/b^2*a*c*x+1/3/b^6*a^2*
sum((-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.97

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$= \frac{420\,b^4fx^{13}+546\,(b^4e-ab^3f)x^{10}+780\,(b^4d-ab^3e+a^2b^2f)x^7+1365\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^4-1}{}$$

input
```
integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/5460*(420*b^4*f*x^13 + 546*(b^4*e - a*b^3*f)*x^10 + 780*(b^4*d - a*b^3*e
 + a^2*b^2*f)*x^7 + 1365*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^4 - 182
0*sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*arctan(1/3*
(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*(a*b^3*c - a^2*b^2*d + a
^3*b*e - a^4*f)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 18
20*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*(-a/b)^(1/3)*log(x - (-a/b)^(1/
3)) - 5460*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5
```

3.235.6 Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.36

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^{10}\left(-\frac{af}{10b^2}+\frac{e}{10b}\right)+x^7\left(\frac{a^2f}{7b^3}-\frac{ae}{7b^2}+\frac{d}{7b}\right)$$
$$+x^4\left(-\frac{a^3f}{4b^4}+\frac{a^2e}{4b^3}-\frac{ad}{4b^2}+\frac{c}{4b}\right)+x\left(\frac{a^4f}{b^5}-\frac{a^3e}{b^4}+\frac{a^2d}{b^3}-\frac{ac}{b^2}\right)$$
$$+\operatorname{RootSum}\left(27t^3b^{16}+a^{13}f^3-3a^{12}bef^2+3a^{11}b^2df^2+3a^{11}b^2e^2f-3a^{10}b^3cf^2-6a^{10}b^3def-a^{10}b^3e^3+6\right.$$
$$+\frac{fx^{13}}{13b}$$

input
```
integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

output
```
x**10*(-a*f/(10*b**2) + e/(10*b)) + x**7*(a**2*f/(7*b**3) - a*e/(7*b**2) +
 d/(7*b)) + x**4*(-a**3*f/(4*b**4) + a**2*e/(4*b**3) - a*d/(4*b**2) + c/(4
*b)) + x*(a**4*f/b**5 - a**3*e/b**4 + a**2*d/b**3 - a*c/b**2) + RootSum(27
*_t**3*b**16 + a**13*f**3 - 3*a**12*b*e*f**2 + 3*a**11*b**2*d*f**2 + 3*a**
11*b**2*e**2*f - 3*a**10*b**3*c*f**2 - 6*a**10*b**3*d*e*f - a**10*b**3*e**
3 + 6*a**9*b**4*c*e*f + 3*a**9*b**4*d**2*f + 3*a**9*b**4*d*e**2 - 6*a**8*b
**5*c*d*f - 3*a**8*b**5*c*e**2 - 3*a**8*b**5*d**2*e + 3*a**7*b**6*c**2*f +
 6*a**7*b**6*c*d*e + a**7*b**6*d**3 - 3*a**6*b**7*c**2*e - 3*a**6*b**7*c*d
**2 + 3*a**5*b**8*c**2*d - a**4*b**9*c**3, Lambda(_t, _t*log(-3*_t*b**5/(a
**4*f - a**3*b*e + a**2*b**2*d - a*b**3*c) + x))) + f*x**13/(13*b)
```

3.235.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{140\,b^4fx^{13}+182\,(b^4e-ab^3f)x^{10}+260\,(b^4d-ab^3e+a^2b^2f)x^7+455\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^4-18}{1820\,b^5}$$
$$+\frac{\sqrt{3}(a^2b^3c-a^3b^2d+a^4be-a^5f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^6\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^6\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^6\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/1820*(140*b^4*f*x^13 + 182*(b^4*e - a*b^3*f)*x^10 + 260*(b^4*d - a*b^3*e
 + a^2*b^2*f)*x^7 + 455*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^4 - 1820
*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5 + 1/3*sqrt(3)*(a^2*b^3*c -
 a^3*b^2*d + a^4*b*e - a^5*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(b^6*(a/b)^(2/3)) - 1/6*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*
log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^6*(a/b)^(2/3)) + 1/3*(a^2*b^3*c
- a^3*b^2*d + a^4*b*e - a^5*f)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(2/3))
```

3.235.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.26

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}\,ab^3c-(-ab^2)^{\frac{1}{3}}\,a^2b^2d+(-ab^2)^{\frac{1}{3}}\,a^3be-(-ab^2)^{\frac{1}{3}}\,a^4f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^6}$$
$$+\frac{\left((-ab^2)^{\frac{1}{3}}\,ab^3c-(-ab^2)^{\frac{1}{3}}\,a^2b^2d+(-ab^2)^{\frac{1}{3}}\,a^3be-(-ab^2)^{\frac{1}{3}}\,a^4f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^6}$$
$$-\frac{(a^2b^{11}c-a^3b^{10}d+a^4b^9e-a^5b^8f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{13}}$$
$$+\frac{140\,b^{12}fx^{13}+182\,b^{12}ex^{10}-182\,ab^{11}fx^{10}+260\,b^{12}dx^7-260\,ab^{11}ex^7+260\,a^2b^{10}fx^7+455\,b^{12}cx^4-455}{1820\,b}$$

input
```
integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*((-a*b^2)^(1/3)*a*b^3*c - (-a*b^2)^(1/3)*a^2*b^2*d + (-a*b^2)^
(1/3)*a^3*b*e - (-a*b^2)^(1/3)*a^4*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/b^6 + 1/6*((-a*b^2)^(1/3)*a*b^3*c - (-a*b^2)^(1/3)*a^2*b
^2*d + (-a*b^2)^(1/3)*a^3*b*e - (-a*b^2)^(1/3)*a^4*f)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))/b^6 - 1/3*(a^2*b^11*c - a^3*b^10*d + a^4*b^9*e - a^5*
b^8*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^13) + 1/1820*(140*b^12
*f*x^13 + 182*b^12*e*x^10 - 182*a*b^11*f*x^10 + 260*b^12*d*x^7 - 260*a*b^1
1*e*x^7 + 260*a^2*b^10*f*x^7 + 455*b^12*c*x^4 - 455*a*b^11*d*x^4 + 455*a^2
*b^10*e*x^4 - 455*a^3*b^9*f*x^4 - 1820*a*b^11*c*x + 1820*a^2*b^10*d*x - 18
20*a^3*b^9*e*x + 1820*a^4*b^8*f*x)/b^13
```

3.235.9 Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^{10}\left(\frac{e}{10\,b}-\frac{a\,f}{10\,b^2}\right)+x^7\left(\frac{d}{7\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{7\,b}\right)+x^4\left(\frac{c}{4\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{4\,b}\right)+\frac{f\,x^{13}}{13\,b}$$
$$+\frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{16/3}}-\frac{a\,x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b}$$
$$+\frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{16/3}}$$
$$-\frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{16/3}}$$

input
```
int((x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

output
```
x^10*(e/(10*b) - (a*f)/(10*b^2)) + x^7*(d/(7*b) - (a*(e/b - (a*f)/b^2))/(7
*b)) + x^4*(c/(4*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(4*b)) + (f*x^13
)/(13*b) + (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^
2*b*e))/(3*b^(16/3)) - (a*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))
/b + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i
)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(16/3)) - (a^(4/3)*lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*
c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(16/3))
```

3.236 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

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3.236.1 Optimal result

Integrand size = 30, antiderivative size = 279

$$\begin{aligned}\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx &= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{2b^4}+\frac{(b^2d-abe+a^2f)\,x^5}{5b^3}+\frac{(be-af)x^8}{8b^2}\\&+\frac{fx^{11}}{11b}+\frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{14/3}}\\&+\frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{14/3}}\\&-\frac{a^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{14/3}}\end{aligned}$$

output
```
1/2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/b^4+1/5*(a^2*f-a*b*e+b^2*d)*x^5/b^3
+1/8*(-a*f+b*e)*x^8/b^2+1/11*f*x^11/b+1/3*a^(2/3)*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(14/3)-1/6*a^(2/3)*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(14/3)+1/3*a^(2/3)*(-a^
3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/b^(14/3)*3^(1/2)
```

3.236.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$= \frac{660b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\,x^2+264b^{5/3}(b^2d-abe+a^2f)\,x^5+165b^{8/3}(be-af)x^8+120b^{11/3}fx^{11}\cdots}{\cdots}$$

input
```
Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(660*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 + 264*b^(5/3)*(b^2*d
- a*b*e + a^2*f)*x^5 + 165*b^(8/3)*(b*e - a*f)*x^8 + 120*b^(11/3)*f*x^11 -
 440*Sqrt[3]*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 440*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e
+ a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*
b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1320*b^(14/3
))
```

3.236.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2375, 27, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{2375}$$

$$\frac{\int \frac{11x^4\left((be-af)x^6+bdx^3+bc\right)}{bx^3+a}dx}{11b}+\frac{fx^{11}}{11b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^4\left((be-af)x^6+bdx^3+bc\right)}{bx^3+a}dx}{b}+\frac{fx^{11}}{11b}$$

$$\Big\downarrow 1812$$

$$\frac{\int \left(\frac{(be-af)x^7}{b}+\frac{(fa^2-bea+b^2d)x^4}{b^2}+\frac{(-fa^3+bea^2-b^2da+b^3c)x}{b^3}+\frac{(fa^4-bea^3+b^2da^2-b^3ca)x}{b^3(bx^3+a)}\right)dx}{b}+\frac{fx^{11}}{11b}$$

$$\Big\downarrow 2009$$

$$\frac{\frac{x^5(a^2f-abe+b^2d)}{5b^2}+\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{2b^3}+\frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}b^{11/3}}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.}{}}{b}$$

$$\frac{fx^{11}}{11b}$$

input
```
Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(f*x^11)/(11*b) + (((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(2*b^3) + ((b
^2*d - a*b*e + a^2*f)*x^5)/(5*b^2) + ((b*e - a*f)*x^8)/(8*b) + (a^(2/3)*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*b^(11/3)) + (a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(11/3)) - (a^(2/3)*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(11/3)
))/b
```

3.236.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.236.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.56

method	result
risch	$\frac{f x^{11}}{11b} - \frac{x^8 f a}{8b^2} + \frac{x^8 e}{8b} + \frac{x^5 f a^2}{5b^3} - \frac{x^5 a e}{5b^2} + \frac{d x^5}{5b} - \frac{x^2 f a^3}{2b^4} + \frac{x^2 a^2 e}{2b^3} - \frac{x^2 a d}{2b^2} + \frac{c x^2}{2b} + \frac{a \left(\sum_{\_R = \text{RootOf}(b\_Z^3+a)} \frac{(f a^3 - a}{} \right.}{3b^5}$
default	$-\frac{-\frac{b^3 f x^{11}}{11} + \frac{(f a b^2 - b^3 e) x^8}{8} + \frac{(-f a^2 b + a b^2 e - b^3 d) x^5}{5} + \frac{x^2 (f a^3 - a^2 b e + a b^2 d - b^3 c)}{2}}{b^4} + \frac{\left(-\frac{\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \cdots \right.}{}$

input
```
int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/11*f*x^11/b-1/8/b^2*x^8*f*a+1/8/b*x^8*e+1/5/b^3*x^5*f*a^2-1/5/b^2*x^5*a*
e+1/5*d*x^5/b-1/2/b^4*x^2*f*a^3+1/2/b^3*x^2*a^2*e-1/2/b^2*x^2*a*d+1/2*c*x^
2/b+1/3/b^5*a*sum((a^3*f-a^2*b*e+a*b^2*d-b^3*c)/_R*ln(x-_R),_R=RootOf(_Z^3
*b+a))
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.01

$$\int \frac{x^4(c + dx^3 + ex^6 + fx^9)}{a + bx^3}\, dx$$
$$= \frac{120\, b^3 f x^{11} + 165\, (b^3 e - ab^2 f)x^8 + 264\, (b^3 d - ab^2 e + a^2 b f)x^5 + 660\, (b^3 c - ab^2 d + a^2 b e - a^3 f)x^2 - 440\, \sqrt{}}{}$$

input
```
integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/1320*(120*b^3*f*x^11 + 165*(b^3*e - a*b^2*f)*x^8 + 264*(b^3*d - a*b^2*e
+ a^2*b*f)*x^5 + 660*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2 - 440*sqrt(3)
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3
)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1
/3)) - 440*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2/b^2)^(1/3)*log(a*x +
b*(-a^2/b^2)^(2/3)))/b^4
```

3.236.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.68

$$\int \frac{x^4(c + dx^3 + ex^6 + fx^9)}{a + bx^3}\, dx$$
$$= x^8\left(-\frac{af}{8b^2} + \frac{e}{8b}\right) + x^5\left(\frac{a^2 f}{5b^3} - \frac{ae}{5b^2} + \frac{d}{5b}\right) + x^2\left(-\frac{a^3 f}{2b^4} + \frac{a^2 e}{2b^3} - \frac{ad}{2b^2} + \frac{c}{2b}\right)$$
$$+ \operatorname{RootSum}\left(27t^3 b^{14} + a^{11} f^3 - 3a^{10} bef^2 + 3a^9 b^2 df^2 + 3a^9 b^2 e^2 f - 3a^8 b^3 cf^2 - 6a^8 b^3 def - a^8 b^3 e^3 + 6a^7 b^4\right.$$
$$+ \frac{fx^{11}}{11b}$$

input
```
integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

output
```
x**8*(-a*f/(8*b**2) + e/(8*b)) + x**5*(a**2*f/(5*b**3) - a*e/(5*b**2) + d/
(5*b)) + x**2*(-a**3*f/(2*b**4) + a**2*e/(2*b**3) - a*d/(2*b**2) + c/(2*b)
) + RootSum(27*_t**3*b**14 + a**11*f**3 - 3*a**10*b*e*f**2 + 3*a**9*b**2*d
*f**2 + 3*a**9*b**2*e**2*f - 3*a**8*b**3*c*f**2 - 6*a**8*b**3*d*e*f - a**8
*b**3*e**3 + 6*a**7*b**4*c*e*f + 3*a**7*b**4*d**2*f + 3*a**7*b**4*d*e**2 -
 6*a**6*b**5*c*d*f - 3*a**6*b**5*c*e**2 - 3*a**6*b**5*d**2*e + 3*a**5*b**6
*c**2*f + 6*a**5*b**6*c*d*e + a**5*b**6*d**3 - 3*a**4*b**7*c**2*e - 3*a**4
*b**7*c*d**2 + 3*a**3*b**8*c**2*d - a**2*b**9*c**3, Lambda(_t, _t*log(9*_t
**2*b**9/(a**7*f**2 - 2*a**6*b*e*f + 2*a**5*b**2*d*f + a**5*b**2*e**2 - 2*
a**4*b**3*c*f - 2*a**4*b**3*d*e + 2*a**3*b**4*c*e + a**3*b**4*d**2 - 2*a**
2*b**5*c*d + a*b**6*c**2) + x))) + f*x**11/(11*b)
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.96

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = -\frac{\sqrt{3}(ab^3c-a^2b^2d+a^3be-a^4f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{40\,b^3fx^{11}+55\,(b^3e-ab^2f)x^8+88\,(b^3d-ab^2e+a^2bf)x^5+220\,(b^3c-ab^2d+a^2be-a^3f)x^2}{440\,b^4} - \frac{(ab^3c-a^2b^2d+a^3be-a^4f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(ab^3c-a^2b^2d+a^3be-a^4f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/440*(40*b^3*f*x^11 +
55*(b^3*e - a*b^2*f)*x^8 + 88*(b^3*d - a*b^2*e + a^2*b*f)*x^5 + 220*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x^2)/b^4 - 1/6*(a*b^3*c - a^2*b^2*d + a^3*b*
e - a^4*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(1/3)) + 1/3*
(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(1
/3))
```

3.236.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.36

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}b^3c-(-ab^2)^{\frac{2}{3}}ab^2d+(-ab^2)^{\frac{2}{3}}a^2be-(-ab^2)^{\frac{2}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^6}$$
$$-\frac{\left((-ab^2)^{\frac{2}{3}}b^3c-(-ab^2)^{\frac{2}{3}}ab^2d+(-ab^2)^{\frac{2}{3}}a^2be-(-ab^2)^{\frac{2}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^6}$$
$$+\frac{\left(ab^{10}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^9d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3b^8e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^4b^7f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{11}}$$
$$+\frac{40b^{10}fx^{11}+55b^{10}ex^8-55ab^9fx^8+88b^{10}dx^5-88ab^9ex^5+88a^2b^8fx^5+220b^{10}cx^2-220ab^9dx^2+2}{440b^{11}}$$

input
```
integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + (-a*b^2)^(2/3
)*a^2*b*e - (-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/
(-a/b)^(1/3))/b^6 - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + (
-a*b^2)^(2/3)*a^2*b*e - (-a*b^2)^(2/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/b^6 + 1/3*(a*b^10*c*(-a/b)^(1/3) - a^2*b^9*d*(-a/b)^(1/3) + a
^3*b^8*e*(-a/b)^(1/3) - a^4*b^7*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/(a*b^11) + 1/440*(40*b^10*f*x^11 + 55*b^10*e*x^8 - 55*a*b^9*
f*x^8 + 88*b^10*d*x^5 - 88*a*b^9*e*x^5 + 88*a^2*b^8*f*x^5 + 220*b^10*c*x^2
 - 220*a*b^9*d*x^2 + 220*a^2*b^8*e*x^2 - 220*a^3*b^7*f*x^2)/b^11
```

3.236.9 Mupad [B] (verification not implemented)

Time = 10.58 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.96

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^8\left(\frac{e}{8\,b}-\frac{a\,f}{8\,b^2}\right)+x^5\left(\frac{d}{5\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{5\,b}\right)+x^2\left(\frac{c}{2\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{2\,b}\right)$$
$$+\frac{f\,x^{11}}{11\,b}+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{14/3}}$$
$$-\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{14/3}}$$
$$+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{14/3}}$$

input
```
int((x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

output
```
x^8*(e/(8*b) - (a*f)/(8*b^2)) + x^5*(d/(5*b) - (a*(e/b - (a*f)/b^2))/(5*b)
) + x^2*(c/(2*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(2*b)) + (f*x^11)/(
11*b) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b
*e))/(3*b^(14/3)) - (a^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3
))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(14/3)
) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)
/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(14/3))
```

3.237 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3} dx$

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3.237.1 Optimal result

Integrand size = 30, antiderivative size = 274

$$\begin{aligned}
&\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3} dx \\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4} + \frac{(b^2d-abe+a^2f)\,x^4}{4b^3} + \frac{(be-af)x^7}{7b^2} \\
&+ \frac{fx^{10}}{10b} + \frac{\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{13/3}} \\
&- \frac{\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{13/3}} \\
&+ \frac{\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{13/3}}
\end{aligned}$$

output
```
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4+1/4*(a^2*f-a*b*e+b^2*d)*x^4/b^3+1/7*(
-a*f+b*e)*x^7/b^2+1/10*f*x^10/b-1/3*a^(1/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
*ln(a^(1/3)+b^(1/3)*x)/b^(13/3)+1/6*a^(1/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(13/3)+1/3*a^(1/3)*(-a^3*f+a^
2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(
13/3)*3^(1/2)
```

3.237.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$= \frac{420\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\,x+105b^{4/3}(b^2d-abe+a^2f)\,x^4+60b^{7/3}(be-af)x^7+42b^{10/3}fx^{10}-14\cdots}{\cdots}$$

input
```
Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(420*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x + 105*b^(4/3)*(b^2*d -
a*b*e + a^2*f)*x^4 + 60*b^(7/3)*(b*e - a*f)*x^7 + 42*b^(10/3)*f*x^10 - 140
*Sqrt[3]*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(
1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^
3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e +
 a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*b^(13/3))
```

3.237.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2375, 27, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{2375}$$

$$\frac{\int \frac{10x^3\left((be-af)x^6+bdx^3+bc\right)}{bx^3+a}dx}{10b}+\frac{fx^{10}}{10b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^3\left((be-af)x^6+bdx^3+bc\right)}{bx^3+a}dx}{b}+\frac{fx^{10}}{10b}$$

$$\Big\downarrow \text{1812}$$

$$\frac{\int \left(\frac{(be-af)x^6}{b}+\frac{(fa^2-bea+b^2d)x^3}{b^2}+c-\frac{a(fa^2-bea+b^2d)}{b^3}+\frac{fa^4-bea^3+b^2da^2-b^3ca}{b^3(bx^3+a)}\right)dx}{b}+\frac{fx^{10}}{10b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{x^4(a^2f-abe+b^2d)}{4b^2}+x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)+\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}b^{10/3}}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-}{3b^{10/3}}}{b}$$

$$\frac{fx^{10}}{10b}$$

input
```
Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(f*x^10)/(10*b) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x + ((b^2*d - a*b
*e + a^2*f)*x^4)/(4*b^2) + ((b*e - a*f)*x^7)/(7*b) + (a^(1/3)*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(Sqrt[3]*b^(10/3)) - (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
1/3) + b^(1/3)*x])/(3*b^(10/3)) + (a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)))/b
```

3.237.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.237.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.53

method	result
risch	$\frac{f x^{10}}{10b} - \frac{x^7 f a}{7b^2} + \frac{x^7 e}{7b} + \frac{x^4 f a^2}{4b^3} - \frac{x^4 a e}{4b^2} + \frac{d x^4}{4b} - \frac{x f a^3}{b^4} + \frac{x a^2 e}{b^3} - \frac{x a d}{b^2} + \frac{c x}{b} + \frac{a\left(\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)} \frac{\left(f a^3 - a^2 b e + \cdots\right.}{\cdots}\right.}{3b^5}$
default	$-\frac{-\frac{1}{10} b^3 f x^{10} + \frac{1}{7} x^7 a b^2 f - \frac{1}{7} x^7 b^3 e - \frac{1}{4} a^2 b f x^4 + \frac{1}{4} a b^2 e x^4 - \frac{1}{4} d x^4 b^3 + f a^3 x - a^2 b e x + a b^2 d x - b^3 c x}{b^4} + \frac{\left(\frac{\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} \cdots\right.}{6b\left(\frac{a}{b} \cdots\right.} \cdots\right.}{\cdots}$

input
```
int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/10*f*x^10/b-1/7/b^2*x^7*f*a+1/7/b*x^7*e+1/4/b^3*x^4*f*a^2-1/4/b^2*x^4*a*
e+1/4*d*x^4/b-1/b^4*x*f*a^3+1/b^3*x*a^2*e-1/b^2*x*a*d+c*x/b+1/3/b^5*a*sum(
(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.91

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{42\,b^3fx^{10}+60\,(b^3e-ab^2f)x^7+105\,(b^3d-ab^2e+a^2bf)x^4-140\,\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\left(\frac{a}{b}\right)^{\frac{1}{3}}\operatorname{arcta}}{}$$

input
```
integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/420*(42*b^3*f*x^10 + 60*(b^3*e - a*b^2*f)*x^7 + 105*(b^3*d - a*b^2*e + a
^2*b*f)*x^4 - 140*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*
arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*(b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) -
140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) +
 420*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4
```

3.237.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.37

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^7\left(-\frac{af}{7b^2}+\frac{e}{7b}\right)+x^4\left(\frac{a^2f}{4b^3}-\frac{ae}{4b^2}+\frac{d}{4b}\right)+x\left(-\frac{a^3f}{b^4}+\frac{a^2e}{b^3}-\frac{ad}{b^2}+\frac{c}{b}\right)$$
$$+\operatorname{RootSum}\left(27t^3b^{13}-a^{10}f^3+3a^9bef^2-3a^8b^2df^2-3a^8b^2e^2f+3a^7b^3cf^2+6a^7b^3def+a^7b^3e^3-6a^6b^4c\right.$$
$$+\frac{fx^{10}}{10b}$$

input
```
integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

output
```
x**7*(-a*f/(7*b**2) + e/(7*b)) + x**4*(a**2*f/(4*b**3) - a*e/(4*b**2) + d/
(4*b)) + x*(-a**3*f/b**4 + a**2*e/b**3 - a*d/b**2 + c/b) + RootSum(27*_t**
3*b**13 - a**10*f**3 + 3*a**9*b*e*f**2 - 3*a**8*b**2*d*f**2 - 3*a**8*b**2*
e**2*f + 3*a**7*b**3*c*f**2 + 6*a**7*b**3*d*e*f + a**7*b**3*e**3 - 6*a**6*
b**4*c*e*f - 3*a**6*b**4*d**2*f - 3*a**6*b**4*d*e**2 + 6*a**5*b**5*c*d*f +
 3*a**5*b**5*c*e**2 + 3*a**5*b**5*d**2*e - 3*a**4*b**6*c**2*f - 6*a**4*b**
6*c*d*e - a**4*b**6*d**3 + 3*a**3*b**7*c**2*e + 3*a**3*b**7*c*d**2 - 3*a**
2*b**8*c**2*d + a*b**9*c**3, Lambda(_t, _t*log(3*_t*b**4/(a**3*f - a**2*b*
e + a*b**2*d - b**3*c) + x))) + f*x**10/(10*b)
```

3.237.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{14\,b^3fx^{10}+20\,(b^3e-ab^2f)x^7+35\,(b^3d-ab^2e+a^2bf)x^4+140\,(b^3c-ab^2d+a^2be-a^3f)x}{140\,b^4}$$
$$-\frac{\sqrt{3}(ab^3c-a^2b^2d+a^3be-a^4f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(ab^3c-a^2b^2d+a^3be-a^4f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(ab^3c-a^2b^2d+a^3be-a^4f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/140*(14*b^3*f*x^10 + 20*(b^3*e - a*b^2*f)*x^7 + 35*(b^3*d - a*b^2*e + a^
2*b*f)*x^4 + 140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 - 1/3*sqrt(3)*
(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1
/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) + 1/6*(a*b^3*c - a^2*b^2*d + a^3*b*e -
 a^4*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(2/3)) - 1/3*(a*
b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(2/3)
)
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.24

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx =$$

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d+\left(-ab^2\right)^{\frac{1}{3}}a^2be-\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^5}$$
$$-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d+\left(-ab^2\right)^{\frac{1}{3}}a^2be-\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^5}$$
$$+\frac{\left(ab^9c-a^2b^8d+a^3b^7e-a^4b^6f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{10}}$$
$$+\frac{14b^9fx^{10}+20b^9ex^7-20ab^8fx^7+35b^9dx^4-35ab^8ex^4+35a^2b^7fx^4+140b^9cx-140ab^8dx+140a^2}{140b^{10}}$$

input
```
integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d + (-a*b^2)^(1/
3)*a^2*b*e - (-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/b^5 - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d +
(-a*b^2)^(1/3)*a^2*b*e - (-a*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/b^5 + 1/3*(a*b^9*c - a^2*b^8*d + a^3*b^7*e - a^4*b^6*f)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*f*x^10 + 20*b
^9*e*x^7 - 20*a*b^8*f*x^7 + 35*b^9*d*x^4 - 35*a*b^8*e*x^4 + 35*a^2*b^7*f*x
^4 + 140*b^9*c*x - 140*a*b^8*d*x + 140*a^2*b^7*e*x - 140*a^3*b^6*f*x)/b^10
```

3.237.9 Mupad [B] (verification not implemented)

Time = 10.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^7\left(\frac{e}{7\,b}-\frac{a\,f}{7\,b^2}\right)+x^4\left(\frac{d}{4\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{4\,b}\right)+x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)$$
$$+\frac{f\,x^{10}}{10\,b}-\frac{a^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{13/3}}$$
$$-\frac{a^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{13/3}}$$
$$+\frac{a^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,b^{13/3}}$$

input
```
int((x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

output
```
x^7*(e/(7*b) - (a*f)/(7*b^2)) + x^4*(d/(4*b) - (a*(e/b - (a*f)/b^2))/(4*b)
) + x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b) + (f*x^10)/(10*b) - (a
^(1/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^
(13/3)) - (a^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/
2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(13/3)) + (a^(1/
3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*b^(13/3))
```

3.238 $\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

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3.238.1 Optimal result

Integrand size = 28, antiderivative size = 245

$$\begin{aligned}\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx &= \frac{(b^2d-abe+a^2f)\,x^2}{2b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^8}{8b}\\ &-\frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{11/3}}\\ &-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{11/3}}\\ &+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{11/3}}\end{aligned}$$

output
```
1/2*(a^2*f-a*b*e+b^2*d)*x^2/b^3+1/5*(-a*f+b*e)*x^5/b^2+1/8*f*x^8/b-1/3*(-a
^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(11/3)+1/6*(-a
^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1
/3)/b^(11/3)-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(11/3)*3^(1/2)
```

3.238.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.94

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$=\frac{60b^{2/3}(b^2d-abe+a^2f)\,x^2+24b^{5/3}(be-af)x^5+15b^{8/3}fx^8+\frac{40\sqrt{3}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\cdots}{120b^{11/3}}$$

input
```
Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(60*b^(2/3)*(b^2*d - a*b*e + a^2*f)*x^2 + 24*b^(5/3)*(b*e - a*f)*x^5 + 15*
b^(8/3)*f*x^8 + (40*Sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-(b^3*c) + a*b^2*d -
a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (20*(b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3)
)/(120*b^(11/3))
```

3.238.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2375, 27, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{2375}$$

$$\frac{\int \frac{8x\left((be-af)x^6+bdx^3+bc\right)}{bx^3+a}dx}{8b}+\frac{fx^8}{8b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x\left((be-af)x^6+bdx^3+bc\right)}{bx^3+a}dx}{b}+\frac{fx^8}{8b}$$

$$\Big\downarrow \text{1812}$$

$$\frac{\int \left(\frac{(be-af)x^4}{b}+\frac{(fa^2-bea+b^2d)x}{b^2}+\frac{(-fa^3+bea^2-b^2da+b^3c)x}{b^2(bx^3+a)}\right)dx}{b}+\frac{fx^8}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{x^2(a^2f-abe+b^2d)}{2b^2}-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}\sqrt[3]{a}b^{8/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3\sqrt[3]{a}b^{8/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{\ldots}\right.}{}}{b}$$

$$\frac{fx^8}{8b}$$

input
```
Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x]
```

output
```
(f*x^8)/(8*b) + (((b^2*d - a*b*e + a^2*f)*x^2)/(2*b^2) + ((b*e - a*f)*x^5)
/(5*b) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(8/3)) - ((b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(8/3)) + ((b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(1/3)*b^(8/3)))/b
```

3.238.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.238. $\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.238.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.46

method	result
risch	$\frac{f x^8}{8b}-\frac{x^5 a f}{5b^2}+\frac{x^5 e}{5b}+\frac{a^2 f x^2}{2b^3}-\frac{a e x^2}{2b^2}+\frac{d x^2}{2b}+\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-f a^3+a^2 b e-a b^2 d+b^3 c\right)\ln\left(x-\_R\right)}{\_R}}{3b^4}$
default	$\frac{\frac{b^2 f x^8}{8}+\frac{\left(-afb+b^2 e\right)x^5}{5}+\frac{\left(a^2 f-aeb+b^2 d\right)x^2}{2}}{b^3}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\left(f a^3-\right.}{b^3}$

input
```
int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*f*x^8/b-1/5/b^2*x^5*a*f+1/5/b*x^5*e+1/2/b^3*a^2*f*x^2-1/2/b^2*a*e*x^2+
1/2*d*x^2/b+1/3/b^4*sum((-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/_R*ln(x-_R),_R=Root
Of(_Z^3*b+a))
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.32

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$

$$= \left[\frac{15\,ab^4fx^8 + 24\,(ab^4e - a^2b^3f)x^5 + 60\,(ab^4d - a^2b^3e + a^3b^2f)x^2 - 60\,\sqrt{\frac{1}{3}}(ab^4c - a^2b^3d + a^3b^2e - a^4bf)\cdots}{\cdots} \right.$$

input
```
integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/120*(15*a*b^4*f*x^8 + 24*(a*b^4*e - a^2*b^3*f)*x^5 + 60*(a*b^4*d - a^2*
b^3*e + a^3*b^2*f)*x^2 - 60*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a
^4*b*f)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x +
 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)
^(2/3)*x)/(b*x^3 + a)) + 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2
/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^5), 1/120
*(15*a*b^4*f*x^8 + 24*(a*b^4*e - a^2*b^3*f)*x^5 + 60*(a*b^4*d - a^2*b^3*e
+ a^3*b^2*f)*x^2 - 120*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*
f)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a
*b^2)^(1/3)/a)/b) + 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*l
og(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^5)]
```

3.238.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.74

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = x^5\left(-\frac{af}{5b^2}+\frac{e}{5b}\right)+x^2\left(\frac{a^2f}{2b^3}-\frac{ae}{2b^2}+\frac{d}{2b}\right)$$

$$+\mathrm{RootSum}\left(27t^3ab^{11} - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4\cdots\right.$$

$$+\frac{fx^8}{8b}$$

input
```
integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

output
```
x**5*(-a*f/(5*b**2) + e/(5*b)) + x**2*(a**2*f/(2*b**3) - a*e/(2*b**2) + d/
(2*b)) + RootSum(27*_t**3*a*b**11 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b
**2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f +
 a**6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e
**2 + 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3
*b**6*c**2*f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3
*a**2*b**7*c*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2
*a*b**7/(a**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a
**3*b**3*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b*
*5*c*d + b**6*c**2) + x))) + f*x**8/(8*b)
```

3.238.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.92

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx = \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5\,b^2fx^8+8\,(b^2e-abf)x^5+20\,(b^2d-abe+a^2f)x^2}{40\,b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) + 1/40*(5*b^2*f*x^8 + 8*(b^2*e
 - a*b*f)*x^5 + 20*(b^2*d - a*b*e + a^2*f)*x^2)/b^3 + 1/6*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3
)) - 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(b^4*(a/
b)^(1/3))
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.17

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}b^3} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}b^3} - \frac{\left(b^8c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^7d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^6e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^5f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^8} + \frac{5\,b^7fx^8+8\,b^7ex^5-8\,ab^6fx^5+20\,b^7dx^2-20\,ab^6ex^2+20\,a^2b^5fx^2}{40\,b^8}$$

input
```
integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^3) - 1/6*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*
b^3) - 1/3*(b^8*c*(-a/b)^(1/3) - a*b^7*d*(-a/b)^(1/3) + a^2*b^6*e*(-a/b)^(
1/3) - a^3*b^5*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*
b^8) + 1/40*(5*b^7*f*x^8 + 8*b^7*e*x^5 - 8*a*b^6*f*x^5 + 20*b^7*d*x^2 - 20
*a*b^6*e*x^2 + 20*a^2*b^5*f*x^2)/b^8
```

3.238.9 Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.92

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{a+bx^3}\,dx$$
$$= x^5\left(\frac{e}{5\,b}-\frac{a\,f}{5\,b^2}\right)+x^2\left(\frac{d}{2\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{2\,b}\right)+\frac{f\,x^8}{8\,b}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{1/3}\,b^{11/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{1/3}\,b^{11/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{1/3}\,b^{11/3}}$$

input
```
int((x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3),x)
```

output
```
x^5*(e/(5*b) - (a*f)/(5*b^2)) + x^2*(d/(2*b) - (a*(e/b - (a*f)/b^2))/(2*b)
) + (f*x^8)/(8*b) - (log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a
^2*b*e))/(3*a^(1/3)*b^(11/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^
(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(1
/3)*b^(11/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)
*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(1/3)*b^(11/3))
```

3.239 $\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx$

3.239.1 Optimal result 1801
3.239.2 Mathematica [A] (verified) 1802
3.239.3 Rubi [A] (verified) 1802
3.239.4 Maple [C] (verified) 1804
3.239.5 Fricas [A] (verification not implemented) 1804
3.239.6 Sympy [A] (verification not implemented) 1805
3.239.7 Maxima [A] (verification not implemented) 1806
3.239.8 Giac [A] (verification not implemented) 1807
3.239.9 Mupad [B] (verification not implemented) 1807

3.239.1 Optimal result

Integrand size = 27, antiderivative size = 240

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx = \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^4}{4b^2}+\frac{fx^7}{7b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}$$

output
```
(a^2*f-a*b*e+b^2*d)*x/b^3+1/4*(-a*f+b*e)*x^4/b^2+1/7*f*x^7/b+1/3*(-a^3*f+a
^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(10/3)-1/6*(-a^3*f+a
^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^
(10/3)-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)
/a^(1/3)*3^(1/2))/a^(2/3)/b^(10/3)*3^(1/2)
```

3.239.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx$$

$$= \frac{84\sqrt[3]{b}(b^2d-abe+a^2f)\,x+21b^{4/3}(be-af)x^4+12b^{7/3}fx^7+\frac{28\sqrt{3}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{28}{}}{84b^{10/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```

output
```
(84*b^(1/3)*(b^2*d - a*b*e + a^2*f)*x + 21*b^(4/3)*(b*e - a*f)*x^4 + 12*b^
(7/3)*f*x^7 + (28*Sqrt[3]*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b^3*c) + a*b^2*d -
a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/
(84*b^(10/3))
```

3.239.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx$$

$$\Big\downarrow \; 2426$$

$$\int \left(\frac{a^2f-abe+b^2d}{b^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{b^3\left(a+bx^3\right)}+\frac{x^3(be-af)}{b^2}+\frac{fx^6}{b}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x(a^2f-abe+b^2d)}{b^3}-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{2/3}b^{10/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{2/3}b^{10/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{2/3}b^{10/3}}+\frac{x^4(be-af)}{4b^2}+\frac{fx^7}{7b}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x]
```

output
```
((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^4)/(4*b^2) + (f*x^7)/(7*b
) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a
^(2/3)*b^(10/3))
```

3.239.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.239.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.43

method	result
risch	$\frac{f x^7}{7b} - \frac{x^4 a f}{4b^2} + \frac{x^4 e}{4b} + \frac{a^2 f x}{b^3} - \frac{a e x}{b^2} + \frac{d x}{b} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(-f a^3+a^2 b e-a b^2 d+b^3 c\right) \ln\left(x-\_R\right)}{\_R^2}}{3b^4}$
default	$\frac{\frac{1}{7}b^2 f x^7-\frac{1}{4}abf x^4+\frac{1}{4}b^2 e x^4+a^2 f x-abex+b^2 dx}{b^3} + \frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(-f}{b^3}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/7*f*x^7/b-1/4/b^2*x^4*a*f+1/4/b*x^4*e+1/b^3*a^2*f*x-1/b^2*a*e*x+d*x/b+1/
3/b^4*sum((-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a)
)
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.50

$$\int \frac{c + dx^3 + ex^6 + fx^9}{a + bx^3}\, dx$$
$$= \left[\frac{12\,a^2b^3fx^7 + 21\,(a^2b^3e - a^3b^2f)x^4 - 42\,\sqrt{\frac{1}{3}}(ab^4c - a^2b^3d + a^3b^2e - a^4bf)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\,(-a^2b)^{\frac{1}{3}}\right.}{}}{}\right.$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/84*(12*a^2*b^3*f*x^7 + 21*(a^2*b^3*e - a^3*b^2*f)*x^4 - 42*sqrt(1/3)*(a
*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*
b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/
3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x
 - (-a^2*b)^(1/3)*a) + 28*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/
3)*log(a*b*x + (-a^2*b)^(2/3)) + 84*(a^2*b^3*d - a^3*b^2*e + a^4*b*f)*x)/(
a^2*b^4), 1/84*(12*a^2*b^3*f*x^7 + 21*(a^2*b^3*e - a^3*b^2*f)*x^4 + 84*sqr
t(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*sqrt(-(-a^2*b)^(1/3)/b)
*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(
1/3)/b)/a^2) - 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*log(a
*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 84*(a^2*b^3*d -
a^3*b^2*e + a^4*b*f)*x)/(a^2*b^4)]
```

3.239.6 Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.42

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx = x^4\left(-\frac{af}{4b^2}+\frac{e}{4b}\right)+x\left(\frac{a^2f}{b^3}-\frac{ae}{b^2}+\frac{d}{b}\right)$$
$$+\mathrm{RootSum}\left(27t^3a^2b^{10}+a^9f^3-3a^8bef^2+3a^7b^2df^2+3a^7b^2e^2f-3a^6b^3cf^2-6a^6b^3def-a^6b^3e^3+6a^5b^4\right.$$
$$+\frac{fx^7}{7b}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a),x)
```

output
```
x**4*(-a*f/(4*b**2) + e/(4*b)) + x*(a**2*f/b**3 - a*e/b**2 + d/b) + RootSu
m(27*_t**3*a**2*b**10 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f**2 +
 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b**3*e
**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a**4
*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*f
 + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c
*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a*b**3/(a**3*
f - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x**7/(7*b)
```

3.239.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx = \frac{4\,b^2fx^7+7\,(b^2e-abf)x^4+28\,(b^2d-abe+a^2f)x}{28\,b^3} + \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/28*(4*b^2*f*x^7 + 7*(b^2*e - a*b*f)*x^4 + 28*(b^2*d - a*b*e + a^2*f)*x)/
b^3 + 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) - 1/6*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3))
 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(b^4*(a/b)
^(2/3))
```

3.239.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx$$
$$= -\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b^2} - \frac{(b^7c-ab^6d+a^2b^5e-a^3b^4f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7} + \frac{4\,b^6fx^7+7\,b^6ex^4-7\,ab^5fx^4+28\,b^6dx-28\,ab^5ex+28\,a^2b^4fx}{28\,b^7}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/6*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)
*b^2) - 1/3*(b^7*c - a*b^6*d + a^2*b^5*e - a^3*b^4*f)*(-a/b)^(1/3)*log(abs
(x - (-a/b)^(1/3)))/(a*b^7) + 1/28*(4*b^6*f*x^7 + 7*b^6*e*x^4 - 7*a*b^5*f*
x^4 + 28*b^6*d*x - 28*a*b^5*e*x + 28*a^2*b^4*f*x)/b^7
```

3.239.9 Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx$$
$$= x^4\left(\frac{e}{4\,b}-\frac{a\,f}{4\,b^2}\right)+x\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)+\frac{f\,x^7}{7\,b} + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{2/3}\,b^{10/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{2/3}\,b^{10/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{2/3}\,b^{10/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3),x)
```

output
```
x^4*(e/(4*b) - (a*f)/(4*b^2)) + x*(d/b - (a*(e/b - (a*f)/b^2))/b) + (f*x^7
)/(7*b) + (log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(
3*a^(2/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3
^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(2/3)*b^(10/
3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1
/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(2/3)*b^(10/3))
```

3.239. $\int \frac{c+dx^3+ex^6+fx^9}{a+bx^3}\,dx$

3.240 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx$

3.240.1 Optimal result . . . 1809
3.240.2 Mathematica [A] (verified) . . . 1810
3.240.3 Rubi [A] (verified) . . . 1810
3.240.4 Maple [A] (verified) . . . 1812
3.240.5 Fricas [A] (verification not implemented) . . . 1812
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3.240.8 Giac [A] (verification not implemented) . . . 1814
3.240.9 Mupad [B] (verification not implemented) . . . 1815

3.240.1 Optimal result

Integrand size = 30, antiderivative size = 227

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx = -\frac{c}{ax}+\frac{(be-af)x^2}{2b^2}+\frac{fx^5}{5b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{8/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{8/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{8/3}}$$

output
```
-c/a/x+1/2*(-a*f+b*e)*x^2/b^2+1/5*f*x^5/b+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(8/3)-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(8/3)+1/3*(-a^3*f+a^
2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
4/3)/b^(8/3)*3^(1/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx$$

$$= \frac{-30\sqrt[3]{a}b^{8/3}c+15a^{4/3}b^{2/3}(be-af)x^3+6a^{4/3}b^{5/3}fx^6+10\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\,x\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{3}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```

output
```
(-30*a^(1/3)*b^(8/3)*c + 15*a^(4/3)*b^(2/3)*(b*e - a*f)*x^3 + 6*a^(4/3)*b^
(5/3)*f*x^6 + 10*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x*Log[a^(1/3) + b^(1/3)*x] - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(4/3)*b^(8/3)*x)
```

3.240.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{x(a^3f-a^2be+ab^2d-b^3c)}{ab^2(a+bx^3)}+\frac{x(be-af)}{b^2}+\frac{c}{ax^2}+\frac{fx^4}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{4/3}b^{8/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{4/3}b^{8/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{4/3}b^{8/3}} + \frac{x^2(be-af)}{2b^2} - \frac{c}{ax} + \frac{fx^5}{5b}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x]
```

output
```
-(c/(a*x)) + ((b*e - a*f)*x^2)/(2*b^2) + (f*x^5)/(5*b) + ((b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(4/3)*b^(8/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3)
+ b^(1/3)*x])/(3*a^(4/3)*b^(8/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(8/3))
```

3.240.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

3.240. $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx$

3.240.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.70

method	result
default	$-\frac{-\frac{bf\,x^5}{5}+\frac{(af-be)x^2}{2}}{b^2}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)}{a\,b^2}-$
risch	$\frac{f\,x^5}{5b}-\frac{x^2af}{2b^2}+\frac{e\,x^2}{2b}-\frac{c}{ax}+\frac{\_R=\mathrm{RootOf}\left(a^9f^3-3a^8be\,f^2+3a^7b^2d\,f^2+3a^7b^2e^2f-3a^6b^3c\,f^2-6a^6b^3def-a^6b^3e^3+6a^5b^4cef+3a^5b^4d^2f\right.}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/5*b*f*x^5+1/2*(a*f-b*e)*x^2)+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^
3*c)/a/b^2-c/a/x
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.47

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\,(a+bx^3)}\,dx$$

$$= \left[\frac{6\,a^2b^3fx^6-30\,ab^4c+15\,(a^2b^3e-a^3b^2f)x^3-15\,\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3}{}\right.}{}\right.$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/30*(6*a^2*b^3*f*x^6 - 30*a*b^4*c + 15*(a^2*b^3*e - a^3*b^2*f)*x^3 - 15*
sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x*sqrt((-a*b^2)^(1/3
)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-
a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a))
- 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b^2*x^2 + (-a
*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*(-a*b^2)^(2/3)*x*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x), 1/30*(6*a^2*b^3*
f*x^6 - 30*a*b^4*c + 15*(a^2*b^3*e - a^3*b^2*f)*x^3 - 30*sqrt(1/3)*(a*b^4*
c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt
(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x
+ (-a*b^2)^(2/3)) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*
x*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x)]
```

3.240.6 Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.80

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx = x^2\left(-\frac{af}{2b^2}+\frac{e}{2b}\right)$$
$$+\text{RootSum}\left(27t^3a^4b^8+a^9f^3-3a^8bef^2+3a^7b^2df^2+3a^7b^2e^2f-3a^6b^3cf^2-6a^6b^3def-a^6b^3e^3+6a^5b^4\right.$$
$$+\frac{fx^5}{5b}-\frac{c}{ax}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a),x)
```

output
```
x**2*(-a*f/(2*b**2) + e/(2*b)) + RootSum(27*_t**3*a**4*b**8 + a**9*f**3 -
3*a**8*b*e*f**2 + 3*a**7*b**2*d*f**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*
f**2 - 6*a**6*b**3*d*e*f - a**6*b**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**
4*d**2*f + 3*a**5*b**4*d*e**2 - 6*a**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3
*a**4*b**5*d**2*e + 3*a**3*b**6*c**2*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**
3 - 3*a**2*b**7*c**2*e - 3*a**2*b**7*c*d**2 + 3*a*b**8*c**2*d - b**9*c**3,
 Lambda(_t, _t*log(9*_t**2*a**3*b**5/(a**6*f**2 - 2*a**5*b*e*f + 2*a**4*b*
*2*d*f + a**4*b**2*e**2 - 2*a**3*b**3*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*
c*e + a**2*b**4*d**2 - 2*a*b**5*c*d + b**6*c**2) + x))) + f*x**5/(5*b) - c
/(a*x)
```

3.240.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx = \frac{2\,bfx^5+5\,(be-af)x^2}{10\,b^2} - \frac{c}{ax} - \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/10*(2*b*f*x^5 + 5*(b*e - a*f)*x^2)/b^2 - c/(a*x) - 1/3*sqrt(3)*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(a*b^3*(a/b)^(1/3)) - 1/6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x^2
 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(1/3)) + 1/3*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(1/3))
```

3.240.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.17

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx = -\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{1}{3}}\,ab^2} - \frac{c}{ax} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{1}{3}}\,ab^2} + \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2b^2} + \frac{2\,b^4fx^5+5\,b^4ex^2-5\,ab^3fx^2}{10\,b^5}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a*b^2) - c/(a*x) + 1/6*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-
a*b^2)^(1/3)*a*b^2) + 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) + a^2
*b*e*(-a/b)^(1/3) - a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1
/3)))/(a^2*b^2) + 1/10*(2*b^4*f*x^5 + 5*b^4*e*x^2 - 5*a*b^3*f*x^2)/b^5
```

3.240.9 Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)}\,dx$$
$$= x^2\left(\frac{e}{2\,b}-\frac{a\,f}{2\,b^2}\right)-\frac{c}{a\,x}+\frac{f\,x^5}{5\,b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{4/3}\,b^{8/3}}$$
$$-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{4/3}\,b^{8/3}}$$
$$+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{4/3}\,b^{8/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)),x)
```

output
```
x^2*(e/(2*b) - (a*f)/(2*b^2)) - c/(a*x) + (f*x^5)/(5*b) + (log(b^(1/3)*x +
 a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(4/3)*b^(8/3)) - (log(
3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c
- a^3*f - a*b^2*d + a^2*b*e))/(3*a^(4/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1
i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d
 + a^2*b*e))/(3*a^(4/3)*b^(8/3))
```

3.241 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx$

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3.241.1 Optimal result

Integrand size = 30, antiderivative size = 224

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx = -\frac{c}{2ax^2}+\frac{(be-af)x}{b^2}+\frac{fx^4}{4b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{7/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{7/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}b^{7/3}}$$

output
```
-1/2*c/a/x^2+(-a*f+b*e)*x/b^2+1/4*f*x^4/b-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)+1/3*(-a^3*f+a^
2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
5/3)/b^(7/3)*3^(1/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)}\,dx = \frac{1}{12}\left(-\frac{6c}{ax^2}+\frac{12(be-af)x}{b^2}+\frac{3fx^4}{b}+\frac{4\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}b^{7/3}}+\frac{4(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}b^{7/3}}+\frac{2(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}b^{7/3}}\right)$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]
```

output
```
((-6*c)/(a*x^2) + (12*(b*e - a*f)*x)/b^2 + (3*f*x^4)/b + (4*Sqrt[3]*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])
/(a^(5/3)*b^(7/3)) + (4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3)
 + b^(1/3)*x])/(a^(5/3)*b^(7/3)) + (2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(5/3)*b^(7/3)))/12
```

3.241.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}dx$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{a^3f-a^2be+ab^2d-b^3c}{ab^2(a+bx^3)}+\frac{be-af}{b^2}+\frac{c}{ax^3}+\frac{fx^3}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{5/3}b^{7/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{5/3}b^{7/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{5/3}b^{7/3}}+\frac{x(be-af)}{b^2}-\frac{c}{2ax^2}+\frac{fx^4}{4b}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*c/(a*x^2) + ((b*e - a*f)*x)/b^2 + (f*x^4)/(4*b) + ((b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*a^(5/3)*b^(7/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) +
b^(1/3)*x]])/(3*a^(5/3)*b^(7/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*b^(7/3))
```

3.241.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.241.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.69

method	result
default	$-\frac{-\frac{1}{4}bf\,x^4+afx-bex}{b^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)}{a\,b^2}-$
risch	$\frac{f\,x^4}{4b}-\frac{afx}{b^2}+\frac{ex}{b}-\frac{c}{2a\,x^2}+\frac{\_R{=}\mathrm{RootOf}\left(-a^9f^3+3a^8be\,f^2-3a^7b^2d\,f^2-3a^7b^2e^2f+3a^6b^3c\,f^2+6a^6b^3def+a^6b^3e^3-6a^5b^4cef-3a^5b^4d^2\right.}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/4*b*f*x^4+a*f*x-b*e*x)+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6
/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a/b^2*(a^3*f-a^2*b*e+a*b^2*d-b^3
*c)-1/2*c/a/x^2
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.52

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^3 (a + bx^3)} \, dx$$
$$= \left[\frac{3\,a^3b^2fx^6 - 6\,a^2b^3c - 6\,\sqrt{\frac{1}{3}}(ab^4c - a^2b^3d + a^3b^2e - a^4bf)x^2\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3 - 3\,(a^2b)^{\frac{1}{3}}ax - a^2 + 3\,\sqrt{\frac{1}{3}}\left(2\,aba\right.}{bx^3\ldots}\right.}{} \right.$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/12*(3*a^3*b^2*f*x^6 - 6*a^2*b^3*c - 6*sqrt(1/3)*(a*b^4*c - a^2*b^3*d +
a^3*b^2*e - a^4*b*f)*x^2*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)
^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3
)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + 2*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a)
 - 4*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x + (a^
2*b)^(2/3)) + 12*(a^3*b^2*e - a^4*b*f)*x^3)/(a^3*b^3*x^2), 1/12*(3*a^3*b^2
*f*x^6 - 6*a^2*b^3*c - 12*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4
*b*f)*x^2*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2
*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) -
 4*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^2*log(a*b*x + (a^2*
b)^(2/3)) + 12*(a^3*b^2*e - a^4*b*f)*x^3)/(a^3*b^3*x^2)]
```

3.241.6 Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.46

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^3 (a + bx^3)} \, dx = x\left(-\frac{af}{b^2} + \frac{e}{b}\right)$$
$$+ \mathrm{RootSum}\left(27t^3a^5b^7 - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4\ldots\right.$$
$$+ \frac{fx^4}{4b} - \frac{c}{2ax^2}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a),x)
```

output
```
x*(-a*f/b**2 + e/b) + RootSum(27*_t**3*a**5*b**7 - a**9*f**3 + 3*a**8*b*e*
f**2 - 3*a**7*b**2*d*f**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a*
*6*b**3*d*e*f + a**6*b**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f -
3*a**5*b**4*d*e**2 + 6*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*
d**2*e - 3*a**3*b**6*c**2*f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*
b**7*c**2*e + 3*a**2*b**7*c*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t,
 _t*log(3*_t*a**2*b**2/(a**3*f - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*
x**4/(4*b) - c/(2*a*x**2)
```

3.241.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx = \frac{bfx^4+4\,(be-af)x}{4\,b^2} - \frac{c}{2\,ax^2} - \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/4*(b*f*x^4 + 4*(b*e - a*f)*x)/b^2 - 1/2*c/(a*x^2) - 1/3*sqrt(3)*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/(a*b^3*(a/b)^(2/3)) + 1/6*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x^
2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) - 1/3*(b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx = \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}ab} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}ab} + \frac{(b^3c-ab^2d+a^2be-a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^2b^2} - \frac{c}{2ax^2} + \frac{b^3fx^4+4b^3ex-4ab^2fx}{4b^4}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) + 1/6*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*
a*b) + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a^2*b^2) - 1/2*c/(a*x^2) + 1/4*(b^3*f*x^4 + 4*b^3*e*x - 4*a*
b^2*f*x)/b^4
```

3.241.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx = x\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)-\frac{c}{2\,a\,x^2}+\frac{f\,x^4}{4\,b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{5/3}\,b^{7/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{5/3}\,b^{7/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{5/3}\,b^{7/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)),x)
```

output
```
x*(e/b - (a*f)/b^2) - c/(2*a*x^2) + (f*x^4)/(4*b) - (log(b^(1/3)*x + a^(1/
3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(5/3)*b^(7/3)) - (log(3^(1/2
)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*
f - a*b^2*d + a^2*b*e))/(3*a^(5/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*
b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2
*b*e))/(3*a^(5/3)*b^(7/3))
```

3.241. $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)}\,dx$

3.242 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx$

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3.242.9 Mupad [B] (verification not implemented) . . . 1830

3.242.1 Optimal result

Integrand size = 30, antiderivative size = 227

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx = -\frac{c}{4ax^4}+\frac{bc-ad}{a^2x}+\frac{fx^2}{2b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}b^{5/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}b^{5/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}b^{5/3}}$$

output
```
-1/4*c/a/x^4+(-a*d+b*c)/a^2/x+1/2*f*x^2/b-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(5/3)+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(5/3)-1/3*(-a^3*f+a^
2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
7/3)/b^(5/3)*3^(1/2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx = \frac{1}{12}\left(-\frac{3c}{ax^4}+\frac{12(bc-ad)}{a^2x}+\frac{6fx^2}{b} + \frac{4\sqrt{3}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}b^{5/3}} + \frac{4(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{7/3}b^{5/3}} + \frac{2(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{7/3}b^{5/3}}\right)$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```

output
```
((-3*c)/(a*x^4) + (12*(b*c - a*d))/(a^2*x) + (6*f*x^2)/b + (4*Sqrt[3]*(-(b
^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]])/(a^(7/3)*b^(5/3)) + (4*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^
(1/3) + b^(1/3)*x])/(a^(7/3)*b^(5/3)) + (2*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(7/3)*b^(5/3)))/12
```

3.242.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}dx$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{ad-bc}{a^2x^2}-\frac{x(a^3f-a^2be+ab^2d-b^3c)}{a^2b(a+bx^3)}+\frac{c}{ax^5}+\frac{fx}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc-ad}{a^2x}-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{7/3}b^{5/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{7/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{7/3}b^{5/3}}-\frac{c}{4ax^4}+\frac{fx^2}{2b}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x]
```

output
```
-1/4*c/(a*x^4) + (b*c - a*d)/(a^2*x) + (f*x^2)/(2*b) - ((b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*a^(7/3)*b^(5/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) +
b^(1/3)*x])/(3*a^(7/3)*b^(5/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*b^(5/3))
```

3.242.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

3.242.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.70

method	result
default	$\frac{f\,x^{2}}{2b}-\frac{c}{4a\,x^{4}}-\frac{ad-bc}{a^{2}x}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)}{a^{2}b}$
risch	$\frac{f\,x^{2}}{2b}+\frac{-\frac{(ad-bc)b\,x^{3}}{a^{2}}-\frac{cb}{4a}}{b\,x^{4}}+\frac{\_R=\text{RootOf}\left(a^{7}b^{2}\_Z^{3}-a^{9}f^{3}+3a^{8}be\,f^{2}-3a^{7}b^{2}d\,f^{2}-3a^{7}b^{2}e^{2}f+3a^{6}b^{3}c\,f^{2}+6a^{6}b^{3}def+a^{6}b^{3}e^{3}-6a^{5}b^{4}cef-3a\right.}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*f*x^2/b-1/4*c/a/x^4-(a*d-b*c)/a^2/x-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^
3*c)/a^2/b
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 556, normalized size of antiderivative = 2.45

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx$$

$$= \left[\frac{6\,a^3b^2fx^6 - 6\,\sqrt{\frac{1}{3}}(ab^4c - a^2b^3d + a^3b^2e - a^4bf)x^4\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab - 3\,\sqrt{\frac{1}{3}}\left(abx + 2\,(ab^2)^{\frac{2}{3}}x^2 - (ab^2)^{\frac{1}{3}}a\right)\sqrt{\cdot}}{bx^3+a}\right.}{} \right.$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/12*(6*a^3*b^2*f*x^6 - 6*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^
4*b*f)*x^4*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*
x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b
^2)^(2/3)*x)/(b*x^3 + a)) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^
(2/3)*x^4*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b*x + (a*b^2)^(1/3)) - 3*a^
2*b^3*c + 12*(a*b^4*c - a^2*b^3*d)*x^3)/(a^3*b^3*x^4), 1/12*(6*a^3*b^2*f*x
^6 - 12*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4*sqrt((a*
b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)
/a)/b) + 2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a*b^2)^(2/3)*x^4*log(b^2*x
^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*(a*b^2)^(2/3)*x^4*log(b*x + (a*b^2)^(1/3)) - 3*a^2*b^3*c + 12*(a*b^4
*c - a^2*b^3*d)*x^3)/(a^3*b^3*x^4)]
```

3.242.6 Sympy [A] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.81

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx$$

$$= \operatorname{RootSum}\left(27t^3a^7b^5 - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4ce\right.$$

$$+ \frac{fx^2}{2b} + \frac{-ac + x^3(-4ad + 4bc)}{4a^2x^4}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**7*b**5 - a**9*f**3 + 3*a**8*b*e*f**2 - 3*a**7*b**2*d*f
**2 - 3*a**7*b**2*e**2*f + 3*a**6*b**3*c*f**2 + 6*a**6*b**3*d*e*f + a**6*b
**3*e**3 - 6*a**5*b**4*c*e*f - 3*a**5*b**4*d**2*f - 3*a**5*b**4*d*e**2 + 6
*a**4*b**5*c*d*f + 3*a**4*b**5*c*e**2 + 3*a**4*b**5*d**2*e - 3*a**3*b**6*c
**2*f - 6*a**3*b**6*c*d*e - a**3*b**6*d**3 + 3*a**2*b**7*c**2*e + 3*a**2*b
**7*c*d**2 - 3*a*b**8*c**2*d + b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**5*b
**3/(a**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*
b**3*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c
*d + b**6*c**2) + x))) + f*x**2/(2*b) + (-a*c + x**3*(-4*a*d + 4*b*c))/(4*
a**2*x**4)
```

3.242.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

$$
\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x^5\,(a+bx^3)}\,dx = {} & \frac{fx^2}{2\,b} + \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\
& + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\
& - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\
& + \frac{4\,(bc-ad)x^3-ac}{4\,a^2x^4}
\end{aligned}
$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/2*f*x^2/b + 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*s
qrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3)) + 1/6*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*
b^2*(a/b)^(1/3)) - 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(
1/3))/(a^2*b^2*(a/b)^(1/3)) + 1/4*(4*(b*c - a*d)*x^3 - a*c)/(a^2*x^4)
```

3.242.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\,(a+bx^3)}\,dx$$
$$= \frac{fx^2}{2\,b} + \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{1}{3}}\,a^2b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{1}{3}}\,a^2b} - \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b} + \frac{4\,bcx^3-4\,adx^3-ac}{4\,a^2x^4}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a),x, algorithm="giac")
```

output
```
1/2*f*x^2/b + 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*s
qrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b) - 1/6*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
((-a*b^2)^(1/3)*a^2*b) - 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) +
a^2*b*e*(-a/b)^(1/3) - a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^3*b) + 1/4*(4*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^2*x^4)
```

3.242.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\,(a+bx^3)}\,dx$$
$$= \frac{f\,x^2}{2\,b} - \frac{\frac{b\,c}{4\,a}+\frac{b\,x^3\,(a\,d-b\,c)}{a^2}}{b\,x^4} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{7/3}\,b^{5/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{7/3}\,b^{5/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{7/3}\,b^{5/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)),x)
```

output
```
(f*x^2)/(2*b) - ((b*c)/(4*a) + (b*x^3*(a*d - b*c))/a^2)/(b*x^4) - (log(b^(
1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(7/3)*b^(5/3))
 + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)
*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(7/3)*b^(5/3)) - (log(3^(1/2)*a
^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f -
 a*b^2*d + a^2*b*e))/(3*a^(7/3)*b^(5/3))
```

3.242. $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)}\,dx$

3.243 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx$

3.243.1 Optimal result 1832
3.243.2 Mathematica [A] (verified) 1833
3.243.3 Rubi [A] (verified) 1833
3.243.4 Maple [A] (verified) 1835
3.243.5 Fricas [A] (verification not implemented) 1835
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3.243.8 Giac [A] (verification not implemented) 1837
3.243.9 Mupad [B] (verification not implemented) 1838

3.243.1 Optimal result

Integrand size = 30, antiderivative size = 225

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx = -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}b^{4/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}b^{4/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}b^{4/3}}$$

output
```
-1/5*c/a/x^5+1/2*(-a*d+b*c)/a^2/x^2+f*x/b+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(4/3)-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(4/3)-1/3*(-a^3*f+a^
2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
8/3)/b^(4/3)*3^(1/2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx = -\frac{c}{5ax^5}+\frac{bc-ad}{2a^2x^2}+\frac{fx}{b} + \frac{(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{8/3}b^{4/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}b^{4/3}} + \frac{(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}b^{4/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```

output
```
-1/5*c/(a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b + ((-(b^3*c) + a*b^2*d
- a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a
^(8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/
3)*x]])/(3*a^(8/3)*b^(4/3)) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/(6*a^(8/3)*b^(4/3))
```

3.243.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx$$

$$\Big\downarrow \text{ 2373}$$

$$\int \left(\frac{ad-bc}{a^2x^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^2b(a+bx^3)}+\frac{c}{ax^6}+\frac{f}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bc-ad}{2a^2x^2}-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{8/3}b^{4/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{8/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{8/3}b^{4/3}}-\frac{c}{5ax^5}+\frac{fx}{b}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x]
```

output
```
-1/5*c/(a*x^5) + (b*c - a*d)/(2*a^2*x^2) + (f*x)/b - ((b^3*c - a*b^2*d + a
^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*a^(8/3)*b^(4/3)) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^
(1/3)*x])/(3*a^(8/3)*b^(4/3)) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*b^(4/3))
```

3.243.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.243.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.69

method	result
default	$\frac{fx}{b}-\frac{c}{5a\,x^{5}}-\frac{ad-bc}{2x^{2}a^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(-f\,a^{3}+a^{2}be-a\,b^{2}d+b^{3}c\right)}{a^{2}b}$
risch	$\frac{fx}{b}+\frac{-\frac{(ad-bc)b\,x^{3}}{2a^{2}}-\frac{cb}{5a}}{b\,x^{5}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a^{8}b\_Z^{3}+a^{9}f^{3}-3a^{8}be\,f^{2}+3a^{7}b^{2}d\,f^{2}+3a^{7}b^{2}e^{2}f-3a^{6}b^{3}c\,f^{2}-6a^{6}b^{3}def-a^{6}b^{3}e^{3}+6a^{5}b^{4}cef+3a^{5}b\right.}}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
f*x/b-1/5*c/a/x^5-1/2*(a*d-b*c)/x^2/a^2+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a^2/b*(-a^3*f+a^2*b*e-a*b^
2*d+b^3*c)
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.60

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)}\,dx$$

$$=\left[\frac{30\,a^{4}bfx^{6}-15\,\sqrt{\frac{1}{3}}(ab^{4}c-a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf)x^{5}\sqrt{\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}+3\,(-a^{2}b)^{\frac{1}{3}}ax-a^{2}-3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+(-a^{2}b)\right.}{bx^{3}+a}\right.}{}\right.$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/30*(30*a^4*b*f*x^6 - 15*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^
4*b*f)*x^5*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x -
a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((
-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a
^2*b)^(2/3)*x^5*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*(b
^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x + (-a^2*b)^
(2/3)) - 6*a^3*b^2*c + 15*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^5), 1/30
*(30*a^4*b*f*x^6 + 30*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f
)*x^5*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2
*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 5*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)
*a) + 10*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a^2*b)^(2/3)*x^5*log(a*b*x
+ (-a^2*b)^(2/3)) - 6*a^3*b^2*c + 15*(a^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2
*x^5)]
```

3.243.6 Sympy [A] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.46

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\,(a+bx^3)}\,dx$$
$$= \mathrm{RootSum}\left(27t^3a^8b^4 + a^9f^3 - 3a^8bef^2 + 3a^7b^2df^2 + 3a^7b^2e^2f - 3a^6b^3cf^2 - 6a^6b^3def - a^6b^3e^3 + 6a^5b^4ce\right.$$
$$+ \frac{fx}{b} + \frac{-2ac + x^3(-5ad+5bc)}{10a^2x^5}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**8*b**4 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*f
**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*b
**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 - 6
*a**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*c
**2*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*b
**7*c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(-3*_t*a**3*b/(
a**3*f - a**2*b*e + a*b**2*d - b**3*c) + x))) + f*x/b + (-2*a*c + x**3*(-5
*a*d + 5*b*c))/(10*a**2*x**5)
```

3.243.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx = \frac{fx}{b} + \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5(bc-ad)x^3-2ac}{10a^2x^5}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="maxima")
```

output
```
f*x/b + 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)
*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) - 1/6*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a
/b)^(2/3)) + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/
(a^2*b^2*(a/b)^(2/3)) + 1/10*(5*(b*c - a*d)*x^3 - 2*a*c)/(a^2*x^5)
```

3.243.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx = \frac{fx}{b} - \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b} + \frac{5bcx^3-5adx^3-2ac}{10a^2x^5}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a),x, algorithm="giac")
```

output
```
f*x/b - 1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/6*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2
)^(2/3)*a^2) - 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a^3*b) + 1/10*(5*b*c*x^3 - 5*a*d*x^3 - 2*a*c)/(a^2*x
^5)
```

3.243.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.92

$$
\begin{aligned}
\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)}\,dx &= \frac{f\,x}{b}-\frac{\frac{b\,c}{5\,a}+\frac{b\,x^3\,(a\,d-b\,c)}{2\,a^2}}{b\,x^5}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{8/3}\,b^{4/3}} \\
&\quad+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{8/3}\,b^{4/3}} \\
&\quad-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{8/3}\,b^{4/3}}
\end{aligned}
$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)),x)
```

output
```
(f*x)/b - ((b*c)/(5*a) + (b*x^3*(a*d - b*c))/(2*a^2))/(b*x^5) + (log(b^(1/
3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(8/3)*b^(4/3)) +
 (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(
b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(8/3)*b^(4/3)) - (log(3^(1/2)*a^(
1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a
*b^2*d + a^2*b*e))/(3*a^(8/3)*b^(4/3))
```

3.244 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)} dx$

3.244.1 Optimal result 1839
3.244.2 Mathematica [A] (verified) 1840
3.244.3 Rubi [A] (verified) 1840
3.244.4 Maple [A] (verified) 1842
3.244.5 Fricas [A] (verification not implemented) 1843
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3.244.7 Maxima [A] (verification not implemented) 1844
3.244.8 Giac [A] (verification not implemented) 1845
3.244.9 Mupad [B] (verification not implemented) 1846

3.244.1 Optimal result

Integrand size = 30, antiderivative size = 242

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)} dx = -\frac{c}{7ax^7}+\frac{bc-ad}{4a^2x^4}-\frac{b^2c-abd+a^2e}{a^3x} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}b^{2/3}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}b^{2/3}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}b^{2/3}}$$

output
```
-1/7*c/a/x^7+1/4*(-a*d+b*c)/a^2/x^4+(-a^2*e+a*b*d-b^2*c)/a^3/x+1/3*(-a^3*f
+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(2/3)-1/6*(-a^3*f
+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)
/b^(2/3)+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(2/3)*3^(1/2)
```

3.244.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$$

$$= \frac{-\frac{12a^{7/3}c}{x^7}+\frac{21a^{4/3}(bc-ad)}{x^4}-\frac{84\sqrt[3]{a}(b^2c-abd+a^2e)}{x}+\frac{28\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{28(b^3c-ab^2d+a^2be-a^3f)}{b^{2/3}}}{84a^{10/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

output
```
((-12*a^(7/3)*c)/x^7 + (21*a^(4/3)*(b*c - a*d))/x^4 - (84*a^(1/3)*(b^2*c -
 a*b*d + a^2*e))/x + (28*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(b^3*c - a*b^2*d + a
^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*(-(b^3*c) + a*b^2*
d - a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/
3))/(84*a^(10/3))
```

3.244.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{ad-bc}{a^2x^5}+\frac{a^2e-abd+b^2c}{a^3x^2}+\frac{x(a^3f-a^2be+ab^2d-b^3c)}{a^3(a+bx^3)}+\frac{c}{ax^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc-ad}{4a^2x^4}-\frac{a^2e-abd+b^2c}{a^3x}+\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{10/3}b^{2/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{10/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{10/3}b^{2/3}}-\frac{c}{7ax^7}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x]
```

output
```
-1/7*c/(a*x^7) + (b*c - a*d)/(4*a^2*x^4) - (b^2*c - a*b*d + a^2*e)/(a^3*x)
 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(Sqrt[3]*a^(10/3)*b^(2/3)) + ((b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)*b^(2/3)) - ((b^3*c - a*b^2*
d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^
(10/3)*b^(2/3))
```

3.244.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

3.244.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.70

method	result
default	$-\frac{c}{7a\,x^{7}}-\frac{ad-bc}{4a^{2}x^{4}}-\frac{a^{2}e-abd+b^{2}c}{a^{3}x}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\left(f\,a^{3}-a^{2}be+\right.}{a^{3}}$
risch	$\frac{-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{a^{3}}-\frac{(ad-bc)x^{3}}{4a^{2}}-\frac{c}{7a}}{x^{7}}+\left(\_R=\mathrm{RootOf}\left(a^{10}b^{2}\_Z^{3}+a^{9}f^{3}-3a^{8}be\,f^{2}+3a^{7}b^{2}d\,f^{2}+3a^{7}b^{2}e^{2}f-3a^{6}b^{3}c\,f^{2}-6a^{6}b^{3}def-a^{6}b^{3}e^{3}+\right.\right.$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/7*c/a/x^7-1/4*(a*d-b*c)/a^2/x^4-(a^2*e-a*b*d+b^2*c)/a^3/x+(-1/3/b/(a/b)
^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a^3*
(a^3*f-a^2*b*e+a*b^2*d-b^3*c)
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.52

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)}\,dx$$

$$= \left[-\frac{42\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^7\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-}{bx^3+a}\right.}{}\right.$$

$$-\frac{84\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^7\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right)+14(b^3c-a}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/84*(42*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^7*sqrt(
(-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(
2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/
(b*x^3 + a)) + 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*l
og(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b*x - (-a*b^2)^(1/3)) + 84*(a*b^4*
c - a^2*b^3*d + a^3*b^2*e)*x^6 + 12*a^3*b^2*c - 21*(a^2*b^3*c - a^3*b^2*d)
*x^3)/(a^4*b^2*x^7), -1/84*(84*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e
- a^4*b*f)*x^7*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^
(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*(-a*b^2)^(2/3)*x^7*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2
8*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a*b^2)^(2/3)*x^7*log(b*x - (-a*b^2
)^(1/3)) + 84*(a*b^4*c - a^2*b^3*d + a^3*b^2*e)*x^6 + 12*a^3*b^2*c - 21*(a
^2*b^3*c - a^3*b^2*d)*x^3)/(a^4*b^2*x^7)]
```

3.244.6 Sympy [A] (verification not implemented)

Time = 43.35 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.79

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$$
$$=\text{RootSum}\left(27t^3a^{10}b^2+a^9f^3-3a^8bef^2+3a^7b^2df^2+3a^7b^2e^2f-3a^6b^3cf^2-6a^6b^3def-a^6b^3e^3+6a^5b^4ce\right.$$
$$+\frac{-4a^2c+x^6(-28a^2e+28abd-28b^2c)+x^3(-7a^2d+7abc)}{28a^3x^7}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**10*b**2 + a**9*f**3 - 3*a**8*b*e*f**2 + 3*a**7*b**2*d*
f**2 + 3*a**7*b**2*e**2*f - 3*a**6*b**3*c*f**2 - 6*a**6*b**3*d*e*f - a**6*
b**3*e**3 + 6*a**5*b**4*c*e*f + 3*a**5*b**4*d**2*f + 3*a**5*b**4*d*e**2 -
6*a**4*b**5*c*d*f - 3*a**4*b**5*c*e**2 - 3*a**4*b**5*d**2*e + 3*a**3*b**6*
c**2*f + 6*a**3*b**6*c*d*e + a**3*b**6*d**3 - 3*a**2*b**7*c**2*e - 3*a**2*
b**7*c*d**2 + 3*a*b**8*c**2*d - b**9*c**3, Lambda(_t, _t*log(9*_t**2*a**7*
b/(a**6*f**2 - 2*a**5*b*e*f + 2*a**4*b**2*d*f + a**4*b**2*e**2 - 2*a**3*b*
*3*c*f - 2*a**3*b**3*d*e + 2*a**2*b**4*c*e + a**2*b**4*d**2 - 2*a*b**5*c*d
 + b**6*c**2) + x))) + (-4*a**2*c + x**6*(-28*a**2*e + 28*a*b*d - 28*b**2*
c) + x**3*(-7*a**2*d + 7*a*b*c))/(28*a**3*x**7)
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx = -\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{28(b^2c-abd+a^2e)x^6-7(abc-a^2d)x^3+4a^2c}{28a^3x^7}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x -
 (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(1/3)) - 1/6*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(1/3))
+ 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^3*b*(a/b
)^(1/3)) - 1/28*(28*(b^2*c - a*b*d + a^2*e)*x^6 - 7*(a*b*c - a^2*d)*x^3 +
4*a^2*c)/(a^3*x^7)
```

3.244.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$$
$$= -\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}a^3}$$
$$+ \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}a^3}$$
$$+ \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^4}$$
$$- \frac{28b^2cx^6-28abdx^6+28a^2ex^6-7abcx^3+7a^2dx^3+4a^2c}{28a^3x^7}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^3) + 1/6*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)
*a^3) + 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*d*(-a/b)^(1/3) + a^2*b*e*(-a/b)^(1
/3) - a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - 1/
28*(28*b^2*c*x^6 - 28*a*b*d*x^6 + 28*a^2*e*x^6 - 7*a*b*c*x^3 + 7*a^2*d*x^3
 + 4*a^2*c)/(a^3*x^7)
```

3.244.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)}\,dx$$
$$= \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{10/3}\,b^{2/3}} - \frac{\frac{c}{7\,a}+\frac{x^3\,(a\,d-b\,c)}{4\,a^2}+\frac{x^6\,(e\,a^2-d\,a\,b+c\,b^2)}{a^3}}{x^7}$$
$$- \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{10/3}\,b^{2/3}}$$
$$+ \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{10/3}\,b^{2/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)),x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(10/3)
*b^(2/3)) - (c/(7*a) + (x^3*(a*d - b*c))/(4*a^2) + (x^6*(b^2*c + a^2*e - a
*b*d))/a^3)/x^7 - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/
2)*1i)/2 + 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(10/3)*b^(2/3))
+ (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*
(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(10/3)*b^(2/3))
```

3.245 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$

3.245.1 Optimal result . . . 1847
3.245.2 Mathematica [A] (verified) . . . 1848
3.245.3 Rubi [A] (verified) . . . 1848
3.245.4 Maple [A] (verified) . . . 1850
3.245.5 Fricas [A] (verification not implemented) . . . 1851
3.245.6 Sympy [F(-1)] . . . 1852
3.245.7 Maxima [A] (verification not implemented) . . . 1852
3.245.8 Giac [A] (verification not implemented) . . . 1853
3.245.9 Mupad [B] (verification not implemented) . . . 1853

3.245.1 Optimal result

Integrand size = 30, antiderivative size = 244

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx = -\frac{c}{8ax^8}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{2a^3x^2} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{11/3}\sqrt[3]{b}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{11/3}\sqrt[3]{b}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{11/3}\sqrt[3]{b}}$$

output
```
-1/8*c/a/x^8+1/5*(-a*d+b*c)/a^2/x^5+1/2*(-a^2*e+a*b*d-b^2*c)/a^3/x^2-1/3*(
-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(1/3)+1/6*(
-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^
(11/3)/b^(1/3)+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(1/3)*3^(1/2)
```

3.245.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$$
$$= \frac{-\frac{15a^{8/3}c}{x^8}+\frac{24a^{5/3}(bc-ad)}{x^5}-\frac{60a^{2/3}(b^2c-abd+a^2e)}{x^2}+\frac{40\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{40(-b^3c+ab^2d-a^2be+a^3f}{\sqrt[3]{b}}}{120a^{11/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

output
```
((-15*a^(8/3)*c)/x^8 + (24*a^(5/3)*(b*c - a*d))/x^5 - (60*a^(2/3)*(b^2*c -
 a*b*d + a^2*e))/x^2 + (40*Sqrt[3]*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Arc
Tan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-(b^3*c) + a*b^2*
d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (20*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(
1/3))/(120*a^(11/3))
```

3.245.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$$
$$\Big\downarrow \text{ 2373}$$
$$\int \left(\frac{ad-bc}{a^2x^6}+\frac{a^2e-abd+b^2c}{a^3x^3}+\frac{a^3f-a^2be+ab^2d-b^3c}{a^3(a+bx^3)}+\frac{c}{ax^9}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{2a^3x^2}+\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{11/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{11/3}\sqrt[3]{b}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{11/3}\sqrt[3]{b}}-\frac{c}{8ax^8}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x]
```

output
```
-1/8*c/(a*x^8) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(2*a^3*
x^2) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)
/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(11/3)*b^(1/3)) - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(11/3)*b^(1/3)) + ((b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(11/3)*b^(1/3))
```

3.245.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.245.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.70

method	result
default	$-\frac{c}{8a\,x^{8}}-\frac{ad-bc}{5a^{2}x^{5}}-\frac{a^{2}e-abd+b^{2}c}{2a^{3}x^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(f\,a^{3}-a^{2}be+a\right.}{a^{3}}$
risch	$\frac{-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{2a^{3}}-\frac{(ad-bc)x^{3}}{5a^{2}}-\frac{c}{8a}}{x^{8}}+\frac{\left(\_R=\text{RootOf}\left(a^{11}b\_Z^{3}-a^{9}f^{3}+3a^{8}be\,f^{2}-3a^{7}b^{2}d\,f^{2}-3a^{7}b^{2}e^{2}f+3a^{6}b^{3}c\,f^{2}+6a^{6}b^{3}def+a^{6}b^{3}e^{3}-\right.\right.}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8*c/a/x^8-1/5*(a*d-b*c)/a^2/x^5-1/2*(a^2*e-a*b*d+b^2*c)/a^3/x^2+(1/3/b/
(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
/a^3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.44

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$$

$$= \left[-\frac{60\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^8\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3-3(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{}}{bx^3+a}\right.}{} \right.$$

$$-\frac{120\sqrt{\frac{1}{3}}(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^8\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-20(b^3c-a}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="fricas")
```

output
```
[-1/120*(60*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8*sqrt
(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3
)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/
(b*x^3 + a)) - 20*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*lo
g(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x + (a^2*b)^(2/3)) + 60*(a^2*b^3*c
 - a^3*b^2*d + a^4*b*e)*x^6 + 15*a^4*b*c - 24*(a^3*b^2*c - a^4*b*d)*x^3)/(
a^5*b*x^8), -1/120*(120*sqrt(1/3)*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b
*f)*x^8*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b
)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*(b^3*c - a*b^2*d + a^2*b*e - a^
3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) +
40*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(a^2*b)^(2/3)*x^8*log(a*b*x + (a^2*
b)^(2/3)) + 60*(a^2*b^3*c - a^3*b^2*d + a^4*b*e)*x^6 + 15*a^4*b*c - 24*(a^
3*b^2*c - a^4*b*d)*x^3)/(a^5*b*x^8)]
```

3.245.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx=\text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a),x)
```

output
```
Timed out
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx=-\frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{20\,(b^2c-abd+a^2e)x^6-8\,(abc-a^2d)x^3+5\,a^2c}{40\,a^3x^8}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x -
 (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/6*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3))
- 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^3*b*(a/b
)^(2/3)) - 1/40*(20*(b^2*c - a*b*d + a^2*e)*x^6 - 8*(a*b*c - a^2*d)*x^3 +
5*a^2*c)/(a^3*x^8)
```

3.245.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.20

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^4}$$
$$-\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}\,b^3c-(-ab^2)^{\frac{1}{3}}\,ab^2d+(-ab^2)^{\frac{1}{3}}\,a^2be-(-ab^2)^{\frac{1}{3}}\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^4b}$$
$$-\frac{\left((-ab^2)^{\frac{1}{3}}\,b^3c-(-ab^2)^{\frac{1}{3}}\,ab^2d+(-ab^2)^{\frac{1}{3}}\,a^2be-(-ab^2)^{\frac{1}{3}}\,a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^4b}$$
$$-\frac{20\,b^2cx^6-20\,abdx^6+20\,a^2ex^6-8\,abcx^3+8\,a^2dx^3+5\,a^2c}{40\,a^3x^8}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1
/3)))/a^4 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d + (
-a*b^2)^(1/3)*a^2*b*e - (-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-
a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(
1/3)*a*b^2*d + (-a*b^2)^(1/3)*a^2*b*e - (-a*b^2)^(1/3)*a^3*f)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/40*(20*b^2*c*x^6 - 20*a*b*d*x^6 +
 20*a^2*e*x^6 - 8*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c)/(a^3*x^8)
```

3.245.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$$
$$=-\frac{\frac{c}{8\,a}+\frac{x^3\,(a\,d-b\,c)}{5\,a^2}+\frac{x^6\,(e\,a^2-d\,a\,b+c\,b^2)}{2\,a^3}}{x^8}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{11/3}\,b^{1/3}}$$
$$-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{11/3}\,b^{1/3}}$$
$$+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{11/3}\,b^{1/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)),x)
```

output
```
(log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(11/3)*b^(1/3)) - (log(b^(1/3)*x +
 a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(11/3)*b^(1/3)) - (log
(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c
 - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(11/3)*b^(1/3)) - (c/(8*a) + (x^3*(a*d
 - b*c))/(5*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(2*a^3))/x^8
```

3.245. $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)}\,dx$

3.246 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$

3.246.1 Optimal result . . . 1855
3.246.2 Mathematica [A] (verified) . . . 1856
3.246.3 Rubi [A] (verified) . . . 1856
3.246.4 Maple [A] (verified) . . . 1858
3.246.5 Fricas [A] (verification not implemented) . . . 1858
3.246.6 Sympy [F(-1)] . . . 1859
3.246.7 Maxima [A] (verification not implemented) . . . 1859
3.246.8 Giac [A] (verification not implemented) . . . 1860
3.246.9 Mupad [B] (verification not implemented) . . . 1861

3.246.1 Optimal result

Integrand size = 30, antiderivative size = 277

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx = -\frac{c}{10ax^{10}}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{4a^3x^4}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}-\frac{\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{13/3}}-\frac{\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{13/3}}+\frac{\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{13/3}}$$

output
```
-1/10*c/a/x^10+1/7*(-a*d+b*c)/a^2/x^7+1/4*(-a^2*e+a*b*d-b^2*c)/a^3/x^4+(-a
^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x-1/3*b^(1/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)+1/6*b^(1/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(13/3)-1/3*b^(1/3)*(-a^3*f+
a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(13/3)*3^(1/2)
```

3.246.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)}\,dx$$

$$= \frac{-\frac{42a^{10/3}c}{x^{10}}+\frac{60a^{7/3}(bc-ad)}{x^7}-\frac{105a^{4/3}(b^2c-abd+a^2e)}{x^4}+\frac{420\sqrt[3]{a}(b^3c-ab^2d+a^2be-a^3f)}{x}-140\sqrt{3}\sqrt[3]{b}(b^3c-ab^2d+a^2be-}{}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```

output
```
((-42*a^(10/3)*c)/x^10 + (60*a^(7/3)*(b*c - a*d))/x^7 - (105*a^(4/3)*(b^2*
c - a*b*d + a^2*e))/x^4 + (420*a^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/x - 140*Sqrt[3]*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*
e + a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(420*a^(13/3))
```

3.246.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)}\,dx$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{ad-bc}{a^2x^8}+\frac{a^2e-abd+b^2c}{a^3x^5}-\frac{bx\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^4\left(a+bx^3\right)}+\frac{a^3f-a^2be+ab^2d-b^3c}{a^4x^2}+\frac{c}{ax^{11}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{4a^3x^4}-\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{13/3}}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{13/3}}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{13/3}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{10ax^{10}}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x]
```

output
```
-1/10*c/(a*x^10) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(4*a^
3*x^4) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (b^(1/3)*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(13/3)) - (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a
^(1/3) + b^(1/3)*x])/(3*a^(13/3)) + (b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(13/3))
```

3.246.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.246.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.74

method	result
default	$-\frac{c}{10a\,x^{10}}-\frac{ad-bc}{7a^2x^7}-\frac{a^2e-abd+b^2c}{4a^3x^4}-\frac{f\,a^3-a^2be+a\,b^2d-b^3c}{a^4x}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}}{}\right.}{3b\left(\frac{a}{b}\right.}\right.}{a^4}$
risch	$\frac{-\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^9}{a^4}-\frac{\left(a^2e-abd+b^2c\right)x^6}{4a^3}-\frac{(ad-bc)x^3}{7a^2}-\frac{c}{10a}}{x^{10}}+\left(\sum_{\_R=\mathrm{RootOf}\left(a^{13}\_Z^3-a^9b\,f^3+3a^8b^2e\,f^2-3a^7b^3d\,f^2-3a^7b^3e^2f+\right.}\right.$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/10*c/a/x^10-1/7*(a*d-b*c)/a^2/x^7-1/4*(a^2*e-a*b*d+b^2*c)/a^3/x^4-(a^3*
f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b
/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4*
b
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)}\,dx$$
$$=\frac{140\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)x^{10}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+70\,(b^3c-ab^2d+a^2be-a^3f)x^{10}}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/420*(140*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^10*(b/a)^(1/3)*ar
ctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^10*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3))
 - 140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^10*(b/a)^(1/3)*log(b*x + a*(b
/a)^(2/3)) + 420*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 105*(a*b^2*c -
a^2*b*d + a^3*e)*x^6 - 42*a^3*c + 60*(a^2*b*c - a^3*d)*x^3)/(a^4*x^10)
```

3.246.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a),x)
```

output
```
Timed out
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx = \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{140\,(b^3c-ab^2d+a^2be-a^3f)x^9-35\,(ab^2c-a^2bd+a^3e)x^6-14\,a^3c+20\,(a^2bc-a^3d)x^3}{140\,a^4x^{10}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^4*(a/b)^(1/3)) + 1/6*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(1/3)) - 1/3
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(1/3)
) + 1/140*(140*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 35*(a*b^2*c - a^2
*b*d + a^3*e)*x^6 - 14*a^3*c + 20*(a^2*b*c - a^3*d)*x^3)/(a^4*x^10)
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.34

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$$
$$= -\frac{\left(b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^5}$$
$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d + \left(-ab^2\right)^{\frac{2}{3}}a^2be - \left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^5b}$$
$$+\frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c - \left(-ab^2\right)^{\frac{2}{3}}ab^2d + \left(-ab^2\right)^{\frac{2}{3}}a^2be - \left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^5b}$$
$$+\frac{140\,b^3cx^9 - 140\,ab^2dx^9 + 140\,a^2bex^9 - 140\,a^3fx^9 - 35\,ab^2cx^6 + 35\,a^2bdx^6 - 35\,a^3ex^6 + 20\,a^2bcx^3 - 20}{140\,a^4x^{10}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*(b^4*c*(-a/b)^(1/3) - a*b^3*d*(-a/b)^(1/3) + a^2*b^2*e*(-a/b)^(1/3) -
 a^3*b*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 1/3*s
qrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + (-a*b^2)^(2/3)*a^2
*b*e - (-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b
)^(1/3))/(a^5*b) + 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + (-
a*b^2)^(2/3)*a^2*b*e - (-a*b^2)^(2/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(a^5*b) + 1/140*(140*b^3*c*x^9 - 140*a*b^2*d*x^9 + 140*a^2*b*e
*x^9 - 140*a^3*f*x^9 - 35*a*b^2*c*x^6 + 35*a^2*b*d*x^6 - 35*a^3*e*x^6 + 20
*a^2*b*c*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(a^4*x^10)
```

3.246.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)}\,dx$$
$$= -\frac{\frac{c}{10\,a}-\frac{x^9\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{a^4}+\frac{x^3\,(a\,d-b\,c)}{7\,a^2}+\frac{x^6\,(e\,a^2-d\,a\,b+c\,b^2)}{4\,a^3}}{x^{10}}$$
$$-\frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{3\,a^{13/3}}$$
$$+\frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{3\,a^{13/3}}$$
$$-\frac{b^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{3\,a^{13/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)),x)
```

output
```
(b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(13/3)) - (b^(1/3)*log(b^(
1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(13/3)) - (c/(
10*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4 + (x^3*(a*d - b*c))/
(7*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(4*a^3))/x^10 - (b^(1/3)*log(3^(1/
2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3
*f - a*b^2*d + a^2*b*e))/(3*a^(13/3))
```

3.247 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$

3.247.1 Optimal result . . . 1862
3.247.2 Mathematica [A] (verified) . . . 1863
3.247.3 Rubi [A] (verified) . . . 1863
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3.247.8 Giac [A] (verification not implemented) . . . 1867
3.247.9 Mupad [B] (verification not implemented) . . . 1868

3.247.1 Optimal result

Integrand size = 30, antiderivative size = 280

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx = -\frac{c}{11ax^{11}}+\frac{bc-ad}{8a^2x^8}-\frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{2a^4x^2} - \frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{14/3}} + \frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{14/3}} - \frac{b^{2/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{14/3}}$$

output
```
-1/11*c/a/x^11+1/8*(-a*d+b*c)/a^2/x^8+1/5*(-a^2*e+a*b*d-b^2*c)/a^3/x^5+1/2
*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^2+1/3*b^(2/3)*(-a^3*f+a^2*b*e-a*b^2*
d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)-1/6*b^(2/3)*(-a^3*f+a^2*b*e-a*b^2*
d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)-1/3*b^(2/3)*(-
a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1
/2))/a^(14/3)*3^(1/2)
```

3.247.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$$

$$= \frac{-\frac{120a^{11/3}c}{x^{11}}+\frac{165a^{8/3}(bc-ad)}{x^8}-\frac{264a^{5/3}(b^2c-abd+a^2e)}{x^5}+\frac{660a^{2/3}(b^3c-ab^2d+a^2be-a^3f)}{x^2}-440\sqrt{3}b^{2/3}(b^3c-ab^2d+a^2be-}{}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```

output
```
((-120*a^(11/3)*c)/x^11 + (165*a^(8/3)*(b*c - a*d))/x^8 - (264*a^(5/3)*(b^
2*c - a*b*d + a^2*e))/x^5 + (660*a^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f))/x^2 - 440*Sqrt[3]*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 440*b^(2/3)*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*b^(2/3)*(-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1320*a^(1
4/3))
```

3.247.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{ad-bc}{a^2x^9}+\frac{a^2e-abd+b^2c}{a^3x^6}-\frac{b(a^3f-a^2be+ab^2d-b^3c)}{a^4(a+bx^3)}+\frac{a^3f-a^2be+ab^2d-b^3c}{a^4x^3}+\frac{c}{ax^{12}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc-ad}{8a^2x^8}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{3}a^{14/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^{14/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^{14/3}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{2a^4x^2}-\frac{c}{11ax^{11}}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x]
```

output
```
-1/11*c/(a*x^11) + (b*c - a*d)/(8*a^2*x^8) - (b^2*c - a*b*d + a^2*e)/(5*a^
3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^4*x^2) - (b^(2/3)*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(14/3)) + (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*L
og[a^(1/3) + b^(1/3)*x])/(3*a^(14/3)) - (b^(2/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(14/3))
```

3.247.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.247.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.73

method	result
default	$-\frac{c}{11a\,x^{11}}-\frac{ad-bc}{8a^2x^8}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{f\,a^3-a^2be+a\,b^2d-b^3c}{2a^4x^2}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\cdots\right.}{\cdots}\right.}{3b\left(\frac{a}{b}\right)\cdots}\right.}{a^4}$
risch	$\frac{-\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^9}{2a^4}-\frac{\left(a^2e-abd+b^2c\right)x^6}{5a^3}-\frac{(ad-bc)x^3}{8a^2}-\frac{c}{11a}}{x^{11}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{14}\_Z^3+a^9b^2f^3-3a^8b^3e\,f^2+3a^7b^4d\,f^2+3a^7b^4e^2f\cdots\right.}\right.}{\cdots}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/11*c/a/x^11-1/8*(a*d-b*c)/a^2/x^8-1/5*(a^2*e-a*b*d+b^2*c)/a^3/x^5-1/2*(
a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^2-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-
1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)
/a^4*b
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.05

$$\int\frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)}\,dx =$$

$$-\frac{440\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)x^{11}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-220\,(b^3c-ab^2d+a^2be-a^3\cdots}{\cdots}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/1320*(440*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(
1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 220*(b^3
*c - a*b^2*d + a^2*b*e - a^3*f)*x^11*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*
(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 440*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*x^11*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 660*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x^9 + 264*(a*b^2*c - a^2*b*d + a^3*e)*x^6 + 1
20*a^3*c - 165*(a^2*b*c - a^3*d)*x^3)/(a^4*x^11)
```

3.247.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a),x)
```

output
```
Timed out
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx = \frac{\sqrt{3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^3c-ab^2d+a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{220(b^3c-ab^2d+a^2be-a^3f)x^9-88(ab^2c-a^2bd+a^3e)x^6-40a^3c+55(a^2bc-a^3d)x^3}{440a^4x^{11}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^4*(a/b)^(2/3)) - 1/6*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) + 1/3
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(2/3)
) + 1/440*(220*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^9 - 88*(a*b^2*c - a^2
*b*d + a^3*e)*x^6 - 40*a^3*c + 55*(a^2*b*c - a^3*d)*x^3)/(a^4*x^11)
```

3.247.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.19

$$
\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)}\,dx \\
&= \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d+\left(-ab^2\right)^{\frac{1}{3}}a^2be-\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^5} \\
&\quad-\frac{\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^5} \\
&\quad+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c-\left(-ab^2\right)^{\frac{1}{3}}ab^2d+\left(-ab^2\right)^{\frac{1}{3}}a^2be-\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^5} \\
&\quad+\frac{220\,b^3cx^9-220\,ab^2dx^9+220\,a^2bex^9-220\,a^3fx^9-88\,ab^2cx^6+88\,a^2bdx^6-88\,a^3ex^6+55\,a^2bcx^3-55}{440\,a^4x^{11}}
\end{aligned}
$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(1/3)*a*b^2*d + (-a*b^2)^(1/3
)*a^2*b*e - (-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/
(-a/b)^(1/3))/a^5 - 1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*(-a/b)^(1/
3)*log(abs(x - (-a/b)^(1/3)))/a^5 + 1/6*((-a*b^2)^(1/3)*b^3*c - (-a*b^2)^(
1/3)*a*b^2*d + (-a*b^2)^(1/3)*a^2*b*e - (-a*b^2)^(1/3)*a^3*f)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/a^5 + 1/440*(220*b^3*c*x^9 - 220*a*b^2*d*x^9
+ 220*a^2*b*e*x^9 - 220*a^3*f*x^9 - 88*a*b^2*c*x^6 + 88*a^2*b*d*x^6 - 88*a
^3*e*x^6 + 55*a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^4*x^11)
```

3.247.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)}\,dx = \frac{b^{2/3}\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{14/3}} - \frac{\frac{c}{11\,a}-\frac{x^9\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{2\,a^4}+\frac{x^3\,(a\,d-b\,c)}{8\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{5\,a^3}}{x^{11}} + \frac{b^{2/3}\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{14/3}} - \frac{b^{2/3}\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{14/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)),x)
```

output
```
(b^(2/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*
a^(14/3)) - (c/(11*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(2*a^4)
+ (x^3*(a*d - b*c))/(8*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(5*a^3))/x^11
+ (b^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2
 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(14/3)) - (b^(2/3)*log(3
^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c -
 a^3*f - a*b^2*d + a^2*b*e))/(3*a^(14/3))
```

3.248 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx$

3.248.1 Optimal result . . . 1869
3.248.2 Mathematica [A] (verified) . . . 1870
3.248.3 Rubi [A] (verified) . . . 1870
3.248.4 Maple [A] (verified) . . . 1872
3.248.5 Fricas [A] (verification not implemented) . . . 1872
3.248.6 Sympy [F(-1)] . . . 1873
3.248.7 Maxima [A] (verification not implemented) . . . 1873
3.248.8 Giac [A] (verification not implemented) . . . 1874
3.248.9 Mupad [B] (verification not implemented) . . . 1875

3.248.1 Optimal result

Integrand size = 30, antiderivative size = 313

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx = -\frac{c}{13ax^{13}}+\frac{bc-ad}{10a^2x^{10}}-\frac{b^2c-abd+a^2e}{7a^3x^7}+\frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x}+\frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{16/3}}+\frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{16/3}}-\frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{16/3}}$$

output
```
-1/13*c/a/x^13+1/10*(-a*d+b*c)/a^2/x^10+1/7*(-a^2*e+a*b*d-b^2*c)/a^3/x^7+1
/4*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^4-b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
/a^5/x+1/3*b^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^
(16/3)-1/6*b^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(16/3)+1/3*b^(4/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(16/3)*3^(1/2)
```

3.248.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx = -\frac{c}{13ax^{13}}+\frac{bc-ad}{10a^2x^{10}}-\frac{b^2c-abd+a^2e}{7a^3x^7}+\frac{b^3c-ab^2d+a^2be-a^3f}{4a^4x^4}+\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x}+\frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{16/3}}+\frac{b^{4/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{16/3}}+\frac{b^{4/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{16/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

output
```
-1/13*c/(a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2*e)/(7*
a^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) + (b*(-(b^3*c)
+ a*b^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(16/3))
 + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
(3*a^(16/3)) + (b^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

3.248.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx$$

$$\Big\downarrow \; 2373$$

$$\int \left(\frac{ad-bc}{a^2x^{11}}+\frac{a^2e-abd+b^2c}{a^3x^8}+\frac{b^2x(a^3f-a^2be+ab^2d-b^3c)}{a^5\,(a+bx^3)}-\frac{b(a^3f-a^2be+ab^2d-b^3c)}{a^5x^2}+\frac{a^3f-a^2be+a}{a^4x^5}\right.$$

$\Big\downarrow$ 2009

$$\frac{bc-ad}{10a^2x^{10}}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{16/3}}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{16/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{16/3}}-\frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{a^5x}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{4a^4x^4}-\frac{c}{13ax^{13}}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x]
```

output
```
-1/13*c/(a*x^13) + (b*c - a*d)/(10*a^2*x^10) - (b^2*c - a*b*d + a^2*e)/(7*
a^3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(4*a^4*x^4) - (b*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f))/(a^5*x) + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(16/
3)) + (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x
])/(3*a^(16/3)) - (b^(4/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3))
```

3.248.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.248. $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx$

3.248.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.76

method	result
default	$-\frac{c}{13a\,x^{13}}-\frac{ad-bc}{10a^{2}x^{10}}-\frac{a^{2}e-abd+b^{2}c}{7a^{3}x^{7}}-\frac{f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c}{4a^{4}x^{4}}+\frac{b\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)}{a^{5}x}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\right.\right.}{6}\right.}{}$
risch	$\frac{\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)b\,x^{12}}{a^{5}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{9}}{4a^{4}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{6}}{7a^{3}}-\frac{(ad-bc)x^{3}}{10a^{2}}-\frac{c}{13a}}{x^{13}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{16}\_Z^{3}+a^{9}b^{4}f^{3}-3a\right.}\right.}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/13*c/a/x^13-1/10*(a*d-b*c)/a^2/x^10-1/7*(a^2*e-a*b*d+b^2*c)/a^3/x^7-1/4
*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^4+b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5
/x+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b^2
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx =$$
$$-\frac{1820\,\sqrt{3}(b^4c-ab^3d+a^2b^2e-a^3bf)x^{13}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)-910\,(b^4c-ab^3d+a^2b^2}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/5460*(1820*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^
(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*(b^4*c - a*b^
3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3)
- a*(-b/a)^(1/3)) + 1820*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^13*(-b/
a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) + 5460*(b^4*c - a*b^3*d + a^2*b^2*e - a
^3*b*f)*x^12 - 1365*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 780*(a^2
*b^2*c - a^3*b*d + a^4*e)*x^6 + 420*a^4*c - 546*(a^3*b*c - a^4*d)*x^3)/(a^
5*x^13)
```

3.248.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a),x)
```

output
```
Timed out
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)}\,dx = -\frac{\sqrt{3}(b^4c-ab^3d+a^2b^2e-a^3bf)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{1820\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}-455\,(ab^3c-a^2b^2d+a^3be-a^4f)x^9+260\,(a^2b^2c-a^3bd+a^4e)x^6}{1820\,a^5x^{13}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(1/3)) - 1/6*(b^4*c - a*b^3*d +
a^2*b^2*e - a^3*b*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(1/
3)) + 1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(x + (a/b)^(1/3))/(a^
5*(a/b)^(1/3)) - 1/1820*(1820*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12
 - 455*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 260*(a^2*b^2*c - a^3*
b*d + a^4*e)*x^6 + 140*a^4*c - 182*(a^3*b*c - a^4*d)*x^3)/(a^5*x^13)
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.32

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx$$
$$= \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d+\left(-ab^2\right)^{\frac{2}{3}}a^2be-\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^6}$$
$$+\frac{\left(b^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^3e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^2f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^6}$$
$$-\frac{\left(\left(-ab^2\right)^{\frac{2}{3}}b^3c-\left(-ab^2\right)^{\frac{2}{3}}ab^2d+\left(-ab^2\right)^{\frac{2}{3}}a^2be-\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^6}$$
$$-\frac{1820\,b^4cx^{12}-1820\,ab^3dx^{12}+1820\,a^2b^2ex^{12}-1820\,a^3bfx^{12}-455\,ab^3cx^9+455\,a^2b^2dx^9-455\,a^3bex^9+}{1820\,a^5x^{13}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + (-a*b^2)^(2/3
)*a^2*b*e - (-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/
(-a/b)^(1/3))/a^6 + 1/3*(b^5*c*(-a/b)^(1/3) - a*b^4*d*(-a/b)^(1/3) + a^2*b
^3*e*(-a/b)^(1/3) - a^3*b^2*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b
)^(1/3)))/a^6 - 1/6*((-a*b^2)^(2/3)*b^3*c - (-a*b^2)^(2/3)*a*b^2*d + (-a*b
^2)^(2/3)*a^2*b*e - (-a*b^2)^(2/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/a^6 - 1/1820*(1820*b^4*c*x^12 - 1820*a*b^3*d*x^12 + 1820*a^2*b^2*
e*x^12 - 1820*a^3*b*f*x^12 - 455*a*b^3*c*x^9 + 455*a^2*b^2*d*x^9 - 455*a^3
*b*e*x^9 + 455*a^4*f*x^9 + 260*a^2*b^2*c*x^6 - 260*a^3*b*d*x^6 + 260*a^4*e
*x^6 - 182*a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^5*x^13)
```

3.248.9 Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)}\,dx$$
$$= \frac{b^{4/3}\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{16/3}} - \frac{\frac{c}{13\,a}-\frac{x^9\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{4\,a^4}+\frac{x^3\left(a\,d-b\,c\right)}{10\,a^2}+\frac{x^6\left(e\,a^2-d\,a\,b+c\,b^2\right)}{7\,a^3}+\frac{b\,x^{12}\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^5}}{x^{13}}$$
$$- \frac{b^{4/3}\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{16/3}}$$
$$+ \frac{b^{4/3}\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{16/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)),x)
```

output
```
(b^(4/3)*log(b^(1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*
a^(16/3)) - (c/(13*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(4*a^4)
+ (x^3*(a*d - b*c))/(10*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(7*a^3) + (b*
x^12*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^5)/x^13 - (b^(4/3)*log(3^(1/2)
*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - a^3*f
 - a*b^2*d + a^2*b*e))/(3*a^(16/3)) + (b^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*
b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2
*b*e))/(3*a^(16/3))
```

3.249 $\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx$

3.249.1 Optimal result . . . 1876
3.249.2 Mathematica [A] (verified) . . . 1877
3.249.3 Rubi [A] (verified) . . . 1877
3.249.4 Maple [A] (verified) . . . 1879
3.249.5 Fricas [A] (verification not implemented) . . . 1879
3.249.6 Sympy [F(-1)] . . . 1880
3.249.7 Maxima [A] (verification not implemented) . . . 1880
3.249.8 Giac [A] (verification not implemented) . . . 1881
3.249.9 Mupad [B] (verification not implemented) . . . 1882

3.249.1 Optimal result

Integrand size = 30, antiderivative size = 315

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx = -\frac{c}{14ax^{14}}+\frac{bc-ad}{11a^2x^{11}}-\frac{b^2c-abd+a^2e}{8a^3x^8} +\frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{2a^5x^2} +\frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{17/3}} -\frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{17/3}} +\frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{17/3}}$$

output
```
-1/14*c/a/x^14+1/11*(-a*d+b*c)/a^2/x^11+1/8*(-a^2*e+a*b*d-b^2*c)/a^3/x^8+1
/5*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^5-1/2*b*(-a^3*f+a^2*b*e-a*b^2*d+b^
3*c)/a^5/x^2-1/3*b^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)
*x)/a^(17/3)+1/6*b^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/a^(17/3)+1/3*b^(5/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(17/3)*3^(1/2)
```

3.249.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx = -\frac{c}{14ax^{14}}+\frac{bc-ad}{11a^2x^{11}}-\frac{b^2c-abd+a^2e}{8a^3x^8} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5}+\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{2a^5x^2} + \frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{17/3}} + \frac{b^{5/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{17/3}} + \frac{b^{5/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{17/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```

output
```
-1/14*c/(a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2*e)/(8*
a^3*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) + (b*(-(b^3*c)
+ a*b^2*d - a^2*b*e + a^3*f))/(2*a^5*x^2) + (b^(5/3)*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(17
/3)) + (b^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/
3)*x])/(3*a^(17/3)) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(17/3))
```

3.249.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx$$

$$\Big\downarrow \; 2373$$

$$\int \left(\frac{ad-bc}{a^2x^{12}}+\frac{a^2e-abd+b^2c}{a^3x^9}+\frac{b^2(a^3f-a^2be+ab^2d-b^3c)}{a^5(a+bx^3)}-\frac{b(a^3f-a^2be+ab^2d-b^3c)}{a^5x^3}+\frac{a^3f-a^2be+ab^2}{a^4x^6}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bc-ad}{11a^2x^{11}}-\frac{a^2e-abd+b^2c}{8a^3x^8}+\frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{17/3}}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{17/3}}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{17/3}}-\frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{2a^5x^2}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5}-\frac{c}{14ax^{14}}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x]
```

output
```
-1/14*c/(a*x^14) + (b*c - a*d)/(11*a^2*x^11) - (b^2*c - a*b*d + a^2*e)/(8*
a^3*x^8) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f))/(2*a^5*x^2) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(17/3)) - (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/
3)*x]])/(3*a^(17/3)) + (b^(5/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/(6*a^(17/3))
```

3.249.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

3.249. $\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx$

3.249.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.76

method	result
default	$-\frac{c}{14a\,x^{14}}-\frac{ad-bc}{11a^2x^{11}}-\frac{a^2e-abd+b^2c}{8a^3x^8}-\frac{f\,a^3-a^2be+a\,b^2d-b^3c}{5a^4x^5}+\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b}{2a^5x^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right.\right.}{6b(}\right.}{}$
risch	$\frac{\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)b\,x^{12}}{2a^5}-\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^9}{5a^4}-\frac{\left(a^2e-abd+b^2c\right)x^6}{8a^3}-\frac{(ad-bc)x^3}{11a^2}-\frac{c}{14a}}{x^{14}}+\frac{\left(\_R=\mathrm{RootOf}\left(a^{17}\_Z^3-a^9b^5f^3+3a\right.\right.}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/14*c/a/x^14-1/11*(a*d-b*c)/a^2/x^11-1/8*(a^2*e-a*b*d+b^2*c)/a^3/x^8-1/5
*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^5+1/2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a
^5*b/x^2+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1)))*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b^2
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}\left(a+bx^3\right)}\,dx =$$

$$-\frac{3080\sqrt{3}(b^4c-ab^3d+a^2b^2e-a^3bf)x^{14}\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-1540\,(b^4c-ab^3d+a^2b^2e-}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/9240*(3080*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^
2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 1540*
(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b^2*x^2 -
 a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 3080*(b^4*c - a*b^3*d + a^
2*b^2*e - a^3*b*f)*x^14*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 462
0*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12 - 1848*(a*b^3*c - a^2*b^2*d
 + a^3*b*e - a^4*f)*x^9 + 1155*(a^2*b^2*c - a^3*b*d + a^4*e)*x^6 + 660*a^4
*c - 840*(a^3*b*c - a^4*d)*x^3)/(a^5*x^14)
```

3.249.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}\left(a+bx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**15/(b*x**3+a),x)
```

output
```
Timed out
```

3.249.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}\left(a+bx^3\right)}\,dx = -\frac{\sqrt{3}(b^4c-ab^3d+a^2b^2e-a^3bf)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{1540\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^{12}-616\,(ab^3c-a^2b^2d+a^3be-a^4f)x^9+385\,(a^2b^2c-a^3bd+a^4e)x^6}{3080\,a^5x^{14}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(2/3)) + 1/6*(b^4*c - a*b^3*d +
a^2*b^2*e - a^3*b*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(2/
3)) - 1/3*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*log(x + (a/b)^(1/3))/(a^
5*(a/b)^(2/3)) - 1/3080*(1540*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^12
 - 616*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^9 + 385*(a^2*b^2*c - a^3*
b*d + a^4*e)*x^6 + 220*a^4*c - 280*(a^3*b*c - a^4*d)*x^3)/(a^5*x^14)
```

3.249.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.23

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx =$$
$$-\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}b^4c-(-ab^2)^{\frac{1}{3}}ab^3d+(-ab^2)^{\frac{1}{3}}a^2b^2e-(-ab^2)^{\frac{1}{3}}a^3bf\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^6}$$
$$+\frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^6}$$
$$-\frac{\left((-ab^2)^{\frac{1}{3}}b^4c-(-ab^2)^{\frac{1}{3}}ab^3d+(-ab^2)^{\frac{1}{3}}a^2b^2e-(-ab^2)^{\frac{1}{3}}a^3bf\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^6}$$
$$-\frac{1540\,b^4cx^{12}-1540\,ab^3dx^{12}+1540\,a^2b^2ex^{12}-1540\,a^3bfx^{12}-616\,ab^3cx^9+616\,a^2b^2dx^9-616\,a^3bex^9+}{3080\,a^5x^{14}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^15/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*((-a*b^2)^(1/3)*b^4*c - (-a*b^2)^(1/3)*a*b^3*d + (-a*b^2)^(1/
3)*a^2*b^2*e - (-a*b^2)^(1/3)*a^3*b*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/a^6 + 1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 - 1/6*((-a*b^2)^(1/3)*b^4*c - (-a
*b^2)^(1/3)*a*b^3*d + (-a*b^2)^(1/3)*a^2*b^2*e - (-a*b^2)^(1/3)*a^3*b*f)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 - 1/3080*(1540*b^4*c*x^12 - 15
40*a*b^3*d*x^12 + 1540*a^2*b^2*e*x^12 - 1540*a^3*b*f*x^12 - 616*a*b^3*c*x^
9 + 616*a^2*b^2*d*x^9 - 616*a^3*b*e*x^9 + 616*a^4*f*x^9 + 385*a^2*b^2*c*x^
6 - 385*a^3*b*d*x^6 + 385*a^4*e*x^6 - 280*a^3*b*c*x^3 + 280*a^4*d*x^3 + 22
0*a^4*c)/(a^5*x^14)
```

3.249.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{15}(a+bx^3)}\,dx$$
$$= -\frac{\frac{c}{14a}-\frac{x^9(-fa^3+ea^2b-dab^2+cb^3)}{5a^4}+\frac{x^3(ad-bc)}{11a^2}+\frac{x^6(ea^2-dab+cb^2)}{8a^3}+\frac{bx^{12}(-fa^3+ea^2b-dab^2+cb^3)}{2a^5}}{x^{14}}$$
$$-\frac{b^{5/3}\ln\left(b^{1/3}x+a^{1/3}\right)(-fa^3+ea^2b-dab^2+cb^3)}{3a^{17/3}}$$
$$-\frac{b^{5/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)(-fa^3+ea^2b-dab^2+cb^3)}{3a^{17/3}}$$
$$+\frac{b^{5/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)(-fa^3+ea^2b-dab^2+cb^3)}{3a^{17/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^15*(a + b*x^3)),x)
```

output
```
(b^(5/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 +
 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(17/3)) - (b^(5/3)*log(b^(
1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(17/3)) - (b^(
5/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(17/3)) - (c/(14*a) - (x^9*(b^
3*c - a^3*f - a*b^2*d + a^2*b*e))/(5*a^4) + (x^3*(a*d - b*c))/(11*a^2) + (
x^6*(b^2*c + a^2*e - a*b*d))/(8*a^3) + (b*x^12*(b^3*c - a^3*f - a*b^2*d +
a^2*b*e))/(2*a^5))/x^14
```

3.250 $\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx$

3.250.1 Optimal result 1883
3.250.2 Mathematica [A] (verified) 1884
3.250.3 Rubi [A] (verified) 1884
3.250.4 Maple [A] (verified) 1886
3.250.5 Fricas [A] (verification not implemented) 1886
3.250.6 Sympy [F(-1)] 1887
3.250.7 Maxima [A] (verification not implemented) 1887
3.250.8 Giac [A] (verification not implemented) 1888
3.250.9 Mupad [B] (verification not implemented) 1889

3.250.1 Optimal result

Integrand size = 30, antiderivative size = 351

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx =& -\frac{c}{16ax^{16}}+\frac{bc-ad}{13a^2x^{13}}-\frac{b^2c-abd+a^2e}{10a^3x^{10}} \\ &+\frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{4a^5x^4} \\ &+\frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x} \\ &-\frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{19/3}} \\ &-\frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{19/3}} \\ &+\frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{19/3}}\end{aligned}$$

output
```
-1/16*c/a/x^16+1/13*(-a*d+b*c)/a^2/x^13+1/10*(-a^2*e+a*b*d-b^2*c)/a^3/x^10
+1/7*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^7-1/4*b*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)/a^5/x^4+b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^6/x-1/3*b^(7/3)*(-a^3*
f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(19/3)+1/6*b^(7/3)*(-a^3*
f+a^2*b*e-a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(19/3
)-1/3*b^(7/3)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)
*x)/a^(1/3)*3^(1/2))/a^(19/3)*3^(1/2)
```

3.250.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx = -\frac{c}{16ax^{16}}+\frac{bc-ad}{13a^2x^{13}}-\frac{b^2c-abd+a^2e}{10a^3x^{10}} + \frac{b^3c-ab^2d+a^2be-a^3f}{7a^4x^7}+\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{4a^5x^4} + \frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x} + \frac{b^{7/3}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{19/3}} + \frac{b^{7/3}(-b^3c+ab^2d-a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{19/3}} + \frac{b^{7/3}(b^3c-ab^2d+a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{19/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```

output
```
-1/16*c/(a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2*e)/(10
*a^3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) + (b*(-(b^3*c
) + a*b^2*d - a^2*b*e + a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*
b*e - a^3*f))/(a^6*x) + (b^(7/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(19/3)) + (b^(7/3)*(
-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3
)) + (b^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(6*a^(19/3))
```

3.250.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{ad-bc}{a^2x^{14}}+\frac{a^2e-abd+b^2c}{a^3x^{11}}-\frac{b^3x(a^3f-a^2be+ab^2d-b^3c)}{a^6(a+bx^3)}+\frac{b^2(a^3f-a^2be+ab^2d-b^3c)}{a^6x^2}-\frac{b(a^3f-a^2be\cdots}{a^5\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc-ad}{13a^2x^{13}}-\frac{a^2e-abd+b^2c}{10a^3x^{10}}-\frac{b^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{3}a^{19/3}}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^{19/3}}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^{19/3}}+\frac{b^2(a^3(-f)+a^2be-ab^2d+b^3c)}{a^6x}-\frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^5x^4}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{7a^4x^7}-\frac{c}{16ax^{16}}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x]
```

output
```
-1/16*c/(a*x^16) + (b*c - a*d)/(13*a^2*x^13) - (b^2*c - a*b*d + a^2*e)/(10
*a^3*x^10) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(7*a^4*x^7) - (b*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f))/(4*a^5*x^4) + (b^2*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f))/(a^6*x) - (b^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(
a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(19/3)) - (b^(7/3)*(
b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(19/3))
+ (b^(7/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(6*a^(19/3))
```

3.250.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.250.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.79

method	result
default	$-\frac{c}{16a\,x^{16}}-\frac{ad-bc}{13a^2x^{13}}-\frac{a^2e-abd+b^2c}{10a^3x^{10}}-\frac{f\,a^3-a^2be+a\,b^2d-b^3c}{7a^4x^7}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b^2}{a^6x}+\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b}{4a^5x^4}$
risch	$\frac{-\frac{c}{16a}-\frac{(ad-bc)x^3}{13a^2}-\frac{(a^2e-abd+b^2c)x^6}{10a^3}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)x^9}{7a^4}+\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b\,x^{12}}{4a^5}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)b^2x^{15}}{a^6}}{x^{16}}+$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/16*c/a/x^16-1/13*(a*d-b*c)/a^2/x^13-1/10*(a^2*e-a*b*d+b^2*c)/a^3/x^10-1
/7*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^7-(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6
*b^2/x+1/4*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5*b/x^4-(-1/3/b/(a/b)^(1/3)*ln(
x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(
1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^3*(a^3*f-a^2
*b*e+a*b^2*d-b^3*c)/a^6
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}\,(a+bx^3)}\,dx$$
$$= \frac{7280\,\sqrt{3}(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{16}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+3640\,(b^5c-ab^4d+a^2b^3e-}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/21840*(7280*sqrt(3)*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)
^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 3640*(b^5*c - a*b
^4*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3)
 + a*(b/a)^(1/3)) - 7280*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^16*(b
/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) + 21840*(b^5*c - a*b^4*d + a^2*b^3*e -
a^3*b^2*f)*x^15 - 5460*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 +
3120*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^9 - 2184*(a^3*b^2*c - a^4
*b*d + a^5*e)*x^6 - 1365*a^5*c + 1680*(a^4*b*c - a^5*d)*x^3)/(a^6*x^16)
```

3.250.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**17/(b*x**3+a),x)
```

output
```
Timed out
```

3.250.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx = \frac{\sqrt{3}(b^5c-ab^4d+a^2b^3e-a^3b^2f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{7280\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{15}-1820\,(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^{12}+1040\,(a^2b^3c-a^3b^2d+}{7280\,a^6x^{16}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^6*(a/b)^(1/3)) + 1/6*(b^5*c - a*b^4*d +
 a^2*b^3*e - a^3*b^2*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^
(1/3)) - 1/3*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*log(x + (a/b)^(1/3)
)/(a^6*(a/b)^(1/3)) + 1/7280*(7280*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*
f)*x^15 - 1820*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^12 + 1040*(a^
2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^9 - 728*(a^3*b^2*c - a^4*b*d + a^
5*e)*x^6 - 455*a^5*c + 560*(a^4*b*c - a^5*d)*x^3)/(a^6*x^16)
```

3.250.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.33

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}(a+bx^3)}\,dx =$$

$$-\frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}b^4c-(-ab^2)^{\frac{2}{3}}ab^3d+(-ab^2)^{\frac{2}{3}}a^2b^2e-(-ab^2)^{\frac{2}{3}}a^3bf\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^7}$$
$$-\frac{\left(b^6c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^5d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^4e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^7}$$
$$+\frac{\left((-ab^2)^{\frac{2}{3}}b^4c-(-ab^2)^{\frac{2}{3}}ab^3d+(-ab^2)^{\frac{2}{3}}a^2b^2e-(-ab^2)^{\frac{2}{3}}a^3bf\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^7}$$
$$+\frac{7280\,b^5cx^{15}-7280\,ab^4dx^{15}+7280\,a^2b^3ex^{15}-7280\,a^3b^2fx^{15}-1820\,ab^4cx^{12}+1820\,a^2b^3dx^{12}-1820\,a^3b}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^17/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*((-a*b^2)^(2/3)*b^4*c - (-a*b^2)^(2/3)*a*b^3*d + (-a*b^2)^(2/
3)*a^2*b^2*e - (-a*b^2)^(2/3)*a^3*b*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/a^7 - 1/3*(b^6*c*(-a/b)^(1/3) - a*b^5*d*(-a/b)^(1/3) +
a^2*b^4*e*(-a/b)^(1/3) - a^3*b^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/a^7 + 1/6*((-a*b^2)^(2/3)*b^4*c - (-a*b^2)^(2/3)*a*b^3*d +
(-a*b^2)^(2/3)*a^2*b^2*e - (-a*b^2)^(2/3)*a^3*b*f)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/a^7 + 1/7280*(7280*b^5*c*x^15 - 7280*a*b^4*d*x^15 + 7280
*a^2*b^3*e*x^15 - 7280*a^3*b^2*f*x^15 - 1820*a*b^4*c*x^12 + 1820*a^2*b^3*d
*x^12 - 1820*a^3*b^2*e*x^12 + 1820*a^4*b*f*x^12 + 1040*a^2*b^3*c*x^9 - 104
0*a^3*b^2*d*x^9 + 1040*a^4*b*e*x^9 - 1040*a^5*f*x^9 - 728*a^3*b^2*c*x^6 +
728*a^4*b*d*x^6 - 728*a^5*e*x^6 + 560*a^4*b*c*x^3 - 560*a^5*d*x^3 - 455*a^
5*c)/(a^6*x^16)
```

3.250.9 Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{17}\,(a+bx^3)}\,dx =$$

$$-\frac{\frac{c}{16\,a}-\frac{x^9\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{7\,a^4}+\frac{x^3\,\left(a\,d-b\,c\right)}{13\,a^2}+\frac{x^6\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{10\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{4\,a^5}-\frac{b^2\,x^{15}\,\left(-f\,a^3+e\,\ldots\right)}{a\ldots}}{x^{16}}$$

$$-\frac{b^{7/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{19/3}}$$

$$+\frac{b^{7/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{19/3}}$$

$$-\frac{b^{7/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^{19/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^17*(a + b*x^3)),x)
```

output
```
(b^(7/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(19/3)) - (b^(7/3)*log(b^(
1/3)*x + a^(1/3))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(19/3)) - (c/(
16*a) - (x^9*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(7*a^4) + (x^3*(a*d - b*
c))/(13*a^2) + (x^6*(b^2*c + a^2*e - a*b*d))/(10*a^3) + (b*x^12*(b^3*c - a
^3*f - a*b^2*d + a^2*b*e))/(4*a^5) - (b^2*x^15*(b^3*c - a^3*f - a*b^2*d +
a^2*b*e))/a^6)/x^16 - (b^(7/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^(19/
3))
```

3.251 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.251.1 Optimal result 1890
3.251.2 Mathematica [A] (verified) 1891
3.251.3 Rubi [A] (verified) 1891
3.251.4 Maple [A] (verified) 1893
3.251.5 Fricas [A] (verification not implemented) 1893
3.251.6 Sympy [A] (verification not implemented) 1894
3.251.7 Maxima [A] (verification not implemented) 1894
3.251.8 Giac [A] (verification not implemented) 1895
3.251.9 Mupad [B] (verification not implemented) 1896

3.251.1 Optimal result

Integrand size = 30, antiderivative size = 220

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x^3}{3b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^6}{6b^5} + \frac{(b^2d-2abe+3a^2f)\,x^9}{9b^4} + \frac{(be-2af)x^{12}}{12b^3} + \frac{fx^{15}}{15b^2} + \frac{a^3(b^3c-ab^2d+a^2be-a^3f)}{3b^7\,(a+bx^3)} + \frac{a^2(3b^3c-4ab^2d+5a^2be-6a^3f)\log(a+bx^3)}{3b^7}$$

output
```
-1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*x^3/b^6+1/6*(-4*a^3*f+3*a^2*
b*e-2*a*b^2*d+b^3*c)*x^6/b^5+1/9*(3*a^2*f-2*a*b*e+b^2*d)*x^9/b^4+1/12*(-2*
a*f+b*e)*x^12/b^3+1/15*f*x^15/b^2+1/3*a^3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b
^7/(b*x^3+a)+1/3*a^2*(-6*a^3*f+5*a^2*b*e-4*a*b^2*d+3*b^3*c)*ln(b*x^3+a)/b^
7
```

3.251.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{60ab(-2b^3c+3ab^2d-4a^2be+5a^3f)\,x^3+30b^2(b^3c-2ab^2d+3a^2be-4a^3f)\,x^6+20b^3(b^2d-2abe+3a^2f}{}$$

input
```
Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(60*a*b*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x^3 + 30*b^2*(b^3*c -
 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6 + 20*b^3*(b^2*d - 2*a*b*e + 3*a^2*f)
*x^9 + 15*b^4*(b*e - 2*a*f)*x^12 + 12*b^5*f*x^15 - (60*a^3*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 60*a^2*(3*b^3*c - 4*a*b^2*d + 5*a^2
*b*e - 6*a^3*f)*Log[a + b*x^3])/(180*b^7)
```

3.251.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{2361}$$

$$\frac{1}{3}\int \frac{x^9\left(fx^9+ex^6+dx^3+c\right)}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{3}\int \left(\frac{fx^{12}}{b^2}+\frac{(be-2af)x^9}{b^3}+\frac{\left(3fa^2-2bea+b^2d\right)x^6}{b^4}+\frac{\left(-4fa^3+3bea^2-2b^2da+b^3c\right)x^3}{b^5}+\frac{a\left(5fa^3-4bea^2+\right.}{b^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{x^9(3a^2f-2abe+b^2d)}{3b^4}+\frac{a^3(a^3(-f)+a^2be-ab^2d+b^3c)}{b^7(a+bx^3)}+\frac{a^2\log(a+bx^3)(-6a^3f+5a^2be-4ab^2d+3b^3c}{b^7}\right.$$

input
```
Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(-((a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x^3)/b^6) + ((b^3*c - 2*
a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^6)/(2*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)
*x^9)/(3*b^4) + ((b*e - 2*a*f)*x^12)/(4*b^3) + (f*x^15)/(5*b^2) + (a^3*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f))/(b^7*(a + b*x^3)) + (a^2*(3*b^3*c - 4*a*
b^2*d + 5*a^2*b*e - 6*a^3*f)*Log[a + b*x^3])/b^7)/3
```

3.251.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.251. $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.251.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.99

method	result
default	$\frac{\frac{f\,x^{15}b^4}{15}+\frac{\left(-2a\,b^3f+b^4e\right)x^{12}}{12}+\frac{\left(3a^2b^2f-2a\,b^3e+b^4d\right)x^9}{9}+\frac{\left(-4a^3bf+3a^2e\,b^2-2a\,b^3d+b^4c\right)x^6}{6}+\frac{\left(5a^4f-4a^3be+3a^2b^2d-2a\,b^3c\right)x^3}{3}}{b^6}-$
norman	$\frac{-\frac{a\left(6f\,a^5-5a^4eb+4a^3d\,b^2-3a^2c\,b^3\right)}{3b^7}+\frac{f\,x^{18}}{15b}-\frac{(6af-5be)x^{15}}{60b^2}+\frac{\left(6a^2f-5aeb+4b^2d\right)x^{12}}{36b^3}-\frac{\left(6f\,a^3-5a^2be+4a\,b^2d-3b^3c\right)x^9}{18b^4}+\frac{a\left(6f\,a^3-5a\right.}{}}{b\,x^3+a}$
risch	$\frac{f\,x^{15}}{15b^2}-\frac{af\,x^{12}}{6b^3}+\frac{e\,x^{12}}{12b^2}+\frac{a^2f\,x^9}{3b^4}-\frac{2ae\,x^9}{9b^3}+\frac{d\,x^9}{9b^2}-\frac{2f\,x^6a^3}{3b^5}+\frac{a^2e\,x^6}{2b^4}-\frac{ad\,x^6}{3b^3}+\frac{c\,x^6}{6b^2}+\frac{5a^4f\,x^3}{3b^6}-\frac{4a^3e\,x^3}{3b^5}$
parallelrisch	$-\frac{-12f\,x^{18}b^6+360a^6f-15x^{15}b^6e-20x^{12}b^6d-30x^9b^6c+360\ln\left(b\,x^3+a\right)a^6f-180a^3b^3c-50x^9a^2b^4e+40x^9a\,b^5d-180x^6a^4b^2f+}{}$

input
```
int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^6*(1/15*f*x^15*b^4+1/12*(-2*a*b^3*f+b^4*e)*x^12+1/9*(3*a^2*b^2*f-2*a*b
^3*e+b^4*d)*x^9+1/6*(-4*a^3*b*f+3*a^2*b^2*e-2*a*b^3*d+b^4*c)*x^6+1/3*(5*a^
4*f-4*a^3*b*e+3*a^2*b^2*d-2*a*b^3*c)*x^3)-1/3*a^2/b^6*((6*a^3*f-5*a^2*b*e+
4*a*b^2*d-3*b^3*c)/b*ln(b*x^3+a)+a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+
a))
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.38

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$=\frac{12\,b^6fx^{18}+3\,(5\,b^6e-6\,ab^5f)x^{15}+5\,(4\,b^6d-5\,ab^5e+6\,a^2b^4f)x^{12}+10\,(3\,b^6c-4\,ab^5d+5\,a^2b^4e-6\,a^3b^3f}{}$$

input
```
integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/180*(12*b^6*f*x^18 + 3*(5*b^6*e - 6*a*b^5*f)*x^15 + 5*(4*b^6*d - 5*a*b^5
*e + 6*a^2*b^4*f)*x^12 + 10*(3*b^6*c - 4*a*b^5*d + 5*a^2*b^4*e - 6*a^3*b^3
*f)*x^9 + 60*a^3*b^3*c - 60*a^4*b^2*d + 60*a^5*b*e - 60*a^6*f - 30*(3*a*b^
5*c - 4*a^2*b^4*d + 5*a^3*b^3*e - 6*a^4*b^2*f)*x^6 - 60*(2*a^2*b^4*c - 3*a
^3*b^3*d + 4*a^4*b^2*e - 5*a^5*b*f)*x^3 + 60*(3*a^3*b^3*c - 4*a^4*b^2*d +
5*a^5*b*e - 6*a^6*f + (3*a^2*b^4*c - 4*a^3*b^3*d + 5*a^4*b^2*e - 6*a^5*b*f
)*x^3)*log(b*x^3 + a))/(b^8*x^3 + a*b^7)
```

3.251.6 Sympy [A] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.07

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{a^2\cdot(6a^3f-5a^2be+4ab^2d-3b^3c)\log(a+bx^3)}{3b^7} + x^{12}\left(-\frac{af}{6b^3}+\frac{e}{12b^2}\right) + x^9\left(\frac{a^2f}{3b^4}-\frac{2ae}{9b^3}+\frac{d}{9b^2}\right) + x^6\left(-\frac{2a^3f}{3b^5}+\frac{a^2e}{2b^4}-\frac{ad}{3b^3}+\frac{c}{6b^2}\right) + x^3\cdot\left(\frac{5a^4f}{3b^6}-\frac{4a^3e}{3b^5}+\frac{a^2d}{b^4}-\frac{2ac}{3b^3}\right) + \frac{-a^6f+a^5be-a^4b^2d+a^3b^3c}{3ab^7+3b^8x^3} + \frac{fx^{15}}{15b^2}$$

input
```
integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
-a**2*(6*a**3*f - 5*a**2*b*e + 4*a*b**2*d - 3*b**3*c)*log(a + b*x**3)/(3*b
**7) + x**12*(-a*f/(6*b**3) + e/(12*b**2)) + x**9*(a**2*f/(3*b**4) - 2*a*e
/(9*b**3) + d/(9*b**2)) + x**6*(-2*a**3*f/(3*b**5) + a**2*e/(2*b**4) - a*d
/(3*b**3) + c/(6*b**2)) + x**3*(5*a**4*f/(3*b**6) - 4*a**3*e/(3*b**5) + a*
*2*d/b**4 - 2*a*c/(3*b**3)) + (-a**6*f + a**5*b*e - a**4*b**2*d + a**3*b**
3*c)/(3*a*b**7 + 3*b**8*x**3) + f*x**15/(15*b**2)
```

3.251.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.01

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{a^3b^3c-a^4b^2d+a^5be-a^6f}{3\,(b^8x^3+ab^7)} + \frac{12\,b^4fx^{15}+15\,(b^4e-2\,ab^3f)x^{12}+20\,(b^4d-2\,ab^3e+3\,a^2b^2f)x^9+30\,(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3bf)x}{180\,b^6} + \frac{(3\,a^2b^3c-4\,a^3b^2d+5\,a^4be-6\,a^5f)\log(bx^3+a)}{3\,b^7}$$

input
```
integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)/(b^8*x^3 + a*b^7) + 1/180*(1
2*b^4*f*x^15 + 15*(b^4*e - 2*a*b^3*f)*x^12 + 20*(b^4*d - 2*a*b^3*e + 3*a^2
*b^2*f)*x^9 + 30*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^6 - 60*(2
*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*x^3)/b^6 + 1/3*(3*a^2*b^3*c
- 4*a^3*b^2*d + 5*a^4*b*e - 6*a^5*f)*log(b*x^3 + a)/b^7
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.33

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{(3\,a^2b^3c-4\,a^3b^2d+5\,a^4be-6\,a^5f)\log\left(|bx^3+a|\right)}{3\,b^7} - \frac{3\,a^2b^4cx^3-4\,a^3b^3dx^3+5\,a^4b^2ex^3-6\,a^5bfx^3+2\,a^3b^3c-3\,a^4b^2d+4\,a^5be-5\,a^6f}{3\,(bx^3+a)b^7} + \frac{12\,b^8fx^{15}+15\,b^8ex^{12}-30\,ab^7fx^{12}+20\,b^8dx^9-40\,ab^7ex^9+60\,a^2b^6fx^9+30\,b^8cx^6-60\,ab^7dx^6+90\,a^2}{180\,b^{10}}$$

input
```
integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*(3*a^2*b^3*c - 4*a^3*b^2*d + 5*a^4*b*e - 6*a^5*f)*log(abs(b*x^3 + a))/
b^7 - 1/3*(3*a^2*b^4*c*x^3 - 4*a^3*b^3*d*x^3 + 5*a^4*b^2*e*x^3 - 6*a^5*b*f
*x^3 + 2*a^3*b^3*c - 3*a^4*b^2*d + 4*a^5*b*e - 5*a^6*f)/((b*x^3 + a)*b^7)
+ 1/180*(12*b^8*f*x^15 + 15*b^8*e*x^12 - 30*a*b^7*f*x^12 + 20*b^8*d*x^9 -
40*a*b^7*e*x^9 + 60*a^2*b^6*f*x^9 + 30*b^8*c*x^6 - 60*a*b^7*d*x^6 + 90*a^2
*b^6*e*x^6 - 120*a^3*b^5*f*x^6 - 120*a*b^7*c*x^3 + 180*a^2*b^6*d*x^3 - 240
*a^3*b^5*e*x^3 + 300*a^4*b^4*f*x^3)/b^10
```

3.251.9 Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.62

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^{12}\left(\frac{e}{12\,b^2}-\frac{a\,f}{6\,b^3}\right) - x^3\left(\frac{2\,a\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)}{3\,b} - \frac{a^2\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b^2}\right) - x^9\left(\frac{a^2\,f}{9\,b^4}-\frac{d}{9\,b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{9\,b}\right) + x^6\left(\frac{c}{6\,b^2} - \frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{6\,b^2} + \frac{a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b}\right) - \frac{\ln\left(b\,x^3+a\right)\,\left(6\,f\,a^5-5\,e\,a^4\,b+4\,d\,a^3\,b^2-3\,c\,a^2\,b^3\right)}{3\,b^7} + \frac{f\,x^{15}}{15\,b^2} - \frac{f\,a^6-e\,a^5\,b+d\,a^4\,b^2-c\,a^3\,b^3}{3\,b\,\left(b^7\,x^3+a\,b^6\right)}$$

input
```
int((x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

output
```
x^12*(e/(12*b^2) - (a*f)/(6*b^3)) - x^3*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*
f)/b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))
/b))/(3*b) - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(
3*b^2)) - x^9*((a^2*f)/(9*b^4) - d/(9*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(
9*b)) + x^6*(c/(6*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(6*b^2) + (a*((a^2*f)
/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b)) - (log(a + b*x^3)*(6
*a^5*f - 3*a^2*b^3*c + 4*a^3*b^2*d - 5*a^4*b*e))/(3*b^7) + (f*x^15)/(15*b^
2) - (a^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e)/(3*b*(a*b^6 + b^7*x^3))
```

3.251. $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.252 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.252.1 Optimal result 1898
3.252.2 Mathematica [A] (verified) 1898
3.252.3 Rubi [A] (verified) 1899
3.252.4 Maple [A] (verified) 1900
3.252.5 Fricas [A] (verification not implemented) 1901
3.252.6 Sympy [A] (verification not implemented) 1901
3.252.7 Maxima [A] (verification not implemented) 1902
3.252.8 Giac [A] (verification not implemented) 1902
3.252.9 Mupad [B] (verification not implemented) 1903

3.252.1 Optimal result

Integrand size = 30, antiderivative size = 180

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^3}{3b^5}+\frac{(b^2d-2abe+3a^2f)\,x^6}{6b^4} + \frac{(be-2af)x^9}{9b^3}+\frac{fx^{12}}{12b^2}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{3b^6\,(a+bx^3)} - \frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\log(a+bx^3)}{3b^6}$$

output
```
1/3*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*x^3/b^5+1/6*(3*a^2*f-2*a*b*e+b^2*
d)*x^6/b^4+1/9*(-2*a*f+b*e)*x^9/b^3+1/12*f*x^12/b^2-1/3*a^2*(-a^3*f+a^2*b*
e-a*b^2*d+b^3*c)/b^6/(b*x^3+a)-1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c
)*ln(b*x^3+a)/b^6
```

3.252.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{12b(b^3c-2ab^2d+3a^2be-4a^3f)\,x^3+6b^2(b^2d-2abe+3a^2f)\,x^6+4b^3(be-2af)x^9+3b^4fx^{12}+\frac{12a^2(-b^3c\cdots}{\cdots}}{36b^6}$$

input
```
Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(12*b*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3 + 6*b^2*(b^2*d - 2*a*b
*e + 3*a^2*f)*x^6 + 4*b^3*(b*e - 2*a*f)*x^9 + 3*b^4*f*x^12 + (12*a^2*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 12*a*(-2*b^3*c + 3*a*b^2*
d - 4*a^2*b*e + 5*a^3*f)*Log[a + b*x^3])/(36*b^6)
```

3.252.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{2361}$$

$$\frac{1}{3}\int \frac{x^6 (fx^9+ex^6+dx^3+c)}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{3}\int \left(\frac{fx^9}{b^2}+\frac{(be-2af)x^6}{b^3}+\frac{(3fa^2-2bea+b^2d)\,x^3}{b^4}+\frac{-4fa^3+3bea^2-2b^2da+b^3c}{b^5}+\frac{a(5fa^3-4bea^2+3b^2da}{b^5\,(bx^3+a)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{x^6(3a^2f-2abe+b^2d)}{2b^4}-\frac{a^2(a^3(-f)+a^2be-ab^2d+b^3c)}{b^6\,(a+bx^3)}-\frac{a\log(a+bx^3)\,(-5a^3f+4a^2be-3ab^2d+2b^3c)}{b^6}\right.$$

input
```
Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/b^5 + ((b^2*d - 2*a*b*e +
 3*a^2*f)*x^6)/(2*b^4) + ((b*e - 2*a*f)*x^9)/(3*b^3) + (f*x^12)/(4*b^2) -
(a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(b^6*(a + b*x^3)) - (a*(2*b^3*c
- 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*Log[a + b*x^3])/b^6)/3
```

3.252.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.252.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.99

method	result
default	$-\frac{-\frac{b^3 f x^{12}}{12}+\frac{\left(2 f a b^2-b^3 e\right)x^9}{9}+\frac{\left(-3 f a^2 b+2 a b^2 e-b^3 d\right)x^6}{6}+\frac{x^3\left(4 f a^3-3 a^2 b e+2 a b^2 d-b^3 c\right)}{3}}{b^5}+\frac{a\left(\frac{\left(5 f a^3-4 a^2 b e+3 a b^2 d-2 b^3 c\right)\ln\left(b\right)}{b}\right)}{3b}$
norman	$\frac{-\frac{\left(5 f a^3-4 a^2 b e+3 a b^2 d-2 b^3 c\right)x^6}{6 b^4}+\frac{f x^{15}}{12 b}-\frac{(5 a f-4 b e)x^{12}}{36 b^2}+\frac{\left(5 a^2 f-4 a e b+3 b^2 d\right)x^9}{18 b^3}-\frac{\left(5 f a^5-4 a^4 e b+3 a^3 d b^2-2 a^2 c b^3\right)x^3}{3 a b^5}}{b x^3+a}+\frac{a(5 f a^3-}{}$
risch	$\frac{f x^{12}}{12 b^2}-\frac{2 a f x^9}{9 b^3}+\frac{e x^9}{9 b^2}+\frac{x^6 f a^2}{2 b^4}-\frac{a e x^6}{3 b^3}+\frac{d x^6}{6 b^2}-\frac{4 f a^3 x^3}{3 b^5}+\frac{a^2 e x^3}{b^4}-\frac{2 a d x^3}{3 b^3}+\frac{c x^3}{3 b^2}+\frac{a^5 f}{3 b^6(b x^3+a)}-\frac{a}{3 b^5(b}$
parallelrisch	$\frac{3 f x^{15} b^5-5 x^{12} a b^4 f+4 x^{12} b^5 e+10 x^9 a^2 b^3 f-8 x^9 a b^4 e+6 x^9 b^5 d-30 x^6 a^3 b^2 f+24 x^6 a^2 b^3 e-18 x^6 a b^4 d+12 x^6 b^5 c+60\ln\left(b x^3+a\right)x}{}$

input
```
int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^5*(-1/12*b^3*f*x^12+1/9*(2*a*b^2*f-b^3*e)*x^9+1/6*(-3*a^2*b*f+2*a*b^2
*e-b^3*d)*x^6+1/3*x^3*(4*a^3*f-3*a^2*b*e+2*a*b^2*d-b^3*c))+1/3*a/b^5*((5*a
^3*f-4*a^2*b*e+3*a*b^2*d-2*b^3*c)/b*ln(b*x^3+a)+a*(a^3*f-a^2*b*e+a*b^2*d-b
^3*c)/b/(b*x^3+a))
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.43

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{3\,b^5fx^{15}+(4\,b^5e-5\,ab^4f)x^{12}+2\,(3\,b^5d-4\,ab^4e+5\,a^2b^3f)x^9+6\,(2\,b^5c-3\,ab^4d+4\,a^2b^3e-5\,a^3b^2f)x^6}{}$$

input
```
integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(3*b^5*f*x^15 + (4*b^5*e - 5*a*b^4*f)*x^12 + 2*(3*b^5*d - 4*a*b^4*e +
 5*a^2*b^3*f)*x^9 + 6*(2*b^5*c - 3*a*b^4*d + 4*a^2*b^3*e - 5*a^3*b^2*f)*x^
6 - 12*a^2*b^3*c + 12*a^3*b^2*d - 12*a^4*b*e + 12*a^5*f + 12*(a*b^4*c - 2*
a^2*b^3*d + 3*a^3*b^2*e - 4*a^4*b*f)*x^3 - 12*(2*a^2*b^3*c - 3*a^3*b^2*d +
 4*a^4*b*e - 5*a^5*f + (2*a*b^4*c - 3*a^2*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)
*x^3)*log(b*x^3 + a))/(b^7*x^3 + a*b^6)
```

3.252.6 Sympy [A] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx &= \frac{a(5a^3f-4a^2be+3ab^2d-2b^3c)\log(a+bx^3)}{3b^6} \\ &+ x^9\left(-\frac{2af}{9b^3}+\frac{e}{9b^2}\right)+x^6\left(\frac{a^2f}{2b^4}-\frac{ae}{3b^3}+\frac{d}{6b^2}\right) \\ &+ x^3\left(-\frac{4a^3f}{3b^5}+\frac{a^2e}{b^4}-\frac{2ad}{3b^3}+\frac{c}{3b^2}\right) \\ &+ \frac{a^5f-a^4be+a^3b^2d-a^2b^3c}{3ab^6+3b^7x^3}+\frac{fx^{12}}{12b^2}\end{aligned}$$

input
```
integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
a*(5*a**3*f - 4*a**2*b*e + 3*a*b**2*d - 2*b**3*c)*log(a + b*x**3)/(3*b**6)
 + x**9*(-2*a*f/(9*b**3) + e/(9*b**2)) + x**6*(a**2*f/(2*b**4) - a*e/(3*b*
*3) + d/(6*b**2)) + x**3*(-4*a**3*f/(3*b**5) + a**2*e/b**4 - 2*a*d/(3*b**3
) + c/(3*b**2)) + (a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**
6 + 3*b**7*x**3) + f*x**12/(12*b**2)
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{a^2b^3c-a^3b^2d+a^4be-a^5f}{3\,(b^7x^3+ab^6)} + \frac{3\,b^3fx^{12}+4\,(b^3e-2\,ab^2f)x^9+6\,(b^3d-2\,ab^2e+3\,a^2bf)x^6+12\,(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f)x^3}{36\,b^5} - \frac{(2\,ab^3c-3\,a^2b^2d+4\,a^3be-5\,a^4f)\log\left(bx^3+a\right)}{3\,b^6}$$

input
```
integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)/(b^7*x^3 + a*b^6) + 1/36*(3
*b^3*f*x^12 + 4*(b^3*e - 2*a*b^2*f)*x^9 + 6*(b^3*d - 2*a*b^2*e + 3*a^2*b*f
)*x^6 + 12*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^3)/b^5 - 1/3*(2*a*b
^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*log(b*x^3 + a)/b^6
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.34

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{(2\,ab^3c-3\,a^2b^2d+4\,a^3be-5\,a^4f)\log\left(|bx^3+a|\right)}{3\,b^6} + \frac{2\,ab^4cx^3-3\,a^2b^3dx^3+4\,a^3b^2ex^3-5\,a^4bfx^3+a^2b^3c-2\,a^3b^2d+3\,a^4be-4\,a^5f}{3\,(bx^3+a)b^6} + \frac{3\,b^6fx^{12}+4\,b^6ex^9-8\,ab^5fx^9+6\,b^6dx^6-12\,ab^5ex^6+18\,a^2b^4fx^6+12\,b^6cx^3-24\,ab^5dx^3+36\,a^2b^4ex^3}{36\,b^8}$$

input
```
integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*(2*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*log(abs(b*x^3 + a))/b
^6 + 1/3*(2*a*b^4*c*x^3 - 3*a^2*b^3*d*x^3 + 4*a^3*b^2*e*x^3 - 5*a^4*b*f*x^
3 + a^2*b^3*c - 2*a^3*b^2*d + 3*a^4*b*e - 4*a^5*f)/((b*x^3 + a)*b^6) + 1/3
6*(3*b^6*f*x^12 + 4*b^6*e*x^9 - 8*a*b^5*f*x^9 + 6*b^6*d*x^6 - 12*a*b^5*e*x
^6 + 18*a^2*b^4*f*x^6 + 12*b^6*c*x^3 - 24*a*b^5*d*x^3 + 36*a^2*b^4*e*x^3 -
 48*a^3*b^3*f*x^3)/b^8
```

3.252.9 Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.29

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^9\left(\frac{e}{9\,b^2}-\frac{2\,a\,f}{9\,b^3}\right)-x^6\left(\frac{a^2\,f}{6\,b^4}-\frac{d}{6\,b^2}+\frac{a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b}\right)+x^3\left(\frac{c}{3\,b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^{12}}{12\,b^2}+\frac{f\,a^5-e\,a^4\,b+d\,a^3\,b^2-c\,a^2\,b^3}{3\,b\,\left(b^6\,x^3+a\,b^5\right)}+\frac{\ln\left(b\,x^3+a\right)\,\left(5\,f\,a^4-4\,e\,a^3\,b+3\,d\,a^2\,b^2-2\,c\,a\,b^3\right)}{3\,b^6}$$

input
```
int((x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

output
```
x^9*(e/(9*b^2) - (2*a*f)/(9*b^3)) - x^6*((a^2*f)/(6*b^4) - d/(6*b^2) + (a*
(e/b^2 - (2*a*f)/b^3))/(3*b)) + x^3*(c/(3*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3
))/(3*b^2) + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(
3*b)) + (f*x^12)/(12*b^2) + (a^5*f - a^2*b^3*c + a^3*b^2*d - a^4*b*e)/(3*b
*(a*b^5 + b^6*x^3)) + (log(a + b*x^3)*(5*a^4*f + 3*a^2*b^2*d - 2*a*b^3*c -
 4*a^3*b*e))/(3*b^6)
```

3.253 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.253.1 Optimal result . . . 1904
3.253.2 Mathematica [A] (verified) . . . 1904
3.253.3 Rubi [A] (verified) . . . 1905
3.253.4 Maple [A] (verified) . . . 1906
3.253.5 Fricas [A] (verification not implemented) . . . 1907
3.253.6 Sympy [A] (verification not implemented) . . . 1907
3.253.7 Maxima [A] (verification not implemented) . . . 1908
3.253.8 Giac [A] (verification not implemented) . . . 1908
3.253.9 Mupad [B] (verification not implemented) . . . 1909

3.253.1 Optimal result

Integrand size = 30, antiderivative size = 140

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{(b^2d-2abe+3a^2f)\,x^3}{3b^4}+\frac{(be-2af)x^6}{6b^3}+\frac{fx^9}{9b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)}{3b^5\,(a+bx^3)}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\log(a+bx^3)}{3b^5}$$

output
```
1/3*(3*a^2*f-2*a*b*e+b^2*d)*x^3/b^4+1/6*(-2*a*f+b*e)*x^6/b^3+1/9*f*x^9/b^2
+1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b^5/(b*x^3+a)+1/3*(-4*a^3*f+3*a^2*b*
e-2*a*b^2*d+b^3*c)*ln(b*x^3+a)/b^5
```

3.253.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{6b(b^2d-2abe+3a^2f)\,x^3+3b^2(be-2af)x^6+2b^3fx^9+\frac{6a(b^3c-ab^2d+a^2be-a^3f)}{a+bx^3}+6(b^3c-2ab^2d+3a^2be-4}{18b^5}$$

input
```
Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(6*b*(b^2*d - 2*a*b*e + 3*a^2*f)*x^3 + 3*b^2*(b*e - 2*a*f)*x^6 + 2*b^3*f*x
^9 + (6*a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3) + 6*(b^3*c - 2*
a*b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/(18*b^5)
```

3.253.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{ 2361}$$

$$\frac{1}{3} \int \frac{x^3 (fx^9+ex^6+dx^3+c)}{(bx^3+a)^2} dx^3$$

$$\Big\downarrow \text{ 2123}$$

$$\frac{1}{3} \int \left(\frac{fx^6}{b^2}+\frac{(be-2af)x^3}{b^3}+\frac{3fa^2-2bea+b^2d}{b^4}+\frac{-4fa^3+3bea^2-2b^2da+b^3c}{b^4 (bx^3+a)}+\frac{a(fa^3-bea^2+b^2da-b^3c)}{b^4 (bx^3+a)^2}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{x^3 (3a^2f-2abe+b^2d)}{b^4}+\frac{a(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5 (a+bx^3)}+\frac{\log (a+bx^3) (-4a^3f+3a^2be-2ab^2d+b^3c)}{b^5}+\right.$$

input
```
Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(((b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/b^4 + ((b*e - 2*a*f)*x^6)/(2*b^3) + (f*
x^9)/(3*b^2) + (a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(b^5*(a + b*x^3)) +
 ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*Log[a + b*x^3])/b^5)/3
```

3.253.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.253.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

method	result
default	$\frac{\frac{b^{2} f \,x^{9}}{9}+\frac{\left(-2 a f b +b^{2} e \right) x^{6}}{6}+\frac{x^{3} \left(3 a^{2} f -2 a e b +b^{2} d \right)}{3}}{b^{4}}-\frac{\frac{\left(4 f \,a^{3}-3 a^{2} b e +2 a \,b^{2} d -b^{3} c \right) \ln \left(b \,x^{3}+a \right)}{b}+\frac{a \left(f \,a^{3}-a^{2} b e +a \,b^{2} d -b^{3} c \right)}{b \left(b \,x^{3}+a \right)}}{3 b^{4}}$
norman	$\frac{\frac{f \,x^{12}}{9 b}-\frac{\left(4 a f -3 b e \right) x^{9}}{18 b^{2}}+\frac{\left(4 a^{2} f -3 a e b +2 b^{2} d \right) x^{6}}{6 b^{3}}+\frac{\left(4 a^{4} f -3 a^{3} b e +2 a^{2} b^{2} d -a \,b^{3} c \right) x^{3}}{3 a \,b^{4}}}{b \,x^{3}+a}-\frac{\left(4 f \,a^{3}-3 a^{2} b e +2 a \,b^{2} d -b^{3} c \right) \ln \left(b \,x^{3}+a \right)}{3 b^{5}}$
risch	$\frac{f \,x^{9}}{9 b^{2}}-\frac{a f \,x^{6}}{3 b^{3}}+\frac{e \,x^{6}}{6 b^{2}}+\frac{a^{2} f \,x^{3}}{b^{4}}-\frac{2 a e \,x^{3}}{3 b^{3}}+\frac{d \,x^{3}}{3 b^{2}}-\frac{a^{4} f}{3 b^{5} \left(b \,x^{3}+a \right)}+\frac{a^{3} e}{3 b^{4} \left(b \,x^{3}+a \right)}-\frac{a^{2} d}{3 b^{3} \left(b \,x^{3}+a \right)}+\frac{a c}{3 b^{2} \left(b \,x^{3}+a \right)}-$
parallelrisch	$-\frac{-2 f \,x^{12} b^{4}+4 x^{9} a \,b^{3} f -3 x^{9} b^{4} e -12 x^{6} a^{2} b^{2} f +9 x^{6} a \,b^{3} e -6 b^{4} d \,x^{6}+24 \ln \left(b \,x^{3}+a \right) x^{3} a^{3} b f -18 \ln \left(b \,x^{3}+a \right) x^{3} a^{2} b^{2} e +12 \ln \left(b \,x^{3}+a \right.}{}$

input
```
int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/9*b^2*f*x^9+1/6*(-2*a*b*f+b^2*e)*x^6+1/3*x^3*(3*a^2*f-2*a*b*e+b^2
*d))-1/3/b^4*((4*a^3*f-3*a^2*b*e+2*a*b^2*d-b^3*c)/b*ln(b*x^3+a)+a*(a^3*f-a
^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a))
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.44

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{2\,b^4fx^{12} + (3\,b^4e - 4\,ab^3f)x^9 + 3\,(2\,b^4d - 3\,ab^3e + 4\,a^2b^2f)x^6 + 6\,ab^3c - 6\,a^2b^2d + 6\,a^3be - 6\,a^4f + 6\,(ab^3}{18\,(}$$

input
```
integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/18*(2*b^4*f*x^12 + (3*b^4*e - 4*a*b^3*f)*x^9 + 3*(2*b^4*d - 3*a*b^3*e +
4*a^2*b^2*f)*x^6 + 6*a*b^3*c - 6*a^2*b^2*d + 6*a^3*b*e - 6*a^4*f + 6*(a*b^
3*d - 2*a^2*b^2*e + 3*a^3*b*f)*x^3 + 6*(a*b^3*c - 2*a^2*b^2*d + 3*a^3*b*e
- 4*a^4*f + (b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^3)*log(b*x^3 +
 a))/(b^6*x^3 + a*b^5)
```

3.253.6 Sympy [A] (verification not implemented)

Time = 11.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^6\left(-\frac{af}{3b^3}+\frac{e}{6b^2}\right)+x^3\left(\frac{a^2f}{b^4}-\frac{2ae}{3b^3}+\frac{d}{3b^2}\right) + \frac{-a^4f+a^3be-a^2b^2d+ab^3c}{3ab^5+3b^6x^3}+\frac{fx^9}{9b^2} - \frac{(4a^3f-3a^2be+2ab^2d-b^3c)\log\left(a+bx^3\right)}{3b^5}$$

input
```
integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
x**6*(-a*f/(3*b**3) + e/(6*b**2)) + x**3*(a**2*f/b**4 - 2*a*e/(3*b**3) + d
/(3*b**2)) + (-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(3*a*b**5 + 3*b
**6*x**3) + f*x**9/(9*b**2) - (4*a**3*f - 3*a**2*b*e + 2*a*b**2*d - b**3*c
)*log(a + b*x**3)/(3*b**5)
```

3.253.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{ab^3c-a^2b^2d+a^3be-a^4f}{3\,(b^6x^3+ab^5)} + \frac{2\,b^2fx^9+3\,(b^2e-2\,abf)x^6+6\,(b^2d-2\,abe+3\,a^2f)x^3}{18\,b^4} + \frac{(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f)\log{(bx^3+a)}}{3\,b^5}$$

input
```
integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)/(b^6*x^3 + a*b^5) + 1/18*(2*b^
2*f*x^9 + 3*(b^2*e - 2*a*b*f)*x^6 + 6*(b^2*d - 2*a*b*e + 3*a^2*f)*x^3)/b^4
 + 1/3*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*log(b*x^3 + a)/b^5
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.51

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{\frac{(bx^3+a)^3\left(2\,f+\frac{3\,(b^2e-4\,abf)}{(bx^3+a)b}+\frac{6\,(b^4d-3\,ab^3e+6\,a^2b^2f)}{(bx^3+a)^2b^2}\right)}{b^4}-\frac{6\,(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f)\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^4}+\frac{6\left(\frac{ab^6c}{bx^3+a}-\frac{a^2b^5d}{bx^3+a}+\frac{a^3b^4e}{bx^3+a}-\right.}{b^7}}{18\,b}$$

input
```
integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/18*((b*x^3 + a)^3*(2*f + 3*(b^2*e - 4*a*b*f)/((b*x^3 + a)*b) + 6*(b^4*d
- 3*a*b^3*e + 6*a^2*b^2*f)/((b*x^3 + a)^2*b^2))/b^4 - 6*(b^3*c - 2*a*b^2*d
 + 3*a^2*b*e - 4*a^3*f)*log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^4 + 6
*(a*b^6*c/(b*x^3 + a) - a^2*b^5*d/(b*x^3 + a) + a^3*b^4*e/(b*x^3 + a) - a^
4*b^3*f/(b*x^3 + a))/b^7)/b
```

3.253.9 Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.11

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^6\left(\frac{e}{6\,b^2}-\frac{a\,f}{3\,b^3}\right)-x^3\left(\frac{a^2\,f}{3\,b^4}-\frac{d}{3\,b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(-4\,f\,a^3+3\,e\,a^2\,b-2\,d\,a\,b^2+c\,b^3\right)}{3\,b^5}-\frac{f\,a^4-e\,a^3\,b+d\,a^2\,b^2-c\,a\,b^3}{3\,b\,\left(b^5\,x^3+a\,b^4\right)}+\frac{f\,x^9}{9\,b^2}$$

input
```
int((x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

output
```
x^6*(e/(6*b^2) - (a*f)/(3*b^3)) - x^3*((a^2*f)/(3*b^4) - d/(3*b^2) + (2*a*
(e/b^2 - (2*a*f)/b^3))/(3*b)) + (log(a + b*x^3)*(b^3*c - 4*a^3*f - 2*a*b^2
*d + 3*a^2*b*e))/(3*b^5) - (a^4*f + a^2*b^2*d - a*b^3*c - a^3*b*e)/(3*b*(a
*b^4 + b^5*x^3)) + (f*x^9)/(9*b^2)
```

3.254 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.254.1 Optimal result . . . 1910
3.254.2 Mathematica [A] (verified) . . . 1910
3.254.3 Rubi [A] (verified) . . . 1911
3.254.4 Maple [A] (verified) . . . 1912
3.254.5 Fricas [A] (verification not implemented) . . . 1912
3.254.6 Sympy [A] (verification not implemented) . . . 1913
3.254.7 Maxima [A] (verification not implemented) . . . 1913
3.254.8 Giac [B] (verification not implemented) . . . 1913
3.254.9 Mupad [B] (verification not implemented) . . . 1914

3.254.1 Optimal result

Integrand size = 30, antiderivative size = 103

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{(be-2af)x^3}{3b^3}+\frac{fx^6}{6b^2}-\frac{b^3c-ab^2d+a^2be-a^3f}{3b^4(a+bx^3)}+\frac{(b^2d-2abe+3a^2f)\log(a+bx^3)}{3b^4}$$

output
```
1/3*(-2*a*f+b*e)*x^3/b^3+1/6*f*x^6/b^2+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b
^4/(b*x^3+a)+1/3*(3*a^2*f-2*a*b*e+b^2*d)*ln(b*x^3+a)/b^4
```

3.254.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{2b(be-2af)x^3+b^2fx^6+\frac{2(-b^3c+ab^2d-a^2be+a^3f)}{a+bx^3}+2(b^2d-2abe+3a^2f)\log(a+bx^3)}{6b^4}$$

input
```
Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(2*b*(b*e - 2*a*f)*x^3 + b^2*f*x^6 + (2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^
3*f))/(a + b*x^3) + 2*(b^2*d - 2*a*b*e + 3*a^2*f)*Log[a + b*x^3])/(6*b^4)
```

3.254.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2359, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx^3+ex^6+fx^9)}{(a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{2359}$$

$$\frac{1}{3} \int \frac{fx^9+ex^6+dx^3+c}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{3} \int \left(\frac{fx^3}{b^2}+\frac{be-2af}{b^3}+\frac{3fa^2-2bea+b^2d}{b^3 (bx^3+a)}+\frac{-fa^3+bea^2-b^2da+b^3c}{b^3 (bx^3+a)^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{\log (a+bx^3) (3a^2f-2abe+b^2d)}{b^4}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{b^4 (a+bx^3)}+\frac{x^3(be-2af)}{b^3}+\frac{fx^6}{2b^2}\right)$$

input
```
Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(((b*e - 2*a*f)*x^3)/b^3 + (f*x^6)/(2*b^2) - (b^3*c - a*b^2*d + a^2*b*e -
a^3*f)/(b^4*(a + b*x^3)) + ((b^2*d - 2*a*b*e + 3*a^2*f)*Log[a + b*x^3])/b^
4)/3
```

3.254.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2359
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/(m
 + 1)   Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^
p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ
[Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.254.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

method	result
norman	$\frac{\frac{3f\,a^3-2a^2be+a\,b^2d-b^3c}{3b^4}+\frac{f\,x^9}{6b}-\frac{(3af-2be)x^6}{6b^2}}{b\,x^3+a}+\frac{(3a^2f-2aeb+b^2d)\ln(b\,x^3+a)}{3b^4}$
default	$\frac{(-f\,x^3b+2af-be)^2}{6b^4f}+\frac{\frac{(3a^2f-2aeb+b^2d)\ln(b\,x^3+a)}{b}-\frac{-f\,a^3+a^2be-a\,b^2d+b^3c}{b(b\,x^3+a)}}{3b^3}$
parallelrisch	$\frac{b^3f\,x^9-3x^6a\,b^2f+2x^6b^3e+6\ln(b\,x^3+a)x^3a^2bf-4\ln(b\,x^3+a)x^3a\,b^2e+2\ln(b\,x^3+a)x^3b^3d+6\ln(b\,x^3+a)a^3f-4\ln(b\,x^3+a)a^2b}{6b^4(b\,x^3+a)}$
risch	$\frac{f\,x^6}{6b^2}-\frac{2fa\,x^3}{3b^3}+\frac{e\,x^3}{3b^2}+\frac{2f\,a^2}{3b^4}-\frac{2ae}{3b^3}+\frac{e^2}{6b^2f}+\frac{f\,a^3}{3b^4(b\,x^3+a)}-\frac{a^2e}{3b^3(b\,x^3+a)}+\frac{ad}{3b^2(b\,x^3+a)}-\frac{c}{3b(b\,x^3+a)}+\frac{\ln(}{}$

input
```
int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3*(3*a^3*f-2*a^2*b*e+a*b^2*d-b^3*c)/b^4+1/6*f*x^9/b-1/6*(3*a*f-2*b*e)/b
^2*x^6)/(b*x^3+a)+1/3*(3*a^2*f-2*a*b*e+b^2*d)*ln(b*x^3+a)/b^4
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.39

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$=\frac{b^3fx^9+(2\,b^3e-3\,ab^2f)x^6-2\,b^3c+2\,ab^2d-2\,a^2be+2\,a^3f+2\,(ab^2e-2\,a^2bf)x^3+2\,(ab^2d-2\,a^2be+3}{6\,(b^5x^3+ab^4)}$$

input
```
integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/6*(b^3*f*x^9 + (2*b^3*e - 3*a*b^2*f)*x^6 - 2*b^3*c + 2*a*b^2*d - 2*a^2*b
*e + 2*a^3*f + 2*(a*b^2*e - 2*a^2*b*f)*x^3 + 2*(a*b^2*d - 2*a^2*b*e + 3*a^
3*f + (b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^3)*log(b*x^3 + a))/(b^5*x^3 + a*b^
4)
```

3.254.6 Sympy [A] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^3\left(-\frac{2af}{3b^3}+\frac{e}{3b^2}\right)+\frac{a^3f-a^2be+ab^2d-b^3c}{3ab^4+3b^5x^3} + \frac{fx^6}{6b^2}+\frac{(3a^2f-2abe+b^2d)\log(a+bx^3)}{3b^4}$$

input
```
integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
x**3*(-2*a*f/(3*b**3) + e/(3*b**2)) + (a**3*f - a**2*b*e + a*b**2*d - b**3
*c)/(3*a*b**4 + 3*b**5*x**3) + f*x**6/(6*b**2) + (3*a**2*f - 2*a*b*e + b**
2*d)*log(a + b*x**3)/(3*b**4)
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{b^3c-ab^2d+a^2be-a^3f}{3(b^5x^3+ab^4)}+\frac{bfx^6+2(be-2af)x^3}{6b^3} + \frac{(b^2d-2abe+3a^2f)\log(bx^3+a)}{3b^4}$$

input
```
integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(b^5*x^3 + a*b^4) + 1/6*(b*f*x^6
+ 2*(b*e - 2*a*f)*x^3)/b^3 + 1/3*(b^2*d - 2*a*b*e + 3*a^2*f)*log(b*x^3 + a
)/b^4
```

3.254.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(95) = 190.

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.99

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= -\frac{1}{6}f\left(\frac{(bx^3+a)^2\left(\frac{6a}{bx^3+a}-1\right)}{b^4}+\frac{6a^2\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^4}-\frac{2a^3}{(bx^3+a)b^4}\right)$$
$$+\frac{1}{3}e\left(\frac{2a\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^3}+\frac{bx^3+a}{b^3}-\frac{a^2}{(bx^3+a)b^3}\right)$$
$$-\frac{d\left(\frac{\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b}-\frac{a}{(bx^3+a)b}\right)}{3b}-\frac{c}{3(bx^3+a)b}$$

input
```
integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/6*f*((b*x^3 + a)^2*(6*a/(b*x^3 + a) - 1)/b^4 + 6*a^2*log(abs(b*x^3 + a)
/((b*x^3 + a)^2*abs(b)))/b^4 - 2*a^3/((b*x^3 + a)*b^4)) + 1/3*e*(2*a*log(a
bs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b^3 + (b*x^3 + a)/b^3 - a^2/((b*x^3
+ a)*b^3)) - 1/3*d*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x
^3 + a)*b))/b - 1/3*c/((b*x^3 + a)*b)
```

3.254.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^3\left(\frac{e}{3b^2}-\frac{2af}{3b^3}\right)+\frac{fx^6}{6b^2}-\frac{-fa^3+ea^2b-dab^2+cb^3}{3b(b^4x^3+ab^3)}$$
$$+\frac{\ln(bx^3+a)(3fa^2-2eab+db^2)}{3b^4}$$

input
```
int((x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

output
```
x^3*(e/(3*b^2) - (2*a*f)/(3*b^3)) + (f*x^6)/(6*b^2) - (b^3*c - a^3*f - a*b
^2*d + a^2*b*e)/(3*b*(a*b^3 + b^4*x^3)) + (log(a + b*x^3)*(b^2*d + 3*a^2*f
 - 2*a*b*e))/(3*b^4)
```

3.255 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2}\,dx$

3.255.1 Optimal result . . . 1915
3.255.2 Mathematica [A] (verified) . . . 1915
3.255.3 Rubi [A] (verified) . . . 1916
3.255.4 Maple [A] (verified) . . . 1917
3.255.5 Fricas [A] (verification not implemented) . . . 1918
3.255.6 Sympy [F(-1)] . . . 1918
3.255.7 Maxima [A] (verification not implemented) . . . 1918
3.255.8 Giac [A] (verification not implemented) . . . 1919
3.255.9 Mupad [B] (verification not implemented) . . . 1919

3.255.1 Optimal result

Integrand size = 30, antiderivative size = 100

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx = \frac{fx^3}{3b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{3ab^3\left(a+bx^3\right)}+\frac{c\log(x)}{a^2} - \frac{\left(b^3c-a^2be+2a^3f\right)\log\left(a+bx^3\right)}{3a^2b^3}$$

output
```
1/3*f*x^3/b^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a/b^3/(b*x^3+a)+c*ln(x)/a
^2-1/3*(2*a^3*f-a^2*b*e+b^3*c)*ln(b*x^3+a)/a^2/b^3
```

3.255.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx = \frac{3c\log(x)+\frac{\frac{a\left(b^3c-a^3f+a^2b\left(e+fx^3\right)+ab^2\left(-d+fx^6\right)\right)}{a+bx^3}+\left(-b^3c+a^2be-2a^3f\right)\log\left(a+bx^3\right)}{b^3}}{3a^2}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
```

output
```
(3*c*Log[x] + ((a*(b^3*c - a^3*f + a^2*b*(e + f*x^3) + a*b^2*(-d + f*x^6))
)/(a + b*x^3) + (-(b^3*c) + a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/b^3)/(3*a^2
)
```

3.255.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^3(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{3}\int \left(\frac{c}{a^2x^3}+\frac{f}{b^2}+\frac{-2fa^3+bea^2-b^3c}{a^2b^2(bx^3+a)}+\frac{fa^3-bea^2+b^2da-b^3c}{ab^2(bx^3+a)^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{c\log(x^3)}{a^2}-\frac{\log(a+bx^3)(2a^3f-a^2be+b^3c)}{a^2b^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{ab^3(a+bx^3)}+\frac{fx^3}{b^2}\right)$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x]
```

output
```
((f*x^3)/b^2 + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a*b^3*(a + b*x^3)) + (
c*Log[x^3])/a^2 - ((b^3*c - a^2*b*e + 2*a^3*f)*Log[a + b*x^3])/(a^2*b^3))/
3
```

3.255.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.255.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

method	result
default	$\frac{f\,x^{3}}{3b^{2}}+\frac{c\ln\left(x\right)}{a^{2}}-\frac{\frac{\left(2f\,a^{3}-a^{2}be+b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{b}+\frac{a\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)}{b\left(b\,x^{3}+a\right)}}{3a^{2}b^{2}}$
norman	$\frac{\frac{f\,x^{6}}{3b}-\frac{2f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c}{3a\,b^{3}}}{b\,x^{3}+a}+\frac{c\ln\left(x\right)}{a^{2}}-\frac{\left(2f\,a^{3}-a^{2}be+b^{3}c\right)\ln\left(b\,x^{3}+a\right)}{3a^{2}b^{3}}$
risch	$\frac{f\,x^{3}}{3b^{2}}-\frac{a^{2}f}{3b^{3}\left(b\,x^{3}+a\right)}+\frac{ae}{3b^{2}\left(b\,x^{3}+a\right)}-\frac{d}{3b\left(b\,x^{3}+a\right)}+\frac{c}{3a\left(b\,x^{3}+a\right)}+\frac{c\ln\left(x\right)}{a^{2}}-\frac{2\ln\left(b\,x^{3}+a\right)af}{3b^{3}}+\frac{\ln\left(b\,x^{3}+a\right)e}{3b^{2}}-c\ln\cdots$
parallelrisch	$\frac{x^{6}a^{2}b^{2}f+3\ln\left(x\right)x^{3}b^{4}c-2\ln\left(b\,x^{3}+a\right)x^{3}a^{3}bf+\ln\left(b\,x^{3}+a\right)x^{3}a^{2}b^{2}e-\ln\left(b\,x^{3}+a\right)x^{3}b^{4}c+3\ln\left(x\right)a\,b^{3}c-2\ln\left(b\,x^{3}+a\right)a^{4}f+\ln\left(b\,x^{3}+\cdots\right.}{3a^{2}b^{3}\left(b\,x^{3}+a\right)}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*f*x^3/b^2+c*ln(x)/a^2-1/3/a^2/b^2*((2*a^3*f-a^2*b*e+b^3*c)/b*ln(b*x^3+
a)+a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a))
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.45

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2}\,dx$$
$$=\frac{a^2b^2fx^6+a^3bfx^3+ab^3c-a^2b^2d+a^3be-a^4f-(ab^3c-a^3be+2\,a^4f+(b^4c-a^2b^2e+2\,a^3bf)x^3)\log(bx}{3\,(a^2b^4x^3+a^3b^3)}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/3*(a^2*b^2*f*x^6 + a^3*b*f*x^3 + a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f -
 (a*b^3*c - a^3*b*e + 2*a^4*f + (b^4*c - a^2*b^2*e + 2*a^3*b*f)*x^3)*log(b
*x^3 + a) + 3*(b^4*c*x^3 + a*b^3*c)*log(x))/(a^2*b^4*x^3 + a^3*b^3)
```

3.255.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.255.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^2}\,dx = \frac{fx^3}{3\,b^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{3\,(ab^4x^3+a^2b^3)}+\frac{c\log(x^3)}{3\,a^2} - \frac{(b^3c-a^2be+2\,a^3f)\log(bx^3+a)}{3\,a^2b^3}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*f*x^3/b^2 + 1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a*b^4*x^3 + a^2*b
^3) + 1/3*c*log(x^3)/a^2 - 1/3*(b^3*c - a^2*b*e + 2*a^3*f)*log(b*x^3 + a)/
(a^2*b^3)
```

3.255.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.23

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx = \frac{fx^3}{3\,b^2}+\frac{c\log\left(|x|\right)}{a^2}-\frac{\left(b^3c-a^2be+2\,a^3f\right)\log\left(|bx^3+a|\right)}{3\,a^2b^3}+\frac{b^4cx^3-a^2b^2ex^3+2\,a^3bfx^3+2\,ab^3c-a^2b^2d+a^4f}{3\,(bx^3+a)a^2b^3}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*f*x^3/b^2 + c*log(abs(x))/a^2 - 1/3*(b^3*c - a^2*b*e + 2*a^3*f)*log(ab
s(b*x^3 + a))/(a^2*b^3) + 1/3*(b^4*c*x^3 - a^2*b^2*e*x^3 + 2*a^3*b*f*x^3 +
 2*a*b^3*c - a^2*b^2*d + a^4*f)/((b*x^3 + a)*a^2*b^3)
```

3.255.9 Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^2}\,dx = \frac{f\,x^3}{3\,b^2}+\frac{c\,\ln\left(x\right)}{a^2}+\frac{-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3}{3\,a\,b\,\left(b^3\,x^3+a\,b^2\right)}-\frac{\ln\left(b\,x^3+a\right)\,\left(2\,f\,a^3-e\,a^2\,b+c\,b^3\right)}{3\,a^2\,b^3}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^2),x)
```

output
```
(f*x^3)/(3*b^2) + (c*log(x))/a^2 + (b^3*c - a^3*f - a*b^2*d + a^2*b*e)/(3*
a*b*(a*b^2 + b^3*x^3)) - (log(a + b*x^3)*(b^3*c + 2*a^3*f - a^2*b*e))/(3*a
^2*b^3)
```

3.256 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^2}\,dx$

3.256.1 Optimal result 1920
3.256.2 Mathematica [A] (verified) 1920
3.256.3 Rubi [A] (verified) 1921
3.256.4 Maple [A] (verified) 1922
3.256.5 Fricas [A] (verification not implemented) 1922
3.256.6 Sympy [F(-1)] 1923
3.256.7 Maxima [A] (verification not implemented) 1923
3.256.8 Giac [A] (verification not implemented) 1924
3.256.9 Mupad [B] (verification not implemented) 1924

3.256.1 Optimal result

Integrand size = 30, antiderivative size = 109

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^2}\,dx = -\frac{c}{3a^2x^3} - \frac{b^3c-ab^2d+a^2be-a^3f}{3a^2b^2(a+bx^3)} - \frac{(2bc-ad)\log(x)}{a^3} + \frac{(2b^3c-ab^2d+a^3f)\log(a+bx^3)}{3a^3b^2}$$

output
```
-1/3*c/a^2/x^3+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^2/b^2/(b*x^3+a)-(-a*d+2
*b*c)*ln(x)/a^3+1/3*(a^3*f-a*b^2*d+2*b^3*c)*ln(b*x^3+a)/a^3/b^2
```

3.256.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^2}\,dx = \frac{-\frac{ac}{x^3} + \frac{a(-b^3c+ab^2d-a^2be+a^3f)}{b^2(a+bx^3)} + 3(-2bc+ad)\log(x) + \frac{(2b^3c-ab^2d+a^3f)\log(a+bx^3)}{b^2}}{3a^3}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
```

output
```
(-((a*c)/x^3) + (a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(a + b*x^3
)) + 3*(-2*b*c + a*d)*Log[x] + ((2*b^3*c - a*b^2*d + a^3*f)*Log[a + b*x^3]
)/b^2)/(3*a^3)
```

3.256.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^6\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{3}\int \left(\frac{c}{a^2x^6}+\frac{fa^3-b^2da+2b^3c}{a^3b\left(bx^3+a\right)}+\frac{-fa^3+bea^2-b^2da+b^3c}{a^2b\left(bx^3+a\right)^2}+\frac{ad-2bc}{a^3x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\log\left(a+bx^3\right)\left(a^3f-ab^2d+2b^3c\right)}{a^3b^2}-\frac{\log\left(x^3\right)(2bc-ad)}{a^3}-\frac{c}{a^2x^3}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^2b^2\left(a+bx^3\right)}\right)$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x]
```

output
```
(-(c/(a^2*x^3)) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^2*b^2*(a + b*x^3)
) - ((2*b*c - a*d)*Log[x^3])/a^3 + ((2*b^3*c - a*b^2*d + a^3*f)*Log[a + b*
x^3])/(a^3*b^2))/3
```

3.256.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.256.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

method	result
default	$-\frac{c}{3a^2x^3}+\frac{(ad-2bc)\ln(x)}{a^3}+\frac{\frac{(f\,a^3-a\,b^2d+2b^3c)\ln(b\,x^3+a)}{b^2}+\frac{a(f\,a^3-a^2be+a\,b^2d-b^3c)}{b^2(b\,x^3+a)}}{3a^3}$
norman	$\frac{-\frac{c}{3a}+\frac{(f\,a^3-a^2be+a\,b^2d-2b^3c)x^3}{3a^2b^2}}{x^3(b\,x^3+a)}+\frac{(ad-2bc)\ln(x)}{a^3}+\frac{(f\,a^3-a\,b^2d+2b^3c)\ln(b\,x^3+a)}{3a^3b^2}$
risch	$\frac{-\frac{c}{3a}+\frac{(f\,a^3-a^2be+a\,b^2d-2b^3c)x^3}{3a^2b^2}}{x^3(b\,x^3+a)}+\frac{d\ln(x)}{a^2}-\frac{2bc\ln(x)}{a^3}+\frac{\ln(-b\,x^3-a)f}{3b^2}-\frac{\ln(-b\,x^3-a)d}{3a^2}+\frac{2b\ln(-b\,x^3-a)c}{3a^3}$
parallelrisch	$\frac{3\ln(x)x^6a\,b^3d-6\ln(x)x^6b^4c+\ln(b\,x^3+a)x^6a^3bf-\ln(b\,x^3+a)x^6a\,b^3d+2\ln(b\,x^3+a)x^6b^4c+3\ln(x)x^3a^2b^2d-6\ln(x)x^3a\,b^3c+\ln}{3a^3b^2x^3(b\,x^3+a)}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c/a^2/x^3+(a*d-2*b*c)/a^3*ln(x)+1/3/a^3*((a^3*f-a*b^2*d+2*b^3*c)/b^2*
ln(b*x^3+a)+a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^2/(b*x^3+a))
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.58

$$\int\frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx=$$
$$-\frac{a^2b^2c+(2\,ab^3c-a^2b^2d+a^3be-a^4f)x^3-((2\,b^4c-ab^3d+a^3bf)x^6+(2\,ab^3c-a^2b^2d+a^4f)x^3)\log\left(bx\right.}{3\left(a^3b^3x^6+a^4b^2x^3\right)}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(a^2*b^2*c + (2*a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^3 - ((2*b^4*
c - a*b^3*d + a^3*b*f)*x^6 + (2*a*b^3*c - a^2*b^2*d + a^4*f)*x^3)*log(b*x^
3 + a) + 3*((2*b^4*c - a*b^3*d)*x^6 + (2*a*b^3*c - a^2*b^2*d)*x^3)*log(x))
/(a^3*b^3*x^6 + a^4*b^2*x^3)
```

3.256.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.256.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^2}\,dx = -\frac{ab^2c+(2\,b^3c-ab^2d+a^2be-a^3f)x^3}{3\left(a^2b^3x^6+a^3b^2x^3\right)} - \frac{(2\,bc-ad)\log\left(x^3\right)}{3\,a^3} + \frac{(2\,b^3c-ab^2d+a^3f)\log\left(bx^3+a\right)}{3\,a^3b^2}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(a*b^2*c + (2*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3)/(a^2*b^3*x^6 +
a^3*b^2*x^3) - 1/3*(2*b*c - a*d)*log(x^3)/a^3 + 1/3*(2*b^3*c - a*b^2*d + a
^3*f)*log(b*x^3 + a)/(a^3*b^2)
```

3.256.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.19

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^2}\,dx = -\frac{(2\,bc-ad)\log(|x|)}{a^3}+\frac{(2\,b^3c-ab^2d+a^3f)\log(|bx^3+a|)}{3\,a^3b^2} - \frac{a^2bfx^6+4\,b^3cx^3-2\,ab^2dx^3+2\,a^2bex^3-a^3fx^3+2\,ab^2c}{6\,(bx^6+ax^3)a^2b^2}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-(2*b*c - a*d)*log(abs(x))/a^3 + 1/3*(2*b^3*c - a*b^2*d + a^3*f)*log(abs(b
*x^3 + a))/(a^3*b^2) - 1/6*(a^2*b*f*x^6 + 4*b^3*c*x^3 - 2*a*b^2*d*x^3 + 2*
a^2*b*e*x^3 - a^3*f*x^3 + 2*a*b^2*c)/((b*x^6 + a*x^3)*a^2*b^2)
```

3.256.9 Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^2}\,dx = \frac{\ln(x)\,(a\,d-2\,b\,c)}{a^3} - \frac{\frac{c}{3\,a}+\frac{x^3\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+2\,c\,b^3)}{3\,a^2\,b^2}}{b\,x^6+a\,x^3} + \frac{\ln(b\,x^3+a)\,(f\,a^3-d\,a\,b^2+2\,c\,b^3)}{3\,a^3\,b^2}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^2),x)
```

output
```
(log(x)*(a*d - 2*b*c))/a^3 - (c/(3*a) + (x^3*(2*b^3*c - a^3*f - a*b^2*d +
a^2*b*e))/(3*a^2*b^2))/(a*x^3 + b*x^6) + (log(a + b*x^3)*(2*b^3*c + a^3*f
- a*b^2*d))/(3*a^3*b^2)
```

3.257 $\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2} dx$

3.257.1 Optimal result . . . 1925
3.257.2 Mathematica [A] (verified) . . . 1925
3.257.3 Rubi [A] (verified) . . . 1926
3.257.4 Maple [A] (verified) . . . 1927
3.257.5 Fricas [A] (verification not implemented) . . . 1928
3.257.6 Sympy [F(-1)] . . . 1928
3.257.7 Maxima [A] (verification not implemented) . . . 1928
3.257.8 Giac [A] (verification not implemented) . . . 1929
3.257.9 Mupad [B] (verification not implemented) . . . 1929

3.257.1 Optimal result

Integrand size = 30, antiderivative size = 130

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx=-\frac{c}{6a^2x^6}+\frac{2bc-ad}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^3b\left(a+bx^3\right)}+\frac{\left(3b^2c-2abd+a^2e\right)\log(x)}{a^4}-\frac{\left(3b^2c-2abd+a^2e\right)\log\left(a+bx^3\right)}{3a^4}$$

output
```
-1/6*c/a^2/x^6+1/3*(-a*d+2*b*c)/a^3/x^3+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
/a^3/b/(b*x^3+a)+(a^2*e-2*a*b*d+3*b^2*c)*ln(x)/a^4-1/3*(a^2*e-2*a*b*d+3*b^
2*c)*ln(b*x^3+a)/a^4
```

3.257.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx=-\frac{\frac{a^2c}{x^6}+\frac{2a(-2bc+ad)}{x^3}+\frac{2a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b(a+bx^3)}-6\left(3b^2c-2abd+a^2e\right)\log(x)+2\left(3b^2c-2abd+a^2e\right)\log\left(a+\right.}{6a^4}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

output
```
-1/6*((a^2*c)/x^6 + (2*a*(-2*b*c + a*d))/x^3 + (2*a*(-(b^3*c) + a*b^2*d -
a^2*b*e + a^3*f))/(b*(a + b*x^3)) - 6*(3*b^2*c - 2*a*b*d + a^2*e)*Log[x] +
 2*(3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/a^4
```

3.257.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{ 2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^9\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{ 2123}$$

$$\frac{1}{3}\int \left(\frac{c}{a^2x^9}-\frac{b\left(ea^2-2bda+3b^2c\right)}{a^4\left(bx^3+a\right)}+\frac{fa^3-bea^2+b^2da-b^3c}{a^3\left(bx^3+a\right)^2}+\frac{ea^2-2bda+3b^2c}{a^4x^3}+\frac{ad-2bc}{a^3x^6}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{2bc-ad}{a^3x^3}-\frac{c}{2a^2x^6}+\frac{\log\left(x^3\right)\left(a^2e-2abd+3b^2c\right)}{a^4}-\frac{\log\left(a+bx^3\right)\left(a^2e-2abd+3b^2c\right)}{a^4}+\frac{a^3(-f)+a^2be-ab}{a^3b\left(a+bx^3\right.}\right.$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x]
```

output
```
(-1/2*c/(a^2*x^6) + (2*b*c - a*d)/(a^3*x^3) + (b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)/(a^3*b*(a + b*x^3)) + ((3*b^2*c - 2*a*b*d + a^2*e)*Log[x^3])/a^4 -
 ((3*b^2*c - 2*a*b*d + a^2*e)*Log[a + b*x^3])/a^4)/3
```

3.257.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.257.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

method	result
default	$-\frac{c}{6a^2x^6}-\frac{ad-2bc}{3a^3x^3}+\frac{(a^2e-2abd+3b^2c)\ln(x)}{a^4}+\frac{(-a^2e+2abd-3b^2c)\ln(bx^3+a)-\frac{a(fa^3-a^2be+ab^2d-b^3c)}{b(bx^3+a)}}{3a^4}$
norman	$\frac{-\frac{c}{6a}-\frac{(2ad-3bc)x^3}{6a^2}+\frac{(fa^3-a^2be+2ab^2d-3b^3c)x^9}{3a^4}}{x^6(bx^3+a)}+\frac{(a^2e-2abd+3b^2c)\ln(x)}{a^4}-\frac{(a^2e-2abd+3b^2c)\ln(bx^3+a)}{3a^4}$
risch	$\frac{-\frac{(fa^3-a^2be+2ab^2d-3b^3c)x^6}{3a^3b}-\frac{(2ad-3bc)x^3}{6a^2}-\frac{c}{6a}}{x^6(bx^3+a)}+\frac{e\ln(x)}{a^2}-\frac{2\ln(x)bd}{a^3}+\frac{3\ln(x)b^2c}{a^4}-\frac{e\ln(bx^3+a)}{3a^2}+\frac{2\ln(bx^3+a)bd}{3a^3}$
parallelrisch	$6\ln(x)x^9a^2be-12\ln(x)x^9ab^2d+18\ln(x)x^9b^3c-2\ln(bx^3+a)x^9a^2be+4\ln(bx^3+a)x^9ab^2d-6\ln(bx^3+a)x^9b^3c+2x^9a^3f-2x^9a$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*c/a^2/x^6-1/3*(a*d-2*b*c)/a^3/x^3+(a^2*e-2*a*b*d+3*b^2*c)*ln(x)/a^4+1
/3/a^4*((-a^2*e+2*a*b*d-3*b^2*c)*ln(b*x^3+a)-a*(a^3*f-a^2*b*e+a*b^2*d-b^3*
c)/b/(b*x^3+a))
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.60

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx$$
$$= \frac{2\,(3\,ab^3c-2\,a^2b^2d+a^3be-a^4f)x^6-a^3bc+(3\,a^2b^2c-2\,a^3bd)x^3-2\,((3\,b^4c-2\,ab^3d+a^2b^2e)x^9+(3\,ab^3}{6\,(a^4b^2x^9}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/6*(2*(3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e - a^4*f)*x^6 - a^3*b*c + (3*a^2*
b^2*c - 2*a^3*b*d)*x^3 - 2*((3*b^4*c - 2*a*b^3*d + a^2*b^2*e)*x^9 + (3*a*b
^3*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(b*x^3 + a) + 6*((3*b^4*c - 2*a*b^3*
d + a^2*b^2*e)*x^9 + (3*a*b^3*c - 2*a^2*b^2*d + a^3*b*e)*x^6)*log(x))/(a^4
*b^2*x^9 + a^5*b*x^6)
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.257.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx = \frac{2\,(3\,b^3c-2\,ab^2d+a^2be-a^3f)x^6-a^2bc+(3\,ab^2c-2\,a^2bd)x^3}{6\,(a^3b^2x^9+a^4bx^6)} - \frac{(3\,b^2c-2\,abd+a^2e)\log\left(bx^3+a\right)}{3\,a^4} + \frac{(3\,b^2c-2\,abd+a^2e)\log\left(x^3\right)}{3\,a^4}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/6*(2*(3*b^3*c - 2*a*b^2*d + a^2*b*e - a^3*f)*x^6 - a^2*b*c + (3*a*b^2*c
- 2*a^2*b*d)*x^3)/(a^3*b^2*x^9 + a^4*b*x^6) - 1/3*(3*b^2*c - 2*a*b*d + a^2
*e)*log(b*x^3 + a)/a^4 + 1/3*(3*b^2*c - 2*a*b*d + a^2*e)*log(x^3)/a^4
```

3.257.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.51

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx = \frac{(3\,b^2c-2\,abd+a^2e)\log\left(|x|\right)}{a^4} - \frac{(3\,b^3c-2\,ab^2d+a^2be)\log\left(|bx^3+a|\right)}{3\,a^4b} + \frac{3\,b^4cx^3-2\,ab^3dx^3+a^2b^2ex^3+4\,ab^3c-3\,a^2b^2d+2\,a^3be-a^4f}{3\,(bx^3+a)a^4b} - \frac{9\,b^2cx^6-6\,abdx^6+3\,a^2ex^6-4\,abcx^3+2\,a^2dx^3+a^2c}{6\,a^4x^6}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
(3*b^2*c - 2*a*b*d + a^2*e)*log(abs(x))/a^4 - 1/3*(3*b^3*c - 2*a*b^2*d + a
^2*b*e)*log(abs(b*x^3 + a))/(a^4*b) + 1/3*(3*b^4*c*x^3 - 2*a*b^3*d*x^3 + a
^2*b^2*e*x^3 + 4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)/((b*x^3 + a)*a
^4*b) - 1/6*(9*b^2*c*x^6 - 6*a*b*d*x^6 + 3*a^2*e*x^6 - 4*a*b*c*x^3 + 2*a^2
*d*x^3 + a^2*c)/(a^4*x^6)
```

3.257.9 Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^2}\,dx = \frac{\ln\left(x\right)\,\left(e\,a^2-2\,d\,a\,b+3\,c\,b^2\right)}{a^4} - \frac{\ln\left(b\,x^3+a\right)\,\left(e\,a^2-2\,d\,a\,b+3\,c\,b^2\right)}{3\,a^4} - \frac{\frac{c}{6\,a}+\frac{x^3\,\left(2\,a\,d-3\,b\,c\right)}{6\,a^2}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-2\,d\,a\,b^2+3\,c\,b^3\right)}{3\,a^3\,b}}{b\,x^9+a\,x^6}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^2),x)
```

output
```
(log(x)*(3*b^2*c + a^2*e - 2*a*b*d))/a^4 - (log(a + b*x^3)*(3*b^2*c + a^2*
e - 2*a*b*d))/(3*a^4) - (c/(6*a) + (x^3*(2*a*d - 3*b*c))/(6*a^2) - (x^6*(3
*b^3*c - a^3*f - 2*a*b^2*d + a^2*b*e))/(3*a^3*b))/(a*x^6 + b*x^9)
```

3.257. $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^2}\,dx$

3.258 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2} dx$

3.258.1 Optimal result . . . 1931
3.258.2 Mathematica [A] (verified) . . . 1932
3.258.3 Rubi [A] (verified) . . . 1932
3.258.4 Maple [A] (verified) . . . 1934
3.258.5 Fricas [A] (verification not implemented) . . . 1934
3.258.6 Sympy [F(-1)] . . . 1935
3.258.7 Maxima [A] (verification not implemented) . . . 1935
3.258.8 Giac [A] (verification not implemented) . . . 1935
3.258.9 Mupad [B] (verification not implemented) . . . 1936

3.258.1 Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^2} dx = -\frac{c}{9a^2x^9}+\frac{2bc-ad}{6a^3x^6}-\frac{3b^2c-2abd+a^2e}{3a^4x^3} - \frac{b^3c-ab^2d+a^2be-a^3f}{3a^4(a+bx^3)} - \frac{(4b^3c-3ab^2d+2a^2be-a^3f)\log(x)}{a^5} + \frac{(4b^3c-3ab^2d+2a^2be-a^3f)\log(a+bx^3)}{3a^5}$$

output
```
-1/9*c/a^2/x^9+1/6*(-a*d+2*b*c)/a^3/x^6+1/3*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x
^3+1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/(b*x^3+a)-(-a^3*f+2*a^2*b*e-3*a*b
^2*d+4*b^3*c)*ln(x)/a^5+1/3*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)*ln(b*x^3+
a)/a^5
```

3.258.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx$$
$$= \frac{-\frac{2a^3c}{x^9}-\frac{3a^2(-2bc+ad)}{x^6}-\frac{6a(3b^2c-2abd+a^2e)}{x^3}+\frac{6a(-b^3c+ab^2d-a^2be+a^3f)}{a+bx^3}+18(-4b^3c+3ab^2d-2a^2be+a^3f)\log(x)}{18a^5}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]
```

output
```
((-2*a^3*c)/x^9 - (3*a^2*(-2*b*c + a*d))/x^6 - (6*a*(3*b^2*c - 2*a*b*d + a
^2*e))/x^3 + (6*a*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) + 18
*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f)*Log[x] + 6*(4*b^3*c - 3*a*b^2*
d + 2*a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*a^5)
```

3.258.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \text{ 2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^{12}\left(bx^3+a\right)^2}dx^3$$

$$\Big\downarrow \text{ 2123}$$

$$\frac{1}{3}\int \left(\frac{c}{a^2x^{12}}-\frac{b\left(fa^3-2bea^2+3b^2da-4b^3c\right)}{a^5\left(bx^3+a\right)}-\frac{b\left(fa^3-bea^2+b^2da-b^3c\right)}{a^4\left(bx^3+a\right)^2}+\frac{fa^3-2bea^2+3b^2da-4b^3c}{a^5x^3}+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{2bc-ad}{2a^3x^6}-\frac{c}{3a^2x^9}-\frac{a^2e-2abd+3b^2c}{a^4x^3}-\frac{\log\left(x^3\right)\left(a^3(-f)+2a^2be-3ab^2d+4b^3c\right)}{a^5}+\frac{\log\left(a+bx^3\right)\left(a^3(-f)\right.}{}\right.$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x]
```

output
```
(-1/3*c/(a^2*x^9) + (2*b*c - a*d)/(2*a^3*x^6) - (3*b^2*c - 2*a*b*d + a^2*e
)/(a^4*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*(a + b*x^3)) - ((4*
b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*Log[x^3])/a^5 + ((4*b^3*c - 3*a*b^2
*d + 2*a^2*b*e - a^3*f)*Log[a + b*x^3])/a^5)/3
```

3.258.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.258.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97

method	result
default	$-\frac{c}{9a^2x^9}-\frac{ad-2bc}{6a^3x^6}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{(f a^3-2a^2be+3a b^2d-4b^3c)\ln(x)}{a^5}-\frac{b\left(\frac{(f a^3-2a^2be+3a b^2d-4b^3c)\ln(bx^3+a)}{b}\right.}{3a^5}$
norman	$\frac{-\frac{c}{9a}-\frac{(3ad-4bc)x^3}{18a^2}-\frac{(2a^2e-3abd+4b^2c)x^6}{6a^3}+\frac{b(-f a^3+2a^2be-3a b^2d+4b^3c)x^{12}}{3a^5}}{x^9(bx^3+a)}+\frac{(f a^3-2a^2be+3a b^2d-4b^3c)\ln(x)}{a^5}-(f a^3-$
risch	$\frac{\frac{(f a^3-2a^2be+3a b^2d-4b^3c)x^9}{3a^4}-\frac{(2a^2e-3abd+4b^2c)x^6}{6a^3}-\frac{(3ad-4bc)x^3}{18a^2}-\frac{c}{9a}}{x^9(bx^3+a)}+\frac{\ln(x)f}{a^2}-\frac{2\ln(x)be}{a^3}+\frac{3\ln(x)b^2d}{a^4}-\frac{4\ln(x)b^3c}{a^5}$
parallelrisch	$-6x^6a^4be+9x^6a^3b^2d-12x^6a^2b^3c-3x^3a^4bd-72\ln(x)x^{12}b^5c+24\ln(bx^3+a)x^{12}b^5c-2a^4bc+4a^3b^2cx^3-36\ln(x)x^{12}a^2b^3e+54\ln$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/9*c/a^2/x^9-1/6*(a*d-2*b*c)/a^3/x^6-1/3*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^3
+(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5*ln(x)-1/3/a^5*b*((a^3*f-2*a^2*b*e
+3*a*b^2*d-4*b^3*c)/b*ln(b*x^3+a)-a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3
+a))
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.49

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx =$$
$$-\frac{6\,(4\,ab^3c-3\,a^2b^2d+2\,a^3be-a^4f)x^9+3\,(4\,a^2b^2c-3\,a^3bd+2\,a^4e)x^6+2\,a^4c-(4\,a^3bc-3\,a^4d)x^3-6\,(}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/18*(6*(4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*x^9 + 3*(4*a^2*b^2*
c - 3*a^3*b*d + 2*a^4*e)*x^6 + 2*a^4*c - (4*a^3*b*c - 3*a^4*d)*x^3 - 6*((4
*b^4*c - 3*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*x^12 + (4*a*b^3*c - 3*a^2*b^2*
d + 2*a^3*b*e - a^4*f)*x^9)*log(b*x^3 + a) + 18*((4*b^4*c - 3*a*b^3*d + 2*
a^2*b^2*e - a^3*b*f)*x^12 + (4*a*b^3*c - 3*a^2*b^2*d + 2*a^3*b*e - a^4*f)*
x^9)*log(x))/(a^5*b*x^12 + a^6*x^9)
```

3.258.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.258.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx = -\frac{6\,(4\,b^3c-3\,ab^2d+2\,a^2be-a^3f)x^9+3\,(4\,ab^2c-3\,a^2bd+2\,a^3e)x^6+2\,a^3c-(4\,a^2bc-3\,a^3d)x^3}{18\,(a^4bx^{12}+a^5x^9)} + \frac{(4\,b^3c-3\,ab^2d+2\,a^2be-a^3f)\log\left(bx^3+a\right)}{3\,a^5} - \frac{(4\,b^3c-3\,ab^2d+2\,a^2be-a^3f)\log\left(x^3\right)}{3\,a^5}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/18*(6*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*x^9 + 3*(4*a*b^2*c - 3*
a^2*b*d + 2*a^3*e)*x^6 + 2*a^3*c - (4*a^2*b*c - 3*a^3*d)*x^3)/(a^4*b*x^12
+ a^5*x^9) + 1/3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*log(b*x^3 + a)/
a^5 - 1/3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*log(x^3)/a^5
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.54

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx = -\frac{(4\,b^3c-3\,ab^2d+2\,a^2be-a^3f)\log\left(|x|\right)}{a^5} + \frac{(4\,b^4c-3\,ab^3d+2\,a^2b^2e-a^3bf)\log\left(|bx^3+a|\right)}{3\,a^5b} - \frac{4\,b^4cx^3-3\,ab^3dx^3+2\,a^2b^2ex^3-a^3bfx^3+5\,ab^3c-4\,a^2b^2d+3\,a^3be-2\,a^4f}{3\,(bx^3+a)a^5} + \frac{44\,b^3cx^9-33\,ab^2dx^9+22\,a^2bex^9-11\,a^3fx^9-18\,ab^2cx^6+12\,a^2bdx^6-6\,a^3ex^6+6\,a^2bcx^3-3\,a^3dx^3-}{18\,a^5x^9}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)*log(abs(x))/a^5 + 1/3*(4*b^4*c
- 3*a*b^3*d + 2*a^2*b^2*e - a^3*b*f)*log(abs(b*x^3 + a))/(a^5*b) - 1/3*(4*
b^4*c*x^3 - 3*a*b^3*d*x^3 + 2*a^2*b^2*e*x^3 - a^3*b*f*x^3 + 5*a*b^3*c - 4*
a^2*b^2*d + 3*a^3*b*e - 2*a^4*f)/((b*x^3 + a)*a^5) + 1/18*(44*b^3*c*x^9 -
33*a*b^2*d*x^9 + 22*a^2*b*e*x^9 - 11*a^3*f*x^9 - 18*a*b^2*c*x^6 + 12*a^2*b
*d*x^6 - 6*a^3*e*x^6 + 6*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)/(a^5*x^9)
```

3.258.9 Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^2}\,dx = \frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+2\,e\,a^2\,b-3\,d\,a\,b^2+4\,c\,b^3\right)}{3\,a^5} - \frac{\frac{c}{9\,a}+\frac{x^9\,\left(-f\,a^3+2\,e\,a^2\,b-3\,d\,a\,b^2+4\,c\,b^3\right)}{3\,a^4}+\frac{x^3\,\left(3\,a\,d-4\,b\,c\right)}{18\,a^2}+\frac{x^6\,\left(2\,e\,a^2-3\,d\,a\,b+4\,c\,b^2\right)}{6\,a^3}}{b\,x^{12}+a\,x^9} - \frac{\ln\left(x\right)\,\left(-f\,a^3+2\,e\,a^2\,b-3\,d\,a\,b^2+4\,c\,b^3\right)}{a^5}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^2),x)
```

output
```
(log(a + b*x^3)*(4*b^3*c - a^3*f - 3*a*b^2*d + 2*a^2*b*e))/(3*a^5) - (c/(9
*a) + (x^9*(4*b^3*c - a^3*f - 3*a*b^2*d + 2*a^2*b*e))/(3*a^4) + (x^3*(3*a*
d - 4*b*c))/(18*a^2) + (x^6*(4*b^2*c + 2*a^2*e - 3*a*b*d))/(6*a^3))/(a*x^9
 + b*x^12) - (log(x)*(4*b^3*c - a^3*f - 3*a*b^2*d + 2*a^2*b*e))/a^5
```

3.259 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx$

3.259.1 Optimal result . . . 1937
3.259.2 Mathematica [A] (verified) . . . 1938
3.259.3 Rubi [A] (verified) . . . 1938
3.259.4 Maple [A] (verified) . . . 1940
3.259.5 Fricas [A] (verification not implemented) . . . 1940
3.259.6 Sympy [F(-1)] . . . 1941
3.259.7 Maxima [A] (verification not implemented) . . . 1941
3.259.8 Giac [A] (verification not implemented) . . . 1942
3.259.9 Mupad [B] (verification not implemented) . . . 1942

3.259.1 Optimal result

Integrand size = 30, antiderivative size = 214

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx = -\frac{c}{12a^2x^{12}}+\frac{2bc-ad}{9a^3x^9}-\frac{3b^2c-2abd+a^2e}{6a^4x^6} +\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}+\frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5\left(a+bx^3\right)} +\frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)\log(x)}{a^6} -\frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)\log\left(a+bx^3\right)}{3a^6}$$

output

```
-1/12*c/a^2/x^12+1/9*(-a*d+2*b*c)/a^3/x^9+1/6*(-a^2*e+2*a*b*d-3*b^2*c)/a^4
/x^6+1/3*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^3+1/3*b*(-a^3*f+a^2*b*
e-a*b^2*d+b^3*c)/a^5/(b*x^3+a)+b*(-2*a^3*f+3*a^2*b*e-4*a*b^2*d+5*b^3*c)*ln
(x)/a^6-1/3*b*(-2*a^3*f+3*a^2*b*e-4*a*b^2*d+5*b^3*c)*ln(b*x^3+a)/a^6
```

3.259.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^2}\,dx = -\frac{\frac{3a^4c}{x^{12}}+\frac{4a^3(-2bc+ad)}{x^9}+\frac{6a^2(3b^2c-2abd+a^2e)}{x^6}+\frac{12a(-4b^3c+3ab^2d-2a^2be+a^3f)}{x^3}+\frac{12ab(-b^3c+ab^2d-a^2be+a^3f)}{a+bx^3}-36b(5b^3c-}{36a^6}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```

output
```
-1/36*((3*a^4*c)/x^12 + (4*a^3*(-2*b*c + a*d))/x^9 + (6*a^2*(3*b^2*c - 2*a
*b*d + a^2*e))/x^6 + (12*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/x^3
 + (12*a*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3) - 36*b*(5*b
^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x] + 12*b*(5*b^3*c - 4*a*b^2*d
 + 3*a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/a^6
```

3.259.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{ 2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^{15}(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{ 2123}$$

$$\frac{1}{3}\int \left(\frac{(2fa^3-3bea^2+4b^2da-5b^3c)b^2}{a^6(bx^3+a)}+\frac{(fa^3-bea^2+b^2da-b^3c)b^2}{a^5(bx^3+a)^2}-\frac{(2fa^3-3bea^2+4b^2da-5b^3c)b}{a^6x^3}+f\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{2bc-ad}{3a^3x^9}-\frac{c}{4a^2x^{12}}-\frac{a^2e-2abd+3b^2c}{2a^4x^6}+\frac{b\log\left(x^3\right)\left(-2a^3f+3a^2be-4ab^2d+5b^3c\right)}{a^6}-\frac{b\log\left(a+bx^3\right)\left(-2a\right.}{}\right.$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x]
```

output
```
(-1/4*c/(a^2*x^12) + (2*b*c - a*d)/(3*a^3*x^9) - (3*b^2*c - 2*a*b*d + a^2*
e)/(2*a^4*x^6) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x^3) + (b*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(a^5*(a + b*x^3)) + (b*(5*b^3*c - 4*a
*b^2*d + 3*a^2*b*e - 2*a^3*f)*Log[x^3])/a^6 - (b*(5*b^3*c - 4*a*b^2*d + 3*
a^2*b*e - 2*a^3*f)*Log[a + b*x^3])/a^6)/3
```

3.259.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.259.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.98

method	result
default	$-\frac{c}{12a^2x^{12}}-\frac{ad-2bc}{9a^3x^9}-\frac{a^2e-2abd+3b^2c}{6a^4x^6}-\frac{f\,a^3-2a^2be+3a\,b^2d-4b^3c}{3a^5x^3}-\frac{b(2f\,a^3-3a^2be+4a\,b^2d-5b^3c)\ln(x)}{a^6}+\frac{b^2\Big(\Big(}{}$
norman	$\frac{-\frac{c}{12a}-\frac{\left(2f\,a^3-3a^2be+4a\,b^2d-5b^3c\right)x^9}{6a^4}-\frac{(4ad-5bc)x^3}{36a^2}-\frac{\left(3a^2e-4abd+5b^2c\right)x^6}{18a^3}+\frac{b\left(2a^3bf-3a^2e\,b^2+4a\,b^3d-5b^4c\right)x^{15}}{3a^6}}{x^{12}(b\,x^3+a)}-\frac{b(2f\,a^3-3a}{}$
risch	$\frac{-\frac{c}{12a}-\frac{(4ad-5bc)x^3}{36a^2}-\frac{\left(3a^2e-4abd+5b^2c\right)x^6}{18a^3}-\frac{\left(2f\,a^3-3a^2be+4a\,b^2d-5b^3c\right)x^9}{6a^4}-\frac{b\left(2f\,a^3-3a^2be+4a\,b^2d-5b^3c\right)x^{12}}{3a^5}}{x^{12}(b\,x^3+a)}-\frac{2b\ln(x)f}{a^3}+$
parallelrisch	$-\frac{3a^5bc-180\ln(x)x^{12}a\,b^5c-24\ln(b\,x^3+a)x^{12}a^4b^2f+36\ln(b\,x^3+a)x^{12}a^3b^3e-48\ln(b\,x^3+a)x^{12}a^2b^4d+60\ln(b\,x^3+a)x^{12}a\,b^5c}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*c/a^2/x^12-1/9*(a*d-2*b*c)/a^3/x^9-1/6*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x
^6-1/3*(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x^3-b*(2*a^3*f-3*a^2*b*e+4*
a*b^2*d-5*b^3*c)/a^6*ln(x)+1/3*b^2/a^6*((2*a^3*f-3*a^2*b*e+4*a*b^2*d-5*b^3
*c)/b*ln(b*x^3+a)-a*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a))
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.45

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx$$
$$=\frac{12\left(5\,ab^4c-4\,a^2b^3d+3\,a^3b^2e-2\,a^4bf\right)x^{12}+6\left(5\,a^2b^3c-4\,a^3b^2d+3\,a^4be-2\,a^5f\right)x^9-2\left(5\,a^3b^2c-4\,a^4bc\right.}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(12*(5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e - 2*a^4*b*f)*x^12 + 6*(5*a
^2*b^3*c - 4*a^3*b^2*d + 3*a^4*b*e - 2*a^5*f)*x^9 - 2*(5*a^3*b^2*c - 4*a^4
*b*d + 3*a^5*e)*x^6 - 3*a^5*c + (5*a^4*b*c - 4*a^5*d)*x^3 - 12*((5*b^5*c -
 4*a*b^4*d + 3*a^2*b^3*e - 2*a^3*b^2*f)*x^15 + (5*a*b^4*c - 4*a^2*b^3*d +
3*a^3*b^2*e - 2*a^4*b*f)*x^12)*log(b*x^3 + a) + 36*((5*b^5*c - 4*a*b^4*d +
 3*a^2*b^3*e - 2*a^3*b^2*f)*x^15 + (5*a*b^4*c - 4*a^2*b^3*d + 3*a^3*b^2*e
- 2*a^4*b*f)*x^12)*log(x))/(a^6*b*x^15 + a^7*x^12)
```

3.259.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.259.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx$$
$$= \frac{12\,(5\,b^4c-4\,ab^3d+3\,a^2b^2e-2\,a^3bf)x^{12}+6\,(5\,ab^3c-4\,a^2b^2d+3\,a^3be-2\,a^4f)x^9-2\,(5\,a^2b^2c-4\,a^3bd+}{36\,(a^5bx^{15}+a^6x^{12})}$$
$$-\frac{(5\,b^4c-4\,ab^3d+3\,a^2b^2e-2\,a^3bf)\log\left(bx^3+a\right)}{3\,a^6}$$
$$+\frac{(5\,b^4c-4\,ab^3d+3\,a^2b^2e-2\,a^3bf)\log\left(x^3\right)}{3\,a^6}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/36*(12*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*x^12 + 6*(5*a*b^3
*c - 4*a^2*b^2*d + 3*a^3*b*e - 2*a^4*f)*x^9 - 2*(5*a^2*b^2*c - 4*a^3*b*d +
 3*a^4*e)*x^6 - 3*a^4*c + (5*a^3*b*c - 4*a^4*d)*x^3)/(a^5*b*x^15 + a^6*x^1
2) - 1/3*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*log(b*x^3 + a)/a^
6 + 1/3*(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*log(x^3)/a^6
```

3.259.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.51

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx = \frac{\left(5\,b^4c-4\,ab^3d+3\,a^2b^2e-2\,a^3bf\right)\log\left(|x|\right)}{a^6} - \frac{\left(5\,b^5c-4\,ab^4d+3\,a^2b^3e-2\,a^3b^2f\right)\log\left(|bx^3+a|\right)}{3\,a^6b} + \frac{5\,b^5cx^3-4\,ab^4dx^3+3\,a^2b^3ex^3-2\,a^3b^2fx^3+6\,ab^4c-5\,a^2b^3d+4\,a^3b^2e-3\,a^4bf}{3\,(bx^3+a)a^6} - \frac{125\,b^4cx^{12}-100\,ab^3dx^{12}+75\,a^2b^2ex^{12}-50\,a^3bfx^{12}-48\,ab^3cx^9+36\,a^2b^2dx^9-24\,a^3bex^9+12\,a^4fx^9 -}{36\,a^6x^{12}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
(5*b^4*c - 4*a*b^3*d + 3*a^2*b^2*e - 2*a^3*b*f)*log(abs(x))/a^6 - 1/3*(5*b
^5*c - 4*a*b^4*d + 3*a^2*b^3*e - 2*a^3*b^2*f)*log(abs(b*x^3 + a))/(a^6*b)
+ 1/3*(5*b^5*c*x^3 - 4*a*b^4*d*x^3 + 3*a^2*b^3*e*x^3 - 2*a^3*b^2*f*x^3 + 6
*a*b^4*c - 5*a^2*b^3*d + 4*a^3*b^2*e - 3*a^4*b*f)/((b*x^3 + a)*a^6) - 1/36
*(125*b^4*c*x^12 - 100*a*b^3*d*x^12 + 75*a^2*b^2*e*x^12 - 50*a^3*b*f*x^12
- 48*a*b^3*c*x^9 + 36*a^2*b^2*d*x^9 - 24*a^3*b*e*x^9 + 12*a^4*f*x^9 + 18*a
^2*b^2*c*x^6 - 12*a^3*b*d*x^6 + 6*a^4*e*x^6 - 8*a^3*b*c*x^3 + 4*a^4*d*x^3
+ 3*a^4*c)/(a^6*x^12)
```

3.259.9 Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^2}\,dx = \frac{\ln\left(x\right)\,\left(-2\,f\,a^3\,b+3\,e\,a^2\,b^2-4\,d\,a\,b^3+5\,c\,b^4\right)}{a^6} - \frac{\ln\left(b\,x^3+a\right)\,\left(-2\,f\,a^3\,b+3\,e\,a^2\,b^2-4\,d\,a\,b^3+5\,c\,b^4\right)}{3\,a^6} - \frac{\frac{c}{12\,a}-\frac{x^9\,\left(-2\,f\,a^3+3\,e\,a^2\,b-4\,d\,a\,b^2+5\,c\,b^3\right)}{6\,a^4}+\frac{x^3\,\left(4\,a\,d-5\,b\,c\right)}{36\,a^2}+\frac{x^6\,\left(3\,e\,a^2-4\,d\,a\,b+5\,c\,b^2\right)}{18\,a^3}-\frac{b\,x^{12}\,\left(-2\,f\,a^3+3\,e\,a^2\,b-4\,d\,a\,b^2+5\,c\,b^3\right)}{3\,a^5}}{b\,x^{15}+a\,x^{12}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^2),x)
```

output
```
(log(x)*(5*b^4*c + 3*a^2*b^2*e - 4*a*b^3*d - 2*a^3*b*f))/a^6 - (log(a + b*
x^3)*(5*b^4*c + 3*a^2*b^2*e - 4*a*b^3*d - 2*a^3*b*f))/(3*a^6) - (c/(12*a)
- (x^9*(5*b^3*c - 2*a^3*f - 4*a*b^2*d + 3*a^2*b*e))/(6*a^4) + (x^3*(4*a*d
- 5*b*c))/(36*a^2) + (x^6*(5*b^2*c + 3*a^2*e - 4*a*b*d))/(18*a^3) - (b*x^1
2*(5*b^3*c - 2*a^3*f - 4*a*b^2*d + 3*a^2*b*e))/(3*a^5))/(a*x^12 + b*x^15)
```

3.259. $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^2}\,dx$

3.260 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.260.1 Optimal result . . . 1944
3.260.2 Mathematica [A] (verified) . . . 1945
3.260.3 Rubi [A] (verified) . . . 1946
3.260.4 Maple [C] (verified) . . . 1947
3.260.5 Fricas [A] (verification not implemented) . . . 1948
3.260.6 Sympy [A] (verification not implemented) . . . 1949
3.260.7 Maxima [A] (verification not implemented) . . . 1950
3.260.8 Giac [A] (verification not implemented) . . . 1951
3.260.9 Mupad [B] (verification not implemented) . . . 1953

3.260.1 Optimal result

Integrand size = 30, antiderivative size = 369

$$\begin{aligned}\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx &= -\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,x}{b^6}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5}\\&\quad+\frac{(b^2d-2abe+3a^2f)\,x^7}{7b^4}+\frac{(be-2af)x^{10}}{10b^3}+\frac{fx^{13}}{13b^2}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^6\,(a+bx^3)}\\&\quad-\frac{a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{19/3}}\\&\quad+\frac{a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{19/3}}\\&\quad-\frac{a^{4/3}(7b^3c-10ab^2d+13a^2be-16a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{19/3}}\end{aligned}$$

output
```
-a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*x/b^6+1/4*(-4*a^3*f+3*a^2*b*e-2*
a*b^2*d+b^3*c)*x^4/b^5+1/7*(3*a^2*f-2*a*b*e+b^2*d)*x^7/b^4+1/10*(-2*a*f+b*
e)*x^10/b^3+1/13*f*x^13/b^2-1/3*a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^6/(
b*x^3+a)+1/9*a^(4/3)*(-16*a^3*f+13*a^2*b*e-10*a*b^2*d+7*b^3*c)*ln(a^(1/3)+
b^(1/3)*x)/b^(19/3)-1/18*a^(4/3)*(-16*a^3*f+13*a^2*b*e-10*a*b^2*d+7*b^3*c)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(19/3)-1/9*a^(4/3)*(-16*a^3*f
+13*a^2*b*e-10*a*b^2*d+7*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3
^(1/2))/b^(19/3)*3^(1/2)
```

3.260.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.99

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{a(-2b^3c+3ab^2d-4a^2be+5a^3f)\,x}{b^6}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^4}{4b^5}$$
$$+\frac{(b^2d-2abe+3a^2f)\,x^7}{7b^4}+\frac{(be-2af)x^{10}}{10b^3}+\frac{fx^{13}}{13b^2}+\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{3b^6\,(a+bx^3)}$$
$$+\frac{a^{4/3}(-7b^3c+10ab^2d-13a^2be+16a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{19/3}}$$
$$-\frac{a^{4/3}(-7b^3c+10ab^2d-13a^2be+16a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{19/3}}$$
$$+\frac{a^{4/3}(-7b^3c+10ab^2d-13a^2be+16a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{19/3}}$$

input
```
Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c - 2*a*b^2
*d + 3*a^2*b*e - 4*a^3*f)*x^4)/(4*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^7)
/(7*b^4) + ((b*e - 2*a*f)*x^10)/(10*b^3) + (f*x^13)/(13*b^2) + (a^2*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(3*b^6*(a + b*x^3)) + (a^(4/3)*(-7*b^3
*c + 10*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/(3*Sqrt[3]*b^(19/3)) - (a^(4/3)*(-7*b^3*c + 10*a*b^2*d - 13*a^
2*b*e + 16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(19/3)) + (a^(4/3)*(-7*b^
3*c + 10*a*b^2*d - 13*a^2*b*e + 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(18*b^(19/3))
```

3.260.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2367, 25, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$\Big\downarrow$ 2367

$$-\frac{\int -\frac{3ab^5fx^{15}+3ab^4(be-af)x^{12}+3ab^3(fa^2-bea+b^2d)x^9+3ab^2(-fa^3+bea^2-b^2da+b^3c)x^6-3a^2b(-fa^3+bea^2-b^2da+b^3c)x^3+a^3(-fa^3+bea^2}{bx^3+a}}{3ab^6}$$
$$\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^6(a+bx^3)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{3ab^5fx^{15}+3ab^4(be-af)x^{12}+3ab^3(fa^2-bea+b^2d)x^9+3ab^2(-fa^3+bea^2-b^2da+b^3c)x^6-3a^2b(-fa^3+bea^2-b^2da+b^3c)x^3+a^3(-fa^3+bea^2-b^2}{bx^3+a}}{3ab^6}$$
$$\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^6(a+bx^3)}$$

$\Big\downarrow$ 2426

$$\frac{\int \Big(3ab^4fx^{12}+3ab^3(be-2af)x^9+3ab^2(3fa^2-2bea+b^2d)x^6+3ab(-4fa^3+3bea^2-2b^2da+b^3c)x^3-3a^2(-}{3ab^6}$$
$$\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^6(a+bx^3)}$$

$\Big\downarrow$ 2009

$$\frac{\frac{3}{7}ab^2x^7(3a^2f-2abe+b^2d)+\frac{3}{4}abx^4(-4a^3f+3a^2be-2ab^2d+b^3c)-3a^2x(-5a^3f+4a^2be-3ab^2d+2b^3c)-\frac{a^7}{}}{}$$
$$\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^6(a+bx^3)}$$

input
```
Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
-1/3*(a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(b^6*(a + b*x^3)) + (-3*a
^2*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*x + (3*a*b*(b^3*c - 2*a*b^2
*d + 3*a^2*b*e - 4*a^3*f)*x^4)/4 + (3*a*b^2*(b^2*d - 2*a*b*e + 3*a^2*f)*x^
7)/7 + (3*a*b^3*(b*e - 2*a*f)*x^10)/10 + (3*a*b^4*f*x^13)/13 - (a^(7/3)*(7
*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3)) + (a^(7/3)*(7*b^3*c - 10*a*b^2*d +
 13*a^2*b*e - 16*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(1/3)) - (a^(7/3)*(
7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(6*b^(1/3)))/(3*a*b^6)
```

3.260.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.260.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.66

method	result
risch	$\frac{f x^{13}}{13b^2} - \frac{x^{10}af}{5b^3} + \frac{x^{10}e}{10b^2} + \frac{3x^7a^2f}{7b^4} - \frac{2x^7ae}{7b^3} + \frac{d x^7}{7b^2} - \frac{a^3f x^4}{b^5} + \frac{3a^2e x^4}{4b^4} - \frac{ad x^4}{2b^3} + \frac{c x^4}{4b^2} + \frac{5a^4fx}{b^6} - \frac{4a^3ex}{b^5} + \frac{3a^2dx}{b^4}$
default	$\frac{\frac{1}{13}f x^{13}b^4 - \frac{1}{5}x^{10}a b^3 f + \frac{1}{10}x^{10}b^4e + \frac{3}{7}x^7a^2b^2f - \frac{2}{7}x^7a b^3e + \frac{1}{7}b^4d x^7 - a^3bf x^4 + \frac{3}{4}a^2b^2e x^4 - \frac{1}{2}a b^3d x^4 + \frac{1}{4}b^4c x^4 + 5a^4fx - 4a^3bex + 3a^2b}{b^6}$

input
```
int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*f*x^13/b^2-1/5/b^3*x^10*a*f+1/10/b^2*x^10*e+3/7/b^4*x^7*a^2*f-2/7/b^3
*x^7*a*e+1/7/b^2*d*x^7-1/b^5*a^3*f*x^4+3/4/b^4*a^2*e*x^4-1/2/b^3*a*d*x^4+1
/4/b^2*c*x^4+5/b^6*a^4*f*x-4/b^5*a^3*e*x+3/b^4*a^2*d*x-2/b^3*a*c*x+(1/3*f*
a^5-1/3*a^4*e*b+1/3*a^3*d*b^2-1/3*a^2*c*b^3)*x/b^6/(b*x^3+a)+1/9/b^7*a^2*s
um((-16*a^3*f+13*a^2*b*e-10*a*b^2*d+7*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*
b+a))
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.32

$$\int \frac{x^9(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} \, dx$$

$$= \frac{1260\, b^5 f x^{16} + 126\, (13\, b^5 e - 16\, a b^4 f) x^{13} + 234\, (10\, b^5 d - 13\, a b^4 e + 16\, a^2 b^3 f) x^{10} + 585\, (7\, b^5 c - 10\, a b^4 d + 1}{}$$

input
```
integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/16380*(1260*b^5*f*x^16 + 126*(13*b^5*e - 16*a*b^4*f)*x^13 + 234*(10*b^5*
d - 13*a*b^4*e + 16*a^2*b^3*f)*x^10 + 585*(7*b^5*c - 10*a*b^4*d + 13*a^2*b
^3*e - 16*a^3*b^2*f)*x^7 - 4095*(7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e -
 16*a^4*b*f)*x^4 - 1820*sqrt(3)*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e -
 16*a^5*f + (7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-
a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 910*(7
*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f + (7*a*b^4*c - 10*a^2*b^
3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3)) - 1820*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5
*f + (7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^3)*(-a/b)^(1
/3)*log(x - (-a/b)^(1/3)) - 5460*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e
- 16*a^5*f)*x)/(b^7*x^3 + a*b^6)
```

3.260.6 Sympy [A] (verification not implemented)

Time = 85.66 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.36

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= x^{10}\left(-\frac{af}{5b^3}+\frac{e}{10b^2}\right)+x^7\cdot\left(\frac{3a^2f}{7b^4}-\frac{2ae}{7b^3}+\frac{d}{7b^2}\right)+x^4\left(-\frac{a^3f}{b^5}+\frac{3a^2e}{4b^4}-\frac{ad}{2b^3}+\frac{c}{4b^2}\right)$$
$$+x\left(\frac{5a^4f}{b^6}-\frac{4a^3e}{b^5}+\frac{3a^2d}{b^4}-\frac{2ac}{b^3}\right)+\frac{x(a^5f-a^4be+a^3b^2d-a^2b^3c)}{3ab^6+3b^7x^3}$$
$$+\mathrm{RootSum}\left(729t^3b^{19}+4096a^{13}f^3-9984a^{12}bef^2+7680a^{11}b^2df^2+8112a^{11}b^2e^2f-5376a^{10}b^3cf^2-124\right.$$
$$+\frac{fx^{13}}{13b^2}$$

input
```
integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
x**10*(-a*f/(5*b**3) + e/(10*b**2)) + x**7*(3*a**2*f/(7*b**4) - 2*a*e/(7*b
**3) + d/(7*b**2)) + x**4*(-a**3*f/b**5 + 3*a**2*e/(4*b**4) - a*d/(2*b**3)
 + c/(4*b**2)) + x*(5*a**4*f/b**6 - 4*a**3*e/b**5 + 3*a**2*d/b**4 - 2*a*c/
b**3) + x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(3*a*b**6 + 3*b*
*7*x**3) + RootSum(729*_t**3*b**19 + 4096*a**13*f**3 - 9984*a**12*b*e*f**2
 + 7680*a**11*b**2*d*f**2 + 8112*a**11*b**2*e**2*f - 5376*a**10*b**3*c*f**
2 - 12480*a**10*b**3*d*e*f - 2197*a**10*b**3*e**3 + 8736*a**9*b**4*c*e*f +
 4800*a**9*b**4*d**2*f + 5070*a**9*b**4*d*e**2 - 6720*a**8*b**5*c*d*f - 35
49*a**8*b**5*c*e**2 - 3900*a**8*b**5*d**2*e + 2352*a**7*b**6*c**2*f + 5460
*a**7*b**6*c*d*e + 1000*a**7*b**6*d**3 - 1911*a**6*b**7*c**2*e - 2100*a**6
*b**7*c*d**2 + 1470*a**5*b**8*c**2*d - 343*a**4*b**9*c**3, Lambda(_t, _t*l
og(-9*_t*b**6/(16*a**4*f - 13*a**3*b*e + 10*a**2*b**2*d - 7*a*b**3*c) + x)
)) + f*x**13/(13*b**2)
```

3.260.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)x}{3\,(b^7x^3+ab^6)} + \frac{140\,b^4fx^{13}+182\,(b^4e-2\,ab^3f)x^{10}+260\,(b^4d-2\,ab^3e+3\,a^2b^2f)x^7+455\,(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a}{1820\,b^6} + \frac{\sqrt{3}(7\,a^2b^3c-10\,a^3b^2d+13\,a^4be-16\,a^5f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(7\,a^2b^3c-10\,a^3b^2d+13\,a^4be-16\,a^5f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(7\,a^2b^3c-10\,a^3b^2d+13\,a^4be-16\,a^5f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x/(b^7*x^3 + a*b^6) + 1/182
0*(140*b^4*f*x^13 + 182*(b^4*e - 2*a*b^3*f)*x^10 + 260*(b^4*d - 2*a*b^3*e
+ 3*a^2*b^2*f)*x^7 + 455*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^4
 - 1820*(2*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*x)/b^6 + 1/9*sqrt(
3)*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f)*arctan(1/3*sqrt(3)
*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)^(2/3)) - 1/18*(7*a^2*b^3*c -
10*a^3*b^2*d + 13*a^4*b*e - 16*a^5*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(b^7*(a/b)^(2/3)) + 1/9*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^4*b*e - 16*a
^5*f)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(2/3))
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.20

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{\sqrt{3}\left(7\,(-ab^2)^{\frac{1}{3}}\,ab^3c-10\,(-ab^2)^{\frac{1}{3}}\,a^2b^2d+13\,(-ab^2)^{\frac{1}{3}}\,a^3be-16\,(-ab^2)^{\frac{1}{3}}\,a^4f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^7}$$
$$-\frac{\left(7\,a^2b^3c-10\,a^3b^2d+13\,a^4be-16\,a^5f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^6}$$
$$+\frac{\left(7\,(-ab^2)^{\frac{1}{3}}\,ab^3c-10\,(-ab^2)^{\frac{1}{3}}\,a^2b^2d+13\,(-ab^2)^{\frac{1}{3}}\,a^3be-16\,(-ab^2)^{\frac{1}{3}}\,a^4f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right.}{18\,b^7}$$
$$-\frac{a^2b^3cx-a^3b^2dx+a^4bex-a^5fx}{3\,(bx^3+a)b^6}$$
$$+\frac{140\,b^{24}fx^{13}+182\,b^{24}ex^{10}-364\,ab^{23}fx^{10}+260\,b^{24}dx^7-520\,ab^{23}ex^7+780\,a^2b^{22}fx^7+455\,b^{24}cx^4-910}{1820}$$

input
```
integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2*d + 13*(
-a*b^2)^(1/3)*a^3*b*e - 16*(-a*b^2)^(1/3)*a^4*f)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/b^7 - 1/9*(7*a^2*b^3*c - 10*a^3*b^2*d + 13*a^
4*b*e - 16*a^5*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^6) + 1/18*(
7*(-a*b^2)^(1/3)*a*b^3*c - 10*(-a*b^2)^(1/3)*a^2*b^2*d + 13*(-a*b^2)^(1/3)
*a^3*b*e - 16*(-a*b^2)^(1/3)*a^4*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/b^7 - 1/3*(a^2*b^3*c*x - a^3*b^2*d*x + a^4*b*e*x - a^5*f*x)/((b*x^3 + a
)*b^6) + 1/1820*(140*b^24*f*x^13 + 182*b^24*e*x^10 - 364*a*b^23*f*x^10 + 2
60*b^24*d*x^7 - 520*a*b^23*e*x^7 + 780*a^2*b^22*f*x^7 + 455*b^24*c*x^4 - 9
10*a*b^23*d*x^4 + 1365*a^2*b^22*e*x^4 - 1820*a^3*b^21*f*x^4 - 3640*a*b^23*
c*x + 5460*a^2*b^22*d*x - 7280*a^3*b^21*e*x + 9100*a^4*b^20*f*x)/b^26
```

3.260. $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.260.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.30

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= x^{10}\left(\frac{e}{10\,b^2}-\frac{a\,f}{5\,b^3}\right)-x\left(\frac{2\,a\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)}{b}-\frac{a^2\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b^2}\right)-x^7\left(\frac{a^2\,f}{7\,b^4}-\frac{d}{7\,b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{7\,b}\right)$$
$$+x^4\left(\frac{c}{4\,b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{4\,b^2}+\frac{a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{2\,b}\right)$$
$$+\frac{f\,x^{13}}{13\,b^2}+\frac{x\left(\frac{f\,a^5}{3}-\frac{e\,a^4\,b}{3}+\frac{d\,a^3\,b^2}{3}-\frac{c\,a^2\,b^3}{3}\right)}{b^7\,x^3+a\,b^6}$$
$$+\frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-16\,f\,a^3+13\,e\,a^2\,b-10\,d\,a\,b^2+7\,c\,b^3\right)}{9\,b^{19/3}}$$
$$+\frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-16\,f\,a^3+13\,e\,a^2\,b-10\,d\,a\,b^2+7\,c\,b^3\right)}{9\,b^{19/3}}$$
$$-\frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-16\,f\,a^3+13\,e\,a^2\,b-10\,d\,a\,b^2+7\,c\,b^3\right)}{9\,b^{19/3}}$$

input
```
int((x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

output
```
x^10*(e/(10*b^2) - (a*f)/(5*b^3)) - x*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*f)
/b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b
))/b - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b^2) -
x^7*((a^2*f)/(7*b^4) - d/(7*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(7*b)) + x^
4*(c/(4*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(4*b^2) + (a*((a^2*f)/b^4 - d/b
^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(2*b)) + (f*x^13)/(13*b^2) + (x*((a^5
*f)/3 - (a^2*b^3*c)/3 + (a^3*b^2*d)/3 - (a^4*b*e)/3))/(a*b^6 + b^7*x^3) +
(a^(4/3)*log(b^(1/3)*x + a^(1/3))*(7*b^3*c - 16*a^3*f - 10*a*b^2*d + 13*a^
2*b*e))/(9*b^(19/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(
1/3))*((3^(1/2)*1i)/2 - 1/2)*(7*b^3*c - 16*a^3*f - 10*a*b^2*d + 13*a^2*b*e
))/(9*b^(19/3)) - (a^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))
*((3^(1/2)*1i)/2 + 1/2)*(7*b^3*c - 16*a^3*f - 10*a*b^2*d + 13*a^2*b*e))/(9
*b^(19/3))
```

3.260. $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.261 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.261.1 Optimal result . . . 1955
3.261.2 Mathematica [A] (verified) . . . 1956
3.261.3 Rubi [A] (verified) . . . 1956
3.261.4 Maple [C] (verified) . . . 1959
3.261.5 Fricas [A] (verification not implemented) . . . 1960
3.261.6 Sympy [F(-1)] . . . 1961
3.261.7 Maxima [A] (verification not implemented) . . . 1961
3.261.8 Giac [A] (verification not implemented) . . . 1962
3.261.9 Mupad [B] (verification not implemented) . . . 1963

3.261.1 Optimal result

Integrand size = 30, antiderivative size = 335

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2}{2b^5}+\frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4} + \frac{(be-2af)x^8}{8b^3}+\frac{fx^{11}}{11b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^5(a+bx^3)} + \frac{a^{2/3}(5b^3c-8ab^2d+11a^2be-14a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{17/3}} + \frac{a^{2/3}(5b^3c-8ab^2d+11a^2be-14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{17/3}} - \frac{a^{2/3}(5b^3c-8ab^2d+11a^2be-14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{17/3}}$$

output
```
1/2*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*x^2/b^5+1/5*(3*a^2*f-2*a*b*e+b^2*
d)*x^5/b^4+1/8*(-2*a*f+b*e)*x^8/b^3+1/11*f*x^11/b^2+1/3*a*(-a^3*f+a^2*b*e-
a*b^2*d+b^3*c)*x^2/b^5/(b*x^3+a)+1/9*a^(2/3)*(-14*a^3*f+11*a^2*b*e-8*a*b^2
*d+5*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(17/3)-1/18*a^(2/3)*(-14*a^3*f+11*a^2*
b*e-8*a*b^2*d+5*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(17/3)+
1/9*a^(2/3)*(-14*a^3*f+11*a^2*b*e-8*a*b^2*d+5*b^3*c)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(17/3)*3^(1/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.95

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \frac{1980b^{2/3}(b^3c-2ab^2d+3a^2be-4a^3f)\,x^2+792b^{5/3}(b^2d-2abe+3a^2f)\,x^5+495b^{8/3}(be-2af)x^8+360b^{1}}{}$$

input
```
Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(1980*b^(2/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2 + 792*b^(5/3)*
(b^2*d - 2*a*b*e + 3*a^2*f)*x^5 + 495*b^(8/3)*(b*e - 2*a*f)*x^8 + 360*b^(1
1/3)*f*x^11 + (1320*a*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a
+ b*x^3) - 440*Sqrt[3]*a^(2/3)*(-5*b^3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3
*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 440*a^(2/3)*(-5*b^3*c +
8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 220*a^(2/3)*
(-5*b^3*c + 8*a*b^2*d - 11*a^2*b*e + 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(3960*b^(17/3))
```

3.261.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2367, 2390, 2375, 27, 2375, 27, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{2367}$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{\int \frac{-3ab^5fx^{13}-3ab^4(be-af)x^{10}-3ab^3\left(fa^2-bea+b^2d\right)x^7-3ab^2\left(-fa^3+bea^2-b^2da+b^3c\right)x^4+2a^2b\left(-fa^3+bea^2-b^2da+b^3c\right)x}{bx^3+a}dx}{3ab^6}$$

$$\Big\downarrow \ \text{2390}$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{\int \frac{x\left(-3ab^5fx^{12}-3ab^4(be-af)x^9-3ab^3\left(fa^2-bea+b^2d\right)x^6-3ab^2\left(-fa^3+bea^2-b^2da+b^3c\right)x^3+2a^2b\left(-fa^3+bea^2-b^2da+b^3c\right)\right)}{bx^3+a}dx}{3ab^6}$$

$$\Big\downarrow \ \text{2375}$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{\frac{\int \frac{11x\left(-3ab^5(be-2af)x^9-3ab^4\left(fa^2-bea+b^2d\right)x^6-3ab^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^3+2a^2b^2\left(-fa^3+bea^2-b^2da+b^3c\right)\right)}{bx^3+a}dx}{11b}-\frac{3}{11}ab^4fx^{11}}{3ab^6}$$

$$\Big\downarrow \ \text{27}$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{\frac{\int \frac{x\left(-3ab^5(be-2af)x^9-3ab^4\left(fa^2-bea+b^2d\right)x^6-3ab^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^3+2a^2b^2\left(-fa^3+bea^2-b^2da+b^3c\right)\right)}{bx^3+a}dx}{b}-\frac{3}{11}ab^4fx^{11}}{3ab^6}$$

$$\Big\downarrow \ \text{2375}$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{\frac{\frac{\int \frac{8x\left(-3ab^5\left(3fa^2-2bea+b^2d\right)x^6-3ab^4\left(-fa^3+bea^2-b^2da+b^3c\right)x^3+2a^2b^3\left(-fa^3+bea^2-b^2da+b^3c\right)\right)}{bx^3+a}dx}{8b}-\frac{3}{8}ab^4x^8(be-2af)}{b}-\frac{3}{11}ab^4fx^{11}}{3ab^6}$$

$$\Big\downarrow \ \text{27}$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{\frac{\frac{\int \frac{x\left(-3ab^5\left(3fa^2-2bea+b^2d\right)x^6-3ab^4\left(-fa^3+bea^2-b^2da+b^3c\right)x^3+2a^2b^3\left(-fa^3+bea^2-b^2da+b^3c\right)\right)}{bx^3+a}dx}{b}-\frac{3}{8}ab^4x^8(be-2af)}{b}-\frac{3}{11}ab^4fx^{11}}{3ab^6}$$

$$\Big\downarrow \ \text{1812}$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{\frac{\frac{\int \left(-3ab^4\left(3fa^2-2bea+b^2d\right)x^4-3ab^3\left(-4fa^3+3bea^2-2b^2da+b^3c\right)x+\frac{\left(5a^2cb^6-8a^3db^5+11a^4eb^4-14a^5fb^3\right)x}{bx^3+a}\right)dx}{b}-\frac{3}{8}ab^4x^8(be-2af)}{b}-\frac{3}{11}ab^4fx^{11}}{3ab^6}$$

$$\Big\downarrow \ \text{2009}$$

3.261. $\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{\frac{\frac{-\frac{3}{5}ab^4x^5\left(3a^2f-2abe+b^2d\right)-\frac{3}{2}ab^3x^2\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)-\frac{a^{5/3}b^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-14a^3f+11a^2be-8ab^2d+5b^3c\right)}{\sqrt{3}}+\frac{1}{6}a^{5/3}b^{7/3}\log\left(a^{2/3}-\right.}{b}}{b}}{3a}$$

input
```
Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*b^5*(a + b*x^3)) - ((-3*a*b
^4*f*x^11)/11 + ((-3*a*b^4*(b*e - 2*a*f)*x^8)/8 + ((-3*a*b^3*(b^3*c - 2*a*
b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/2 - (3*a*b^4*(b^2*d - 2*a*b*e + 3*a^2*f)
*x^5)/5 - (a^(5/3)*b^(7/3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (a^(5/3)*b^(7/
3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])
/3 + (a^(5/3)*b^(7/3)*(5*b^3*c - 8*a*b^2*d + 11*a^2*b*e - 14*a^3*f)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/6)/b)/b)/(3*a*b^6)
```

3.261.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.261. $\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2390
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

3.261.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.61

method	result
risch	$\frac{f x^{11}}{11 b^2} - \frac{x^8 f a}{4 b^3} + \frac{x^8 e}{8 b^2} + \frac{3 x^5 f a^2}{5 b^4} - \frac{2 x^5 a e}{5 b^3} + \frac{d x^5}{5 b^2} - \frac{2 x^2 f a^3}{b^5} + \frac{3 x^2 a^2 e}{2 b^4} - \frac{x^2 a d}{b^3} + \frac{x^2 c}{2 b^2} + \frac{\left(-\frac{1}{3} a^4 f + \frac{1}{3} a^3 b e - \frac{1}{3} a^2 b^2 d + \frac{1}{3} c \ldots\right)}{b^5 (b x^3 + a)}$
default	$-\frac{-\frac{b^3 f x^{11}}{11} + \frac{(2 f a b^2 - b^3 e) x^8}{8} + \frac{(-3 f a^2 b + 2 a b^2 e - b^3 d) x^5}{5} + \frac{x^2 (4 f a^3 - 3 a^2 b e + 2 a b^2 d - b^3 c)}{2}}{b^5} + \frac{a \left(\frac{\left(-\frac{1}{3} f a^3 + \frac{1}{3} a^2 b e - \frac{1}{3} a b^2 d + \frac{1}{3} b^3 c\right) x^2}{b x^3 + a} + (\ldots \right.}{\ldots}$

input
```
int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*f*x^11/b^2-1/4/b^3*x^8*f*a+1/8/b^2*x^8*e+3/5/b^4*x^5*f*a^2-2/5/b^3*x^
5*a*e+1/5/b^2*d*x^5-2/b^5*x^2*f*a^3+3/2/b^4*x^2*a^2*e-1/b^3*x^2*a*d+1/2/b^
2*x^2*c+(-1/3*a^4*f+1/3*a^3*b*e-1/3*a^2*b^2*d+1/3*a*b^3*c)*x^2/b^5/(b*x^3+
a)+1/9/b^6*a*sum((14*a^3*f-11*a^2*b*e+8*a*b^2*d-5*b^3*c)/_R*ln(x-_R),_R=Ro
otOf(_Z^3*b+a))
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.36

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \frac{360\,b^4fx^{14}+45\,(11\,b^4e-14\,ab^3f)x^{11}+99\,(8\,b^4d-11\,ab^3e+14\,a^2b^2f)x^8+396\,(5\,b^4c-8\,ab^3d+11\,a^2b^2e}{}$$

input
```
integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/3960*(360*b^4*f*x^14 + 45*(11*b^4*e - 14*a*b^3*f)*x^11 + 99*(8*b^4*d - 1
1*a*b^3*e + 14*a^2*b^2*f)*x^8 + 396*(5*b^4*c - 8*a*b^3*d + 11*a^2*b^2*e -
14*a^3*b*f)*x^5 + 660*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f)*x^
2 - 440*sqrt(3)*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f + (5*b^4*
c - 8*a*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f)*x^3)*(-a^2/b^2)^(1/3)*arctan(1/
3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*(5*a*b^3*c - 8*a^2
*b^2*d + 11*a^3*b*e - 14*a^4*f + (5*b^4*c - 8*a*b^3*d + 11*a^2*b^2*e - 14*
a^3*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/
b^2)^(1/3)) - 440*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f + (5*b^
4*c - 8*a*b^3*d + 11*a^2*b^2*e - 14*a^3*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x
 + b*(-a^2/b^2)^(2/3)))/(b^6*x^3 + a*b^5)
```

3.261.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx &= \frac{(ab^3c-a^2b^2d+a^3be-a^4f)x^2}{3\,(b^6x^3+ab^5)} \\ &- \frac{\sqrt{3}(5\,ab^3c-8\,a^2b^2d+11\,a^3be-14\,a^4f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &+ \frac{40\,b^3fx^{11}+55\,(b^3e-2\,ab^2f)x^8+88\,(b^3d-2\,ab^2e+3\,a^2bf)x^5+220\,(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f)x^2}{440\,b^5} \\ &- \frac{(5\,ab^3c-8\,a^2b^2d+11\,a^3be-14\,a^4f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &+ \frac{(5\,ab^3c-8\,a^2b^2d+11\,a^3be-14\,a^4f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}\end{aligned}$$

input
```
integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^2/(b^6*x^3 + a*b^5) - 1/9*sq
rt(3)*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f)*arctan(1/3*sqrt(3)
*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(1/3)) + 1/440*(40*b^3*f*x^11
 + 55*(b^3*e - 2*a*b^2*f)*x^8 + 88*(b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^5 + 2
20*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x^2)/b^5 - 1/18*(5*a*b^3*c -
8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(b^6*(a/b)^(1/3)) + 1/9*(5*a*b^3*c - 8*a^2*b^2*d + 11*a^3*b*e - 14*a^4*f
)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(1/3))
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.30

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{\left(5\,ab^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 8\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 11\,a^3be\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 14\,a^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}$$
$$+ \frac{\sqrt{3}\left(5\,(-ab^2)^{\frac{2}{3}}\,b^3c - 8\,(-ab^2)^{\frac{2}{3}}\,ab^2d + 11\,(-ab^2)^{\frac{2}{3}}\,a^2be - 14\,(-ab^2)^{\frac{2}{3}}\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^7}$$
$$+ \frac{ab^3cx^2 - a^2b^2dx^2 + a^3bex^2 - a^4fx^2}{3\,(bx^3+a)b^5}$$
$$- \frac{\left(5\,(-ab^2)^{\frac{2}{3}}\,b^3c - 8\,(-ab^2)^{\frac{2}{3}}\,ab^2d + 11\,(-ab^2)^{\frac{2}{3}}\,a^2be - 14\,(-ab^2)^{\frac{2}{3}}\,a^3f\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^7}$$
$$+ \frac{40\,b^{20}fx^{11} + 55\,b^{20}ex^8 - 110\,ab^{19}fx^8 + 88\,b^{20}dx^5 - 176\,ab^{19}ex^5 + 264\,a^2b^{18}fx^5 + 220\,b^{20}cx^2 - 440\,ab^{19}}{440\,b^{22}}$$

input
```
integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*(5*a*b^3*c*(-a/b)^(1/3) - 8*a^2*b^2*d*(-a/b)^(1/3) + 11*a^3*b*e*(-a/b)
^(1/3) - 14*a^4*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a
*b^5) + 1/9*sqrt(3)*(5*(-a*b^2)^(2/3)*b^3*c - 8*(-a*b^2)^(2/3)*a*b^2*d + 1
1*(-a*b^2)^(2/3)*a^2*b*e - 14*(-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*
x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/3*(a*b^3*c*x^2 - a^2*b^2*d*x^2 + a
^3*b*e*x^2 - a^4*f*x^2)/((b*x^3 + a)*b^5) - 1/18*(5*(-a*b^2)^(2/3)*b^3*c -
 8*(-a*b^2)^(2/3)*a*b^2*d + 11*(-a*b^2)^(2/3)*a^2*b*e - 14*(-a*b^2)^(2/3)*
a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^7 + 1/440*(40*b^20*f*x^1
1 + 55*b^20*e*x^8 - 110*a*b^19*f*x^8 + 88*b^20*d*x^5 - 176*a*b^19*e*x^5 +
264*a^2*b^18*f*x^5 + 220*b^20*c*x^2 - 440*a*b^19*d*x^2 + 660*a^2*b^18*e*x^
2 - 880*a^3*b^17*f*x^2)/b^22
```

3.261.9 Mupad [B] (verification not implemented)

Time = 10.37 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.08

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^8\left(\frac{e}{8\,b^2}-\frac{a\,f}{4\,b^3}\right)-x^5\left(\frac{a^2\,f}{5\,b^4}-\frac{d}{5\,b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{5\,b}\right)$$
$$+x^2\left(\frac{c}{2\,b^2}-\frac{a^2\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{2\,b^2}+\frac{a\,\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)$$
$$+\frac{f\,x^{11}}{11\,b^2}-\frac{x^2\,\left(\frac{f\,a^4}{3}-\frac{e\,a^3\,b}{3}+\frac{d\,a^2\,b^2}{3}-\frac{c\,a\,b^3}{3}\right)}{b^6\,x^3+a\,b^5}$$
$$+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-14\,f\,a^3+11\,e\,a^2\,b-8\,d\,a\,b^2+5\,c\,b^3\right)}{9\,b^{17/3}}$$
$$-\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+11\,e\,a^2\,b-8\,d\,a\,b^2+5\,c\,b^3\right)}{9\,b^{17/3}}$$
$$+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+11\,e\,a^2\,b-8\,d\,a\,b^2+5\,c\,b^3\right)}{9\,b^{17/3}}$$

input
```
int((x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

output
```
x^8*(e/(8*b^2) - (a*f)/(4*b^3)) - x^5*((a^2*f)/(5*b^4) - d/(5*b^2) + (2*a*
(e/b^2 - (2*a*f)/b^3))/(5*b)) + x^2*(c/(2*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3
))/(2*b^2) + (a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b)
+ (f*x^11)/(11*b^2) - (x^2*((a^4*f)/3 + (a^2*b^2*d)/3 - (a*b^3*c)/3 - (a^3
*b*e)/3))/(a*b^5 + b^6*x^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*b^3*c -
 14*a^3*f - 8*a*b^2*d + 11*a^2*b*e))/(9*b^(17/3)) - (a^(2/3)*log(3^(1/2)*a
^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c - 14*a^
3*f - 8*a*b^2*d + 11*a^2*b*e))/(9*b^(17/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)
*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c - 14*a^3*f -
8*a*b^2*d + 11*a^2*b*e))/(9*b^(17/3))
```

3.262 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.262.1 Optimal result . . . 1964
3.262.2 Mathematica [A] (verified) . . . 1965
3.262.3 Rubi [A] (verified) . . . 1965
3.262.4 Maple [C] (verified) . . . 1967
3.262.5 Fricas [A] (verification not implemented) . . . 1968
3.262.6 Sympy [A] (verification not implemented) . . . 1968
3.262.7 Maxima [A] (verification not implemented) . . . 1969
3.262.8 Giac [A] (verification not implemented) . . . 1970
3.262.9 Mupad [B] (verification not implemented) . . . 1971

3.262.1 Optimal result

Integrand size = 30, antiderivative size = 328

$$\begin{aligned}\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx &= \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5}+\frac{(b^2d-2abe+3a^2f)\,x^4}{4b^4}\\&+\frac{(be-2af)x^7}{7b^3}+\frac{fx^{10}}{10b^2}+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^5\,(a+bx^3)}\\&+\frac{\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{16/3}}\\&-\frac{\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{16/3}}\\&+\frac{\sqrt[3]{a}(4b^3c-7ab^2d+10a^2be-13a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{16/3}}\end{aligned}$$

output
```
(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*x/b^5+1/4*(3*a^2*f-2*a*b*e+b^2*d)*x^4
/b^4+1/7*(-2*a*f+b*e)*x^7/b^3+1/10*f*x^10/b^2+1/3*a*(-a^3*f+a^2*b*e-a*b^2*
d+b^3*c)*x/b^5/(b*x^3+a)-1/9*a^(1/3)*(-13*a^3*f+10*a^2*b*e-7*a*b^2*d+4*b^3
*c)*ln(a^(1/3)+b^(1/3)*x)/b^(16/3)+1/18*a^(1/3)*(-13*a^3*f+10*a^2*b*e-7*a*
b^2*d+4*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(16/3)+1/9*a^(1
/3)*(-13*a^3*f+10*a^2*b*e-7*a*b^2*d+4*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)
*x)/a^(1/3)*3^(1/2))/b^(16/3)*3^(1/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.96

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \frac{1260\sqrt[3]{b}(b^3c-2ab^2d+3a^2be-4a^3f)\,x+315b^{4/3}(b^2d-2abe+3a^2f)\,x^4+180b^{7/3}(be-2af)x^7+126b^{10/}}{}$$

input
```
Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(1260*b^(1/3)*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x + 315*b^(4/3)*(b
^2*d - 2*a*b*e + 3*a^2*f)*x^4 + 180*b^(7/3)*(b*e - 2*a*f)*x^7 + 126*b^(10/
3)*f*x^10 + (420*a*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a + b*x
^3) - 140*Sqrt[3]*a^(1/3)*(-4*b^3*c + 7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*(-4*b^3*c + 7*a*b
^2*d - 10*a^2*b*e + 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x] - 70*a^(1/3)*(-4*b^
3*c + 7*a*b^2*d - 10*a^2*b*e + 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(1260*b^(16/3))
```

3.262.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2367, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \text{2367}$$

$$\frac{ax\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-\frac{\int \frac{-3ab^4fx^{12}-3ab^3(be-af)x^9-3ab^2\left(fa^2-bea+b^2d\right)x^6-3ab\left(-fa^3+bea^2-b^2da+b^3c\right)x^3+a^2\left(-fa^3+bea^2-b^2da+b^3c\right)}{bx^3+a}dx}{3ab^5}$$

$$\Big\downarrow \; 2426$$

$$\frac{ax\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-$$
$$\frac{\int \left(-3ab^3fx^9-3ab^2(be-2af)x^6-3ab\left(3fa^2-2bea+b^2d\right)x^3-3a\left(-4fa^3+3bea^2-2b^2da+b^3c\right)+\frac{-13fa^5+10b\ldots}{\ldots}\right.}{3ab^5}$$

$$\Big\downarrow \; 2009$$

$$\frac{ax\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5\left(a+bx^3\right)}-$$
$$\frac{-\frac{3}{4}abx^4\left(3a^2f-2abe+b^2d\right)-3ax\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)-\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-13a^3f+10a^2be-7ab^2d+\ldots\right.}{\sqrt{3}\sqrt[3]{b}}}{\ldots}$$

input
```
Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*b^5*(a + b*x^3)) - (-3*a*(b^3
*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x - (3*a*b*(b^2*d - 2*a*b*e + 3*a^2*
f)*x^4)/4 - (3*a*b^2*(b*e - 2*a*f)*x^7)/7 - (3*a*b^3*f*x^10)/10 - (a^(4/3)
*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3)) + (a^(4/3)*(4*b^3*c - 7*a*b^2*d
+ 10*a^2*b*e - 13*a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/(3*b^(1/3)) - (a^(4/3)*
(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(6*b^(1/3)))/(3*a*b^5)
```

3.262.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

3.262. $\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.262.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.59

method	result
risch	$\frac{f\,x^{10}}{10b^2}-\frac{2x^7 fa}{7b^3}+\frac{x^7 e}{7b^2}+\frac{3x^4 f\,a^2}{4b^4}-\frac{x^4 ae}{2b^3}+\frac{d\,x^4}{4b^2}-\frac{4xf\,a^3}{b^5}+\frac{3x\,a^2 e}{b^4}-\frac{2xad}{b^3}+\frac{xc}{b^2}+\frac{\left(-\frac{1}{3}a^4 f+\frac{1}{3}a^3 be-\frac{1}{3}a^2 b^2 d+\frac{1}{3}a\,b^3\right.}{b^5(b\,x^3+a)}$
default	$-\frac{-\frac{1}{10}b^3 f\,x^{10}+\frac{2}{7}x^7 a\,b^2 f-\frac{1}{7}x^7 b^3 e-\frac{3}{4}a^2 bf\,x^4+\frac{1}{2}a\,b^2 e\,x^4-\frac{1}{4}d\,x^4 b^3+4f\,a^3 x-3a^2 bex+2a\,b^2 dx-b^3 cx}{b^5}+\frac{a\left(\frac{\left(-\frac{1}{3}f\,a^3+\frac{1}{3}a^2 be-\frac{1}{3}a\,b^2 d\right.}{b\,x^3+a}\right.}{}$

input
```
int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*f*x^10/b^2-2/7/b^3*x^7*f*a+1/7/b^2*x^7*e+3/4/b^4*x^4*f*a^2-1/2/b^3*x^
4*a*e+1/4/b^2*d*x^4-4/b^5*x*f*a^3+3/b^4*x*a^2*e-2/b^3*x*a*d+1/b^2*x*c+(-1/
3*a^4*f+1/3*a^3*b*e-1/3*a^2*b^2*d+1/3*a*b^3*c)*x/b^5/(b*x^3+a)+1/9/b^6*a*s
um((13*a^3*f-10*a^2*b*e+7*a*b^2*d-4*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+
a))
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.29

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{126\,b^4fx^{13}+18\,(10\,b^4e-13\,ab^3f)x^{10}+45\,(7\,b^4d-10\,ab^3e+13\,a^2b^2f)x^7+315\,(4\,b^4c-7\,ab^3d+10\,a^2b^2e}{}$$

input
```
integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/1260*(126*b^4*f*x^13 + 18*(10*b^4*e - 13*a*b^3*f)*x^10 + 45*(7*b^4*d - 1
0*a*b^3*e + 13*a^2*b^2*f)*x^7 + 315*(4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e -
13*a^3*b*f)*x^4 - 140*sqrt(3)*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a
^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*
arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*(4*a*b^3*c - 7*
a^2*b^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e -
13*a^3*b*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 140*
(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f + (4*b^4*c - 7*a*b^3*d +
10*a^2*b^2*e - 13*a^3*b*f)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*(4*
a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f)*x)/(b^6*x^3 + a*b^5)
```

3.262.6 Sympy [A] (verification not implemented)

Time = 76.44 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.37

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = x^7\left(-\frac{2af}{7b^3}+\frac{e}{7b^2}\right)+x^4\cdot\left(\frac{3a^2f}{4b^4}-\frac{ae}{2b^3}+\frac{d}{4b^2}\right)$$
$$+x\left(-\frac{4a^3f}{b^5}+\frac{3a^2e}{b^4}-\frac{2ad}{b^3}+\frac{c}{b^2}\right)+\frac{x(-a^4f+a^3be-a^2b^2d+ab^3c)}{3ab^5+3b^6x^3}$$
$$+\mathrm{RootSum}\left(729t^3b^{16}-2197a^{10}f^3+5070a^9bef^2-3549a^8b^2df^2-3900a^8b^2e^2f+2028a^7b^3cf^2+5460a^7\right.$$
$$+\frac{fx^{10}}{10b^2}$$

input
```
integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
x**7*(-2*a*f/(7*b**3) + e/(7*b**2)) + x**4*(3*a**2*f/(4*b**4) - a*e/(2*b**
3) + d/(4*b**2)) + x*(-4*a**3*f/b**5 + 3*a**2*e/b**4 - 2*a*d/b**3 + c/b**2
) + x*(-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(3*a*b**5 + 3*b**6*x**
3) + RootSum(729*_t**3*b**16 - 2197*a**10*f**3 + 5070*a**9*b*e*f**2 - 3549
*a**8*b**2*d*f**2 - 3900*a**8*b**2*e**2*f + 2028*a**7*b**3*c*f**2 + 5460*a
**7*b**3*d*e*f + 1000*a**7*b**3*e**3 - 3120*a**6*b**4*c*e*f - 1911*a**6*b*
*4*d**2*f - 2100*a**6*b**4*d*e**2 + 2184*a**5*b**5*c*d*f + 1200*a**5*b**5*
c*e**2 + 1470*a**5*b**5*d**2*e - 624*a**4*b**6*c**2*f - 1680*a**4*b**6*c*d
*e - 343*a**4*b**6*d**3 + 480*a**3*b**7*c**2*e + 588*a**3*b**7*c*d**2 - 33
6*a**2*b**8*c**2*d + 64*a*b**9*c**3, Lambda(_t, _t*log(9*_t*b**5/(13*a**3*
f - 10*a**2*b*e + 7*a*b**2*d - 4*b**3*c) + x))) + f*x**10/(10*b**2)
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.98

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{(ab^3c-a^2b^2d+a^3be-a^4f)x}{3\,(b^6x^3+ab^5)} + \frac{14\,b^3fx^{10}+20\,(b^3e-2\,ab^2f)x^7+35\,(b^3d-2\,ab^2e+3\,a^2bf)x^4+140\,(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f)x}{140\,b^5} - \frac{\sqrt{3}(4\,ab^3c-7\,a^2b^2d+10\,a^3be-13\,a^4f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(4\,ab^3c-7\,a^2b^2d+10\,a^3be-13\,a^4f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(4\,ab^3c-7\,a^2b^2d+10\,a^3be-13\,a^4f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x/(b^6*x^3 + a*b^5) + 1/140*(1
4*b^3*f*x^10 + 20*(b^3*e - 2*a*b^2*f)*x^7 + 35*(b^3*d - 2*a*b^2*e + 3*a^2*
b*f)*x^4 + 140*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 - 1/9*sqrt
(3)*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f)*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(2/3)) + 1/18*(4*a*b^3*c - 7*a^
2*b^2*d + 10*a^3*b*e - 13*a^4*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b
^6*(a/b)^(2/3)) - 1/9*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e - 13*a^4*f)*lo
g(x + (a/b)^(1/3))/(b^6*(a/b)^(2/3))
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.18

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx =$$

$$-\frac{\sqrt{3}\left(4\left(-ab^2\right)^{\frac{1}{3}}b^3c-7\left(-ab^2\right)^{\frac{1}{3}}ab^2d+10\left(-ab^2\right)^{\frac{1}{3}}a^2be-13\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^6}$$
$$+\frac{\left(4ab^3c-7a^2b^2d+10a^3be-13a^4f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9ab^5}$$
$$-\frac{\left(4\left(-ab^2\right)^{\frac{1}{3}}b^3c-7\left(-ab^2\right)^{\frac{1}{3}}ab^2d+10\left(-ab^2\right)^{\frac{1}{3}}a^2be-13\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^6}$$
$$+\frac{ab^3cx-a^2b^2dx+a^3bex-a^4fx}{3\left(bx^3+a\right)b^5}$$
$$+\frac{14b^{18}fx^{10}+20b^{18}ex^7-40ab^{17}fx^7+35b^{18}dx^4-70ab^{17}ex^4+105a^2b^{16}fx^4+140b^{18}cx-280ab^{17}dx}{140b^{20}}$$

input
```
integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(4*(-a*b^2)^(1/3)*b^3*c - 7*(-a*b^2)^(1/3)*a*b^2*d + 10*(-a*b
^2)^(1/3)*a^2*b*e - 13*(-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/b^6 + 1/9*(4*a*b^3*c - 7*a^2*b^2*d + 10*a^3*b*e -
 13*a^4*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(4*(-a*b
^2)^(1/3)*b^3*c - 7*(-a*b^2)^(1/3)*a*b^2*d + 10*(-a*b^2)^(1/3)*a^2*b*e - 1
3*(-a*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^6 + 1/3
*(a*b^3*c*x - a^2*b^2*d*x + a^3*b*e*x - a^4*f*x)/((b*x^3 + a)*b^5) + 1/140
*(14*b^18*f*x^10 + 20*b^18*e*x^7 - 40*a*b^17*f*x^7 + 35*b^18*d*x^4 - 70*a*
b^17*e*x^4 + 105*a^2*b^16*f*x^4 + 140*b^18*c*x - 280*a*b^17*d*x + 420*a^2*
b^16*e*x - 560*a^3*b^15*f*x)/b^20
```

3.262.9 Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.09

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= x^7\left(\frac{e}{7\,b^2}-\frac{2\,a\,f}{7\,b^3}\right)+x\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)$$
$$-x^4\left(\frac{a^2\,f}{4\,b^4}-\frac{d}{4\,b^2}+\frac{a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{2\,b}\right)-\frac{x\left(\frac{f\,a^4}{3}-\frac{e\,a^3\,b}{3}+\frac{d\,a^2\,b^2}{3}-\frac{c\,a\,b^3}{3}\right)}{b^6\,x^3+a\,b^5}$$
$$+\frac{f\,x^{10}}{10\,b^2}-\frac{a^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-13\,f\,a^3+10\,e\,a^2\,b-7\,d\,a\,b^2+4\,c\,b^3\right)}{9\,b^{16/3}}$$
$$-\frac{a^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-13\,f\,a^3+10\,e\,a^2\,b-7\,d\,a\,b^2+4\,c\,b^3\right)}{9\,b^{16/3}}$$
$$+\frac{a^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-13\,f\,a^3+10\,e\,a^2\,b-7\,d\,a\,b^2+4\,c\,b^3\right)}{9\,b^{16/3}}$$

input
```
int((x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

output
```
x^7*(e/(7*b^2) - (2*a*f)/(7*b^3)) + x*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/b^3))
/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b) - x^
4*((a^2*f)/(4*b^4) - d/(4*b^2) + (a*(e/b^2 - (2*a*f)/b^3))/(2*b)) - (x*((a
^4*f)/3 + (a^2*b^2*d)/3 - (a*b^3*c)/3 - (a^3*b*e)/3))/(a*b^5 + b^6*x^3) +
(f*x^10)/(10*b^2) - (a^(1/3)*log(b^(1/3)*x + a^(1/3))*(4*b^3*c - 13*a^3*f
- 7*a*b^2*d + 10*a^2*b*e))/(9*b^(16/3)) - (a^(1/3)*log(3^(1/2)*a^(1/3)*1i
+ 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*b^3*c - 13*a^3*f - 7*a*
b^2*d + 10*a^2*b*e))/(9*b^(16/3)) + (a^(1/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^
(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(4*b^3*c - 13*a^3*f - 7*a*b^2*d
+ 10*a^2*b*e))/(9*b^(16/3))
```

3.262. $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.263 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.263.1 Optimal result 1973
3.263.2 Mathematica [A] (verified) 1974
3.263.3 Rubi [A] (verified) 1974
3.263.4 Maple [C] (verified) 1977
3.263.5 Fricas [A] (verification not implemented) 1978
3.263.6 Sympy [F(-1)] 1979
3.263.7 Maxima [A] (verification not implemented) 1980
3.263.8 Giac [A] (verification not implemented) 1981
3.263.9 Mupad [B] (verification not implemented) 1982

3.263.1 Optimal result

Integrand size = 30, antiderivative size = 298

$$\begin{aligned}\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx &= \frac{(b^2d-2abe+3a^2f)\,x^2}{2b^4}+\frac{(be-2af)x^5}{5b^3}+\frac{fx^8}{8b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3b^4\,(a+bx^3)}\\&\quad-\frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{14/3}}\\&\quad-\frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{14/3}}\\&\quad+\frac{(2b^3c-5ab^2d+8a^2be-11a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{14/3}}\end{aligned}$$

output
```
1/2*(3*a^2*f-2*a*b*e+b^2*d)*x^2/b^4+1/5*(-2*a*f+b*e)*x^5/b^3+1/8*f*x^8/b^2
-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/b^4/(b*x^3+a)-1/9*(-11*a^3*f+8*a^2
*b*e-5*a*b^2*d+2*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(14/3)+1/18*(-11*a
^3*f+8*a^2*b*e-5*a*b^2*d+2*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/a^(1/3)/b^(14/3)-1/9*(-11*a^3*f+8*a^2*b*e-5*a*b^2*d+2*b^3*c)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(14/3)*3^(1/2)
```

3.263.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.95

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \frac{180b^{2/3}(b^2d-2abe+3a^2f)\,x^2+72b^{5/3}(be-2af)x^5+45b^{8/3}fx^8-\frac{120b^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{a+bx^3}+\frac{40\sqrt{3}(-2b^3c}{}}{}$$

input
```
Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(180*b^(2/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x^2 + 72*b^(5/3)*(b*e - 2*a*f)*x^
5 + 45*b^(8/3)*f*x^8 - (120*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^
2)/(a + b*x^3) + (40*Sqrt[3]*(-2*b^3*c + 5*a*b^2*d - 8*a^2*b*e + 11*a^3*f)
*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-2*b^3*c + 5*
a*b^2*d - 8*a^2*b*e + 11*a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/a^(1/3) + (20*(2
*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/a^(1/3))/(360*b^(14/3))
```

3.263.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2367, 25, 2029, 2375, 27, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \ 2367$$

$$-\frac{\int -\frac{3ab^4fx^{10}+3ab^3(be-af)x^7+3ab^2(fa^2-bea+b^2d)x^4+2ab(-fa^3+bea^2-b^2da+b^3c)x}{bx^3+a}dx}{3ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\,(a+bx^3)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3ab^4fx^{10}+3ab^3(be-af)x^7+3ab^2\left(fa^2-bea+b^2d\right)x^4+2ab\left(-fa^3+bea^2-b^2da+b^3c\right)x}{bx^3+a}dx}{3ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 2029$$

$$\frac{\int \frac{x\left(3ab^4fx^9+3ab^3(be-af)x^6+3ab^2\left(fa^2-bea+b^2d\right)x^3+2ab\left(-fa^3+bea^2-b^2da+b^3c\right)\right)}{bx^3+a}dx}{3ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 2375$$

$$\frac{\frac{\int \frac{8x\left(3ab^4(be-2af)x^6+3ab^3\left(fa^2-bea+b^2d\right)x^3+2ab^2\left(-fa^3+bea^2-b^2da+b^3c\right)\right)}{bx^3+a}dx}{8b}+\frac{3}{8}ab^3fx^8}{3ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{x\left(3ab^4(be-2af)x^6+3ab^3\left(fa^2-bea+b^2d\right)x^3+2ab^2\left(-fa^3+bea^2-b^2da+b^3c\right)\right)}{bx^3+a}dx}{b}+\frac{3}{8}ab^3fx^8}{3ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1812$$

$$\frac{\frac{\int \left(3ab^3(be-2af)x^4+3ab^2\left(3fa^2-2bea+b^2d\right)x+\frac{\left(2acb^5-5a^2db^4+8a^3eb^3-11a^4fb^2\right)x}{bx^3+a}\right)dx}{b}+\frac{3}{8}ab^3fx^8}{3ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{3}{2}ab^2x^2\left(3a^2f-2abe+b^2d\right)-\frac{a^{2/3}b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-11a^3f+8a^2be-5ab^2d+2b^3c\right)}{\sqrt{3}}+\frac{1}{6}a^{2/3}b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-11a^3f+8a\right.}{b}}{3ab^5}$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}$$

3.263. $\displaystyle\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

input
```
Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
-1/3*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(b^4*(a + b*x^3)) + ((3*a*b
^3*f*x^8)/8 + ((3*a*b^2*(b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/2 + (3*a*b^3*(b*e
 - 2*a*f)*x^5)/5 - (a^(2/3)*b^(4/3)*(2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (a^(2/
3)*b^(4/3)*(2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*Log[a^(1/3) + b^(1
/3)*x])/3 + (a^(2/3)*b^(4/3)*(2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/6)/b)/(3*a*b^5)
```

3.263.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

3.263. $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.263.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.53

method	result
risch	$\frac{f x^8}{8b^2} - \frac{2x^5 a f}{5b^3} + \frac{x^5 e}{5b^2} + \frac{3a^2 f x^2}{2b^4} - \frac{a e x^2}{b^3} + \frac{d x^2}{2b^2} + \frac{\left(\frac{1}{3} f a^3 - \frac{1}{3} a^2 b e + \frac{1}{3} a b^2 d - \frac{1}{3} b^3 c\right) x^2}{b^4 (b x^3 + a)} + \frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)} \frac{(-11 f a^3+8}{}}{9b^5}$
default	$\frac{\frac{b^2 f x^8}{8} + \frac{(-2afb+b^2 e) x^5}{5} + \frac{x^2 (3a^2 f - 2aeb + b^2 d)}{2}}{b^4} - \frac{\frac{\left(-\frac{1}{3} f a^3 + \frac{1}{3} a^2 b e - \frac{1}{3} a b^2 d + \frac{1}{3} b^3 c\right) x^2}{b x^3 + a} + \left(\frac{11}{3} f a^3 - \frac{8}{3} a^2 b e + \frac{5}{3} a b^2 d - \frac{2}{3} b^3 c\right) \left(- \frac{\ln\left(x + \left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right)} \right.}{b^4}$

input
```
int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*f*x^8/b^2-2/5/b^3*x^5*a*f+1/5/b^2*x^5*e+3/2/b^4*a^2*f*x^2-1/b^3*a*e*x^
2+1/2*d*x^2/b^2+(1/3*f*a^3-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x^2/b^4/(b*x
^3+a)+1/9/b^5*sum((-11*a^3*f+8*a^2*b*e-5*a*b^2*d+2*b^3*c)/_R*ln(x-_R),_R=R
ootOf(_Z^3*b+a))
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 920, normalized size of antiderivative = 3.09

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{45\,ab^5fx^{11}+9\,(8\,ab^5e-11\,a^2b^4f)x^8+36\,(5\,ab^5d-8\,a^2b^4e+11\,a^3b^3f)x^5-60\,(2\,ab^5c-5\,a^2b^4d+8\,a^3b^3}{}\right.$$

input
```
integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/360*(45*a*b^5*f*x^11 + 9*(8*a*b^5*e - 11*a^2*b^4*f)*x^8 + 36*(5*a*b^5*d
 - 8*a^2*b^4*e + 11*a^3*b^3*f)*x^5 - 60*(2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b
^3*e - 11*a^4*b^2*f)*x^2 - 60*sqrt(1/3)*(2*a^2*b^4*c - 5*a^3*b^3*d + 8*a^4
*b^2*e - 11*a^5*b*f + (2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*
f)*x^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x +
 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)
^(2/3)*x)/(b*x^3 + a)) + 20*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*
f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*lo
g(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(2*a*b^3*c - 5*a^2*b^2
*d + 8*a^3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*
f)*x^3)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^7*x^3 + a^2*b^6), 1/3
60*(45*a*b^5*f*x^11 + 9*(8*a*b^5*e - 11*a^2*b^4*f)*x^8 + 36*(5*a*b^5*d - 8
*a^2*b^4*e + 11*a^3*b^3*f)*x^5 - 60*(2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e
 - 11*a^4*b^2*f)*x^2 - 120*sqrt(1/3)*(2*a^2*b^4*c - 5*a^3*b^3*d + 8*a^4*b^
2*e - 11*a^5*b*f + (2*a*b^5*c - 5*a^2*b^4*d + 8*a^3*b^3*e - 11*a^4*b^2*f)*
x^3)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt(
(a*b^2)^(1/3)/a)/b) + 20*(2*a*b^3*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f +
 (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*f)*x^3)*(a*b^2)^(2/3)*log(b
^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*(2*a*b^3*c - 5*a^2*b^2*d
+ 8*a^3*b*e - 11*a^4*f + (2*b^4*c - 5*a*b^3*d + 8*a^2*b^2*e - 11*a^3*b*...
```

3.263.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.93

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= -\frac{(b^3c-ab^2d+a^2be-a^3f)x^2}{3\,(b^5x^3+ab^4)} + \frac{\sqrt{3}(2\,b^3c-5\,ab^2d+8\,a^2be-11\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5\,b^2fx^8+8\,(b^2e-2\,abf)x^5+20\,(b^2d-2\,abe+3\,a^2f)x^2}{40\,b^4} + \frac{(2\,b^3c-5\,ab^2d+8\,a^2be-11\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(2\,b^3c-5\,ab^2d+8\,a^2be-11\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2/(b^5*x^3 + a*b^4) + 1/9*sqrt(
3)*(2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/40*(5*b^2*f*x^8 + 8*(b^2*e
 - 2*a*b*f)*x^5 + 20*(b^2*d - 2*a*b*e + 3*a^2*f)*x^2)/b^4 + 1/18*(2*b^3*c
- 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(b^5*(a/b)^(1/3)) - 1/9*(2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*log(
x + (a/b)^(1/3))/(b^5*(a/b)^(1/3))
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.13

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{\sqrt{3}(2\,b^3c-5\,ab^2d+8\,a^2be-11\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}b^4} - \frac{(2\,b^3c-5\,ab^2d+8\,a^2be-11\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}b^4} - \frac{\left(2\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+8\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-11\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4} - \frac{b^3cx^2-ab^2dx^2+a^2bex^2-a^3fx^2}{3\,(bx^3+a)b^4} + \frac{5\,b^{14}fx^8+8\,b^{14}ex^5-16\,ab^{13}fx^5+20\,b^{14}dx^2-40\,ab^{13}ex^2+60\,a^2b^{12}fx^2}{40\,b^{16}}$$

input
```
integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(2*b^3*c - 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^4) - 1/18*(2*b^3*c
- 5*a*b^2*d + 8*a^2*b*e - 11*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/((-a*b^2)^(1/3)*b^4) - 1/9*(2*b^3*c*(-a/b)^(1/3) - 5*a*b^2*d*(-a/b)^(1/
3) + 8*a^2*b*e*(-a/b)^(1/3) - 11*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(
x - (-a/b)^(1/3)))/(a*b^4) - 1/3*(b^3*c*x^2 - a*b^2*d*x^2 + a^2*b*e*x^2 -
a^3*f*x^2)/((b*x^3 + a)*b^4) + 1/40*(5*b^14*f*x^8 + 8*b^14*e*x^5 - 16*a*b^
13*f*x^5 + 20*b^14*d*x^2 - 40*a*b^13*e*x^2 + 60*a^2*b^12*f*x^2)/b^16
```

3.263.9 Mupad [B] (verification not implemented)

Time = 10.32 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.96

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= x^5\left(\frac{e}{5\,b^2}-\frac{2\,a\,f}{5\,b^3}\right)-x^2\left(\frac{a^2\,f}{2\,b^4}-\frac{d}{2\,b^2}+\frac{a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)$$
$$+\frac{f\,x^8}{8\,b^2}-\frac{x^2\left(-\frac{f\,a^3}{3}+\frac{e\,a^2\,b}{3}-\frac{d\,a\,b^2}{3}+\frac{c\,b^3}{3}\right)}{b^5\,x^3+a\,b^4}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-11\,f\,a^3+8\,e\,a^2\,b-5\,d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{1/3}\,b^{14/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-11\,f\,a^3+8\,e\,a^2\,b-5\,d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{1/3}\,b^{14/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-11\,f\,a^3+8\,e\,a^2\,b-5\,d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{1/3}\,b^{14/3}}$$

input
```
int((x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

output
```
x^5*(e/(5*b^2) - (2*a*f)/(5*b^3)) - x^2*((a^2*f)/(2*b^4) - d/(2*b^2) + (a*
(e/b^2 - (2*a*f)/b^3))/b) + (f*x^8)/(8*b^2) - (x^2*((b^3*c)/3 - (a^3*f)/3
- (a*b^2*d)/3 + (a^2*b*e)/3))/(a*b^4 + b^5*x^3) - (log(b^(1/3)*x + a^(1/3)
)*(2*b^3*c - 11*a^3*f - 5*a*b^2*d + 8*a^2*b*e))/(9*a^(1/3)*b^(14/3)) + (lo
g(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*b^
3*c - 11*a^3*f - 5*a*b^2*d + 8*a^2*b*e))/(9*a^(1/3)*b^(14/3)) - (log(3^(1/
2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*b^3*c - 1
1*a^3*f - 5*a*b^2*d + 8*a^2*b*e))/(9*a^(1/3)*b^(14/3))
```

3.264 $\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$

3.264.1 Optimal result 1983
3.264.2 Mathematica [A] (verified) 1984
3.264.3 Rubi [A] (verified) 1984
3.264.4 Maple [C] (verified) 1986
3.264.5 Fricas [A] (verification not implemented) 1987
3.264.6 Sympy [A] (verification not implemented) 1988
3.264.7 Maxima [A] (verification not implemented) 1989
3.264.8 Giac [A] (verification not implemented) 1990
3.264.9 Mupad [B] (verification not implemented) 1991

3.264.1 Optimal result

Integrand size = 30, antiderivative size = 288

$$\begin{aligned}\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx &= \frac{(b^2d-2abe+3a^2f)\,x}{b^4}+\frac{(be-2af)x^4}{4b^3}+\frac{fx^7}{7b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3b^4\,(a+bx^3)}\\ &\quad-\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{13/3}}\\ &\quad+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{13/3}}\\ &\quad-\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{13/3}}\end{aligned}$$

output
```
(3*a^2*f-2*a*b*e+b^2*d)*x/b^4+1/4*(-2*a*f+b*e)*x^4/b^3+1/7*f*x^7/b^2-1/3*(
-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4/(b*x^3+a)+1/9*(-10*a^3*f+7*a^2*b*e-4*a
*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(13/3)-1/18*(-10*a^3*f+7*a^2
*b*e-4*a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^
(13/3)-1/9*(-10*a^3*f+7*a^2*b*e-4*a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(13/3)*3^(1/2)
```

3.264.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \frac{252\sqrt[3]{b}(b^2d-2abe+3a^2f)\,x+63b^{4/3}(be-2af)x^4+36b^{7/3}fx^7-\frac{84\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)x}{a+bx^3}+\frac{28\sqrt{3}(-b^3c+4ab}{\quad}}{\quad}$$

input
```
Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(252*b^(1/3)*(b^2*d - 2*a*b*e + 3*a^2*f)*x + 63*b^(4/3)*(b*e - 2*a*f)*x^4
+ 36*b^(7/3)*f*x^7 - (84*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a
 + b*x^3) + (28*Sqrt[3]*(-(b^3*c) + 4*a*b^2*d - 7*a^2*b*e + 10*a^3*f)*ArcT
an[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(b^3*c - 4*a*b^2*d
+ 7*a^2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b^3*c)
+ 4*a*b^2*d - 7*a^2*b*e + 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/a^(2/3))/(252*b^(13/3))
```

3.264.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2367, 25, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \; 2367$$

$$-\frac{\int -\frac{3ab^3fx^9+3ab^2(be-af)x^6+3ab\left(fa^2-bea+b^2d\right)x^3+a\left(-fa^3+bea^2-b^2da+b^3c\right)}{bx^3+a}dx}{3ab^4}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3ab^3fx^9+3ab^2(be-af)x^6+3ab(fa^2-bea+b^2d)x^3+a(-fa^3+bea^2-b^2da+b^3c)}{bx^3+a}dx}{3ab^4}-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4(a+bx^3)}$$

$$\Big\downarrow \; 2426$$

$$\frac{\int \left(3ab^2fx^6+3ab(be-2af)x^3+3a(3fa^2-2bea+b^2d)+\frac{-10fa^4+7bea^3-4b^2da^2+b^3ca}{bx^3+a}\right)dx}{3ab^4}-\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4(a+bx^3)}$$

$$\Big\downarrow \; 2009$$

$$\frac{3ax(3a^2f-2abe+b^2d)-\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-10a^3f+7a^2be-4ab^2d+b^3c)}{\sqrt{3}\sqrt[3]{b}}+\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(-10a^3f+7a^2be-4ab^2d}{3\sqrt[3]{b}}}{3ab^4}$$

$$\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4(a+bx^3)}$$

input
```
Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
-1/3*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(b^4*(a + b*x^3)) + (3*a*(b^2
*d - 2*a*b*e + 3*a^2*f)*x + (3*a*b*(b*e - 2*a*f)*x^4)/4 + (3*a*b^2*f*x^7)/
7 - (a^(1/3)*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3)) + (a^(1/3)*(b^3*c - 4*a
*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(1/3)) - (a^
(1/3)*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(6*b^(1/3)))/(3*a*b^4)
```

3.264.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.264.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.51

method	result
risch	$\frac{f x^7}{7b^2} - \frac{x^4 a f}{2b^3} + \frac{x^4 e}{4b^2} + \frac{3a^2 f x}{b^4} - \frac{2aex}{b^3} + \frac{dx}{b^2} + \frac{\left(\frac{1}{3} f a^3 - \frac{1}{3} a^2 b e + \frac{1}{3} a b^2 d - \frac{1}{3} b^3 c\right) x}{b^4 (b x^3 + a)} + \frac{\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)} \frac{\left(-10 f a^3 + 7a^2 b e - \cdots\right.}{\cdots}}{9b^5}$
default	$\frac{\frac{1}{7} b^2 f x^7 - \frac{1}{2} a b f x^4 + \frac{1}{4} b^2 e x^4 + 3a^2 f x - 2abex + b^2 dx}{b^4} - \frac{\frac{\left(-\frac{1}{3} f a^3 + \frac{1}{3} a^2 b e - \frac{1}{3} a b^2 d + \frac{1}{3} b^3 c\right) x}{b x^3 + a} + \left(10 f a^3 - 7a^2 b e + 4a b^2 d - b^3 c\right) \left(\frac{\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b \left(\frac{a}{b}\right)^{\frac{2}{3}}} \cdots\right)}{b^4}$

input
```
int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*f*x^7/b^2-1/2/b^3*x^4*a*f+1/4/b^2*x^4*e+3/b^4*a^2*f*x-2/b^3*a*e*x+d*x/
b^2+(1/3*f*a^3-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x/b^4/(b*x^3+a)+1/9/b^5*
sum((-10*a^3*f+7*a^2*b*e-4*a*b^2*d+b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a
))
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 946, normalized size of antiderivative = 3.28

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{36\,a^2b^4fx^{10} + 9\,(7\,a^2b^4e - 10\,a^3b^3f)x^7 + 63\,(4\,a^2b^4d - 7\,a^3b^3e + 10\,a^4b^2f)x^4 - 42\,\sqrt{\frac{1}{3}}(a^2b^4c - 4\,a^3b^3d + \cdots}{\cdots}\right.$$

input
```
integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/252*(36*a^2*b^4*f*x^10 + 9*(7*a^2*b^4*e - 10*a^3*b^3*f)*x^7 + 63*(4*a^2
*b^4*d - 7*a^3*b^3*e + 10*a^4*b^2*f)*x^4 - 42*sqrt(1/3)*(a^2*b^4*c - 4*a^3
*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f + (a*b^5*c - 4*a^2*b^4*d + 7*a^3*b^3*e -
 10*a^4*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/
3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*
a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(a*b^3*c - 4*a^2*b^2*d + 7*a^
3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*(-a
^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^3
*c - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e
 - 10*a^3*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(a^2*b
^4*c - 4*a^3*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f)*x)/(a^2*b^6*x^3 + a^3*b^5),
 1/252*(36*a^2*b^4*f*x^10 + 9*(7*a^2*b^4*e - 10*a^3*b^3*f)*x^7 + 63*(4*a^2
*b^4*d - 7*a^3*b^3*e + 10*a^4*b^2*f)*x^4 + 84*sqrt(1/3)*(a^2*b^4*c - 4*a^3
*b^3*d + 7*a^4*b^2*e - 10*a^5*b*f + (a*b^5*c - 4*a^2*b^4*d + 7*a^3*b^3*e -
 10*a^4*b^2*f)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(
2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 14*(a*b^3*c - 4*
a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a
^3*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3
)*a) + 28*(a*b^3*c - 4*a^2*b^2*d + 7*a^3*b*e - 10*a^4*f + (b^4*c - 4*a*b^3
*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)...
```

3.264.6 Sympy [A] (verification not implemented)

Time = 67.31 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.39

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= x^4\left(-\frac{af}{2b^3}+\frac{e}{4b^2}\right)+x\left(\frac{3a^2f}{b^4}-\frac{2ae}{b^3}+\frac{d}{b^2}\right)+\frac{x(a^3f-a^2be+ab^2d-b^3c)}{3ab^4+3b^5x^3}$$
$$+\text{RootSum}\left(729t^3a^2b^{13}+1000a^9f^3-2100a^8bef^2+1200a^7b^2df^2+1470a^7b^2e^2f-300a^6b^3cf^2-1680a^6\right.$$
$$+\frac{fx^7}{7b^2}$$

input
```
integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
x**4*(-a*f/(2*b**3) + e/(4*b**2)) + x*(3*a**2*f/b**4 - 2*a*e/b**3 + d/b**2
) + x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(3*a*b**4 + 3*b**5*x**3) + R
ootSum(729*_t**3*a**2*b**13 + 1000*a**9*f**3 - 2100*a**8*b*e*f**2 + 1200*a
**7*b**2*d*f**2 + 1470*a**7*b**2*e**2*f - 300*a**6*b**3*c*f**2 - 1680*a**6
*b**3*d*e*f - 343*a**6*b**3*e**3 + 420*a**5*b**4*c*e*f + 480*a**5*b**4*d**
2*f + 588*a**5*b**4*d*e**2 - 240*a**4*b**5*c*d*f - 147*a**4*b**5*c*e**2 -
336*a**4*b**5*d**2*e + 30*a**3*b**6*c**2*f + 168*a**3*b**6*c*d*e + 64*a**3
*b**6*d**3 - 21*a**2*b**7*c**2*e - 48*a**2*b**7*c*d**2 + 12*a*b**8*c**2*d
- b**9*c**3, Lambda(_t, _t*log(-9*_t*a*b**4/(10*a**3*f - 7*a**2*b*e + 4*a*
b**2*d - b**3*c) + x))) + f*x**7/(7*b**2)
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.94

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = -\frac{(b^3c-ab^2d+a^2be-a^3f)x}{3\,(b^5x^3+ab^4)}+\frac{4\,b^2fx^7+7\,(b^2e-2\,abf)x^4+28\,(b^2d-2\,abe+3\,a^2f)x}{28\,b^4} + \frac{\sqrt{3}(b^3c-4\,ab^2d+7\,a^2be-10\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^3c-4\,ab^2d+7\,a^2be-10\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^3c-4\,ab^2d+7\,a^2be-10\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(b^5*x^3 + a*b^4) + 1/28*(4*b^2
*f*x^7 + 7*(b^2*e - 2*a*b*f)*x^4 + 28*(b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 +
 1/9*sqrt(3)*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*arctan(1/3*sqrt(3)
*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) - 1/18*(b^3*c - 4*a*b^
2*d + 7*a^2*b*e - 10*a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5*(a
/b)^(2/3)) + 1/9*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*log(x + (a/b)^
(1/3))/(b^5*(a/b)^(2/3))
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= -\frac{\sqrt{3}(b^3c-4\,ab^2d+7\,a^2be-10\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}b^3} - \frac{(b^3c-4\,ab^2d+7\,a^2be-10\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}b^3} - \frac{(b^3c-4\,ab^2d+7\,a^2be-10\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4} - \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{3\,(bx^3+a)b^4} + \frac{4\,b^{12}fx^7+7\,b^{12}ex^4-14\,ab^{11}fx^4+28\,b^{12}dx-56\,ab^{11}ex+84\,a^2b^{10}fx}{28\,b^{14}}$$

input
```
integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^3) - 1/18*(b^3*c - 4
*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
((-a*b^2)^(2/3)*b^3) - 1/9*(b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 1/3*(b^3*c*x - a*b^2*d*x + a
^2*b*e*x - a^3*f*x)/((b*x^3 + a)*b^4) + 1/28*(4*b^12*f*x^7 + 7*b^12*e*x^4
- 14*a*b^11*f*x^4 + 28*b^12*d*x - 56*a*b^11*e*x + 84*a^2*b^10*f*x)/b^14
```

3.264.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= x^4\left(\frac{e}{4\,b^2}-\frac{a\,f}{2\,b^3}\right)-x\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)-\frac{x\,\left(-\frac{f\,a^3}{3}+\frac{e\,a^2\,b}{3}-\frac{d\,a\,b^2}{3}+\frac{c\,b^3}{3}\right)}{b^5\,x^3+a\,b^4}$$
$$+\frac{f\,x^7}{7\,b^2}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-10\,f\,a^3+7\,e\,a^2\,b-4\,d\,a\,b^2+c\,b^3\right)}{9\,a^{2/3}\,b^{13/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-10\,f\,a^3+7\,e\,a^2\,b-4\,d\,a\,b^2+c\,b^3\right)}{9\,a^{2/3}\,b^{13/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-10\,f\,a^3+7\,e\,a^2\,b-4\,d\,a\,b^2+c\,b^3\right)}{9\,a^{2/3}\,b^{13/3}}$$

input
```
int((x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

output
```
x^4*(e/(4*b^2) - (a*f)/(2*b^3)) - x*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (
2*a*f)/b^3))/b) - (x*((b^3*c)/3 - (a^3*f)/3 - (a*b^2*d)/3 + (a^2*b*e)/3))/
(a*b^4 + b^5*x^3) + (f*x^7)/(7*b^2) + (log(b^(1/3)*x + a^(1/3))*(b^3*c - 1
0*a^3*f - 4*a*b^2*d + 7*a^2*b*e))/(9*a^(2/3)*b^(13/3)) + (log(3^(1/2)*a^(1
/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c - 10*a^3*f -
 4*a*b^2*d + 7*a^2*b*e))/(9*a^(2/3)*b^(13/3)) - (log(3^(1/2)*a^(1/3)*1i -
2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c - 10*a^3*f - 4*a*b^2*
d + 7*a^2*b*e))/(9*a^(2/3)*b^(13/3))
```

3.265 $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

3.265.1 Optimal result . . . 1992
3.265.2 Mathematica [A] (verified) . . . 1993
3.265.3 Rubi [A] (verified) . . . 1993
3.265.4 Maple [C] (verified) . . . 1995
3.265.5 Fricas [A] (verification not implemented) . . . 1996
3.265.6 Sympy [F(-1)] . . . 1997
3.265.7 Maxima [A] (verification not implemented) . . . 1998
3.265.8 Giac [A] (verification not implemented) . . . 1999
3.265.9 Mupad [B] (verification not implemented) . . . 2000

3.265.1 Optimal result

Integrand size = 28, antiderivative size = 271

$$\begin{aligned}\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx &= \frac{(be-2af)x^2}{2b^3}+\frac{fx^5}{5b^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3ab^3(a+bx^3)} \\ &\quad -\frac{(b^3c+2ab^2d-5a^2be+8a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{11/3}} \\ &\quad -\frac{(b^3c+2ab^2d-5a^2be+8a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{11/3}} \\ &\quad +\frac{(b^3c+2ab^2d-5a^2be+8a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{11/3}}\end{aligned}$$

output
```
1/2*(-2*a*f+b*e)*x^2/b^3+1/5*f*x^5/b^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
x^2/a/b^3/(b*x^3+a)-1/9*(8*a^3*f-5*a^2*b*e+2*a*b^2*d+b^3*c)*ln(a^(1/3)+b^(
1/3)*x)/a^(4/3)/b^(11/3)+1/18*(8*a^3*f-5*a^2*b*e+2*a*b^2*d+b^3*c)*ln(a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(11/3)-1/9*(8*a^3*f-5*a^2*b*e+
2*a*b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)
/b^(11/3)*3^(1/2)
```

3.265.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.94

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \frac{45b^{2/3}(be-2af)x^2+18b^{5/3}fx^5+\frac{30b^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{a(a+bx^3)}-\frac{10\sqrt{3}(b^3c+2ab^2d-5a^2be+8a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}-}{90b^{11/3}}$$

input
```
Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
(45*b^(2/3)*(b*e - 2*a*f)*x^2 + 18*b^(5/3)*f*x^5 + (30*b^(2/3)*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*x^2)/(a*(a + b*x^3)) - (10*Sqrt[3]*(b^3*c + 2*a*b
^2*d - 5*a^2*b*e + 8*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a
^(4/3) - (10*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(1/3) + b^(1/
3)*x]])/a^(4/3) + (5*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(90*b^(11/3))
```

3.265.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2367, 25, 2028, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \text{ 2367}$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}-\frac{\int -\frac{3ab^3fx^7+3ab^2(be-af)x^4+b(2fa^3-2bea^2+2b^2da+b^3c)x}{bx^3+a}dx}{3ab^4}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3ab^3fx^7+3ab^2(be-af)x^4+b(2fa^3-2bea^2+2b^2da+b^3c)x}{bx^3+a}dx}{3ab^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 2028}$$

$$\frac{\int \frac{x(3ab^3fx^6+3ab^2(be-af)x^3+b(2fa^3-2bea^2+2b^2da+b^3c))}{bx^3+a}dx}{3ab^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 1812}$$

$$\frac{\int \left(3ab^2fx^4+3ab(be-2af)x+\frac{(cb^4+2adb^3-5a^2eb^2+8a^3fb)x}{bx^3+a}\right)dx}{3ab^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}+$$

$$\frac{-\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(8a^3f-5a^2be+2ab^2d+b^3c)}{\sqrt{3}\sqrt[3]{a}}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(8a^3f-5a^2be+2ab^2d+b^3c)}{3\sqrt[3]{a}}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b\right.}{}}{3ab^4}$$

input
```
Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x]
```

output
```
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a*b^3*(a + b*x^3)) + ((3*a*b*
(b*e - 2*a*f)*x^2)/2 + (3*a*b^2*f*x^5)/5 - (b^(1/3)*(b^3*c + 2*a*b^2*d - 5
*a^2*b*e + 8*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(1/3)) - (b^(1/3)*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*Log[a^
(1/3) + b^(1/3)*x])/(3*a^(1/3)) + (b^(1/3)*(b^3*c + 2*a*b^2*d - 5*a^2*b*e
+ 8*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)))/(3
*a*b^4)
```

3.265.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.265. $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

3.265.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.48

method	result
risch	$\frac{f x^5}{5b^2}-\frac{x^2 a f}{b^3}+\frac{e x^2}{2b^2}-\frac{(f a^3-a^2 be+a b^2 d-b^3 c)x^2}{3a b^3(b x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(8f a^3-5a^2 be+2a b^2 d+b^3 c)\ln(x-\_R)}{\_R}}{9b^4 a}$
default	$-\frac{-\frac{bf x^5}{5}+\frac{(2af-be)x^2}{2}}{b^3}+\frac{-\frac{(f a^3-a^2be+a b^2 d-b^3 c)x^2}{3a(b x^3+a)}+\frac{(8f a^3-5a^2be+2a b^2 d+b^3 c)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\ a}{}\right.}{3a}}{b^3}$

input
```
int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*f*x^5/b^2-1/b^3*x^2*a*f+1/2/b^2*e*x^2-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c
)/a*x^2/b^3/(b*x^3+a)+1/9/b^4/a*sum((8*a^3*f-5*a^2*b*e+2*a*b^2*d+b^3*c)/_R
*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 874, normalized size of antiderivative = 3.23

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{18\,a^2b^4fx^8 + 9\,(5\,a^2b^4e - 8\,a^3b^3f)x^5 + 15\,(2\,ab^5c - 2\,a^2b^4d + 5\,a^3b^3e - 8\,a^4b^2f)x^2 + 15\,\sqrt{\frac{1}{3}}(a^2b^4c + 2\,a^3}{}\right.$$

input
```
integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/90*(18*a^2*b^4*f*x^8 + 9*(5*a^2*b^4*e - 8*a^3*b^3*f)*x^5 + 15*(2*a*b^5*
c - 2*a^2*b^4*d + 5*a^3*b^3*e - 8*a^4*b^2*f)*x^2 + 15*sqrt(1/3)*(a^2*b^4*c
 + 2*a^3*b^3*d - 5*a^4*b^2*e + 8*a^5*b*f + (a*b^5*c + 2*a^2*b^4*d - 5*a^3*
b^3*e + 8*a^4*b^2*f)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*
sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^
(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 5*(a*b^3*c + 2*a^2*b^2*d - 5
*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-
a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*(a*b^
3*c + 2*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e
 + 8*a^3*b*f)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^3
+ a^3*b^5), 1/90*(18*a^2*b^4*f*x^8 + 9*(5*a^2*b^4*e - 8*a^3*b^3*f)*x^5 + 1
5*(2*a*b^5*c - 2*a^2*b^4*d + 5*a^3*b^3*e - 8*a^4*b^2*f)*x^2 + 30*sqrt(1/3)
*(a^2*b^4*c + 2*a^3*b^3*d - 5*a^4*b^2*e + 8*a^5*b*f + (a*b^5*c + 2*a^2*b^4
*d - 5*a^3*b^3*e + 8*a^4*b^2*f)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1
/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 5*(a*b^3*c + 2*a
^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d - 5*a^2*b^2*e + 8*a^3*
b*f)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)
) - 10*(a*b^3*c + 2*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f + (b^4*c + 2*a*b^3*d -
 5*a^2*b^2*e + 8*a^3*b*f)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(
a^2*b^6*x^3 + a^3*b^5)]
```

3.265.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.265.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.96

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)x^2}{3\,(ab^4x^3+a^2b^3)} + \frac{2\,bfx^5+5\,(be-2\,af)x^2}{10\,b^3} + \frac{\sqrt{3}(b^3c+2\,ab^2d-5\,a^2be+8\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(b^3c+2\,ab^2d-5\,a^2be+8\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(b^3c+2\,ab^2d-5\,a^2be+8\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2/(a*b^4*x^3 + a^2*b^3) + 1/10*(
2*b*f*x^5 + 5*(b*e - 2*a*f)*x^2)/b^3 + 1/9*sqrt(3)*(b^3*c + 2*a*b^2*d - 5*
a^2*b*e + 8*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*
b^4*(a/b)^(1/3)) + 1/18*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^4*(a/b)^(1/3)) - 1/9*(b^3*c + 2*a*b^2*
d - 5*a^2*b*e + 8*a^3*f)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(1/3))
```

3.265.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.15

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= \frac{\sqrt{3}(b^3c+2\,ab^2d-5\,a^2be+8\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}ab^3} - \frac{(b^3c+2\,ab^2d-5\,a^2be+8\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}ab^3} - \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+8\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3} + \frac{b^3cx^2-ab^2dx^2+a^2bex^2-a^3fx^2}{3\,(bx^3+a)ab^3} + \frac{2\,b^8fx^5+5\,b^8ex^2-10\,ab^7fx^2}{10\,b^{10}}$$

input
```
integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(b^3*c + 2*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a*b^3) - 1/18*(b^3*c + 2
*a*b^2*d - 5*a^2*b*e + 8*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(
(-a*b^2)^(1/3)*a*b^3) - 1/9*(b^3*c*(-a/b)^(1/3) + 2*a*b^2*d*(-a/b)^(1/3) -
 5*a^2*b*e*(-a/b)^(1/3) + 8*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/(a^2*b^3) + 1/3*(b^3*c*x^2 - a*b^2*d*x^2 + a^2*b*e*x^2 - a^3
*f*x^2)/((b*x^3 + a)*a*b^3) + 1/10*(2*b^8*f*x^5 + 5*b^8*e*x^2 - 10*a*b^7*f
*x^2)/b^10
```

3.265.9 Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.91

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^2}\,dx$$
$$= x^2\left(\frac{e}{2\,b^2}-\frac{a\,f}{b^3}\right)+\frac{f\,x^5}{5\,b^2}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(8\,f\,a^3-5\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{9\,a^{4/3}\,b^{11/3}}$$
$$+\frac{x^2\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a\,\left(b^4\,x^3+a\,b^3\right)}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(8\,f\,a^3-5\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{9\,a^{4/3}\,b^{11/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(8\,f\,a^3-5\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{9\,a^{4/3}\,b^{11/3}}$$

input
```
int((x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^2,x)
```

output
```
x^2*(e/(2*b^2) - (a*f)/b^3) + (f*x^5)/(5*b^2) - (log(b^(1/3)*x + a^(1/3))*
(b^3*c + 8*a^3*f + 2*a*b^2*d - 5*a^2*b*e))/(9*a^(4/3)*b^(11/3)) + (x^2*(b^
3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a*(a*b^3 + b^4*x^3)) + (log(3^(1/2)*a
^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*c + 8*a^3*f
 + 2*a*b^2*d - 5*a^2*b*e))/(9*a^(4/3)*b^(11/3)) - (log(3^(1/2)*a^(1/3)*1i
- 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(b^3*c + 8*a^3*f + 2*a*b^2
*d - 5*a^2*b*e))/(9*a^(4/3)*b^(11/3))
```

3.266 $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx$

3.266.1 Optimal result 2001
3.266.2 Mathematica [A] (verified) 2002
3.266.3 Rubi [A] (verified) 2002
3.266.4 Maple [C] (verified) 2007
3.266.5 Fricas [A] (verification not implemented) 2008
3.266.6 Sympy [A] (verification not implemented) 2009
3.266.7 Maxima [A] (verification not implemented) 2010
3.266.8 Giac [A] (verification not implemented) 2011
3.266.9 Mupad [B] (verification not implemented) 2011

3.266.1 Optimal result

Integrand size = 27, antiderivative size = 264

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx = \frac{(be-2af)x}{b^3}+\frac{fx^4}{4b^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3ab^3\,(a+bx^3)} - \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}} + \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} - \frac{(2b^3c+ab^2d-4a^2be+7a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}$$

output
```
(-2*a*f+b*e)*x/b^3+1/4*f*x^4/b^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a/b^
3/(b*x^3+a)+1/9*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/
a^(5/3)/b^(10/3)-1/18*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)*ln(a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(10/3)-1/9*(7*a^3*f-4*a^2*b*e+a*b^2*d+
2*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(10/3
)*3^(1/2)
```

3.266.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx$$

$$= \frac{36\sqrt[3]{b}(be-2af)x+9b^{4/3}fx^4+\frac{12\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)x}{a(a+bx^3)}-\frac{4\sqrt{3}(2b^3c+ab^2d-4a^2be+7a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{4(2b^3\cdots}{\cdots}}{36b^{10/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

output
```
(36*b^(1/3)*(b*e - 2*a*f)*x + 9*b^(4/3)*f*x^4 + (12*b^(1/3)*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*x)/(a*(a + b*x^3)) - (4*Sqrt[3]*(2*b^3*c + a*b^2*d -
 4*a^2*b*e + 7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3)
 + (4*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/
a^(5/3) - (2*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b^(10/3))
```

3.266.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.82, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {2397, 25, 1741, 27, 913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \ 2397$$

$$\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3ab^3(a+bx^3)}-\frac{\int -\frac{3ab^2fx^6+3ab(be-af)x^3+2b^3c+ab^2d-a^2be+a^3f}{bx^3+a}dx}{3ab^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{3ab^2fx^6+3ab(be-af)x^3+2b^3c+ab^2d-a^2be+a^3f}{bx^3+a}dx}{3ab^3}+\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{3ab^3(a+bx^3)}$$

$$\Big\downarrow \text{1741}$$

$$\frac{\frac{\int \frac{4b\left(fa^3-bea^2+3b(be-2af)x^3a+b^2da+2b^3c\right)}{bx^3+a}dx}{4b}+\frac{3}{4}abfx^4}{3ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{fa^3-bea^2+3b(be-2af)x^3a+b^2da+2b^3c}{bx^3+a}dx+\frac{3}{4}abfx^4}{3ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{913}$$

$$\frac{\left(7a^3f-4a^2be+ab^2d+2b^3c\right)\int \frac{1}{bx^3+a}dx+3ax(be-2af)+\frac{3}{4}abfx^4}{3ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{750}$$

$$\frac{\left(7a^3f-4a^2be+ab^2d+2b^3c\right)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)+3ax(be-2af)+\frac{3}{4}abfx^4}{3ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{16}$$

$$\frac{\left(7a^3f-4a^2be+ab^2d+2b^3c\right)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+3ax(be-2af)+\frac{3}{4}abfx^4}{3ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1142}$$

3.266. $\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^2}\,dx$

$$\frac{(7a^3f-4a^2be+ab^2d+2b^3c)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+3ax(\ldots}{3ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$\Big\downarrow$ 25

$$\frac{(7a^3f-4a^2be+ab^2d+2b^3c)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+3ax(be\ldots}{3ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$\Big\downarrow$ 27

$$\frac{(7a^3f-4a^2be+ab^2d+2b^3c)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+3ax(\ldots}{3ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$\Big\downarrow$ 1082

3.266. $\int\frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^2}\,dx$

$$\frac{\left(7a^3f-4a^2be+ab^2d+2b^3c\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+3ax}{3ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$\downarrow$ 217

$$\frac{\left(7a^3f-4a^2be+ab^2d+2b^3c\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+3ax(be-2af)}{3ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}$$

$\downarrow$ 1103

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3ab^3\left(a+bx^3\right)}+$$

$$\frac{\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)\left(7a^3f-4a^2be+ab^2d+2b^3c\right)+3ax(be-2af)}{3ab^3}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x]
```

3.266. $\int\frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^2}\,dx$

output
```
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a*b^3*(a + b*x^3)) + (3*a*(b*e
- 2*a*f)*x + (3*a*b*f*x^4)/4 + (2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*(
Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(3*a*b^3)
```

3.266.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.266. $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2} dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

3.266.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.47

method	result
risch	$\frac{f x^4}{4b^2} - \frac{2xaf}{b^3} + \frac{ex}{b^2} - \frac{(f a^3-a^2be+a b^2d-b^3c)x}{3a b^3(b x^3+a)} + \frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)} \frac{(7f a^3-4a^2be+a b^2d+2b^3c)\ln(x-\_R)}{\_R^2}}{9b^4a}$
default	$-\frac{-\frac{1}{4}bf x^4+2afx-bex}{b^3} + \frac{-\frac{(f a^3-a^2be+a b^2d-b^3c)x}{3a(b x^3+a)} + \frac{(7f a^3-4a^2be+a b^2d+2b^3c)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\,\mathrm{arct}}{}\right.}{3a}}{b^3}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*f*x^4/b^2-2/b^3*x*a*f+1/b^2*e*x-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a*x/
b^3/(b*x^3+a)+1/9/b^4/a*sum((7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)/_R^2*ln(x-
_R),_R=RootOf(_Z^3*b+a))
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 861, normalized size of antiderivative = 3.26

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{9\,a^3b^3fx^7 + 9\,(4\,a^3b^3e - 7\,a^4b^2f)x^4 + 6\,\sqrt{\frac{1}{3}}(2\,a^2b^4c + a^3b^3d - 4\,a^4b^2e + 7\,a^5bf + (2\,ab^5c + a^2b^4d - 4\,a^3b}{}\right.$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/36*(9*a^3*b^3*f*x^7 + 9*(4*a^3*b^3*e - 7*a^4*b^2*f)*x^4 + 6*sqrt(1/3)*(
2*a^2*b^4*c + a^3*b^3*d - 4*a^4*b^2*e + 7*a^5*b*f + (2*a*b^5*c + a^2*b^4*d
 - 4*a^3*b^3*e + 7*a^4*b^2*f)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 -
 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a
^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 2*(2*a*b^3*c + a^2*b
^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)
*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(
2*a*b^3*c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b
^2*e + 7*a^3*b*f)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*
b^4*c - a^3*b^3*d + 4*a^4*b^2*e - 7*a^5*b*f)*x)/(a^3*b^5*x^3 + a^4*b^4), 1
/36*(9*a^3*b^3*f*x^7 + 9*(4*a^3*b^3*e - 7*a^4*b^2*f)*x^4 + 12*sqrt(1/3)*(2
*a^2*b^4*c + a^3*b^3*d - 4*a^4*b^2*e + 7*a^5*b*f + (2*a*b^5*c + a^2*b^4*d
- 4*a^3*b^3*e + 7*a^4*b^2*f)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(
2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(2*a*b
^3*c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3*d - 4*a^2*b^2*e
+ 7*a^3*b*f)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1
/3)*a) + 4*(2*a*b^3*c + a^2*b^2*d - 4*a^3*b*e + 7*a^4*f + (2*b^4*c + a*b^3
*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)
) + 12*(a^2*b^4*c - a^3*b^3*d + 4*a^4*b^2*e - 7*a^5*b*f)*x)/(a^3*b^5*x^3 +
 a^4*b^4)]
```

3.266.6 Sympy [A] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.43

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx = x\left(-\frac{2af}{b^3}+\frac{e}{b^2}\right)+\frac{x(-a^3f+a^2be-ab^2d+b^3c)}{3a^2b^3+3ab^4x^3}$$
$$+\text{RootSum}\left(729t^3a^5b^{10}-343a^9f^3+588a^8bef^2-147a^7b^2df^2-336a^7b^2e^2f-294a^6b^3cf^2+168a^6b^3def\right.$$
$$+\frac{fx^4}{4b^2}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
x*(-2*a*f/b**3 + e/b**2) + x*(-a**3*f + a**2*b*e - a*b**2*d + b**3*c)/(3*a
**2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**3*a**5*b**10 - 343*a**9*f**3 +
 588*a**8*b*e*f**2 - 147*a**7*b**2*d*f**2 - 336*a**7*b**2*e**2*f - 294*a**
6*b**3*c*f**2 + 168*a**6*b**3*d*e*f + 64*a**6*b**3*e**3 + 336*a**5*b**4*c*
e*f - 21*a**5*b**4*d**2*f - 48*a**5*b**4*d*e**2 - 84*a**4*b**5*c*d*f - 96*
a**4*b**5*c*e**2 + 12*a**4*b**5*d**2*e - 84*a**3*b**6*c**2*f + 48*a**3*b**
6*c*d*e - a**3*b**6*d**3 + 48*a**2*b**7*c**2*e - 6*a**2*b**7*c*d**2 - 12*a
*b**8*c**2*d - 8*b**9*c**3, Lambda(_t, _t*log(9*_t*a**2*b**3/(7*a**3*f - 4
*a**2*b*e + a*b**2*d + 2*b**3*c) + x))) + f*x**4/(4*b**2)
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)x}{3\,(ab^4x^3+a^2b^3)} + \frac{bfx^4+4\,(be-2\,af)x}{4\,b^3} + \frac{\sqrt{3}(2\,b^3c+ab^2d-4\,a^2be+7\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2\,b^3c+ab^2d-4\,a^2be+7\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2\,b^3c+ab^2d-4\,a^2be+7\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(a*b^4*x^3 + a^2*b^3) + 1/4*(b*f
*x^4 + 4*(b*e - 2*a*f)*x)/b^3 + 1/9*sqrt(3)*(2*b^3*c + a*b^2*d - 4*a^2*b*e
 + 7*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4*(a/
b)^(2/3)) - 1/18*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*log(x^2 - x*(a/
b)^(1/3) + (a/b)^(2/3))/(a*b^4*(a/b)^(2/3)) + 1/9*(2*b^3*c + a*b^2*d - 4*a
^2*b*e + 7*a^3*f)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(2\,b^3c+ab^2d-4\,a^2be+7\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^2} - \frac{(2\,b^3c+ab^2d-4\,a^2be+7\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}ab^2} - \frac{(2\,b^3c+ab^2d-4\,a^2be+7\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3} + \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{3\,(bx^3+a)ab^3} + \frac{b^6fx^4+4\,b^6ex-8\,ab^5fx}{4\,b^8}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1/18*(2*b^3*c
+ a*b^2*d - 4*a^2*b*e + 7*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
((-a*b^2)^(2/3)*a*b^2) - 1/9*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3) + 1/3*(b^3*c*x - a*b^2*d*x
+ a^2*b*e*x - a^3*f*x)/((b*x^3 + a)*a*b^3) + 1/4*(b^6*f*x^4 + 4*b^6*e*x -
8*a*b^5*f*x)/b^8
```

3.266.9 Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx = x\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)+\frac{f\,x^4}{4\,b^2}+\frac{x\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a\,\left(b^4\,x^3+a\,b^3\right)} + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(7\,f\,a^3-4\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{5/3}\,b^{10/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,f\,a^3-4\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{5/3}\,b^{10/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,f\,a^3-4\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{9\,a^{5/3}\,b^{10/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^2,x)
```

output
```
x*(e/b^2 - (2*a*f)/b^3) + (f*x^4)/(4*b^2) + (x*(b^3*c - a^3*f - a*b^2*d +
a^2*b*e))/(3*a*(a*b^3 + b^4*x^3)) + (log(b^(1/3)*x + a^(1/3))*(2*b^3*c + 7
*a^3*f + a*b^2*d - 4*a^2*b*e))/(9*a^(5/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)
*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*b^3*c + 7*a^3*f + a
*b^2*d - 4*a^2*b*e))/(9*a^(5/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^
(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*b^3*c + 7*a^3*f + a*b^2*d - 4
*a^2*b*e))/(9*a^(5/3)*b^(10/3))
```

3.266. $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^2}\,dx$

3.267 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx$

3.267.1 Optimal result 2013
3.267.2 Mathematica [A] (verified) 2014
3.267.3 Rubi [A] (verified) 2015
3.267.4 Maple [A] (verified) 2016
3.267.5 Fricas [A] (verification not implemented) 2017
3.267.6 Sympy [A] (verification not implemented) 2018
3.267.7 Maxima [A] (verification not implemented) 2019
3.267.8 Giac [A] (verification not implemented) 2020
3.267.9 Mupad [B] (verification not implemented) 2021

3.267.1 Optimal result

Integrand size = 30, antiderivative size = 265

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx = -\frac{c}{a^2x}+\frac{fx^2}{2b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^2b^2(a+bx^3)} + \frac{(4b^3c-ab^2d-2a^2be+5a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{8/3}} + \frac{(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{8/3}} - \frac{(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{8/3}}$$

output
```
-c/a^2/x+1/2*f*x^2/b^2-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^2/b^2/(b*x
^3+a)+1/9*(5*a^3*f-2*a^2*b*e-a*b^2*d+4*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3
)/b^(8/3)-1/18*(5*a^3*f-2*a^2*b*e-a*b^2*d+4*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(8/3)+1/9*(5*a^3*f-2*a^2*b*e-a*b^2*d+4*b^3*c)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(8/3)*3^(1/2)
```

3.267.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx$$
$$=\frac{1}{18}\left(-\frac{18c}{a^2x}+\frac{9fx^2}{b^2}+\frac{6(-b^3c+ab^2d-a^2be+a^3f)\,x^2}{a^2b^2(a+bx^3)}+\frac{2\sqrt{3}(4b^3c-ab^2d-2a^2be+5a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}b^{8/3}}+\frac{2(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{7/3}b^{8/3}}-\frac{(4b^3c-ab^2d-2a^2be+5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{7/3}b^{8/3}}\right)$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```

output
```
((-18*c)/(a^2*x) + (9*f*x^2)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*x^2)/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(4*b^3*c - a*b^2*d - 2*a^2*b*e
+ 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(7/3)*b^(8/3))
+ (2*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(
a^(7/3)*b^(8/3)) - ((4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(7/3)*b^(8/3)))/18
```

3.267.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2368, 25, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^2}dx$$

$$\Big\downarrow \ 2368$$

$$-\frac{\int -\frac{3ab^2fx^6-b\left(\frac{cb^3}{a}-db^2-2aeb+2a^2f\right)x^3+3b^3c}{x^2(bx^3+a)}dx}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{3ab^2fx^6-b\left(\frac{cb^3}{a}-db^2-2aeb+2a^2f\right)x^3+3b^3c}{x^2(bx^3+a)}dx}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}$$

$$\Big\downarrow \ 1812$$

$$\frac{\int \left(\frac{3cb^3}{ax^2}+3afxb-\frac{(5fa^3-2bea^2-b^2da+4b^3c)xb}{a(bx^3+a)}\right)dx}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(5a^3f-2a^2be-ab^2d+4b^3c)}{\sqrt{3}a^{4/3}}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)(5a^3f-2a^2be-ab^2d+4b^3c)}{3a^{4/3}}-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\ldots\right.}{6a\ldots}}{3ab^3}$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2\left(a+bx^3\right)}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x]
```

output
```
-1/3*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a^2*b^2*(a + b*x^3)) + ((-
3*b^3*c)/(a*x) + (3*a*b*f*x^2)/2 + (b^(1/3)*(4*b^3*c - a*b^2*d - 2*a^2*b*e
 + 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(4/3)) + (b^(1/3)*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*Log[a^(1/3) +
b^(1/3)*x])/(3*a^(4/3)) - (b^(1/3)*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*
f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)))/(3*a*b^3)
```

3.267.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.267.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.71

method	result
default	$\frac{f\,x^2}{2b^2}-\frac{c}{a^2x}-\frac{\frac{\left(-\frac{1}{3}f\,a^3+\frac{1}{3}a^2be-\frac{1}{3}a\,b^2d+\frac{1}{3}b^3c\right)x^2}{b\,x^3+a}+\left(\frac{5}{3}f\,a^3-\frac{1}{3}a\,b^2d+\frac{4}{3}b^3c-\frac{2}{3}a^2be\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\cdots\right.}{a^2b^2}$
risch	$\frac{f\,x^2}{2b^2}+\frac{\frac{\left(f\,a^3-a^2be+a\,b^2d-4b^3c\right)x^3}{3a^2}-\frac{b^2c}{a}}{b^2x(b\,x^3+a)}+\frac{\_R=\text{RootOf}\left(a^7b^2\_Z^3-125a^9f^3+150a^8be\,f^2+75a^7b^2d\,f^2-60a^7b^2e^2f-300a^6b^3c\,f^2-60a^6b^3\cdots\right.}{\cdots}$

3.267. $\int\frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*f*x^2/b^2-c/a^2/x-1/a^2/b^2*((-1/3*f*a^3+1/3*a^2*b*e-1/3*a*b^2*d+1/3*b
^3*c)*x^2/(b*x^3+a)+(5/3*f*a^3-1/3*a*b^2*d+4/3*b^3*c-2/3*a^2*b*e)*(-1/3/b/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
)
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 860, normalized size of antiderivative = 3.25

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^2 (a + bx^3)^2} \, dx$$

$$= \left[\frac{9 a^3b^3fx^6 - 18 a^2b^4c - 3 (8 ab^5c - 2 a^2b^4d + 2 a^3b^3e - 5 a^4b^2f)x^3 + 3 \sqrt{\frac{1}{3}}((4 ab^5c - a^2b^4d - 2 a^3b^3e + 5 a}{} \right.$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/18*(9*a^3*b^3*f*x^6 - 18*a^2*b^4*c - 3*(8*a*b^5*c - 2*a^2*b^4*d + 2*a^3
*b^3*e - 5*a^4*b^2*f)*x^3 + 3*sqrt(1/3)*((4*a*b^5*c - a^2*b^4*d - 2*a^3*b^
3*e + 5*a^4*b^2*f)*x^4 + (4*a^2*b^4*c - a^3*b^3*d - 2*a^4*b^2*e + 5*a^5*b*
f)*x)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2
*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(
2/3)*x)/(b*x^3 + a)) - ((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4
+ (4*a*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b^2*x
^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*((4*b^4*c - a*b^3*d - 2*a^2*b^
2*e + 5*a^3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a
*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^5*x^4 + a^4*b^4*x), 1/18*(9*a
^3*b^3*f*x^6 - 18*a^2*b^4*c - 3*(8*a*b^5*c - 2*a^2*b^4*d + 2*a^3*b^3*e - 5
*a^4*b^2*f)*x^3 + 6*sqrt(1/3)*((4*a*b^5*c - a^2*b^4*d - 2*a^3*b^3*e + 5*a^
4*b^2*f)*x^4 + (4*a^2*b^4*c - a^3*b^3*d - 2*a^4*b^2*e + 5*a^5*b*f)*x)*sqrt
((a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(
1/3)/a)/b) - ((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^3*b*f)*x^4 + (4*a*b^3
*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^
2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*((4*b^4*c - a*b^3*d - 2*a^2*b^2*e + 5*a^
3*b*f)*x^4 + (4*a*b^3*c - a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)*(a*b^2)^(2/3
)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^5*x^4 + a^4*b^4*x)]
```

3.267.6 Sympy [A] (verification not implemented)

Time = 63.97 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.72

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx = \frac{-3ab^2c+x^3(a^3f-a^2be+ab^2d-4b^3c)}{3a^3b^2x+3a^2b^3x^4}$$
$$+\operatorname{RootSum}\Big(729t^3a^7b^8-125a^9f^3+150a^8bef^2+75a^7b^2df^2-60a^7b^2e^2f-300a^6b^3cf^2-60a^6b^3def+8a$$
$$+\frac{fx^2}{2b^2}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**2,x)
```

output
```
(-3*a*b**2*c + x**3*(a**3*f - a**2*b*e + a*b**2*d - 4*b**3*c))/(3*a**3*b**
2*x + 3*a**2*b**3*x**4) + RootSum(729*_t**3*a**7*b**8 - 125*a**9*f**3 + 15
0*a**8*b*e*f**2 + 75*a**7*b**2*d*f**2 - 60*a**7*b**2*e**2*f - 300*a**6*b**
3*c*f**2 - 60*a**6*b**3*d*e*f + 8*a**6*b**3*e**3 + 240*a**5*b**4*c*e*f - 1
5*a**5*b**4*d**2*f + 12*a**5*b**4*d*e**2 + 120*a**4*b**5*c*d*f - 48*a**4*b
**5*c*e**2 + 6*a**4*b**5*d**2*e - 240*a**3*b**6*c**2*f - 48*a**3*b**6*c*d*
e + a**3*b**6*d**3 + 96*a**2*b**7*c**2*e - 12*a**2*b**7*c*d**2 + 48*a*b**8
*c**2*d - 64*b**9*c**3, Lambda(_t, _t*log(81*_t**2*a**5*b**5/(25*a**6*f**2
 - 20*a**5*b*e*f - 10*a**4*b**2*d*f + 4*a**4*b**2*e**2 + 40*a**3*b**3*c*f
+ 4*a**3*b**3*d*e - 16*a**2*b**4*c*e + a**2*b**4*d**2 - 8*a*b**5*c*d + 16*
b**6*c**2) + x))) + f*x**2/(2*b**2)
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\,(a+bx^3)^2}\,dx = \frac{fx^2}{2\,b^2} - \frac{3\,ab^2c+(4\,b^3c-ab^2d+a^2be-a^3f)x^3}{3\,(a^2b^3x^4+a^3b^2x)} - \frac{\sqrt{3}(4\,b^3c-ab^2d-2\,a^2be+5\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(4\,b^3c-ab^2d-2\,a^2be+5\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(4\,b^3c-ab^2d-2\,a^2be+5\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/2*f*x^2/b^2 - 1/3*(3*a*b^2*c + (4*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^3
)/(a^2*b^3*x^4 + a^3*b^2*x) - 1/9*sqrt(3)*(4*b^3*c - a*b^2*d - 2*a^2*b*e +
 5*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/
b)^(1/3)) - 1/18*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*log(x^2 - x*(a/
b)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(1/3)) + 1/9*(4*b^3*c - a*b^2*d - 2
*a^2*b*e + 5*a^3*f)*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(1/3))
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^2}\,dx$$
$$= \frac{fx^2}{2\,b^2} - \frac{\sqrt{3}(4\,b^3c-ab^2d-2\,a^2be+5\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{1}{3}}\,a^2b^2}$$
$$+ \frac{(4\,b^3c-ab^2d-2\,a^2be+5\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{1}{3}}\,a^2b^2}$$
$$+ \frac{\left(4\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^2}$$
$$- \frac{4\,b^3cx^3-ab^2dx^3+a^2bex^3-a^3fx^3+3\,ab^2c}{3\,(bx^4+ax)a^2b^2}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/2*f*x^2/b^2 - 1/9*sqrt(3)*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b^2)
 + 1/18*(4*b^3*c - a*b^2*d - 2*a^2*b*e + 5*a^3*f)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b^2) + 1/9*(4*b^3*c*(-a/b)^(1/3) - a*
b^2*d*(-a/b)^(1/3) - 2*a^2*b*e*(-a/b)^(1/3) + 5*a^3*f*(-a/b)^(1/3))*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2) - 1/3*(4*b^3*c*x^3 - a*b^2*d*x
^3 + a^2*b*e*x^3 - a^3*f*x^3 + 3*a*b^2*c)/((b*x^4 + a*x)*a^2*b^2)
```

3.267.9 Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^2}\,dx$$
$$= \frac{f\,x^2}{2\,b^2} - \frac{\frac{x^3\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{3\,a^2}+\frac{b^2\,c}{a}}{b^3\,x^4+a\,b^2\,x}$$
$$+ \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a^{7/3}\,b^{8/3}}$$
$$- \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a^{7/3}\,b^{8/3}}$$
$$+ \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3\right)}{9\,a^{7/3}\,b^{8/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^2),x)
```

output
```
(f*x^2)/(2*b^2) - ((x^3*(4*b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^2) + (
b^2*c)/a)/(b^3*x^4 + a*b^2*x) + (log(b^(1/3)*x + a^(1/3))*(4*b^3*c + 5*a^3
*f - a*b^2*d - 2*a^2*b*e))/(9*a^(7/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*1i +
 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(4*b^3*c + 5*a^3*f - a*b^2*
d - 2*a^2*b*e))/(9*a^(7/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*
x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*b^3*c + 5*a^3*f - a*b^2*d - 2*a^2*b
*e))/(9*a^(7/3)*b^(8/3))
```

3.268 $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2}\,dx$

3.268.1 Optimal result . . . 2022
3.268.2 Mathematica [A] (verified) . . . 2023
3.268.3 Rubi [A] (verified) . . . 2024
3.268.4 Maple [A] (verified) . . . 2025
3.268.5 Fricas [A] (verification not implemented) . . . 2026
3.268.6 Sympy [F(-1)] . . . 2027
3.268.7 Maxima [A] (verification not implemented) . . . 2028
3.268.8 Giac [A] (verification not implemented) . . . 2029
3.268.9 Mupad [B] (verification not implemented) . . . 2030

3.268.1 Optimal result

Integrand size = 30, antiderivative size = 260

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2}\,dx =& -\frac{c}{2a^2x^2}+\frac{fx}{b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^2b^2(a+bx^3)}\\ &+\frac{(5b^3c-2ab^2d-a^2be+4a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}b^{7/3}}\\ &-\frac{(5b^3c-2ab^2d-a^2be+4a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}b^{7/3}}\\ &+\frac{(5b^3c-2ab^2d-a^2be+4a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}b^{7/3}}\end{aligned}$$

output
```
-1/2*c/a^2/x^2+f*x/b^2-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^2/b^2/(b*x^3
+a)-1/9*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/
b^(7/3)+1/18*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/a^(8/3)/b^(7/3)+1/9*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*a
rctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(7/3)*3^(1/2)
```

3.268.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}\,dx$$

$$=\frac{1}{18}\left(-\frac{9c}{a^2x^2}+\frac{18fx}{b^2}+\frac{6(-b^3c+ab^2d-a^2be+a^3f)\,x}{a^2b^2\,(a+bx^3)}+\frac{2\sqrt{3}(5b^3c-2ab^2d-a^2be+4a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}b^{7/3}}-\frac{2(5b^3c-2ab^2d-a^2be+4a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}b^{7/3}}+\frac{(5b^3c-2ab^2d-a^2be+4a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{8/3}b^{7/3}}\right)$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

output
```
((-9*c)/(a^2*x^2) + (18*f*x)/b^2 + (6*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*x)/(a^2*b^2*(a + b*x^3)) + (2*Sqrt[3]*(5*b^3*c - 2*a*b^2*d - a^2*b*e +
4*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(8/3)*b^(7/3)) -
(2*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(a^
(8/3)*b^(7/3)) + ((5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(8/3)*b^(7/3)))/18
```

3.268.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2368, 25, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3 (a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{2368}$$

$$-\frac{\int -\frac{3ab^2fx^6-b\left(\frac{2cb^3}{a}-2db^2-aeb+a^2f\right)x^3+3b^3c}{x^3(bx^3+a)}dx}{3ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2(a+bx^3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3ab^2fx^6-b\left(\frac{2cb^3}{a}-2db^2-aeb+a^2f\right)x^3+3b^3c}{x^3(bx^3+a)}dx}{3ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2(a+bx^3)}$$

$$\Big\downarrow \text{1812}$$

$$\frac{\int \left(\frac{3cb^3}{ax^3}+3afb-\frac{(4fa^3-bea^2-2b^2da+5b^3c)b}{a(bx^3+a)}\right)dx}{3ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(4a^3f-a^2be-2ab^2d+5b^3c)}{\sqrt{3}a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(4a^3f-a^2be-2ab^2d+5b^3c)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{\ldots}\right.}{\ldots}}{3ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^2b^2(a+bx^3)}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x]
```

output
```
-1/3*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a^2*b^2*(a + b*x^3)) + ((-3*
b^3*c)/(2*a*x^2) + 3*a*b*f*x + (b^(2/3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4
*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3
)) - (b^(2/3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(5/3)) + (b^(2/3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)))/(3*a*b^3)
```

3.268. $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2} \, dx$

3.268.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.268.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.70

method	result
default	$\frac{fx}{b^2}-\frac{c}{2a^2x^2}-\frac{\frac{\left(-\frac{1}{3}fa^3+\frac{1}{3}a^2be-\frac{1}{3}ab^2d+\frac{1}{3}b^3c\right)x}{bx^3+a}+\frac{\left(4fa^3-a^2be-2ab^2d+5b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\cdots}{\cdots}\right.}{3}}{a^2b^2}$
risch	$\frac{fx}{b^2}+\frac{\frac{\left(2fa^3-2a^2be+2ab^2d-5b^3c\right)x^3}{6a^2}-\frac{b^2c}{2a}}{b^2x^2\left(bx^3+a\right)}+\frac{\_R=\text{RootOf}\left(a^8b\_Z^3+64a^9f^3-48a^8bef^2-96a^7b^2df^2+12a^7b^2e^2f+240a^6b^3cf^2+48a^6b^3de\cdots\right.}{\cdots}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
f*x/b^2-1/2*c/a^2/x^2-1/a^2/b^2*((-1/3*f*a^3+1/3*a^2*b*e-1/3*a*b^2*d+1/3*b
^3*c)*x/(b*x^3+a)+1/3*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*(1/3/b/(a/b)^(2/
3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1
/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 902, normalized size of antiderivative = 3.47

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}\,dx$$

$$= \left[\frac{18\,a^4b^2fx^6-9\,a^3b^3c-3\,(5\,a^2b^4c-2\,a^3b^3d+2\,a^4b^2e-8\,a^5bf)x^3+3\,\sqrt{\tfrac{1}{3}}((5\,ab^5c-2\,a^2b^4d-a^3b^3e+4\,a}{}\right.$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/18*(18*a^4*b^2*f*x^6 - 9*a^3*b^3*c - 3*(5*a^2*b^4*c - 2*a^3*b^3*d + 2*a
^4*b^2*e - 8*a^5*b*f)*x^3 + 3*sqrt(1/3)*((5*a*b^5*c - 2*a^2*b^4*d - a^3*b^
3*e + 4*a^4*b^2*f)*x^5 + (5*a^2*b^4*c - 2*a^3*b^3*d - a^4*b^2*e + 4*a^5*b*
f)*x^2)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2
 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^
2*b)^(1/3)/b))/(b*x^3 + a)) + ((5*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*
f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)
*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((5*b^4*c - 2*a*b^
3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*
a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^4*x^5 + a^5
*b^3*x^2), 1/18*(18*a^4*b^2*f*x^6 - 9*a^3*b^3*c - 3*(5*a^2*b^4*c - 2*a^3*b
^3*d + 2*a^4*b^2*e - 8*a^5*b*f)*x^3 - 6*sqrt(1/3)*((5*a*b^5*c - 2*a^2*b^4*
d - a^3*b^3*e + 4*a^4*b^2*f)*x^5 + (5*a^2*b^4*c - 2*a^3*b^3*d - a^4*b^2*e
+ 4*a^5*b*f)*x^2)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/
3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((5*b^4*c - 2*a*b^
3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c - 2*a^2*b^2*d - a^3*b*e + 4*
a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)
*a) - 2*((5*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^5 + (5*a*b^3*c -
2*a^2*b^2*d - a^3*b*e + 4*a^4*f)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^
(2/3)))/(a^4*b^4*x^5 + a^5*b^3*x^2)]
```

3.268.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2}\,dx = -\frac{3ab^2c+(5b^3c-2ab^2d+2a^2be-2a^3f)x^3}{6(a^2b^3x^5+a^3b^2x^2)}+\frac{fx}{b^2} - \frac{\sqrt{3}(5b^3c-2ab^2d-a^2be+4a^3f)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(5b^3c-2ab^2d-a^2be+4a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(5b^3c-2ab^2d-a^2be+4a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*a*b^2*c + (5*b^3*c - 2*a*b^2*d + 2*a^2*b*e - 2*a^3*f)*x^3)/(a^2*b^
3*x^5 + a^3*b^2*x^2) + f*x/b^2 - 1/9*sqrt(3)*(5*b^3*c - 2*a*b^2*d - a^2*b*
e + 4*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*
(a/b)^(2/3)) + 1/18*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(2/3)) - 1/9*(5*b^3*c - 2*a*b^2*
d - a^2*b*e + 4*a^3*f)*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^2}\,dx = \frac{fx}{b^2} + \frac{\sqrt{3}(5\,b^3c-2\,ab^2d-a^2be+4\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a^2b} + \frac{(5\,b^3c-2\,ab^2d-a^2be+4\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a^2b} + \frac{(5\,b^3c-2\,ab^2d-a^2be+4\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^2} - \frac{c}{2\,a^2x^2} - \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{3\,(bx^3+a)a^2b^2}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
f*x/b^2 + 1/9*sqrt(3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b) + 1/18*
(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/((-a*b^2)^(2/3)*a^2*b) + 1/9*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a
^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2) - 1/2*c/(a^2*x^2)
- 1/3*(b^3*c*x - a*b^2*d*x + a^2*b*e*x - a^3*f*x)/((b*x^3 + a)*a^2*b^2)
```

3.268.9 Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^2}\,dx$$
$$=\frac{f\,x}{b^2}-\frac{\frac{x^3\,\left(-2\,f\,a^3+2\,e\,a^2\,b-2\,d\,a\,b^2+5\,c\,b^3\right)}{6\,a^2}+\frac{b^2\,c}{2\,a}}{b^3\,x^5+a\,b^2\,x^2}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+5\,c\,b^3\right)}{9\,a^{8/3}\,b^{7/3}}$$
$$-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+5\,c\,b^3\right)}{9\,a^{8/3}\,b^{7/3}}$$
$$+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+5\,c\,b^3\right)}{9\,a^{8/3}\,b^{7/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^2),x)
```

output
```
(f*x)/b^2 - ((x^3*(5*b^3*c - 2*a^3*f - 2*a*b^2*d + 2*a^2*b*e))/(6*a^2) + (
b^2*c)/(2*a))/(b^3*x^5 + a*b^2*x^2) - (log(b^(1/3)*x + a^(1/3))*(5*b^3*c +
 4*a^3*f - 2*a*b^2*d - a^2*b*e))/(9*a^(8/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3
)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c + 4*a^3*f -
2*a*b^2*d - a^2*b*e))/(9*a^(8/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^
(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c + 4*a^3*f - 2*a*b^2*d -
 a^2*b*e))/(9*a^(8/3)*b^(7/3))
```

3.269 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2}\,dx$

3.269.1 Optimal result 2031
3.269.2 Mathematica [A] (verified) 2032
3.269.3 Rubi [A] (verified) 2032
3.269.4 Maple [A] (verified) 2034
3.269.5 Fricas [A] (verification not implemented) 2035
3.269.6 Sympy [F(-1)] 2036
3.269.7 Maxima [A] (verification not implemented) 2037
3.269.8 Giac [A] (verification not implemented) 2038
3.269.9 Mupad [B] (verification not implemented) 2039

3.269.1 Optimal result

Integrand size = 30, antiderivative size = 269

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2}\,dx = -\frac{c}{4a^2x^4}+\frac{2bc-ad}{a^3x}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^3b\,(a+bx^3)} - \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}b^{5/3}} - \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}b^{5/3}} + \frac{(7b^3c-4ab^2d+a^2be+2a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{10/3}b^{5/3}}$$

output
```
-1/4*c/a^2/x^4+(-a*d+2*b*c)/a^3/x+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a
^3/b/(b*x^3+a)-1/9*(2*a^3*f+a^2*b*e-4*a*b^2*d+7*b^3*c)*ln(a^(1/3)+b^(1/3)*
x)/a^(10/3)/b^(5/3)+1/18*(2*a^3*f+a^2*b*e-4*a*b^2*d+7*b^3*c)*ln(a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(5/3)-1/9*(2*a^3*f+a^2*b*e-4*a*b^2
*d+7*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(
5/3)*3^(1/2)
```

3.269.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{9a^{4/3}c}{x^4} - \frac{36\sqrt[3]{a}(-2bc+ad)}{x} - \frac{12\sqrt[3]{a}(-b^3c+ab^2d-a^2be+a^3f)x^2}{b(a+bx^3)} - \frac{4\sqrt{3}(7b^3c-4ab^2d+a^2be+2a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} - \frac{4(7b^3c-4}{}}{36a^{10/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]
```

output
```
((-9*a^(4/3)*c)/x^4 - (36*a^(1/3)*(-2*b*c + a*d))/x - (12*a^(1/3)*(-(b^3*c
) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4*Sqrt[3]*(7*b^3*c
- 4*a*b^2*d + a^2*b*e + 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/b^(5/3) - (4*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b
^(1/3)*x])/b^(5/3) + (2*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3))/(36*a^(10/3))
```

3.269.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2368, 25, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)} - \frac{\int -\frac{b^2\left(\frac{cb^3}{a^2}-\frac{db^2}{a}+eb+2af\right)x^6-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^5(bx^3+a)}dx}{3ab^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2\left(\frac{cb^3}{a^2}-\frac{db^2}{a}+eb+2af\right)x^6-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^5(bx^3+a)}dx}{3ab^3}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}$$

$\downarrow$ 1812

$$\frac{\int \left(\frac{3(ad-2bc)b^3}{a^2x^2}+\frac{3cb^3}{ax^5}+\frac{(2fa^3+bea^2-4b^2da+7b^3c)xb^2}{a^2(bx^3+a)}\right)dx}{3ab^3}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}$$

$\downarrow$ 2009

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}+$$

$$\frac{\frac{3b^3(2bc-ad)}{a^2x}-\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(2a^3f+a^2be-4ab^2d+7b^3c)}{\sqrt{3}a^{7/3}}+\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(2a^3f+a^2be-4ab^2d+7b^3c)}{6a^{7/3}}-\ldots}{3ab^3}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x]
```

output
```
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^3*b*(a + b*x^3)) + ((-3*b^3
*c)/(4*a*x^4) + (3*b^3*(2*b*c - a*d))/(a^2*x) - (b^(4/3)*(7*b^3*c - 4*a*b^
2*d + a^2*b*e + 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(7/3)) - (b^(4/3)*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*L
og[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)) + (b^(4/3)*(7*b^3*c - 4*a*b^2*d + a^2
*b*e + 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)
))/(3*a*b^3)
```

3.269.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.269. $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2}\,dx$

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.269.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.72

method	result
default	$-\frac{c}{4a^{2}x^{4}}-\frac{ad-2bc}{a^{3}x}+\frac{-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{2}}{3b\left(b\,x^{3}+a\right)}+\frac{\left(2f\,a^{3}+a^{2}be-4a\,b^{2}d+7b^{3}c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\mathrm{arct}}{}\right.}{3b}}{a^{3}}$
risch	$\frac{-\frac{\left(f\,a^{3}-a^{2}be+4a\,b^{2}d-7b^{3}c\right)x^{6}}{3a^{3}b}-\frac{(4ad-7bc)x^{3}}{4a^{2}}-\frac{c}{4a}}{x^{4}\left(b\,x^{3}+a\right)}+\left(\underline{}R=\mathrm{RootOf}\left(a^{10}b^{5}\underline{}Z^{3}+8a^{9}f^{3}+12a^{8}be\,f^{2}-48a^{7}b^{2}d\,f^{2}+6a^{7}b^{2}e^{2}f+84a^{6}b^{3}c\,f^{2}-48\right.\right.$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*c/a^2/x^4-(a*d-2*b*c)/a^3/x+1/a^3*(-1/3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)
/b*x^2/(b*x^3+a)+1/3*(2*a^3*f+a^2*b*e-4*a*b^2*d+7*b^3*c)/b*(-1/3/b/(a/b)^(
1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 902, normalized size of antiderivative = 3.35

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^5 (a + bx^3)^2} \, dx$$

$$= \left[-\frac{9\, a^3 b^3 c - 12\, (7\, ab^5 c - 4\, a^2 b^4 d + a^3 b^3 e - a^4 b^2 f) x^6 - 9\, (7\, a^2 b^4 c - 4\, a^3 b^3 d) x^3 - 6\, \sqrt{\frac{1}{3}} ((7\, ab^5 c - 4\, a^2 b^4 d +}{} \right.$$

$$-\frac{9\, a^3 b^3 c - 12\, (7\, ab^5 c - 4\, a^2 b^4 d + a^3 b^3 e - a^4 b^2 f) x^6 - 9\, (7\, a^2 b^4 c - 4\, a^3 b^3 d) x^3 - 12\, \sqrt{\frac{1}{3}} ((7\, ab^5 c - 4\, a^2 b^4 d}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/36*(9*a^3*b^3*c - 12*(7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e - a^4*b^2*f)
*x^6 - 9*(7*a^2*b^4*c - 4*a^3*b^3*d)*x^3 - 6*sqrt(1/3)*((7*a*b^5*c - 4*a^2
*b^4*d + a^3*b^3*e + 2*a^4*b^2*f)*x^7 + (7*a^2*b^4*c - 4*a^3*b^3*d + a^4*b
^2*e + 2*a^5*b*f)*x^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqr
t(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/
3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 2*((7*b^4*c - 4*a*b^3*d + a^2*b
^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)
*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 4*((7
*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d
 + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*
b^4*x^7 + a^5*b^3*x^4), -1/36*(9*a^3*b^3*c - 12*(7*a*b^5*c - 4*a^2*b^4*d +
 a^3*b^3*e - a^4*b^2*f)*x^6 - 9*(7*a^2*b^4*c - 4*a^3*b^3*d)*x^3 - 12*sqrt(
1/3)*((7*a*b^5*c - 4*a^2*b^4*d + a^3*b^3*e + 2*a^4*b^2*f)*x^7 + (7*a^2*b^4
*c - 4*a^3*b^3*d + a^4*b^2*e + 2*a^5*b*f)*x^4)*sqrt(-(-a*b^2)^(1/3)/a)*arc
tan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 2*((7*
b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 + (7*a*b^3*c - 4*a^2*b^2*d
+ a^3*b*e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x
+ (-a*b^2)^(2/3)) + 4*((7*b^4*c - 4*a*b^3*d + a^2*b^2*e + 2*a^3*b*f)*x^7 +
 (7*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^4)*(-a*b^2)^(2/3)*log(b*x
 - (-a*b^2)^(1/3)))/(a^4*b^4*x^7 + a^5*b^3*x^4)]
```

3.269.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^5 (a + bx^3)^2} \, dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^2}\,dx = \frac{4\,(7\,b^3c-4\,ab^2d+a^2be-a^3f)x^6-3\,a^2bc+3\,(7\,ab^2c-4\,a^2bd)x^3}{12\,(a^3b^2x^7+a^4bx^4)} + \frac{\sqrt{3}(7\,b^3c-4\,ab^2d+a^2be+2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(7\,b^3c-4\,ab^2d+a^2be+2\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(7\,b^3c-4\,ab^2d+a^2be+2\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/12*(4*(7*b^3*c - 4*a*b^2*d + a^2*b*e - a^3*f)*x^6 - 3*a^2*b*c + 3*(7*a*b
^2*c - 4*a^2*b*d)*x^3)/(a^3*b^2*x^7 + a^4*b*x^4) + 1/9*sqrt(3)*(7*b^3*c -
4*a*b^2*d + a^2*b*e + 2*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(a^3*b^2*(a/b)^(1/3)) + 1/18*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a
^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(1/3)) - 1/9*(
7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*log(x + (a/b)^(1/3))/(a^3*b^2*(a/
b)^(1/3))
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^2}\,dx$$
$$= \frac{\sqrt{3}(7\,b^3c-4\,ab^2d+a^2be+2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^3b} - \frac{(7\,b^3c-4\,ab^2d+a^2be+2\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^3b} - \frac{\left(7\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4b} + \frac{b^3cx^2-ab^2dx^2+a^2bex^2-a^3fx^2}{3\,(bx^3+a)a^3b} + \frac{8\,bcx^3-4\,adx^3-ac}{4\,a^3x^4}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(7*b^3*c - 4*a*b^2*d + a^2*b*e + 2*a^3*f)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^3*b) - 1/18*(7*b^3*c -
 4*a*b^2*d + a^2*b*e + 2*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(
(-a*b^2)^(1/3)*a^3*b) - 1/9*(7*b^3*c*(-a/b)^(1/3) - 4*a*b^2*d*(-a/b)^(1/3)
 + a^2*b*e*(-a/b)^(1/3) + 2*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/(a^4*b) + 1/3*(b^3*c*x^2 - a*b^2*d*x^2 + a^2*b*e*x^2 - a^3*f
*x^2)/((b*x^3 + a)*a^3*b) + 1/4*(8*b*c*x^3 - 4*a*d*x^3 - a*c)/(a^3*x^4)
```

3.269.9 Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^2}\,dx$$
$$= -\frac{\frac{c}{4a}+\frac{x^3\,(4\,a\,d-7\,b\,c)}{4\,a^2}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-4\,d\,a\,b^2+7\,c\,b^3\right)}{3\,a^3\,b}}{b\,x^7+a\,x^4}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(2\,f\,a^3+e\,a^2\,b-4\,d\,a\,b^2+7\,c\,b^3\right)}{9\,a^{10/3}\,b^{5/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,f\,a^3+e\,a^2\,b-4\,d\,a\,b^2+7\,c\,b^3\right)}{9\,a^{10/3}\,b^{5/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,f\,a^3+e\,a^2\,b-4\,d\,a\,b^2+7\,c\,b^3\right)}{9\,a^{10/3}\,b^{5/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^2),x)
```

output
```
(log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(7
*b^3*c + 2*a^3*f - 4*a*b^2*d + a^2*b*e))/(9*a^(10/3)*b^(5/3)) - (log(b^(1/
3)*x + a^(1/3))*(7*b^3*c + 2*a^3*f - 4*a*b^2*d + a^2*b*e))/(9*a^(10/3)*b^(
5/3)) - (c/(4*a) + (x^3*(4*a*d - 7*b*c))/(4*a^2) - (x^6*(7*b^3*c - a^3*f -
 4*a*b^2*d + a^2*b*e))/(3*a^3*b))/(a*x^4 + b*x^7) - (log(3^(1/2)*a^(1/3)*1
i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(7*b^3*c + 2*a^3*f - 4*a
*b^2*d + a^2*b*e))/(9*a^(10/3)*b^(5/3))
```

3.270 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2}\,dx$

3.270.1 Optimal result . . . 2040
3.270.2 Mathematica [A] (verified) . . . 2041
3.270.3 Rubi [A] (verified) . . . 2041
3.270.4 Maple [A] (verified) . . . 2043
3.270.5 Fricas [A] (verification not implemented) . . . 2044
3.270.6 Sympy [F(-1)] . . . 2045
3.270.7 Maxima [A] (verification not implemented) . . . 2046
3.270.8 Giac [A] (verification not implemented) . . . 2047
3.270.9 Mupad [B] (verification not implemented) . . . 2047

3.270.1 Optimal result

Integrand size = 30, antiderivative size = 270

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2}\,dx = -\frac{c}{5a^2x^5}+\frac{2bc-ad}{2a^3x^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^3b\,(a+bx^3)} - \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}b^{4/3}} + \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}b^{4/3}} - \frac{(8b^3c-5ab^2d+2a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{11/3}b^{4/3}}$$

output
```
-1/5*c/a^2/x^5+1/2*(-a*d+2*b*c)/a^3/x^2+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
*x/a^3/b/(b*x^3+a)+1/9*(a^3*f+2*a^2*b*e-5*a*b^2*d+8*b^3*c)*ln(a^(1/3)+b^(1
/3)*x)/a^(11/3)/b^(4/3)-1/18*(a^3*f+2*a^2*b*e-5*a*b^2*d+8*b^3*c)*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(4/3)-1/9*(a^3*f+2*a^2*b*e-5*a
*b^2*d+8*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)
/b^(4/3)*3^(1/2)
```

3.270.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{18a^{5/3}c}{x^5} - \frac{45a^{2/3}(-2bc+ad)}{x^2} - \frac{30a^{2/3}(-b^3c+ab^2d-a^2be+a^3f)x}{b(a+bx^3)} - \frac{10\sqrt{3}(8b^3c-5ab^2d+2a^2be+a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}} + 10(8b^3c- \cdots}{90a^{11/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

output
```
((-18*a^(5/3)*c)/x^5 - (45*a^(2/3)*(-2*b*c + a*d))/x^2 - (30*a^(2/3)*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)) - (10*Sqrt[3]*(8*b^3*
c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]])/b^(4/3) + (10*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a^(1/3)
+ b^(1/3)*x])/b^(4/3) - (5*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3))/(90*a^(11/3))
```

3.270.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2368, 25, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \; 2368$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)} - \frac{\int -\frac{b^2\left(\frac{2cb^3}{a^2}-\frac{2db^2}{a}+2eb+af\right)x^6-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^6\left(bx^3+a\right)}dx}{3ab^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b^2\left(\frac{2cb^3}{a^2}-\frac{2db^2}{a}+2eb+af\right)x^6-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^6(bx^3+a)}dx}{3ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1812$$

$$\frac{\int \left(\frac{3(ad-2bc)b^3}{a^2x^3}+\frac{3cb^3}{ax^6}+\frac{(fa^3+2bea^2-5b^2da+8b^3c)b^2}{a^2(bx^3+a)}\right)dx}{3ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^3b\left(a+bx^3\right)}+$$

$$\frac{\frac{3b^3(2bc-ad)}{2a^2x^2}-\frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{\sqrt{3}a^{8/3}}-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{6a^{8/3}}+\cdots}{3ab^3}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x]
```

output
```
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^3*b*(a + b*x^3)) + ((-3*b^3*c
)/(5*a*x^5) + (3*b^3*(2*b*c - a*d))/(2*a^2*x^2) - (b^(5/3)*(8*b^3*c - 5*a*
b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(8/3)) + (b^(5/3)*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)
*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)) - (b^(5/3)*(8*b^3*c - 5*a*b^2*d + 2
*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/
3)))/(3*a*b^3)
```

3.270.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.270.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.71

method	result
default	$-\frac{c}{5a^2x^5}-\frac{ad-2bc}{2a^3x^2}+\frac{-\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x}{3b\left(b\,x^3+a\right)}+\frac{\left(f\,a^3+2a^2be-5a\,b^2d+8b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan}{}\right.}{3b}}{a^3}$
risch	$\frac{-\frac{\left(2f\,a^3-2a^2be+5a\,b^2d-8b^3c\right)x^6}{6a^3b}-\frac{(5ad-8bc)x^3}{10a^2}-\frac{c}{5a}}{x^5\left(b\,x^3+a\right)}+\left(\_R=\mathrm{RootOf}\left(a^{11}b^4\_Z^3-a^9f^3-6a^8be\,f^2+15a^7b^2d\,f^2-12a^7b^2e^2f-24a^6b^3c\,f^2+\right.\right.$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*c/a^2/x^5-1/2*(a*d-2*b*c)/a^3/x^2+1/a^3*(-1/3*(a^3*f-a^2*b*e+a*b^2*d-
b^3*c)/b*x/(b*x^3+a)+1/3*(a^3*f+2*a^2*b*e-5*a*b^2*d+8*b^3*c)/b*(1/3/b/(a/b
)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 897, normalized size of antiderivative = 3.32

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^6 \left(a + bx^3\right)^2} \, dx$$

$$= \left[-\frac{18\,a^4b^2c - 15\,(8\,a^2b^4c - 5\,a^3b^3d + 2\,a^4b^2e - 2\,a^5bf)x^6 - 9\,(8\,a^3b^3c - 5\,a^4b^2d)x^3 - 15\,\sqrt{\frac{1}{3}}((8\,ab^5c - 5\,a}{} \right.$$

$$-\frac{18\,a^4b^2c - 15\,(8\,a^2b^4c - 5\,a^3b^3d + 2\,a^4b^2e - 2\,a^5bf)x^6 - 9\,(8\,a^3b^3c - 5\,a^4b^2d)x^3 - 30\,\sqrt{\frac{1}{3}}((8\,ab^5c - 5\,a^2}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/90*(18*a^4*b^2*c - 15*(8*a^2*b^4*c - 5*a^3*b^3*d + 2*a^4*b^2*e - 2*a^5
*b*f)*x^6 - 9*(8*a^3*b^3*c - 5*a^4*b^2*d)*x^3 - 15*sqrt(1/3)*((8*a*b^5*c -
 5*a^2*b^4*d + 2*a^3*b^3*e + a^4*b^2*f)*x^8 + (8*a^2*b^4*c - 5*a^3*b^3*d +
 2*a^4*b^2*e + a^5*b*f)*x^5)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^
2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^
(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + 5*((8*b^4*c - 5*a*b^3*d +
2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)
*x^5)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 10*
((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^
2*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a
^5*b^3*x^8 + a^6*b^2*x^5), -1/90*(18*a^4*b^2*c - 15*(8*a^2*b^4*c - 5*a^3*b
^3*d + 2*a^4*b^2*e - 2*a^5*b*f)*x^6 - 9*(8*a^3*b^3*c - 5*a^4*b^2*d)*x^3 -
30*sqrt(1/3)*((8*a*b^5*c - 5*a^2*b^4*d + 2*a^3*b^3*e + a^4*b^2*f)*x^8 + (8
*a^2*b^4*c - 5*a^3*b^3*d + 2*a^4*b^2*e + a^5*b*f)*x^5)*sqrt((a^2*b)^(1/3)/
b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/
3)/b)/a^2) + 5*((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^8 + (8*a*b
^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x^2 - (
a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 10*((8*b^4*c - 5*a*b^3*d + 2*a^2*b^2*e
 + a^3*b*f)*x^8 + (8*a*b^3*c - 5*a^2*b^2*d + 2*a^3*b*e + a^4*f)*x^5)*(a^2*
b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^3*x^8 + a^6*b^2*x^5)]
```

3.270.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^6 (a + bx^3)^2} \, dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2}\,dx$$
$$= \frac{5\,(8\,b^3c-5\,ab^2d+2\,a^2be-2\,a^3f)x^6-6\,a^2bc+3\,(8\,ab^2c-5\,a^2bd)x^3}{30\,(a^3b^2x^8+a^4bx^5)}$$
$$+\frac{\sqrt{3}(8\,b^3c-5\,ab^2d+2\,a^2be+a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(8\,b^3c-5\,ab^2d+2\,a^2be+a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(8\,b^3c-5\,ab^2d+2\,a^2be+a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/30*(5*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e - 2*a^3*f)*x^6 - 6*a^2*b*c + 3*(8
*a*b^2*c - 5*a^2*b*d)*x^3)/(a^3*b^2*x^8 + a^4*b*x^5) + 1/9*sqrt(3)*(8*b^3*
c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) - 1/18*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e
 + a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) + 1
/9*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*log(x + (a/b)^(1/3))/(a^3*b^2
*(a/b)^(2/3))
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(8\,b^3c-5\,ab^2d+2\,a^2be+a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a^3} - \frac{(8\,b^3c-5\,ab^2d+2\,a^2be+a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a^3} - \frac{(8\,b^3c-5\,ab^2d+2\,a^2be+a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4b} + \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{3\,(bx^3+a)a^3b} + \frac{10\,bcx^3-5\,adx^3-2\,ac}{10\,a^3x^5}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/18*(8*b^3*c -
5*a*b^2*d + 2*a^2*b*e + a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((
-a*b^2)^(2/3)*a^3) - 1/9*(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*(-a/b)^
(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b) + 1/3*(b^3*c*x - a*b^2*d*x + a^2*
b*e*x - a^3*f*x)/((b*x^3 + a)*a^3*b) + 1/10*(10*b*c*x^3 - 5*a*d*x^3 - 2*a*
c)/(a^3*x^5)
```

3.270.9 Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{9\,a^{11/3}\,b^{4/3}} - \frac{\frac{c}{5\,a}+\frac{x^3\,(5\,a\,d-8\,b\,c)}{10\,a^2}-\frac{x^6\,\left(-2\,f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{6\,a^3\,b}}{b\,x^8+a\,x^5} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{9\,a^{11/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(f\,a^3+2\,e\,a^2\,b-5\,d\,a\,b^2+8\,c\,b^3\right)}{9\,a^{11/3}\,b^{4/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^2),x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(8*b^3*c + a^3*f - 5*a*b^2*d + 2*a^2*b*e))/(9*a^
(11/3)*b^(4/3)) - (c/(5*a) + (x^3*(5*a*d - 8*b*c))/(10*a^2) - (x^6*(8*b^3*
c - 2*a^3*f - 5*a*b^2*d + 2*a^2*b*e))/(6*a^3*b))/(a*x^5 + b*x^8) + (log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(8*b^3*c
+ a^3*f - 5*a*b^2*d + 2*a^2*b*e))/(9*a^(11/3)*b^(4/3)) - (log(3^(1/2)*a^(1
/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(8*b^3*c + a^3*f -
5*a*b^2*d + 2*a^2*b*e))/(9*a^(11/3)*b^(4/3))
```

3.270. $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^2} \, dx$

3.271 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx$

3.271.1 Optimal result . . . 2049
3.271.2 Mathematica [A] (verified) . . . 2050
3.271.3 Rubi [A] (verified) . . . 2050
3.271.4 Maple [A] (verified) . . . 2052
3.271.5 Fricas [A] (verification not implemented) . . . 2053
3.271.6 Sympy [F(-1)] . . . 2053
3.271.7 Maxima [A] (verification not implemented) . . . 2054
3.271.8 Giac [A] (verification not implemented) . . . 2055
3.271.9 Mupad [B] (verification not implemented) . . . 2056

3.271.1 Optimal result

Integrand size = 30, antiderivative size = 297

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx = -\frac{c}{7a^2x^7}+\frac{2bc-ad}{4a^3x^4}-\frac{3b^2c-2abd+a^2e}{a^4x} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^4(a+bx^3)} + \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{13/3}b^{2/3}} + \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{13/3}b^{2/3}} - \frac{(10b^3c-7ab^2d+4a^2be-a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{13/3}b^{2/3}}$$

output
```
-1/7*c/a^2/x^7+1/4*(-a*d+2*b*c)/a^3/x^4+(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x-1/3
*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^4/(b*x^3+a)+1/9*(-a^3*f+4*a^2*b*e-7*
a*b^2*d+10*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)/b^(2/3)-1/18*(-a^3*f+4*a^
2*b*e-7*a*b^2*d+10*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(13/
3)/b^(2/3)+1/9*(-a^3*f+4*a^2*b*e-7*a*b^2*d+10*b^3*c)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)/b^(2/3)*3^(1/2)
```

3.271.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{36a^{7/3}c}{x^7}-\frac{63a^{4/3}(-2bc+ad)}{x^4}-\frac{252\sqrt[3]{a}(3b^2c-2abd+a^2e)}{x}+\frac{84\sqrt[3]{a}(-b^3c+ab^2d-a^2be+a^3f)x^2}{a+bx^3}+\frac{28\sqrt{3}(10b^3c-7ab^2d+4a^2be-a^3f)\,a}{b^{2/3}}}{252a}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```

output
```
((-36*a^(7/3)*c)/x^7 - (63*a^(4/3)*(-2*b*c + a*d))/x^4 - (252*a^(1/3)*(3*b
^2*c - 2*a*b*d + a^2*e))/x + (84*a^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a
^3*f)*x^2)/(a + b*x^3) + (28*Sqrt[3]*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a
^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (28*(10*b^3*c
 - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (14*
(-10*b^3*c + 7*a*b^2*d - 4*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/b^(2/3))/(252*a^(13/3))
```

3.271.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 2368$$

$$-\frac{\int -\frac{-\frac{b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^8(bx^3+a)}dx}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{b^3(-fa^3+bea^2-b^2da+b^3c)x^9}{a^3}+\frac{3b^3(ea^2-bda+b^2c)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^8(bx^3+a)}dx}{3ab^3}-\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^4(a+bx^3)}$$

$$\Big\downarrow \text{2373}$$

$$\frac{\int \left(\frac{(fa^3-4bea^2+7b^2da-10b^3c)xb^3}{a^3(bx^3+a)}+\frac{3(ea^2-2bda+3b^2c)b^3}{a^3x^2}+\frac{3(ad-2bc)b^3}{a^2x^5}+\frac{3cb^3}{ax^8}\right)dx}{3ab^3}-\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^4(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3b^3(2bc-ad)}{4a^2x^4}-\frac{3b^3(a^2e-2abd+3b^2c)}{a^3x}+\frac{b^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^3(-f)+4a^2be-7ab^2d+10b^3c)}{\sqrt{3}a^{10/3}}-\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(a}{6a^{10/3}}}{3ab^3}$$
$$\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^4(a+bx^3)}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x]
```

output
```
-1/3*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a^4*(a + b*x^3)) + ((-3*b^
3*c)/(7*a*x^7) + (3*b^3*(2*b*c - a*d))/(4*a^2*x^4) - (3*b^3*(3*b^2*c - 2*a
*b*d + a^2*e))/(a^3*x) + (b^(7/3)*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*
f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(10/3)) +
 (b^(7/3)*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Log[a^(1/3) + b^(1/3)
*x])/(3*a^(10/3)) - (b^(7/3)*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(10/3)))/(3*a*b^3)
```

3.271.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.271. $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx$

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.271.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.72

method	result
default	$-\frac{c}{7a^2x^7}-\frac{ad-2bc}{4a^3x^4}-\frac{a^2e-2abd+3b^2c}{a^4x}+\frac{\frac{\left(\frac{1}{3}fa^3-\frac{1}{3}a^2be+\frac{1}{3}ab^2d-\frac{1}{3}b^3c\right)x^2}{bx^3+a}+\left(-\frac{4}{3}a^2be+\frac{7}{3}ab^2d-\frac{10}{3}b^3c+\frac{1}{3}fa^3\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{a^4}$
risch	$\frac{\frac{\left(fa^3-4a^2be+7ab^2d-10b^3c\right)x^9}{3a^4}-\frac{\left(4a^2e-7abd+10b^2c\right)x^6}{4a^3}-\frac{(7ad-10bc)x^3}{28a^2}-\frac{c}{7a}}{x^7(bx^3+a)}+\frac{\left(\_R=\mathrm{RootOf}\left(a^{13}b^2\_Z^3+a^9f^3-12a^8bef^2+21a^7b^2df^2+\right.\right.}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/7*c/a^2/x^7-1/4*(a*d-2*b*c)/a^3/x^4-(a^2*e-2*a*b*d+3*b^2*c)/a^4/x+1/a^4
*((1/3*f*a^3-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x^2/(b*x^3+a)+(-4/3*a^2*b*
e+7/3*a*b^2*d-10/3*b^3*c+1/3*f*a^3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+
1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 982, normalized size of antiderivative = 3.31

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^8 (a + bx^3)^2} dx = \text{Too large to display}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/252*(84*(10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x^9 + 36*
a^4*b^2*c + 63*(10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e)*x^6 - 9*(10*a^3*
b^3*c - 7*a^4*b^2*d)*x^3 + 42*sqrt(1/3)*((10*a*b^5*c - 7*a^2*b^4*d + 4*a^3
*b^3*e - a^4*b^2*f)*x^10 + (10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2*e - a^5
*b*f)*x^7)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*
x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-
a*b^2)^(2/3)*x)/(b*x^3 + a)) + 14*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e - a
^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e - a^4*f)*x^7)*(-a*b^2
)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((10*b^4*c
 - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4
*a^3*b*e - a^4*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^3*
x^10 + a^6*b^2*x^7), -1/252*(84*(10*a*b^5*c - 7*a^2*b^4*d + 4*a^3*b^3*e -
a^4*b^2*f)*x^9 + 36*a^4*b^2*c + 63*(10*a^2*b^4*c - 7*a^3*b^3*d + 4*a^4*b^2
*e)*x^6 - 9*(10*a^3*b^3*c - 7*a^4*b^2*d)*x^3 + 84*sqrt(1/3)*((10*a*b^5*c -
 7*a^2*b^4*d + 4*a^3*b^3*e - a^4*b^2*f)*x^10 + (10*a^2*b^4*c - 7*a^3*b^3*d
 + 4*a^4*b^2*e - a^5*b*f)*x^7)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2
*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*((10*b^4*c - 7*a*b^
3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a*b^3*c - 7*a^2*b^2*d + 4*a^3*b*e
- a^4*f)*x^7)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(
2/3)) - 28*((10*b^4*c - 7*a*b^3*d + 4*a^2*b^2*e - a^3*b*f)*x^10 + (10*a...
```

3.271.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^8 (a + bx^3)^2} dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx = -\frac{28\,(10\,b^3c-7\,ab^2d+4\,a^2be-a^3f)x^9+21\,(10\,ab^2c-7\,a^2bd+4\,a^3e)x^6+12\,a^3c-3\,(10\,a^2bc-7\,a^3d)x^3}{84\,(a^4bx^{10}+a^5x^7)} - \frac{\sqrt{3}(10\,b^3c-7\,ab^2d+4\,a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(10\,b^3c-7\,ab^2d+4\,a^2be-a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(10\,b^3c-7\,ab^2d+4\,a^2be-a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^4b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/84*(28*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*x^9 + 21*(10*a*b^2*c
- 7*a^2*b*d + 4*a^3*e)*x^6 + 12*a^3*c - 3*(10*a^2*b*c - 7*a^3*d)*x^3)/(a^4
*b*x^10 + a^5*x^7) - 1/9*sqrt(3)*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
- 1/18*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(a^4*b*(a/b)^(1/3)) + 1/9*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e
 - a^3*f)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
```

3.271.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.10

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^2}\,dx$$
$$= -\frac{\sqrt{3}(10\,b^3c-7\,ab^2d+4\,a^2be-a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^4}$$
$$+\frac{(10\,b^3c-7\,ab^2d+4\,a^2be-a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^4}$$
$$+\frac{\left(10\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5}$$
$$-\frac{b^3cx^2-ab^2dx^2+a^2bex^2-a^3fx^2}{3\,(bx^3+a)a^4}$$
$$-\frac{84\,b^2cx^6-56\,abdx^6+28\,a^2ex^6-14\,abcx^3+7\,a^2dx^3+4\,a^2c}{28\,a^4x^7}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(10*b^3*c - 7*a*b^2*d + 4*a^2*b*e - a^3*f)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^4) + 1/18*(10*b^3*c
- 7*a*b^2*d + 4*a^2*b*e - a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
((-a*b^2)^(1/3)*a^4) + 1/9*(10*b^3*c*(-a/b)^(1/3) - 7*a*b^2*d*(-a/b)^(1/3)
 + 4*a^2*b*e*(-a/b)^(1/3) - a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/a^5 - 1/3*(b^3*c*x^2 - a*b^2*d*x^2 + a^2*b*e*x^2 - a^3*f*x^2
)/((b*x^3 + a)*a^4) - 1/28*(84*b^2*c*x^6 - 56*a*b*d*x^6 + 28*a^2*e*x^6 - 1
4*a*b*c*x^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^4*x^7)
```

3.271.9 Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^2}\,dx$$
$$= \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-f\,a^3+4\,e\,a^2\,b-7\,d\,a\,b^2+10\,c\,b^3\right)}{9\,a^{13/3}\,b^{2/3}}$$
$$-\frac{\frac{c}{7\,a}+\frac{x^9\,\left(-f\,a^3+4\,e\,a^2\,b-7\,d\,a\,b^2+10\,c\,b^3\right)}{3\,a^4}+\frac{x^3\,(7\,a\,d-10\,b\,c)}{28\,a^2}+\frac{x^6\,\left(4\,e\,a^2-7\,d\,a\,b+10\,c\,b^2\right)}{4\,a^3}}{b\,x^{10}+a\,x^7}$$
$$-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+4\,e\,a^2\,b-7\,d\,a\,b^2+10\,c\,b^3\right)}{9\,a^{13/3}\,b^{2/3}}$$
$$+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-f\,a^3+4\,e\,a^2\,b-7\,d\,a\,b^2+10\,c\,b^3\right)}{9\,a^{13/3}\,b^{2/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^2),x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e))/(9*a
^(13/3)*b^(2/3)) - (c/(7*a) + (x^9*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b
*e))/(3*a^4) + (x^3*(7*a*d - 10*b*c))/(28*a^2) + (x^6*(10*b^2*c + 4*a^2*e
- 7*a*b*d))/(4*a^3))/(a*x^7 + b*x^10) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3
)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^
2*b*e))/(9*a^(13/3)*b^(2/3)) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(
1/3))*((3^(1/2)*1i)/2 - 1/2)*(10*b^3*c - a^3*f - 7*a*b^2*d + 4*a^2*b*e))/(
9*a^(13/3)*b^(2/3))
```

3.272 $\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx$

3.272.1 Optimal result . . . 2057
3.272.2 Mathematica [A] (verified) . . . 2058
3.272.3 Rubi [A] (verified) . . . 2058
3.272.4 Maple [A] (verified) . . . 2060
3.272.5 Fricas [A] (verification not implemented) . . . 2061
3.272.6 Sympy [F(-1)] . . . 2061
3.272.7 Maxima [A] (verification not implemented) . . . 2062
3.272.8 Giac [A] (verification not implemented) . . . 2063
3.272.9 Mupad [B] (verification not implemented) . . . 2064

3.272.1 Optimal result

Integrand size = 30, antiderivative size = 297

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx &= -\frac{c}{8a^2x^8}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{2a^4x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{3a^4\left(a+bx^3\right)} \\ &+\frac{\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{14/3}\sqrt[3]{b}} \\ &-\frac{\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{14/3}\sqrt[3]{b}} \\ &+\frac{\left(11b^3c-8ab^2d+5a^2be-2a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{14/3}\sqrt[3]{b}}\end{aligned}$$

output
```
-1/8*c/a^2/x^8+1/5*(-a*d+2*b*c)/a^3/x^5+1/2*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x
^2-1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^3+a)-1/9*(-2*a^3*f+5*a^2*
b*e-8*a*b^2*d+11*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)/b^(1/3)+1/18*(-2*a^
3*f+5*a^2*b*e-8*a*b^2*d+11*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/a^(14/3)/b^(1/3)+1/9*(-2*a^3*f+5*a^2*b*e-8*a*b^2*d+11*b^3*c)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)/b^(1/3)*3^(1/2)
```

3.272.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx$$

$$=\frac{-\frac{45a^{8/3}c}{x^8}-\frac{72a^{5/3}(-2bc+ad)}{x^5}-\frac{180a^{2/3}(3b^2c-2abd+a^2e)}{x^2}+\frac{120a^{2/3}(-b^3c+ab^2d-a^2be+a^3f)x}{a+bx^3}+\frac{40\sqrt{3}(11b^3c-8ab^2d+5a^2be-2a^3f)\ a}{\sqrt[3]{b}}}{360}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```

output
```
((-45*a^(8/3)*c)/x^8 - (72*a^(5/3)*(-2*b*c + a*d))/x^5 - (180*a^(2/3)*(3*b
^2*c - 2*a*b*d + a^2*e))/x^2 + (120*a^(2/3)*(-(b^3*c) + a*b^2*d - a^2*b*e
+ a^3*f)*x)/(a + b*x^3) + (40*Sqrt[3]*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e -
2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (40*(-11*b
^3*c + 8*a*b^2*d - 5*a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
+ (20*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(360*a^(14/3))
```

3.272.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \ 2368$$

$$-\frac{\int -\frac{-\frac{2b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^9\left(bx^3+a\right)}dx}{3ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{-\frac{2b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^9(bx^3+a)}dx}{3ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 2373$$

$$\frac{\int \left(\frac{\left(2fa^3-5bea^2+8b^2da-11b^3c\right)b^3}{a^3(bx^3+a)}+\frac{3\left(ea^2-2bda+3b^2c\right)b^3}{a^3x^3}+\frac{3(ad-2bc)b^3}{a^2x^6}+\frac{3cb^3}{ax^9}\right)dx}{3ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{3b^3(2bc-ad)}{5a^2x^5}-\frac{3b^3\left(a^2e-2abd+3b^2c\right)}{2a^3x^2}+\frac{b^{8/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{\sqrt{3}a^{11/3}}+\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(-\cdots}{6a^{11/3}}\cdots}{3ab^3}$$
$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^4\left(a+bx^3\right)}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x]
```

output
```
-1/3*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a^4*(a + b*x^3)) + ((-3*b^3*
c)/(8*a*x^8) + (3*b^3*(2*b*c - a*d))/(5*a^2*x^5) - (3*b^3*(3*b^2*c - 2*a*b
*d + a^2*e))/(2*a^3*x^2) + (b^(8/3)*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*
a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(11/3
)) - (b^(8/3)*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(11/3)) + (b^(8/3)*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a
^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(11/3)))/(3*a*b
^3)
```

3.272.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.272.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.72

method	result
default	$-\frac{c}{8a^2x^8}-\frac{ad-2bc}{5a^3x^5}-\frac{a^2e-2abd+3b^2c}{2a^4x^2}+\frac{\frac{\left(\frac{1}{3}fa^3-\frac{1}{3}a^2be+\frac{1}{3}ab^2d-\frac{1}{3}b^3c\right)x}{bx^3+a}+\frac{\left(2fa^3-5a^2be+8ab^2d-11b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\ldots\right.}{\ldots}\right.}{3}}{a^4}$
risch	$\frac{\frac{\left(2fa^3-5a^2be+8ab^2d-11b^3c\right)x^9}{6a^4}-\frac{\left(5a^2e-8abd+11b^2c\right)x^6}{10a^3}-\frac{(8ad-11bc)x^3}{40a^2}-\frac{c}{8a}}{x^8(bx^3+a)}+\left(\sum_{\_R=\mathrm{RootOf}\left(a^{14}b\_Z^3-8a^9f^3+60a^8bef^2-96a^7b^2df^2\ldots\right.}\right.$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*c/a^2/x^8-1/5*(a*d-2*b*c)/a^3/x^5-1/2*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^2
+1/a^4*((1/3*f*a^3-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x/(b*x^3+a)+1/3*(2*a
^3*f-5*a^2*b*e+8*a*b^2*d-11*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/
6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 959, normalized size of antiderivative = 3.23

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^9 (a + bx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/360*(60*(11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5*b*f)*x^9 + 4
5*a^5*b*c + 36*(11*a^3*b^3*c - 8*a^4*b^2*d + 5*a^5*b*e)*x^6 - 9*(11*a^4*b^
2*c - 8*a^5*b*d)*x^3 + 60*sqrt(1/3)*((11*a*b^5*c - 8*a^2*b^4*d + 5*a^3*b^3
*e - 2*a^4*b^2*f)*x^11 + (11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*e - 2*a^5
*b*f)*x^8)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a
^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^
2*b)^(1/3)/b))/(b*x^3 + a)) - 20*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*
a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2
*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*((11*b^4*c
 - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^2*d +
 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*
b^2*x^11 + a^7*b*x^8), -1/360*(60*(11*a^2*b^4*c - 8*a^3*b^3*d + 5*a^4*b^2*
e - 2*a^5*b*f)*x^9 + 45*a^5*b*c + 36*(11*a^3*b^3*c - 8*a^4*b^2*d + 5*a^5*b
*e)*x^6 - 9*(11*a^4*b^2*c - 8*a^5*b*d)*x^3 + 120*sqrt(1/3)*((11*a*b^5*c -
8*a^2*b^4*d + 5*a^3*b^3*e - 2*a^4*b^2*f)*x^11 + (11*a^2*b^4*c - 8*a^3*b^3*
d + 5*a^4*b^2*e - 2*a^5*b*f)*x^8)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(
2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*((11*
b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*b*f)*x^11 + (11*a*b^3*c - 8*a^2*b^
2*d + 5*a^3*b*e - 2*a^4*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*
x + (a^2*b)^(1/3)*a) + 40*((11*b^4*c - 8*a*b^3*d + 5*a^2*b^2*e - 2*a^3*...
```

3.272.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^9 (a + bx^3)^2} \, dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx =$$
$$-\frac{20\,(11\,b^3c-8\,ab^2d+5\,a^2be-2\,a^3f)x^9+12\,(11\,ab^2c-8\,a^2bd+5\,a^3e)x^6+15\,a^3c-3\,(11\,a^2bc-8\,a^3d)}{120\,(a^4bx^{11}+a^5x^8)}$$
$$-\frac{\sqrt{3}(11\,b^3c-8\,ab^2d+5\,a^2be-2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(11\,b^3c-8\,ab^2d+5\,a^2be-2\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(11\,b^3c-8\,ab^2d+5\,a^2be-2\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/120*(20*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*x^9 + 12*(11*a*b^2
*c - 8*a^2*b*d + 5*a^3*e)*x^6 + 15*a^3*c - 3*(11*a^2*b*c - 8*a^3*d)*x^3)/(
a^4*b*x^11 + a^5*x^8) - 1/9*sqrt(3)*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*
a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(2
/3)) + 1/18*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*log(x^2 - x*(a/b)
^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) - 1/9*(11*b^3*c - 8*a*b^2*d + 5*
a^2*b*e - 2*a^3*f)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.15

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx = \frac{\left(11\,b^3c-8\,ab^2d+5\,a^2be-2\,a^3f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5}$$
$$-\frac{\sqrt{3}\left(11\left(-ab^2\right)^{\frac{1}{3}}b^3c-8\left(-ab^2\right)^{\frac{1}{3}}ab^2d+5\left(-ab^2\right)^{\frac{1}{3}}a^2be-2\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^5b}$$
$$-\frac{b^3cx-ab^2dx+a^2bex-a^3fx}{3\left(bx^3+a\right)a^4}$$
$$-\frac{\left(11\left(-ab^2\right)^{\frac{1}{3}}b^3c-8\left(-ab^2\right)^{\frac{1}{3}}ab^2d+5\left(-ab^2\right)^{\frac{1}{3}}a^2be-2\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^5b}$$
$$-\frac{60\,b^2cx^6-40\,abdx^6+20\,a^2ex^6-16\,abcx^3+8\,a^2dx^3+5\,a^2c}{40\,a^4x^8}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/a^5 - 1/9*sqrt(3)*(11*(-a*b^2)^(1/3)*b^3*c - 8*(-a*b^2)^(1/
3)*a*b^2*d + 5*(-a*b^2)^(1/3)*a^2*b*e - 2*(-a*b^2)^(1/3)*a^3*f)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 1/3*(b^3*c*x - a*b^2
*d*x + a^2*b*e*x - a^3*f*x)/((b*x^3 + a)*a^4) - 1/18*(11*(-a*b^2)^(1/3)*b^
3*c - 8*(-a*b^2)^(1/3)*a*b^2*d + 5*(-a*b^2)^(1/3)*a^2*b*e - 2*(-a*b^2)^(1/
3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/40*(60*b^2*
c*x^6 - 40*a*b*d*x^6 + 20*a^2*e*x^6 - 16*a*b*c*x^3 + 8*a^2*d*x^3 + 5*a^2*c
)/(a^4*x^8)
```

3.272.9 Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^2}\,dx$$
$$= -\frac{\frac{c}{8a}+\frac{x^9\left(-2fa^3+5ea^2b-8dab^2+11cb^3\right)}{6a^4}+\frac{x^3\left(8ad-11bc\right)}{40a^2}+\frac{x^6\left(5ea^2-8dab+11cb^2\right)}{10a^3}}{bx^{11}+ax^8}$$
$$-\frac{\ln\left(b^{1/3}x+a^{1/3}\right)\left(-2fa^3+5ea^2b-8dab^2+11cb^3\right)}{9a^{14/3}b^{1/3}}$$
$$-\frac{\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)\left(-2fa^3+5ea^2b-8dab^2+11cb^3\right)}{9a^{14/3}b^{1/3}}$$
$$+\frac{\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)\left(-2fa^3+5ea^2b-8dab^2+11cb^3\right)}{9a^{14/3}b^{1/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^2),x)
```

output
```
(log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(1
1*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(9*a^(14/3)*b^(1/3)) - (log(b^
(1/3)*x + a^(1/3))*(11*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(9*a^(14/
3)*b^(1/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 - 1/2)*(11*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(9*a^(14/3)*b^(1
/3)) - (c/(8*a) + (x^9*(11*b^3*c - 2*a^3*f - 8*a*b^2*d + 5*a^2*b*e))/(6*a^
4) + (x^3*(8*a*d - 11*b*c))/(40*a^2) + (x^6*(11*b^2*c + 5*a^2*e - 8*a*b*d)
)/(10*a^3))/(a*x^8 + b*x^11)
```

3.273 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^2}\,dx$

3.273.1 Optimal result . . . 2065
3.273.2 Mathematica [A] (verified) . . . 2066
3.273.3 Rubi [A] (verified) . . . 2066
3.273.4 Maple [A] (verified) . . . 2068
3.273.5 Fricas [A] (verification not implemented) . . . 2069
3.273.6 Sympy [F(-1)] . . . 2070
3.273.7 Maxima [A] (verification not implemented) . . . 2070
3.273.8 Giac [A] (verification not implemented) . . . 2071
3.273.9 Mupad [B] (verification not implemented) . . . 2072

3.273.1 Optimal result

Integrand size = 30, antiderivative size = 334

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^2}\,dx &= -\frac{c}{10a^2x^{10}}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{4a^4x^4} \\ &\quad +\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^5(a+bx^3)} \\ &\quad -\frac{\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{16/3}} \\ &\quad -\frac{\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{16/3}} \\ &\quad +\frac{\sqrt[3]{b}(13b^3c-10ab^2d+7a^2be-4a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{16/3}}\end{aligned}$$

output
```
-1/10*c/a^2/x^10+1/7*(-a*d+2*b*c)/a^3/x^7+1/4*(-a^2*e+2*a*b*d-3*b^2*c)/a^4
/x^4+(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x+1/3*b*(-a^3*f+a^2*b*e-a*b^
2*d+b^3*c)*x^2/a^5/(b*x^3+a)-1/9*b^(1/3)*(-4*a^3*f+7*a^2*b*e-10*a*b^2*d+13
*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(16/3)+1/18*b^(1/3)*(-4*a^3*f+7*a^2*b*e-10
*a*b^2*d+13*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(16/3)-1/9*
b^(1/3)*(-4*a^3*f+7*a^2*b*e-10*a*b^2*d+13*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(16/3)*3^(1/2)
```

3.273.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.96

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{11}\left(a + bx^3\right)^2}\,dx$$

$$= \frac{-\frac{126a^{10/3}c}{x^{10}} - \frac{180a^{7/3}(-2bc+ad)}{x^7} - \frac{315a^{4/3}(3b^2c-2abd+a^2e)}{x^4} - \frac{1260\sqrt[3]{a}(-4b^3c+3ab^2d-2a^2be+a^3f)}{x} - \frac{420\sqrt[3]{a}b(-b^3c+ab^2d-a^2be\cdots}{a+bx^3}}{}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```

output
```
((-126*a^(10/3)*c)/x^10 - (180*a^(7/3)*(-2*b*c + a*d))/x^7 - (315*a^(4/3)*
(3*b^2*c - 2*a*b*d + a^2*e))/x^4 - (1260*a^(1/3)*(-4*b^3*c + 3*a*b^2*d - 2
*a^2*b*e + a^3*f))/x - (420*a^(1/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*x^2)/(a + b*x^3) - 140*Sqrt[3]*b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b
*e - 4*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*b^(1/3)*(-
13*b^3*c + 10*a*b^2*d - 7*a^2*b*e + 4*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 70
*b^(1/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(1260*a^(16/3))
```

3.273.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{11}\left(a + bx^3\right)^2}\,dx$$

$$\Big\downarrow \; 2368$$

$$\frac{bx^2\left(a^3(-f) + a^2be - ab^2d + b^3c\right)}{3a^5\left(a + bx^3\right)} - \frac{\int -\frac{\frac{b^4\left(-fa^3+bea^2-b^2da+b^3c\right)x^{12}}{a^4} - \frac{3b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3} + \frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2} - 3b^3\left(\frac{bc}{a}-d\right)x^3 + 3b^3c}{x^{11}\left(bx^3+a\right)}dx}{3ab^3}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{\frac{b^4(-fa^3+bea^2-b^2da+b^3c)x^{12}}{a^4}-\frac{3b^3(-fa^3+bea^2-b^2da+b^3c)x^9}{a^3}+\frac{3b^3(ea^2-bda+b^2c)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^{11}(bx^3+a)}dx}{3ab^3}+\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}$$

$$\Big\downarrow 2373$$

$$\frac{\int \left(-\frac{(4fa^3-7bea^2+10b^2da-13b^3c)xb^4}{a^4(bx^3+a)}+\frac{3(fa^3-2bea^2+3b^2da-4b^3c)b^3}{a^4x^2}+\frac{3(ea^2-2bda+3b^2c)b^3}{a^3x^5}+\frac{3(ad-2bc)b^3}{a^2x^8}+\frac{3cb^3}{ax^{11}}\right)dx}{3ab^3}+\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}$$

$$\Big\downarrow 2009$$

$$\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}+$$

$$\frac{\frac{3b^3(2bc-ad)}{7a^2x^7}-\frac{3b^3(a^2e-2abd+3b^2c)}{4a^3x^4}-\frac{b^{10/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-4a^3f+7a^2be-10ab^2d+13b^3c)}{\sqrt{3}a^{13/3}}+\frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{13}}}{3a}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x]
```

output
```
(b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(3*a^5*(a + b*x^3)) + ((-3*b^3
*c)/(10*a*x^10) + (3*b^3*(2*b*c - a*d))/(7*a^2*x^7) - (3*b^3*(3*b^2*c - 2*
a*b*d + a^2*e))/(4*a^3*x^4) + (3*b^3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^
3*f))/(a^4*x) - (b^(10/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Ar
cTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(13/3)) - (b^(
10/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(13/3)) + (b^(10/3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(13/3)))/(3*a*b^3)
```

3.273. $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^2}\,dx$

3.273.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.273.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.75

method	result
default	$-\frac{c}{10a^{2}x^{10}}-\frac{ad-2bc}{7a^{3}x^{7}}-\frac{a^{2}e-2abd+3b^{2}c}{4a^{4}x^{4}}-\frac{f\,a^{3}-2a^{2}be+3a\,b^{2}d-4b^{3}c}{a^{5}x}-\frac{b\left(\frac{\left(\frac{1}{3}f\,a^{3}-\frac{1}{3}a^{2}be+\frac{1}{3}a\,b^{2}d-\frac{1}{3}b^{3}c\right)x^{2}}{b\,x^{3}+a}+\left(\frac{4}{3}f\,a^{3}-\frac{7}{3}a^{2}be\right.\right.}{}$
risch	$\frac{-\frac{b\left(4f\,a^{3}-7a^{2}be+10a\,b^{2}d-13b^{3}c\right)x^{12}}{3a^{5}}-\frac{\left(4f\,a^{3}-7a^{2}be+10a\,b^{2}d-13b^{3}c\right)x^{9}}{4a^{4}}-\frac{\left(7a^{2}e-10abd+13b^{2}c\right)x^{6}}{28a^{3}}-\frac{(10ad-13bc)x^{3}}{70a^{2}}-\frac{c}{10a}}{x^{10}(b\,x^{3}+a)}+\frac{\left(\_R=\mathrm{Root}\right.}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/10*c/a^2/x^10-1/7*(a*d-2*b*c)/a^3/x^7-1/4*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x
^4-(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x-1/a^5*b*((1/3*f*a^3-1/3*a^2*b
*e+1/3*a*b^2*d-1/3*b^3*c)*x^2/(b*x^3+a)+(4/3*f*a^3-7/3*a^2*b*e+10/3*a*b^2*
d-13/3*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))))
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.32

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx$$
$$= \frac{420\,(13\,b^4c-10\,ab^3d+7\,a^2b^2e-4\,a^3bf)x^{12}+315\,(13\,ab^3c-10\,a^2b^2d+7\,a^3be-4\,a^4f)x^9-45\,(13\,a^2b^2c}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/1260*(420*(13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^12 + 315*(
13*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^9 - 45*(13*a^2*b^2*c -
10*a^3*b*d + 7*a^4*e)*x^6 - 126*a^4*c + 18*(13*a^3*b*c - 10*a^4*d)*x^3 + 1
40*sqrt(3)*((13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^13 + (13*a
*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^10)*(b/a)^(1/3)*arctan(2/3*
sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*((13*b^4*c - 10*a*b^3*d + 7*a^2*
b^2*e - 4*a^3*b*f)*x^13 + (13*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f
)*x^10)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 140*((1
3*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^13 + (13*a*b^3*c - 10*a^
2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^10)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)))
/(a^5*b*x^13 + a^6*x^10)
```

3.273.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.273.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx$$
$$= \frac{140\,(13\,b^4c-10\,ab^3d+7\,a^2b^2e-4\,a^3bf)x^{12}+105\,(13\,ab^3c-10\,a^2b^2d+7\,a^3be-4\,a^4f)x^9-15\,(13\,a^2b^2c}{420\,(a^5bx^{13}+a^6x^{10})}$$
$$+\frac{\sqrt{3}(13\,b^3c-10\,ab^2d+7\,a^2be-4\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^5\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{(13\,b^3c-10\,ab^2d+7\,a^2be-4\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^5\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{(13\,b^3c-10\,ab^2d+7\,a^2be-4\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^5\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/420*(140*(13*b^4*c - 10*a*b^3*d + 7*a^2*b^2*e - 4*a^3*b*f)*x^12 + 105*(1
3*a*b^3*c - 10*a^2*b^2*d + 7*a^3*b*e - 4*a^4*f)*x^9 - 15*(13*a^2*b^2*c - 1
0*a^3*b*d + 7*a^4*e)*x^6 - 42*a^4*c + 6*(13*a^3*b*c - 10*a^4*d)*x^3)/(a^5*
b*x^13 + a^6*x^10) + 1/9*sqrt(3)*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^
3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(1/3))
 + 1/18*(13*b^3*c - 10*a*b^2*d + 7*a^2*b*e - 4*a^3*f)*log(x^2 - x*(a/b)^(1
/3) + (a/b)^(2/3))/(a^5*(a/b)^(1/3)) - 1/9*(13*b^3*c - 10*a*b^2*d + 7*a^2*
b*e - 4*a^3*f)*log(x + (a/b)^(1/3))/(a^5*(a/b)^(1/3))
```

3.273.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.29

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^2}\,dx = -\frac{\left(13\,b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-10\,ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+7\,a^2b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^6} - \frac{\sqrt{3}\left(13\left(-ab^2\right)^{\frac{2}{3}}b^3c-10\left(-ab^2\right)^{\frac{2}{3}}ab^2d+7\left(-ab^2\right)^{\frac{2}{3}}a^2be-4\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^6b} + \frac{b^4cx^2-ab^3dx^2+a^2b^2ex^2-a^3bfx^2}{3\,(bx^3+a)a^5} + \frac{\left(13\left(-ab^2\right)^{\frac{2}{3}}b^3c-10\left(-ab^2\right)^{\frac{2}{3}}ab^2d+7\left(-ab^2\right)^{\frac{2}{3}}a^2be-4\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^6b} + \frac{560\,b^3cx^9-420\,ab^2dx^9+280\,a^2bex^9-140\,a^3fx^9-105\,ab^2cx^6+70\,a^2bdx^6-35\,a^3ex^6+40\,a^2bcx^3-20}{140\,a^5x^{10}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*(13*b^4*c*(-a/b)^(1/3) - 10*a*b^3*d*(-a/b)^(1/3) + 7*a^2*b^2*e*(-a/b)
^(1/3) - 4*a^3*b*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a
^6 - 1/9*sqrt(3)*(13*(-a*b^2)^(2/3)*b^3*c - 10*(-a*b^2)^(2/3)*a*b^2*d + 7*
(-a*b^2)^(2/3)*a^2*b*e - 4*(-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a^6*b) + 1/3*(b^4*c*x^2 - a*b^3*d*x^2 + a^2*
b^2*e*x^2 - a^3*b*f*x^2)/((b*x^3 + a)*a^5) + 1/18*(13*(-a*b^2)^(2/3)*b^3*c
 - 10*(-a*b^2)^(2/3)*a*b^2*d + 7*(-a*b^2)^(2/3)*a^2*b*e - 4*(-a*b^2)^(2/3)
*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) + 1/140*(560*b^3*
c*x^9 - 420*a*b^2*d*x^9 + 280*a^2*b*e*x^9 - 140*a^3*f*x^9 - 105*a*b^2*c*x^
6 + 70*a^2*b*d*x^6 - 35*a^3*e*x^6 + 40*a^2*b*c*x^3 - 20*a^3*d*x^3 - 14*a^3
*c)/(a^5*x^10)
```

3.273.9 Mupad [B] (verification not implemented)

Time = 9.80 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^2}\,dx =$$

$$-\frac{\frac{c}{10\,a}-\frac{x^9\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{4\,a^4}+\frac{x^3\,\left(10\,a\,d-13\,b\,c\right)}{70\,a^2}+\frac{x^6\,\left(7\,e\,a^2-10\,d\,a\,b+13\,c\,b^2\right)}{28\,a^3}-\frac{b\,x^{12}\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+\dots}{3\,a^5}}{b\,x^{13}+a\,x^{10}}$$
$$-\frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{9\,a^{16/3}}$$
$$+\frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{9\,a^{16/3}}$$
$$-\frac{b^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-4\,f\,a^3+7\,e\,a^2\,b-10\,d\,a\,b^2+13\,c\,b^3\right)}{9\,a^{16/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^2),x)
```

output
```
(b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
 1/2)*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e))/(9*a^(16/3)) - (b^(1/
3)*log(b^(1/3)*x + a^(1/3))*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*b*e))
/(9*a^(16/3)) - (c/(10*a) - (x^9*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2*
b*e))/(4*a^4) + (x^3*(10*a*d - 13*b*c))/(70*a^2) + (x^6*(13*b^2*c + 7*a^2*
e - 10*a*b*d))/(28*a^3) - (b*x^12*(13*b^3*c - 4*a^3*f - 10*a*b^2*d + 7*a^2
*b*e))/(3*a^5))/(a*x^10 + b*x^13) - (b^(1/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^
(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(13*b^3*c - 4*a^3*f - 10*a*b^2*d
 + 7*a^2*b*e))/(9*a^(16/3))
```

3.274 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^2}\,dx$

3.274.1 Optimal result 2073
3.274.2 Mathematica [A] (verified) 2074
3.274.3 Rubi [A] (verified) 2074
3.274.4 Maple [A] (verified) 2076
3.274.5 Fricas [A] (verification not implemented) 2077
3.274.6 Sympy [F(-1)] 2078
3.274.7 Maxima [A] (verification not implemented) 2079
3.274.8 Giac [A] (verification not implemented) 2080
3.274.9 Mupad [B] (verification not implemented) 2081

3.274.1 Optimal result

Integrand size = 30, antiderivative size = 335

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^2}\,dx &= -\frac{c}{11a^2x^{11}}+\frac{2bc-ad}{8a^3x^8}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}\\ &\quad+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{2a^5x^2}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{3a^5(a+bx^3)}\\ &\quad-\frac{b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{17/3}}\\ &\quad+\frac{b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{17/3}}\\ &\quad-\frac{b^{2/3}(14b^3c-11ab^2d+8a^2be-5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{17/3}}\end{aligned}$$

output
```
-1/11*c/a^2/x^11+1/8*(-a*d+2*b*c)/a^3/x^8+1/5*(-a^2*e+2*a*b*d-3*b^2*c)/a^4
/x^5+1/2*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^2+1/3*b*(-a^3*f+a^2*b*
e-a*b^2*d+b^3*c)*x/a^5/(b*x^3+a)+1/9*b^(2/3)*(-5*a^3*f+8*a^2*b*e-11*a*b^2*
d+14*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(17/3)-1/18*b^(2/3)*(-5*a^3*f+8*a^2*b*
e-11*a*b^2*d+14*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(17/3)-
1/9*b^(2/3)*(-5*a^3*f+8*a^2*b*e-11*a*b^2*d+14*b^3*c)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(17/3)*3^(1/2)
```

3.274.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{360a^{11/3}c}{x^{11}}-\frac{495a^{8/3}(-2bc+ad)}{x^8}-\frac{792a^{5/3}\left(3b^2c-2abd+a^2e\right)}{x^5}-\frac{1980a^{2/3}\left(-4b^3c+3ab^2d-2a^2be+a^3f\right)}{x^2}-\frac{1320a^{2/3}b\left(-b^3c+ab^2d-a^2be\right.}{a+bx^3}}{}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]
```

output
```
((-360*a^(11/3)*c)/x^11 - (495*a^(8/3)*(-2*b*c + a*d))/x^8 - (792*a^(5/3)*
(3*b^2*c - 2*a*b*d + a^2*e))/x^5 - (1980*a^(2/3)*(-4*b^3*c + 3*a*b^2*d - 2
*a^2*b*e + a^3*f))/x^2 - (1320*a^(2/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a
^3*f)*x)/(a + b*x^3) - 440*Sqrt[3]*b^(2/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*
b*e - 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 440*b^(2/3)*(
14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x] + 22
0*b^(2/3)*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3960*a^(17/3))
```

3.274.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5\left(a+bx^3\right)}-\frac{\int -\frac{\frac{2b^4\left(-fa^3+bea^2-b^2da+b^3c\right)x^{12}}{a^4}-\frac{3b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^{12}\left(bx^3+a\right)}dx}{3ab^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{2b^4(-fa^3+bea^2-b^2da+b^3c)x^{12}}{a^4}-\frac{3b^3(-fa^3+bea^2-b^2da+b^3c)x^9}{a^3}+\frac{3b^3(ea^2-bda+b^2c)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^{12}(bx^3+a)}dx}{3ab^3}+\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5(a+bx^3)}$$

$$\Big\downarrow \; 2373$$

$$\frac{\int \left(-\frac{(5fa^3-8bea^2+11b^2da-14b^3c)b^4}{a^4(bx^3+a)}+\frac{3(fa^3-2bea^2+3b^2da-4b^3c)b^3}{a^4x^3}+\frac{3(ea^2-2bda+3b^2c)b^3}{a^3x^6}+\frac{3(ad-2bc)b^3}{a^2x^9}+\frac{3cb^3}{ax^{12}}\right)dx}{3ab^3}+\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5(a+bx^3)}$$

$$\Big\downarrow \; 2009$$

$$\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5(a+bx^3)}+$$

$$\frac{\frac{3b^3(2bc-ad)}{8a^2x^8}-\frac{3b^3(a^2e-2abd+3b^2c)}{5a^3x^5}-\frac{b^{11/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-5a^3f+8a^2be-11ab^2d+14b^3c)}{\sqrt{3}a^{14/3}}-\frac{b^{11/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{14}}}{3a}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x]
```

output
```
(b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(3*a^5*(a + b*x^3)) + ((-3*b^3*c
)/(11*a*x^11) + (3*b^3*(2*b*c - a*d))/(8*a^2*x^8) - (3*b^3*(3*b^2*c - 2*a*
b*d + a^2*e))/(5*a^3*x^5) + (3*b^3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*
f))/(2*a^4*x^2) - (b^(11/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(14/3)) + (b
^(11/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*Log[a^(1/3) + b^(1/3
)*x])/(3*a^(14/3)) - (b^(11/3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*
f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(14/3)))/(3*a*b^3)
```

3.274. $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^2}\,dx$

3.274.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.274.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.75

method	result
default	$-\frac{c}{11a^2x^{11}}-\frac{ad-2bc}{8a^3x^8}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{f\,a^3-2a^2be+3a\,b^2d-4b^3c}{2a^5x^2}-\frac{b\left(\frac{\left(\frac{1}{3}f\,a^3-\frac{1}{3}a^2be+\frac{1}{3}a\,b^2d-\frac{1}{3}b^3c\right)x}{b\,x^3+a}+\frac{\left(5f\,a^3-8a^2be+11\ldots\right.}{}\right)}{}$
risch	$\frac{-\frac{b\left(5f\,a^3-8a^2be+11a\,b^2d-14b^3c\right)x^{12}}{6a^5}-\frac{\left(5f\,a^3-8a^2be+11a\,b^2d-14b^3c\right)x^9}{10a^4}-\frac{\left(8a^2e-11abd+14b^2c\right)x^6}{40a^3}-\frac{(11ad-14bc)x^3}{88a^2}-\frac{c}{11a}}{x^{11}(b\,x^3+a)}+\frac{\left(\sum_{\_R=\mathrm{Root}\ldots}\right.}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/11*c/a^2/x^11-1/8*(a*d-2*b*c)/a^3/x^8-1/5*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x
^5-1/2*(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x^2-1/a^5*b*((1/3*f*a^3-1/3
*a^2*b*e+1/3*a*b^2*d-1/3*b^3*c)*x/(b*x^3+a)+1/3*(5*a^3*f-8*a^2*b*e+11*a*b^
2*d-14*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))))
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.42

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx$$

$$=\frac{660\,(14\,b^4c-11\,ab^3d+8\,a^2b^2e-5\,a^3bf)x^{12}+396\,(14\,ab^3c-11\,a^2b^2d+8\,a^3be-5\,a^4f)x^9-99\,(14\,a^2b^2c\ldots}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/3960*(660*(14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^12 + 396*(
14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^9 - 99*(14*a^2*b^2*c -
11*a^3*b*d + 8*a^4*e)*x^6 - 360*a^4*c + 45*(14*a^3*b*c - 11*a^4*d)*x^3 - 4
40*sqrt(3)*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a
*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*arctan
(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 220*((14*b^4*c - 11
*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*
a^3*b*e - 5*a^4*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(
1/3) + a^2*(-b^2/a^2)^(2/3)) - 440*((14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e -
 5*a^3*b*f)*x^14 + (14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^11)
*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^5*b*x^14 + a^6*x^11)
```

3.274.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.96

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^{12}(a + bx^3)^2}\, dx$$
$$= \frac{220\,(14\,b^4c - 11\,ab^3d + 8\,a^2b^2e - 5\,a^3bf)x^{12} + 132\,(14\,ab^3c - 11\,a^2b^2d + 8\,a^3be - 5\,a^4f)x^9 - 33\,(14\,a^2b^2c}{1320\,(a^5bx^{14} + a^6x^{11})}$$
$$+ \frac{\sqrt{3}(14\,b^3c - 11\,ab^2d + 8\,a^2be - 5\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{(14\,b^3c - 11\,ab^2d + 8\,a^2be - 5\,a^3f)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+ \frac{(14\,b^3c - 11\,ab^2d + 8\,a^2be - 5\,a^3f)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/1320*(220*(14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5*a^3*b*f)*x^12 + 132*(
14*a*b^3*c - 11*a^2*b^2*d + 8*a^3*b*e - 5*a^4*f)*x^9 - 33*(14*a^2*b^2*c -
11*a^3*b*d + 8*a^4*e)*x^6 - 120*a^4*c + 15*(14*a^3*b*c - 11*a^4*d)*x^3)/(a
^5*b*x^14 + a^6*x^11) + 1/9*sqrt(3)*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5
*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^5*(a/b)^(2/
3)) - 1/18*(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*log(x^2 - x*(a/b)
^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(2/3)) + 1/9*(14*b^3*c - 11*a*b^2*d + 8*a
^2*b*e - 5*a^3*f)*log(x + (a/b)^(1/3))/(a^5*(a/b)^(2/3))
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.15

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^2}\,dx$$

$$= \frac{\sqrt{3}\left(14\left(-ab^2\right)^{\frac{1}{3}}b^3c-11\left(-ab^2\right)^{\frac{1}{3}}ab^2d+8\left(-ab^2\right)^{\frac{1}{3}}a^2be-5\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^6}$$
$$-\frac{\left(14b^4c-11ab^3d+8a^2b^2e-5a^3bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^6}$$
$$+\frac{\left(14\left(-ab^2\right)^{\frac{1}{3}}b^3c-11\left(-ab^2\right)^{\frac{1}{3}}ab^2d+8\left(-ab^2\right)^{\frac{1}{3}}a^2be-5\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^6}$$
$$+\frac{b^4cx-ab^3dx+a^2b^2ex-a^3bfx}{3(bx^3+a)a^5}$$
$$+\frac{880b^3cx^9-660ab^2dx^9+440a^2bex^9-220a^3fx^9-264ab^2cx^6+176a^2bdx^6-88a^3ex^6+110a^2bcx^3-}{440a^5x^{11}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(14*(-a*b^2)^(1/3)*b^3*c - 11*(-a*b^2)^(1/3)*a*b^2*d + 8*(-a*b
^2)^(1/3)*a^2*b*e - 5*(-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/a^6 - 1/9*(14*b^4*c - 11*a*b^3*d + 8*a^2*b^2*e - 5
*a^3*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^6 + 1/18*(14*(-a*b^2)^
(1/3)*b^3*c - 11*(-a*b^2)^(1/3)*a*b^2*d + 8*(-a*b^2)^(1/3)*a^2*b*e - 5*(-a
*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^6 + 1/3*(b^4
*c*x - a*b^3*d*x + a^2*b^2*e*x - a^3*b*f*x)/((b*x^3 + a)*a^5) + 1/440*(880
*b^3*c*x^9 - 660*a*b^2*d*x^9 + 440*a^2*b*e*x^9 - 220*a^3*f*x^9 - 264*a*b^2
*c*x^6 + 176*a^2*b*d*x^6 - 88*a^3*e*x^6 + 110*a^2*b*c*x^3 - 55*a^3*d*x^3 -
 40*a^3*c)/(a^5*x^11)
```

3.274.9 Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^2}\,dx = \frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-5\,f\,a^3+8\,e\,a^2\,b-11\,d\,a\,b^2+14\,c\,b^3\right)}{9\,a^{17/3}}$$
$$-\frac{\frac{c}{11\,a}-\frac{x^9\,\left(-5\,f\,a^3+8\,e\,a^2\,b-11\,d\,a\,b^2+14\,c\,b^3\right)}{10\,a^4}+\frac{x^3\,\left(11\,a\,d-14\,b\,c\right)}{88\,a^2}+\frac{x^6\,\left(8\,e\,a^2-11\,d\,a\,b+14\,c\,b^2\right)}{40\,a^3}-\frac{b\,x^{12}\,\left(-5\,f\,a^3+8\,e\,a^2\,b-11\,d\,a\,b^2+\right.}{6\,a^5}}{b\,x^{14}+a\,x^{11}}$$
$$+\frac{b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-5\,f\,a^3+8\,e\,a^2\,b-11\,d\,a\,b^2+14\,c\,b^3\right)}{9\,a^{17/3}}$$
$$-\frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-5\,f\,a^3+8\,e\,a^2\,b-11\,d\,a\,b^2+14\,c\,b^3\right)}{9\,a^{17/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^2),x)
```

output
```
(b^(2/3)*log(b^(1/3)*x + a^(1/3))*(14*b^3*c - 5*a^3*f - 11*a*b^2*d + 8*a^2
*b*e))/(9*a^(17/3)) - (c/(11*a) - (x^9*(14*b^3*c - 5*a^3*f - 11*a*b^2*d +
8*a^2*b*e))/(10*a^4) + (x^3*(11*a*d - 14*b*c))/(88*a^2) + (x^6*(14*b^2*c +
 8*a^2*e - 11*a*b*d))/(40*a^3) - (b*x^12*(14*b^3*c - 5*a^3*f - 11*a*b^2*d
+ 8*a^2*b*e))/(6*a^5))/(a*x^11 + b*x^14) + (b^(2/3)*log(3^(1/2)*a^(1/3)*1i
 + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(14*b^3*c - 5*a^3*f - 11*
a*b^2*d + 8*a^2*b*e))/(9*a^(17/3)) - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b
^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(14*b^3*c - 5*a^3*f - 11*a*b^2*
d + 8*a^2*b*e))/(9*a^(17/3))
```

3.275 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^2}\,dx$

3.275.1 Optimal result . . . 2082
3.275.2 Mathematica [A] (verified) . . . 2083
3.275.3 Rubi [A] (verified) . . . 2084
3.275.4 Maple [A] (verified) . . . 2086
3.275.5 Fricas [A] (verification not implemented) . . . 2086
3.275.6 Sympy [F(-1)] . . . 2087
3.275.7 Maxima [A] (verification not implemented) . . . 2088
3.275.8 Giac [A] (verification not implemented) . . . 2089
3.275.9 Mupad [B] (verification not implemented) . . . 2090

3.275.1 Optimal result

Integrand size = 30, antiderivative size = 375

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^2}\,dx &= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4}\\&\quad-\frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x}-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{3a^6(a+bx^3)}\\&\quad+\frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{19/3}}\\&\quad+\frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{19/3}}\\&\quad-\frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{19/3}}\end{aligned}$$

output
```
-1/13*c/a^2/x^13+1/10*(-a*d+2*b*c)/a^3/x^10+1/7*(-a^2*e+2*a*b*d-3*b^2*c)/a
^4/x^7+1/4*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^4-b*(-2*a^3*f+3*a^2*
b*e-4*a*b^2*d+5*b^3*c)/a^6/x-1/3*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^
6/(b*x^3+a)+1/9*b^(4/3)*(-7*a^3*f+10*a^2*b*e-13*a*b^2*d+16*b^3*c)*ln(a^(1/
3)+b^(1/3)*x)/a^(19/3)-1/18*b^(4/3)*(-7*a^3*f+10*a^2*b*e-13*a*b^2*d+16*b^3
*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(19/3)+1/9*b^(4/3)*(-7*a^3
*f+10*a^2*b*e-13*a*b^2*d+16*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3
)*3^(1/2))/a^(19/3)*3^(1/2)
```

3.275.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.99

$$
\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^2}\,dx \\
&= -\frac{c}{13a^2x^{13}}+\frac{2bc-ad}{10a^3x^{10}}-\frac{3b^2c-2abd+a^2e}{7a^4x^7}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{4a^5x^4} \\
&\quad+\frac{b(-5b^3c+4ab^2d-3a^2be+2a^3f)}{a^6x}+\frac{b^2(-b^3c+ab^2d-a^2be+a^3f)\,x^2}{3a^6(a+bx^3)} \\
&\quad+\frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{19/3}} \\
&\quad+\frac{b^{4/3}(16b^3c-13ab^2d+10a^2be-7a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{19/3}} \\
&\quad+\frac{b^{4/3}(-16b^3c+13ab^2d-10a^2be+7a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{19/3}}
\end{aligned}
$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```

output
```
-1/13*c/(a^2*x^13) + (2*b*c - a*d)/(10*a^3*x^10) - (3*b^2*c - 2*a*b*d + a^
2*e)/(7*a^4*x^7) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(4*a^5*x^4) +
 (b*(-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a^6*x) + (b^2*(-(b^3*c)
 + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(3*a^6*(a + b*x^3)) + (b^(4/3)*(16*b^3*
c - 13*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/(3*Sqrt[3]*a^(19/3)) + (b^(4/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*
b*e - 7*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(19/3)) + (b^(4/3)*(-16*b^3*
c + 13*a*b^2*d - 10*a^2*b*e + 7*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(18*a^(19/3))
```

3.275.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \ 2368$$

$$-\frac{\int -\frac{-\frac{b^5\left(-fa^3+bea^2-b^2da+b^3c\right)x^{15}}{a^5}+\frac{3b^4\left(-fa^3+bea^2-b^2da+b^3c\right)x^{12}}{a^4}-\frac{3b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3}{x^{14}(bx^3+a)}}{3ab^3}$$

$$\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^6\left(a+bx^3\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-\frac{b^5\left(-fa^3+bea^2-b^2da+b^3c\right)x^{15}}{a^5}+\frac{3b^4\left(-fa^3+bea^2-b^2da+b^3c\right)x^{12}}{a^4}-\frac{3b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^{14}(bx^3+a)}dx}{3ab^3}$$

$$\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^6\left(a+bx^3\right)}$$

$$\Big\downarrow \ 2373$$

$$\frac{\int \left(\frac{(7fa^3-10bea^2+13b^2da-16b^3c)xb^5}{a^5(bx^3+a)}-\frac{3(2fa^3-3bea^2+4b^2da-5b^3c)b^4}{a^5x^2}+\frac{3(fa^3-2bea^2+3b^2da-4b^3c)b^3}{a^4x^5}+\frac{3(ea^2-2bda+3b^2c)b^3}{a^3x^8}+\frac{3(ad}{a}\right.}{3ab^3}$$

$$\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^6\left(a+bx^3\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{3b^3(2bc-ad)}{10a^2x^{10}}-\frac{3b^3(a^2e-2abd+3b^2c)}{7a^3x^7}+\frac{b^{13/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-7a^3f+10a^2be-13ab^2d+16b^3c)}{\sqrt{3}a^{16/3}}-\frac{b^{13/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right.}{6a^1}}{}$$

$$\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^6\left(a+bx^3\right)}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x]
```

output
```
-1/3*(b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a^6*(a + b*x^3)) + ((-
3*b^3*c)/(13*a*x^13) + (3*b^3*(2*b*c - a*d))/(10*a^2*x^10) - (3*b^3*(3*b^2
*c - 2*a*b*d + a^2*e))/(7*a^3*x^7) + (3*b^3*(4*b^3*c - 3*a*b^2*d + 2*a^2*b
*e - a^3*f))/(4*a^4*x^4) - (3*b^4*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3
*f))/(a^5*x) + (b^(13/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*Ar
cTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(16/3)) + (b^(
13/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e - 7*a^3*f)*Log[a^(1/3) + b^(1/3)
*x])/(3*a^(16/3)) - (b^(13/3)*(16*b^3*c - 13*a*b^2*d + 10*a^2*b*e - 7*a^3*
f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(16/3)))/(3*a*b^3)
```

3.275.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.275.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.77

method	result
default	$-\frac{c}{13a^2x^{13}}-\frac{ad-2bc}{10a^3x^{10}}-\frac{a^2e-2abd+3b^2c}{7a^4x^7}-\frac{f\,a^3-2a^2be+3a\,b^2d-4b^3c}{4a^5x^4}+\frac{b(2f\,a^3-3a^2be+4a\,b^2d-5b^3c)}{a^6x}+\frac{b^2\left(\frac{\left(\frac{1}{3}f\,a^3-\frac{1}{3}a^2\right.}{}\right.}{}$
risch	$\frac{-\frac{c}{13a}-\frac{(13ad-16bc)x^3}{130a^2}-\frac{\left(10a^2e-13abd+16b^2c\right)x^6}{70a^3}-\frac{\left(7f\,a^3-10a^2be+13a\,b^2d-16b^3c\right)x^9}{28a^4}+\frac{b\left(7f\,a^3-10a^2be+13a\,b^2d-16b^3c\right)x^{12}}{4a^5}+\frac{b^2\left(7f\,a^3-10\right.}{}}{x^{13}(b\,x^3+a)}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/13*c/a^2/x^13-1/10*(a*d-2*b*c)/a^3/x^10-1/7*(a^2*e-2*a*b*d+3*b^2*c)/a^4
/x^7-1/4*(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x^4+b*(2*a^3*f-3*a^2*b*e+
4*a*b^2*d-5*b^3*c)/a^6/x+b^2/a^6*((1/3*f*a^3-1/3*a^2*b*e+1/3*a*b^2*d-1/3*b
^3*c)*x^2/(b*x^3+a)+(7/3*f*a^3-10/3*a^2*b*e+13/3*a*b^2*d-16/3*b^3*c)*(-1/3
/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))))
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.35

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx =$$
$$-\frac{5460\,(16\,b^5c-13\,ab^4d+10\,a^2b^3e-7\,a^3b^2f)x^{15}+4095\,(16\,ab^4c-13\,a^2b^3d+10\,a^3b^2e-7\,a^4bf)x^{12}-58}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/16380*(5460*(16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^15 +
 4095*(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^12 - 585*(1
6*a^2*b^3*c - 13*a^3*b^2*d + 10*a^4*b*e - 7*a^5*f)*x^9 + 234*(16*a^3*b^2*c
 - 13*a^4*b*d + 10*a^5*e)*x^6 + 1260*a^5*c - 126*(16*a^4*b*c - 13*a^5*d)*x
^3 + 1820*sqrt(3)*((16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^
16 + (16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(
1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*((16*b^5*c - 1
3*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^16 + (16*a*b^4*c - 13*a^2*b^3*d
+ 10*a^3*b^2*e - 7*a^4*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3
) - a*(-b/a)^(1/3)) + 1820*((16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*
b^2*f)*x^16 + (16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^13)
*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^6*b*x^16 + a^7*x^13)
```

3.275.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx =$$

$$-\frac{1820\,(16\,b^5c-13\,ab^4d+10\,a^2b^3e-7\,a^3b^2f)x^{15}+1365\,(16\,ab^4c-13\,a^2b^3d+10\,a^3b^2e-7\,a^4bf)x^{12}-19\cdots}{546\cdots}$$

$$-\frac{\sqrt{3}(16\,b^4c-13\,ab^3d+10\,a^2b^2e-7\,a^3bf)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

$$-\frac{(16\,b^4c-13\,ab^3d+10\,a^2b^2e-7\,a^3bf)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

$$+\frac{(16\,b^4c-13\,ab^3d+10\,a^2b^2e-7\,a^3bf)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/5460*(1820*(16*b^5*c - 13*a*b^4*d + 10*a^2*b^3*e - 7*a^3*b^2*f)*x^15 +
1365*(16*a*b^4*c - 13*a^2*b^3*d + 10*a^3*b^2*e - 7*a^4*b*f)*x^12 - 195*(16
*a^2*b^3*c - 13*a^3*b^2*d + 10*a^4*b*e - 7*a^5*f)*x^9 + 78*(16*a^3*b^2*c -
 13*a^4*b*d + 10*a^5*e)*x^6 + 420*a^5*c - 42*(16*a^4*b*c - 13*a^5*d)*x^3)/
(a^6*b*x^16 + a^7*x^13) - 1/9*sqrt(3)*(16*b^4*c - 13*a*b^3*d + 10*a^2*b^2*
e - 7*a^3*b*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^6*(a
/b)^(1/3)) - 1/18*(16*b^4*c - 13*a*b^3*d + 10*a^2*b^2*e - 7*a^3*b*f)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(1/3)) + 1/9*(16*b^4*c - 13*a
*b^3*d + 10*a^2*b^2*e - 7*a^3*b*f)*log(x + (a/b)^(1/3))/(a^6*(a/b)^(1/3))
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.27

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx$$
$$= \frac{\sqrt{3}\left(16\left(-ab^2\right)^{\frac{2}{3}}b^3c-13\left(-ab^2\right)^{\frac{2}{3}}ab^2d+10\left(-ab^2\right)^{\frac{2}{3}}a^2be-7\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^7}$$
$$+\frac{\left(16b^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-13ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+10a^2b^3e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7a^3b^2f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^7}$$
$$-\frac{\left(16\left(-ab^2\right)^{\frac{2}{3}}b^3c-13\left(-ab^2\right)^{\frac{2}{3}}ab^2d+10\left(-ab^2\right)^{\frac{2}{3}}a^2be-7\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^7}$$
$$-\frac{b^5cx^2-ab^4dx^2+a^2b^3ex^2-a^3b^2fx^2}{3\left(bx^3+a\right)a^6}$$
$$-\frac{9100b^4cx^{12}-7280ab^3dx^{12}+5460a^2b^2ex^{12}-3640a^3bfx^{12}-1820ab^3cx^9+1365a^2b^2dx^9-910a^3bex^9}{1820a^6x^{13}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(16*(-a*b^2)^(2/3)*b^3*c - 13*(-a*b^2)^(2/3)*a*b^2*d + 10*(-a*
b^2)^(2/3)*a^2*b*e - 7*(-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/a^7 + 1/9*(16*b^5*c*(-a/b)^(1/3) - 13*a*b^4*d*(-a
/b)^(1/3) + 10*a^2*b^3*e*(-a/b)^(1/3) - 7*a^3*b^2*f*(-a/b)^(1/3))*(-a/b)^(
1/3)*log(abs(x - (-a/b)^(1/3)))/a^7 - 1/18*(16*(-a*b^2)^(2/3)*b^3*c - 13*(
-a*b^2)^(2/3)*a*b^2*d + 10*(-a*b^2)^(2/3)*a^2*b*e - 7*(-a*b^2)^(2/3)*a^3*f
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7 - 1/3*(b^5*c*x^2 - a*b^4*d*
x^2 + a^2*b^3*e*x^2 - a^3*b^2*f*x^2)/((b*x^3 + a)*a^6) - 1/1820*(9100*b^4*
c*x^12 - 7280*a*b^3*d*x^12 + 5460*a^2*b^2*e*x^12 - 3640*a^3*b*f*x^12 - 182
0*a*b^3*c*x^9 + 1365*a^2*b^2*d*x^9 - 910*a^3*b*e*x^9 + 455*a^4*f*x^9 + 780
*a^2*b^2*c*x^6 - 520*a^3*b*d*x^6 + 260*a^4*e*x^6 - 364*a^3*b*c*x^3 + 182*a
^4*d*x^3 + 140*a^4*c)/(a^6*x^13)
```

3.275.9 Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^2}\,dx = \frac{b^{4/3}\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^{19/3}} - \frac{\frac{c}{13\,a}-\frac{x^9\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{28\,a^4}+\frac{x^3\,\left(13\,a\,d-16\,b\,c\right)}{130\,a^2}+\frac{x^6\,\left(10\,e\,a^2-13\,d\,a\,b+16\,c\,b^2\right)}{70\,a^3}+\frac{b\,x^{12}\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a}{4\,a^5}}{b\,x^{16}+a\,x^{13}} - \frac{b^{4/3}\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^{19/3}} + \frac{b^{4/3}\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-7\,f\,a^3+10\,e\,a^2\,b-13\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^{19/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^2),x)
```

output
```
(b^(4/3)*log(b^(1/3)*x + a^(1/3))*(16*b^3*c - 7*a^3*f - 13*a*b^2*d + 10*a^
2*b*e))/(9*a^(19/3)) - (c/(13*a) - (x^9*(16*b^3*c - 7*a^3*f - 13*a*b^2*d +
 10*a^2*b*e))/(28*a^4) + (x^3*(13*a*d - 16*b*c))/(130*a^2) + (x^6*(16*b^2*
c + 10*a^2*e - 13*a*b*d))/(70*a^3) + (b*x^12*(16*b^3*c - 7*a^3*f - 13*a*b^
2*d + 10*a^2*b*e))/(4*a^5) + (b^2*x^15*(16*b^3*c - 7*a^3*f - 13*a*b^2*d +
10*a^2*b*e))/(3*a^6))/(a*x^13 + b*x^16) - (b^(4/3)*log(3^(1/2)*a^(1/3)*1i
+ 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(16*b^3*c - 7*a^3*f - 13*a
*b^2*d + 10*a^2*b*e))/(9*a^(19/3)) + (b^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b
^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(16*b^3*c - 7*a^3*f - 13*a*b^2*
d + 10*a^2*b*e))/(9*a^(19/3))
```

3.276 $\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$

3.276.1 Optimal result . . . 2091
3.276.2 Mathematica [A] (verified) . . . 2092
3.276.3 Rubi [A] (verified) . . . 2092
3.276.4 Maple [A] (verified) . . . 2094
3.276.5 Fricas [A] (verification not implemented) . . . 2094
3.276.6 Sympy [F(-1)] . . . 2095
3.276.7 Maxima [A] (verification not implemented) . . . 2095
3.276.8 Giac [A] (verification not implemented) . . . 2096
3.276.9 Mupad [B] (verification not implemented) . . . 2097

3.276.1 Optimal result

Integrand size = 30, antiderivative size = 266

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx = -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x^3}{3b^7} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^6}{6b^6} + \frac{(b^2d-3abe+6a^2f)\,x^9}{9b^5} + \frac{(be-3af)x^{12}}{12b^4} + \frac{fx^{15}}{15b^3} - \frac{a^4(b^3c-ab^2d+a^2be-a^3f)}{6b^8\,(a+bx^3)^2} + \frac{a^3(4b^3c-5ab^2d+6a^2be-7a^3f)}{3b^8\,(a+bx^3)} + \frac{a^2(6b^3c-10ab^2d+15a^2be-21a^3f)\log(a+bx^3)}{3b^8}$$

output
```
-1/3*a*(-15*a^3*f+10*a^2*b*e-6*a*b^2*d+3*b^3*c)*x^3/b^7+1/6*(-10*a^3*f+6*a
^2*b*e-3*a*b^2*d+b^3*c)*x^6/b^6+1/9*(6*a^2*f-3*a*b*e+b^2*d)*x^9/b^5+1/12*(
-3*a*f+b*e)*x^12/b^4+1/15*f*x^15/b^3-1/6*a^4*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)/b^8/(b*x^3+a)^2+1/3*a^3*(-7*a^3*f+6*a^2*b*e-5*a*b^2*d+4*b^3*c)/b^8/(b*x^
3+a)+1/3*a^2*(-21*a^3*f+15*a^2*b*e-10*a*b^2*d+6*b^3*c)*ln(b*x^3+a)/b^8
```

3.276.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.92

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{60ab(-3b^3c+6ab^2d-10a^2be+15a^3f)\,x^3+30b^2(b^3c-3ab^2d+6a^2be-10a^3f)\,x^6+20b^3(b^2d-3abe+6\ldots}{\ldots}$$

input
```
Integrate[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(60*a*b*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x^3 + 30*b^2*(b^3*c
 - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6 + 20*b^3*(b^2*d - 3*a*b*e + 6*a^2
*f)*x^9 + 15*b^4*(b*e - 3*a*f)*x^12 + 12*b^5*f*x^15 + (30*a^4*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 - (60*a^3*(-4*b^3*c + 5*a*b^2*d
- 6*a^2*b*e + 7*a^3*f))/(a + b*x^3) + 60*a^2*(6*b^3*c - 10*a*b^2*d + 15*a^
2*b*e - 21*a^3*f)*Log[a + b*x^3])/(180*b^8)
```

3.276.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$\Big\downarrow \text{2361}$$
$$\frac{1}{3}\int \frac{x^{12}(fx^9+ex^6+dx^3+c)}{(bx^3+a)^3}dx^3$$
$$\Big\downarrow \text{2123}$$
$$\frac{1}{3}\int \left(\frac{fx^{12}}{b^3}+\frac{(be-3af)x^9}{b^4}+\frac{(6fa^2-3bea+b^2d)\,x^6}{b^5}+\frac{(-10fa^3+6bea^2-3b^2da+b^3c)\,x^3}{b^6}+\frac{a(15fa^3-10bea\ldots}{\ldots}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{x^9(6a^2f-3abe+b^2d)}{3b^5}+\frac{a^3(-7a^3f+6a^2be-5ab^2d+4b^3c)}{b^8(a+bx^3)}+\frac{a^2\log(a+bx^3)(-21a^3f+15a^2be-10ab^2d\right.$$

input
```
Int[(x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(-((a*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x^3)/b^7) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^6)/(2*b^6) + ((b^2*d - 3*a*b*e + 6*a^2
*f)*x^9)/(3*b^5) + ((b*e - 3*a*f)*x^12)/(4*b^4) + (f*x^15)/(5*b^3) - (a^4*
(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(2*b^8*(a + b*x^3)^2) + (a^3*(4*b^3*c
 - 5*a*b^2*d + 6*a^2*b*e - 7*a^3*f))/(b^8*(a + b*x^3)) + (a^2*(6*b^3*c - 1
0*a*b^2*d + 15*a^2*b*e - 21*a^3*f)*Log[a + b*x^3])/b^8)/3
```

3.276.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.276.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.98

method	result
norman	$\frac{-\frac{a^2\left(21f\,a^5-15a^4eb+10a^3d\,b^2-6a^2c\,b^3\right)}{2b^8}+\frac{f\,x^{21}}{15b}-\frac{(7af-5be)x^{18}}{60b^2}+\frac{\left(21a^2f-15aeb+10b^2d\right)x^{15}}{90b^3}-\frac{\left(21f\,a^3-15a^2be+10a\,b^2d-6b^3c\right)x^{12}}{36b^4}+}{\left(b\,x^3+a\right)^2}$
default	$\frac{\frac{f\,x^{15}b^4}{15}+\frac{\left(-3a\,b^3f+b^4e\right)x^{12}}{12}+\frac{\left(6a^2b^2f-3a\,b^3e+b^4d\right)x^9}{9}+\frac{\left(-10a^3bf+6a^2e\,b^2-3a\,b^3d+b^4c\right)x^6}{6}+\frac{\left(15a^4f-10a^3be+6a^2b^2d-3a\,b^3c\right)x^3}{3}}{b^7}$
risch	$\frac{f\,x^{15}}{15b^3}-\frac{x^{12}af}{4b^4}+\frac{e\,x^{12}}{12b^3}+\frac{2x^9a^2f}{3b^5}-\frac{ae\,x^9}{3b^4}+\frac{x^9d}{9b^3}-\frac{5x^6a^3f}{3b^6}+\frac{a^2e\,x^6}{b^5}-\frac{x^6ad}{2b^4}+\frac{x^6c}{6b^3}+\frac{5a^4f\,x^3}{b^7}-\frac{10a^3e\,x^3}{3b^6}+$
parallelrisch	$-\frac{1200x^3a^4b^3d-720x^3a^3b^4c+21x^{18}a\,b^6f-42x^{15}a^2b^5f+30x^{15}a\,b^6e+1260\ln\left(b\,x^3+a\right)x^6a^5b^2f-900\ln\left(b\,x^3+a\right)x^6a^4b^3e+600}{}$

input
```
int(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*a^2*(21*a^5*f-15*a^4*b*e+10*a^3*b^2*d-6*a^2*b^3*c)/b^8+1/15*f/b*x^21
-1/60*(7*a*f-5*b*e)/b^2*x^18+1/90*(21*a^2*f-15*a*b*e+10*b^2*d)/b^3*x^15-1/
36*(21*a^3*f-15*a^2*b*e+10*a*b^2*d-6*b^3*c)/b^4*x^12+1/9*a/b^5*(21*a^3*f-1
5*a^2*b*e+10*a*b^2*d-6*b^3*c)*x^9-2/3*a*(21*a^5*f-15*a^4*b*e+10*a^3*b^2*d-
6*a^2*b^3*c)/b^7*x^3)/(b*x^3+a)^2-1/3*a^2*(21*a^3*f-15*a^2*b*e+10*a*b^2*d-
6*b^3*c)/b^8*ln(b*x^3+a)
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.49

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$=\frac{12\,b^7fx^{21}+3\,(5\,b^7e-7\,ab^6f)x^{18}+2\,(10\,b^7d-15\,ab^6e+21\,a^2b^5f)x^{15}+5\,(6\,b^7c-10\,ab^6d+15\,a^2b^5e-21}{}$$

input
```
integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/180*(12*b^7*f*x^21 + 3*(5*b^7*e - 7*a*b^6*f)*x^18 + 2*(10*b^7*d - 15*a*b
^6*e + 21*a^2*b^5*f)*x^15 + 5*(6*b^7*c - 10*a*b^6*d + 15*a^2*b^5*e - 21*a^
3*b^4*f)*x^12 - 20*(6*a*b^6*c - 10*a^2*b^5*d + 15*a^3*b^4*e - 21*a^4*b^3*f
)*x^9 + 210*a^4*b^3*c - 270*a^5*b^2*d + 330*a^6*b*e - 390*a^7*f - 30*(11*a
^2*b^5*c - 21*a^3*b^4*d + 34*a^4*b^3*e - 50*a^5*b^2*f)*x^6 + 60*(a^3*b^4*c
 + a^4*b^3*d - 4*a^5*b^2*e + 8*a^6*b*f)*x^3 + 60*(6*a^4*b^3*c - 10*a^5*b^2
*d + 15*a^6*b*e - 21*a^7*f + (6*a^2*b^5*c - 10*a^3*b^4*d + 15*a^4*b^3*e -
21*a^5*b^2*f)*x^6 + 2*(6*a^3*b^4*c - 10*a^4*b^3*d + 15*a^5*b^2*e - 21*a^6*
b*f)*x^3)*log(b*x^3 + a))/(b^10*x^6 + 2*a*b^9*x^3 + a^2*b^8)
```

3.276.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**14*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.03

$$\begin{aligned}&\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx \\ &= \frac{7\,a^4b^3c-9\,a^5b^2d+11\,a^6be-13\,a^7f+2\,(4\,a^3b^4c-5\,a^4b^3d+6\,a^5b^2e-7\,a^6bf)x^3}{6\,(b^{10}x^6+2\,ab^9x^3+a^2b^8)} \\ &\quad+\frac{12\,b^4fx^{15}+15\,(b^4e-3\,ab^3f)x^{12}+20\,(b^4d-3\,ab^3e+6\,a^2b^2f)x^9+30\,(b^4c-3\,ab^3d+6\,a^2b^2e-10\,a^3bf}{180\,b^7} \\ &\quad+\frac{(6\,a^2b^3c-10\,a^3b^2d+15\,a^4be-21\,a^5f)\log(bx^3+a)}{3\,b^8}\end{aligned}$$

input
```
integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(7*a^4*b^3*c - 9*a^5*b^2*d + 11*a^6*b*e - 13*a^7*f + 2*(4*a^3*b^4*c -
5*a^4*b^3*d + 6*a^5*b^2*e - 7*a^6*b*f)*x^3)/(b^10*x^6 + 2*a*b^9*x^3 + a^2*
b^8) + 1/180*(12*b^4*f*x^15 + 15*(b^4*e - 3*a*b^3*f)*x^12 + 20*(b^4*d - 3*
a*b^3*e + 6*a^2*b^2*f)*x^9 + 30*(b^4*c - 3*a*b^3*d + 6*a^2*b^2*e - 10*a^3*
b*f)*x^6 - 60*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f)*x^3)/b^7 +
 1/3*(6*a^2*b^3*c - 10*a^3*b^2*d + 15*a^4*b*e - 21*a^5*f)*log(b*x^3 + a)/b
^8
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.28

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(6\,a^2b^3c-10\,a^3b^2d+15\,a^4be-21\,a^5f)\log(|bx^3+a|)}{3\,b^8} - \frac{18\,a^2b^5cx^6-30\,a^3b^4dx^6+45\,a^4b^3ex^6-63\,a^5b^2fx^6+28\,a^3b^4cx^3-50\,a^4b^3dx^3+78\,a^5b^2ex^3-112\,a^6bfx}{6\,(bx^3+a)^2b^8} + \frac{12\,b^{12}fx^{15}+15\,b^{12}ex^{12}-45\,ab^{11}fx^{12}+20\,b^{12}dx^9-60\,ab^{11}ex^9+120\,a^2b^{10}fx^9+30\,b^{12}cx^6-90\,ab^{11}dx^6}{180\,b^{15}}$$

input
```
integrate(x^14*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/3*(6*a^2*b^3*c - 10*a^3*b^2*d + 15*a^4*b*e - 21*a^5*f)*log(abs(b*x^3 + a
))/b^8 - 1/6*(18*a^2*b^5*c*x^6 - 30*a^3*b^4*d*x^6 + 45*a^4*b^3*e*x^6 - 63*
a^5*b^2*f*x^6 + 28*a^3*b^4*c*x^3 - 50*a^4*b^3*d*x^3 + 78*a^5*b^2*e*x^3 - 1
12*a^6*b*f*x^3 + 11*a^4*b^3*c - 21*a^5*b^2*d + 34*a^6*b*e - 50*a^7*f)/((b*
x^3 + a)^2*b^8) + 1/180*(12*b^12*f*x^15 + 15*b^12*e*x^12 - 45*a*b^11*f*x^1
2 + 20*b^12*d*x^9 - 60*a*b^11*e*x^9 + 120*a^2*b^10*f*x^9 + 30*b^12*c*x^6 -
 90*a*b^11*d*x^6 + 180*a^2*b^10*e*x^6 - 300*a^3*b^9*f*x^6 - 180*a*b^11*c*x
^3 + 360*a^2*b^10*d*x^3 - 600*a^3*b^9*e*x^3 + 900*a^4*b^8*f*x^3)/b^15
```

3.276.9 Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.69

$$\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= x^{12}\left(\frac{e}{12\,b^3}-\frac{a\,f}{4\,b^4}\right)+x^6\left(\frac{c}{6\,b^3}-\frac{a^3\,f}{6\,b^6}-\frac{a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{2\,b^2}+\frac{a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{2\,b}\right)$$
$$-x^9\left(\frac{a^2\,f}{3\,b^5}-\frac{d}{9\,b^3}+\frac{a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{3\,b}\right)$$
$$-\frac{\frac{13\,f\,a^7-11\,e\,a^6\,b+9\,d\,a^5\,b^2-7\,c\,a^4\,b^3}{6\,b}+x^3\left(\frac{7\,f\,a^6}{3}-2\,e\,a^5\,b+\frac{5\,d\,a^4\,b^2}{3}-\frac{4\,c\,a^3\,b^3}{3}\right)}{a^2\,b^7+2\,a\,b^8\,x^3+b^9\,x^6}$$
$$-x^3\left(\frac{a\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)}{b}-\frac{a^2\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b^2}+\frac{a^3\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{3\,b^3}\right)$$
$$-\frac{\ln\left(b\,x^3+a\right)\,\left(21\,f\,a^5-15\,e\,a^4\,b+10\,d\,a^3\,b^2-6\,c\,a^2\,b^3\right)}{3\,b^8}+\frac{f\,x^{15}}{15\,b^3}$$

input
```
int((x^14*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
x^12*(e/(12*b^3) - (a*f)/(4*b^4)) + x^6*(c/(6*b^3) - (a^3*f)/(6*b^6) - (a^
2*(e/b^3 - (3*a*f)/b^4))/(2*b^2) + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3
 - (3*a*f)/b^4))/b))/(2*b)) - x^9*((a^2*f)/(3*b^5) - d/(9*b^3) + (a*(e/b^3
 - (3*a*f)/b^4))/(3*b)) - ((13*a^7*f - 7*a^4*b^3*c + 9*a^5*b^2*d - 11*a^6*
b*e)/(6*b) + x^3*((7*a^6*f)/3 - (4*a^3*b^3*c)/3 + (5*a^4*b^2*d)/3 - 2*a^5*
b*e))/(a^2*b^7 + b^9*x^6 + 2*a*b^8*x^3) - x^3*((a*(c/b^3 - (a^3*f)/b^6 - (
3*a^2*(e/b^3 - (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b
^3 - (3*a*f)/b^4))/b))/b))/b - (a^2*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 -
 (3*a*f)/b^4))/b))/b^2 + (a^3*(e/b^3 - (3*a*f)/b^4))/(3*b^3)) - (log(a + b
*x^3)*(21*a^5*f - 6*a^2*b^3*c + 10*a^3*b^2*d - 15*a^4*b*e))/(3*b^8) + (f*x
^15)/(15*b^3)
```

3.276. $\int \frac{x^{14}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

3.277 $\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

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3.277.1 Optimal result

Integrand size = 30, antiderivative size = 226

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^3}{3b^6} + \frac{(b^2d-3abe+6a^2f)\,x^6}{6b^5} + \frac{(be-3af)x^9}{9b^4} + \frac{fx^{12}}{12b^3} + \frac{a^3(b^3c-ab^2d+a^2be-a^3f)}{6b^7\,(a+bx^3)^2} - \frac{a^2(3b^3c-4ab^2d+5a^2be-6a^3f)}{3b^7\,(a+bx^3)} - \frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\log(a+bx^3)}{3b^7}$$

output
```
1/3*(-10*a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*x^3/b^6+1/6*(6*a^2*f-3*a*b*e+b^2
*d)*x^6/b^5+1/9*(-3*a*f+b*e)*x^9/b^4+1/12*f*x^12/b^3+1/6*a^3*(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)/b^7/(b*x^3+a)^2-1/3*a^2*(-6*a^3*f+5*a^2*b*e-4*a*b^2*d+3*
b^3*c)/b^7/(b*x^3+a)-1/3*a*(-15*a^3*f+10*a^2*b*e-6*a*b^2*d+3*b^3*c)*ln(b*x
^3+a)/b^7
```

3.277.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.92

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$=\frac{12b(b^3c-3ab^2d+6a^2be-10a^3f)\,x^3+6b^2(b^2d-3abe+6a^2f)\,x^6+4b^3(be-3af)x^9+3b^4fx^{12}+\frac{6a^3(b^3c-}{}}{36b^7}$$

input
```
Integrate[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(12*b*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3 + 6*b^2*(b^2*d - 3*a*
b*e + 6*a^2*f)*x^6 + 4*b^3*(b*e - 3*a*f)*x^9 + 3*b^4*f*x^12 + (6*a^3*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f))/(a + b*x^3)^2 + (12*a^2*(-3*b^3*c + 4*a*b^
2*d - 5*a^2*b*e + 6*a^3*f))/(a + b*x^3) + 12*a*(-3*b^3*c + 6*a*b^2*d - 10*
a^2*b*e + 15*a^3*f)*Log[a + b*x^3])/(36*b^7)
```

3.277.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx$$
$$\Big\downarrow \text{2361}$$
$$\frac{1}{3}\int \frac{x^9\left(fx^9+ex^6+dx^3+c\right)}{\left(bx^3+a\right)^3}dx^3$$
$$\Big\downarrow \text{2123}$$
$$\frac{1}{3}\int \left(\frac{fx^9}{b^3}+\frac{(be-3af)x^6}{b^4}+\frac{\left(6fa^2-3bea+b^2d\right)x^3}{b^5}+\frac{-10fa^3+6bea^2-3b^2da+b^3c}{b^6}+\frac{a\left(15fa^3-10bea^2+6\right.}{b^6\left(bx^3+a\right.}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{x^6(6a^2f-3abe+b^2d)}{2b^5}-\frac{a^2(-6a^3f+5a^2be-4ab^2d+3b^3c)}{b^7(a+bx^3)}+\frac{a^3(a^3(-f)+a^2be-ab^2d+b^3c)}{2b^7(a+bx^3)^2}-\frac{a\log(a+\right.$$

input
```
Int[(x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/b^6 + ((b^2*d - 3*a*b*e
+ 6*a^2*f)*x^6)/(2*b^5) + ((b*e - 3*a*f)*x^9)/(3*b^4) + (f*x^12)/(4*b^3) +
 (a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(2*b^7*(a + b*x^3)^2) - (a^2*(3
*b^3*c - 4*a*b^2*d + 5*a^2*b*e - 6*a^3*f))/(b^7*(a + b*x^3)) - (a*(3*b^3*c
 - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*Log[a + b*x^3])/b^7)/3
```

3.277.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.277.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.96

method	result
norman	$\frac{\frac{a^2(15a^4f-10a^3be+6a^2b^2d-3a\,b^3c)}{2b^7}-\frac{(15f\,a^3-10a^2be+6a\,b^2d-3b^3c)x^9}{9b^4}+\frac{f\,x^{18}}{12b}-\frac{(3af-2be)x^{15}}{18b^2}+\frac{(15a^2f-10aeb+6b^2d)x^{12}}{36b^3}+\frac{2a(15a^4 \cdots}{\cdots}}{(b\,x^3+a)^2}$
default	$-\frac{-\frac{b^3f\,x^{12}}{12}+\frac{(3fa\,b^2-b^3e)x^9}{9}+\frac{(-6f\,a^2b+3a\,b^2e-b^3d)x^6}{6}+\frac{(10f\,a^3-6a^2be+3a\,b^2d-b^3c)x^3}{3}}{b^6}+\frac{a\left(\frac{(15f\,a^3-10a^2be+6a\,b^2d-3b^3c)\ln \cdots}{b}\cdots\right.}{\cdots}$
risch	$\frac{f\,x^{12}}{12b^3}-\frac{af\,x^9}{3b^4}+\frac{e\,x^9}{9b^3}+\frac{x^6f\,a^2}{b^5}-\frac{ae\,x^6}{2b^4}+\frac{d\,x^6}{6b^3}-\frac{10f\,a^3x^3}{3b^6}+\frac{2a^2e\,x^3}{b^5}-\frac{ad\,x^3}{b^4}+\frac{c\,x^3}{3b^3}+\frac{(2f\,a^5-\frac{5}{3}a^4eb+\frac{4}{3}a^3d \cdots}{\cdots}$
parallelrisch	$\frac{3f\,x^{18}b^6+270a^6f+4x^{15}b^6e+6x^{12}b^6d+12x^9b^6c+180\ln(b\,x^3+a)a^6f-36\ln(b\,x^3+a)x^6a\,b^5c-54a^3b^3c-120\ln(b\,x^3+a)x^6a^3b^3e \cdots}{\cdots}$

input
```
int(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2*a^2*(15*a^4*f-10*a^3*b*e+6*a^2*b^2*d-3*a*b^3*c)/b^7-1/9/b^4*(15*a^3*f
-10*a^2*b*e+6*a*b^2*d-3*b^3*c)*x^9+1/12*f*x^18/b-1/18*(3*a*f-2*b*e)/b^2*x^
15+1/36*(15*a^2*f-10*a*b*e+6*b^2*d)/b^3*x^12+2/3*a*(15*a^4*f-10*a^3*b*e+6*
a^2*b^2*d-3*a*b^3*c)/b^6*x^3)/(b*x^3+a)^2+1/3*a*(15*a^3*f-10*a^2*b*e+6*a*b
^2*d-3*b^3*c)/b^7*ln(b*x^3+a)
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.56

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$=\frac{3\,b^6fx^{18}+2\,(2\,b^6e-3\,ab^5f)x^{15}+(6\,b^6d-10\,ab^5e+15\,a^2b^4f)x^{12}+4\,(3\,b^6c-6\,ab^5d+10\,a^2b^4e-15\,a^3b^3 \cdots}{\cdots}$$

input
```
integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/36*(3*b^6*f*x^18 + 2*(2*b^6*e - 3*a*b^5*f)*x^15 + (6*b^6*d - 10*a*b^5*e
+ 15*a^2*b^4*f)*x^12 + 4*(3*b^6*c - 6*a*b^5*d + 10*a^2*b^4*e - 15*a^3*b^3*
f)*x^9 - 30*a^3*b^3*c + 42*a^4*b^2*d - 54*a^5*b*e + 66*a^6*f + 6*(4*a*b^5*
c - 11*a^2*b^4*d + 21*a^3*b^3*e - 34*a^4*b^2*f)*x^6 - 12*(2*a^2*b^4*c - a^
3*b^3*d - a^4*b^2*e + 4*a^5*b*f)*x^3 - 12*(3*a^3*b^3*c - 6*a^4*b^2*d + 10*
a^5*b*e - 15*a^6*f + (3*a*b^5*c - 6*a^2*b^4*d + 10*a^3*b^3*e - 15*a^4*b^2*
f)*x^6 + 2*(3*a^2*b^4*c - 6*a^3*b^3*d + 10*a^4*b^2*e - 15*a^5*b*f)*x^3)*lo
g(b*x^3 + a))/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

3.277.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**11*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.03

$$\begin{aligned}&\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx\\&=-\frac{5\,a^3b^3c-7\,a^4b^2d+9\,a^5be-11\,a^6f+2\,(3\,a^2b^4c-4\,a^3b^3d+5\,a^4b^2e-6\,a^5bf)x^3}{6\,(b^9x^6+2\,ab^8x^3+a^2b^7)}\\&\quad+\frac{3\,b^3fx^{12}+4\,(b^3e-3\,ab^2f)x^9+6\,(b^3d-3\,ab^2e+6\,a^2bf)x^6+12\,(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f)x^3}{36\,b^6}\\&\quad-\frac{(3\,ab^3c-6\,a^2b^2d+10\,a^3be-15\,a^4f)\log(bx^3+a)}{3\,b^7}\end{aligned}$$

input
```
integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(5*a^3*b^3*c - 7*a^4*b^2*d + 9*a^5*b*e - 11*a^6*f + 2*(3*a^2*b^4*c -
4*a^3*b^3*d + 5*a^4*b^2*e - 6*a^5*b*f)*x^3)/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b
^7) + 1/36*(3*b^3*f*x^12 + 4*(b^3*e - 3*a*b^2*f)*x^9 + 6*(b^3*d - 3*a*b^2*
e + 6*a^2*b*f)*x^6 + 12*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/b^
6 - 1/3*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f)*log(b*x^3 + a)/b
^7
```

3.277.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.29

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = -\frac{(3\,ab^3c-6\,a^2b^2d+10\,a^3be-15\,a^4f)\log(|bx^3+a|)}{3\,b^7}$$
$$+\frac{9\,ab^5cx^6-18\,a^2b^4dx^6+30\,a^3b^3ex^6-45\,a^4b^2fx^6+12\,a^2b^4cx^3-28\,a^3b^3dx^3+50\,a^4b^2ex^3-78\,a^5bfx^3+}{6\,(bx^3+a)^2b^7}$$
$$+\frac{3\,b^9fx^{12}+4\,b^9ex^9-12\,ab^8fx^9+6\,b^9dx^6-18\,ab^8ex^6+36\,a^2b^7fx^6+12\,b^9cx^3-36\,ab^8dx^3+72\,a^2b^7ex^3}{36\,b^{12}}$$

input
```
integrate(x^11*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/3*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*a^4*f)*log(abs(b*x^3 + a))
/b^7 + 1/6*(9*a*b^5*c*x^6 - 18*a^2*b^4*d*x^6 + 30*a^3*b^3*e*x^6 - 45*a^4*b
^2*f*x^6 + 12*a^2*b^4*c*x^3 - 28*a^3*b^3*d*x^3 + 50*a^4*b^2*e*x^3 - 78*a^5
*b*f*x^3 + 4*a^3*b^3*c - 11*a^4*b^2*d + 21*a^5*b*e - 34*a^6*f)/((b*x^3 + a
)^2*b^7) + 1/36*(3*b^9*f*x^12 + 4*b^9*e*x^9 - 12*a*b^8*f*x^9 + 6*b^9*d*x^6
 - 18*a*b^8*e*x^6 + 36*a^2*b^7*f*x^6 + 12*b^9*c*x^3 - 36*a*b^8*d*x^3 + 72*
a^2*b^7*e*x^3 - 120*a^3*b^6*f*x^3)/b^12
```

3.277.9 Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.30

$$\int \frac{x^{11}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x^9\left(\frac{e}{9\,b^3}-\frac{a\,f}{3\,b^4}\right)+x^3\left(\frac{c}{3\,b^3}-\frac{a^3\,f}{3\,b^6}-\frac{a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)$$
$$-x^6\left(\frac{a^2\,f}{2\,b^5}-\frac{d}{6\,b^3}+\frac{a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{2\,b}\right)$$
$$+\frac{\frac{11\,f\,a^6-9\,e\,a^5\,b+7\,d\,a^4\,b^2-5\,c\,a^3\,b^3}{6\,b}+x^3\left(2\,f\,a^5-\frac{5\,e\,a^4\,b}{3}+\frac{4\,d\,a^3\,b^2}{3}-c\,a^2\,b^3\right)}{a^2\,b^6+2\,a\,b^7\,x^3+b^8\,x^6}$$
$$+\frac{f\,x^{12}}{12\,b^3}+\frac{\ln\left(b\,x^3+a\right)\,\left(15\,f\,a^4-10\,e\,a^3\,b+6\,d\,a^2\,b^2-3\,c\,a\,b^3\right)}{3\,b^7}$$

input
```
int((x^11*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
x^9*(e/(9*b^3) - (a*f)/(3*b^4)) + x^3*(c/(3*b^3) - (a^3*f)/(3*b^6) - (a^2*
(e/b^3 - (3*a*f)/b^4))/b^2 + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*
a*f)/b^4))/b))/b) - x^6*((a^2*f)/(2*b^5) - d/(6*b^3) + (a*(e/b^3 - (3*a*f)
/b^4))/(2*b)) + ((11*a^6*f - 5*a^3*b^3*c + 7*a^4*b^2*d - 9*a^5*b*e)/(6*b)
+ x^3*(2*a^5*f - a^2*b^3*c + (4*a^3*b^2*d)/3 - (5*a^4*b*e)/3))/(a^2*b^6 +
b^8*x^6 + 2*a*b^7*x^3) + (f*x^12)/(12*b^3) + (log(a + b*x^3)*(15*a^4*f + 6
*a^2*b^2*d - 3*a*b^3*c - 10*a^3*b*e))/(3*b^7)
```

3.278 $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

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3.278.1 Optimal result

Integrand size = 30, antiderivative size = 186

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(b^2d-3abe+6a^2f)\,x^3}{3b^5}+\frac{(be-3af)x^6}{6b^4}+\frac{fx^9}{9b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{6b^6\,(a+bx^3)^2}+\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)}{3b^6\,(a+bx^3)}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\log(a+bx^3)}{3b^6}$$

output
```
1/3*(6*a^2*f-3*a*b*e+b^2*d)*x^3/b^5+1/6*(-3*a*f+b*e)*x^6/b^4+1/9*f*x^9/b^3
-1/6*a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b^6/(b*x^3+a)^2+1/3*a*(-5*a^3*f+4*
a^2*b*e-3*a*b^2*d+2*b^3*c)/b^6/(b*x^3+a)+1/3*(-10*a^3*f+6*a^2*b*e-3*a*b^2*
d+b^3*c)*ln(b*x^3+a)/b^6
```

3.278.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.91

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{6b(b^2d-3abe+6a^2f)\,x^3+3b^2(be-3af)x^6+2b^3fx^9+\frac{3a^2(-b^3c+ab^2d-a^2be+a^3f)}{(a+bx^3)^2}-\frac{6a(-2b^3c+3ab^2d-4a^2be+5a^3f)}{a+bx^3}}{18b^6}$$

input
```
Integrate[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(6*b*(b^2*d - 3*a*b*e + 6*a^2*f)*x^3 + 3*b^2*(b*e - 3*a*f)*x^6 + 2*b^3*f*x
^9 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2 - (6*a*(
-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f))/(a + b*x^3) + 6*(b^3*c - 3*a*
b^2*d + 6*a^2*b*e - 10*a^3*f)*Log[a + b*x^3])/(18*b^6)
```

3.278.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$\Big\downarrow \text{2361}$$
$$\frac{1}{3}\int \frac{x^6(fx^9+ex^6+dx^3+c)}{(bx^3+a)^3}dx^3$$
$$\Big\downarrow \text{2123}$$
$$\frac{1}{3}\int \left(\frac{fx^6}{b^3}+\frac{(be-3af)x^3}{b^4}+\frac{6fa^2-3bea+b^2d}{b^5}+\frac{-10fa^3+6bea^2-3b^2da+b^3c}{b^5\,(bx^3+a)}+\frac{a(5fa^3-4bea^2+3b^2da-}{b^5\,(bx^3+a)^2}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{x^3(6a^2f-3abe+b^2d)}{b^5}+\frac{a(-5a^3f+4a^2be-3ab^2d+2b^3c)}{b^6(a+bx^3)}-\frac{a^2(a^3(-f)+a^2be-ab^2d+b^3c)}{2b^6(a+bx^3)^2}+\frac{\log(a+bx$$

input
```
Int[(x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(((b^2*d - 3*a*b*e + 6*a^2*f)*x^3)/b^5 + ((b*e - 3*a*f)*x^6)/(2*b^4) + (f*
x^9)/(3*b^3) - (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(2*b^6*(a + b*x^3
)^2) + (a*(2*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f))/(b^6*(a + b*x^3)) +
 ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*Log[a + b*x^3])/b^6)/3
```

3.278.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.278.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.95

method	result
norman	$\frac{-\frac{a^2\left(10 f a^3-6a^2 be+3a b^2 d-b^3 c\right)}{2b^6}+\frac{f x^{15}}{9b}-\frac{(5af-3be)x^{12}}{18b^2}+\frac{\left(10a^2f-6aeb+3b^2d\right)x^9}{9b^3}-\frac{2a\left(10fa^3-6a^2be+3ab^2d-b^3c\right)x^3}{3b^5}}{\left(bx^3+a\right)^2}-\frac{(10fa^3-\cdots}{}$
default	$\frac{\frac{b^2 f x^9}{9}+\frac{\left(-3afb+b^2e\right)x^6}{6}+\frac{\left(6a^2f-3aeb+b^2d\right)x^3}{3}}{b^5}-\frac{\frac{\left(10fa^3-6a^2be+3ab^2d-b^3c\right)\ln\left(bx^3+a\right)}{b}-\frac{a^2\left(fa^3-a^2be+ab^2d-b^3c\right)}{2b\left(bx^3+a\right)^2}+\frac{a(5fa^3\cdots}{}}{3b^5}$
risch	$\frac{fx^9}{9b^3}-\frac{afx^6}{2b^4}+\frac{ex^6}{6b^3}+\frac{2a^2fx^3}{b^5}-\frac{aex^3}{b^4}+\frac{dx^3}{3b^3}+\frac{\left(-\frac{5}{3}a^4f+\frac{4}{3}a^3be-a^2b^2d+\frac{2}{3}ab^3c\right)x^3-\frac{a^2\left(9fa^3-7a^2be+5ab^2d-3b^3c\right)}{6b}}{b^5\left(bx^3+a\right)^2}\cdots$
parallelrisch	$-\frac{-2fx^{15}b^5+90fa^5-72a^3b^2ex^3+36a^2b^3dx^3-12ab^4cx^3-9a^2cb^3+27a^3db^2-54a^4eb+120a^4bfx^3+120\ln\left(bx^3+a\right)x^3a^4bf-\cdots}{\cdots}$

input
```
int(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*a^2*(10*a^3*f-6*a^2*b*e+3*a*b^2*d-b^3*c)/b^6+1/9*f*x^15/b-1/18*(5*a*
f-3*b*e)/b^2*x^12+1/9*(10*a^2*f-6*a*b*e+3*b^2*d)/b^3*x^9-2/3*a*(10*a^3*f-6
*a^2*b*e+3*a*b^2*d-b^3*c)/b^5*x^3)/(b*x^3+a)^2-1/3*(10*a^3*f-6*a^2*b*e+3*a
*b^2*d-b^3*c)/b^6*ln(b*x^3+a)
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.59

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{2\,b^5fx^{15}+(3\,b^5e-5\,ab^4f)x^{12}+2\,(3\,b^5d-6\,ab^4e+10\,a^2b^3f)x^9+3\,(4\,ab^4d-11\,a^2b^3e+21\,a^3b^2f)x^6+9\,a\cdots}{\cdots}$$

input
```
integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/18*(2*b^5*f*x^15 + (3*b^5*e - 5*a*b^4*f)*x^12 + 2*(3*b^5*d - 6*a*b^4*e +
 10*a^2*b^3*f)*x^9 + 3*(4*a*b^4*d - 11*a^2*b^3*e + 21*a^3*b^2*f)*x^6 + 9*a
^2*b^3*c - 15*a^3*b^2*d + 21*a^4*b*e - 27*a^5*f + 6*(2*a*b^4*c - 2*a^2*b^3
*d + a^3*b^2*e + a^4*b*f)*x^3 + 6*((b^5*c - 3*a*b^4*d + 6*a^2*b^3*e - 10*a
^3*b^2*f)*x^6 + a^2*b^3*c - 3*a^3*b^2*d + 6*a^4*b*e - 10*a^5*f + 2*(a*b^4*
c - 3*a^2*b^3*d + 6*a^3*b^2*e - 10*a^4*b*f)*x^3)*log(b*x^3 + a))/(b^8*x^6
+ 2*a*b^7*x^3 + a^2*b^6)
```

3.278.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= \frac{3\,a^2b^3c-5\,a^3b^2d+7\,a^4be-9\,a^5f+2\,(2\,ab^4c-3\,a^2b^3d+4\,a^3b^2e-5\,a^4bf)x^3}{6\,(b^8x^6+2\,ab^7x^3+a^2b^6)} \\ &\quad+\frac{2\,b^2fx^9+3\,(b^2e-3\,abf)x^6+6\,(b^2d-3\,abe+6\,a^2f)x^3}{18\,b^5} \\ &\quad+\frac{(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f)\log(bx^3+a)}{3\,b^6}\end{aligned}$$

input
```
integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(3*a^2*b^3*c - 5*a^3*b^2*d + 7*a^4*b*e - 9*a^5*f + 2*(2*a*b^4*c - 3*a^
2*b^3*d + 4*a^3*b^2*e - 5*a^4*b*f)*x^3)/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6)
+ 1/18*(2*b^2*f*x^9 + 3*(b^2*e - 3*a*b*f)*x^6 + 6*(b^2*d - 3*a*b*e + 6*a^2
*f)*x^3)/b^5 + 1/3*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*log(b*x^3 +
a)/b^6
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.24

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f)\log\left(|bx^3+a|\right)}{3\,b^6} - \frac{3\,b^5cx^6-9\,ab^4dx^6+18\,a^2b^3ex^6-30\,a^3b^2fx^6+2\,ab^4cx^3-12\,a^2b^3dx^3+28\,a^3b^2ex^3-50\,a^4bfx^3-4\,a^3b}{6\,(bx^3+a)^2b^6} + \frac{2\,b^6fx^9+3\,b^6ex^6-9\,ab^5fx^6+6\,b^6dx^3-18\,ab^5ex^3+36\,a^2b^4fx^3}{18\,b^9}$$

input
```
integrate(x^8*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/3*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*log(abs(b*x^3 + a))/b^6 - 1
/6*(3*b^5*c*x^6 - 9*a*b^4*d*x^6 + 18*a^2*b^3*e*x^6 - 30*a^3*b^2*f*x^6 + 2*
a*b^4*c*x^3 - 12*a^2*b^3*d*x^3 + 28*a^3*b^2*e*x^3 - 50*a^4*b*f*x^3 - 4*a^3
*b^2*d + 11*a^4*b*e - 21*a^5*f)/((b*x^3 + a)^2*b^6) + 1/18*(2*b^6*f*x^9 +
3*b^6*e*x^6 - 9*a*b^5*f*x^6 + 6*b^6*d*x^3 - 18*a*b^5*e*x^3 + 36*a^2*b^4*f*
x^3)/b^9
```

3.278.9 Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10

$$\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = x^6\left(\frac{e}{6\,b^3}-\frac{a\,f}{2\,b^4}\right) - \frac{x^3\left(\frac{5\,f\,a^4}{3}-\frac{4\,e\,a^3\,b}{3}+d\,a^2\,b^2-\frac{2\,c\,a\,b^3}{3}\right)+\frac{9\,f\,a^5-7\,e\,a^4\,b+5\,d\,a^3\,b^2-3\,c\,a^2\,b^3}{6\,b}}{a^2\,b^5+2\,a\,b^6\,x^3+b^7\,x^6} - x^3\left(\frac{a^2\,f}{b^5}-\frac{d}{3\,b^3}+\frac{a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right) + \frac{\ln\left(b\,x^3+a\right)\,\left(-10\,f\,a^3+6\,e\,a^2\,b-3\,d\,a\,b^2+c\,b^3\right)}{3\,b^6} + \frac{f\,x^9}{9\,b^3}$$

input
```
int((x^8*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
x^6*(e/(6*b^3) - (a*f)/(2*b^4)) - (x^3*((5*a^4*f)/3 + a^2*b^2*d - (2*a*b^3
*c)/3 - (4*a^3*b*e)/3) + (9*a^5*f - 3*a^2*b^3*c + 5*a^3*b^2*d - 7*a^4*b*e)
/(6*b))/(a^2*b^5 + b^7*x^6 + 2*a*b^6*x^3) - x^3*((a^2*f)/b^5 - d/(3*b^3) +
 (a*(e/b^3 - (3*a*f)/b^4))/b) + (log(a + b*x^3)*(b^3*c - 10*a^3*f - 3*a*b^
2*d + 6*a^2*b*e))/(3*b^6) + (f*x^9)/(9*b^3)
```

3.278. $\int \frac{x^8(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

3.279 $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

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3.279.1 Optimal result

Integrand size = 30, antiderivative size = 146

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(be-3af)x^3}{3b^4}+\frac{fx^6}{6b^3}+\frac{a(b^3c-ab^2d+a^2be-a^3f)}{6b^5(a+bx^3)^2} - \frac{b^3c-2ab^2d+3a^2be-4a^3f}{3b^5(a+bx^3)} + \frac{(b^2d-3abe+6a^2f)\log(a+bx^3)}{3b^5}$$

output
```
1/3*(-3*a*f+b*e)*x^3/b^4+1/6*f*x^6/b^3+1/6*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c
)/b^5/(b*x^3+a)^2+1/3*(4*a^3*f-3*a^2*b*e+2*a*b^2*d-b^3*c)/b^5/(b*x^3+a)+1/
3*(6*a^2*f-3*a*b*e+b^2*d)*ln(b*x^3+a)/b^5
```

3.279.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{7a^4f+a^3b(-5e+2fx^3)+a^2b^2(3d-4ex^3-11fx^6)+b^4x^3(-2c+2ex^6+fx^9)-ab^3(c-4x^3(d+ex^3-}{6b^5(a+bx^3)^2}$$

input
```
Integrate[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(7*a^4*f + a^3*b*(-5*e + 2*f*x^3) + a^2*b^2*(3*d - 4*e*x^3 - 11*f*x^6) + b
^4*x^3*(-2*c + 2*e*x^6 + f*x^9) - a*b^3*(c - 4*x^3*(d + e*x^3 - f*x^6)) +
2*(b^2*d - 3*a*b*e + 6*a^2*f)*(a + b*x^3)^2*Log[a + b*x^3])/(6*b^5*(a + b*
x^3)^2)
```

3.279.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}dx$$

$$\Big\downarrow \text{2361}$$

$$\frac{1}{3}\int \frac{x^3(fx^9+ex^6+dx^3+c)}{(bx^3+a)^3}dx^3$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{3}\int \left(\frac{fx^3}{b^3}+\frac{be-3af}{b^4}+\frac{6fa^2-3bea+b^2d}{b^4(bx^3+a)}+\frac{-4fa^3+3bea^2-2b^2da+b^3c}{b^4(bx^3+a)^2}+\frac{a(fa^3-bea^2+b^2da-b^3c)}{b^4(bx^3+a)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\log(a+bx^3)(6a^2f-3abe+b^2d)}{b^5}-\frac{-4a^3f+3a^2be-2ab^2d+b^3c}{b^5(a+bx^3)}+\frac{a(a^3(-f)+a^2be-ab^2d+b^3c)}{2b^5(a+bx^3)^2}+\frac{x^3(be\right.$$

input
```
Int[(x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(((b*e - 3*a*f)*x^3)/b^4 + (f*x^6)/(2*b^3) + (a*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f))/(2*b^5*(a + b*x^3)^2) - (b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*
f)/(b^5*(a + b*x^3)) + ((b^2*d - 3*a*b*e + 6*a^2*f)*Log[a + b*x^3])/b^5)/3
```

3.279.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.279.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94

method	result
norman	$\frac{\frac{a\left(18 f\,a^{3}-9 a^{2} b e+3 a\,b^{2} d-b^{3} c\right)}{6 b^{5}}+\frac{f\,x^{12}}{6 b}-\frac{(2 a f-b e) x^{9}}{3 b^{2}}+\frac{\left(12 f\,a^{3}-6 a^{2} b e+2 a\,b^{2} d-b^{3} c\right) x^{3}}{3 b^{4}}}{\left(b\,x^{3}+a\right)^{2}}+\frac{\left(6 a^{2} f-3 a e b+b^{2} d\right) \ln \left(b\,x^{3}+a\right)}{3 b^{5}}$
default	$\frac{\left(-f\,x^{3} b+3 a f-b e\right)^{2}}{6 b^{5} f}+\frac{\frac{\left(6 a^{2} f-3 a e b+b^{2} d\right) \ln \left(b\,x^{3}+a\right)}{b}-\frac{a\left(f\,a^{3}-a^{2} b e+a\,b^{2} d-b^{3} c\right)}{2 b \left(b\,x^{3}+a\right)^{2}}-\frac{-4 f\,a^{3}+3 a^{2} b e-2 a\,b^{2} d+b^{3} c}{b \left(b\,x^{3}+a\right)}}{3 b^{4}}$
risch	$\frac{f\,x^{6}}{6 b^{3}}-\frac{f a\,x^{3}}{b^{4}}+\frac{e\,x^{3}}{3 b^{3}}+\frac{3 f\,a^{2}}{2 b^{5}}-\frac{a e}{b^{4}}+\frac{e^{2}}{6 b^{3} f}+\frac{\left(\frac{4}{3} f\,a^{3}-a^{2} b e+\frac{2}{3} a\,b^{2} d-\frac{1}{3} b^{3} c\right) x^{3}+\frac{a\left(7 f\,a^{3}-5 a^{2} b e+3 a\,b^{2} d-b^{3} c\right)}{6 b}}{b^{4} (b\,x^{3}+a)^{2}}+\frac{2 \ln (b$
parallelrisch	$\frac{f\,x^{12} b^{4}-4 x^{9} a\,b^{3} f+2 x^{9} b^{4} e+12 \ln \left(b\,x^{3}+a\right) x^{6} a^{2} b^{2} f-6 \ln \left(b\,x^{3}+a\right) x^{6} a\,b^{3} e+2 \ln \left(b\,x^{3}+a\right) x^{6} b^{4} d+24 \ln \left(b\,x^{3}+a\right) x^{3} a^{3} b f-12 \ln (b}{}$

input
```
int(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/6*a*(18*a^3*f-9*a^2*b*e+3*a*b^2*d-b^3*c)/b^5+1/6*f*x^12/b-1/3*(2*a*f-b*
e)/b^2*x^9+1/3*(12*a^3*f-6*a^2*b*e+2*a*b^2*d-b^3*c)/b^4*x^3)/(b*x^3+a)^2+1
/3*(6*a^2*f-3*a*b*e+b^2*d)*ln(b*x^3+a)/b^5
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.54

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{b^4fx^{12}+2\,(b^4e-2\,ab^3f)x^9+(4\,ab^3e-11\,a^2b^2f)x^6-ab^3c+3\,a^2b^2d-5\,a^3be+7\,a^4f-2\,(b^4c-2\,ab^3d+}{6\,(b^7}$$

input
```
integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(b^4*f*x^12 + 2*(b^4*e - 2*a*b^3*f)*x^9 + (4*a*b^3*e - 11*a^2*b^2*f)*x
^6 - a*b^3*c + 3*a^2*b^2*d - 5*a^3*b*e + 7*a^4*f - 2*(b^4*c - 2*a*b^3*d +
2*a^2*b^2*e - a^3*b*f)*x^3 + 2*((b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^6 + a^
2*b^2*d - 3*a^3*b*e + 6*a^4*f + 2*(a*b^3*d - 3*a^2*b^2*e + 6*a^3*b*f)*x^3)
*log(b*x^3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)
```

3.279.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= -\frac{ab^3c-3\,a^2b^2d+5\,a^3be-7\,a^4f+2\,(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3bf)x^3}{6\,(b^7x^6+2\,ab^6x^3+a^2b^5)} + \frac{bfx^6+2\,(be-3\,af)x^3}{6\,b^4} + \frac{(b^2d-3\,abe+6\,a^2f)\log(bx^3+a)}{3\,b^5}$$

input
```
integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a^4*f + 2*(b^4*c - 2*a*b^3*d +
 3*a^2*b^2*e - 4*a^3*b*f)*x^3)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5) + 1/6*(b*
f*x^6 + 2*(b*e - 3*a*f)*x^3)/b^4 + 1/3*(b^2*d - 3*a*b*e + 6*a^2*f)*log(b*x
^3 + a)/b^5
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(b^2d-3\,abe+6\,a^2f)\log\left(|bx^3+a|\right)}{3\,b^5}+\frac{b^3fx^6+2\,b^3ex^3-6\,ab^2fx^3}{6\,b^6} - \frac{ab^3c-3\,a^2b^2d+5\,a^3be-7\,a^4f+2\,(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3bf)x^3}{6\,(bx^3+a)^2b^5}$$

input
```
integrate(x^5*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/3*(b^2*d - 3*a*b*e + 6*a^2*f)*log(abs(b*x^3 + a))/b^5 + 1/6*(b^3*f*x^6 +
 2*b^3*e*x^3 - 6*a*b^2*f*x^3)/b^6 - 1/6*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e
 - 7*a^4*f + 2*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a^3*b*f)*x^3)/((b*x^3
+ a)^2*b^5)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.04

$$\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = x^3\left(\frac{e}{3\,b^3}-\frac{a\,f}{b^4}\right)+\frac{\frac{7\,f\,a^4-5\,e\,a^3\,b+3\,d\,a^2\,b^2-c\,a\,b^3}{6\,b}-x^3\left(-\frac{4\,f\,a^3}{3}+e\,a^2\,b-\frac{2\,d\,a\,b^2}{3}+\frac{c\,b^3}{3}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6} + \frac{f\,x^6}{6\,b^3}+\frac{\ln\left(b\,x^3+a\right)\,\left(6\,f\,a^2-3\,e\,a\,b+d\,b^2\right)}{3\,b^5}$$

input
```
int((x^5*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
x^3*(e/(3*b^3) - (a*f)/b^4) + ((7*a^4*f + 3*a^2*b^2*d - a*b^3*c - 5*a^3*b*
e)/(6*b) - x^3*((b^3*c)/3 - (4*a^3*f)/3 - (2*a*b^2*d)/3 + a^2*b*e))/(a^2*b
^4 + b^6*x^6 + 2*a*b^5*x^3) + (f*x^6)/(6*b^3) + (log(a + b*x^3)*(b^2*d + 6
*a^2*f - 3*a*b*e))/(3*b^5)
```

3.279. $\int \frac{x^5(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

3.280 $\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

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3.280.2 Mathematica [A] (verified) 2119
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3.280.1 Optimal result

Integrand size = 30, antiderivative size = 109

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{fx^3}{3b^3} - \frac{b^3c-ab^2d+a^2be-a^3f}{6b^4(a+bx^3)^2} - \frac{b^2d-2abe+3a^2f}{3b^4(a+bx^3)} + \frac{(be-3af)\log(a+bx^3)}{3b^4}$$

output
```
1/3*f*x^3/b^3+1/6*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b^4/(b*x^3+a)^2+1/3*(-3*a^
2*f+2*a*b*e-b^2*d)/b^4/(b*x^3+a)+1/3*(-3*a*f+b*e)*ln(b*x^3+a)/b^4
```

3.280.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{-5a^3f+a^2b(3e-4fx^3)+ab^2(-d+4ex^3+4fx^6)-b^3(c+2dx^3-2fx^9)+2(be-3af)(a+bx^3)^2\log(a}{6b^4(a+bx^3)^2}$$

input
```
Integrate[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(-5*a^3*f + a^2*b*(3*e - 4*f*x^3) + a*b^2*(-d + 4*e*x^3 + 4*f*x^6) - b^3*(
c + 2*d*x^3 - 2*f*x^9) + 2*(b*e - 3*a*f)*(a + b*x^3)^2*Log[a + b*x^3])/(6*
b^4*(a + b*x^3)^2)
```

3.280.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2359, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}dx$$

$$\Big\downarrow \text{2359}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{(bx^3+a)^3}dx^3$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{3}\int \left(\frac{f}{b^3}+\frac{be-3af}{b^3(bx^3+a)}+\frac{3fa^2-2bea+b^2d}{b^3(bx^3+a)^2}+\frac{-fa^3+bea^2-b^2da+b^3c}{b^3(bx^3+a)^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{3a^2f-2abe+b^2d}{b^4(a+bx^3)}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{2b^4(a+bx^3)^2}+\frac{(be-3af)\log(a+bx^3)}{b^4}+\frac{fx^3}{b^3}\right)$$

input
```
Int[(x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
((f*x^3)/b^3 - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*b^4*(a + b*x^3)^2) -
 (b^2*d - 2*a*b*e + 3*a^2*f)/(b^4*(a + b*x^3)) + ((b*e - 3*a*f)*Log[a + b*
x^3])/b^4)/3
```

3.280.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2359
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/(m
 + 1)   Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^
p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ
[Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.280.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

method	result
norman	$\frac{-\frac{9fa^3-3a^2be+ab^2d+b^3c}{6b^4}+\frac{fx^9}{3b}-\frac{\left(6a^2f-2aeb+b^2d\right)x^3}{3b^3}}{\left(bx^3+a\right)^2}-\frac{\left(3af-be\right)\ln\left(bx^3+a\right)}{3b^4}$
risch	$\frac{fx^3}{3b^3}+\frac{\left(-a^2f+\frac{2}{3}aeb-\frac{1}{3}b^2d\right)x^3-\frac{5fa^3-3a^2be+ab^2d+b^3c}{6b}}{b^3\left(bx^3+a\right)^2}-\frac{\ln\left(bx^3+a\right)af}{b^4}+\frac{\ln\left(bx^3+a\right)e}{3b^3}$
default	$\frac{fx^3}{3b^3}-\frac{\frac{\left(3af-be\right)\ln\left(bx^3+a\right)}{b}-\frac{fa^3-a^2be+ab^2d-b^3c}{2b\left(bx^3+a\right)^2}-\frac{-3a^2f+2aeb-b^2d}{b\left(bx^3+a\right)}}{3b^3}$
parallelrisch	$-\frac{-2b^3fx^9+6\ln\left(bx^3+a\right)x^6ab^2f-2\ln\left(bx^3+a\right)x^6b^3e+12\ln\left(bx^3+a\right)x^3a^2bf-4\ln\left(bx^3+a\right)x^3ab^2e+12a^2bfx^3-4ab^2ex^3+2b}{6b^4\left(bx^3+a\right)^2}$

input
```
int(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/6*(9*a^3*f-3*a^2*b*e+a*b^2*d+b^3*c)/b^4+1/3*f*x^9/b-1/3*(6*a^2*f-2*a*b
*e+b^2*d)/b^3*x^3)/(b*x^3+a)^2-1/3*(3*a*f-b*e)/b^4*ln(b*x^3+a)
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.45

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$=\frac{2\,b^3fx^9+4\,ab^2fx^6-b^3c-ab^2d+3\,a^2be-5\,a^3f-2\,(b^3d-2\,ab^2e+2\,a^2bf)x^3+2\,((b^3e-3\,ab^2f)x^6+a^2}{6\,(b^6x^6+2\,ab^5x^3+a^2b^4)}$$

input
```
integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*b^3*f*x^9 + 4*a*b^2*f*x^6 - b^3*c - a*b^2*d + 3*a^2*b*e - 5*a^3*f -
 2*(b^3*d - 2*a*b^2*e + 2*a^2*b*f)*x^3 + 2*((b^3*e - 3*a*b^2*f)*x^6 + a^2*
b*e - 3*a^3*f + 2*(a*b^2*e - 3*a^2*b*f)*x^3)*log(b*x^3 + a))/(b^6*x^6 + 2*
a*b^5*x^3 + a^2*b^4)
```

3.280.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{fx^3}{3\,b^3} - \frac{b^3c+ab^2d-3\,a^2be+5\,a^3f+2\,(b^3d-2\,ab^2e+3\,a^2bf)x^3}{6\,(b^6x^6+2\,ab^5x^3+a^2b^4)} + \frac{(be-3\,af)\log\left(bx^3+a\right)}{3\,b^4}$$

input
```
integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/3*f*x^3/b^3 - 1/6*(b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f + 2*(b^3*d - 2*
a*b^2*e + 3*a^2*b*f)*x^3)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) + 1/3*(b*e - 3
*a*f)*log(b*x^3 + a)/b^4
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{fx^3}{3\,b^3} + \frac{(be-3\,af)\log\left(|bx^3+a|\right)}{3\,b^4} - \frac{b^3c+ab^2d-3\,a^2be+5\,a^3f+2\,(b^3d-2\,ab^2e+3\,a^2bf)x^3}{6\,(bx^3+a)^2b^4}$$

input
```
integrate(x^2*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/3*f*x^3/b^3 + 1/3*(b*e - 3*a*f)*log(abs(b*x^3 + a))/b^4 - 1/6*(b^3*c + a
*b^2*d - 3*a^2*b*e + 5*a^3*f + 2*(b^3*d - 2*a*b^2*e + 3*a^2*b*f)*x^3)/((b*
x^3 + a)^2*b^4)
```

3.280.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.03

$$\int \frac{x^2(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{f\,x^3}{3\,b^3} - \frac{x^3\,\left(f\,a^2-\frac{2\,e\,a\,b}{3}+\frac{d\,b^2}{3}\right)+\frac{5\,f\,a^3-3\,e\,a^2\,b+d\,a\,b^2+c\,b^3}{6\,b}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6} - \frac{\ln\left(b\,x^3+a\right)\,\left(3\,a\,f-b\,e\right)}{3\,b^4}$$

input
```
int((x^2*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
(f*x^3)/(3*b^3) - (x^3*((b^2*d)/3 + a^2*f - (2*a*b*e)/3) + (b^3*c + 5*a^3*
f + a*b^2*d - 3*a^2*b*e)/(6*b))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) - (log(a
 + b*x^3)*(3*a*f - b*e))/(3*b^4)
```

3.281 $\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^3}\,dx$

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3.281.8 Giac [A] (verification not implemented) . . . 2128
3.281.9 Mupad [B] (verification not implemented) . . . 2128

3.281.1 Optimal result

Integrand size = 30, antiderivative size = 114

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^3}\,dx=\frac{b^3c-ab^2d+a^2be-a^3f}{6ab^3(a+bx^3)^2}+\frac{b^3c-a^2be+2a^3f}{3a^2b^3(a+bx^3)}+\frac{c\log(x)}{a^3}-\frac{1}{3}\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)$$

output
```
1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a/b^3/(b*x^3+a)^2+1/3*(2*a^3*f-a^2*b*e+
b^3*c)/a^2/b^3/(b*x^3+a)+c*ln(x)/a^3-1/3*(c/a^3-f/b^3)*ln(b*x^3+a)
```

3.281.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^3}\,dx=\frac{6c\log(x)+\frac{\frac{a\left(3ab^3c+3a^4f+2b^4cx^3-a^2b^2(d+2ex^3)-a^3b(e-4fx^3)\right)}{(a+bx^3)^2}+2(-b^3c+a^3f)\log(a+bx^3)}{b^3}}{6a^3}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

output
```
(6*c*Log[x] + ((a*(3*a*b^3*c + 3*a^4*f + 2*b^4*c*x^3 - a^2*b^2*(d + 2*e*x^
3) - a^3*b*(e - 4*f*x^3)))/(a + b*x^3)^2 + 2*(-(b^3*c) + a^3*f)*Log[a + b*
x^3])/b^3)/(6*a^3)
```

3.281.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{ 2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^3\left(bx^3+a\right)^3}dx^3$$

$$\Big\downarrow \text{ 2123}$$

$$\frac{1}{3}\int \left(\frac{c}{a^3x^3}+\frac{a^3f-b^3c}{a^3b^2\left(bx^3+a\right)}+\frac{-2fa^3+bea^2-b^3c}{a^2b^2\left(bx^3+a\right)^2}+\frac{fa^3-bea^2+b^2da-b^3c}{ab^2\left(bx^3+a\right)^3}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\left(\frac{c}{a^3}-\frac{f}{b^3}\right)\log\left(a+bx^3\right)+\frac{c\log\left(x^3\right)}{a^3}+\frac{2a^3f-a^2be+b^3c}{a^2b^3\left(a+bx^3\right)}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{2ab^3\left(a+bx^3\right)^2}\right)$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x]
```

output
```
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a*b^3*(a + b*x^3)^2) + (b^3*c - a^
2*b*e + 2*a^3*f)/(a^2*b^3*(a + b*x^3)) + (c*Log[x^3])/a^3 - (c/a^3 - f/b^3
)*Log[a + b*x^3])/3
```

3.281.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.281.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

method	result
norman	$\frac{\frac{3 f a^{3}-a^{2} b e-a b^{2} d+3 b^{3} c}{6 a b^{3}}+\frac{\left(2 f a^{3}-a^{2} b e+b^{3} c\right) x^{3}}{3 a^{2} b^{2}}}{\left(b x^{3}+a\right)^{2}}+\frac{c \ln \left(x\right)}{a^{3}}+\frac{\left(f a^{3}-b^{3} c\right) \ln \left(b x^{3}+a\right)}{3 a^{3} b^{3}}$
default	$\frac{c \ln \left(x\right)}{a^{3}}+\frac{\frac{\left(f a^{3}-b^{3} c\right) \ln \left(b x^{3}+a\right)}{b^{3}}-\frac{a^{2} \left(f a^{3}-a^{2} b e+a b^{2} d-b^{3} c\right)}{2 b^{3} \left(b x^{3}+a\right)^{2}}+\frac{a \left(2 f a^{3}-a^{2} b e+b^{3} c\right)}{b^{3} \left(b x^{3}+a\right)}}{3 a^{3}}$
risch	$\frac{\frac{3 f a^{3}-a^{2} b e-a b^{2} d+3 b^{3} c}{6 a b^{3}}+\frac{\left(2 f a^{3}-a^{2} b e+b^{3} c\right) x^{3}}{3 a^{2} b^{2}}}{\left(b x^{3}+a\right)^{2}}+\frac{c \ln \left(x\right)}{a^{3}}+\frac{\ln \left(-b x^{3}-a\right) f}{3 b^{3}}-\frac{\ln \left(-b x^{3}-a\right) c}{3 a^{3}}$
parallelrisch	$\frac{6 \ln \left(x\right) x^{6} b^{5} c+2 \ln \left(b x^{3}+a\right) x^{6} a^{3} b^{2} f-2 \ln \left(b x^{3}+a\right) x^{6} b^{5} c+12 \ln \left(x\right) x^{3} a b^{4} c+4 \ln \left(b x^{3}+a\right) x^{3} a^{4} b f-4 \ln \left(b x^{3}+a\right) x^{3} a b^{4} c+4 a^{4} b f x}{6 a^{3} b^{3} \left(b x^{3}+a\right)^{2}}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/6*(3*a^3*f-a^2*b*e-a*b^2*d+3*b^3*c)/a/b^3+1/3*(2*a^3*f-a^2*b*e+b^3*c)/a
^2/b^2*x^3)/(b*x^3+a)^2+c*ln(x)/a^3+1/3*(a^3*f-b^3*c)/a^3/b^3*ln(b*x^3+a)
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.64

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}\,dx$$
$$= \frac{3\,a^2b^3c - a^3b^2d - a^4be + 3\,a^5f + 2\,(ab^4c - a^3b^2e + 2\,a^4bf)x^3 - 2\,((b^5c - a^3b^2f)x^6 + a^2b^3c - a^5f + 2\,(ab^4}{6\,(a^3b^5x^6 + 2\,a^4b^4x^3 + a^5b^3)}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(3*a^2*b^3*c - a^3*b^2*d - a^4*b*e + 3*a^5*f + 2*(a*b^4*c - a^3*b^2*e
+ 2*a^4*b*f)*x^3 - 2*((b^5*c - a^3*b^2*f)*x^6 + a^2*b^3*c - a^5*f + 2*(a*b
^4*c - a^4*b*f)*x^3)*log(b*x^3 + a) + 6*(b^5*c*x^6 + 2*a*b^4*c*x^3 + a^2*b
^3*c)*log(x))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)
```

3.281.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.281.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int \frac{c+dx^3+ex^6+fx^9}{x\left(a+bx^3\right)^3}\,dx = \frac{3\,ab^3c - a^2b^2d - a^3be + 3\,a^4f + 2\,(b^4c - a^2b^2e + 2\,a^3bf)x^3}{6\,(a^2b^5x^6 + 2\,a^3b^4x^3 + a^4b^3)} + \frac{c\log\left(x^3\right)}{3\,a^3} - \frac{(b^3c - a^3f)\log\left(bx^3 + a\right)}{3\,a^3b^3}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(3*a*b^3*c - a^2*b^2*d - a^3*b*e + 3*a^4*f + 2*(b^4*c - a^2*b^2*e + 2*
a^3*b*f)*x^3)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3) + 1/3*c*log(x^3)/a^3
 - 1/3*(b^3*c - a^3*f)*log(b*x^3 + a)/(a^3*b^3)
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^3}\,dx = \frac{c\log(|x|)}{a^3} - \frac{(b^3c-a^3f)\log(|bx^3+a|)}{3\,a^3b^3} + \frac{3\,b^4cx^6-3\,a^3bfx^6+8\,ab^3cx^3-2\,a^3bex^3-2\,a^4fx^3+6\,a^2b^2c-a^3bd-a^4e}{6\,(bx^3+a)^2a^3b^2}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
c*log(abs(x))/a^3 - 1/3*(b^3*c - a^3*f)*log(abs(b*x^3 + a))/(a^3*b^3) + 1/
6*(3*b^4*c*x^6 - 3*a^3*b*f*x^6 + 8*a*b^3*c*x^3 - 2*a^3*b*e*x^3 - 2*a^4*f*x
^3 + 6*a^2*b^2*c - a^3*b*d - a^4*e)/((b*x^3 + a)^2*a^3*b^2)
```

3.281.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{c+dx^3+ex^6+fx^9}{x(a+bx^3)^3}\,dx = \frac{\frac{3\,f\,a^3-e\,a^2\,b-d\,a\,b^2+3\,c\,b^3}{6\,a\,b^3}+\frac{x^3\,\left(2\,f\,a^3-e\,a^2\,b+c\,b^3\right)}{3\,a^2\,b^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \frac{c\,\ln(x)}{a^3} - \frac{\ln\left(b\,x^3+a\right)\,\left(b^3\,c-a^3\,f\right)}{3\,a^3\,b^3}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x*(a + b*x^3)^3),x)
```

output
```
((3*b^3*c + 3*a^3*f - a*b^2*d - a^2*b*e)/(6*a*b^3) + (x^3*(b^3*c + 2*a^3*f
 - a^2*b*e))/(3*a^2*b^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (c*log(x))/a^3 - (
log(a + b*x^3)*(b^3*c - a^3*f))/(3*a^3*b^3)
```

3.282 $\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3} dx$

3.282.1 Optimal result . . . 2129
3.282.2 Mathematica [A] (verified) . . . 2129
3.282.3 Rubi [A] (verified) . . . 2130
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3.282.1 Optimal result

Integrand size = 30, antiderivative size = 134

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3} dx = -\frac{c}{3a^3x^3} - \frac{b^3c-ab^2d+a^2be-a^3f}{6a^2b^2(a+bx^3)^2} - \frac{2b^3c-ab^2d+a^3f}{3a^3b^2(a+bx^3)} - \frac{(3bc-ad)\log(x)}{a^4} + \frac{(3bc-ad)\log(a+bx^3)}{3a^4}$$

output
```
-1/3*c/a^3/x^3+1/6*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^2/b^2/(b*x^3+a)^2+1/3*(
-a^3*f+a*b^2*d-2*b^3*c)/a^3/b^2/(b*x^3+a)-(-a*d+3*b*c)*ln(x)/a^4+1/3*(-a*d
+3*b*c)*ln(b*x^3+a)/a^4
```

3.282.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4(a+bx^3)^3} dx = \frac{-\frac{2ac}{x^3} + \frac{a^2(-b^3c+ab^2d-a^2be+a^3f)}{b^2(a+bx^3)^2} - \frac{2a(2b^3c-ab^2d+a^3f)}{b^2(a+bx^3)} + 6(-3bc+ad)\log(x) + 2(3bc-ad)\log(a+bx^3)}{6a^4}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

output
```
((-2*a*c)/x^3 + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(b^2*(a + b*x
^3)^2) - (2*a*(2*b^3*c - a*b^2*d + a^3*f))/(b^2*(a + b*x^3)) + 6*(-3*b*c +
 a*d)*Log[x] + 2*(3*b*c - a*d)*Log[a + b*x^3])/(6*a^4)
```

3.282.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^6\left(bx^3+a\right)^3}dx^3$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{3}\int \left(\frac{c}{a^3x^6}-\frac{b(ad-3bc)}{a^4\left(bx^3+a\right)}+\frac{fa^3-b^2da+2b^3c}{a^3b\left(bx^3+a\right)^2}+\frac{ad-3bc}{a^4x^3}+\frac{-fa^3+bea^2-b^2da+b^3c}{a^2b\left(bx^3+a\right)^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\log\left(x^3\right)(3bc-ad)}{a^4}+\frac{(3bc-ad)\log\left(a+bx^3\right)}{a^4}-\frac{a^3f-ab^2d+2b^3c}{a^3b^2\left(a+bx^3\right)}-\frac{c}{a^3x^3}-\frac{a^3(-f)+a^2be-ab^2d+b^3c}{2a^2b^2\left(a+bx^3\right)^2}\right)$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x]
```

output
```
(-(c/(a^3*x^3)) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^2*b^2*(a + b*x^
3)^2) - (2*b^3*c - a*b^2*d + a^3*f)/(a^3*b^2*(a + b*x^3)) - ((3*b*c - a*d)
*Log[x^3])/a^4 + ((3*b*c - a*d)*Log[a + b*x^3])/a^4)/3
```

3.282.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.282.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

method	result
norman	$\frac{-\frac{c}{3a}+\frac{\left(a^{2}e-2abd+6b^{2}c\right)x^{6}}{3a^{3}}+\frac{\left(f\,a^{3}+a^{2}be-3a\,b^{2}d+9b^{3}c\right)x^{9}}{6a^{4}}}{x^{3}\left(b\,x^{3}+a\right)^{2}}+\frac{\left(ad-3bc\right)\ln\left(x\right)}{a^{4}}-\frac{\left(ad-3bc\right)\ln\left(b\,x^{3}+a\right)}{3a^{4}}$
default	$-\frac{c}{3a^{3}x^{3}}+\frac{\left(ad-3bc\right)\ln\left(x\right)}{a^{4}}+\frac{\left(-ad+3bc\right)\ln\left(b\,x^{3}+a\right)+\frac{a^{2}\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)}{2b^{2}\left(b\,x^{3}+a\right)^{2}}-\frac{a\left(f\,a^{3}-a\,b^{2}d+2b^{3}c\right)}{b^{2}\left(b\,x^{3}+a\right)}}{3a^{4}}$
risch	$\frac{-\frac{\left(f\,a^{3}-a\,b^{2}d+3b^{3}c\right)x^{6}}{3a^{3}b}-\frac{\left(f\,a^{3}+a^{2}be-3a\,b^{2}d+9b^{3}c\right)x^{3}}{6a^{2}b^{2}}-\frac{c}{3a}}{x^{3}\left(b\,x^{3}+a\right)^{2}}+\frac{d\ln\left(x\right)}{a^{3}}-\frac{3bc\ln\left(x\right)}{a^{4}}-\frac{d\ln\left(b\,x^{3}+a\right)}{3a^{3}}+\frac{bc\ln\left(b\,x^{3}+a\right)}{a^{4}}$
parallelrisch	$\frac{6\ln\left(x\right)x^{9}a\,b^{2}d-18\ln\left(x\right)x^{9}b^{3}c-2\ln\left(b\,x^{3}+a\right)x^{9}a\,b^{2}d+6\ln\left(b\,x^{3}+a\right)x^{9}b^{3}c+x^{9}a^{3}f+x^{9}a^{2}be-3x^{9}a\,b^{2}d+9b^{3}c\,x^{9}+12\ln\left(x\right)x^{6}a^{2}bd-}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*c/a+1/3*(a^2*e-2*a*b*d+6*b^2*c)/a^3*x^6+1/6*(a^3*f+a^2*b*e-3*a*b^2*d
+9*b^3*c)/a^4*x^9)/x^3/(b*x^3+a)^2+(a*d-3*b*c)/a^4*ln(x)-1/3*(a*d-3*b*c)/a
^4*ln(b*x^3+a)
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.87

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^3}\,dx = -\frac{2\left(3\,ab^4c-a^2b^3d+a^4bf\right)x^6+2\,a^3b^2c+\left(9\,a^2b^3c-3\,a^3b^2d+a^4be+a^5f\right)x^3-2\left(\left(3\,b^5c-ab^4d\right)x^9+2\left(3\right.\right.}{6\,(a}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/6*(2*(3*a*b^4*c - a^2*b^3*d + a^4*b*f)*x^6 + 2*a^3*b^2*c + (9*a^2*b^3*c
 - 3*a^3*b^2*d + a^4*b*e + a^5*f)*x^3 - 2*((3*b^5*c - a*b^4*d)*x^9 + 2*(3*
a*b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c - a^3*b^2*d)*x^3)*log(b*x^3 + a) +
 6*((3*b^5*c - a*b^4*d)*x^9 + 2*(3*a*b^4*c - a^2*b^3*d)*x^6 + (3*a^2*b^3*c
 - a^3*b^2*d)*x^3)*log(x))/(a^4*b^4*x^9 + 2*a^5*b^3*x^6 + a^6*b^2*x^3)
```

3.282.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**4/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.282.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^3}\,dx = -\frac{2\left(3\,b^4c-ab^3d+a^3bf\right)x^6+2\,a^2b^2c+\left(9\,ab^3c-3\,a^2b^2d+a^3be+a^4f\right)x^3}{6\left(a^3b^4x^9+2\,a^4b^3x^6+a^5b^2x^3\right)} + \frac{\left(3\,bc-ad\right)\log\left(bx^3+a\right)}{3\,a^4} - \frac{\left(3\,bc-ad\right)\log\left(x^3\right)}{3\,a^4}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(2*(3*b^4*c - a*b^3*d + a^3*b*f)*x^6 + 2*a^2*b^2*c + (9*a*b^3*c - 3*a
^2*b^2*d + a^3*b*e + a^4*f)*x^3)/(a^3*b^4*x^9 + 2*a^4*b^3*x^6 + a^5*b^2*x^
3) + 1/3*(3*b*c - a*d)*log(b*x^3 + a)/a^4 - 1/3*(3*b*c - a*d)*log(x^3)/a^4
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.28

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^3}\,dx = -\frac{(3\,bc-ad)\log\left(|x|\right)}{a^4}+\frac{\left(3\,b^2c-abd\right)\log\left(|bx^3+a|\right)}{3\,a^4b}+\frac{3\,bcx^3-adx^3-ac}{3\,a^4x^3} - \frac{9\,b^5cx^6-3\,ab^4dx^6+22\,ab^4cx^3-8\,a^2b^3dx^3+2\,a^4bfx^3+14\,a^2b^3c-6\,a^3b^2d+a^4be+a^5f}{6\left(bx^3+a\right)^2a^4b^2}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-(3*b*c - a*d)*log(abs(x))/a^4 + 1/3*(3*b^2*c - a*b*d)*log(abs(b*x^3 + a))
/(a^4*b) + 1/3*(3*b*c*x^3 - a*d*x^3 - a*c)/(a^4*x^3) - 1/6*(9*b^5*c*x^6 -
3*a*b^4*d*x^6 + 22*a*b^4*c*x^3 - 8*a^2*b^3*d*x^3 + 2*a^4*b*f*x^3 + 14*a^2*
b^3*c - 6*a^3*b^2*d + a^4*b*e + a^5*f)/((b*x^3 + a)^2*a^4*b^2)
```

3.282.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^4\left(a+bx^3\right)^3}\,dx = \frac{\ln\left(x\right)\,\left(a\,d-3\,b\,c\right)}{a^4}-\frac{\ln\left(b\,x^3+a\right)\,\left(a\,d-3\,b\,c\right)}{3\,a^4} - \frac{\frac{c}{3\,a}+\frac{x^6\,\left(f\,a^3-d\,a\,b^2+3\,c\,b^3\right)}{3\,a^3\,b}+\frac{x^3\,\left(f\,a^3+e\,a^2\,b-3\,d\,a\,b^2+9\,c\,b^3\right)}{6\,a^2\,b^2}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^4*(a + b*x^3)^3),x)
```

output
```
(log(x)*(a*d - 3*b*c))/a^4 - (log(a + b*x^3)*(a*d - 3*b*c))/(3*a^4) - (c/(
3*a) + (x^6*(3*b^3*c + a^3*f - a*b^2*d))/(3*a^3*b) + (x^3*(9*b^3*c + a^3*f
 - 3*a*b^2*d + a^2*b*e))/(6*a^2*b^2))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6)
```

3.283 $\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3} dx$

3.283.1 Optimal result . . . 2134
3.283.2 Mathematica [A] (verified) . . . 2134
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3.283.1 Optimal result

Integrand size = 30, antiderivative size = 163

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3} dx = -\frac{c}{6a^3x^6}+\frac{3bc-ad}{3a^4x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{6a^3b(a+bx^3)^2}+\frac{3b^2c-2abd+a^2e}{3a^4(a+bx^3)}+\frac{(6b^2c-3abd+a^2e)\log(x)}{a^5}-\frac{(6b^2c-3abd+a^2e)\log(a+bx^3)}{3a^5}$$

output
```
-1/6*c/a^3/x^6+1/3*(-a*d+3*b*c)/a^4/x^3+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
/a^3/b/(b*x^3+a)^2+1/3*(a^2*e-2*a*b*d+3*b^2*c)/a^4/(b*x^3+a)+(a^2*e-3*a*b*
d+6*b^2*c)*ln(x)/a^5-1/3*(a^2*e-3*a*b*d+6*b^2*c)*ln(b*x^3+a)/a^5
```

3.283.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7(a+bx^3)^3} dx = \frac{-\frac{a^2c}{x^6}-\frac{2a(-3bc+ad)}{x^3}+\frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{b(a+bx^3)^2}+\frac{2a(3b^2c-2abd+a^2e)}{a+bx^3}+6(6b^2c-3abd+a^2e)\log(x)-2(6b^2c-3ab}{6a^5}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

output
```
(-((a^2*c)/x^6) - (2*a*(-3*b*c + a*d))/x^3 + (a^2*(b^3*c - a*b^2*d + a^2*b
*e - a^3*f))/(b*(a + b*x^3)^2) + (2*a*(3*b^2*c - 2*a*b*d + a^2*e))/(a + b*
x^3) + 6*(6*b^2*c - 3*a*b*d + a^2*e)*Log[x] - 2*(6*b^2*c - 3*a*b*d + a^2*e
)*Log[a + b*x^3])/(6*a^5)
```

3.283.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \; 2361$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^9\left(bx^3+a\right)^3}dx^3$$

$$\Big\downarrow \; 2123$$

$$\frac{1}{3}\int \left(\frac{c}{a^3x^9}-\frac{b\left(ea^2-3bda+6b^2c\right)}{a^5\left(bx^3+a\right)}-\frac{b\left(ea^2-2bda+3b^2c\right)}{a^4\left(bx^3+a\right)^2}+\frac{ea^2-3bda+6b^2c}{a^5x^3}+\frac{fa^3-bea^2+b^2da-b^3c}{a^3\left(bx^3+a\right)^3}+\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(\frac{3bc-ad}{a^4x^3}-\frac{c}{2a^3x^6}+\frac{\log\left(x^3\right)\left(a^2e-3abd+6b^2c\right)}{a^5}-\frac{\log\left(a+bx^3\right)\left(a^2e-3abd+6b^2c\right)}{a^5}+\frac{a^2e-2abd+3b^2c}{a^4\left(a+bx^3\right)}+\right.$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x]
```

output
```
(-1/2*c/(a^3*x^6) + (3*b*c - a*d)/(a^4*x^3) + (b^3*c - a*b^2*d + a^2*b*e -
 a^3*f)/(2*a^3*b*(a + b*x^3)^2) + (3*b^2*c - 2*a*b*d + a^2*e)/(a^4*(a + b*
x^3)) + ((6*b^2*c - 3*a*b*d + a^2*e)*Log[x^3])/a^5 - ((6*b^2*c - 3*a*b*d +
 a^2*e)*Log[a + b*x^3])/a^5)/3
```

3.283.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.283.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.94

method	result
default	$-\frac{c}{6a^3x^6}-\frac{ad-3bc}{3a^4x^3}+\frac{(a^2e-3abd+6b^2c)\ln(x)}{a^5}+\frac{(-a^2e+3abd-6b^2c)\ln(bx^3+a)-\frac{a^2(fa^3-a^2be+ab^2d-b^3c)}{2b(bx^3+a)^2}+\frac{a(a^2e-2ab}{bx^3+}}{3a^5}$
norman	$\frac{-\frac{c}{6a}-\frac{(ad-2bc)x^3}{3a^2}+\frac{(fa^3-2a^2be+6ab^2d-12b^3c)x^9}{3a^4}+\frac{b(fa^3-3a^2be+9ab^2d-18b^3c)x^{12}}{6a^5}}{x^6(bx^3+a)^2}+\frac{(a^2e-3abd+6b^2c)\ln(x)}{a^5}-(a^2e-3ab$
risch	$\frac{\frac{b(a^2e-3abd+6b^2c)x^9}{3a^4}-\frac{(fa^3-3a^2be+9ab^2d-18b^3c)x^6}{6a^3b}-\frac{(ad-2bc)x^3}{3a^2}-\frac{c}{6a}}{x^6(bx^3+a)^2}+\frac{e\ln(x)}{a^3}-\frac{3\ln(x)bd}{a^4}+\frac{6\ln(x)b^2c}{a^5}-\frac{e\ln(bx^3+a}{3a^3}$
parallelrisch	$-4a^3bex^9+2a^4fx^9+12a^2b^2dx^9-18b^4cx^{12}+36\ln(x)x^6a^2b^2c+6\ln(bx^3+a)x^6a^3bd-12\ln(bx^3+a)x^6a^2b^2c-a^4c-2a^4dx^3+4a$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*c/a^3/x^6-1/3*(a*d-3*b*c)/a^4/x^3+(a^2*e-3*a*b*d+6*b^2*c)*ln(x)/a^5+1
/3/a^5*((-a^2*e+3*a*b*d-6*b^2*c)*ln(b*x^3+a)-1/2*a^2*(a^3*f-a^2*b*e+a*b^2*
d-b^3*c)/b/(b*x^3+a)^2+a*(a^2*e-2*a*b*d+3*b^2*c)/(b*x^3+a))
```

3.283.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(153) = 306$.

Time = 0.28 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.94

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^7 (a + bx^3)^3} \, dx$$
$$= \frac{2 (6 ab^4c - 3 a^2b^3d + a^3b^2e)x^9 + (18 a^2b^3c - 9 a^3b^2d + 3 a^4be - a^5 f)x^6 - a^4bc + 2 (2 a^3b^2c - a^4bd)x^3 - 2 (}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*(6*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e)*x^9 + (18*a^2*b^3*c - 9*a^3*b
^2*d + 3*a^4*b*e - a^5*f)*x^6 - a^4*b*c + 2*(2*a^3*b^2*c - a^4*b*d)*x^3 -
2*((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^12 + 2*(6*a*b^4*c - 3*a^2*b^3*d + a
^3*b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*log(b*x^3 + a)
+ 6*((6*b^5*c - 3*a*b^4*d + a^2*b^3*e)*x^12 + 2*(6*a*b^4*c - 3*a^2*b^3*d +
 a^3*b^2*e)*x^9 + (6*a^2*b^3*c - 3*a^3*b^2*d + a^4*b*e)*x^6)*log(x))/(a^5*
b^3*x^12 + 2*a^6*b^2*x^9 + a^7*b*x^6)
```

3.283.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^7 (a + bx^3)^3} \, dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.283.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^3}\,dx$$
$$= \frac{2\left(6\,b^4c-3\,ab^3d+a^2b^2e\right)x^9+\left(18\,ab^3c-9\,a^2b^2d+3\,a^3be-a^4f\right)x^6-a^3bc+2\left(2\,a^2b^2c-a^3bd\right)x^3}{6\left(a^4b^3x^{12}+2\,a^5b^2x^9+a^6bx^6\right)}$$
$$-\frac{\left(6\,b^2c-3\,abd+a^2e\right)\log\left(bx^3+a\right)}{3\,a^5}+\frac{\left(6\,b^2c-3\,abd+a^2e\right)\log\left(x^3\right)}{3\,a^5}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(2*(6*b^4*c - 3*a*b^3*d + a^2*b^2*e)*x^9 + (18*a*b^3*c - 9*a^2*b^2*d +
 3*a^3*b*e - a^4*f)*x^6 - a^3*b*c + 2*(2*a^2*b^2*c - a^3*b*d)*x^3)/(a^4*b^
3*x^12 + 2*a^5*b^2*x^9 + a^6*b*x^6) - 1/3*(6*b^2*c - 3*a*b*d + a^2*e)*log(
b*x^3 + a)/a^5 + 1/3*(6*b^2*c - 3*a*b*d + a^2*e)*log(x^3)/a^5
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.13

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^3}\,dx$$
$$= \frac{\left(6\,b^2c-3\,abd+a^2e\right)\log\left(|x|\right)}{a^5}-\frac{\left(6\,b^3c-3\,ab^2d+a^2be\right)\log\left(|bx^3+a|\right)}{3\,a^5b}$$
$$+\frac{12\,b^4cx^9-6\,ab^3dx^9+2\,a^2b^2ex^9+18\,ab^3cx^6-9\,a^2b^2dx^6+3\,a^3bex^6-a^4fx^6+4\,a^2b^2cx^3-2\,a^3bdx^3-a}{6\left(bx^6+ax^3\right)^2a^4b}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^7/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
(6*b^2*c - 3*a*b*d + a^2*e)*log(abs(x))/a^5 - 1/3*(6*b^3*c - 3*a*b^2*d + a
^2*b*e)*log(abs(b*x^3 + a))/(a^5*b) + 1/6*(12*b^4*c*x^9 - 6*a*b^3*d*x^9 +
2*a^2*b^2*e*x^9 + 18*a*b^3*c*x^6 - 9*a^2*b^2*d*x^6 + 3*a^3*b*e*x^6 - a^4*f
*x^6 + 4*a^2*b^2*c*x^3 - 2*a^3*b*d*x^3 - a^3*b*c)/((b*x^6 + a*x^3)^2*a^4*b
)
```

3.283.9 Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{x^7\left(a+bx^3\right)^3}\,dx = \frac{\ln\left(x\right)\,\left(e\,a^2-3\,d\,a\,b+6\,c\,b^2\right)}{a^5}-\frac{\ln\left(b\,x^3+a\right)\,\left(e\,a^2-3\,d\,a\,b+6\,c\,b^2\right)}{3\,a^5} - \frac{\frac{c}{6\,a}+\frac{x^3\,\left(a\,d-2\,b\,c\right)}{3\,a^2}-\frac{b\,x^9\,\left(e\,a^2-3\,d\,a\,b+6\,c\,b^2\right)}{3\,a^4}-\frac{x^6\,\left(-f\,a^3+3\,e\,a^2\,b-9\,d\,a\,b^2+18\,c\,b^3\right)}{6\,a^3\,b}}{a^2\,x^6+2\,a\,b\,x^9+b^2\,x^{12}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^7*(a + b*x^3)^3),x)
```

output
```
(log(x)*(6*b^2*c + a^2*e - 3*a*b*d))/a^5 - (log(a + b*x^3)*(6*b^2*c + a^2*
e - 3*a*b*d))/(3*a^5) - (c/(6*a) + (x^3*(a*d - 2*b*c))/(3*a^2) - (b*x^9*(6
*b^2*c + a^2*e - 3*a*b*d))/(3*a^4) - (x^6*(18*b^3*c - a^3*f - 9*a*b^2*d +
3*a^2*b*e))/(6*a^3*b))/(a^2*x^6 + b^2*x^12 + 2*a*b*x^9)
```

3.284 $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx$

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3.284.1 Optimal result

Integrand size = 30, antiderivative size = 218

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx = -\frac{c}{9a^3x^9}+\frac{3bc-ad}{6a^4x^6}-\frac{6b^2c-3abd+a^2e}{3a^5x^3} -\frac{b^3c-ab^2d+a^2be-a^3f}{6a^4\left(a+bx^3\right)^2}-\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5\left(a+bx^3\right)} -\frac{\left(10b^3c-6ab^2d+3a^2be-a^3f\right)\log(x)}{a^6} +\frac{\left(10b^3c-6ab^2d+3a^2be-a^3f\right)\log\left(a+bx^3\right)}{3a^6}$$

output
```
-1/9*c/a^3/x^9+1/6*(-a*d+3*b*c)/a^4/x^6+1/3*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x
^3+1/6*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/(b*x^3+a)^2+1/3*(a^3*f-2*a^2*b*e+
3*a*b^2*d-4*b^3*c)/a^5/(b*x^3+a)-(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)*ln(
x)/a^6+1/3*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)*ln(b*x^3+a)/a^6
```

3.284.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^3}\,dx = \frac{-\frac{2a^3c}{x^9}-\frac{3a^2(-3bc+ad)}{x^6}-\frac{6a(6b^2c-3abd+a^2e)}{x^3}+\frac{3a^2(-b^3c+ab^2d-a^2be+a^3f)}{(a+bx^3)^2}+\frac{6a(-4b^3c+3ab^2d-2a^2be+a^3f)}{a+bx^3}+18(-10b^3c+}{18a^6}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```

output
```
((-2*a^3*c)/x^9 - (3*a^2*(-3*b*c + a*d))/x^6 - (6*a*(6*b^2*c - 3*a*b*d + a
^2*e))/x^3 + (3*a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f))/(a + b*x^3)^2
+ (6*a*(-4*b^3*c + 3*a*b^2*d - 2*a^2*b*e + a^3*f))/(a + b*x^3) + 18*(-10*b
^3*c + 6*a*b^2*d - 3*a^2*b*e + a^3*f)*Log[x] + 6*(10*b^3*c - 6*a*b^2*d + 3
*a^2*b*e - a^3*f)*Log[a + b*x^3])/(18*a^6)
```

3.284.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^{12}(bx^3+a)^3}dx^3$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{3}\int \left(\frac{c}{a^3x^{12}}-\frac{b(fa^3-3bea^2+6b^2da-10b^3c)}{a^6(bx^3+a)}-\frac{b(fa^3-2bea^2+3b^2da-4b^3c)}{a^5(bx^3+a)^2}+\frac{fa^3-3bea^2+6b^2da-10b^3}{a^6x^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3bc-ad}{2a^4x^6}-\frac{c}{3a^3x^9}-\frac{a^2e-3abd+6b^2c}{a^5x^3}-\frac{\log\left(x^3\right)\left(a^3(-f)+3a^2be-6ab^2d+10b^3c\right)}{a^6}+\frac{\log\left(a+bx^3\right)\left(a^3(-f\right.}{}\right.$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x]
```

output
```
(-1/3*c/(a^3*x^9) + (3*b*c - a*d)/(2*a^4*x^6) - (6*b^2*c - 3*a*b*d + a^2*e
)/(a^5*x^3) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(2*a^4*(a + b*x^3)^2) -
(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*(a + b*x^3)) - ((10*b^3*c -
 6*a*b^2*d + 3*a^2*b*e - a^3*f)*Log[x^3])/a^6 + ((10*b^3*c - 6*a*b^2*d + 3
*a^2*b*e - a^3*f)*Log[a + b*x^3])/a^6)/3
```

3.284.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.284.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.97

method	result
default	$-\frac{c}{9a^3x^9}-\frac{ad-3bc}{6a^4x^6}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{(f\,a^3-3a^2be+6a\,b^2d-10b^3c)\ln(x)}{a^6}-\frac{b\left(\frac{\left(f\,a^3-3a^2be+6a\,b^2d-10b^3c\right)\ln\left(b\,x^3+a\right)}{b}\right.}{}$
norman	$\frac{-\frac{c}{9a}-\frac{(3ad-5bc)x^3}{18a^2}-\frac{\left(3a^2e-6abd+10b^2c\right)x^6}{9a^3}+\frac{\left(a^3b^2f-3a^2b^3e+6a\,b^4d-10b^5c\right)x^9}{2a^4b^2}+\frac{\left(a^3b^2f-3a^2b^3e+6a\,b^4d-10b^5c\right)x^{12}}{3a^5b}}{x^9(b\,x^3+a)^2}+\frac{(f\,a^3-3a}{}$
risch	$\frac{-\frac{c}{9a}-\frac{(3ad-5bc)x^3}{18a^2}-\frac{\left(3a^2e-6abd+10b^2c\right)x^6}{9a^3}+\frac{\left(f\,a^3-3a^2be+6a\,b^2d-10b^3c\right)x^9}{2a^4}+\frac{b\left(f\,a^3-3a^2be+6a\,b^2d-10b^3c\right)x^{12}}{3a^5}}{x^9(b\,x^3+a)^2}+\frac{\ln(x)f}{a^3}-\frac{3\,l}{}$
parallelrisch	$\frac{-6x^6a^5b^2e+12x^6a^4b^3d-20x^6a^3b^4c-90x^9a^2b^5c+9x^9a^5b^2f-27x^9a^4b^3e-3x^3a^5b^2d+5x^3a^4b^3c+54x^9a^3b^4d-12\ln\left(b\,x^3+a\right)x^{12}}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/9*c/a^3/x^9-1/6*(a*d-3*b*c)/a^4/x^6-1/3*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^3
+(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6*ln(x)-1/3*b/a^6*((a^3*f-3*a^2*b*
e+6*a*b^2*d-10*b^3*c)/b*ln(b*x^3+a)-1/2*a^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/
b/(b*x^3+a)^2-a*(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/b/(b*x^3+a))
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.82

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx =$$
$$-\frac{6\,(10\,ab^4c-6\,a^2b^3d+3\,a^3b^2e-a^4bf)x^{12}+9\,(10\,a^2b^3c-6\,a^3b^2d+3\,a^4be-a^5f)x^9+2\,(10\,a^3b^2c-6\,a^4}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/18*(6*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12 + 9*(10*a
^2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9 + 2*(10*a^3*b^2*c - 6*a^4*
b*d + 3*a^5*e)*x^6 + 2*a^5*c - (5*a^4*b*c - 3*a^5*d)*x^3 - 6*((10*b^5*c -
6*a*b^4*d + 3*a^2*b^3*e - a^3*b^2*f)*x^15 + 2*(10*a*b^4*c - 6*a^2*b^3*d +
3*a^3*b^2*e - a^4*b*f)*x^12 + (10*a^2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^
5*f)*x^9)*log(b*x^3 + a) + 18*((10*b^5*c - 6*a*b^4*d + 3*a^2*b^3*e - a^3*b
^2*f)*x^15 + 2*(10*a*b^4*c - 6*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^12 + (
10*a^2*b^3*c - 6*a^3*b^2*d + 3*a^4*b*e - a^5*f)*x^9)*log(x))/(a^6*b^2*x^15
 + 2*a^7*b*x^12 + a^8*x^9)
```

3.284.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^3}\,dx=\text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**10/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^3}\,dx= -\frac{6\,(10\,b^4c-6\,ab^3d+3\,a^2b^2e-a^3bf)x^{12}+9\,(10\,ab^3c-6\,a^2b^2d+3\,a^3be-a^4f)x^9+2\,(10\,a^2b^2c-6\,a^3bd}{18\,(a^5b^2x^{15}+2\,a^6bx^{12}+a^7x^9)} + \frac{(10\,b^3c-6\,ab^2d+3\,a^2be-a^3f)\log\left(bx^3+a\right)}{3\,a^6}-\frac{(10\,b^3c-6\,ab^2d+3\,a^2be-a^3f)\log\left(x^3\right)}{3\,a^6}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*(6*(10*b^4*c - 6*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^12 + 9*(10*a*b^3
*c - 6*a^2*b^2*d + 3*a^3*b*e - a^4*f)*x^9 + 2*(10*a^2*b^2*c - 6*a^3*b*d +
3*a^4*e)*x^6 + 2*a^4*c - (5*a^3*b*c - 3*a^4*d)*x^3)/(a^5*b^2*x^15 + 2*a^6*
b*x^12 + a^7*x^9) + 1/3*(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*log(b*x
^3 + a)/a^6 - 1/3*(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*log(x^3)/a^6
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.45

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx = -\frac{(10\,b^3c-6\,ab^2d+3\,a^2be-a^3f)\log\left(|x|\right)}{a^6} + \frac{(10\,b^4c-6\,ab^3d+3\,a^2b^2e-a^3bf)\log\left(|bx^3+a|\right)}{3\,a^6b} - \frac{30\,b^5cx^6-18\,ab^4dx^6+9\,a^2b^3ex^6-3\,a^3b^2fx^6+68\,ab^4cx^3-42\,a^2b^3dx^3+22\,a^3b^2ex^3-8\,a^4bfx^3+39\,a^2}{6\left(bx^3+a\right)^2a^6} + \frac{110\,b^3cx^9-66\,ab^2dx^9+33\,a^2bex^9-11\,a^3fx^9-36\,ab^2cx^6+18\,a^2bdx^6-6\,a^3ex^6+9\,a^2bcx^3-3\,a^3dx^3}{18\,a^6x^9}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^10/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)*log(abs(x))/a^6 + 1/3*(10*b^4*
c - 6*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*log(abs(b*x^3 + a))/(a^6*b) - 1/6*(
30*b^5*c*x^6 - 18*a*b^4*d*x^6 + 9*a^2*b^3*e*x^6 - 3*a^3*b^2*f*x^6 + 68*a*b
^4*c*x^3 - 42*a^2*b^3*d*x^3 + 22*a^3*b^2*e*x^3 - 8*a^4*b*f*x^3 + 39*a^2*b^
3*c - 25*a^3*b^2*d + 14*a^4*b*e - 6*a^5*f)/((b*x^3 + a)^2*a^6) + 1/18*(110
*b^3*c*x^9 - 66*a*b^2*d*x^9 + 33*a^2*b*e*x^9 - 11*a^3*f*x^9 - 36*a*b^2*c*x
^6 + 18*a^2*b*d*x^6 - 6*a^3*e*x^6 + 9*a^2*b*c*x^3 - 3*a^3*d*x^3 - 2*a^3*c)
/(a^6*x^9)
```

3.284.9 Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{10}\left(a+bx^3\right)^3}\,dx = \frac{\ln\left(b\,x^3+a\right)\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{3\,a^6} - \frac{\frac{c}{9\,a}+\frac{x^9\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{2\,a^4}+\frac{x^3\,\left(3\,a\,d-5\,b\,c\right)}{18\,a^2}+\frac{x^6\,\left(3\,e\,a^2-6\,d\,a\,b+10\,c\,b^2\right)}{9\,a^3}+\frac{b\,x^{12}\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{3\,a^5}}{a^2\,x^9+2\,a\,b\,x^{12}+b^2\,x^{15}} - \frac{\ln\left(x\right)\,\left(-f\,a^3+3\,e\,a^2\,b-6\,d\,a\,b^2+10\,c\,b^3\right)}{a^6}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^10*(a + b*x^3)^3),x)
```

output
```
(log(a + b*x^3)*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/(3*a^6) - (c/(
9*a) + (x^9*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/(2*a^4) + (x^3*(3*
a*d - 5*b*c))/(18*a^2) + (x^6*(10*b^2*c + 3*a^2*e - 6*a*b*d))/(9*a^3) + (b
*x^12*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/(3*a^5))/(a^2*x^9 + b^2*
x^15 + 2*a*b*x^12) - (log(x)*(10*b^3*c - a^3*f - 6*a*b^2*d + 3*a^2*b*e))/a
^6
```

3.284. $\int \frac{c+dx^3+ex^6+fx^9}{x^{10}(a+bx^3)^3}\,dx$

3.285 $\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx$

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3.285.1 Optimal result

Integrand size = 30, antiderivative size = 258

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx = &-\frac{c}{12a^3x^{12}}+\frac{3bc-ad}{9a^4x^9}-\frac{6b^2c-3abd+a^2e}{6a^5x^6}\\ &+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3}+\frac{b(b^3c-ab^2d+a^2be-a^3f)}{6a^5\left(a+bx^3\right)^2}\\ &+\frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{3a^6\left(a+bx^3\right)}\\ &+\frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)\log(x)}{a^7}\\ &-\frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)\log\left(a+bx^3\right)}{3a^7}\end{aligned}$$

output
```
-1/12*c/a^3/x^12+1/9*(-a*d+3*b*c)/a^4/x^9+1/6*(-a^2*e+3*a*b*d-6*b^2*c)/a^5
/x^6+1/3*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^3+1/6*b*(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)/a^5/(b*x^3+a)^2+1/3*b*(-2*a^3*f+3*a^2*b*e-4*a*b^2*d+5*b^
3*c)/a^6/(b*x^3+a)+b*(-3*a^3*f+6*a^2*b*e-10*a*b^2*d+15*b^3*c)*ln(x)/a^7-1/
3*b*(-3*a^3*f+6*a^2*b*e-10*a*b^2*d+15*b^3*c)*ln(b*x^3+a)/a^7
```

3.285.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx = \frac{-\frac{a\left(-180b^5cx^{15}+30ab^4x^{12}\left(-9c+4dx^3\right)-12a^2b^3x^9\left(5c-15dx^3+6ex^6\right)-2a^4bx^3\left(3c+5dx^3+12ex^6-27fx^9\right)+a^5\left(3c+4dx^3+6ex^6+12fx^9\right)+a^3b^2x}{x^{12}\left(a+bx^3\right)^2}}{}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```

output
```
(-((a*(-180*b^5*c*x^15 + 30*a*b^4*x^12*(-9*c + 4*d*x^3) - 12*a^2*b^3*x^9*(
5*c - 15*d*x^3 + 6*e*x^6) - 2*a^4*b*x^3*(3*c + 5*d*x^3 + 12*e*x^6 - 27*f*x
^9) + a^5*(3*c + 4*d*x^3 + 6*e*x^6 + 12*f*x^9) + a^3*b^2*x^6*(15*c + 40*d*
x^3 - 108*e*x^6 + 36*f*x^9)))/(x^12*(a + b*x^3)^2)) + 36*b*(15*b^3*c - 10*
a*b^2*d + 6*a^2*b*e - 3*a^3*f)*Log[x] + 12*b*(-15*b^3*c + 10*a*b^2*d - 6*a
^2*b*e + 3*a^3*f)*Log[a + b*x^3])/(36*a^7)
```

3.285.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \text{ 2361}$$

$$\frac{1}{3}\int \frac{fx^9+ex^6+dx^3+c}{x^{15}\left(bx^3+a\right)^3}dx^3$$

$$\Big\downarrow \text{ 2123}$$

$$\frac{1}{3}\int \left(\frac{\left(3fa^3-6bea^2+10b^2da-15b^3c\right)b^2}{a^7\left(bx^3+a\right)}+\frac{\left(2fa^3-3bea^2+4b^2da-5b^3c\right)b^2}{a^6\left(bx^3+a\right)^2}+\frac{\left(fa^3-bea^2+b^2da-b^3c\right)b^2}{a^5\left(bx^3+a\right)^3}-$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{3bc-ad}{3a^4x^9}-\frac{c}{4a^3x^{12}}-\frac{a^2e-3abd+6b^2c}{2a^5x^6}+\frac{b\log\left(x^3\right)\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{a^7}-\frac{b\log\left(a+bx^3\right)\left(-\right.}{}\right.$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x]
```

output
```
(-1/4*c/(a^3*x^12) + (3*b*c - a*d)/(3*a^4*x^9) - (6*b^2*c - 3*a*b*d + a^2*
e)/(2*a^5*x^6) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x^3) + (b
*(b^3*c - a*b^2*d + a^2*b*e - a^3*f))/(2*a^5*(a + b*x^3)^2) + (b*(5*b^3*c
- 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6*(a + b*x^3)) + (b*(15*b^3*c - 10*
a*b^2*d + 6*a^2*b*e - 3*a^3*f)*Log[x^3])/a^7 - (b*(15*b^3*c - 10*a*b^2*d +
 6*a^2*b*e - 3*a^3*f)*Log[a + b*x^3])/a^7)/3
```

3.285.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.285.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.98

method	result
default	$-\frac{c}{12a^3x^{12}}-\frac{ad-3bc}{9a^4x^9}-\frac{a^2e-3abd+6b^2c}{6a^5x^6}-\frac{f\,a^3-3a^2be+6a\,b^2d-10b^3c}{3a^6x^3}-\frac{b(3f\,a^3-6a^2be+10a\,b^2d-15b^3c)\ln(x)}{a^7}+\frac{b^2}{}$
norman	$\frac{-\frac{c}{12a}-\frac{(2ad-3bc)x^3}{18a^2}-\frac{\left(6a^2e-10abd+15b^2c\right)x^6}{36a^3}-\frac{\left(3f\,a^3-6a^2be+10a\,b^2d-15b^3c\right)x^9}{9a^4}+\frac{\left(-3a^3b^3f+6a^2b^4e-10a\,b^5d+15b^6c\right)x^{12}}{2a^5b^2}+\left(-3a^3\right.}{x^{12}(b\,x^3+a)^2}$
risch	$\frac{-\frac{c}{12a}-\frac{(2ad-3bc)x^3}{18a^2}-\frac{\left(6a^2e-10abd+15b^2c\right)x^6}{36a^3}-\frac{\left(3f\,a^3-6a^2be+10a\,b^2d-15b^3c\right)x^9}{9a^4}-\frac{b\left(3f\,a^3-6a^2be+10a\,b^2d-15b^3c\right)x^{12}}{2a^5}-\frac{b^2\left(3f\,a^3-\right.}{}}{x^{12}(b\,x^3+a)^2}$
parallelrisch	$-\frac{-60x^9a^3b^5c+6x^6a^6b^2e-10x^6a^5b^3d+15x^6a^4b^4c+4x^3a^6b^2d-6x^3a^5b^3c-540\ln(x)x^{18}b^8c+180\ln(b\,x^3+a)x^{18}b^8c+3a^6b^2c+1}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/12*c/a^3/x^12-1/9*(a*d-3*b*c)/a^4/x^9-1/6*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x
^6-1/3*(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x^3-b*(3*a^3*f-6*a^2*b*e+1
0*a*b^2*d-15*b^3*c)/a^7*ln(x)+1/3*b^2/a^7*((3*a^3*f-6*a^2*b*e+10*a*b^2*d-1
5*b^3*c)/b*ln(b*x^3+a)-1/2*a^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b/(b*x^3+a)^2
-a*(2*a^3*f-3*a^2*b*e+4*a*b^2*d-5*b^3*c)/b/(b*x^3+a))
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.74

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx$$
$$=\frac{12\,(15\,ab^5c-10\,a^2b^4d+6\,a^3b^3e-3\,a^4b^2f)x^{15}+18\,(15\,a^2b^4c-10\,a^3b^3d+6\,a^4b^2e-3\,a^5bf)x^{12}+4\,(15\,a^3b}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/36*(12*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3*e - 3*a^4*b^2*f)*x^15 + 18
*(15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^5*b*f)*x^12 + 4*(15*a^3*
b^3*c - 10*a^4*b^2*d + 6*a^5*b*e - 3*a^6*f)*x^9 - 3*a^6*c - (15*a^4*b^2*c
- 10*a^5*b*d + 6*a^6*e)*x^6 + 2*(3*a^5*b*c - 2*a^6*d)*x^3 - 12*((15*b^6*c
- 10*a*b^5*d + 6*a^2*b^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^
4*d + 6*a^3*b^3*e - 3*a^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a
^4*b^2*e - 3*a^5*b*f)*x^12)*log(b*x^3 + a) + 36*((15*b^6*c - 10*a*b^5*d +
6*a^2*b^4*e - 3*a^3*b^3*f)*x^18 + 2*(15*a*b^5*c - 10*a^2*b^4*d + 6*a^3*b^3
*e - 3*a^4*b^2*f)*x^15 + (15*a^2*b^4*c - 10*a^3*b^3*d + 6*a^4*b^2*e - 3*a^
5*b*f)*x^12)*log(x))/(a^7*b^2*x^18 + 2*a^8*b*x^15 + a^9*x^12)
```

3.285.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**13/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.09

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx = \frac{12\,(15\,b^5c-10\,ab^4d+6\,a^2b^3e-3\,a^3b^2f)x^{15}+18\,(15\,ab^4c-10\,a^2b^3d+6\,a^3b^2e-3\,a^4bf)x^{12}+4\,(15\,a^2b^3c}{36\,(a^6b^2x^{18}+2\,a^7bx^{15}} - \frac{(15\,b^4c-10\,ab^3d+6\,a^2b^2e-3\,a^3bf)\log\left(bx^3+a\right)}{3\,a^7} + \frac{(15\,b^4c-10\,ab^3d+6\,a^2b^2e-3\,a^3bf)\log\left(x^3\right)}{3\,a^7}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/36*(12*(15*b^5*c - 10*a*b^4*d + 6*a^2*b^3*e - 3*a^3*b^2*f)*x^15 + 18*(15
*a*b^4*c - 10*a^2*b^3*d + 6*a^3*b^2*e - 3*a^4*b*f)*x^12 + 4*(15*a^2*b^3*c
- 10*a^3*b^2*d + 6*a^4*b*e - 3*a^5*f)*x^9 - (15*a^3*b^2*c - 10*a^4*b*d + 6
*a^5*e)*x^6 - 3*a^5*c + 2*(3*a^4*b*c - 2*a^5*d)*x^3)/(a^6*b^2*x^18 + 2*a^7
*b*x^15 + a^8*x^12) - 1/3*(15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f
)*log(b*x^3 + a)/a^7 + 1/3*(15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*
f)*log(x^3)/a^7
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.44

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}\left(a+bx^3\right)^3}\,dx = \frac{\left(15\,b^4c-10\,ab^3d+6\,a^2b^2e-3\,a^3bf\right)\log\left(|x|\right)}{a^7} - \frac{\left(15\,b^5c-10\,ab^4d+6\,a^2b^3e-3\,a^3b^2f\right)\log\left(|bx^3+a|\right)}{3\,a^7b} + \frac{45\,b^6cx^6-30\,ab^5dx^6+18\,a^2b^4ex^6-9\,a^3b^3fx^6+100\,ab^5cx^3-68\,a^2b^4dx^3+42\,a^3b^3ex^3-22\,a^4b^2fx^3+\cdots}{6\left(bx^3+a\right)^2a^7} - \frac{375\,b^4cx^{12}-250\,ab^3dx^{12}+150\,a^2b^2ex^{12}-75\,a^3bfx^{12}-120\,ab^3cx^9+72\,a^2b^2dx^9-36\,a^3bex^9+12\,a^4fx\cdots}{36\,a^7x^{12}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^13/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
(15*b^4*c - 10*a*b^3*d + 6*a^2*b^2*e - 3*a^3*b*f)*log(abs(x))/a^7 - 1/3*(1
5*b^5*c - 10*a*b^4*d + 6*a^2*b^3*e - 3*a^3*b^2*f)*log(abs(b*x^3 + a))/(a^7
*b) + 1/6*(45*b^6*c*x^6 - 30*a*b^5*d*x^6 + 18*a^2*b^4*e*x^6 - 9*a^3*b^3*f*
x^6 + 100*a*b^5*c*x^3 - 68*a^2*b^4*d*x^3 + 42*a^3*b^3*e*x^3 - 22*a^4*b^2*f
*x^3 + 56*a^2*b^4*c - 39*a^3*b^3*d + 25*a^4*b^2*e - 14*a^5*b*f)/((b*x^3 +
a)^2*a^7) - 1/36*(375*b^4*c*x^12 - 250*a*b^3*d*x^12 + 150*a^2*b^2*e*x^12 -
 75*a^3*b*f*x^12 - 120*a*b^3*c*x^9 + 72*a^2*b^2*d*x^9 - 36*a^3*b*e*x^9 + 1
2*a^4*f*x^9 + 36*a^2*b^2*c*x^6 - 18*a^3*b*d*x^6 + 6*a^4*e*x^6 - 12*a^3*b*c
*x^3 + 4*a^4*d*x^3 + 3*a^4*c)/(a^7*x^12)
```

3.285.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{13}(a+bx^3)^3}\,dx = \frac{\ln(x)\left(-3\,f\,a^3\,b+6\,e\,a^2\,b^2-10\,d\,a\,b^3+15\,c\,b^4\right)}{a^7}$$
$$-\frac{\ln\left(b\,x^3+a\right)\left(-3\,f\,a^3\,b+6\,e\,a^2\,b^2-10\,d\,a\,b^3+15\,c\,b^4\right)}{3\,a^7}$$
$$-\frac{\frac{c}{12\,a}-\frac{x^9\left(-3\,f\,a^3+6\,e\,a^2\,b-10\,d\,a\,b^2+15\,c\,b^3\right)}{9\,a^4}+\frac{x^3\,(2\,a\,d-3\,b\,c)}{18\,a^2}+\frac{x^6\left(6\,e\,a^2-10\,d\,a\,b+15\,c\,b^2\right)}{36\,a^3}-\frac{b\,x^{12}\left(-3\,f\,a^3+6\,e\,a^2\,b-10\,d\,a\,b^2+15\right.}{2\,a^5}}{a^2\,x^{12}+2\,a\,b\,x^{15}+b^2\,x^{18}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^13*(a + b*x^3)^3),x)
```

output
```
(log(x)*(15*b^4*c + 6*a^2*b^2*e - 10*a*b^3*d - 3*a^3*b*f))/a^7 - (log(a +
b*x^3)*(15*b^4*c + 6*a^2*b^2*e - 10*a*b^3*d - 3*a^3*b*f))/(3*a^7) - (c/(12
*a) - (x^9*(15*b^3*c - 3*a^3*f - 10*a*b^2*d + 6*a^2*b*e))/(9*a^4) + (x^3*(
2*a*d - 3*b*c))/(18*a^2) + (x^6*(15*b^2*c + 6*a^2*e - 10*a*b*d))/(36*a^3)
- (b*x^12*(15*b^3*c - 3*a^3*f - 10*a*b^2*d + 6*a^2*b*e))/(2*a^5) - (b^2*x^
15*(15*b^3*c - 3*a^3*f - 10*a*b^2*d + 6*a^2*b*e))/(3*a^6))/(a^2*x^12 + b^2
*x^18 + 2*a*b*x^15)
```

3.286 $\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

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3.286.1 Optimal result

Integrand size = 30, antiderivative size = 416

$$\begin{aligned}\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= -\frac{a(3b^3c-6ab^2d+10a^2be-15a^3f)\,x}{b^7}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^4}{4b^6}\\
&+\frac{(b^2d-3abe+6a^2f)\,x^7}{7b^5}+\frac{(be-3af)x^{10}}{10b^4}+\frac{fx^{13}}{13b^3}\\
&+\frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2}-\frac{a^2(19b^3c-25ab^2d+31a^2be-37a^3f)\,x}{18b^7\,(a+bx^3)}\\
&-\frac{a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{22/3}}\\
&+\frac{a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{22/3}}\\
&-\frac{a^{4/3}(35b^3c-65ab^2d+104a^2be-152a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{22/3}}\end{aligned}$$

output
```
-a*(-15*a^3*f+10*a^2*b*e-6*a*b^2*d+3*b^3*c)*x/b^7+1/4*(-10*a^3*f+6*a^2*b*e
-3*a*b^2*d+b^3*c)*x^4/b^6+1/7*(6*a^2*f-3*a*b*e+b^2*d)*x^7/b^5+1/10*(-3*a*f
+b*e)*x^10/b^4+1/13*f*x^13/b^3+1/6*a^3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^
7/(b*x^3+a)^2-1/18*a^2*(-37*a^3*f+31*a^2*b*e-25*a*b^2*d+19*b^3*c)*x/b^7/(b
*x^3+a)+1/27*a^(4/3)*(-152*a^3*f+104*a^2*b*e-65*a*b^2*d+35*b^3*c)*ln(a^(1/
3)+b^(1/3)*x)/b^(22/3)-1/54*a^(4/3)*(-152*a^3*f+104*a^2*b*e-65*a*b^2*d+35*
b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(22/3)-1/27*a^(4/3)*(-1
52*a^3*f+104*a^2*b*e-65*a*b^2*d+35*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)
/a^(1/3)*3^(1/2))/b^(22/3)*3^(1/2)
```

3.286.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.99

$$\begin{aligned}
&\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx \\
&= \frac{a(-3b^3c+6ab^2d-10a^2be+15a^3f)\,x}{b^7}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^4}{4b^6} \\
&+\frac{(b^2d-3abe+6a^2f)\,x^7}{7b^5}+\frac{(be-3af)x^{10}}{10b^4}+\frac{fx^{13}}{13b^3} \\
&+\frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^7\,(a+bx^3)^2}+\frac{a^2(-19b^3c+25ab^2d-31a^2be+37a^3f)\,x}{18b^7\,(a+bx^3)} \\
&+\frac{a^{4/3}(-35b^3c+65ab^2d-104a^2be+152a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{22/3}} \\
&-\frac{a^{4/3}(-35b^3c+65ab^2d-104a^2be+152a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{22/3}} \\
&+\frac{a^{4/3}(-35b^3c+65ab^2d-104a^2be+152a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{22/3}}
\end{aligned}$$

input
```
Integrate[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*c - 3*a*b
^2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/(4*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x
^7)/(7*b^5) + ((b*e - 3*a*f)*x^10)/(10*b^4) + (f*x^13)/(13*b^3) + (a^3*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^7*(a + b*x^3)^2) + (a^2*(-19*b^3*
c + 25*a*b^2*d - 31*a^2*b*e + 37*a^3*f)*x)/(18*b^7*(a + b*x^3)) + (a^(4/3)
*(-35*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(22/3)) - (a^(4/3)*(-35*b^3*c + 65*a*b
^2*d - 104*a^2*b*e + 152*a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/(27*b^(22/3)) +
(a^(4/3)*(-35*b^3*c + 65*a*b^2*d - 104*a^2*b*e + 152*a^3*f)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/(54*b^(22/3))
```

3.286.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2367, 2397, 27, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \text{2367}$$

$$\frac{a^3x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{\int \frac{-6ab^6fx^{18}-6ab^5(be-af)x^{15}-6ab^4\left(fa^2-bea+b^2d\right)x^{12}-6ab^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9+6a^2b^2\left(-fa^3+bea^2-b^2da+b^3c\right)x^6-6a^3b\left(-fa^3+be\cdots\right.}{\left(bx^3+a\right)^2}}{6ab^7}$$

$$\Big\downarrow \text{2397}$$

$$\frac{a^3x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{\frac{a^3x\left(-37a^3f+31a^2be-25ab^2d+19b^3c\right)}{3\left(a+bx^3\right)}-\frac{\int \frac{2\left(9a^2b^{11}fx^{15}+9a^2b^{10}(be-2af)x^{12}+9a^2b^9\left(3fa^2-2bea+b^2d\right)x^9+9a^2b^8\left(-4fa^3+3bea^2-2b^2da+b^3c\right)x^6-9a^3b\cdots\right.}{bx^3+a}}{3ab^6}}{6ab^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^7\left(a+bx^3\right)^2}-\frac{\frac{a^3x\left(-37a^3f+31a^2be-25ab^2d+19b^3c\right)}{3\left(a+bx^3\right)}-\frac{2\int \frac{9a^2b^{11}fx^{15}+9a^2b^{10}(be-2af)x^{12}+9a^2b^9\left(3fa^2-2bea+b^2d\right)x^9+9a^2b^8\left(-4fa^3+3bea^2-2b^2da+b^3c\right)x^6-9a^3b^7\cdots}{bx^3+a}}{3ab^6}}{6ab^7}$$

$\downarrow$ 2426

$$\frac{a^3x(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^7(a+bx^3)^2}-\frac{\frac{a^3x(-37a^3f+31a^2be-25ab^2d+19b^3c)}{3(a+bx^3)}-\frac{2\int\left(9a^2b^{10}fx^{12}+9a^2b^9(be-3af)x^9+9a^2b^8(6fa^2-3bea+b^2d)x^6+9a^2b^7(-10fa^3+6bea^2-3b^2da+b^3c)\right.}{3ab^6}}{6ab^7}$$

$\downarrow$ 2009

$$\frac{a^3x(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^7(a+bx^3)^2}-\frac{a^3x(-37a^3f+31a^2be-25ab^2d+19b^3c)}{3(a+bx^3)}-\frac{2\left(\frac{9}{13}a^2b^{10}fx^{13}+\frac{9}{10}a^2b^9x^{10}(be-3af)+\frac{9}{7}a^2b^8x^7(6a^2f-3abe+b^2d)+\frac{9}{4}a^2b^7x^4(-10a^3f+6a^2be-3ab^2d\right.}{}$$

input
```
Int[(x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(a^3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^7*(a + b*x^3)^2) - ((a^3*
(19*b^3*c - 25*a*b^2*d + 31*a^2*b*e - 37*a^3*f)*x)/(3*(a + b*x^3)) - (2*(-
9*a^3*b^6*(3*b^3*c - 6*a*b^2*d + 10*a^2*b*e - 15*a^3*f)*x + (9*a^2*b^7*(b^
3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^4)/4 + (9*a^2*b^8*(b^2*d - 3*a*b
*e + 6*a^2*f)*x^7)/7 + (9*a^2*b^9*(b*e - 3*a*f)*x^10)/10 + (9*a^2*b^10*f*x
^13)/13 - (a^(10/3)*b^(17/3)*(35*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^
3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/Sqrt[3] + (a^(10/3
)*b^(17/3)*(35*b^3*c - 65*a*b^2*d + 104*a^2*b*e - 152*a^3*f)*Log[a^(1/3) +
 b^(1/3)*x])/3 - (a^(10/3)*b^(17/3)*(35*b^3*c - 65*a*b^2*d + 104*a^2*b*e -
 152*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/6))/(3*a*b^6))
/(6*a*b^7)
```

3.286.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.286. $\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.286.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.68

method	result
risch	$\frac{f x^{13}}{13b^3}-\frac{3x^{10}af}{10b^4}+\frac{x^{10}e}{10b^3}+\frac{6x^7a^2f}{7b^5}-\frac{3x^7ae}{7b^4}+\frac{dx^7}{7b^3}-\frac{5a^3fx^4}{2b^6}+\frac{3a^2ex^4}{2b^5}-\frac{3adx^4}{4b^4}+\frac{cx^4}{4b^3}+\frac{15a^4fx}{b^7}-\frac{10a^3ex}{b^6}+$
default	$\frac{\frac{1}{13}fx^{13}b^4-\frac{3}{10}x^{10}ab^3f+\frac{1}{10}x^{10}b^4e+\frac{6}{7}x^7a^2b^2f-\frac{3}{7}x^7ab^3e+\frac{1}{7}b^4dx^7-\frac{5}{2}a^3bfx^4+\frac{3}{2}a^2b^2ex^4-\frac{3}{4}ab^3dx^4+\frac{1}{4}b^4cx^4+15a^4fx-10a^3bex+}{b^7}$

input
```
int(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*f*x^13/b^3-3/10/b^4*x^10*a*f+1/10/b^3*x^10*e+6/7/b^5*x^7*a^2*f-3/7/b^
4*x^7*a*e+1/7/b^3*d*x^7-5/2/b^6*a^3*f*x^4+3/2/b^5*a^2*e*x^4-3/4/b^4*a*d*x^
4+1/4/b^3*c*x^4+15/b^7*a^4*f*x-10/b^6*a^3*e*x+6/b^5*a^2*d*x-3/b^4*a*c*x+((
37/18*a^5*b*f-31/18*a^4*e*b^2+25/18*a^3*d*b^3-19/18*a^2*c*b^4)*x^4+1/9*a^3
*(17*a^3*f-14*a^2*b*e+11*a*b^2*d-8*b^3*c)*x)/b^7/(b*x^3+a)^2-1/27/b^8*a^2*
sum((152*a^3*f-104*a^2*b*e+65*a*b^2*d-35*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z
^3*b+a))
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.60

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{3780\,b^6fx^{19}+378\,(13\,b^6e-19\,ab^5f)x^{16}+108\,(65\,b^6d-104\,ab^5e+152\,a^2b^4f)x^{13}+351\,(35\,b^6c-65\,ab^5d}{}$$

input
```
integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/49140*(3780*b^6*f*x^19 + 378*(13*b^6*e - 19*a*b^5*f)*x^16 + 108*(65*b^6*
d - 104*a*b^5*e + 152*a^2*b^4*f)*x^13 + 351*(35*b^6*c - 65*a*b^5*d + 104*a
^2*b^4*e - 152*a^3*b^3*f)*x^10 - 3510*(35*a*b^5*c - 65*a^2*b^4*d + 104*a^3
*b^3*e - 152*a^4*b^2*f)*x^7 - 9555*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*
b^2*e - 152*a^5*b*f)*x^4 - 1820*sqrt(3)*(35*a^3*b^3*c - 65*a^4*b^2*d + 104
*a^5*b*e - 152*a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^
4*b^2*f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*
f)*x^3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a
) + 910*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^5*b*e - 152*a^6*f + (35*a*b^5
*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b^2*f)*x^6 + 2*(35*a^2*b^4*c -
 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*x^3)*(-a/b)^(1/3)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3)) - 1820*(35*a^3*b^3*c - 65*a^4*b^2*d + 104*a^
5*b*e - 152*a^6*f + (35*a*b^5*c - 65*a^2*b^4*d + 104*a^3*b^3*e - 152*a^4*b
^2*f)*x^6 + 2*(35*a^2*b^4*c - 65*a^3*b^3*d + 104*a^4*b^2*e - 152*a^5*b*f)*
x^3)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 5460*(35*a^3*b^3*c - 65*a^4*b^2*
d + 104*a^5*b*e - 152*a^6*f)*x)/(b^9*x^6 + 2*a*b^8*x^3 + a^2*b^7)
```

3.286.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**12*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.02

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx =$$
$$-\frac{(19\,a^2b^4c-25\,a^3b^3d+31\,a^4b^2e-37\,a^5bf)x^4+2\,(8\,a^3b^3c-11\,a^4b^2d+14\,a^5be-17\,a^6f)x}{18\,(b^9x^6+2\,ab^8x^3+a^2b^7)}$$
$$+\frac{140\,b^4fx^{13}+182\,(b^4e-3\,ab^3f)x^{10}+260\,(b^4d-3\,ab^3e+6\,a^2b^2f)x^7+455\,(b^4c-3\,ab^3d+6\,a^2b^2e-10}{1820\,b^7}$$
$$+\frac{\sqrt{3}(35\,a^2b^3c-65\,a^3b^2d+104\,a^4be-152\,a^5f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^8\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(35\,a^2b^3c-65\,a^3b^2d+104\,a^4be-152\,a^5f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^8\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(35\,a^2b^3c-65\,a^3b^2d+104\,a^4be-152\,a^5f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^8\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*((19*a^2*b^4*c - 25*a^3*b^3*d + 31*a^4*b^2*e - 37*a^5*b*f)*x^4 + 2*(
8*a^3*b^3*c - 11*a^4*b^2*d + 14*a^5*b*e - 17*a^6*f)*x)/(b^9*x^6 + 2*a*b^8*
x^3 + a^2*b^7) + 1/1820*(140*b^4*f*x^13 + 182*(b^4*e - 3*a*b^3*f)*x^10 + 2
60*(b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^7 + 455*(b^4*c - 3*a*b^3*d + 6*a^2*
b^2*e - 10*a^3*b*f)*x^4 - 1820*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3*b*e - 15*
a^4*f)*x)/b^7 + 1/27*sqrt(3)*(35*a^2*b^3*c - 65*a^3*b^2*d + 104*a^4*b*e -
152*a^5*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^8*(a/b)^
(2/3)) - 1/54*(35*a^2*b^3*c - 65*a^3*b^2*d + 104*a^4*b*e - 152*a^5*f)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^8*(a/b)^(2/3)) + 1/27*(35*a^2*b^3*c
- 65*a^3*b^2*d + 104*a^4*b*e - 152*a^5*f)*log(x + (a/b)^(1/3))/(b^8*(a/b)^
(2/3))
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.18

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{\sqrt{3}\left(35\left(-ab^2\right)^{\frac{1}{3}}ab^3c-65\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d+104\left(-ab^2\right)^{\frac{1}{3}}a^3be-152\left(-ab^2\right)^{\frac{1}{3}}a^4f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^8}$$
$$-\frac{\left(35a^2b^3c-65a^3b^2d+104a^4be-152a^5f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27ab^7}$$
$$+\frac{\left(35\left(-ab^2\right)^{\frac{1}{3}}ab^3c-65\left(-ab^2\right)^{\frac{1}{3}}a^2b^2d+104\left(-ab^2\right)^{\frac{1}{3}}a^3be-152\left(-ab^2\right)^{\frac{1}{3}}a^4f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)\right.}{54b^8}$$
$$-\frac{19a^2b^4cx^4-25a^3b^3dx^4+31a^4b^2ex^4-37a^5bfx^4+16a^3b^3cx-22a^4b^2dx+28a^5bex-34a^6fx}{18\left(bx^3+a\right)^2b^7}$$
$$+\frac{140b^{36}fx^{13}+182b^{36}ex^{10}-546ab^{35}fx^{10}+260b^{36}dx^7-780ab^{35}ex^7+1560a^2b^{34}fx^7+455b^{36}cx^4-13}{1}$$

input
```
integrate(x^12*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/27*sqrt(3)*(35*(-a*b^2)^(1/3)*a*b^3*c - 65*(-a*b^2)^(1/3)*a^2*b^2*d + 10
4*(-a*b^2)^(1/3)*a^3*b*e - 152*(-a*b^2)^(1/3)*a^4*f)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^8 - 1/27*(35*a^2*b^3*c - 65*a^3*b^2*d +
 104*a^4*b*e - 152*a^5*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7)
+ 1/54*(35*(-a*b^2)^(1/3)*a*b^3*c - 65*(-a*b^2)^(1/3)*a^2*b^2*d + 104*(-a*
b^2)^(1/3)*a^3*b*e - 152*(-a*b^2)^(1/3)*a^4*f)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/b^8 - 1/18*(19*a^2*b^4*c*x^4 - 25*a^3*b^3*d*x^4 + 31*a^4*b^2
*e*x^4 - 37*a^5*b*f*x^4 + 16*a^3*b^3*c*x - 22*a^4*b^2*d*x + 28*a^5*b*e*x -
 34*a^6*f*x)/((b*x^3 + a)^2*b^7) + 1/1820*(140*b^36*f*x^13 + 182*b^36*e*x^
10 - 546*a*b^35*f*x^10 + 260*b^36*d*x^7 - 780*a*b^35*e*x^7 + 1560*a^2*b^34
*f*x^7 + 455*b^36*c*x^4 - 1365*a*b^35*d*x^4 + 2730*a^2*b^34*e*x^4 - 4550*a
^3*b^33*f*x^4 - 5460*a*b^35*c*x + 10920*a^2*b^34*d*x - 18200*a^3*b^33*e*x
+ 27300*a^4*b^32*f*x)/b^39
```

3.286.9 Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.38

$$\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = x^{10}\left(\frac{e}{10\,b^3}-\frac{3\,a\,f}{10\,b^4}\right) + x^4\left(\frac{c}{4\,b^3}-\frac{a^3\,f}{4\,b^6}-\frac{3\,a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{4\,b^2}+\frac{3\,a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{4\,b}\right) + \frac{x\left(\frac{17\,f\,a^6}{9}-\frac{14\,e\,a^5\,b}{9}+\frac{11\,d\,a^4\,b^2}{9}-\frac{8\,c\,a^3\,b^3}{9}\right)-x^4\left(-\frac{37\,f\,a^5\,b}{18}+\frac{31\,e\,a^4\,b^2}{18}-\frac{25\,d\,a^3\,b^3}{18}+\frac{19\,c\,a^2\,b^4}{18}\right)}{a^2\,b^7+2\,a\,b^8\,x^3+b^9\,x^6} - x\left(\frac{3\,a\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)}{b} - \frac{3\,a^2\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b^2}+\frac{a^3\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^3}\right) - x^7\left(\frac{3\,a^2\,f}{7\,b^5}-\frac{d}{7\,b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{7\,b}\right)+\frac{f\,x^{13}}{13\,b^3} + \frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-152\,f\,a^3+104\,e\,a^2\,b-65\,d\,a\,b^2+35\,c\,b^3\right)}{27\,b^{22/3}} + \frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-152\,f\,a^3+104\,e\,a^2\,b-65\,d\,a\,b^2+35\,c\,b^3\right)}{27\,b^{22/3}} - \frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-152\,f\,a^3+104\,e\,a^2\,b-65\,d\,a\,b^2+35\,c\,b^3\right)}{27\,b^{22/3}}$$

input
```
int((x^12*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
x^10*(e/(10*b^3) - (3*a*f)/(10*b^4)) + x^4*(c/(4*b^3) - (a^3*f)/(4*b^6) -
(3*a^2*(e/b^3 - (3*a*f)/b^4))/(4*b^2) + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a
*(e/b^3 - (3*a*f)/b^4))/b))/(4*b)) + (x*((17*a^6*f)/9 - (8*a^3*b^3*c)/9 +
(11*a^4*b^2*d)/9 - (14*a^5*b*e)/9) - x^4*((19*a^2*b^4*c)/18 - (25*a^3*b^3*
d)/18 + (31*a^4*b^2*e)/18 - (37*a^5*b*f)/18))/(a^2*b^7 + b^9*x^6 + 2*a*b^8
*x^3) - x*((3*a*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3 - (3*a*f)/b^4))/b^2 +
 (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b))/b - (3*
a^2*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b^2 + (a^3*(e
/b^3 - (3*a*f)/b^4))/b^3) - x^7*((3*a^2*f)/(7*b^5) - d/(7*b^3) + (3*a*(e/b
^3 - (3*a*f)/b^4))/(7*b)) + (f*x^13)/(13*b^3) + (a^(4/3)*log(b^(1/3)*x + a
^(1/3))*(35*b^3*c - 152*a^3*f - 65*a*b^2*d + 104*a^2*b*e))/(27*b^(22/3)) +
 (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2
- 1/2)*(35*b^3*c - 152*a^3*f - 65*a*b^2*d + 104*a^2*b*e))/(27*b^(22/3)) -
(a^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 +
 1/2)*(35*b^3*c - 152*a^3*f - 65*a*b^2*d + 104*a^2*b*e))/(27*b^(22/3))
```

3.286. $\int \frac{x^{12}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

3.287 $\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

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3.287.1 Optimal result

Integrand size = 30, antiderivative size = 384

$$\begin{aligned}\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6}+\frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5}+\frac{(be-3af)x^8}{8b^4}\\&\quad+\frac{fx^{11}}{11b^3}-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^6\,(a+bx^3)^2}+\frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)}\\&\quad+\frac{a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{20/3}}\\&\quad+\frac{a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{20/3}}\\&\quad-\frac{a^{2/3}(20b^3c-44ab^2d+77a^2be-119a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{20/3}}\end{aligned}$$

output
```
1/2*(-10*a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*x^2/b^6+1/5*(6*a^2*f-3*a*b*e+b^2
*d)*x^5/b^5+1/8*(-3*a*f+b*e)*x^8/b^4+1/11*f*x^11/b^3-1/6*a^2*(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)*x^2/b^6/(b*x^3+a)^2+1/9*a*(-16*a^3*f+13*a^2*b*e-10*a*b^2
*d+7*b^3*c)*x^2/b^6/(b*x^3+a)+1/27*a^(2/3)*(-119*a^3*f+77*a^2*b*e-44*a*b^2
*d+20*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(20/3)-1/54*a^(2/3)*(-119*a^3*f+77*a^
2*b*e-44*a*b^2*d+20*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(20
/3)+1/27*a^(2/3)*(-119*a^3*f+77*a^2*b*e-44*a*b^2*d+20*b^3*c)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(20/3)*3^(1/2)
```

3.287.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.99

$$\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^2}{2b^6}+\frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5}+\frac{(be-3af)x^8}{8b^4}+\frac{fx^{11}}{11b^3}$$
$$+\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)\,x^2}{6b^6\,(a+bx^3)^2}+\frac{a(7b^3c-10ab^2d+13a^2be-16a^3f)\,x^2}{9b^6\,(a+bx^3)}$$
$$-\frac{a^{2/3}(-20b^3c+44ab^2d-77a^2be+119a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{20/3}}$$
$$-\frac{a^{2/3}(-20b^3c+44ab^2d-77a^2be+119a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{20/3}}$$
$$+\frac{a^{2/3}(-20b^3c+44ab^2d-77a^2be+119a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{20/3}}$$

input
```
Integrate[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/(2*b^6) + ((b^2*d - 3*a*b
*e + 6*a^2*f)*x^5)/(5*b^5) + ((b*e - 3*a*f)*x^8)/(8*b^4) + (f*x^11)/(11*b^
3) + (a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6*b^6*(a + b*x^3)^2
) + (a*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(9*b^6*(a + b*x
^3)) - (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*ArcTan[(
1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(20/3)) - (a^(2/3)*(-20*
b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27
*b^(20/3)) + (a^(2/3)*(-20*b^3*c + 44*a*b^2*d - 77*a^2*b*e + 119*a^3*f)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(20/3))
```

3.287.3 Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {2367, 27, 2390, 2367, 2390, 2375, 27, 2375, 27, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{2367}$$

$$-\frac{\int -\frac{2(3ab^6fx^{16}+3ab^5(be-af)x^{13}+3ab^4(fa^2-bea+b^2d)x^{10}+3ab^3(-fa^3+bea^2-b^2da+b^3c)x^7-3a^2b^2(-fa^3+bea^2-b^2da+b^3c)x^4+a^3b(-fa^3}{(bx^3+a)^2}}{6ab^7}$$
$$\frac{a^2x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6(a+bx^3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3ab^6fx^{16}+3ab^5(be-af)x^{13}+3ab^4(fa^2-bea+b^2d)x^{10}+3ab^3(-fa^3+bea^2-b^2da+b^3c)x^7-3a^2b^2(-fa^3+bea^2-b^2da+b^3c)x^4+a^3b(-fa^3+bea^2-}{(bx^3+a)^2}}{3ab^7}$$
$$\frac{a^2x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6(a+bx^3)^2}$$

$$\Big\downarrow \text{2390}$$

$$\frac{\int \frac{x(3ab^6fx^{15}+3ab^5(be-af)x^{12}+3ab^4(fa^2-bea+b^2d)x^9+3ab^3(-fa^3+bea^2-b^2da+b^3c)x^6-3a^2b^2(-fa^3+bea^2-b^2da+b^3c)x^3+a^3b(-fa^3+bea^2}{(bx^3+a)^2}}{3ab^7}$$
$$\frac{a^2x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6(a+bx^3)^2}$$

$$\Big\downarrow \text{2367}$$

$$\frac{\frac{a^2bx^2(-16a^3f+13a^2be-10ab^2d+7b^3c)}{3(a+bx^3)}-\frac{\int \frac{-9a^2b^{11}fx^{13}-9a^2b^{10}(be-2af)x^{10}-9a^2b^9(3fa^2-2bea+b^2d)x^7-9a^2b^8(-4fa^3+3bea^2-2b^2da+b^3c)x^4+a^3b^7}{bx^3+a}}{3ab^6}}{3ab^7}$$
$$\frac{a^2x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^6(a+bx^3)^2}$$

$$\Big\downarrow \text{2390}$$

$$\frac{\frac{a^2bx^2(-16a^3f+13a^2be-10ab^2d+7b^3c)}{3(a+bx^3)}-\frac{\int \frac{x\left(-9a^2b^{11}fx^{12}-9a^2b^{10}(be-2af)x^9-9a^2b^9\left(3fa^2-2bea+b^2d\right)x^6-9a^2b^8\left(-4fa^3+3bea^2-2b^2da+b^3c\right)x^3+a^3\cdots}{bx^3+a}}{3ab^6}}{3ab^7}$$

$$\frac{a^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

$\downarrow$ 2375

$$\frac{\frac{a^2bx^2(-16a^3f+13a^2be-10ab^2d+7b^3c)}{3(a+bx^3)}-\frac{\frac{\int \frac{11x\left(-9a^2(be-3af)x^9b^{11}-9a^2\left(3fa^2-2bea+b^2d\right)x^6b^{10}-9a^2\left(-4fa^3+3bea^2-2b^2da+b^3c\right)x^3b^9+a^3\left(-29fa^3+\cdots\right.}{bx^3+a}}{11b}}{3ab^6}}{3ab^7}$$

$$\frac{a^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{a^2bx^2(-16a^3f+13a^2be-10ab^2d+7b^3c)}{3(a+bx^3)}-\frac{\frac{\int \frac{x\left(-9a^2(be-3af)x^9b^{11}-9a^2\left(3fa^2-2bea+b^2d\right)x^6b^{10}-9a^2\left(-4fa^3+3bea^2-2b^2da+b^3c\right)x^3b^9+a^3\left(-29fa^3+23\cdots\right.}{bx^3+a}}{b}}{3ab^6}}{3ab^7}$$

$$\frac{a^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

$\downarrow$ 2375

$$\frac{\frac{a^2bx^2(-16a^3f+13a^2be-10ab^2d+7b^3c)}{3(a+bx^3)}-\frac{\frac{\frac{\int \frac{8x\left(-9a^2\left(6fa^2-3bea+b^2d\right)x^6b^{11}-9a^2\left(-4fa^3+3bea^2-2b^2da+b^3c\right)x^3b^{10}+a^3\left(-29fa^3+23bea^2-17b^2da+11\cdots\right.}{bx^3+a}}{8b}}{b}}{3ab^6}}{3ab^7}$$

$$\frac{a^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{a^2bx^2(-16a^3f+13a^2be-10ab^2d+7b^3c)}{3(a+bx^3)}-\frac{\frac{\frac{\int \frac{x\left(-9a^2\left(6fa^2-3bea+b^2d\right)x^6b^{11}-9a^2\left(-4fa^3+3bea^2-2b^2da+b^3c\right)x^3b^{10}+a^3\left(-29fa^3+23bea^2-17b^2da+11b^3\cdots\right.}{bx^3+a}}{b}}{b}}{3ab^6}}{3ab^7}$$

$$\frac{a^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

$\downarrow$ 1812

3.287. $\int \frac{x^{10}\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{a^2bx^2(-16a^3f+13a^2be-10ab^2d+7b^3c)}{3(a+bx^3)} - \frac{\frac{\frac{\int\left(-9a^2\left(6fa^2-3bea+b^2d\right)x^4b^{10}-9a^2\left(-10fa^3+6bea^2-3b^2da+b^3c\right)xb^9+\frac{\left(20a^3cb^{12}-44a^4db^{11}+77a^5eb^{10}-11\cdots\right.}{bx^3+a}\right.}{b}}{b}}{3ab^6}}{3ab^7}$$

$$\frac{a^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{a^2bx^2(-16a^3f+13a^2be-10ab^2d+7b^3c)}{3(a+bx^3)} - \frac{-\frac{9}{5}a^2b^{10}x^5\left(6a^2f-3abe+b^2d\right)-\frac{9}{2}a^2b^9x^2\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)-\frac{a^{8/3}b^{25/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\cdots\right.}{\sqrt{3}\cdots}}{\cdots}}{\cdots}$$

$$\frac{a^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

input
```
Int[(x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
-1/6*(a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(b^6*(a + b*x^3)^2) + (
(a^2*b*(7*b^3*c - 10*a*b^2*d + 13*a^2*b*e - 16*a^3*f)*x^2)/(3*(a + b*x^3))
 - ((-9*a^2*b^10*f*x^11)/11 + ((-9*a^2*b^10*(b*e - 3*a*f)*x^8)/8 + ((-9*a^
2*b^9*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^2)/2 - (9*a^2*b^10*(b^2
*d - 3*a*b*e + 6*a^2*f)*x^5)/5 - (a^(8/3)*b^(25/3)*(20*b^3*c - 44*a*b^2*d
+ 77*a^2*b*e - 119*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/Sqrt[3] - (a^(8/3)*b^(25/3)*(20*b^3*c - 44*a*b^2*d + 77*a^2*b*e - 119*a
^3*f)*Log[a^(1/3) + b^(1/3)*x])/3 + (a^(8/3)*b^(25/3)*(20*b^3*c - 44*a*b^2
*d + 77*a^2*b*e - 119*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/6)/b)/b)/(3*a*b^6))/(3*a*b^7)
```

3.287.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
 + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2390
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

3.287.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.64

method	result
risch	$\frac{f x^{11}}{11b^3} - \frac{3x^8 fa}{8b^4} + \frac{x^8 e}{8b^3} + \frac{6x^5 f a^2}{5b^5} - \frac{3x^5 ae}{5b^4} + \frac{d x^5}{5b^3} - \frac{5x^2 f a^3}{b^6} + \frac{3x^2 a^2 e}{b^5} - \frac{3x^2 ad}{2b^4} + \frac{x^2 c}{2b^3} + \frac{\left(-\frac{16}{9}a^4 bf + \frac{13}{9}a^3 b^2 e - \frac{10}{9}a^2 \cdots\right.}{\cdots}$
default	$-\frac{-\frac{b^3 f x^{11}}{11} + \frac{\left(3fa b^2 - b^3 e\right)x^8}{8} + \frac{\left(-6f a^2 b + 3a b^2 e - b^3 d\right)x^5}{5} + \frac{\left(10f a^3 - 6a^2 be + 3a b^2 d - b^3 c\right)x^2}{2}}{b^6} + \frac{a\left(\frac{\left(-\frac{16}{9}a^3 bf + \frac{13}{9}a^2 e b^2 - \frac{10}{9}a b^3 d + \frac{7}{9}b^4 c\right)\cdots}{\left(b x \cdots\right.}\cdots\right.}{\cdots}$

input
```
int(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/11*f*x^11/b^3-3/8/b^4*x^8*f*a+1/8/b^3*x^8*e+6/5/b^5*x^5*f*a^2-3/5/b^4*x^
5*a*e+1/5/b^3*d*x^5-5/b^6*x^2*f*a^3+3/b^5*x^2*a^2*e-3/2/b^4*x^2*a*d+1/2/b^
3*x^2*c+((-16/9*a^4*b*f+13/9*a^3*b^2*e-10/9*a^2*b^3*d+7/9*a*b^4*c)*x^5-1/1
8*a^2*(29*a^3*f-23*a^2*b*e+17*a*b^2*d-11*b^3*c)*x^2)/b^6/(b*x^3+a)^2+1/27/
b^7*a*sum((119*a^3*f-77*a^2*b*e+44*a*b^2*d-20*b^3*c)/_R*ln(x-_R),_R=RootOf
(_Z^3*b+a))
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.65

$$\int \frac{x^{10}(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3}\, dx$$

$$= \frac{1080\, b^5 f x^{17} + 135\,(11\, b^5 e - 17\, ab^4 f)x^{14} + 54\,(44\, b^5 d - 77\, ab^4 e + 119\, a^2 b^3 f)x^{11} + 297\,(20\, b^5 c - 44\, ab^4 d + \cdots}{\cdots}$$

input
```
integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/11880*(1080*b^5*f*x^17 + 135*(11*b^5*e - 17*a*b^4*f)*x^14 + 54*(44*b^5*d
 - 77*a*b^4*e + 119*a^2*b^3*f)*x^11 + 297*(20*b^5*c - 44*a*b^4*d + 77*a^2*
b^3*e - 119*a^3*b^2*f)*x^8 + 1056*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*
e - 119*a^4*b*f)*x^5 + 660*(20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119
*a^5*f)*x^2 - 440*sqrt(3)*((20*b^5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119*a^3
*b^2*f)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5*f + 2*(20
*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 119*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3
)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) + 220*((20*b^
5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119*a^3*b^2*f)*x^6 + 20*a^2*b^3*c - 44*a
^3*b^2*d + 77*a^4*b*e - 119*a^5*f + 2*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*
b^2*e - 119*a^4*b*f)*x^3)*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3
) - a*(-a^2/b^2)^(1/3)) - 440*((20*b^5*c - 44*a*b^4*d + 77*a^2*b^3*e - 119
*a^3*b^2*f)*x^6 + 20*a^2*b^3*c - 44*a^3*b^2*d + 77*a^4*b*e - 119*a^5*f + 2
*(20*a*b^4*c - 44*a^2*b^3*d + 77*a^3*b^2*e - 119*a^4*b*f)*x^3)*(-a^2/b^2)^
(1/3)*log(a*x + b*(-a^2/b^2)^(2/3)))/(b^8*x^6 + 2*a*b^7*x^3 + a^2*b^6)
```

3.287.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**10*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.287.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.99

$$\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{2\,(7\,ab^4c-10\,a^2b^3d+13\,a^3b^2e-16\,a^4bf)x^5+(11\,a^2b^3c-17\,a^3b^2d+23\,a^4be-29\,a^5f)x^2}{18\,(b^8x^6+2\,ab^7x^3+a^2b^6)}$$
$$-\frac{\sqrt{3}(20\,ab^3c-44\,a^2b^2d+77\,a^3be-119\,a^4f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{40\,b^3fx^{11}+55\,(b^3e-3\,ab^2f)x^8+88\,(b^3d-3\,ab^2e+6\,a^2bf)x^5+220\,(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f)x^2}{440\,b^6}$$
$$-\frac{(20\,ab^3c-44\,a^2b^2d+77\,a^3be-119\,a^4f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{(20\,ab^3c-44\,a^2b^2d+77\,a^3be-119\,a^4f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(2*(7*a*b^4*c - 10*a^2*b^3*d + 13*a^3*b^2*e - 16*a^4*b*f)*x^5 + (11*a
^2*b^3*c - 17*a^3*b^2*d + 23*a^4*b*e - 29*a^5*f)*x^2)/(b^8*x^6 + 2*a*b^7*x
^3 + a^2*b^6) - 1/27*sqrt(3)*(20*a*b^3*c - 44*a^2*b^2*d + 77*a^3*b*e - 119
*a^4*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)^(1/
3)) + 1/440*(40*b^3*f*x^11 + 55*(b^3*e - 3*a*b^2*f)*x^8 + 88*(b^3*d - 3*a*
b^2*e + 6*a^2*b*f)*x^5 + 220*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x^
2)/b^6 - 1/54*(20*a*b^3*c - 44*a^2*b^2*d + 77*a^3*b*e - 119*a^4*f)*log(x^2
 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^7*(a/b)^(1/3)) + 1/27*(20*a*b^3*c - 44*
a^2*b^2*d + 77*a^3*b*e - 119*a^4*f)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(1/3))
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.26

$$\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{\left(20\,ab^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 44\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 77\,a^3be\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 119\,a^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^6}$$
$$+ \frac{\sqrt{3}\left(20\,(-ab^2)^{\frac{2}{3}}\,b^3c - 44\,(-ab^2)^{\frac{2}{3}}\,ab^2d + 77\,(-ab^2)^{\frac{2}{3}}\,a^2be - 119\,(-ab^2)^{\frac{2}{3}}\,a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^8}$$
$$- \frac{\left(20\,(-ab^2)^{\frac{2}{3}}\,b^3c - 44\,(-ab^2)^{\frac{2}{3}}\,ab^2d + 77\,(-ab^2)^{\frac{2}{3}}\,a^2be - 119\,(-ab^2)^{\frac{2}{3}}\,a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^8}$$
$$+ \frac{14\,ab^4cx^5 - 20\,a^2b^3dx^5 + 26\,a^3b^2ex^5 - 32\,a^4bfx^5 + 11\,a^2b^3cx^2 - 17\,a^3b^2dx^2 + 23\,a^4bex^2 - 29\,a^5fx^2}{18\,(bx^3+a)^2b^6}$$
$$+ \frac{40\,b^{30}fx^{11} + 55\,b^{30}ex^8 - 165\,ab^{29}fx^8 + 88\,b^{30}dx^5 - 264\,ab^{29}ex^5 + 528\,a^2b^{28}fx^5 + 220\,b^{30}cx^2 - 660\,ab^{29}dx^2 + \ldots}{440\,b^{33}}$$

input
```
integrate(x^10*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/27*(20*a*b^3*c*(-a/b)^(1/3) - 44*a^2*b^2*d*(-a/b)^(1/3) + 77*a^3*b*e*(-a
/b)^(1/3) - 119*a^4*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3))
)/(a*b^6) + 1/27*sqrt(3)*(20*(-a*b^2)^(2/3)*b^3*c - 44*(-a*b^2)^(2/3)*a*b^
2*d + 77*(-a*b^2)^(2/3)*a^2*b*e - 119*(-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^8 - 1/54*(20*(-a*b^2)^(2/3)*b^3*
c - 44*(-a*b^2)^(2/3)*a*b^2*d + 77*(-a*b^2)^(2/3)*a^2*b*e - 119*(-a*b^2)^(
2/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^8 + 1/18*(14*a*b^4*
c*x^5 - 20*a^2*b^3*d*x^5 + 26*a^3*b^2*e*x^5 - 32*a^4*b*f*x^5 + 11*a^2*b^3*
c*x^2 - 17*a^3*b^2*d*x^2 + 23*a^4*b*e*x^2 - 29*a^5*f*x^2)/((b*x^3 + a)^2*b
^6) + 1/440*(40*b^30*f*x^11 + 55*b^30*e*x^8 - 165*a*b^29*f*x^8 + 88*b^30*d
*x^5 - 264*a*b^29*e*x^5 + 528*a^2*b^28*f*x^5 + 220*b^30*c*x^2 - 660*a*b^29
*d*x^2 + 1320*a^2*b^28*e*x^2 - 2200*a^3*b^27*f*x^2)/b^33
```

3.287.9 Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.11

$$\int \frac{x^{10}(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x^8\left(\frac{e}{8\,b^3}-\frac{3\,a\,f}{8\,b^4}\right)+x^2\left(\frac{c}{2\,b^3}-\frac{a^3\,f}{2\,b^6}-\frac{3\,a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{2\,b^2}+\frac{3\,a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{2\,b}\right)$$
$$-\frac{\left(\frac{16\,f\,a^4\,b}{9}-\frac{13\,e\,a^3\,b^2}{9}+\frac{10\,d\,a^2\,b^3}{9}-\frac{7\,c\,a\,b^4}{9}\right)x^5+\left(\frac{29\,f\,a^5}{18}-\frac{23\,e\,a^4\,b}{18}+\frac{17\,d\,a^3\,b^2}{18}-\frac{11\,c\,a^2\,b^3}{18}\right)x^2}{a^2\,b^6+2\,a\,b^7\,x^3+b^8\,x^6}$$
$$-x^5\left(\frac{3\,a^2\,f}{5\,b^5}-\frac{d}{5\,b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{5\,b}\right)+\frac{f\,x^{11}}{11\,b^3}$$
$$+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-119\,f\,a^3+77\,e\,a^2\,b-44\,d\,a\,b^2+20\,c\,b^3\right)}{27\,b^{20/3}}$$
$$-\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-119\,f\,a^3+77\,e\,a^2\,b-44\,d\,a\,b^2+20\,c\,b^3\right)}{27\,b^{20/3}}$$
$$+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-119\,f\,a^3+77\,e\,a^2\,b-44\,d\,a\,b^2+20\,c\,b^3\right)}{27\,b^{20/3}}$$

input
```
int((x^10*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
x^8*(e/(8*b^3) - (3*a*f)/(8*b^4)) + x^2*(c/(2*b^3) - (a^3*f)/(2*b^6) - (3*
a^2*(e/b^3 - (3*a*f)/b^4))/(2*b^2) + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e
/b^3 - (3*a*f)/b^4))/b))/(2*b)) - (x^2*((29*a^5*f)/18 - (11*a^2*b^3*c)/18
+ (17*a^3*b^2*d)/18 - (23*a^4*b*e)/18) + x^5*((10*a^2*b^3*d)/9 - (13*a^3*b
^2*e)/9 - (7*a*b^4*c)/9 + (16*a^4*b*f)/9))/(a^2*b^6 + b^8*x^6 + 2*a*b^7*x^
3) - x^5*((3*a^2*f)/(5*b^5) - d/(5*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))/(5*b
)) + (f*x^11)/(11*b^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(20*b^3*c - 119
*a^3*f - 44*a*b^2*d + 77*a^2*b*e))/(27*b^(20/3)) - (a^(2/3)*log(3^(1/2)*a^
(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(20*b^3*c - 119*a
^3*f - 44*a*b^2*d + 77*a^2*b*e))/(27*b^(20/3)) + (a^(2/3)*log(3^(1/2)*a^(1
/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(20*b^3*c - 119*a^3
*f - 44*a*b^2*d + 77*a^2*b*e))/(27*b^(20/3))
```

3.288 $\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

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3.288.1 Optimal result

Integrand size = 30, antiderivative size = 375

$$\begin{aligned}\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6}+\frac{(b^2d-3abe+6a^2f)\,x^4}{4b^5}+\frac{(be-3af)x^7}{7b^4}+\frac{fx^{10}}{10b^3}\\&-\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^6\,(a+bx^3)^2}+\frac{a(13b^3c-19ab^2d+25a^2be-31a^3f)\,x}{18b^6\,(a+bx^3)}\\&+\frac{\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{19/3}}\\&-\frac{\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{19/3}}\\&+\frac{\sqrt[3]{a}(14b^3c-35ab^2d+65a^2be-104a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{19/3}}\end{aligned}$$

output
```
(-10*a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*x/b^6+1/4*(6*a^2*f-3*a*b*e+b^2*d)*x^
4/b^5+1/7*(-3*a*f+b*e)*x^7/b^4+1/10*f*x^10/b^3-1/6*a^2*(-a^3*f+a^2*b*e-a*b
^2*d+b^3*c)*x/b^6/(b*x^3+a)^2+1/18*a*(-31*a^3*f+25*a^2*b*e-19*a*b^2*d+13*b
^3*c)*x/b^6/(b*x^3+a)-1/27*a^(1/3)*(-104*a^3*f+65*a^2*b*e-35*a*b^2*d+14*b^
3*c)*ln(a^(1/3)+b^(1/3)*x)/b^(19/3)+1/54*a^(1/3)*(-104*a^3*f+65*a^2*b*e-35
*a*b^2*d+14*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(19/3)+1/27
*a^(1/3)*(-104*a^3*f+65*a^2*b*e-35*a*b^2*d+14*b^3*c)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(19/3)*3^(1/2)
```

3.288.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.97

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{3780\sqrt[3]{b}(b^3c-3ab^2d+6a^2be-10a^3f)\,x+945b^{4/3}(b^2d-3abe+6a^2f)\,x^4+540b^{7/3}(be-3af)x^7+378b^{10}\cdots}{}$$

input
```
Integrate[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(3780*b^(1/3)*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x + 945*b^(4/3)*(
b^2*d - 3*a*b*e + 6*a^2*f)*x^4 + 540*b^(7/3)*(b*e - 3*a*f)*x^7 + 378*b^(10
/3)*f*x^10 + (630*a^2*b^(1/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(a
 + b*x^3)^2 + (210*a*b^(1/3)*(13*b^3*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*
f)*x)/(a + b*x^3) - 140*Sqrt[3]*a^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b
*e + 104*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 140*a^(1/3)*
(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(1/3) + b^(1/3)*x]
 - 70*a^(1/3)*(-14*b^3*c + 35*a*b^2*d - 65*a^2*b*e + 104*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3780*b^(19/3))
```

3.288.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2367, 25, 2397, 27, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9\left(c+dx^3+ex^6+fx^9\right)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \; 2367$$

$$-\frac{\int -\frac{6ab^5fx^{15}+6ab^4(be-af)x^{12}+6ab^3(fa^2-bea+b^2d)x^9+6ab^2(-fa^3+bea^2-b^2da+b^3c)x^6-6a^2b(-fa^3+bea^2-b^2da+b^3c)x^3+a^3(-fa^3+bea^2}{(bx^3+a)^2}}{6ab^6}$$

$$\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

$\downarrow$ 25

$$\frac{\int \frac{6ab^5fx^{15}+6ab^4(be-af)x^{12}+6ab^3(fa^2-bea+b^2d)x^9+6ab^2(-fa^3+bea^2-b^2da+b^3c)x^6-6a^2b(-fa^3+bea^2-b^2da+b^3c)x^3+a^3(-fa^3+bea^2-b^2}{(bx^3+a)^2}}{6ab^6}$$

$$\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

$\downarrow$ 2397

$$\frac{\frac{a^2x(-31a^3f+25a^2be-19ab^2d+13b^3c)}{3(a+bx^3)}-\frac{\int \frac{2\left(-9a^2b^9fx^{12}-9a^2b^8(be-2af)x^9-9a^2b^7\left(3fa^2-2bea+b^2d\right)x^6-9a^2b^6\left(-4fa^3+3bea^2-2b^2da+b^3c\right)x^3+a^3b^5\left(\right.}{bx^3+a}}{3ab^5}}{6ab^6}$$

$$\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{a^2x(-31a^3f+25a^2be-19ab^2d+13b^3c)}{3(a+bx^3)}-\frac{2\int \frac{-9a^2b^9fx^{12}-9a^2b^8(be-2af)x^9-9a^2b^7\left(3fa^2-2bea+b^2d\right)x^6-9a^2b^6\left(-4fa^3+3bea^2-2b^2da+b^3c\right)x^3+a^3b^5\left(\right.}{bx^3+a}}{3ab^5}}{6ab^6}$$

$$\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

$\downarrow$ 2426

$$\frac{\frac{a^2x(-31a^3f+25a^2be-19ab^2d+13b^3c)}{3(a+bx^3)}-\frac{2\int \left(-9a^2b^8fx^9-9a^2b^7(be-3af)x^6-9a^2b^6(6fa^2-3bea+b^2d)x^3-9a^2b^5(-10fa^3+6bea^2-3b^2da+b^3c)\right.}{3ab^5}}{6ab^6}$$

$$\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

$\downarrow$ 2009

3.288. $\int \frac{x^9\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{a^2x(-31a^3f+25a^2be-19ab^2d+13b^3c)}{3(a+bx^3)}-\frac{2\left(-\frac{9}{10}a^2b^8fx^{10}-\frac{9}{7}a^2b^7x^7(be-3af)-\frac{9}{4}a^2b^6x^4(6a^2f-3abe+b^2d)-9a^2b^5x(-10a^3f+6a^2be-3ab^2d+b^3\ldots\right.}{\ldots}}{\ldots}$$

$$\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^6\left(a+bx^3\right)^2}$$

input
```
Int[(x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
-1/6*(a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(b^6*(a + b*x^3)^2) + ((a
^2*(13*b^3*c - 19*a*b^2*d + 25*a^2*b*e - 31*a^3*f)*x)/(3*(a + b*x^3)) - (2
*(-9*a^2*b^5*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x - (9*a^2*b^6*(b^
2*d - 3*a*b*e + 6*a^2*f)*x^4)/4 - (9*a^2*b^7*(b*e - 3*a*f)*x^7)/7 - (9*a^2
*b^8*f*x^10)/10 - (a^(7/3)*b^(14/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e -
104*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/Sqrt[3] + (a
^(7/3)*b^(14/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e - 104*a^3*f)*Log[a^(1/
3) + b^(1/3)*x])/3 - (a^(7/3)*b^(14/3)*(14*b^3*c - 35*a*b^2*d + 65*a^2*b*e
 - 104*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/6))/(3*a*b^5
))/(6*a*b^6)
```

3.288.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.288.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.62

method	result
risch	$\frac{f\,x^{10}}{10b^3}-\frac{3x^7fa}{7b^4}+\frac{x^7e}{7b^3}+\frac{3x^4fa^2}{2b^5}-\frac{3x^4ae}{4b^4}+\frac{d\,x^4}{4b^3}-\frac{10xf\,a^3}{b^6}+\frac{6x\,a^2e}{b^5}-\frac{3xad}{b^4}+\frac{xc}{b^3}+\frac{\left(-\frac{31}{18}a^4bf+\frac{25}{18}a^3b^2e-\frac{19}{18}a^2b^3c\right.}{}$
default	$-\frac{-\frac{1}{10}b^3f\,x^{10}+\frac{3}{7}x^7a\,b^2f-\frac{1}{7}x^7b^3e-\frac{3}{2}a^2bf\,x^4+\frac{3}{4}a\,b^2e\,x^4-\frac{1}{4}d\,x^4b^3+10f\,a^3x-6a^2bex+3a\,b^2dx-b^3cx}{b^6}+\frac{a\left(\frac{\left(-\frac{31}{18}a^3bf+\frac{25}{18}a^2e\,b^2-\right.}{}\right.}{}$

input
```
int(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*f*x^10/b^3-3/7/b^4*x^7*f*a+1/7/b^3*x^7*e+3/2/b^5*x^4*f*a^2-3/4/b^4*x^
4*a*e+1/4/b^3*d*x^4-10/b^6*x*f*a^3+6/b^5*x*a^2*e-3/b^4*x*a*d+1/b^3*x*c+((-
31/18*a^4*b*f+25/18*a^3*b^2*e-19/18*a^2*b^3*d+13/18*a*b^4*c)*x^4-1/9*a^2*(
14*a^3*f-11*a^2*b*e+8*a*b^2*d-5*b^3*c)*x)/b^6/(b*x^3+a)^2+1/27/b^7*a*sum((
104*a^3*f-65*a^2*b*e+35*a*b^2*d-14*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a
))
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.61

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$=\frac{378\,b^5fx^{16}+108\,(5\,b^5e-8\,ab^4f)x^{13}+27\,(35\,b^5d-65\,ab^4e+104\,a^2b^3f)x^{10}+270\,(14\,b^5c-35\,ab^4d+65\,a}{}$$

input
```
integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/3780*(378*b^5*f*x^16 + 108*(5*b^5*e - 8*a*b^4*f)*x^13 + 27*(35*b^5*d - 6
5*a*b^4*e + 104*a^2*b^3*f)*x^10 + 270*(14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*
e - 104*a^3*b^2*f)*x^7 + 735*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 1
04*a^4*b*f)*x^4 - 140*sqrt(3)*((14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e - 104
*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b*e - 104*a^5*f + 2
*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e - 104*a^4*b*f)*x^3)*(a/b)^(1/3)
*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 70*((14*b^5*c - 3
5*a*b^4*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3*c - 35*a^3*b^2*
d + 65*a^4*b*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d + 65*a^3*b^2*e -
 104*a^4*b*f)*x^3)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 14
0*((14*b^5*c - 35*a*b^4*d + 65*a^2*b^3*e - 104*a^3*b^2*f)*x^6 + 14*a^2*b^3
*c - 35*a^3*b^2*d + 65*a^4*b*e - 104*a^5*f + 2*(14*a*b^4*c - 35*a^2*b^3*d
+ 65*a^3*b^2*e - 104*a^4*b*f)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 420*
(14*a^2*b^3*c - 35*a^3*b^2*d + 65*a^4*b*e - 104*a^5*f)*x)/(b^8*x^6 + 2*a*b
^7*x^3 + a^2*b^6)
```

3.288.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^9(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**9*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.288.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{(13\,ab^4c-19\,a^2b^3d+25\,a^3b^2e-31\,a^4bf)x^4+2\,(5\,a^2b^3c-8\,a^3b^2d+11\,a^4be-14\,a^5f)x}{18\,(b^8x^6+2\,ab^7x^3+a^2b^6)}$$
$$+\frac{14\,b^3fx^{10}+20\,(b^3e-3\,ab^2f)x^7+35\,(b^3d-3\,ab^2e+6\,a^2bf)x^4+140\,(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f)x}{140\,b^6}$$
$$-\frac{\sqrt{3}(14\,ab^3c-35\,a^2b^2d+65\,a^3be-104\,a^4f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(14\,ab^3c-35\,a^2b^2d+65\,a^3be-104\,a^4f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(14\,ab^3c-35\,a^2b^2d+65\,a^3be-104\,a^4f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^7\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*((13*a*b^4*c - 19*a^2*b^3*d + 25*a^3*b^2*e - 31*a^4*b*f)*x^4 + 2*(5*a
^2*b^3*c - 8*a^3*b^2*d + 11*a^4*b*e - 14*a^5*f)*x)/(b^8*x^6 + 2*a*b^7*x^3
+ a^2*b^6) + 1/140*(14*b^3*f*x^10 + 20*(b^3*e - 3*a*b^2*f)*x^7 + 35*(b^3*d
 - 3*a*b^2*e + 6*a^2*b*f)*x^4 + 140*(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^
3*f)*x)/b^6 - 1/27*sqrt(3)*(14*a*b^3*c - 35*a^2*b^2*d + 65*a^3*b*e - 104*a
^4*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^7*(a/b)^(2/3)
) + 1/54*(14*a*b^3*c - 35*a^2*b^2*d + 65*a^3*b*e - 104*a^4*f)*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(b^7*(a/b)^(2/3)) - 1/27*(14*a*b^3*c - 35*a^2*b
^2*d + 65*a^3*b*e - 104*a^4*f)*log(x + (a/b)^(1/3))/(b^7*(a/b)^(2/3))
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.16

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx =$$

$$-\frac{\sqrt{3}\left(14\left(-ab^2\right)^{\frac{1}{3}}b^3c-35\left(-ab^2\right)^{\frac{1}{3}}ab^2d+65\left(-ab^2\right)^{\frac{1}{3}}a^2be-104\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^7}$$
$$+\frac{\left(14ab^3c-35a^2b^2d+65a^3be-104a^4f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27ab^6}$$
$$-\frac{\left(14\left(-ab^2\right)^{\frac{1}{3}}b^3c-35\left(-ab^2\right)^{\frac{1}{3}}ab^2d+65\left(-ab^2\right)^{\frac{1}{3}}a^2be-104\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54b^7}$$
$$+\frac{13ab^4cx^4-19a^2b^3dx^4+25a^3b^2ex^4-31a^4bfx^4+10a^2b^3cx-16a^3b^2dx+22a^4bex-28a^5fx}{18\left(bx^3+a\right)^2b^6}$$
$$+\frac{14b^{27}fx^{10}+20b^{27}ex^7-60ab^{26}fx^7+35b^{27}dx^4-105ab^{26}ex^4+210a^2b^{25}fx^4+140b^{27}cx-420ab^{26}dx}{140b^{30}}$$

input
```
integrate(x^9*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(14*(-a*b^2)^(1/3)*b^3*c - 35*(-a*b^2)^(1/3)*a*b^2*d + 65*(-
a*b^2)^(1/3)*a^2*b*e - 104*(-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/b^7 + 1/27*(14*a*b^3*c - 35*a^2*b^2*d + 65*a^
3*b*e - 104*a^4*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^6) - 1/54*
(14*(-a*b^2)^(1/3)*b^3*c - 35*(-a*b^2)^(1/3)*a*b^2*d + 65*(-a*b^2)^(1/3)*a
^2*b*e - 104*(-a*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/b^7 + 1/18*(13*a*b^4*c*x^4 - 19*a^2*b^3*d*x^4 + 25*a^3*b^2*e*x^4 - 31*a^
4*b*f*x^4 + 10*a^2*b^3*c*x - 16*a^3*b^2*d*x + 22*a^4*b*e*x - 28*a^5*f*x)/(
(b*x^3 + a)^2*b^6) + 1/140*(14*b^27*f*x^10 + 20*b^27*e*x^7 - 60*a*b^26*f*x
^7 + 35*b^27*d*x^4 - 105*a*b^26*e*x^4 + 210*a^2*b^25*f*x^4 + 140*b^27*c*x
- 420*a*b^26*d*x + 840*a^2*b^25*e*x - 1400*a^3*b^24*f*x)/b^30
```

3.288.9 Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.12

$$\int \frac{x^9(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x^7\left(\frac{e}{7\,b^3}-\frac{3\,a\,f}{7\,b^4}\right)+x\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)$$
$$-x^4\left(\frac{3\,a^2\,f}{4\,b^5}-\frac{d}{4\,b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{4\,b}\right)$$
$$-\frac{\left(\frac{31\,f\,a^4\,b}{18}-\frac{25\,e\,a^3\,b^2}{18}+\frac{19\,d\,a^2\,b^3}{18}-\frac{13\,c\,a\,b^4}{18}\right)\,x^4+\left(\frac{14\,f\,a^5}{9}-\frac{11\,e\,a^4\,b}{9}+\frac{8\,d\,a^3\,b^2}{9}-\frac{5\,c\,a^2\,b^3}{9}\right)\,x}{a^2\,b^6+2\,a\,b^7\,x^3+b^8\,x^6}$$
$$+\frac{f\,x^{10}}{10\,b^3}-\frac{a^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-104\,f\,a^3+65\,e\,a^2\,b-35\,d\,a\,b^2+14\,c\,b^3\right)}{27\,b^{19/3}}$$
$$-\frac{a^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-104\,f\,a^3+65\,e\,a^2\,b-35\,d\,a\,b^2+14\,c\,b^3\right)}{27\,b^{19/3}}$$
$$+\frac{a^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-104\,f\,a^3+65\,e\,a^2\,b-35\,d\,a\,b^2+14\,c\,b^3\right)}{27\,b^{19/3}}$$

input
```
int((x^9*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
x^7*(e/(7*b^3) - (3*a*f)/(7*b^4)) + x*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3
 - (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)
/b^4))/b))/b) - x^4*((3*a^2*f)/(4*b^5) - d/(4*b^3) + (3*a*(e/b^3 - (3*a*f)
/b^4))/(4*b)) - (x*((14*a^5*f)/9 - (5*a^2*b^3*c)/9 + (8*a^3*b^2*d)/9 - (11
*a^4*b*e)/9) + x^4*((19*a^2*b^3*d)/18 - (25*a^3*b^2*e)/18 - (13*a*b^4*c)/1
8 + (31*a^4*b*f)/18))/(a^2*b^6 + b^8*x^6 + 2*a*b^7*x^3) + (f*x^10)/(10*b^3
) - (a^(1/3)*log(b^(1/3)*x + a^(1/3))*(14*b^3*c - 104*a^3*f - 35*a*b^2*d +
 65*a^2*b*e))/(27*b^(19/3)) - (a^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*
x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(14*b^3*c - 104*a^3*f - 35*a*b^2*d + 6
5*a^2*b*e))/(27*b^(19/3)) + (a^(1/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
+ a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(14*b^3*c - 104*a^3*f - 35*a*b^2*d + 65*
a^2*b*e))/(27*b^(19/3))
```

3.289 $\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

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3.289.1 Optimal result

Integrand size = 30, antiderivative size = 345

$$\begin{aligned}\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= \frac{(b^2d-3abe+6a^2f)\,x^2}{2b^5}+\frac{(be-3af)x^5}{5b^4}+\frac{fx^8}{8b^3}\\&+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^5\,(a+bx^3)^2}-\frac{(4b^3c-7ab^2d+10a^2be-13a^3f)\,x^2}{9b^5\,(a+bx^3)}\\&-\frac{(5b^3c-20ab^2d+44a^2be-77a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}b^{17/3}}\\&-\frac{(5b^3c-20ab^2d+44a^2be-77a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{17/3}}\\&+\frac{(5b^3c-20ab^2d+44a^2be-77a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{17/3}}\end{aligned}$$

output
```
1/2*(6*a^2*f-3*a*b*e+b^2*d)*x^2/b^5+1/5*(-3*a*f+b*e)*x^5/b^4+1/8*f*x^8/b^3
+1/6*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/b^5/(b*x^3+a)^2-1/9*(-13*a^3*f+1
0*a^2*b*e-7*a*b^2*d+4*b^3*c)*x^2/b^5/(b*x^3+a)-1/27*(-77*a^3*f+44*a^2*b*e-
20*a*b^2*d+5*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(17/3)+1/54*(-77*a^3*f
+44*a^2*b*e-20*a*b^2*d+5*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
a^(1/3)/b^(17/3)-1/27*(-77*a^3*f+44*a^2*b*e-20*a*b^2*d+5*b^3*c)*arctan(1/3
*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(17/3)*3^(1/2)
```

3.289.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.95

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{540b^{2/3}(b^2d-3abe+6a^2f)\,x^2+216b^{5/3}(be-3af)x^5+135b^{8/3}fx^8+\frac{180ab^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{(a+bx^3)^2}-\frac{120b^{2/3}}{}}{}$$

input
```
Integrate[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(540*b^(2/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x^2 + 216*b^(5/3)*(b*e - 3*a*f)*x
^5 + 135*b^(8/3)*f*x^8 + (180*a*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f
)*x^2)/(a + b*x^3)^2 - (120*b^(2/3)*(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13
*a^3*f)*x^2)/(a + b*x^3) + (40*Sqrt[3]*(-5*b^3*c + 20*a*b^2*d - 44*a^2*b*e
 + 77*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (40*(-
5*b^3*c + 20*a*b^2*d - 44*a^2*b*e + 77*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^
(1/3) + (20*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(1080*b^(17/3))
```

3.289.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2367, 27, 2390, 2367, 2029, 2375, 27, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{ 2367}$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\frac{\int \frac{2\left(-3ab^5fx^{13}-3ab^4(be-af)x^{10}-3ab^3\left(fa^2-bea+b^2d\right)x^7-3ab^2\left(-fa^3+bea^2-b^2da+b^3c\right)x^4+a^2b\left(-fa^3+bea^2-b^2da+b^3c\right)x\right)}{(bx^3+a)^2}dx}{6ab^6}$$

$$\Big\downarrow \; 27$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\frac{\int \frac{-3ab^5fx^{13}-3ab^4(be-af)x^{10}-3ab^3\left(fa^2-bea+b^2d\right)x^7-3ab^2\left(-fa^3+bea^2-b^2da+b^3c\right)x^4+a^2b\left(-fa^3+bea^2-b^2da+b^3c\right)x}{\left(bx^3+a\right)^2}dx}{3ab^6}$$

$$\Big\downarrow \; 2390$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\frac{\int \frac{x\left(-3ab^5fx^{12}-3ab^4(be-af)x^9-3ab^3\left(fa^2-bea+b^2d\right)x^6-3ab^2\left(-fa^3+bea^2-b^2da+b^3c\right)x^3+a^2b\left(-fa^3+bea^2-b^2da+b^3c\right)\right)}{\left(bx^3+a\right)^2}dx}{3ab^6}$$

$$\Big\downarrow \; 2367$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\frac{\frac{abx^2\left(-13a^3f+10a^2be-7ab^2d+4b^3c\right)}{3(a+bx^3)}-\frac{\int \frac{9a^2b^9fx^{10}+9a^2b^8(be-2af)x^7+9a^2b^7\left(3fa^2-2bea+b^2d\right)x^4+a^2b^6\left(-23fa^3+17bea^2-11b^2da+5b^3c\right)x}{bx^3+a}dx}{3ab^5}}{3ab^6}$$

$$\Big\downarrow \; 2029$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\frac{\frac{abx^2\left(-13a^3f+10a^2be-7ab^2d+4b^3c\right)}{3(a+bx^3)}-\frac{\int \frac{x\left(9a^2b^9fx^9+9a^2b^8(be-2af)x^6+9a^2b^7\left(3fa^2-2bea+b^2d\right)x^3+a^2b^6\left(-23fa^3+17bea^2-11b^2da+5b^3c\right)\right)}{bx^3+a}dx}{3ab^5}}{3ab^6}$$

$$\Big\downarrow \; 2375$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\frac{\frac{abx^2\left(-13a^3f+10a^2be-7ab^2d+4b^3c\right)}{3(a+bx^3)}-\frac{\frac{\int \frac{8x\left(9a^2(be-3af)x^6b^9+9a^2\left(3fa^2-2bea+b^2d\right)x^3b^8+a^2\left(-23fa^3+17bea^2-11b^2da+5b^3c\right)b^7\right)}{bx^3+a}dx}{8b}+\frac{9}{8}a^2b^8fx^8}{3ab^5}}{3ab^6}$$

$$\Big\downarrow \; 27$$

$$\frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\frac{\frac{abx^2\left(-13a^3f+10a^2be-7ab^2d+4b^3c\right)}{3(a+bx^3)}-\frac{\frac{\int \frac{x\left(9a^2(be-3af)x^6b^9+9a^2\left(3fa^2-2bea+b^2d\right)x^3b^8+a^2\left(-23fa^3+17bea^2-11b^2da+5b^3c\right)b^7\right)}{bx^3+a}dx}{b}+\frac{9}{8}a^2b^8fx^8}{3ab^5}}{3ab^6}$$

$$\Big\downarrow \; 1812$$

3.289. $\int \frac{x^7\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}-\frac{\frac{abx^2(-13a^3f+10a^2be-7ab^2d+4b^3c)}{3(a+bx^3)}-\frac{\frac{\int\left(9a^2(be-3af)x^4b^8+9a^2\left(6fa^2-3bea+b^2d\right)xb^7+\frac{\left(5a^2cb^{10}-20a^3db^9+44a^4eb^8-77a^5fb^7\right)x}{bx^3+a}\right)dx}{b}+\frac{9}{8}a^2b^8fx^8}{3ab^5}}{3ab^6}}{}$$

$\downarrow$ 2009

$$\frac{ax^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}-\frac{\frac{abx^2(-13a^3f+10a^2be-7ab^2d+4b^3c)}{3(a+bx^3)}-\frac{\frac{9}{8}a^2b^8fx^8+\frac{\frac{9}{5}a^2b^8x^5(be-3af)+\frac{9}{2}a^2b^7x^2\left(6a^2f-3abe+b^2d\right)-\frac{a^{5/3}b^{19/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-77a^3f+44a^2\right.}{\sqrt{3}}}{}}{}}{}$$

input
```
Int[(x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*b^5*(a + b*x^3)^2) - ((a*b*
(4*b^3*c - 7*a*b^2*d + 10*a^2*b*e - 13*a^3*f)*x^2)/(3*(a + b*x^3)) - ((9*a
^2*b^8*f*x^8)/8 + ((9*a^2*b^7*(b^2*d - 3*a*b*e + 6*a^2*f)*x^2)/2 + (9*a^2*
b^8*(b*e - 3*a*f)*x^5)/5 - (a^(5/3)*b^(19/3)*(5*b^3*c - 20*a*b^2*d + 44*a^
2*b*e - 77*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/Sqrt[
3] - (a^(5/3)*b^(19/3)*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*Log[
a^(1/3) + b^(1/3)*x])/3 + (a^(5/3)*b^(19/3)*(5*b^3*c - 20*a*b^2*d + 44*a^2
*b*e - 77*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/6)/b)/(3*
a*b^5))/(3*a*b^6)
```

3.289.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2390
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

3.289.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.57

method	result
risch	$\frac{f\,x^{8}}{8b^{3}}-\frac{3x^{5}af}{5b^{4}}+\frac{x^{5}e}{5b^{3}}+\frac{3a^{2}f\,x^{2}}{b^{5}}-\frac{3ae\,x^{2}}{2b^{4}}+\frac{x^{2}d}{2b^{3}}+\frac{\left(\frac{13}{9}a^{3}bf-\frac{10}{9}a^{2}e\,b^{2}+\frac{7}{9}a\,b^{3}d-\frac{4}{9}b^{4}c\right)x^{5}+\frac{a\left(23f\,a^{3}-17a^{2}be+11a\,b^{2}d-5b^{3}c\right)x}{18}}{b^{5}\left(b\,x^{3}+a\right)^{2}}$
	$\frac{\left(-\frac{13}{9}a^{3}bf+\frac{10}{9}a^{2}e\,b^{2}-\frac{7}{9}a\,b^{3}d+\frac{4}{9}b^{4}c\right)x^{5}-\frac{a\left(23f\,a^{3}-17a^{2}be+11a\,b^{2}d-5b^{3}c\right)x^{2}}{18}}{\left(b\,x^{3}+a\right)^{2}}+\left(\frac{77}{9}f\right.$
default	$\frac{\frac{b^{2}f\,x^{8}}{8}+\frac{\left(-3afb+b^{2}e\right)x^{5}}{5}+\frac{\left(6a^{2}f-3aeb+b^{2}d\right)x^{2}}{2}}{b^{5}}-$

input

```
int(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output

```
1/8*f*x^8/b^3-3/5/b^4*x^5*a*f+1/5/b^3*x^5*e+3/b^5*a^2*f*x^2-3/2/b^4*a*e*x^
2+1/2/b^3*x^2*d+((13/9*a^3*b*f-10/9*a^2*e*b^2+7/9*a*b^3*d-4/9*b^4*c)*x^5+1
/18*a*(23*a^3*f-17*a^2*b*e+11*a*b^2*d-5*b^3*c)*x^2)/b^5/(b*x^3+a)^2+1/27/b
^6*sum((-77*a^3*f+44*a^2*b*e-20*a*b^2*d+5*b^3*c)/_R*ln(x-_R),_R=RootOf(_Z^
3*b+a))
```

3.289.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(298) = 596$.

Time = 0.29 (sec) , antiderivative size = 1278, normalized size of antiderivative = 3.70

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input

```
integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/1080*(135*a*b^6*f*x^14 + 54*(4*a*b^6*e - 7*a^2*b^5*f)*x^11 + 27*(20*a*b
^6*d - 44*a^2*b^5*e + 77*a^3*b^4*f)*x^8 - 96*(5*a*b^6*c - 20*a^2*b^5*d + 4
4*a^3*b^4*e - 77*a^4*b^3*f)*x^5 - 60*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*
b^3*e - 77*a^5*b^2*f)*x^2 - 60*sqrt(1/3)*(5*a^3*b^4*c - 20*a^4*b^3*d + 44*
a^5*b^2*e - 77*a^6*b*f + (5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e - 77*a^4
*b^3*f)*x^6 + 2*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*a^4*b^3*e - 77*a^5*b^2*f)
*x^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2
*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(
2/3)*x)/(b*x^3 + a)) + 20*((5*b^5*c - 20*a*b^4*d + 44*a^2*b^3*e - 77*a^3*b
^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e - 77*a^5*f + 2*(5*a*b^
4*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*(a*b^2)^(2/3)*log(b^2
*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 40*((5*b^5*c - 20*a*b^4*d + 44
*a^2*b^3*e - 77*a^3*b^2*f)*x^6 + 5*a^2*b^3*c - 20*a^3*b^2*d + 44*a^4*b*e -
 77*a^5*f + 2*(5*a*b^4*c - 20*a^2*b^3*d + 44*a^3*b^2*e - 77*a^4*b*f)*x^3)*
(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a*b^9*x^6 + 2*a^2*b^8*x^3 + a^3*b
^7), 1/1080*(135*a*b^6*f*x^14 + 54*(4*a*b^6*e - 7*a^2*b^5*f)*x^11 + 27*(20
*a*b^6*d - 44*a^2*b^5*e + 77*a^3*b^4*f)*x^8 - 96*(5*a*b^6*c - 20*a^2*b^5*d
 + 44*a^3*b^4*e - 77*a^4*b^3*f)*x^5 - 60*(5*a^2*b^5*c - 20*a^3*b^4*d + 44*
a^4*b^3*e - 77*a^5*b^2*f)*x^2 - 120*sqrt(1/3)*(5*a^3*b^4*c - 20*a^4*b^3*d
+ 44*a^5*b^2*e - 77*a^6*b*f + (5*a*b^6*c - 20*a^2*b^5*d + 44*a^3*b^4*e ...
```

3.289.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.289.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.96

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= -\frac{2\,(4\,b^4c-7\,ab^3d+10\,a^2b^2e-13\,a^3bf)x^5+(5\,ab^3c-11\,a^2b^2d+17\,a^3be-23\,a^4f)x^2}{18\,(b^7x^6+2\,ab^6x^3+a^2b^5)}$$
$$+\frac{\sqrt{3}(5\,b^3c-20\,ab^2d+44\,a^2be-77\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{5\,b^2fx^8+8\,(b^2e-3\,abf)x^5+20\,(b^2d-3\,abe+6\,a^2f)x^2}{40\,b^5}$$
$$+\frac{(5\,b^3c-20\,ab^2d+44\,a^2be-77\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{(5\,b^3c-20\,ab^2d+44\,a^2be-77\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^6\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*(2*(4*b^4*c - 7*a*b^3*d + 10*a^2*b^2*e - 13*a^3*b*f)*x^5 + (5*a*b^3*
c - 11*a^2*b^2*d + 17*a^3*b*e - 23*a^4*f)*x^2)/(b^7*x^6 + 2*a*b^6*x^3 + a^
2*b^5) + 1/27*sqrt(3)*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*arcta
n(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(1/3)) + 1/40*(5
*b^2*f*x^8 + 8*(b^2*e - 3*a*b*f)*x^5 + 20*(b^2*d - 3*a*b*e + 6*a^2*f)*x^2)
/b^5 + 1/54*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*log(x^2 - x*(a/
b)^(1/3) + (a/b)^(2/3))/(b^6*(a/b)^(1/3)) - 1/27*(5*b^3*c - 20*a*b^2*d + 4
4*a^2*b*e - 77*a^3*f)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(1/3))
```

3.289.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.11

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{\sqrt{3}(5\,b^3c-20\,ab^2d+44\,a^2be-77\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{1}{3}}\,b^5}$$
$$- \frac{(5\,b^3c-20\,ab^2d+44\,a^2be-77\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{1}{3}}\,b^5}$$
$$- \frac{\left(5\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+44\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-77\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^5}$$
$$- \frac{8\,b^4cx^5-14\,ab^3dx^5+20\,a^2b^2ex^5-26\,a^3bfx^5+5\,ab^3cx^2-11\,a^2b^2dx^2+17\,a^3bex^2-23\,a^4fx^2}{18\,(bx^3+a)^2b^5}$$
$$+ \frac{5\,b^{21}fx^8+8\,b^{21}ex^5-24\,ab^{20}fx^5+20\,b^{21}dx^2-60\,ab^{20}ex^2+120\,a^2b^{19}fx^2}{40\,b^{24}}$$

input
```
integrate(x^7*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/27*sqrt(3)*(5*b^3*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^5) - 1/54*(5*b^3
*c - 20*a*b^2*d + 44*a^2*b*e - 77*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)
^(2/3))/((-a*b^2)^(1/3)*b^5) - 1/27*(5*b^3*c*(-a/b)^(1/3) - 20*a*b^2*d*(-a
/b)^(1/3) + 44*a^2*b*e*(-a/b)^(1/3) - 77*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*
log(abs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(8*b^4*c*x^5 - 14*a*b^3*d*x^5 +
20*a^2*b^2*e*x^5 - 26*a^3*b*f*x^5 + 5*a*b^3*c*x^2 - 11*a^2*b^2*d*x^2 + 17*
a^3*b*e*x^2 - 23*a^4*f*x^2)/((b*x^3 + a)^2*b^5) + 1/40*(5*b^21*f*x^8 + 8*b
^21*e*x^5 - 24*a*b^20*f*x^5 + 20*b^21*d*x^2 - 60*a*b^20*e*x^2 + 120*a^2*b^
19*f*x^2)/b^24
```

3.289.9 Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.98

$$\int \frac{x^7(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x^5\left(\frac{e}{5\,b^3}-\frac{3\,a\,f}{5\,b^4}\right)$$
$$+\frac{x^2\left(\frac{23\,f\,a^4}{18}-\frac{17\,e\,a^3\,b}{18}+\frac{11\,d\,a^2\,b^2}{18}-\frac{5\,c\,a\,b^3}{18}\right)-x^5\left(-\frac{13\,f\,a^3\,b}{9}+\frac{10\,e\,a^2\,b^2}{9}-\frac{7\,d\,a\,b^3}{9}+\frac{4\,c\,b^4}{9}\right)}{a^2\,b^5+2\,a\,b^6\,x^3+b^7\,x^6}$$
$$-x^2\left(\frac{3\,a^2\,f}{2\,b^5}-\frac{d}{2\,b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{2\,b}\right)+\frac{f\,x^8}{8\,b^3}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(-77\,f\,a^3+44\,e\,a^2\,b-20\,d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{1/3}\,b^{17/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-77\,f\,a^3+44\,e\,a^2\,b-20\,d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{1/3}\,b^{17/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-77\,f\,a^3+44\,e\,a^2\,b-20\,d\,a\,b^2+5\,c\,b^3\right)}{27\,a^{1/3}\,b^{17/3}}$$

input
```
int((x^7*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
x^5*(e/(5*b^3) - (3*a*f)/(5*b^4)) + (x^2*((23*a^4*f)/18 + (11*a^2*b^2*d)/1
8 - (5*a*b^3*c)/18 - (17*a^3*b*e)/18) - x^5*((4*b^4*c)/9 + (10*a^2*b^2*e)/
9 - (7*a*b^3*d)/9 - (13*a^3*b*f)/9))/(a^2*b^5 + b^7*x^6 + 2*a*b^6*x^3) - x
^2*((3*a^2*f)/(2*b^5) - d/(2*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))/(2*b)) + (
f*x^8)/(8*b^3) - (log(b^(1/3)*x + a^(1/3))*(5*b^3*c - 77*a^3*f - 20*a*b^2*
d + 44*a^2*b*e))/(27*a^(1/3)*b^(17/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/
3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c - 77*a^3*f - 20*a*b^2*d +
44*a^2*b*e))/(27*a^(1/3)*b^(17/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
 + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c - 77*a^3*f - 20*a*b^2*d + 44*a
^2*b*e))/(27*a^(1/3)*b^(17/3))
```

3.290 $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$

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3.290.1 Optimal result

Integrand size = 30, antiderivative size = 336

$$\begin{aligned}\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx &= \frac{(b^2d-3abe+6a^2f)\,x}{b^5}+\frac{(be-3af)x^4}{4b^4}+\frac{fx^7}{7b^3}\\ &+\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^5\,(a+bx^3)^2}-\frac{(7b^3c-13ab^2d+19a^2be-25a^3f)\,x}{18b^5\,(a+bx^3)}\\ &-\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{16/3}}\\ &+\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{16/3}}\\ &-\frac{(2b^3c-14ab^2d+35a^2be-65a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{2/3}b^{16/3}}\end{aligned}$$

output
```
(6*a^2*f-3*a*b*e+b^2*d)*x/b^5+1/4*(-3*a*f+b*e)*x^4/b^4+1/7*f*x^7/b^3+1/6*a
*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^5/(b*x^3+a)^2-1/18*(-25*a^3*f+19*a^2*b
*e-13*a*b^2*d+7*b^3*c)*x/b^5/(b*x^3+a)+1/27*(-65*a^3*f+35*a^2*b*e-14*a*b^2
*d+2*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(16/3)-1/54*(-65*a^3*f+35*a^2*
b*e-14*a*b^2*d+2*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/
b^(16/3)-1/27*(-65*a^3*f+35*a^2*b*e-14*a*b^2*d+2*b^3*c)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(16/3)*3^(1/2)
```

3.290.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.96

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{756\sqrt[3]{b}(b^2d-3abe+6a^2f)\,x+189b^{4/3}(be-3af)x^4+108b^{7/3}fx^7+\frac{126a\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)x}{(a+bx^3)^2}-\frac{42\sqrt[3]{b}(7b^3}{}}{}$$

input
```
Integrate[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(756*b^(1/3)*(b^2*d - 3*a*b*e + 6*a^2*f)*x + 189*b^(4/3)*(b*e - 3*a*f)*x^4
 + 108*b^(7/3)*f*x^7 + (126*a*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x)/(a + b*x^3)^2 - (42*b^(1/3)*(7*b^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3
*f)*x)/(a + b*x^3) + (28*Sqrt[3]*(-2*b^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*
a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (28*(2*b^3*c
 - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) +
 (14*(-2*b^3*c + 14*a*b^2*d - 35*a^2*b*e + 65*a^3*f)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(756*b^(16/3))
```

3.290.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2367, 2397, 27, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \; 2367$$

$$\frac{ax\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^5\left(a+bx^3\right)^2}-\frac{\int \frac{-6ab^4fx^{12}-6ab^3(be-af)x^9-6ab^2\left(fa^2-bea+b^2d\right)x^6-6ab\left(-fa^3+bea^2-b^2da+b^3c\right)x^3+a^2\left(-fa^3+bea^2-b^2da+b^3c\right)}{\left(bx^3+a\right)^2}dx}{6ab^5}$$

$$\Big\downarrow \; 2397$$

$$\frac{\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}-\frac{\frac{ax(-25a^3f+19a^2be-13ab^2d+7b^3c)}{3(a+bx^3)}-\frac{\int \frac{2\left(9a^2b^7fx^9+9a^2b^6(be-2af)x^6+9a^2b^5\left(3fa^2-2bea+b^2d\right)x^3+a^2b^4\left(-11fa^3+8bea^2-5b^2da+2b^3c\right)\right)}{bx^3+a}dx}{3ab^4}}{6ab^5}}{}$$

$$\Big\downarrow \; 27$$

$$\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}-\frac{\frac{ax(-25a^3f+19a^2be-13ab^2d+7b^3c)}{3(a+bx^3)}-\frac{2\int \frac{9a^2b^7fx^9+9a^2b^6(be-2af)x^6+9a^2b^5\left(3fa^2-2bea+b^2d\right)x^3+a^2b^4\left(-11fa^3+8bea^2-5b^2da+2b^3c\right)}{bx^3+a}dx}{3ab^4}}{6ab^5}$$

$$\Big\downarrow \; 2426$$

$$\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}-\frac{\frac{ax(-25a^3f+19a^2be-13ab^2d+7b^3c)}{3(a+bx^3)}-\frac{2\int \left(9a^2b^6fx^6+9a^2b^5(be-3af)x^3+9a^2b^4(6fa^2-3bea+b^2d)+\frac{2a^2cb^7-14a^3db^6+35a^4eb^5-65a^5fb^4}{bx^3+a}\right)dx}{3ab^4}}{6ab^5}$$

$$\Big\downarrow \; 2009$$

$$\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^5(a+bx^3)^2}-\frac{\frac{ax(-25a^3f+19a^2be-13ab^2d+7b^3c)}{3(a+bx^3)}-\frac{2\left(\frac{9}{7}a^2b^6fx^7+\frac{9}{4}a^2b^5x^4(be-3af)+9a^2b^4x(6a^2f-3abe+b^2d)-\frac{a^{4/3}b^{11/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(-65a^3f+}{\sqrt{3}}\right.}{}}{}$$

input
```
Int[(x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

3.290. $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

output
```
(a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*b^5*(a + b*x^3)^2) - ((a*(7*b
^3*c - 13*a*b^2*d + 19*a^2*b*e - 25*a^3*f)*x)/(3*(a + b*x^3)) - (2*(9*a^2*
b^4*(b^2*d - 3*a*b*e + 6*a^2*f)*x + (9*a^2*b^5*(b*e - 3*a*f)*x^4)/4 + (9*a
^2*b^6*f*x^7)/7 - (a^(4/3)*b^(11/3)*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 6
5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/Sqrt[3] + (a^(
4/3)*b^(11/3)*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*Log[a^(1/3) +
 b^(1/3)*x])/3 - (a^(4/3)*b^(11/3)*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65
*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/6))/(3*a*b^4))/(6*
a*b^5)
```

3.290.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.290. $\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

3.290.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.55

method	result
risch	$\frac{f x^7}{7b^3} - \frac{3x^4 af}{4b^4} + \frac{x^4 e}{4b^3} + \frac{6a^2 fx}{b^5} - \frac{3aex}{b^4} + \frac{xd}{b^3} + \frac{(\frac{25}{18}a^3bf - \frac{19}{18}a^2eb^2 + \frac{13}{18}ab^3d - \frac{7}{18}b^4c)x^4 + \frac{a(11fa^3-8a^2be+5ab^2d-2b^3c)x}{9}}{b^5(bx^3+a)^2} +$ ... $(65$
default	$\frac{\frac{1}{7}b^2fx^7 - \frac{3}{4}abfx^4 + \frac{1}{4}b^2ex^4 + 6a^2fx - 3abex + b^2dx}{b^5} - \frac{\frac{(-\frac{25}{18}a^3bf + \frac{19}{18}a^2eb^2 - \frac{13}{18}ab^3d + \frac{7}{18}b^4c)x^4 - \frac{a(11fa^3-8a^2be+5ab^2d-2b^3c)x}{9}}{(bx^3+a)^2} + \dots}{\dots}$

input
```
int(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*f*x^7/b^3-3/4/b^4*x^4*a*f+1/4/b^3*x^4*e+6/b^5*a^2*f*x-3/b^4*a*e*x+1/b^
3*x*d+((25/18*a^3*b*f-19/18*a^2*e*b^2+13/18*a*b^3*d-7/18*b^4*c)*x^4+1/9*a*
(11*a^3*f-8*a^2*b*e+5*a*b^2*d-2*b^3*c)*x)/b^5/(b*x^3+a)^2+1/27/b^6*sum((-6
5*a^3*f+35*a^2*b*e-14*a*b^2*d+2*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.290.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(291) = 582.

Time = 0.29 (sec) , antiderivative size = 1318, normalized size of antiderivative = 3.92

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/756*(108*a^2*b^5*f*x^13 + 27*(7*a^2*b^5*e - 13*a^3*b^4*f)*x^10 + 54*(14
*a^2*b^5*d - 35*a^3*b^4*e + 65*a^4*b^3*f)*x^7 - 147*(2*a^2*b^5*c - 14*a^3*
b^4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^4 - 42*sqrt(1/3)*(2*a^3*b^4*c - 14*
a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f + (2*a*b^6*c - 14*a^2*b^5*d + 35*a^3
*b^4*e - 65*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3*e
- 65*a^5*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1
/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)
*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*((2*b^5*c - 14*a*b^4*d + 35*
a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a^3*b^2*d + 35*a^4*b*e -
65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^2*e - 65*a^4*b*f)*x^3)*(
-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*((2*
b^5*c - 14*a*b^4*d + 35*a^2*b^3*e - 65*a^3*b^2*f)*x^6 + 2*a^2*b^3*c - 14*a
^3*b^2*d + 35*a^4*b*e - 65*a^5*f + 2*(2*a*b^4*c - 14*a^2*b^3*d + 35*a^3*b^
2*e - 65*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) - 84*(2*
a^3*b^4*c - 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f)*x)/(a^2*b^8*x^6 + 2*
a^3*b^7*x^3 + a^4*b^6), 1/756*(108*a^2*b^5*f*x^13 + 27*(7*a^2*b^5*e - 13*a
^3*b^4*f)*x^10 + 54*(14*a^2*b^5*d - 35*a^3*b^4*e + 65*a^4*b^3*f)*x^7 - 147
*(2*a^2*b^5*c - 14*a^3*b^4*d + 35*a^4*b^3*e - 65*a^5*b^2*f)*x^4 + 84*sqrt(
1/3)*(2*a^3*b^4*c - 14*a^4*b^3*d + 35*a^5*b^2*e - 65*a^6*b*f + (2*a*b^6*c
- 14*a^2*b^5*d + 35*a^3*b^4*e - 65*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c - 14...
```

3.290.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.290.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.97

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= -\frac{(7\,b^4c-13\,ab^3d+19\,a^2b^2e-25\,a^3bf)x^4+2\,(2\,ab^3c-5\,a^2b^2d+8\,a^3be-11\,a^4f)x}{18\,(b^7x^6+2\,ab^6x^3+a^2b^5)}$$
$$+\frac{4\,b^2fx^7+7\,(b^2e-3\,abf)x^4+28\,(b^2d-3\,abe+6\,a^2f)x}{28\,b^5}$$
$$+\frac{\sqrt{3}(2\,b^3c-14\,ab^2d+35\,a^2be-65\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(2\,b^3c-14\,ab^2d+35\,a^2be-65\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(2\,b^3c-14\,ab^2d+35\,a^2be-65\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*((7*b^4*c - 13*a*b^3*d + 19*a^2*b^2*e - 25*a^3*b*f)*x^4 + 2*(2*a*b^3
*c - 5*a^2*b^2*d + 8*a^3*b*e - 11*a^4*f)*x)/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b
^5) + 1/28*(4*b^2*f*x^7 + 7*(b^2*e - 3*a*b*f)*x^4 + 28*(b^2*d - 3*a*b*e +
6*a^2*f)*x)/b^5 + 1/27*sqrt(3)*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3
*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^6*(a/b)^(2/3))
- 1/54*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*log(x^2 - x*(a/b)^(1
/3) + (a/b)^(2/3))/(b^6*(a/b)^(2/3)) + 1/27*(2*b^3*c - 14*a*b^2*d + 35*a^2
*b*e - 65*a^3*f)*log(x + (a/b)^(1/3))/(b^6*(a/b)^(2/3))
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.01

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= -\frac{\sqrt{3}(2\,b^3c-14\,ab^2d+35\,a^2be-65\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}b^4} - \frac{(2\,b^3c-14\,ab^2d+35\,a^2be-65\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}b^4} - \frac{(2\,b^3c-14\,ab^2d+35\,a^2be-65\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^5} - \frac{7\,b^4cx^4-13\,ab^3dx^4+19\,a^2b^2ex^4-25\,a^3bfx^4+4\,ab^3cx-10\,a^2b^2dx+16\,a^3bex-22\,a^4fx}{18\left(bx^3+a\right)^2b^5} + \frac{4\,b^{18}fx^7+7\,b^{18}ex^4-21\,ab^{17}fx^4+28\,b^{18}dx-84\,ab^{17}ex+168\,a^2b^{16}fx}{28\,b^{21}}$$

input
```
integrate(x^6*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^4) - 1/54*(2*b^
3*c - 14*a*b^2*d + 35*a^2*b*e - 65*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/((-a*b^2)^(2/3)*b^4) - 1/27*(2*b^3*c - 14*a*b^2*d + 35*a^2*b*e -
65*a^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) - 1/18*(7*b^4*c*
x^4 - 13*a*b^3*d*x^4 + 19*a^2*b^2*e*x^4 - 25*a^3*b*f*x^4 + 4*a*b^3*c*x - 1
0*a^2*b^2*d*x + 16*a^3*b*e*x - 22*a^4*f*x)/((b*x^3 + a)^2*b^5) + 1/28*(4*b
^18*f*x^7 + 7*b^18*e*x^4 - 21*a*b^17*f*x^4 + 28*b^18*d*x - 84*a*b^17*e*x +
 168*a^2*b^16*f*x)/b^21
```

3.290.9 Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00

$$\int \frac{x^6(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x^4\left(\frac{e}{4\,b^3}-\frac{3\,a\,f}{4\,b^4}\right)-x\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)$$
$$-\frac{x^4\left(-\frac{25\,f\,a^3\,b}{18}+\frac{19\,e\,a^2\,b^2}{18}-\frac{13\,d\,a\,b^3}{18}+\frac{7\,c\,b^4}{18}\right)-x\left(\frac{11\,f\,a^4}{9}-\frac{8\,e\,a^3\,b}{9}+\frac{5\,d\,a^2\,b^2}{9}-\frac{2\,c\,a\,b^3}{9}\right)}{a^2\,b^5+2\,a\,b^6\,x^3+b^7\,x^6}$$
$$+\frac{f\,x^7}{7\,b^3}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-65\,f\,a^3+35\,e\,a^2\,b-14\,d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{2/3}\,b^{16/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-65\,f\,a^3+35\,e\,a^2\,b-14\,d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{2/3}\,b^{16/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-65\,f\,a^3+35\,e\,a^2\,b-14\,d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{2/3}\,b^{16/3}}$$

input
```
int((x^6*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
x^4*(e/(4*b^3) - (3*a*f)/(4*b^4)) - x*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3
 - (3*a*f)/b^4))/b) - (x^4*((7*b^4*c)/18 + (19*a^2*b^2*e)/18 - (13*a*b^3*d
)/18 - (25*a^3*b*f)/18) - x*((11*a^4*f)/9 + (5*a^2*b^2*d)/9 - (2*a*b^3*c)/
9 - (8*a^3*b*e)/9))/(a^2*b^5 + b^7*x^6 + 2*a*b^6*x^3) + (f*x^7)/(7*b^3) +
(log(b^(1/3)*x + a^(1/3))*(2*b^3*c - 65*a^3*f - 14*a*b^2*d + 35*a^2*b*e))/
(27*a^(2/3)*b^(16/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*(
(3^(1/2)*1i)/2 - 1/2)*(2*b^3*c - 65*a^3*f - 14*a*b^2*d + 35*a^2*b*e))/(27*
a^(2/3)*b^(16/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(
1/2)*1i)/2 + 1/2)*(2*b^3*c - 65*a^3*f - 14*a*b^2*d + 35*a^2*b*e))/(27*a^(2
/3)*b^(16/3))
```

3.291 $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

3.291.1 Optimal result 2205
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3.291.3 Rubi [A] (verified) 2206
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3.291.5 Fricas [B] (verification not implemented) 2210
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3.291.9 Mupad [B] (verification not implemented) 2213

3.291.1 Optimal result

Integrand size = 30, antiderivative size = 316

$$\begin{aligned}
&\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx \\
&= \frac{(be-3af)x^2}{2b^4}+\frac{fx^5}{5b^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6b^4\,(a+bx^3)^2}+\frac{(b^3c-4ab^2d+7a^2be-10a^3f)\,x^2}{9ab^4\,(a+bx^3)} \\
&\quad -\frac{(b^3c+5ab^2d-20a^2be+44a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{14/3}} \\
&\quad -\frac{(b^3c+5ab^2d-20a^2be+44a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{14/3}} \\
&\quad +\frac{(b^3c+5ab^2d-20a^2be+44a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{14/3}}
\end{aligned}$$

output
```
1/2*(-3*a*f+b*e)*x^2/b^4+1/5*f*x^5/b^3-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
x^2/b^4/(b*x^3+a)^2+1/9*(-10*a^3*f+7*a^2*b*e-4*a*b^2*d+b^3*c)*x^2/a/b^4/(b
*x^3+a)-1/27*(44*a^3*f-20*a^2*b*e+5*a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a
^(4/3)/b^(14/3)+1/54*(44*a^3*f-20*a^2*b*e+5*a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(14/3)-1/27*(44*a^3*f-20*a^2*b*e+5*a*
b^2*d+b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(
14/3)*3^(1/2)
```

3.291.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.95

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{135b^{2/3}(be-3af)x^2+54b^{5/3}fx^5-\frac{45b^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{(a+bx^3)^2}+\frac{30b^{2/3}(b^3c-4ab^2d+7a^2be-10a^3f)x^2}{a(a+bx^3)}-\frac{10\sqrt{3}(b^3c+5ab}{}}{}$$

input

```
Integrate[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output

```
(135*b^(2/3)*(b*e - 3*a*f)*x^2 + 54*b^(5/3)*f*x^5 - (45*b^(2/3)*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*x^2)/(a + b*x^3)^2 + (30*b^(2/3)*(b^3*c - 4*a*b^
2*d + 7*a^2*b*e - 10*a^3*f)*x^2)/(a*(a + b*x^3)) - (10*Sqrt[3]*(b^3*c + 5*
a*b^2*d - 20*a^2*b*e + 44*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/a^(4/3) - (10*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(1/3)
+ b^(1/3)*x])/a^(4/3) + (5*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(270*b^(14/3))
```

3.291.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2367, 27, 2029, 2367, 25, 2028, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \text{2367}$$

$$-\frac{\int -\frac{2\left(3ab^4fx^{10}+3ab^3(be-af)x^7+3ab^2\left(fa^2-bea+b^2d\right)x^4+ab\left(-fa^3+bea^2-b^2da+b^3c\right)x\right)}{\left(bx^3+a\right)^2}dx}{6ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3ab^4fx^{10}+3ab^3(be-af)x^7+3ab^2\left(fa^2-bea+b^2d\right)x^4+ab\left(-fa^3+bea^2-b^2da+b^3c\right)x}{\left(bx^3+a\right)^2}dx}{3ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2029$$

$$\frac{\int \frac{x\left(3ab^4fx^9+3ab^3(be-af)x^6+3ab^2\left(fa^2-bea+b^2d\right)x^3+ab\left(-fa^3+bea^2-b^2da+b^3c\right)\right)}{\left(bx^3+a\right)^2}dx}{3ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2367$$

$$\frac{\frac{bx^2\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{3\left(a+bx^3\right)}-\frac{\int -\frac{9a^2b^7fx^7+9a^2b^6(be-2af)x^4+ab^5\left(17fa^3-11bea^2+5b^2da+b^3c\right)x}{bx^3+a}dx}{3ab^4}}{3ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{9a^2b^7fx^7+9a^2b^6(be-2af)x^4+ab^5\left(17fa^3-11bea^2+5b^2da+b^3c\right)x}{bx^3+a}dx}{3ab^4}+\frac{bx^2\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{3\left(a+bx^3\right)}}{3ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2028$$

$$\frac{\frac{\int \frac{x\left(9a^2fx^6b^7+9a^2(be-2af)x^3b^6+a\left(17fa^3-11bea^2+5b^2da+b^3c\right)b^5\right)}{bx^3+a}dx}{3ab^4}+\frac{bx^2\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{3\left(a+bx^3\right)}}{3ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1812$$

$$\frac{\frac{\int \left(9a^2fx^4b^6+9a^2(be-3af)xb^5+\frac{\left(acb^8+5a^2db^7-20a^3eb^6+44a^4fb^5\right)x}{bx^3+a}\right)dx}{3ab^4}+\frac{bx^2\left(-10a^3f+7a^2be-4ab^2d+b^3c\right)}{3\left(a+bx^3\right)}}{3ab^5}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2009$$

3.291. $\int \frac{x^4\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{bx^2(-10a^3f+7a^2be-4ab^2d+b^3c)}{3(a+bx^3)}+\frac{\frac{9}{5}a^2b^6fx^5+\frac{9}{2}a^2b^5x^2(be-3af)-\frac{a^{2/3}b^{13/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(44a^3f-20a^2be+5ab^2d+b^3c)}{\sqrt{3}}+\frac{1}{6}a^{2/3}b^{13/3}\log}{}}{3ab^5}$$

$$\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^4(a+bx^3)^2}$$

input
```
Int[(x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
-1/6*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(b^4*(a + b*x^3)^2) + ((b*(
b^3*c - 4*a*b^2*d + 7*a^2*b*e - 10*a^3*f)*x^2)/(3*(a + b*x^3)) + ((9*a^2*b
^5*(b*e - 3*a*f)*x^2)/2 + (9*a^2*b^6*f*x^5)/5 - (a^(2/3)*b^(13/3)*(b^3*c +
 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/Sqrt[3] - (a^(2/3)*b^(13/3)*(b^3*c + 5*a*b^2*d - 20*a^2*b*e +
 44*a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/3 + (a^(2/3)*b^(13/3)*(b^3*c + 5*a*b^
2*d - 20*a^2*b*e + 44*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/6)/(3*a*b^4))/(3*a*b^5)
```

3.291.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.291. $\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

3.291.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.53

method	result
risch	$\frac{f x^5}{5b^3} - \frac{3x^2 a f}{2b^4} + \frac{e x^2}{2b^3} + \frac{-\frac{b\left(10 f a^3 - 7a^2 b e + 4 a b^2 d - b^3 c\right) x^5}{9a} + \left(-\frac{17}{18} f a^3 + \frac{11}{18} a^2 b e - \frac{5}{18} a b^2 d - \frac{1}{18} b^3 c\right) x^2}{b^4 (b x^3 + a)^2} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \cdots}{\cdots}$ … $\left(44 f a^3 - 20a^2 b e + 5 a b^2 d + b^3 c\right)$ …
default	$-\frac{-\frac{b f x^5}{5} + \frac{(3af - be)x^2}{2}}{b^4} + \frac{\frac{-\frac{b\left(10 f a^3 - 7a^2 b e + 4 a b^2 d - b^3 c\right) x^5}{9a} + \left(-\frac{17}{18} f a^3 + \frac{11}{18} a^2 b e - \frac{5}{18} a b^2 d - \frac{1}{18} b^3 c\right) x^2}{\left(b x^3 + a\right)^2} + \cdots}{b^4}$

input
```
int(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*f*x^5/b^3-3/2/b^4*x^2*a*f+1/2/b^3*e*x^2+(-1/9*b*(10*a^3*f-7*a^2*b*e+4*
a*b^2*d-b^3*c)/a*x^5+(-17/18*f*a^3+11/18*a^2*b*e-5/18*a*b^2*d-1/18*b^3*c)*
x^2)/b^4/(b*x^3+a)^2+1/27/b^5/a*sum((44*a^3*f-20*a^2*b*e+5*a*b^2*d+b^3*c)/
_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.291.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(271) = 542$.

Time = 0.29 (sec) , antiderivative size = 1224, normalized size of antiderivative = 3.87

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/270*(54*a^2*b^5*f*x^11 + 27*(5*a^2*b^5*e - 11*a^3*b^4*f)*x^8 + 6*(5*a*b
^6*c - 20*a^2*b^5*d + 80*a^3*b^4*e - 176*a^4*b^3*f)*x^5 - 15*(a^2*b^5*c +
5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^2 + 15*sqrt(1/3)*(a^3*b^4*c +
 5*a^4*b^3*d - 20*a^5*b^2*e + 44*a^6*b*f + (a*b^6*c + 5*a^2*b^5*d - 20*a^3
*b^4*e + 44*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 4
4*a^5*b^2*f)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3
)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a)
 - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 5*((b^5*c + 5*a*b^4*d - 20*a^2*b^3*e
 + 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e + 44*a^5*f + 2
*(a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(-a*b^2)^(2/3)*l
og(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 10*((b^5*c + 5*a*b^4*d
 - 20*a^2*b^3*e + 44*a^3*b^2*f)*x^6 + a^2*b^3*c + 5*a^3*b^2*d - 20*a^4*b*e
 + 44*a^5*f + 2*(a*b^4*c + 5*a^2*b^3*d - 20*a^3*b^2*e + 44*a^4*b*f)*x^3)*(
-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^8*x^6 + 2*a^3*b^7*x^3 + a^
4*b^6), 1/270*(54*a^2*b^5*f*x^11 + 27*(5*a^2*b^5*e - 11*a^3*b^4*f)*x^8 + 6
*(5*a*b^6*c - 20*a^2*b^5*d + 80*a^3*b^4*e - 176*a^4*b^3*f)*x^5 - 15*(a^2*b
^5*c + 5*a^3*b^4*d - 20*a^4*b^3*e + 44*a^5*b^2*f)*x^2 + 30*sqrt(1/3)*(a^3*
b^4*c + 5*a^4*b^3*d - 20*a^5*b^2*e + 44*a^6*b*f + (a*b^6*c + 5*a^2*b^5*d -
 20*a^3*b^4*e + 44*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 5*a^3*b^4*d - 20*a^4*b^
3*e + 44*a^5*b^2*f)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*...
```

3.291.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.291.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.98

$$\begin{aligned}
&\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx \\
&=\frac{2\,(b^4c-4\,ab^3d+7\,a^2b^2e-10\,a^3bf)x^5-(ab^3c+5\,a^2b^2d-11\,a^3be+17\,a^4f)x^2}{18\,(ab^6x^6+2\,a^2b^5x^3+a^3b^4)} \\
&\quad+\frac{2\,bfx^5+5\,(be-3\,af)x^2}{10\,b^4} \\
&\quad+\frac{\sqrt{3}(b^3c+5\,ab^2d-20\,a^2be+44\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\
&\quad+\frac{(b^3c+5\,ab^2d-20\,a^2be+44\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\
&\quad-\frac{(b^3c+5\,ab^2d-20\,a^2be+44\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}
\end{aligned}$$

input
```
integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(2*(b^4*c - 4*a*b^3*d + 7*a^2*b^2*e - 10*a^3*b*f)*x^5 - (a*b^3*c + 5*
a^2*b^2*d - 11*a^3*b*e + 17*a^4*f)*x^2)/(a*b^6*x^6 + 2*a^2*b^5*x^3 + a^3*b
^4) + 1/10*(2*b*f*x^5 + 5*(b*e - 3*a*f)*x^2)/b^4 + 1/27*sqrt(3)*(b^3*c + 5
*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(
a/b)^(1/3))/(a*b^5*(a/b)^(1/3)) + 1/54*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 4
4*a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(1/3)) - 1/27
*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*log(x + (a/b)^(1/3))/(a*b^5*(
a/b)^(1/3))
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.14

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{\sqrt{3}(b^3c+5\,ab^2d-20\,a^2be+44\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}\,ab^4} - \frac{(b^3c+5\,ab^2d-20\,a^2be+44\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}\,ab^4} - \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+44\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^4} + \frac{2\,b^4cx^5-8\,ab^3dx^5+14\,a^2b^2ex^5-20\,a^3bfx^5-ab^3cx^2-5\,a^2b^2dx^2+11\,a^3bex^2-17\,a^4fx^2}{18\,(bx^3+a)^2ab^4} + \frac{2\,b^{12}fx^5+5\,b^{12}ex^2-15\,ab^{11}fx^2}{10\,b^{15}}$$

input
```
integrate(x^4*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/27*sqrt(3)*(b^3*c + 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a*b^4) - 1/54*(b^3*c
+ 5*a*b^2*d - 20*a^2*b*e + 44*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/
3))/((-a*b^2)^(1/3)*a*b^4) - 1/27*(b^3*c*(-a/b)^(1/3) + 5*a*b^2*d*(-a/b)^(
1/3) - 20*a^2*b*e*(-a/b)^(1/3) + 44*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a^2*b^4) + 1/18*(2*b^4*c*x^5 - 8*a*b^3*d*x^5 + 14*a
^2*b^2*e*x^5 - 20*a^3*b*f*x^5 - a*b^3*c*x^2 - 5*a^2*b^2*d*x^2 + 11*a^3*b*e
*x^2 - 17*a^4*f*x^2)/((b*x^3 + a)^2*a*b^4) + 1/10*(2*b^12*f*x^5 + 5*b^12*e
*x^2 - 15*a*b^11*f*x^2)/b^15
```

3.291.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.93

$$\int \frac{x^4(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x^2\left(\frac{e}{2\,b^3}-\frac{3\,a\,f}{2\,b^4}\right)-\frac{x^2\left(\frac{17\,f\,a^3}{18}-\frac{11\,e\,a^2\,b}{18}+\frac{5\,d\,a\,b^2}{18}+\frac{c\,b^3}{18}\right)-\frac{x^5\,(-10\,f\,a^3\,b+7\,e\,a^2\,b^2-4\,d\,a\,b^3+c\,b^4)}{9\,a}}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}$$
$$+\frac{f\,x^5}{5\,b^3}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(44\,f\,a^3-20\,e\,a^2\,b+5\,d\,a\,b^2+c\,b^3)}{27\,a^{4/3}\,b^{14/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(44\,f\,a^3-20\,e\,a^2\,b+5\,d\,a\,b^2+c\,b^3)}{27\,a^{4/3}\,b^{14/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(44\,f\,a^3-20\,e\,a^2\,b+5\,d\,a\,b^2+c\,b^3)}{27\,a^{4/3}\,b^{14/3}}$$

input
```
int((x^4*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
x^2*(e/(2*b^3) - (3*a*f)/(2*b^4)) - (x^2*((b^3*c)/18 + (17*a^3*f)/18 + (5*
a*b^2*d)/18 - (11*a^2*b*e)/18) - (x^5*(b^4*c + 7*a^2*b^2*e - 4*a*b^3*d - 1
0*a^3*b*f))/(9*a))/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + (f*x^5)/(5*b^3) - (
log(b^(1/3)*x + a^(1/3))*(b^3*c + 44*a^3*f + 5*a*b^2*d - 20*a^2*b*e))/(27*
a^(4/3)*b^(14/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(
1/2)*1i)/2 + 1/2)*(b^3*c + 44*a^3*f + 5*a*b^2*d - 20*a^2*b*e))/(27*a^(4/3)
*b^(14/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i
)/2 - 1/2)*(b^3*c + 44*a^3*f + 5*a*b^2*d - 20*a^2*b*e))/(27*a^(4/3)*b^(14/
3))
```

3.292 $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

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3.292.1 Optimal result

Integrand size = 30, antiderivative size = 307

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \frac{(be-3af)x}{b^4}+\frac{fx^4}{4b^3}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6b^4(a+bx^3)^2}+\frac{(b^3c-7ab^2d+13a^2be-19a^3f)\,x}{18ab^4(a+bx^3)}$$
$$-\frac{(b^3c+2ab^2d-14a^2be+35a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{13/3}}$$
$$+\frac{(b^3c+2ab^2d-14a^2be+35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{13/3}}$$
$$-\frac{(b^3c+2ab^2d-14a^2be+35a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{13/3}}$$

output
```
(-3*a*f+b*e)*x/b^4+1/4*f*x^4/b^3-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4/
(b*x^3+a)^2+1/18*(-19*a^3*f+13*a^2*b*e-7*a*b^2*d+b^3*c)*x/a/b^4/(b*x^3+a)+
1/27*(35*a^3*f-14*a^2*b*e+2*a*b^2*d+b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b
^(13/3)-1/54*(35*a^3*f-14*a^2*b*e+2*a*b^2*d+b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(13/3)-1/27*(35*a^3*f-14*a^2*b*e+2*a*b^2*d+b^
3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(13/3)*3^
(1/2)
```

3.292.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{108\sqrt[3]{b}(be-3af)x+27b^{4/3}fx^4-\frac{18\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)x}{(a+bx^3)^2}+\frac{6\sqrt[3]{b}(b^3c-7ab^2d+13a^2be-19a^3f)x}{a(a+bx^3)}-\frac{4\sqrt{3}(b^3c+2ab^2d-1}{}}{}$$

input
```
Integrate[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(108*b^(1/3)*(b*e - 3*a*f)*x + 27*b^(4/3)*f*x^4 - (18*b^(1/3)*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*x)/(a + b*x^3)^2 + (6*b^(1/3)*(b^3*c - 7*a*b^2*d +
 13*a^2*b*e - 19*a^3*f)*x)/(a*(a + b*x^3)) - (4*Sqrt[3]*(b^3*c + 2*a*b^2*d
 - 14*a^2*b*e + 35*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(
5/3) + (4*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(1/3) + b^(1/3
)*x])/a^(5/3) - (2*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(108*b^(13/3))
```

3.292.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.90, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2367, 25, 2397, 27, 1741, 27, 913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{2367}$$

$$-\frac{\int -\frac{6ab^3fx^9+6ab^2(be-af)x^6+6ab(fa^2-bea+b^2d)x^3+a\left(-fa^3+bea^2-b^2da+b^3c\right)}{(bx^3+a)^2}dx}{6ab^4}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{6ab^3fx^9+6ab^2(be-af)x^6+6ab\left(fa^2-bea+b^2d\right)x^3+a\left(-fa^3+bea^2-b^2da+b^3c\right)}{\left(bx^3+a\right)^2}dx}{6ab^4}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2397$$

$$\frac{\frac{x\left(-19a^3f+13a^2be-7ab^2d+b^3c\right)}{3\left(a+bx^3\right)}-\frac{\int -\frac{2\left(9a^2b^5fx^6+9a^2b^4(be-2af)x^3+ab^3\left(8fa^3-5bea^2+2b^2da+b^3c\right)\right)}{bx^3+a}dx}{3ab^3}}{6ab^4}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{9a^2b^5fx^6+9a^2b^4(be-2af)x^3+ab^3\left(8fa^3-5bea^2+2b^2da+b^3c\right)}{bx^3+a}dx}{3ab^3}+\frac{x\left(-19a^3f+13a^2be-7ab^2d+b^3c\right)}{3\left(a+bx^3\right)}}{6ab^4}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1741$$

$$\frac{\frac{2\left(\frac{\int \frac{4ab^4\left(8fa^3-5bea^2+9b(be-3af)x^3a+2b^2da+b^3c\right)}{bx^3+a}dx}{4b}+\frac{9}{4}a^2b^4fx^4\right)}{3ab^3}+\frac{x\left(-19a^3f+13a^2be-7ab^2d+b^3c\right)}{3\left(a+bx^3\right)}}{6ab^4}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(ab^3\int \frac{8fa^3-5bea^2+9b(be-3af)x^3a+2b^2da+b^3c}{bx^3+a}dx+\frac{9}{4}a^2b^4fx^4\right)}{3ab^3}+\frac{x\left(-19a^3f+13a^2be-7ab^2d+b^3c\right)}{3\left(a+bx^3\right)}}{6ab^4}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{2\left(ab^3\left(\left(35a^3f-14a^2be+2ab^2d+b^3c\right)\int \frac{1}{bx^3+a}dx+9ax(be-3af)\right)+\frac{9}{4}a^2b^4fx^4\right)}{3ab^3}+\frac{x\left(-19a^3f+13a^2be-7ab^2d+b^3c\right)}{3\left(a+bx^3\right)}}{6ab^4}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 750$$

3.292. $\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{2\left(ab^3\left((35a^3f-14a^2be+2ab^2d+b^3c)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)+9ax(be-3af)\right)+\frac{9}{4}a^2b^4fx^4\right)}{3ab^3}+\frac{x(-19a^3f+13a^2be-}{3(a+bx^3}}{6ab^4}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$\big\downarrow$ 16

$$\frac{\frac{2\left(ab^3\left((35a^3f-14a^2be+2ab^2d+b^3c)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+9ax(be-3af)\right)+\frac{9}{4}a^2b^4fx^4\right)}{3ab^3}+\frac{x(-19a^3f+13a^2be}{3(a+bx}}{6ab^4}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$\big\downarrow$ 1142

$$\frac{\frac{2\left(ab^3\left((35a^3f-14a^2be+2ab^2d+b^3c)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+9ax(be-3af)\right.}{3ab^3}}{6ab^4}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$\big\downarrow$ 25

$$\frac{\frac{2\left(ab^3\left((35a^3f-14a^2be+2ab^2d+b^3c)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+9ax(be-3af)\right)}{3ab^3}}{6ab^4}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

3.292. $\int \frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

$\downarrow$ 27

$$\frac{2\left(ab^3\left((35a^3f-14a^2be+2ab^2d+b^3c)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+9ax(be-3af)\right.\right.}{3ab^3}$$
$$6ab^4$$
$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$\downarrow$ 1082

$$\frac{2\left(ab^3\left((35a^3f-14a^2be+2ab^2d+b^3c)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+9ax(be-3af\right.\right.}{3ab^3}$$
$$6ab^4$$
$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$\downarrow$ 217

$$\frac{2\left(ab^3\left((35a^3f-14a^2be+2ab^2d+b^3c)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+9ax(be-3af)\right)+\frac{9}{4}a^2b^4fx\right.}{3ab^3}$$
$$6ab^4$$
$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6b^4\left(a+bx^3\right)^2}$$

$\downarrow$ 1103

3.292. $\int\frac{x^3\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{x(-19a^3f+13a^2be-7ab^2d+b^3c)}{3(a+bx^3)}+\frac{2\left(\frac{9}{4}a^2b^4fx^4+ab^3\left(\left(\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)(35a^3f\right.\right.\right.}{3ab^3}}{6ab^4}$$

$$\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6b^4(a+bx^3)^2}$$

input
```
Int[(x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
-1/6*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(b^4*(a + b*x^3)^2) + (((b^3*
c - 7*a*b^2*d + 13*a^2*b*e - 19*a^3*f)*x)/(3*(a + b*x^3)) + (2*((9*a^2*b^4
*f*x^4)/4 + a*b^3*(9*a*(b*e - 3*a*f)*x + (b^3*c + 2*a*b^2*d - 14*a^2*b*e +
 35*a^3*f)*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*Arc
Tan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))))/(3*a*b^3))/(6*a*b^4
)
```

3.292.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

3.292. $\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

3.292.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.52

method	result
risch	$\frac{f\,x^{4}}{4b^{3}}-\frac{3xaf}{b^{4}}+\frac{ex}{b^{3}}+\frac{-\frac{b\left(19f\,a^{3}-13a^{2}be+7a\,b^{2}d-b^{3}c\right)x^{4}}{18a}+\left(-\frac{8}{9}f\,a^{3}+\frac{5}{9}a^{2}be-\frac{2}{9}a\,b^{2}d-\frac{1}{9}b^{3}c\right)x}{b^{4}\left(b\,x^{3}+a\right)^{2}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(35f\,a^{3}-\right.}{}}{27b}$
default	$-\frac{-\frac{1}{4}bf\,x^{4}+3afx-bex}{b^{4}}+\frac{\frac{-\frac{b\left(19f\,a^{3}-13a^{2}be+7a\,b^{2}d-b^{3}c\right)x^{4}}{18a}+\left(-\frac{8}{9}f\,a^{3}+\frac{5}{9}a^{2}be-\frac{2}{9}a\,b^{2}d-\frac{1}{9}b^{3}c\right)x}{\left(b\,x^{3}+a\right)^{2}}+\frac{\left(35f\,a^{3}-14a^{2}be+2a\,b^{2}d+b^{3}c\right)\left(\frac{\ln\left(x\right.}{3b}\right.}{}}{b^{4}}$

input
```
int(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*f*x^4/b^3-3/b^4*x*a*f+1/b^3*e*x+(-1/18*b*(19*a^3*f-13*a^2*b*e+7*a*b^2*
d-b^3*c)/a*x^4+(-8/9*f*a^3+5/9*a^2*b*e-2/9*a*b^2*d-1/9*b^3*c)*x)/b^4/(b*x^
3+a)^2+1/27/b^5/a*sum((35*a^3*f-14*a^2*b*e+2*a*b^2*d+b^3*c)/_R^2*ln(x-_R),
_R=RootOf(_Z^3*b+a))
```

3.292.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(264) = 528$.

Time = 0.31 (sec) , antiderivative size = 1213, normalized size of antiderivative = 3.95

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/108*(27*a^3*b^4*f*x^10 + 54*(2*a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 3*(2*a^2*
b^5*c - 14*a^3*b^4*d + 98*a^4*b^3*e - 245*a^5*b^2*f)*x^4 + 6*sqrt(1/3)*(a^
3*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e + 35*a^6*b*f + (a*b^6*c + 2*a^2*b^5*d
 - 14*a^3*b^4*e + 35*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 2*a^3*b^4*d - 14*a^4*
b^3*e + 35*a^5*b^2*f)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*
b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1
/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 2*((b^5*c + 2*a*b^4*d - 14*a
^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4*b*e + 35*a
^5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^3)*(a^2*b)^
(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*((b^5*c + 2*a*b
^4*d - 14*a^2*b^3*e + 35*a^3*b^2*f)*x^6 + a^2*b^3*c + 2*a^3*b^2*d - 14*a^4
*b*e + 35*a^5*f + 2*(a*b^4*c + 2*a^2*b^3*d - 14*a^3*b^2*e + 35*a^4*b*f)*x^
3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 12*(a^3*b^4*c + 2*a^4*b^3*d
- 14*a^5*b^2*e + 35*a^6*b*f)*x)/(a^3*b^7*x^6 + 2*a^4*b^6*x^3 + a^5*b^5), 1
/108*(27*a^3*b^4*f*x^10 + 54*(2*a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 3*(2*a^2*b^
5*c - 14*a^3*b^4*d + 98*a^4*b^3*e - 245*a^5*b^2*f)*x^4 + 12*sqrt(1/3)*(a^3
*b^4*c + 2*a^4*b^3*d - 14*a^5*b^2*e + 35*a^6*b*f + (a*b^6*c + 2*a^2*b^5*d
- 14*a^3*b^4*e + 35*a^4*b^3*f)*x^6 + 2*(a^2*b^5*c + 2*a^3*b^4*d - 14*a^4*b
^3*e + 35*a^5*b^2*f)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b
)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*((b^5*c + 2...
```

3.292.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= \frac{(b^4c-7\,ab^3d+13\,a^2b^2e-19\,a^3bf)x^4-2\,(ab^3c+2\,a^2b^2d-5\,a^3be+8\,a^4f)x}{18\,(ab^6x^6+2\,a^2b^5x^3+a^3b^4)} \\ &+ \frac{bfx^4+4\,(be-3\,af)x}{4\,b^4} + \frac{\sqrt{3}(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &- \frac{(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &+ \frac{(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

input
```
integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*((b^4*c - 7*a*b^3*d + 13*a^2*b^2*e - 19*a^3*b*f)*x^4 - 2*(a*b^3*c + 2
*a^2*b^2*d - 5*a^3*b*e + 8*a^4*f)*x)/(a*b^6*x^6 + 2*a^2*b^5*x^3 + a^3*b^4)
 + 1/4*(b*f*x^4 + 4*(b*e - 3*a*f)*x)/b^4 + 1/27*sqrt(3)*(b^3*c + 2*a*b^2*d
 - 14*a^2*b*e + 35*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(a*b^5*(a/b)^(2/3)) - 1/54*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(2/3)) + 1/27*(b^3*c
+ 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*log(x + (a/b)^(1/3))/(a*b^5*(a/b)^(2/
3))
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.02

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= -\frac{\sqrt{3}(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab^3}$$
$$-\frac{(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab^3}$$
$$-\frac{(b^3c+2\,ab^2d-14\,a^2be+35\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^4}$$
$$+\frac{b^4cx^4-7\,ab^3dx^4+13\,a^2b^2ex^4-19\,a^3bfx^4-2\,ab^3cx-4\,a^2b^2dx+10\,a^3bex-16\,a^4fx}{18\left(bx^3+a\right)^2ab^4}$$
$$+\frac{b^9fx^4+4\,b^9ex-12\,ab^8fx}{4\,b^{12}}$$

input
```
integrate(x^3*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^3) - 1/54*(b^3*c
 + 2*a*b^2*d - 14*a^2*b*e + 35*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/((-a*b^2)^(2/3)*a*b^3) - 1/27*(b^3*c + 2*a*b^2*d - 14*a^2*b*e + 35*a^
3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4) + 1/18*(b^4*c*x^4 -
 7*a*b^3*d*x^4 + 13*a^2*b^2*e*x^4 - 19*a^3*b*f*x^4 - 2*a*b^3*c*x - 4*a^2*b
^2*d*x + 10*a^3*b*e*x - 16*a^4*f*x)/((b*x^3 + a)^2*a*b^4) + 1/4*(b^9*f*x^4
 + 4*b^9*e*x - 12*a*b^8*f*x)/b^12
```

3.292.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.94

$$\int \frac{x^3(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= x\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)-\frac{x\left(\frac{8\,f\,a^3}{9}-\frac{5\,e\,a^2\,b}{9}+\frac{2\,d\,a\,b^2}{9}+\frac{c\,b^3}{9}\right)-\frac{x^4\,\left(-19\,f\,a^3\,b+13\,e\,a^2\,b^2-7\,d\,a\,b^3+c\,b^4\right)}{18\,a}}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}$$
$$+\frac{f\,x^4}{4\,b^3}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(35\,f\,a^3-14\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{27\,a^{5/3}\,b^{13/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(35\,f\,a^3-14\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{27\,a^{5/3}\,b^{13/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(35\,f\,a^3-14\,e\,a^2\,b+2\,d\,a\,b^2+c\,b^3\right)}{27\,a^{5/3}\,b^{13/3}}$$

input
```
int((x^3*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
x*(e/b^3 - (3*a*f)/b^4) - (x*((b^3*c)/9 + (8*a^3*f)/9 + (2*a*b^2*d)/9 - (5
*a^2*b*e)/9) - (x^4*(b^4*c + 13*a^2*b^2*e - 7*a*b^3*d - 19*a^3*b*f))/(18*a
))/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + (f*x^4)/(4*b^3) + (log(b^(1/3)*x +
a^(1/3))*(b^3*c + 35*a^3*f + 2*a*b^2*d - 14*a^2*b*e))/(27*a^(5/3)*b^(13/3)
) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(b^3*c + 35*a^3*f + 2*a*b^2*d - 14*a^2*b*e))/(27*a^(5/3)*b^(13/3)) - (lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(b^3*
c + 35*a^3*f + 2*a*b^2*d - 14*a^2*b*e))/(27*a^(5/3)*b^(13/3))
```

3.293 $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

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3.293.1 Optimal result

Integrand size = 28, antiderivative size = 301

$$\begin{aligned}\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx &= \frac{fx^2}{2b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6ab^3\,(a+bx^3)^2}+\frac{(2b^3c+ab^2d-4a^2be+7a^3f)\,x^2}{9a^2b^3\,(a+bx^3)}\\&-\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{11/3}}\\&-\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{11/3}}\\&+\frac{(2b^3c+ab^2d+5a^2be-20a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{11/3}}\end{aligned}$$

output
```
1/2*f*x^2/b^3+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a/b^3/(b*x^3+a)^2+1/9
*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)*x^2/a^2/b^3/(b*x^3+a)-1/27*(-20*a^3*f
+5*a^2*b*e+a*b^2*d+2*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(11/3)+1/54*(-
20*a^3*f+5*a^2*b*e+a*b^2*d+2*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/a^(7/3)/b^(11/3)-1/27*(-20*a^3*f+5*a^2*b*e+a*b^2*d+2*b^3*c)*arctan(1/3
*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(11/3)*3^(1/2)
```

3.293.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.94

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{27b^{2/3}fx^2 + \frac{9b^{2/3}(b^3c-ab^2d+a^2be-a^3f)x^2}{a(a+bx^3)^2} + \frac{6b^{2/3}(2b^3c+ab^2d-4a^2be+7a^3f)x^2}{a^2(a+bx^3)} - \frac{2\sqrt{3}(2b^3c+ab^2d+5a^2be-20a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{a^{7/3}}}{54b^{11/3}}$$

input
```
Integrate[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

output
```
(27*b^(2/3)*f*x^2 + (9*b^(2/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a
*(a + b*x^3)^2) + (6*b^(2/3)*(2*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2
)/(a^2*(a + b*x^3)) - (2*Sqrt[3]*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(7/3) - (2*(2*b^3*c + a*b
^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/a^(7/3) + ((2*b^3*c
 + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/a^(7/3))/(54*b^(11/3))
```

3.293.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.87, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {2367, 27, 2028, 1806, 25, 27, 959, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{ 2367}$$

$$\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6ab^3(a+bx^3)^2} - \frac{\int -\frac{2(3ab^3fx^7+3ab^2(be-af)x^4+b(fa^3-bea^2+b^2da+2b^3c)x)}{(bx^3+a)^2}dx}{6ab^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3ab^3fx^7+3ab^2(be-af)x^4+b(fa^3-bea^2+b^2da+2b^3c)x}{(bx^3+a)^2}dx}{3ab^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{2028}$$

$$\frac{\int \frac{x\left(3ab^3fx^6+3ab^2(be-af)x^3+b\left(fa^3-bea^2+b^2da+2b^3c\right)\right)}{(bx^3+a)^2}dx}{3ab^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1806}$$

$$\frac{\frac{bx^2(7a^3f-4a^2be+ab^2d+2b^3c)}{3a(a+bx^3)}-\frac{\int -\frac{b^3x\left(\frac{2cb^3}{a}+db^2+9afx^3b+5aeb-11a^2f\right)}{bx^3+a}dx}{3b^2}}{3ab^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b^3x\left(\frac{2cb^3}{a}+db^2+9afx^3b+5aeb-11a^2f\right)}{bx^3+a}dx}{3b^2}+\frac{bx^2(7a^3f-4a^2be+ab^2d+2b^3c)}{3a(a+bx^3)}}{3ab^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}b\int \frac{x\left(\frac{2cb^3}{a}+db^2+9afx^3b+5aeb-11a^2f\right)}{bx^3+a}dx+\frac{bx^2(7a^3f-4a^2be+ab^2d+2b^3c)}{3a(a+bx^3)}}{3ab^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{959}$$

$$\frac{\frac{1}{3}b\left(\left(-20a^2f+\frac{2b^3c}{a}+5abe+b^2d\right)\int \frac{x}{bx^3+a}dx+\frac{9}{2}afx^2\right)+\frac{bx^2(7a^3f-4a^2be+ab^2d+2b^3c)}{3a(a+bx^3)}}{3ab^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{821}$$

$$\frac{\frac{1}{3}b\left(\left(-20a^2f+\frac{2b^3c}{a}+5abe+b^2d\right)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)+\frac{9}{2}afx^2\right)+\frac{bx^2(7a^3f-4a^2be+ab^2d+}{3a(a+bx^3)}}{3ab^4}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{16}$$

3.293. $\int \frac{x\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{1}{3}b\left(\left(-20a^2f+\frac{2b^3c}{a}+5abe+b^2d\right)\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)+\frac{9}{2}afx^2\right)+\frac{bx^2(7a^3f-4a^2be+ab^2d}{3a(a+bx^3)}}{3ab^4}$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$\downarrow$ 1142

$$\frac{\frac{1}{3}b\left(\left(-20a^2f+\frac{2b^3c}{a}+5abe+b^2d\right)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)+}{3ab^4}$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$\downarrow$ 25

$$\frac{\frac{1}{3}b\left(\left(-20a^2f+\frac{2b^3c}{a}+5abe+b^2d\right)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)+\frac{9}{2}}{3ab^4}$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{1}{3}b\left(\left(-20a^2f+\frac{2b^3c}{a}+5abe+b^2d\right)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)+}{3ab^4}$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$\downarrow$ 1082

3.293. $\int\frac{x\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{1}{3}b\left(\left(-20a^2f+\frac{2b^3c}{a}+5abe+b^2d\right)\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)\right.}{3ab^4}$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$\downarrow$ 217

$$\frac{\frac{1}{3}b\left(\left(-20a^2f+\frac{2b^3c}{a}+5abe+b^2d\right)\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)+\frac{9}{2}afx^2\right.}{3ab^4}$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$\downarrow$ 1103

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}+$$

$$\frac{\frac{1}{3}b\left(\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)\left(-20a^2f+\frac{2b^3c}{a}+5abe+b^2d\right)+\frac{9}{2}afx^2\right.}{3ab^4}$$

input
```
Int[(x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x]
```

3.293. $\int\frac{x\left(c+dx^3+ex^6+fx^9\right)}{\left(a+bx^3\right)^3}\,dx$

output
```
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a*b^3*(a + b*x^3)^2) + ((b*(2
*b^3*c + a*b^2*d - 4*a^2*b*e + 7*a^3*f)*x^2)/(3*a*(a + b*x^3)) + (b*((9*a*
f*x^2)/2 + ((2*b^3*c)/a + b^2*d + 5*a*b*e - 20*a^2*f)*(-1/3*Log[a^(1/3) +
b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2
*b^(1/3)))/(3*a^(1/3)*b^(1/3)))))/3)/(3*a*b^4)
```

3.293.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

3.293. $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3} dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1806
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
 Mod[m, n])/n)*(q + 1))), x] + Simp[1/(n*e^(2*p + (m - Mod[m, n])/n)*(q + 1
))   Int[x^Mod[m, n]*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n
))*(n*e^(2*p + (m - Mod[m, n])/n)*(q + 1)*x^(m - Mod[m, n])*(a + b*x^n + c*
x^(2*n))^p - (-d)^((m - Mod[m, n])/n - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d*(Mod
[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IG
tQ[p, 0] && ILtQ[q, -1] && IGtQ[m, 0]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

3.293. $\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$

3.293.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.50

method	result
risch	$\frac{f x^2}{2b^3}+\frac{\frac{b(7f a^3-4a^2be+a b^2d+2b^3c)x^5}{9a^2}+\frac{(11f a^3-5a^2be-a b^2d+7b^3c)x^2}{18a}}{b^3(b x^3+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(20f a^3-5a^2be-a b^2d-2b^3c)\ln(x-\_R)}{\_R}}{27b^4a^2}$
default	$\frac{f x^2}{2b^3}-\frac{\frac{-\frac{b(7f a^3-4a^2be+a b^2d+2b^3c)x^5}{9a^2}-\frac{(11f a^3-5a^2be-a b^2d+7b^3c)x^2}{18a}}{(b x^3+a)^2}+\frac{(20f a^3-5a^2be-a b^2d-2b^3c)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)\cdots\right.}{6b\left(\cdots\right.}\cdots\right.}{9a^2}}{b^3}$

input
```
int(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*f*x^2/b^3+(1/9*b*(7*a^3*f-4*a^2*b*e+a*b^2*d+2*b^3*c)/a^2*x^5+1/18*(11*
a^3*f-5*a^2*b*e-a*b^2*d+7*b^3*c)/a*x^2)/b^3/(b*x^3+a)^2-1/27/b^4/a^2*sum((
20*a^3*f-5*a^2*b*e-a*b^2*d-2*b^3*c)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.293.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(258) = 516.

Time = 0.29 (sec) , antiderivative size = 1158, normalized size of antiderivative = 3.85

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(27*a^3*b^4*f*x^8 + 6*(2*a*b^6*c + a^2*b^5*d - 4*a^3*b^4*e + 16*a^4*
b^3*f)*x^5 + 3*(7*a^2*b^5*c - a^3*b^4*d - 5*a^4*b^3*e + 20*a^5*b^2*f)*x^2
- 3*sqrt(1/3)*(2*a^3*b^4*c + a^4*b^3*d + 5*a^5*b^2*e - 20*a^6*b*f + (2*a*b
^6*c + a^2*b^5*d + 5*a^3*b^4*e - 20*a^4*b^3*f)*x^6 + 2*(2*a^2*b^5*c + a^3*
b^4*d + 5*a^4*b^3*e - 20*a^5*b^2*f)*x^3)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b^2
*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*s
qrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((2*b^5*c + a*b^
4*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d + 5*a^4*b*
e - 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f)*x^3)*(
a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 2*((2*b^5*
c + a*b^4*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*b^2*d +
5*a^4*b*e - 20*a^5*f + 2*(2*a*b^4*c + a^2*b^3*d + 5*a^3*b^2*e - 20*a^4*b*f
)*x^3)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^7*x^6 + 2*a^4*b^6*x^
3 + a^5*b^5), 1/54*(27*a^3*b^4*f*x^8 + 6*(2*a*b^6*c + a^2*b^5*d - 4*a^3*b^
4*e + 16*a^4*b^3*f)*x^5 + 3*(7*a^2*b^5*c - a^3*b^4*d - 5*a^4*b^3*e + 20*a^
5*b^2*f)*x^2 - 6*sqrt(1/3)*(2*a^3*b^4*c + a^4*b^3*d + 5*a^5*b^2*e - 20*a^6
*b*f + (2*a*b^6*c + a^2*b^5*d + 5*a^3*b^4*e - 20*a^4*b^3*f)*x^6 + 2*(2*a^2
*b^5*c + a^3*b^4*d + 5*a^4*b^3*e - 20*a^5*b^2*f)*x^3)*sqrt((a*b^2)^(1/3)/a
)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + ((2
*b^5*c + a*b^4*d + 5*a^2*b^3*e - 20*a^3*b^2*f)*x^6 + 2*a^2*b^3*c + a^3*...
```

3.293.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.98

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$=\frac{2\,(2\,b^4c+ab^3d-4\,a^2b^2e+7\,a^3bf)x^5+(7\,ab^3c-a^2b^2d-5\,a^3be+11\,a^4f)x^2}{18\,(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3)}$$
$$+\frac{fx^2}{2\,b^3}+\frac{\sqrt{3}(2\,b^3c+ab^2d+5\,a^2be-20\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{(2\,b^3c+ab^2d+5\,a^2be-20\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{(2\,b^3c+ab^2d+5\,a^2be-20\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(2*(2*b^4*c + a*b^3*d - 4*a^2*b^2*e + 7*a^3*b*f)*x^5 + (7*a*b^3*c - a
^2*b^2*d - 5*a^3*b*e + 11*a^4*f)*x^2)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b
^3) + 1/2*f*x^2/b^3 + 1/27*sqrt(3)*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3
*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4*(a/b)^(1/
3)) + 1/54*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*log(x^2 - x*(a/b)^(1
/3) + (a/b)^(2/3))/(a^2*b^4*(a/b)^(1/3)) - 1/27*(2*b^3*c + a*b^2*d + 5*a^2
*b*e - 20*a^3*f)*log(x + (a/b)^(1/3))/(a^2*b^4*(a/b)^(1/3))
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.11

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$

$$= \frac{fx^2}{2\,b^3} + \frac{\sqrt{3}(2\,b^3c+ab^2d+5\,a^2be-20\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{1}{3}}\,a^2b^3}$$
$$- \frac{(2\,b^3c+ab^2d+5\,a^2be-20\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{1}{3}}\,a^2b^3}$$
$$- \frac{\left(2\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-20\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^3}$$
$$+ \frac{4\,b^4cx^5+2\,ab^3dx^5-8\,a^2b^2ex^5+14\,a^3bfx^5+7\,ab^3cx^2-a^2b^2dx^2-5\,a^3bex^2+11\,a^4fx^2}{18\,(bx^3+a)^2a^2b^3}$$

input
```
integrate(x*(f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/2*f*x^2/b^3 + 1/27*sqrt(3)*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*ar
ctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b^
3) - 1/54*(2*b^3*c + a*b^2*d + 5*a^2*b*e - 20*a^3*f)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b^3) - 1/27*(2*b^3*c*(-a/b)^(1/3)
+ a*b^2*d*(-a/b)^(1/3) + 5*a^2*b*e*(-a/b)^(1/3) - 20*a^3*f*(-a/b)^(1/3))*(
-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3) + 1/18*(4*b^4*c*x^5 + 2*a
*b^3*d*x^5 - 8*a^2*b^2*e*x^5 + 14*a^3*b*f*x^5 + 7*a*b^3*c*x^2 - a^2*b^2*d*
x^2 - 5*a^3*b*e*x^2 + 11*a^4*f*x^2)/((b*x^3 + a)^2*a^2*b^3)
```

3.293.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.93

$$\int \frac{x(c+dx^3+ex^6+fx^9)}{(a+bx^3)^3}\,dx$$
$$= \frac{\frac{x^2\,(11\,f\,a^3-5\,e\,a^2\,b-d\,a\,b^2+7\,c\,b^3)}{18\,a}+\frac{x^5\,(7\,f\,a^3\,b-4\,e\,a^2\,b^2+d\,a\,b^3+2\,c\,b^4)}{9\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{f\,x^2}{2\,b^3}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-20\,f\,a^3+5\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{7/3}\,b^{11/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-20\,f\,a^3+5\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{7/3}\,b^{11/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-20\,f\,a^3+5\,e\,a^2\,b+d\,a\,b^2+2\,c\,b^3\right)}{27\,a^{7/3}\,b^{11/3}}$$

input
```
int((x*(c + d*x^3 + e*x^6 + f*x^9))/(a + b*x^3)^3,x)
```

output
```
((x^2*(7*b^3*c + 11*a^3*f - a*b^2*d - 5*a^2*b*e))/(18*a) + (x^5*(2*b^4*c -
 4*a^2*b^2*e + a*b^3*d + 7*a^3*b*f))/(9*a^2))/(a^2*b^3 + b^5*x^6 + 2*a*b^4
*x^3) + (f*x^2)/(2*b^3) - (log(b^(1/3)*x + a^(1/3))*(2*b^3*c - 20*a^3*f +
a*b^2*d + 5*a^2*b*e))/(27*a^(7/3)*b^(11/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*
b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*b^3*c - 20*a^3*f + a*b^2*d
+ 5*a^2*b*e))/(27*a^(7/3)*b^(11/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*
x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*b^3*c - 20*a^3*f + a*b^2*d + 5*a^2*
b*e))/(27*a^(7/3)*b^(11/3))
```

3.294 $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3} dx$

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3.294.1 Optimal result

Integrand size = 27, antiderivative size = 292

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx &= \frac{fx}{b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6ab^3(a+bx^3)^2} \\ &+ \frac{(5b^3c+ab^2d-7a^2be+13a^3f)\,x}{18a^2b^3(a+bx^3)} \\ &- \frac{(5b^3c+ab^2d+2a^2be-14a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{10/3}} \\ &+ \frac{(5b^3c+ab^2d+2a^2be-14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{10/3}} \\ &- \frac{(5b^3c+ab^2d+2a^2be-14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{10/3}}\end{aligned}$$

output
```
f*x/b^3+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a/b^3/(b*x^3+a)^2+1/18*(13*a^
3*f-7*a^2*b*e+a*b^2*d+5*b^3*c)*x/a^2/b^3/(b*x^3+a)+1/27*(-14*a^3*f+2*a^2*b
*e+a*b^2*d+5*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(10/3)-1/54*(-14*a^3*f
+2*a^2*b*e+a*b^2*d+5*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8
/3)/b^(10/3)-1/27*(-14*a^3*f+2*a^2*b*e+a*b^2*d+5*b^3*c)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(10/3)*3^(1/2)
```

3.294.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx$$

$$= \frac{54\sqrt[3]{b}fx + \frac{9\sqrt[3]{b}(b^3c-ab^2d+a^2be-a^3f)x}{a(a+bx^3)^2} + \frac{3\sqrt[3]{b}(5b^3c+ab^2d-7a^2be+13a^3f)x}{a^2(a+bx^3)} - \frac{2\sqrt{3}(5b^3c+ab^2d+2a^2be-14a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}}}{54b^{10/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```

output
```
(54*b^(1/3)*f*x + (9*b^(1/3)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a*(a
+ b*x^3)^2) + (3*b^(1/3)*(5*b^3*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(a^
2*(a + b*x^3)) - (2*Sqrt[3]*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*Arc
Tan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) + (2*(5*b^3*c + a*b^2*d
+ 2*a^2*b*e - 14*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - ((5*b^3*c + a*
b^2*d + 2*a^2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/a^(8/3))/(54*b^(10/3))
```

3.294.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {2397, 25, 1739, 27, 913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \; 2397$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}-\frac{\int -\frac{6ab^2fx^6+6ab(be-af)x^3+5b^3c+ab^2d-a^2be+a^3f}{(bx^3+a)^2}dx}{6ab^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{6ab^2fx^6+6ab(be-af)x^3+5b^3c+ab^2d-a^2be+a^3f}{(bx^3+a)^2}dx}{6ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 1739$$

$$\frac{\frac{x(13a^3f-7a^2be+ab^2d+5b^3c)}{3a(a+bx^3)}-\frac{\int -\frac{2b^2\left(-5fa^3+9bfx^3a^2+2bea^2+b^2da+5b^3c\right)}{bx^3+a}dx}{3ab^2}}{6ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2\int \frac{-5fa^3+9bfx^3a^2+2bea^2+b^2da+5b^3c}{bx^3+a}dx}{3a}+\frac{x(13a^3f-7a^2be+ab^2d+5b^3c)}{3a(a+bx^3)}}{6ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 913$$

$$\frac{\frac{2\left((-14a^3f+2a^2be+ab^2d+5b^3c)\int \frac{1}{bx^3+a}dx+9a^2fx\right)}{3a}+\frac{x(13a^3f-7a^2be+ab^2d+5b^3c)}{3a(a+bx^3)}}{6ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 750$$

$$\frac{\frac{2\left((-14a^3f+2a^2be+ab^2d+5b^3c)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)+9a^2fx\right)}{3a}+\frac{x(13a^3f-7a^2be+ab^2d+5b^3c)}{3a(a+bx^3)}}{6ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{2\left((-14a^3f+2a^2be+ab^2d+5b^3c)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+9a^2fx\right)}{3a}+\frac{x(13a^3f-7a^2be+ab^2d+5b^3c)}{3a(a+bx^3)}}{6ab^3}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 1142$$

3.294. $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}dx$

$$\frac{\frac{2\left((-14a^3f+2a^2be+ab^2d+5b^3c)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}(\sqrt[3]{a}-2\sqrt[3]{b}x)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3a^{2/3}\sqrt[3]{b}}\right)+9a^2fx\right)}{3a}+\frac{x(13a}{}}{6ab^3}$$

$$\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6ab^3(a+bx^3)^2}$$

$\downarrow$ 25

$$\frac{\frac{2\left((-14a^3f+2a^2be+ab^2d+5b^3c)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}(\sqrt[3]{a}-2\sqrt[3]{b}x)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3a^{2/3}\sqrt[3]{b}}\right)+9a^2fx\right)}{3a}+\frac{x(13a^3}{}}{6ab^3}$$

$$\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6ab^3(a+bx^3)^2}$$

$\downarrow$ 27

$$\frac{\frac{2\left((-14a^3f+2a^2be+ab^2d+5b^3c)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3a^{2/3}\sqrt[3]{b}}\right)+9a^2fx\right)}{3a}+\frac{x(13a}{}}{6ab^3}$$

$$\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6ab^3(a+bx^3)^2}$$

$\downarrow$ 1082

3.294. $\int\frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}dx$

$$\frac{\frac{2\left(\left(-14a^3f+2a^2be+ab^2d+5b^3c\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+9a^2fx\right)}{3a}+\frac{x(13}{}}{6ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{2\left(\left(-14a^3f+2a^2be+ab^2d+5b^3c\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)+9a^2fx\right)}{3a}+\frac{x\left(13a^3f-7a^2be+\right.}{3a(a+bx}}{6ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6ab^3\left(a+bx^3\right)^2}+$$

$$\frac{\frac{x\left(13a^3f-7a^2be+ab^2d+5b^3c\right)}{3a\left(a+bx^3\right)}+\frac{2\left(9a^2fx+\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)\left(-14a^3f+2a^2be+a\right.\right.}{3a}}{6ab^3}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x]
```

3.294. $\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^3}\,dx$

output
```
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a*b^3*(a + b*x^3)^2) + (((5*b^3
*c + a*b^2*d - 7*a^2*b*e + 13*a^3*f)*x)/(3*a*(a + b*x^3)) + (2*(9*a^2*f*x
+ (5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*(Log[a^(1/3) + b^(1/3)*x]/(3*
a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
)/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(
3*a^(2/3)))))/(3*a))/(6*a*b^3)
```

3.294.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.294. $\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3} dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

3.294.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.50

method	result
risch	$\frac{fx}{b^3}+\frac{\frac{b\left(13fa^3-7a^2be+ab^2d+5b^3c\right)x^4}{18a^2}+\frac{\left(5fa^3-2a^2be-ab^2d+4b^3c\right)x}{9a}}{b^3(bx^3+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(14fa^3-2a^2be-ab^2d-5b^3c\right)\ln\left(x-\_R\right)}{\_R^2}}{27b^4a^2}$
default	$\frac{fx}{b^3}-\frac{\frac{-\frac{b\left(13fa^3-7a^2be+ab^2d+5b^3c\right)x^4}{18a^2}-\frac{\left(5fa^3-2a^2be-ab^2d+4b^3c\right)x}{9a}}{\left(bx^3+a\right)^2}+\frac{\left(14fa^3-2a^2be-ab^2d-5b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{9a^2}}{b^3}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
f*x/b^3+(1/18*b*(13*a^3*f-7*a^2*b*e+a*b^2*d+5*b^3*c)/a^2*x^4+1/9*(5*a^3*f-
2*a^2*b*e-a*b^2*d+4*b^3*c)/a*x)/b^3/(b*x^3+a)^2-1/27/b^4/a^2*sum((14*a^3*f
-2*a^2*b*e-a*b^2*d-5*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.294.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(251) = 502$.

Time = 0.28 (sec) , antiderivative size = 1184, normalized size of antiderivative = 4.05

$$\int \frac{c+dx^3+ex^6+fx^9}{\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(54*a^4*b^3*f*x^7 + 3*(5*a^2*b^5*c + a^3*b^4*d - 7*a^4*b^3*e + 49*a^
5*b^2*f)*x^4 - 3*sqrt(1/3)*(5*a^3*b^4*c + a^4*b^3*d + 2*a^5*b^2*e - 14*a^6
*b*f + (5*a*b^6*c + a^2*b^5*d + 2*a^3*b^4*e - 14*a^4*b^3*f)*x^6 + 2*(5*a^2
*b^5*c + a^3*b^4*d + 2*a^4*b^3*e - 14*a^5*b^2*f)*x^3)*sqrt((-a^2*b)^(1/3)/
b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 +
(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a))
- ((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a^
3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a^3*b^2*e -
14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^
(1/3)*a) + 2*((5*b^5*c + a*b^4*d + 2*a^2*b^3*e - 14*a^3*b^2*f)*x^6 + 5*a^2
*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f + 2*(5*a*b^4*c + a^2*b^3*d + 2*a
^3*b^2*e - 14*a^4*b*f)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 6
*(4*a^3*b^4*c - a^4*b^3*d - 2*a^5*b^2*e + 14*a^6*b*f)*x)/(a^4*b^6*x^6 + 2*
a^5*b^5*x^3 + a^6*b^4), 1/54*(54*a^4*b^3*f*x^7 + 3*(5*a^2*b^5*c + a^3*b^4*
d - 7*a^4*b^3*e + 49*a^5*b^2*f)*x^4 + 6*sqrt(1/3)*(5*a^3*b^4*c + a^4*b^3*d
 + 2*a^5*b^2*e - 14*a^6*b*f + (5*a*b^6*c + a^2*b^5*d + 2*a^3*b^4*e - 14*a^
4*b^3*f)*x^6 + 2*(5*a^2*b^5*c + a^3*b^4*d + 2*a^4*b^3*e - 14*a^5*b^2*f)*x^
3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)
^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - ((5*b^5*c + a*b^4*d + 2*a^2*b^3*e
 - 14*a^3*b^2*f)*x^6 + 5*a^2*b^3*c + a^3*b^2*d + 2*a^4*b*e - 14*a^5*f +...
```

3.294.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{(a + bx^3)^3} \, dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00

$$\int \frac{c + dx^3 + ex^6 + fx^9}{(a + bx^3)^3} \, dx$$
$$= \frac{(5\,b^4c + ab^3d - 7\,a^2b^2e + 13\,a^3bf)x^4 + 2\,(4\,ab^3c - a^2b^2d - 2\,a^3be + 5\,a^4f)x}{18\,(a^2b^5x^6 + 2\,a^3b^4x^3 + a^4b^3)} + \frac{fx}{b^3} + \frac{\sqrt{3}(5\,b^3c + ab^2d + 2\,a^2be - 14\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(5\,b^3c + ab^2d + 2\,a^2be - 14\,a^3f)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(5\,b^3c + ab^2d + 2\,a^2be - 14\,a^3f)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*((5*b^4*c + a*b^3*d - 7*a^2*b^2*e + 13*a^3*b*f)*x^4 + 2*(4*a*b^3*c -
a^2*b^2*d - 2*a^3*b*e + 5*a^4*f)*x)/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3
) + f*x/b^3 + 1/27*sqrt(3)*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^4*(a/b)^(2/3)) - 1/
54*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(a^2*b^4*(a/b)^(2/3)) + 1/27*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 1
4*a^3*f)*log(x + (a/b)^(1/3))/(a^2*b^4*(a/b)^(2/3))
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx$$
$$= \frac{fx}{b^3} - \frac{\sqrt{3}(5\,b^3c+ab^2d+2\,a^2be-14\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^2b^2}$$
$$- \frac{(5\,b^3c+ab^2d+2\,a^2be-14\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^2b^2}$$
$$- \frac{(5\,b^3c+ab^2d+2\,a^2be-14\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^3}$$
$$+ \frac{5\,b^4cx^4+ab^3dx^4-7\,a^2b^2ex^4+13\,a^3bfx^4+8\,ab^3cx-2\,a^2b^2dx-4\,a^3bex+10\,a^4fx}{18\,(bx^3+a)^2a^2b^3}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
f*x/b^3 - 1/27*sqrt(3)*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*arctan(1
/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b^2) - 1
/54*(5*b^3*c + a*b^2*d + 2*a^2*b*e - 14*a^3*f)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b^2) - 1/27*(5*b^3*c + a*b^2*d + 2*a^2*b
*e - 14*a^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3) + 1/18*(5
*b^4*c*x^4 + a*b^3*d*x^4 - 7*a^2*b^2*e*x^4 + 13*a^3*b*f*x^4 + 8*a*b^3*c*x
- 2*a^2*b^2*d*x - 4*a^3*b*e*x + 10*a^4*f*x)/((b*x^3 + a)^2*a^2*b^3)
```

3.294.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{(a+bx^3)^3}\,dx$$
$$= \frac{\frac{x\,(5\,f\,a^3-2\,e\,a^2\,b-d\,a\,b^2+4\,c\,b^3)}{9\,a}+\frac{x^4\,(13\,f\,a^3\,b-7\,e\,a^2\,b^2+d\,a\,b^3+5\,c\,b^4)}{18\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{f\,x}{b^3}$$
$$+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(-14\,f\,a^3+2\,e\,a^2\,b+d\,a\,b^2+5\,c\,b^3)}{27\,a^{8/3}\,b^{10/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(-14\,f\,a^3+2\,e\,a^2\,b+d\,a\,b^2+5\,c\,b^3)}{27\,a^{8/3}\,b^{10/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(-14\,f\,a^3+2\,e\,a^2\,b+d\,a\,b^2+5\,c\,b^3)}{27\,a^{8/3}\,b^{10/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(a + b*x^3)^3,x)
```

output
```
((x*(4*b^3*c + 5*a^3*f - a*b^2*d - 2*a^2*b*e))/(9*a) + (x^4*(5*b^4*c - 7*a
^2*b^2*e + a*b^3*d + 13*a^3*b*f))/(18*a^2))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x
^3) + (f*x)/b^3 + (log(b^(1/3)*x + a^(1/3))*(5*b^3*c - 14*a^3*f + a*b^2*d
+ 2*a^2*b*e))/(27*a^(8/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*
x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*b^3*c - 14*a^3*f + a*b^2*d + 2*a^2*
b*e))/(27*a^(8/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1
/3))*((3^(1/2)*1i)/2 + 1/2)*(5*b^3*c - 14*a^3*f + a*b^2*d + 2*a^2*b*e))/(2
7*a^(8/3)*b^(10/3))
```

3.295 $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx$

3.295.1 Optimal result 2250
3.295.2 Mathematica [A] (verified) 2251
3.295.3 Rubi [A] (verified) 2251
3.295.4 Maple [A] (verified) 2257
3.295.5 Fricas [B] (verification not implemented) 2257
3.295.6 Sympy [F(-1)] 2258
3.295.7 Maxima [A] (verification not implemented) 2259
3.295.8 Giac [A] (verification not implemented) 2260
3.295.9 Mupad [B] (verification not implemented) 2261

3.295.1 Optimal result

Integrand size = 30, antiderivative size = 303

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx = -\frac{c}{a^3x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^2b^2(a+bx^3)^2} -\frac{(5b^3c-2ab^2d-a^2be+4a^3f)\,x^2}{9a^3b^2(a+bx^3)} +\frac{(14b^3c-2ab^2d-a^2be-5a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{8/3}} +\frac{(14b^3c-2ab^2d-a^2be-5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{8/3}} -\frac{(14b^3c-2ab^2d-a^2be-5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}b^{8/3}}$$

output
```
-c/a^3/x-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^2/b^2/(b*x^3+a)^2-1/9*(4
*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)*x^2/a^3/b^2/(b*x^3+a)+1/27*(-5*a^3*f-a^2
*b*e-2*a*b^2*d+14*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(8/3)-1/54*(-5*a
^3*f-a^2*b*e-2*a*b^2*d+14*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(10/3)/b^(8/3)+1/27*(-5*a^3*f-a^2*b*e-2*a*b^2*d+14*b^3*c)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(8/3)*3^(1/2)
```

3.295.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{54\sqrt[3]{a}c}{x} + \frac{9a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)x^2}{b^2(a+bx^3)^2} - \frac{6\sqrt[3]{a}(5b^3c-2ab^2d-a^2be+4a^3f)x^2}{b^2(a+bx^3)} + \frac{2\sqrt{3}(14b^3c-2ab^2d-a^2be-5a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{b^{8/3}}}{54a^{10/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```

output
```
((-54*a^(1/3)*c)/x + (9*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2
)/(b^2*(a + b*x^3)^2) - (6*a^(1/3)*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*
f)*x^2)/(b^2*(a + b*x^3)) + (2*Sqrt[3]*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5
*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(8/3) - (2*(-14*b^3
*c + 2*a*b^2*d + a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(8/3) + ((
-14*b^3*c + 2*a*b^2*d + a^2*b*e + 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/b^(8/3))/(54*a^(10/3))
```

3.295.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.89, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2368, 27, 1808, 25, 27, 955, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx$$

$$\Big\downarrow \; 2368$$

$$-\frac{\int -\frac{2\left(3ab^2fx^6-b\left(\frac{2cb^3}{a}-2db^2-aeb+a^2f\right)x^3+3b^3c\right)}{x^2(bx^3+a)^2}dx}{6ab^3} - \frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^2b^2(a+bx^3)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3ab^2fx^6-b\left(\frac{2cb^3}{a}-2db^2-aeb+a^2f\right)x^3+3b^3c}{x^2(bx^3+a)^2}dx}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 1808}$$

$$\frac{-\frac{\int -\frac{b^3\left(9ab^2c-\left(-5fa^3-bea^2-2b^2da+5b^3c\right)x^3\right)}{x^2\left(bx^3+a\right)}dx}{3a^2b^2}-\frac{bx^2\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{3a^2(a+bx^3)}}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{b^3\left(9ab^2c-\left(-5fa^3-bea^2-2b^2da+5b^3c\right)x^3\right)}{x^2\left(bx^3+a\right)}dx}{3a^2b^2}-\frac{bx^2\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{3a^2(a+bx^3)}}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{b\int \frac{9ab^2c-\left(-5fa^3-bea^2-2b^2da+5b^3c\right)x^3}{x^2\left(bx^3+a\right)}dx}{3a^2}-\frac{bx^2\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{3a^2(a+bx^3)}}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 955}$$

$$\frac{\frac{b\left(-\left(-5a^3f-a^2be-2ab^2d+14b^3c\right)\int \frac{x}{bx^3+a}dx-\frac{9b^2c}{x}\right)}{3a^2}-\frac{bx^2\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{3a^2(a+bx^3)}}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{\frac{b\left(-\left(-5a^3f-a^2be-2ab^2d+14b^3c\right)\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)-\frac{9b^2c}{x}\right)}{3a^2}-\frac{bx^2\left(4a^3f-a^2be-2ab^2d+5b^3c\right)}{3a^2(a+bx^3)}}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 16}$$

3.295. $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx$

$$\frac{\frac{b\left(-(-5a^3f-a^2be-2ab^2d+14b^3c)\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{9b^2c}{x}\right)}{3a^2}-\frac{bx^2(4a^3f-a^2be-2ab^2d+5b^3c)}{3a^2(a+bx^3)}}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{\frac{b\left(-(-5a^3f-a^2be-2ab^2d+14b^3c)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{9b^2c}{x}\right)}{3a^2}-\frac{bx^2(4a^3f-a^2be-2ab^2d+5b^3c)}{3a^2(a+bx^3)}}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{b\left(-(-5a^3f-a^2be-2ab^2d+14b^3c)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{9b^2c}{x}\right)}{3a^2}-\frac{bx^2(4a^3f-a^2be-2ab^2d+5b^3c)}{3a^2(a+bx^3)}}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{b\left(-(-5a^3f-a^2be-2ab^2d+14b^3c)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{9b^2c}{x}\right)}{3a^2}-\frac{bx^2(4a^3f-a^2be-2ab^2d+5b^3c)}{3a^2(a+bx^3)}}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

3.295. $\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx$

$$\downarrow \text{1082}$$

$$\frac{\frac{b\left(-(-5a^3f-a^2be-2ab^2d+14b^3c)\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{9b^2c}{x}\right)}{3a^2}-\frac{bx^2(\cdots}{\cdots}}{3ab^3}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\downarrow \text{217}$$

$$\frac{\frac{b\left(-(-5a^3f-a^2be-2ab^2d+14b^3c)\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{9b^2c}{x}\right)}{3a^2}-\frac{bx^2\left(4a^3f-a^2be-\cdots\right.}{3a^2(a+\cdots}}{3ab^3}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\downarrow \text{1103}$$

$$\frac{\frac{b\left(-\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)(-5a^3f-a^2be-2ab^2d+14b^3c)-\frac{9b^2c}{x}\right)}{3a^2}-\frac{bx^2\left(4a^3f-a^2be-2\cdots\right.}{3a^2(a+b\cdots}}{3ab^3}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

3.295. $\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x]
```

output
```
-1/6*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a^2*b^2*(a + b*x^3)^2) + (
-1/3*(b*(5*b^3*c - 2*a*b^2*d - a^2*b*e + 4*a^3*f)*x^2)/(a^2*(a + b*x^3)) +
 (b*((-9*b^2*c)/x - (14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*(-1/3*Log[a
^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3)))))/(3*a^2))/(3*a*b^3)
```

3.295.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1808
```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1))), x] + Simp[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1))   Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n))
*(n*(-d)^(-(m - Mod[m, n])/n + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p
 - ((c*d^2 - b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d
*(Mod[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.295. $\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx$

3.295.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.72

method	result
default	$-\frac{c}{a^3 x}+\frac{\frac{-\frac{\left(4 f\,a^3-a^2 b e-2 a\,b^2 d+5 b^3 c\right) x^5}{9 b}-\frac{a\left(5 f\,a^3+a^2 b e-7 a\,b^2 d+13 b^3 c\right) x^2}{18 b^2}}{\left(b\,x^3+a\right)^2}+\frac{\left(5 f\,a^3+a^2 b e+2 a\,b^2 d-14 b^3 c\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right.\right.}{6b}\right.}{9 b^2}}{a^3}$
risch	$\frac{-\frac{\left(4 f\,a^3-a^2 b e-2 a\,b^2 d+14 b^3 c\right) x^6}{9 a^3 b}-\frac{\left(5 f\,a^3+a^2 b e-7 a\,b^2 d+49 b^3 c\right) x^3}{18 a^2 b^2}-\frac{c}{a}}{x\left(b\,x^3+a\right)^2}+\frac{\left(\_R=\text{RootOf}\left(a^{10} b^8 \_Z^3+125 a^9 f^3+75 a^8 b e\,f^2+150 a^7 b^2 d\,f^2+15\right.\right.}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-c/a^3/x+1/a^3*((-1/9*(4*a^3*f-a^2*b*e-2*a*b^2*d+5*b^3*c)/b*x^5-1/18*a*(5*
a^3*f+a^2*b*e-7*a*b^2*d+13*b^3*c)/b^2*x^2)/(b*x^3+a)^2+1/9*(5*a^3*f+a^2*b*
e+2*a*b^2*d-14*b^3*c)/b^2*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b
)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.295.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(262) = 524.

Time = 0.31 (sec) , antiderivative size = 1206, normalized size of antiderivative = 3.98

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/54*(54*a^3*b^4*c + 6*(14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e + 4*a^4*b^3
*f)*x^6 + 3*(49*a^2*b^5*c - 7*a^3*b^4*d + a^4*b^3*e + 5*a^5*b^2*f)*x^3 + 3
*sqrt(1/3)*((14*a*b^6*c - 2*a^2*b^5*d - a^3*b^4*e - 5*a^4*b^3*f)*x^7 + 2*(
14*a^2*b^5*c - 2*a^3*b^4*d - a^4*b^3*e - 5*a^5*b^2*f)*x^4 + (14*a^3*b^4*c
 - 2*a^4*b^3*d - a^5*b^2*e - 5*a^6*b*f)*x)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^
2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a
)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((14*b^5*c -
 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c - 2*a^2*b^3*d -
a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d - a^4*b*e - 5*a^5
*f)*x)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) -
 2*((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*x^7 + 2*(14*a*b^4*c -
 2*a^2*b^3*d - a^3*b^2*e - 5*a^4*b*f)*x^4 + (14*a^2*b^3*c - 2*a^3*b^2*d -
a^4*b*e - 5*a^5*f)*x)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^6*x
^7 + 2*a^5*b^5*x^4 + a^6*b^4*x), -1/54*(54*a^3*b^4*c + 6*(14*a*b^6*c - 2*a
^2*b^5*d - a^3*b^4*e + 4*a^4*b^3*f)*x^6 + 3*(49*a^2*b^5*c - 7*a^3*b^4*d +
a^4*b^3*e + 5*a^5*b^2*f)*x^3 + 6*sqrt(1/3)*((14*a*b^6*c - 2*a^2*b^5*d - a^
3*b^4*e - 5*a^4*b^3*f)*x^7 + 2*(14*a^2*b^5*c - 2*a^3*b^4*d - a^4*b^3*e - 5
*a^5*b^2*f)*x^4 + (14*a^3*b^4*c - 2*a^4*b^3*d - a^5*b^2*e - 5*a^6*b*f)*x)*
sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-
a*b^2)^(1/3)/a)/b) + ((14*b^5*c - 2*a*b^4*d - a^2*b^3*e - 5*a^3*b^2*f)*...
```

3.295.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2(a+bx^3)^3}\,dx=\text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**2/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{2\,(14\,b^4c-2\,ab^3d-a^2b^2e+4\,a^3bf)x^6+18\,a^2b^2c+(49\,ab^3c-7\,a^2b^2d+a^3be+5\,a^4f)x^3}{18\,(a^3b^4x^7+2\,a^4b^3x^4+a^5b^2x)}$$
$$-\frac{\sqrt{3}(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*(2*(14*b^4*c - 2*a*b^3*d - a^2*b^2*e + 4*a^3*b*f)*x^6 + 18*a^2*b^2*c
 + (49*a*b^3*c - 7*a^2*b^2*d + a^3*b*e + 5*a^4*f)*x^3)/(a^3*b^4*x^7 + 2*a^
4*b^3*x^4 + a^5*b^2*x) - 1/27*sqrt(3)*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*
a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^3*(a/b)^
(1/3)) - 1/54*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*log(x^2 - x*(a/b)
^(1/3) + (a/b)^(2/3))/(a^3*b^3*(a/b)^(1/3)) + 1/27*(14*b^3*c - 2*a*b^2*d -
 a^2*b*e - 5*a^3*f)*log(x + (a/b)^(1/3))/(a^3*b^3*(a/b)^(1/3))
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{\sqrt{3}(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^3b^2}-\frac{c}{a^3x}$$
$$+\frac{(14\,b^3c-2\,ab^2d-a^2be-5\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{1}{3}}a^3b^2}$$
$$+\frac{\left(14\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4b^2}$$
$$-\frac{10\,b^4cx^5-4\,ab^3dx^5-2\,a^2b^2ex^5+8\,a^3bfx^5+13\,ab^3cx^2-7\,a^2b^2dx^2+a^3bex^2+5\,a^4fx^2}{18\left(bx^3+a\right)^2a^3b^2}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^3*b^2) - c/(a^3*x)
+ 1/54*(14*b^3*c - 2*a*b^2*d - a^2*b*e - 5*a^3*f)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3*b^2) + 1/27*(14*b^3*c*(-a/b)^(1/3) -
2*a*b^2*d*(-a/b)^(1/3) - a^2*b*e*(-a/b)^(1/3) - 5*a^3*f*(-a/b)^(1/3))*(-a/
b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b^2) - 1/18*(10*b^4*c*x^5 - 4*a*b
^3*d*x^5 - 2*a^2*b^2*e*x^5 + 8*a^3*b*f*x^5 + 13*a*b^3*c*x^2 - 7*a^2*b^2*d*
x^2 + a^3*b*e*x^2 + 5*a^4*f*x^2)/((b*x^3 + a)^2*a^3*b^2)
```

3.295.9 Mupad [B] (verification not implemented)

Time = 11.93 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^2\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{\frac{c}{a}+\frac{x^6\,\left(4\,f\,a^3-e\,a^2\,b-2\,d\,a\,b^2+14\,c\,b^3\right)}{9\,a^3\,b}+\frac{x^3\,\left(5\,f\,a^3+e\,a^2\,b-7\,d\,a\,b^2+49\,c\,b^3\right)}{18\,a^2\,b^2}}{a^2\,x+2\,a\,b\,x^4+b^2\,x^7}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(5\,f\,a^3+e\,a^2\,b+2\,d\,a\,b^2-14\,c\,b^3\right)}{27\,a^{10/3}\,b^{8/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,f\,a^3+e\,a^2\,b+2\,d\,a\,b^2-14\,c\,b^3\right)}{27\,a^{10/3}\,b^{8/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,f\,a^3+e\,a^2\,b+2\,d\,a\,b^2-14\,c\,b^3\right)}{27\,a^{10/3}\,b^{8/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^2*(a + b*x^3)^3),x)
```

output
```
(log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5
*a^3*f - 14*b^3*c + 2*a*b^2*d + a^2*b*e))/(27*a^(10/3)*b^(8/3)) - (log(b^(
1/3)*x + a^(1/3))*(5*a^3*f - 14*b^3*c + 2*a*b^2*d + a^2*b*e))/(27*a^(10/3)
*b^(8/3)) - (c/a + (x^6*(14*b^3*c + 4*a^3*f - 2*a*b^2*d - a^2*b*e))/(9*a^3
*b) + (x^3*(49*b^3*c + 5*a^3*f - 7*a*b^2*d + a^2*b*e))/(18*a^2*b^2))/(a^2*
x + b^2*x^7 + 2*a*b*x^4) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(5*a^3*f - 14*b^3*c + 2*a*b^2*d + a^2*b*e))/(27*a
^(10/3)*b^(8/3))
```

3.296 $\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx$

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3.296.1 Optimal result

Integrand size = 30, antiderivative size = 301

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx =& -\frac{c}{2a^3x^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{6a^2b^2\left(a+bx^3\right)^2}\\&-\frac{\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{18a^3b^2\left(a+bx^3\right)}\\&+\frac{\left(20b^3c-5ab^2d-a^2be-2a^3f\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}b^{7/3}}\\&-\frac{\left(20b^3c-5ab^2d-a^2be-2a^3f\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}b^{7/3}}\\&+\frac{\left(20b^3c-5ab^2d-a^2be-2a^3f\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}b^{7/3}}\end{aligned}$$

output
```
-1/2*c/a^3/x^2-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^2/b^2/(b*x^3+a)^2-1/
18*(7*a^3*f-a^2*b*e-5*a*b^2*d+11*b^3*c)*x/a^3/b^2/(b*x^3+a)-1/27*(-2*a^3*f
-a^2*b*e-5*a*b^2*d+20*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(7/3)+1/54*(
-2*a^3*f-a^2*b*e-5*a*b^2*d+20*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/a^(11/3)/b^(7/3)+1/27*(-2*a^3*f-a^2*b*e-5*a*b^2*d+20*b^3*c)*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(7/3)*3^(1/2)
```

3.296.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{27a^{2/3}c}{x^2}+\frac{9a^{5/3}\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{b^2(a+bx^3)^2}-\frac{3a^{2/3}\left(11b^3c-5ab^2d-a^2be+7a^3f\right)x}{b^2(a+bx^3)}+\frac{2\sqrt{3}\left(20b^3c-5ab^2d-a^2be-2a^3f\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{7/3}}}{54a^{11/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

output
```
((-27*a^(2/3)*c)/x^2 + (9*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x
)/(b^2*(a + b*x^3)^2) - (3*a^(2/3)*(11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3
*f)*x)/(b^2*(a + b*x^3)) + (2*Sqrt[3]*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*
a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(7/3) + (2*(-20*b^3*
c + 5*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/b^(7/3) - ((-
20*b^3*c + 5*a*b^2*d + a^2*b*e + 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/b^(7/3))/(54*a^(11/3))
```

3.296.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {2368, 25, 1808, 27, 955, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \; 2368$$

$$-\frac{\int -\frac{6ab^2fx^6-b\left(\frac{5cb^3}{a}-5db^2-aeb+a^2f\right)x^3+6b^3c}{x^3(bx^3+a)^2}dx}{6ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{6ab^2fx^6-b\left(\frac{5cb^3}{a}-5db^2-aeb+a^2f\right)x^3+6b^3c}{x^3(bx^3+a)^2}dx}{6ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 1808$$

$$\frac{-\frac{\int -\frac{2b^3\left(9ab^2c-\left(-2fa^3-bea^2-5b^2da+11b^3c\right)x^3\right)}{x^3\left(bx^3+a\right)}dx}{3a^2b^2}-\frac{bx(7a^3f-a^2be-5ab^2d+11b^3c)}{3a^2(a+bx^3)}}{6ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2b\int \frac{9ab^2c-\left(-2fa^3-bea^2-5b^2da+11b^3c\right)x^3}{x^3\left(bx^3+a\right)}dx}{3a^2}-\frac{bx(7a^3f-a^2be-5ab^2d+11b^3c)}{3a^2(a+bx^3)}}{6ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 955$$

$$\frac{\frac{2b\left(-(-2a^3f-a^2be-5ab^2d+20b^3c)\int \frac{1}{bx^3+a}dx-\frac{9b^2c}{2x^2}\right)}{3a^2}-\frac{bx(7a^3f-a^2be-5ab^2d+11b^3c)}{3a^2(a+bx^3)}}{6ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 750$$

$$\frac{\frac{2b\left(-(-2a^3f-a^2be-5ab^2d+20b^3c)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)-\frac{9b^2c}{2x^2}\right)}{3a^2}-\frac{bx(7a^3f-a^2be-5ab^2d+11b^3c)}{3a^2(a+bx^3)}}{6ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{2b\left(-(-2a^3f-a^2be-5ab^2d+20b^3c)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{9b^2c}{2x^2}\right)}{3a^2}-\frac{bx(7a^3f-a^2be-5ab^2d+11b^3c)}{3a^2(a+bx^3)}}{6ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

3.296. $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^3}\,dx$

$\Big\downarrow$ 1142

$$\frac{\frac{2b\left(-\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{9b^2c}{2x^2}\right)}{3a^2}-\frac{bx(7a}{}}{6ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{2b\left(-\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{9b^2c}{2x^2}\right)}{3a^2}-\frac{bx(7a^3}{}}{6ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{2b\left(-\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{9b^2c}{2x^2}\right)}{3a^2}-\frac{bx(7a}{}}{6ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1082

3.296. $\int\frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{2b\left(-\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{9b^2c}{2x^2}\right)}{3a^2}-\frac{bx(}{}}{6ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$\downarrow$ 217

$$\frac{\frac{2b\left(-\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{9b^2c}{2x^2}\right)}{3a^2}-\frac{bx\left(7a^3f-a^2be-5\right.}{3a^2(a+}}{6ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

$\downarrow$ 1103

$$\frac{\frac{2b\left(-\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)\left(-2a^3f-a^2be-5ab^2d+20b^3c\right)-\frac{9b^2c}{2x^2}\right)}{3a^2}-\frac{bx\left(7a^3f-a^2be-\right.}{3a^2(a-}}{6ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^2b^2\left(a+bx^3\right)^2}$$

3.296. $\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x]
```

output
```
-1/6*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a^2*b^2*(a + b*x^3)^2) + (-1
/3*(b*(11*b^3*c - 5*a*b^2*d - a^2*b*e + 7*a^3*f)*x)/(a^2*(a + b*x^3)) + (2
*b*((-9*b^2*c)/(2*x^2) - (20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*(Log[a
^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/
3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3)))))/(3*a^2))/(6*a*b^3)
```

3.296.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1808
```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1))), x] + Simp[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1))   Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n))
*(n*(-d)^(-(m - Mod[m, n])/n + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p
 - ((c*d^2 - b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d
*(Mod[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.296. $\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^3}\,dx$

3.296.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.72

method	result
default	$-\frac{c}{2a^3x^2}+\frac{\frac{-\frac{\left(7f\,a^3-a^2be-5a\,b^2d+11b^3c\right)x^4}{18b}-\frac{a\left(2f\,a^3+a^2be-4a\,b^2d+7b^3c\right)x}{9b^2}}{\left(b\,x^3+a\right)^2}+\frac{\left(2f\,a^3+a^2be+5a\,b^2d-20b^3c\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right.\right.}{6b}\right.}{9b^2}}{a^3}$
risch	$\frac{-\frac{\left(7f\,a^3-a^2be-5a\,b^2d+20b^3c\right)x^6}{18a^3b}-\frac{\left(2f\,a^3+a^2be-4a\,b^2d+16b^3c\right)x^3}{9a^2b^2}-\frac{c}{2a}}{x^2\left(b\,x^3+a\right)^2}+\frac{\left(\_R=\text{RootOf}\left(a^{11}b^7\_Z^3-8a^9f^3-12a^8be\,f^2-60a^7b^2d\,f^2-6a^7b\right.\right.}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/a^3/x^2+1/a^3*((-1/18*(7*a^3*f-a^2*b*e-5*a*b^2*d+11*b^3*c)/b*x^4-1/
9*a*(2*a^3*f+a^2*b*e-4*a*b^2*d+7*b^3*c)/b^2*x)/(b*x^3+a)^2+1/9*(2*a^3*f+a^
2*b*e+5*a*b^2*d-20*b^3*c)/b^2*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.296.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(258) = 516.

Time = 0.29 (sec) , antiderivative size = 1217, normalized size of antiderivative = 4.04

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/54*(27*a^4*b^3*c + 3*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e + 7*a^5*b
^2*f)*x^6 + 6*(16*a^3*b^4*c - 4*a^4*b^3*d + a^5*b^2*e + 2*a^6*b*f)*x^3 + 3
*sqrt(1/3)*((20*a*b^6*c - 5*a^2*b^5*d - a^3*b^4*e - 2*a^4*b^3*f)*x^8 + 2*(
20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e - 2*a^5*b^2*f)*x^5 + (20*a^3*b^4*c
- 5*a^4*b^3*d - a^5*b^2*e - 2*a^6*b*f)*x^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*
a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/
3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - ((20*b^5*c
- 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c - 5*a^2*b^3*d -
 a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d - a^4*b*e - 2*a^
5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) +
 2*((20*b^5*c - 5*a*b^4*d - a^2*b^3*e - 2*a^3*b^2*f)*x^8 + 2*(20*a*b^4*c -
 5*a^2*b^3*d - a^3*b^2*e - 2*a^4*b*f)*x^5 + (20*a^2*b^3*c - 5*a^3*b^2*d -
a^4*b*e - 2*a^5*f)*x^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^5
*x^8 + 2*a^6*b^4*x^5 + a^7*b^3*x^2), -1/54*(27*a^4*b^3*c + 3*(20*a^2*b^5*c
 - 5*a^3*b^4*d - a^4*b^3*e + 7*a^5*b^2*f)*x^6 + 6*(16*a^3*b^4*c - 4*a^4*b^
3*d + a^5*b^2*e + 2*a^6*b*f)*x^3 + 6*sqrt(1/3)*((20*a*b^6*c - 5*a^2*b^5*d
- a^3*b^4*e - 2*a^4*b^3*f)*x^8 + 2*(20*a^2*b^5*c - 5*a^3*b^4*d - a^4*b^3*e
 - 2*a^5*b^2*f)*x^5 + (20*a^3*b^4*c - 5*a^4*b^3*d - a^5*b^2*e - 2*a^6*b*f)
*x^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^
(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((20*b^5*c - 5*a*b^4*d - a^2*b^3*...
```

3.296.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^3 (a + bx^3)^3} \, dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^3}\,dx = -\frac{(20\,b^4c-5\,ab^3d-a^2b^2e+7\,a^3bf)x^6+9\,a^2b^2c+2\,(16\,ab^3c-4\,a^2b^2d+a^3be+2\,a^4f)x^3}{18\,(a^3b^4x^8+2\,a^4b^3x^5+a^5b^2x^2)} - \frac{\sqrt{3}(20\,b^3c-5\,ab^2d-a^2be-2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(20\,b^3c-5\,ab^2d-a^2be-2\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(20\,b^3c-5\,ab^2d-a^2be-2\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*((20*b^4*c - 5*a*b^3*d - a^2*b^2*e + 7*a^3*b*f)*x^6 + 9*a^2*b^2*c +
2*(16*a*b^3*c - 4*a^2*b^2*d + a^3*b*e + 2*a^4*f)*x^3)/(a^3*b^4*x^8 + 2*a^4
*b^3*x^5 + a^5*b^2*x^2) - 1/27*sqrt(3)*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2
*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^3*(a/b)
^(2/3)) + 1/54*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(a^3*b^3*(a/b)^(2/3)) - 1/27*(20*b^3*c - 5*a*b^2*d
- a^2*b*e - 2*a^3*f)*log(x + (a/b)^(1/3))/(a^3*b^3*(a/b)^(2/3))
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3(a+bx^3)^3}\,dx = \frac{\sqrt{3}(20\,b^3c-5\,ab^2d-a^2be-2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^3b} + \frac{(20\,b^3c-5\,ab^2d-a^2be-2\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^3b} + \frac{(20\,b^3c-5\,ab^2d-a^2be-2\,a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4b^2} - \frac{20\,b^4cx^6-5\,ab^3dx^6-a^2b^2ex^6+7\,a^3bfx^6+32\,ab^3cx^3-8\,a^2b^2dx^3+2\,a^3bex^3+4\,a^4fx^3+9\,a^2b^2c}{18\,(bx^4+ax)^2a^3b^2}$$

input

```
integrate((f*x^9+e*x^6+d*x^3+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

output

```
1/27*sqrt(3)*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3*b) + 1/54*(20*b^3*
c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/((-a*b^2)^(2/3)*a^3*b) + 1/27*(20*b^3*c - 5*a*b^2*d - a^2*b*e - 2*a^3*f)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b^2) - 1/18*(20*b^4*c*x^6 -
5*a*b^3*d*x^6 - a^2*b^2*e*x^6 + 7*a^3*b*f*x^6 + 32*a*b^3*c*x^3 - 8*a^2*b^2
*d*x^3 + 2*a^3*b*e*x^3 + 4*a^4*f*x^3 + 9*a^2*b^2*c)/((b*x^4 + a*x)^2*a^3*b
^2)
```

3.296.9 Mupad [B] (verification not implemented)

Time = 10.05 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^3\left(a+bx^3\right)^3}\,dx$$
$$=\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(2\,f\,a^3+e\,a^2\,b+5\,d\,a\,b^2-20\,c\,b^3\right)}{27\,a^{11/3}\,b^{7/3}}$$
$$-\frac{\frac{c}{2\,a}+\frac{x^3\,\left(2\,f\,a^3+e\,a^2\,b-4\,d\,a\,b^2+16\,c\,b^3\right)}{9\,a^2\,b^2}+\frac{x^6\,\left(7\,f\,a^3-e\,a^2\,b-5\,d\,a\,b^2+20\,c\,b^3\right)}{18\,a^3\,b}}{a^2\,x^2+2\,a\,b\,x^5+b^2\,x^8}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,f\,a^3+e\,a^2\,b+5\,d\,a\,b^2-20\,c\,b^3\right)}{27\,a^{11/3}\,b^{7/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,f\,a^3+e\,a^2\,b+5\,d\,a\,b^2-20\,c\,b^3\right)}{27\,a^{11/3}\,b^{7/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^3*(a + b*x^3)^3),x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(2*a^3*f - 20*b^3*c + 5*a*b^2*d + a^2*b*e))/(27*
a^(11/3)*b^(7/3)) - (c/(2*a) + (x^3*(16*b^3*c + 2*a^3*f - 4*a*b^2*d + a^2*
b*e))/(9*a^2*b^2) + (x^6*(20*b^3*c + 7*a^3*f - 5*a*b^2*d - a^2*b*e))/(18*a
^3*b))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/
3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*a^3*f - 20*b^3*c + 5*a*b^2*d + a
^2*b*e))/(27*a^(11/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a
^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*a^3*f - 20*b^3*c + 5*a*b^2*d + a^2*b*e))
/(27*a^(11/3)*b^(7/3))
```

3.297 $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^3}\,dx$

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3.297.1 Optimal result

Integrand size = 30, antiderivative size = 317

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^3}\,dx = -\frac{c}{4a^3x^4}+\frac{3bc-ad}{a^4x}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^3b(a+bx^3)^2}+\frac{(8b^3c-5ab^2d+2a^2be+a^3f)\,x^2}{9a^4b(a+bx^3)} - \frac{(35b^3c-14ab^2d+2a^2be+a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{13/3}b^{5/3}} - \frac{(35b^3c-14ab^2d+2a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{13/3}b^{5/3}} + \frac{(35b^3c-14ab^2d+2a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{13/3}b^{5/3}}$$

output
```
-1/4*c/a^3/x^4+(-a*d+3*b*c)/a^4/x+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a
^3/b/(b*x^3+a)^2+1/9*(a^3*f+2*a^2*b*e-5*a*b^2*d+8*b^3*c)*x^2/a^4/b/(b*x^3+
a)-1/27*(a^3*f+2*a^2*b*e-14*a*b^2*d+35*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(13/
3)/b^(5/3)+1/54*(a^3*f+2*a^2*b*e-14*a*b^2*d+35*b^3*c)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/a^(13/3)/b^(5/3)-1/27*(a^3*f+2*a^2*b*e-14*a*b^2*d+35
*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)/b^(5/3)
*3^(1/2)
```

3.297.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{27a^{4/3}c}{x^4} - \frac{108\sqrt[3]{a}(-3bc+ad)}{x} - \frac{18a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)x^2}{b(a+bx^3)^2} + \frac{12\sqrt[3]{a}(8b^3c-5ab^2d+2a^2be+a^3f)x^2}{b(a+bx^3)} - \frac{4\sqrt{3}(35b^3c-14ab^2d+2a}{}}{}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]
```

output
```
((-27*a^(4/3)*c)/x^4 - (108*a^(1/3)*(-3*b*c + a*d))/x - (18*a^(4/3)*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)^2) + (12*a^(1/3)*(8*b
^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*x^2)/(b*(a + b*x^3)) - (4*Sqrt[3]*(3
5*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/b^(5/3) - (4*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*Log[
a^(1/3) + b^(1/3)*x])/b^(5/3) + (2*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^
3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3))/(108*a^(13/3
))
```

3.297.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2368, 27, 1808, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \; 2368$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}-\frac{\int -\frac{2\left(b^2\left(\frac{2cb^3}{a^2}-\frac{2db^2}{a}+2eb+af\right)x^6-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c\right)}{x^5\left(bx^3+a\right)^2}dx}{6ab^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{b^2\left(\frac{2cb^3}{a^2}-\frac{2db^2}{a}+2eb+af\right)x^6-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^5(bx^3+a)^2}dx}{3ab^3}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1808$$

$$\frac{\frac{\int \frac{b^4\left(fa^3+2bea^2-5b^2da+8b^3c\right)x^6-9ab^5(2bc-ad)x^3+9a^2b^5c}{x^5\left(bx^3+a\right)}dx}{3a^3b^2}+\frac{b^2x^2\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{3a^3(a+bx^3)}}{3ab^3}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1812$$

$$\frac{\frac{\int\left(-\frac{9(3bc-ad)b^5}{x^2}+\frac{9acb^5}{x^5}+\frac{\left(fa^3+2bea^2-14b^2da+35b^3c\right)xb^4}{bx^3+a}\right)dx}{3a^3b^2}+\frac{b^2x^2\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{3a^3(a+bx^3)}}{3ab^3}+\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}+\frac{\frac{b^2x^2\left(a^3f+2a^2be-5ab^2d+8b^3c\right)}{3a^3(a+bx^3)}+\frac{-\frac{b^{10/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^3f+2a^2be-14ab^2d+35b^3c\right)}{3\sqrt[3]{a}}}{3a^3b^2}}{3ab^3}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x]
```

output
```
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^3*b*(a + b*x^3)^2) + ((b^2*
(8*b^3*c - 5*a*b^2*d + 2*a^2*b*e + a^3*f)*x^2)/(3*a^3*(a + b*x^3)) + ((-9*
a*b^5*c)/(4*x^4) + (9*b^5*(3*b*c - a*d))/x - (b^(10/3)*(35*b^3*c - 14*a*b^
2*d + 2*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(1/3)) - (b^(10/3)*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f
)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)) + (b^(10/3)*(35*b^3*c - 14*a*b^2*d
 + 2*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a
^(1/3)))/(3*a^3*b^2))/(3*a*b^3)
```

3.297. $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^3}\,dx$

3.297.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1808
```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1))), x] + Simp[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1))   Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n))
*(n*(-d)^(-(m - Mod[m, n])/n + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p
 - ((c*d^2 - b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d
*(Mod[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.297. $\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^3}\,dx$

3.297.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.73

method	result
default	$-\frac{c}{4a^{3}x^{4}}-\frac{ad-3bc}{a^{4}x}+\frac{\frac{\left(\frac{1}{9}fa^{3}+\frac{2}{9}a^{2}be-\frac{5}{9}ab^{2}d+\frac{8}{9}b^{3}c\right)x^{5}-\frac{a\left(fa^{3}-7a^{2}be+13ab^{2}d-19b^{3}c\right)x^{2}}{18b}}{\left(bx^{3}+a\right)^{2}}+\frac{\left(fa^{3}+2a^{2}be-14ab^{2}d+35b^{3}c\right)\left(-\frac{\ln\left(x+\left(\cdots\right.\right.}{3b\left(\frac{a}{b}\cdots\right.}\cdots\right.}{\cdots}}{a^{4}}$
risch	$\frac{\frac{\left(fa^{3}+2a^{2}be-14ab^{2}d+35b^{3}c\right)x^{9}}{9a^{4}}-\frac{\left(2fa^{3}-14a^{2}be+98ab^{2}d-245b^{3}c\right)x^{6}}{36a^{3}b}-\frac{(2ad-5bc)x^{3}}{2a^{2}}-\frac{c}{4a}}{x^{4}\left(bx^{3}+a\right)^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{13}b^{5}\_Z^{3}+a^{9}f^{3}+6a^{8}bef^{2}-\cdots\right.}\cdots\right.}{\cdots}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*c/a^3/x^4-(a*d-3*b*c)/a^4/x+1/a^4*(((1/9*f*a^3+2/9*a^2*b*e-5/9*a*b^2*
d+8/9*b^3*c)*x^5-1/18*a*(a^3*f-7*a^2*b*e+13*a*b^2*d-19*b^3*c)/b*x^2)/(b*x^
3+a)^2+1/9*(a^3*f+2*a^2*b*e-14*a*b^2*d+35*b^3*c)/b*(-1/3/b/(a/b)^(1/3)*ln(
x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(
1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.297.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. 2(274) = 548.

Time = 0.29 (sec) , antiderivative size = 1254, normalized size of antiderivative = 3.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/108*(12*(35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^9 - 27*
a^4*b^3*c + 3*(245*a^2*b^5*c - 98*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*
x^6 + 54*(5*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 6*sqrt(1/3)*((35*a*b^6*c - 14*a
^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^10 + 2*(35*a^2*b^5*c - 14*a^3*b^4*d
+ 2*a^4*b^3*e + a^5*b^2*f)*x^7 + (35*a^3*b^4*c - 14*a^4*b^3*d + 2*a^5*b^2*
e + a^6*b*f)*x^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3
)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a)
 - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((35*b^5*c - 14*a*b^4*d + 2*a^2*b^
3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d + 2*a^3*b^2*e + a^4*b
*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e + a^5*f)*x^4)*(-a*b^2)^
(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*((35*b^5*c -
14*a*b^4*d + 2*a^2*b^3*e + a^3*b^2*f)*x^10 + 2*(35*a*b^4*c - 14*a^2*b^3*d
+ 2*a^3*b^2*e + a^4*b*f)*x^7 + (35*a^2*b^3*c - 14*a^3*b^2*d + 2*a^4*b*e +
a^5*f)*x^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^5*x^10 + 2*a^
6*b^4*x^7 + a^7*b^3*x^4), 1/108*(12*(35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4
*e + a^4*b^3*f)*x^9 - 27*a^4*b^3*c + 3*(245*a^2*b^5*c - 98*a^3*b^4*d + 14*
a^4*b^3*e - 2*a^5*b^2*f)*x^6 + 54*(5*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 12*sqr
t(1/3)*((35*a*b^6*c - 14*a^2*b^5*d + 2*a^3*b^4*e + a^4*b^3*f)*x^10 + 2*(35
*a^2*b^5*c - 14*a^3*b^4*d + 2*a^4*b^3*e + a^5*b^2*f)*x^7 + (35*a^3*b^4*c -
 14*a^4*b^3*d + 2*a^5*b^2*e + a^6*b*f)*x^4)*sqrt(-(-a*b^2)^(1/3)/a)*arc...
```

3.297.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^3}\,dx$$
$$= \frac{4\,(35\,b^4c-14\,ab^3d+2\,a^2b^2e+a^3bf)x^9+(245\,ab^3c-98\,a^2b^2d+14\,a^3be-2\,a^4f)x^6-9\,a^3bc+18\,(5\,a^2b^2c}{36\,(a^4b^3x^{10}+2\,a^5b^2x^7+a^6bx^4)}$$
$$+\frac{\sqrt{3}(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/36*(4*(35*b^4*c - 14*a*b^3*d + 2*a^2*b^2*e + a^3*b*f)*x^9 + (245*a*b^3*c
 - 98*a^2*b^2*d + 14*a^3*b*e - 2*a^4*f)*x^6 - 9*a^3*b*c + 18*(5*a^2*b^2*c
- 2*a^3*b*d)*x^3)/(a^4*b^3*x^10 + 2*a^5*b^2*x^7 + a^6*b*x^4) + 1/27*sqrt(3
)*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a
/b)^(1/3))/(a/b)^(1/3))/(a^4*b^2*(a/b)^(1/3)) + 1/54*(35*b^3*c - 14*a*b^2*
d + 2*a^2*b*e + a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b^2*(a/
b)^(1/3)) - 1/27*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*log(x + (a/b)
^(1/3))/(a^4*b^2*(a/b)^(1/3))
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5\left(a+bx^3\right)^3}\,dx = \frac{\sqrt{3}(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^4b} - \frac{(35\,b^3c-14\,ab^2d+2\,a^2be+a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{1}{3}}a^4b} - \frac{\left(35\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5b} + \frac{16\,b^4cx^5-10\,ab^3dx^5+4\,a^2b^2ex^5+2\,a^3bfx^5+19\,ab^3cx^2-13\,a^2b^2dx^2+7\,a^3bex^2-a^4fx^2}{18\left(bx^3+a\right)^2a^4b} + \frac{12\,bcx^3-4\,adx^3-ac}{4\,a^4x^4}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^5/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/27*sqrt(3)*(35*b^3*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^4*b) - 1/54*(35*b^3
*c - 14*a*b^2*d + 2*a^2*b*e + a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/
3))/((-a*b^2)^(1/3)*a^4*b) - 1/27*(35*b^3*c*(-a/b)^(1/3) - 14*a*b^2*d*(-a/
b)^(1/3) + 2*a^2*b*e*(-a/b)^(1/3) + a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a^5*b) + 1/18*(16*b^4*c*x^5 - 10*a*b^3*d*x^5 + 4*a^
2*b^2*e*x^5 + 2*a^3*b*f*x^5 + 19*a*b^3*c*x^2 - 13*a^2*b^2*d*x^2 + 7*a^3*b*
e*x^2 - a^4*f*x^2)/((b*x^3 + a)^2*a^4*b) + 1/4*(12*b*c*x^3 - 4*a*d*x^3 - a
*c)/(a^4*x^4)
```

3.297.9 Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3+ex^6+fx^9}{x^5(a+bx^3)^3}\,dx$$
$$= -\frac{\frac{c}{4a}-\frac{x^9\,(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3)}{9\,a^4}+\frac{x^3\,(2\,a\,d-5\,b\,c)}{2\,a^2}-\frac{x^6\,(-2\,f\,a^3+14\,e\,a^2\,b-98\,d\,a\,b^2+245\,c\,b^3)}{36\,a^3\,b}}{a^2\,x^4+2\,a\,b\,x^7+b^2\,x^{10}}$$
$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3)}{27\,a^{13/3}\,b^{5/3}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3)}{27\,a^{13/3}\,b^{5/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(f\,a^3+2\,e\,a^2\,b-14\,d\,a\,b^2+35\,c\,b^3)}{27\,a^{13/3}\,b^{5/3}}$$

input

```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^5*(a + b*x^3)^3),x)
```

output

```
(log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(3
5*b^3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))/(27*a^(13/3)*b^(5/3)) - (log(b^
(1/3)*x + a^(1/3))*(35*b^3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))/(27*a^(13/
3)*b^(5/3)) - (c/(4*a) - (x^9*(35*b^3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))
/(9*a^4) + (x^3*(2*a*d - 5*b*c))/(2*a^2) - (x^6*(245*b^3*c - 2*a^3*f - 98*
a*b^2*d + 14*a^2*b*e))/(36*a^3*b))/(a^2*x^4 + b^2*x^10 + 2*a*b*x^7) - (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(35*b^
3*c + a^3*f - 14*a*b^2*d + 2*a^2*b*e))/(27*a^(13/3)*b^(5/3))
```

3.298 $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^3}\,dx$

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3.298.1 Optimal result

Integrand size = 30, antiderivative size = 316

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^3}\,dx = -\frac{c}{5a^3x^5}+\frac{3bc-ad}{2a^4x^2}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^3b(a+bx^3)^2}+\frac{(17b^3c-11ab^2d+5a^2be+a^3f)\,x}{18a^4b(a+bx^3)} - \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{14/3}b^{4/3}} + \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}b^{4/3}} - \frac{(44b^3c-20ab^2d+5a^2be+a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{14/3}b^{4/3}}$$

output
```
-1/5*c/a^3/x^5+1/2*(-a*d+3*b*c)/a^4/x^2+1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
*x/a^3/b/(b*x^3+a)^2+1/18*(a^3*f+5*a^2*b*e-11*a*b^2*d+17*b^3*c)*x/a^4/b/(b
*x^3+a)+1/27*(a^3*f+5*a^2*b*e-20*a*b^2*d+44*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a
^(14/3)/b^(4/3)-1/54*(a^3*f+5*a^2*b*e-20*a*b^2*d+44*b^3*c)*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)/b^(4/3)-1/27*(a^3*f+5*a^2*b*e-20*a*b^2
*d+44*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)/b^
(4/3)*3^(1/2)
```

3.298.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{54a^{5/3}c}{x^5}-\frac{135a^{2/3}(-3bc+ad)}{x^2}-\frac{45a^{5/3}(-b^3c+ab^2d-a^2be+a^3f)x}{b(a+bx^3)^2}+\frac{15a^{2/3}(17b^3c-11ab^2d+5a^2be+a^3f)x}{b(a+bx^3)}-\frac{10\sqrt{3}(44b^3c-20ab^2d+5a\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```

output
```
((-54*a^(5/3)*c)/x^5 - (135*a^(2/3)*(-3*b*c + a*d))/x^2 - (45*a^(5/3)*(-(b
^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)^2) + (15*a^(2/3)*(17*
b^3*c - 11*a*b^2*d + 5*a^2*b*e + a^3*f)*x)/(b*(a + b*x^3)) - (10*Sqrt[3]*(
44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/b^(4/3) + (10*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*Lo
g[a^(1/3) + b^(1/3)*x])/b^(4/3) - (5*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e +
a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3))/(270*a^(14
/3))
```

3.298.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2368, 25, 1808, 27, 1812, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^3b\left(a+bx^3\right)^2}-\frac{\int -\frac{b^2\left(\frac{5cb^3}{a^2}-\frac{5db^2}{a}+5eb+af\right)x^6-6b^3\left(\frac{bc}{a}-d\right)x^3+6b^3c}{x^6(bx^3+a)^2}dx}{6ab^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2\left(\frac{5cb^3}{a^2}-\frac{5db^2}{a}+5eb+af\right)x^6-6b^3\left(\frac{bc}{a}-d\right)x^3+6b^3c}{x^6(bx^3+a)^2}dx}{6ab^3}+\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^3b(a+bx^3)^2}$$

$$\Big\downarrow \text{1808}$$

$$\frac{\frac{\int \frac{2\left(b^4\left(fa^3+5bea^2-11b^2da+17b^3c\right)x^6-9ab^5(2bc-ad)x^3+9a^2b^5c\right)}{x^6\left(bx^3+a\right)}dx}{3a^3b^2}+\frac{b^2x(a^3f+5a^2be-11ab^2d+17b^3c)}{3a^3(a+bx^3)}}{6ab^3}+\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^3b(a+bx^3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{b^4\left(fa^3+5bea^2-11b^2da+17b^3c\right)x^6-9ab^5(2bc-ad)x^3+9a^2b^5c}{x^6\left(bx^3+a\right)}dx}{3a^3b^2}+\frac{b^2x(a^3f+5a^2be-11ab^2d+17b^3c)}{3a^3(a+bx^3)}}{6ab^3}+\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^3b(a+bx^3)^2}$$

$$\Big\downarrow \text{1812}$$

$$\frac{\frac{2\int \left(-\frac{9(3bc-ad)b^5}{x^3}+\frac{9acb^5}{x^6}+\frac{\left(fa^3+5bea^2-20b^2da+44b^3c\right)b^4}{bx^3+a}\right)dx}{3a^3b^2}+\frac{b^2x(a^3f+5a^2be-11ab^2d+17b^3c)}{3a^3(a+bx^3)}}{6ab^3}+\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^3b(a+bx^3)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{6a^3b(a+bx^3)^2}+\frac{\frac{b^2x(a^3f+5a^2be-11ab^2d+17b^3c)}{3a^3(a+bx^3)}+\frac{2\left(-\frac{b^{11/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^3f+5a^2be-20ab^2d+44b^3c\right)}{\sqrt{3}a^{2/3}}-\frac{b^{11/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^3f+5a^2b\right.}{6a^{2/3}}\right.}{3a^3b^2}}{6ab^3}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x]
```

3.298. $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^3}\,dx$

output
```
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^3*b*(a + b*x^3)^2) + ((b^2*(1
7*b^3*c - 11*a*b^2*d + 5*a^2*b*e + a^3*f)*x)/(3*a^3*(a + b*x^3)) + (2*((-9
*a*b^5*c)/(5*x^5) + (9*b^5*(3*b*c - a*d))/(2*x^2) - (b^(11/3)*(44*b^3*c -
20*a*b^2*d + 5*a^2*b*e + a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)) + (b^(11/3)*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e
+ a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)) - (b^(11/3)*(44*b^3*c - 20*
a*b^2*d + 5*a^2*b*e + a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(6*a^(2/3))))/(3*a^3*b^2))/(6*a*b^3)
```

3.298.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1808
```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e
_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-d)^((m - Mod[m, n])/n - 1)*(c*d^2
- b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*((d + e*x^n)^(q + 1)/(n*e^(2*p + (m -
Mod[m, n])/n)*(q + 1))), x] + Simp[(-d)^((m - Mod[m, n])/n - 1)/(n*e^(2*p)*
(q + 1))   Int[x^m*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^n))
*(n*(-d)^(-(m - Mod[m, n])/n + 1)*e^(2*p)*(q + 1)*(a + b*x^n + c*x^(2*n))^p
 - ((c*d^2 - b*d*e + a*e^2)^p/(e^((m - Mod[m, n])/n)*x^(m - Mod[m, n])))*(d
*(Mod[m, n] + 1) + e*(Mod[m, n] + n*(q + 1) + 1)*x^n))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m, 0]
```

rule 1812
```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.298. $\int \frac{c+dx^3+ex^6+fx^9}{x^6(a+bx^3)^3}\,dx$

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.298.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.72

method	result
default	$-\frac{c}{5a^3x^5}-\frac{ad-3bc}{2a^4x^2}+\frac{\frac{\left(\frac{1}{18}fa^3+\frac{5}{18}a^2be-\frac{11}{18}ab^2d+\frac{17}{18}b^3c\right)x^4-\frac{a\left(fa^3-4a^2be+7ab^2d-10b^3c\right)x}{9b}}{\left(bx^3+a\right)^2}+\left(fa^3+5a^2be-20ab^2d+44b^3c\right)\left(\frac{\ln\left(x+\left(\ldots\right.\right.}{3b\left(\ldots\right.}\ldots\right.}{a^4}$ …
risch	$\frac{\frac{\left(fa^3+5a^2be-20ab^2d+44b^3c\right)x^9}{18a^4}-\frac{\left(5fa^3-20a^2be+80ab^2d-176b^3c\right)x^6}{45a^3b}-\frac{(5ad-11bc)x^3}{10a^2}-\frac{c}{5a}}{x^5\left(bx^3+a\right)^2}+\left(\sum_{\_R=\mathrm{RootOf}\left(a^{14}b^4\_Z^3-a^9f^3-15a^8bef^2\ldots\right.}\ldots\right.$ …

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*c/a^3/x^5-1/2*(a*d-3*b*c)/a^4/x^2+1/a^4*(((1/18*f*a^3+5/18*a^2*b*e-11
/18*a*b^2*d+17/18*b^3*c)*x^4-1/9*a*(a^3*f-4*a^2*b*e+7*a*b^2*d-10*b^3*c)/b*
x)/(b*x^3+a)^2+1/9*(a^3*f+5*a^2*b*e-20*a*b^2*d+44*b^3*c)/b*(1/3/b/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+
1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.298.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(271) = 542$.

Time = 0.30 (sec) , antiderivative size = 1247, normalized size of antiderivative = 3.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/270*(15*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^9 - 5
4*a^5*b^2*c + 6*(176*a^3*b^4*c - 80*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*
x^6 + 27*(11*a^4*b^3*c - 5*a^5*b^2*d)*x^3 + 15*sqrt(1/3)*((44*a*b^6*c - 20
*a^2*b^5*d + 5*a^3*b^4*e + a^4*b^3*f)*x^11 + 2*(44*a^2*b^5*c - 20*a^3*b^4*
d + 5*a^4*b^3*e + a^5*b^2*f)*x^8 + (44*a^3*b^4*c - 20*a^4*b^3*d + 5*a^5*b^
2*e + a^6*b*f)*x^5)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3
)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*((44*b^5*c - 20*a*b^4*d + 5*a^2*b
^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d + 5*a^3*b^2*e + a^4*
b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a^5*f)*x^5)*(a^2*b)^
(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*((44*b^5*c - 2
0*a*b^4*d + 5*a^2*b^3*e + a^3*b^2*f)*x^11 + 2*(44*a*b^4*c - 20*a^2*b^3*d +
 5*a^3*b^2*e + a^4*b*f)*x^8 + (44*a^2*b^3*c - 20*a^3*b^2*d + 5*a^4*b*e + a
^5*f)*x^5)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^4*x^11 + 2*a^7
*b^3*x^8 + a^8*b^2*x^5), 1/270*(15*(44*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^
3*e + a^5*b^2*f)*x^9 - 54*a^5*b^2*c + 6*(176*a^3*b^4*c - 80*a^4*b^3*d + 20
*a^5*b^2*e - 5*a^6*b*f)*x^6 + 27*(11*a^4*b^3*c - 5*a^5*b^2*d)*x^3 + 30*sqr
t(1/3)*((44*a*b^6*c - 20*a^2*b^5*d + 5*a^3*b^4*e + a^4*b^3*f)*x^11 + 2*(44
*a^2*b^5*c - 20*a^3*b^4*d + 5*a^4*b^3*e + a^5*b^2*f)*x^8 + (44*a^3*b^4*c -
 20*a^4*b^3*d + 5*a^5*b^2*e + a^6*b*f)*x^5)*sqrt((a^2*b)^(1/3)/b)*arcta...
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**6/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.01

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^6 (a + bx^3)^3} \, dx$$
$$= \frac{5 (44 b^4 c - 20 ab^3 d + 5 a^2 b^2 e + a^3 bf) x^9 + 2 (176 ab^3 c - 80 a^2 b^2 d + 20 a^3 be - 5 a^4 f) x^6 - 18 a^3 bc + 9 (11 a^2}{90 (a^4 b^3 x^{11} + 2 a^5 b^2 x^8 + a^6 bx^5)}$$
$$+ \frac{\sqrt{3} (44 b^3 c - 20 ab^2 d + 5 a^2 be + a^3 f) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27 a^4 b^2 \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{(44 b^3 c - 20 ab^2 d + 5 a^2 be + a^3 f) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 a^4 b^2 \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+ \frac{(44 b^3 c - 20 ab^2 d + 5 a^2 be + a^3 f) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 a^4 b^2 \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/90*(5*(44*b^4*c - 20*a*b^3*d + 5*a^2*b^2*e + a^3*b*f)*x^9 + 2*(176*a*b^3
*c - 80*a^2*b^2*d + 20*a^3*b*e - 5*a^4*f)*x^6 - 18*a^3*b*c + 9*(11*a^2*b^2
*c - 5*a^3*b*d)*x^3)/(a^4*b^3*x^11 + 2*a^5*b^2*x^8 + a^6*b*x^5) + 1/27*sqr
t(3)*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*arctan(1/3*sqrt(3)*(2*x -
 (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b^2*(a/b)^(2/3)) - 1/54*(44*b^3*c - 20*a*b
^2*d + 5*a^2*b*e + a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b^2*
(a/b)^(2/3)) + 1/27*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*log(x + (a
/b)^(1/3))/(a^4*b^2*(a/b)^(2/3))
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx$$

$$= -\frac{\sqrt{3}(44\,b^3c-20\,ab^2d+5\,a^2be+a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^4} - \frac{(44\,b^3c-20\,ab^2d+5\,a^2be+a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^4} - \frac{(44\,b^3c-20\,ab^2d+5\,a^2be+a^3f)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5b} + \frac{17\,b^4cx^4-11\,ab^3dx^4+5\,a^2b^2ex^4+a^3bfx^4+20\,ab^3cx-14\,a^2b^2dx+8\,a^3bex-2\,a^4fx}{18\left(bx^3+a\right)^2a^4b} + \frac{15\,bcx^3-5\,adx^3-2\,ac}{10\,a^4x^5}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^6/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^4) - 1/54*(44*b^3*
c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/((-a*b^2)^(2/3)*a^4) - 1/27*(44*b^3*c - 20*a*b^2*d + 5*a^2*b*e + a^3*f)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b) + 1/18*(17*b^4*c*x^4 - 11
*a*b^3*d*x^4 + 5*a^2*b^2*e*x^4 + a^3*b*f*x^4 + 20*a*b^3*c*x - 14*a^2*b^2*d
*x + 8*a^3*b*e*x - 2*a^4*f*x)/((b*x^3 + a)^2*a^4*b) + 1/10*(15*b*c*x^3 - 5
*a*d*x^3 - 2*a*c)/(a^4*x^5)
```

3.298.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3+ex^6+fx^9}{x^6\left(a+bx^3\right)^3}\,dx$$
$$=\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{27\,a^{14/3}\,b^{4/3}}$$
$$-\frac{\frac{c}{5\,a}-\frac{x^9\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{18\,a^4}+\frac{x^3\,\left(5\,a\,d-11\,b\,c\right)}{10\,a^2}-\frac{x^6\,\left(-5\,f\,a^3+20\,e\,a^2\,b-80\,d\,a\,b^2+176\,c\,b^3\right)}{45\,a^3\,b}}{a^2\,x^5+2\,a\,b\,x^8+b^2\,x^{11}}$$
$$+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{27\,a^{14/3}\,b^{4/3}}$$
$$-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(f\,a^3+5\,e\,a^2\,b-20\,d\,a\,b^2+44\,c\,b^3\right)}{27\,a^{14/3}\,b^{4/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^6*(a + b*x^3)^3),x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(44*b^3*c + a^3*f - 20*a*b^2*d + 5*a^2*b*e))/(27
*a^(14/3)*b^(4/3)) - (c/(5*a) - (x^9*(44*b^3*c + a^3*f - 20*a*b^2*d + 5*a^
2*b*e))/(18*a^4) + (x^3*(5*a*d - 11*b*c))/(10*a^2) - (x^6*(176*b^3*c - 5*a
^3*f - 80*a*b^2*d + 20*a^2*b*e))/(45*a^3*b))/(a^2*x^5 + b^2*x^11 + 2*a*b*x
^8) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1
/2)*(44*b^3*c + a^3*f - 20*a*b^2*d + 5*a^2*b*e))/(27*a^(14/3)*b^(4/3)) - (
log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(44
*b^3*c + a^3*f - 20*a*b^2*d + 5*a^2*b*e))/(27*a^(14/3)*b^(4/3))
```

3.299 $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^3} dx$

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3.299.1 Optimal result

Integrand size = 30, antiderivative size = 343

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^3} dx = -\frac{c}{7a^3x^7}+\frac{3bc-ad}{4a^4x^4}-\frac{6b^2c-3abd+a^2e}{a^5x} - \frac{(b^3c-ab^2d+a^2be-a^3f)x^2}{6a^4(a+bx^3)^2}-\frac{(11b^3c-8ab^2d+5a^2be-2a^3f)x^2}{9a^5(a+bx^3)} + \frac{(65b^3c-35ab^2d+14a^2be-2a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{16/3}b^{2/3}} + \frac{(65b^3c-35ab^2d+14a^2be-2a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{16/3}b^{2/3}} - \frac{(65b^3c-35ab^2d+14a^2be-2a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{16/3}b^{2/3}}$$

output
```
-1/7*c/a^3/x^7+1/4*(-a*d+3*b*c)/a^4/x^4+(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x-1/6
*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2/a^4/(b*x^3+a)^2-1/9*(-2*a^3*f+5*a^2*b*
e-8*a*b^2*d+11*b^3*c)*x^2/a^5/(b*x^3+a)+1/27*(-2*a^3*f+14*a^2*b*e-35*a*b^2
*d+65*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(16/3)/b^(2/3)-1/54*(-2*a^3*f+14*a^2*
b*e-35*a*b^2*d+65*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(16/3
)/b^(2/3)+1/27*(-2*a^3*f+14*a^2*b*e-35*a*b^2*d+65*b^3*c)*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(16/3)/b^(2/3)*3^(1/2)
```

3.299.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8 (a+bx^3)^3} \, dx$$

$$= \frac{-\frac{108a^{7/3}c}{x^7} - \frac{189a^{4/3}(-3bc+ad)}{x^4} - \frac{756\sqrt[3]{a}(6b^2c-3abd+a^2e)}{x} + \frac{126a^{4/3}(-b^3c+ab^2d-a^2be+a^3f)x^2}{(a+bx^3)^2} + \frac{84\sqrt[3]{a}(-11b^3c+8ab^2d-5a^2be+}{a+bx^3}}{}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
```

output
```
((-108*a^(7/3)*c)/x^7 - (189*a^(4/3)*(-3*b*c + a*d))/x^4 - (756*a^(1/3)*(6
*b^2*c - 3*a*b*d + a^2*e))/x + (126*a^(4/3)*(-(b^3*c) + a*b^2*d - a^2*b*e
+ a^3*f)*x^2)/(a + b*x^3)^2 + (84*a^(1/3)*(-11*b^3*c + 8*a*b^2*d - 5*a^2*b
*e + 2*a^3*f)*x^2)/(a + b*x^3) + (28*Sqrt[3]*(65*b^3*c - 35*a*b^2*d + 14*a
^2*b*e - 2*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (
28*(65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Log[a^(1/3) + b^(1/3)*x]
)/b^(2/3) + (14*(-65*b^3*c + 35*a*b^2*d - 14*a^2*b*e + 2*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(756*a^(16/3))
```

3.299.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2368, 27, 2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8 (a+bx^3)^3} \, dx$$

$$\Big\downarrow \text{2368}$$

$$-\frac{\int -\frac{2\left(-\frac{2b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c\right)}{x^8(bx^3+a)^2}dx}{6ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{-\frac{2b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^8(bx^3+a)^2}dx}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 2368

$$-\frac{-\frac{\int -\frac{-\frac{b^6\left(-2fa^3+5bea^2-8b^2da+11b^3c\right)x^9}{a^3}+\frac{9b^6\left(ea^2-2bda+3b^2c\right)x^6}{a^2}-9b^6\left(\frac{2bc}{a}-d\right)x^3+9b^6c}{x^8\left(bx^3+a\right)}dx}{3ab^3}-\frac{b^3x^2\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{3a^4(a+bx^3)}}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{-\frac{b^6\left(-2fa^3+5bea^2-8b^2da+11b^3c\right)x^9}{a^3}+\frac{9b^6\left(ea^2-2bda+3b^2c\right)x^6}{a^2}-9b^6\left(\frac{2bc}{a}-d\right)x^3+9b^6c}{x^8\left(bx^3+a\right)}dx}{3ab^3}-\frac{b^3x^2\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{3a^4(a+bx^3)}}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 2373

$$\frac{\frac{\int \left(\frac{\left(2fa^3-14bea^2+35b^2da-65b^3c\right)xb^6}{a^3\left(bx^3+a\right)}+\frac{9\left(ea^2-3bda+6b^2c\right)b^6}{a^3x^2}+\frac{9(ad-3bc)b^6}{a^2x^5}+\frac{9cb^6}{ax^8}\right)dx}{3ab^3}-\frac{b^3x^2\left(-2a^3f+5a^2be-8ab^2d+11b^3c\right)}{3a^4(a+bx^3)}}{3ab^3}-\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 2009

3.299. $\int \frac{c+dx^3+ex^6+fx^9}{x^8(a+bx^3)^3}\,dx$

$$\frac{\frac{\frac{9b^6(3bc-ad)}{4a^2x^4}-\frac{9b^6\left(a^2e-3abd+6b^2c\right)}{a^3x}+\frac{b^{16/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^3f+14a^2be-35ab^2d+65b^3c\right)}{\sqrt{3}a^{10/3}}-\frac{b^{16/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-2a^3f+14a^2\cdots\right)}{6a^{10/3}}}{3ab^3}}{3ab^3}$$

$$\frac{x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x]
```

output
```
-1/6*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a^4*(a + b*x^3)^2) + (-1/3
*(b^3*(11*b^3*c - 8*a*b^2*d + 5*a^2*b*e - 2*a^3*f)*x^2)/(a^4*(a + b*x^3))
+ ((-9*b^6*c)/(7*a*x^7) + (9*b^6*(3*b*c - a*d))/(4*a^2*x^4) - (9*b^6*(6*b^
2*c - 3*a*b*d + a^2*e))/(a^3*x) + (b^(16/3)*(65*b^3*c - 35*a*b^2*d + 14*a^
2*b*e - 2*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[
3]*a^(10/3)) + (b^(16/3)*(65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*Lo
g[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)) - (b^(16/3)*(65*b^3*c - 35*a*b^2*d +
14*a^2*b*e - 2*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a
^(10/3)))/(3*a*b^3))/(3*a*b^3)
```

3.299.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.299.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.74

method	result
default	$-\frac{c}{7a^3x^7}-\frac{ad-3bc}{4a^4x^4}-\frac{a^2e-3abd+6b^2c}{a^5x}+\frac{\frac{\left(\frac{2}{9}a^3bf-\frac{5}{9}a^2eb^2+\frac{8}{9}ab^3d-\frac{11}{9}b^4c\right)x^5+\frac{a\left(7fa^3-13a^2be+19ab^2d-25b^3c\right)x^2}{18}}{\left(bx^3+a\right)^2}+\left(\frac{2}{9}fa^3-\frac{14}{9}a\right.}{}$
risch	$\frac{\frac{b\left(2fa^3-14a^2be+35ab^2d-65b^3c\right)x^{12}}{9a^5}+\frac{7\left(2fa^3-14a^2be+35ab^2d-65b^3c\right)x^9}{36a^4}-\frac{\left(14a^2e-35abd+65b^2c\right)x^6}{14a^3}-\frac{(7ad-13bc)x^3}{28a^2}-\frac{c}{7a}}{x^7(bx^3+a)^2}+\frac{\left(\sum_{\_R=\mathrm{Root}}\right.}{}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/7*c/a^3/x^7-1/4*(a*d-3*b*c)/a^4/x^4-(a^2*e-3*a*b*d+6*b^2*c)/a^5/x+1/a^5
*(((2/9*a^3*b*f-5/9*a^2*e*b^2+8/9*a*b^3*d-11/9*b^4*c)*x^5+1/18*a*(7*a^3*f-
13*a^2*b*e+19*a*b^2*d-25*b^3*c)*x^2)/(b*x^3+a)^2+(2/9*f*a^3-14/9*a^2*b*e+3
5/9*a*b^2*d-65/9*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^
(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.299.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(298) = 596.

Time = 0.30 (sec) , antiderivative size = 1340, normalized size of antiderivative = 3.91

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/756*(84*(65*a*b^6*c - 35*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3*f)*x^12
+ 147*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^9 + 108
*a^5*b^2*c + 54*(65*a^3*b^4*c - 35*a^4*b^3*d + 14*a^5*b^2*e)*x^6 - 27*(13*
a^4*b^3*c - 7*a^5*b^2*d)*x^3 + 42*sqrt(1/3)*((65*a*b^6*c - 35*a^2*b^5*d +
14*a^3*b^4*e - 2*a^4*b^3*f)*x^13 + 2*(65*a^2*b^5*c - 35*a^3*b^4*d + 14*a^4
*b^3*e - 2*a^5*b^2*f)*x^10 + (65*a^3*b^4*c - 35*a^4*b^3*d + 14*a^5*b^2*e -
 2*a^6*b*f)*x^7)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)
*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a)
- 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 14*((65*b^5*c - 35*a*b^4*d + 14*a^2*b
^3*e - 2*a^3*b^2*f)*x^13 + 2*(65*a*b^4*c - 35*a^2*b^3*d + 14*a^3*b^2*e - 2
*a^4*b*f)*x^10 + (65*a^2*b^3*c - 35*a^3*b^2*d + 14*a^4*b*e - 2*a^5*f)*x^7)
*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*((
65*b^5*c - 35*a*b^4*d + 14*a^2*b^3*e - 2*a^3*b^2*f)*x^13 + 2*(65*a*b^4*c -
 35*a^2*b^3*d + 14*a^3*b^2*e - 2*a^4*b*f)*x^10 + (65*a^2*b^3*c - 35*a^3*b^
2*d + 14*a^4*b*e - 2*a^5*f)*x^7)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))
/(a^6*b^4*x^13 + 2*a^7*b^3*x^10 + a^8*b^2*x^7), -1/756*(84*(65*a*b^6*c - 3
5*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3*f)*x^12 + 147*(65*a^2*b^5*c - 35*a^
3*b^4*d + 14*a^4*b^3*e - 2*a^5*b^2*f)*x^9 + 108*a^5*b^2*c + 54*(65*a^3*b^4
*c - 35*a^4*b^3*d + 14*a^5*b^2*e)*x^6 - 27*(13*a^4*b^3*c - 7*a^5*b^2*d)*x^
3 + 84*sqrt(1/3)*((65*a*b^6*c - 35*a^2*b^5*d + 14*a^3*b^4*e - 2*a^4*b^3...
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**8/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx = -\frac{28\,(65\,b^4c-35\,ab^3d+14\,a^2b^2e-2\,a^3bf)x^{12}+49\,(65\,ab^3c-35\,a^2b^2d+14\,a^3be-2\,a^4f)x^9+18\,(65\,a^2b^2\cdots}{252\,(a^5b^2x^{13}+2\,a^6bx^{10}+a^7x^7)} - \frac{\sqrt{3}(65\,b^3c-35\,ab^2d+14\,a^2be-2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(65\,b^3c-35\,ab^2d+14\,a^2be-2\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^5b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(65\,b^3c-35\,ab^2d+14\,a^2be-2\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^5b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/252*(28*(65*b^4*c - 35*a*b^3*d + 14*a^2*b^2*e - 2*a^3*b*f)*x^12 + 49*(6
5*a*b^3*c - 35*a^2*b^2*d + 14*a^3*b*e - 2*a^4*f)*x^9 + 18*(65*a^2*b^2*c -
35*a^3*b*d + 14*a^4*e)*x^6 + 36*a^4*c - 9*(13*a^3*b*c - 7*a^4*d)*x^3)/(a^5
*b^2*x^13 + 2*a^6*b*x^10 + a^7*x^7) - 1/27*sqrt(3)*(65*b^3*c - 35*a*b^2*d
+ 14*a^2*b*e - 2*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a^5*b*(a/b)^(1/3)) - 1/54*(65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*b*(a/b)^(1/3)) + 1/27*(65*b^
3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*log(x + (a/b)^(1/3))/(a^5*b*(a/b)
^(1/3))
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.09

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{\sqrt{3}(65\,b^3c-35\,ab^2d+14\,a^2be-2\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^5}$$
$$+\frac{(65\,b^3c-35\,ab^2d+14\,a^2be-2\,a^3f)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{1}{3}}a^5}$$
$$+\frac{\left(65\,b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-35\,ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+14\,a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^6}$$
$$-\frac{22\,b^4cx^5-16\,ab^3dx^5+10\,a^2b^2ex^5-4\,a^3bfx^5+25\,ab^3cx^2-19\,a^2b^2dx^2+13\,a^3bex^2-7\,a^4fx^2}{18\left(bx^3+a\right)^2a^5}$$
$$-\frac{168\,b^2cx^6-84\,abdx^6+28\,a^2ex^6-21\,abcx^3+7\,a^2dx^3+4\,a^2c}{28\,a^5x^7}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^8/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(65*b^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*arctan(1/3*sq
rt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^5) + 1/54*(65*b
^3*c - 35*a*b^2*d + 14*a^2*b*e - 2*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/((-a*b^2)^(1/3)*a^5) + 1/27*(65*b^3*c*(-a/b)^(1/3) - 35*a*b^2*d*(
-a/b)^(1/3) + 14*a^2*b*e*(-a/b)^(1/3) - 2*a^3*f*(-a/b)^(1/3))*(-a/b)^(1/3)
*log(abs(x - (-a/b)^(1/3)))/a^6 - 1/18*(22*b^4*c*x^5 - 16*a*b^3*d*x^5 + 10
*a^2*b^2*e*x^5 - 4*a^3*b*f*x^5 + 25*a*b^3*c*x^2 - 19*a^2*b^2*d*x^2 + 13*a^
3*b*e*x^2 - 7*a^4*f*x^2)/((b*x^3 + a)^2*a^5) - 1/28*(168*b^2*c*x^6 - 84*a*
b*d*x^6 + 28*a^2*e*x^6 - 21*a*b*c*x^3 + 7*a^2*d*x^3 + 4*a^2*c)/(a^5*x^7)
```

3.299.9 Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^8\left(a+bx^3\right)^3}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{27\,a^{16/3}\,b^{2/3}}$$
$$-\frac{\frac{c}{7\,a}+\frac{7\,x^9\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{36\,a^4}+\frac{x^3\,\left(7\,a\,d-13\,b\,c\right)}{28\,a^2}+\frac{x^6\,\left(14\,e\,a^2-35\,d\,a\,b+65\,c\,b^2\right)}{14\,a^3}+\frac{b\,x^{12}\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b}{9\,a^5}}{a^2\,x^7+2\,a\,b\,x^{10}+b^2\,x^{13}}$$
$$-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{27\,a^{16/3}\,b^{2/3}}$$
$$+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-2\,f\,a^3+14\,e\,a^2\,b-35\,d\,a\,b^2+65\,c\,b^3\right)}{27\,a^{16/3}\,b^{2/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^8*(a + b*x^3)^3),x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(65*b^3*c - 2*a^3*f - 35*a*b^2*d + 14*a^2*b*e))/
(27*a^(16/3)*b^(2/3)) - (c/(7*a) + (7*x^9*(65*b^3*c - 2*a^3*f - 35*a*b^2*d
 + 14*a^2*b*e))/(36*a^4) + (x^3*(7*a*d - 13*b*c))/(28*a^2) + (x^6*(65*b^2*
c + 14*a^2*e - 35*a*b*d))/(14*a^3) + (b*x^12*(65*b^3*c - 2*a^3*f - 35*a*b^
2*d + 14*a^2*b*e))/(9*a^5))/(a^2*x^7 + b^2*x^13 + 2*a*b*x^10) - (log(3^(1/
2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(65*b^3*c -
2*a^3*f - 35*a*b^2*d + 14*a^2*b*e))/(27*a^(16/3)*b^(2/3)) + (log(3^(1/2)*a
^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(65*b^3*c - 2*a^
3*f - 35*a*b^2*d + 14*a^2*b*e))/(27*a^(16/3)*b^(2/3))
```

3.300 $\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^3}\,dx$

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3.300.1 Optimal result

Integrand size = 30, antiderivative size = 341

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^9(a+bx^3)^3}\,dx &= -\frac{c}{8a^3x^8}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{2a^5x^2}\\ &\quad-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^4(a+bx^3)^2}-\frac{(23b^3c-17ab^2d+11a^2be-5a^3f)\,x}{18a^5(a+bx^3)}\\ &\quad+\frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{17/3}\sqrt[3]{b}}\\ &\quad-\frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{17/3}\sqrt[3]{b}}\\ &\quad+\frac{(77b^3c-44ab^2d+20a^2be-5a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{17/3}\sqrt[3]{b}}\end{aligned}$$

output
```
-1/8*c/a^3/x^8+1/5*(-a*d+3*b*c)/a^4/x^5+1/2*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x
^2-1/6*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^3+a)^2-1/18*(-5*a^3*f+11*
a^2*b*e-17*a*b^2*d+23*b^3*c)*x/a^5/(b*x^3+a)-1/27*(-5*a^3*f+20*a^2*b*e-44*
a*b^2*d+77*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(17/3)/b^(1/3)+1/54*(-5*a^3*f+20
*a^2*b*e-44*a*b^2*d+77*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^
(17/3)/b^(1/3)+1/27*(-5*a^3*f+20*a^2*b*e-44*a*b^2*d+77*b^3*c)*arctan(1/3*(
a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(17/3)/b^(1/3)*3^(1/2)
```

3.300.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{135a^{8/3}c}{x^8}-\frac{216a^{5/3}(-3bc+ad)}{x^5}-\frac{540a^{2/3}(6b^2c-3abd+a^2e)}{x^2}+\frac{180a^{5/3}(-b^3c+ab^2d-a^2be+a^3f)x}{(a+bx^3)^2}+\frac{60a^{2/3}(-23b^3c+17ab^2d-11a^2be\cdots}{a+bx^3}}{\cdots}$$

input

```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```

output

```
((-135*a^(8/3)*c)/x^8 - (216*a^(5/3)*(-3*b*c + a*d))/x^5 - (540*a^(2/3)*(6
*b^2*c - 3*a*b*d + a^2*e))/x^2 + (180*a^(5/3)*(-(b^3*c) + a*b^2*d - a^2*b*
e + a^3*f)*x)/(a + b*x^3)^2 + (60*a^(2/3)*(-23*b^3*c + 17*a*b^2*d - 11*a^2
*b*e + 5*a^3*f)*x)/(a + b*x^3) + (40*Sqrt[3]*(77*b^3*c - 44*a*b^2*d + 20*a
^2*b*e - 5*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (
40*(-77*b^3*c + 44*a*b^2*d - 20*a^2*b*e + 5*a^3*f)*Log[a^(1/3) + b^(1/3)*x
])/b^(1/3) + (20*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(1080*a^(17/3))
```

3.300.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \text{ 2368}$$

$$-\frac{\int -\frac{-\frac{5b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{6b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-6b^3\left(\frac{bc}{a}-d\right)x^3+6b^3c}{x^9\left(bx^3+a\right)^2}dx}{6ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-\frac{5b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{6b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-6b^3\left(\frac{bc}{a}-d\right)x^3+6b^3c}{x^9\left(bx^3+a\right)^2}dx}{6ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2368$$

$$-\frac{-\frac{\int -\frac{2\left(-\frac{b^6\left(-5fa^3+11bea^2-17b^2da+23b^3c\right)x^9}{a^3}+\frac{9b^6\left(ea^2-2bda+3b^2c\right)x^6}{a^2}-9b^6\left(\frac{2bc}{a}-d\right)x^3+9b^6c\right)}{x^9\left(bx^3+a\right)}dx}{3ab^3}-\frac{b^3x\left(-5a^3f+11a^2be-17ab^2d+23b^3c\right)}{3a^4\left(a+bx^3\right)}}{6ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{2\int \frac{-\frac{b^6\left(-5fa^3+11bea^2-17b^2da+23b^3c\right)x^9}{a^3}+\frac{9b^6\left(ea^2-2bda+3b^2c\right)x^6}{a^2}-9b^6\left(\frac{2bc}{a}-d\right)x^3+9b^6c}{x^9\left(bx^3+a\right)}dx}{3ab^3}-\frac{b^3x\left(-5a^3f+11a^2be-17ab^2d+23b^3c\right)}{3a^4\left(a+bx^3\right)}}{6ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2373$$

$$-\frac{\frac{2\int \left(\frac{\left(5fa^3-20bea^2+44b^2da-77b^3c\right)b^6}{a^3\left(bx^3+a\right)}+\frac{9\left(ea^2-3bda+6b^2c\right)b^6}{a^3x^3}+\frac{9(ad-3bc)b^6}{a^2x^6}+\frac{9cb^6}{ax^9}\right)dx}{3ab^3}-\frac{b^3x\left(-5a^3f+11a^2be-17ab^2d+23b^3c\right)}{3a^4\left(a+bx^3\right)}}{6ab^3}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2009$$

3.300. $\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{2\left(\frac{9b^6(3bc-ad)}{5a^2x^5}-\frac{9b^6\left(a^2e-3abd+6b^2c\right)}{2a^3x^2}+\frac{b^{17/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^3f+20a^2be-44ab^2d+77b^3c\right)}{\sqrt{3}a^{11/3}}+\frac{b^{17/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-5a^3f+2\cdots\right.}{6a^{11/3}}}{3ab^3}}{6ab^3}$$

$$\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^4\left(a+bx^3\right)^2}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x]
```

output
```
-1/6*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a^4*(a + b*x^3)^2) + (-1/3*(
b^3*(23*b^3*c - 17*a*b^2*d + 11*a^2*b*e - 5*a^3*f)*x)/(a^4*(a + b*x^3)) +
(2*((-9*b^6*c)/(8*a*x^8) + (9*b^6*(3*b*c - a*d))/(5*a^2*x^5) - (9*b^6*(6*b
^2*c - 3*a*b*d + a^2*e))/(2*a^3*x^2) + (b^(17/3)*(77*b^3*c - 44*a*b^2*d +
20*a^2*b*e - 5*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(11/3)) - (b^(17/3)*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*
f)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(11/3)) + (b^(17/3)*(77*b^3*c - 44*a*b^2
*d + 20*a^2*b*e - 5*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(6*a^(11/3))))/(3*a*b^3))/(6*a*b^3)
```

3.300.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.300.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.74

method	result
default	$-\frac{c}{8a^3x^8}-\frac{ad-3bc}{5a^4x^5}-\frac{a^2e-3abd+6b^2c}{2a^5x^2}+\frac{\left(\frac{5}{18}a^3bf-\frac{11}{18}a^2eb^2+\frac{17}{18}ab^3d-\frac{23}{18}b^4c\right)x^4+\frac{a\left(4fa^3-7a^2be+10ab^2d-13b^3c\right)x}{9}}{\left(bx^3+a\right)^2}+$ ... $\left(5fa^3-20a^2\right.$ ...
risch	$\frac{\frac{b\left(5fa^3-20a^2be+44ab^2d-77b^3c\right)x^{12}}{18a^5}+\frac{4\left(5fa^3-20a^2be+44ab^2d-77b^3c\right)x^9}{45a^4}-\frac{\left(20a^2e-44abd+77b^2c\right)x^6}{40a^3}-\frac{(4ad-7bc)x^3}{20a^2}-\frac{c}{8a}}{x^8\left(bx^3+a\right)^2}+$ ... $\left(\_R=\text{RootO}\right.$ ...

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*c/a^3/x^8-1/5*(a*d-3*b*c)/a^4/x^5-1/2*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^2
+1/a^5*(((5/18*a^3*b*f-11/18*a^2*e*b^2+17/18*a*b^3*d-23/18*b^4*c)*x^4+1/9*
a*(4*a^3*f-7*a^2*b*e+10*a*b^2*d-13*b^3*c)*x)/(b*x^3+a)^2+1/9*(5*a^3*f-20*a
^2*b*e+44*a*b^2*d-77*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/
b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

3.300.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(294) = 588$.

Time = 0.28 (sec) , antiderivative size = 1317, normalized size of antiderivative = 3.86

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^9 (a + bx^3)^3} \, dx = \text{Too large to display}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/1080*(60*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e - 5*a^5*b^2*f)*x^
12 + 96*(77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*b*f)*x^9 + 135
*a^6*b*c + 27*(77*a^4*b^3*c - 44*a^5*b^2*d + 20*a^6*b*e)*x^6 - 54*(7*a^5*b
^2*c - 4*a^6*b*d)*x^3 + 60*sqrt(1/3)*((77*a*b^6*c - 44*a^2*b^5*d + 20*a^3*
b^4*e - 5*a^4*b^3*f)*x^14 + 2*(77*a^2*b^5*c - 44*a^3*b^4*d + 20*a^4*b^3*e
- 5*a^5*b^2*f)*x^11 + (77*a^3*b^4*c - 44*a^4*b^3*d + 20*a^5*b^2*e - 5*a^6*
b*f)*x^8)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2
*b)^(1/3)/b))/(b*x^3 + a)) - 20*((77*b^5*c - 44*a*b^4*d + 20*a^2*b^3*e - 5
*a^3*b^2*f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3*d + 20*a^3*b^2*e - 5*a^4*b*f
)*x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^4*b*e - 5*a^5*f)*x^8)*(a^2*b)
^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*((77*b^5*c -
44*a*b^4*d + 20*a^2*b^3*e - 5*a^3*b^2*f)*x^14 + 2*(77*a*b^4*c - 44*a^2*b^3
*d + 20*a^3*b^2*e - 5*a^4*b*f)*x^11 + (77*a^2*b^3*c - 44*a^3*b^2*d + 20*a^
4*b*e - 5*a^5*f)*x^8)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^7*b^3*x
^14 + 2*a^8*b^2*x^11 + a^9*b*x^8), -1/1080*(60*(77*a^2*b^5*c - 44*a^3*b^4*
d + 20*a^4*b^3*e - 5*a^5*b^2*f)*x^12 + 96*(77*a^3*b^4*c - 44*a^4*b^3*d + 2
0*a^5*b^2*e - 5*a^6*b*f)*x^9 + 135*a^6*b*c + 27*(77*a^4*b^3*c - 44*a^5*b^2
*d + 20*a^6*b*e)*x^6 - 54*(7*a^5*b^2*c - 4*a^6*b*d)*x^3 + 120*sqrt(1/3)*((
77*a*b^6*c - 44*a^2*b^5*d + 20*a^3*b^4*e - 5*a^4*b^3*f)*x^14 + 2*(77*a^...
```

3.300.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^3}\,dx=$$

$$-\frac{20\,(77\,b^4c-44\,ab^3d+20\,a^2b^2e-5\,a^3bf)x^{12}+32\,(77\,ab^3c-44\,a^2b^2d+20\,a^3be-5\,a^4f)x^9+9\,(77\,a^2b^2c}{360\,(a^5b^2x^{14}+2\,a^6bx^{11}+a^7x^8)}$$

$$-\frac{\sqrt{3}(77\,b^3c-44\,ab^2d+20\,a^2be-5\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$+\frac{(77\,b^3c-44\,ab^2d+20\,a^2be-5\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^5b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$-\frac{(77\,b^3c-44\,ab^2d+20\,a^2be-5\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^5b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/360*(20*(77*b^4*c - 44*a*b^3*d + 20*a^2*b^2*e - 5*a^3*b*f)*x^12 + 32*(7
7*a*b^3*c - 44*a^2*b^2*d + 20*a^3*b*e - 5*a^4*f)*x^9 + 9*(77*a^2*b^2*c - 4
4*a^3*b*d + 20*a^4*e)*x^6 + 45*a^4*c - 18*(7*a^3*b*c - 4*a^4*d)*x^3)/(a^5*
b^2*x^14 + 2*a^6*b*x^11 + a^7*x^8) - 1/27*sqrt(3)*(77*b^3*c - 44*a*b^2*d +
 20*a^2*b*e - 5*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a^5*b*(a/b)^(2/3)) + 1/54*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*b*(a/b)^(2/3)) - 1/27*(77*b^3
*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*log(x + (a/b)^(1/3))/(a^5*b*(a/b)^
(2/3))
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\left(a+bx^3\right)^3}\,dx = \frac{\left(77\,b^3c-44\,ab^2d+20\,a^2be-5\,a^3f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^6} - \frac{\sqrt{3}\left(77\left(-ab^2\right)^{\frac{1}{3}}b^3c-44\left(-ab^2\right)^{\frac{1}{3}}ab^2d+20\left(-ab^2\right)^{\frac{1}{3}}a^2be-5\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^6b} - \frac{\left(77\left(-ab^2\right)^{\frac{1}{3}}b^3c-44\left(-ab^2\right)^{\frac{1}{3}}ab^2d+20\left(-ab^2\right)^{\frac{1}{3}}a^2be-5\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^6b} - \frac{23\,b^4cx^4-17\,ab^3dx^4+11\,a^2b^2ex^4-5\,a^3bfx^4+26\,ab^3cx-20\,a^2b^2dx+14\,a^3bex-8\,a^4fx}{18\left(bx^3+a\right)^2a^5} - \frac{120\,b^2cx^6-60\,abdx^6+20\,a^2ex^6-24\,abcx^3+8\,a^2dx^3+5\,a^2c}{40\,a^5x^8}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^9/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/27*(77*b^3*c - 44*a*b^2*d + 20*a^2*b*e - 5*a^3*f)*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/a^6 - 1/27*sqrt(3)*(77*(-a*b^2)^(1/3)*b^3*c - 44*(-a*b^2
)^(1/3)*a*b^2*d + 20*(-a*b^2)^(1/3)*a^2*b*e - 5*(-a*b^2)^(1/3)*a^3*f)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^6*b) - 1/54*(77*(-a*b
^2)^(1/3)*b^3*c - 44*(-a*b^2)^(1/3)*a*b^2*d + 20*(-a*b^2)^(1/3)*a^2*b*e -
5*(-a*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^6*b) -
 1/18*(23*b^4*c*x^4 - 17*a*b^3*d*x^4 + 11*a^2*b^2*e*x^4 - 5*a^3*b*f*x^4 +
26*a*b^3*c*x - 20*a^2*b^2*d*x + 14*a^3*b*e*x - 8*a^4*f*x)/((b*x^3 + a)^2*a
^5) - 1/40*(120*b^2*c*x^6 - 60*a*b*d*x^6 + 20*a^2*e*x^6 - 24*a*b*c*x^3 + 8
*a^2*d*x^3 + 5*a^2*c)/(a^5*x^8)
```

3.300.9 Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^9\,(a+bx^3)^3}\,dx =$$

$$-\frac{\frac{c}{8\,a}+\frac{4\,x^9\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{45\,a^4}+\frac{x^3\,\left(4\,a\,d-7\,b\,c\right)}{20\,a^2}+\frac{x^6\,\left(20\,e\,a^2-44\,d\,a\,b+77\,c\,b^2\right)}{40\,a^3}+\frac{b\,x^{12}\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2}{18\,a^5}}{a^2\,x^8+2\,a\,b\,x^{11}+b^2\,x^{14}}$$

$$-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{27\,a^{17/3}\,b^{1/3}}$$

$$-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{27\,a^{17/3}\,b^{1/3}}$$

$$+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-5\,f\,a^3+20\,e\,a^2\,b-44\,d\,a\,b^2+77\,c\,b^3\right)}{27\,a^{17/3}\,b^{1/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^9*(a + b*x^3)^3),x)
```

output
```
(log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(7
7*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e))/(27*a^(17/3)*b^(1/3)) - (log
(b^(1/3)*x + a^(1/3))*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e))/(27*
a^(17/3)*b^(1/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(
1/2)*1i)/2 - 1/2)*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^2*b*e))/(27*a^(1
7/3)*b^(1/3)) - (c/(8*a) + (4*x^9*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*a^
2*b*e))/(45*a^4) + (x^3*(4*a*d - 7*b*c))/(20*a^2) + (x^6*(77*b^2*c + 20*a^
2*e - 44*a*b*d))/(40*a^3) + (b*x^12*(77*b^3*c - 5*a^3*f - 44*a*b^2*d + 20*
a^2*b*e))/(18*a^5))/(a^2*x^8 + b^2*x^14 + 2*a*b*x^11)
```

3.301 $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^3}\,dx$

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3.301.1 Optimal result

Integrand size = 30, antiderivative size = 381

$$\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^3}\,dx \\
&= -\frac{c}{10a^3x^{10}}+\frac{3bc-ad}{7a^4x^7}-\frac{6b^2c-3abd+a^2e}{4a^5x^4}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} \\
&\quad+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^5(a+bx^3)^2}+\frac{b(14b^3c-11ab^2d+8a^2be-5a^3f)\,x^2}{9a^6(a+bx^3)} \\
&\quad-\frac{\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{19/3}} \\
&\quad-\frac{\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{19/3}} \\
&\quad+\frac{\sqrt[3]{b}(104b^3c-65ab^2d+35a^2be-14a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{19/3}}
\end{aligned}$$

output
```
-1/10*c/a^3/x^10+1/7*(-a*d+3*b*c)/a^4/x^7+1/4*(-a^2*e+3*a*b*d-6*b^2*c)/a^5
/x^4+(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x+1/6*b*(-a^3*f+a^2*b*e-a*b
^2*d+b^3*c)*x^2/a^5/(b*x^3+a)^2+1/9*b*(-5*a^3*f+8*a^2*b*e-11*a*b^2*d+14*b^
3*c)*x^2/a^6/(b*x^3+a)-1/27*b^(1/3)*(-14*a^3*f+35*a^2*b*e-65*a*b^2*d+104*b
^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(19/3)+1/54*b^(1/3)*(-14*a^3*f+35*a^2*b*e-65
*a*b^2*d+104*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(19/3)-1/2
7*b^(1/3)*(-14*a^3*f+35*a^2*b*e-65*a*b^2*d+104*b^3*c)*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(19/3)*3^(1/2)
```

3.301.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{378a^{10/3}c}{x^{10}} - \frac{540a^{7/3}(-3bc+ad)}{x^7} - \frac{945a^{4/3}(6b^2c-3abd+a^2e)}{x^4} - \frac{3780\sqrt[3]{a}(-10b^3c+6ab^2d-3a^2be+a^3f)}{x} - \frac{630a^{4/3}b(-b^3c+ab^2d-a^2be}{(a+bx^3)^2}}{}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

output
```
((-378*a^(10/3)*c)/x^10 - (540*a^(7/3)*(-3*b*c + a*d))/x^7 - (945*a^(4/3)*
(6*b^2*c - 3*a*b*d + a^2*e))/x^4 - (3780*a^(1/3)*(-10*b^3*c + 6*a*b^2*d -
3*a^2*b*e + a^3*f))/x - (630*a^(4/3)*b*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3
*f)*x^2)/(a + b*x^3)^2 - (420*a^(1/3)*b*(-14*b^3*c + 11*a*b^2*d - 8*a^2*b*
e + 5*a^3*f)*x^2)/(a + b*x^3) - 140*Sqrt[3]*b^(1/3)*(104*b^3*c - 65*a*b^2*
d + 35*a^2*b*e - 14*a^3*f)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 1
40*b^(1/3)*(-104*b^3*c + 65*a*b^2*d - 35*a^2*b*e + 14*a^3*f)*Log[a^(1/3) +
 b^(1/3)*x] + 70*b^(1/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3780*a^(19/3))
```

3.301.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2368, 27, 2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \; 2368$$

$$\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}-\frac{\int -\frac{2\left(\frac{2b^4\left(-fa^3+bea^2-b^2da+b^3c\right)x^{12}}{a^4}-\frac{3b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c\right)}{x^{11}(bx^3+a)^2}dx}{6ab^3}$$

$\downarrow$ 27

$$\frac{\int \frac{\frac{2b^4\left(-fa^3+bea^2-b^2da+b^3c\right)x^{12}}{a^4}-\frac{3b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^{11}(bx^3+a)^2}dx}{3ab^3}+\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

$\downarrow$ 2368

$$\frac{\frac{b^4x^2\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)}{3a^5(a+bx^3)}-\frac{\int -\frac{\frac{b^8\left(-5fa^3+8bea^2-11b^2da+14b^3c\right)x^{12}}{a^4}-\frac{9b^7\left(-fa^3+2bea^2-3b^2da+4b^3c\right)x^9}{a^3}+\frac{9b^7\left(ea^2-2bda+3b^2c\right)x^6}{a^2}-9b^7\left(\cdots\right.}{x^{11}\left(bx^3+a\right)}\cdots}{3ab^4}}{3ab^3}+\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{\frac{b^8\left(-5fa^3+8bea^2-11b^2da+14b^3c\right)x^{12}}{a^4}-\frac{9b^7\left(-fa^3+2bea^2-3b^2da+4b^3c\right)x^9}{a^3}+\frac{9b^7\left(ea^2-2bda+3b^2c\right)x^6}{a^2}-9b^7\left(\frac{2bc}{a}-d\right)x^3+9b^7c}{x^{11}\left(bx^3+a\right)}dx}{3ab^4}+\frac{b^4x^2\left(-5a^3f+8a^2be-\cdots\right.}{3a^5(a+b\cdots}}{3ab^3}+\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

$\downarrow$ 2373

$$\frac{\frac{\int \left(-\frac{\left(14fa^3-35bea^2+65b^2da-104b^3c\right)xb^8}{a^4\left(bx^3+a\right)}+\frac{9\left(fa^3-3bea^2+6b^2da-10b^3c\right)b^7}{a^4x^2}+\frac{9\left(ea^2-3bda+6b^2c\right)b^7}{a^3x^5}+\frac{9(ad-3bc)b^7}{a^2x^8}+\frac{9cb^7}{ax^{11}}\right)dx}{3ab^4}+\frac{b^4x^2\left(-5a^3f+8a^2be-\cdots\right.}{3a^5(a+ba\cdots}}{3ab^3}+\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

$\downarrow$ 2009

3.301. $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^3}\,dx$

$$\frac{bx^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}+$$

$$\frac{b^4x^2\left(-5a^3f+8a^2be-11ab^2d+14b^3c\right)}{3a^5(a+bx^3)}+\frac{\frac{9b^7(3bc-ad)}{7a^2x^7}-\frac{9b^7\left(a^2e-3abd+6b^2c\right)}{4a^3x^4}-\frac{b^{22/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-14a^3f+35a^2be-65ab^2d+104b^3c\right)}{\sqrt{3}a^{13/3}}+\frac{b^{22/3}}{}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x]
```

output
```
(b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(6*a^5*(a + b*x^3)^2) + ((b^4*
(14*b^3*c - 11*a*b^2*d + 8*a^2*b*e - 5*a^3*f)*x^2)/(3*a^5*(a + b*x^3)) + (
(-9*b^7*c)/(10*a*x^10) + (9*b^7*(3*b*c - a*d))/(7*a^2*x^7) - (9*b^7*(6*b^2
*c - 3*a*b*d + a^2*e))/(4*a^3*x^4) + (9*b^7*(10*b^3*c - 6*a*b^2*d + 3*a^2*
b*e - a^3*f))/(a^4*x) - (b^(22/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 1
4*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(13
/3)) - (b^(22/3)*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*Log[a^(1
/3) + b^(1/3)*x])/(3*a^(13/3)) + (b^(22/3)*(104*b^3*c - 65*a*b^2*d + 35*a^
2*b*e - 14*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(13
/3)))/(3*a*b^4))/(3*a*b^3)
```

3.301.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.301. $\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx$

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.301.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.76

method	result
default	$-\frac{c}{10a^{3}x^{10}}-\frac{ad-3bc}{7a^{4}x^{7}}-\frac{a^{2}e-3abd+6b^{2}c}{4a^{5}x^{4}}-\frac{f\,a^{3}-3a^{2}be+6a\,b^{2}d-10b^{3}c}{a^{6}x}-\frac{b\left(\frac{\frac{b\left(5f\,a^{3}-8a^{2}be+11a\,b^{2}d-14b^{3}c\right)x^{5}}{9}+\left(\frac{13}{18}a^{4}f-\frac{19}{18}a^{3}\right.}{\left(b\,x^{3}+a\right)^{2}}\right.}{}$
risch	$\frac{-\frac{c}{10a}-\frac{(5ad-8bc)x^{3}}{35a^{2}}-\frac{\left(35a^{2}e-65abd+104b^{2}c\right)x^{6}}{140a^{3}}-\frac{\left(14f\,a^{3}-35a^{2}be+65a\,b^{2}d-104b^{3}c\right)x^{9}}{14a^{4}}-\frac{7b\left(14f\,a^{3}-35a^{2}be+65a\,b^{2}d-104b^{3}c\right)x^{12}}{36a^{5}}-\frac{b^{2}\left(14f\right.}{}}{x^{10}\left(b\,x^{3}+a\right)^{2}}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/10*c/a^3/x^10-1/7*(a*d-3*b*c)/a^4/x^7-1/4*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x
^4-(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x-b/a^6*((1/9*b*(5*a^3*f-8*a^2
*b*e+11*a*b^2*d-14*b^3*c)*x^5+(13/18*a^4*f-19/18*a^3*b*e+25/18*a^2*b^2*d-3
1/18*a*b^3*c)*x^2)/(b*x^3+a)^2+(14/9*f*a^3-35/9*a^2*b*e+65/9*a*b^2*d-104/9
*b^3*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))))
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.63

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx$$
$$= \frac{420\,(104\,b^5c-65\,ab^4d+35\,a^2b^3e-14\,a^3b^2f)x^{15}+735\,(104\,ab^4c-65\,a^2b^3d+35\,a^3b^2e-14\,a^4bf)x^{12}+27}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/3780*(420*(104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^15 +
735*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^12 + 270*(1
04*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^9 - 27*(104*a^3*b^2
*c - 65*a^4*b*d + 35*a^5*e)*x^6 - 378*a^5*c + 108*(8*a^4*b*c - 5*a^5*d)*x^
3 + 140*sqrt(3)*((104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^
16 + 2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^13 + (10
4*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^10)*(b/a)^(1/3)*arct
an(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 70*((104*b^5*c - 65*a*b^4*d
+ 35*a^2*b^3*e - 14*a^3*b^2*f)*x^16 + 2*(104*a*b^4*c - 65*a^2*b^3*d + 35*a
^3*b^2*e - 14*a^4*b*f)*x^13 + (104*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e -
 14*a^5*f)*x^10)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3))
- 140*((104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^16 + 2*(10
4*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^13 + (104*a^2*b^3*
c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^10)*(b/a)^(1/3)*log(b*x + a*(b
/a)^(2/3)))/(a^6*b^2*x^16 + 2*a^7*b*x^13 + a^8*x^10)
```

3.301.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**11/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx$$
$$= \frac{140\,(104\,b^5c-65\,ab^4d+35\,a^2b^3e-14\,a^3b^2f)x^{15}+245\,(104\,ab^4c-65\,a^2b^3d+35\,a^3b^2e-14\,a^4bf)x^{12}+90}{1260\,(a^6b^2x}$$
$$+\frac{\sqrt{3}(104\,b^3c-65\,ab^2d+35\,a^2be-14\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^6\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$+\frac{(104\,b^3c-65\,ab^2d+35\,a^2be-14\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^6\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{(104\,b^3c-65\,ab^2d+35\,a^2be-14\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^6\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/1260*(140*(104*b^5*c - 65*a*b^4*d + 35*a^2*b^3*e - 14*a^3*b^2*f)*x^15 +
245*(104*a*b^4*c - 65*a^2*b^3*d + 35*a^3*b^2*e - 14*a^4*b*f)*x^12 + 90*(10
4*a^2*b^3*c - 65*a^3*b^2*d + 35*a^4*b*e - 14*a^5*f)*x^9 - 9*(104*a^3*b^2*c
 - 65*a^4*b*d + 35*a^5*e)*x^6 - 126*a^5*c + 36*(8*a^4*b*c - 5*a^5*d)*x^3)/
(a^6*b^2*x^16 + 2*a^7*b*x^13 + a^8*x^10) + 1/27*sqrt(3)*(104*b^3*c - 65*a*
b^2*d + 35*a^2*b*e - 14*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(a^6*(a/b)^(1/3)) + 1/54*(104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 1
4*a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(1/3)) - 1/27*(
104*b^3*c - 65*a*b^2*d + 35*a^2*b*e - 14*a^3*f)*log(x + (a/b)^(1/3))/(a^6*
(a/b)^(1/3))
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.26

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{\left(104\,b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-65\,ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+35\,a^2b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-14\,a^3bf\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7}$$

$$-\frac{\sqrt{3}\left(104\left(-ab^2\right)^{\frac{2}{3}}b^3c-65\left(-ab^2\right)^{\frac{2}{3}}ab^2d+35\left(-ab^2\right)^{\frac{2}{3}}a^2be-14\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^7b}$$

$$+\frac{\left(104\left(-ab^2\right)^{\frac{2}{3}}b^3c-65\left(-ab^2\right)^{\frac{2}{3}}ab^2d+35\left(-ab^2\right)^{\frac{2}{3}}a^2be-14\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^7b}$$

$$+\frac{28\,b^5cx^5-22\,ab^4dx^5+16\,a^2b^3ex^5-10\,a^3b^2fx^5+31\,ab^4cx^2-25\,a^2b^3dx^2+19\,a^3b^2ex^2-13\,a^4bfx^2}{18\left(bx^3+a\right)^2a^6}$$

$$+\frac{1400\,b^3cx^9-840\,ab^2dx^9+420\,a^2bex^9-140\,a^3fx^9-210\,ab^2cx^6+105\,a^2bdx^6-35\,a^3ex^6+60\,a^2bcx^3-}{140\,a^6x^{10}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^11/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*(104*b^4*c*(-a/b)^(1/3) - 65*a*b^3*d*(-a/b)^(1/3) + 35*a^2*b^2*e*(-a
/b)^(1/3) - 14*a^3*b*f*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)
))/a^7 - 1/27*sqrt(3)*(104*(-a*b^2)^(2/3)*b^3*c - 65*(-a*b^2)^(2/3)*a*b^2*
d + 35*(-a*b^2)^(2/3)*a^2*b*e - 14*(-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^7*b) + 1/54*(104*(-a*b^2)^(2/3)*b^
3*c - 65*(-a*b^2)^(2/3)*a*b^2*d + 35*(-a*b^2)^(2/3)*a^2*b*e - 14*(-a*b^2)^
(2/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^7*b) + 1/18*(28*b
^5*c*x^5 - 22*a*b^4*d*x^5 + 16*a^2*b^3*e*x^5 - 10*a^3*b^2*f*x^5 + 31*a*b^4
*c*x^2 - 25*a^2*b^3*d*x^2 + 19*a^3*b^2*e*x^2 - 13*a^4*b*f*x^2)/((b*x^3 + a
)^2*a^6) + 1/140*(1400*b^3*c*x^9 - 840*a*b^2*d*x^9 + 420*a^2*b*e*x^9 - 140
*a^3*f*x^9 - 210*a*b^2*c*x^6 + 105*a^2*b*d*x^6 - 35*a^3*e*x^6 + 60*a^2*b*c
*x^3 - 20*a^3*d*x^3 - 14*a^3*c)/(a^6*x^10)
```

3.301.9 Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{11}(a+bx^3)^3}\,dx =$$

$$-\frac{\frac{c}{10\,a}-\frac{x^9\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{14\,a^4}+\frac{x^3\,\left(5\,a\,d-8\,b\,c\right)}{35\,a^2}+\frac{x^6\,\left(35\,e\,a^2-65\,d\,a\,b+104\,c\,b^2\right)}{140\,a^3}-\frac{7\,b\,x^{12}\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\right.}{36\,a^5}}{a^2\,x^{10}+2\,a\,b\,x^{13}+b^2\,x^{16}}$$

$$-\frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{27\,a^{19/3}}$$

$$+\frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{27\,a^{19/3}}$$

$$-\frac{b^{1/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-14\,f\,a^3+35\,e\,a^2\,b-65\,d\,a\,b^2+104\,c\,b^3\right)}{27\,a^{19/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^11*(a + b*x^3)^3),x)
```

output
```
(b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 +
 1/2)*(104*b^3*c - 14*a^3*f - 65*a*b^2*d + 35*a^2*b*e))/(27*a^(19/3)) - (b
^(1/3)*log(b^(1/3)*x + a^(1/3))*(104*b^3*c - 14*a^3*f - 65*a*b^2*d + 35*a^
2*b*e))/(27*a^(19/3)) - (c/(10*a) - (x^9*(104*b^3*c - 14*a^3*f - 65*a*b^2*
d + 35*a^2*b*e))/(14*a^4) + (x^3*(5*a*d - 8*b*c))/(35*a^2) + (x^6*(104*b^2
*c + 35*a^2*e - 65*a*b*d))/(140*a^3) - (7*b*x^12*(104*b^3*c - 14*a^3*f - 6
5*a*b^2*d + 35*a^2*b*e))/(36*a^5) - (b^2*x^15*(104*b^3*c - 14*a^3*f - 65*a
*b^2*d + 35*a^2*b*e))/(9*a^6))/(a^2*x^10 + b^2*x^16 + 2*a*b*x^13) - (b^(1/
3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*
(104*b^3*c - 14*a^3*f - 65*a*b^2*d + 35*a^2*b*e))/(27*a^(19/3))
```

3.302 $\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^3} dx$

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3.302.2 Mathematica [A] (verified) . . . 2320
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3.302.9 Mupad [B] (verification not implemented) . . . 2327

3.302.1 Optimal result

Integrand size = 30, antiderivative size = 380

$$\begin{aligned}\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^3}\,dx &= -\frac{c}{11a^3x^{11}}+\frac{3bc-ad}{8a^4x^8}-\frac{6b^2c-3abd+a^2e}{5a^5x^5}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2}\\ &+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5(a+bx^3)^2}+\frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6(a+bx^3)}\\ &-\frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{20/3}}\\ &+\frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{20/3}}\\ &-\frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{20/3}}\end{aligned}$$

output
```
-1/11*c/a^3/x^11+1/8*(-a*d+3*b*c)/a^4/x^8+1/5*(-a^2*e+3*a*b*d-6*b^2*c)/a^5
/x^5+1/2*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^2+1/6*b*(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)*x/a^5/(b*x^3+a)^2+1/18*b*(-11*a^3*f+17*a^2*b*e-23*a*b^2*
d+29*b^3*c)*x/a^6/(b*x^3+a)+1/27*b^(2/3)*(-20*a^3*f+44*a^2*b*e-77*a*b^2*d+
119*b^3*c)*ln(a^(1/3)+b^(1/3)*x)/a^(20/3)-1/54*b^(2/3)*(-20*a^3*f+44*a^2*b
*e-77*a*b^2*d+119*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(20/3
)-1/27*b^(2/3)*(-20*a^3*f+44*a^2*b*e-77*a*b^2*d+119*b^3*c)*arctan(1/3*(a^(
1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(20/3)*3^(1/2)
```

3.302.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^3}\,dx$$
$$= -\frac{c}{11a^3x^{11}} + \frac{3bc-ad}{8a^4x^8} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{2a^6x^2} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{6a^5(a+bx^3)^2} + \frac{b(29b^3c-23ab^2d+17a^2be-11a^3f)\,x}{18a^6(a+bx^3)} + \frac{b^{2/3}(-119b^3c+77ab^2d-44a^2be+20a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{20/3}} + \frac{b^{2/3}(119b^3c-77ab^2d+44a^2be-20a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{20/3}} + \frac{b^{2/3}(-119b^3c+77ab^2d-44a^2be+20a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{20/3}}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```

output
```
-1/11*c/(a^3*x^11) + (3*b*c - a*d)/(8*a^4*x^8) - (6*b^2*c - 3*a*b*d + a^2*
e)/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(2*a^6*x^2) +
(b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + (b*(29*b
^3*c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(18*a^6*(a + b*x^3)) + (b^(2
/3)*(-119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*ArcTan[(1 - (2*b^(1/
3)*x)/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*a^(20/3)) + (b^(2/3)*(119*b^3*c - 77*a
*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(20/3)) +
(b^(2/3)*(-119*b^3*c + 77*a*b^2*d - 44*a^2*b*e + 20*a^3*f)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(20/3))
```

3.302.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}-\frac{\int -\frac{\frac{5b^4\left(-fa^3+bea^2-b^2da+b^3c\right)x^{12}}{a^4}-\frac{6b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{6b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-6b^3\left(\frac{bc}{a}-d\right)x^3+6b^3c}{x^{12}\left(bx^3+a\right)^2}dx}{6ab^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{5b^4\left(-fa^3+bea^2-b^2da+b^3c\right)x^{12}}{a^4}-\frac{6b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{6b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-6b^3\left(\frac{bc}{a}-d\right)x^3+6b^3c}{x^{12}\left(bx^3+a\right)^2}dx}{6ab^3}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{2368}$$

$$\frac{\frac{b^4x\left(-11a^3f+17a^2be-23ab^2d+29b^3c\right)}{3a^5\left(a+bx^3\right)}-\frac{\int -\frac{2\left(\frac{b^8\left(-11fa^3+17bea^2-23b^2da+29b^3c\right)x^{12}}{a^4}-\frac{9b^7\left(-fa^3+2bea^2-3b^2da+4b^3c\right)x^9}{a^3}+\frac{9b^7\left(ea^2-2bda+3b^2c\right)x^6}{a^2}\cdots\right.}{x^{12}\left(bx^3+a\right)}}{3ab^4}}{6ab^3}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{\frac{b^8\left(-11fa^3+17bea^2-23b^2da+29b^3c\right)x^{12}}{a^4}-\frac{9b^7\left(-fa^3+2bea^2-3b^2da+4b^3c\right)x^9}{a^3}+\frac{9b^7\left(ea^2-2bda+3b^2c\right)x^6}{a^2}-9b^7\left(\frac{2bc}{a}-d\right)x^3+9b^7c}{x^{12}\left(bx^3+a\right)}dx}{3ab^4}+\frac{b^4x\left(-11a^3f+17a\cdots\right.}{3a^5\left(\cdots\right.}}{6ab^3}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{2373}$$

$$\frac{\frac{2\int\left(-\frac{\left(20fa^3-44bea^2+77b^2da-119b^3c\right)b^8}{a^4\left(bx^3+a\right)}+\frac{9\left(fa^3-3bea^2+6b^2da-10b^3c\right)b^7}{a^4x^3}+\frac{9\left(ea^2-3bda+6b^2c\right)b^7}{a^3x^6}+\frac{9(ad-3bc)b^7}{a^2x^9}+\frac{9cb^7}{ax^{12}}\right)dx}{3ab^4}+\frac{b^4x(-11a^3f+17a^2be\cdots}{3a^5(a+b\cdots}}{6ab^3}$$

$$\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 2009

$$\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^5\left(a+bx^3\right)^2}+$$

$$\frac{\frac{b^4x\left(-11a^3f+17a^2be-23ab^2d+29b^3c\right)}{3a^5(a+bx^3)}+\frac{2\left(\frac{9b^7(3bc-ad)}{8a^2x^8}-\frac{9b^7\left(a^2e-3abd+6b^2c\right)}{5a^3x^5}-\frac{b^{23/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-20a^3f+44a^2be-77ab^2d+119b^3c\right)}{\sqrt{3}a^{14/3}}-\cdots\right)}{\cdots}}{\cdots}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x]
```

output
```
(b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(6*a^5*(a + b*x^3)^2) + ((b^4*(2
9*b^3*c - 23*a*b^2*d + 17*a^2*b*e - 11*a^3*f)*x)/(3*a^5*(a + b*x^3)) + (2*
((-9*b^7*c)/(11*a*x^11) + (9*b^7*(3*b*c - a*d))/(8*a^2*x^8) - (9*b^7*(6*b^
2*c - 3*a*b*d + a^2*e))/(5*a^3*x^5) + (9*b^7*(10*b^3*c - 6*a*b^2*d + 3*a^2
*b*e - a^3*f))/(2*a^4*x^2) - (b^(23/3)*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*
e - 20*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(14/3)) + (b^(23/3)*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*Log
[a^(1/3) + b^(1/3)*x])/(3*a^(14/3)) - (b^(23/3)*(119*b^3*c - 77*a*b^2*d +
44*a^2*b*e - 20*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*
a^(14/3))))/(3*a*b^4))/(6*a*b^3)
```

3.302.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.302.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.76

method	result
default	$-\frac{c}{11a^3x^{11}}-\frac{ad-3bc}{8a^4x^8}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{f\,a^3-3a^2be+6a\,b^2d-10b^3c}{2a^6x^2}-\frac{b\left(\frac{\left(\frac{11}{18}a^3bf-\frac{17}{18}a^2e\,b^2+\frac{23}{18}a\,b^3d-\frac{29}{18}b^4c\right)x^4+\frac{a\left(7f\,a^3-\right.}{}}{\left(b\,x^3+a\right)^2}\right.}{}$
risch	$\frac{-\frac{c}{11a}-\frac{(11ad-17bc)x^3}{88a^2}-\frac{\left(44a^2e-77abd+119b^2c\right)x^6}{220a^3}-\frac{\left(20f\,a^3-44a^2be+77a\,b^2d-119b^3c\right)x^9}{40a^4}-\frac{4b\left(20f\,a^3-44a^2be+77a\,b^2d-119b^3c\right)x^{12}}{45a^5}-\frac{b^2\left(20\right.}{}}{x^{11}\left(b\,x^3+a\right)^2}$

input
```
int((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/11*c/a^3/x^11-1/8*(a*d-3*b*c)/a^4/x^8-1/5*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x
^5-1/2*(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x^2-b/a^6*(((11/18*a^3*b*f
-17/18*a^2*e*b^2+23/18*a*b^3*d-29/18*b^4*c)*x^4+1/9*a*(7*a^3*f-10*a^2*b*e+
13*a*b^2*d-16*b^3*c)*x)/(b*x^3+a)^2+1/9*(20*a^3*f-44*a^2*b*e+77*a*b^2*d-11
9*b^3*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))))
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.72

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx$$

$$= \frac{660\,(119\,b^5c-77\,ab^4d+44\,a^2b^3e-20\,a^3b^2f)x^{15}+1056\,(119\,ab^4c-77\,a^2b^3d+44\,a^3b^2e-20\,a^4bf)x^{12}+2}{}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/11880*(660*(119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^15 +
 1056*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^12 + 297*
(119*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^9 - 54*(119*a^3*b
^2*c - 77*a^4*b*d + 44*a^5*e)*x^6 - 1080*a^5*c + 135*(17*a^4*b*c - 11*a^5*
d)*x^3 - 440*sqrt(3)*((119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*
f)*x^17 + 2*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^14
+ (119*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^
(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 220*((1
19*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c
 - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^
3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*
x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 440*((119*b^5*c - 77*a*b^4*d
+ 44*a^2*b^3*e - 20*a^3*b^2*f)*x^17 + 2*(119*a*b^4*c - 77*a^2*b^3*d + 44*a
^3*b^2*e - 20*a^4*b*f)*x^14 + (119*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e -
 20*a^5*f)*x^11)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^6*b^2*
x^17 + 2*a^7*b*x^14 + a^8*x^11)
```

3.302.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**12/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.302.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^3}\,dx = \frac{220\,(119\,b^5c-77\,ab^4d+44\,a^2b^3e-20\,a^3b^2f)x^{15}+352\,(119\,ab^4c-77\,a^2b^3d+44\,a^3b^2e-20\,a^4bf)x^{12}+99}{3960\,(a^6b^2}$$
$$+\frac{\sqrt{3}(119\,b^3c-77\,ab^2d+44\,a^2be-20\,a^3f)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(119\,b^3c-77\,ab^2d+44\,a^2be-20\,a^3f)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(119\,b^3c-77\,ab^2d+44\,a^2be-20\,a^3f)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/3960*(220*(119*b^5*c - 77*a*b^4*d + 44*a^2*b^3*e - 20*a^3*b^2*f)*x^15 +
352*(119*a*b^4*c - 77*a^2*b^3*d + 44*a^3*b^2*e - 20*a^4*b*f)*x^12 + 99*(11
9*a^2*b^3*c - 77*a^3*b^2*d + 44*a^4*b*e - 20*a^5*f)*x^9 - 18*(119*a^3*b^2*
c - 77*a^4*b*d + 44*a^5*e)*x^6 - 360*a^5*c + 45*(17*a^4*b*c - 11*a^5*d)*x^
3)/(a^6*b^2*x^17 + 2*a^7*b*x^14 + a^8*x^11) + 1/27*sqrt(3)*(119*b^3*c - 77
*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(
a/b)^(1/3))/(a^6*(a/b)^(2/3)) - 1/54*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e
- 20*a^3*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^6*(a/b)^(2/3)) + 1/2
7*(119*b^3*c - 77*a*b^2*d + 44*a^2*b*e - 20*a^3*f)*log(x + (a/b)^(1/3))/(a
^6*(a/b)^(2/3))
```

3.302.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{12}(a+bx^3)^3}\,dx$$
$$= \frac{\sqrt{3}\left(119\left(-ab^2\right)^{\frac{1}{3}}b^3c-77\left(-ab^2\right)^{\frac{1}{3}}ab^2d+44\left(-ab^2\right)^{\frac{1}{3}}a^2be-20\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^7}$$
$$- \frac{\left(119b^4c-77ab^3d+44a^2b^2e-20a^3bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^7}$$
$$+ \frac{\left(119\left(-ab^2\right)^{\frac{1}{3}}b^3c-77\left(-ab^2\right)^{\frac{1}{3}}ab^2d+44\left(-ab^2\right)^{\frac{1}{3}}a^2be-20\left(-ab^2\right)^{\frac{1}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^7}$$
$$+ \frac{29b^5cx^4-23ab^4dx^4+17a^2b^3ex^4-11a^3b^2fx^4+32ab^4cx-26a^2b^3dx+20a^3b^2ex-14a^4bfx}{18\left(bx^3+a\right)^2a^6}$$
$$+ \frac{2200b^3cx^9-1320ab^2dx^9+660a^2bex^9-220a^3fx^9-528ab^2cx^6+264a^2bdx^6-88a^3ex^6+165a^2bcx^3}{440a^6x^{11}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^12/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/27*sqrt(3)*(119*(-a*b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*d + 44*(-
a*b^2)^(1/3)*a^2*b*e - 20*(-a*b^2)^(1/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/a^7 - 1/27*(119*b^4*c - 77*a*b^3*d + 44*a^2*b^
2*e - 20*a^3*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^7 + 1/54*(119*
(-a*b^2)^(1/3)*b^3*c - 77*(-a*b^2)^(1/3)*a*b^2*d + 44*(-a*b^2)^(1/3)*a^2*b
*e - 20*(-a*b^2)^(1/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^7
 + 1/18*(29*b^5*c*x^4 - 23*a*b^4*d*x^4 + 17*a^2*b^3*e*x^4 - 11*a^3*b^2*f*x
^4 + 32*a*b^4*c*x - 26*a^2*b^3*d*x + 20*a^3*b^2*e*x - 14*a^4*b*f*x)/((b*x^
3 + a)^2*a^6) + 1/440*(2200*b^3*c*x^9 - 1320*a*b^2*d*x^9 + 660*a^2*b*e*x^9
 - 220*a^3*f*x^9 - 528*a*b^2*c*x^6 + 264*a^2*b*d*x^6 - 88*a^3*e*x^6 + 165*
a^2*b*c*x^3 - 55*a^3*d*x^3 - 40*a^3*c)/(a^6*x^11)
```

3.302.9 Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.94

$$
\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^{12}\left(a+bx^3\right)^3}\,dx \\
&= \frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{27\,a^{20/3}} \\
&\quad - \frac{\frac{c}{11\,a}-\frac{x^9\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{40\,a^4}+\frac{x^3\,\left(11\,a\,d-17\,b\,c\right)}{88\,a^2}+\frac{x^6\,\left(44\,e\,a^2-77\,d\,a\,b+119\,c\,b^2\right)}{220\,a^3}-\frac{4\,b\,x^{12}\,\left(-20\,f\,a^3+44\,e\,a^2\,b-\right)}{45\,a^5}}{a^2\,x^{11}+2\,a\,b\,x^{14}+b^2\,x^{17}} \\
&\quad + \frac{b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{27\,a^{20/3}} \\
&\quad - \frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-20\,f\,a^3+44\,e\,a^2\,b-77\,d\,a\,b^2+119\,c\,b^3\right)}{27\,a^{20/3}}
\end{aligned}
$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^12*(a + b*x^3)^3),x)
```

output
```
(b^(2/3)*log(b^(1/3)*x + a^(1/3))*(119*b^3*c - 20*a^3*f - 77*a*b^2*d + 44*
a^2*b*e))/(27*a^(20/3)) - (c/(11*a) - (x^9*(119*b^3*c - 20*a^3*f - 77*a*b^
2*d + 44*a^2*b*e))/(40*a^4) + (x^3*(11*a*d - 17*b*c))/(88*a^2) + (x^6*(119
*b^2*c + 44*a^2*e - 77*a*b*d))/(220*a^3) - (4*b*x^12*(119*b^3*c - 20*a^3*f
 - 77*a*b^2*d + 44*a^2*b*e))/(45*a^5) - (b^2*x^15*(119*b^3*c - 20*a^3*f -
77*a*b^2*d + 44*a^2*b*e))/(18*a^6))/(a^2*x^11 + b^2*x^17 + 2*a*b*x^14) + (
b^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 -
1/2)*(119*b^3*c - 20*a^3*f - 77*a*b^2*d + 44*a^2*b*e))/(27*a^(20/3)) - (b^
(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/
2)*(119*b^3*c - 20*a^3*f - 77*a*b^2*d + 44*a^2*b*e))/(27*a^(20/3))
```

3.303 $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^3} dx$

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3.303.1 Optimal result

Integrand size = 30, antiderivative size = 424

$$\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^3}\,dx \\
&= -\frac{c}{13a^3x^{13}}+\frac{3bc-ad}{10a^4x^{10}}-\frac{6b^2c-3abd+a^2e}{7a^5x^7} \\
&\quad+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4}-\frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)}{a^7x} \\
&\quad-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x^2}{6a^6(a+bx^3)^2}-\frac{b^2(17b^3c-14ab^2d+11a^2be-8a^3f)\,x^2}{9a^7(a+bx^3)} \\
&\quad+\frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{22/3}} \\
&\quad+\frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{22/3}} \\
&\quad-\frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{22/3}}
\end{aligned}$$

output
```
-1/13*c/a^3/x^13+1/10*(-a*d+3*b*c)/a^4/x^10+1/7*(-a^2*e+3*a*b*d-6*b^2*c)/a
^5/x^7+1/4*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^4-b*(-3*a^3*f+6*a^2
*b*e-10*a*b^2*d+15*b^3*c)/a^7/x-1/6*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^2
/a^6/(b*x^3+a)^2-1/9*b^2*(-8*a^3*f+11*a^2*b*e-14*a*b^2*d+17*b^3*c)*x^2/a^7
/(b*x^3+a)+1/27*b^(4/3)*(-35*a^3*f+65*a^2*b*e-104*a*b^2*d+152*b^3*c)*ln(a^
(1/3)+b^(1/3)*x)/a^(22/3)-1/54*b^(4/3)*(-35*a^3*f+65*a^2*b*e-104*a*b^2*d+1
52*b^3*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(22/3)+1/27*b^(4/3)*
(-35*a^3*f+65*a^2*b*e-104*a*b^2*d+152*b^3*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)
*x)/a^(1/3)*3^(1/2))/a^(22/3)*3^(1/2)
```

3.303.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.99

$$\begin{aligned}
&\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx \\
&= -\frac{c}{13a^3x^{13}}+\frac{3bc-ad}{10a^4x^{10}}-\frac{6b^2c-3abd+a^2e}{7a^5x^7}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{4a^6x^4} \\
&\quad+\frac{b(-15b^3c+10ab^2d-6a^2be+3a^3f)}{a^7x}+\frac{b^2(-b^3c+ab^2d-a^2be+a^3f)\,x^2}{6a^6\left(a+bx^3\right)^2} \\
&\quad+\frac{b^2(-17b^3c+14ab^2d-11a^2be+8a^3f)\,x^2}{9a^7\left(a+bx^3\right)} \\
&\quad+\frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{22/3}} \\
&\quad+\frac{b^{4/3}(152b^3c-104ab^2d+65a^2be-35a^3f)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{22/3}} \\
&\quad+\frac{b^{4/3}(-152b^3c+104ab^2d-65a^2be+35a^3f)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{22/3}}
\end{aligned}$$

input
```
Integrate[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
```

output
```
-1/13*c/(a^3*x^13) + (3*b*c - a*d)/(10*a^4*x^10) - (6*b^2*c - 3*a*b*d + a^
2*e)/(7*a^5*x^7) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(4*a^6*x^4)
+ (b*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/(a^7*x) + (b^2*(-(b^3
*c) + a*b^2*d - a^2*b*e + a^3*f)*x^2)/(6*a^6*(a + b*x^3)^2) + (b^2*(-17*b^
3*c + 14*a*b^2*d - 11*a^2*b*e + 8*a^3*f)*x^2)/(9*a^7*(a + b*x^3)) + (b^(4/
3)*(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]])/(9*Sqrt[3]*a^(22/3)) + (b^(4/3)*(152*b^3*c - 104*a
*b^2*d + 65*a^2*b*e - 35*a^3*f)*Log[a^(1/3) + b^(1/3)*x]])/(27*a^(22/3)) +
(b^(4/3)*(-152*b^3*c + 104*a*b^2*d - 65*a^2*b*e + 35*a^3*f)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/(54*a^(22/3))
```

3.303.3 Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2368, 27, 2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14} \left(a+bx^3\right)^3} \, dx$$

$$\Big\downarrow \ 2368$$

$$-\frac{\int -\frac{2\left(-\frac{2b^5\left(-fa^3+bea^2-b^2da+b^3c\right)x^{15}}{a^5}+\frac{3b^4\left(-fa^3+bea^2-b^2da+b^3c\right)x^{12}}{a^4}-\frac{3b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3-}{x^{14}(bx^3+a)^2}}{6ab^3}$$

$$\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^6\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{-\frac{2b^5\left(-fa^3+bea^2-b^2da+b^3c\right)x^{15}}{a^5}+\frac{3b^4\left(-fa^3+bea^2-b^2da+b^3c\right)x^{12}}{a^4}-\frac{3b^3\left(-fa^3+bea^2-b^2da+b^3c\right)x^9}{a^3}+\frac{3b^3\left(ea^2-bda+b^2c\right)x^6}{a^2}-3b^3\left(\frac{bc}{a}-d\right)x^3+3b^3c}{x^{14}(bx^3+a)^2}d}{3ab^3}$$

$$\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^6\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 2368$$

$$-\frac{\frac{\int -\frac{-\frac{b^{10}\left(-8fa^3+11bea^2-14b^2da+17b^3c\right)x^{15}}{a^5}+\frac{9b^9\left(-2fa^3+3bea^2-4b^2da+5b^3c\right)x^{12}}{a^4}-\frac{9b^8\left(-fa^3+2bea^2-3b^2da+4b^3c\right)x^9}{a^3}+\frac{9b^8\left(ea^2-2bda+3b^2c\right)x^6}{a^2}-9b^8(\ldots}{x^{14}\left(bx^3+a\right)}}{3ab^5}}{3ab^3}$$

$$\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^6\left(a+bx^3\right)^2}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{-\frac{b^{10}\left(-8fa^3+11bea^2-14b^2da+17b^3c\right)x^{15}}{a^5}+\frac{9b^9\left(-2fa^3+3bea^2-4b^2da+5b^3c\right)x^{12}}{a^4}-\frac{9b^8\left(-fa^3+2bea^2-3b^2da+4b^3c\right)x^9}{a^3}+\frac{9b^8\left(ea^2-2bda+3b^2c\right)x^6}{a^2}-9b^8\left(\frac{2bc}{a}\ldots\right.}{x^{14}\left(bx^3+a\right)}}{3ab^5}}{3ab^3}$$

$$\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^6\left(a+bx^3\right)^2}$$

$\downarrow$ 2373

$$\frac{\frac{\int\left(\frac{\left(35fa^3-65bea^2+104b^2da-152b^3c\right)xb^{10}}{a^5\left(bx^3+a\right)}-\frac{9\left(3fa^3-6bea^2+10b^2da-15b^3c\right)b^9}{a^5x^2}+\frac{9\left(fa^3-3bea^2+6b^2da-10b^3c\right)b^8}{a^4x^5}+\frac{9\left(ea^2-3bda+6b^2c\right)b^8}{a^3x^8}+\frac{9(ad-3bc)b^8}{a^2x^{11}}+\frac{9c\ldots}{ax\ldots}\right.}{3ab^5}}{3ab^3}$$

$$\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^6\left(a+bx^3\right)^2}$$

$\downarrow$ 2009

$$\frac{\frac{9b^8(3bc-ad)}{10a^2x^{10}}-\frac{9b^8\left(a^2e-3abd+6b^2c\right)}{7a^3x^7}+\frac{b^{28/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-35a^3f+65a^2be-104ab^2d+152b^3c\right)}{\sqrt{3}a^{16/3}}-\frac{b^{28/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-35a^3f+\ldots\right)}{6a^{16/3}}\ldots}{\ldots}$$

$$\frac{b^2x^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{6a^6\left(a+bx^3\right)^2}$$

input
```
Int[(c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x]
```

3.303. $\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^3}\,dx$

output
```
-1/6*(b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a^6*(a + b*x^3)^2) + (
-1/3*(b^5*(17*b^3*c - 14*a*b^2*d + 11*a^2*b*e - 8*a^3*f)*x^2)/(a^6*(a + b*
x^3)) + ((-9*b^8*c)/(13*a*x^13) + (9*b^8*(3*b*c - a*d))/(10*a^2*x^10) - (9
*b^8*(6*b^2*c - 3*a*b*d + a^2*e))/(7*a^3*x^7) + (9*b^8*(10*b^3*c - 6*a*b^2
*d + 3*a^2*b*e - a^3*f))/(4*a^4*x^4) - (9*b^9*(15*b^3*c - 10*a*b^2*d + 6*a
^2*b*e - 3*a^3*f))/(a^5*x) + (b^(28/3)*(152*b^3*c - 104*a*b^2*d + 65*a^2*b
*e - 35*a^3*f)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]
*a^(16/3)) + (b^(28/3)*(152*b^3*c - 104*a*b^2*d + 65*a^2*b*e - 35*a^3*f)*L
og[a^(1/3) + b^(1/3)*x])/(3*a^(16/3)) - (b^(28/3)*(152*b^3*c - 104*a*b^2*d
 + 65*a^2*b*e - 35*a^3*f)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(6*a^(16/3)))/(3*a*b^5))/(3*a*b^3)
```

3.303.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.303.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.77

method	result
default	$-\frac{c}{13a^3x^{13}}-\frac{ad-3bc}{10a^4x^{10}}-\frac{a^2e-3abd+6b^2c}{7a^5x^7}-\frac{f a^3-3a^2be+6a b^2d-10b^3c}{4a^6x^4}+\frac{b(3f a^3-6a^2be+10a b^2d-15b^3c)}{a^7x}+\frac{b^2\left(\frac{b(8f a^3-\ldots}{\ldots}\right.}{\ldots}$
risch	$\frac{-\frac{c}{13a}-\frac{(13ad-19bc)x^3}{130a^2}-\frac{(65a^2e-104abd+152b^2c)x^6}{455a^3}-\frac{(35f a^3-65a^2be+104a b^2d-152b^3c)x^9}{140a^4}+\frac{b(35f a^3-65a^2be+104a b^2d-152b^3c)x^{12}}{14a^5}+\frac{7b^2\ldots}{\ldots}}{x^{13}(b x^3+a)^2}$

input

```
int((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/13*c/a^3/x^13-1/10*(a*d-3*b*c)/a^4/x^10-1/7*(a^2*e-3*a*b*d+6*b^2*c)/a^5
/x^7-1/4*(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x^4+b*(3*a^3*f-6*a^2*b*e
+10*a*b^2*d-15*b^3*c)/a^7/x+b^2/a^7*((1/9*b*(8*a^3*f-11*a^2*b*e+14*a*b^2*d
-17*b^3*c)*x^5+(19/18*a^4*f-25/18*a^3*b*e+31/18*a^2*b^2*d-37/18*a*b^3*c)*x
^2)/(b*x^3+a)^2+(35/9*f*a^3-65/9*a^2*b*e+104/9*a*b^2*d-152/9*b^3*c)*(-1/3/
b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/
b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)))
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.62

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{5460\,(152\,b^6c-104\,ab^5d+65\,a^2b^4e-35\,a^3b^3f)x^{18}+9555\,(152\,ab^5c-104\,a^2b^4d+65\,a^3b^3e-35\,a^4b^2f)\ldots}{\ldots}$$

input

```
integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/49140*(5460*(152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^1
8 + 9555*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^15
+ 3510*(152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^12 -
351*(152*a^3*b^3*c - 104*a^4*b^2*d + 65*a^5*b*e - 35*a^6*f)*x^9 + 3780*a^6
*c + 108*(152*a^4*b^2*c - 104*a^5*b*d + 65*a^6*e)*x^6 - 378*(19*a^5*b*c -
13*a^6*d)*x^3 + 1820*sqrt(3)*((152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35
*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*
b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*
x^13)*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 910*
((152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^19 + 2*(152*a*b
^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^16 + (152*a^2*b^4*c
- 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^13)*(-b/a)^(1/3)*log(b*x^2
- a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 1820*((152*b^6*c - 104*a*b^5*d + 65
*a^2*b^4*e - 35*a^3*b^3*f)*x^19 + 2*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*
b^3*e - 35*a^4*b^2*f)*x^16 + (152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e
 - 35*a^5*b*f)*x^13)*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^7*b^2*x^19
 + 2*a^8*b*x^16 + a^9*x^13)
```

3.303.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((f*x**9+e*x**6+d*x**3+c)/x**14/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}(a+bx^3)^3}\,dx = -\frac{1820\,(152\,b^6c-104\,ab^5d+65\,a^2b^4e-35\,a^3b^3f)x^{18}+3185\,(152\,ab^5c-104\,a^2b^4d+65\,a^3b^3e-35\,a^4b^2f)\ldots}{\ldots}$$

$$-\frac{\sqrt{3}(152\,b^4c-104\,ab^3d+65\,a^2b^2e-35\,a^3bf)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

$$-\frac{(152\,b^4c-104\,ab^3d+65\,a^2b^2e-35\,a^3bf)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

$$+\frac{(152\,b^4c-104\,ab^3d+65\,a^2b^2e-35\,a^3bf)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^7\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/16380*(1820*(152*b^6*c - 104*a*b^5*d + 65*a^2*b^4*e - 35*a^3*b^3*f)*x^1
8 + 3185*(152*a*b^5*c - 104*a^2*b^4*d + 65*a^3*b^3*e - 35*a^4*b^2*f)*x^15
+ 1170*(152*a^2*b^4*c - 104*a^3*b^3*d + 65*a^4*b^2*e - 35*a^5*b*f)*x^12 -
117*(152*a^3*b^3*c - 104*a^4*b^2*d + 65*a^5*b*e - 35*a^6*f)*x^9 + 1260*a^6
*c + 36*(152*a^4*b^2*c - 104*a^5*b*d + 65*a^6*e)*x^6 - 126*(19*a^5*b*c - 1
3*a^6*d)*x^3)/(a^7*b^2*x^19 + 2*a^8*b*x^16 + a^9*x^13) - 1/27*sqrt(3)*(152
*b^4*c - 104*a*b^3*d + 65*a^2*b^2*e - 35*a^3*b*f)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(a^7*(a/b)^(1/3)) - 1/54*(152*b^4*c - 104*a*b^
3*d + 65*a^2*b^2*e - 35*a^3*b*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
^7*(a/b)^(1/3)) + 1/27*(152*b^4*c - 104*a*b^3*d + 65*a^2*b^2*e - 35*a^3*b*
f)*log(x + (a/b)^(1/3))/(a^7*(a/b)^(1/3))
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.23

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx$$
$$= \frac{\sqrt{3}\left(152\left(-ab^2\right)^{\frac{2}{3}}b^3c-104\left(-ab^2\right)^{\frac{2}{3}}ab^2d+65\left(-ab^2\right)^{\frac{2}{3}}a^2be-35\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^8}$$
$$+ \frac{\left(152b^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-104ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+65a^2b^3e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-35a^3b^2f\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^8}$$
$$- \frac{\left(152\left(-ab^2\right)^{\frac{2}{3}}b^3c-104\left(-ab^2\right)^{\frac{2}{3}}ab^2d+65\left(-ab^2\right)^{\frac{2}{3}}a^2be-35\left(-ab^2\right)^{\frac{2}{3}}a^3f\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right.}{54a^8}$$
$$- \frac{34b^6cx^5-28ab^5dx^5+22a^2b^4ex^5-16a^3b^3fx^5+37ab^5cx^2-31a^2b^4dx^2+25a^3b^3ex^2-19a^4b^2fx^2}{18\left(bx^3+a\right)^2a^7}$$
$$- \frac{27300b^4cx^{12}-18200ab^3dx^{12}+10920a^2b^2ex^{12}-5460a^3bfx^{12}-4550ab^3cx^9+2730a^2b^2dx^9-1365a^3}{1820a^7x}$$

input
```
integrate((f*x^9+e*x^6+d*x^3+c)/x^14/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/27*sqrt(3)*(152*(-a*b^2)^(2/3)*b^3*c - 104*(-a*b^2)^(2/3)*a*b^2*d + 65*(
-a*b^2)^(2/3)*a^2*b*e - 35*(-a*b^2)^(2/3)*a^3*f)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/a^8 + 1/27*(152*b^5*c*(-a/b)^(1/3) - 104*a*b^
4*d*(-a/b)^(1/3) + 65*a^2*b^3*e*(-a/b)^(1/3) - 35*a^3*b^2*f*(-a/b)^(1/3))*
(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^8 - 1/54*(152*(-a*b^2)^(2/3)*b^3
*c - 104*(-a*b^2)^(2/3)*a*b^2*d + 65*(-a*b^2)^(2/3)*a^2*b*e - 35*(-a*b^2)^
(2/3)*a^3*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^8 - 1/18*(34*b^6*c
*x^5 - 28*a*b^5*d*x^5 + 22*a^2*b^4*e*x^5 - 16*a^3*b^3*f*x^5 + 37*a*b^5*c*x
^2 - 31*a^2*b^4*d*x^2 + 25*a^3*b^3*e*x^2 - 19*a^4*b^2*f*x^2)/((b*x^3 + a)^
2*a^7) - 1/1820*(27300*b^4*c*x^12 - 18200*a*b^3*d*x^12 + 10920*a^2*b^2*e*x
^12 - 5460*a^3*b*f*x^12 - 4550*a*b^3*c*x^9 + 2730*a^2*b^2*d*x^9 - 1365*a^3
*b*e*x^9 + 455*a^4*f*x^9 + 1560*a^2*b^2*c*x^6 - 780*a^3*b*d*x^6 + 260*a^4*
e*x^6 - 546*a^3*b*c*x^3 + 182*a^4*d*x^3 + 140*a^4*c)/(a^7*x^13)
```

3.303.9 Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^3+ex^6+fx^9}{x^{14}\left(a+bx^3\right)^3}\,dx$$
$$=\frac{b^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{27\,a^{22/3}}$$
$$-\frac{\frac{c}{13\,a}-\frac{x^9\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{140\,a^4}+\frac{x^3\,\left(13\,a\,d-19\,b\,c\right)}{130\,a^2}+\frac{x^6\,\left(65\,e\,a^2-104\,d\,a\,b+152\,c\,b^2\right)}{455\,a^3}+\frac{b\,x^{12}\,\left(-35\,f\,a^3+65\,e\,a^2\,b-\right.}{14\,a^5}}{a^2\,x^{13}+2\,a\,b\,x^{16}-}$$
$$-\frac{b^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{27\,a^{22/3}}$$
$$+\frac{b^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-35\,f\,a^3+65\,e\,a^2\,b-104\,d\,a\,b^2+152\,c\,b^3\right)}{27\,a^{22/3}}$$

input
```
int((c + d*x^3 + e*x^6 + f*x^9)/(x^14*(a + b*x^3)^3),x)
```

output
```
(b^(4/3)*log(b^(1/3)*x + a^(1/3))*(152*b^3*c - 35*a^3*f - 104*a*b^2*d + 65
*a^2*b*e))/(27*a^(22/3)) - (c/(13*a) - (x^9*(152*b^3*c - 35*a^3*f - 104*a*
b^2*d + 65*a^2*b*e))/(140*a^4) + (x^3*(13*a*d - 19*b*c))/(130*a^2) + (x^6*
(152*b^2*c + 65*a^2*e - 104*a*b*d))/(455*a^3) + (b*x^12*(152*b^3*c - 35*a^
3*f - 104*a*b^2*d + 65*a^2*b*e))/(14*a^5) + (7*b^2*x^15*(152*b^3*c - 35*a^
3*f - 104*a*b^2*d + 65*a^2*b*e))/(36*a^6) + (b^3*x^18*(152*b^3*c - 35*a^3*
f - 104*a*b^2*d + 65*a^2*b*e))/(9*a^7))/(a^2*x^13 + b^2*x^19 + 2*a*b*x^16)
 - (b^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/
2 + 1/2)*(152*b^3*c - 35*a^3*f - 104*a*b^2*d + 65*a^2*b*e))/(27*a^(22/3))
+ (b^(4/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2
 - 1/2)*(152*b^3*c - 35*a^3*f - 104*a*b^2*d + 65*a^2*b*e))/(27*a^(22/3))
```

3.304 $\int \frac{(1-x)x^4}{1+x^3}\,dx$

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3.304.2 Mathematica [A] (verified) 2338
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3.304.8 Giac [A] (verification not implemented) 2341
3.304.9 Mupad [B] (verification not implemented) 2342

3.304.1 Optimal result

Integrand size = 16, antiderivative size = 54

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = \frac{x^2}{2} - \frac{x^3}{3} + \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

output
```
1/2*x^2-1/3*x^3+2/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2)
)*3^(1/2)
```

3.304.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = \frac{1}{6}\left(3x^2 - 2x^3 - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 2\log(1+x) - \log\left(1-x+x^2\right) + 2\log\left(1+x^3\right)\right)$$

input
```
Integrate[((1 - x)*x^4)/(1 + x^3),x]
```

output
```
(3*x^2 - 2*x^3 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log
[1 - x + x^2] + 2*Log[1 + x^3])/6
```

3.304.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)x^4}{x^3+1} \, dx$$

$$\Big\downarrow \text{2426}$$

$$\int \left(\frac{(x-1)x}{x^3+1}-x^2+x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{x^3}{3}+\frac{x^2}{2}+\frac{1}{6}\log\left(x^2-x+1\right)+\frac{2}{3}\log(x+1)$$

input
```
Int[((1 - x)*x^4)/(1 + x^3),x]
```

output
```
x^2/2 - x^3/3 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log
[1 - x + x^2]/6
```

3.304.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.304.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	45
risch	$-\frac{x^3}{3}+\frac{x^2}{2}+\frac{\ln(4x^2-4x+4)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}+\frac{2\ln(1+x)}{3}$	47
meijerg	$\frac{x^2}{2}-\frac{x^2\left(-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{(x^3)^{\frac{2}{3}}}+\frac{\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2(x^3)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{(x^3)^{\frac{2}{3}}}\right)}{3}-\frac{x^3}{3}+\frac{\ln(x^3+1)}{3}$	94

input
```
int((1-x)*x^4/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3+1/2*x^2+2/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(-1+2*
x)*3^(1/2))
```

3.304.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = -\frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{6}\,\log\left(x^2-x+1\right)+\frac{2}{3}\,\log\left(x+1\right)$$

input
```
integrate((1-x)*x^4/(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x
^2 - x + 1) + 2/3*log(x + 1)
```

3.304.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = -\frac{x^3}{3} + \frac{x^2}{2} + \frac{2\log{(x+1)}}{3} + \frac{\log{(x^2-x+1)}}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((1-x)*x**4/(x**3+1),x)
```

output
```
-x**3/3 + x**2/2 + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*s
qrt(3)*x/3 - sqrt(3)/3)/3
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = -\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{6}\,\log\left(x^2-x+1\right) + \frac{2}{3}\,\log\left(x+1\right)$$

input
```
integrate((1-x)*x^4/(x^3+1),x, algorithm="maxima")
```

output
```
-1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x
^2 - x + 1) + 2/3*log(x + 1)
```

3.304.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = -\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{6}\,\log\left(x^2-x+1\right) + \frac{2}{3}\,\log\left(|x+1|\right)$$

input
```
integrate((1-x)*x^4/(x^3+1),x, algorithm="giac")
```

output
```
-1/3*x^3 + 1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x
^2 - x + 1) + 2/3*log(abs(x + 1))
```

3.304.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{(1-x)x^4}{1+x^3}\,dx = \frac{2\ln(x+1)}{3} + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \frac{x^2}{2} - \frac{x^3}{3}$$

input
```
int(-(x^4*(x - 1))/(x^3 + 1),x)
```

output
```
(2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) -
log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) + x^2/2 - x^3/3
```

3.305 $\int \frac{(1-x)x^3}{1+x^3} dx$

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3.305.2 Mathematica [A] (verified) 2343
3.305.3 Rubi [A] (verified) 2344
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3.305.5 Fricas [A] (verification not implemented) 2345
3.305.6 Sympy [A] (verification not implemented) 2346
3.305.7 Maxima [A] (verification not implemented) 2346
3.305.8 Giac [A] (verification not implemented) 2346
3.305.9 Mupad [B] (verification not implemented) 2347

3.305.1 Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{(1-x)x^3}{1+x^3} dx = x - \frac{x^2}{2} - \frac{2}{3}\log(1+x) + \frac{1}{3}\log\left(1-x+x^2\right)$$

output
```
x-1/2*x^2-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

3.305.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(1-x)x^3}{1+x^3} dx = x - \frac{x^2}{2} - \frac{2}{3}\log(1+x) + \frac{1}{3}\log\left(1-x+x^2\right)$$

input
```
Integrate[((1 - x)*x^3)/(1 + x^3),x]
```

output
```
x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

3.305.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)x^3}{x^3+1} \, dx$$

$$\Big\downarrow \text{ 2426}$$

$$\int \left(-\frac{1-x}{x^3+1}-x+1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{x^2}{2}+\frac{1}{3}\log\left(x^2-x+1\right)+x-\frac{2}{3}\log(x+1)$$

input
```
Int[((1 - x)*x^3)/(1 + x^3),x]
```

output
```
x - x^2/2 - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

3.305.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.305.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result
default	$x-\frac{x^2}{2}-\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{3}$
norman	$x-\frac{x^2}{2}-\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{3}$
risch	$x-\frac{x^2}{2}-\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{3}$
parallelrisch	$x-\frac{x^2}{2}-\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{3}$
meijerg	$x-\frac{x\left(\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{\left(x^3\right)^{\frac{1}{3}}}-\frac{\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2\left(x^3\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2-\left(x^3\right)^{\frac{1}{3}}}\right)}{\left(x^3\right)^{\frac{1}{3}}}\right)}{3}-\frac{x^2}{2}+\frac{x^2\left(-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}\right)}{\left(x^3\right)^{\frac{2}{3}}}+\frac{\ln\left(1-\left(x^3\right)^{\frac{1}{3}}+\left(x\right.\right.}{2\left(x^3\right)^{\frac{2}{3}}}\right.}{3}$

input
```
int((1-x)*x^3/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
x-1/2*x^2-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

3.305.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(1-x)x^3}{1+x^3}\,dx = -\frac{1}{2}\,x^2+x+\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{2}{3}\,\log\left(x+1\right)$$

input
```
integrate((1-x)*x^3/(x^3+1),x, algorithm="fricas")
```

output
```
-1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

3.305.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(1-x)x^3}{1+x^3}\,dx = -\frac{x^2}{2} + x - \frac{2\log(x+1)}{3} + \frac{\log(x^2-x+1)}{3}$$

input
```
integrate((1-x)*x**3/(x**3+1),x)
```

output
```
-x**2/2 + x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3
```

3.305.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(1-x)x^3}{1+x^3}\,dx = -\frac{1}{2}x^2 + x + \frac{1}{3}\log(x^2-x+1) - \frac{2}{3}\log(x+1)$$

input
```
integrate((1-x)*x^3/(x^3+1),x, algorithm="maxima")
```

output
```
-1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

3.305.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{(1-x)x^3}{1+x^3}\,dx = -\frac{1}{2}x^2 + x + \frac{1}{3}\log(x^2-x+1) - \frac{2}{3}\log(|x+1|)$$

input
```
integrate((1-x)*x^3/(x^3+1),x, algorithm="giac")
```

output
```
-1/2*x^2 + x + 1/3*log(x^2 - x + 1) - 2/3*log(abs(x + 1))
```

3.305.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(1-x)x^3}{1+x^3}\,dx = x - \frac{2\,\ln(x+1)}{3} + \frac{\ln(x^2-x+1)}{3} - \frac{x^2}{2}$$

input
```
int(-(x^3*(x - 1))/(x^3 + 1),x)
```

output
```
x - (2*log(x + 1))/3 + log(x^2 - x + 1)/3 - x^2/2
```

3.306 $\int \frac{(1-x)x^2}{1+x^3}\,dx$

3.306.1 Optimal result 2348
3.306.2 Mathematica [A] (verified) 2348
3.306.3 Rubi [A] (verified) 2349
3.306.4 Maple [A] (verified) 2350
3.306.5 Fricas [A] (verification not implemented) 2350
3.306.6 Sympy [A] (verification not implemented) 2351
3.306.7 Maxima [A] (verification not implemented) 2351
3.306.8 Giac [A] (verification not implemented) 2351
3.306.9 Mupad [B] (verification not implemented) 2352

3.306.1 Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = -x-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

output
```
-x+2/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.306.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = -x+\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)+\frac{1}{3}\log\left(1+x^3\right)$$

input
```
Integrate[((1 - x)*x^2)/(1 + x^3),x]
```

output
```
-x + ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x]/3 - Log[1 - x + x^2]/
6 + Log[1 + x^3]/3
```

3.306.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)x^2}{x^3+1} \, dx$$

$$\Big\downarrow \text{ 2426}$$

$$\int \left(\frac{x^2+1}{x^3+1}-1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^2-x+1\right)-x+\frac{2}{3}\log(x+1)$$

input
```
Int[((1 - x)*x^2)/(1 + x^3),x]
```

output
```
-x - ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + (2*Log[1 + x])/3 + Log[1 - x + x^
2]/6
```

3.306.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.306.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

method	result	size
risch	$-x+\frac{2\ln\left(1+x\right)}{3}+\frac{\ln\left(x^{2}-x+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	36
default	$-x+\frac{2\ln\left(1+x\right)}{3}+\frac{\ln\left(x^{2}-x+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(-1+2x\right)\sqrt{3}}{3}\right)}{3}$	38
meijerg	$\frac{\ln\left(x^{3}+1\right)}{3}-x+\frac{x\left(\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}-\frac{\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2\left(x^{3}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{3}\right)^{\frac{1}{3}}}{2-\left(x^{3}\right)^{\frac{1}{3}}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}\right)}{3}$	84

input
```
int((1-x)*x^2/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-x+2/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

3.306.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-x+\frac{1}{6}\,\log\left(x^2-x+1\right)+\frac{2}{3}\,\log\left(x+1\right)$$

input
```
integrate((1-x)*x^2/(x^3+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3
*log(x + 1)
```

3.306.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = -x + \frac{2\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((1-x)*x**2/(x**3+1),x)
```

output
```
-x + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - s
qrt(3)/3)/3
```

3.306.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - x + \frac{1}{6}\log\left(x^2-x+1\right) + \frac{2}{3}\log(x+1)$$

input
```
integrate((1-x)*x^2/(x^3+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3
*log(x + 1)
```

3.306.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - x + \frac{1}{6}\log\left(x^2-x+1\right) + \frac{2}{3}\log(|x+1|)$$

input
```
integrate((1-x)*x^2/(x^3+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/6*log(x^2 - x + 1) + 2/3
*log(abs(x + 1))
```

3.306.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{(1-x)x^2}{1+x^3}\,dx = \frac{2\,\ln(x+1)}{3} - x - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(-(x^2*(x - 1))/(x^3 + 1),x)
```

output
```
(2*log(x + 1))/3 - x - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6
) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6)
```

3.307 $\int \frac{(1-x)x}{1+x^3} dx$

3.307.1 Optimal result 2353
3.307.2 Mathematica [A] (verified) 2353
3.307.3 Rubi [A] (verified) 2354
3.307.4 Maple [A] (verified) 2356
3.307.5 Fricas [A] (verification not implemented) 2356
3.307.6 Sympy [A] (verification not implemented) 2356
3.307.7 Maxima [A] (verification not implemented) 2357
3.307.8 Giac [A] (verification not implemented) 2357
3.307.9 Mupad [B] (verification not implemented) 2358

3.307.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{(1-x)x}{1+x^3} dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log(1+x) - \frac{1}{6}\log\left(1-x+x^2\right)$$

output
```
-2/3*ln(1+x)-1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.307.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{(1-x)x}{1+x^3} dx = \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right) - \frac{1}{3}\log\left(1+x^3\right)$$

input
```
Integrate[((1 - x)*x)/(1 + x^3),x]
```

output
```
ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6 - L
og[1 + x^3]/3
```

3.307.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2413, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)x}{x^3+1}dx$$

$$\Big\downarrow \text{2413}$$

$$\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{2}{3}\int \frac{1}{x+1}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{2}{3}\log(x+1)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)-\frac{2}{3}\log(x+1)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{2}{3}\log(x+1)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)-\frac{2}{3}\log(x+1)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{2}{3}\log(x+1)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)-\frac{2}{3}\log(x+1)$$

input
```
Int[((1 - x)*x)/(1 + x^3),x]
```

output
```
(-2*Log[1 + x])/3 + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - Log[1 - x + x^2]
/2)/3
```

3.307.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2413
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Simp[q*((A - B*q
+ C*q^2)/(3*a))   Int[1/(q + x), x], x] + Simp[q/(3*a)   Int[(q*(2*A + B*q
- C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 -
b*A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2
] && GtQ[a/b, 0]
```

3.307. $\int \frac{(1-x)x}{1+x^3} \, dx$

3.307.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$-\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	35
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}-\frac{\ln(x^3+1)}{3}$	88

input
```
int((1-x)*x/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-2/3*ln(1+x)-1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

3.307.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{(1-x)x}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)$$

input
```
integrate((1-x)*x/(x^3+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(x + 1)
```

3.307.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{(1-x)x}{1+x^3}\,dx = -\frac{2\log(x+1)}{3}-\frac{\log(x^2-x+1)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((1-x)*x/(x**3+1),x)
```

output
```
-2*log(x + 1)/3 - log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(
3)/3)/3
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{(1-x)x}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) - \frac{2}{3}\log\left(x+1\right)$$

input
```
integrate((1-x)*x/(x^3+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(x + 1)
```

3.307.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(1-x)x}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) - \frac{2}{3}\log\left(|x+1|\right)$$

input
```
integrate((1-x)*x/(x^3+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*log
(abs(x + 1))
```

3.307.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \frac{(1-x)x}{1+x^3}\,dx = -\frac{\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6} - \frac{\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6} - \frac{2\,\ln\left(x+1\right)}{3} - \frac{\sqrt{3}\,\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6} + \frac{\sqrt{3}\,\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}$$

input
```
int(-(x*(x - 1))/(x^3 + 1),x)
```

output
```
(3^(1/2)*log(x + (3^(1/2)*1i)/2 - 1/2)*1i)/6 - log(x + (3^(1/2)*1i)/2 - 1/
2)/6 - (2*log(x + 1))/3 - (3^(1/2)*log(x - (3^(1/2)*1i)/2 - 1/2)*1i)/6 - l
og(x - (3^(1/2)*1i)/2 - 1/2)/6
```

3.308 $\int \frac{1-x}{x(1+x^3)}\,dx$

3.308.1 Optimal result 2359
3.308.2 Mathematica [A] (verified) 2359
3.308.3 Rubi [A] (verified) 2360
3.308.4 Maple [A] (verified) 2361
3.308.5 Fricas [A] (verification not implemented) 2361
3.308.6 Sympy [A] (verification not implemented) 2361
3.308.7 Maxima [A] (verification not implemented) 2362
3.308.8 Giac [A] (verification not implemented) 2362
3.308.9 Mupad [B] (verification not implemented) 2363

3.308.1 Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{1-x}{x\left(1+x^3\right)}\,dx=\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{2}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)$$

output
```
ln(x)-2/3*ln(1+x)-1/6*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.308.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{1-x}{x\left(1+x^3\right)}\,dx=-\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{3}\log\left(1+x^3\right)$$

input
```
Integrate[(1 - x)/(x*(1 + x^3)),x]
```

output
```
-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 + x]/3 + Log[1 - x
+ x^2]/6 - Log[1 + x^3]/3
```

3.308.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{x(x^3+1)} dx$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{-x-1}{3(x^2-x+1)}+\frac{1}{x}-\frac{2}{3(x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^2-x+1\right)+\log(x)-\frac{2}{3}\log(x+1)$$

input
```
Int[(1 - x)/(x*(1 + x^3)),x]
```

output
```
ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[x] - (2*Log[1 + x])/3 - Log[1 - x
+ x^2]/6
```

3.308.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

3.308.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{2\ln(1+x)}{3}+\ln(x)-\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	35
default	$\ln(x)-\frac{2\ln(1+x)}{3}-\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	37
meijerg	$\ln(x)-\frac{\ln(x^3+1)}{3}-\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}-\frac{x\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}$	84

input
```
int((1-x)/x/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-2/3*ln(1+x)+ln(x)-1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

3.308.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1-x}{x(1+x^3)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{6}\log\left(x^2-x+1\right)-\frac{2}{3}\log\left(x+1\right)+\log\left(x\right)$$

input
```
integrate((1-x)/x/(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*lo
g(x + 1) + log(x)
```

3.308.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{1-x}{x(1+x^3)}\,dx = \log(x)-\frac{2\log(x+1)}{3}-\frac{\log(x^2-x+1)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((1-x)/x/(x**3+1),x)
```

output
```
log(x) - 2*log(x + 1)/3 - log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3
 - sqrt(3)/3)/3
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1-x}{x(1+x^3)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) - \frac{2}{3}\log\left(x+1\right) + \log\left(x\right)$$

input
```
integrate((1-x)/x/(x^3+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*lo
g(x + 1) + log(x)
```

3.308.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1-x}{x(1+x^3)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{6}\log\left(x^2-x+1\right) - \frac{2}{3}\log\left(|x+1|\right) + \log\left(|x|\right)$$

input
```
integrate((1-x)/x/(x^3+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) - 2/3*lo
g(abs(x + 1)) + log(abs(x))
```

3.308.9 Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{1-x}{x\left(1+x^3\right)}\,dx = \ln\left(x\right)-\frac{2\ \ln\left(x+1\right)}{3}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(-(x - 1)/(x*(x^3 + 1)),x)
```

output
```
log(x) - (2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6
- 1/6) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6)
```

3.309 $\int \frac{1-x}{x^2(1+x^3)}\,dx$

3.309.1 Optimal result 2364
3.309.2 Mathematica [A] (verified) 2364
3.309.3 Rubi [A] (verified) 2365
3.309.4 Maple [A] (verified) 2366
3.309.5 Fricas [A] (verification not implemented) 2366
3.309.6 Sympy [A] (verification not implemented) 2367
3.309.7 Maxima [A] (verification not implemented) 2367
3.309.8 Giac [A] (verification not implemented) 2367
3.309.9 Mupad [B] (verification not implemented) 2368

3.309.1 Optimal result

Integrand size = 16, antiderivative size = 49

$$\int \frac{1-x}{x^2(1+x^3)}\,dx=-\frac{1}{x}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(x)+\frac{2}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

output
```
-1/x-ln(x)+2/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(
1/2)
```

3.309.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{1-x}{x^2(1+x^3)}\,dx=-\frac{1}{x}-\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\log(x)+\frac{1}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)+\frac{1}{3}\log\left(1+x^3\right)$$

input
```
Integrate[(1 - x)/(x^2*(1 + x^3)),x]
```

output
```
-x^(-1) - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + Log[1 + x]/3 - Log
[1 - x + x^2]/6 + Log[1 + x^3]/3
```

3.309.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{x^2 (x^3+1)} \, dx$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{x-2}{3 (x^2-x+1)}+\frac{1}{x^2}-\frac{1}{x}+\frac{2}{3(x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{x}-\log(x)+\frac{2}{3}\log(x+1)$$

input
```
Int[(1 - x)/(x^2*(1 + x^3)),x]
```

output
```
-x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[x] + (2*Log[1 + x])/3 +
Log[1 - x + x^2]/6
```

3.309.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

3.309.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
risch	$-\frac{1}{x}+\frac{2\ln(1+x)}{3}-\ln(x)+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	42
default	$-\frac{1}{x}-\ln(x)+\frac{2\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	44
meijerg	$-\frac{1}{x}-\frac{x^2\left(-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{(x^3)^{\frac{2}{3}}}+\frac{\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2(x^3)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{(x^3)^{\frac{2}{3}}}\right)}{3}-\ln(x)+\frac{\ln(x^3+1)}{3}$	93

input
```
int((1-x)/x^2/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+2/3*ln(1+x)-ln(x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1
/2))
```

3.309.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{1-x}{x^2(1+x^3)}\,dx = -\frac{2\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-x\log\left(x^2-x+1\right)-4x\log\left(x+1\right)+6x\log\left(x\right)+6}{6x}$$

input
```
integrate((1-x)/x^2/(x^3+1),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x - 1)) - x*log(x^2 - x + 1) - 4*x
*log(x + 1) + 6*x*log(x) + 6)/x
```

3.309.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{x^2(1+x^3)}\,dx = -\log(x) + \frac{2\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{1}{x}$$

input
```
integrate((1-x)/x**2/(x**3+1),x)
```

output
```
-log(x) + 2*log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/
3 - sqrt(3)/3)/3 - 1/x
```

3.309.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{1-x}{x^2(1+x^3)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{x} + \frac{1}{6}\log\left(x^2-x+1\right) + \frac{2}{3}\log(x+1) - \log(x)$$

input
```
integrate((1-x)/x^2/(x^3+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x + 1/6*log(x^2 - x + 1) +
2/3*log(x + 1) - log(x)
```

3.309.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{1-x}{x^2(1+x^3)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{x} + \frac{1}{6}\log\left(x^2-x+1\right) + \frac{2}{3}\log(|x+1|) - \log(|x|)$$

input
```
integrate((1-x)/x^2/(x^3+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/x + 1/6*log(x^2 - x + 1) +
2/3*log(abs(x + 1)) - log(abs(x))
```

3.309.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{1-x}{x^2\left(1+x^3\right)}\,dx = \frac{2\ln\left(x+1\right)}{3} - \ln\left(x\right) + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \frac{1}{x}$$

input
```
int(-(x - 1)/(x^2*(x^3 + 1)),x)
```

output
```
(2*log(x + 1))/3 - log(x) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6
+ 1/6) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - 1/x
```

3.310 $\int \frac{1-x}{x^3(1+x^3)} dx$

3.310.1 Optimal result 2369
3.310.2 Mathematica [A] (verified) 2369
3.310.3 Rubi [A] (verified) 2370
3.310.4 Maple [A] (verified) 2371
3.310.5 Fricas [A] (verification not implemented) 2371
3.310.6 Sympy [A] (verification not implemented) 2372
3.310.7 Maxima [A] (verification not implemented) 2372
3.310.8 Giac [A] (verification not implemented) 2372
3.310.9 Mupad [B] (verification not implemented) 2373

3.310.1 Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{1-x}{x^3(1+x^3)}\,dx = -\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

output
```
-1/2/x^2+1/x-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

3.310.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{x^3(1+x^3)}\,dx = -\frac{1}{2x^2}+\frac{1}{x}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 - x)/(x^3*(1 + x^3)),x]
```

output
```
-1/2*1/x^2 + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

3.310.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{x^3\left(x^3+1\right)}dx$$
$$\Big\downarrow \text{2373}$$
$$\int \left(\frac{1}{x^3}+\frac{2x-1}{3\left(x^2-x+1\right)}-\frac{1}{x^2}-\frac{2}{3(x+1)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{2x^2}+\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{x}-\frac{2}{3}\log(x+1)$$

input
```
Int[(1 - x)/(x^3*(1 + x^3)),x]
```

output
```
-1/2*1/x^2 + x^(-1) - (2*Log[1 + x])/3 + Log[1 - x + x^2]/3
```

3.310.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

3.310.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result
norman	$\frac{x-\frac{1}{2}}{x^{2}}-\frac{2\ln(1+x)}{3}+\frac{\ln(x^{2}-x+1)}{3}$
risch	$\frac{x-\frac{1}{2}}{x^{2}}-\frac{2\ln(1+x)}{3}+\frac{\ln(x^{2}-x+1)}{3}$
default	$-\frac{1}{2x^{2}}+\frac{1}{x}-\frac{2\ln(1+x)}{3}+\frac{\ln(x^{2}-x+1)}{3}$
parallelrisch	$-\frac{4\ln(1+x)x^{2}-2\ln(x^{2}-x+1)x^{2}+3-6x}{6x^{2}}$
meijerg	$-\frac{1}{2x^{2}}-\frac{x\left(\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}-\frac{\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2\left(x^{3}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{2-\left(x^{3}\right)^{\frac{1}{3}}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}\right)}{3}+\frac{1}{x}+\frac{x^{2}\left(-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{\left(x^{3}\right)^{\frac{2}{3}}}+\frac{\ln\left(1-\left(x^{3}\right.\right.}{2\left(x^{3}\right.}\right.}{}$

input
```
int((1-x)/x^3/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
(x-1/2)/x^2-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

3.310.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1-x}{x^{3}\left(1+x^{3}\right)}\,dx=\frac{2\,x^{2}\log\left(x^{2}-x+1\right)-4\,x^{2}\log\left(x+1\right)+6\,x-3}{6\,x^{2}}$$

input
```
integrate((1-x)/x^3/(x^3+1),x, algorithm="fricas")
```

output
```
1/6*(2*x^2*log(x^2 - x + 1) - 4*x^2*log(x + 1) + 6*x - 3)/x^2
```

3.310.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{x^3(1+x^3)}\,dx = -\frac{2\log(x+1)}{3} + \frac{\log(x^2-x+1)}{3} - \frac{1-2x}{2x^2}$$

input
```
integrate((1-x)/x**3/(x**3+1),x)
```

output
```
-2*log(x + 1)/3 + log(x**2 - x + 1)/3 - (1 - 2*x)/(2*x**2)
```

3.310.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1-x}{x^3(1+x^3)}\,dx = \frac{2\,x-1}{2\,x^2} + \frac{1}{3}\,\log\left(x^2-x+1\right) - \frac{2}{3}\,\log\left(x+1\right)$$

input
```
integrate((1-x)/x^3/(x^3+1),x, algorithm="maxima")
```

output
```
1/2*(2*x - 1)/x^2 + 1/3*log(x^2 - x + 1) - 2/3*log(x + 1)
```

3.310.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{1-x}{x^3(1+x^3)}\,dx = \frac{2\,x-1}{2\,x^2} + \frac{1}{3}\,\log\left(x^2-x+1\right) - \frac{2}{3}\,\log\left(|x+1|\right)$$

input
```
integrate((1-x)/x^3/(x^3+1),x, algorithm="giac")
```

output
```
1/2*(2*x - 1)/x^2 + 1/3*log(x^2 - x + 1) - 2/3*log(abs(x + 1))
```

3.310.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{1-x}{x^3\left(1+x^3\right)}\,dx=\frac{\ln\left(x^2-x+1\right)}{3}-\frac{2\,\ln\left(x+1\right)}{3}+\frac{x-\frac{1}{2}}{x^2}$$

input
```
int(-(x - 1)/(x^3*(x^3 + 1)),x)
```

output
```
log(x^2 - x + 1)/3 - (2*log(x + 1))/3 + (x - 1/2)/x^2
```

3.311 $\int \frac{x(1+2x)}{1+x^3} dx$

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3.311.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{x(1+2x)}{1+x^3} dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1+x) + \frac{5}{6}\log\left(1-x+x^2\right)$$

output
```
1/3*ln(1+x)+5/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.311.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{x(1+2x)}{1+x^3} dx = \frac{1}{6}\left(2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\log(1+x) + \log\left(1-x+x^2\right) + 4\log\left(1+x^3\right)\right)$$

input
```
Integrate[(x*(1 + 2*x))/(1 + x^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + Log[1 - x + x^2] +
4*Log[1 + x^3])/6
```

3.311.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2413, 16, 25, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2x+1)}{x^3+1}\,dx$$

$$\Big\downarrow \text{ 2413}$$

$$\frac{1}{3}\int -\frac{1-5x}{x^2-x+1}dx+\frac{1}{3}\int \frac{1}{x+1}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\int -\frac{1-5x}{x^2-x+1}dx+\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\log(x+1)-\frac{1}{3}\int \frac{1-5x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{5}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{5}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{3}\left(-\frac{5}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{5}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{5}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{3}\log(x+1)$$

input
```
Int[(x*(1 + 2*x))/(1 + x^3),x]
```

output
```
Log[1 + x]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + (5*Log[1 - x + x^2])/
2)/3
```

3.311.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2413
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Simp[q*((A - B*q
+ C*q^2)/(3*a))   Int[1/(q + x), x], x] + Simp[q/(3*a)   Int[(q*(2*A + B*q
- C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 -
b*A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2
] && GtQ[a/b, 0]
```

3.311.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln(1+x)}{3}+\frac{5\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}$	35
risch	$\frac{5\ln(4x^2-4x+4)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}+\frac{\ln(1+x)}{3}$	37
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{2\ln(x^3+1)}{3}$	88

input
```
int(x*(1+2*x)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(1+x)+5/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

3.311.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x(1+2x)}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{5}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(x+1\right)$$

input
```
integrate(x*(1+2*x)/(x^3+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log
(x + 1)
```

3.311.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{x(1+2x)}{1+x^3}\,dx = \frac{\log(x+1)}{3}+\frac{5\log(x^2-x+1)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x*(1+2*x)/(x**3+1),x)
```

output
```
log(x + 1)/3 + 5*log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3
)/3)/3
```

3.311.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x(1+2x)}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{5}{6}\log\left(x^2-x+1\right) + \frac{1}{3}\log\left(x+1\right)$$

input
```
integrate(x*(1+2*x)/(x^3+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log
(x + 1)
```

3.311.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x(1+2x)}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{5}{6}\log\left(x^2-x+1\right) + \frac{1}{3}\log\left(|x+1|\right)$$

input
```
integrate(x*(1+2*x)/(x^3+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 5/6*log(x^2 - x + 1) + 1/3*log
(abs(x + 1))
```

3.311.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \frac{x(1+2x)}{1+x^3}\,dx = \frac{5\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6} + \frac{5\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6} + \frac{\ln\left(x+1\right)}{3} - \frac{\sqrt{3}\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6} + \frac{\sqrt{3}\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}$$

input
```
int((x*(2*x + 1))/(x^3 + 1),x)
```

output
```
(5*log(x - (3^(1/2)*1i)/2 - 1/2))/6 + (5*log(x + (3^(1/2)*1i)/2 - 1/2))/6
+ log(x + 1)/3 - (3^(1/2)*log(x - (3^(1/2)*1i)/2 - 1/2)*1i)/6 + (3^(1/2)*l
og(x + (3^(1/2)*1i)/2 - 1/2)*1i)/6
```

3.311. $\int \frac{x(1+2x)}{1+x^3}\,dx$

3.312 $\int \frac{x(1+2x)}{1-x^3} \, dx$

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3.312.1 Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{x(1+2x)}{1-x^3} \, dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log(1-x) - \frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
-ln(1-x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.312.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \frac{x(1+2x)}{1-x^3} \, dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1-x) + \frac{1}{6}\log\left(1+x+x^2\right) - \frac{2}{3}\log\left(1-x^3\right)$$

input
```
Integrate[(x*(1 + 2*x))/(1 - x^3),x]
```

output
```
-(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x]/3 + Log[1 + x + x^2]/6 -
 (2*Log[1 - x^3])/3
```

3.312.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2414, 16, 27, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2x+1)}{1-x^3}dx$$

$$\Big\downarrow \text{ 2414}$$

$$\frac{1}{3}\int -\frac{3(x+1)}{x^2+x+1}dx+\int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\int -\frac{3(x+1)}{x^2+x+1}dx-\log(1-x)$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{x+1}{x^2+x+1}dx-\log(1-x)$$

$$\Big\downarrow \text{ 1142}$$

$$-\frac{1}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\log(1-x)$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx+\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\log(1-x)$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\log(1-x)$$

$$\Big\downarrow \text{ 1103}$$

$$-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)$$

input
```
Int[(x*(1 + 2*x))/(1 - x^3),x]
```

output
```
-(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 - x] - Log[1 + x + x^2]/2
```

3.312.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2414
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Simp[q*((A + B*q
 + C*q^2)/(3*a))   Int[1/(q - x), x], x] + Simp[q/(3*a)   Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

3.312. $\int \frac{x(1+2x)}{1-x^3} dx$

3.312.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$-\ln \left(-1+x \right)-\frac{\ln \left(x^{2}+x +1\right)}{2}-\frac{\arctan \left(\frac{\left(1+2 x \right) \sqrt{3}}{3}\right) \sqrt{3}}{3}$	33
risch	$-\frac{\arctan \left(\frac{\left(1+2 x \right) \sqrt{3}}{3}\right) \sqrt{3}}{3}-\frac{\ln \left(4 x^{2}+4 x +4\right)}{2}-\ln \left(-1+x \right)$	37
meijerg	$-\frac{x^{2} \left(\ln \left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln \left(1+\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(x^{3}\right)^{\frac{1}{3}}}{2+\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{3 \left(x^{3}\right)^{\frac{2}{3}}}-\frac{2 \ln \left(-x^{3}+1\right)}{3}$	74

input
```
int(x*(1+2*x)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(-1+x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x(1+2x)}{1-x^3}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{2}\,\log\left(x^2+x+1\right) - \log\left(x-1\right)$$

input
```
integrate(x*(1+2*x)/(-x^3+1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(x
- 1)
```

3.312.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{x(1+2x)}{1-x^3}\,dx = -\log\left(x-1\right) - \frac{\log\left(x^2+x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x*(1+2*x)/(-x**3+1),x)
```

output
```
-log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3
```

3.312.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x(1+2x)}{1-x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) - \frac{1}{2}\log\left(x^2+x+1\right) - \log\left(x-1\right)$$

input
```
integrate(x*(1+2*x)/(-x^3+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(x
- 1)
```

3.312.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{x(1+2x)}{1-x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) - \frac{1}{2}\log\left(x^2+x+1\right) - \log\left(|x-1|\right)$$

input
```
integrate(x*(1+2*x)/(-x^3+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1) - log(ab
s(x - 1))
```

3.312.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{x(1+2x)}{1-x^3}\,dx = -\frac{\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2} - \frac{\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2} - \ln(x-1) + \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6} - \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{6}$$

input
```
int(-(x*(2*x + 1))/(x^3 - 1),x)
```

output
```
(3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/6 - log(x + (3^(1/2)*1i)/2 + 1/
2)/2 - log(x - 1) - log(x - (3^(1/2)*1i)/2 + 1/2)/2 - (3^(1/2)*log(x + (3^
(1/2)*1i)/2 + 1/2)*1i)/6
```

3.313 $\int x^2(c+dx+ex^2)(a+bx^3)\,dx$

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3.313.1 Optimal result

Integrand size = 21, antiderivative size = 55

$$\int x^2(c+dx+ex^2)(a+bx^3)\,dx=\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

output
```
1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*b*c*x^6+1/7*b*d*x^7+1/8*b*e*x^8
```

3.313.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^2(c+dx+ex^2)(a+bx^3)\,dx=\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

input
```
Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
```

output
```
(a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e
*x^8)/8
```

3.313.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx^3) (c+dx+ex^2) \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int (acx^2+adx^3+aex^4+bcx^5+bdx^6+bex^7) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}bcx^6+\frac{1}{7}bdx^7+\frac{1}{8}bex^8$$

input
```
Int[x^2*(c + d*x + e*x^2)*(a + b*x^3),x]
```

output
```
(a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*c*x^6)/6 + (b*d*x^7)/7 + (b*e
*x^8)/8
```

3.313.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.313.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{3}ac\,x^3+\frac{1}{4}ad\,x^4+\frac{1}{5}ae\,x^5+\frac{1}{6}bc\,x^6+\frac{1}{7}bd\,x^7+\frac{1}{8}be\,x^8$	44
default	$\frac{1}{3}ac\,x^3+\frac{1}{4}ad\,x^4+\frac{1}{5}ae\,x^5+\frac{1}{6}bc\,x^6+\frac{1}{7}bd\,x^7+\frac{1}{8}be\,x^8$	44
norman	$\frac{1}{3}ac\,x^3+\frac{1}{4}ad\,x^4+\frac{1}{5}ae\,x^5+\frac{1}{6}bc\,x^6+\frac{1}{7}bd\,x^7+\frac{1}{8}be\,x^8$	44
risch	$\frac{1}{3}ac\,x^3+\frac{1}{4}ad\,x^4+\frac{1}{5}ae\,x^5+\frac{1}{6}bc\,x^6+\frac{1}{7}bd\,x^7+\frac{1}{8}be\,x^8$	44
parallelrisch	$\frac{1}{3}ac\,x^3+\frac{1}{4}ad\,x^4+\frac{1}{5}ae\,x^5+\frac{1}{6}bc\,x^6+\frac{1}{7}bd\,x^7+\frac{1}{8}be\,x^8$	44

input
```
int(x^2*(e*x^2+d*x+c)*(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*b*c*x^6+1/7*b*d*x^7+1/8*b*e*x^8
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{1}{8}\,bex^8+\frac{1}{7}\,bdx^7+\frac{1}{6}\,bcx^6+\frac{1}{5}\,aex^5+\frac{1}{4}\,adx^4+\frac{1}{3}\,acx^3$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
```

output
```
1/8*b*e*x^8 + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*e*x^5 + 1/4*a*d*x^4 + 1/3*
a*c*x^3
```

3.313.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{acx^3}{3}+\frac{adx^4}{4}+\frac{aex^5}{5}+\frac{bcx^6}{6}+\frac{bdx^7}{7}+\frac{bex^8}{8}$$

input
```
integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a),x)
```

output
```
a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*c*x**6/6 + b*d*x**7/7 + b*e*x**8/
8
```

3.313.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)\,dx = \frac{1}{8}bex^8+\frac{1}{7}bdx^7+\frac{1}{6}bcx^6+\frac{1}{5}aex^5+\frac{1}{4}adx^4+\frac{1}{3}acx^3$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
```

output
```
1/8*b*e*x^8 + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*e*x^5 + 1/4*a*d*x^4 + 1/3*
a*c*x^3
```

3.313.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)\,dx = \frac{1}{8}bex^8+\frac{1}{7}bdx^7+\frac{1}{6}bcx^6+\frac{1}{5}aex^5+\frac{1}{4}adx^4+\frac{1}{3}acx^3$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
```

output
```
1/8*b*e*x^8 + 1/7*b*d*x^7 + 1/6*b*c*x^6 + 1/5*a*e*x^5 + 1/4*a*d*x^4 + 1/3*
a*c*x^3
```

3.313.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)\,dx = \frac{b\,e\,x^8}{8}+\frac{b\,d\,x^7}{7}+\frac{b\,c\,x^6}{6}+\frac{a\,e\,x^5}{5}+\frac{a\,d\,x^4}{4}+\frac{a\,c\,x^3}{3}$$

input
```
int(x^2*(a + b*x^3)*(c + d*x + e*x^2),x)
```

output
```
(a*c*x^3)/3 + (a*d*x^4)/4 + (b*c*x^6)/6 + (a*e*x^5)/5 + (b*d*x^7)/7 + (b*e
*x^8)/8
```

3.314 $\int x(c+dx+ex^2)(a+bx^3)\,dx$

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3.314.6 Sympy [A] (verification not implemented) 2392
3.314.7 Maxima [A] (verification not implemented) 2393
3.314.8 Giac [A] (verification not implemented) 2393
3.314.9 Mupad [B] (verification not implemented) 2393

3.314.1 Optimal result

Integrand size = 19, antiderivative size = 55

$$\int x(c+dx+ex^2)(a+bx^3)\,dx=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

output
```
1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x^7
```

3.314.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x(c+dx+ex^2)(a+bx^3)\,dx=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

input
```
Integrate[x*(c + d*x + e*x^2)*(a + b*x^3),x]
```

output
```
(a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e
*x^7)/7
```

3.314.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)\left(c+dx+ex^2\right)\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(acx+adx^2+aex^3+bcx^4+bdx^5+bex^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7$$

input
```
Int[x*(c + d*x + e*x^2)*(a + b*x^3),x]
```

output
```
(a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/6 + (b*e
*x^7)/7
```

3.314.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.314.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{2}ac\,x^2+\frac{1}{3}ad\,x^3+\frac{1}{4}ae\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7$	44
default	$\frac{1}{2}ac\,x^2+\frac{1}{3}ad\,x^3+\frac{1}{4}ae\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7$	44
norman	$\frac{1}{2}ac\,x^2+\frac{1}{3}ad\,x^3+\frac{1}{4}ae\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7$	44
risch	$\frac{1}{2}ac\,x^2+\frac{1}{3}ad\,x^3+\frac{1}{4}ae\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7$	44
parallelrisch	$\frac{1}{2}ac\,x^2+\frac{1}{3}ad\,x^3+\frac{1}{4}ae\,x^4+\frac{1}{5}bc\,x^5+\frac{1}{6}bd\,x^6+\frac{1}{7}be\,x^7$	44

input
```
int(x*(e*x^2+d*x+c)*(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x^7
```

3.314.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,aex^4+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
```

output
```
1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*e*x^4 + 1/3*a*d*x^3 + 1/2*
a*c*x^2
```

3.314.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int x(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{acx^2}{2}+\frac{adx^3}{3}+\frac{aex^4}{4}+\frac{bcx^5}{5}+\frac{bdx^6}{6}+\frac{bex^7}{7}$$

input
```
integrate(x*(e*x**2+d*x+c)*(b*x**3+a),x)
```

output
```
a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b*e*x**7/
7
```

3.314.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,aex^4+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
```

output
```
1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*e*x^4 + 1/3*a*d*x^3 + 1/2*
a*c*x^2
```

3.314.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,aex^4+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
```

output
```
1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*e*x^4 + 1/3*a*d*x^3 + 1/2*
a*c*x^2
```

3.314.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x(c+dx+ex^2)\left(a+bx^3\right)\,dx=\frac{b\,e\,x^7}{7}+\frac{b\,d\,x^6}{6}+\frac{b\,c\,x^5}{5}+\frac{a\,e\,x^4}{4}+\frac{a\,d\,x^3}{3}+\frac{a\,c\,x^2}{2}$$

input
```
int(x*(a + b*x^3)*(c + d*x + e*x^2),x)
```

output
```
(a*c*x^2)/2 + (a*d*x^3)/3 + (b*c*x^5)/5 + (a*e*x^4)/4 + (b*d*x^6)/6 + (b*e
*x^7)/7
```

3.315 $\int (c+dx+ex^2)(a+bx^3)\,dx$

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3.315.8 Giac [A] (verification not implemented) 2397
3.315.9 Mupad [B] (verification not implemented) 2397

3.315.1 Optimal result

Integrand size = 18, antiderivative size = 50

$$\int (c+dx+ex^2)(a+bx^3)\,dx = acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

output
```
a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*b*c*x^4+1/5*b*d*x^5+1/6*b*e*x^6
```

3.315.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (c+dx+ex^2)(a+bx^3)\,dx = acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

input
```
Integrate[(c + d*x + e*x^2)*(a + b*x^3),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/
6
```

3.315.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)\left(c+dx+ex^2\right)\,dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(ac+adx+aex^2+bcx^3+bdx^4+bex^5\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}bcx^4+\frac{1}{5}bdx^5+\frac{1}{6}bex^6$$

input
```
Int[(c + d*x + e*x^2)*(a + b*x^3),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*c*x^4)/4 + (b*d*x^5)/5 + (b*e*x^6)/
6
```

3.315.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.315.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

method	result	size
gosper	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}ae\,x^{3}+\frac{1}{4}bc\,x^{4}+\frac{1}{5}bd\,x^{5}+\frac{1}{6}be\,x^{6}$	41
default	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}ae\,x^{3}+\frac{1}{4}bc\,x^{4}+\frac{1}{5}bd\,x^{5}+\frac{1}{6}be\,x^{6}$	41
norman	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}ae\,x^{3}+\frac{1}{4}bc\,x^{4}+\frac{1}{5}bd\,x^{5}+\frac{1}{6}be\,x^{6}$	41
risch	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}ae\,x^{3}+\frac{1}{4}bc\,x^{4}+\frac{1}{5}bd\,x^{5}+\frac{1}{6}be\,x^{6}$	41
parallelrisch	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}ae\,x^{3}+\frac{1}{4}bc\,x^{4}+\frac{1}{5}bd\,x^{5}+\frac{1}{6}be\,x^{6}$	41

input
```
int((e*x^2+d*x+c)*(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*b*c*x^4+1/5*b*d*x^5+1/6*b*e*x^6
```

3.315.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx=\frac{1}{6}\,bex^6+\frac{1}{5}\,bdx^5+\frac{1}{4}\,bcx^4+\frac{1}{3}\,aex^3+\frac{1}{2}\,adx^2+acx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*b*e*x^6 + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*
x
```

3.315.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)\,dx=acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{bcx^4}{4}+\frac{bdx^5}{5}+\frac{bex^6}{6}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a),x)
```

output
```
a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*c*x**4/4 + b*d*x**5/5 + b*e*x**6/6
```

3.315.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (c+dx+ex^2)(a+bx^3)\,dx = \frac{1}{6}bex^6 + \frac{1}{5}bdx^5 + \frac{1}{4}bcx^4 + \frac{1}{3}aex^3 + \frac{1}{2}adx^2 + acx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="maxima")
```

output
```
1/6*b*e*x^6 + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*
x
```

3.315.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (c+dx+ex^2)(a+bx^3)\,dx = \frac{1}{6}bex^6 + \frac{1}{5}bdx^5 + \frac{1}{4}bcx^4 + \frac{1}{3}aex^3 + \frac{1}{2}adx^2 + acx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a),x, algorithm="giac")
```

output
```
1/6*b*e*x^6 + 1/5*b*d*x^5 + 1/4*b*c*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*
x
```

3.315.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (c+dx+ex^2)(a+bx^3)\,dx = \frac{b\,e\,x^6}{6} + \frac{b\,d\,x^5}{5} + \frac{b\,c\,x^4}{4} + \frac{a\,e\,x^3}{3} + \frac{a\,d\,x^2}{2} + a\,c\,x$$

input
```
int((a + b*x^3)*(c + d*x + e*x^2),x)
```

output
```
a*c*x + (a*d*x^2)/2 + (b*c*x^4)/4 + (a*e*x^3)/3 + (b*d*x^5)/5 + (b*e*x^6)/
6
```

3.316 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx$

3.316.1 Optimal result 2398
3.316.2 Mathematica [A] (verified) 2398
3.316.3 Rubi [A] (verified) 2399
3.316.4 Maple [A] (verified) 2400
3.316.5 Fricas [A] (verification not implemented) 2400
3.316.6 Sympy [A] (verification not implemented) 2400
3.316.7 Maxima [A] (verification not implemented) 2401
3.316.8 Giac [A] (verification not implemented) 2401
3.316.9 Mupad [B] (verification not implemented) 2401

3.316.1 Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx = adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5 + ac\log(x)$$

output
```
a*d*x+1/2*a*e*x^2+1/3*b*c*x^3+1/4*b*d*x^4+1/5*b*e*x^5+a*c*ln(x)
```

3.316.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x} dx = adx + \frac{1}{2}aex^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 + \frac{1}{5}bex^5 + ac\log(x)$$

input
```
Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x,x]
```

output
```
a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]
```

3.316.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(c+dx+ex^2)}{x}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{ac}{x}+ad+aex+bcx^2+bdx^3+bex^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4+\frac{1}{5}bex^5$$

input
```
Int[((c + d*x + e*x^2)*(a + b*x^3))/x,x]
```

output
```
a*d*x + (a*e*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4 + (b*e*x^5)/5 + a*c*Log[x]
```

3.316.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.316.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
default	$adx+\frac{ae\,x^{2}}{2}+\frac{bc\,x^{3}}{3}+\frac{bd\,x^{4}}{4}+\frac{be\,x^{5}}{5}+ac\ln\left(x\right)$	39
norman	$adx+\frac{ae\,x^{2}}{2}+\frac{bc\,x^{3}}{3}+\frac{bd\,x^{4}}{4}+\frac{be\,x^{5}}{5}+ac\ln\left(x\right)$	39
risch	$adx+\frac{ae\,x^{2}}{2}+\frac{bc\,x^{3}}{3}+\frac{bd\,x^{4}}{4}+\frac{be\,x^{5}}{5}+ac\ln\left(x\right)$	39
parallelrisch	$adx+\frac{ae\,x^{2}}{2}+\frac{bc\,x^{3}}{3}+\frac{bd\,x^{4}}{4}+\frac{be\,x^{5}}{5}+ac\ln\left(x\right)$	39

input
```
int((e*x^2+d*x+c)*(b*x^3+a)/x,x,method=_RETURNVERBOSE)
```

output
```
a*d*x+1/2*a*e*x^2+1/3*b*c*x^3+1/4*b*d*x^4+1/5*b*e*x^5+a*c*ln(x)
```

3.316.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x}\,dx = \frac{1}{5}\,bex^5+\frac{1}{4}\,bdx^4+\frac{1}{3}\,bcx^3+\frac{1}{2}\,aex^2+adx+ac\log\left(x\right)$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="fricas")
```

output
```
1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*log(x)
```

3.316.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x}\,dx = ac\log\left(x\right)+adx+\frac{aex^2}{2}+\frac{bcx^3}{3}+\frac{bdx^4}{4}+\frac{bex^5}{5}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)/x,x)
```

output
```
a*c*log(x) + a*d*x + a*e*x**2/2 + b*c*x**3/3 + b*d*x**4/4 + b*e*x**5/5
```

3.316.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x}\,dx = \frac{1}{5}bex^5 + \frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}aex^2 + adx + ac\log(x)$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="maxima")
```

output
```
1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*log(x)
```

3.316.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x}\,dx = \frac{1}{5}bex^5 + \frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}aex^2 + adx + ac\log(|x|)$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)/x,x, algorithm="giac")
```

output
```
1/5*b*e*x^5 + 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*e*x^2 + a*d*x + a*c*log(ab
s(x))
```

3.316.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x}\,dx = a\,c\,\ln(x) + a\,d\,x + \frac{b\,c\,x^3}{3} + \frac{a\,e\,x^2}{2} + \frac{b\,d\,x^4}{4} + \frac{b\,e\,x^5}{5}$$

input
```
int(((a + b*x^3)*(c + d*x + e*x^2))/x,x)
```

output
```
a*c*log(x) + a*d*x + (b*c*x^3)/3 + (a*e*x^2)/2 + (b*d*x^4)/4 + (b*e*x^5)/5
```

3.317 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx$

3.317.1 Optimal result 2402
3.317.2 Mathematica [A] (verified) 2402
3.317.3 Rubi [A] (verified) 2403
3.317.4 Maple [A] (verified) 2404
3.317.5 Fricas [A] (verification not implemented) 2404
3.317.6 Sympy [A] (verification not implemented) 2404
3.317.7 Maxima [A] (verification not implemented) 2405
3.317.8 Giac [A] (verification not implemented) 2405
3.317.9 Mupad [B] (verification not implemented) 2405

3.317.1 Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx = -\frac{ac}{x}+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+ad\log(x)$$

output
```
-a*c/x+a*e*x+1/2*b*c*x^2+1/3*b*d*x^3+1/4*b*e*x^4+a*d*ln(x)
```

3.317.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx = -\frac{ac}{x}+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+ad\log(x)$$

input
```
Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x^2,x]
```

output
```
-((a*c)/x) + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*Log[x]
```

3.317.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(c+dx+ex^2)}{x^2}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{ac}{x^2}+\frac{ad}{x}+ae+bcx+bdx^2+bex^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{2}bcx^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4$$

input
```
Int[((c + d*x + e*x^2)*(a + b*x^3))/x^2,x]
```

output
```
-((a*c)/x) + a*e*x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + a*d*Log[x]
```

3.317.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.317.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{ac}{x}+aex+\frac{cb\,x^{2}}{2}+\frac{bd\,x^{3}}{3}+\frac{be\,x^{4}}{4}+ad\ln\left(x\right)$	39
risch	$-\frac{ac}{x}+aex+\frac{cb\,x^{2}}{2}+\frac{bd\,x^{3}}{3}+\frac{be\,x^{4}}{4}+ad\ln\left(x\right)$	39
norman	$\frac{ae\,x^{2}-ac+\frac{1}{2}bc\,x^{3}+\frac{1}{3}bd\,x^{4}+\frac{1}{4}be\,x^{5}}{x}+ad\ln\left(x\right)$	43
parallelrisch	$\frac{3be\,x^{5}+4bd\,x^{4}+6bc\,x^{3}+12ad\ln\left(x\right)x+12ae\,x^{2}-12ac}{12x}$	46

input
```
int((e*x^2+d*x+c)*(b*x^3+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a*c/x+a*e*x+1/2*c*b*x^2+1/3*b*d*x^3+1/4*b*e*x^4+a*d*ln(x)
```

3.317.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)}{x^2}\,dx=\frac{3\,bex^5+4\,bdx^4+6\,bcx^3+12\,aex^2+12\,adx\log\left(x\right)-12\,ac}{12\,x}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="fricas")
```

output
```
1/12*(3*b*e*x^5 + 4*b*d*x^4 + 6*b*c*x^3 + 12*a*e*x^2 + 12*a*d*x*log(x) - 1
2*a*c)/x
```

3.317.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)}{x^2}\,dx=-\frac{ac}{x}+ad\log\left(x\right)+aex+\frac{bcx^2}{2}+\frac{bdx^3}{3}+\frac{bex^4}{4}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)/x**2,x)
```

output
```
-a*c/x + a*d*log(x) + a*e*x + b*c*x**2/2 + b*d*x**3/3 + b*e*x**4/4
```

3.317.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx = \frac{1}{4}\,bex^4 + \frac{1}{3}\,bdx^3 + \frac{1}{2}\,bcx^2 + aex + ad\log\left(x\right) - \frac{ac}{x}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="maxima")
```

output
```
1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*b*c*x^2 + a*e*x + a*d*log(x) - a*c/x
```

3.317.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx = \frac{1}{4}\,bex^4 + \frac{1}{3}\,bdx^3 + \frac{1}{2}\,bcx^2 + aex + ad\log\left(|x|\right) - \frac{ac}{x}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)/x^2,x, algorithm="giac")
```

output
```
1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*b*c*x^2 + a*e*x + a*d*log(abs(x)) - a*c/x
```

3.317.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^2}\,dx = a\,d\,\ln\left(x\right) + a\,e\,x - \frac{a\,c}{x} + \frac{b\,c\,x^2}{2} + \frac{b\,d\,x^3}{3} + \frac{b\,e\,x^4}{4}$$

input
```
int(((a + b*x^3)*(c + d*x + e*x^2))/x^2,x)
```

output
```
a*d*log(x) + a*e*x - (a*c)/x + (b*c*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4
```

3.318 $\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3} dx$

3.318.1 Optimal result 2406
3.318.2 Mathematica [A] (verified) 2406
3.318.3 Rubi [A] (verified) 2407
3.318.4 Maple [A] (verified) 2408
3.318.5 Fricas [A] (verification not implemented) 2408
3.318.6 Sympy [A] (verification not implemented) 2408
3.318.7 Maxima [A] (verification not implemented) 2409
3.318.8 Giac [A] (verification not implemented) 2409
3.318.9 Mupad [B] (verification not implemented) 2409

3.318.1 Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3} dx = -\frac{ac}{2x^2} - \frac{ad}{x} + bcx + \frac{1}{2}bdx^2 + \frac{1}{3}bex^3 + ae\log(x)$$

output
```
-1/2*a*c/x^2-a*d/x+b*c*x+1/2*b*d*x^2+1/3*b*e*x^3+a*e*ln(x)
```

3.318.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3} dx = -\frac{ac}{2x^2} - \frac{ad}{x} + bcx + \frac{1}{2}bdx^2 + \frac{1}{3}bex^3 + ae\log(x)$$

input
```
Integrate[((c + d*x + e*x^2)*(a + b*x^3))/x^3,x]
```

output
```
-1/2*(a*c)/x^2 - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*Log[x]
```

3.318.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(c+dx+ex^2)}{x^3}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{ac}{x^3}+\frac{ad}{x^2}+\frac{ae}{x}+bc+bdx+bex^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+bcx+\frac{1}{2}bdx^2+\frac{1}{3}bex^3$$

input
```
Int[((c + d*x + e*x^2)*(a + b*x^3))/x^3,x]
```

output
```
-1/2*(a*c)/x^2 - (a*d)/x + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3 + a*e*Log[x]
```

3.318.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.318.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{ac}{2x^2}-\frac{ad}{x}+bcx+\frac{bd\,x^2}{2}+\frac{be\,x^3}{3}+ae\ln\left(x\right)$	39
risch	$\frac{be\,x^3}{3}+\frac{bd\,x^2}{2}+bcx+\frac{-adx-\frac{1}{2}ac}{x^2}+ae\ln\left(x\right)$	39
norman	$\frac{bc\,x^3-\frac{1}{2}ac-adx+\frac{1}{2}bd\,x^4+\frac{1}{3}be\,x^5}{x^2}+ae\ln\left(x\right)$	41
parallelrisch	$\frac{2be\,x^5+3bd\,x^4+6ae\ln(x)x^2+6bc\,x^3-6adx-3ac}{6x^2}$	46

input
```
int((e*x^2+d*x+c)*(b*x^3+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*c/x^2-a*d/x+b*c*x+1/2*b*d*x^2+1/3*b*e*x^3+a*e*ln(x)
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)}{x^3}\,dx=\frac{2\,bex^5+3\,bdx^4+6\,bcx^3+6\,aex^2\log\left(x\right)-6\,adx-3\,ac}{6\,x^2}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="fricas")
```

output
```
1/6*(2*b*e*x^5 + 3*b*d*x^4 + 6*b*c*x^3 + 6*a*e*x^2*log(x) - 6*a*d*x - 3*a*
c)/x^2
```

3.318.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)}{x^3}\,dx=ae\log\left(x\right)+bcx+\frac{bdx^2}{2}+\frac{bex^3}{3}+\frac{-ac-2adx}{2x^2}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)/x**3,x)
```

output
```
a*e*log(x) + b*c*x + b*d*x**2/2 + b*e*x**3/3 + (-a*c - 2*a*d*x)/(2*x**2)
```

3.318.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx = \frac{1}{3}\,bex^3+\frac{1}{2}\,bdx^2+bcx+ae\log{(x)}-\frac{2\,adx+ac}{2\,x^2}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="maxima")
```

output
```
1/3*b*e*x^3 + 1/2*b*d*x^2 + b*c*x + a*e*log(x) - 1/2*(2*a*d*x + a*c)/x^2
```

3.318.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx = \frac{1}{3}\,bex^3+\frac{1}{2}\,bdx^2+bcx+ae\log{(|x|)}-\frac{2\,adx+ac}{2\,x^2}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)/x^3,x, algorithm="giac")
```

output
```
1/3*b*e*x^3 + 1/2*b*d*x^2 + b*c*x + a*e*log(abs(x)) - 1/2*(2*a*d*x + a*c)/
x^2
```

3.318.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)}{x^3}\,dx = a\,e\,\ln{(x)}-\frac{\frac{a\,c}{2}+a\,d\,x}{x^2}+b\,c\,x+\frac{b\,d\,x^2}{2}+\frac{b\,e\,x^3}{3}$$

input
```
int(((a + b*x^3)*(c + d*x + e*x^2))/x^3,x)
```

output
```
a*e*log(x) - ((a*c)/2 + a*d*x)/x^2 + b*c*x + (b*d*x^2)/2 + (b*e*x^3)/3
```

3.319 $\int x^2(c+dx+ex^2)(a+bx^3)^2 \, dx$

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3.319.2 Mathematica [A] (verified) . . . 2410
3.319.3 Rubi [A] (verified) . . . 2411
3.319.4 Maple [A] (verified) . . . 2412
3.319.5 Fricas [A] (verification not implemented) . . . 2412
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3.319.8 Giac [A] (verification not implemented) . . . 2413
3.319.9 Mupad [B] (verification not implemented) . . . 2414

3.319.1 Optimal result

Integrand size = 23, antiderivative size = 82

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx=\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{c(a+bx^3)^3}{9b}$$

output
```
1/4*a^2*d*x^4+1/5*a^2*e*x^5+2/7*a*b*d*x^7+1/4*a*b*e*x^8+1/10*b^2*d*x^10+1/
11*b^2*e*x^11+1/9*c*(b*x^3+a)^3/b
```

3.319.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx=\frac{1}{3}a^2cx^3+\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{3}abcx^6+\frac{2}{7}abdx^7+\frac{1}{4}abex^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

input
```
Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

output
```
(a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a*b*c*x^6)/3 + (2*a*b*d*x
^7)/7 + (a*b*e*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11
```

3.319.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b x^3\right)^2 \left(c+d x+e x^2\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(b x^3+a\right)^2 \left(x^2 \left(e x^2+d x+c\right)-c x^2\right) \, dx+\frac{c \left(a+b x^3\right)^3}{9 b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^2 e x^{10}+b^2 d x^9+2 a b e x^7+2 a b d x^6+a^2 e x^4+a^2 d x^3\right) \, dx+\frac{c \left(a+b x^3\right)^3}{9 b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} a^2 d x^4+\frac{1}{5} a^2 e x^5+\frac{c \left(a+b x^3\right)^3}{9 b}+\frac{2}{7} a b d x^7+\frac{1}{4} a b e x^8+\frac{1}{10} b^2 d x^{10}+\frac{1}{11} b^2 e x^{11}$$

input
```
Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

output
```
(a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4 + (b^2*d*x
^10)/10 + (b^2*e*x^11)/11 + (c*(a + b*x^3)^3)/(9*b)
```

3.319.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.319.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result
gosper	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abe\,x^8+\frac{2}{7}ad\,x^7b+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x$
default	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abe\,x^8+\frac{2}{7}ad\,x^7b+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x$
norman	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abe\,x^8+\frac{2}{7}ad\,x^7b+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x$
risch	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abe\,x^8+\frac{2}{7}ad\,x^7b+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x$
parallelrisch	$\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abe\,x^8+\frac{2}{7}ad\,x^7b+\frac{1}{3}abc\,x^6+\frac{1}{5}a^2e\,x^5+\frac{1}{4}a^2d\,x^4+\frac{1}{3}a^2c\,x$

input
```
int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*a*b*e*x^8+2/7*a*d*x^7*b+
1/3*a*b*c*x^6+1/5*a^2*e*x^5+1/4*a^2*d*x^4+1/3*a^2*c*x^3
```

3.319.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abex^8+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2ex^5+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*e*x^8 + 2/7*a*
b*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*e*x^5 + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

3.319.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{a^2cx^3}{3}+\frac{a^2dx^4}{4}+\frac{a^2ex^5}{5}+\frac{abcx^6}{3}+\frac{2abdx^7}{7}+\frac{abex^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}$$

input
```
integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

output
```
a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + a*b*c*x**6/3 + 2*a*b*d*x**
7/7 + a*b*e*x**8/4 + b**2*c*x**9/9 + b**2*d*x**10/10 + b**2*e*x**11/11
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abex^8+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2ex^5+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*e*x^8 + 2/7*a*
b*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*e*x^5 + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

3.319.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abex^8+\frac{2}{7}\,abdx^7+\frac{1}{3}\,abcx^6+\frac{1}{5}\,a^2ex^5+\frac{1}{4}\,a^2dx^4+\frac{1}{3}\,a^2cx^3$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*a*b*e*x^8 + 2/7*a*
b*d*x^7 + 1/3*a*b*c*x^6 + 1/5*a^2*e*x^5 + 1/4*a^2*d*x^4 + 1/3*a^2*c*x^3
```

3.319.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^2\,dx=\frac{e\,a^2\,x^5}{5}+\frac{d\,a^2\,x^4}{4}+\frac{c\,a^2\,x^3}{3}+\frac{e\,a\,b\,x^8}{4}+\frac{2\,d\,a\,b\,x^7}{7}+\frac{c\,a\,b\,x^6}{3}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

input
```
int(x^2*(a + b*x^3)^2*(c + d*x + e*x^2),x)
```

output
```
(a^2*c*x^3)/3 + (a^2*d*x^4)/4 + (b^2*c*x^9)/9 + (a^2*e*x^5)/5 + (b^2*d*x^1
0)/10 + (b^2*e*x^11)/11 + (a*b*c*x^6)/3 + (2*a*b*d*x^7)/7 + (a*b*e*x^8)/4
```

3.320 $\int x(c+dx+ex^2)(a+bx^3)^2 \, dx$

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3.320.2 Mathematica [A] (verified) . . . 2415
3.320.3 Rubi [A] (verified) . . . 2416
3.320.4 Maple [A] (verified) . . . 2417
3.320.5 Fricas [A] (verification not implemented) . . . 2417
3.320.6 Sympy [A] (verification not implemented) . . . 2418
3.320.7 Maxima [A] (verification not implemented) . . . 2418
3.320.8 Giac [A] (verification not implemented) . . . 2418
3.320.9 Mupad [B] (verification not implemented) . . . 2419

3.320.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int x(c+dx+ex^2)(a+bx^3)^2 \, dx = \frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}+\frac{d(a+bx^3)^3}{9b}$$

output
```
1/2*a^2*c*x^2+1/4*a^2*e*x^4+2/5*a*b*c*x^5+2/7*a*b*e*x^7+1/8*b^2*c*x^8+1/10
*b^2*e*x^10+1/9*d*(b*x^3+a)^3/b
```

3.320.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int x(c+dx+ex^2)(a+bx^3)^2 \, dx = \frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{9}b^2dx^9+\frac{1}{10}b^2ex^{10}$$

input
```
Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

output
```
(a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 + (a*b*d*x
^6)/3 + (2*a*b*e*x^7)/7 + (b^2*c*x^8)/8 + (b^2*d*x^9)/9 + (b^2*e*x^10)/10
```

3.320.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^2\left(c+dx+ex^2\right)\,dx$$

$$\Big\downarrow \ 2017$$

$$\int \left(bx^3+a\right)^2\left(x\left(ex^2+dx+c\right)-dx^2\right)dx+\frac{d\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \ 2389$$

$$\int \left(b^2ex^9+b^2cx^7+2abex^6+2abcx^4+a^2ex^3+a^2cx\right)dx+\frac{d\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{2}{5}abcx^5+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{2}{7}abex^7+\frac{1}{8}b^2cx^8+\frac{1}{10}b^2ex^{10}$$

input
```
Int[x*(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

output
```
(a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (2*a*b*c*x^5)/5 + (2*a*b*e*x^7)/7 + (b^2*c
*x^8)/8 + (b^2*e*x^10)/10 + (d*(a + b*x^3)^3)/(9*b)
```

3.320.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.320.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result
gosper	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$
default	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$
norman	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$
risch	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$
parallelrisch	$\frac{1}{10}b^2e\,x^{10}+\frac{1}{9}b^2d\,x^9+\frac{1}{8}b^2c\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2e\,x^4+\frac{1}{3}a^2d\,x^3+\frac{1}{2}a^2c\,x^2$

input
```
int(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*b^2*e*x^10+1/9*b^2*d*x^9+1/8*b^2*c*x^8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/
5*a*b*c*x^5+1/4*a^2*e*x^4+1/3*a^2*d*x^3+1/2*a^2*c*x^2
```

3.320.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{10}\,b^2ex^{10}+\frac{1}{9}\,b^2dx^9+\frac{1}{8}\,b^2cx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2ex^4+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/10*b^2*e*x^10 + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*
d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*e*x^4 + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

3.320.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{a^2cx^2}{2}+\frac{a^2dx^3}{3}+\frac{a^2ex^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{b^2cx^8}{8}+\frac{b^2dx^9}{9}+\frac{b^2ex^{10}}{10}$$

input
```
integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

output
```
a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + 2*a*b*c*x**5/5 + a*b*d*x**
6/3 + 2*a*b*e*x**7/7 + b**2*c*x**8/8 + b**2*d*x**9/9 + b**2*e*x**10/10
```

3.320.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{10}\,b^2ex^{10}+\frac{1}{9}\,b^2dx^9+\frac{1}{8}\,b^2cx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2ex^4+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/10*b^2*e*x^10 + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*
d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*e*x^4 + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

3.320.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{10}\,b^2ex^{10}+\frac{1}{9}\,b^2dx^9+\frac{1}{8}\,b^2cx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2ex^4+\frac{1}{3}\,a^2dx^3+\frac{1}{2}\,a^2cx^2$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/10*b^2*e*x^10 + 1/9*b^2*d*x^9 + 1/8*b^2*c*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*
d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*e*x^4 + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

3.320.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{e\,a^2\,x^4}{4}+\frac{d\,a^2\,x^3}{3}+\frac{c\,a^2\,x^2}{2}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{e\,b^2\,x^{10}}{10}+\frac{d\,b^2\,x^9}{9}+\frac{c\,b^2\,x^8}{8}$$

input
```
int(x*(a + b*x^3)^2*(c + d*x + e*x^2),x)
```

output
```
(a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (b^2*c*x^8)/8 + (a^2*e*x^4)/4 + (b^2*d*x^9
)/9 + (b^2*e*x^10)/10 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7
```

3.321 $\int (c+dx+ex^2)(a+bx^3)^2 \, dx$

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3.321.1 Optimal result

Integrand size = 20, antiderivative size = 77

$$\int (c+dx+ex^2)(a+bx^3)^2 \, dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{e(a+bx^3)^3}{9b}$$

output
```
a^2*c*x+1/2*a^2*d*x^2+1/2*a*b*c*x^4+2/5*a*b*d*x^5+1/7*b^2*c*x^7+1/8*b^2*d*
x^8+1/9*e*(b*x^3+a)^3/b
```

3.321.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int (c+dx+ex^2)(a+bx^3)^2 \, dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{3}abex^6+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{1}{9}b^2ex^9$$

input
```
Integrate[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5
+ (a*b*e*x^6)/3 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + (b^2*e*x^9)/9
```

3.321.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)^2 (c+dx+ex^2)\,dx$$

$$\Big\downarrow \text{2017}$$

$$\int (c+dx)(bx^3+a)^2dx+\frac{e(a+bx^3)^3}{9b}$$

$$\Big\downarrow \text{2389}$$

$$\int (b^2dx^7+b^2cx^6+2abdx^4+2abcx^3+a^2dx+a^2c)\,dx+\frac{e(a+bx^3)^3}{9b}$$

$$\Big\downarrow \text{2009}$$

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{e(a+bx^3)^3}{9b}+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

input
```
Int[(c + d*x + e*x^2)*(a + b*x^3)^2,x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7
+ (b^2*d*x^8)/8 + (e*(a + b*x^3)^3)/(9*b)
```

3.321.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.321.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}aeb\,x^6+\frac{2}{5}x^5dba+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77
default	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}aeb\,x^6+\frac{2}{5}x^5dba+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77
norman	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}aeb\,x^6+\frac{2}{5}x^5dba+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77
risch	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}aeb\,x^6+\frac{2}{5}x^5dba+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77
parallelrisch	$\frac{1}{9}b^2e\,x^9+\frac{1}{8}b^2d\,x^8+\frac{1}{7}b^2c\,x^7+\frac{1}{3}aeb\,x^6+\frac{2}{5}x^5dba+\frac{1}{2}abc\,x^4+\frac{1}{3}a^2e\,x^3+\frac{1}{2}a^2d\,x^2+a^2cx$	77

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^2*e*x^9+1/8*b^2*d*x^8+1/7*b^2*c*x^7+1/3*a*e*b*x^6+2/5*x^5*d*b*a+1/2*
a*b*c*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x
```

3.321.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2)\left(a+bx^3\right)^2\,dx=\frac{1}{9}\,b^2ex^9+\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{1}{3}\,abex^6+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/9*b^2*e*x^9 + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*e*x^6 + 2/5*a*b*d*
x^5 + 1/2*a*b*c*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

3.321.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int (c + dx + ex^2)(a + bx^3)^2 \, dx = a^2cx + \frac{a^2dx^2}{2} + \frac{a^2ex^3}{3} + \frac{abcx^4}{2} + \frac{2abdx^5}{5} + \frac{abex^6}{3} + \frac{b^2cx^7}{7} + \frac{b^2dx^8}{8} + \frac{b^2ex^9}{9}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**2,x)
```

output
```
a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 +
 a*b*e*x**6/3 + b**2*c*x**7/7 + b**2*d*x**8/8 + b**2*e*x**9/9
```

3.321.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (c + dx + ex^2)(a + bx^3)^2 \, dx = \frac{1}{9}b^2ex^9 + \frac{1}{8}b^2dx^8 + \frac{1}{7}b^2cx^7 + \frac{1}{3}abex^6 + \frac{2}{5}abdx^5 + \frac{1}{2}abcx^4 + \frac{1}{3}a^2ex^3 + \frac{1}{2}a^2dx^2 + a^2cx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/9*b^2*e*x^9 + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*e*x^6 + 2/5*a*b*d*
x^5 + 1/2*a*b*c*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

3.321.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (c + dx + ex^2)(a + bx^3)^2 \, dx = \frac{1}{9}b^2ex^9 + \frac{1}{8}b^2dx^8 + \frac{1}{7}b^2cx^7 + \frac{1}{3}abex^6 + \frac{2}{5}abdx^5 + \frac{1}{2}abcx^4 + \frac{1}{3}a^2ex^3 + \frac{1}{2}a^2dx^2 + a^2cx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*b^2*e*x^9 + 1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 1/3*a*b*e*x^6 + 2/5*a*b*d*
x^5 + 1/2*a*b*c*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

3.321.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2)(a+bx^3)^2 \, dx = \frac{e\,a^2\,x^3}{3} + \frac{d\,a^2\,x^2}{2} + c\,a^2\,x + \frac{e\,a\,b\,x^6}{3} + \frac{2\,d\,a\,b\,x^5}{5} + \frac{c\,a\,b\,x^4}{2} + \frac{e\,b^2\,x^9}{9} + \frac{d\,b^2\,x^8}{8} + \frac{c\,b^2\,x^7}{7}$$

input
```
int((a + b*x^3)^2*(c + d*x + e*x^2),x)
```

output
```
(a^2*d*x^2)/2 + (b^2*c*x^7)/7 + (a^2*e*x^3)/3 + (b^2*d*x^8)/8 + (b^2*e*x^9
)/9 + a^2*c*x + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (a*b*e*x^6)/3
```

3.322 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x} dx$

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3.322.2 Mathematica [A] (verified) 2425
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3.322.9 Mupad [B] (verification not implemented) 2429

3.322.1 Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x} dx = a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}abcx^3 + \frac{1}{2}abdx^4 + \frac{2}{5}abex^5 + \frac{1}{6}b^2cx^6 + \frac{1}{7}b^2dx^7 + \frac{1}{8}b^2ex^8 + a^2c\log(x)$$

output
```
a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*c*x^3+1/2*a*b*d*x^4+2/5*a*b*e*x^5+1/6*b^2*c*
x^6+1/7*b^2*d*x^7+1/8*b^2*e*x^8+a^2*c*ln(x)
```

3.322.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x} dx = a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}abcx^3 + \frac{1}{2}abdx^4 + \frac{2}{5}abex^5 + \frac{1}{6}b^2cx^6 + \frac{1}{7}b^2dx^7 + \frac{1}{8}b^2ex^8 + a^2c\log(x)$$

input
```
Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]
```

output
```
a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/
5 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]
```

3.322.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2(c+dx+ex^2)}{x}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{a^2c}{x}+a^2d+a^2ex+2abcx^2+2abdx^3+2abex^4+b^2cx^5+b^2dx^6+b^2ex^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{2}abdx^4+\frac{2}{5}abex^5+\frac{1}{6}b^2cx^6+\frac{1}{7}b^2dx^7+\frac{1}{8}b^2ex^8$$

input
```
Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x,x]
```

output
```
a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/
5 + (b^2*c*x^6)/6 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*Log[x]
```

3.322.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.322.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

method	result	size
default	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abc\,x^3}{3}+\frac{abd\,x^4}{2}+\frac{2abe\,x^5}{5}+\frac{b^2c\,x^6}{6}+\frac{b^2d\,x^7}{7}+\frac{b^2e\,x^8}{8}+a^2c\ln\left(x\right)$	75
norman	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abc\,x^3}{3}+\frac{abd\,x^4}{2}+\frac{2abe\,x^5}{5}+\frac{b^2c\,x^6}{6}+\frac{b^2d\,x^7}{7}+\frac{b^2e\,x^8}{8}+a^2c\ln\left(x\right)$	75
risch	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abc\,x^3}{3}+\frac{abd\,x^4}{2}+\frac{2abe\,x^5}{5}+\frac{b^2c\,x^6}{6}+\frac{b^2d\,x^7}{7}+\frac{b^2e\,x^8}{8}+a^2c\ln\left(x\right)$	75
parallelrisch	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abc\,x^3}{3}+\frac{abd\,x^4}{2}+\frac{2abe\,x^5}{5}+\frac{b^2c\,x^6}{6}+\frac{b^2d\,x^7}{7}+\frac{b^2e\,x^8}{8}+a^2c\ln\left(x\right)$	75

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*c*x^3+1/2*a*b*d*x^4+2/5*a*b*e*x^5+1/6*b^2*c*
x^6+1/7*b^2*d*x^7+1/8*b^2*e*x^8+a^2*c*ln(x)
```

3.322.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)^2}{x}\,dx = \frac{1}{8}\,b^2ex^8+\frac{1}{7}\,b^2dx^7+\frac{1}{6}\,b^2cx^6+\frac{2}{5}\,abex^5+\frac{1}{2}\,abdx^4 + \frac{2}{3}\,abcx^3+\frac{1}{2}\,a^2ex^2+a^2dx+a^2c\log\left(x\right)$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="fricas")
```

output
```
1/8*b^2*e*x^8 + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*e*x^5 + 1/2*a*b*d*
x^4 + 2/3*a*b*c*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(x)
```

3.322.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x}\,dx = a^2c\log(x) + a^2dx + \frac{a^2ex^2}{2} + \frac{2abcx^3}{3} + \frac{abdx^4}{2} + \frac{2abex^5}{5} + \frac{b^2cx^6}{6} + \frac{b^2dx^7}{7} + \frac{b^2ex^8}{8}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x,x)
```

output
```
a**2*c*log(x) + a**2*d*x + a**2*e*x**2/2 + 2*a*b*c*x**3/3 + a*b*d*x**4/2 +
 2*a*b*e*x**5/5 + b**2*c*x**6/6 + b**2*d*x**7/7 + b**2*e*x**8/8
```

3.322.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x}\,dx = \frac{1}{8}b^2ex^8 + \frac{1}{7}b^2dx^7 + \frac{1}{6}b^2cx^6 + \frac{2}{5}abex^5 + \frac{1}{2}abdx^4 + \frac{2}{3}abcx^3 + \frac{1}{2}a^2ex^2 + a^2dx + a^2c\log(x)$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="maxima")
```

output
```
1/8*b^2*e*x^8 + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*e*x^5 + 1/2*a*b*d*
x^4 + 2/3*a*b*c*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(x)
```

3.322.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x}\,dx = \frac{1}{8}b^2ex^8 + \frac{1}{7}b^2dx^7 + \frac{1}{6}b^2cx^6 + \frac{2}{5}abex^5 + \frac{1}{2}abdx^4 + \frac{2}{3}abcx^3 + \frac{1}{2}a^2ex^2 + a^2dx + a^2c\log(|x|)$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x,x, algorithm="giac")
```

output
```
1/8*b^2*e*x^8 + 1/7*b^2*d*x^7 + 1/6*b^2*c*x^6 + 2/5*a*b*e*x^5 + 1/2*a*b*d*
x^4 + 2/3*a*b*c*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(abs(x))
```

3.322.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x}\,dx = \frac{b^2\,c\,x^6}{6} + \frac{a^2\,e\,x^2}{2} + \frac{b^2\,d\,x^7}{7} + \frac{b^2\,e\,x^8}{8} + a^2\,c\,\ln(x) + a^2\,d\,x + \frac{2\,a\,b\,c\,x^3}{3} + \frac{a\,b\,d\,x^4}{2} + \frac{2\,a\,b\,e\,x^5}{5}$$

input
```
int(((a + b*x^3)^2*(c + d*x + e*x^2))/x,x)
```

output
```
(b^2*c*x^6)/6 + (a^2*e*x^2)/2 + (b^2*d*x^7)/7 + (b^2*e*x^8)/8 + a^2*c*log(
x) + a^2*d*x + (2*a*b*c*x^3)/3 + (a*b*d*x^4)/2 + (2*a*b*e*x^5)/5
```

3.323 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} \, dx$

3.323.1 Optimal result 2430
3.323.2 Mathematica [A] (verified) 2430
3.323.3 Rubi [A] (verified) 2431
3.323.4 Maple [A] (verified) 2432
3.323.5 Fricas [A] (verification not implemented) 2432
3.323.6 Sympy [A] (verification not implemented) 2433
3.323.7 Maxima [A] (verification not implemented) 2433
3.323.8 Giac [A] (verification not implemented) 2433
3.323.9 Mupad [B] (verification not implemented) 2434

3.323.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} \, dx = -\frac{a^2c}{x} + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7 + a^2d\log(x)$$

output
```
-a^2*c/x+a^2*e*x+a*b*c*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*b^2*c*x^5+1/6*b
^2*d*x^6+1/7*b^2*e*x^7+a^2*d*ln(x)
```

3.323.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2} \, dx = -\frac{a^2c}{x} + a^2ex + abcx^2 + \frac{2}{3}abdx^3 + \frac{1}{2}abex^4 + \frac{1}{5}b^2cx^5 + \frac{1}{6}b^2dx^6 + \frac{1}{7}b^2ex^7 + a^2d\log(x)$$

input
```
Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]
```

output
```
-((a^2*c)/x) + a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (b^
2*c*x^5)/5 + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*Log[x]
```

3.323.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2(c+dx+ex^2)}{x^2}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{a^2c}{x^2}+\frac{a^2d}{x}+a^2e+2abcx+2abdx^2+2abex^3+b^2cx^4+b^2dx^5+b^2ex^6\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+abcx^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{1}{5}b^2cx^5+\frac{1}{6}b^2dx^6+\frac{1}{7}b^2ex^7$$

input
```
Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^2,x]
```

output
```
-((a^2*c)/x) + a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (b^
2*c*x^5)/5 + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*Log[x]
```

3.323.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.323.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2c}{x}+a^2ex+abc\,x^2+\frac{2x^3abd}{3}+\frac{abe\,x^4}{2}+\frac{b^2c\,x^5}{5}+\frac{b^2d\,x^6}{6}+\frac{b^2e\,x^7}{7}+a^2d\ln\left(x\right)$	74
risch	$-\frac{a^2c}{x}+a^2ex+abc\,x^2+\frac{2x^3abd}{3}+\frac{abe\,x^4}{2}+\frac{b^2c\,x^5}{5}+\frac{b^2d\,x^6}{6}+\frac{b^2e\,x^7}{7}+a^2d\ln\left(x\right)$	74
norman	$\frac{a^2e\,x^2+abc\,x^3-a^2c+\frac{1}{5}b^2c\,x^6+\frac{1}{6}b^2d\,x^7+\frac{1}{7}b^2e\,x^8+\frac{2}{3}abd\,x^4+\frac{1}{2}abe\,x^5}{x}+a^2d\ln\left(x\right)$	78
parallelrisch	$\frac{30b^2e\,x^8+35b^2d\,x^7+42b^2c\,x^6+105abe\,x^5+140abd\,x^4+210abc\,x^3+210a^2d\ln(x)x+210a^2e\,x^2-210a^2c}{210x}$	82

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*c/x+a^2*e*x+a*b*c*x^2+2/3*x^3*a*b*d+1/2*a*b*e*x^4+1/5*b^2*c*x^5+1/6*b
^2*d*x^6+1/7*b^2*e*x^7+a^2*d*ln(x)
```

3.323.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int\frac{\left(c+dx+ex^2\right)\left(a+bx^3\right)^2}{x^2}\,dx$$
$$=\frac{30\,b^2ex^8+35\,b^2dx^7+42\,b^2cx^6+105\,abex^5+140\,abdx^4+210\,abcx^3+210\,a^2ex^2+210\,a^2dx\log\left(x\right)-210}{210\,x}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="fricas")
```

output
```
1/210*(30*b^2*e*x^8 + 35*b^2*d*x^7 + 42*b^2*c*x^6 + 105*a*b*e*x^5 + 140*a*
b*d*x^4 + 210*a*b*c*x^3 + 210*a^2*e*x^2 + 210*a^2*d*x*log(x) - 210*a^2*c)/
x
```

3.323.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2}\,dx = -\frac{a^2c}{x} + a^2d\log(x) + a^2ex + abcx^2 + \frac{2abdx^3}{3} + \frac{abex^4}{2} + \frac{b^2cx^5}{5} + \frac{b^2dx^6}{6} + \frac{b^2ex^7}{7}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x**2,x)
```

output
```
-a**2*c/x + a**2*d*log(x) + a**2*e*x + a*b*c*x**2 + 2*a*b*d*x**3/3 + a*b*e
*x**4/2 + b**2*c*x**5/5 + b**2*d*x**6/6 + b**2*e*x**7/7
```

3.323.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2}\,dx = \frac{1}{7}b^2ex^7 + \frac{1}{6}b^2dx^6 + \frac{1}{5}b^2cx^5 + \frac{1}{2}abex^4 + \frac{2}{3}abdx^3 + abcx^2 + a^2ex + a^2d\log(x) - \frac{a^2c}{x}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="maxima")
```

output
```
1/7*b^2*e*x^7 + 1/6*b^2*d*x^6 + 1/5*b^2*c*x^5 + 1/2*a*b*e*x^4 + 2/3*a*b*d*
x^3 + a*b*c*x^2 + a^2*e*x + a^2*d*log(x) - a^2*c/x
```

3.323.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2}\,dx = \frac{1}{7}b^2ex^7 + \frac{1}{6}b^2dx^6 + \frac{1}{5}b^2cx^5 + \frac{1}{2}abex^4 + \frac{2}{3}abdx^3 + abcx^2 + a^2ex + a^2d\log(|x|) - \frac{a^2c}{x}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^2,x, algorithm="giac")
```

output
```
1/7*b^2*e*x^7 + 1/6*b^2*d*x^6 + 1/5*b^2*c*x^5 + 1/2*a*b*e*x^4 + 2/3*a*b*d*
x^3 + a*b*c*x^2 + a^2*e*x + a^2*d*log(abs(x)) - a^2*c/x
```

3.323.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^2}\,dx = \frac{b^2\,c\,x^5}{5}-\frac{a^2\,c}{x}+\frac{b^2\,d\,x^6}{6}+\frac{b^2\,e\,x^7}{7}+a^2\,d\,\ln(x)+a^2\,e\,x+a\,b\,c\,x^2+\frac{2\,a\,b\,d\,x^3}{3}+\frac{a\,b\,e\,x^4}{2}$$

input
```
int(((a + b*x^3)^2*(c + d*x + e*x^2))/x^2,x)
```

output
```
(b^2*c*x^5)/5 - (a^2*c)/x + (b^2*d*x^6)/6 + (b^2*e*x^7)/7 + a^2*d*log(x) +
 a^2*e*x + a*b*c*x^2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2
```

3.324 $\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx$

3.324.1 Optimal result 2435
3.324.2 Mathematica [A] (verified) 2435
3.324.3 Rubi [A] (verified) 2436
3.324.4 Maple [A] (verified) 2437
3.324.5 Fricas [A] (verification not implemented) 2437
3.324.6 Sympy [A] (verification not implemented) 2438
3.324.7 Maxima [A] (verification not implemented) 2438
3.324.8 Giac [A] (verification not implemented) 2438
3.324.9 Mupad [B] (verification not implemented) 2439

3.324.1 Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx = -\frac{a^2c}{2x^2}-\frac{a^2d}{x}+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6+a^2e\log(x)$$

output
```
-1/2*a^2*c/x^2-a^2*d/x+2*a*b*c*x+a*b*d*x^2+2/3*a*b*e*x^3+1/4*b^2*c*x^4+1/5
*b^2*d*x^5+1/6*b^2*e*x^6+a^2*e*ln(x)
```

3.324.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx = -\frac{a^2c}{2x^2}-\frac{a^2d}{x}+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6+a^2e\log(x)$$

input
```
Integrate[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]
```

output
```
-1/2*(a^2*c)/x^2 - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (
b^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*Log[x]
```

3.324.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2(c+dx+ex^2)}{x^3}\,dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(\frac{a^2c}{x^3}+\frac{a^2d}{x^2}+\frac{a^2e}{x}+2abc+2abdx+2abex^2+b^2cx^3+b^2dx^4+b^2ex^5\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+2abcx+abdx^2+\frac{2}{3}abex^3+\frac{1}{4}b^2cx^4+\frac{1}{5}b^2dx^5+\frac{1}{6}b^2ex^6$$

input
```
Int[((c + d*x + e*x^2)*(a + b*x^3)^2)/x^3,x]
```

output
```
-1/2*(a^2*c)/x^2 - (a^2*d)/x + 2*a*b*c*x + a*b*d*x^2 + (2*a*b*e*x^3)/3 + (
b^2*c*x^4)/4 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6 + a^2*e*Log[x]
```

3.324.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.324.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^{2}c}{2x^{2}}-\frac{a^{2}d}{x}+2abcx+x^{2}abd+\frac{2abe\,x^{3}}{3}+\frac{b^{2}c\,x^{4}}{4}+\frac{b^{2}d\,x^{5}}{5}+\frac{b^{2}e\,x^{6}}{6}+a^{2}e\ln\left(x\right)$	75
risch	$\frac{b^{2}e\,x^{6}}{6}+\frac{b^{2}d\,x^{5}}{5}+\frac{b^{2}c\,x^{4}}{4}+\frac{2abe\,x^{3}}{3}+x^{2}abd+2abcx+\frac{-a^{2}dx-\frac{1}{2}a^{2}c}{x^{2}}+a^{2}e\ln\left(x\right)$	75
norman	$\frac{abd\,x^{4}-\frac{1}{2}a^{2}c-a^{2}dx+\frac{1}{4}b^{2}c\,x^{6}+\frac{1}{5}b^{2}d\,x^{7}+\frac{1}{6}b^{2}e\,x^{8}+2abc\,x^{3}+\frac{2}{3}abe\,x^{5}}{x^{2}}+a^{2}e\ln\left(x\right)$	77
parallelrisch	$\frac{10b^{2}e\,x^{8}+12b^{2}d\,x^{7}+15b^{2}c\,x^{6}+40abe\,x^{5}+60abd\,x^{4}+60a^{2}e\ln\left(x\right)x^{2}+120abc\,x^{3}-60a^{2}dx-30a^{2}c}{60x^{2}}$	82

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*c/x^2-a^2*d/x+2*a*b*c*x+x^2*a*b*d+2/3*a*b*e*x^3+1/4*b^2*c*x^4+1/5
*b^2*d*x^5+1/6*b^2*e*x^6+a^2*e*ln(x)
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int\frac{(c+dx+ex^2)\,(a+bx^3)^2}{x^3}\,dx$$
$$=\frac{10\,b^2ex^8+12\,b^2dx^7+15\,b^2cx^6+40\,abex^5+60\,abdx^4+120\,abcx^3+60\,a^2ex^2\log\left(x\right)-60\,a^2dx-30\,a^2c}{60\,x^2}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="fricas")
```

output
```
1/60*(10*b^2*e*x^8 + 12*b^2*d*x^7 + 15*b^2*c*x^6 + 40*a*b*e*x^5 + 60*a*b*d
*x^4 + 120*a*b*c*x^3 + 60*a^2*e*x^2*log(x) - 60*a^2*d*x - 30*a^2*c)/x^2
```

3.324.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx = a^2e\log(x)+2abcx+abdx^2+\frac{2abex^3}{3}+\frac{b^2cx^4}{4}+\frac{b^2dx^5}{5}+\frac{b^2ex^6}{6}+\frac{-a^2c-2a^2dx}{2x^2}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**2/x**3,x)
```

output
```
a**2*e*log(x) + 2*a*b*c*x + a*b*d*x**2 + 2*a*b*e*x**3/3 + b**2*c*x**4/4 +
b**2*d*x**5/5 + b**2*e*x**6/6 + (-a**2*c - 2*a**2*d*x)/(2*x**2)
```

3.324.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx = \frac{1}{6}\,b^2ex^6+\frac{1}{5}\,b^2dx^5+\frac{1}{4}\,b^2cx^4+\frac{2}{3}\,abex^3+abdx^2+2\,abcx+a^2e\log(x)-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="maxima")
```

output
```
1/6*b^2*e*x^6 + 1/5*b^2*d*x^5 + 1/4*b^2*c*x^4 + 2/3*a*b*e*x^3 + a*b*d*x^2
+ 2*a*b*c*x + a^2*e*log(x) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

3.324.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)^2}{x^3}\,dx = \frac{1}{6}\,b^2ex^6+\frac{1}{5}\,b^2dx^5+\frac{1}{4}\,b^2cx^4+\frac{2}{3}\,abex^3+abdx^2+2\,abcx+a^2e\log(|x|)-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^2/x^3,x, algorithm="giac")
```

output
```
1/6*b^2*e*x^6 + 1/5*b^2*d*x^5 + 1/4*b^2*c*x^4 + 2/3*a*b*e*x^3 + a*b*d*x^2
+ 2*a*b*c*x + a^2*e*log(abs(x)) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

3.324.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)^2}{x^3}\,dx = \frac{b^2\,c\,x^4}{4} - \frac{\frac{a^2\,c}{2}+a^2\,d\,x}{x^2} + \frac{b^2\,d\,x^5}{5} + \frac{b^2\,e\,x^6}{6} + a^2\,e\,\ln(x) + a\,b\,d\,x^2 + \frac{2\,a\,b\,e\,x^3}{3} + 2\,a\,b\,c\,x$$

input
```
int(((a + b*x^3)^2*(c + d*x + e*x^2))/x^3,x)
```

output
```
(b^2*c*x^4)/4 - ((a^2*c)/2 + a^2*d*x)/x^2 + (b^2*d*x^5)/5 + (b^2*e*x^6)/6
+ a^2*e*log(x) + a*b*d*x^2 + (2*a*b*e*x^3)/3 + 2*a*b*c*x
```

3.325 $\int x^2(c+dx+ex^2)(a+bx^3)^3\,dx$

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3.325.1 Optimal result

Integrand size = 23, antiderivative size = 110

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}+\frac{c(a+bx^3)^4}{12b}$$

output
```
1/4*a^3*d*x^4+1/5*a^3*e*x^5+3/7*a^2*b*d*x^7+3/8*a^2*b*e*x^8+3/10*a*b^2*d*x
^10+3/11*a*b^2*e*x^11+1/13*b^3*d*x^13+1/14*b^3*e*x^14+1/12*c*(b*x^3+a)^4/b
```

3.325.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.26

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{2}a^2bcx^6+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{12}b^3cx^{12}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

input
```
Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

output
```
(a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*b*c*x^6)/2 + (3*a^2*b
*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3
*a*b^2*e*x^11)/11 + (b^3*c*x^12)/12 + (b^3*d*x^13)/13 + (b^3*e*x^14)/14
```

3.325.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx^3)^3 (c+dx+ex^2) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int (bx^3+a)^3 \left(x^2 (ex^2+dx+c)-cx^2\right) dx+\frac{c(a+bx^3)^4}{12b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^3ex^{13}+b^3dx^{12}+3ab^2ex^{10}+3ab^2dx^9+3a^2bex^7+3a^2bdx^6+a^3ex^4+a^3dx^3\right) dx+\frac{c(a+bx^3)^4}{12b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{3}{7}a^2bdx^7+\frac{3}{8}a^2bex^8+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{c(a+bx^3)^4}{12b}+\frac{1}{13}b^3dx^{13}+\frac{1}{14}b^3ex^{14}$$

input
```
Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

output
```
(a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (3*a^2*b*d*x^7)/7 + (3*a^2*b*e*x^8)/8 + (3
*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (b^3*d*x^13)/13 + (b^3*e*x^14)/1
4 + (c*(a + b*x^3)^4)/(12*b)
```

3.325.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.325.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

method	result
gosper	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}a^2bc\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}$
default	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}a^2bc\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}$
norman	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}a^2bc\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}$
risch	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}a^2bc\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}$
parallelrisch	$\frac{1}{3}c\,a^3x^3+\frac{1}{4}a^3d\,x^4+\frac{1}{5}a^3e\,x^5+\frac{1}{2}a^2bc\,x^6+\frac{3}{7}a^2bd\,x^7+\frac{3}{8}a^2be\,x^8+\frac{1}{3}a\,b^2c\,x^9+\frac{3}{10}a\,b^2d\,x^{10}+\frac{3}{11}$

input
```
int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*c*a^3*x^3+1/4*a^3*d*x^4+1/5*a^3*e*x^5+1/2*a^2*b*c*x^6+3/7*a^2*b*d*x^7+
3/8*a^2*b*e*x^8+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^11+1/12*b
^3*c*x^12+1/13*b^3*d*x^13+1/14*b^3*e*x^14
```

3.325.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{1}{14}\,b^3ex^{14}+\frac{1}{13}\,b^3dx^{13}+\frac{1}{12}\,b^3cx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bex^8+\frac{3}{7}\,a^2bdx^7+\frac{1}{2}\,a^2bcx^6+\frac{1}{5}\,a^3ex^5+\frac{1}{4}\,a^3dx^4+\frac{1}{3}\,a^3cx^3$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/14*b^3*e*x^14 + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*e*x^11 +
3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*e*x^8 + 3/7*a^2*b*d*x^7 +
1/2*a^2*b*c*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

3.325.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{a^3cx^3}{3}+\frac{a^3dx^4}{4}+\frac{a^3ex^5}{5}+\frac{a^2bcx^6}{2}+\frac{3a^2bdx^7}{7}+\frac{3a^2bex^8}{8}+\frac{ab^2cx^9}{3}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{b^3cx^{12}}{12}+\frac{b^3dx^{13}}{13}+\frac{b^3ex^{14}}{14}$$

input
```
integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

output
```
a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + a**2*b*c*x**6/2 + 3*a**2*b
*d*x**7/7 + 3*a**2*b*e*x**8/8 + a*b**2*c*x**9/3 + 3*a*b**2*d*x**10/10 + 3*
a*b**2*e*x**11/11 + b**3*c*x**12/12 + b**3*d*x**13/13 + b**3*e*x**14/14
```

3.325.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{1}{14}\,b^3ex^{14}+\frac{1}{13}\,b^3dx^{13}+\frac{1}{12}\,b^3cx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bex^8+\frac{3}{7}\,a^2bdx^7+\frac{1}{2}\,a^2bcx^6+\frac{1}{5}\,a^3ex^5+\frac{1}{4}\,a^3dx^4+\frac{1}{3}\,a^3cx^3$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/14*b^3*e*x^14 + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*e*x^11 +
3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*e*x^8 + 3/7*a^2*b*d*x^7 +
1/2*a^2*b*c*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

3.325.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{1}{14}\,b^3ex^{14}+\frac{1}{13}\,b^3dx^{13}+\frac{1}{12}\,b^3cx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bex^8+\frac{3}{7}\,a^2bdx^7+\frac{1}{2}\,a^2bcx^6+\frac{1}{5}\,a^3ex^5+\frac{1}{4}\,a^3dx^4+\frac{1}{3}\,a^3cx^3$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/14*b^3*e*x^14 + 1/13*b^3*d*x^13 + 1/12*b^3*c*x^12 + 3/11*a*b^2*e*x^11 +
3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 3/8*a^2*b*e*x^8 + 3/7*a^2*b*d*x^7 +
1/2*a^2*b*c*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

3.325.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{e\,a^3\,x^5}{5}+\frac{d\,a^3\,x^4}{4}+\frac{c\,a^3\,x^3}{3}+\frac{3\,e\,a^2\,b\,x^8}{8}+\frac{3\,d\,a^2\,b\,x^7}{7}+\frac{c\,a^2\,b\,x^6}{2}+\frac{3\,e\,a\,b^2\,x^{11}}{11}+\frac{3\,d\,a\,b^2\,x^{10}}{10}+\frac{c\,a\,b^2\,x^9}{3}+\frac{e\,b^3\,x^{14}}{14}+\frac{d\,b^3\,x^{13}}{13}+\frac{c\,b^3\,x^{12}}{12}$$

input
```
int(x^2*(a + b*x^3)^3*(c + d*x + e*x^2),x)
```

output
```
(a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (b^3*c*x^12)/12 + (a^3*e*x^5)/5 + (b^3*d*x
^13)/13 + (b^3*e*x^14)/14 + (a^2*b*c*x^6)/2 + (a*b^2*c*x^9)/3 + (3*a^2*b*d
*x^7)/7 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^8)/8 + (3*a*b^2*e*x^11)/11
```

3.326 $\int x(c+dx+ex^2)(a+bx^3)^3\,dx$

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3.326.2 Mathematica [A] (verified) . . . 2446
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3.326.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int x(c+dx+ex^2)(a+bx^3)^3\,dx=\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{1}{11}b^3cx^{11}+\frac{1}{13}b^3ex^{13}+\frac{d(a+bx^3)^4}{12b}$$

output
```
1/2*a^3*c*x^2+1/4*a^3*e*x^4+3/5*a^2*b*c*x^5+3/7*a^2*b*e*x^7+3/8*a*b^2*c*x^
8+3/10*a*b^2*e*x^10+1/11*b^3*c*x^11+1/13*b^3*e*x^13+1/12*d*(b*x^3+a)^4/b
```

3.326.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.26

$$\int x(c+dx+ex^2)(a+bx^3)^3\,dx=\frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{1}{3}ab^2dx^9+\frac{3}{10}ab^2ex^{10}+\frac{1}{11}b^3cx^{11}+\frac{1}{12}b^3dx^{12}+\frac{1}{13}b^3ex^{13}$$

input
```
Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

output
```
(a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (a^2*b
*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*c*x^8)/8 + (a*b^2*d*x^9)/3 + (3*a
*b^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*d*x^12)/12 + (b^3*e*x^13)/13
```

3.326.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^3\left(c+dx+ex^2\right)\,dx$$

$$\Big\downarrow \; 2017$$

$$\int \left(bx^3+a\right)^3\left(x\left(ex^2+dx+c\right)-dx^2\right)dx+\frac{d\left(a+bx^3\right)^4}{12b}$$

$$\Big\downarrow \; 2389$$

$$\int \left(b^3ex^{12}+b^3cx^{10}+3ab^2ex^9+3ab^2cx^7+3a^2bex^6+3a^2bcx^4+a^3ex^3+a^3cx\right)dx+\frac{d\left(a+bx^3\right)^4}{12b}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{3}{5}a^2bcx^5+\frac{3}{7}a^2bex^7+\frac{3}{8}ab^2cx^8+\frac{3}{10}ab^2ex^{10}+\frac{d\left(a+bx^3\right)^4}{12b}+\frac{1}{11}b^3cx^{11}+\frac{1}{13}b^3ex^{13}$$

input
```
Int[x*(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

output
```
(a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (3*a^2*b*c*x^5)/5 + (3*a^2*b*e*x^7)/7 + (3
*a*b^2*c*x^8)/8 + (3*a*b^2*e*x^10)/10 + (b^3*c*x^11)/11 + (b^3*e*x^13)/13
+ (d*(a + b*x^3)^4)/(12*b)
```

3.326.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.326.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

method	result
gosper	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}x^9a\,b^2d+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6$
default	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}x^9a\,b^2d+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6$
norman	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}x^9a\,b^2d+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6$
risch	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}x^9a\,b^2d+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6$
parallelrisch	$\frac{1}{13}b^3e\,x^{13}+\frac{1}{12}b^3d\,x^{12}+\frac{1}{11}b^3c\,x^{11}+\frac{3}{10}a\,b^2e\,x^{10}+\frac{1}{3}x^9a\,b^2d+\frac{3}{8}a\,b^2c\,x^8+\frac{3}{7}a^2be\,x^7+\frac{1}{2}a^2bd\,x^6$

input
```
int(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^3*e*x^13+1/12*b^3*d*x^12+1/11*b^3*c*x^11+3/10*a*b^2*e*x^10+1/3*x^9*
a*b^2*d+3/8*a*b^2*c*x^8+3/7*a^2*b*e*x^7+1/2*a^2*b*d*x^6+3/5*a^2*b*c*x^5+1/
4*a^3*e*x^4+1/3*a^3*d*x^3+1/2*a^3*c*x^2
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x(c+dx+ex^2)(a+bx^3)^3\,dx = \frac{1}{13}b^3ex^{13}+\frac{1}{12}b^3dx^{12}+\frac{1}{11}b^3cx^{11}+\frac{3}{10}ab^2ex^{10}+\frac{1}{3}ab^2dx^9+\frac{3}{8}ab^2cx^8+\frac{3}{7}a^2bex^7+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3ex^4+\frac{1}{3}a^3dx^3+\frac{1}{2}a^3cx^2$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/13*b^3*e*x^13 + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*e*x^10 +
1/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/
5*a^2*b*c*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

3.326.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

$$\int x(c+dx+ex^2)(a+bx^3)^3\,dx = \frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3ab^2cx^8}{8}+\frac{ab^2dx^9}{3}+\frac{3ab^2ex^{10}}{10}+\frac{b^3cx^{11}}{11}+\frac{b^3dx^{12}}{12}+\frac{b^3ex^{13}}{13}$$

input
```
integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

output
```
a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + 3*a**2*b*c*x**5/5 + a**2*b
*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a*b**2*c*x**8/8 + a*b**2*d*x**9/3 + 3*a*
b**2*e*x**10/10 + b**3*c*x**11/11 + b**3*d*x**12/12 + b**3*e*x**13/13
```

3.326.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^3\,dx = \frac{1}{13}b^3ex^{13}+\frac{1}{12}b^3dx^{12}+\frac{1}{11}b^3cx^{11}+\frac{3}{10}ab^2ex^{10}+\frac{1}{3}ab^2dx^9+\frac{3}{8}ab^2cx^8+\frac{3}{7}a^2bex^7+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3ex^4+\frac{1}{3}a^3dx^3+\frac{1}{2}a^3cx^2$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/13*b^3*e*x^13 + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*e*x^10 +
1/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/
5*a^2*b*c*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

3.326.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^3\,dx = \frac{1}{13}b^3ex^{13}+\frac{1}{12}b^3dx^{12}+\frac{1}{11}b^3cx^{11}+\frac{3}{10}ab^2ex^{10}+\frac{1}{3}ab^2dx^9+\frac{3}{8}ab^2cx^8+\frac{3}{7}a^2bex^7+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3ex^4+\frac{1}{3}a^3dx^3+\frac{1}{2}a^3cx^2$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/13*b^3*e*x^13 + 1/12*b^3*d*x^12 + 1/11*b^3*c*x^11 + 3/10*a*b^2*e*x^10 +
1/3*a*b^2*d*x^9 + 3/8*a*b^2*c*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/
5*a^2*b*c*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

3.326.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{e\,a^3\,x^4}{4}+\frac{d\,a^3\,x^3}{3}+\frac{c\,a^3\,x^2}{2}+\frac{3\,e\,a^2\,b\,x^7}{7}+\frac{d\,a^2\,b\,x^6}{2}+\frac{3\,c\,a^2\,b\,x^5}{5}+\frac{3\,e\,a\,b^2\,x^{10}}{10}+\frac{d\,a\,b^2\,x^9}{3}+\frac{3\,c\,a\,b^2\,x^8}{8}+\frac{e\,b^3\,x^{13}}{13}+\frac{d\,b^3\,x^{12}}{12}+\frac{c\,b^3\,x^{11}}{11}$$

input
```
int(x*(a + b*x^3)^3*(c + d*x + e*x^2),x)
```

output
```
(a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (b^3*c*x^11)/11 + (a^3*e*x^4)/4 + (b^3*d*x
^12)/12 + (b^3*e*x^13)/13 + (3*a^2*b*c*x^5)/5 + (3*a*b^2*c*x^8)/8 + (a^2*b
*d*x^6)/2 + (a*b^2*d*x^9)/3 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*e*x^10)/10
```

3.327 $\int (c+dx+ex^2)(a+bx^3)^3\,dx$

3.327.1 Optimal result 2452
3.327.2 Mathematica [A] (verified) 2452
3.327.3 Rubi [A] (verified) 2453
3.327.4 Maple [A] (verified) 2454
3.327.5 Fricas [A] (verification not implemented) 2455
3.327.6 Sympy [A] (verification not implemented) 2455
3.327.7 Maxima [A] (verification not implemented) 2456
3.327.8 Giac [A] (verification not implemented) 2456
3.327.9 Mupad [B] (verification not implemented) 2457

3.327.1 Optimal result

Integrand size = 20, antiderivative size = 105

$$\int (c+dx+ex^2)(a+bx^3)^3\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}+\frac{e(a+bx^3)^4}{12b}$$

output
```
a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c*x^7+3/8*
a*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11+1/12*e*(b*x^3+a)^4/b
```

3.327.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.28

$$\int (c+dx+ex^2)(a+bx^3)^3\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{1}{2}a^2bex^6+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{3}ab^2ex^9+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}+\frac{1}{12}b^3ex^{12}$$

input
```
Integrate[(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x
^5)/5 + (a^2*b*e*x^6)/2 + (3*a*b^2*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (a*b^2*e
*x^9)/3 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (b^3*e*x^12)/12
```

3.327.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^3 \left(c+dx+ex^2\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int (c+dx) \left(bx^3+a\right)^3 dx+\frac{e\left(a+bx^3\right)^4}{12b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^3dx^{10}+b^3cx^9+3ab^2dx^7+3ab^2cx^6+3a^2bdx^4+3a^2bcx^3+a^3dx+a^3c\right) dx+\frac{e\left(a+bx^3\right)^4}{12b}$$

$$\Big\downarrow \text{2009}$$

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{e\left(a+bx^3\right)^4}{12b}+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

input
```
Int[(c + d*x + e*x^2)*(a + b*x^3)^3,x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2
*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + (e*(a
+ b*x^3)^4)/(12*b)
```

3.327.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.327.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

method	result
gosper	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}x^8b^2da+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}x^5bd\,a^2+$
default	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}x^8b^2da+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}x^5bd\,a^2+$
norman	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}x^8b^2da+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}x^5bd\,a^2+$
risch	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}x^8b^2da+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}x^5bd\,a^2+$
parallelrisch	$\frac{1}{12}b^3e\,x^{12}+\frac{1}{11}b^3d\,x^{11}+\frac{1}{10}b^3c\,x^{10}+\frac{1}{3}a\,b^2e\,x^9+\frac{3}{8}x^8b^2da+\frac{3}{7}a\,b^2c\,x^7+\frac{1}{2}a^2be\,x^6+\frac{3}{5}x^5bd\,a^2+$

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^3*e*x^12+1/11*b^3*d*x^11+1/10*b^3*c*x^10+1/3*a*b^2*e*x^9+3/8*x^8*b^
2*d*a+3/7*a*b^2*c*x^7+1/2*a^2*b*e*x^6+3/5*x^5*b*d*a^2+3/4*a^2*b*c*x^4+1/3*
a^3*e*x^3+1/2*a^3*d*x^2+a^3*c*x
```

3.327.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int (c+dx+ex^2)\left(a+bx^3\right)^3\,dx = \frac{1}{12}\,b^3ex^{12}+\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{1}{3}\,ab^2ex^9+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{1}{2}\,a^2bex^6+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{3}\,a^3ex^3+\frac{1}{2}\,a^3dx^2+a^3cx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/12*b^3*e*x^12 + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*e*x^9 + 3/
8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*e*x^6 + 3/5*a^2*b*d*x^5 + 3/4*
a^2*b*c*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.327.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.28

$$\int (c+dx+ex^2)\left(a+bx^3\right)^3\,dx = a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{3a^2bcx^4}{4}+\frac{3a^2bdx^5}{5}+\frac{a^2bex^6}{2}+\frac{3ab^2cx^7}{7}+\frac{3ab^2dx^8}{8}+\frac{ab^2ex^9}{3}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}+\frac{b^3ex^{12}}{12}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**3,x)
```

output
```
a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*
x**5/5 + a**2*b*e*x**6/2 + 3*a*b**2*c*x**7/7 + 3*a*b**2*d*x**8/8 + a*b**2*
e*x**9/3 + b**3*c*x**10/10 + b**3*d*x**11/11 + b**3*e*x**12/12
```

3.327.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int (c+dx+ex^2)(a+bx^3)^3\,dx = \frac{1}{12}b^3ex^{12} + \frac{1}{11}b^3dx^{11} + \frac{1}{10}b^3cx^{10} + \frac{1}{3}ab^2ex^9 + \frac{3}{8}ab^2dx^8 + \frac{3}{7}ab^2cx^7 + \frac{1}{2}a^2bex^6 + \frac{3}{5}a^2bdx^5 + \frac{3}{4}a^2bcx^4 + \frac{1}{3}a^3ex^3 + \frac{1}{2}a^3dx^2 + a^3cx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/12*b^3*e*x^12 + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*e*x^9 + 3/
8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*e*x^6 + 3/5*a^2*b*d*x^5 + 3/4*
a^2*b*c*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.327.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int (c+dx+ex^2)(a+bx^3)^3\,dx = \frac{1}{12}b^3ex^{12} + \frac{1}{11}b^3dx^{11} + \frac{1}{10}b^3cx^{10} + \frac{1}{3}ab^2ex^9 + \frac{3}{8}ab^2dx^8 + \frac{3}{7}ab^2cx^7 + \frac{1}{2}a^2bex^6 + \frac{3}{5}a^2bdx^5 + \frac{3}{4}a^2bcx^4 + \frac{1}{3}a^3ex^3 + \frac{1}{2}a^3dx^2 + a^3cx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/12*b^3*e*x^12 + 1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 1/3*a*b^2*e*x^9 + 3/
8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 1/2*a^2*b*e*x^6 + 3/5*a^2*b*d*x^5 + 3/4*
a^2*b*c*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.327.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int (c+dx+ex^2)\left(a+bx^3\right)^3\,dx=\frac{e\,a^3\,x^3}{3}+\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{e\,a^2\,b\,x^6}{2}+\frac{3\,d\,a^2\,b\,x^5}{5}+\frac{3\,c\,a^2\,b\,x^4}{4}+\frac{e\,a\,b^2\,x^9}{3}+\frac{3\,d\,a\,b^2\,x^8}{8}+\frac{3\,c\,a\,b^2\,x^7}{7}+\frac{e\,b^3\,x^{12}}{12}+\frac{d\,b^3\,x^{11}}{11}+\frac{c\,b^3\,x^{10}}{10}$$

input
```
int((a + b*x^3)^3*(c + d*x + e*x^2),x)
```

output
```
(a^3*d*x^2)/2 + (b^3*c*x^10)/10 + (a^3*e*x^3)/3 + (b^3*d*x^11)/11 + (b^3*e
*x^12)/12 + a^3*c*x + (3*a^2*b*c*x^4)/4 + (3*a*b^2*c*x^7)/7 + (3*a^2*b*d*x
^5)/5 + (3*a*b^2*d*x^8)/8 + (a^2*b*e*x^6)/2 + (a*b^2*e*x^9)/3
```

3.328 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx$

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3.328.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx = a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5 +\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9 +\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex^{11}+a^3c\log(x)$$

output
```
a^3*d*x+1/2*a^3*e*x^2+a^2*b*c*x^3+3/4*a^2*b*d*x^4+3/5*a^2*b*e*x^5+1/2*a*b^
2*c*x^6+3/7*a*b^2*d*x^7+3/8*a*b^2*e*x^8+1/9*b^3*c*x^9+1/10*b^3*d*x^10+1/11
*b^3*e*x^11+a^3*c*ln(x)
```

3.328.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx = a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5 +\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9 +\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex^{11}+a^3c\log(x)$$

input
```
Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]
```

output
```
a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5
)/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9
)/9 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*Log[x]
```

3.328.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2)}{x} \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{a^3c}{x}+a^3d+a^3ex+3a^2bcx^2+3a^2bdx^3+3a^2bex^4+3ab^2cx^5+3ab^2dx^6+3ab^2ex^7+b^3cx^8+b^3dx^9+b^3ex^{10}\right)$$

$$\Big\downarrow \text{2009}$$

$$a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{3}{4}a^2bdx^4+\frac{3}{5}a^2bex^5+\frac{1}{2}ab^2cx^6+\frac{3}{7}ab^2dx^7+\frac{3}{8}ab^2ex^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^3dx^{10}+\frac{1}{11}b^3ex^{11}$$

input
```
Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x,x]
```

output
```
a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (3*a^2*b*d*x^4)/4 + (3*a^2*b*e*x^5
)/5 + (a*b^2*c*x^6)/2 + (3*a*b^2*d*x^7)/7 + (3*a*b^2*e*x^8)/8 + (b^3*c*x^9
)/9 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*Log[x]
```

3.328.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.328.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

method	result
default	$a^3dx+\frac{a^3e\,x^2}{2}+a^2x^3bc+\frac{3a^2bd\,x^4}{4}+\frac{3a^2be\,x^5}{5}+\frac{a\,b^2c\,x^6}{2}+\frac{3a\,b^2d\,x^7}{7}+\frac{3a\,b^2e\,x^8}{8}+\frac{b^3c\,x^9}{9}+\frac{b^3d\,x^{10}}{10}+b^3$
norman	$a^3dx+\frac{a^3e\,x^2}{2}+a^2x^3bc+\frac{3a^2bd\,x^4}{4}+\frac{3a^2be\,x^5}{5}+\frac{a\,b^2c\,x^6}{2}+\frac{3a\,b^2d\,x^7}{7}+\frac{3a\,b^2e\,x^8}{8}+\frac{b^3c\,x^9}{9}+\frac{b^3d\,x^{10}}{10}+b^3$
risch	$a^3dx+\frac{a^3e\,x^2}{2}+a^2x^3bc+\frac{3a^2bd\,x^4}{4}+\frac{3a^2be\,x^5}{5}+\frac{a\,b^2c\,x^6}{2}+\frac{3a\,b^2d\,x^7}{7}+\frac{3a\,b^2e\,x^8}{8}+\frac{b^3c\,x^9}{9}+\frac{b^3d\,x^{10}}{10}+b^3$
parallelrisch	$a^3dx+\frac{a^3e\,x^2}{2}+a^2x^3bc+\frac{3a^2bd\,x^4}{4}+\frac{3a^2be\,x^5}{5}+\frac{a\,b^2c\,x^6}{2}+\frac{3a\,b^2d\,x^7}{7}+\frac{3a\,b^2e\,x^8}{8}+\frac{b^3c\,x^9}{9}+\frac{b^3d\,x^{10}}{10}+b^3$

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^3*d*x+1/2*a^3*e*x^2+a^2*x^3*b*c+3/4*a^2*b*d*x^4+3/5*a^2*b*e*x^5+1/2*a*b^
2*c*x^6+3/7*a*b^2*d*x^7+3/8*a*b^2*e*x^8+1/9*b^3*c*x^9+1/10*b^3*d*x^10+1/11
*b^3*e*x^11+a^3*c*ln(x)
```

3.328.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx = \frac{1}{11}\,b^3ex^{11}+\frac{1}{10}\,b^3dx^{10}+\frac{1}{9}\,b^3cx^9+\frac{3}{8}\,ab^2ex^8 + \frac{3}{7}\,ab^2dx^7+\frac{1}{2}\,ab^2cx^6+\frac{3}{5}\,a^2bex^5+\frac{3}{4}\,a^2bdx^4 + a^2bcx^3+\frac{1}{2}\,a^3ex^2+a^3dx+a^3c\log(x)$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="fricas")
```

output
```
1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*
a*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*
c*x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(x)
```

3.328.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx = a^3c\log(x)+a^3dx+\frac{a^3ex^2}{2}+a^2bcx^3+\frac{3a^2bdx^4}{4}+\frac{3a^2bex^5}{5}+\frac{ab^2cx^6}{2}+\frac{3ab^2dx^7}{7}+\frac{3ab^2ex^8}{8}+\frac{b^3cx^9}{9}+\frac{b^3dx^{10}}{10}+\frac{b^3ex^{11}}{11}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x,x)
```

output
```
a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + a**2*b*c*x**3 + 3*a**2*b*d*x**4
/4 + 3*a**2*b*e*x**5/5 + a*b**2*c*x**6/2 + 3*a*b**2*d*x**7/7 + 3*a*b**2*e*
x**8/8 + b**3*c*x**9/9 + b**3*d*x**10/10 + b**3*e*x**11/11
```

3.328.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx = \frac{1}{11}\,b^3ex^{11}+\frac{1}{10}\,b^3dx^{10}+\frac{1}{9}\,b^3cx^9+\frac{3}{8}\,ab^2ex^8+\frac{3}{7}\,ab^2dx^7+\frac{1}{2}\,ab^2cx^6+\frac{3}{5}\,a^2bex^5+\frac{3}{4}\,a^2bdx^4+a^2bcx^3+\frac{1}{2}\,a^3ex^2+a^3dx+a^3c\log(x)$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="maxima")
```

output
```
1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*
a*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*
c*x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(x)
```

3.328.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx = \frac{1}{11}b^3ex^{11} + \frac{1}{10}b^3dx^{10} + \frac{1}{9}b^3cx^9 + \frac{3}{8}ab^2ex^8 + \frac{3}{7}ab^2dx^7 + \frac{1}{2}ab^2cx^6 + \frac{3}{5}a^2bex^5 + \frac{3}{4}a^2bdx^4 + a^2bcx^3 + \frac{1}{2}a^3ex^2 + a^3dx + a^3c\log(|x|)$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x,x, algorithm="giac")
```

output
```
1/11*b^3*e*x^11 + 1/10*b^3*d*x^10 + 1/9*b^3*c*x^9 + 3/8*a*b^2*e*x^8 + 3/7*
a*b^2*d*x^7 + 1/2*a*b^2*c*x^6 + 3/5*a^2*b*e*x^5 + 3/4*a^2*b*d*x^4 + a^2*b*
c*x^3 + 1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(abs(x))
```

3.328.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x}\,dx = \frac{b^3\,c\,x^9}{9} + \frac{a^3\,e\,x^2}{2} + \frac{b^3\,d\,x^{10}}{10} + \frac{b^3\,e\,x^{11}}{11} + a^3\,c\,\ln(x) + a^3\,d\,x + a^2\,b\,c\,x^3 + \frac{a\,b^2\,c\,x^6}{2} + \frac{3\,a^2\,b\,d\,x^4}{4} + \frac{3\,a\,b^2\,d\,x^7}{7} + \frac{3\,a^2\,b\,e\,x^5}{5} + \frac{3\,a\,b^2\,e\,x^8}{8}$$

input
```
int(((a + b*x^3)^3*(c + d*x + e*x^2))/x,x)
```

output
```
(b^3*c*x^9)/9 + (a^3*e*x^2)/2 + (b^3*d*x^10)/10 + (b^3*e*x^11)/11 + a^3*c*
log(x) + a^3*d*x + a^2*b*c*x^3 + (a*b^2*c*x^6)/2 + (3*a^2*b*d*x^4)/4 + (3*
a*b^2*d*x^7)/7 + (3*a^2*b*e*x^5)/5 + (3*a*b^2*e*x^8)/8
```

3.329 $\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx$

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3.329.1 Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx=-\frac{a^3c}{x}+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8+\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^{10}+a^3d\log(x)$$

output
```
-a^3*c/x+a^3*e*x+3/2*a^2*b*c*x^2+a^2*b*d*x^3+3/4*a^2*b*e*x^4+3/5*a*b^2*c*x
^5+1/2*a*b^2*d*x^6+3/7*a*b^2*e*x^7+1/8*b^3*c*x^8+1/9*b^3*d*x^9+1/10*b^3*e*
x^10+a^3*d*ln(x)
```

3.329.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx=-\frac{a^3c}{x}+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8+\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^{10}+a^3d\log(x)$$

input
```
Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]
```

output
```
-((a^3*c)/x) + a^3*e*x + (3*a^2*b*c*x^2)/2 + a^2*b*d*x^3 + (3*a^2*b*e*x^4)
/4 + (3*a*b^2*c*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b^2*e*x^7)/7 + (b^3*c*x^8)
/8 + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*Log[x]
```

3.329.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2)}{x^2} \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{a^3c}{x^2}+\frac{a^3d}{x}+a^3e+3a^2bcx+3a^2bdx^2+3a^2bex^3+3ab^2cx^4+3ab^2dx^5+3ab^2ex^6+b^3cx^7+b^3dx^8+b^3ex^9\right) d$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{3}{2}a^2bcx^2+a^2bdx^3+\frac{3}{4}a^2bex^4+\frac{3}{5}ab^2cx^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab^2ex^7+\frac{1}{8}b^3cx^8+\frac{1}{9}b^3dx^9+\frac{1}{10}b^3ex^{10}$$

input
```
Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^2,x]
```

output
```
-((a^3*c)/x) + a^3*e*x + (3*a^2*b*c*x^2)/2 + a^2*b*d*x^3 + (3*a^2*b*e*x^4)
/4 + (3*a*b^2*c*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b^2*e*x^7)/7 + (b^3*c*x^8)
/8 + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*Log[x]
```

3.329.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.329.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a^{3}c}{x}+a^{3}ex+\frac{3a^{2}bc\,x^{2}}{2}+a^{2}bd\,x^{3}+\frac{3a^{2}be\,x^{4}}{4}+\frac{3a\,b^{2}c\,x^{5}}{5}+\frac{a\,b^{2}d\,x^{6}}{2}+\frac{3a\,b^{2}e\,x^{7}}{7}+\frac{b^{3}c\,x^{8}}{8}+\frac{b^{3}d\,x^{9}}{9}+b^{3}e$
risch	$-\frac{a^{3}c}{x}+a^{3}ex+\frac{3a^{2}bc\,x^{2}}{2}+a^{2}bd\,x^{3}+\frac{3a^{2}be\,x^{4}}{4}+\frac{3a\,b^{2}c\,x^{5}}{5}+\frac{a\,b^{2}d\,x^{6}}{2}+\frac{3a\,b^{2}e\,x^{7}}{7}+\frac{b^{3}c\,x^{8}}{8}+\frac{b^{3}d\,x^{9}}{9}+b^{3}e$
norman	$\frac{a^{3}e\,x^{2}+a^{2}bd\,x^{4}-c\,a^{3}+\frac{1}{8}b^{3}c\,x^{9}+\frac{1}{9}b^{3}d\,x^{10}+\frac{1}{10}b^{3}e\,x^{11}+\frac{3}{5}a\,b^{2}c\,x^{6}+\frac{1}{2}a\,b^{2}d\,x^{7}+\frac{3}{7}a\,b^{2}e\,x^{8}+\frac{3}{4}a^{2}be\,x^{5}+\frac{3}{2}a^{2}x^{3}bc}{x}+a^{3}d\ln\left(x\right)$
parallelrisch	$\frac{252b^{3}e\,x^{11}+280b^{3}d\,x^{10}+315b^{3}c\,x^{9}+1080a\,b^{2}e\,x^{8}+1260a\,b^{2}d\,x^{7}+1512a\,b^{2}c\,x^{6}+1890a^{2}be\,x^{5}+2520a^{2}bd\,x^{4}+3780a^{2}x^{3}bc+2520}{2520x}$

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3*c/x+a^3*e*x+3/2*a^2*b*c*x^2+a^2*b*d*x^3+3/4*a^2*b*e*x^4+3/5*a*b^2*c*x
^5+1/2*a*b^2*d*x^6+3/7*a*b^2*e*x^7+1/8*b^3*c*x^8+1/9*b^3*d*x^9+1/10*b^3*e*
x^10+a^3*d*ln(x)
```

3.329.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)^3}{x^2}\,dx$$
$$=\frac{252\,b^3ex^{11}+280\,b^3dx^{10}+315\,b^3cx^9+1080\,ab^2ex^8+1260\,ab^2dx^7+1512\,ab^2cx^6+1890\,a^2bex^5+2520\,a^2b}{2520\,x}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="fricas")
```

output
```
1/2520*(252*b^3*e*x^11 + 280*b^3*d*x^10 + 315*b^3*c*x^9 + 1080*a*b^2*e*x^8
 + 1260*a*b^2*d*x^7 + 1512*a*b^2*c*x^6 + 1890*a^2*b*e*x^5 + 2520*a^2*b*d*x
^4 + 3780*a^2*b*c*x^3 + 2520*a^3*e*x^2 + 2520*a^3*d*x*log(x) - 2520*a^3*c)
/x
```

3.329.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx = -\frac{a^3c}{x} + a^3d\log(x) + a^3ex + \frac{3a^2bcx^2}{2} + a^2bdx^3 + \frac{3a^2bex^4}{4} + \frac{3ab^2cx^5}{5} + \frac{ab^2dx^6}{2} + \frac{3ab^2ex^7}{7} + \frac{b^3cx^8}{8} + \frac{b^3dx^9}{9} + \frac{b^3ex^{10}}{10}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x**2,x)
```

output
```
-a**3*c/x + a**3*d*log(x) + a**3*e*x + 3*a**2*b*c*x**2/2 + a**2*b*d*x**3 +
 3*a**2*b*e*x**4/4 + 3*a*b**2*c*x**5/5 + a*b**2*d*x**6/2 + 3*a*b**2*e*x**7
/7 + b**3*c*x**8/8 + b**3*d*x**9/9 + b**3*e*x**10/10
```

3.329.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx = \frac{1}{10}b^3ex^{10} + \frac{1}{9}b^3dx^9 + \frac{1}{8}b^3cx^8 + \frac{3}{7}ab^2ex^7 + \frac{1}{2}ab^2dx^6 + \frac{3}{5}ab^2cx^5 + \frac{3}{4}a^2bex^4 + a^2bdx^3 + \frac{3}{2}a^2bcx^2 + a^3ex + a^3d\log(x) - \frac{a^3c}{x}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/10*b^3*e*x^10 + 1/9*b^3*d*x^9 + 1/8*b^3*c*x^8 + 3/7*a*b^2*e*x^7 + 1/2*a*
b^2*d*x^6 + 3/5*a*b^2*c*x^5 + 3/4*a^2*b*e*x^4 + a^2*b*d*x^3 + 3/2*a^2*b*c*
x^2 + a^3*e*x + a^3*d*log(x) - a^3*c/x
```

3.329.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx = \frac{1}{10}b^3ex^{10} + \frac{1}{9}b^3dx^9 + \frac{1}{8}b^3cx^8 + \frac{3}{7}ab^2ex^7 + \frac{1}{2}ab^2dx^6 + \frac{3}{5}ab^2cx^5 + \frac{3}{4}a^2bex^4 + a^2bdx^3 + \frac{3}{2}a^2bcx^2 + a^3ex + a^3d\log(|x|) - \frac{a^3c}{x}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^2,x, algorithm="giac")
```

output
```
1/10*b^3*e*x^10 + 1/9*b^3*d*x^9 + 1/8*b^3*c*x^8 + 3/7*a*b^2*e*x^7 + 1/2*a*
b^2*d*x^6 + 3/5*a*b^2*c*x^5 + 3/4*a^2*b*e*x^4 + a^2*b*d*x^3 + 3/2*a^2*b*c*
x^2 + a^3*e*x + a^3*d*log(abs(x)) - a^3*c/x
```

3.329.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^2}\,dx = \frac{b^3\,c\,x^8}{8} - \frac{a^3\,c}{x} + \frac{b^3\,d\,x^9}{9} + \frac{b^3\,e\,x^{10}}{10} + a^3\,d\,\ln(x) + a^3\,e\,x + \frac{3\,a^2\,b\,c\,x^2}{2} + \frac{3\,a\,b^2\,c\,x^5}{5} + a^2\,b\,d\,x^3 + \frac{a\,b^2\,d\,x^6}{2} + \frac{3\,a^2\,b\,e\,x^4}{4} + \frac{3\,a\,b^2\,e\,x^7}{7}$$

input
```
int(((a + b*x^3)^3*(c + d*x + e*x^2))/x^2,x)
```

output
```
(b^3*c*x^8)/8 - (a^3*c)/x + (b^3*d*x^9)/9 + (b^3*e*x^10)/10 + a^3*d*log(x)
 + a^3*e*x + (3*a^2*b*c*x^2)/2 + (3*a*b^2*c*x^5)/5 + a^2*b*d*x^3 + (a*b^2*
d*x^6)/2 + (3*a^2*b*e*x^4)/4 + (3*a*b^2*e*x^7)/7
```

3.330 $\int \frac{(c+dx+ex^2)\left(a+bx^3\right)^3}{x^3}\,dx$

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3.330.1 Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{(c+dx+ex^2)\left(a+bx^3\right)^3}{x^3}\,dx = -\frac{a^3c}{2x^2}-\frac{a^3d}{x}+3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7+\frac{1}{8}b^3dx^8+\frac{1}{9}b^3ex^9+a^3e\log(x)$$

output
```
-1/2*a^3*c/x^2-a^3*d/x+3*a^2*b*c*x+3/2*a^2*b*d*x^2+a^2*b*e*x^3+3/4*a*b^2*c
*x^4+3/5*a*b^2*d*x^5+1/2*a*b^2*e*x^6+1/7*b^3*c*x^7+1/8*b^3*d*x^8+1/9*b^3*e
*x^9+a^3*e*ln(x)
```

3.330.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)\left(a+bx^3\right)^3}{x^3}\,dx = -\frac{a^3c}{2x^2}-\frac{a^3d}{x}+3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7+\frac{1}{8}b^3dx^8+\frac{1}{9}b^3ex^9+a^3e\log(x)$$

input
```
Integrate[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3,x]
```

output
```
-1/2*(a^3*c)/x^2 - (a^3*d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x
^3 + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)
/7 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*Log[x]
```

3.330.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3(c+dx+ex^2)}{x^3}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{a^3c}{x^3}+\frac{a^3d}{x^2}+\frac{a^3e}{x}+3a^2bc+3a^2bdx+3a^2bex^2+3ab^2cx^3+3ab^2dx^4+3ab^2ex^5+b^3cx^6+b^3dx^7+b^3ex^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+3a^2bcx+\frac{3}{2}a^2bdx^2+a^2bex^3+\frac{3}{4}ab^2cx^4+\frac{3}{5}ab^2dx^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^3cx^7+\frac{1}{8}b^3dx^8+\frac{1}{9}b^3ex^9$$

input
```
Int[((c + d*x + e*x^2)*(a + b*x^3)^3)/x^3,x]
```

output
```
-1/2*(a^3*c)/x^2 - (a^3*d)/x + 3*a^2*b*c*x + (3*a^2*b*d*x^2)/2 + a^2*b*e*x
^3 + (3*a*b^2*c*x^4)/4 + (3*a*b^2*d*x^5)/5 + (a*b^2*e*x^6)/2 + (b^3*c*x^7)
/7 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9 + a^3*e*Log[x]
```

3.330.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.330.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+3a^2bcx+\frac{3a^2bd\,x^2}{2}+a^2be\,x^3+\frac{3a\,b^2c\,x^4}{4}+\frac{3a\,b^2d\,x^5}{5}+\frac{a\,b^2e\,x^6}{2}+\frac{b^3c\,x^7}{7}+\frac{b^3d\,x^8}{8}+\frac{b^3e\,x}{9}$
risch	$\frac{b^3e\,x^9}{9}+\frac{b^3d\,x^8}{8}+\frac{b^3c\,x^7}{7}+\frac{a\,b^2e\,x^6}{2}+\frac{3a\,b^2d\,x^5}{5}+\frac{3a\,b^2c\,x^4}{4}+a^2be\,x^3+\frac{3a^2bd\,x^2}{2}+3a^2bcx+\frac{-a^3dx-\frac{1}{2}c\,a}{x^2}$
norman	$\frac{a^2be\,x^5-\frac{1}{2}c\,a^3-a^3dx+\frac{1}{7}b^3c\,x^9+\frac{1}{8}b^3d\,x^{10}+\frac{1}{9}b^3e\,x^{11}+\frac{3}{4}a\,b^2c\,x^6+\frac{3}{5}a\,b^2d\,x^7+\frac{1}{2}a\,b^2e\,x^8+\frac{3}{2}a^2bd\,x^4+3a^2x^3bc}{x^2}+a^3e\ln(x)$
parallelrisch	$\frac{280b^3e\,x^{11}+315b^3d\,x^{10}+360b^3c\,x^9+1260a\,b^2e\,x^8+1512a\,b^2d\,x^7+1890a\,b^2c\,x^6+2520a^2be\,x^5+3780a^2bd\,x^4+2520e\,a^3\ln(x)x^2+}{2520x^2}$

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^3*c/x^2-a^3*d/x+3*a^2*b*c*x+3/2*a^2*b*d*x^2+a^2*b*e*x^3+3/4*a*b^2*c
*x^4+3/5*a*b^2*d*x^5+1/2*a*b^2*e*x^6+1/7*b^3*c*x^7+1/8*b^3*d*x^8+1/9*b^3*e
*x^9+a^3*e*ln(x)
```

3.330.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93

$$\int\frac{(c+dx+ex^2)\,(a+bx^3)^3}{x^3}\,dx$$
$$=\frac{280\,b^3ex^{11}+315\,b^3dx^{10}+360\,b^3cx^9+1260\,ab^2ex^8+1512\,ab^2dx^7+1890\,ab^2cx^6+2520\,a^2bex^5+3780\,a^2b}{2520\,x^2}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="fricas")
```

output
```
1/2520*(280*b^3*e*x^11 + 315*b^3*d*x^10 + 360*b^3*c*x^9 + 1260*a*b^2*e*x^8
 + 1512*a*b^2*d*x^7 + 1890*a*b^2*c*x^6 + 2520*a^2*b*e*x^5 + 3780*a^2*b*d*x
^4 + 7560*a^2*b*c*x^3 + 2520*a^3*e*x^2*log(x) - 2520*a^3*d*x - 1260*a^3*c)
/x^2
```

3.330.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx = a^3e\log{(x)} + 3a^2bcx + \frac{3a^2bdx^2}{2} + a^2bex^3 + \frac{3ab^2cx^4}{4} + \frac{3ab^2dx^5}{5} + \frac{ab^2ex^6}{2} + \frac{b^3cx^7}{7} + \frac{b^3dx^8}{8} + \frac{b^3ex^9}{9} + \frac{-a^3c-2a^3dx}{2x^2}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**3/x**3,x)
```

output
```
a**3*e*log(x) + 3*a**2*b*c*x + 3*a**2*b*d*x**2/2 + a**2*b*e*x**3 + 3*a*b**
2*c*x**4/4 + 3*a*b**2*d*x**5/5 + a*b**2*e*x**6/2 + b**3*c*x**7/7 + b**3*d*
x**8/8 + b**3*e*x**9/9 + (-a**3*c - 2*a**3*d*x)/(2*x**2)
```

3.330.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx = \frac{1}{9}b^3ex^9 + \frac{1}{8}b^3dx^8 + \frac{1}{7}b^3cx^7 + \frac{1}{2}ab^2ex^6 + \frac{3}{5}ab^2dx^5 + \frac{3}{4}ab^2cx^4 + a^2bex^3 + \frac{3}{2}a^2bdx^2 + 3\,a^2bcx + a^3e\log{(x)} - \frac{2\,a^3dx + a^3c}{2\,x^2}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="maxima")
```

output
```
1/9*b^3*e*x^9 + 1/8*b^3*d*x^8 + 1/7*b^3*c*x^7 + 1/2*a*b^2*e*x^6 + 3/5*a*b^
2*d*x^5 + 3/4*a*b^2*c*x^4 + a^2*b*e*x^3 + 3/2*a^2*b*d*x^2 + 3*a^2*b*c*x +
a^3*e*log(x) - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

3.330.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx = \frac{1}{9}\,b^3ex^9 + \frac{1}{8}\,b^3dx^8 + \frac{1}{7}\,b^3cx^7 + \frac{1}{2}\,ab^2ex^6 + \frac{3}{5}\,ab^2dx^5 + \frac{3}{4}\,ab^2cx^4 + a^2bex^3 + \frac{3}{2}\,a^2bdx^2 + 3\,a^2bcx + a^3e\log(|x|) - \frac{2\,a^3dx + a^3c}{2\,x^2}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^3/x^3,x, algorithm="giac")
```

output
```
1/9*b^3*e*x^9 + 1/8*b^3*d*x^8 + 1/7*b^3*c*x^7 + 1/2*a*b^2*e*x^6 + 3/5*a*b^
2*d*x^5 + 3/4*a*b^2*c*x^4 + a^2*b*e*x^3 + 3/2*a^2*b*d*x^2 + 3*a^2*b*c*x +
a^3*e*log(abs(x)) - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

3.330.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^3}{x^3}\,dx = \frac{b^3\,c\,x^7}{7} - \frac{\frac{a^3\,c}{2} + a^3\,d\,x}{x^2} + \frac{b^3\,d\,x^8}{8} + \frac{b^3\,e\,x^9}{9} + a^3\,e\,\ln(x) + 3\,a^2\,b\,c\,x + \frac{3\,a\,b^2\,c\,x^4}{4} + \frac{3\,a^2\,b\,d\,x^2}{2} + \frac{3\,a\,b^2\,d\,x^5}{5} + a^2\,b\,e\,x^3 + \frac{a\,b^2\,e\,x^6}{2}$$

input
```
int(((a + b*x^3)^3*(c + d*x + e*x^2))/x^3,x)
```

output
```
(b^3*c*x^7)/7 - ((a^3*c)/2 + a^3*d*x)/x^2 + (b^3*d*x^8)/8 + (b^3*e*x^9)/9
+ a^3*e*log(x) + 3*a^2*b*c*x + (3*a*b^2*c*x^4)/4 + (3*a^2*b*d*x^2)/2 + (3*
a*b^2*d*x^5)/5 + a^2*b*e*x^3 + (a*b^2*e*x^6)/2
```

3.331 $\int x^2(c+dx+ex^2)\left(a+bx^3\right)^4\,dx$

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3.331.1 Optimal result

Integrand size = 23, antiderivative size = 138

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx=\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{1}{16}b^4dx^{16}+\frac{1}{17}b^4ex^{17}+\frac{c\left(a+bx^3\right)^5}{15b}$$

output
```
1/4*a^4*d*x^4+1/5*a^4*e*x^5+4/7*a^3*b*d*x^7+1/2*a^3*b*e*x^8+3/5*a^2*b^2*d*
x^10+6/11*a^2*b^2*e*x^11+4/13*a*b^3*d*x^13+2/7*a*b^3*e*x^14+1/16*b^4*d*x^1
6+1/17*b^4*e*x^17+1/15*c*(b*x^3+a)^5/b
```

3.331.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.31

$$\int x^2\left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx=\frac{1}{3}a^4cx^3+\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{2}{3}a^3bcx^6+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{1}{3}ab^3cx^{12}+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{1}{15}b^4cx^{15}+\frac{1}{16}b^4dx^{16}+\frac{1}{17}b^4ex^{17}$$

input
```
Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

output
```
(a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (2*a^3*b*c*x^6)/3 + (4*a^3
*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5
 + (6*a^2*b^2*e*x^11)/11 + (a*b^3*c*x^12)/3 + (4*a*b^3*d*x^13)/13 + (2*a*b
^3*e*x^14)/7 + (b^4*c*x^15)/15 + (b^4*d*x^16)/16 + (b^4*e*x^17)/17
```

3.331.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right)^4 \left(c+dx+ex^2\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^3+a\right)^4 \left(x^2 \left(ex^2+dx+c\right)-cx^2\right) dx+\frac{c\left(a+bx^3\right)^5}{15b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^4ex^{16}+b^4dx^{15}+4ab^3ex^{13}+4ab^3dx^{12}+6a^2b^2ex^{10}+6a^2b^2dx^9+4a^3bex^7+4a^3bdx^6+a^4ex^4+a^4dx^3\right)dx+\frac{c\left(a+bx^3\right)^5}{15b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^4dx^4+\frac{1}{5}a^4ex^5+\frac{4}{7}a^3bdx^7+\frac{1}{2}a^3bex^8+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3dx^{13}+\frac{2}{7}ab^3ex^{14}+\frac{c\left(a+bx^3\right)^5}{15b}+\frac{1}{16}b^4dx^{16}+\frac{1}{17}b^4ex^{17}$$

input
```
Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

output
```
(a^4*d*x^4)/4 + (a^4*e*x^5)/5 + (4*a^3*b*d*x^7)/7 + (a^3*b*e*x^8)/2 + (3*a
^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (4*a*b^3*d*x^13)/13 + (2*a*b^3*
e*x^14)/7 + (b^4*d*x^16)/16 + (b^4*e*x^17)/17 + (c*(a + b*x^3)^5)/(15*b)
```

3.331.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.331.4 Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10

method	result
gosper	$\frac{1}{3}a^4c\,x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}a^3bc\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}$
default	$\frac{1}{3}a^4c\,x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}a^3bc\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}$
norman	$\frac{1}{3}a^4c\,x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}a^3bc\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}$
risch	$\frac{1}{3}a^4c\,x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}a^3bc\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}$
parallelrisch	$\frac{1}{3}a^4c\,x^3+\frac{1}{4}a^4d\,x^4+\frac{1}{5}a^4e\,x^5+\frac{2}{3}a^3bc\,x^6+\frac{4}{7}a^3bd\,x^7+\frac{1}{2}a^3be\,x^8+\frac{2}{3}a^2b^2c\,x^9+\frac{3}{5}a^2b^2d\,x^{10}+\frac{6}{11}$

input
```
int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^4*c*x^3+1/4*a^4*d*x^4+1/5*a^4*e*x^5+2/3*a^3*b*c*x^6+4/7*a^3*b*d*x^7+
1/2*a^3*b*e*x^8+2/3*a^2*b^2*c*x^9+3/5*a^2*b^2*d*x^10+6/11*a^2*b^2*e*x^11+1
/3*a*b^3*c*x^12+4/13*a*b^3*d*x^13+2/7*a*b^3*e*x^14+1/15*b^4*c*x^15+1/16*b^
4*d*x^16+1/17*b^4*e*x^17
```

3.331.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{1}{17}b^4ex^{17}+\frac{1}{16}b^4dx^{16}+\frac{1}{15}b^4cx^{15}+\frac{2}{7}ab^3ex^{14}+\frac{4}{13}ab^3dx^{13}+\frac{1}{3}ab^3cx^{12}+\frac{6}{11}a^2b^2ex^{11}+\frac{3}{5}a^2b^2dx^{10}+\frac{2}{3}a^2b^2cx^9+\frac{1}{2}a^3bex^8+\frac{4}{7}a^3bdx^7+\frac{2}{3}a^3bcx^6+\frac{1}{5}a^4ex^5+\frac{1}{4}a^4dx^4+\frac{1}{3}a^4cx^3$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

output
```
1/17*b^4*e*x^17 + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*e*x^14 + 4
/13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*
x^10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*e*x^8 + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c
*x^6 + 1/5*a^4*e*x^5 + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

3.331.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.33

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{a^4cx^3}{3}+\frac{a^4dx^4}{4}+\frac{a^4ex^5}{5}+\frac{2a^3bcx^6}{3}+\frac{4a^3bdx^7}{7}+\frac{a^3bex^8}{2}+\frac{2a^2b^2cx^9}{3}+\frac{3a^2b^2dx^{10}}{5}+\frac{6a^2b^2ex^{11}}{11}+\frac{ab^3cx^{12}}{3}+\frac{4ab^3dx^{13}}{13}+\frac{2ab^3ex^{14}}{7}+\frac{b^4cx^{15}}{15}+\frac{b^4dx^{16}}{16}+\frac{b^4ex^{17}}{17}$$

input
```
integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

output
```
a**4*c*x**3/3 + a**4*d*x**4/4 + a**4*e*x**5/5 + 2*a**3*b*c*x**6/3 + 4*a**3
*b*d*x**7/7 + a**3*b*e*x**8/2 + 2*a**2*b**2*c*x**9/3 + 3*a**2*b**2*d*x**10
/5 + 6*a**2*b**2*e*x**11/11 + a*b**3*c*x**12/3 + 4*a*b**3*d*x**13/13 + 2*a
*b**3*e*x**14/7 + b**4*c*x**15/15 + b**4*d*x**16/16 + b**4*e*x**17/17
```

3.331.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^4\,dx = \frac{1}{17}b^4ex^{17}+\frac{1}{16}b^4dx^{16}+\frac{1}{15}b^4cx^{15}+\frac{2}{7}ab^3ex^{14}+\frac{4}{13}ab^3dx^{13}+\frac{1}{3}ab^3cx^{12}+\frac{6}{11}a^2b^2ex^{11}+\frac{3}{5}a^2b^2dx^{10}+\frac{2}{3}a^2b^2cx^9+\frac{1}{2}a^3bex^8+\frac{4}{7}a^3bdx^7+\frac{2}{3}a^3bcx^6+\frac{1}{5}a^4ex^5+\frac{1}{4}a^4dx^4+\frac{1}{3}a^4cx^3$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

output
```
1/17*b^4*e*x^17 + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*e*x^14 + 4
/13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*
x^10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*e*x^8 + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c
*x^6 + 1/5*a^4*e*x^5 + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

3.331.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^4\,dx = \frac{1}{17}b^4ex^{17}+\frac{1}{16}b^4dx^{16}+\frac{1}{15}b^4cx^{15}+\frac{2}{7}ab^3ex^{14}+\frac{4}{13}ab^3dx^{13}+\frac{1}{3}ab^3cx^{12}+\frac{6}{11}a^2b^2ex^{11}+\frac{3}{5}a^2b^2dx^{10}+\frac{2}{3}a^2b^2cx^9+\frac{1}{2}a^3bex^8+\frac{4}{7}a^3bdx^7+\frac{2}{3}a^3bcx^6+\frac{1}{5}a^4ex^5+\frac{1}{4}a^4dx^4+\frac{1}{3}a^4cx^3$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

output
```
1/17*b^4*e*x^17 + 1/16*b^4*d*x^16 + 1/15*b^4*c*x^15 + 2/7*a*b^3*e*x^14 + 4
/13*a*b^3*d*x^13 + 1/3*a*b^3*c*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*
x^10 + 2/3*a^2*b^2*c*x^9 + 1/2*a^3*b*e*x^8 + 4/7*a^3*b*d*x^7 + 2/3*a^3*b*c
*x^6 + 1/5*a^4*e*x^5 + 1/4*a^4*d*x^4 + 1/3*a^4*c*x^3
```

3.331.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^4\,dx = \frac{e\,a^4\,x^5}{5}+\frac{d\,a^4\,x^4}{4}+\frac{c\,a^4\,x^3}{3}+\frac{e\,a^3\,b\,x^8}{2}+\frac{4\,d\,a^3\,b\,x^7}{7}+\frac{2\,c\,a^3\,b\,x^6}{3}+\frac{6\,e\,a^2\,b^2\,x^{11}}{11}+\frac{3\,d\,a^2\,b^2\,x^{10}}{5}+\frac{2\,c\,a^2\,b^2\,x^9}{3}+\frac{2\,e\,a\,b^3\,x^{14}}{7}+\frac{4\,d\,a\,b^3\,x^{13}}{13}+\frac{c\,a\,b^3\,x^{12}}{3}+\frac{e\,b^4\,x^{17}}{17}+\frac{d\,b^4\,x^{16}}{16}+\frac{c\,b^4\,x^{15}}{15}$$

input
```
int(x^2*(a + b*x^3)^4*(c + d*x + e*x^2),x)
```

output
```
(a^4*c*x^3)/3 + (a^4*d*x^4)/4 + (b^4*c*x^15)/15 + (a^4*e*x^5)/5 + (b^4*d*x
^16)/16 + (b^4*e*x^17)/17 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (
6*a^2*b^2*e*x^11)/11 + (2*a^3*b*c*x^6)/3 + (a*b^3*c*x^12)/3 + (4*a^3*b*d*x
^7)/7 + (4*a*b^3*d*x^13)/13 + (a^3*b*e*x^8)/2 + (2*a*b^3*e*x^14)/7
```

3.332 $\int x(c+dx+ex^2)(a+bx^3)^4\,dx$

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3.332.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{1}{2}a^4cx^2+\frac{1}{4}a^4ex^4+\frac{4}{5}a^3bcx^5+\frac{4}{7}a^3bex^7+\frac{3}{4}a^2b^2cx^8+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{11}ab^3cx^{11}+\frac{4}{13}ab^3ex^{13}+\frac{1}{14}b^4cx^{14}+\frac{1}{16}b^4ex^{16}+\frac{d(a+bx^3)^5}{15b}$$

output
```
1/2*a^4*c*x^2+1/4*a^4*e*x^4+4/5*a^3*b*c*x^5+4/7*a^3*b*e*x^7+3/4*a^2*b^2*c*
x^8+3/5*a^2*b^2*e*x^10+4/11*a*b^3*c*x^11+4/13*a*b^3*e*x^13+1/14*b^4*c*x^14
+1/16*b^4*e*x^16+1/15*d*(b*x^3+a)^5/b
```

3.332.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.31

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{1}{2}a^4cx^2+\frac{1}{3}a^4dx^3+\frac{1}{4}a^4ex^4+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{3}{4}a^2b^2cx^8+\frac{2}{3}a^2b^2dx^9+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{11}ab^3cx^{11}+\frac{1}{3}ab^3dx^{12}+\frac{4}{13}ab^3ex^{13}+\frac{1}{14}b^4cx^{14}+\frac{1}{15}b^4dx^{15}+\frac{1}{16}b^4ex^{16}$$

input
```
Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

output
```
(a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5 + (2*a^3
*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (3*a^2*b^2*c*x^8)/4 + (2*a^2*b^2*d*x^9)/
3 + (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11 + (a*b^3*d*x^12)/3 + (4*a*b
^3*e*x^13)/13 + (b^4*c*x^14)/14 + (b^4*d*x^15)/15 + (b^4*e*x^16)/16
```

3.332.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^4\left(c+dx+ex^2\right)\,dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^3+a\right)^4\left(x\left(ex^2+dx+c\right)-dx^2\right)dx+\frac{d\left(a+bx^3\right)^5}{15b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^4ex^{15}+b^4cx^{13}+4ab^3ex^{12}+4ab^3cx^{10}+6a^2b^2ex^9+6a^2b^2cx^7+4a^3bex^6+4a^3bcx^4+a^4ex^3+a^4cx\right)dx+\frac{d\left(a+bx^3\right)^5}{15b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^4cx^2+\frac{1}{4}a^4ex^4+\frac{4}{5}a^3bcx^5+\frac{4}{7}a^3bex^7+\frac{3}{4}a^2b^2cx^8+\frac{3}{5}a^2b^2ex^{10}+\frac{4}{11}ab^3cx^{11}+\frac{4}{13}ab^3ex^{13}+\frac{d\left(a+bx^3\right)^5}{15b}+\frac{1}{14}b^4cx^{14}+\frac{1}{16}b^4ex^{16}$$

input
```
Int[x*(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

output
```
(a^4*c*x^2)/2 + (a^4*e*x^4)/4 + (4*a^3*b*c*x^5)/5 + (4*a^3*b*e*x^7)/7 + (3
*a^2*b^2*c*x^8)/4 + (3*a^2*b^2*e*x^10)/5 + (4*a*b^3*c*x^11)/11 + (4*a*b^3*
e*x^13)/13 + (b^4*c*x^14)/14 + (b^4*e*x^16)/16 + (d*(a + b*x^3)^5)/(15*b)
```

3.332.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.332.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10

method	result
gosper	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}a^4d\,x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a$
default	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}a^4d\,x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a$
norman	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}a^4d\,x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a$
risch	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}a^4d\,x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a$
parallelrisch	$\frac{1}{2}a^4c\,x^2+\frac{1}{3}a^4d\,x^3+\frac{1}{4}a^4e\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{3}{4}a^2b^2c\,x^8+\frac{2}{3}a^2b^2d\,x^9+\frac{3}{5}a$

input
```
int(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^4*c*x^2+1/3*a^4*d*x^3+1/4*a^4*e*x^4+4/5*a^3*b*c*x^5+2/3*a^3*b*d*x^6+
4/7*a^3*b*e*x^7+3/4*a^2*b^2*c*x^8+2/3*a^2*b^2*d*x^9+3/5*a^2*b^2*e*x^10+4/1
1*a*b^3*c*x^11+1/3*x^12*a*b^3*d+4/13*a*b^3*e*x^13+1/14*b^4*c*x^14+1/15*d*b
^4*x^15+1/16*b^4*e*x^16
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^4\,dx = \frac{1}{16}b^4ex^{16}+\frac{1}{15}b^4dx^{15}+\frac{1}{14}b^4cx^{14}+\frac{4}{13}ab^3ex^{13}+\frac{1}{3}ab^3dx^{12}+\frac{4}{11}ab^3cx^{11}+\frac{3}{5}a^2b^2ex^{10}+\frac{2}{3}a^2b^2dx^9+\frac{3}{4}a^2b^2cx^8+\frac{4}{7}a^3bex^7+\frac{2}{3}a^3bdx^6+\frac{4}{5}a^3bcx^5+\frac{1}{4}a^4ex^4+\frac{1}{3}a^4dx^3+\frac{1}{2}a^4cx^2$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

output
```
1/16*b^4*e*x^16 + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*e*x^13 +
1/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*e*x^10 + 2/3*a^2*b^2*d*
x^9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*
x^5 + 1/4*a^4*e*x^4 + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

3.332.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.34

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^4\,dx = \frac{a^4cx^2}{2}+\frac{a^4dx^3}{3}+\frac{a^4ex^4}{4}+\frac{4a^3bcx^5}{5}+\frac{2a^3bdx^6}{3}+\frac{4a^3bex^7}{7}+\frac{3a^2b^2cx^8}{4}+\frac{2a^2b^2dx^9}{3}+\frac{3a^2b^2ex^{10}}{5}+\frac{4ab^3cx^{11}}{11}+\frac{ab^3dx^{12}}{3}+\frac{4ab^3ex^{13}}{13}+\frac{b^4cx^{14}}{14}+\frac{b^4dx^{15}}{15}+\frac{b^4ex^{16}}{16}$$

input
```
integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

output
```
a**4*c*x**2/2 + a**4*d*x**3/3 + a**4*e*x**4/4 + 4*a**3*b*c*x**5/5 + 2*a**3
*b*d*x**6/3 + 4*a**3*b*e*x**7/7 + 3*a**2*b**2*c*x**8/4 + 2*a**2*b**2*d*x**
9/3 + 3*a**2*b**2*e*x**10/5 + 4*a*b**3*c*x**11/11 + a*b**3*d*x**12/3 + 4*a
*b**3*e*x**13/13 + b**4*c*x**14/14 + b**4*d*x**15/15 + b**4*e*x**16/16
```

3.332.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^4\,dx = \frac{1}{16}b^4ex^{16}+\frac{1}{15}b^4dx^{15}+\frac{1}{14}b^4cx^{14}+\frac{4}{13}ab^3ex^{13}+\frac{1}{3}ab^3dx^{12}+\frac{4}{11}ab^3cx^{11}+\frac{3}{5}a^2b^2ex^{10}+\frac{2}{3}a^2b^2dx^9+\frac{3}{4}a^2b^2cx^8+\frac{4}{7}a^3bex^7+\frac{2}{3}a^3bdx^6+\frac{4}{5}a^3bcx^5+\frac{1}{4}a^4ex^4+\frac{1}{3}a^4dx^3+\frac{1}{2}a^4cx^2$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

output
```
1/16*b^4*e*x^16 + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*e*x^13 +
1/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*e*x^10 + 2/3*a^2*b^2*d*
x^9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*
x^5 + 1/4*a^4*e*x^4 + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

3.332.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^4\,dx = \frac{1}{16}b^4ex^{16}+\frac{1}{15}b^4dx^{15}+\frac{1}{14}b^4cx^{14}+\frac{4}{13}ab^3ex^{13}+\frac{1}{3}ab^3dx^{12}+\frac{4}{11}ab^3cx^{11}+\frac{3}{5}a^2b^2ex^{10}+\frac{2}{3}a^2b^2dx^9+\frac{3}{4}a^2b^2cx^8+\frac{4}{7}a^3bex^7+\frac{2}{3}a^3bdx^6+\frac{4}{5}a^3bcx^5+\frac{1}{4}a^4ex^4+\frac{1}{3}a^4dx^3+\frac{1}{2}a^4cx^2$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

output
```
1/16*b^4*e*x^16 + 1/15*b^4*d*x^15 + 1/14*b^4*c*x^14 + 4/13*a*b^3*e*x^13 +
1/3*a*b^3*d*x^12 + 4/11*a*b^3*c*x^11 + 3/5*a^2*b^2*e*x^10 + 2/3*a^2*b^2*d*
x^9 + 3/4*a^2*b^2*c*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*b*c*
x^5 + 1/4*a^4*e*x^4 + 1/3*a^4*d*x^3 + 1/2*a^4*c*x^2
```

3.332.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{e\,a^4\,x^4}{4}+\frac{d\,a^4\,x^3}{3}+\frac{c\,a^4\,x^2}{2}+\frac{4\,e\,a^3\,b\,x^7}{7}+\frac{2\,d\,a^3\,b\,x^6}{3}+\frac{4\,c\,a^3\,b\,x^5}{5}+\frac{3\,e\,a^2\,b^2\,x^{10}}{5}+\frac{2\,d\,a^2\,b^2\,x^9}{3}+\frac{3\,c\,a^2\,b^2\,x^8}{4}+\frac{4\,e\,a\,b^3\,x^{13}}{13}+\frac{d\,a\,b^3\,x^{12}}{3}+\frac{4\,c\,a\,b^3\,x^{11}}{11}+\frac{e\,b^4\,x^{16}}{16}+\frac{d\,b^4\,x^{15}}{15}+\frac{c\,b^4\,x^{14}}{14}$$

input
```
int(x*(a + b*x^3)^4*(c + d*x + e*x^2),x)
```

output
```
(a^4*c*x^2)/2 + (a^4*d*x^3)/3 + (b^4*c*x^14)/14 + (a^4*e*x^4)/4 + (b^4*d*x
^15)/15 + (b^4*e*x^16)/16 + (3*a^2*b^2*c*x^8)/4 + (2*a^2*b^2*d*x^9)/3 + (3
*a^2*b^2*e*x^10)/5 + (4*a^3*b*c*x^5)/5 + (4*a*b^3*c*x^11)/11 + (2*a^3*b*d*
x^6)/3 + (a*b^3*d*x^12)/3 + (4*a^3*b*e*x^7)/7 + (4*a*b^3*e*x^13)/13
```

3.333 $\int (c+dx+ex^2) (a+bx^3)^4 \, dx$

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3.333.7 Maxima [A] (verification not implemented) . . . 2489
3.333.8 Giac [A] (verification not implemented) . . . 2489
3.333.9 Mupad [B] (verification not implemented) . . . 2490

3.333.1 Optimal result

Integrand size = 20, antiderivative size = 130

$$\int (c+dx+ex^2) (a+bx^3)^4 \, dx = a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}+\frac{e(a+bx^3)^5}{15b}$$

output
```
a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x^7+3/4*a^
2*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*c*x^13+1/14*b^4*d*
x^14+1/15*e*(b*x^3+a)^5/b
```

3.333.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

$$\int (c+dx+ex^2) (a+bx^3)^4 \, dx = a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{2}{3}a^3bex^6+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{3}a^2b^2ex^9+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{3}ab^3ex^{12}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}+\frac{1}{15}b^4ex^{15}$$

input
```
Integrate[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

output
```
a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5
+ (2*a^3*b*e*x^6)/3 + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a^2*b
^2*e*x^9)/3 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (a*b^3*e*x^12)/3
+ (b^4*c*x^13)/13 + (b^4*d*x^14)/14 + (b^4*e*x^15)/15
```

3.333.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^4 \left(c+dx+ex^2\right) \, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int (c+dx) \left(bx^3+a\right)^4 dx+\frac{e \left(a+bx^3\right)^5}{15 b}$$

$$\Big\downarrow \text{ 2389}$$

$$\int \left(b^4 d x^{13}+b^4 c x^{12}+4 a b^3 d x^{10}+4 a b^3 c x^9+6 a^2 b^2 d x^7+6 a^2 b^2 c x^6+4 a^3 b d x^4+4 a^3 b c x^3+a^4 d x+a^4 c\right) dx+\frac{e \left(a+bx^3\right)^5}{15 b}$$

$$\Big\downarrow \text{ 2009}$$

$$a^4 c x+\frac{1}{2} a^4 d x^2+a^3 b c x^4+\frac{4}{5} a^3 b d x^5+\frac{6}{7} a^2 b^2 c x^7+\frac{3}{4} a^2 b^2 d x^8+\frac{2}{5} a b^3 c x^{10}+\frac{4}{11} a b^3 d x^{11}+\frac{e \left(a+bx^3\right)^5}{15 b}+\frac{1}{13} b^4 c x^{13}+\frac{1}{14} b^4 d x^{14}$$

input
```
Int[(c + d*x + e*x^2)*(a + b*x^3)^4,x]
```

output
```
a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x
^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (
b^4*c*x^13)/13 + (b^4*d*x^14)/14 + (e*(a + b*x^3)^5)/(15*b)
```

3.333.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.333.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.14

method	result
gosper	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{3}a^2e\,b$
default	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{3}a^2e\,b$
norman	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{3}a^2e\,b$
risch	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{3}a^2e\,b$
parallelrisch	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}a^4e\,x^3+a^3bc\,x^4+\frac{4}{5}d\,x^5b\,a^3+\frac{2}{3}a^3be\,x^6+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}x^8b^2d\,a^2+\frac{2}{3}a^2e\,b$

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*c*x+1/2*a^4*d*x^2+1/3*a^4*e*x^3+a^3*b*c*x^4+4/5*d*x^5*b*a^3+2/3*a^3*b*
e*x^6+6/7*a^2*b^2*c*x^7+3/4*x^8*b^2*d*a^2+2/3*a^2*e*b^2*x^9+2/5*a*b^3*c*x^
10+4/11*x^11*d*b^3*a+1/3*a*b^3*e*x^12+1/13*b^4*c*x^13+1/14*b^4*d*x^14+1/15
*e*b^4*x^15
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.13

$$\int (c+dx+ex^2)\left(a+bx^3\right)^4\,dx = \frac{1}{15}\,b^4ex^{15}+\frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{1}{3}\,ab^3ex^{12}+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{2}{3}\,a^2b^2ex^9+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{2}{3}\,a^3bex^6+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{3}\,a^4ex^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="fricas")
```

output
```
1/15*b^4*e*x^15 + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*e*x^12 + 4
/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*e*x^9 + 3/4*a^2*b^2*d*x^
8 + 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*e*x^6 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 +
1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

3.333.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.37

$$\int (c+dx+ex^2)\left(a+bx^3\right)^4\,dx = a^4cx+\frac{a^4dx^2}{2}+\frac{a^4ex^3}{3}+a^3bcx^4+\frac{4a^3bdx^5}{5}+\frac{2a^3bex^6}{3}+\frac{6a^2b^2cx^7}{7}+\frac{3a^2b^2dx^8}{4}+\frac{2a^2b^2ex^9}{3}+\frac{2ab^3cx^{10}}{5}+\frac{4ab^3dx^{11}}{11}+\frac{ab^3ex^{12}}{3}+\frac{b^4cx^{13}}{13}+\frac{b^4dx^{14}}{14}+\frac{b^4ex^{15}}{15}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**4,x)
```

output
```
a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**3*b*c*x**4 + 4*a**3*b*d*x**5
/5 + 2*a**3*b*e*x**6/3 + 6*a**2*b**2*c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a
**2*b**2*e*x**9/3 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/11 + a*b**3*e*x*
*12/3 + b**4*c*x**13/13 + b**4*d*x**14/14 + b**4*e*x**15/15
```

3.333.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.13

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx = \frac{1}{15}\,b^4ex^{15}+\frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{1}{3}\,ab^3ex^{12}+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{2}{3}\,a^2b^2ex^9+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{2}{3}\,a^3bex^6+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{3}\,a^4ex^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="maxima")
```

output
```
1/15*b^4*e*x^15 + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*e*x^12 + 4
/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*e*x^9 + 3/4*a^2*b^2*d*x^
8 + 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*e*x^6 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 +
1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

3.333.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.13

$$\int \left(c+dx+ex^2\right)\left(a+bx^3\right)^4\,dx = \frac{1}{15}\,b^4ex^{15}+\frac{1}{14}\,b^4dx^{14}+\frac{1}{13}\,b^4cx^{13}+\frac{1}{3}\,ab^3ex^{12}+\frac{4}{11}\,ab^3dx^{11}+\frac{2}{5}\,ab^3cx^{10}+\frac{2}{3}\,a^2b^2ex^9+\frac{3}{4}\,a^2b^2dx^8+\frac{6}{7}\,a^2b^2cx^7+\frac{2}{3}\,a^3bex^6+\frac{4}{5}\,a^3bdx^5+a^3bcx^4+\frac{1}{3}\,a^4ex^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^4,x, algorithm="giac")
```

output
```
1/15*b^4*e*x^15 + 1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 1/3*a*b^3*e*x^12 + 4
/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 + 2/3*a^2*b^2*e*x^9 + 3/4*a^2*b^2*d*x^
8 + 6/7*a^2*b^2*c*x^7 + 2/3*a^3*b*e*x^6 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 +
1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

3.333.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.13

$$\int (c+dx+ex^2)\left(a+bx^3\right)^4\,dx=\frac{e\,a^4\,x^3}{3}+\frac{d\,a^4\,x^2}{2}+c\,a^4\,x+\frac{2\,e\,a^3\,b\,x^6}{3}+\frac{4\,d\,a^3\,b\,x^5}{5}+c\,a^3\,b\,x^4+\frac{2\,e\,a^2\,b^2\,x^9}{3}+\frac{3\,d\,a^2\,b^2\,x^8}{4}+\frac{6\,c\,a^2\,b^2\,x^7}{7}+\frac{e\,a\,b^3\,x^{12}}{3}+\frac{4\,d\,a\,b^3\,x^{11}}{11}+\frac{2\,c\,a\,b^3\,x^{10}}{5}+\frac{e\,b^4\,x^{15}}{15}+\frac{d\,b^4\,x^{14}}{14}+\frac{c\,b^4\,x^{13}}{13}$$

input
```
int((a + b*x^3)^4*(c + d*x + e*x^2),x)
```

output
```
(a^4*d*x^2)/2 + (b^4*c*x^13)/13 + (a^4*e*x^3)/3 + (b^4*d*x^14)/14 + (b^4*e
*x^15)/15 + a^4*c*x + (6*a^2*b^2*c*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a^2*b
^2*e*x^9)/3 + a^3*b*c*x^4 + (2*a*b^3*c*x^10)/5 + (4*a^3*b*d*x^5)/5 + (4*a*
b^3*d*x^11)/11 + (2*a^3*b*e*x^6)/3 + (a*b^3*e*x^12)/3
```

3.334 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x} dx$

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3.334.1 Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x} dx = a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \frac{4}{9}ab^3cx^9 + \frac{2}{5}ab^3dx^{10} + \frac{4}{11}ab^3ex^{11} + \frac{1}{12}b^4cx^{12} + \frac{1}{13}b^4dx^{13} + \frac{1}{14}b^4ex^{14} + a^4c\log(x)$$

output
```
a^4*d*x+1/2*a^4*e*x^2+4/3*a^3*b*c*x^3+a^3*b*d*x^4+4/5*a^3*b*e*x^5+a^2*b^2*
c*x^6+6/7*a^2*b^2*d*x^7+3/4*a^2*b^2*e*x^8+4/9*a*b^3*c*x^9+2/5*a*b^3*d*x^10
+4/11*a*b^3*e*x^11+1/12*b^4*c*x^12+1/13*b^4*d*x^13+1/14*b^4*e*x^14+a^4*c*l
n(x)
```

3.334.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x} dx = a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \frac{4}{9}ab^3cx^9 + \frac{2}{5}ab^3dx^{10} + \frac{4}{11}ab^3ex^{11} + \frac{1}{12}b^4cx^{12} + \frac{1}{13}b^4dx^{13} + \frac{1}{14}b^4ex^{14} + a^4c\log(x)$$

input
```
Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]
```

output
```
a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5
)/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3
*c*x^9)/9 + (2*a*b^3*d*x^10)/5 + (4*a*b^3*e*x^11)/11 + (b^4*c*x^12)/12 + (
b^4*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c*Log[x]
```

3.334.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^4 (c+dx+ex^2)}{x} \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{a^4c}{x} + a^4d + a^4ex + 4a^3bcx^2 + 4a^3bdx^3 + 4a^3bex^4 + 6a^2b^2cx^5 + 6a^2b^2dx^6 + 6a^2b^2ex^7 + 4ab^3cx^8 + 4ab^3dx^9 +\right.$$

$$\Big\downarrow \text{2009}$$

$$a^4c\log(x) + a^4dx + \frac{1}{2}a^4ex^2 + \frac{4}{3}a^3bcx^3 + a^3bdx^4 + \frac{4}{5}a^3bex^5 + a^2b^2cx^6 + \frac{6}{7}a^2b^2dx^7 + \frac{3}{4}a^2b^2ex^8 + \frac{4}{9}ab^3cx^9 + \frac{2}{5}ab^3dx^{10} + \frac{4}{11}ab^3ex^{11} + \frac{1}{12}b^4cx^{12} + \frac{1}{13}b^4dx^{13} + \frac{1}{14}b^4ex^{14}$$

input
```
Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x,x]
```

output
```
a^4*d*x + (a^4*e*x^2)/2 + (4*a^3*b*c*x^3)/3 + a^3*b*d*x^4 + (4*a^3*b*e*x^5
)/5 + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^8)/4 + (4*a*b^3
*c*x^9)/9 + (2*a*b^3*d*x^10)/5 + (4*a*b^3*e*x^11)/11 + (b^4*c*x^12)/12 + (
b^4*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*c*Log[x]
```

3.334.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.334.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.87

method	result
default	$a^4dx+\frac{a^4e\,x^2}{2}+\frac{4a^3bc\,x^3}{3}+a^3bd\,x^4+\frac{4a^3be\,x^5}{5}+a^2b^2c\,x^6+\frac{6a^2b^2d\,x^7}{7}+\frac{3a^2b^2e\,x^8}{4}+\frac{4a\,b^3c\,x^9}{9}+\frac{2a\,b^3d}{5}$
norman	$a^4dx+\frac{a^4e\,x^2}{2}+\frac{4a^3bc\,x^3}{3}+a^3bd\,x^4+\frac{4a^3be\,x^5}{5}+a^2b^2c\,x^6+\frac{6a^2b^2d\,x^7}{7}+\frac{3a^2b^2e\,x^8}{4}+\frac{4a\,b^3c\,x^9}{9}+\frac{2a\,b^3d}{5}$
risch	$a^4dx+\frac{a^4e\,x^2}{2}+\frac{4a^3bc\,x^3}{3}+a^3bd\,x^4+\frac{4a^3be\,x^5}{5}+a^2b^2c\,x^6+\frac{6a^2b^2d\,x^7}{7}+\frac{3a^2b^2e\,x^8}{4}+\frac{4a\,b^3c\,x^9}{9}+\frac{2a\,b^3d}{5}$
parallelrisch	$a^4dx+\frac{a^4e\,x^2}{2}+\frac{4a^3bc\,x^3}{3}+a^3bd\,x^4+\frac{4a^3be\,x^5}{5}+a^2b^2c\,x^6+\frac{6a^2b^2d\,x^7}{7}+\frac{3a^2b^2e\,x^8}{4}+\frac{4a\,b^3c\,x^9}{9}+\frac{2a\,b^3d}{5}$

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^4/x,x,method=_RETURNVERBOSE)
```

output
```
a^4*d*x+1/2*a^4*e*x^2+4/3*a^3*b*c*x^3+a^3*b*d*x^4+4/5*a^3*b*e*x^5+a^2*b^2*
c*x^6+6/7*a^2*b^2*d*x^7+3/4*a^2*b^2*e*x^8+4/9*a*b^3*c*x^9+2/5*a*b^3*d*x^10
+4/11*a*b^3*e*x^11+1/12*b^4*c*x^12+1/13*b^4*d*x^13+1/14*b^4*e*x^14+a^4*c*l
n(x)
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx=\frac{1}{14}\,b^4ex^{14}+\frac{1}{13}\,b^4dx^{13}+\frac{1}{12}\,b^4cx^{12}+\frac{4}{11}\,ab^3ex^{11}+\frac{2}{5}\,ab^3dx^{10}+\frac{4}{9}\,ab^3cx^9+\frac{3}{4}\,a^2b^2ex^8+\frac{6}{7}\,a^2b^2dx^7+a^2b^2cx^6+\frac{4}{5}\,a^3bex^5+a^3bdx^4+\frac{4}{3}\,a^3bcx^3+\frac{1}{2}\,a^4ex^2+a^4dx+a^4c\log(x)$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="fricas")
```

output
```
1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 +
2/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7
 + a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a
^4*e*x^2 + a^4*d*x + a^4*c*log(x)
```

3.334.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx = a^4c\log{(x)} + a^4dx + \frac{a^4ex^2}{2} + \frac{4a^3bcx^3}{3} + a^3bdx^4 + \frac{4a^3bex^5}{5} + a^2b^2cx^6 + \frac{6a^2b^2dx^7}{7} + \frac{3a^2b^2ex^8}{4} + \frac{4ab^3cx^9}{9} + \frac{2ab^3dx^{10}}{5} + \frac{4ab^3ex^{11}}{11} + \frac{b^4cx^{12}}{12} + \frac{b^4dx^{13}}{13} + \frac{b^4ex^{14}}{14}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x,x)
```

output
```
a**4*c*log(x) + a**4*d*x + a**4*e*x**2/2 + 4*a**3*b*c*x**3/3 + a**3*b*d*x*
*4 + 4*a**3*b*e*x**5/5 + a**2*b**2*c*x**6 + 6*a**2*b**2*d*x**7/7 + 3*a**2*
b**2*e*x**8/4 + 4*a*b**3*c*x**9/9 + 2*a*b**3*d*x**10/5 + 4*a*b**3*e*x**11/
11 + b**4*c*x**12/12 + b**4*d*x**13/13 + b**4*e*x**14/14
```

3.334.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx = \frac{1}{14}\,b^4ex^{14} + \frac{1}{13}\,b^4dx^{13} + \frac{1}{12}\,b^4cx^{12} + \frac{4}{11}\,ab^3ex^{11} + \frac{2}{5}\,ab^3dx^{10} + \frac{4}{9}\,ab^3cx^9 + \frac{3}{4}\,a^2b^2ex^8 + \frac{6}{7}\,a^2b^2dx^7 + a^2b^2cx^6 + \frac{4}{5}\,a^3bex^5 + a^3bdx^4 + \frac{4}{3}\,a^3bcx^3 + \frac{1}{2}\,a^4ex^2 + a^4dx + a^4c\log{(x)}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="maxima")
```

output
```
1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 +
2/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7
 + a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a
^4*e*x^2 + a^4*d*x + a^4*c*log(x)
```

3.334.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx = \frac{1}{14}b^4ex^{14} + \frac{1}{13}b^4dx^{13} + \frac{1}{12}b^4cx^{12} + \frac{4}{11}ab^3ex^{11} + \frac{2}{5}ab^3dx^{10} + \frac{4}{9}ab^3cx^9 + \frac{3}{4}a^2b^2ex^8 + \frac{6}{7}a^2b^2dx^7 + a^2b^2cx^6 + \frac{4}{5}a^3bex^5 + a^3bdx^4 + \frac{4}{3}a^3bcx^3 + \frac{1}{2}a^4ex^2 + a^4dx + a^4c\log(|x|)$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x,x, algorithm="giac")
```

output
```
1/14*b^4*e*x^14 + 1/13*b^4*d*x^13 + 1/12*b^4*c*x^12 + 4/11*a*b^3*e*x^11 +
2/5*a*b^3*d*x^10 + 4/9*a*b^3*c*x^9 + 3/4*a^2*b^2*e*x^8 + 6/7*a^2*b^2*d*x^7
 + a^2*b^2*c*x^6 + 4/5*a^3*b*e*x^5 + a^3*b*d*x^4 + 4/3*a^3*b*c*x^3 + 1/2*a
^4*e*x^2 + a^4*d*x + a^4*c*log(abs(x))
```

3.334.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x}\,dx = \frac{b^4\,c\,x^{12}}{12} + \frac{a^4\,e\,x^2}{2} + \frac{b^4\,d\,x^{13}}{13} + \frac{b^4\,e\,x^{14}}{14} + a^4\,c\,\ln(x) + a^4\,d\,x + a^2\,b^2\,c\,x^6 + \frac{6\,a^2\,b^2\,d\,x^7}{7} + \frac{3\,a^2\,b^2\,e\,x^8}{4} + \frac{4\,a^3\,b\,c\,x^3}{3} + \frac{4\,a\,b^3\,c\,x^9}{9} + a^3\,b\,d\,x^4 + \frac{2\,a\,b^3\,d\,x^{10}}{5} + \frac{4\,a^3\,b\,e\,x^5}{5} + \frac{4\,a\,b^3\,e\,x^{11}}{11}$$

input
```
int(((a + b*x^3)^4*(c + d*x + e*x^2))/x,x)
```

output
```
(b^4*c*x^12)/12 + (a^4*e*x^2)/2 + (b^4*d*x^13)/13 + (b^4*e*x^14)/14 + a^4*
c*log(x) + a^4*d*x + a^2*b^2*c*x^6 + (6*a^2*b^2*d*x^7)/7 + (3*a^2*b^2*e*x^
8)/4 + (4*a^3*b*c*x^3)/3 + (4*a*b^3*c*x^9)/9 + a^3*b*d*x^4 + (2*a*b^3*d*x^
10)/5 + (4*a^3*b*e*x^5)/5 + (4*a*b^3*e*x^11)/11
```

3.335 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2} dx$

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3.335.1 Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2} dx = -\frac{a^4c}{x} + a^4ex + 2a^3bcx^2 + \frac{4}{3}a^3bdx^3 + a^3bex^4 + \frac{6}{5}a^2b^2cx^5 + a^2b^2dx^6 + \frac{6}{7}a^2b^2ex^7 + \frac{1}{2}ab^3cx^8 + \frac{4}{9}ab^3dx^9 + \frac{2}{5}ab^3ex^{10} + \frac{1}{11}b^4cx^{11} + \frac{1}{12}b^4dx^{12} + \frac{1}{13}b^4ex^{13} + a^4d\log(x)$$

output
```
-a^4*c/x+a^4*e*x+2*a^3*b*c*x^2+4/3*a^3*b*d*x^3+a^3*b*e*x^4+6/5*a^2*b^2*c*x
^5+a^2*b^2*d*x^6+6/7*a^2*b^2*e*x^7+1/2*a*b^3*c*x^8+4/9*a*b^3*d*x^9+2/5*a*b
^3*e*x^10+1/11*b^4*c*x^11+1/12*b^4*d*x^12+1/13*b^4*e*x^13+a^4*d*ln(x)
```

3.335.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2} dx = -\frac{a^4c}{x} + a^4ex + 2a^3bcx^2 + \frac{4}{3}a^3bdx^3 + a^3bex^4 + \frac{6}{5}a^2b^2cx^5 + a^2b^2dx^6 + \frac{6}{7}a^2b^2ex^7 + \frac{1}{2}ab^3cx^8 + \frac{4}{9}ab^3dx^9 + \frac{2}{5}ab^3ex^{10} + \frac{1}{11}b^4cx^{11} + \frac{1}{12}b^4dx^{12} + \frac{1}{13}b^4ex^{13} + a^4d\log(x)$$

input
```
Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2,x]
```

output
```
-((a^4*c)/x) + a^4*e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 +
 (6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)
/2 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^1
2)/12 + (b^4*e*x^13)/13 + a^4*d*Log[x]
```

3.335.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^4 (c+dx+ex^2)}{x^2} dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(\frac{a^4c}{x^2}+\frac{a^4d}{x}+a^4e+4a^3bcx+4a^3bdx^2+4a^3bex^3+6a^2b^2cx^4+6a^2b^2dx^5+6a^2b^2ex^6+4ab^3cx^7+4ab^3dx^8+4$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4c}{x}+a^4d\log(x)+a^4ex+2a^3bcx^2+\frac{4}{3}a^3bdx^3+a^3bex^4+\frac{6}{5}a^2b^2cx^5+a^2b^2dx^6+\frac{6}{7}a^2b^2ex^7+\frac{1}{2}ab^3cx^8+\frac{4}{9}ab^3dx^9+\frac{2}{5}ab^3ex^{10}+\frac{1}{11}b^4cx^{11}+\frac{1}{12}b^4dx^{12}+\frac{1}{13}b^4ex^{13}$$

input
```
Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^2,x]
```

output
```
-((a^4*c)/x) + a^4*e*x + 2*a^3*b*c*x^2 + (4*a^3*b*d*x^3)/3 + a^3*b*e*x^4 +
 (6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7 + (a*b^3*c*x^8)
/2 + (4*a*b^3*d*x^9)/9 + (2*a*b^3*e*x^10)/5 + (b^4*c*x^11)/11 + (b^4*d*x^1
2)/12 + (b^4*e*x^13)/13 + a^4*d*Log[x]
```

3.335.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.335.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^4c}{x}+a^4ex+2a^3bc\,x^2+\frac{4a^3bd\,x^3}{3}+a^3be\,x^4+\frac{6a^2b^2c\,x^5}{5}+a^2b^2d\,x^6+\frac{6a^2b^2e\,x^7}{7}+\frac{a\,b^3c\,x^8}{2}+\frac{4a\,b^3d}{9}$
risch	$-\frac{a^4c}{x}+a^4ex+2a^3bc\,x^2+\frac{4a^3bd\,x^3}{3}+a^3be\,x^4+\frac{6a^2b^2c\,x^5}{5}+a^2b^2d\,x^6+\frac{6a^2b^2e\,x^7}{7}+\frac{a\,b^3c\,x^8}{2}+\frac{4a\,b^3d}{9}$
norman	$\frac{a^4e\,x^2+a^2b^2d\,x^7+a^3be\,x^5-a^4c+\frac{1}{11}b^4c\,x^{12}+\frac{1}{12}b^4d\,x^{13}+\frac{1}{13}b^4e\,x^{14}+\frac{1}{2}a\,b^3c\,x^9+\frac{4}{9}a\,b^3d\,x^{10}+\frac{2}{5}a\,b^3e\,x^{11}+\frac{6}{5}a^2b^2c\,x^6+\frac{6}{7}a^2b^2e\,x^8}{x}$
parallelrisch	$\frac{13860b^4e\,x^{14}+15015b^4d\,x^{13}+16380b^4c\,x^{12}+72072a\,b^3e\,x^{11}+80080a\,b^3d\,x^{10}+90090a\,b^3c\,x^9+154440a^2b^2e\,x^8+180180a^2b^2d\,x^7}{180180x}$

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^4*c/x+a^4*e*x+2*a^3*b*c*x^2+4/3*a^3*b*d*x^3+a^3*b*e*x^4+6/5*a^2*b^2*c*x
^5+a^2*b^2*d*x^6+6/7*a^2*b^2*e*x^7+1/2*a*b^3*c*x^8+4/9*a*b^3*d*x^9+2/5*a*b
^3*e*x^10+1/11*b^4*c*x^11+1/12*b^4*d*x^12+1/13*b^4*e*x^13+a^4*d*ln(x)
```

3.335.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx+ex^2)\,(a+bx^3)^4}{x^2}\,dx$$
$$= \frac{13860\,b^4ex^{14}+15015\,b^4dx^{13}+16380\,b^4cx^{12}+72072\,ab^3ex^{11}+80080\,ab^3dx^{10}+90090\,ab^3cx^9+154440\,a^2}{}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="fricas")
```

output
```
1/180180*(13860*b^4*e*x^14 + 15015*b^4*d*x^13 + 16380*b^4*c*x^12 + 72072*a
*b^3*e*x^11 + 80080*a*b^3*d*x^10 + 90090*a*b^3*c*x^9 + 154440*a^2*b^2*e*x^
8 + 180180*a^2*b^2*d*x^7 + 216216*a^2*b^2*c*x^6 + 180180*a^3*b*e*x^5 + 240
240*a^3*b*d*x^4 + 360360*a^3*b*c*x^3 + 180180*a^4*e*x^2 + 180180*a^4*d*x*l
og(x) - 180180*a^4*c)/x
```

3.335.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx = -\frac{a^4c}{x} + a^4d\log(x) + a^4ex + 2a^3bcx^2 + \frac{4a^3bdx^3}{3} + a^3bex^4 + \frac{6a^2b^2cx^5}{5} + a^2b^2dx^6 + \frac{6a^2b^2ex^7}{7} + \frac{ab^3cx^8}{2} + \frac{4ab^3dx^9}{9} + \frac{2ab^3ex^{10}}{5} + \frac{b^4cx^{11}}{11} + \frac{b^4dx^{12}}{12} + \frac{b^4ex^{13}}{13}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x**2,x)
```

output
```
-a**4*c/x + a**4*d*log(x) + a**4*e*x + 2*a**3*b*c*x**2 + 4*a**3*b*d*x**3/3
 + a**3*b*e*x**4 + 6*a**2*b**2*c*x**5/5 + a**2*b**2*d*x**6 + 6*a**2*b**2*e
*x**7/7 + a*b**3*c*x**8/2 + 4*a*b**3*d*x**9/9 + 2*a*b**3*e*x**10/5 + b**4*
c*x**11/11 + b**4*d*x**12/12 + b**4*e*x**13/13
```

3.335.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx = \frac{1}{13}\,b^4ex^{13} + \frac{1}{12}\,b^4dx^{12} + \frac{1}{11}\,b^4cx^{11} + \frac{2}{5}\,ab^3ex^{10} + \frac{4}{9}\,ab^3dx^9 + \frac{1}{2}\,ab^3cx^8 + \frac{6}{7}\,a^2b^2ex^7 + a^2b^2dx^6 + \frac{6}{5}\,a^2b^2cx^5 + a^3bex^4 + \frac{4}{3}\,a^3bdx^3 + 2\,a^3bcx^2 + a^4ex + a^4d\log(x) - \frac{a^4c}{x}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="maxima")
```

output
```
1/13*b^4*e*x^13 + 1/12*b^4*d*x^12 + 1/11*b^4*c*x^11 + 2/5*a*b^3*e*x^10 + 4
/9*a*b^3*d*x^9 + 1/2*a*b^3*c*x^8 + 6/7*a^2*b^2*e*x^7 + a^2*b^2*d*x^6 + 6/5
*a^2*b^2*c*x^5 + a^3*b*e*x^4 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + a^4*e*x +
 a^4*d*log(x) - a^4*c/x
```

3.335.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx = \frac{1}{13}\,b^4ex^{13} + \frac{1}{12}\,b^4dx^{12} + \frac{1}{11}\,b^4cx^{11} + \frac{2}{5}\,ab^3ex^{10} + \frac{4}{9}\,ab^3dx^9 + \frac{1}{2}\,ab^3cx^8 + \frac{6}{7}\,a^2b^2ex^7 + a^2b^2dx^6 + \frac{6}{5}\,a^2b^2cx^5 + a^3bex^4 + \frac{4}{3}\,a^3bdx^3 + 2\,a^3bcx^2 + a^4ex + a^4d\log(|x|) - \frac{a^4c}{x}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^2,x, algorithm="giac")
```

output
```
1/13*b^4*e*x^13 + 1/12*b^4*d*x^12 + 1/11*b^4*c*x^11 + 2/5*a*b^3*e*x^10 + 4
/9*a*b^3*d*x^9 + 1/2*a*b^3*c*x^8 + 6/7*a^2*b^2*e*x^7 + a^2*b^2*d*x^6 + 6/5
*a^2*b^2*c*x^5 + a^3*b*e*x^4 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + a^4*e*x +
 a^4*d*log(abs(x)) - a^4*c/x
```

3.335.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx = \frac{b^4\,c\,x^{11}}{11} - \frac{a^4\,c}{x} + \frac{b^4\,d\,x^{12}}{12} + \frac{b^4\,e\,x^{13}}{13} + a^4\,d\,\ln(x) + a^4\,e\,x + \frac{6\,a^2\,b^2\,c\,x^5}{5} + a^2\,b^2\,d\,x^6 + \frac{6\,a^2\,b^2\,e\,x^7}{7} + 2\,a^3\,b\,c\,x^2 + \frac{a\,b^3\,c\,x^8}{2} + \frac{4\,a^3\,b\,d\,x^3}{3} + \frac{4\,a\,b^3\,d\,x^9}{9} + a^3\,b\,e\,x^4 + \frac{2\,a\,b^3\,e\,x^{10}}{5}$$

input
```
int(((a + b*x^3)^4*(c + d*x + e*x^2))/x^2,x)
```

output
```
(b^4*c*x^11)/11 - (a^4*c)/x + (b^4*d*x^12)/12 + (b^4*e*x^13)/13 + a^4*d*lo
g(x) + a^4*e*x + (6*a^2*b^2*c*x^5)/5 + a^2*b^2*d*x^6 + (6*a^2*b^2*e*x^7)/7
 + 2*a^3*b*c*x^2 + (a*b^3*c*x^8)/2 + (4*a^3*b*d*x^3)/3 + (4*a*b^3*d*x^9)/9
 + a^3*b*e*x^4 + (2*a*b^3*e*x^10)/5
```

3.335. $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^2}\,dx$

3.336 $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx$

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3.336.1 Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = -\frac{a^4c}{2x^2}-\frac{a^4d}{x}+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3ex^9+\frac{1}{10}b^4cx^{10}+\frac{1}{11}b^4dx^{11}+\frac{1}{12}b^4ex^{12}+a^4e\log(x)$$

output
```
-1/2*a^4*c/x^2-a^4*d/x+4*a^3*b*c*x+2*a^3*b*d*x^2+4/3*a^3*b*e*x^3+3/2*a^2*b
^2*c*x^4+6/5*a^2*b^2*d*x^5+a^2*b^2*e*x^6+4/7*a*b^3*c*x^7+1/2*a*b^3*d*x^8+4
/9*a*b^3*e*x^9+1/10*b^4*c*x^10+1/11*b^4*d*x^11+1/12*b^4*e*x^12+a^4*e*ln(x)
```

3.336.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = -\frac{a^4c}{2x^2}-\frac{a^4d}{x}+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3ex^9+\frac{1}{10}b^4cx^{10}+\frac{1}{11}b^4dx^{11}+\frac{1}{12}b^4ex^{12}+a^4e\log(x)$$

input
```
Integrate[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3,x]
```

output
```
-1/2*(a^4*c)/x^2 - (a^4*d)/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + (4*a^3*b*e*x^
3)/3 + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e*x^6 + (4*a*b^
3*c*x^7)/7 + (a*b^3*d*x^8)/2 + (4*a*b^3*e*x^9)/9 + (b^4*c*x^10)/10 + (b^4*
d*x^11)/11 + (b^4*e*x^12)/12 + a^4*e*Log[x]
```

3.336.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^4\,(c+dx+ex^2)}{x^3}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{a^4c}{x^3}+\frac{a^4d}{x^2}+\frac{a^4e}{x}+4a^3bc+4a^3bdx+4a^3bex^2+6a^2b^2cx^3+6a^2b^2dx^4+6a^2b^2ex^5+4ab^3cx^6+4ab^3dx^7+4a$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+a^4e\log(x)+4a^3bcx+2a^3bdx^2+\frac{4}{3}a^3bex^3+\frac{3}{2}a^2b^2cx^4+\frac{6}{5}a^2b^2dx^5+a^2b^2ex^6+\frac{4}{7}ab^3cx^7+\frac{1}{2}ab^3dx^8+\frac{4}{9}ab^3ex^9+\frac{1}{10}b^4cx^{10}+\frac{1}{11}b^4dx^{11}+\frac{1}{12}b^4ex^{12}$$

input
```
Int[((c + d*x + e*x^2)*(a + b*x^3)^4)/x^3,x]
```

output
```
-1/2*(a^4*c)/x^2 - (a^4*d)/x + 4*a^3*b*c*x + 2*a^3*b*d*x^2 + (4*a^3*b*e*x^
3)/3 + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2*e*x^6 + (4*a*b^
3*c*x^7)/7 + (a*b^3*d*x^8)/2 + (4*a*b^3*e*x^9)/9 + (b^4*c*x^10)/10 + (b^4*
d*x^11)/11 + (b^4*e*x^12)/12 + a^4*e*Log[x]
```

3.336.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.336.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^4c}{2x^2}-\frac{a^4d}{x}+4a^3bcx+2a^3bd\,x^2+\frac{4a^3be\,x^3}{3}+\frac{3a^2b^2c\,x^4}{2}+\frac{6a^2b^2d\,x^5}{5}+a^2b^2e\,x^6+\frac{4a\,b^3c\,x^7}{7}+\frac{a\,b^3d\,x^{8}}{2}$
risch	$\frac{b^4e\,x^{12}}{12}+\frac{b^4d\,x^{11}}{11}+\frac{b^4c\,x^{10}}{10}+\frac{4a\,b^3e\,x^9}{9}+\frac{a\,b^3d\,x^8}{2}+\frac{4a\,b^3c\,x^7}{7}+a^2b^2e\,x^6+\frac{6a^2b^2d\,x^5}{5}+\frac{3a^2b^2c\,x^4}{2}+\frac{4a^3be}{3}$
norman	$\frac{a^2b^2e\,x^8-\frac{1}{2}a^4c-a^4dx+\frac{1}{10}b^4c\,x^{12}+\frac{1}{11}b^4d\,x^{13}+\frac{1}{12}b^4e\,x^{14}+\frac{4}{7}a\,b^3c\,x^9+\frac{1}{2}a\,b^3d\,x^{10}+\frac{4}{9}a\,b^3e\,x^{11}+\frac{3}{2}a^2b^2c\,x^6+\frac{6}{5}a^2b^2d\,x^7+4a^3bc}{x^2}$
parallelrisch	$\frac{1155b^4e\,x^{14}+1260b^4d\,x^{13}+1386b^4c\,x^{12}+6160a\,b^3e\,x^{11}+6930a\,b^3d\,x^{10}+7920a\,b^3c\,x^9+13860a^2b^2e\,x^8+16632a^2b^2d\,x^7+20790a}{13860x^2}$

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^4*c/x^2-a^4*d/x+4*a^3*b*c*x+2*a^3*b*d*x^2+4/3*a^3*b*e*x^3+3/2*a^2*b
^2*c*x^4+6/5*a^2*b^2*d*x^5+a^2*b^2*e*x^6+4/7*a*b^3*c*x^7+1/2*a*b^3*d*x^8+4
/9*a*b^3*e*x^9+1/10*b^4*c*x^10+1/11*b^4*d*x^11+1/12*b^4*e*x^12+a^4*e*ln(x)
```

3.336.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = \frac{1155\,b^4ex^{14}+1260\,b^4dx^{13}+1386\,b^4cx^{12}+6160\,ab^3ex^{11}+6930\,ab^3dx^{10}+7920\,ab^3cx^9+13860\,a^2b^2ex^8+}{}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="fricas")
```

output
```
1/13860*(1155*b^4*e*x^14 + 1260*b^4*d*x^13 + 1386*b^4*c*x^12 + 6160*a*b^3*
e*x^11 + 6930*a*b^3*d*x^10 + 7920*a*b^3*c*x^9 + 13860*a^2*b^2*e*x^8 + 1663
2*a^2*b^2*d*x^7 + 20790*a^2*b^2*c*x^6 + 18480*a^3*b*e*x^5 + 27720*a^3*b*d*
x^4 + 55440*a^3*b*c*x^3 + 13860*a^4*e*x^2*log(x) - 13860*a^4*d*x - 6930*a^
4*c)/x^2
```

3.336.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = a^4e\log{(x)} + 4a^3bcx + 2a^3bdx^2 + \frac{4a^3bex^3}{3} + \frac{3a^2b^2cx^4}{2} + \frac{6a^2b^2dx^5}{5} + a^2b^2ex^6 + \frac{4ab^3cx^7}{7} + \frac{ab^3dx^8}{2} + \frac{4ab^3ex^9}{9} + \frac{b^4cx^{10}}{10} + \frac{b^4dx^{11}}{11} + \frac{b^4ex^{12}}{12} + \frac{-a^4c-2a^4dx}{2x^2}$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**4/x**3,x)
```

output
```
a**4*e*log(x) + 4*a**3*b*c*x + 2*a**3*b*d*x**2 + 4*a**3*b*e*x**3/3 + 3*a**
2*b**2*c*x**4/2 + 6*a**2*b**2*d*x**5/5 + a**2*b**2*e*x**6 + 4*a*b**3*c*x**
7/7 + a*b**3*d*x**8/2 + 4*a*b**3*e*x**9/9 + b**4*c*x**10/10 + b**4*d*x**11
/11 + b**4*e*x**12/12 + (-a**4*c - 2*a**4*d*x)/(2*x**2)
```

3.336.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = \frac{1}{12}\,b^4ex^{12} + \frac{1}{11}\,b^4dx^{11} + \frac{1}{10}\,b^4cx^{10} + \frac{4}{9}\,ab^3ex^9 + \frac{1}{2}\,ab^3dx^8 + \frac{4}{7}\,ab^3cx^7 + a^2b^2ex^6 + \frac{6}{5}\,a^2b^2dx^5 + \frac{3}{2}\,a^2b^2cx^4 + \frac{4}{3}\,a^3bex^3 + 2\,a^3bdx^2 + 4\,a^3bcx + a^4e\log{(x)} - \frac{2\,a^4dx + a^4c}{2\,x^2}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="maxima")
```

output
```
1/12*b^4*e*x^12 + 1/11*b^4*d*x^11 + 1/10*b^4*c*x^10 + 4/9*a*b^3*e*x^9 + 1/
2*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b^2*e*x^6 + 6/5*a^2*b^2*d*x^5 + 3/2*
a^2*b^2*c*x^4 + 4/3*a^3*b*e*x^3 + 2*a^3*b*d*x^2 + 4*a^3*b*c*x + a^4*e*log(
x) - 1/2*(2*a^4*d*x + a^4*c)/x^2
```

3.336.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = \frac{1}{12}\,b^4ex^{12} + \frac{1}{11}\,b^4dx^{11} + \frac{1}{10}\,b^4cx^{10} + \frac{4}{9}\,ab^3ex^9 + \frac{1}{2}\,ab^3dx^8 + \frac{4}{7}\,ab^3cx^7 + a^2b^2ex^6 + \frac{6}{5}\,a^2b^2dx^5 + \frac{3}{2}\,a^2b^2cx^4 + \frac{4}{3}\,a^3bex^3 + 2\,a^3bdx^2 + 4\,a^3bcx + a^4e\log(|x|) - \frac{2\,a^4dx + a^4c}{2\,x^2}$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^4/x^3,x, algorithm="giac")
```

output
```
1/12*b^4*e*x^12 + 1/11*b^4*d*x^11 + 1/10*b^4*c*x^10 + 4/9*a*b^3*e*x^9 + 1/
2*a*b^3*d*x^8 + 4/7*a*b^3*c*x^7 + a^2*b^2*e*x^6 + 6/5*a^2*b^2*d*x^5 + 3/2*
a^2*b^2*c*x^4 + 4/3*a^3*b*e*x^3 + 2*a^3*b*d*x^2 + 4*a^3*b*c*x + a^4*e*log(
abs(x)) - 1/2*(2*a^4*d*x + a^4*c)/x^2
```

3.336.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3}\,dx = \frac{b^4\,c\,x^{10}}{10} - \frac{\frac{a^4\,c}{2} + a^4\,d\,x}{x^2} + \frac{b^4\,d\,x^{11}}{11} + \frac{b^4\,e\,x^{12}}{12} + a^4\,e\,\ln(x) + \frac{3\,a^2\,b^2\,c\,x^4}{2} + \frac{6\,a^2\,b^2\,d\,x^5}{5} + a^2\,b^2\,e\,x^6 + 4\,a^3\,b\,c\,x + \frac{4\,a\,b^3\,c\,x^7}{7} + 2\,a^3\,b\,d\,x^2 + \frac{a\,b^3\,d\,x^8}{2} + \frac{4\,a^3\,b\,e\,x^3}{3} + \frac{4\,a\,b^3\,e\,x^9}{9}$$

input
```
int(((a + b*x^3)^4*(c + d*x + e*x^2))/x^3,x)
```

output
```
(b^4*c*x^10)/10 - ((a^4*c)/2 + a^4*d*x)/x^2 + (b^4*d*x^11)/11 + (b^4*e*x^1
2)/12 + a^4*e*log(x) + (3*a^2*b^2*c*x^4)/2 + (6*a^2*b^2*d*x^5)/5 + a^2*b^2
*e*x^6 + 4*a^3*b*c*x + (4*a*b^3*c*x^7)/7 + 2*a^3*b*d*x^2 + (a*b^3*d*x^8)/2
 + (4*a^3*b*e*x^3)/3 + (4*a*b^3*e*x^9)/9
```

3.336. $\int \frac{(c+dx+ex^2)(a+bx^3)^4}{x^3} \, dx$

3.337 $\int \frac{x^3(c+dx+ex^2)}{a+bx^3} dx$

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3.337.9 Mupad [B] (verification not implemented) . . . 2514

3.337.1 Optimal result

Integrand size = 23, antiderivative size = 205

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3} dx = \frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} + \frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}} - \frac{ae\log\left(a+bx^3\right)}{3b^2}$$

output
```
c*x/b+1/2*d*x^2/b+1/3*e*x^3/b-1/3*a^(1/3)*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)
+b^(1/3)*x)/b^(5/3)+1/6*a^(1/3)*(c-a^(1/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/b^(4/3)-1/3*a*e*ln(b*x^3+a)/b^2+1/3*a^(1/3)*(b^(1/3)
*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^
(1/2)
```

3.337.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.93

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx$$
$$= \frac{6bcx+3bdx^2+2bex^3+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{b}\left(-\sqrt[3]{a}\sqrt[3]{b}c+a^{2/3}d\right)\log\left(\sqrt[3]{a}\right.}{6b^2}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

output
```
(6*b*c*x + 3*b*d*x^2 + 2*b*e*x^3 + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c +
a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(-(a^(1
/3)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^(1/3)*b^
(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*
e*Log[a + b*x^3])/(6*b^2)
```

3.337.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx+ex^2\right)}{a+bx^3}\,dx$$
$$\Big\downarrow \text{2426}$$
$$\int \left(-\frac{ac+adx+aex^2}{b\left(a+bx^3\right)}+\frac{c}{b}+\frac{dx}{b}+\frac{ex^2}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[3]{a}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}+\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt{3}b^{5/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}-\frac{ae\log\left(a+bx^3\right)}{3b^2}+\frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}$$

input
```
Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3),x]
```

output
```
(c*x)/b + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (a^(1/3)*(b^(1/3)*c + a^(1/3)*d)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a
^(1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^
(1/3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(6*b^(4/3)) - (a*e*Log[a + b*x^3])/(3*b^2)
```

3.337.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.337.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.33

method	result
risch	$\frac{e\,x^3}{3b}+\frac{d\,x^2}{2b}+\frac{cx}{b}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-\_R^2 e-\_R d-c\right)\ln\left(x-\_R\right)}{\_R^2}\right)}{3b^2}$
default	$\frac{\frac{1}{3}e\,x^3+\frac{1}{2}d\,x^2+cx}{b}-\frac{\left(\left(c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\cdots\right.\right.}{6\cdots}\cdots\right.\right.\right.}{b}$

input
```
int(x^3*(e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*e*x^3/b+1/2*d*x^2/b+c*x/b+1/3/b^2*a*sum((-_R^2*e-_R*d-c)/_R^2*ln(x-_R)
,_R=RootOf(_Z^3*b+a))
```

3.337.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 4798, normalized size of antiderivative = 23.40

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/36*(12*b*e*x^3 + 18*b*d*x^2 - 2*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*
d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(
a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a
^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*
d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3
- (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^2)*b^2*log(1/36*((-I*sqrt(3)
 + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b
*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 +
 a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*a^
3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6
- 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 6*a*e/b^
2)^2*b^4*d + 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 + 1/6*(b^3*c^2 - 2*a*b^2*
d*e)*((-I*sqrt(3) + 1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*
e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 -
1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)^(1/3) + 9*(I*sqrt(
3) + 1)*(-1/27*a^3*e^3/b^6 + 1/54*(b*c^3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d +
a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 + a^3*e^3 - (d^3 - 3*c*d*e)*a^2*b)/b^6)
^(1/3) + 6*a*e/b^2) + (b^2*c^3 + a*b*d^3)*x) + 36*b*c*x + (((-I*sqrt(3) +
1)*(a^2*e^2/b^4 - (a*b*c*d + a^2*e^2)/b^4)/(-1/27*a^3*e^3/b^6 + 1/54*(b*c^
3 + a*d^3)*a/b^5 + 1/18*(a*b*c*d + a^2*e^2)*a*e/b^6 - 1/54*(a*b^2*c^3 +...
```

3.337.6 Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.87

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx$$
$$=\operatorname{RootSum}\left(27t^3b^6+27t^2ab^4e+t\left(9a^2b^2e^2+9ab^3cd\right)+a^3e^3+3a^2bcde-a^2bd^3+ab^2c^3,\left(t\mapsto t\log\left(x+\right.\right.\right.$$
$$+\frac{cx}{b}+\frac{dx^2}{2b}+\frac{ex^3}{3b}$$

input
```
integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*b**6 + 27*_t**2*a*b**4*e + _t*(9*a**2*b**2*e**2 + 9*a*b**
3*c*d) + a**3*e**3 + 3*a**2*b*c*d*e - a**2*b*d**3 + a*b**2*c**3, Lambda(_t
, _t*log(x + (9*_t**2*b**4*d + 6*_t*a*b**2*d*e - 3*_t*b**3*c**2 + a**2*d*e
**2 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*d**3 + b**2*c**3)))) + c*x/b + d*x**
2/(2*b) + e*x**3/(3*b)
```

3.337.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.93

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx = \frac{2\,ex^3+3\,dx^2+6\,cx}{6\,b} - \frac{\sqrt{3}\left(abd\left(\frac{a}{b}\right)^{\frac{2}{3}}+abc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} - \frac{\left(2\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad\left(\frac{a}{b}\right)^{\frac{1}{3}}-ac\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(ae\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}+ac\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/6*(2*e*x^3 + 3*d*x^2 + 6*c*x)/b - 1/3*sqrt(3)*(a*b*d*(a/b)^(2/3) + a*b*c
*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2)
- 1/6*(2*a*e*(a/b)^(2/3) + a*d*(a/b)^(1/3) - a*c)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b^2*(a/b)^(2/3)) - 1/3*(a*e*(a/b)^(2/3) - a*d*(a/b)^(1/3)
+ a*c)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

3.337.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx = -\frac{ae\log\left(|bx^3+a|\right)}{3\,b^2} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc-\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3} + \frac{2\,b^2ex^3+3\,b^2dx^2+6\,b^2cx}{6\,b^3} + \frac{\left(ab^6d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^6c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7}$$

input
```
integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*a*e*log(abs(b*x^3 + a))/b^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b
^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 - 1
/6*((-a*b^2)^(1/3)*b*c + (-a*b^2)^(2/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3))/b^3 + 1/6*(2*b^2*e*x^3 + 3*b^2*d*x^2 + 6*b^2*c*x)/b^3 + 1/3*(a*b
^6*d*(-a/b)^(1/3) + a*b^6*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^
7)
```

3.337.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.56

$$\int \frac{x^3(c+dx+ex^2)}{a+bx^3}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\left(27\,b^6\,z^3+27\,a\,b^4\,e\,z^2+9\,a\,b^3\,c\,d\,z+9\,a^2\,b^2\,e^2\,z+3\,a^2\,b\,c\,d\,e+a\,b^2\,c^3+a^3\,e^3-a^2\,b\,d^3,z,k\right)\,\left(6\,a^2\right.\right.\right.$$
$$\left.\left. + \frac{a^3\,e^2+b\,c\,d\,a^2}{b^2} + \frac{x\,\left(a^2\,d^2-a^2\,c\,e\right)}{b}\right)\,\mathrm{root}\left(27\,b^6\,z^3+27\,a\,b^4\,e\,z^2+9\,a\,b^3\,c\,d\,z\right.$$
$$\left.\left. + 9\,a^2\,b^2\,e^2\,z+3\,a^2\,b\,c\,d\,e+a\,b^2\,c^3+a^3\,e^3-a^2\,b\,d^3,z,k\right)\right) + \frac{d\,x^2}{2\,b} + \frac{e\,x^3}{3\,b} + \frac{c\,x}{b}$$

input
```
int((x^3*(c + d*x + e*x^2))/(a + b*x^3),x)
```

output
```
symsum(log(root(27*b^6*z^3 + 27*a*b^4*e*z^2 + 9*a*b^3*c*d*z + 9*a^2*b^2*e^
2*z + 3*a^2*b*c*d*e + a*b^2*c^3 + a^3*e^3 - a^2*b*d^3, z, k)*(6*a^2*e + 9*
root(27*b^6*z^3 + 27*a*b^4*e*z^2 + 9*a*b^3*c*d*z + 9*a^2*b^2*e^2*z + 3*a^2
*b*c*d*e + a*b^2*c^3 + a^3*e^3 - a^2*b*d^3, z, k)*a*b^2 - 3*a*b*c*x) + (a^
3*e^2 + a^2*b*c*d)/b^2 + (x*(a^2*d^2 - a^2*c*e))/b)*root(27*b^6*z^3 + 27*a
*b^4*e*z^2 + 9*a*b^3*c*d*z + 9*a^2*b^2*e^2*z + 3*a^2*b*c*d*e + a*b^2*c^3 +
 a^3*e^3 - a^2*b*d^3, z, k), k, 1, 3) + (d*x^2)/(2*b) + (e*x^3)/(3*b) + (c
*x)/b
```

3.338 $\int \frac{x^2(c+dx+ex^2)}{a+bx^3} dx$

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3.338.1 Optimal result

Integrand size = 23, antiderivative size = 193

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3} dx = \frac{dx}{b}+\frac{ex^2}{2b}+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}} + \frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}+\frac{c\log(a+bx^3)}{3b}$$

output
```
d*x/b+1/2*e*x^2/b-1/3*a^(1/3)*(b^(1/3)*d-a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/
b^(5/3)+1/6*a^(1/3)*(d-a^(1/3)*e/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/b^(4/3)+1/3*c*ln(b*x^3+a)/b+1/3*a^(1/3)*(b^(1/3)*d+a^(1/3)*e)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^(1/2)
```

3.338.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx$$

$$= \frac{6b^{2/3}dx+3b^{2/3}ex^2+2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(-\sqrt[3]{a}\sqrt[3]{b}d+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{6b^{5/3}}$$

input
```
Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```

output
```
(6*b^(2/3)*d*x + 3*b^(2/3)*e*x^2 + 2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*
e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-(a^(1/3)*b^(1/3)*d) +
 a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] - (-(a^(1/3)*b^(1/3)*d) + a^(2/3)*e)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c*Log[a + b*x^3
])/(6*b^(5/3))
```

3.338.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx$$

$$\Big\downarrow \ 2426$$

$$\int \left(-\frac{ad+aex-bcx^2}{b(a+bx^3)}+\frac{d}{b}+\frac{ex}{b}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt[3]{a}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}+\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)}{\sqrt{3}b^{5/3}}-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{c\log\left(a+bx^3\right)}{3b}+\frac{dx}{b}+\frac{ex^2}{2b}$$

input
```
Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3),x]
```

output
```
(d*x)/b + (e*x^2)/(2*b) + (a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)*(b^(1/3)*
d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/3)*(d - (a^(1
/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)
) + (c*Log[a + b*x^3])/(3*b)
```

3.338.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.338.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.31

method	result
risch	$\frac{e\,x^2}{2b}+\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_R^2bc-\_Rae-ad\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^2}$
default	$\frac{\frac{1}{2}e\,x^2+dx}{b}+\frac{-ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-ae\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\right.}{6b\left(\frac{a}{b}\right)}\right.}{b}$

input
```
int(x^2*(e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*e*x^2/b+d*x/b+1/3/b^2*sum((_R^2*b*c-_R*a*e-a*d)/_R^2*ln(x-_R),_R=RootO
f(_Z^3*b+a))
```

3.338.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 4261, normalized size of antiderivative = 22.08

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/12*(6*e*x^2 - 2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*
e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^
2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3)
 + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*
c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b*log(1/4*(2*(1/2
)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b
*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 -
3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c
^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*
c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)^2*b^3*e + b*c*d^2 + b*c^2*e + 2*a*d*e^2 +
1/2*(b^2*d^2 + 2*b^2*c*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^
2 + a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b
^5 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5
 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b) + (b*d^3
+ a*e^3)*x) + 12*d*x + ((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(c^2/b^2 - (b*c^2
+ a*d*e)/b^3)/(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5
 + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*(2*c^3/b^3 - 3*(b*c^2 + a*d*e)*c/b^4 + (b*d^3 + a*e^3)*a/b^5 +
 (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/b^5)^(1/3) - 2*c/b)*b + 3*sq...
```

3.338.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.78

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx$$
$$= \text{RootSum}\left(27t^3b^5 - 27t^2b^4c + t\left(9ab^2de + 9b^3c^2\right) - a^2e^3 - 3abcde + abd^3 - b^2c^3, \left(t \mapsto t\log\left(x + \frac{9t^2b^3e\ \cdots}{}\right.\right.\right.$$
$$+ \frac{dx}{b} + \frac{ex^2}{2b}$$

input
```
integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c + _t*(9*a*b**2*d*e + 9*b**3*c**2)
- a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3, Lambda(_t, _t*log(x + (9
*_t**2*b**3*e - 6*_t*b**2*c*e - 3*_t*b**2*d**2 + 2*a*d*e**2 + b*c**2*e + b
*c*d**2)/(a*e**3 + b*d**3)))) + d*x/b + e*x**2/(2*b)
```

3.338.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab}+\frac{ex^2+2dx}{2b}+\frac{\left(2bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*(a*e*(a/b)^(2/3) + a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x -
 (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/2*(e*x^2 + 2*d*x)/b + 1/6*(2*b*c*(a/b
)^(2/3) - a*e*(a/b)^(1/3) + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b
^2*(a/b)^(2/3)) + 1/3*(b*c*(a/b)^(2/3) + a*e*(a/b)^(1/3) - a*d)*log(x + (a
/b)^(1/3))/(b^2*(a/b)^(2/3))
```

3.338.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.99

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx = \frac{c\log\left(\left|bx^3+a\right|\right)}{3b}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bd-\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3}+\frac{bex^2+2bdx}{2b^2}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bd+\left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^3}+\frac{\left(ab^4e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^4d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^5}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*c*log(abs(b*x^3 + a))/b - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*d - (-a*b^2)^(
2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/2*(b
*e*x^2 + 2*b*d*x)/b^2 - 1/6*((-a*b^2)^(1/3)*b*d + (-a*b^2)^(2/3)*e)*log(x^
2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 1/3*(a*b^4*e*(-a/b)^(1/3) + a*b^4
*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

3.338.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.76

$$\int \frac{x^2(c+dx+ex^2)}{a+bx^3}\,dx$$

$$= \left(\sum_{k=1}^{3} \ln\left(\frac{a\left(b\,c^2+\mathrm{root}(27\,b^5\,z^3-27\,b^4\,c\,z^2+9\,a\,b^2\,d\,e\,z+9\,b^3\,c^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,\right.}{}\right.\right.$$

$$\left.\left.-27\,b^4\,c\,z^2+9\,a\,b^2\,d\,e\,z+9\,b^3\,c^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k)\right)+\frac{e\,x^2}{2\,b}+\frac{d\,x}{b}$$

input
```
int((x^2*(c + d*x + e*x^2))/(a + b*x^3),x)
```

output
```
symsum(log((a*(b*c^2 + 9*root(27*b^5*z^3 - 27*b^4*c*z^2 + 9*a*b^2*d*e*z +
9*b^3*c^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)^2*b^3 + a*d
*e - 6*root(27*b^5*z^3 - 27*b^4*c*z^2 + 9*a*b^2*d*e*z + 9*b^3*c^2*z - 3*a*
b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*b^2*c + a*e^2*x + b*c*d*x - 3
*root(27*b^5*z^3 - 27*b^4*c*z^2 + 9*a*b^2*d*e*z + 9*b^3*c^2*z - 3*a*b*c*d*
e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*b^2*d*x))/b)*root(27*b^5*z^3 - 27*b
^4*c*z^2 + 9*a*b^2*d*e*z + 9*b^3*c^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 -
 b^2*c^3, z, k), k, 1, 3) + (e*x^2)/(2*b) + (d*x)/b
```

3.339 $\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx$

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3.339.1 Optimal result

Integrand size = 21, antiderivative size = 183

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx = \frac{ex}{b} - \frac{\left(b^{2/3}c-a^{2/3}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}} - \frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}} + \frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}} + \frac{d\log\left(a+bx^3\right)}{3b}$$

output
```
e*x/b-1/3*(b^(2/3)*c+a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(4/3)+1/6*
(b^(2/3)*c+a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^
(4/3)+1/3*d*ln(b*x^3+a)/b-1/3*(b^(2/3)*c-a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*
b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(4/3)*3^(1/2)
```

3.339.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.09

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx = \frac{ex}{b} + \frac{\left(a^{2/3}bc-a^{4/3}\sqrt[3]{b}e\right)\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}ab^{5/3}} + \frac{\left(-a^{2/3}bc-a^{4/3}\sqrt[3]{b}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3ab^{5/3}} - \frac{\left(-a^{2/3}bc-a^{4/3}\sqrt[3]{b}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6ab^{5/3}} + \frac{d\log\left(a+bx^3\right)}{3b}$$

input
```
Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

output
```
(e*x)/b + ((a^(2/3)*b*c - a^(4/3)*b^(1/3)*e)*ArcTan[(-a^(1/3) + 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a*b^(5/3)) + ((-(a^(2/3)*b*c) - a^(4/3)*b^
(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a*b^(5/3)) - ((-(a^(2/3)*b*c) - a^(4
/3)*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a*b^(5/3
)) + (d*Log[a + b*x^3])/(3*b)
```

3.339.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx+ex^2\right)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{ 2426}$$

$$\int \left(\frac{e}{b}-\frac{ae-bcx-bdx^2}{b\left(a+bx^3\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(b^{2/3}c-a^{2/3}e\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}}+\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{4/3}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{4/3}}+\frac{d\log\left(a+bx^3\right)}{3b}+\frac{ex}{b}$$

input
```
Int[(x*(c + d*x + e*x^2))/(a + b*x^3),x]
```

output
```
(e*x)/b - ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)) - ((b^(2/3)*c + a^(2/3)*e)*Log[a^(1/
3) + b^(1/3)*x])/(3*a^(1/3)*b^(4/3)) + ((b^(2/3)*c + a^(2/3)*e)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(4/3)) + (d*Log[a + b*x
^3])/(3*b)
```

3.339.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.339.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.27

method	result
risch	$\frac{ex}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_R^2bd+\_Rbc-ae\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^2}$
default	$\frac{ex}{b}+\frac{-ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{b}$

input
```
int(x*(e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
e*x/b+1/3/b^2*sum((_R^2*b*d+_R*b*c-a*e)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.339.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 4628, normalized size of antiderivative = 25.29

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^
3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a
*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(
2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b
)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)*b*log(-1/4*(2*(1/2
)^(2/3)*(-I*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 -
 c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3
 - a^2*e^3)/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d
^2 - c*e)*d/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2
*c^3 - a^2*e^3)/(a*b^4))^(1/3) - 2*d/b)^2*a*b^3*c - a*b*c*d^2 + 2*a*b*c^2*
e + a^2*d*e^2 - 1/2*(2*a*b^2*c*d - a^2*b*e^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^2*c^
3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^4))^
(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3 - (b^
2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)/(a*b^
4))^(1/3) - 2*d/b) - (b^2*c^3 - a^2*e^3)*x) - 12*e*x - ((2*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(d^2/b^2 - (d^2 - c*e)/b^2)/(2*d^3/b^3 - 3*(d^2 - c*e)*d/b^3
 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a^2*e^3)
/(a*b^4))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*d^3/b^3 - 3*(d^2 - c*e)*d
/b^3 - (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a*b^4) - (b^2*c^3 - a...
```

3.339.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.87

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx$$
$$= \operatorname{RootSum}\left(27t^3ab^4 - 27t^2ab^3d + t(-9ab^2ce + 9ab^2d^2) + a^2e^3 + 3abcde - abd^3 + b^2c^3, \left(t \mapsto t\log\left(x + \frac{-}{}\right.\right.\right.$$
$$+ \frac{ex}{b}$$

input
```
integrate(x*(e*x**2+d*x+c)/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a*b**4 - 27*_t**2*a*b**3*d + _t*(-9*a*b**2*c*e + 9*a*b**2
*d**2) + a**2*e**3 + 3*a*b*c*d*e - a*b*d**3 + b**2*c**3, Lambda(_t, _t*log
(x + (-9*_t**2*a*b**3*c - 3*_t*a**2*b*e**2 + 6*_t*a*b**2*c*d + a**2*d*e**2
 + 2*a*b*c**2*e - a*b*c*d**2)/(a**2*e**3 - b**2*c**3)))) + e*x/b
```

3.339.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.95

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx = \frac{ex}{b} + \frac{\sqrt{3}\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}} - ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab} + \frac{\left(2\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}} + bc\left(\frac{a}{b}\right)^{\frac{1}{3}} + ae\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}} - bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
e*x/b + 1/3*sqrt(3)*(b*c*(a/b)^(2/3) - a*e*(a/b)^(1/3))*arctan(1/3*sqrt(3)
*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*b*d*(a/b)^(2/3) + b*c*(a/
b)^(1/3) + a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) +
 1/3*(b*d*(a/b)^(2/3) - b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(b^2*(
a/b)^(2/3))
```

3.339.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.95

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx = \frac{\sqrt{3}\left(ae + (-ab^2)^{\frac{1}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} + \frac{\left(ae - (-ab^2)^{\frac{1}{3}}c\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} + \frac{ex}{b} + \frac{d\log\left(|bx^3+a|\right)}{3\,b} - \frac{\left(b^3c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2e\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^3}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(a*e + (-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(-a*b^2)^(2/3) + 1/6*(a*e - (-a*b^2)^(1/3)*c)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) + e*x/b + 1/3*d*log(abs(b*x^3
 + a))/b - 1/3*(b^3*c*(-a/b)^(1/3) - a*b^2*e)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a*b^3)
```

3.339.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.45

$$\int \frac{x(c+dx+ex^2)}{a+bx^3}\,dx = \left(\sum_{k=1}^{3}\ln\left(x\left(b\,c^2+a\,d\,e\right)\right.\right.$$
$$-\mathrm{root}\left(27\,a\,b^4\,z^3-27\,a\,b^3\,d\,z^2-9\,a\,b^2\,c\,e\,z+9\,a\,b^2\,d^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\,\left(6\,a\,b\,d-\mathrm{root}\left(\right.\right.$$
$$+a\,d^2-a\,c\,e\Big)\;\mathrm{root}\left(27\,a\,b^4\,z^3-27\,a\,b^3\,d\,z^2-9\,a\,b^2\,c\,e\,z+9\,a\,b^2\,d^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3\right.$$
$$\left.\left.+a^2\,e^3+b^2\,c^3,z,k\right)\right)+\frac{e\,x}{b}$$

input
```
int((x*(c + d*x + e*x^2))/(a + b*x^3),x)
```

output
```
symsum(log(x*(b*c^2 + a*d*e) - root(27*a*b^4*z^3 - 27*a*b^3*d*z^2 - 9*a*b^
2*c*e*z + 9*a*b^2*d^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)
*(6*a*b*d - 9*root(27*a*b^4*z^3 - 27*a*b^3*d*z^2 - 9*a*b^2*c*e*z + 9*a*b^2
*d^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a*b^2 + 3*a*b*e*
x) + a*d^2 - a*c*e)*root(27*a*b^4*z^3 - 27*a*b^3*d*z^2 - 9*a*b^2*c*e*z + 9
*a*b^2*d^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k), k, 1, 3)
+ (e*x)/b
```

3.340 $\int \frac{c+dx+ex^2}{a+bx^3} dx$

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3.340.1 Optimal result

Integrand size = 20, antiderivative size = 177

$$\int \frac{c+dx+ex^2}{a+bx^3} dx = -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} - \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{e\log\left(a+bx^3\right)}{3b}$$

output
```
1/3*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-1/6*(c-a^(
1/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)+
1/3*e*ln(b*x^3+a)/b-1/3*(b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3
)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(2/3)*3^(1/2)
```

3.340.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx = \frac{-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}c\right.}{6ab}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^3),x]
```

output
```
(-2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(1
/3) + b^(1/3)*x] - b^(1/3)*(a^(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*e*Log[a + b*x^3])/(6*a*b)
```

3.340.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2410, 792, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx$$

$$\Big\downarrow \text{ 2410}$$

$$\int \frac{c+dx}{bx^3+a}dx+e\int \frac{x^2}{bx^3+a}dx$$

$$\Big\downarrow \text{ 792}$$

$$\int \frac{c+dx}{bx^3+a}dx+\frac{e\log\left(a+bx^3\right)}{3b}$$

$$\Big\downarrow \text{ 2399}$$

$$\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{e\log\left(a+bx^3\right)}{3b}$$

$$\Big\downarrow \; 16$$

$$\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{e\log\left(a+bx^3\right)}{3b}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{e\log\left(a+bx^3\right)}{3b}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{e\log\left(a+bx^3\right)}{3b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{e\log\left(a+bx^3\right)}{3b}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{1}{2}\sqrt[3]{b}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{e\log\left(a+bx^3\right)}{3b}$$

3.340. $\int \frac{c+dx+ex^2}{a+bx^3}\,dx$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}\sqrt[3]{b}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{e\log\left(a+bx^3\right)}{3b}$$

$$\Big\downarrow \; 1103$$

$$\frac{-\frac{1}{2}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{e\log\left(a+bx^3\right)}{3b}$$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^3),x]
```

output
```
((c - (a^(1/3)*d)/b^(1/3))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) +
 (-((Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(1/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)*b^(1/3)) + (e*Log[a + b*x^3])/(3*b)
```

3.340.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.340. $\int \frac{c+dx+ex^2}{a+bx^3} \, dx$

3.340.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.21

method	result
risch	$\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_R^2e+\_Rd+c\right)\ln\left(x-\_R\right)}{\_R^2}}{3b}$
default	$c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x\right.}{6b\left(\frac{a}{b}\right)}\right.$

input
```
int((e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*sum((_R^2*e+_R*d+c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.340.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 4671, normalized size of antiderivative = 26.39

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2
))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) +
(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(
I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/
(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) - 2
*e/b)*b*log(1/4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a*e^2)
/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/(a^2*
b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (1/2)^
(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 +
a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1
/3) - 2*e/b)^2*a^2*b^2*d + 2*a*b*c*d^2 - a*b*c^2*e + a^2*d*e^2 - 1/2*(a*b^
2*c^2 - 2*a^2*b*d*e)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(e^2/b^2 - (b*c*d + a
*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c^3 + a*d^3)/
(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3))^(1/3) + (
1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(a*b^3) + (b*c
^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^2*b^3
))^(1/3) - 2*e/b) + (b^2*c^3 + a*b*d^3)*x) - ((2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)*(e^2/b^2 - (b*c*d + a*e^2)/(a*b^2))/(2*e^3/b^3 - 3*(b*c*d + a*e^2)*e/(
a*b^3) + (b*c^3 + a*d^3)/(a^2*b^2) + (b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*
a*b)/(a^2*b^3))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(2*e^3/b^3 - 3*(b*c...
```

3.340.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx$$
$$=\operatorname{RootSum}\left(27t^3a^2b^3-27t^2a^2b^2e+t\left(9a^2be^2+9ab^2cd\right)-a^2e^3-3abcde+abd^3-b^2c^3,\left(t\mapsto t\log\left(x+\frac{9\right.\right.\right.$$

input
```
integrate((e*x**2+d*x+c)/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**2*b**3 - 27*_t**2*a**2*b**2*e + _t*(9*a**2*b*e**2 + 9*
a*b**2*c*d) - a**2*e**3 - 3*a*b*c*d*e + a*b*d**3 - b**2*c**3, Lambda(_t, _
t*log(x + (9*_t**2*a**2*b**2*d - 6*_t*a**2*b*d*e + 3*_t*a*b**2*c**2 + a**2
*d*e**2 - a*b*c**2*e + 2*a*b*c*d**2)/(a*b*d**3 + b**2*c**3))))
```

3.340.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx = \frac{\sqrt{3}\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab} + \frac{\left(2\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}+d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(e\left(\frac{a}{b}\right)^{\frac{2}{3}}-d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(b*d*(a/b)^(2/3) + b*c*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*e*(a/b)^(2/3) + d*(a/b)^(1/3) - c
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(e*(a/b)^(2
/3) - d*(a/b)^(1/3) + c)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

3.340.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(bc-\left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(bc+\left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} + \frac{e\log\left(\left|bx^3+a\right|\right)}{3\,b} - \frac{\left(bd\left(-\frac{a}{b}\right)^{\frac{1}{3}}+bc\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) + 1/3*e*log(abs(b*x^3 + a))/
b - 1/3*(b*d*(-a/b)^(1/3) + b*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(
a*b)
```

3.340.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.55

$$\int \frac{c+dx+ex^2}{a+bx^3}\,dx=\sum_{k=1}^{3}\ln\left(x\left(b\,d^2-b\,c\,e\right)+\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,a^2\,b^2\,e\,z^2+9\,a\,b^2\,c\,d\,z+9\,a^2\,b\,e^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k\right)\left(-6\,a\,b\,e+\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,a^2\,b^2\,e\,z^2+9\,a\,b^2\,c\,d\,z+9\,a^2\,b\,e^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k\right)\,a\,b^2\,9+3\,b^2\,c\,x\right)+a\,e^2+b\,c\,d\right)\,\mathrm{root}\left(27\,a^2\,b^3\,z^3-27\,a^2\,b^2\,e\,z^2+9\,a\,b^2\,c\,d\,z+9\,a^2\,b\,e^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k\right)$$

input
```
int((c + d*x + e*x^2)/(a + b*x^3),x)
```

output
```
symsum(log(x*(b*d^2 - b*c*e) + root(27*a^2*b^3*z^3 - 27*a^2*b^2*e*z^2 + 9*
a*b^2*c*d*z + 9*a^2*b*e^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z
, k)*(9*root(27*a^2*b^3*z^3 - 27*a^2*b^2*e*z^2 + 9*a*b^2*c*d*z + 9*a^2*b*e
^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a*b^2 - 6*a*b*e +
3*b^2*c*x) + a*e^2 + b*c*d)*root(27*a^2*b^3*z^3 - 27*a^2*b^2*e*z^2 + 9*a*b
^2*c*d*z + 9*a^2*b*e^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k
), k, 1, 3)
```

3.341 $\int \frac{c+dx+ex^2}{x(a+bx^3)}\,dx$

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3.341.1 Optimal result

Integrand size = 23, antiderivative size = 184

$$\int \frac{c+dx+ex^2}{x(a+bx^3)}\,dx=-\frac{\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}+\frac{c\log(x)}{a}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}-\frac{c\log\left(a+bx^3\right)}{3a}$$

output
```
c*ln(x)/a+1/3*(b^(1/3)*d-a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(2/3)-
1/6*(d-a^(1/3)*e/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3
)/b^(1/3)-1/3*c*ln(b*x^3+a)/a-1/3*(b^(1/3)*d+a^(1/3)*e)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(2/3)*3^(1/2)
```

3.341.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x(a+bx^3)}\,dx$$
$$= \frac{-2\sqrt{3}\sqrt[3]{a}\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6b^{2/3}c\log(x)+2\left(\sqrt[3]{a}\sqrt[3]{b}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\left(-\right.}{6ab^{2/3}}$$

input
```
Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

output
```
(-2*Sqrt[3]*a^(1/3)*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]] + 6*b^(2/3)*c*Log[x] + 2*(a^(1/3)*b^(1/3)*d - a^(2/3)*e)*Log
[a^(1/3) + b^(1/3)*x] + (-(a^(1/3)*b^(1/3)*d) + a^(2/3)*e)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b^(2/3)*c*Log[a + b*x^3])/(6*a*b^(2/3)
)
```

3.341.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x(a+bx^3)}\,dx$$
$$\Big\downarrow \text{2373}$$
$$\int \left(\frac{ad+aex-bcx^2}{a(a+bx^3)}+\frac{c}{ax}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)}{\sqrt{3}a^{2/3}b^{2/3}}-\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}-\frac{c\log\left(a+bx^3\right)}{3a}+\frac{c\log(x)}{a}$$

input
```
Int[(c + d*x + e*x^2)/(x*(a + b*x^3)),x]
```

output
```
-(((b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(2/3)*b^(2/3))) + (c*Log[x])/a + ((b^(1/3)*d - a^(1/3)*e)*L
og[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((d - (a^(1/3)*e)/b^(1/3))*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)) - (c*L
og[a + b*x^3])/(3*a)
```

3.341.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

3.341.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.03

method	result
risch	$\frac{c\ln(-x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^3b^2\_Z^3+3a^2b^2c\_Z^2+\left(3a^2bde+3b^2c^2a\right)\_Z+a^2e^3+3abcde-ab\,d^3+b^2c^3\right)}\_R\ln\left(\left(-4\_R^3a^2b^2-8\_R^2a\,b \cdots\right.\right.\right.}{3}$
default	$\frac{c\ln(x)}{a}+\frac{ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+ae\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right. \cdots\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}} \cdots\right.}{a}$

input
```
int((e*x^2+d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
c/a*ln(-x)+1/3*sum(_R*ln((-4*_R^3*a^2*b^2-8*_R^2*a*b^2*c+(-10*a*b*d*e-4*b^
2*c^2)*_R-3*a*e^3-6*b*c*d*e+3*b*d^3)*x+a^2*b*e*_R^2+(-2*a*b*c*e-a*b*d^2)*_
R-3*b*c^2*e+3*b*c*d^2),_R=RootOf(a^3*b^2*_Z^3+3*a^2*b^2*c*_Z^2+(3*a^2*b*d*
e+3*a*b^2*c^2)*_Z+a^2*e^3+3*a*b*c*d*e-a*b*d^3+b^2*c^3))
```

3.341.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 4588, normalized size of antiderivative = 24.93

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/36*(2*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/
a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/
54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(
3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*
e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))
^(1/3) + 6*c/a)*a*log(1/36*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a
^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e
^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^
(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b)
+ 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*
e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a)^2*a^2*b*e + b*c*d^2 + b*c^2*e + 2*a*d*e^
2 - 1/6*(a*b*d^2 + 2*a*b*c*e)*((-I*sqrt(3) + 1)*(c^2/a^2 - (b*c^2 + a*d*e)
/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*b) + 1/54*(b*d^3 +
a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^3*b^2
))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*d*e)*c/(a^3*
b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c
*d*e)*a*b)/(a^3*b^2))^(1/3) + 6*c/a) + (b*d^3 + a*e^3)*x) - (((-I*sqrt(3)
+ 1)*(c^2/a^2 - (b*c^2 + a*d*e)/(a^2*b))/(-1/27*c^3/a^3 + 1/18*(b*c^2 + a*
d*e)*c/(a^3*b) + 1/54*(b*d^3 + a*e^3)/(a^2*b^2) - 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/(a^3*b^2))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*c^3/...
```

3.341.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx=\text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x/(b*x**3+a),x)
```

output
```
Timed out
```

3.341.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx = \frac{c\log\left(x\right)}{a} + \frac{\sqrt{3}\left(ae\left(\frac{a}{b}\right)^{\frac{2}{3}} + ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2} - \frac{\left(2\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}} - ae\left(\frac{a}{b}\right)^{\frac{1}{3}} + ad\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}} + ae\left(\frac{a}{b}\right)^{\frac{1}{3}} - ad\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
c*log(x)/a + 1/3*sqrt(3)*(a*e*(a/b)^(2/3) + a*d*(a/b)^(1/3))*arctan(1/3*sq
rt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^2 - 1/6*(2*b*c*(a/b)^(2/3) - a*e*
(a/b)^(1/3) + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)
) - 1/3*(b*c*(a/b)^(2/3) + a*e*(a/b)^(1/3) - a*d)*log(x + (a/b)^(1/3))/(a*
b*(a/b)^(2/3))
```

3.341.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)}\,dx = -\frac{\sqrt{3}\left(bd - \left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(bd + \left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} - \frac{c\log\left(\left|bx^3+a\right|\right)}{3\,a} + \frac{c\log\left(\left|x\right|\right)}{a} - \frac{\left(a^2be\left(-\frac{a}{b}\right)^{\frac{1}{3}} + a^2bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b}$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b*d - (-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*d + (-a*b^2)^(1/3)*e)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*c*log(abs(b*x^3 + a))/
a + c*log(abs(x))/a - 1/3*(a^2*b*e*(-a/b)^(1/3) + a^2*b*d)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a^3*b)
```

3.341.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 716, normalized size of antiderivative = 3.89

$$\int \frac{c+dx+ex^2}{x(a+bx^3)}\,dx = \left(\sum_{k=1}^{3}\ln\left(b^2\,c\,d^2-b^2\,c^2\,e+b^2\,d^3\,x\right.\right.$$
$$-\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)^3\,a^2\,b^3\,x$$
$$-a\,b\,e^3\,x$$
$$-\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\,a\,b^2\,d^2$$
$$-\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\,b^3\,c^2\,x\,4$$
$$+\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)^2\,a^2\,b^2\,e$$
$$-\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)^2\,a\,b^3\,c\,x$$
$$-\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\,a\,b^2\,c\,e\,2$$
$$-2\,b^2\,c\,d\,e\,x$$
$$\left.-\mathrm{root}\left(27\,a^3\,b^2\,z^3+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\,a\,b^2\,d\,e\,x\,10\right)$$
$$\left.+27\,a^2\,b^2\,c\,z^2+9\,a^2\,b\,d\,e\,z+9\,a\,b^2\,c^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k)\right)+\frac{c\,\ln(x)}{a}$$

input
```
int((c + d*x + e*x^2)/(x*(a + b*x^3)),x)
```

output
```
symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - 36*root(27*a^3*b^2*z^3 + 27
*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a
^2*e^3 + b^2*c^3, z, k)^3*a^2*b^3*x - a*b*e^3*x - root(27*a^3*b^2*z^3 + 27
*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a
^2*e^3 + b^2*c^3, z, k)*a*b^2*d^2 - 4*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z
^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2
*c^3, z, k)*b^3*c^2*x + 3*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b
*d*e*z + 9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)^
2*a^2*b^2*e - 24*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z +
9*a*b^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)^2*a*b^3*c
*x - 2*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^
2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a*b^2*c*e - 2*b^2*c
*d*e*x - 10*root(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b
^2*c^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a*b^2*d*e*x)*r
oot(27*a^3*b^2*z^3 + 27*a^2*b^2*c*z^2 + 9*a^2*b*d*e*z + 9*a*b^2*c^2*z + 3*
a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k), k, 1, 3) + (c*log(x))/a
```

3.341. $\int \frac{c+dx+ex^2}{x(a+bx^3)}\,dx$

3.342 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx$

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3.342.1 Optimal result

Integrand size = 23, antiderivative size = 192

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = -\frac{c}{ax}+\frac{\left(b^{2/3}c-a^{2/3}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}}+\frac{d\log(x)}{a}+\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}}-\frac{\left(b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}}-\frac{d\log\left(a+bx^3\right)}{3a}$$

output
```
-c/a/x+d*ln(x)/a+1/3*(b^(2/3)*c+a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b
^(1/3)-1/6*(b^(2/3)*c+a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(4/3)/b^(1/3)-1/3*d*ln(b*x^3+a)/a+1/3*(b^(2/3)*c-a^(2/3)*e)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(1/3)*3^(1/2)
```

3.342.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = -\frac{\frac{6ac}{x} + \frac{2\sqrt{3}a^{2/3}(-b^{2/3}c+a^{2/3}e)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - 6ad\log(x) - \frac{2(a^{2/3}b^{2/3}c+a^{4/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{(a^{2/3}b^{2/3}c+a^{4/3}e\cdots}{\cdots}}{6a^2}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

output
```
-1/6*((6*a*c)/x + (2*Sqrt[3]*a^(2/3)*(-(b^(2/3)*c) + a^(2/3)*e)*ArcTan[(1
- (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - 6*a*d*Log[x] - (2*(a^(2/3)*b^
(2/3)*c + a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((a^(2/3)*b^(2/3)
*c + a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) +
2*a*d*Log[a + b*x^3])/a^2
```

3.342.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx$$

$$\Big\downarrow \ 2373$$

$$\int \left(\frac{ae-bcx-bdx^2}{a(a+bx^3)}+\frac{c}{ax^2}+\frac{d}{ax}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(b^{2/3}c-a^{2/3}e\right)}{\sqrt{3}a^{4/3}\sqrt[3]{b}}-\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt[3]{b}}+\frac{\left(a^{2/3}e+b^{2/3}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt[3]{b}}-\frac{d\log\left(a+bx^3\right)}{3a}-\frac{c}{ax}+\frac{d\log(x)}{a}$$

input
```
Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)),x]
```

output
```
-(c/(a*x)) + ((b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt
[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(1/3)) + (d*Log[x])/a + ((b^(2/3)*c + a^
(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*b^(1/3)) - ((b^(2/3)*c + a^(
2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(1/3)
) - (d*Log[a + b*x^3])/(3*a)
```

3.342.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.342.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{c}{ax}+\frac{d\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^4b\_Z^3+3a^3bd\_Z^2+\left(-3a^2bce+3a^2bd^2\right)\_Z-a^2e^3-3abcde+abd^3-b^2c^3\right)}\_R\ln\left(\left(-4\_R^3a^4b-8\_\right.\right.\right.}{}$
default	$-\frac{c}{ax}+\frac{d\ln(x)}{a}+\frac{ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right.\right.}{6b}\right.}{a}$

input
```
int((e*x^2+d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-c/a/x+d*ln(x)/a+1/3*sum(_R*ln((-4*_R^3*a^4*b-8*_R^2*a^3*b*d+(10*a^2*b*c*e
-4*a^2*b*d^2)*_R+3*a^2*e^3+6*a*b*c*d*e+3*b^2*c^3)*x-a^3*b*c*_R^2+(-a^3*e^2
+2*a^2*b*c*d)*_R+3*a^2*d*e^2+3*a*b*c*d^2),_R=RootOf(a^4*b*_Z^3+3*a^3*b*d*_
Z^2+(-3*a^2*b*c*e+3*a^2*b*d^2)*_Z-a^2*e^3-3*a*b*c*d*e+a*b*d^3-b^2*c^3))
```

3.342.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 4524, normalized size of antiderivative = 23.56

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/36*(2*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/
18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4
*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d
^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)*a*x*log(
-1/36*((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*
(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b)
 - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/
a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a
*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a)^2*a^3*b*c -
 a*b*c*d^2 + 2*a*b*c^2*e + a^2*d*e^2 + 1/6*(2*a^2*b*c*d - a^3*e^2)*((-I*sq
rt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d
/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) - 1/54*(b^2*
c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*d^3/a^3 + 1/18*(d
^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/(a^4*b) -
 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 6*d/a) - (b^2*c^3 - a^2*e^3)*x)
 - 36*d*x*log(x) - (((-I*sqrt(3) + 1)*(d^2/a^2 - (d^2 - c*e)/a^2)/(-1/27*d
^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e
)*a*b)/(a^4*b) - 1/54*(b^2*c^3 - a^2*e^3)/(a^4*b))^(1/3) + 9*(I*sqrt(3) +
1)*(-1/27*d^3/a^3 + 1/18*(d^2 - c*e)*d/a^3 + 1/54*(b^2*c^3 + a^2*e^3 - ...
```

3.342.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x**2/(b*x**3+a),x)
```

output
```
Timed out
```

3.342.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.97

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = \frac{d\log(x)}{a} - \frac{\sqrt{3}\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}} - ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2} - \frac{\left(2bd\left(\frac{a}{b}\right)^{\frac{2}{3}} + bc\left(\frac{a}{b}\right)^{\frac{1}{3}} + ae\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}} - bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{c}{ax}$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
d*log(x)/a - 1/3*sqrt(3)*(b*c*(a/b)^(2/3) - a*e*(a/b)^(1/3))*arctan(1/3*sq
rt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^2 - 1/6*(2*b*d*(a/b)^(2/3) + b*c*
(a/b)^(1/3) + a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)
) - 1/3*(b*d*(a/b)^(2/3) - b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(a*
b*(a/b)^(2/3)) - c/(a*x)
```

3.342.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = -\frac{d\log(|bx^3+a|)}{3a} + \frac{d\log(|x|)}{a} + \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}ae + (-ab^2)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b} + \frac{\left((-ab^2)^{\frac{1}{3}}ae - (-ab^2)^{\frac{2}{3}}c\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b} + \frac{\left(ab^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b} - \frac{c}{ax}$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*d*log(abs(b*x^3 + a))/a + d*log(abs(x))/a + 1/3*sqrt(3)*((-a*b^2)^(1/
3)*a*e + (-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/(a^2*b) - c/(a*x) + 1/6*((-a*b^2)^(1/3)*a*e - (-a*b^2)^(2/3)*c)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) + 1/3*(a*b^2*c*(-a/b)^(1/3)
- a^2*b*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b)
```

3.342.9 Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 723, normalized size of antiderivative = 3.77

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{b^4\,c^3\,x+a^2\,b^2\,d\,e^2-\mathrm{root}(27\,a^4\,b\,z^3+27\,a^3\,b\,d\,z^2-9\,a^2\,b\,c\,e\,z+9\,a^2\,b\,d^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-\cdots}{\cdots}\right) \cdots +27\,a^3\,b\,d\,z^2-9\,a^2\,b\,c\,e\,z+9\,a^2\,b\,d^2\,z-3\,a\,b\,c\,d\,e+a\,b\,d^3-a^2\,e^3-b^2\,c^3,z,k)\right) - \frac{c}{a\,x}+\frac{d\,\ln(x)}{a}$$

input
```
int((c + d*x + e*x^2)/(x^2*(a + b*x^3)),x)
```

output
```
symsum(log((b^4*c^3*x + a^2*b^2*d*e^2 - 36*root(27*a^4*b*z^3 + 27*a^3*b*d*
z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^
2*c^3, z, k)^3*a^4*b^3*x + a^2*b^2*e^3*x + a*b^3*c*d^2 - 3*root(27*a^4*b*z
^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^
3 - a^2*e^3 - b^2*c^3, z, k)^2*a^3*b^3*c - root(27*a^4*b*z^3 + 27*a^3*b*d*
z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^
2*c^3, z, k)*a^3*b^2*e^2 - 4*root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*
c*e*z + 9*a^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a
^2*b^3*d^2*x - 24*root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a
^2*b*d^2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)^2*a^3*b^3*d*
x + 2*root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z -
 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a^2*b^3*c*d + 2*a*b^3*c*
d*e*x + 10*root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^
2*z - 3*a*b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k)*a^2*b^3*c*e*x)/a^2)
*root(27*a^4*b*z^3 + 27*a^3*b*d*z^2 - 9*a^2*b*c*e*z + 9*a^2*b*d^2*z - 3*a*
b*c*d*e + a*b*d^3 - a^2*e^3 - b^2*c^3, z, k), k, 1, 3) - c/(a*x) + (d*log(
x))/a
```

3.343 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx$

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3.343.1 Optimal result

Integrand size = 23, antiderivative size = 203

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx = -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}+\frac{e\log(x)}{a}-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}+\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{e\log(a+bx^3)}{3a}$$

output
```
-1/2*c/a/x^2-d/a/x+e*ln(x)/a-1/3*b^(1/3)*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+
b^(1/3)*x)/a^(5/3)+1/6*b^(2/3)*(c-a^(1/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/a^(5/3)-1/3*e*ln(b*x^3+a)/a+1/3*b^(1/3)*(b^(1/3)*c+a^
(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

3.343.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx$$

$$= \frac{-\frac{3ac}{x^2}-\frac{6ad}{x}+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6ae\log(x)+2\sqrt[3]{b}\left(-\sqrt[3]{a}\sqrt[3]{b}c+a^{2/3}d\right)\log\left(\cdots\right.}{6a^2}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```

output
```
((-3*a*c)/x^2 - (6*a*d)/x + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(b^(1/3)*c + a^(1/3)
*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 6*a*e*Log[x] + 2*b^(1/3)
*(-(a^(1/3)*b^(1/3)*c) + a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(a^
(1/3)*b^(1/3)*c - a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
] - 2*a*e*Log[a + b*x^3])/(6*a^2)
```

3.343.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx$$

$$\Big\downarrow \ 2373$$

$$\int \left(-\frac{b(c+dx+ex^2)}{a(a+bx^3)}+\frac{c}{ax^3}+\frac{d}{ax^2}+\frac{e}{ax}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt{3}a^{5/3}}+\frac{b^{2/3}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}-\frac{e\log\left(a+bx^3\right)}{3a}-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}$$

input
```
Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*c/(a*x^2) - d/(a*x) + (b^(1/3)*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (e*Log[x])/a - (b
^(1/3)*(b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (b^
(2/3)*(c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(6*a^(5/3)) - (e*Log[a + b*x^3])/(3*a)
```

3.343.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

3.343.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.68 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.03

method	result
risch	$\frac{-\frac{xd}{a}-\frac{c}{2a}}{x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^3+3a^4e\_Z^2+\left(3a^3e^2+3a^2bcd\right)\_Z+a^2e^3+3abcde-ab\,d^3+b^2c^3\right)}\_R\ln\left(\left(-4\_R^3a^5-8\_R^2a^4e+(-\right.\right.}{3}$
default	$-\frac{c}{2a\,x^2}-\frac{d}{ax}+\frac{e\ln(x)}{a}-\frac{\left(\left(c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.\right.\right.}{a}$

input
```
int((e*x^2+d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
(-1/a*x*d-1/2*c/a)/x^2+1/3*sum(_R*ln((-4*_R^3*a^5-8*_R^2*a^4*e+(-4*a^3*e^2
-10*a^2*b*c*d)*_R-6*a*b*c*d*e+3*a*b*d^3-3*b^2*c^3)*x-a^4*d*_R^2+(2*a^3*d*e
-a^2*b*c^2)*_R+3*a^2*d*e^2+3*a*b*c^2*e),_R=RootOf(a^5*_Z^3+3*a^4*e*_Z^2+(3
*a^3*e^2+3*a^2*b*c*d)*_Z+a^2*e^3+3*a*b*c*d*e-a*b*d^3+b^2*c^3))+1/a*e*ln(-x
)
```

3.343.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 4279, normalized size of antiderivative = 21.08

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/36*(2*((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3
+ 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3
+ a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3
/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2
*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)*a*x^2*log(1/36*(
(-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*
c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3
- (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/1
8*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2
*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 6*e/a)^2*a^4*d + 2*a*b*c*d^2 - a*
b*c^2*e + a^2*d*e^2 + 1/6*(a^2*b*c^2 - 2*a^3*d*e)*((-I*sqrt(3) + 1)*(e^2/a
^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/
54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*a*b)/
a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^3 + 1/18*(b*c*d + a*e^2)*e/a^4
 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a^2*e^3 - (d^3 - 3*c*d*e)*
a*b)/a^5)^(1/3) + 6*e/a) + (b^2*c^3 + a*b*d^3)*x) - 36*e*x^2*log(x) + 36*d
*x - (((-I*sqrt(3) + 1)*(e^2/a^2 - (b*c*d + a*e^2)/a^3)/(-1/27*e^3/a^3 + 1
/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2*c^3 + a
^2*e^3 - (d^3 - 3*c*d*e)*a*b)/a^5)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27*e^3/a^
3 + 1/18*(b*c*d + a*e^2)*e/a^4 + 1/54*(b*c^3 + a*d^3)*b/a^5 - 1/54*(b^2...
```

3.343.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2}{x^3 (a + bx^3)} \, dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x**3/(b*x**3+a),x)
```

output
```
Timed out
```

3.343.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.87

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx = \frac{e\log(x)}{a} - \frac{\sqrt{3}\left(bd\left(\frac{a}{b}\right)^{\frac{2}{3}} + bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2} - \frac{\left(2e\left(\frac{a}{b}\right)^{\frac{2}{3}} + d\left(\frac{a}{b}\right)^{\frac{1}{3}} - c\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(e\left(\frac{a}{b}\right)^{\frac{2}{3}} - d\left(\frac{a}{b}\right)^{\frac{1}{3}} + c\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{2dx+c}{2ax^2}$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
e*log(x)/a - 1/3*sqrt(3)*(b*d*(a/b)^(2/3) + b*c*(a/b)^(1/3))*arctan(1/3*sq
rt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^2 - 1/6*(2*e*(a/b)^(2/3) + d*(a/b
)^(1/3) - c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*
(e*(a/b)^(2/3) - d*(a/b)^(1/3) + c)*log(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) -
 1/2*(2*d*x + c)/(a*x^2)
```

3.343.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx = -\frac{e\log(|bx^3+a|)}{3a} + \frac{e\log(|x|)}{a} - \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}bc - (-ab^2)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b} - \frac{\left((-ab^2)^{\frac{1}{3}}bc + (-ab^2)^{\frac{2}{3}}d\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b} + \frac{\left(ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ab^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b} - \frac{2dx+c}{2ax^2}$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*e*log(abs(b*x^3 + a))/a + e*log(abs(x))/a - 1/3*sqrt(3)*((-a*b^2)^(1/
3)*b*c - (-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/(a^2*b) - 1/6*((-a*b^2)^(1/3)*b*c + (-a*b^2)^(2/3)*d)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) + 1/3*(a*b^2*d*(-a/b)^(1/3) + a*b^2*c)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) - 1/2*(2*d*x + c)/(a*x^2)
```

3.343.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 701, normalized size of antiderivative = 3.45

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(-\frac{b^5\,c^3\,x-a^2\,b^3\,d\,e^2+\mathrm{root}(27\,a^5\,z^3+27\,a^4\,e\,z^2+9\,a^2\,b\,c\,d\,z+9\,a^3\,e^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2}{}\right.\right.$$
$$\left.\left. +27\,a^4\,e\,z^2+9\,a^2\,b\,c\,d\,z+9\,a^3\,e^2\,z+3\,a\,b\,c\,d\,e-a\,b\,d^3+a^2\,e^3+b^2\,c^3,z,k\right)\right)$$
$$-\frac{c}{2\,a\,x^2}-\frac{d}{a\,x}+\frac{e\,\ln(x)}{a}$$

input
```
int((c + d*x + e*x^2)/(x^3*(a + b*x^3)),x)
```

output
```
symsum(log(-(b^5*c^3*x - a^2*b^3*d*e^2 + 36*root(27*a^5*z^3 + 27*a^4*e*z^2
 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3
, z, k)^3*a^5*b^3*x - a*b^4*c^2*e - a*b^4*d^3*x + root(27*a^5*z^3 + 27*a^4
*e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b
^2*c^3, z, k)*a^2*b^4*c^2 + 3*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c*d
*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)^2*a^4*
b^3*d + 4*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3
*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k)*a^3*b^3*e^2*x + 24*root(27
*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*
d^3 + a^2*e^3 + b^2*c^3, z, k)^2*a^4*b^3*e*x - 2*root(27*a^5*z^3 + 27*a^4*
e*z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^
2*c^3, z, k)*a^3*b^3*d*e + 2*a*b^4*c*d*e*x + 10*root(27*a^5*z^3 + 27*a^4*e
*z^2 + 9*a^2*b*c*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2
*c^3, z, k)*a^2*b^4*c*d*x)/a^3)*root(27*a^5*z^3 + 27*a^4*e*z^2 + 9*a^2*b*c
*d*z + 9*a^3*e^2*z + 3*a*b*c*d*e - a*b*d^3 + a^2*e^3 + b^2*c^3, z, k), k,
1, 3) - c/(2*a*x^2) - d/(a*x) + (e*log(x))/a
```

3.344 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2} dx$

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3.344.1 Optimal result

Integrand size = 23, antiderivative size = 190

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2} dx = -\frac{c+dx+ex^2}{3b(a+bx^3)} - \frac{\left(\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{5/3}} + \frac{\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{5/3}} - \frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}$$

output
```
1/3*(-e*x^2-d*x-c)/b/(b*x^3+a)+1/9*(b^(1/3)*d-2*a^(1/3)*e)*ln(a^(1/3)+b^(1
/3)*x)/a^(2/3)/b^(5/3)-1/18*(d-2*a^(1/3)*e/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/9*(b^(1/3)*d+2*a^(1/3)*e)*arctan(1/3
*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(5/3)*3^(1/2)
```

3.344.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx$$
$$=\frac{-\frac{6b^{2/3}(c+x(d+ex))}{a+bx^3}-\frac{2\sqrt{3}\left(\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{2\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}+\frac{\left(-\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}}{18b^{5/3}}$$

input
```
Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

output
```
((-6*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*(b^(1/3)*d + 2*a^
(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(b^(1/3
)*d - 2*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(b^(1/3)*d) + 2*
a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^
(5/3))
```

3.344.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2363, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx$$
$$\Big\downarrow \; 2363$$
$$\frac{\int \frac{d+2ex}{bx^3+a}dx}{3b}-\frac{c+dx+ex^2}{3b(a+bx^3)}$$
$$\Big\downarrow \; 2399$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)-\sqrt[3]{b}\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3b}-\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)-\sqrt[3]{b}\left(\sqrt[3]{b}d-2\sqrt[3]{a}e\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1082$$

3.344. $\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{-\frac{1}{2}\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2\sqrt[3]{a}e+\sqrt[3]{b}d\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{2\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx+ex^2}{3b\left(a+bx^3\right)}$$

input
```
Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

output
```
-1/3*(c + d*x + e*x^2)/(b*(a + b*x^3)) + (((d - (2*a^(1/3)*e)/b^(1/3))*Log
[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*(b^(1/3)*d + 2*a^
(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - ((d - (2*
a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/2)/(3*
a^(2/3)*b^(1/3)))/(3*b)
```

3.344. $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx$

3.344.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

3.344. $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx$

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.344.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.35

method	result
risch	$\frac{-\frac{e\,x^{2}}{3b}-\frac{dx}{3b}-\frac{c}{3b}}{b\,x^{3}+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(2e\_R+d\right)\ln\left(x-\_R\right)}{\_R^{2}}}{9b^{2}}$
default	$\frac{-\frac{e\,x^{2}}{3b}-\frac{dx}{3b}-\frac{c}{3b}}{b\,x^{3}+a}+\frac{d\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+2e\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)\right.}{6b\left(\right.}\right.}{3b}$

input
```
int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*e*x^2/b-1/3*d*x/b-1/3*c/b)/(b*x^3+a)+1/9/b^2*sum((2*_R*e+d)/_R^2*ln(
x-_R),_R=RootOf(_Z^3*b+a))
```

3.344.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 2077, normalized size of antiderivative = 10.93

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/36*(12*e*x^2 + 2*(b^2*x^3 + a*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 +
 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d
*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)
/(a^2*b^5))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3
)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sq
rt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^
5))^(1/3)))^2*a^2*b^3*e - 1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e
^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*
sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*
b^5))^(1/3)))*a*b^2*d^2 + 8*a*d*e^2 + (b*d^3 + 8*a*e^3)*x) + 12*d*x - ((b^
2*x^3 + a*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (
b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b
^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3))) + 3
*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + 8
*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3) + 4*(1/2)^(2/3)*d*e
*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(
a^2*b^5))^(1/3)))^2*a*b^3 + 32*d*e)/(a*b^3)))*log(-1/2*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((b*d^3 + 8*a*e^3)/(a^2*b^5) + (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3
) + 4*(1/2)^(2/3)*d*e*(I*sqrt(3) - 1)/(a*b^3*((b*d^3 + 8*a*e^3)/(a^2*b^5)
+ (b*d^3 - 8*a*e^3)/(a^2*b^5))^(1/3)))^2*a^2*b^3*e + 1/2*((1/2)^(1/3)*(...
```

3.344.6 Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.58

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx$$
$$=\text{RootSum}\left(729t^3a^2b^5+54tab^2de+8ae^3-bd^3,\left(t\mapsto t\log\left(x+\frac{162t^2a^2b^3e+9tab^2d^2+8ade^2}{8ae^3+bd^3}\right)\right)\right)$$
$$+\frac{-c-dx-ex^2}{3ab+3b^2x^3}$$

input
```
integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

output
```
RootSum(729*_t**3*a**2*b**5 + 54*_t*a*b**2*d*e + 8*a*e**3 - b*d**3, Lambda
(_t, _t*log(x + (162*_t**2*a**2*b**3*e + 9*_t*a*b**2*d**2 + 8*a*d*e**2)/(8
*a*e**3 + b*d**3)))) + (-c - d*x - e*x**2)/(3*a*b + 3*b**2*x**3)
```

3.344.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.86

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx = -\frac{ex^2+dx+c}{3\,(b^2x^3+ab)} + \frac{\sqrt{3}\left(2\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}+d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(2\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}-d\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(2\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}-d\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(e*x^2 + d*x + c)/(b^2*x^3 + a*b) + 1/9*sqrt(3)*(2*e*(a/b)^(1/3) + d)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) + 1
/18*(2*e*(a/b)^(1/3) - d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b
)^(2/3)) - 1/9*(2*e*(a/b)^(1/3) - d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3)
)
```

3.344.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.93

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(bd-2\left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}b} - \frac{\left(bd+2\left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}b} - \frac{\left(2\,e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab} - \frac{ex^2+dx+c}{3\,(bx^3+a)b}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(b*d - 2*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/18*(b*d + 2*(-a*b^2)^(1/3)*e)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/9*(2*e*(-a/
b)^(1/3) + d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 1/3*(e*x^2 +
 d*x + c)/((b*x^3 + a)*b)
```

3.344.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^2}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\frac{2\,d\,e+4\,e^2\,x+\mathrm{root}(729\,a^2\,b^5\,z^3+54\,a\,b^2\,d\,e\,z+8\,a\,e^3-b\,d^3,z,k)^2\,a\,b^3\,81+\mathrm{root}(729\,a^2\,b^5\,z^3+\cdots}{b\,9}\right.\right.$$
$$\left.\left. +54\,a\,b^2\,d\,e\,z+8\,a\,e^3-b\,d^3,z,k)\right)-\frac{\frac{c}{3\,b}+\frac{e\,x^2}{3\,b}+\frac{d\,x}{3\,b}}{b\,x^3+a}$$

input
```
int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^2,x)
```

output
```
symsum(log((2*d*e + 4*e^2*x + 81*root(729*a^2*b^5*z^3 + 54*a*b^2*d*e*z + 8
*a*e^3 - b*d^3, z, k)^2*a*b^3 + 9*root(729*a^2*b^5*z^3 + 54*a*b^2*d*e*z +
8*a*e^3 - b*d^3, z, k)*b^2*d*x)/(9*b))*root(729*a^2*b^5*z^3 + 54*a*b^2*d*e
*z + 8*a*e^3 - b*d^3, z, k), k, 1, 3) - (c/(3*b) + (e*x^2)/(3*b) + (d*x)/(
3*b))/(a + b*x^3)
```

3.345 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2} dx$

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3.345.1 Optimal result

Integrand size = 21, antiderivative size = 200

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2} dx = -\frac{x(ae-bcx-bdx^2)}{3ab(a+bx^3)} - \frac{(b^{2/3}c+a^{2/3}e)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{4/3}} - \frac{(b^{2/3}c-a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{4/3}} + \frac{(b^{2/3}c-a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{4/3}}$$

output
```
-1/3*x*(-b*d*x^2-b*c*x+a*e)/a/b/(b*x^3+a)-1/9*(b^(2/3)*c-a^(2/3)*e)*ln(a^(
1/3)+b^(1/3)*x)/a^(4/3)/b^(4/3)+1/18*(b^(2/3)*c-a^(2/3)*e)*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(4/3)-1/9*(b^(2/3)*c+a^(2/3)*e)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(4/3)*3^(1/2)
```

3.345.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.93

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx = \frac{-\frac{6ab^{2/3}(-bcx^2+a(d+ex))}{a+bx^3}-2\sqrt{3}\left(a^{2/3}bc+a^{4/3}\sqrt[3]{b}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(-a^{2/3}bc+a^{4/3}\sqrt[3]{b}e\right)\log\left(\sqrt[3]{a}+\right.}{18a^2b^{5/3}}$$

input
```
Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

output
```
((-6*a*b^(2/3)*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) - 2*Sqrt[3]*(a^(2/3
)*b*c + a^(4/3)*b^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2
*(-(a^(2/3)*b*c) + a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] - (-(a^(2/3
)*b*c) + a^(4/3)*b^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(18*a^2*b^(5/3))
```

3.345.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2367, 25, 2399, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{2367}$$

$$-\frac{\int -\frac{ae+bcx}{bx^3+a}dx}{3ab}-\frac{x(ae-bcx-bdx^2)}{3ab(a+bx^3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ae+bcx}{bx^3+a}dx}{3ab}-\frac{x(ae-bcx-bdx^2)}{3ab(a+bx^3)}$$

$$\Big\downarrow \text{2399}$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}\left(b^{2/3}c+2a^{2/3}e\right)+\sqrt[3]{b}\left(b^{2/3}c-a^{2/3}e\right)x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}}}{3ab}-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 16

$$\frac{\frac{\int \frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}\left(b^{2/3}c+2a^{2/3}e\right)+\sqrt[3]{b}\left(b^{2/3}c-a^{2/3}e\right)x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3ab}-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(b^{2/3}c+2a^{2/3}e\right)+\sqrt[3]{b}\left(b^{2/3}c-a^{2/3}e\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3ab}-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 1142

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{2/3}e+b^{2/3}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\left(b^{2/3}c-a^{2/3}e\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3ab}-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{2/3}e+b^{2/3}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\left(b^{2/3}c-a^{2/3}e\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3ab}-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}$$

$\Big\downarrow$ 27

3.345. $\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^2}\,dx$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{2/3}e+b^{2/3}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(b^{2/3}c-a^{2/3}e\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3ab}-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\frac{3\left(a^{2/3}e+b^{2/3}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\left(b^{2/3}c-a^{2/3}e\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3ab}-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{-\frac{1}{2}\left(b^{2/3}c-a^{2/3}e\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{2/3}e+b^{2/3}c\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3ab}-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{2/3}e+b^{2/3}c\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\left(b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3ab}-\frac{x\left(ae-bcx-bdx^2\right)}{3ab\left(a+bx^3\right)}$$

input

```
Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^2,x]
```

3.345. $\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^2}\,dx$

output
```
-1/3*(x*(a*e - b*c*x - b*d*x^2))/(a*b*(a + b*x^3)) + (-1/3*((b^(2/3)*c - a
^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*b^(1/3)) + (-((Sqrt[3]*(b^(2/
3)*c + a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) +
((b^(2/3)*c - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
2*b^(1/3)))/(3*a^(1/3)))/(3*a*b)
```

3.345.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.345. $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2} dx$

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.345.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.58 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\frac{c x^2}{3a}-\frac{e x}{3b}-\frac{d}{3b}}{b x^3+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\frac{c\_R}{a}+\frac{e}{b}\right)\ln\left(x-\_R\right)}{\_R^2}}{9b}$
default	$\frac{\frac{c x^2}{3a}-\frac{e x}{3b}-\frac{d}{3b}}{b x^3+a}+\frac{ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{6b\left(\frac{a}{b}\right.}\right.}{3ba}$

input
```
int(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3*c/a*x^2-1/3*e*x/b-1/3*d/b)/(b*x^3+a)+1/9/b*sum((c/a*_R+1/b*e)/_R^2*ln
(x-_R),_R=RootOf(_Z^3*b+a))
```

3.345.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 2358, normalized size of antiderivative = 11.79

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(12*b*c*x^2 - 12*a*e*x - 2*(a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/
3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4
*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^
(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(
a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))^2*a^3*b^3*c - 1/2*((1/2)
^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3
)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3
 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3)))*a^3*b*e^2 +
 2*a*b*c^2*e + (b^2*c^3 + a^2*e^3)*x) - 12*a*d + ((a*b^2*x^3 + a^2*b)*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e
^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c
^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3))) + 3*sqrt(
1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a
^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c
*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a
^2*e^3)/(a^4*b^4))^(1/3)))^2*a^2*b^2 + 16*c*e)/(a^2*b^2)))*log(-1/4*((1/2)
^(1/3)*(I*sqrt(3) + 1)*((b^2*c^3 + a^2*e^3)/(a^4*b^4) - (b^2*c^3 - a^2*e^3
)/(a^4*b^4))^(1/3) - 2*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^2*b^2*((b^2*...
```

3.345.6 Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.62

$$\begin{aligned}&\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx\\&=\operatorname{RootSum}\left(729t^3a^4b^4+27ta^2b^2ce-a^2e^3+b^2c^3,\left(t\mapsto t\log\left(x+\frac{81t^2a^3b^3c+9ta^3be^2+2abc^2e}{a^2e^3+b^2c^3}\right)\right)\right)\\&\quad+\frac{-ad-aex+bcx^2}{3a^2b+3ab^2x^3}\end{aligned}$$

input
```
integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

output
```
RootSum(729*_t**3*a**4*b**4 + 27*_t*a**2*b**2*c*e - a**2*e**3 + b**2*c**3,
 Lambda(_t, _t*log(x + (81*_t**2*a**3*b**3*c + 9*_t*a**3*b*e**2 + 2*a*b*c*
*2*e)/(a**2*e**3 + b**2*c**3)))) + (-a*d - a*e*x + b*c*x**2)/(3*a**2*b + 3
*a*b**2*x**3)
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.92

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx = \frac{bcx^2-aex-ad}{3\,(ab^2x^3+a^2b)} + \frac{\sqrt{3}\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b*c*x^2 - a*e*x - a*d)/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(b*c*(a/b)^(
1/3) + a*e)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/
b)^(2/3)) + 1/18*(b*c*(a/b)^(1/3) - a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3))/(a*b^2*(a/b)^(2/3)) - 1/9*(b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3
))/(a*b^2*(a/b)^(2/3))
```

3.345.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.93

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(ae-(-ab^2)^{\frac{1}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{\left(ae+(-ab^2)^{\frac{1}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{\left(bc\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} + \frac{bcx^2-aex-ad}{3\,(bx^3+a)ab}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(a*e - (-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(a*e + (-a*b^2)^(1/3)*c)*log(x
^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(b*c*(-a/b)^(
1/3) + a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/3*(b*c*x^2
 - a*e*x - a*d)/((b*x^3 + a)*a*b)
```

3.345.9 Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx = \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(729\,a^4\,b^4\,z^3+27\,a^2\,b^2\,c\,e\,z+b^2\,c^3-a^2\,e^3,z,k\right)\,\left(b\,e\,x+\mathrm{root}\left(729\,a^4\,b^4\,z^3+27\,a^2\,b^2\,c\,e\,z+b^2\,c^3-\right.\right.\right.\right.$$
$$\left.\left.+\frac{c\,e}{9\,a}+\frac{b\,c^2\,x}{9\,a^2}\right)\mathrm{root}\left(729\,a^4\,b^4\,z^3+27\,a^2\,b^2\,c\,e\,z+b^2\,c^3-a^2\,e^3,z,k\right)\right)-\frac{\frac{d}{3\,b}-\frac{c\,x^2}{3\,a}+\frac{e\,x}{3\,b}}{b\,x^3+a}$$

input
```
int((x*(c + d*x + e*x^2))/(a + b*x^3)^2,x)
```

output
```
symsum(log(root(729*a^4*b^4*z^3 + 27*a^2*b^2*c*e*z + b^2*c^3 - a^2*e^3, z,
 k)*(b*e*x + 9*root(729*a^4*b^4*z^3 + 27*a^2*b^2*c*e*z + b^2*c^3 - a^2*e^3
, z, k)*a*b^2) + (c*e)/(9*a) + (b*c^2*x)/(9*a^2))*root(729*a^4*b^4*z^3 + 2
7*a^2*b^2*c*e*z + b^2*c^3 - a^2*e^3, z, k), k, 1, 3) - (d/(3*b) - (c*x^2)/
(3*a) + (e*x)/(3*b))/(a + b*x^3)
```

3.345. $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^2}\,dx$

3.346 $\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx$

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3.346.1 Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = -\frac{ae-bx(c+dx)}{3ab(a+bx^3)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}$$

output
```
1/3*(-a*e+b*x*(d*x+c))/a/b/(b*x^3+a)+1/9*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3
)+b^(1/3)*x)/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/9*(2*b^(1/3)*c+a^(1/3)*d)*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

3.346.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx$$
$$=\frac{\frac{6a(-ae+bx(c+dx))}{a+bx^3}-2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(4\sqrt[3]{a}b^{2/3}c-2a^{2/3}\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\ldots}{18a^2b}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```

output
```
((6*a*(-(a*e) + b*x*(c + d*x)))/(a + b*x^3) - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(2
*b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + (4*a
^(1/3)*b^(2/3)*c - 2*a^(2/3)*b^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)
*b^(1/3)*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(18*a^2*b)
```

3.346.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2393, 25, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx$$
$$\Big\downarrow \text{ 2393}$$
$$-\frac{\int -\frac{2c+dx}{bx^3+a}dx}{3a}-\frac{ae-bx(c+dx)}{3ab\,(a+bx^3)}$$
$$\Big\downarrow \text{ 25}$$
$$\frac{\int \frac{2c+dx}{bx^3+a}dx}{3a}-\frac{ae-bx(c+dx)}{3ab\,(a+bx^3)}$$
$$\Big\downarrow \text{ 2399}$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3a}-\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 16

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}-\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 1142

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}-\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 25

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}-\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}-\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 1082

3.346. $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^2}\,dx$

$$\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}-\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}-\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{-\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}-\frac{ae-bx(c+dx)}{3ab\left(a+bx^3\right)}$$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^3)^2,x]
```

output
```
-1/3*(a*e - b*x*(c + d*x))/(a*b*(a + b*x^3)) + (((2*c - (a^(1/3)*d)/b^(1/3
))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*(2*b^(1/3)*
c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - ((2
*c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
2)/(3*a^(2/3)*b^(1/3)))/(3*a)
```

3.346. $\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx$

3.346.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

3.346. $\int \frac{c+dx+ex^2}{(a+bx^3)^2} dx$

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.346.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\frac{d x^2}{3a}+\frac{cx}{3a}-\frac{e}{3b}}{b x^3+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\_Rd+2c\right)\ln\left(x-\_R\right)}{\_R^2}}{9ba}$
default	$c\left(\frac{x}{3a(b x^3+a)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\right)+d\left(\frac{x^2}{3a(b x^3+a)}+\frac{-\frac{\ln\left(x+\left(\frac{a}{b}\right.\right.}{3b\left(\frac{a}{b}\right.}\right.$

input
```
int((e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3*d/a*x^2+1/3*c/a*x-1/3/b*e)/(b*x^3+a)+1/9/b/a*sum((_R*d+2*c)/_R^2*ln(x
-_R),_R=RootOf(_Z^3*b+a))
```

3.346.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 2118, normalized size of antiderivative = 10.64

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(12*b*d*x^2 + 12*b*c*x - 2*(a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) +
 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (
8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(
(8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)
^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3
- a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8
*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(
2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 -
a*d^3)/(a^5*b^2))^(1/3)))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) - 1
2*a*e + ((a*b^2*x^3 + a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^
3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*s
qrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b
^2))^(1/3))) + 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt
(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)
 + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) +
 (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4
*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*
d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3
 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + ...
```

3.346.6 Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.58

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx$$
$$= \operatorname{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right)$$
$$+ \frac{-ae + bcx + bdx^2}{3a^2b + 3ab^2x^3}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

output
```
RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda
(_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d*
*3 + 8*b*c**3)))) + (-a*e + b*c*x + b*d*x**2)/(3*a**2*b + 3*a*b**2*x**3)
```

3.346.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = \frac{bdx^2+bcx-ae}{3\,(ab^2x^3+a^2b)} + \frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b*d*x^2 + b*c*x - a*e)/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(d*(a/b)^(1/
3) + 2*c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(
2/3)) + 1/18*(d*(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/
(a*b*(a/b)^(2/3)) - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a
/b)^(2/3))
```

3.346.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.92

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(2\,bc-\left(-ab^2\right)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}\,a} - \frac{\left(2\,bc+\left(-ab^2\right)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}\,a} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{bdx^2+bcx-ae}{3\left(bx^3+a\right)ab}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)
^(1/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(b*d*x^2 +
 b*c*x - a*e)/((b*x^3 + a)*a*b)
```

3.346.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.88

$$\int \frac{c+dx+ex^2}{(a+bx^3)^2}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{b\left(2\,c\,d+d^2\,x+\mathrm{root}(729\,a^5\,b^2\,z^3+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k)^2\,a^3\,b\,81+\mathrm{root}(729\,a^5\,b^2\,z^3}{a^2\,9} + 54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k)\right)\right) + \frac{\frac{d\,x^2}{3\,a}-\frac{e}{3\,b}+\frac{c\,x}{3\,a}}{b\,x^3+a}$$

input
```
int((c + d*x + e*x^2)/(a + b*x^3)^2,x)
```

output
```
symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z
- 8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*
b*c*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) - e/(3*b) + (c
*x)/(3*a))/(a + b*x^3)
```

3.346. $\int \frac{c+dx+ex^2}{(a+bx^3)^2} \, dx$

3.347 $\int \frac{c+dx+ex^2}{x(a+bx^3)^2}\,dx$

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3.347.1 Optimal result

Integrand size = 23, antiderivative size = 222

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx = \frac{x(ad+aex-bcx^2)}{3a^2\left(a+bx^3\right)} - \frac{\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{c\log(x)}{a^2} + \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}} - \frac{c\log\left(a+bx^3\right)}{3a^2}$$

output
```
1/3*x*(-b*c*x^2+a*e*x+a*d)/a^2/(b*x^3+a)+c*ln(x)/a^2+1/9*(2*b^(1/3)*d-a^(1
/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*d-a^(1/3)*e)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/3*c*ln(b*x^3+a
)/a^2-1/9*(2*b^(1/3)*d+a^(1/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)
*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

3.347.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx$$
$$= \frac{\frac{6a(c+x(d+ex))}{a+bx^3} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + 18c\log(x) + \frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}d-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{(-2\cdots}{\cdots}}{18a^2}$$

input
```
Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```

output
```
((6*a*(c + x*(d + e*x)))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*d + a
^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 18*c*Log[
x] + (2*(2*a^(1/3)*b^(1/3)*d - a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3
) + ((-2*a^(1/3)*b^(1/3)*d + a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/b^(2/3) - 6*c*Log[a + b*x^3])/(18*a^2)
```

3.347.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \; 2368$$

$$\frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)} - \frac{\int -\frac{bex^2+2bdx+3bc}{x(bx^3+a)}dx}{3ab}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{bex^2+2bdx+3bc}{x(bx^3+a)}dx}{3ab} + \frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2373}$$

$$\frac{\int \left(\frac{3bc}{ax}+\frac{b(-3bcx^2+aex+2ad)}{a(bx^3+a)}\right)dx}{3ab}+\frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)}{\sqrt{3}a^{2/3}}-\frac{\sqrt[3]{b}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}}+\frac{\sqrt[3]{b}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\ldots}\right.}{3a^{2/3}}}{3ab}$$

$$\frac{x\left(ad+aex-bcx^2\right)}{3a^2\left(a+bx^3\right)}$$

input
```
Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^2),x]
```

output
```
(x*(a*d + a*e*x - b*c*x^2))/(3*a^2*(a + b*x^3)) + (-((b^(1/3)*(2*b^(1/3)*d
 + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(2/3))) + (3*b*c*Log[x])/a + (b^(1/3)*(2*b^(1/3)*d - a^(1/3)*e)*Log[a^(1
/3) + b^(1/3)*x])/(3*a^(2/3)) - (b^(1/3)*(2*b^(1/3)*d - a^(1/3)*e)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)) - (b*c*Log[a + b*x^3]
)/a)/(3*a*b)
```

3.347.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.347.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.06

method	result
risch	$\frac{\frac{e\,x^2}{3a}+\frac{xd}{3a}+\frac{c}{3a}}{b\,x^3+a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^6b^2\_Z^3+9a^4b^2c\_Z^2+\left(6a^3bde+27a^2b^2c^2\right)\_Z+a^2e^3+18abcde-8ab\,d^3+27b^2c^3\right)}\_R\ln\left(\left(-4\_R^3a^5b^2\cdots\right.\right.\right.}{\cdots}$
default	$\frac{c\ln(x)}{a^2}+\frac{\frac{\frac{1}{3}ae\,x^2+\frac{1}{3}adx+\frac{1}{3}ac}{b\,x^3+a}+\frac{2ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}+\frac{ae\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\cdots\right)}{\cdots}}{a^2}$

input
```
int((e*x^2+d*x+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3/a*e*x^2+1/3/a*x*d+1/3*c/a)/(b*x^3+a)+1/9*sum(_R*ln((-4*_R^3*a^5*b^2-2
4*_R^2*a^3*b^2*c+(-20*a^2*b*d*e-36*a*b^2*c^2)*_R-3*a*e^3-36*b*c*d*e+24*b*d
^3)*x+a^4*b*e*_R^2+(-6*a^2*b*c*e-4*a^2*b*d^2)*_R-27*b*c^2*e+36*b*c*d^2),_R
=RootOf(a^6*b^2*_Z^3+9*a^4*b^2*c*_Z^2+(6*a^3*b*d*e+27*a^2*b^2*c^2)*_Z+a^2*
e^3+18*a*b*c*d*e-8*a*b*d^3+27*b^2*c^3))+1/a^2*c*ln(-x)
```

3.347.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 5018, normalized size of antiderivative = 22.60

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.347.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.347.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx &= \frac{ex^2+dx+c}{3\left(abx^3+a^2\right)} + \frac{c\log\left(x\right)}{a^2} \\ &+ \frac{\sqrt{3}\left(ae\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3} \\ &- \frac{\left(6\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}} - ae\left(\frac{a}{b}\right)^{\frac{1}{3}} + 2\,ad\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &- \frac{\left(3\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}} + ae\left(\frac{a}{b}\right)^{\frac{1}{3}} - 2\,ad\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(e*x^2 + d*x + c)/(a*b*x^3 + a^2) + c*log(x)/a^2 + 1/9*sqrt(3)*(a*e*(a
/b)^(2/3) + 2*a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/a^3 - 1/18*(6*b*c*(a/b)^(2/3) - a*e*(a/b)^(1/3) + 2*a*d)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/9*(3*b*c*(a/b)^(2/3
) + a*e*(a/b)^(1/3) - 2*a*d)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

3.347.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx = -\frac{\sqrt{3}\left(2\,bd-\left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{\left(2\,bd+\left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{c\log\left(\left|bx^3+a\right|\right)}{3\,a^2} + \frac{c\log\left(\left|x\right|\right)}{a^2} + \frac{aex^2+adx+ac}{3\left(bx^3+a\right)a^2} - \frac{\left(a^3be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^3bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b}$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b*d - (-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*d + (-a*b^2)^(1/3)*e)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/3*c*log(abs
(b*x^3 + a))/a^2 + c*log(abs(x))/a^2 + 1/3*(a*e*x^2 + a*d*x + a*c)/((b*x^3
 + a)*a^2) - 1/9*(a^3*b*e*(-a/b)^(1/3) + 2*a^3*b*d)*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a^5*b)
```

3.347.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.21

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^2}\,dx=\frac{\frac{c}{3\,a}+\frac{e\,x^2}{3\,a}+\frac{d\,x}{3\,a}}{b\,x^3+a}+\left(\sum_{k=1}^{3}\ln\left(\frac{4\,b^2\,c\,d^2-3\,b^2\,c^2\,e}{9\,a^3}-\mathrm{root}\left(729\,a^6\,b^2\,z^3+729\,a^4\,b^2\,c\,z^2+54\,a^3\,b\,d\,e\,z+243\,a^2\,b^2\,c^2\,z+18\,a\,b\,c\,d\,e-8\,a\,b\,d^3+27\,b^2\,c^3+a^2\,e^3,z,k\right)-\frac{x\left(-8\,b^2\,d^3+12\,c\,b^2\,d\,e+a\,b\,e^3\right)}{27\,a^3}\right)\mathrm{root}\left(729\,a^6\,b^2\,z^3+729\,a^4\,b^2\,c\,z^2+54\,a^3\,b\,d\,e\,z+243\,a^2\,b^2\,c^2\,z+18\,a\,b\,c\,d\,e-8\,a\,b\,d^3+27\,b^2\,c^3+a^2\,e^3,z,k\right)\right)+\frac{c\,\ln\left(x\right)}{a^2}$$

input
```
int((c + d*x + e*x^2)/(x*(a + b*x^3)^2),x)
```

output
```
(c/(3*a) + (e*x^2)/(3*a) + (d*x)/(3*a))/(a + b*x^3) + symsum(log((4*b^2*c*
d^2 - 3*b^2*c^2*e)/(9*a^3) - root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54
*a^3*b*d*e*z + 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 +
 a^2*e^3, z, k)*(root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a^3*b*d*e*z
 + 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^2*e^3, z,
 k)*(24*b^3*c*x - a*b^2*e + 36*root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 +
54*a^3*b*d*e*z + 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3
 + a^2*e^3, z, k)*a^2*b^3*x) + (4*a^2*b^2*d^2 + 6*a^2*b^2*c*e)/(9*a^3) + (
x*(108*a*b^3*c^2 + 60*a^2*b^2*d*e))/(27*a^3)) - (x*(a*b*e^3 - 8*b^2*d^3 +
12*b^2*c*d*e))/(27*a^3))*root(729*a^6*b^2*z^3 + 729*a^4*b^2*c*z^2 + 54*a^3
*b*d*e*z + 243*a^2*b^2*c^2*z + 18*a*b*c*d*e - 8*a*b*d^3 + 27*b^2*c^3 + a^2
*e^3, z, k), k, 1, 3) + (c*log(x))/a^2
```

3.348 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx$

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3.348.1 Optimal result

Integrand size = 23, antiderivative size = 231

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx = &-\frac{c}{a^2x}+\frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)}+\frac{2\left(2b^{2/3}c-a^{2/3}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{b}}\\&+\frac{d\log(x)}{a^2}+\frac{2\left(2b^{2/3}c+a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}\sqrt[3]{b}}\\&-\frac{\left(2b^{2/3}c+a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{7/3}\sqrt[3]{b}}-\frac{d\log(a+bx^3)}{3a^2}\end{aligned}$$

output
```
-c/a^2/x+1/3*x*(-b*d*x^2-b*c*x+a*e)/a^2/(b*x^3+a)+d*ln(x)/a^2+2/9*(2*b^(2/
3)*c+a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(1/3)-1/9*(2*b^(2/3)*c+a^(
2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(1/3)-1/3*d*ln
(b*x^3+a)/a^2+2/9*(2*b^(2/3)*c-a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)
/a^(1/3)*3^(1/2))/a^(7/3)/b^(1/3)*3^(1/2)
```

3.348.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.92

$$\int \frac{c+dx+ex^2}{x^2 (a+bx^3)^2} \, dx =$$

$$-\frac{\frac{9ac}{x} - \frac{3a(-bcx^2+a(d+ex))}{a+bx^3} + \frac{2\sqrt{3}a^{2/3}(-2b^{2/3}c+a^{2/3}e)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - 9ad\log(x) - \frac{2(2a^{2/3}b^{2/3}c+a^{4/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{\sqrt[3]{b}}}{9a^3}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```

output
```
-1/9*((9*a*c)/x - (3*a*(-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3) + (2*Sqrt[3
]*a^(2/3)*(-2*b^(2/3)*c + a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(1/3) - 9*a*d*Log[x] - (2*(2*a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[
a^(1/3) + b^(1/3)*x])/b^(1/3) + ((2*a^(2/3)*b^(2/3)*c + a^(4/3)*e)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 3*a*d*Log[a + b*x^3])/a
^3
```

3.348.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^2 (a+bx^3)^2} \, dx$$

$$\Big\downarrow \; 2368$$

$$\frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)} - \frac{\int -\frac{-\frac{b^2cx^3}{a}+2bex^2+3bdx+3bc}{x^2(bx^3+a)}dx}{3ab}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-\frac{b^2cx^3}{a}+2bex^2+3bdx+3bc}{x^2(bx^3+a)}dx}{3ab}+\frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)}$$

$$\Big\downarrow \text{2373}$$

$$\frac{\int \left(\frac{3bc}{ax^2}+\frac{3bd}{ax}+\frac{b(-3bdx^2-4bcx+2ae)}{a(bx^3+a)}\right)dx}{3ab}+\frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(2b^{2/3}c-a^{2/3}e)}{\sqrt{3}a^{4/3}}-\frac{b^{2/3}(a^{2/3}e+2b^{2/3}c)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{3a^{4/3}}+\frac{2b^{2/3}(a^{2/3}e+2b^{2/3}c)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}}}{3ab}$$

$$\frac{x(ae-bcx-bdx^2)}{3a^2(a+bx^3)}$$

input
```
Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x]
```

output
```
(x*(a*e - b*c*x - b*d*x^2))/(3*a^2*(a + b*x^3)) + ((-3*b*c)/(a*x) + (2*b^(
2/3)*(2*b^(2/3)*c - a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(4/3)) + (3*b*d*Log[x])/a + (2*b^(2/3)*(2*b^(2/3)*c + a
^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)) - (b^(2/3)*(2*b^(2/3)*c +
a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3*a^(4/3)) - (
b*d*Log[a + b*x^3])/a)/(3*a*b)
```

3.348.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.348. $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx$

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.348.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.06

method	result
risch	$\frac{-\frac{4bc x^3}{3a^2}+\frac{e x^2}{3a}+\frac{xd}{3a}-\frac{c}{a}}{x(b x^3+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^7 b \_Z^3+9a^5 bd \_Z^2+\left(-24a^3bce+27a^3bd^2\right)\_Z-8a^2e^3-72abcde+27abd^3-64b^2c^3\right)} \_R\ln\left(\left(-\ldots\right.\right.\right.}{\ldots}$
default	$-\frac{c}{a^2 x}+\frac{d\ln(x)}{a^2}+\frac{\frac{-\frac{1}{3}cbx^2+\frac{1}{3}aex+\frac{1}{3}ad}{bx^3+a}+\frac{2ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}-\frac{4bc\left(-\frac{\ln\ldots}{\ldots}\right.}{\ldots}}{a^2}$

input
```
int((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4/3*b*c/a^2*x^3+1/3/a*e*x^2+1/3/a*x*d-c/a)/x/(b*x^3+a)+1/9*sum(_R*ln((-_
R^3*a^7*b-6*_R^2*a^5*b*d+(20*a^3*b*c*e-9*a^3*b*d^2)*_R+6*a^2*e^3+36*a*b*c*
d*e+48*b^2*c^3)*x-a^5*b*c*_R^2+(-a^4*e^2+6*a^3*b*c*d)*_R+9*a^2*d*e^2+27*a*
b*c*d^2),_R=RootOf(a^7*b*_Z^3+9*a^5*b*d*_Z^2+(-24*a^3*b*c*e+27*a^3*b*d^2)*
_Z-8*a^2*e^3-72*a*b*c*d*e+27*a*b*d^3-64*b^2*c^3))+d*ln(x)/a^2
```

3.348.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 4976, normalized size of antiderivative = 21.54

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/324*(432*b*c*x^3 - 108*a*e*x^2 - 108*a*d*x + 2*(a^2*b*x^4 + a^3*x)*((-I
*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*
d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*
a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3)
+ 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8
*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/
(a^7*b))^(1/3) + 54*d/a^2)*log(-1/324*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^
2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*
b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3
- a^2*e^3)/(a^7*b))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d
^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a
*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2)^2*a^5
*b*c - 9*a*b*c*d^2 + 16*a*b*c^2*e + 3*a^2*d*e^2 + 1/18*(6*a^3*b*c*d - a^4*
e^2)*((-I*sqrt(3) + 1)*(9*d^2/a^4 - (9*d^2 - 8*c*e)/a^4)/(-1/27*d^3/a^6 +
1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^2*c^3 + 8*a^2*e^3 - 9*(3*d^3 -
8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 - a^2*e^3)/(a^7*b))^(1/3) + 81*(I
*sqrt(3) + 1)*(-1/27*d^3/a^6 + 1/162*(9*d^2 - 8*c*e)*d/a^6 + 1/1458*(64*b^
2*c^3 + 8*a^2*e^3 - 9*(3*d^3 - 8*c*d*e)*a*b)/(a^7*b) - 4/729*(8*b^2*c^3 -
a^2*e^3)/(a^7*b))^(1/3) + 54*d/a^2) - 2*(8*b^2*c^3 - a^2*e^3)*x) + 324*a*c
 + (162*b*d*x^4 + 162*a*d*x - (a^2*b*x^4 + a^3*x)*((-I*sqrt(3) + 1)*(9*...
```

3.348.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx = -\frac{4\,bcx^3-aex^2-adx+3\,ac}{3\,(a^2bx^4+a^3x)} + \frac{d\log(x)}{a^2} - \frac{2\,\sqrt{3}\left(2\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3} - \frac{\left(3\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(3\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(4*b*c*x^3 - a*e*x^2 - a*d*x + 3*a*c)/(a^2*b*x^4 + a^3*x) + d*log(x)/
a^2 - 2/9*sqrt(3)*(2*b*c*(a/b)^(2/3) - a*e*(a/b)^(1/3))*arctan(1/3*sqrt(3)
*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^3 - 1/9*(3*b*d*(a/b)^(2/3) + 2*b*c*(a/
b)^(1/3) + a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3))
 - 1/9*(3*b*d*(a/b)^(2/3) - 4*b*c*(a/b)^(1/3) - 2*a*e)*log(x + (a/b)^(1/3)
)/(a^2*b*(a/b)^(2/3))
```

3.348.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx = -\frac{d\log\left(|bx^3+a|\right)}{3\,a^2} + \frac{d\log\left(|x|\right)}{a^2} + \frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}ae + 2\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} - \frac{4\,bcx^3 - aex^2 - adx + 3\,ac}{3\,(bx^4+ax)a^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}ae - 2\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3b} + \frac{2\left(2\,a^2b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^3be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b}$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*d*log(abs(b*x^3 + a))/a^2 + d*log(abs(x))/a^2 + 2/9*sqrt(3)*((-a*b^2)
^(1/3)*a*e + 2*(-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/(a^3*b) - 1/3*(4*b*c*x^3 - a*e*x^2 - a*d*x + 3*a*c)/((b*x^4 +
 a*x)*a^2) + 1/9*((-a*b^2)^(1/3)*a*e - 2*(-a*b^2)^(2/3)*c)*log(x^2 + x*(-a
/b)^(1/3) + (-a/b)^(2/3))/(a^3*b) + 2/9*(2*a^2*b^2*c*(-a/b)^(1/3) - a^3*b*
e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b)
```

3.348.9 Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.11

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx = \left(\sum_{k=1}^{3}\ln\left(-\mathrm{root}\left(729\,a^7\,b\,z^3+729\,a^5\,b\,d\,z^2-216\,a^3\,b\,c\,e\,z+243\,a^3\,b\,d^2\,z-72\,a\,b\,c\,d\,e+27\,a\,b\,d^3-8\,a^2\,e^3-64\,b\right.\right.\right.$$
$$\left. + \frac{4\,(3\,c\,b^3\,d^2 + a\,b^2\,d\,e^2)}{9\,a^4} + \frac{4\,x\,(2\,a^2\,b^2\,e^3 + 12\,d\,a\,b^3\,c\,e + 16\,b^4\,c^3)}{27\,a^5}\right)\mathrm{root}\left(729\,a^7\,b\,z^3\right.$$
$$\left.\left. +729\,a^5\,b\,d\,z^2-216\,a^3\,b\,c\,e\,z+243\,a^3\,b\,d^2\,z-72\,a\,b\,c\,d\,e+27\,a\,b\,d^3-8\,a^2\,e^3-64\,b^2\,c^3,z,k\right)\right)$$
$$-\frac{\frac{c}{a} - \frac{e\,x^2}{3\,a} - \frac{d\,x}{3\,a} + \frac{4\,b\,c\,x^3}{3\,a^2}}{b\,x^4 + a\,x} + \frac{d\,\ln(x)}{a^2}$$

input
```
int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^2),x)
```

output
```
symsum(log((4*(3*b^3*c*d^2 + a*b^2*d*e^2))/(9*a^4) - root(729*a^7*b*z^3 +
729*a^5*b*d*z^2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*
b*d^3 - 8*a^2*e^3 - 64*b^2*c^3, z, k)*(root(729*a^7*b*z^3 + 729*a^5*b*d*z^
2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*d^3 - 8*a^2*
e^3 - 64*b^2*c^3, z, k)*(4*b^3*c + 24*b^3*d*x + 36*root(729*a^7*b*z^3 + 72
9*a^5*b*d*z^2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e + 27*a*b*
d^3 - 8*a^2*e^3 - 64*b^2*c^3, z, k)*a^2*b^3*x) + (4*(a^3*b^2*e^2 - 6*a^2*b
^3*c*d))/(9*a^4) + (4*x*(27*a^3*b^3*d^2 - 60*a^3*b^3*c*e))/(27*a^5)) + (4*
x*(16*b^4*c^3 + 2*a^2*b^2*e^3 + 12*a*b^3*c*d*e))/(27*a^5))*root(729*a^7*b*
z^3 + 729*a^5*b*d*z^2 - 216*a^3*b*c*e*z + 243*a^3*b*d^2*z - 72*a*b*c*d*e +
 27*a*b*d^3 - 8*a^2*e^3 - 64*b^2*c^3, z, k), k, 1, 3) - (c/a - (e*x^2)/(3*
a) - (d*x)/(3*a) + (4*b*c*x^3)/(3*a^2))/(a*x + b*x^4) + (d*log(x))/a^2
```

3.348. $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^2}\,dx$

3.349 $\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx$

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3.349.1 Optimal result

Integrand size = 23, antiderivative size = 242

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx=-\frac{c}{2a^2x^2}-\frac{d}{a^2x}-\frac{x(bc+bdx+bex^2)}{3a^2\left(a+bx^3\right)}+\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c+4\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}}+\frac{e\log(x)}{a^2}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}+\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{e\log\left(a+bx^3\right)}{3a^2}$$

output
```
-1/2*c/a^2/x^2-d/a^2/x-1/3*x*(b*e*x^2+b*d*x+b*c)/a^2/(b*x^3+a)+e*ln(x)/a^2
-1/9*b^(1/3)*(5*b^(1/3)*c-4*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)+1/18*
b^(1/3)*(5*b^(1/3)*c-4*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/a^(8/3)-1/3*e*ln(b*x^3+a)/a^2+1/9*b^(1/3)*(5*b^(1/3)*c+4*a^(1/3)*d)*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

3.349.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.91

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{9ac}{x^2}-\frac{18ad}{x}+\frac{6a(ae-bx(c+dx))}{a+bx^3}+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(5\sqrt[3]{b}c+4\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+18ae\log(x)+2\sqrt[3]{b}\left(-5\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```

output
```
((-9*a*c)/x^2 - (18*a*d)/x + (6*a*(a*e - b*x*(c + d*x)))/(a + b*x^3) + 2*S
qrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]] + 18*a*e*Log[x] + 2*b^(1/3)*(-5*a^(1/3)*b^(1/3)*c + 4
*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(5*a^(1/3)*b^(1/3)*c - 4*a^
(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 6*a*e*Log[a + b*
x^3])/(18*a^3)
```

3.349.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^2}\,dx$$

$$\Big\downarrow \ 2368$$

$$-\frac{\int -\frac{-\frac{b^2dx^4}{a}-\frac{2b^2cx^3}{a}+3bex^2+3bdx+3bc}{x^3(bx^3+a)}dx}{3ab}-\frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-\frac{b^2dx^4}{a}-\frac{2b^2cx^3}{a}+3bex^2+3bdx+3bc}{x^3(bx^3+a)}dx}{3ab}-\frac{x(bc+bdx+bex^2)}{3a^2(a+bx^3)}$$

$$\Big\downarrow \text{2373}$$

$$\frac{\int \left(-\frac{(3ex^2+4dx+5c)b^2}{a(bx^3+a)}+\frac{3eb}{ax}+\frac{3db}{ax^2}+\frac{3cb}{ax^3}\right)dx}{3ab}-\frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(4\sqrt[3]{a}d+5\sqrt[3]{b}c\right)}{\sqrt{3}a^{5/3}}+\frac{b^{4/3}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{b^{4/3}\left(5\sqrt[3]{b}c-4\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\right.}{3a^{5/3}}}{3ab}$$

$$\frac{x\left(bc+bdx+bex^2\right)}{3a^2\left(a+bx^3\right)}$$

input
```
Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x]
```

output
```
-1/3*(x*(b*c + b*d*x + b*e*x^2))/(a^2*(a + b*x^3)) + ((-3*b*c)/(2*a*x^2) -
 (3*b*d)/(a*x) + (b^(4/3)*(5*b^(1/3)*c + 4*a^(1/3)*d)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (3*b*e*Log[x])/a - (b^(
4/3)*(5*b^(1/3)*c - 4*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (
b^(4/3)*(5*b^(1/3)*c - 4*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(6*a^(5/3)) - (b*e*Log[a + b*x^3])/a)/(3*a*b)
```

3.349.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.349. $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^2}\,dx$

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.349.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.61 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.03

method	result
risch	$\frac{-\frac{4bd\,x^4}{3a^2}-\frac{5bc\,x^3}{6a^2}+\frac{e\,x^2}{3a}-\frac{xd}{a}-\frac{c}{2a}}{x^2(b\,x^3+a)}+\frac{e\ln(-x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^8\_Z^3+9a^6e\_Z^2+\left(27a^4e^2+60a^3bcd\right)\_Z+27a^2e^3+180abcde-64ab\,d^3+1\right.}\right.}{}$
default	$-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\ln(x)}{a^2}-\frac{b\left(\frac{\frac{d\,x^2}{3}+\frac{cx}{3}-\frac{ae}{3b}}{b\,x^3+a}+\frac{5c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}+\cdots 4a\right.}{a^2}$

input
```
int((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4/3*b*d/a^2*x^4-5/6*b*c/a^2*x^3+1/3/a*e*x^2-1/a*x*d-1/2*c/a)/x^2/(b*x^3+
a)+1/a^2*e*ln(-x)+1/9*sum(_R*ln((-4*_R^3*a^8-24*_R^2*a^6*e+(-36*a^4*e^2-20
0*a^3*b*c*d)*_R-360*a*b*c*d*e+192*a*b*d^3-375*b^2*c^3)*x-4*a^6*d*_R^2+(24*
a^4*d*e-25*a^3*b*c^2)*_R+108*a^2*d*e^2+225*a*b*c^2*e),_R=RootOf(a^8*_Z^3+9
*a^6*e*_Z^2+(27*a^4*e^2+60*a^3*b*c*d)*_Z+27*a^2*e^3+180*a*b*c*d*e-64*a*b*d
^3+125*b^2*c^3))
```

3.349.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 4774, normalized size of antiderivative = 19.73

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/324*(432*b*d*x^4 + 270*b*c*x^3 - 108*a*e*x^2 + 324*a*d*x + 2*(a^2*b*x^5
 + a^3*x^2)*((-I*sqrt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/2
7*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^
3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a
^8)^(1/3) + 81*(I*sqrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)
*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^
2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)*log(1/81*((-I*sq
rt(3) + 1)*(9*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(
20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(
125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*s
qrt(3) + 1)*(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(12
5*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 -
 45*c*d*e)*a*b)/a^8)^(1/3) + 54*e/a^2)^2*a^6*d + 160*a*b*c*d^2 - 75*a*b*c^
2*e + 36*a^2*d*e^2 + 1/18*(25*a^3*b*c^2 - 24*a^4*d*e)*((-I*sqrt(3) + 1)*(9
*e^2/a^4 - (20*b*c*d + 9*a*e^2)/a^5)/(-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*
a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 +
 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*b)/a^8)^(1/3) + 81*(I*sqrt(3) + 1)*(
-1/27*e^3/a^6 + 1/162*(20*b*c*d + 9*a*e^2)*e/a^7 + 1/1458*(125*b*c^3 + 64*
a*d^3)*b/a^8 - 1/1458*(125*b^2*c^3 + 27*a^2*e^3 - 4*(16*d^3 - 45*c*d*e)*a*
b)/a^8)^(1/3) + 54*e/a^2) + (125*b^2*c^3 + 64*a*b*d^3)*x) + 162*a*c + (...
```

3.349.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.349.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.91

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^2}\,dx = -\frac{8\,bdx^4+5\,bcx^3-2\,aex^2+6\,adx+3\,ac}{6\,(a^2bx^5+a^3x^2)}+\frac{e\log(x)}{a^2} - \frac{\sqrt{3}\left(4\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3} - \frac{\left(6\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(3\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(8*b*d*x^4 + 5*b*c*x^3 - 2*a*e*x^2 + 6*a*d*x + 3*a*c)/(a^2*b*x^5 + a^
3*x^2) + e*log(x)/a^2 - 1/9*sqrt(3)*(4*b*d*(a/b)^(2/3) + 5*b*c*(a/b)^(1/3)
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^3 - 1/18*(6*e*(a/b
)^(2/3) + 4*d*(a/b)^(1/3) - 5*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
^2*(a/b)^(2/3)) - 1/9*(3*e*(a/b)^(2/3) - 4*d*(a/b)^(1/3) + 5*c)*log(x + (a
/b)^(1/3))/(a^2*(a/b)^(2/3))
```

3.349.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^2}\,dx = -\frac{e\log(|bx^3+a|)}{3\,a^2}+\frac{e\log(|x|)}{a^2} - \frac{\sqrt{3}\left(5\,(-ab^2)^{\frac{1}{3}}\,bc-4\,(-ab^2)^{\frac{2}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} - \frac{\left(5\,(-ab^2)^{\frac{1}{3}}\,bc+4\,(-ab^2)^{\frac{2}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b} + \frac{\left(4\,a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^5b} - \frac{8\,bdx^4+5\,bcx^3-2\,aex^2+6\,adx+3\,ac}{6\,(bx^3+a)a^2x^2}$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*e*log(abs(b*x^3 + a))/a^2 + e*log(abs(x))/a^2 - 1/9*sqrt(3)*(5*(-a*b^
2)^(1/3)*b*c - 4*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))/(a^3*b) - 1/18*(5*(-a*b^2)^(1/3)*b*c + 4*(-a*b^2)^(2/3)*d)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b) + 1/9*(4*a^2*b^2*d*(-a/b)
^(1/3) + 5*a^2*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b) - 1/
6*(8*b*d*x^4 + 5*b*c*x^3 - 2*a*e*x^2 + 6*a*d*x + 3*a*c)/((b*x^3 + a)*a^2*x
^2)
```

3.349.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 733, normalized size of antiderivative = 3.03

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^2}\,dx$$

$$= \left(\sum_{k=1}^{3}\ln\left(-\frac{b^3\left(\mathrm{root}(729\,a^8\,z^3+729\,a^6\,e\,z^2+540\,a^3\,b\,c\,d\,z+243\,a^4\,e^2\,z+180\,a\,b\,c\,d\,e-64\,a\,b\,d^3+27\,a^2\,e\cdots\right.}{\cdots}\right.\right.$$

$$+729\,a^6\,e\,z^2+540\,a^3\,b\,c\,d\,z+243\,a^4\,e^2\,z+180\,a\,b\,c\,d\,e-64\,a\,b\,d^3+27\,a^2\,e^3$$

$$\left.\left.+125\,b^2\,c^3,z,k\right)\right)-\frac{\frac{c}{2\,a}-\frac{e\,x^2}{3\,a}+\frac{d\,x}{a}+\frac{5\,b\,c\,x^3}{6\,a^2}+\frac{4\,b\,d\,x^4}{3\,a^2}}{b\,x^5+a\,x^2}+\frac{e\,\ln\left(x\right)}{a^2}$$

input
```
int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^2),x)
```

output
```
symsum(log(-(b^3*(108*root(729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z +
 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z,
 k)^2*a^6*d - 36*a^2*d*e^2 + 972*root(729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^
3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*
b^2*c^3, z, k)^3*a^8*x + 125*b^2*c^3*x - 72*root(729*a^8*z^3 + 729*a^6*e*z
^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2
*e^3 + 125*b^2*c^3, z, k)*a^4*d*e - 75*a*b*c^2*e - 64*a*b*d^3*x + 75*root(
729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*
d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k)*a^3*b*c^2 + 108*root(72
9*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*
e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k)*a^4*e^2*x + 648*root(729*
a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e
- 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k)^2*a^6*e*x + 600*root(729*a^
8*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*e^2*z + 180*a*b*c*d*e -
64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k)*a^3*b*c*d*x + 120*a*b*c*d*e*x
))/(27*a^6))*root(729*a^8*z^3 + 729*a^6*e*z^2 + 540*a^3*b*c*d*z + 243*a^4*
e^2*z + 180*a*b*c*d*e - 64*a*b*d^3 + 27*a^2*e^3 + 125*b^2*c^3, z, k), k, 1
, 3) - (c/(2*a) - (e*x^2)/(3*a) + (d*x)/a + (5*b*c*x^3)/(6*a^2) + (4*b*d*x
^4)/(3*a^2))/(a*x^2 + b*x^5) + (e*log(x))/a^2
```

3.349. $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^2}\,dx$

3.350 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx$

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3.350.1 Optimal result

Integrand size = 23, antiderivative size = 262

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx = -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{3a^2(a+bx^3)} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}} - \frac{2bc\log(x)}{a^3}-\frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}} + \frac{\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}} + \frac{2bc\log(a+bx^3)}{3a^3}$$

output
```
-1/3*c/a^2/x^3-1/2*d/a^2/x^2-e/a^2/x-1/3*x*(b*d+b*e*x-b^2*c*x^2/a)/a^2/(b*
x^3+a)-2*b*c*ln(x)/a^3-1/9*b^(1/3)*(5*b^(1/3)*d-4*a^(1/3)*e)*ln(a^(1/3)+b^
(1/3)*x)/a^(8/3)+1/18*b^(1/3)*(5*b^(1/3)*d-4*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)+2/3*b*c*ln(b*x^3+a)/a^3+1/9*b^(1/3)*(5*b^(
1/3)*d+4*a^(1/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8
/3)*3^(1/2)
```

3.350.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.86

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{6ac}{x^3}-\frac{9ad}{x^2}-\frac{18ae}{x}-\frac{6ab(c+x(d+ex))}{a+bx^3}+2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-36bc\log(x)+2\sqrt[3]{b}\cdots}{\cdots}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

output
```
((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x - (6*a*b*(c + x*(d + e*x)))/(a +
b*x^3) + 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 36*b*c*Log[x] + 2*b^(1/3)*(-5*a^(1/3)*b
^(1/3)*d + 4*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*(5*a^(1/3)*b^(1
/3)*d - 4*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 12*b
*c*Log[a + b*x^3])/(18*a^3)
```

3.350.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \; 2368$$

$$-\frac{\int -\frac{-\frac{b^2ex^5}{a}-\frac{2b^2dx^4}{a}-\frac{3b^2cx^3}{a}+3bex^2+3bdx+3bc}{x^4\left(bx^3+a\right)}dx}{3ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{3a^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-\frac{b^2ex^5}{a}-\frac{2b^2dx^4}{a}-\frac{3b^2cx^3}{a}+3bex^2+3bdx+3bc}{x^4\left(bx^3+a\right)}dx}{3ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{3a^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2373}$$

$$\frac{\int \left(-\frac{6cb^2}{a^2x}-\frac{(-6bcx^2+4aex+5ad)b^2}{a^2(bx^3+a)}+\frac{3eb}{ax^2}+\frac{3db}{ax^3}+\frac{3cb}{ax^4}\right)dx}{3ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{3a^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)}{\sqrt{3}a^{5/3}}+\frac{b^{4/3}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{b^{4/3}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\right.}{3a^{5/3}}}{3ab}$$

$$\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{3a^2\left(a+bx^3\right)}$$

input
```
Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x]
```

output
```
-1/3*(x*(b*d + b*e*x - (b^2*c*x^2)/a))/(a^2*(a + b*x^3)) + (-((b*c)/(a*x^3
)) - (3*b*d)/(2*a*x^2) - (3*b*e)/(a*x) + (b^(4/3)*(5*b^(1/3)*d + 4*a^(1/3)
*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) -
 (6*b^2*c*Log[x])/a^2 - (b^(4/3)*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) +
 b^(1/3)*x])/(3*a^(5/3)) + (b^(4/3)*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)) + (2*b^2*c*Log[a + b*x^3
])/a^2)/(3*a*b)
```

3.350.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.350.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.03

method	result
default	$-\frac{e}{a^2 x}-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{2bc\ln(x)}{a^3}-\frac{b\left(\frac{\frac{1}{3}ae\,x^2+\frac{1}{3}adx+\frac{1}{3}ac}{b\,x^3+a}+\frac{5ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\cdots\right.}{\cdots}\right.}{3b\left(\frac{a}{b}\right.\cdots}\cdots}{3}\cdots$
risch	$\frac{-\frac{4be\,x^5}{3a^2}-\frac{5bd\,x^4}{6a^2}-\frac{2bc\,x^3}{3a^2}-\frac{e\,x^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3(b\,x^3+a)}-\frac{2bc\ln(x)}{a^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^9\_Z^3-18a^6bc\_Z^2+\left(60a^4bde+108a^3b^2c^2\right)\_Z-64a^2b\,e^3-360a\cdots\right.}\cdots\right.}{\cdots}$

input
```
int((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-e/a^2/x-1/3*c/a^2/x^3-1/2*d/a^2/x^2-2*b*c*ln(x)/a^3-1/a^3*b*((1/3*a*e*x^2
+1/3*a*d*x+1/3*a*c)/(b*x^3+a)+5/3*a*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))
-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+4/3*a*e*(-1/3/b/(a/b)^(1/3)*
ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*
3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-2/3*c*ln(b*
x^3+a))
```

3.350.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 5373, normalized size of antiderivative = 20.51

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.350.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx &= -\frac{8\,bex^5+5\,bdx^4+4\,bcx^3+6\,aex^2+3\,adx+2\,ac}{6\,(a^2bx^6+a^3x^3)} - \frac{2\,bc\log(x)}{a^3} \\ &\quad - \frac{\sqrt{3}\left(4\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4} \\ &\quad + \frac{\left(12\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &\quad + \frac{\left(6\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

input
```
integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(8*b*e*x^5 + 5*b*d*x^4 + 4*b*c*x^3 + 6*a*e*x^2 + 3*a*d*x + 2*a*c)/(a^
2*b*x^6 + a^3*x^3) - 2*b*c*log(x)/a^3 - 1/9*sqrt(3)*(4*a*e*(a/b)^(2/3) + 5
*a*d*(a/b)^(1/3))*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^
4 + 1/18*(12*b*c*(a/b)^(2/3) - 4*a*e*(a/b)^(1/3) + 5*a*d)*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/3)) + 1/9*(6*b*c*(a/b)^(2/3) + 4*a*e*
(a/b)^(1/3) - 5*a*d)*log(x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

3.350.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^2}\,dx = \frac{2\,bc\log\left(|bx^3+a|\right)}{3\,a^3} - \frac{2\,bc\log\left(|x|\right)}{a^3} - \frac{\sqrt{3}\left(5\,(-ab^2)^{\frac{1}{3}}\,bd - 4\,(-ab^2)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} - \frac{\left(5\,(-ab^2)^{\frac{1}{3}}\,bd + 4\,(-ab^2)^{\frac{2}{3}}\,e\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b} + \frac{\left(4\,a^4b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,a^4b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^7b} - \frac{8\,abex^5 + 5\,abdx^4 + 4\,abcx^3 + 6\,a^2ex^2 + 3\,a^2dx + 2\,a^2c}{6\,(bx^3+a)a^3x^3}$$

input
```
integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
2/3*b*c*log(abs(b*x^3 + a))/a^3 - 2*b*c*log(abs(x))/a^3 - 1/9*sqrt(3)*(5*(
-a*b^2)^(1/3)*b*d - 4*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a^3*b) - 1/18*(5*(-a*b^2)^(1/3)*b*d + 4*(-a*b^2)^(2/3
)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b) + 1/9*(4*a^4*b^2*e*(
-a/b)^(1/3) + 5*a^4*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
 - 1/6*(8*a*b*e*x^5 + 5*a*b*d*x^4 + 4*a*b*c*x^3 + 6*a^2*e*x^2 + 3*a^2*d*x
+ 2*a^2*c)/((b*x^3 + a)*a^3*x^3)
```

3.350.9 Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 537, normalized size of antiderivative = 2.05

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^2}\,dx = \left(\sum_{k=1}^{3}\ln\left(-\frac{50\,b^5\,c\,d^2-48\,b^5\,c^2\,e}{9\,a^6}\right.\right.$$

$$-\mathrm{root}\big(729\,a^9\,z^3-1458\,a^6\,b\,c\,z^2+540\,a^4\,b\,d\,e\,z+972\,a^3\,b^2\,c^2\,z-360\,a\,b^2\,c\,d\,e-64\,a^2\,b\,e^3+125\,a\,b^2\,d^3-216\,b^3$$

$$\left.+\frac{x\left(-125\,b^5\,d^3+240\,c\,b^5\,d\,e+64\,a\,b^4\,e^3\right)}{27\,a^6}\right)\mathrm{root}\big(729\,a^9\,z^3-1458\,a^6\,b\,c\,z^2+540\,a^4\,b\,d\,e\,z$$

$$\left.+972\,a^3\,b^2\,c^2\,z-360\,a\,b^2\,c\,d\,e-64\,a^2\,b\,e^3+125\,a\,b^2\,d^3-216\,b^3\,c^3,z,k\big)\right)$$

$$-\frac{\frac{c}{3\,a}+\frac{e\,x^2}{a}+\frac{d\,x}{2\,a}+\frac{2\,b\,c\,x^3}{3\,a^2}+\frac{5\,b\,d\,x^4}{6\,a^2}+\frac{4\,b\,e\,x^5}{3\,a^2}}{b\,x^6+a\,x^3}-\frac{2\,b\,c\,\ln\left(x\right)}{a^3}$$

input

```
int((c + d*x + e*x^2)/(x^4*(a + b*x^3)^2),x)
```

output

```
symsum(log((x*(64*a*b^4*e^3 - 125*b^5*d^3 + 240*b^5*c*d*e))/(27*a^6) - roo
t(729*a^9*z^3 - 1458*a^6*b*c*z^2 + 540*a^4*b*d*e*z + 972*a^3*b^2*c^2*z - 3
60*a*b^2*c*d*e - 64*a^2*b*e^3 + 125*a*b^2*d^3 - 216*b^3*c^3, z, k)*((25*a^
3*b^4*d^2 + 48*a^3*b^4*c*e)/(9*a^6) + root(729*a^9*z^3 - 1458*a^6*b*c*z^2
+ 540*a^4*b*d*e*z + 972*a^3*b^2*c^2*z - 360*a*b^2*c*d*e - 64*a^2*b*e^3 + 1
25*a*b^2*d^3 - 216*b^3*c^3, z, k)*(4*b^3*e + 36*root(729*a^9*z^3 - 1458*a^
6*b*c*z^2 + 540*a^4*b*d*e*z + 972*a^3*b^2*c^2*z - 360*a*b^2*c*d*e - 64*a^2
*b*e^3 + 125*a*b^2*d^3 - 216*b^3*c^3, z, k)*a^2*b^3*x - (48*b^4*c*x)/a) +
(x*(432*a^2*b^5*c^2 + 600*a^3*b^4*d*e))/(27*a^6)) - (50*b^5*c*d^2 - 48*b^5
*c^2*e)/(9*a^6))*root(729*a^9*z^3 - 1458*a^6*b*c*z^2 + 540*a^4*b*d*e*z + 9
72*a^3*b^2*c^2*z - 360*a*b^2*c*d*e - 64*a^2*b*e^3 + 125*a*b^2*d^3 - 216*b^
3*c^3, z, k), k, 1, 3) - (c/(3*a) + (e*x^2)/a + (d*x)/(2*a) + (2*b*c*x^3)/
(3*a^2) + (5*b*d*x^4)/(6*a^2) + (4*b*e*x^5)/(3*a^2))/(a*x^3 + b*x^6) - (2*
b*c*log(x))/a^3
```

3.351 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3} dx$

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3.351.1 Optimal result

Integrand size = 23, antiderivative size = 215

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3} dx = -\frac{c+dx+ex^2}{6b(a+bx^3)^2} + \frac{x(d+2ex)}{18ab(a+bx^3)} - \frac{\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{5/3}} + \frac{\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{5/3}} - \frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}$$

output
```
1/6*(-e*x^2-d*x-c)/b/(b*x^3+a)^2+1/18*x*(2*e*x+d)/a/b/(b*x^3+a)+1/27*(b^(1
/3)*d-a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(5/3)-1/54*(d-a^(1/3)*e/b
^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(4/3)-1/27*(b^
(1/3)*d+a^(1/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/
3)/b^(5/3)*3^(1/2)
```

3.351.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.92

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx$$

$$= \frac{\frac{3b^{2/3}x(d+2ex)}{a(a+bx^3)} - \frac{9b^{2/3}(c+x(d+ex))}{(a+bx^3)^2} - \frac{2\sqrt{3}\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} + \frac{\left(-\sqrt[3]{b}d+\cdots\right.}{}}{54b^{5/3}}$$

input
```
Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

output
```
((3*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)) - (9*b^(2/3)*(c + x*(d + e*x)))
/(a + b*x^3)^2 - (2*Sqrt[3]*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*(b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) +
b^(1/3)*x])/a^(5/3) + ((-(b^(1/3)*d) + a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(54*b^(5/3))
```

3.351.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2363, 2394, 27, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c+dx+ex^2\right)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \ 2363$$

$$\frac{\int \frac{d+2ex}{(bx^3+a)^2}dx}{6b} - \frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 2394$$

$$\frac{\frac{x(d+2ex)}{3a(a+bx^3)} - \frac{\int -\frac{2(d+ex)}{bx^3+a}dx}{3a}}{6b} - \frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{2\int \frac{d+ex}{bx^3+a}dx}{3a}+\frac{x(d+2ex)}{3a(a+bx^3)}}{6b}-\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 2399

$$\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)-\sqrt[3]{b}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x(d+2ex)}{3a(a+bx^3)}}{6b}-\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 16

$$\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)-\sqrt[3]{b}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(d+2ex)}{3a(a+bx^3)}}{6b}-\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 1142

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(d+2ex)}{3a(a+bx^3)}}{6b}-\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 25

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(d+2ex)}{3a(a+bx^3)}}{6b}-\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

3.351. $\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^3}dx$

$\Big\downarrow$ 27

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(d\cdots}{3a(a\cdots}}{6b}$$

$$\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{2\left(\frac{\frac{1}{2}\sqrt[3]{b}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(\cdots}{3a\cdots}}{6b}$$

$$\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 217

$$\frac{2\left(\frac{\frac{1}{2}\sqrt[3]{b}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(d+2ex)}{3a(a+bx^3)}}{6b}-$$

$$\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1103

3.351. $\int\frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx$

$$\frac{\frac{2\left(\frac{-\frac{1}{2}\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(d+2ex)}{3a(a+bx^3)}}{6b}-\frac{c+dx+ex^2}{6b\left(a+bx^3\right)^2}$$

input
```
Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

output
```
-1/6*(c + d*x + e*x^2)/(b*(a + b*x^3)^2) + ((x*(d + 2*e*x))/(3*a*(a + b*x^
3)) + (2*(((d - (a^(1/3)*e)/b^(1/3))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*
b^(1/3)) + (-((Sqrt[3]*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]])/b^(1/3)) - ((d - (a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)*b^(1/3))))/(3*a))/(6*b)
```

3.351.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.351. $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx$

3.351.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{e\,x^5}{9a}+\frac{d\,x^4}{18a}-\frac{e\,x^2}{18b}-\frac{dx}{9b}-\frac{c}{6b}}{\left(b\,x^3+a\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(e\_R+d\right)\ln\left(x-\_R\right)}{\_R^2}}{27a\,b^2}$
default	$\frac{\frac{e\,x^5}{9a}+\frac{d\,x^4}{18a}-\frac{e\,x^2}{18b}-\frac{dx}{9b}-\frac{c}{6b}}{\left(b\,x^3+a\right)^2}+\frac{d\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+e\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln(}{}\right.}{9ba}$

input
```
int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/9/a*e*x^5+1/18*d/a*x^4-1/18*e*x^2/b-1/9*d*x/b-1/6*c/b)/(b*x^3+a)^2+1/27
/a/b^2*sum((_R*e+d)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.351.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 2163, normalized size of antiderivative = 10.06

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/108*(12*b*e*x^5 + 6*b*d*x^4 - 6*a*e*x^2 - 12*a*d*x - 2*(a*b^3*x^6 + 2*a^
2*b^2*x^3 + a^3*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5)
 + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(
a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*lo
g(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a
*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d
^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))^2*a^4*b^3*e - 1
/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^
3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3
+ a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)))*a^2*b^2*d^2 + 2*a*
d*e^2 + (b*d^3 + a*e^3)*x) - 18*a*c + ((a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)
*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)
/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 +
a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3))) + 3*sqrt(1/3)*(a*b^3
*x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*d^3
+ a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3) - 2*(1/2)^(2/3)*d*e*
(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^
5*b^5))^(1/3)))^2*a^3*b^3 + 16*d*e)/(a^3*b^3)))*log(-1/4*((1/2)^(1/3)*(I*s
qrt(3) + 1)*((b*d^3 + a*e^3)/(a^5*b^5) + (b*d^3 - a*e^3)/(a^5*b^5))^(1/3)
- 2*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^3*b^3*((b*d^3 + a*e^3)/(a^5*b^5...
```

3.351.6 Sympy [A] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.69

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx$$
$$=\text{RootSum}\left(19683t^3a^5b^5+81ta^2b^2de+ae^3-bd^3,\left(t\mapsto t\log\left(x+\frac{729t^2a^4b^3e+27ta^2b^2d^2+2ade^2}{ae^3+bd^3}\right)\right)\right)+\frac{-3ac-2adx-aex^2+bdx^4+2bex^5}{18a^3b+36a^2b^2x^3+18ab^3x^6}$$

input
```
integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

output
```
RootSum(19683*_t**3*a**5*b**5 + 81*_t*a**2*b**2*d*e + a*e**3 - b*d**3, Lam
bda(_t, _t*log(x + (729*_t**2*a**4*b**3*e + 27*_t*a**2*b**2*d**2 + 2*a*d*e
**2)/(a*e**3 + b*d**3)))) + (-3*a*c - 2*a*d*x - a*e*x**2 + b*d*x**4 + 2*b*
e*x**5)/(18*a**3*b + 36*a**2*b**2*x**3 + 18*a*b**3*x**6)
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx = \frac{2\,bex^5+bdx^4-aex^2-2\,adx-3\,ac}{18\,(ab^3x^6+2\,a^2b^2x^3+a^3b)} + \frac{\sqrt{3}\left(e\left(\frac{a}{b}\right)^{\frac{1}{3}}+d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(e\left(\frac{a}{b}\right)^{\frac{1}{3}}-d\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(e\left(\frac{a}{b}\right)^{\frac{1}{3}}-d\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(2*b*e*x^5 + b*d*x^4 - a*e*x^2 - 2*a*d*x - 3*a*c)/(a*b^3*x^6 + 2*a^2*
b^2*x^3 + a^3*b) + 1/27*sqrt(3)*(e*(a/b)^(1/3) + d)*arctan(1/3*sqrt(3)*(2*
x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) + 1/54*(e*(a/b)^(1/3) -
d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) - 1/27*(e*(a
/b)^(1/3) - d)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

3.351.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}\left(bd-\left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{\left(bd+\left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{\left(e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b} + \frac{2\,bex^5+bdx^4-aex^2-2\,adx-3\,ac}{18\,(bx^3+a)^2ab}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(b*d - (-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/54*(b*d + (-a*b^2)^(1/3)*e)*lo
g(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) - 1/27*(e*(-a/
b)^(1/3) + d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/18*(2*b*
e*x^5 + b*d*x^4 - a*e*x^2 - 2*a*d*x - 3*a*c)/((b*x^3 + a)^2*a*b)
```

3.351.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^3}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\frac{d\,e + e^2\,x + \mathrm{root}(19683\,a^5\,b^5\,z^3 + 81\,a^2\,b^2\,d\,e\,z + a\,e^3 - b\,d^3, z, k)^2\,a^3\,b^3\,729 + \mathrm{root}(19683\,a^5\,b^5\,z \cdots}{a^2\,b\,81}\right. \right.$$
$$\left.\left. + 81\,a^2\,b^2\,d\,e\,z + a\,e^3 - b\,d^3, z, k)\right)\right) - \frac{\frac{c}{6\,b} - \frac{d\,x^4}{18\,a} - \frac{e\,x^5}{9\,a} + \frac{e\,x^2}{18\,b} + \frac{d\,x}{9\,b}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}$$

input
```
int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^3,x)
```

output
```
symsum(log((d*e + e^2*x + 729*root(19683*a^5*b^5*z^3 + 81*a^2*b^2*d*e*z +
a*e^3 - b*d^3, z, k)^2*a^3*b^3 + 27*root(19683*a^5*b^5*z^3 + 81*a^2*b^2*d*
e*z + a*e^3 - b*d^3, z, k)*a*b^2*d*x)/(81*a^2*b))*root(19683*a^5*b^5*z^3 +
 81*a^2*b^2*d*e*z + a*e^3 - b*d^3, z, k), k, 1, 3) - (c/(6*b) - (d*x^4)/(1
8*a) - (e*x^5)/(9*a) + (e*x^2)/(18*b) + (d*x)/(9*b))/(a^2 + b^2*x^6 + 2*a*
b*x^3)
```

3.352 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3} dx$

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3.352.1 Optimal result

Integrand size = 21, antiderivative size = 239

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3} dx = -\frac{x(ae-bcx-bdx^2)}{6ab(a+bx^3)^2} - \frac{3ad-x(ae+4bcx)}{18a^2b(a+bx^3)} - \frac{\left(2b^{2/3}c+a^{2/3}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{4/3}} - \frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{4/3}} + \frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{4/3}}$$

output
```
-1/6*x*(-b*d*x^2-b*c*x+a*e)/a/b/(b*x^3+a)^2+1/18*(-3*a*d+x*(4*b*c*x+a*e))/
a^2/b/(b*x^3+a)-1/27*(2*b^(2/3)*c-a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)
/b^(4/3)+1/54*(2*b^(2/3)*c-a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/a^(7/3)/b^(4/3)-1/27*(2*b^(2/3)*c+a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(4/3)*3^(1/2)
```

3.352.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.90

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx$$
$$= \frac{\frac{3ab^{2/3}(4b^2cx^5-a^2(3d+2ex)+abx^2(7c+ex^2))}{(a+bx^3)^2}-2\sqrt{3}a^{2/3}\sqrt[3]{b}\left(2b^{2/3}c+a^{2/3}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(-2a^{2/3}bc+a^{4/3}\right.}{54a^3b^{5/3}}$$

input
```
Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

output
```
((3*a*b^(2/3)*(4*b^2*c*x^5 - a^2*(3*d + 2*e*x) + a*b*x^2*(7*c + e*x^2)))/(
a + b*x^3)^2 - 2*Sqrt[3]*a^(2/3)*b^(1/3)*(2*b^(2/3)*c + a^(2/3)*e)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-2*a^(2/3)*b*c + a^(4/3)*b^(1/3)
*e)*Log[a^(1/3) + b^(1/3)*x] + (2*a^(2/3)*b*c - a^(4/3)*b^(1/3)*e)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^3*b^(5/3))
```

3.352.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {2367, 25, 2393, 27, 2399, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx$$
$$\Big\downarrow \text{ 2367}$$
$$-\frac{\int -\frac{3bdx^2+4bcx+ae}{(bx^3+a)^2}dx}{6ab}-\frac{x(ae-bcx-bdx^2)}{6ab(a+bx^3)^2}$$
$$\Big\downarrow \text{ 25}$$
$$\frac{\int \frac{3bdx^2+4bcx+ae}{(bx^3+a)^2}dx}{6ab}-\frac{x(ae-bcx-bdx^2)}{6ab(a+bx^3)^2}$$
$$\Big\downarrow \text{ 2393}$$

$$-\frac{-\frac{\int -\frac{2(ae+2bcx)}{bx^3+a}dx}{3a}-\frac{3ad-x(ae+4bcx)}{3a(a+bx^3)}}{6ab}-\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{2\int \frac{ae+2bcx}{bx^3+a}dx}{3a}-\frac{3ad-x(ae+4bcx)}{3a(a+bx^3)}}{6ab}-\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 2399

$$\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\sqrt[3]{b}\left(2\sqrt[3]{a}\left(b^{2/3}c+a^{2/3}e\right)+\sqrt[3]{b}\left(2b^{2/3}c-a^{2/3}e\right)x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}}\right)}{3a}-\frac{3ad-x(ae+4bcx)}{3a(a+bx^3)}}{6ab}-\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 16

$$\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\sqrt[3]{b}\left(2\sqrt[3]{a}\left(b^{2/3}c+a^{2/3}e\right)+\sqrt[3]{b}\left(2b^{2/3}c-a^{2/3}e\right)x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}-\frac{3ad-x(ae+4bcx)}{3a(a+bx^3)}}{6ab}-\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}\left(b^{2/3}c+a^{2/3}e\right)+\sqrt[3]{b}\left(2b^{2/3}c-a^{2/3}e\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}-\frac{3ad-x(ae+4bcx)}{3a(a+bx^3)}}{6ab}-\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 1142

3.352. $\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^3}dx$

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{2/3}e+2b^{2/3}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\left(2b^{2/3}c-a^{2/3}e\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}-\frac{3a}{}}{6ab}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{2/3}e+2b^{2/3}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}-\frac{3ad}{}}{6ab}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{2/3}e+2b^{2/3}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(2b^{2/3}c-a^{2/3}e\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}-\frac{3a}{}}{6ab}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1082

3.352. $\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{2\left(\frac{\frac{3\left(a^{2/3}e+2b^{2/3}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\left(2b^{2/3}c-a^{2/3}e\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a} - \cdots$$
$$\frac{\cdots}{6ab}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 217

$$\frac{2\left(\frac{-\frac{1}{2}\left(2b^{2/3}c-a^{2/3}e\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{2/3}e+2b^{2/3}c\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a} - \frac{3ad-x(ae+4\cdots}{3a(a+bx^3\cdots}$$
$$\frac{\cdots}{6ab}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 1103

$$\frac{2\left(\frac{\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{2/3}e+2b^{2/3}c\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\left(2b^{2/3}c-a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a} - \frac{3ad-x(ae+4b\cdots}{3a(a+bx^3)\cdots}$$
$$\frac{\cdots}{6ab}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{6ab\left(a+bx^3\right)^2}$$

3.352. $\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^3}\,dx$

input
```
Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^3,x]
```

output
```
-1/6*(x*(a*e - b*c*x - b*d*x^2))/(a*b*(a + b*x^3)^2) + (-1/3*(3*a*d - x*(a
*e + 4*b*c*x))/(a*(a + b*x^3)) + (2*(-1/3*((2*b^(2/3)*c - a^(2/3)*e)*Log[a
^(1/3) + b^(1/3)*x])/(a^(1/3)*b^(1/3)) + (-((Sqrt[3]*(2*b^(2/3)*c + a^(2/3
)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + ((2*b^(2/3)*c
 - a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)))
/(3*a^(1/3))))/(3*a))/(6*a*b)
```

3.352.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.352.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{2bc\,x^5}{9a^2}+\frac{e\,x^4}{18a}+\frac{7c\,x^2}{18a}-\frac{ex}{9b}-\frac{d}{6b}}{(b\,x^3+a)^2}+\frac{\sum\limits_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(\frac{2c\_R}{a}+\frac{e}{b}\right)\ln\left(x-\_R\right)}{\_R^2}}{27ba}$
default	$\frac{\frac{2bc\,x^5}{9a^2}+\frac{e\,x^4}{18a}+\frac{7c\,x^2}{18a}-\frac{ex}{9b}-\frac{d}{6b}}{(b\,x^3+a)^2}+\frac{ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+2bc\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{9a^2b}$

input
```
int(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(2/9*b*c/a^2*x^5+1/18/a*e*x^4+7/18*c/a*x^2-1/9*e*x/b-1/6*d/b)/(b*x^3+a)^2+
1/27/b/a*sum((2*c/a*_R+1/b*e)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.352.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 2519, normalized size of antiderivative = 10.54

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/108*(24*b^2*c*x^5 + 6*a*b*e*x^4 + 42*a*b*c*x^2 - 12*a^2*e*x - 18*a^2*d -
 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*
b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*
(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4)
- (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3)
 + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^
(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/
(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))^2*a^5*b^3*c - 1/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 -
a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*b^2*((8
*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3)))*a
^4*b*e^2 + 8*a*b*c^2*e + (8*b^2*c^3 + a^2*e^3)*x) + ((a^2*b^3*x^6 + 2*a^3*
b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*
b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(
3) - 1)/(a^4*b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/
(a^7*b^4))^(1/3))) + 3*sqrt(1/3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2
*c^3 - a^2*e^3)/(a^7*b^4))^(1/3) + 4*(1/2)^(2/3)*c*e*(I*sqrt(3) - 1)/(a^4*
b^2*((8*b^2*c^3 + a^2*e^3)/(a^7*b^4) - (8*b^2*c^3 - a^2*e^3)/(a^7*b^4))^(1
/3)))^2*a^4*b^2 + 32*c*e)/(a^4*b^2)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) ...
```

3.352.6 Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.71

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx$$
$$= \mathrm{RootSum}\left(19683t^3a^7b^4+162ta^3b^2ce-a^2e^3+8b^2c^3,\left(t\mapsto t\log\left(x+\frac{1458t^2a^5b^3c+27ta^4be^2+8abc^2e}{a^2e^3+8b^2c^3}\right)\right)\right)+\frac{-3a^2d-2a^2ex+7abcx^2+abex^4+4b^2cx^5}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}$$

input
```
integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

output
```
RootSum(19683*_t**3*a**7*b**4 + 162*_t*a**3*b**2*c*e - a**2*e**3 + 8*b**2*
c**3, Lambda(_t, _t*log(x + (1458*_t**2*a**5*b**3*c + 27*_t*a**4*b*e**2 +
8*a*b*c**2*e)/(a**2*e**3 + 8*b**2*c**3)))) + (-3*a**2*d - 2*a**2*e*x + 7*a
*b*c*x**2 + a*b*e*x**4 + 4*b**2*c*x**5)/(18*a**4*b + 36*a**3*b**2*x**3 + 1
8*a**2*b**3*x**6)
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx = \frac{4\,b^2cx^5+abex^4+7\,abcx^2-2\,a^2ex-3\,a^2d}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} + \frac{\sqrt{3}\left(2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(2\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(4*b^2*c*x^5 + a*b*e*x^4 + 7*a*b*c*x^2 - 2*a^2*e*x - 3*a^2*d)/(a^2*b^
3*x^6 + 2*a^3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(2*b*c*(a/b)^(1/3) + a*e)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) +
1/54*(2*b*c*(a/b)^(1/3) - a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2
*b^2*(a/b)^(2/3)) - 1/27*(2*b*c*(a/b)^(1/3) - a*e)*log(x + (a/b)^(1/3))/(a
^2*b^2*(a/b)^(2/3))
```

3.352.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.88

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}\left(ae-2\,(-ab^2)^{\frac{1}{3}}\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{\left(ae+2\,(-ab^2)^{\frac{1}{3}}\,c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{\left(2\,bc\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b} + \frac{4\,b^2cx^5+abex^4+7\,abcx^2-2\,a^2ex-3\,a^2d}{18\,(bx^3+a)^2a^2b}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(a*e - 2*(-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(a*e + 2*(-a*b^2)^(1/3)*c
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(2*
b*c*(-a/b)^(1/3) + a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) +
1/18*(4*b^2*c*x^5 + a*b*e*x^4 + 7*a*b*c*x^2 - 2*a^2*e*x - 3*a^2*d)/((b*x^3
 + a)^2*a^2*b)
```

3.352.9 Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.97

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^3}\,dx = \frac{\frac{7\,c\,x^2}{18\,a}-\frac{d}{6\,b}+\frac{e\,x^4}{18\,a}-\frac{e\,x}{9\,b}+\frac{2\,b\,c\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$
$$+\left(\sum_{k=1}^{3}\ln\left(\frac{2\,a\,c\,e+\mathrm{root}(19683\,a^7\,b^4\,z^3+162\,a^3\,b^2\,c\,e\,z+8\,b^2\,c^3-a^2\,e^3,z,k)^2\,a^5\,b^2\,729+4\,b\,c^2\,x+\mathrm{root}(}{a^4\,81}\right.\right.$$
$$\left.\left.+162\,a^3\,b^2\,c\,e\,z+8\,b^2\,c^3-a^2\,e^3,z,k\right)\right)$$

input
```
int((x*(c + d*x + e*x^2))/(a + b*x^3)^3,x)
```

output
```
((7*c*x^2)/(18*a) - d/(6*b) + (e*x^4)/(18*a) - (e*x)/(9*b) + (2*b*c*x^5)/(
9*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((2*a*c*e + 729*root(19683
*a^7*b^4*z^3 + 162*a^3*b^2*c*e*z + 8*b^2*c^3 - a^2*e^3, z, k)^2*a^5*b^2 +
4*b*c^2*x + 27*root(19683*a^7*b^4*z^3 + 162*a^3*b^2*c*e*z + 8*b^2*c^3 - a^
2*e^3, z, k)*a^3*b*e*x)/(81*a^4))*root(19683*a^7*b^4*z^3 + 162*a^3*b^2*c*e
*z + 8*b^2*c^3 - a^2*e^3, z, k), k, 1, 3)
```

3.353 $\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx$

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3.353.1 Optimal result

Integrand size = 20, antiderivative size = 225

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx=\frac{x(5c+4dx)}{18a^2(a+bx^3)}-\frac{ae-bx(c+dx)}{6ab(a+bx^3)^2}-\frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}}+\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}}-\frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}$$

output
```
1/18*x*(4*d*x+5*c)/a^2/(b*x^3+a)+1/6*(-a*e+b*x*(d*x+c))/a/b/(b*x^3+a)^2+1/
27*(5*b^(1/3)*c-2*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(2/3)-1/54*(5
*b^(1/3)*c-2*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/
b^(2/3)-1/27*(5*b^(1/3)*c+2*a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/a^(8/3)/b^(2/3)*3^(1/2)
```

3.353.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx$$
$$= \frac{\frac{3a(-3a^2e+b^2x^4(5c+4dx)+abx(8c+7dx))}{(a+bx^3)^2} - 2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\sqrt[3]{b}\left(5\sqrt[3]{a}\sqrt[3]{b}c-2a^{2/\cdots}\right.}{54a^3b}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

output
```
((3*a*(-3*a^2*e + b^2*x^4*(5*c + 4*d*x) + a*b*x*(8*c + 7*d*x)))/(a + b*x^3
)^2 - 2*Sqrt[3]*a^(1/3)*b^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(5*a^(1/3)*b^(1/3)*c - 2*a^(2/3)
*d)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*b^(1/3)*(-5*b^(1/3)*c + 2*a^(1/3)*d
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^3*b)
```

3.353.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2393, 25, 2394, 27, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{2393}$$

$$-\frac{\int -\frac{5c+4dx}{(bx^3+a)^2}dx}{6a} - \frac{ae-bx(c+dx)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5c+4dx}{(bx^3+a)^2}dx}{6a} - \frac{ae-bx(c+dx)}{6ab\,(a+bx^3)^2}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(5c+4dx)}{3a(a+bx^3)}-\frac{\int -\frac{2(5c+2dx)}{bx^3+a}dx}{3a}}{6a}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\int \frac{5c+2dx}{bx^3+a}dx}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 2399

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}\left(5\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 16

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}\left(5\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\cdots}{6a}-\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 25

3.353. $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^3}\,dx$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5}{3a}}{6a}$$

$$\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 27

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+}{6a}$$

$$\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{2\left(\frac{\frac{1}{2}\sqrt[3]{b}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}}{6a}$$

$$\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 217

3.353. $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{2\left(\frac{\frac{1}{2}\sqrt[3]{b}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}$$

$$\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

$\big\downarrow$ 1103

$$\frac{\frac{2\left(\frac{-\frac{1}{2}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}$$

$$\frac{ae-bx(c+dx)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^3)^3,x]
```

output
```
-1/6*(a*e - b*x*(c + d*x))/(a*b*(a + b*x^3)^2) + ((x*(5*c + 4*d*x))/(3*a*(
a + b*x^3)) + (2*(((5*c - (2*a^(1/3)*d)/b^(1/3))*Log[a^(1/3) + b^(1/3)*x])
/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - ((5*c - (2*a^(1/3)*d)/b^(1/3)
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)*b^(1/3))))
/(3*a))/(6*a)
```

3.353. $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^3}\,dx$

3.353.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

3.353. $\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx$

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.353.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.41

method	result
risch	$\frac{\frac{2bd\,x^5}{9a^2}+\frac{5bc\,x^4}{18a^2}+\frac{7d\,x^2}{18a}+\frac{4cx}{9a}-\frac{e}{6b}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)}\frac{\left(2\_R d+5c\right)\ln\left(x-\_R\right)}{\_R^2}}{27a^2b}$
default	$c\left(\frac{x}{6a(b\,x^3+a)^2}+\frac{\frac{5x}{18a\left(b\,x^3+a\right)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6a}}{a}\right)+d\left(\frac{x}{6a(b\,x^3\right.$

input
```
int((e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(2/9*b*d/a^2*x^5+5/18*b*c/a^2*x^4+7/18*d/a*x^2+4/9*c/a*x-1/6/b*e)/(b*x^3+a
)^2+1/27/a^2/b*sum((2*_R*d+5*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.353.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 2251, normalized size of antiderivative = 10.00

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/108*(24*b^2*d*x^5 + 30*b^2*c*x^4 + 42*a*b*d*x^2 + 48*a*b*c*x - 18*a^2*e
- 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((1
25*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 2
0*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2)
 + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))
^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)
/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d - 25/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 -
8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((1
25*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a
^3*b*c^2 + 40*a*c*d^2 + (125*b*c^3 + 8*a*d^3)*x) + ((a^2*b^3*x^6 + 2*a^3*b
^2*x^3 + a^4*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b
^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt
(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(
a^8*b^2))^(1/3))) + 3*sqrt(1/3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sqrt
(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*
c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5
*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/
3)))^2*a^5*b + 160*c*d)/(a^5*b)))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)...
```

3.353.6 Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.72

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx$$
$$= \text{RootSum}\left(19683t^3a^8b^2+810ta^3bcd+8ad^3-125bc^3, \left(t\mapsto t\log\left(x+\frac{1458t^2a^6bd+675ta^3bc^2+40acd^2}{8ad^3+125bc^3}\right.\right.\right.$$
$$+\frac{-3a^2e+8abcx+7abdx^2+5b^2cx^4+4b^2dx^5}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

output
```
RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3,
 Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*
d**2)/(8*a*d**3 + 125*b*c**3)))) + (-3*a**2*e + 8*a*b*c*x + 7*a*b*d*x**2 +
 5*b**2*c*x**4 + 4*b**2*d*x**5)/(18*a**4*b + 36*a**3*b**2*x**3 + 18*a**2*b
**3*x**6)
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.97

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx = \frac{4\,b^2dx^5+5\,b^2cx^4+7\,abdx^2+8\,abcx-3\,a^2e}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}$$
$$+\frac{\sqrt{3}\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(4*b^2*d*x^5 + 5*b^2*c*x^4 + 7*a*b*d*x^2 + 8*a*b*c*x - 3*a^2*e)/(a^2*
b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(2*d*(a/b)^(1/3) + 5*c)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) + 1/
54*(2*d*(a/b)^(1/3) - 5*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(
a/b)^(2/3)) - 1/27*(2*d*(a/b)^(1/3) - 5*c)*log(x + (a/b)^(1/3))/(a^2*b*(a/
b)^(2/3))
```

3.353.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}\left(5\,bc-2\,(-ab^2)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{\left(5\,bc+2\,(-ab^2)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3} + \frac{4\,b^2dx^5+5\,b^2cx^4+7\,abdx^2+8\,abcx-3\,a^2e}{18\,(bx^3+a)^2a^2b}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(5*b*c - 2*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*c + 2*(-a*b^2)^(1/
3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27
*(2*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/
18*(4*b^2*d*x^5 + 5*b^2*c*x^4 + 7*a*b*d*x^2 + 8*a*b*c*x - 3*a^2*e)/((b*x^3
 + a)^2*a^2*b)
```

3.353.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.94

$$\int \frac{c+dx+ex^2}{(a+bx^3)^3}\,dx = \frac{\frac{7\,d\,x^2}{18\,a}-\frac{e}{6\,b}+\frac{4\,c\,x}{9\,a}+\frac{5\,b\,c\,x^4}{18\,a^2}+\frac{2\,b\,d\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$
$$+\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(10\,c\,d+4\,d^2\,x+\mathrm{root}(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)^2\,a^5\,b\,729+\mathrm{roo}\right.}{a^4\,81}\right.\right.$$
$$\left.+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)\right)$$

input
```
int((c + d*x + e*x^2)/(a + b*x^3)^3,x)
```

output
```
((7*d*x^2)/(18*a) - e/(6*b) + (4*c*x)/(9*a) + (5*b*c*x^4)/(18*a^2) + (2*b*
d*x^5)/(9*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((b*(10*c*d + 4*d^
2*x + 729*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3,
z, k)^2*a^5*b + 135*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 +
 8*a*d^3, z, k)*a^2*b*c*x))/(81*a^4))*root(19683*a^8*b^2*z^3 + 810*a^3*b*c
*d*z - 125*b*c^3 + 8*a*d^3, z, k), k, 1, 3)
```

3.354 $\int \frac{c+dx+ex^2}{x(a+bx^3)^3} dx$

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3.354.1 Optimal result

Integrand size = 23, antiderivative size = 257

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^3}\,dx = \frac{x(ad+aex-bcx^2)}{6a^2(a+bx^3)^2} + \frac{x(5ad+4aex-9bcx^2)}{18a^3(a+bx^3)} - \frac{\left(5\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} + \frac{c\log(x)}{a^3} + \frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}} - \frac{c\log(a+bx^3)}{3a^3}$$

output
```
1/6*x*(-b*c*x^2+a*e*x+a*d)/a^2/(b*x^3+a)^2+1/18*x*(-9*b*c*x^2+4*a*e*x+5*a*
d)/a^3/(b*x^3+a)+c*ln(x)/a^3+1/27*(5*b^(1/3)*d-2*a^(1/3)*e)*ln(a^(1/3)+b^(
1/3)*x)/a^(8/3)/b^(2/3)-1/54*(5*b^(1/3)*d-2*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(2/3)-1/3*c*ln(b*x^3+a)/a^3-1/27*(5*b^(1/
3)*d+2*a^(1/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3
)/b^(2/3)*3^(1/2)
```

3.354.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.89

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx$$

$$= \frac{\frac{9a^2(c+x(d+ex))}{(a+bx^3)^2} + \frac{3a(6c+x(5d+4ex))}{a+bx^3} - \frac{2\sqrt{3}\sqrt[3]{a}\left(5\sqrt[3]{b}d+2\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + 54c\log(x) + \frac{2\left(5\sqrt[3]{a}\sqrt[3]{b}d-2a^{2/3}e\right)\,\mathrm{l}}{b^{2/3}}}{54a^3}$$

input
```
Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

output
```
((9*a^2*(c + x*(d + e*x)))/(a + b*x^3)^2 + (3*a*(6*c + x*(5*d + 4*e*x)))/(
a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(5*b^(1/3)*d + 2*a^(1/3)*e)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 54*c*Log[x] + (2*(5*a^(1/3)*b^(1/
3)*d - 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/
3)*d + 2*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3
) - 18*c*Log[a + b*x^3])/(54*a^3)
```

3.354.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \text{ 2368}$$

$$\frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2} - \frac{\int -\frac{-\frac{3b^2cx^3}{a}+4bex^2+5bdx+6bc}{x\left(bx^3+a\right)^2}dx}{6ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-\frac{3b^2cx^3}{a}+4bex^2+5bdx+6bc}{x(bx^3+a)^2}dx}{6ab}+\frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{2368}$$

$$\frac{\frac{x(5abd+4abex-9b^2cx^2)}{3a^2(a+bx^3)}-\frac{\int -\frac{2\left(2ex^2b^2+9cb^2+5dxb^2\right)}{x\left(bx^3+a\right)}dx}{3ab}}{6ab}+\frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{2ex^2b^2+9cb^2+5dxb^2}{x\left(bx^3+a\right)}dx}{3ab}+\frac{x(5abd+4abex-9b^2cx^2)}{3a^2(a+bx^3)}}{6ab}+\frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{2373}$$

$$\frac{\frac{2\int \left(\frac{9cb^2}{ax}+\frac{\left(-9bcx^2+2aex+5ad\right)b^2}{a\left(bx^3+a\right)}\right)dx}{3ab}+\frac{x(5abd+4abex-9b^2cx^2)}{3a^2(a+bx^3)}}{6ab}+\frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(ad+aex-bcx^2\right)}{6a^2\left(a+bx^3\right)^2}+$$

$$\frac{\frac{2\left(-\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2\sqrt[3]{a}e+5\sqrt[3]{b}d\right)}{\sqrt{3}a^{2/3}}-\frac{b^{4/3}\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}}+\frac{b^{4/3}\left(5\sqrt[3]{b}d-2\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}}\right.}{3ab}}{6ab}$$

input
```
Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^3),x]
```

output
```
(x*(a*d + a*e*x - b*c*x^2))/(6*a^2*(a + b*x^3)^2) + ((x*(5*a*b*d + 4*a*b*e
*x - 9*b^2*c*x^2))/(3*a^2*(a + b*x^3)) + (2*(-((b^(4/3)*(5*b^(1/3)*d + 2*a
^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3))) + (9*b^2*c*Log[x])/a + (b^(4/3)*(5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a^(1
/3) + b^(1/3)*x])/(3*a^(2/3)) - (b^(4/3)*(5*b^(1/3)*d - 2*a^(1/3)*e)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)) - (3*b^2*c*Log[a +
b*x^3])/a))/(3*a*b))/(6*a*b)
```

3.354. $\int \frac{c+dx+ex^2}{x(a+bx^3)^3}\,dx$

3.354.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.354.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.05

method	result
risch	$\frac{\frac{2be\,x^5}{9a^2}+\frac{5bd\,x^4}{18a^2}+\frac{bc\,x^3}{3a^2}+\frac{7e\,x^2}{18a}+\frac{4xd}{9a}+\frac{c}{2a}}{(b\,x^3+a)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^9b^2\_Z^3+27a^6b^2c\_Z^2+\left(30a^4bde+243a^3b^2c^2\right)\_Z+8a^2e^3+270abcde-125ab\,d^3 \cdots\right.}\right.}{\cdots}$
default	$\frac{c\ln(x)}{a^3}+\frac{\frac{\frac{2}{9}abe\,x^5+\frac{5}{18}abd\,x^4+\frac{1}{3}abc\,x^3+\frac{7}{18}a^2e\,x^2+\frac{4}{9}a^2dx+\frac{1}{2}a^2c}{\left(b\,x^3+a\right)^2}+\frac{5ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\cdots\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\cdots\right)}{9}}{a^3}$

input
```
int((e*x^2+d*x+c)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(2/9*b*e/a^2*x^5+5/18*b*d/a^2*x^4+1/3*b*c/a^2*x^3+7/18/a*e*x^2+4/9/a*x*d+1
/2*c/a)/(b*x^3+a)^2+1/27*sum(_R*ln((-4*_R^3*a^8*b^2-72*_R^2*a^5*b^2*c+(-10
0*a^3*b*d*e-324*a^2*b^2*c^2)*_R-24*a*e^3-540*b*c*d*e+375*b*d^3)*x+2*a^6*b*
e*_R^2+(-36*a^3*b*c*e-25*a^3*b*d^2)*_R-486*b*c^2*e+675*b*c*d^2),_R=RootOf(
a^9*b^2*_Z^3+27*a^6*b^2*c*_Z^2+(30*a^4*b*d*e+243*a^3*b^2*c^2)*_Z+8*a^2*e^3
+270*a*b*c*d*e-125*a*b*d^3+729*b^2*c^3))+c/a^3*ln(-x)
```

3.354.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 5229, normalized size of antiderivative = 20.35

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.354.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^3}\,dx = \frac{4\,bex^5+5\,bdx^4+6\,bcx^3+7\,aex^2+8\,adx+9\,ac}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)}+\frac{c\log(x)}{a^3} + \frac{\sqrt{3}\left(2\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4} - \frac{\left(18\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(9\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(4*b*e*x^5 + 5*b*d*x^4 + 6*b*c*x^3 + 7*a*e*x^2 + 8*a*d*x + 9*a*c)/(a^
2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + c*log(x)/a^3 + 1/27*sqrt(3)*(2*a*e*(a/b)^
(2/3) + 5*a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/a^4 - 1/54*(18*b*c*(a/b)^(2/3) - 2*a*e*(a/b)^(1/3) + 5*a*d)*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) - 1/27*(9*b*c*(a/b)^(2/3
) + 2*a*e*(a/b)^(1/3) - 5*a*d)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

3.354.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx = -\frac{\sqrt{3}\left(5\,bd-2\left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{\left(5\,bd+2\left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2} - \frac{c\log\left(\left|bx^3+a\right|\right)}{3\,a^3} + \frac{c\log\left(\left|x\right|\right)}{a^3} + \frac{4\,abex^5+5\,abdx^4+6\,abcx^3+7\,a^2ex^2+8\,a^2dx+9\,a^2c}{18\left(bx^3+a\right)^2a^3} - \frac{\left(2\,a^4be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^4bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(5*b*d - 2*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*d + 2*(-a*b^2)^(1/
3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/3*
c*log(abs(b*x^3 + a))/a^3 + c*log(abs(x))/a^3 + 1/18*(4*a*b*e*x^5 + 5*a*b*
d*x^4 + 6*a*b*c*x^3 + 7*a^2*e*x^2 + 8*a^2*d*x + 9*a^2*c)/((b*x^3 + a)^2*a^
3) - 1/27*(2*a^4*b*e*(-a/b)^(1/3) + 5*a^4*b*d)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a^7*b)
```

3.354.9 Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.10

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^3}\,dx$$
$$= \frac{\frac{c}{2\,a}+\frac{7\,e\,x^2}{18\,a}+\frac{4\,d\,x}{9\,a}+\frac{b\,c\,x^3}{3\,a^2}+\frac{5\,b\,d\,x^4}{18\,a^2}+\frac{2\,b\,e\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\left(\sum_{k=1}^{3}\ln\left(\frac{25\,b^2\,c\,d^2-18\,b^2\,c^2\,e}{81\,a^6}\right.\right.$$
$$-\mathrm{root}\left(19683\,a^9\,b^2\,z^3+19683\,a^6\,b^2\,c\,z^2+810\,a^4\,b\,d\,e\,z+6561\,a^3\,b^2\,c^2\,z+270\,a\,b\,c\,d\,e-125\,a\,b\,d^3+8\,a^2\,e^3+72\right.$$
$$\left.\left.-\frac{x\left(-125\,b^2\,d^3+180\,c\,b^2\,d\,e+8\,a\,b\,e^3\right)}{729\,a^6}\right)\mathrm{root}\left(19683\,a^9\,b^2\,z^3+19683\,a^6\,b^2\,c\,z^2\right.$$
$$\left.\left.+810\,a^4\,b\,d\,e\,z+6561\,a^3\,b^2\,c^2\,z+270\,a\,b\,c\,d\,e-125\,a\,b\,d^3+8\,a^2\,e^3+729\,b^2\,c^3,z,k\right)\right)$$
$$+\frac{c\ln(x)}{a^3}$$

input
```
int((c + d*x + e*x^2)/(x*(a + b*x^3)^3),x)
```

output
```
(c/(2*a) + (7*e*x^2)/(18*a) + (4*d*x)/(9*a) + (b*c*x^3)/(3*a^2) + (5*b*d*x
^4)/(18*a^2) + (2*b*e*x^5)/(9*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(l
og((25*b^2*c*d^2 - 18*b^2*c^2*e)/(81*a^6) - root(19683*a^9*b^2*z^3 + 19683
*a^6*b^2*c*z^2 + 810*a^4*b*d*e*z + 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 12
5*a*b*d^3 + 8*a^2*e^3 + 729*b^2*c^3, z, k)*((25*a^3*b^2*d^2 + 36*a^3*b^2*c
*e)/(81*a^6) + root(19683*a^9*b^2*z^3 + 19683*a^6*b^2*c*z^2 + 810*a^4*b*d*
e*z + 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*b*d^3 + 8*a^2*e^3 + 729*b
^2*c^3, z, k)*(36*root(19683*a^9*b^2*z^3 + 19683*a^6*b^2*c*z^2 + 810*a^4*b
*d*e*z + 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*b*d^3 + 8*a^2*e^3 + 72
9*b^2*c^3, z, k)*a^2*b^3*x - (2*b^2*e)/3 + (24*b^3*c*x)/a) + (x*(2916*a^2*
b^3*c^2 + 900*a^3*b^2*d*e))/(729*a^6)) - (x*(8*a*b*e^3 - 125*b^2*d^3 + 180
*b^2*c*d*e))/(729*a^6))*root(19683*a^9*b^2*z^3 + 19683*a^6*b^2*c*z^2 + 810
*a^4*b*d*e*z + 6561*a^3*b^2*c^2*z + 270*a*b*c*d*e - 125*a*b*d^3 + 8*a^2*e^
3 + 729*b^2*c^3, z, k), k, 1, 3) + (c*log(x))/a^3
```

3.355 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx$

3.355.1 Optimal result 2659
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3.355.1 Optimal result

Integrand size = 23, antiderivative size = 267

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx = -\frac{c}{a^3x}+\frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2}+\frac{x(5ae-10bcx-9bdx^2)}{18a^3(a+bx^3)} + \frac{(14b^{2/3}c-5a^{2/3}e)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt[3]{b}} + \frac{d\log(x)}{a^3}+\frac{(14b^{2/3}c+5a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}\sqrt[3]{b}} - \frac{(14b^{2/3}c+5a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}\sqrt[3]{b}}-\frac{d\log(a+bx^3)}{3a^3}$$

output
```
-c/a^3/x+1/6*x*(-b*d*x^2-b*c*x+a*e)/a^2/(b*x^3+a)^2+1/18*x*(-9*b*d*x^2-10*
b*c*x+5*a*e)/a^3/(b*x^3+a)+d*ln(x)/a^3+1/27*(14*b^(2/3)*c+5*a^(2/3)*e)*ln(
a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(1/3)-1/54*(14*b^(2/3)*c+5*a^(2/3)*e)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(1/3)-1/3*d*ln(b*x^3+a)/a^3
+1/27*(14*b^(2/3)*c-5*a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*
3^(1/2))/a^(10/3)/b^(1/3)*3^(1/2)
```

3.355.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx$$
$$= \frac{-\frac{54ac}{x} + \frac{3a(6ad+5aex-10bcx^2)}{a+bx^3} + \frac{9a^2(-bcx^2+a(d+ex))}{(a+bx^3)^2} - \frac{2\sqrt{3}a^{2/3}(-14b^{2/3}c+5a^{2/3}e)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + 54ad\log(x) + \cdots}{54a^4}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

output
```
((-54*a*c)/x + (3*a*(6*a*d + 5*a*e*x - 10*b*c*x^2))/(a + b*x^3) + (9*a^2*(
-(b*c*x^2) + a*(d + e*x)))/(a + b*x^3)^2 - (2*Sqrt[3]*a^(2/3)*(-14*b^(2/3)
*c + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + 5
4*a*d*Log[x] + (2*(14*a^(2/3)*b^(2/3)*c + 5*a^(4/3)*e)*Log[a^(1/3) + b^(1/
3)*x])/b^(1/3) - ((14*a^(2/3)*b^(2/3)*c + 5*a^(4/3)*e)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) - 18*a*d*Log[a + b*x^3])/(54*a^4)
```

3.355.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{ 2368}$$

$$\frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2} - \frac{\int -\frac{-\frac{3b^2dx^4}{a}-\frac{4b^2cx^3}{a}+5bex^2+6bdx+6bc}{x^2(bx^3+a)^2}dx}{6ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int -\frac{-\frac{3b^2dx^4}{a}-\frac{4b^2cx^3}{a}+5bex^2+6bdx+6bc}{x^2(bx^3+a)^2}dx}{6ab}+\frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2}$$

$\Big\downarrow$ 2368

$$\frac{\frac{x(5abe-10b^2cx-9b^2dx^2)}{3a^2(a+bx^3)}-\frac{\int -\frac{2\left(-\frac{5cx^3b^4}{a}+5ex^2b^3+9cb^3+9dxb^3\right)}{x^2(bx^3+a)}dx}{3ab^2}}{6ab}+\frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\int \frac{-\frac{5cx^3b^4}{a}+5ex^2b^3+9cb^3+9dxb^3}{x^2(bx^3+a)}dx}{3ab^2}+\frac{x(5abe-10b^2cx-9b^2dx^2)}{3a^2(a+bx^3)}}{6ab}+\frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2}$$

$\Big\downarrow$ 2373

$$\frac{\frac{2\int \left(\frac{9db^3}{ax}+\frac{(-9bdx^2-14bcx+5ae)b^3}{a(bx^3+a)}+\frac{9cb^3}{ax^2}\right)dx}{3ab^2}+\frac{x(5abe-10b^2cx-9b^2dx^2)}{3a^2(a+bx^3)}}{6ab}+\frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2}$$

$\Big\downarrow$ 2009

$$\frac{x(ae-bcx-bdx^2)}{6a^2(a+bx^3)^2}+$$

$$\frac{2\left(\frac{b^{8/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(14b^{2/3}c-5a^{2/3}e)}{\sqrt{3}a^{4/3}}-\frac{b^{8/3}(5a^{2/3}e+14b^{2/3}c)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}}+\frac{b^{8/3}(5a^{2/3}e+14b^{2/3}c)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}}-\right.}{3ab^2}$$
$$\overline{\qquad 6ab \qquad}$$

input
```
Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x]
```

output
```
(x*(a*e - b*c*x - b*d*x^2))/(6*a^2*(a + b*x^3)^2) + ((x*(5*a*b*e - 10*b^2*
c*x - 9*b^2*d*x^2))/(3*a^2*(a + b*x^3)) + (2*((-9*b^3*c)/(a*x) + (b^(8/3)*
(14*b^(2/3)*c - 5*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(Sqrt[3]*a^(4/3)) + (9*b^3*d*Log[x])/a + (b^(8/3)*(14*b^(2/3)*c + 5*
a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)) - (b^(8/3)*(14*b^(2/3)*c
+ 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3))
 - (3*b^3*d*Log[a + b*x^3])/a))/(3*a*b^2))/(6*a*b)
```

3.355. $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx$

3.355.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.355.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.04

method	result
risch	$\frac{-\frac{14c\,b^{2}x^{6}}{9a^{3}}+\frac{5be\,x^{5}}{18a^{2}}+\frac{bd\,x^{4}}{3a^{2}}-\frac{49bc\,x^{3}}{18a^{2}}+\frac{4e\,x^{2}}{9a}+\frac{xd}{2a}-\frac{c}{a}}{x(b\,x^{3}+a)^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}b\,\_Z^{3}+27a^{7}bd\,\_Z^{2}+\left(-210a^{4}bce+243a^{4}b\,d^{2}\right)\_Z-125a^{2}e^{3}-189\right.}}{}$
default	$-\frac{c}{a^{3}x}+\frac{d\ln(x)}{a^{3}}+\frac{\frac{-\frac{5}{9}b^{2}c\,x^{5}+\frac{5}{18}abe\,x^{4}+\frac{1}{3}x^{3}abd-\frac{13}{18}abc\,x^{2}+\frac{4}{9}a^{2}ex+\frac{1}{2}a^{2}d}{\left(b\,x^{3}+a\right)^{2}}+\frac{5ae\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan}{}\right.}{9}}{}$

input
```
int((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-14/9*c/a^3*b^2*x^6+5/18*b*e/a^2*x^5+1/3*b*d/a^2*x^4-49/18*b*c/a^2*x^3+4/
9/a*e*x^2+1/2/a*x*d-c/a)/x/(b*x^3+a)^2+1/27*sum(_R*ln((-4*_R^3*a^10*b-72*_
R^2*a^7*b*d+(700*a^4*b*c*e-324*a^4*b*d^2)*_R+375*a^2*e^3+3780*a*b*c*d*e+82
32*b^2*c^3)*x-14*a^7*b*c*_R^2+(-25*a^5*e^2+252*a^4*b*c*d)*_R+675*a^2*d*e^2
+3402*a*b*c*d^2),_R=RootOf(a^10*b*_Z^3+27*a^7*b*d*_Z^2+(-210*a^4*b*c*e+243
*a^4*b*d^2)*_Z-125*a^2*e^3-1890*a*b*c*d*e+729*a*b*d^3-2744*b^2*c^3))+d*ln(
x)/a^3
```

3.355.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 5112, normalized size of antiderivative = 19.15

$$\int \frac{c+dx+ex^2}{x^2\,(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.355.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.355.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx = -\frac{28\,b^2cx^6-5\,abex^5-6\,abdx^4+49\,abcx^3-8\,a^2ex^2-9\,a^2dx+18\,a^2c}{18\,(a^3b^2x^7+2\,a^4bx^4+a^5x)} + \frac{d\log(x)}{a^3} - \frac{\sqrt{3}\left(14\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4} - \frac{\left(18\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(9\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*(28*b^2*c*x^6 - 5*a*b*e*x^5 - 6*a*b*d*x^4 + 49*a*b*c*x^3 - 8*a^2*e*x
^2 - 9*a^2*d*x + 18*a^2*c)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x) + d*log(x)/
a^3 - 1/27*sqrt(3)*(14*b*c*(a/b)^(2/3) - 5*a*e*(a/b)^(1/3))*arctan(1/3*sqr
t(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^4 - 1/54*(18*b*d*(a/b)^(2/3) + 14*
b*c*(a/b)^(1/3) + 5*a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/
b)^(2/3)) - 1/27*(9*b*d*(a/b)^(2/3) - 14*b*c*(a/b)^(1/3) - 5*a*e)*log(x +
(a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

3.355.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx = -\frac{d\log(|bx^3+a|)}{3a^3} + \frac{d\log(|x|)}{a^3} + \frac{\sqrt{3}\left(5(-ab^2)^{\frac{1}{3}}ae + 14(-ab^2)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^4b} + \frac{\left(5(-ab^2)^{\frac{1}{3}}ae - 14(-ab^2)^{\frac{2}{3}}c\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^4b} - \frac{28b^2cx^6 - 5abex^5 - 6abdx^4 + 49abcx^3 - 8a^2ex^2 - 9a^2dx + 18a^2c}{18(bx^3+a)^2a^3x} + \frac{\left(14a^3b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5a^4be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^7b}$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/3*d*log(abs(b*x^3 + a))/a^3 + d*log(abs(x))/a^3 + 1/27*sqrt(3)*(5*(-a*b
^2)^(1/3)*a*e + 14*(-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(a^4*b) + 1/54*(5*(-a*b^2)^(1/3)*a*e - 14*(-a*b^2)^(2/3)*
c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/18*(28*b^2*c*x^6 -
 5*a*b*e*x^5 - 6*a*b*d*x^4 + 49*a*b*c*x^3 - 8*a^2*e*x^2 - 9*a^2*d*x + 18*a
^2*c)/((b*x^3 + a)^2*a^3*x) + 1/27*(14*a^3*b^2*c*(-a/b)^(1/3) - 5*a^4*b*e)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

3.355.9 Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 793, normalized size of antiderivative = 2.97

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx = \frac{\frac{4\,e\,x^2}{9\,a} - \frac{c}{a} + \frac{d\,x}{2\,a} - \frac{14\,b^2\,c\,x^6}{9\,a^3} - \frac{49\,b\,c\,x^3}{18\,a^2} + \frac{b\,d\,x^4}{3\,a^2} + \frac{5\,b\,e\,x^5}{18\,a^2}}{a^2\,x + 2\,a\,b\,x^4 + b^2\,x^7} + \left(\sum_{k=1}^{3} \ln\left(\frac{b^2\left(-\mathrm{root}(19683\,a^{10}\,b\,z^3 + 19683\,a^7\,b\,d\,z^2 - 5670\,a^4\,b\,c\,e\,z + 6561\,a^4\,b\,d^2\,z - 1890\,a\,b\,c\,d\,e + 7\right.}{}\right.\right.$$

$$+ 19683\,a^7\,b\,d\,z^2 - 5670\,a^4\,b\,c\,e\,z + 6561\,a^4\,b\,d^2\,z - 1890\,a\,b\,c\,d\,e + 729\,a\,b\,d^3 - 125\,a^2\,e^3 - 2744\,b^2\,c^3, z, k)\Bigg)\Bigg) + \frac{d\,\ln(x)}{a^3}$$

input
```
int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^3),x)
```

output
```
((4*e*x^2)/(9*a) - c/a + (d*x)/(2*a) - (14*b^2*c*x^6)/(9*a^3) - (49*b*c*x^
3)/(18*a^2) + (b*d*x^4)/(3*a^2) + (5*b*e*x^5)/(18*a^2))/(a^2*x + b^2*x^7 +
 2*a*b*x^4) + symsum(log((b^2*(225*a^2*d*e^2 - 225*root(19683*a^10*b*z^3 +
 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e
+ 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)*a^5*e^2 + 2744*b^2*c^3*x
 + 125*a^2*e^3*x + 1134*a*b*c*d^2 - 3402*root(19683*a^10*b*z^3 + 19683*a^7
*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*
d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)^2*a^7*b*c - 26244*root(19683*a^10*
b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b
*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)^3*a^10*b*x - 2916
*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b
*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)*
a^4*b*d^2*x - 17496*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670*a^4*b
*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3 - 125*a^2*e^3 - 2
744*b^2*c^3, z, k)^2*a^7*b*d*x + 2268*root(19683*a^10*b*z^3 + 19683*a^7*b*
d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d*e + 729*a*b*d^3
 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)*a^4*b*c*d + 6300*root(19683*a^10*b*z^
3 + 19683*a^7*b*d*z^2 - 5670*a^4*b*c*e*z + 6561*a^4*b*d^2*z - 1890*a*b*c*d
*e + 729*a*b*d^3 - 125*a^2*e^3 - 2744*b^2*c^3, z, k)*a^4*b*c*e*x + 1260*a*
b*c*d*e*x))/(729*a^8))*root(19683*a^10*b*z^3 + 19683*a^7*b*d*z^2 - 5670...
```

3.355. $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^3}\,dx$

3.356 $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^3}\,dx$

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3.356.1 Optimal result

Integrand size = 23, antiderivative size = 276

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^3}\,dx = -\frac{c}{2a^3x^2}-\frac{d}{a^3x}-\frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2}-\frac{x(11bc+10bdx+9bex^2)}{18a^3(a+bx^3)}$$
$$+\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}}$$
$$+\frac{e\log(x)}{a^3}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}}$$
$$+\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}$$
$$-\frac{e\log(a+bx^3)}{3a^3}$$

output
```
-1/2*c/a^3/x^2-d/a^3/x-1/6*x*(b*e*x^2+b*d*x+b*c)/a^2/(b*x^3+a)^2-1/18*x*(9
*b*e*x^2+10*b*d*x+11*b*c)/a^3/(b*x^3+a)+e*ln(x)/a^3-2/27*b^(1/3)*(10*b^(1/
3)*c-7*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)+1/27*b^(1/3)*(10*b^(1/3)*
c-7*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)-1/3*e*ln
(b*x^3+a)/a^3+2/27*b^(1/3)*(10*b^(1/3)*c+7*a^(1/3)*d)*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)*3^(1/2)
```

3.356.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.92

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{27ac}{x^2}-\frac{54ad}{x}+\frac{9a^2(ae-bx(c+dx))}{(a+bx^3)^2}+\frac{3a(6ae-bx(11c+10dx))}{a+bx^3}+4\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(10\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+ \cdots}{\cdots}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]
```

output
```
((-27*a*c)/x^2 - (54*a*d)/x + (9*a^2*(a*e - b*x*(c + d*x)))/(a + b*x^3)^2
+ (3*a*(6*a*e - b*x*(11*c + 10*d*x)))/(a + b*x^3) + 4*Sqrt[3]*a^(1/3)*b^(1
/3)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 54*a*e*Log[x] + 4*b^(1/3)*(-10*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a
^(1/3) + b^(1/3)*x] + 2*b^(1/3)*(10*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 18*a*e*Log[a + b*x^3])/(54*a^4
)
```

3.356.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \; 2368$$

$$-\frac{\int -\frac{-\frac{3b^2ex^5}{a}-\frac{4b^2dx^4}{a}-\frac{5b^2cx^3}{a}+6bex^2+6bdx+6bc}{x^3\left(bx^3+a\right)^2}dx}{6ab}-\frac{x\left(bc+bdx+bex^2\right)}{6a^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-\frac{3b^2ex^5}{a}-\frac{4b^2dx^4}{a}-\frac{5b^2cx^3}{a}+6bex^2+6bdx+6bc}{x^3(bx^3+a)^2}dx}{6ab}-\frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2}$$

$$\Big\downarrow \text{2368}$$

$$\frac{-\frac{\int -\frac{2\left(-\frac{5dx^4b^4}{a}-\frac{11cx^3b^4}{a}+9ex^2b^3+9cb^3+9dxb^3\right)}{x^3\left(bx^3+a\right)}dx}{3ab^2}-\frac{x(11b^2c+10b^2dx+9b^2ex^2)}{3a^2(a+bx^3)}}{6ab}-\frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{-\frac{5dx^4b^4}{a}-\frac{11cx^3b^4}{a}+9ex^2b^3+9cb^3+9dxb^3}{x^3\left(bx^3+a\right)}dx}{3ab^2}-\frac{x(11b^2c+10b^2dx+9b^2ex^2)}{3a^2(a+bx^3)}}{6ab}-\frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2}$$

$$\Big\downarrow \text{2373}$$

$$\frac{\frac{2\int \left(-\frac{(9ex^2+14dx+20c)b^4}{a\left(bx^3+a\right)}+\frac{9eb^3}{ax}+\frac{9db^3}{ax^2}+\frac{9cb^3}{ax^3}\right)dx}{3ab^2}-\frac{x(11b^2c+10b^2dx+9b^2ex^2)}{3a^2(a+bx^3)}}{6ab}-\frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2\left(\frac{2b^{10/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}d+10\sqrt[3]{b}c\right)}{\sqrt{3}a^{5/3}}+\frac{b^{10/3}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{3a^{5/3}}-\frac{2b^{10/3}\left(10\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}\cdots\right)}{3ab^2}\cdots}{6ab}$$

$$\frac{x(bc+bdx+bex^2)}{6a^2(a+bx^3)^2}$$

input
```
Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x]
```

output
```
-1/6*(x*(b*c + b*d*x + b*e*x^2))/(a^2*(a + b*x^3)^2) + (-1/3*(x*(11*b^2*c
+ 10*b^2*d*x + 9*b^2*e*x^2))/(a^2*(a + b*x^3)) + (2*((-9*b^3*c)/(2*a*x^2)
- (9*b^3*d)/(a*x) + (2*b^(10/3)*(10*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (9*b^3*e*Log[x])
/a - (2*b^(10/3)*(10*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3
*a^(5/3)) + (b^(10/3)*(10*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(3*a^(5/3)) - (3*b^3*e*Log[a + b*x^3])/a))/(3*a*b
^2))/(6*a*b)
```

3.356. $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^3}\,dx$

3.356.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.356.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.03

method	result
risch	$\frac{-\frac{14 d b^{2} x^{7}}{9 a^{3}}-\frac{10 c b^{2} x^{6}}{9 a^{3}}+\frac{b e x^{5}}{3 a^{2}}-\frac{49 b d x^{4}}{18 a^{2}}-\frac{16 b c x^{3}}{9 a^{2}}+\frac{e x^{2}}{2 a}-\frac{x d}{a}-\frac{c}{2 a}}{x^{2}\left(b x^{3}+a\right)^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^{3}+27 a^{8} e\_Z^{2}+\left(243 a^{5} e^{2}+840 a^{4} b c d\right)\_Z+729 a^{2} e\right.}\right.}{}$
default	$-\frac{c}{2 a^{3} x^{2}}-\frac{d}{a^{3} x}+\frac{e \ln \left(x\right)}{a^{3}}-\frac{b\left(\frac{\frac{5 b d x^{5}}{9}+\frac{11 b c x^{4}}{18}-\frac{a e x^{3}}{3}+\frac{13 a d x^{2}}{18}+\frac{7 a c x}{9}-\frac{a^{2} e}{2 b}}{\left(b x^{3}+a\right)^{2}}+\frac{20 c\left(\frac{\ln \left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\ \cdots}{\cdots}\right.}{9}\right.}{}$

input
```
int((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-14/9*d/a^3*b^2*x^7-10/9*c/a^3*b^2*x^6+1/3*b*e/a^2*x^5-49/18*b*d/a^2*x^4-
16/9*b*c/a^2*x^3+1/2/a*e*x^2-1/a*x*d-1/2*c/a)/x^2/(b*x^3+a)^2+1/27*sum(_R*
ln((-2*_R^3*a^11-36*_R^2*a^8*e+(-162*a^5*e^2-1400*a^4*b*c*d)*_R-7560*a*b*c
*d*e+4116*a*b*d^3-12000*b^2*c^3)*x-7*a^8*d*_R^2+(126*a^5*d*e-200*a^4*b*c^2
)*_R+1701*a^2*d*e^2+5400*a*b*c^2*e),_R=RootOf(a^11*_Z^3+27*a^8*e*_Z^2+(243
*a^5*e^2+840*a^4*b*c*d)*_Z+729*a^2*e^3+7560*a*b*c*d*e-2744*a*b*d^3+8000*b^
2*c^3))+e/a^3*ln(-x)
```

3.356.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 4911, normalized size of antiderivative = 17.79

$$\int \frac{c+dx+ex^2}{x^3 (a+bx^3)^3} \, dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/2916*(4536*b^2*d*x^7 + 3240*b^2*c*x^6 - 972*a*b*e*x^5 + 7938*a*b*d*x^4
+ 5184*a*b*c*x^3 - 1458*a^2*e*x^2 + 2916*a^2*d*x + 1458*a^2*c + 2*(a^3*b^2
*x^8 + 2*a^4*b*x^5 + a^5*x^2)*((-I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d +
 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/
19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^
3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*
e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343
*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*
d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)*log(7/2916*((-I*sqrt(3) + 1)*(81*e^2/a^
6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*
e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*
c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 729*(I*sqrt
(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(
1000*b*c^3 + 343*a*d^3)*b/a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*
(49*d^3 - 135*c*d*e)*a*b)/a^11)^(1/3) + 486*e/a^3)^2*a^8*d + 3920*a*b*c*d^
2 - 1800*a*b*c^2*e + 567*a^2*d*e^2 + 1/27*(100*a^4*b*c^2 - 63*a^5*d*e)*((-
I*sqrt(3) + 1)*(81*e^2/a^6 - (280*b*c*d + 81*a*e^2)/a^7)/(-1/27*e^3/a^9 +
1/1458*(280*b*c*d + 81*a*e^2)*e/a^10 + 4/19683*(1000*b*c^3 + 343*a*d^3)*b/
a^11 - 1/39366*(8000*b^2*c^3 + 729*a^2*e^3 - 56*(49*d^3 - 135*c*d*e)*a*b)/
a^11)^(1/3) + 729*(I*sqrt(3) + 1)*(-1/27*e^3/a^9 + 1/1458*(280*b*c*d + ...
```

3.356.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2}{x^3 (a + bx^3)^3} \, dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^3}\,dx$$
$$= -\frac{28\,b^2dx^7+20\,b^2cx^6-6\,abex^5+49\,abdx^4+32\,abcx^3-9\,a^2ex^2+18\,a^2dx+9\,a^2c}{18\,(a^3b^2x^8+2\,a^4bx^5+a^5x^2)}$$
$$+\frac{e\log(x)}{a^3}-\frac{2\sqrt{3}\left(7\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+10\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4}$$
$$-\frac{\left(9\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}+7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}-10\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(9\,e\left(\frac{a}{b}\right)^{\frac{2}{3}}-14\,d\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*(28*b^2*d*x^7 + 20*b^2*c*x^6 - 6*a*b*e*x^5 + 49*a*b*d*x^4 + 32*a*b*c
*x^3 - 9*a^2*e*x^2 + 18*a^2*d*x + 9*a^2*c)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^
5*x^2) + e*log(x)/a^3 - 2/27*sqrt(3)*(7*b*d*(a/b)^(2/3) + 10*b*c*(a/b)^(1/
3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^4 - 1/27*(9*e*(a
/b)^(2/3) + 7*d*(a/b)^(1/3) - 10*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a^3*(a/b)^(2/3)) - 1/27*(9*e*(a/b)^(2/3) - 14*d*(a/b)^(1/3) + 20*c)*log(
x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

3.356.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^3}\,dx$$
$$= -\frac{e\log\left(|bx^3+a|\right)}{3\,a^3} + \frac{e\log\left(|x|\right)}{a^3} - \frac{2\sqrt{3}\left(10\left(-ab^2\right)^{\frac{1}{3}}bc - 7\left(-ab^2\right)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b} - \frac{\left(10\left(-ab^2\right)^{\frac{1}{3}}bc + 7\left(-ab^2\right)^{\frac{2}{3}}d\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4b} - \frac{28\,b^2dx^7 + 20\,b^2cx^6 - 6\,abex^5 + 49\,abdx^4 + 32\,abcx^3 - 9\,a^2ex^2 + 18\,a^2dx + 9\,a^2c}{18\left(bx^4+ax\right)^2a^3} + \frac{2\left(7\,a^3b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 10\,a^3b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/3*e*log(abs(b*x^3 + a))/a^3 + e*log(abs(x))/a^3 - 2/27*sqrt(3)*(10*(-a*
b^2)^(1/3)*b*c - 7*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/(a^4*b) - 1/27*(10*(-a*b^2)^(1/3)*b*c + 7*(-a*b^2)^(2/3)*
d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/18*(28*b^2*d*x^7 +
 20*b^2*c*x^6 - 6*a*b*e*x^5 + 49*a*b*d*x^4 + 32*a*b*c*x^3 - 9*a^2*e*x^2 +
18*a^2*d*x + 9*a^2*c)/((b*x^4 + a*x)^2*a^3) + 2/27*(7*a^3*b^2*d*(-a/b)^(1/
3) + 10*a^3*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

3.356.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 778, normalized size of antiderivative = 2.82

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^3}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(-\frac{b^3\left(\mathrm{root}(19683\,a^{11}\,z^3+19683\,a^8\,e\,z^2+22680\,a^4\,b\,c\,d\,z+6561\,a^5\,e^2\,z+7560\,a\,b\,c\,d\,e-2744\,a\right.}{}\right.\right.$$
$$+19683\,a^8\,e\,z^2+22680\,a^4\,b\,c\,d\,z+6561\,a^5\,e^2\,z+7560\,a\,b\,c\,d\,e-2744\,a\,b\,d^3+729\,a^2\,e^3$$
$$\left.\left.+8000\,b^2\,c^3,z,k\right)\right)$$
$$-\frac{\frac{c}{2\,a}-\frac{e\,x^2}{2\,a}+\frac{d\,x}{a}+\frac{10\,b^2\,c\,x^6}{9\,a^3}+\frac{14\,b^2\,d\,x^7}{9\,a^3}+\frac{16\,b\,c\,x^3}{9\,a^2}+\frac{49\,b\,d\,x^4}{18\,a^2}-\frac{b\,e\,x^5}{3\,a^2}}{a^2\,x^2+2\,a\,b\,x^5+b^2\,x^8}+\frac{e\,\ln(x)}{a^3}$$

input
```
int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^3),x)
```

output
```
symsum(log(-(2*b^3*(1701*root(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4
*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 +
8000*b^2*c^3, z, k)^2*a^8*d - 567*a^2*d*e^2 + 13122*root(19683*a^11*z^3 +
19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 27
44*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)^3*a^11*x + 4000*b^2*c^3*x -
 1134*root(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5
*e^2*z + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)
*a^5*d*e - 1800*a*b*c^2*e - 1372*a*b*d^3*x + 1800*root(19683*a^11*z^3 + 19
683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 2744
*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^4*b*c^2 + 1458*root(19683*a
^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*
c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)*a^5*e^2*x + 8748*
root(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z
 + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k)^2*a^8
*e*x + 12600*root(19683*a^11*z^3 + 19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6
561*a^5*e^2*z + 7560*a*b*c*d*e - 2744*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3
, z, k)*a^4*b*c*d*x + 2520*a*b*c*d*e*x))/(729*a^9))*root(19683*a^11*z^3 +
19683*a^8*e*z^2 + 22680*a^4*b*c*d*z + 6561*a^5*e^2*z + 7560*a*b*c*d*e - 27
44*a*b*d^3 + 729*a^2*e^3 + 8000*b^2*c^3, z, k), k, 1, 3) - (c/(2*a) - (e*x
^2)/(2*a) + (d*x)/a + (10*b^2*c*x^6)/(9*a^3) + (14*b^2*d*x^7)/(9*a^3) +...
```

3.357 $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx$

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3.357.1 Optimal result

Integrand size = 23, antiderivative size = 298

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx =& -\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{6a^2(a+bx^3)^2}\\ &-\frac{x\left(11bd+10bex-\frac{15b^2cx^2}{a}\right)}{18a^3(a+bx^3)}\\ &+\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}}\\ &-\frac{3bc\log(x)}{a^4}-\frac{2\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}}\\ &+\frac{\sqrt[3]{b}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}}\\ &+\frac{bc\log(a+bx^3)}{a^4}\end{aligned}$$

output
```
-1/3*c/a^3/x^3-1/2*d/a^3/x^2-e/a^3/x-1/6*x*(b*d+b*e*x-b^2*c*x^2/a)/a^2/(b*
x^3+a)^2-1/18*x*(11*b*d+10*b*e*x-15*b^2*c*x^2/a)/a^3/(b*x^3+a)-3*b*c*ln(x)
/a^4-2/27*b^(1/3)*(10*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3
)+1/27*b^(1/3)*(10*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/a^(11/3)+b*c*ln(b*x^3+a)/a^4+2/27*b^(1/3)*(10*b^(1/3)*d+7*a^(1/3
)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)*3^(1/2)
```

3.357.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.86

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{\frac{18ac}{x^3}+\frac{27ad}{x^2}+\frac{54ae}{x}+\frac{9a^2b(c+x(d+ex))}{(a+bx^3)^2}+\frac{3ab(12c+x(11d+10ex))}{a+bx^3}-4\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(10\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```

output
```
-1/54*((18*a*c)/x^3 + (27*a*d)/x^2 + (54*a*e)/x + (9*a^2*b*(c + x*(d + e*x
)))/(a + b*x^3)^2 + (3*a*b*(12*c + x*(11*d + 10*e*x)))/(a + b*x^3) - 4*Sqr
t[3]*a^(1/3)*b^(1/3)*(10*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]] + 162*b*c*Log[x] + 4*b^(1/3)*(10*a^(1/3)*b^(1/3)*d - 7
*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x] - 2*b^(1/3)*(10*a^(1/3)*b^(1/3)*d - 7
*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 54*b*c*Log[a
+ b*x^3])/a^4
```

3.357.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \; 2368$$

$$-\frac{\int -\frac{\frac{3b^3cx^6}{a^2}-\frac{4b^2ex^5}{a}-\frac{5b^2dx^4}{a}-\frac{6b^2cx^3}{a}+6bex^2+6bdx+6bc}{x^4\left(bx^3+a\right)^2}dx}{6ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{3b^3cx^6}{a^2}-\frac{4b^2ex^5}{a}-\frac{5b^2dx^4}{a}-\frac{6b^2cx^3}{a}+6bex^2+6bdx+6bc}{x^4(bx^3+a)^2}dx}{6ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2(a+bx^3)^2}$$

$$\Big\downarrow \text{ 2368}$$

$$-\frac{-\frac{\int -\frac{2\left(-\frac{5b^4ex^5}{a}-\frac{11b^4dx^4}{a}-\frac{18b^4cx^3}{a}+9b^3ex^2+9b^3dx+9b^3c\right)}{x^4(bx^3+a)}dx}{3ab^2}-\frac{x\left(-\frac{15b^3cx^2}{a}+11b^2d+10b^2ex\right)}{3a^2(a+bx^3)}}{6ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2(a+bx^3)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2\int \frac{-\frac{5b^4ex^5}{a}-\frac{11b^4dx^4}{a}-\frac{18b^4cx^3}{a}+9b^3ex^2+9b^3dx+9b^3c}{x^4(bx^3+a)}dx}{3ab^2}-\frac{x\left(-\frac{15b^3cx^2}{a}+11b^2d+10b^2ex\right)}{3a^2(a+bx^3)}}{6ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2(a+bx^3)^2}$$

$$\Big\downarrow \text{ 2373}$$

$$\frac{\frac{2\int \left(-\frac{27cb^4}{a^2x}-\frac{(-27bcx^2+14aex+20ad)b^4}{a^2(bx^3+a)}+\frac{9eb^3}{ax^2}+\frac{9db^3}{ax^3}+\frac{9cb^3}{ax^4}\right)dx}{3ab^2}-\frac{x\left(-\frac{15b^3cx^2}{a}+11b^2d+10b^2ex\right)}{3a^2(a+bx^3)}}{6ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2(a+bx^3)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2\left(\frac{2b^{10/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}e+10\sqrt[3]{b}d\right)}{\sqrt{3}a^{5/3}}+\frac{b^{10/3}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{3a^{5/3}}-\frac{2b^{10/3}\left(10\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}\right.}{3ab^2}}{6ab}$$

$$\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{6a^2(a+bx^3)^2}$$

input
```
Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x]
```

3.357. $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx$

output
```
-1/6*(x*(b*d + b*e*x - (b^2*c*x^2)/a))/(a^2*(a + b*x^3)^2) + (-1/3*(x*(11*
b^2*d + 10*b^2*e*x - (15*b^3*c*x^2)/a))/(a^2*(a + b*x^3)) + (2*((-3*b^3*c)
/(a*x^3) - (9*b^3*d)/(2*a*x^2) - (9*b^3*e)/(a*x) + (2*b^(10/3)*(10*b^(1/3)
*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*a^(5/3)) - (27*b^4*c*Log[x])/a^2 - (2*b^(10/3)*(10*b^(1/3)*d - 7*a^(1/
3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (b^(10/3)*(10*b^(1/3)*d - 7*
a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3*a^(5/3)) + (
9*b^4*c*Log[a + b*x^3])/a^2))/(3*a*b^2))/(6*a*b)
```

3.357.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.357. $\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx$

3.357.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.01

method	result
default	$-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{3bc\ln(x)}{a^4}-\frac{b\left(\frac{\frac{5}{9}abe\,x^5+\frac{11}{18}abd\,x^4+\frac{2}{3}abc\,x^3+\frac{13}{18}a^2e\,x^2+\frac{7}{9}a^2dx+\frac{5}{6}a^2c}{\left(b\,x^3+a\right)^2}+\frac{20ad\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{-\frac{14e\,b^2x^8}{9a^3}-\frac{10d\,b^2x^7}{9a^3}-\frac{c\,b^2x^6}{a^3}-\frac{49be\,x^5}{18a^2}-\frac{16bd\,x^4}{9a^2}-\frac{3bc\,x^3}{2a^2}-\frac{e\,x^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3\left(b\,x^3+a\right)^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^{12}\_Z^3-81a^8bc\_Z^2+\left(840a^5bde+2187a^4b^2c\cdots\right.\right.}\right.}{\cdots}$

input
```
int((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c/a^3/x^3-1/2*d/a^3/x^2-e/a^3/x-3*b*c*ln(x)/a^4-1/a^4*b*((5/9*a*b*e*x
^5+11/18*a*b*d*x^4+2/3*a*b*c*x^3+13/18*a^2*e*x^2+7/9*a^2*d*x+5/6*a^2*c)/(b
*x^3+a)^2+20/9*a*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1)))+14/9*a*e*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+
1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-c*ln(b*x^3+a))
```

3.357.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 5550, normalized size of antiderivative = 18.62

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.357.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.357.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx = -\frac{28\,b^2ex^8+20\,b^2dx^7+18\,b^2cx^6+49\,abex^5+32\,abdx^4+27\,abcx^3+18\,a^2ex^2+9\,a^2dx+6\,a^2c}{18\,(a^3b^2x^9+2\,a^4bx^6+a^5x^3)} - \frac{3\,bc\log(x)}{a^4} - \frac{2\,\sqrt{3}\left(7\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+10\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5} + \frac{\left(27\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-7\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+10\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(27\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*(28*b^2*e*x^8 + 20*b^2*d*x^7 + 18*b^2*c*x^6 + 49*a*b*e*x^5 + 32*a*b*
d*x^4 + 27*a*b*c*x^3 + 18*a^2*e*x^2 + 9*a^2*d*x + 6*a^2*c)/(a^3*b^2*x^9 +
2*a^4*b*x^6 + a^5*x^3) - 3*b*c*log(x)/a^4 - 2/27*sqrt(3)*(7*a*e*(a/b)^(2/3
) + 10*a*d*(a/b)^(1/3))*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/a^5 + 1/27*(27*b*c*(a/b)^(2/3) - 7*a*e*(a/b)^(1/3) + 10*a*d)*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) + 1/27*(27*b*c*(a/b)^(2/3)
 + 14*a*e*(a/b)^(1/3) - 20*a*d)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(2/3))
```

3.357.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^3}\,dx = \frac{bc\log(|bx^3+a|)}{a^4} - \frac{3\,bc\log(|x|)}{a^4} - \frac{2\sqrt{3}\left(10\,(-ab^2)^{\frac{1}{3}}\,bd - 7\,(-ab^2)^{\frac{2}{3}}\,e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b} - \frac{\left(10\,(-ab^2)^{\frac{1}{3}}\,bd + 7\,(-ab^2)^{\frac{2}{3}}\,e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4b} + \frac{2\left(7\,a^5b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+10\,a^5b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^9b} - \frac{28\,ab^2ex^8+20\,ab^2dx^7+18\,ab^2cx^6+49\,a^2bex^5+32\,a^2bdx^4+27\,a^2bcx^3+18\,a^3ex^2+9\,a^3dx+6\,a^3c}{18\,(bx^3+a)^2a^4x^3}$$

input
```
integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
b*c*log(abs(b*x^3 + a))/a^4 - 3*b*c*log(abs(x))/a^4 - 2/27*sqrt(3)*(10*(-a
*b^2)^(1/3)*b*d - 7*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^4*b) - 1/27*(10*(-a*b^2)^(1/3)*b*d + 7*(-a*b^2)^(2/3)
*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) + 2/27*(7*a^5*b^2*e*(
-a/b)^(1/3) + 10*a^5*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b
) - 1/18*(28*a*b^2*e*x^8 + 20*a*b^2*d*x^7 + 18*a*b^2*c*x^6 + 49*a^2*b*e*x^
5 + 32*a^2*b*d*x^4 + 27*a^2*b*c*x^3 + 18*a^3*e*x^2 + 9*a^3*d*x + 6*a^3*c)/
((b*x^3 + a)^2*a^4*x^3)
```

3.357.9 Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 870, normalized size of antiderivative = 2.92

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^3}\,dx$$

$$= \left(\sum_{k=1}^{3}\ln\left(-\frac{b^3\left(\mathrm{root}(19683\,a^{12}\,z^3-59049\,a^8\,b\,c\,z^2+22680\,a^5\,b\,d\,e\,z+59049\,a^4\,b^2\,c^2\,z-22680\,a\,b^2\,c\,d\,e-\right.}{}\right.\right.$$

$$-59049\,a^8\,b\,c\,z^2+22680\,a^5\,b\,d\,e\,z+59049\,a^4\,b^2\,c^2\,z-22680\,a\,b^2\,c\,d\,e-2744\,a^2\,b\,e^3$$

$$\left.+8000\,a\,b^2\,d^3-19683\,b^3\,c^3,z,k\right)\Bigg)$$

$$-\frac{\frac{c}{3a}+\frac{e\,x^2}{a}+\frac{d\,x}{2a}+\frac{b^2\,c\,x^6}{a^3}+\frac{10\,b^2\,d\,x^7}{9\,a^3}+\frac{14\,b^2\,e\,x^8}{9\,a^3}+\frac{3\,b\,c\,x^3}{2\,a^2}+\frac{16\,b\,d\,x^4}{9\,a^2}+\frac{49\,b\,e\,x^5}{18\,a^2}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9}-\frac{3\,b\,c\,\ln(x)}{a^4}$$

input
```
int((c + d*x + e*x^2)/(x^4*(a + b*x^3)^3),x)
```

output
```
symsum(log(-(2*b^3*(1701*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a
^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 80
00*a*b^2*d^3 - 19683*b^3*c^3, z, k)^2*a^8*e + 5400*b^2*c*d^2 - 5103*b^2*c^
2*e + 13122*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z +
59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3
- 19683*b^3*c^3, z, k)^3*a^11*x + 4000*b^2*d^3*x - 1372*a*b*e^3*x + 1800*r
oot(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2
*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c
^3, z, k)*a^4*b*d^2 - 26244*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 2268
0*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 +
 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)^2*a^7*b*c*x + 13122*root(19683*a^12
*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680
*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k)*a^3*
b^2*c^2*x + 3402*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e
*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2
*d^3 - 19683*b^3*c^3, z, k)*a^4*b*c*e - 7560*b^2*c*d*e*x + 12600*root(1968
3*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z -
 22680*a*b^2*c*d*e - 2744*a^2*b*e^3 + 8000*a*b^2*d^3 - 19683*b^3*c^3, z, k
)*a^4*b*d*e*x))/(729*a^9))*root(19683*a^12*z^3 - 59049*a^8*b*c*z^2 + 22680
*a^5*b*d*e*z + 59049*a^4*b^2*c^2*z - 22680*a*b^2*c*d*e - 2744*a^2*b*e^3...
```

3.358 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx$

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3.358.1 Optimal result

Integrand size = 23, antiderivative size = 248

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx = -\frac{c+dx+ex^2}{9b(a+bx^3)^3}+\frac{x(d+2ex)}{54ab(a+bx^3)^2}+\frac{x(5d+8ex)}{162a^2b(a+bx^3)} - \frac{\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{8/3}b^{5/3}} + \frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{8/3}b^{5/3}} - \frac{\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{8/3}b^{5/3}}$$

output
```
1/9*(-e*x^2-d*x-c)/b/(b*x^3+a)^3+1/54*x*(2*e*x+d)/a/b/(b*x^3+a)^2+1/162*x*
(8*e*x+5*d)/a^2/b/(b*x^3+a)+1/243*(5*b^(1/3)*d-4*a^(1/3)*e)*ln(a^(1/3)+b^(
1/3)*x)/a^(8/3)/b^(5/3)-1/486*(5*b^(1/3)*d-4*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(5/3)-1/243*(5*b^(1/3)*d+4*a^(1/3)*e)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(5/3)*3^(1/2)
```

3.358.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.93

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$

$$= \frac{\frac{9b^{2/3}x(d+2ex)}{a(a+bx^3)^2}+\frac{3b^{2/3}x(5d+8ex)}{a^2(a+bx^3)}-\frac{54b^{2/3}(c+x(d+ex))}{(a+bx^3)^3}-\frac{2\sqrt{3}\left(5\sqrt[3]{b}d+4\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}}+\frac{2\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)\log\left(\sqrt[3]{\cdots}\right.}{a^{8/3}}}{486b^{5/3}}$$

input
```
Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

output
```
((9*b^(2/3)*x*(d + 2*e*x))/(a*(a + b*x^3)^2) + (3*b^(2/3)*x*(5*d + 8*e*x))
/(a^2*(a + b*x^3)) - (54*b^(2/3)*(c + x*(d + e*x)))/(a + b*x^3)^3 - (2*Sqr
t[3]*(5*b^(1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/a^(8/3) + (2*(5*b^(1/3)*d - 4*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x]])/a^(
8/3) + ((-5*b^(1/3)*d + 4*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(8/3))/(486*b^(5/3))
```

3.358.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2363, 2394, 25, 2394, 27, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$

$$\Big\downarrow \ 2363$$

$$\frac{\int \frac{d+2ex}{(bx^3+a)^3}dx}{9b}-\frac{c+dx+ex^2}{9b(a+bx^3)^3}$$

$$\Big\downarrow \ 2394$$

$$\frac{\frac{x(d+2ex)}{6a(a+bx^3)^2}-\frac{\int -\frac{5d+8ex}{(bx^3+a)^2}dx}{6a}}{9b}-\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{5d+8ex}{(bx^3+a)^2}dx}{6a}+\frac{x(d+2ex)}{6a(a+bx^3)^2}}{9b}-\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}$$

$$\Big\downarrow \ 2394$$

$$\frac{\frac{\frac{x(5d+8ex)}{3a\left(a+bx^3\right)}-\frac{\int -\frac{2(5d+4ex)}{bx^3+a}dx}{3a}}{6a}+\frac{x(d+2ex)}{6a(a+bx^3)^2}}{9b}-\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{2\int \frac{5d+4ex}{bx^3+a}dx}{3a}+\frac{x(5d+8ex)}{3a\left(a+bx^3\right)}}{6a}+\frac{x(d+2ex)}{6a(a+bx^3)^2}}{9b}-\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}$$

$$\Big\downarrow \ 2399$$

$$\frac{\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}\left(5\sqrt[3]{b}d+2\sqrt[3]{a}e\right)-\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x(5d+8ex)}{3a\left(a+bx^3\right)}}{6a}+\frac{x(d+2ex)}{6a(a+bx^3)^2}}{9b}-\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}\left(5\sqrt[3]{b}d+2\sqrt[3]{a}e\right)-\sqrt[3]{b}\left(5\sqrt[3]{b}d-4\sqrt[3]{a}e\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5d+8ex)}{3a\left(a+bx^3\right)}}{6a}+\frac{x(d+2ex)}{6a(a+bx^3)^2}}{9b}-\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}$$

$$\Big\downarrow \ 1142$$

3.358. $\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^4}\,dx$

$$\frac{\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5\ldots}{3a(\ldots}}{6a}\ldots}{9b}$$

$$\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5d+\ldots}{3a(a+\ldots}}{6a}\ldots}{9b}$$

$$\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\ldots}{3\ldots}}{6a}\ldots}{9b}$$

$$\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 1082$$

3.358. $\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^4}\,dx$

$$\frac{\frac{\frac{2\left(\frac{\frac{1}{2}\sqrt[3]{b}\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+}{6a}}{9b}$$

$$\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\frac{2\left(\frac{\frac{1}{2}\sqrt[3]{b}\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5d+8ex)}{3a\left(a+bx^3\right)}}{6a}+}{9b}$$

$$\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\frac{2\left(\frac{-\frac{1}{2}\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(4\sqrt[3]{a}e+5\sqrt[3]{b}d\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5d-\frac{4\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5d+8ex)}{3a\left(a+bx^3\right)}}{6a}+}{9b}$$

$$\frac{c+dx+ex^2}{9b\left(a+bx^3\right)^3}$$

3.358. $\int \frac{x^2\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^4}\,dx$

input
```
Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

output
```
-1/9*(c + d*x + e*x^2)/(b*(a + b*x^3)^3) + ((x*(d + 2*e*x))/(6*a*(a + b*x^
3)^2) + ((x*(5*d + 8*e*x))/(3*a*(a + b*x^3)) + (2*(((5*d - (4*a^(1/3)*e)/b
^(1/3))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*(5*b^(
1/3)*d + 4*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)
) - ((5*d - (4*a^(1/3)*e)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/2)/(3*a^(2/3)*b^(1/3))))/(3*a))/(6*a))/(9*b)
```

3.358.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.358.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\frac{4be\,x^8}{81a^2}+\frac{5bd\,x^7}{162a^2}+\frac{11e\,x^5}{81a}+\frac{13d\,x^4}{162a}-\frac{2e\,x^2}{81b}-\frac{5dx}{81b}-\frac{c}{9b}}{(b\,x^3+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(4e\_R+5d\right)\ln\left(x-\_R\right)}{\_R^2}}{243a^2b^2}$
default	$\frac{\frac{4be\,x^8}{81a^2}+\frac{5bd\,x^7}{162a^2}+\frac{11e\,x^5}{81a}+\frac{13d\,x^4}{162a}-\frac{2e\,x^2}{81b}-\frac{5dx}{81b}-\frac{c}{9b}}{(b\,x^3+a)^3}+\frac{5d\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+}{81c}$

input
```
int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(4/81*b*e/a^2*x^8+5/162*b*d/a^2*x^7+11/81/a*e*x^5+13/162*d/a*x^4-2/81*e*x^
2/b-5/81*d*x/b-1/9*c/b)/(b*x^3+a)^3+1/243/a^2/b^2*sum((4*_R*e+5*d)/_R^2*ln
(x-_R),_R=RootOf(_Z^3*b+a))
```

3.358.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 2364, normalized size of antiderivative = 9.53

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

output
```
1/972*(48*b^2*e*x^8 + 30*b^2*d*x^7 + 132*a*b*e*x^5 + 78*a*b*d*x^4 - 24*a^2
*e*x^2 - 60*a^2*d*x - 108*a^2*c - 2*(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b
^2*x^3 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*
b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sq
rt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*
e^3)/(a^8*b^5))^(1/3)))*log(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*
a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3
)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b
*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))^2*a^6*b^3*e - 25/2*((1/2)^(1/3)*(I*sqr
t(3) + 1)*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*
b^5))^(1/3) - 40*(1/2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 6
4*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3)))*a^3*b^2*d^2
 + 160*a*d*e^2 + (125*b*d^3 + 64*a*e^3)*x) + ((a^2*b^4*x^9 + 3*a^3*b^3*x^6
 + 3*a^4*b^2*x^3 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^3 + 64*a*
e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/2)^(2/3)*
d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5) + (125*b*d
^3 - 64*a*e^3)/(a^8*b^5))^(1/3))) + 3*sqrt(1/3)*(a^2*b^4*x^9 + 3*a^3*b^3*x
^6 + 3*a^4*b^2*x^3 + a^5*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*d^
3 + 64*a*e^3)/(a^8*b^5) + (125*b*d^3 - 64*a*e^3)/(a^8*b^5))^(1/3) - 40*(1/
2)^(2/3)*d*e*(-I*sqrt(3) + 1)/(a^5*b^3*((125*b*d^3 + 64*a*e^3)/(a^8*b^5...
```

3.358.6 Sympy [A] (verification not implemented)

Time = 156.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.81

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$
$$= \operatorname{RootSum}\left(14348907t^3a^8b^5 + 14580ta^3b^2de + 64ae^3 - 125bd^3, \left(t \mapsto t\log\left(x + \frac{236196t^2a^6b^3e + 6075ta^3b\right.\right.\right.$$
$$\frac{}{64ae^3 + 125b}$$
$$+ \frac{-18a^2c - 10a^2dx - 4a^2ex^2 + 13abdx^4 + 22abex^5 + 5b^2dx^7 + 8b^2ex^8}{162a^5b + 486a^4b^2x^3 + 486a^3b^3x^6 + 162a^2b^4x^9}$$

input
```
integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

output
```
RootSum(14348907*_t**3*a**8*b**5 + 14580*_t*a**3*b**2*d*e + 64*a*e**3 - 12
5*b*d**3, Lambda(_t, _t*log(x + (236196*_t**2*a**6*b**3*e + 6075*_t*a**3*b
**2*d**2 + 160*a*d*e**2)/(64*a*e**3 + 125*b*d**3)))) + (-18*a**2*c - 10*a*
*2*d*x - 4*a**2*e*x**2 + 13*a*b*d*x**4 + 22*a*b*e*x**5 + 5*b**2*d*x**7 + 8
*b**2*e*x**8)/(162*a**5*b + 486*a**4*b**2*x**3 + 486*a**3*b**3*x**6 + 162*
a**2*b**4*x**9)
```

3.358.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx = \frac{8\,b^2ex^8+5\,b^2dx^7+22\,abex^5+13\,abdx^4-4\,a^2ex^2-10\,a^2dx-18\,a^2c}{162\,(a^2b^4x^9+3\,a^3b^3x^6+3\,a^4b^2x^3+a^5b)} + \frac{\sqrt{3}\left(4\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(4\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,d\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(4\,e\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,d\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

output
```
1/162*(8*b^2*e*x^8 + 5*b^2*d*x^7 + 22*a*b*e*x^5 + 13*a*b*d*x^4 - 4*a^2*e*x
^2 - 10*a^2*d*x - 18*a^2*c)/(a^2*b^4*x^9 + 3*a^3*b^3*x^6 + 3*a^4*b^2*x^3 +
 a^5*b) + 1/243*sqrt(3)*(4*e*(a/b)^(1/3) + 5*d)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) + 1/486*(4*e*(a/b)^(1/3) -
 5*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) - 1/243
*(4*e*(a/b)^(1/3) - 5*d)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

3.358.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$
$$= -\frac{\sqrt{3}\left(5\,bd-4\,(-ab^2)^{\frac{1}{3}}\,e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,(-ab^2)^{\frac{2}{3}}\,a^2b} - \frac{\left(5\,bd+4\,(-ab^2)^{\frac{1}{3}}\,e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,(-ab^2)^{\frac{2}{3}}\,a^2b} - \frac{\left(4\,e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^3b} + \frac{8\,b^2ex^8+5\,b^2dx^7+22\,abex^5+13\,abdx^4-4\,a^2ex^2-10\,a^2dx-18\,a^2c}{162\,(bx^3+a)^3a^2b}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

output
```
-1/243*sqrt(3)*(5*b*d - 4*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b) - 1/486*(5*b*d + 4*(-a*b^2)
^(1/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b)
- 1/243*(4*e*(-a/b)^(1/3) + 5*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(
a^3*b) + 1/162*(8*b^2*e*x^8 + 5*b^2*d*x^7 + 22*a*b*e*x^5 + 13*a*b*d*x^4 -
4*a^2*e*x^2 - 10*a^2*d*x - 18*a^2*c)/((b*x^3 + a)^3*a^2*b)
```

3.358.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.02

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\frac{20\,d\,e+16\,e^2\,x+\mathrm{root}(14348907\,a^8\,b^5\,z^3+14580\,a^3\,b^2\,d\,e\,z-125\,b\,d^3+64\,a\,e^3,z,k)^2\,a^5\,b^3\,5904}{a^4\,b\,6561} + 14580\,a^3\,b^2\,d\,e\,z-125\,b\,d^3+64\,a\,e^3,z,k)\right) + \frac{\frac{13\,d\,x^4}{162\,a}-\frac{c}{9\,b}+\frac{11\,e\,x^5}{81\,a}-\frac{2\,e\,x^2}{81\,b}-\frac{5\,d\,x}{81\,b}+\frac{5\,b\,d\,x^7}{162\,a^2}+\frac{4\,b\,e\,x^8}{81\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}$$

input
```
int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^4,x)
```

output
```
symsum(log((20*d*e + 16*e^2*x + 59049*root(14348907*a^8*b^5*z^3 + 14580*a^
3*b^2*d*e*z - 125*b*d^3 + 64*a*e^3, z, k)^2*a^5*b^3 + 1215*root(14348907*a
^8*b^5*z^3 + 14580*a^3*b^2*d*e*z - 125*b*d^3 + 64*a*e^3, z, k)*a^2*b^2*d*x
)/(6561*a^4*b))*root(14348907*a^8*b^5*z^3 + 14580*a^3*b^2*d*e*z - 125*b*d^
3 + 64*a*e^3, z, k), k, 1, 3) + ((13*d*x^4)/(162*a) - c/(9*b) + (11*e*x^5)
/(81*a) - (2*e*x^2)/(81*b) - (5*d*x)/(81*b) + (5*b*d*x^7)/(162*a^2) + (4*b
*e*x^8)/(81*a^2))/(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6)
```

3.358. $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^4}\,dx$

3.359 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx$

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3.359.2 Mathematica [A] (verified) 2697
3.359.3 Rubi [A] (verified) 2697
3.359.4 Maple [C] (verified) 2703
3.359.5 Fricas [C] (verification not implemented) 2704
3.359.6 Sympy [A] (verification not implemented) 2705
3.359.7 Maxima [A] (verification not implemented) 2705
3.359.8 Giac [A] (verification not implemented) 2706
3.359.9 Mupad [B] (verification not implemented) 2707

3.359.1 Optimal result

Integrand size = 21, antiderivative size = 270

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx = -\frac{x(ae-bcx-bdx^2)}{9ab(a+bx^3)^3}+\frac{x(5ae+28bcx)}{162a^3b(a+bx^3)}-\frac{6ad-x(ae+7bcx)}{54a^2b(a+bx^3)^2}-\frac{(14b^{2/3}c+5a^{2/3}e)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{10/3}b^{4/3}}-\frac{(14b^{2/3}c-5a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{10/3}b^{4/3}}+\frac{(14b^{2/3}c-5a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{486a^{10/3}b^{4/3}}$$

output
```
-1/9*x*(-b*d*x^2-b*c*x+a*e)/a/b/(b*x^3+a)^3+1/162*x*(28*b*c*x+5*a*e)/a^3/b
/(b*x^3+a)+1/54*(-6*a*d+x*(7*b*c*x+a*e))/a^2/b/(b*x^3+a)^2-1/243*(14*b^(2/
3)*c-5*a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(4/3)+1/486*(14*b^(2/3)
*c-5*a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(4/3)
-1/243*(14*b^(2/3)*c+5*a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)
*3^(1/2))/a^(10/3)/b^(4/3)*3^(1/2)
```

3.359.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.89

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$

$$= \frac{\frac{3ab^{2/3}(28b^3cx^8-2a^3(9d+5ex)+ab^2x^5(77c+5ex^2)+a^2bx^2(67c+13ex^2))}{(a+bx^3)^3} - 2\sqrt{3}a^{2/3}\sqrt[3]{b}(14b^{2/3}c+5a^{2/3}e)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{486a}$$

input
```
Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

output
```
((3*a*b^(2/3)*(28*b^3*c*x^8 - 2*a^3*(9*d + 5*e*x) + a*b^2*x^5*(77*c + 5*e*
x^2) + a^2*b*x^2*(67*c + 13*e*x^2)))/(a + b*x^3)^3 - 2*Sqrt[3]*a^(2/3)*b^(
1/3)*(14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] + 2*(-14*a^(2/3)*b*c + 5*a^(4/3)*b^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x] +
 a^(2/3)*b^(1/3)*(14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(486*a^4*b^(5/3))
```

3.359.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2367, 25, 2393, 25, 2394, 27, 2399, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$

$$\Big\downarrow \text{2367}$$

$$-\frac{\int -\frac{6bdx^2+7bcx+ae}{(bx^3+a)^3}dx}{9ab}-\frac{x(ae-bcx-bdx^2)}{9ab(a+bx^3)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6bdx^2+7bcx+ae}{(bx^3+a)^3}dx}{9ab}-\frac{x(ae-bcx-bdx^2)}{9ab(a+bx^3)^3}$$

$$\Big\downarrow \text{2393}$$

$$\frac{-\frac{\int -\frac{5ae+28bcx}{(bx^3+a)^2}dx}{6a}-\frac{6ad-x(ae+7bcx)}{6a(a+bx^3)^2}}{9ab}-\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{5ae+28bcx}{(bx^3+a)^2}dx}{6a}-\frac{6ad-x(ae+7bcx)}{6a(a+bx^3)^2}}{9ab}-\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x(5ae+28bcx)}{3a(a+bx^3)}-\frac{\int -\frac{2(5ae+14bcx)}{bx^3+a}dx}{3a}}{6a}-\frac{6ad-x(ae+7bcx)}{6a(a+bx^3)^2}}{9ab}-\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{2\int \frac{5ae+14bcx}{bx^3+a}dx}{3a}+\frac{x(5ae+28bcx)}{3a(a+bx^3)}}{6a}-\frac{6ad-x(ae+7bcx)}{6a(a+bx^3)^2}}{9ab}-\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}$$

$$\Big\downarrow \text{2399}$$

$$\frac{\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\sqrt[3]{b}\left(2\sqrt[3]{a}\left(7b^{2/3}c+5a^{2/3}e\right)+\sqrt[3]{b}\left(14b^{2/3}c-5a^{2/3}e\right)x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}}\right)}{3a}+\frac{x(5ae+28bcx)}{3a(a+bx^3)}}{6a}-\frac{6ad-x(ae+7bcx)}{6a(a+bx^3)^2}}{9ab}-\frac{x(ae-bcx-bdx^2)}{9ab\,(a+bx^3)^3}$$

$$\Big\downarrow \text{16}$$

3.359. $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx$

$$\frac{\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\sqrt[3]{b}\left(2\sqrt[3]{a}\left(7b^{2/3}c+5a^{2/3}e\right)+\sqrt[3]{b}\left(14b^{2/3}c-5a^{2/3}e\right)x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}+\frac{x(5ae+28bcx)}{3a\left(a+bx^3\right)}}{6a}-\frac{6ad-x(ae+7bcx)}{6a\left(a+bx^3\right)^2}}{9ab}-$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}\left(7b^{2/3}c+5a^{2/3}e\right)+\sqrt[3]{b}\left(14b^{2/3}c-5a^{2/3}e\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}+\frac{x(5ae+28bcx)}{3a\left(a+bx^3\right)}}{6a}-\frac{6ad-x(ae+7bcx)}{6a\left(a+bx^3\right)^2}}{9ab}-$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(5a^{2/3}e+14b^{2/3}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}}{6a}}{9ab}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 25$$

3.359. $\int \frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^4}\,dx$

$$\frac{\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(5a^{2/3}e+14b^{2/3}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}+}{6a}}{9ab}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3}$$

$\downarrow$ 27

$$\frac{\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(5a^{2/3}e+14b^{2/3}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(14b^{2/3}c-5a^{2/3}e\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}+}{6a}}{9ab}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3}$$

$\downarrow$ 1082

$$\frac{\frac{\frac{2\left(\frac{\frac{3\left(5a^{2/3}e+14b^{2/3}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\left(14b^{2/3}c-5a^{2/3}e\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}}{6a}}{9ab}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3}$$

$\downarrow$ 217

3.359. $\int\frac{x\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^4}\,dx$

$$\frac{\frac{\frac{2\left(\frac{-\frac{1}{2}\left(14b^{2/3}c-5a^{2/3}e\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{2/3}e+14b^{2/3}c\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}+\frac{x(5ae+28b}{3a\left(a+bx^3\right.}}{6a}}{9ab}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\frac{2\left(\frac{\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{2/3}e+14b^{2/3}c\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\left(14b^{2/3}c-5a^{2/3}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}+\frac{x(5ae+28bc}{3a\left(a+bx^3\right)}}{6a}}{9ab}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{9ab\left(a+bx^3\right)^3}$$

input
```
Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^4,x]
```

output
```
-1/9*(x*(a*e - b*c*x - b*d*x^2))/(a*b*(a + b*x^3)^3) + (-1/6*(6*a*d - x*(a
*e + 7*b*c*x))/(a*(a + b*x^3)^2) + ((x*(5*a*e + 28*b*c*x))/(3*a*(a + b*x^3
)) + (2*(-1/3*((14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(a^(
1/3)*b^(1/3)) + (-((Sqrt[3]*(14*b^(2/3)*c + 5*a^(2/3)*e)*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + ((14*b^(2/3)*c - 5*a^(2/3)*e)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)))/(3*a^(1/3))))/(3*a
))/(6*a))/(9*a*b)
```

3.359. $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx$

3.359.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

3.359. $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4} \, dx$

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.359.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\frac{14 c\, b^2 x^8}{81 a^3}+\frac{5 b e\, x^7}{162 a^2}+\frac{77 b c\, x^5}{162 a^2}+\frac{13 e\, x^4}{162 a}+\frac{67 c\, x^2}{162 a}-\frac{5 e x}{81 b}-\frac{d}{9 b}}{(b\, x^3+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\left(\frac{14 c\_R}{a}+\frac{5 e}{b}\right)\ln\left(x-\_R\right)}{\_R^2}}{243 a^2 b}$
default	$\frac{\frac{14 c\, b^2 x^8}{81 a^3}+\frac{5 b e\, x^7}{162 a^2}+\frac{77 b c\, x^5}{162 a^2}+\frac{13 e\, x^4}{162 a}+\frac{67 c\, x^2}{162 a}-\frac{5 e x}{81 b}-\frac{d}{9 b}}{(b\, x^3+a)^3}+\frac{5 a e \left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$

input
```
int(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(14/81*c/a^3*b^2*x^8+5/162*b*e/a^2*x^7+77/162*b*c/a^2*x^5+13/162/a*e*x^4+6
7/162*c/a*x^2-5/81*e*x/b-1/9*d/b)/(b*x^3+a)^3+1/243/a^2/b*sum((14*c/a*_R+5
/b*e)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.359.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 2646, normalized size of antiderivative = 9.80

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

output
```
1/972*(168*b^3*c*x^8 + 30*a*b^2*e*x^7 + 462*a*b^2*c*x^5 + 78*a^2*b*e*x^4 +
 402*a^2*b*c*x^2 - 60*a^3*e*x - 108*a^3*d - 2*(a^3*b^4*x^9 + 3*a^4*b^3*x^6
 + 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 12
5*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 1
40*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)
/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3)))*log(7/2*((1
/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744
*b^2*c^3 - 125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3
) + 1)/(a^6*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 -
 125*a^2*e^3)/(a^10*b^4))^(1/3)))^2*a^7*b^3*c - 25/2*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2
*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6*b^2*((
2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^3)/(a^1
0*b^4))^(1/3)))*a^5*b*e^2 + 1960*a*b*c^2*e + (2744*b^2*c^3 + 125*a^2*e^3)*
x) + ((a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*((1/2)^(1/3)*(
I*sqrt(3) + 1)*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 -
125*a^2*e^3)/(a^10*b^4))^(1/3) - 140*(1/2)^(2/3)*c*e*(-I*sqrt(3) + 1)/(a^6
*b^2*((2744*b^2*c^3 + 125*a^2*e^3)/(a^10*b^4) - (2744*b^2*c^3 - 125*a^2*e^
3)/(a^10*b^4))^(1/3))) + 3*sqrt(1/3)*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*
b^2*x^3 + a^6*b)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((2744*b^2*c^3 + 1...
```

3.359.6 Sympy [A] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.79

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$
$$= \text{RootSum}\left(14348907t^3a^{10}b^4+51030ta^4b^2ce-125a^2e^3+2744b^2c^3,\left(t\mapsto t\log\left(x+\frac{826686t^2a^7b^3c+607}{125a^2e^3+}\right.\right.\right.$$
$$+\frac{-18a^3d-10a^3ex+67a^2bcx^2+13a^2bex^4+77ab^2cx^5+5ab^2ex^7+28b^3cx^8}{162a^6b+486a^5b^2x^3+486a^4b^3x^6+162a^3b^4x^9}$$

input
```
integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

output
```
RootSum(14348907*_t**3*a**10*b**4 + 51030*_t*a**4*b**2*c*e - 125*a**2*e**3
 + 2744*b**2*c**3, Lambda(_t, _t*log(x + (826686*_t**2*a**7*b**3*c + 6075*
_t*a**5*b*e**2 + 1960*a*b*c**2*e)/(125*a**2*e**3 + 2744*b**2*c**3)))) + (-
18*a**3*d - 10*a**3*e*x + 67*a**2*b*c*x**2 + 13*a**2*b*e*x**4 + 77*a*b**2*
c*x**5 + 5*a*b**2*e*x**7 + 28*b**3*c*x**8)/(162*a**6*b + 486*a**5*b**2*x**
3 + 486*a**4*b**3*x**6 + 162*a**3*b**4*x**9)
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.96

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$
$$= \frac{28\,b^3cx^8+5\,ab^2ex^7+77\,ab^2cx^5+13\,a^2bex^4+67\,a^2bcx^2-10\,a^3ex-18\,a^3d}{162\,(a^3b^4x^9+3\,a^4b^3x^6+3\,a^5b^2x^3+a^6b)}$$
$$+\frac{\sqrt{3}\left(14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ae\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(14\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

output
```
1/162*(28*b^3*c*x^8 + 5*a*b^2*e*x^7 + 77*a*b^2*c*x^5 + 13*a^2*b*e*x^4 + 67
*a^2*b*c*x^2 - 10*a^3*e*x - 18*a^3*d)/(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5
*b^2*x^3 + a^6*b) + 1/243*sqrt(3)*(14*b*c*(a/b)^(1/3) + 5*a*e)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) + 1/486*(14
*b*c*(a/b)^(1/3) - 5*a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*
(a/b)^(2/3)) - 1/243*(14*b*c*(a/b)^(1/3) - 5*a*e)*log(x + (a/b)^(1/3))/(a^
3*b^2*(a/b)^(2/3))
```

3.359.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.88

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx$$
$$= -\frac{\sqrt{3}\left(5\,ae-14\,(-ab^2)^{\frac{1}{3}}\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{\left(5\,ae+14\,(-ab^2)^{\frac{1}{3}}\,c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{486\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{\left(14\,bc\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,ae\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4b} + \frac{28\,b^3cx^8+5\,ab^2ex^7+77\,ab^2cx^5+13\,a^2bex^4+67\,a^2bcx^2-10\,a^3ex-18\,a^3d}{162\,(bx^3+a)^3a^3b}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

output
```
-1/243*sqrt(3)*(5*a*e - 14*(-a*b^2)^(1/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/486*(5*a*e + 14*(-a*b^2)
^(1/3)*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) -
1/243*(14*b*c*(-a/b)^(1/3) + 5*a*e)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3))
)/(a^4*b) + 1/162*(28*b^3*c*x^8 + 5*a*b^2*e*x^7 + 77*a*b^2*c*x^5 + 13*a^2*
b*e*x^4 + 67*a^2*b*c*x^2 - 10*a^3*e*x - 18*a^3*d)/((b*x^3 + a)^3*a^3*b)
```

3.359.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.98

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^4}\,dx = \frac{\frac{67\,c\,x^2}{162\,a}-\frac{d}{9\,b}+\frac{13\,e\,x^4}{162\,a}-\frac{5\,e\,x}{81\,b}+\frac{14\,b^2\,c\,x^8}{81\,a^3}+\frac{77\,b\,c\,x^5}{162\,a^2}+\frac{5\,b\,e\,x^7}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9} + \left(\sum_{k=1}^{3}\ln\left(\frac{70\,a\,c\,e+\mathrm{root}(14348907\,a^{10}\,b^4\,z^3+51030\,a^4\,b^2\,c\,e\,z-125\,a^2\,e^3+2744\,b^2\,c^3,z,k)^2\,a^7\,b^2\,59049}{a}\right.\right.$$

$$\left.+51030\,a^4\,b^2\,c\,e\,z-125\,a^2\,e^3+2744\,b^2\,c^3,z,k)\right)$$

input
```
int((x*(c + d*x + e*x^2))/(a + b*x^3)^4,x)
```

output
```
((67*c*x^2)/(162*a) - d/(9*b) + (13*e*x^4)/(162*a) - (5*e*x)/(81*b) + (14*
b^2*c*x^8)/(81*a^3) + (77*b*c*x^5)/(162*a^2) + (5*b*e*x^7)/(162*a^2))/(a^3
 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6) + symsum(log((70*a*c*e + 59049*roo
t(14348907*a^10*b^4*z^3 + 51030*a^4*b^2*c*e*z - 125*a^2*e^3 + 2744*b^2*c^3
, z, k)^2*a^7*b^2 + 196*b*c^2*x + 1215*root(14348907*a^10*b^4*z^3 + 51030*
a^4*b^2*c*e*z - 125*a^2*e^3 + 2744*b^2*c^3, z, k)*a^4*b*e*x)/(6561*a^6))*r
oot(14348907*a^10*b^4*z^3 + 51030*a^4*b^2*c*e*z - 125*a^2*e^3 + 2744*b^2*c
^3, z, k), k, 1, 3)
```

3.360 $\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$

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3.360.1 Optimal result

Integrand size = 20, antiderivative size = 250

$$\begin{aligned}\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx = {} & \frac{x(8c+7dx)}{54a^2(a+bx^3)^2}+\frac{2x(10c+7dx)}{81a^3(a+bx^3)}-\frac{ae-bx(c+dx)}{9ab(a+bx^3)^3} \\ & -\frac{2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}} \\ & +\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} \\ & -\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}\end{aligned}$$

output
```
1/54*x*(7*d*x+8*c)/a^2/(b*x^3+a)^2+2/81*x*(7*d*x+10*c)/a^3/(b*x^3+a)+1/9*(
-a*e+b*x*(d*x+c))/a/b/(b*x^3+a)^3+2/243*(20*b^(1/3)*c-7*a^(1/3)*d)*ln(a^(1
/3)+b^(1/3)*x)/a^(11/3)/b^(2/3)-1/243*(20*b^(1/3)*c-7*a^(1/3)*d)*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(2/3)-2/243*(20*b^(1/3)*c+7*a^
(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3
)*3^(1/2)
```

3.360.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$$

$$= \frac{\frac{9a^2x(8c+7dx)}{(a+bx^3)^2}+\frac{12ax(10c+7dx)}{a+bx^3}-\frac{54a^3(ae-bx(c+dx))}{b(a+bx^3)^3}-\frac{4\sqrt{3}\sqrt[3]{a}\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}c\right.}{b^{2/}}}{486a^4}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

output
```
((9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^2 + (12*a*x*(10*c + 7*d*x))/(a + b*x^
3) - (54*a^3*(a*e - b*x*(c + d*x)))/(b*(a + b*x^3)^3) - (4*Sqrt[3]*a^(1/3)
*(20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])
/b^(2/3) + (4*(20*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x
])/b^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(486*a^4)
```

3.360.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2393, 25, 2394, 27, 2394, 25, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$$

$$\Big\downarrow \text{ 2393}$$

$$-\frac{\int -\frac{8c+7dx}{(bx^3+a)^3}dx}{9a}-\frac{ae-bx(c+dx)}{9ab(a+bx^3)^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{8c+7dx}{(bx^3+a)^3}dx}{9a}-\frac{ae-bx(c+dx)}{9ab(a+bx^3)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(8c+7dx)}{6a(a+bx^3)^2}-\frac{\int -\frac{4(10c+7dx)}{\left(bx^3+a\right)^2}dx}{6a}}{9a}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{10c+7dx}{\left(bx^3+a\right)^2}dx}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{2\left(\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}-\frac{\int -\frac{20c+7dx}{bx^3+a}dx}{3a}\right)}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2\left(\frac{\int \frac{20c+7dx}{bx^3+a}dx}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{2399}$$

$$\frac{\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{a}\left(40\sqrt[3]{b}c+7\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{16}$$

3.360. $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^4}\,dx$

$$\frac{\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{a}\left(40\sqrt[3]{b}c+7\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}-\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

$\big\downarrow$ 1142

$$\frac{\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(}{3a}\right.}{3a}}{9a}$$

$$\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

$\big\downarrow$ 25

$$\frac{\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(10}{3a(}\right.}{3a}}{9a}$$

$$\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

$\big\downarrow$ 27

3.360. $\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$

$$\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\cdots\right)}{3a}}{9a} \cdots$$

$$\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

$\big\downarrow$ 1082

$$\frac{2\left(\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}\cdots\right)}{3a}}{9a} \cdots$$

$$\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

$\big\downarrow$ 217

$$\frac{2\left(\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\cdots\right)}{3a}}{9a} \cdots$$

$$\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

$\big\downarrow$ 1103

3.360. $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^4}\,dx$

$$\frac{\frac{2\left(\frac{\frac{-\frac{1}{2}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\right)}{3a}}{9a}$$

$$\frac{ae-bx(c+dx)}{9ab\left(a+bx^3\right)^3}$$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^3)^4,x]
```

output
```
-1/9*(a*e - b*x*(c + d*x))/(a*b*(a + b*x^3)^3) + ((x*(8*c + 7*d*x))/(6*a*(
a + b*x^3)^2) + (2*((x*(10*c + 7*d*x))/(3*a*(a + b*x^3)) + (((20*c - (7*a^
(1/3)*d)/b^(1/3))*Log[a^(1/3) + b^(1/3)*x]])/(3*a^(2/3)*b^(1/3)) + (-((Sqrt
[3]*(20*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/b^(1/3)) - ((20*c - (7*a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)*b^(1/3)))/(3*a)))/(3*a))/(9*a)
```

3.360.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.360. $\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$

3.360.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.46

method	result
risch	$\frac{\frac{14 d \,b^{2} x^{8}}{81 a^{3}}+\frac{20 c \,b^{2} x^{7}}{81 a^{3}}+\frac{77 b d \,x^{5}}{162 a^{2}}+\frac{52 b c \,x^{4}}{81 a^{2}}+\frac{67 d \,x^{2}}{162 a}+\frac{41 c x}{81 a}-\frac{e}{9 b}}{\left(b \,x^{3}+a \right)^{3}}+\frac{2 \left(\sum_{\textit{\_R} =\operatorname{RootOf}\left(b \,\textit{\_Z}^{3}+a \right)}\frac{\left(7 \textit{\_R} d +20 c \right) \ln \left(x -\textit{\_R} \right)}{\textit{\_R}^{2}}\right)}{243 a^{3} b}$
default	$c \left(\frac{x}{9 a \left(b \,x^{3}+a \right)^{3}}+\frac{\frac{4 x}{27 a \left(b \,x^{3}+a \right)^{2}}+\frac{8 \left(\frac{5 x}{18 a \left(b \,x^{3}+a \right)}+\frac{5 \left(\frac{2 \ln \left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6 a}\right)}{9 a}}{a}$

3.360. $\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$

input
```
int((e*x^2+d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(14/81*d/a^3*b^2*x^8+20/81*c/a^3*b^2*x^7+77/162*b*d/a^2*x^5+52/81*b*c/a^2*
x^4+67/162*d/a*x^2+41/81*c/a*x-1/9/b*e)/(b*x^3+a)^3+2/243/a^3/b*sum((7*_R*
d+20*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.360.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 2344, normalized size of antiderivative = 9.38

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

output
```
1/972*(168*b^3*d*x^8 + 240*b^3*c*x^7 + 462*a*b^2*d*x^5 + 624*a*b^2*c*x^4 +
 402*a^2*b*d*x^2 + 492*a^2*b*c*x - 108*a^3*e - 2*(a^3*b^4*x^9 + 3*a^4*b^3*
x^6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a
*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3
)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000
*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*log(7/4*(4^(1/3)*(I*sqrt(3) + 1)*(
(8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))
^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)
/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d - 400
*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c
^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*
b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^
2))^(1/3)))*a^4*b*c^2 + 7840*a*c*d^2 + 4*(8000*b*c^3 + 343*a*d^3)*x) + ((a
^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*(4^(1/3)*(I*sqrt(3) +
1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b
^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*
d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3))) + 3*sqrt(1/
3)*(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)*sqrt(-((4^(1/3)*(
I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*
d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000...
```

3.360.6 Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.81

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$$
$$= \text{RootSum}\left(14348907t^3a^{11}b^2 + 408240ta^4bcd + 2744ad^3 - 64000bc^3, \left(t \mapsto t\log\left(x + \frac{413343t^2a^8bd + 194\ldots}{1372ad^3\ldots}\right.\right.\right.$$
$$+ \frac{-18a^3e + 82a^2bcx + 67a^2bdx^2 + 104ab^2cx^4 + 77ab^2dx^5 + 40b^3cx^7 + 28b^3dx^8}{162a^6b + 486a^5b^2x^3 + 486a^4b^3x^6 + 162a^3b^4x^9}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**3+a)**4,x)
```

output
```
RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 6
4000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4
*b*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (-18*a**3*e + 8
2*a**2*b*c*x + 67*a**2*b*d*x**2 + 104*a*b**2*c*x**4 + 77*a*b**2*d*x**5 + 4
0*b**3*c*x**7 + 28*b**3*d*x**8)/(162*a**6*b + 486*a**5*b**2*x**3 + 486*a**
4*b**3*x**6 + 162*a**3*b**4*x**9)
```

3.360.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.02

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$$
$$= \frac{28\,b^3dx^8 + 40\,b^3cx^7 + 77\,ab^2dx^5 + 104\,ab^2cx^4 + 67\,a^2bdx^2 + 82\,a^2bcx - 18\,a^3e}{162\,(a^3b^4x^9 + 3\,a^4b^3x^6 + 3\,a^5b^2x^3 + a^6b)}$$
$$+ \frac{2\,\sqrt{3}\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}} + 20\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+ \frac{\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}} - 20\,c\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{2\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}} - 20\,c\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

output
```
1/162*(28*b^3*d*x^8 + 40*b^3*c*x^7 + 77*a*b^2*d*x^5 + 104*a*b^2*c*x^4 + 67
*a^2*b*d*x^2 + 82*a^2*b*c*x - 18*a^3*e)/(a^3*b^4*x^9 + 3*a^4*b^3*x^6 + 3*a
^5*b^2*x^3 + a^6*b) + 2/243*sqrt(3)*(7*d*(a/b)^(1/3) + 20*c)*arctan(1/3*sq
rt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/243*(7*d*(a
/b)^(1/3) - 20*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3
)) - 2/243*(7*d*(a/b)^(1/3) - 20*c)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3
))
```

3.360.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx$$
$$= -\frac{2\sqrt{3}\left(20\,bc-7\left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\left(-ab^2\right)^{\frac{2}{3}}a^3} - \frac{\left(20\,bc+7\left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(-ab^2\right)^{\frac{2}{3}}a^3} - \frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4} + \frac{28\,b^3dx^8+40\,b^3cx^7+77\,ab^2dx^5+104\,ab^2cx^4+67\,a^2bdx^2+82\,a^2bcx-18\,a^3e}{162\left(bx^3+a\right)^3a^3b}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

output
```
-2/243*sqrt(3)*(20*b*c - 7*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*c + 7*(-a*b^2)
^(1/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) -
2/243*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^
4 + 1/162*(28*b^3*d*x^8 + 40*b^3*c*x^7 + 77*a*b^2*d*x^5 + 104*a*b^2*c*x^4
+ 67*a^2*b*d*x^2 + 82*a^2*b*c*x - 18*a^3*e)/((b*x^3 + a)^3*a^3*b)
```

3.360.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2}{(a+bx^3)^4}\,dx = \frac{\frac{67\,d\,x^2}{162\,a}-\frac{e}{9\,b}+\frac{41\,c\,x}{81\,a}+\frac{20\,b^2\,c\,x^7}{81\,a^3}+\frac{14\,b^2\,d\,x^8}{81\,a^3}+\frac{52\,b\,c\,x^4}{81\,a^2}+\frac{77\,b\,d\,x^5}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}$$
$$+\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(560\,c\,d+196\,d^2\,x+\mathrm{root}(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,\right.}{}\right.\right.$$
$$\left.+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k)\right)$$

input
```
int((c + d*x + e*x^2)/(a + b*x^3)^4,x)
```

output
```
((67*d*x^2)/(162*a) - e/(9*b) + (41*c*x)/(81*a) + (20*b^2*c*x^7)/(81*a^3)
+ (14*b^2*d*x^8)/(81*a^3) + (52*b*c*x^4)/(81*a^2) + (77*b*d*x^5)/(162*a^2)
)/(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2*x^6) + symsum(log((b*(560*c*d + 1
96*d^2*x + 59049*root(14348907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b
*c^3 + 2744*a*d^3, z, k)^2*a^7*b + 9720*root(14348907*a^11*b^2*z^3 + 40824
0*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)*a^3*b*c*x))/(6561*a^6))*ro
ot(14348907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3,
z, k), k, 1, 3)
```

3.361 $\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx$

3.361.1 Optimal result 2720
3.361.2 Mathematica [A] (verified) 2721
3.361.3 Rubi [A] (verified) 2721
3.361.4 Maple [C] (verified) 2724
3.361.5 Fricas [C] (verification not implemented) 2725
3.361.6 Sympy [F(-1)] 2725
3.361.7 Maxima [A] (verification not implemented) 2725
3.361.8 Giac [A] (verification not implemented) 2726
3.361.9 Mupad [B] (verification not implemented) 2727

3.361.1 Optimal result

Integrand size = 23, antiderivative size = 291

$$\begin{aligned}\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx &= \frac{x(ad+aex-bcx^2)}{9a^2\left(a+bx^3\right)^3}+\frac{x(8ad+7aex-15bcx^2)}{54a^3\left(a+bx^3\right)^2} \\ &\quad+\frac{x(40ad+28aex-99bcx^2)}{162a^4\left(a+bx^3\right)} \\ &\quad-\frac{2\left(20\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}} \\ &\quad+\frac{c\log(x)}{a^4}+\frac{2\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} \\ &\quad-\frac{\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}-\frac{c\log\left(a+bx^3\right)}{3a^4}\end{aligned}$$

output
```
1/9*x*(-b*c*x^2+a*e*x+a*d)/a^2/(b*x^3+a)^3+1/54*x*(-15*b*c*x^2+7*a*e*x+8*a
*d)/a^3/(b*x^3+a)^2+1/162*x*(-99*b*c*x^2+28*a*e*x+40*a*d)/a^4/(b*x^3+a)+c*
ln(x)/a^4+2/243*(20*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/
b^(2/3)-1/243*(20*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/a^(11/3)/b^(2/3)-1/3*c*ln(b*x^3+a)/a^4-2/243*(20*b^(1/3)*d+7*a^(1
/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*
3^(1/2)
```

3.361.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.89

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx$$

$$= \frac{\frac{54a^3(c+x(d+ex))}{(a+bx^3)^3}+\frac{9a^2(9c+x(8d+7ex))}{(a+bx^3)^2}+\frac{6a(27c+2x(10d+7ex))}{a+bx^3}-\frac{4\sqrt{3}\sqrt[3]{a}\left(20\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+486c\log(x}{48}$$

input
```
Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```

output
```
((54*a^3*(c + x*(d + e*x)))/(a + b*x^3)^3 + (9*a^2*(9*c + x*(8*d + 7*e*x))
)/(a + b*x^3)^2 + (6*a*(27*c + 2*x*(10*d + 7*e*x)))/(a + b*x^3) - (4*Sqrt[
3]*a^(1/3)*(20*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]])/b^(2/3) + 486*c*Log[x] + (4*(20*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e
)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-20*a^(1/3)*b^(1/3)*d + 7*a^(2/3
)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 162*c*Log[a
 + b*x^3])/(486*a^4)
```

3.361.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2368, 25, 2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3}-\frac{\int -\frac{-\frac{6b^2cx^3}{a}+7bex^2+8bdx+9bc}{x(bx^3+a)^3}dx}{9ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{6b^2cx^3}{a}+7bex^2+8bdx+9bc}{x(bx^3+a)^3}dx}{9ab}+\frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3}$$

$\Big\downarrow$ 2368

$$\frac{\frac{x(8abd+7abex-15b^2cx^2)}{6a^2(a+bx^3)^2}-\frac{\int -\frac{-\frac{45b^3cx^3}{a}+28b^2ex^2+40b^2dx+54b^2c}{x\left(bx^3+a\right)^2}dx}{6ab}}{9ab}+\frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{-\frac{45b^3cx^3}{a}+28b^2ex^2+40b^2dx+54b^2c}{x\left(bx^3+a\right)^2}dx}{6ab}+\frac{x(8abd+7abex-15b^2cx^2)}{6a^2(a+bx^3)^2}}{9ab}+\frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3}$$

$\Big\downarrow$ 2368

$$\frac{\frac{\frac{x\left(40ab^2d+28ab^2ex-99b^3cx^2\right)}{3a^2\left(a+bx^3\right)}-\frac{\int -\frac{2\left(14ex^2b^3+81cb^3+40dxb^3\right)}{x\left(bx^3+a\right)}dx}{3ab}}{6ab}+\frac{x(8abd+7abex-15b^2cx^2)}{6a^2(a+bx^3)^2}}{9ab}+\frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{2\int \frac{14ex^2b^3+81cb^3+40dxb^3}{x\left(bx^3+a\right)}dx}{3ab}+\frac{x\left(40ab^2d+28ab^2ex-99b^3cx^2\right)}{3a^2\left(a+bx^3\right)}}{6ab}+\frac{x(8abd+7abex-15b^2cx^2)}{6a^2(a+bx^3)^2}}{9ab}+\frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3}$$

$\Big\downarrow$ 2373

$$\frac{\frac{\frac{2\int \left(\frac{81cb^3}{ax}+\frac{\left(-81bcx^2+14aex+40ad\right)b^3}{a\left(bx^3+a\right)}\right)dx}{3ab}+\frac{x\left(40ab^2d+28ab^2ex-99b^3cx^2\right)}{3a^2\left(a+bx^3\right)}}{6ab}+\frac{x(8abd+7abex-15b^2cx^2)}{6a^2(a+bx^3)^2}}{9ab}+\frac{x\left(ad+aex-bcx^2\right)}{9a^2\left(a+bx^3\right)^3}$$

$\Big\downarrow$ 2009

3.361. $\int \frac{c+dx+ex^2}{x(a+bx^3)^4}\,dx$

$$\frac{\frac{x(ad+aex-bcx^2)}{9a^2(a+bx^3)^3}+\frac{\frac{x(8abd+7abex-15b^2cx^2)}{6a^2(a+bx^3)^2}+\frac{2\left(-\frac{2b^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}e+20\sqrt[3]{b}d\right)}{\sqrt{3}a^{2/3}}-\frac{b^{7/3}\left(20\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{3a^{2/3}}+\frac{2b^{7/3}\left(2\cdots\right.}{\cdots}\cdots\right)}{3ab}}{6ab}}{9ab}$$

input
```
Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^4),x]
```

output
```
(x*(a*d + a*e*x - b*c*x^2))/(9*a^2*(a + b*x^3)^3) + ((x*(8*a*b*d + 7*a*b*e
*x - 15*b^2*c*x^2))/(6*a^2*(a + b*x^3)^2) + ((x*(40*a*b^2*d + 28*a*b^2*e*x
 - 99*b^3*c*x^2))/(3*a^2*(a + b*x^3)) + (2*((-2*b^(7/3)*(20*b^(1/3)*d + 7*
a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(
2/3)) + (81*b^3*c*Log[x])/a + (2*b^(7/3)*(20*b^(1/3)*d - 7*a^(1/3)*e)*Log[
a^(1/3) + b^(1/3)*x])/(3*a^(2/3)) - (b^(7/3)*(20*b^(1/3)*d - 7*a^(1/3)*e)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3*a^(2/3)) - (27*b^3*c*Lo
g[a + b*x^3])/a))/(3*a*b))/(6*a*b))/(9*a*b)
```

3.361.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.361. $\int \frac{c+dx+ex^2}{x(a+bx^3)^4} \, dx$

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.361.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.05

method	result
risch	$\frac{\frac{14 e b^2 x^8}{81 a^3}+\frac{20 d b^2 x^7}{81 a^3}+\frac{c b^2 x^6}{3 a^3}+\frac{77 b e x^5}{162 a^2}+\frac{52 b d x^4}{81 a^2}+\frac{5 b c x^3}{6 a^2}+\frac{67 e x^2}{162 a}+\frac{41 x d}{81 a}+\frac{11 c}{18 a}}{\left(b x^3+a\right)^3}+\frac{\left(\_R=\operatorname{RootOf}\left(a^{12} b^2 \_Z^3+243 a^8 b^2 c \_Z^2+\left(1680 a^5 b d e+1 \dots\right.\right.\right.}{\dots}$
default	$\frac{c\ln(x)}{a^4}+\frac{\frac{\frac{14}{81} a b^2 e x^8+\frac{20}{81} a b^2 d x^7+\frac{1}{3} a b^2 c x^6+\frac{77}{162} a^2 b e x^5+\frac{52}{81} a^2 b d x^4+\frac{5}{6} a^2 x^3 b c+\frac{67}{162} a^3 e x^2+\frac{41}{81} a^3 d x+\frac{11}{18} c a^3}{\left(b x^3+a\right)^3}+\frac{40 a d\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\dots\right.}{\dots}\dots\right.}{\dots}}{\dots}$

input
```
int((e*x^2+d*x+c)/x/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(14/81*e/a^3*b^2*x^8+20/81*d/a^3*b^2*x^7+1/3*c/a^3*b^2*x^6+77/162*b*e/a^2*
x^5+52/81*b*d/a^2*x^4+5/6*b*c/a^2*x^3+67/162/a*e*x^2+41/81/a*x*d+11/18*c/a
)/(b*x^3+a)^3+1/243*sum(_R*ln((-2*_R^3*a^11*b^2-324*_R^2*a^7*b^2*c+(-2800*
a^4*b*d*e-13122*a^3*b^2*c^2)*_R-4116*a*e^3-136080*b*c*d*e+96000*b*d^3)*x+7
*a^8*b*e*_R^2+(-1134*a^4*b*c*e-800*a^4*b*d^2)*_R-137781*b*c^2*e+194400*b*c
*d^2),_R=RootOf(a^12*b^2*_Z^3+243*a^8*b^2*c*_Z^2+(1680*a^5*b*d*e+19683*a^4
*b^2*c^2)*_Z+2744*a^2*e^3+136080*a*b*c*d*e-64000*a*b*d^3+531441*b^2*c^3))+
c/a^4*ln(-x)
```

3.361.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 5370, normalized size of antiderivative = 18.45

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.361.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x/(b*x**3+a)**4,x)
```

output
```
Timed out
```

3.361.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^4}\,dx$$
$$= \frac{28\,b^2ex^8+40\,b^2dx^7+54\,b^2cx^6+77\,abex^5+104\,abdx^4+135\,abcx^3+67\,a^2ex^2+82\,a^2dx+99\,a^2c}{162\,(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6)}$$
$$+\frac{c\log(x)}{a^4}+\frac{2\sqrt{3}\left(7\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+20\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5}$$
$$-\frac{\left(81\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-7\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(81\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-40\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="maxima")
```

output
```
1/162*(28*b^2*e*x^8 + 40*b^2*d*x^7 + 54*b^2*c*x^6 + 77*a*b*e*x^5 + 104*a*b
*d*x^4 + 135*a*b*c*x^3 + 67*a^2*e*x^2 + 82*a^2*d*x + 99*a^2*c)/(a^3*b^3*x^
9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6) + c*log(x)/a^4 + 2/243*sqrt(3)*(7*a
*e*(a/b)^(2/3) + 20*a*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/a^5 - 1/243*(81*b*c*(a/b)^(2/3) - 7*a*e*(a/b)^(1/3) + 20*a*
d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) - 1/243*(81*
b*c*(a/b)^(2/3) + 14*a*e*(a/b)^(1/3) - 40*a*d)*log(x + (a/b)^(1/3))/(a^4*b
*(a/b)^(2/3))
```

3.361.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^4}\,dx = -\frac{2\sqrt{3}\left(20\,bd-7\left(-ab^2\right)^{\frac{1}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\left(-ab^2\right)^{\frac{2}{3}}a^3} - \frac{\left(20\,bd+7\left(-ab^2\right)^{\frac{1}{3}}e\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\left(-ab^2\right)^{\frac{2}{3}}a^3} - \frac{c\log\left(\left|bx^3+a\right|\right)}{3\,a^4} + \frac{c\log\left(\left|x\right|\right)}{a^4} + \frac{28\,ab^2ex^8+40\,ab^2dx^7+54\,ab^2cx^6+77\,a^2bex^5+104\,a^2bdx^4+135\,a^2bcx^3+67\,a^3ex^2+82\,a^3dx+99\,a^3}{162\left(bx^3+a\right)^3a^4} - \frac{2\left(7\,a^5be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,a^5bd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^9b}$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^4,x, algorithm="giac")
```

output
```
-2/243*sqrt(3)*(20*b*d - 7*(-a*b^2)^(1/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*d + 7*(-a*b^2)
^(1/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) -
1/3*c*log(abs(b*x^3 + a))/a^4 + c*log(abs(x))/a^4 + 1/162*(28*a*b^2*e*x^8
+ 40*a*b^2*d*x^7 + 54*a*b^2*c*x^6 + 77*a^2*b*e*x^5 + 104*a^2*b*d*x^4 + 135
*a^2*b*c*x^3 + 67*a^3*e*x^2 + 82*a^3*d*x + 99*a^3*c)/((b*x^3 + a)^3*a^4) -
 2/243*(7*a^5*b*e*(-a/b)^(1/3) + 20*a^5*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/(a^9*b)
```

3.361.9 Mupad [B] (verification not implemented)

Time = 12.95 (sec) , antiderivative size = 871, normalized size of antiderivative = 2.99

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^4}\,dx$$
$$= \frac{\frac{11\,c}{18\,a}+\frac{67\,e\,x^2}{162\,a}+\frac{41\,d\,x}{81\,a}+\frac{b^2\,c\,x^6}{3\,a^3}+\frac{20\,b^2\,d\,x^7}{81\,a^3}+\frac{14\,b^2\,e\,x^8}{81\,a^3}+\frac{5\,b\,c\,x^3}{6\,a^2}+\frac{52\,b\,d\,x^4}{81\,a^2}+\frac{77\,b\,e\,x^5}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}$$
$$+\left(\sum_{k=1}^{3}\ln\left(-\frac{b\left(-64800\,b\,c\,d^2+45927\,b\,c^2\,e+1372\,a\,e^3\,x-32000\,b\,d^3\,x+\mathrm{root}(14348907\,a^{12}\,b^2\,z^3+143\right.}{}\right.\right.$$
$$+14348907\,a^8\,b^2\,c\,z^2+408240\,a^5\,b\,d\,e\,z+4782969\,a^4\,b^2\,c^2\,z+136080\,a\,b\,c\,d\,e$$
$$\left.-64000\,a\,b\,d^3+2744\,a^2\,e^3+531441\,b^2\,c^3,z,k)\right)+\frac{c\,\ln(x)}{a^4}$$

input
```
int((c + d*x + e*x^2)/(x*(a + b*x^3)^4),x)
```

output
```
((11*c)/(18*a) + (67*e*x^2)/(162*a) + (41*d*x)/(81*a) + (b^2*c*x^6)/(3*a^3
) + (20*b^2*d*x^7)/(81*a^3) + (14*b^2*e*x^8)/(81*a^3) + (5*b*c*x^3)/(6*a^2
) + (52*b*d*x^4)/(81*a^2) + (77*b*e*x^5)/(162*a^2))/(a^3 + b^3*x^9 + 3*a^2
*b*x^3 + 3*a*b^2*x^6) + symsum(log(-(2*b*(45927*b*c^2*e - 64800*b*c*d^2 +
1372*a*e^3*x - 32000*b*d^3*x + 9565938*root(14348907*a^12*b^2*z^3 + 143489
07*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b
*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)^3*a^11*b^2*x
 + 64800*root(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*
b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*
a^2*e^3 + 531441*b^2*c^3, z, k)*a^4*b*d^2 - 137781*root(14348907*a^12*b^2*
z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z
+ 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)^
2*a^8*b*e + 45360*b*c*d*e*x + 1062882*root(14348907*a^12*b^2*z^3 + 1434890
7*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b*
c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3, z, k)*a^3*b^2*c^2*x
 + 6377292*root(14348907*a^12*b^2*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^
5*b*d*e*z + 4782969*a^4*b^2*c^2*z + 136080*a*b*c*d*e - 64000*a*b*d^3 + 274
4*a^2*e^3 + 531441*b^2*c^3, z, k)^2*a^7*b^2*c*x + 91854*root(14348907*a^12
*b^2*z^3 + 14348907*a^8*b^2*c*z^2 + 408240*a^5*b*d*e*z + 4782969*a^4*b^2*c
^2*z + 136080*a*b*c*d*e - 64000*a*b*d^3 + 2744*a^2*e^3 + 531441*b^2*c^3...
```

3.362 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^4} dx$

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3.362.1 Optimal result

Integrand size = 23, antiderivative size = 301

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^2(a+bx^3)^4}\,dx = &-\frac{c}{a^4x}+\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}+\frac{x(8ae-16bcx-15bdx^2)}{54a^3(a+bx^3)^2}\\&+\frac{x(40ae-118bcx-99bdx^2)}{162a^4(a+bx^3)}\\&+\frac{20(7b^{2/3}c-2a^{2/3}e)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{13/3}\sqrt[3]{b}}\\&+\frac{d\log(x)}{a^4}+\frac{20(7b^{2/3}c+2a^{2/3}e)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{13/3}\sqrt[3]{b}}\\&-\frac{10(7b^{2/3}c+2a^{2/3}e)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{13/3}\sqrt[3]{b}}-\frac{d\log(a+bx^3)}{3a^4}\end{aligned}$$

output
```
-c/a^4/x+1/9*x*(-b*d*x^2-b*c*x+a*e)/a^2/(b*x^3+a)^3+1/54*x*(-15*b*d*x^2-16
*b*c*x+8*a*e)/a^3/(b*x^3+a)^2+1/162*x*(-99*b*d*x^2-118*b*c*x+40*a*e)/a^4/(
b*x^3+a)+d*ln(x)/a^4+20/243*(7*b^(2/3)*c+2*a^(2/3)*e)*ln(a^(1/3)+b^(1/3)*x
)/a^(13/3)/b^(1/3)-10/243*(7*b^(2/3)*c+2*a^(2/3)*e)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(13/3)/b^(1/3)-1/3*d*ln(b*x^3+a)/a^4+20/243*(7*b^(2/
3)*c-2*a^(2/3)*e)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/
3)/b^(1/3)*3^(1/2)
```

3.362.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx$$

$$= \frac{-\frac{486ac}{x}+\frac{9a^2(9ad+8aex-16bcx^2)}{(a+bx^3)^2}+\frac{6a(27ad+20aex-59bcx^2)}{a+bx^3}+\frac{54a^3(-bcx^2+a(d+ex))}{(a+bx^3)^3}-\frac{40\sqrt{3}a^{2/3}(-7b^{2/3}c+2a^{2/3}e)\arctan\left(\frac{1-\frac{2}{\cdots}}{\cdots}\right)}{\sqrt[3]{b}}\cdots}{\cdots}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```

output
```
((-486*a*c)/x + (9*a^2*(9*a*d + 8*a*e*x - 16*b*c*x^2))/(a + b*x^3)^2 + (6*
a*(27*a*d + 20*a*e*x - 59*b*c*x^2))/(a + b*x^3) + (54*a^3*(-(b*c*x^2) + a*
(d + e*x)))/(a + b*x^3)^3 - (40*Sqrt[3]*a^(2/3)*(-7*b^(2/3)*c + 2*a^(2/3)*
e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + 486*a*d*Log[x] +
 (40*(7*a^(2/3)*b^(2/3)*c + 2*a^(4/3)*e)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
 - (20*(7*a^(2/3)*b^(2/3)*c + 2*a^(4/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/b^(1/3) - 162*a*d*Log[a + b*x^3])/(486*a^5)
```

3.362.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2368, 25, 2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{x\left(ae-bcx-bdx^2\right)}{9a^2\left(a+bx^3\right)^3}-\frac{\int -\frac{-\frac{6b^2dx^4}{a}-\frac{7b^2cx^3}{a}+8bex^2+9bdx+9bc}{x^2\left(bx^3+a\right)^3}dx}{9ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{6b^2dx^4}{a}-\frac{7b^2cx^3}{a}+8bex^2+9bdx+9bc}{x^2(bx^3+a)^3}dx}{9ab}+\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}$$

$\Big\downarrow$ 2368

$$\frac{\frac{x(8abe-16b^2cx-15b^2dx^2)}{6a^2(a+bx^3)^2}-\frac{\int -\frac{-\frac{45dx^4b^4}{a}-\frac{64cx^3b^4}{a}+40ex^2b^3+54cb^3+54dxb^3}{x^2(bx^3+a)^2}dx}{6ab^2}}{9ab}+\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{-\frac{45dx^4b^4}{a}-\frac{64cx^3b^4}{a}+40ex^2b^3+54cb^3+54dxb^3}{x^2(bx^3+a)^2}dx}{6ab^2}+\frac{x(8abe-16b^2cx-15b^2dx^2)}{6a^2(a+bx^3)^2}}{9ab}+\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}$$

$\Big\downarrow$ 2368

$$\frac{\frac{\frac{x(40ab^3e-118b^4cx-99b^4dx^2)}{3a^2(a+bx^3)}-\frac{\int -\frac{2\left(-\frac{59cx^3b^6}{a}+40ex^2b^5+81cb^5+81dxb^5\right)}{x^2(bx^3+a)}dx}{3ab^2}}{6ab^2}+\frac{x(8abe-16b^2cx-15b^2dx^2)}{6a^2(a+bx^3)^2}}{9ab}+\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{2\int \frac{-\frac{59cx^3b^6}{a}+40ex^2b^5+81cb^5+81dxb^5}{x^2(bx^3+a)}dx}{3ab^2}+\frac{x(40ab^3e-118b^4cx-99b^4dx^2)}{3a^2(a+bx^3)}}{6ab^2}+\frac{x(8abe-16b^2cx-15b^2dx^2)}{6a^2(a+bx^3)^2}}{9ab}+\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}$$

$\Big\downarrow$ 2373

$$\frac{\frac{\frac{2\int \left(\frac{81db^5}{ax}+\frac{(-81bdx^2-140bcx+40ae)b^5}{a(bx^3+a)}+\frac{81cb^5}{ax^2}\right)dx}{3ab^2}+\frac{x(40ab^3e-118b^4cx-99b^4dx^2)}{3a^2(a+bx^3)}}{6ab^2}+\frac{x(8abe-16b^2cx-15b^2dx^2)}{6a^2(a+bx^3)^2}}{9ab}+\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}$$

$\Big\downarrow$ 2009

3.362. $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^4}\,dx$

$$\frac{\frac{x(ae-bcx-bdx^2)}{9a^2(a+bx^3)^3}+\frac{\frac{x(8abe-16b^2cx-15b^2dx^2)}{6a^2(a+bx^3)^2}+\frac{\frac{2\left(\frac{20b^{14/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(7b^{2/3}c-2a^{2/3}e)}{\sqrt{3}a^{4/3}}-\frac{10b^{14/3}(2a^{2/3}e+7b^{2/3}c)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{3a^{4/3}}+\frac{20b^{14/3}}{}\cdots\right)}{3ab^2}}{6ab^2}}{9ab}$$

input
```
Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x]
```

output
```
(x*(a*e - b*c*x - b*d*x^2))/(9*a^2*(a + b*x^3)^3) + ((x*(8*a*b*e - 16*b^2*
c*x - 15*b^2*d*x^2))/(6*a^2*(a + b*x^3)^2) + ((x*(40*a*b^3*e - 118*b^4*c*x
 - 99*b^4*d*x^2))/(3*a^2*(a + b*x^3)) + (2*((-81*b^5*c)/(a*x) + (20*b^(14/
3)*(7*b^(2/3)*c - 2*a^(2/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(4/3)) + (81*b^5*d*Log[x])/a + (20*b^(14/3)*(7*b^(2/3)*
c + 2*a^(2/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)) - (10*b^(14/3)*(7*b
^(2/3)*c + 2*a^(2/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3
*a^(4/3)) - (27*b^5*d*Log[a + b*x^3])/a))/(3*a*b^2))/(6*a*b^2))/(9*a*b)
```

3.362.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.362.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.04

method	result
risch	$\frac{-\frac{140b^3c\,x^9}{81a^4}+\frac{20e\,b^2x^8}{81a^3}+\frac{d\,b^2x^7}{3a^3}-\frac{385c\,b^2x^6}{81a^3}+\frac{52be\,x^5}{81a^2}+\frac{5bd\,x^4}{6a^2}-\frac{335bc\,x^3}{81a^2}+\frac{41e\,x^2}{81a}+\frac{11xd}{18a}-\frac{c}{a}}{x(b\,x^3+a)^3}+\frac{\left(\_R=\mathrm{RootOf}\left(a^{13}b\,\_Z^3+243a^9bd\,\_Z^2+\left(\ldots\right.\right.\right.}{\ldots}$
default	$-\frac{c}{a^4x}+\frac{d\ln(x)}{a^4}+\frac{-\frac{59}{81}b^3c\,x^8+\frac{20}{81}a\,b^2e\,x^7+\frac{1}{3}a\,b^2d\,x^6-\frac{142}{81}a\,b^2c\,x^5+\frac{52}{81}a^2be\,x^4+\frac{5}{6}a^2bd\,x^3-\frac{92}{81}a^2bc\,x^2+\frac{41}{81}a^3ex+\frac{11}{18}a^3d}{\left(b\,x^3+a\right)^3}+\frac{40ae\left(\frac{\ln\left(x+\left(\ldots\right.\right.}{3b\left(\frac{a}{b}\ldots\right.}\ldots\right.}{\ldots}$

input
```
int((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(-140/81/a^4*b^3*c*x^9+20/81*e/a^3*b^2*x^8+1/3*d/a^3*b^2*x^7-385/81*c/a^3*
b^2*x^6+52/81*b*e/a^2*x^5+5/6*b*d/a^2*x^4-335/81*b*c/a^2*x^3+41/81/a*e*x^2
+11/18/a*x*d-c/a)/x/(b*x^3+a)^3+1/243*sum(_R*ln((-_R^3*a^13*b-162*_R^2*a^9
*b*d+(14000*a^5*b*c*e-6561*a^5*b*d^2)*_R+48000*a^2*e^3+680400*a*b*c*d*e+20
58000*b^2*c^3)*x-35*a^9*b*c*_R^2+(-400*a^6*e^2+5670*a^5*b*c*d)*_R+97200*a^
2*d*e^2+688905*a*b*c*d^2),_R=RootOf(a^13*b*_Z^3+243*a^9*b*d*_Z^2+(-16800*a
^5*b*c*e+19683*a^5*b*d^2)*_Z-64000*a^2*e^3-1360800*a*b*c*d*e+531441*a*b*d^
3-2744000*b^2*c^3))+d*ln(x)/a^4
```

3.362.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 5250, normalized size of antiderivative = 17.44

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.362.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**4,x)
```

output
```
Timed out
```

3.362.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^4}\,dx =$$
$$-\frac{280\,b^3cx^9-40\,ab^2ex^8-54\,ab^2dx^7+770\,ab^2cx^6-104\,a^2bex^5-135\,a^2bdx^4+670\,a^2bcx^3-82\,a^3ex^2-9}{162\,\left(a^4b^3x^{10}+3\,a^5b^2x^7+3\,a^6bx^4+a^7x\right)}$$
$$+\frac{d\log\left(x\right)}{a^4}-\frac{20\,\sqrt{3}\left(7\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5}$$
$$-\frac{\left(81\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}+70\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,ae\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(81\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-140\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-40\,ae\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="maxima")
```

output
```
-1/162*(280*b^3*c*x^9 - 40*a*b^2*e*x^8 - 54*a*b^2*d*x^7 + 770*a*b^2*c*x^6
- 104*a^2*b*e*x^5 - 135*a^2*b*d*x^4 + 670*a^2*b*c*x^3 - 82*a^3*e*x^2 - 99*
a^3*d*x + 162*a^3*c)/(a^4*b^3*x^10 + 3*a^5*b^2*x^7 + 3*a^6*b*x^4 + a^7*x)
+ d*log(x)/a^4 - 20/243*sqrt(3)*(7*b*c*(a/b)^(2/3) - 2*a*e*(a/b)^(1/3))*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 - 1/243*(81*b*d*(a/b
)^(2/3) + 70*b*c*(a/b)^(1/3) + 20*a*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/
3))/(a^4*b*(a/b)^(2/3)) - 1/243*(81*b*d*(a/b)^(2/3) - 140*b*c*(a/b)^(1/3)
- 40*a*e)*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

3.362.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^2(a+bx^3)^4}\,dx &= -\frac{d\log\left(|bx^3+a|\right)}{3\,a^4} + \frac{d\log\left(|x|\right)}{a^4} \\ &+ \frac{20\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}ae + 7\left(-ab^2\right)^{\frac{2}{3}}c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5b} \\ &+ \frac{10\left(2\left(-ab^2\right)^{\frac{1}{3}}ae - 7\left(-ab^2\right)^{\frac{2}{3}}c\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b} \\ &- \frac{280\,b^3cx^9 - 40\,ab^2ex^8 - 54\,ab^2dx^7 + 770\,ab^2cx^6 - 104\,a^2bex^5 - 135\,a^2bdx^4 + 670\,a^2bcx^3 - 82\,a^3ex^2 - 9}{162\left(bx^3+a\right)^3a^4x} \\ &+ \frac{20\left(7\,a^4b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2\,a^5be\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^9b}\end{aligned}$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^4,x, algorithm="giac")
```

output
```
-1/3*d*log(abs(b*x^3 + a))/a^4 + d*log(abs(x))/a^4 + 20/243*sqrt(3)*(2*(-a
*b^2)^(1/3)*a*e + 7*(-a*b^2)^(2/3)*c)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^5*b) + 10/243*(2*(-a*b^2)^(1/3)*a*e - 7*(-a*b^2)^(2/3
)*c)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/162*(280*b^3*c*x
^9 - 40*a*b^2*e*x^8 - 54*a*b^2*d*x^7 + 770*a*b^2*c*x^6 - 104*a^2*b*e*x^5 -
 135*a^2*b*d*x^4 + 670*a^2*b*c*x^3 - 82*a^3*e*x^2 - 99*a^3*d*x + 162*a^3*c
)/((b*x^3 + a)^3*a^4*x) + 20/243*(7*a^4*b^2*c*(-a/b)^(1/3) - 2*a^5*b*e)*(-
a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b)
```

3.362.9 Mupad [B] (verification not implemented)

Time = 11.85 (sec) , antiderivative size = 840, normalized size of antiderivative = 2.79

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^4}\,dx$$
$$= \frac{\frac{41\,e\,x^2}{81\,a}-\frac{c}{a}+\frac{11\,d\,x}{18\,a}-\frac{385\,b^2\,c\,x^6}{81\,a^3}-\frac{140\,b^3\,c\,x^9}{81\,a^4}+\frac{b^2\,d\,x^7}{3\,a^3}+\frac{20\,b^2\,e\,x^8}{81\,a^3}-\frac{335\,b\,c\,x^3}{81\,a^2}+\frac{5\,b\,d\,x^4}{6\,a^2}+\frac{52\,b\,e\,x^5}{81\,a^2}}{a^3\,x+3\,a^2\,b\,x^4+3\,a\,b^2\,x^7+b^3\,x^{10}}$$
$$+\left(\sum_{k=1}^{3}\ln\left(\frac{b^2\left(-\mathrm{root}(14348907\,a^{13}\,b\,z^3+14348907\,a^9\,b\,d\,z^2-4082400\,a^5\,b\,c\,e\,z+4782969\,a^5\,b\,d^2\,z-13\right.}{}\right.\right.$$
$$+14348907\,a^9\,b\,d\,z^2-4082400\,a^5\,b\,c\,e\,z+4782969\,a^5\,b\,d^2\,z-1360800\,a\,b\,c\,d\,e$$
$$\left.+531441\,a\,b\,d^3-64000\,a^2\,e^3-2744000\,b^2\,c^3,z,k)\right)+\frac{d\,\ln(x)}{a^4}$$

input
```
int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^4),x)
```

output
```
((41*e*x^2)/(81*a) - c/a + (11*d*x)/(18*a) - (385*b^2*c*x^6)/(81*a^3) - (1
40*b^3*c*x^9)/(81*a^4) + (b^2*d*x^7)/(3*a^3) + (20*b^2*e*x^8)/(81*a^3) - (
335*b*c*x^3)/(81*a^2) + (5*b*d*x^4)/(6*a^2) + (52*b*e*x^5)/(81*a^2))/(a^3*
x + b^3*x^10 + 3*a^2*b*x^4 + 3*a*b^2*x^7) + symsum(log((4*b^2*(32400*a^2*d
*e^2 - 32400*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5
*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 6400
0*a^2*e^3 - 2744000*b^2*c^3, z, k)*a^6*e^2 + 686000*b^2*c^3*x + 16000*a^2*
e^3*x + 229635*a*b*c*d^2 - 688905*root(14348907*a^13*b*z^3 + 14348907*a^9*
b*d*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e +
531441*a*b*d^3 - 64000*a^2*e^3 - 2744000*b^2*c^3, z, k)^2*a^9*b*c - 478296
9*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z +
4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3 -
 2744000*b^2*c^3, z, k)^3*a^13*b*x - 531441*root(14348907*a^13*b*z^3 + 143
48907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*
b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3 - 2744000*b^2*c^3, z, k)*a^5*b*d^
2*x - 3188646*root(14348907*a^13*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^
5*b*c*e*z + 4782969*a^5*b*d^2*z - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 640
00*a^2*e^3 - 2744000*b^2*c^3, z, k)^2*a^9*b*d*x + 459270*root(14348907*a^1
3*b*z^3 + 14348907*a^9*b*d*z^2 - 4082400*a^5*b*c*e*z + 4782969*a^5*b*d^2*z
 - 1360800*a*b*c*d*e + 531441*a*b*d^3 - 64000*a^2*e^3 - 2744000*b^2*c^3...
```

3.363 $\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^4}\,dx$

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3.363.1 Optimal result

Integrand size = 23, antiderivative size = 310

$$\begin{aligned}\int \frac{c+dx+ex^2}{x^3\left(a+bx^3\right)^4}\,dx = &-\frac{c}{2a^4x^2}-\frac{d}{a^4x}-\frac{x(bc+bdx+bex^2)}{9a^2\left(a+bx^3\right)^3}\\
&-\frac{x(17bc+16bdx+15bex^2)}{54a^3\left(a+bx^3\right)^2}-\frac{x(139bc+118bdx+99bex^2)}{162a^4\left(a+bx^3\right)}\\
&+\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{14/3}}\\
&+\frac{e\log(x)}{a^4}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}}\\
&+\frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}}\\
&-\frac{e\log\left(a+bx^3\right)}{3a^4}\end{aligned}$$

output
```
-1/2*c/a^4/x^2-d/a^4/x-1/9*x*(b*e*x^2+b*d*x+b*c)/a^2/(b*x^3+a)^3-1/54*x*(1
5*b*e*x^2+16*b*d*x+17*b*c)/a^3/(b*x^3+a)^2-1/162*x*(99*b*e*x^2+118*b*d*x+1
39*b*c)/a^4/(b*x^3+a)+e*ln(x)/a^4-20/243*b^(1/3)*(11*b^(1/3)*c-7*a^(1/3)*d
)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)+10/243*b^(1/3)*(11*b^(1/3)*c-7*a^(1/3)*d)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)-1/3*e*ln(b*x^3+a)/a^4+
20/243*b^(1/3)*(11*b^(1/3)*c+7*a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)
/a^(1/3)*3^(1/2))/a^(14/3)*3^(1/2)
```

3.363.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.92

$$\int \frac{c+dx+ex^2}{x^3 (a+bx^3)^4} \, dx$$

$$= \frac{-\frac{243ac}{x^2} - \frac{486ad}{x} + \frac{54a^3(ae-bx(c+dx))}{(a+bx^3)^3} + \frac{9a^2(9ae-bx(17c+16dx))}{(a+bx^3)^2} + \frac{3a(54ae-bx(139c+118dx))}{a+bx^3} + 40\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(11\sqrt[3]{b}c + 7\sqrt[3]{}\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```

output
```
((-243*a*c)/x^2 - (486*a*d)/x + (54*a^3*(a*e - b*x*(c + d*x)))/(a + b*x^3)
^3 + (9*a^2*(9*a*e - b*x*(17*c + 16*d*x)))/(a + b*x^3)^2 + (3*a*(54*a*e -
b*x*(139*c + 118*d*x)))/(a + b*x^3) + 40*Sqrt[3]*a^(1/3)*b^(1/3)*(11*b^(1/
3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 486*a*e*
Log[x] + 40*b^(1/3)*(-11*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(1/3) + b^
(1/3)*x] + 20*b^(1/3)*(11*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 162*a*e*Log[a + b*x^3])/(486*a^5)
```

3.363.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2368, 25, 2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^3 (a+bx^3)^4} \, dx$$

$$\Big\downarrow \text{ 2368}$$

$$-\frac{\int -\frac{-\frac{6b^2ex^5}{a}-\frac{7b^2dx^4}{a}-\frac{8b^2cx^3}{a}+9bex^2+9bdx+9bc}{x^3(bx^3+a)^3}dx}{9ab} - \frac{x\left(bc+bdx+bex^2\right)}{9a^2 (a+bx^3)^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-\frac{6b^2ex^5}{a}-\frac{7b^2dx^4}{a}-\frac{8b^2cx^3}{a}+9bex^2+9bdx+9bc}{x^3(bx^3+a)^3}dx}{9ab}-\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}$$

$\Big\downarrow$ 2368

$$\frac{-\frac{\int -\frac{-\frac{45b^4ex^5}{a}-\frac{64b^4dx^4}{a}-\frac{85b^4cx^3}{a}+54b^3ex^2+54b^3dx+54b^3c}{x^3\left(bx^3+a\right)^2}dx}{6ab^2}-\frac{x(17b^2c+16b^2dx+15b^2ex^2)}{6a^2(a+bx^3)^2}}{9ab}-\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{-\frac{45b^4ex^5}{a}-\frac{64b^4dx^4}{a}-\frac{85b^4cx^3}{a}+54b^3ex^2+54b^3dx+54b^3c}{x^3\left(bx^3+a\right)^2}dx}{6ab^2}-\frac{x(17b^2c+16b^2dx+15b^2ex^2)}{6a^2(a+bx^3)^2}}{9ab}-\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}$$

$\Big\downarrow$ 2368

$$\frac{\frac{-\frac{\int -\frac{2\left(-\frac{59dx^4b^6}{a}-\frac{139cx^3b^6}{a}+81ex^2b^5+81cb^5+81dxb^5\right)}{x^3\left(bx^3+a\right)}dx}{3ab^2}-\frac{x\left(139b^4c+118b^4dx+99b^4ex^2\right)}{3a^2\left(a+bx^3\right)}}{6ab^2}-\frac{x(17b^2c+16b^2dx+15b^2ex^2)}{6a^2(a+bx^3)^2}}{9ab}-\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{2\int \frac{-\frac{59dx^4b^6}{a}-\frac{139cx^3b^6}{a}+81ex^2b^5+81cb^5+81dxb^5}{x^3\left(bx^3+a\right)}dx}{3ab^2}-\frac{x\left(139b^4c+118b^4dx+99b^4ex^2\right)}{3a^2\left(a+bx^3\right)}}{6ab^2}-\frac{x(17b^2c+16b^2dx+15b^2ex^2)}{6a^2(a+bx^3)^2}}{9ab}-\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}$$

$\Big\downarrow$ 2373

$$\frac{\frac{\frac{2\int \left(-\frac{\left(81ex^2+140dx+220c\right)b^6}{a\left(bx^3+a\right)}+\frac{81eb^5}{ax}+\frac{81db^5}{ax^2}+\frac{81cb^5}{ax^3}\right)dx}{3ab^2}-\frac{x\left(139b^4c+118b^4dx+99b^4ex^2\right)}{3a^2\left(a+bx^3\right)}}{6ab^2}-\frac{x(17b^2c+16b^2dx+15b^2ex^2)}{6a^2(a+bx^3)^2}}{9ab}-\frac{x(bc+bdx+bex^2)}{9a^2(a+bx^3)^3}$$

$\Big\downarrow$ 2009

3.363. $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^4}\,dx$

$$\frac{\frac{\frac{2\left(\frac{20b^{16/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}d+11\sqrt[3]{b}c\right)}{\sqrt{3}a^{5/3}}+\frac{10b^{16/3}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{3a^{5/3}}-\frac{20b^{16/3}\left(11\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}\right.}{3a^{5/3}}}{3ab^2}}{6ab^2}}{9ab}$$

$$\frac{x\left(bc+bdx+bex^2\right)}{9a^2\left(a+bx^3\right)^3}$$

input
```
Int[(c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x]
```

output
```
-1/9*(x*(b*c + b*d*x + b*e*x^2))/(a^2*(a + b*x^3)^3) + (-1/6*(x*(17*b^2*c
+ 16*b^2*d*x + 15*b^2*e*x^2))/(a^2*(a + b*x^3)^2) + (-1/3*(x*(139*b^4*c +
118*b^4*d*x + 99*b^4*e*x^2))/(a^2*(a + b*x^3)) + (2*((-81*b^5*c)/(2*a*x^2)
 - (81*b^5*d)/(a*x) + (20*b^(16/3)*(11*b^(1/3)*c + 7*a^(1/3)*d)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (81*b^5*e*Log
[x])/a - (20*b^(16/3)*(11*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x
])/(3*a^(5/3)) + (10*b^(16/3)*(11*b^(1/3)*c - 7*a^(1/3)*d)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3*a^(5/3)) - (27*b^5*e*Log[a + b*x^3])/a
))/(3*a*b^2))/(6*a*b^2))/(9*a*b)
```

3.363.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.363. $\int \frac{c+dx+ex^2}{x^3(a+bx^3)^4}\,dx$

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.363.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.03

method	result
risch	$\frac{-\frac{140b^3 d x^{10}}{81a^4}-\frac{110b^3 c x^9}{81a^4}+\frac{e b^2 x^8}{3a^3}-\frac{385 d b^2 x^7}{81a^3}-\frac{286 c b^2 x^6}{81a^3}+\frac{5 b e x^5}{6a^2}-\frac{335 b d x^4}{81a^2}-\frac{451 b c x^3}{162a^2}+\frac{11 e x^2}{18a}-\frac{x d}{a}-\frac{c}{2a}}{x^2\left(b x^3+a\right)^3}+\frac{e\ln(-x)}{a^4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}} \right.}{}$
default	$-\frac{c}{2a^4x^2}-\frac{d}{a^4x}+\frac{e\ln(x)}{a^4}-\frac{b\left(\frac{\frac{59b^2 d x^8}{81}+\frac{139b^2 c x^7}{162}-\frac{a e b x^6}{3}+\frac{142x^5 d b a}{81}+\frac{329 a b c x^4}{162}-\frac{5a^2 e x^3}{6}+\frac{92a^2 d x^2}{81}+\frac{104a^2 c x}{81}-\frac{11 e a^3}{18b}}{\left(b x^3+a\right)^3}+\frac{220c\left(\ln \right.}{}\right.}{}$

input
```
int((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(-140/81/a^4*b^3*d*x^10-110/81/a^4*b^3*c*x^9+1/3*e/a^3*b^2*x^8-385/81*d/a^
3*b^2*x^7-286/81*c/a^3*b^2*x^6+5/6*b*e/a^2*x^5-335/81*b*d/a^2*x^4-451/162*
b*c/a^2*x^3+11/18/a*e*x^2-1/a*x*d-1/2*c/a)/x^2/(b*x^3+a)^3+e/a^4*ln(-x)+1/
243*sum(_R*ln((-_R^3*a^14-162*_R^2*a^10*e+(-6561*a^6*e^2-77000*a^5*b*c*d)*
_R-3742200*a*b*c*d*e+2058000*a*b*d^3-7986000*b^2*c^3)*x-35*a^10*d*_R^2+(56
70*a^6*d*e-12100*a^5*b*c^2)*_R+688905*a^2*d*e^2+2940300*a*b*c^2*e),_R=Root
Of(a^14*_Z^3+243*a^10*e*_Z^2+(19683*a^6*e^2+92400*a^5*b*c*d)*_Z+531441*a^2
*e^3+7484400*a*b*c*d*e-2744000*a*b*d^3+10648000*b^2*c^3))
```

3.363.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 5049, normalized size of antiderivative = 16.29

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.363.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**4,x)
```

output
```
Timed out
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2}{x^3\,(a+bx^3)^4}\,dx =$$
$$-\frac{280\,b^3dx^{10}+220\,b^3cx^9-54\,ab^2ex^8+770\,ab^2dx^7+572\,ab^2cx^6-135\,a^2bex^5+670\,a^2bdx^4+451\,a^2bcx^3}{162\,(a^4b^3x^{11}+3\,a^5b^2x^8+3\,a^6bx^5+a^7x^2)}$$
$$+\frac{e\log(x)}{a^4}-\frac{20\,\sqrt{3}\Big(7\,bd\big(\frac{a}{b}\big)^{\frac{2}{3}}+11\,bc\big(\frac{a}{b}\big)^{\frac{1}{3}}\Big)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5}$$
$$-\frac{\Big(81\,e\big(\frac{a}{b}\big)^{\frac{2}{3}}+70\,d\big(\frac{a}{b}\big)^{\frac{1}{3}}-110\,c\Big)\log\Big(x^2-x\big(\frac{a}{b}\big)^{\frac{1}{3}}+\big(\frac{a}{b}\big)^{\frac{2}{3}}\Big)}{243\,a^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\Big(81\,e\big(\frac{a}{b}\big)^{\frac{2}{3}}-140\,d\big(\frac{a}{b}\big)^{\frac{1}{3}}+220\,c\Big)\log\Big(x+\big(\frac{a}{b}\big)^{\frac{1}{3}}\Big)}{243\,a^4\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="maxima")
```

output
```
-1/162*(280*b^3*d*x^10 + 220*b^3*c*x^9 - 54*a*b^2*e*x^8 + 770*a*b^2*d*x^7
+ 572*a*b^2*c*x^6 - 135*a^2*b*e*x^5 + 670*a^2*b*d*x^4 + 451*a^2*b*c*x^3 -
99*a^3*e*x^2 + 162*a^3*d*x + 81*a^3*c)/(a^4*b^3*x^11 + 3*a^5*b^2*x^8 + 3*a
^6*b*x^5 + a^7*x^2) + e*log(x)/a^4 - 20/243*sqrt(3)*(7*b*d*(a/b)^(2/3) + 1
1*b*c*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5
 - 1/243*(81*e*(a/b)^(2/3) + 70*d*(a/b)^(1/3) - 110*c)*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) - 1/243*(81*e*(a/b)^(2/3) - 140*d*(a
/b)^(1/3) + 220*c)*log(x + (a/b)^(1/3))/(a^4*(a/b)^(2/3))
```

3.363.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.02

$$\int \frac{c+dx+ex^2}{x^3(a+bx^3)^4}\,dx = -\frac{e\log(|bx^3+a|)}{3a^4} + \frac{e\log(|x|)}{a^4} - \frac{20\sqrt{3}\left(11(-ab^2)^{\frac{1}{3}}bc - 7(-ab^2)^{\frac{2}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243a^5b} - \frac{10\left(11(-ab^2)^{\frac{1}{3}}bc + 7(-ab^2)^{\frac{2}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243a^5b} + \frac{20\left(7a^4b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+11a^4b^2c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243a^9b} - \frac{280b^3dx^{10}+220b^3cx^9-54ab^2ex^8+770ab^2dx^7+572ab^2cx^6-135a^2bex^5+670a^2bdx^4+451a^2bcx^3}{162(bx^3+a)^3a^4x^2}$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^4,x, algorithm="giac")
```

output
```
-1/3*e*log(abs(b*x^3 + a))/a^4 + e*log(abs(x))/a^4 - 20/243*sqrt(3)*(11*(-
a*b^2)^(1/3)*b*c - 7*(-a*b^2)^(2/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/(a^5*b) - 10/243*(11*(-a*b^2)^(1/3)*b*c + 7*(-a*b^2)^(2
/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) + 20/243*(7*a^4*b^
2*d*(-a/b)^(1/3) + 11*a^4*b^2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(
a^9*b) - 1/162*(280*b^3*d*x^10 + 220*b^3*c*x^9 - 54*a*b^2*e*x^8 + 770*a*b^
2*d*x^7 + 572*a*b^2*c*x^6 - 135*a^2*b*e*x^5 + 670*a^2*b*d*x^4 + 451*a^2*b*
c*x^3 - 99*a^3*e*x^2 + 162*a^3*d*x + 81*a^3*c)/((b*x^3 + a)^3*a^4*x^2)
```

3.363.9 Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.66

$$\int \frac{c+dx+ex^2}{x^3 (a+bx^3)^4} \, dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(-\frac{b^3 \left(\mathrm{root}(14348907\,a^{14}\,z^3+14348907\,a^{10}\,e\,z^2+22453200\,a^5\,b\,c\,d\,z+4782969\,a^6\,e^2\,z+748440\ldots\right.}{\ldots}\right.\right.$$
$$+14348907\,a^{10}\,e\,z^2+22453200\,a^5\,b\,c\,d\,z+4782969\,a^6\,e^2\,z+7484400\,a\,b\,c\,d\,e$$
$$\left.\left.-2744000\,a\,b\,d^3+531441\,a^2\,e^3+10648000\,b^2\,c^3,z,k)\right)\right)$$
$$-\frac{\frac{c}{2a}-\frac{11ex^2}{18a}+\frac{dx}{a}+\frac{286b^2cx^6}{81a^3}+\frac{110b^3cx^9}{81a^4}+\frac{385b^2dx^7}{81a^3}+\frac{140b^3dx^{10}}{81a^4}-\frac{b^2ex^8}{3a^3}+\frac{451bcx^3}{162a^2}+\frac{335bdx^4}{81a^2}-\frac{5bex^5}{6a^2}}{a^3x^2+3a^2bx^5+3ab^2x^8+b^3x^{11}}$$
$$+\frac{e\ln(x)}{a^4}$$

input

```
int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^4),x)
```

output

```
symsum(log(-(4*b^3*(688905*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 +
22453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b
*d^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k)^2*a^10*d - 229635*a^2*d*e^
2 + 4782969*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*
c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a
^2*e^3 + 10648000*b^2*c^3, z, k)^3*a^14*x + 2662000*b^2*c^3*x - 459270*roo
t(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z + 4782969
*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3 + 106480
00*b^2*c^3, z, k)*a^6*d*e - 980100*a*b*c^2*e - 686000*a*b*d^3*x + 980100*r
oot(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z + 47829
69*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3 + 1064
8000*b^2*c^3, z, k)*a^5*b*c^2 + 531441*root(14348907*a^14*z^3 + 14348907*a
^10*e*z^2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*b*c*d*e -
 2744000*a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k)*a^6*e^2*x + 31
88646*root(14348907*a^14*z^3 + 14348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z
+ 4782969*a^6*e^2*z + 7484400*a*b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3
 + 10648000*b^2*c^3, z, k)^2*a^10*e*x + 6237000*root(14348907*a^14*z^3 + 1
4348907*a^10*e*z^2 + 22453200*a^5*b*c*d*z + 4782969*a^6*e^2*z + 7484400*a*
b*c*d*e - 2744000*a*b*d^3 + 531441*a^2*e^3 + 10648000*b^2*c^3, z, k)*a^5*b
*c*d*x + 1247400*a*b*c*d*e*x))/(531441*a^12))*root(14348907*a^14*z^3 + ...
```

3.364 $\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx$

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3.364.1 Optimal result

Integrand size = 23, antiderivative size = 340

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx = -\frac{c}{3a^4x^3}-\frac{d}{2a^4x^2}-\frac{e}{a^4x}-\frac{x\left(bd+bex-\frac{b^2cx^2}{a}\right)}{9a^2\left(a+bx^3\right)^3} - \frac{x\left(17bd+16bex-\frac{24b^2cx^2}{a}\right)}{54a^3\left(a+bx^3\right)^2}-\frac{x\left(139bd+118bex-\frac{234b^2cx^2}{a}\right)}{162a^4\left(a+bx^3\right)} + \frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}d+7\sqrt[3]{a}e\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{14/3}} - \frac{4bc\log(x)}{a^5}-\frac{20\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{14/3}} + \frac{10\sqrt[3]{b}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{14/3}} + \frac{4bc\log\left(a+bx^3\right)}{3a^5}$$

output
```
-1/3*c/a^4/x^3-1/2*d/a^4/x^2-e/a^4/x-1/9*x*(b*d+b*e*x-b^2*c*x^2/a)/a^2/(b*
x^3+a)^3-1/54*x*(17*b*d+16*b*e*x-24*b^2*c*x^2/a)/a^3/(b*x^3+a)^2-1/162*x*(
139*b*d+118*b*e*x-234*b^2*c*x^2/a)/a^4/(b*x^3+a)-4*b*c*ln(x)/a^5-20/243*b^
(1/3)*(11*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)+10/243*b^(
1/3)*(11*b^(1/3)*d-7*a^(1/3)*e)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
a^(14/3)+4/3*b*c*ln(b*x^3+a)/a^5+20/243*b^(1/3)*(11*b^(1/3)*d+7*a^(1/3)*e)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)*3^(1/2)
```

3.364.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.84

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^4}\,dx =$$

$$-\frac{\frac{162ac}{x^3}+\frac{243ad}{x^2}+\frac{486ae}{x}+\frac{54a^3b(c+x(d+ex))}{(a+bx^3)^3}+\frac{9a^2b(18c+x(17d+16ex))}{(a+bx^3)^2}+\frac{3ab(162c+x(139d+118ex))}{a+bx^3}-40\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(11\sqrt[3]{\cdots}\right.}{\cdots}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```

output
```
-1/486*((162*a*c)/x^3 + (243*a*d)/x^2 + (486*a*e)/x + (54*a^3*b*(c + x*(d
+ e*x)))/(a + b*x^3)^3 + (9*a^2*b*(18*c + x*(17*d + 16*e*x)))/(a + b*x^3)^
2 + (3*a*b*(162*c + x*(139*d + 118*e*x)))/(a + b*x^3) - 40*Sqrt[3]*a^(1/3)
*b^(1/3)*(11*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]] + 1944*b*c*Log[x] + 40*b^(1/3)*(11*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*e
)*Log[a^(1/3) + b^(1/3)*x] - 20*b^(1/3)*(11*a^(1/3)*b^(1/3)*d - 7*a^(2/3)*
e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 648*b*c*Log[a + b*x^3]
)/a^5
```

3.364.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2368, 25, 2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^4}dx$$

$$\Big\downarrow \ 2368$$

$$-\frac{\int -\frac{\frac{6b^3cx^6}{a^2}-\frac{7b^2ex^5}{a}-\frac{8b^2dx^4}{a}-\frac{9b^2cx^3}{a}+9bex^2+9bdx+9bc}{x^4(bx^3+a)^3}dx}{9ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2(a+bx^3)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\frac{6b^3cx^6}{a^2}-\frac{7b^2ex^5}{a}-\frac{8b^2dx^4}{a}-\frac{9b^2cx^3}{a}+9bex^2+9bdx+9bc}{x^4(bx^3+a)^3}dx}{9ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2(a+bx^3)^3}$$

↓ 2368

$$\frac{-\frac{\int -\frac{\frac{72b^5cx^6}{a^2}-\frac{64b^4ex^5}{a}-\frac{85b^4dx^4}{a}-\frac{108b^4cx^3}{a}+54b^3ex^2+54b^3dx+54b^3c}{x^4\left(bx^3+a\right)^2}dx}{6ab^2}-\frac{x\left(-\frac{24b^3cx^2}{a}+17b^2d+16b^2ex\right)}{6a^2(a+bx^3)^2}}{9ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2(a+bx^3)^3}$$

↓ 25

$$\frac{\frac{\int \frac{\frac{72b^5cx^6}{a^2}-\frac{64b^4ex^5}{a}-\frac{85b^4dx^4}{a}-\frac{108b^4cx^3}{a}+54b^3ex^2+54b^3dx+54b^3c}{x^4\left(bx^3+a\right)^2}dx}{6ab^2}-\frac{x\left(-\frac{24b^3cx^2}{a}+17b^2d+16b^2ex\right)}{6a^2(a+bx^3)^2}}{9ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2(a+bx^3)^3}$$

↓ 2368

$$\frac{-\frac{\frac{\int -\frac{2\left(-\frac{59ex^5b^6}{a}-\frac{139dx^4b^6}{a}-\frac{243cx^3b^6}{a}+81ex^2b^5+81cb^5+81dxb^5\right)}{x^4\left(bx^3+a\right)}dx}{3ab^2}-\frac{x\left(-\frac{234b^5cx^2}{a}+139b^4d+118b^4ex\right)}{3a^2\left(a+bx^3\right)}}{6ab^2}-\frac{x\left(-\frac{24b^3cx^2}{a}+17b^2d+16b^2ex\right)}{6a^2(a+bx^3)^2}}{9ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2(a+bx^3)^3}$$

↓ 27

$$\frac{\frac{\frac{2\int \frac{-\frac{59ex^5b^6}{a}-\frac{139dx^4b^6}{a}-\frac{243cx^3b^6}{a}+81ex^2b^5+81cb^5+81dxb^5}{x^4\left(bx^3+a\right)}dx}{3ab^2}-\frac{x\left(-\frac{234b^5cx^2}{a}+139b^4d+118b^4ex\right)}{3a^2\left(a+bx^3\right)}}{6ab^2}-\frac{x\left(-\frac{24b^3cx^2}{a}+17b^2d+16b^2ex\right)}{6a^2(a+bx^3)^2}}{9ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2(a+bx^3)^3}$$

↓ 2373

3.364. $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^4}\,dx$

$$\frac{\frac{\frac{2\int \left(-\frac{324cb^6}{a^2x}-\frac{4\left(-81bcx^2+35aex+55ad\right)b^6}{a^2\left(bx^3+a\right)}+\frac{81eb^5}{ax^2}+\frac{81db^5}{ax^3}+\frac{81cb^5}{ax^4}\right)dx}{3ab^2}-\frac{x\left(-\frac{234b^5cx^2}{a}+139b^4d+118b^4ex\right)}{3a^2\left(a+bx^3\right)}}{6ab^2}-\frac{x\left(-\frac{24b^3cx^2}{a}+17b^2d+16b^2ex\right)}{6a^2(a+bx^3)^2}}{9ab}-\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{2\left(\frac{20b^{16/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(7\sqrt[3]{a}e+11\sqrt[3]{b}d\right)}{\sqrt{3}a^{5/3}}+\frac{10b^{16/3}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{3a^{5/3}}-\frac{20b^{16/3}\left(11\sqrt[3]{b}d-7\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}\right.}{3a^{5/3}}\right.}{3ab^2}}{6ab^2}}{9ab}$$

$$\frac{x\left(-\frac{b^2cx^2}{a}+bd+bex\right)}{9a^2\left(a+bx^3\right)^3}$$

input
```
Int[(c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x]
```

output
```
-1/9*(x*(b*d + b*e*x - (b^2*c*x^2)/a))/(a^2*(a + b*x^3)^3) + (-1/6*(x*(17*
b^2*d + 16*b^2*e*x - (24*b^3*c*x^2)/a))/(a^2*(a + b*x^3)^2) + (-1/3*(x*(13
9*b^4*d + 118*b^4*e*x - (234*b^5*c*x^2)/a))/(a^2*(a + b*x^3)) + (2*((-27*b
^5*c)/(a*x^3) - (81*b^5*d)/(2*a*x^2) - (81*b^5*e)/(a*x) + (20*b^(16/3)*(11
*b^(1/3)*d + 7*a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(Sqrt[3]*a^(5/3)) - (324*b^6*c*Log[x])/a^2 - (20*b^(16/3)*(11*b^(1/3)*d
 - 7*a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (10*b^(16/3)*(11*b
^(1/3)*d - 7*a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(3
*a^(5/3)) + (108*b^6*c*Log[a + b*x^3])/a^2))/(3*a*b^2))/(6*a*b^2))/(9*a*b)
```

3.364.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.364.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.99

method	result
default	$-\frac{c}{3a^4x^3}-\frac{d}{2a^4x^2}-\frac{e}{a^4x}-\frac{4bc\ln(x)}{a^5}-\frac{b\left(\frac{\frac{59}{81}ab^2e\,x^8+\frac{139}{162}ab^2d\,x^7+ab^2c\,x^6+\frac{142}{81}a^2be\,x^5+\frac{329}{162}a^2bd\,x^4+\frac{7}{3}a^2x^3bc+\frac{92}{81}a^3e\,x^2+\frac{104}{81}a^3\cdots}{(b\,x^3+a)^3}\cdots\right.}{\cdots}$
risch	$\frac{-\frac{140b^3e\,x^{11}}{81a^4}-\frac{110b^3d\,x^{10}}{81a^4}-\frac{4b^3c\,x^9}{3a^4}-\frac{385e\,b^2x^8}{81a^3}-\frac{286d\,b^2x^7}{81a^3}-\frac{10c\,b^2x^6}{3a^3}-\frac{335be\,x^5}{81a^2}-\frac{451bd\,x^4}{162a^2}-\frac{22bc\,x^3}{9a^2}-\frac{e\,x^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3(b\,x^3+a)^3}+\frac{4\left(\sum_{\_R=\mathrm{RootOf}(\cdots}\cdots\right.}{\cdots}$

input
```
int((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c/a^4/x^3-1/2*d/a^4/x^2-e/a^4/x-4*b*c*ln(x)/a^5-1/a^5*b*((59/81*a*b^2
*e*x^8+139/162*a*b^2*d*x^7+a*b^2*c*x^6+142/81*a^2*b*e*x^5+329/162*a^2*b*d*
x^4+7/3*a^2*x^3*b*c+92/81*a^3*e*x^2+104/81*a^3*d*x+13/9*c*a^3)/(b*x^3+a)^3
+220/81*a*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1)))+140/81*a*e*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b
/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-4/3*c*ln(b*x^3+a))
```

3.364. $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^4}\,dx$

3.364.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 5670, normalized size of antiderivative = 16.68

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.364.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d*x+c)/x**4/(b*x**3+a)**4,x)
```

output
```
Timed out
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.97

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx = -\frac{280\,b^3ex^{11}+220\,b^3dx^{10}+216\,b^3cx^9+770\,ab^2ex^8+572\,ab^2dx^7+540\,ab^2cx^6+670\,a^2bex^5+451\,a^2bdx^4}{162\,\left(a^4b^3x^{12}+3\,a^5b^2x^9+3\,a^6bx^6+a^7x^3\right)}$$
$$-\frac{4\,bc\log\left(x\right)}{a^5}-\frac{20\,\sqrt{3}\left(7\,ae\left(\frac{a}{b}\right)^{\frac{2}{3}}+11\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^6}$$
$$+\frac{2\,\left(162\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-35\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+55\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{4\,\left(81\,bc\left(\frac{a}{b}\right)^{\frac{2}{3}}+35\,ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-55\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="maxima")
```

output
```
-1/162*(280*b^3*e*x^11 + 220*b^3*d*x^10 + 216*b^3*c*x^9 + 770*a*b^2*e*x^8
+ 572*a*b^2*d*x^7 + 540*a*b^2*c*x^6 + 670*a^2*b*e*x^5 + 451*a^2*b*d*x^4 +
396*a^2*b*c*x^3 + 162*a^3*e*x^2 + 81*a^3*d*x + 54*a^3*c)/(a^4*b^3*x^12 + 3
*a^5*b^2*x^9 + 3*a^6*b*x^6 + a^7*x^3) - 4*b*c*log(x)/a^5 - 20/243*sqrt(3)*
(7*a*e*(a/b)^(2/3) + 11*a*d*(a/b)^(1/3))*b*arctan(1/3*sqrt(3)*(2*x - (a/b)
^(1/3))/(a/b)^(1/3))/a^6 + 2/243*(162*b*c*(a/b)^(2/3) - 35*a*e*(a/b)^(1/3)
 + 55*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(a/b)^(2/3)) + 4/24
3*(81*b*c*(a/b)^(2/3) + 35*a*e*(a/b)^(1/3) - 55*a*d)*log(x + (a/b)^(1/3))/
(a^5*(a/b)^(2/3))
```

3.364.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2}{x^4\left(a+bx^3\right)^4}\,dx = \frac{4\,bc\log\left(|bx^3+a|\right)}{3\,a^5} - \frac{4\,bc\log\left(|x|\right)}{a^5} - \frac{20\,\sqrt{3}\left(11\left(-ab^2\right)^{\frac{1}{3}}bd - 7\left(-ab^2\right)^{\frac{2}{3}}e\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^5b} - \frac{10\left(11\left(-ab^2\right)^{\frac{1}{3}}bd + 7\left(-ab^2\right)^{\frac{2}{3}}e\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^5b} - \frac{280\,b^3ex^{11} + 220\,b^3dx^{10} + 216\,b^3cx^9 + 770\,ab^2ex^8 + 572\,ab^2dx^7 + 540\,ab^2cx^6 + 670\,a^2bex^5 + 451\,a^2bdx^4}{162\left(bx^4+ax\right)^3a^4} + \frac{20\left(7\,a^6b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 11\,a^6b^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^{11}b}$$

input
```
integrate((e*x^2+d*x+c)/x^4/(b*x^3+a)^4,x, algorithm="giac")
```

output
```
4/3*b*c*log(abs(b*x^3 + a))/a^5 - 4*b*c*log(abs(x))/a^5 - 20/243*sqrt(3)*(
11*(-a*b^2)^(1/3)*b*d - 7*(-a*b^2)^(2/3)*e)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 10/243*(11*(-a*b^2)^(1/3)*b*d + 7*(-a*b^
2)^(2/3)*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^5*b) - 1/162*(280*
b^3*e*x^11 + 220*b^3*d*x^10 + 216*b^3*c*x^9 + 770*a*b^2*e*x^8 + 572*a*b^2*
d*x^7 + 540*a*b^2*c*x^6 + 670*a^2*b*e*x^5 + 451*a^2*b*d*x^4 + 396*a^2*b*c*
x^3 + 162*a^3*e*x^2 + 81*a^3*d*x + 54*a^3*c)/((b*x^4 + a*x)^3*a^4) + 20/24
3*(7*a^6*b^2*e*(-a/b)^(1/3) + 11*a^6*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b
)^(1/3)))/(a^11*b)
```

3.364.9 Mupad [B] (verification not implemented)

Time = 12.23 (sec) , antiderivative size = 918, normalized size of antiderivative = 2.70

$$\int \frac{c+dx+ex^2}{x^4(a+bx^3)^4}\,dx$$

$$= \left(\sum_{k=1}^{3} \ln\left(-\frac{b^3\left(\mathrm{root}(14348907\,a^{15}\,z^3 - 57395628\,a^{10}\,b\,c\,z^2 + 22453200\,a^6\,b\,d\,e\,z + 76527504\,a^5\,b^2\,c^2\,z - 29\right.}{}\right.\right.$$

$$- 57395628\,a^{10}\,b\,c\,z^2 + 22453200\,a^6\,b\,d\,e\,z + 76527504\,a^5\,b^2\,c^2\,z - 29937600\,a\,b^2\,c\,d\,e$$

$$\left. - 2744000\,a^2\,b\,e^3 + 10648000\,a\,b^2\,d^3 - 34012224\,b^3\,c^3, z, k)\right)$$

$$- \frac{\frac{c}{3\,a} + \frac{e\,x^2}{a} + \frac{d\,x}{2\,a} + \frac{10\,b^2\,c\,x^6}{3\,a^3} + \frac{4\,b^3\,c\,x^9}{3\,a^4} + \frac{286\,b^2\,d\,x^7}{81\,a^3} + \frac{110\,b^3\,d\,x^{10}}{81\,a^4} + \frac{385\,b^2\,e\,x^8}{81\,a^3} + \frac{140\,b^3\,e\,x^{11}}{81\,a^4} + \frac{22\,b\,c\,x^3}{9\,a^2} + \frac{451\,b\,d\,x^4}{162\,a^2} + 3}{a^3\,x^3 + 3\,a^2\,b\,x^6 + 3\,a\,b^2\,x^9 + b^3\,x^{12}}$$

$$- \frac{4\,b\,c\,\ln(x)}{a^5}$$

input
```
int((c + d*x + e*x^2)/(x^4*(a + b*x^3)^4),x)
```

output
```
symsum(log(-(4*b^3*(688905*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2
+ 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2
744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)^2*a^10*e +
 3920400*b^2*c*d^2 - 3674160*b^2*c^2*e + 4782969*root(14348907*a^15*z^3 -
57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29
937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3
*c^3, z, k)^3*a^14*x + 2662000*b^2*d^3*x - 686000*a*b*e^3*x + 980100*root(
14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 7652750
4*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^
2*d^3 - 34012224*b^3*c^3, z, k)*a^5*b*d^2 - 12754584*root(14348907*a^15*z^
3 - 57395628*a^10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z
- 29937600*a*b^2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224
*b^3*c^3, z, k)^2*a^9*b*c*x + 8503056*root(14348907*a^15*z^3 - 57395628*a^
10*b*c*z^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^
2*c*d*e - 2744000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)
*a^4*b^2*c^2*x + 1837080*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z^2 +
22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d*e - 274
4000*a^2*b*e^3 + 10648000*a*b^2*d^3 - 34012224*b^3*c^3, z, k)*a^5*b*c*e -
4989600*b^2*c*d*e*x + 6237000*root(14348907*a^15*z^3 - 57395628*a^10*b*c*z
^2 + 22453200*a^6*b*d*e*z + 76527504*a^5*b^2*c^2*z - 29937600*a*b^2*c*d...
```

3.364. $\int \frac{c+dx+ex^2}{x^4(a+bx^3)^4}\,dx$

3.365 $\int \frac{2ax-x^2}{a^3+x^3} \, dx$

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3.365.1 Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{2ax-x^2}{a^3+x^3} \, dx = -\frac{2\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a+x)$$

output
```
-ln(a+x)-2/3*arctan(1/3*(a-2*x)/a*3^(1/2))*3^(1/2)
```

3.365.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

$$\int \frac{2ax-x^2}{a^3+x^3} \, dx = \frac{1}{3}\left(2\sqrt{3}\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right) - 2\log(a+x) + \log\left(a^2-ax+x^2\right) - \log\left(a^3+x^3\right)\right)$$

input
```
Integrate[(2*a*x - x^2)/(a^3 + x^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x +
 x^2] - Log[a^3 + x^3])/3
```

3.365.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2027, 2407, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2ax-x^2}{a^3+x^3}\,dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x(2a-x)}{a^3+x^3}dx$$

$$\Big\downarrow \text{ 2407}$$

$$a\int \frac{1}{a^2-xa+x^2}dx-\int \frac{1}{a+x}dx$$

$$\Big\downarrow \text{ 16}$$

$$a\int \frac{1}{a^2-xa+x^2}dx-\log(a+x)$$

$$\Big\downarrow \text{ 1082}$$

$$2\int \frac{1}{-\left(1-\frac{2x}{a}\right)^2-3}d\left(1-\frac{2x}{a}\right)-\log(a+x)$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{2\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{\sqrt{3}}-\log(a+x)$$

input
```
Int[(2*a*x - x^2)/(a^3 + x^3),x]
```

output
```
(-2*ArcTan[(1 - (2*x)/a)/Sqrt[3]])/Sqrt[3] - Log[a + x]
```

3.365.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2407
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Simp[C/b
Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]]
 /; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && Pol
yQ[P2, x, 2]
```

3.365.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3}-\ln\left(a+x\right)$	29
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3}-\ln\left(a+x\right)$	29

input
```
int((2*a*x-x^2)/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)/a)-ln(a+x)
```

3.365.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{2ax - x^2}{a^3 + x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a - 2x)}{3a}\right) - \log(a + x)$$

input
```
integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

3.365.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{2ax - x^2}{a^3 + x^3}\,dx = -\log(a + x) - \frac{\sqrt{3}i\log\left(-\frac{a}{2} - \frac{\sqrt{3}ia}{2} + x\right)}{3} + \frac{\sqrt{3}i\log\left(-\frac{a}{2} + \frac{\sqrt{3}ia}{2} + x\right)}{3}$$

input
```
integrate((2*a*x-x**2)/(a**3+x**3),x)
```

output
```
-log(a + x) - sqrt(3)*I*log(-a/2 - sqrt(3)*I*a/2 + x)/3 + sqrt(3)*I*log(-a
/2 + sqrt(3)*I*a/2 + x)/3
```

3.365.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{2ax - x^2}{a^3 + x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a - 2x)}{3a}\right) - \log(a + x)$$

input
```
integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

3.365.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{2ax-x^2}{a^3+x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) - \log\left(|a+x|\right)$$

input
```
integrate((2*a*x-x^2)/(a^3+x^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(abs(a + x))
```

3.365.9 Mupad [B] (verification not implemented)

Time = 11.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{2ax-x^2}{a^3+x^3}\,dx = -\ln(a+x) - \frac{2\sqrt{3}\,\mathrm{atan}\left(-\frac{\sqrt{3}\,a}{a-2x}\right)}{3}$$

input
```
int((2*a*x - x^2)/(a^3 + x^3),x)
```

output
```
- log(a + x) - (2*3^(1/2)*atan(-(3^(1/2)*a)/(a - 2*x)))/3
```

3.366 $\int \frac{(2a-x)x}{a^3+x^3}\,dx$

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3.366.7 Maxima [A] (verification not implemented) . . . 2763
3.366.8 Giac [A] (verification not implemented) . . . 2763
3.366.9 Mupad [B] (verification not implemented) . . . 2764

3.366.1 Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx = -\frac{2\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a+x)$$

output
```
-ln(a+x)-2/3*arctan(1/3*(a-2*x)/a*3^(1/2))*3^(1/2)
```

3.366.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx = \frac{1}{3}\left(2\sqrt{3}\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right) - 2\log(a+x) + \log\left(a^2-ax+x^2\right) - \log\left(a^3+x^3\right)\right)$$

input
```
Integrate[((2*a - x)*x)/(a^3 + x^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x +
 x^2] - Log[a^3 + x^3])/3
```

3.366.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2407, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2a-x)}{a^3+x^3}\,dx$$

$$\Big\downarrow \; 2407$$

$$a\int \frac{1}{a^2-xa+x^2}dx-\int \frac{1}{a+x}dx$$

$$\Big\downarrow \; 16$$

$$a\int \frac{1}{a^2-xa+x^2}dx-\log(a+x)$$

$$\Big\downarrow \; 1082$$

$$2\int \frac{1}{-\left(1-\frac{2x}{a}\right)^2-3}d\left(1-\frac{2x}{a}\right)-\log(a+x)$$

$$\Big\downarrow \; 217$$

$$-\frac{2\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{\sqrt{3}}-\log(a+x)$$

input
```
Int[((2*a - x)*x)/(a^3 + x^3),x]
```

output
```
(-2*ArcTan[(1 - (2*x)/a)/Sqrt[3]])/Sqrt[3] - Log[a + x]
```

3.366.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2407
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Simp[C/b
Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]]
 /; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && Pol
yQ[P2, x, 2]
```

3.366.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3}-\ln\left(a+x\right)$	29
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{(-a+2x)\sqrt{3}}{3a}\right)}{3}-\ln\left(a+x\right)$	29

input
```
int((2*a-x)*x/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)*arctan(1/3*(-a+2*x)*3^(1/2)/a)-ln(a+x)
```

3.366.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx=\frac{2}{3}\,\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)-\log\left(a+x\right)$$

input
```
integrate((2*a-x)*x/(a^3+x^3),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

3.366.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx = -\log{(a+x)} - \frac{\sqrt{3}i\log{\left(-\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}}{3} + \frac{\sqrt{3}i\log{\left(-\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}}{3}$$

input
```
integrate((2*a-x)*x/(a**3+x**3),x)
```

output
```
-log(a + x) - sqrt(3)*I*log(-a/2 - sqrt(3)*I*a/2 + x)/3 + sqrt(3)*I*log(-a
/2 + sqrt(3)*I*a/2 + x)/3
```

3.366.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right) - \log\left(a+x\right)$$

input
```
integrate((2*a-x)*x/(a^3+x^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a + x)
```

3.366.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right) - \log\left(|a+x|\right)$$

input
```
integrate((2*a-x)*x/(a^3+x^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(abs(a + x))
```

3.366.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(2a-x)x}{a^3+x^3}\,dx = -\ln(a+x) - \frac{2\sqrt{3}\,\mathrm{atan}\left(-\frac{\sqrt{3}\,a}{a-2\,x}\right)}{3}$$

input
```
int((x*(2*a - x))/(a^3 + x^3),x)
```

output
```
- log(a + x) - (2*3^(1/2)*atan(-(3^(1/2)*a)/(a - 2*x)))/3
```

3.367 $\int \frac{2ax+x^2}{a^3-x^3}\,dx$

3.367.1 Optimal result 2765
3.367.2 Mathematica [A] (verified) 2765
3.367.3 Rubi [A] (verified) 2766
3.367.4 Maple [A] (verified) 2767
3.367.5 Fricas [A] (verification not implemented) 2768
3.367.6 Sympy [C] (verification not implemented) 2768
3.367.7 Maxima [A] (verification not implemented) 2768
3.367.8 Giac [A] (verification not implemented) 2769
3.367.9 Mupad [B] (verification not implemented) 2769

3.367.1 Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx=-\frac{2\arctan\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}}-\log(a-x)$$

output
```
-ln(a-x)-2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)
```

3.367.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx=\frac{1}{3}\left(-2\sqrt{3}\arctan\left(\frac{a+2x}{\sqrt{3}a}\right)-2\log(-a+x)+\log\left(a^2+ax+x^2\right)-\log\left(-a^3+x^3\right)\right)$$

input
```
Integrate[(2*a*x + x^2)/(a^3 - x^3),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a^2 + a*x
+ x^2] - Log[-a^3 + x^3])/3
```

3.367.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2027, 2407, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x(2a+x)}{a^3-x^3}dx$$

$$\Big\downarrow \text{2407}$$

$$-a\int \frac{1}{a^2+xa+x^2}dx-\int \frac{1}{x-a}dx$$

$$\Big\downarrow \text{16}$$

$$-a\int \frac{1}{a^2+xa+x^2}dx-\log(a-x)$$

$$\Big\downarrow \text{1082}$$

$$2\int \frac{1}{-\left(\frac{2x}{a}+1\right)^2-3}d\left(\frac{2x}{a}+1\right)-\log(a-x)$$

$$\Big\downarrow \text{217}$$

$$-\frac{2\arctan\left(\frac{\frac{2x}{a}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\log(a-x)$$

input
```
Int[(2*a*x + x^2)/(a^3 - x^3),x]
```

output
```
(-2*ArcTan[(1 + (2*x)/a)/Sqrt[3]])/Sqrt[3] - Log[a - x]
```

3.367.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2407
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Simp[C/b
Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]]
 /; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && Pol
yQ[P2, x, 2]
```

3.367.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$-\ln\left(a-x\right)-\frac{2\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3}$	29
risch	$-\frac{2\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3}-\ln\left(-a+x\right)$	29

input
```
int((2*a*x+x^2)/(a^3-x^3),x,method=_RETURNVERBOSE)
```

output
```
-ln(a-x)-2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)
```

3.367.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2\,x)}{3\,a}\right) - \log\left(-a+x\right)$$

input
```
integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="fricas")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

3.367.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx = -\log\left(-a+x\right) + \frac{\sqrt{3}i\log\left(\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3} - \frac{\sqrt{3}i\log\left(\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

input
```
integrate((2*a*x+x**2)/(a**3-x**3),x)
```

output
```
-log(-a + x) + sqrt(3)*I*log(a/2 - sqrt(3)*I*a/2 + x)/3 - sqrt(3)*I*log(a/
2 + sqrt(3)*I*a/2 + x)/3
```

3.367.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2\,x)}{3\,a}\right) - \log\left(-a+x\right)$$

input
```
integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="maxima")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

3.367.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx = -\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2\,x)}{3\,a}\right) - \log\left(|-a+x|\right)$$

input
```
integrate((2*a*x+x^2)/(a^3-x^3),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(abs(-a + x))
```

3.367.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{2ax+x^2}{a^3-x^3}\,dx = \frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,a}{a+2\,x}\right)}{3} - \ln\left(x-a\right)$$

input
```
int((2*a*x + x^2)/(a^3 - x^3),x)
```

output
```
(2*3^(1/2)*atan((3^(1/2)*a)/(a + 2*x)))/3 - log(x - a)
```

3.368 $\int \frac{x(2a+x)}{a^3-x^3} dx$

3.368.1 Optimal result 2770
3.368.2 Mathematica [A] (verified) 2770
3.368.3 Rubi [A] (verified) 2771
3.368.4 Maple [A] (verified) 2772
3.368.5 Fricas [A] (verification not implemented) 2772
3.368.6 Sympy [C] (verification not implemented) 2773
3.368.7 Maxima [A] (verification not implemented) 2773
3.368.8 Giac [A] (verification not implemented) 2773
3.368.9 Mupad [B] (verification not implemented) 2774

3.368.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{x(2a+x)}{a^3-x^3} dx = -\frac{2\arctan\left(\frac{a+2x}{\sqrt{3}a}\right)}{\sqrt{3}} - \log(a-x)$$

output
```
-ln(a-x)-2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)
```

3.368.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \frac{x(2a+x)}{a^3-x^3} dx = \frac{1}{3}\left(-2\sqrt{3}\arctan\left(\frac{a+2x}{\sqrt{3}a}\right) - 2\log(-a+x) + \log\left(a^2+ax+x^2\right) - \log\left(-a^3+x^3\right)\right)$$

input
```
Integrate[(x*(2*a + x))/(a^3 - x^3),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(a + 2*x)/(Sqrt[3]*a)] - 2*Log[-a + x] + Log[a^2 + a*x
+ x^2] - Log[-a^3 + x^3])/3
```

3.368.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2407, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2a+x)}{a^3-x^3}dx$$

$$\Big\downarrow \text{2407}$$

$$-a\int \frac{1}{a^2+xa+x^2}dx-\int \frac{1}{x-a}dx$$

$$\Big\downarrow \text{16}$$

$$-a\int \frac{1}{a^2+xa+x^2}dx-\log(a-x)$$

$$\Big\downarrow \text{1082}$$

$$2\int \frac{1}{-\left(\frac{2x}{a}+1\right)^2-3}d\left(\frac{2x}{a}+1\right)-\log(a-x)$$

$$\Big\downarrow \text{217}$$

$$-\frac{2\arctan\left(\frac{\frac{2x}{a}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\log(a-x)$$

input
```
Int[(x*(2*a + x))/(a^3 - x^3),x]
```

output
```
(-2*ArcTan[(1 + (2*x)/a)/Sqrt[3]])/Sqrt[3] - Log[a - x]
```

3.368.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2407
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Simp[C/b
Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]]
 /; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && Pol
yQ[P2, x, 2]
```

3.368.4 Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$-\ln\left(a-x\right)-\frac{2\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3}$	29
risch	$-\frac{2\arctan\left(\frac{(a+2x)\sqrt{3}}{3a}\right)\sqrt{3}}{3}-\ln\left(-a+x\right)$	29

input
```
int(x*(2*a+x)/(a^3-x^3),x,method=_RETURNVERBOSE)
```

output
```
-ln(a-x)-2/3*arctan(1/3*(a+2*x)/a*3^(1/2))*3^(1/2)
```

3.368.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx = -\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2\,x)}{3\,a}\right)-\log\left(-a+x\right)$$

input
```
integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="fricas")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

3.368.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx = -\log(-a+x) + \frac{\sqrt{3}i\log\left(\frac{a}{2}-\frac{\sqrt{3}ia}{2}+x\right)}{3} - \frac{\sqrt{3}i\log\left(\frac{a}{2}+\frac{\sqrt{3}ia}{2}+x\right)}{3}$$

input
```
integrate(x*(2*a+x)/(a**3-x**3),x)
```

output
```
-log(-a + x) + sqrt(3)*I*log(a/2 - sqrt(3)*I*a/2 + x)/3 - sqrt(3)*I*log(a/
2 + sqrt(3)*I*a/2 + x)/3
```

3.368.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right) - \log(-a+x)$$

input
```
integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="maxima")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(-a + x)
```

3.368.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(a+2x)}{3a}\right) - \log(|-a+x|)$$

input
```
integrate(x*(2*a+x)/(a^3-x^3),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*x)/a) - log(abs(-a + x))
```

3.368.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{x(2a+x)}{a^3-x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,a}{a+2\,x}\right)}{3} - \ln\left(x-a\right)$$

input
```
int((x*(2*a + x))/(a^3 - x^3),x)
```

output
```
(2*3^(1/2)*atan((3^(1/2)*a)/(a + 2*x)))/3 - log(x - a)
```

3.369 $\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$

3.369.1 Optimal result . . . 2775
3.369.2 Mathematica [B] (verified) . . . 2775
3.369.3 Rubi [A] (verified) . . . 2776
3.369.4 Maple [B] (verified) . . . 2778
3.369.5 Fricas [A] (verification not implemented) . . . 2778
3.369.6 Sympy [C] (verification not implemented) . . . 2779
3.369.7 Maxima [A] (verification not implemented) . . . 2779
3.369.8 Giac [C] (verification not implemented) . . . 2780
3.369.9 Mupad [B] (verification not implemented) . . . 2780

3.369.1 Optimal result

Integrand size = 27, antiderivative size = 50

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

output
```
C*ln((a/b)^(1/3)+x)/b+2/3*C*arctan(1/3*(1-2*x/(a/b)^(1/3))*3^(1/2))/b*3^(1
/2)
```

3.369.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(50) = 100.

Time = 0.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.92

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$$
$$=\frac{C\left(2\sqrt{3}\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{3\sqrt[3]{a}b}$$

input
```
Integrate[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

output
```
(C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] - (a/b)^(1/3)*b^(1/3)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a + b*x^3]))
/(3*a^(1/3)*b)
```

3.369.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2406, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(Cx-2C\sqrt[3]{\frac{a}{b}}\right)}{a+bx^3}dx$$

$$\Big\downarrow \text{ 2406}$$

$$\frac{C\int \frac{1}{x+\sqrt[3]{\frac{a}{b}}}dx}{b}-\frac{C\sqrt[3]{\frac{a}{b}}\int \frac{1}{x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}}dx}{b}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{C\sqrt[3]{\frac{a}{b}}\int \frac{1}{x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}}dx}{b}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}-\frac{2C\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}\right)}{b}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{\frac{a}{b}}+x\right)}{b}$$

input
```
Int[(x*(-2*(a/b)^(1/3)*C + C*x))/(a + b*x^3),x]
```

output
```
(2*C*ArcTan[(1 - (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(a/b)^(
1/3) + x])/b
```

3.369.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2406
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Simp[C/b
 Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]
] /; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Po
lyQ[P2, x, 2]
```

3.369.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(43) = 86$.

Time = 1.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.32

method	result	size
default	$C\left(-2\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+\frac{\ln\left(b\,x^{3}+a\right)}{3b}\right)$	116

input
```
int(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*(-2*(a/b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*ln(b*x^3+a)/b)
```

3.369.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=-\frac{2\,\sqrt{3}C\,\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)-3\,C\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

input
```
integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) -
3*C*log(x + (a/b)^(1/3)))/b
```

3.369.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.00

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C + Cx\right)}{a + bx^3}\,dx = \frac{C\left(\log\left(\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + x\right) + \frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + x\right)}{3} - \frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + x\right)}{3}\right)}{b}$$

input
```
integrate(x*(-2*(a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

output
```
C*(log(a/(b*(a/b)**(2/3)) + x) + sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) - sqr
t(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(-a/(2*b*(a/b)**(2/3)) +
 sqrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

3.369.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C + Cx\right)}{a + bx^3}\,dx = -\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b} + \frac{C\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

input
```
integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/b + C*l
og(x + (a/b)^(1/3))/b
```

3.369.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.98

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{\left(Cb\left(-\frac{a}{b}\right)^{\frac{2}{3}}-2\left(ab^2\right)^{\frac{1}{3}}C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}+\frac{\left(3ab^2-i\sqrt{3}\sqrt{a^2b^4}\right)C\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^3}$$

input
```
integrate(x*(-2*(a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/
3*(C*b*(-a/b)^(2/3) - 2*(a*b^2)^(1/3)*C*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs
(x - (-a/b)^(1/3)))/(a*b) + 1/6*(3*a*b^2 - I*sqrt(3)*sqrt(a^2*b^4))*C*log(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

3.369.9 Mupad [B] (verification not implemented)

Time = 11.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.08

$$\int \frac{x\left(-2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=\sum_{k=1}^{3}\ln\left(\frac{C^2\,a+\mathrm{root}(27\,a\,b^3\,z^3-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k)^2\,a\,b^2\,9-C\,\mathrm{root}(27\,a\,b^3\,z^3-27\,C\,a}{b^3}\right.$$
$$\left.-27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z-9\,C^3\,a,z,k\right)$$

input
```
int((x*(C*x - 2*C*(a/b)^(1/3)))/(a + b*x^3),x)
```

output
```
symsum(log((C^2*a + 9*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z - 9
*C^3*a, z, k)^2*a*b^2 - 6*C*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b
*z - 9*C^3*a, z, k)*a*b + 4*C^2*b*x*(a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 -
27*C*a*b^2*z^2 + 9*C^2*a*b*z - 9*C^3*a, z, k), k, 1, 3)
```

3.370 $\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

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3.370.1 Optimal result

Integrand size = 29, antiderivative size = 53

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=-\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

output
```
-C*ln((-a/b)^(1/3)+x)/b-2/3*C*arctan(1/3*(1-2*x/(-a/b)^(1/3))*3^(1/2))/b*3
^(1/2)
```

3.370.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 149 vs. 2(53) = 106.

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.81

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=$$
$$-\frac{C\left(-2\sqrt{3}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x\right.\right.}{3\sqrt[3]{a}b}$$

input
```
Integrate[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

output
```
-1/3*(C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] + (-(a/b)
)^(1/3)*b^(1/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*L
og[a - b*x^3]))/(a^(1/3)*b)
```

3.370.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2406, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(Cx-2C\sqrt[3]{-\frac{a}{b}}\right)}{a-bx^3}dx$$

$$\Big\downarrow \text{ 2406}$$

$$\frac{C\sqrt[3]{-\frac{a}{b}}\int \frac{1}{x^2-\sqrt[3]{-\frac{a}{b}}x+\left(-\frac{a}{b}\right)^{2/3}}dx}{b}-\frac{C\int \frac{1}{x+\sqrt[3]{-\frac{a}{b}}}dx}{b}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{C\sqrt[3]{-\frac{a}{b}}\int \frac{1}{x^2-\sqrt[3]{-\frac{a}{b}}x+\left(-\frac{a}{b}\right)^{2/3}}dx}{b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2C\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}\right)}{b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{2C\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}+x\right)}{b}$$

input
```
Int[(x*(-2*(-(a/b))^(1/3)*C + C*x))/(a - b*x^3),x]
```

output
```
(-2*C*ArcTan[(1 - (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(-(
a/b))^(1/3) + x])/b
```

3.370.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2406
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Simp[C/b
 Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]
] /; EqQ[A - (a/b)^(1/3)*B - 2*(a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] && Po
lyQ[P2, x, 2]
```

3.370.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(46) = 92.

Time = 1.50 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

method	result	s
default	$C\left(-2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-\frac{\ln\left(-b\,x^{3}+a\right)}{3b}\right)$	1

input
```
int(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*(-2*(-a/b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(1/3)
*ln(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*(1
+2/(a/b)^(1/3)*x)*3^(1/2)))-1/3*ln(-b*x^3+a)/b)
```

3.370.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int\frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^{3}}\,dx=-\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right)+3\,C\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

input
```
integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(3)*a)/a) +
 3*C*log(x + (-a/b)^(1/3)))/b
```

3.370.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.08

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{C\left(\log\left(-\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

input
```
integrate(x*(-2*(-a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
```

output
```
-C*(log(-a/(b*(-a/b)**(2/3)) + x) - sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3)) -
sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(a/(2*b*(-a/b)**(2/3
)) + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

3.370.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(46) = 92$.

Time = 0.35 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.13

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}+C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\sqrt{3}\left(Ca-\left(3\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

input
```
integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*(C*(a/b)^(1/3) + C*(-a/b)^(1/3))*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3
))/(b*(a/b)^(1/3)) - 1/3*(C*(a/b)^(1/3) - 2*C*(-a/b)^(1/3))*log(x - (a/b)^
(1/3))/(b*(a/b)^(1/3)) - 2/9*sqrt(3)*(C*a - (3*C*(a/b)^(2/3)*(-a/b)^(1/3)
+ C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

3.370.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.15

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{\left(Cb\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\left(-ab^2\right)^{\frac{1}{3}}C\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab} + \frac{\sqrt{3}\left(ab^2+i\sqrt{3}\sqrt{a^2b^4}\right)C\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}$$

input
```
integrate(x*(-2*(-a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*(C*b*(a/b)^(2/3) - 2*(-a*b^2)^(1/3)*C*(a/b)^(1/3))*(a/b)^(1/3)*log(ab
s(x - (a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(a*b^2 + I*sqrt(3)*sqrt(a^2*b^4))*
C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3)
```

3.370.9 Mupad [B] (verification not implemented)

Time = 11.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.94

$$\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$$
$$=\sum_{k=1}^{3}\ln\left(-\frac{C^2\,a+\mathrm{root}(27\,a\,b^3\,z^3+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k)^2\,a\,b^2\,9+C\,\mathrm{root}(27\,a\,b^3\,z^3+27\,C}{b^3}\right.$$
$$+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k\Big)$$

input
```
int((x*(C*x - 2*C*(-a/b)^(1/3)))/(a - b*x^3),x)
```

output
```
symsum(log(-(C^2*a + 9*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*z +
9*C^3*a, z, k)^2*a*b^2 + 6*C*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*
b*z + 9*C^3*a, z, k)*a*b - 4*C^2*b*x*(-a/b)^(2/3))/b^3)*root(27*a*b^3*z^3
+ 27*C*a*b^2*z^2 + 9*C^2*a*b*z + 9*C^3*a, z, k), k, 1, 3)
```

3.370. $\int \frac{x\left(-2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

3.371 $\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$

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3.371.1 Optimal result

Integrand size = 28, antiderivative size = 54

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=\frac{2C\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{-\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

output
```
C*ln((-a/b)^(1/3)-x)/b+2/3*C*arctan(1/3*(1+2*x/(-a/b)^(1/3))*3^(1/2))/b*3^
(1/2)
```

3.371.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(54) = 108.

Time = 0.05 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.74

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$$
$$=\frac{C\left(-2\sqrt{3}\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\sqrt[3]{-\frac{a}{b}}\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b\right.}{3\sqrt[3]{a}b}$$

input
```
Integrate[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

output
```
(C*(-2*Sqrt[3]*(-(a/b))^(1/3)*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]] - 2*(-(a/b))^(1/3)*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] + (-(a/b))^(1/
3)*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a
+ b*x^3]))/(3*a^(1/3)*b)
```

3.371.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2408, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(2C\sqrt[3]{-\frac{a}{b}}+Cx\right)}{a+bx^3}dx$$

$$\Big\downarrow \text{ 2408}$$

$$\frac{C\sqrt[3]{-\frac{a}{b}}\int \frac{1}{x^2+\sqrt[3]{-\frac{a}{b}}x+\left(-\frac{a}{b}\right)^{2/3}}dx}{b}-\frac{C\int \frac{1}{\sqrt[3]{-\frac{a}{b}}-x}dx}{b}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{C\sqrt[3]{-\frac{a}{b}}\int \frac{1}{x^2+\sqrt[3]{-\frac{a}{b}}x+\left(-\frac{a}{b}\right)^{2/3}}dx}{b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}-\frac{2C\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1\right)}{b}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2C\arctan\left(\frac{\frac{2x}{\sqrt[3]{-\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}+\frac{C\log\left(\sqrt[3]{-\frac{a}{b}}-x\right)}{b}$$

input
```
Int[(x*(2*(-(a/b))^(1/3)*C + C*x))/(a + b*x^3),x]
```

output
```
(2*C*ArcTan[(1 + (2*x)/(-(a/b))^(1/3))/Sqrt[3]])/(Sqrt[3]*b) + (C*Log[(-(a
/b))^(1/3) - x])/b
```

3.371.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2408
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Simp[-C/b
   Int[1/(q - x), x], x] + Simp[(B - C*q)/b   Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &
& PolyQ[P2, x, 2]
```

3.371. $\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$

3.371.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(47) = 94.

Time = 1.52 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.17

method	result	size
default	$C\left(2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+\frac{\ln\left(b\,x^{3}+a\right)}{3b}\right)$	117

input
```
int(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*(2*(-a/b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*ln(b*x^3+a)/b)
```

3.371.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx=-\frac{2\,\sqrt{3}C\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)-3\,C\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

input
```
integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) -
 3*C*log(x - (-a/b)^(1/3)))/b
```

3.371.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.02

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C + Cx\right)}{a+bx^3}\,dx = \frac{C\left(\log\left(\frac{a}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x\right) + \frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x\right)}{3} - \frac{\sqrt{3}i\log\left(-\frac{a}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}ia}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + x\right)}{3}\right)}{b}$$

input
```
integrate(x*(2*(-a/b)**(1/3)*C+C*x)/(b*x**3+a),x)
```

output
```
C*(log(a/(b*(-a/b)**(2/3)) + x) + sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3)) - s
qrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3 - sqrt(3)*I*log(-a/(2*b*(-a/b)**(2/3
)) + sqrt(3)*I*a/(2*b*(-a/b)**(2/3)) + x)/3)/b
```

3.371.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(47) = 94$.

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.09

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C + Cx\right)}{a+bx^3}\,dx = \frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}} + C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}} - 2\,C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\sqrt{3}\left(Ca - \left(3\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \frac{Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

input
```
integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*(C*(a/b)^(1/3) + C*(-a/b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(b*(a/b)^(1/3)) + 1/3*(C*(a/b)^(1/3) - 2*C*(-a/b)^(1/3))*log(x + (a/b)^(
1/3))/(b*(a/b)^(1/3)) - 2/9*sqrt(3)*(C*a - (3*C*(a/b)^(2/3)*(-a/b)^(1/3) +
 C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

3.371.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(47) = 94$.

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.80

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C + Cx\right)}{a+bx^3}\,dx = \frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b} - \frac{\left(Cb\left(-\frac{a}{b}\right)^{\frac{2}{3}} + 2\left(-ab^2\right)^{\frac{1}{3}}C\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}$$

input
```
integrate(x*(2*(-a/b)^(1/3)*C+C*x)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b - 1/
3*(C*b*(-a/b)^(2/3) + 2*(-a*b^2)^(1/3)*C*(-a/b)^(1/3))*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a*b)
```

3.371.9 Mupad [B] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.87

$$\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C + Cx\right)}{a+bx^3}\,dx = \sum_{k=1}^{3}\ln\left(\frac{C^2\,a + \mathrm{root}(27\,a\,b^3\,z^3 - 27\,C\,a\,b^2\,z^2 + 9\,C^2\,a\,b\,z - 9\,C^3\,a, z, k)^2\,a\,b^2\,9 - C\,\mathrm{root}(27\,a\,b^3\,z^3 - 27\,C\,a}{b^3} - 27\,C\,a\,b^2\,z^2 + 9\,C^2\,a\,b\,z - 9\,C^3\,a, z, k\right)$$

input
```
int((x*(C*x + 2*C*(-a/b)^(1/3)))/(a + b*x^3),x)
```

output
```
symsum(log((C^2*a + 9*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b*z - 9
*C^3*a, z, k)^2*a*b^2 - 6*C*root(27*a*b^3*z^3 - 27*C*a*b^2*z^2 + 9*C^2*a*b
*z - 9*C^3*a, z, k)*a*b + 4*C^2*b*x*(-a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 -
 27*C*a*b^2*z^2 + 9*C^2*a*b*z - 9*C^3*a, z, k), k, 1, 3)
```

3.371. $\int \frac{x\left(2\sqrt[3]{-\frac{a}{b}}C+Cx\right)}{a+bx^3}\,dx$

3.372 $\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

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3.372.1 Optimal result

Integrand size = 28, antiderivative size = 53

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=-\frac{2C\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{a}{b}}}}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

output
```
-C*ln((a/b)^(1/3)-x)/b-2/3*C*arctan(1/3*(1+2*x/(a/b)^(1/3))*3^(1/2))/b*3^(
1/2)
```

3.372.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 147 vs. 2(53) = 106.

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.77

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=-\frac{C\left(2\sqrt{3}\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)-\sqrt[3]{\frac{a}{b}}\sqrt[3]{b}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right.}{3\sqrt[3]{a}b}$$

input
```
Integrate[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

output
```
-1/3*(C*(2*Sqrt[3]*(a/b)^(1/3)*b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]] + 2*(a/b)^(1/3)*b^(1/3)*Log[a^(1/3) - b^(1/3)*x] - (a/b)^(1/3)*b^
(1/3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(1/3)*Log[a - b*x
^3]))/(a^(1/3)*b)
```

3.372.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2408, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(2C\sqrt[3]{\frac{a}{b}}+Cx\right)}{a-bx^3}dx$$

$$\Big\downarrow \text{2408}$$

$$\frac{C\int \frac{1}{\sqrt[3]{\frac{a}{b}}-x}dx}{b}-\frac{C\sqrt[3]{\frac{a}{b}}\int \frac{1}{x^2+\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}}dx}{b}$$

$$\Big\downarrow \text{16}$$

$$-\frac{C\sqrt[3]{\frac{a}{b}}\int \frac{1}{x^2+\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{2/3}}dx}{b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2C\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1\right)}{b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2C\arctan\left(\frac{\frac{2x}{\sqrt[3]{\frac{a}{b}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{C\log\left(\sqrt[3]{\frac{a}{b}}-x\right)}{b}$$

input
```
Int[(x*(2*(a/b)^(1/3)*C + C*x))/(a - b*x^3),x]
```

output
```
(-2*C*ArcTan[(1 + (2*x)/(a/b)^(1/3))/Sqrt[3]])/(Sqrt[3]*b) - (C*Log[(a/b)^
(1/3) - x])/b
```

3.372.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2408
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (-a/b)^(1/3)}, Simp[-C/b
   Int[1/(q - x), x], x] + Simp[(B - C*q)/b   Int[1/(q^2 + q*x + x^2), x],
x]] /; EqQ[A + (-a/b)^(1/3)*B - 2*(-a/b)^(2/3)*C, 0]] /; FreeQ[{a, b}, x] &
& PolyQ[P2, x, 2]
```

3.372. $\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx$

3.372.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(46) = 92$.

Time = 1.52 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.23

method	result	size
default	$C\left(2\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\left(1+\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-\frac{\ln\left(-b\,x^{3}+a\right)}{3b}\right)$	118

input
```
int(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*(2*(a/b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*l
n(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*(1+2
/(a/b)^(1/3)*x)*3^(1/2)))-1/3*ln(-b*x^3+a)/b)
```

3.372.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx=-\frac{2\sqrt{3}C\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right)+3\,C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b}$$

input
```
integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(3)*C*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) + sqrt(3)*a)/a) +
3*C*log(x - (a/b)^(1/3)))/b
```

3.372.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{C\left(\log\left(-\frac{a}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)-\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}+\frac{\sqrt{3}i\log\left(\frac{a}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}ia}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+x\right)}{3}\right)}{b}$$

input
```
integrate(x*(2*(a/b)**(1/3)*C+C*x)/(-b*x**3+a),x)
```

output
```
-C*(log(-a/(b*(a/b)**(2/3)) + x) - sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) - sq
rt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3 + sqrt(3)*I*log(a/(2*b*(a/b)**(2/3)) +
 sqrt(3)*I*a/(2*b*(a/b)**(2/3)) + x)/3)/b
```

3.372.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b}-\frac{C\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b}$$

input
```
integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - C*l
og(x - (a/b)^(1/3))/b
```

3.372.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = -\frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b} - \frac{\left(Cb\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\left(ab^2\right)^{\frac{1}{3}}C\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}$$

input
```
integrate(x*(2*(a/b)^(1/3)*C+C*x)/(-b*x^3+a),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b - 1/3
*(C*b*(a/b)^(2/3) + 2*(a*b^2)^(1/3)*C*(a/b)^(1/3))*(a/b)^(1/3)*log(abs(x -
 (a/b)^(1/3)))/(a*b)
```

3.372.9 Mupad [B] (verification not implemented)

Time = 10.96 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.92

$$\int \frac{x\left(2\sqrt[3]{\frac{a}{b}}C+Cx\right)}{a-bx^3}\,dx = \sum_{k=1}^{3}\ln\left(-\frac{C^2\,a+\mathrm{root}(27\,a\,b^3\,z^3+27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k)^2\,a\,b^2\,9+C\,\mathrm{root}(27\,a\,b^3\,z^3+27\,C}{b^3}\right. + 27\,C\,a\,b^2\,z^2+9\,C^2\,a\,b\,z+9\,C^3\,a,z,k)$$

input
```
int((x*(C*x + 2*C*(a/b)^(1/3)))/(a - b*x^3),x)
```

output
```
symsum(log(-(C^2*a + 9*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*b*z +
9*C^3*a, z, k)^2*a*b^2 + 6*C*root(27*a*b^3*z^3 + 27*C*a*b^2*z^2 + 9*C^2*a*
b*z + 9*C^3*a, z, k)*a*b - 4*C^2*b*x*(a/b)^(2/3))/b^3)*root(27*a*b^3*z^3 +
 27*C*a*b^2*z^2 + 9*C^2*a*b*z + 9*C^3*a, z, k), k, 1, 3)
```

3.373 $\int x^4(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

3.373.1 Optimal result 2801
3.373.2 Mathematica [A] (verified) 2801
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3.373.8 Giac [A] (verification not implemented) 2805
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3.373.1 Optimal result

Integrand size = 36, antiderivative size = 97

$$\begin{aligned}&\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\&=\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{8}(bc+af)x^8+\frac{1}{9}(bd+ag)x^9\\&\quad+\frac{1}{10}(be+ah)x^{10}+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}\end{aligned}$$

output
```
1/5*a*c*x^5+1/6*a*d*x^6+1/7*a*e*x^7+1/8*(a*f+b*c)*x^8+1/9*(a*g+b*d)*x^9+1/
10*(a*h+b*e)*x^10+1/11*b*f*x^11+1/12*b*g*x^12+1/13*b*h*x^13
```

3.373.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\&=\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{8}(bc+af)x^8+\frac{1}{9}(bd+ag)x^9\\&\quad+\frac{1}{10}(be+ah)x^{10}+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}\end{aligned}$$

input
```
Integrate[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*
g)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x
^13)/13
```

3.373.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$

$$\Big\downarrow \text{2360}$$

$$\int (x^7(af+bc)+x^8(ag+bd)+x^9(ah+be)+acx^4+adx^5+aex^6+bfx^{10}+bgx^{11}+bhx^{12})\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}x^8(af+bc)+\frac{1}{9}x^9(ag+bd)+\frac{1}{10}x^{10}(ah+be)+\frac{1}{5}acx^5+\frac{1}{6}adx^6+\frac{1}{7}aex^7+\frac{1}{11}bfx^{11}+\frac{1}{12}bgx^{12}+\frac{1}{13}bhx^{13}$$

input
```
Int[x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + ((b*c + a*f)*x^8)/8 + ((b*d + a*
g)*x^9)/9 + ((b*e + a*h)*x^10)/10 + (b*f*x^11)/11 + (b*g*x^12)/12 + (b*h*x
^13)/13
```

3.373.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.373.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

method	result
default	$\frac{ac\,x^5}{5}+\frac{ad\,x^6}{6}+\frac{ae\,x^7}{7}+\frac{(af+bc)x^8}{8}+\frac{(ag+bd)x^9}{9}+\frac{(ah+be)x^{10}}{10}+\frac{bf\,x^{11}}{11}+\frac{bg\,x^{12}}{12}+\frac{bh\,x^{13}}{13}$
norman	$\frac{bh\,x^{13}}{13}+\frac{bg\,x^{12}}{12}+\frac{bf\,x^{11}}{11}+\left(\frac{ah}{10}+\frac{be}{10}\right)x^{10}+\left(\frac{ag}{9}+\frac{bd}{9}\right)x^9+\left(\frac{af}{8}+\frac{bc}{8}\right)x^8+\frac{ae\,x^7}{7}+\frac{ad\,x^6}{6}+\frac{ac\,x^5}{5}$
gosper	$\frac{1}{13}bh\,x^{13}+\frac{1}{12}bg\,x^{12}+\frac{1}{11}bf\,x^{11}+\frac{1}{10}x^{10}ah+\frac{1}{10}be\,x^{10}+\frac{1}{9}x^9ag+\frac{1}{9}bd\,x^9+\frac{1}{8}x^8af+\frac{1}{8}bc\,x^8+$
risch	$\frac{1}{13}bh\,x^{13}+\frac{1}{12}bg\,x^{12}+\frac{1}{11}bf\,x^{11}+\frac{1}{10}x^{10}ah+\frac{1}{10}be\,x^{10}+\frac{1}{9}x^9ag+\frac{1}{9}bd\,x^9+\frac{1}{8}x^8af+\frac{1}{8}bc\,x^8+$
parallelrisch	$\frac{1}{13}bh\,x^{13}+\frac{1}{12}bg\,x^{12}+\frac{1}{11}bf\,x^{11}+\frac{1}{10}x^{10}ah+\frac{1}{10}be\,x^{10}+\frac{1}{9}x^9ag+\frac{1}{9}bd\,x^9+\frac{1}{8}x^8af+\frac{1}{8}bc\,x^8+$

input
```
int(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*c*x^5+1/6*a*d*x^6+1/7*a*e*x^7+1/8*(a*f+b*c)*x^8+1/9*(a*g+b*d)*x^9+1/
10*(a*h+b*e)*x^10+1/11*b*f*x^11+1/12*b*g*x^12+1/13*b*h*x^13
```

3.373.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\begin{aligned}\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\ &=\frac{1}{13}\,bhx^{13}+\frac{1}{12}\,bgx^{12}+\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,(be+ah)x^{10}\\ &\quad+\frac{1}{9}\,(bd+ag)x^9+\frac{1}{7}\,aex^7+\frac{1}{8}\,(bc+af)x^8+\frac{1}{6}\,adx^6+\frac{1}{5}\,acx^5\end{aligned}$$

input
```
integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fric
as")
```

output
```
1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*(b*e + a*h)*x^10 + 1/
9*(b*d + a*g)*x^9 + 1/7*a*e*x^7 + 1/8*(b*c + a*f)*x^8 + 1/6*a*d*x^6 + 1/5*
a*c*x^5
```

3.373.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{acx^5}{5}+\frac{adx^6}{6}+\frac{aex^7}{7}+\frac{bfx^{11}}{11}+\frac{bgx^{12}}{12}+\frac{bhx^{13}}{13} + x^{10}\left(\frac{ah}{10}+\frac{be}{10}\right)+x^9\left(\frac{ag}{9}+\frac{bd}{9}\right)+x^8\left(\frac{af}{8}+\frac{bc}{8}\right)$$

input
```
integrate(x**4*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a*c*x**5/5 + a*d*x**6/6 + a*e*x**7/7 + b*f*x**11/11 + b*g*x**12/12 + b*h*x
**13/13 + x**10*(a*h/10 + b*e/10) + x**9*(a*g/9 + b*d/9) + x**8*(a*f/8 + b
*c/8)
```

3.373.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{13}bhx^{13}+\frac{1}{12}bgx^{12}+\frac{1}{11}bfx^{11}+\frac{1}{10}(be+ah)x^{10} + \frac{1}{9}(bd+ag)x^9+\frac{1}{7}aex^7+\frac{1}{8}(bc+af)x^8+\frac{1}{6}adx^6+\frac{1}{5}acx^5$$

input
```
integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxi
ma")
```

output
```
1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*(b*e + a*h)*x^10 + 1/
9*(b*d + a*g)*x^9 + 1/7*a*e*x^7 + 1/8*(b*c + a*f)*x^8 + 1/6*a*d*x^6 + 1/5*
a*c*x^5
```

3.373.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{13}\,bhx^{13}+\frac{1}{12}\,bgx^{12}+\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bex^{10}+\frac{1}{10}\,ahx^{10}+\frac{1}{9}\,bdx^9+\frac{1}{9}\,agx^9+\frac{1}{8}\,bcx^8+\frac{1}{8}\,afx^8+\frac{1}{7}\,aex^7+\frac{1}{6}\,adx^6+\frac{1}{5}\,acx^5$$

input
```
integrate(x^4*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac
")
```

output
```
1/13*b*h*x^13 + 1/12*b*g*x^12 + 1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/10*a*h*x
^10 + 1/9*b*d*x^9 + 1/9*a*g*x^9 + 1/8*b*c*x^8 + 1/8*a*f*x^8 + 1/7*a*e*x^7
+ 1/6*a*d*x^6 + 1/5*a*c*x^5
```

3.373.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int x^4(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{b\,h\,x^{13}}{13}+\frac{b\,g\,x^{12}}{12}+\frac{b\,f\,x^{11}}{11}+\left(\frac{b\,e}{10}+\frac{a\,h}{10}\right)\,x^{10}+\left(\frac{b\,d}{9}+\frac{a\,g}{9}\right)\,x^9+\left(\frac{b\,c}{8}+\frac{a\,f}{8}\right)\,x^8+\frac{a\,e\,x^7}{7}+\frac{a\,d\,x^6}{6}+\frac{a\,c\,x^5}{5}$$

input
```
int(x^4*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^8*((b*c)/8 + (a*f)/8) + x^9*((b*d)/9 + (a*g)/9) + x^10*((b*e)/10 + (a*h)
/10) + (b*h*x^13)/13 + (a*c*x^5)/5 + (a*d*x^6)/6 + (a*e*x^7)/7 + (b*f*x^11
)/11 + (b*g*x^12)/12
```

3.374 $\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

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3.374.2 Mathematica [A] (verified) . . . 2806
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3.374.5 Fricas [A] (verification not implemented) . . . 2808
3.374.6 Sympy [A] (verification not implemented) . . . 2809
3.374.7 Maxima [A] (verification not implemented) . . . 2809
3.374.8 Giac [A] (verification not implemented) . . . 2810
3.374.9 Mupad [B] (verification not implemented) . . . 2810

3.374.1 Optimal result

Integrand size = 36, antiderivative size = 97

$$\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}(bc+af)x^7+\frac{1}{8}(bd+ag)x^8 + \frac{1}{9}(be+ah)x^9+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

output
```
1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*(a*f+b*c)*x^7+1/8*(a*g+b*d)*x^8+1/
9*(a*h+b*e)*x^9+1/10*b*f*x^10+1/11*b*g*x^11+1/12*b*h*x^12
```

3.374.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}(bc+af)x^7+\frac{1}{8}(bd+ag)x^8 + \frac{1}{9}(be+ah)x^9+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

input
```
Integrate[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*
g)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^1
2)/12
```

3.374.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^3\right)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx$$

$$\Big\downarrow \text{2360}$$

$$\int \left(x^6(af+bc)+x^7(ag+bd)+x^8(ah+be)+acx^3+adx^4+aex^5+bfx^9+bgx^{10}+bhx^{11}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7(af+bc)+\frac{1}{8}x^8(ag+bd)+\frac{1}{9}x^9(ah+be)+\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{10}bfx^{10}+\frac{1}{11}bgx^{11}+\frac{1}{12}bhx^{12}$$

input
```
Int[x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + ((b*c + a*f)*x^7)/7 + ((b*d + a*
g)*x^8)/8 + ((b*e + a*h)*x^9)/9 + (b*f*x^10)/10 + (b*g*x^11)/11 + (b*h*x^1
2)/12
```

3.374.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.374.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

method	result
default	$\frac{a\,x^{4} c}{4}+\frac{a d\,x^{5}}{5}+\frac{a e\,x^{6}}{6}+\frac{\left(a f +b c \right) x^{7}}{7}+\frac{\left(a g +b d \right) x^{8}}{8}+\frac{\left(a h +b e \right) x^{9}}{9}+\frac{b f\,x^{10}}{10}+\frac{b g\,x^{11}}{11}+\frac{b h\,x^{12}}{12}$
norman	$\frac{b h\,x^{12}}{12}+\frac{b g\,x^{11}}{11}+\frac{b f\,x^{10}}{10}+\left(\frac{a h}{9}+\frac{b e}{9}\right) x^{9}+\left(\frac{a g}{8}+\frac{b d}{8}\right) x^{8}+\left(\frac{a f}{7}+\frac{b c}{7}\right) x^{7}+\frac{a e\,x^{6}}{6}+\frac{a d\,x^{5}}{5}+\frac{a\,x^{4} c}{4}$
gosper	$\frac{1}{12}b h\,x^{12}+\frac{1}{11}b g\,x^{11}+\frac{1}{10}b f\,x^{10}+\frac{1}{9}x^{9} a h+\frac{1}{9}x^{9} b e+\frac{1}{8}x^{8} a g+\frac{1}{8}b d\,x^{8}+\frac{1}{7}a f\,x^{7}+\frac{1}{7}b\,x^{7} c+\frac{1}{6}a e$
risch	$\frac{1}{12}b h\,x^{12}+\frac{1}{11}b g\,x^{11}+\frac{1}{10}b f\,x^{10}+\frac{1}{9}x^{9} a h+\frac{1}{9}x^{9} b e+\frac{1}{8}x^{8} a g+\frac{1}{8}b d\,x^{8}+\frac{1}{7}a f\,x^{7}+\frac{1}{7}b\,x^{7} c+\frac{1}{6}a e$
parallelrisch	$\frac{1}{12}b h\,x^{12}+\frac{1}{11}b g\,x^{11}+\frac{1}{10}b f\,x^{10}+\frac{1}{9}x^{9} a h+\frac{1}{9}x^{9} b e+\frac{1}{8}x^{8} a g+\frac{1}{8}b d\,x^{8}+\frac{1}{7}a f\,x^{7}+\frac{1}{7}b\,x^{7} c+\frac{1}{6}a e$

input
```
int(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4*c+1/5*a*d*x^5+1/6*a*e*x^6+1/7*(a*f+b*c)*x^7+1/8*(a*g+b*d)*x^8+1/
9*(a*h+b*e)*x^9+1/10*b*f*x^10+1/11*b*g*x^11+1/12*b*h*x^12
```

3.374.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{12}\,bhx^{12}+\frac{1}{11}\,bgx^{11}+\frac{1}{10}\,bfx^{10}+\frac{1}{9}\,(be+ah)x^9$$
$$+\frac{1}{8}\,(bd+ag)x^8+\frac{1}{6}\,aex^6+\frac{1}{7}\,(bc+af)x^7+\frac{1}{5}\,adx^5+\frac{1}{4}\,acx^4$$

input
```
integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fric
as")
```

output
```
1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*(b*e + a*h)*x^9 + 1/8*
(b*d + a*g)*x^8 + 1/6*a*e*x^6 + 1/7*(b*c + a*f)*x^7 + 1/5*a*d*x^5 + 1/4*a*
c*x^4
```

3.374.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{acx^4}{4}+\frac{adx^5}{5}+\frac{aex^6}{6}+\frac{bfx^{10}}{10}+\frac{bgx^{11}}{11}+\frac{bhx^{12}}{12}+x^9\left(\frac{ah}{9}+\frac{be}{9}\right)+x^8\left(\frac{ag}{8}+\frac{bd}{8}\right)+x^7\left(\frac{af}{7}+\frac{bc}{7}\right)$$

input
```
integrate(x**3*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + b*f*x**10/10 + b*g*x**11/11 + b*h*x
**12/12 + x**9*(a*h/9 + b*e/9) + x**8*(a*g/8 + b*d/8) + x**7*(a*f/7 + b*c/
7)
```

3.374.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{12}bhx^{12}+\frac{1}{11}bgx^{11}+\frac{1}{10}bfx^{10}+\frac{1}{9}(be+ah)x^9+\frac{1}{8}(bd+ag)x^8+\frac{1}{6}aex^6+\frac{1}{7}(bc+af)x^7+\frac{1}{5}adx^5+\frac{1}{4}acx^4$$

input
```
integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxi
ma")
```

output
```
1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*(b*e + a*h)*x^9 + 1/8*
(b*d + a*g)*x^8 + 1/6*a*e*x^6 + 1/7*(b*c + a*f)*x^7 + 1/5*a*d*x^5 + 1/4*a*
c*x^4
```

3.374.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{12}bhx^{12}+\frac{1}{11}bgx^{11}+\frac{1}{10}bfx^{10}+\frac{1}{9}bex^9+\frac{1}{9}ahx^9+\frac{1}{8}bdx^8 + \frac{1}{8}agx^8+\frac{1}{7}bcx^7+\frac{1}{7}afx^7+\frac{1}{6}aex^6+\frac{1}{5}adx^5+\frac{1}{4}acx^4$$

input
```
integrate(x^3*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac
")
```

output
```
1/12*b*h*x^12 + 1/11*b*g*x^11 + 1/10*b*f*x^10 + 1/9*b*e*x^9 + 1/9*a*h*x^9
+ 1/8*b*d*x^8 + 1/8*a*g*x^8 + 1/7*b*c*x^7 + 1/7*a*f*x^7 + 1/6*a*e*x^6 + 1/
5*a*d*x^5 + 1/4*a*c*x^4
```

3.374.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int x^3(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{b\,h\,x^{12}}{12}+\frac{b\,g\,x^{11}}{11}+\frac{b\,f\,x^{10}}{10}+\left(\frac{b\,e}{9}+\frac{a\,h}{9}\right)x^9 + \left(\frac{b\,d}{8}+\frac{a\,g}{8}\right)x^8+\left(\frac{b\,c}{7}+\frac{a\,f}{7}\right)x^7+\frac{a\,e\,x^6}{6}+\frac{a\,d\,x^5}{5}+\frac{a\,c\,x^4}{4}$$

input
```
int(x^3*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^7*((b*c)/7 + (a*f)/7) + x^8*((b*d)/8 + (a*g)/8) + x^9*((b*e)/9 + (a*h)/9
) + (b*h*x^12)/12 + (a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (b*f*x^10)/1
0 + (b*g*x^11)/11
```

3.375 $\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

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3.375.1 Optimal result

Integrand size = 36, antiderivative size = 97

$$\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}(bc+af)x^6+\frac{1}{7}(bd+ag)x^7 + \frac{1}{8}(be+ah)x^8+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

output
```
1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*(a*f+b*c)*x^6+1/7*(a*g+b*d)*x^7+1/
8*(a*h+b*e)*x^8+1/9*b*f*x^9+1/10*b*g*x^10+1/11*b*h*x^11
```

3.375.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{6}(bc+af)x^6+\frac{1}{7}(bd+ag)x^7 + \frac{1}{8}(be+ah)x^8+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

input
```
Integrate[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*
g)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)
/11
```

3.375.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx$$

$$\Big\downarrow \text{ 2360}$$

$$\int \left(x^5(af+bc)+x^6(ag+bd)+x^7(ah+be)+acx^2+adx^3+aex^4+bfx^8+bgx^9+bhx^{10}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}x^6(af+bc)+\frac{1}{7}x^7(ag+bd)+\frac{1}{8}x^8(ah+be)+\frac{1}{3}acx^3+\frac{1}{4}adx^4+\frac{1}{5}aex^5+\frac{1}{9}bfx^9+\frac{1}{10}bgx^{10}+\frac{1}{11}bhx^{11}$$

input
```
Int[x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + ((b*c + a*f)*x^6)/6 + ((b*d + a*
g)*x^7)/7 + ((b*e + a*h)*x^8)/8 + (b*f*x^9)/9 + (b*g*x^10)/10 + (b*h*x^11)
/11
```

3.375.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.375.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

method	result
default	$\frac{ac\,x^{3}}{3}+\frac{ad\,x^{4}}{4}+\frac{ae\,x^{5}}{5}+\frac{(af+bc)x^{6}}{6}+\frac{(ag+bd)x^{7}}{7}+\frac{(ah+be)x^{8}}{8}+\frac{bf\,x^{9}}{9}+\frac{bg\,x^{10}}{10}+\frac{bh\,x^{11}}{11}$
norman	$\frac{bh\,x^{11}}{11}+\frac{bg\,x^{10}}{10}+\frac{bf\,x^{9}}{9}+\left(\frac{ah}{8}+\frac{be}{8}\right)x^{8}+\left(\frac{ag}{7}+\frac{bd}{7}\right)x^{7}+\left(\frac{af}{6}+\frac{bc}{6}\right)x^{6}+\frac{ae\,x^{5}}{5}+\frac{ad\,x^{4}}{4}+\frac{ac\,x^{3}}{3}$
gosper	$\frac{1}{11}bh\,x^{11}+\frac{1}{10}bg\,x^{10}+\frac{1}{9}bf\,x^{9}+\frac{1}{8}x^{8}ah+\frac{1}{8}be\,x^{8}+\frac{1}{7}x^{7}ag+\frac{1}{7}bd\,x^{7}+\frac{1}{6}x^{6}af+\frac{1}{6}bc\,x^{6}+\frac{1}{5}ae\,x$
risch	$\frac{1}{11}bh\,x^{11}+\frac{1}{10}bg\,x^{10}+\frac{1}{9}bf\,x^{9}+\frac{1}{8}x^{8}ah+\frac{1}{8}be\,x^{8}+\frac{1}{7}x^{7}ag+\frac{1}{7}bd\,x^{7}+\frac{1}{6}x^{6}af+\frac{1}{6}bc\,x^{6}+\frac{1}{5}ae\,x$
parallelrisch	$\frac{1}{11}bh\,x^{11}+\frac{1}{10}bg\,x^{10}+\frac{1}{9}bf\,x^{9}+\frac{1}{8}x^{8}ah+\frac{1}{8}be\,x^{8}+\frac{1}{7}x^{7}ag+\frac{1}{7}bd\,x^{7}+\frac{1}{6}x^{6}af+\frac{1}{6}bc\,x^{6}+\frac{1}{5}ae\,x$

input
```
int(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*c*x^3+1/4*a*d*x^4+1/5*a*e*x^5+1/6*(a*f+b*c)*x^6+1/7*(a*g+b*d)*x^7+1/
8*(a*h+b*e)*x^8+1/9*b*f*x^9+1/10*b*g*x^10+1/11*b*h*x^11
```

3.375.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{11}\,bhx^{11}+\frac{1}{10}\,bgx^{10}+\frac{1}{9}\,bfx^{9}+\frac{1}{8}\,(be+ah)x^{8}$$
$$+\frac{1}{7}\,(bd+ag)x^{7}+\frac{1}{5}\,aex^{5}+\frac{1}{6}\,(bc+af)x^{6}+\frac{1}{4}\,adx^{4}+\frac{1}{3}\,acx^{3}$$

input
```
integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fric
as")
```

output
```
1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*(b*e + a*h)*x^8 + 1/7*(b
*d + a*g)*x^7 + 1/5*a*e*x^5 + 1/6*(b*c + a*f)*x^6 + 1/4*a*d*x^4 + 1/3*a*c*
x^3
```

3.375.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{acx^3}{3}+\frac{adx^4}{4}+\frac{aex^5}{5}+\frac{bfx^9}{9}+\frac{bgx^{10}}{10}+\frac{bhx^{11}}{11} + x^8\left(\frac{ah}{8}+\frac{be}{8}\right)+x^7\left(\frac{ag}{7}+\frac{bd}{7}\right)+x^6\left(\frac{af}{6}+\frac{bc}{6}\right)$$

input
```
integrate(x**2*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a*c*x**3/3 + a*d*x**4/4 + a*e*x**5/5 + b*f*x**9/9 + b*g*x**10/10 + b*h*x**
11/11 + x**8*(a*h/8 + b*e/8) + x**7*(a*g/7 + b*d/7) + x**6*(a*f/6 + b*c/6)
```

3.375.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{11}bhx^{11}+\frac{1}{10}bgx^{10}+\frac{1}{9}bfx^9+\frac{1}{8}(be+ah)x^8 + \frac{1}{7}(bd+ag)x^7+\frac{1}{5}aex^5+\frac{1}{6}(bc+af)x^6+\frac{1}{4}adx^4+\frac{1}{3}acx^3$$

input
```
integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxi
ma")
```

output
```
1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*(b*e + a*h)*x^8 + 1/7*(b
*d + a*g)*x^7 + 1/5*a*e*x^5 + 1/6*(b*c + a*f)*x^6 + 1/4*a*d*x^4 + 1/3*a*c*
x^3
```

3.375.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{11}bhx^{11}+\frac{1}{10}bgx^{10}+\frac{1}{9}bfx^9+\frac{1}{8}bex^8+\frac{1}{8}ahx^8+\frac{1}{7}bdx^7+\frac{1}{7}agx^7+\frac{1}{6}bcx^6+\frac{1}{6}afx^6+\frac{1}{5}aex^5+\frac{1}{4}adx^4+\frac{1}{3}acx^3$$

input
```
integrate(x^2*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac
")
```

output
```
1/11*b*h*x^11 + 1/10*b*g*x^10 + 1/9*b*f*x^9 + 1/8*b*e*x^8 + 1/8*a*h*x^8 +
1/7*b*d*x^7 + 1/7*a*g*x^7 + 1/6*b*c*x^6 + 1/6*a*f*x^6 + 1/5*a*e*x^5 + 1/4*
a*d*x^4 + 1/3*a*c*x^3
```

3.375.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int x^2(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{b\,h\,x^{11}}{11}+\frac{b\,g\,x^{10}}{10}+\frac{b\,f\,x^9}{9}+\left(\frac{b\,e}{8}+\frac{a\,h}{8}\right)x^8+\left(\frac{b\,d}{7}+\frac{a\,g}{7}\right)x^7+\left(\frac{b\,c}{6}+\frac{a\,f}{6}\right)x^6+\frac{a\,e\,x^5}{5}+\frac{a\,d\,x^4}{4}+\frac{a\,c\,x^3}{3}$$

input
```
int(x^2*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^6*((b*c)/6 + (a*f)/6) + x^7*((b*d)/7 + (a*g)/7) + x^8*((b*e)/8 + (a*h)/8
) + (b*h*x^11)/11 + (a*c*x^3)/3 + (a*d*x^4)/4 + (a*e*x^5)/5 + (b*f*x^9)/9
+ (b*g*x^10)/10
```

3.376 $\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$

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3.376.1 Optimal result

Integrand size = 34, antiderivative size = 97

$$\begin{aligned}&\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\&=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}(bc+af)x^5+\frac{1}{6}(bd+ag)x^6\\&\quad+\frac{1}{7}(be+ah)x^7+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}\end{aligned}$$

output
```
1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*(a*f+b*c)*x^5+1/6*(a*g+b*d)*x^6+1/
7*(a*h+b*e)*x^7+1/8*b*f*x^8+1/9*b*g*x^9+1/10*b*h*x^10
```

3.376.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\&=\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{5}(bc+af)x^5+\frac{1}{6}(bd+ag)x^6\\&\quad+\frac{1}{7}(be+ah)x^7+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}\end{aligned}$$

input
```
Integrate[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*
g)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/1
0
```

3.376.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$

$$\Big\downarrow \text{2360}$$

$$\int \left(x^4(af+bc)+x^5(ag+bd)+x^6(ah+be)+acx+adx^2+aex^3+bfx^7+bgx^8+bhx^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(af+bc)+\frac{1}{6}x^6(ag+bd)+\frac{1}{7}x^7(ah+be)+\frac{1}{2}acx^2+\frac{1}{3}adx^3+\frac{1}{4}aex^4+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9+\frac{1}{10}bhx^{10}$$

input
```
Int[x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + ((b*c + a*f)*x^5)/5 + ((b*d + a*
g)*x^6)/6 + ((b*e + a*h)*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9 + (b*h*x^10)/1
0
```

3.376.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.376.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

method	result
default	$\frac{ac\,x^{2}}{2}+\frac{ad\,x^{3}}{3}+\frac{ae\,x^{4}}{4}+\frac{(af+bc)x^{5}}{5}+\frac{(ag+bd)x^{6}}{6}+\frac{(ah+be)x^{7}}{7}+\frac{bf\,x^{8}}{8}+\frac{bg\,x^{9}}{9}+\frac{bh\,x^{10}}{10}$
norman	$\frac{bh\,x^{10}}{10}+\frac{bg\,x^{9}}{9}+\frac{bf\,x^{8}}{8}+\left(\frac{ah}{7}+\frac{be}{7}\right)x^{7}+\left(\frac{ag}{6}+\frac{bd}{6}\right)x^{6}+\left(\frac{af}{5}+\frac{bc}{5}\right)x^{5}+\frac{ae\,x^{4}}{4}+\frac{ad\,x^{3}}{3}+\frac{ac\,x^{2}}{2}$
gosper	$\frac{1}{10}bh\,x^{10}+\frac{1}{9}bg\,x^{9}+\frac{1}{8}bf\,x^{8}+\frac{1}{7}x^{7}ah+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}ag+\frac{1}{6}bd\,x^{6}+\frac{1}{5}x^{5}af+\frac{1}{5}bc\,x^{5}+\frac{1}{4}ae\,x^{4}$
risch	$\frac{1}{10}bh\,x^{10}+\frac{1}{9}bg\,x^{9}+\frac{1}{8}bf\,x^{8}+\frac{1}{7}x^{7}ah+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}ag+\frac{1}{6}bd\,x^{6}+\frac{1}{5}x^{5}af+\frac{1}{5}bc\,x^{5}+\frac{1}{4}ae\,x^{4}$
parallelrisch	$\frac{1}{10}bh\,x^{10}+\frac{1}{9}bg\,x^{9}+\frac{1}{8}bf\,x^{8}+\frac{1}{7}x^{7}ah+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}ag+\frac{1}{6}bd\,x^{6}+\frac{1}{5}x^{5}af+\frac{1}{5}bc\,x^{5}+\frac{1}{4}ae\,x^{4}$

input
```
int(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*c*x^2+1/3*a*d*x^3+1/4*a*e*x^4+1/5*(a*f+b*c)*x^5+1/6*(a*g+b*d)*x^6+1/
7*(a*h+b*e)*x^7+1/8*b*f*x^8+1/9*b*g*x^9+1/10*b*h*x^10
```

3.376.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\&=\frac{1}{10}\,bhx^{10}+\frac{1}{9}\,bgx^9+\frac{1}{8}\,bfx^8+\frac{1}{7}\,(be+ah)x^7+\frac{1}{6}\,(bd+ag)x^6\\&\quad+\frac{1}{4}\,aex^4+\frac{1}{5}\,(bc+af)x^5+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2\end{aligned}$$

input
```
integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas
")
```

output
```
1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*(b*e + a*h)*x^7 + 1/6*(b*d
 + a*g)*x^6 + 1/4*a*e*x^4 + 1/5*(b*c + a*f)*x^5 + 1/3*a*d*x^3 + 1/2*a*c*x^
2
```

3.376.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{acx^2}{2}+\frac{adx^3}{3}+\frac{aex^4}{4}+\frac{bfx^8}{8}+\frac{bgx^9}{9}+\frac{bhx^{10}}{10} + x^7\left(\frac{ah}{7}+\frac{be}{7}\right)+x^6\left(\frac{ag}{6}+\frac{bd}{6}\right)+x^5\left(\frac{af}{5}+\frac{bc}{5}\right)$$

input
```
integrate(x*(b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a*c*x**2/2 + a*d*x**3/3 + a*e*x**4/4 + b*f*x**8/8 + b*g*x**9/9 + b*h*x**10
/10 + x**7*(a*h/7 + b*e/7) + x**6*(a*g/6 + b*d/6) + x**5*(a*f/5 + b*c/5)
```

3.376.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{10}\,bhx^{10}+\frac{1}{9}\,bgx^9+\frac{1}{8}\,bfx^8+\frac{1}{7}\,(be+ah)x^7+\frac{1}{6}\,(bd+ag)x^6 + \frac{1}{4}\,aex^4+\frac{1}{5}\,(bc+af)x^5+\frac{1}{3}\,adx^3+\frac{1}{2}\,acx^2$$

input
```
integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima
")
```

output
```
1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*(b*e + a*h)*x^7 + 1/6*(b*d
 + a*g)*x^6 + 1/4*a*e*x^4 + 1/5*(b*c + a*f)*x^5 + 1/3*a*d*x^3 + 1/2*a*c*x^
2
```

3.376.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{10}bhx^{10}+\frac{1}{9}bgx^9+\frac{1}{8}bfx^8+\frac{1}{7}bex^7+\frac{1}{7}ahx^7+\frac{1}{6}bdx^6 + \frac{1}{6}agx^6+\frac{1}{5}bcx^5+\frac{1}{5}afx^5+\frac{1}{4}aex^4+\frac{1}{3}adx^3+\frac{1}{2}acx^2$$

input
```
integrate(x*(b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

output
```
1/10*b*h*x^10 + 1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/7*a*h*x^7 + 1/
6*b*d*x^6 + 1/6*a*g*x^6 + 1/5*b*c*x^5 + 1/5*a*f*x^5 + 1/4*a*e*x^4 + 1/3*a*
d*x^3 + 1/2*a*c*x^2
```

3.376.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int x(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{b\,h\,x^{10}}{10}+\frac{b\,g\,x^9}{9}+\frac{b\,f\,x^8}{8}+\left(\frac{b\,e}{7}+\frac{a\,h}{7}\right)x^7+\left(\frac{b\,d}{6}+\frac{a\,g}{6}\right)x^6 + \left(\frac{b\,c}{5}+\frac{a\,f}{5}\right)x^5+\frac{a\,e\,x^4}{4}+\frac{a\,d\,x^3}{3}+\frac{a\,c\,x^2}{2}$$

input
```
int(x*(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^5*((b*c)/5 + (a*f)/5) + x^6*((b*d)/6 + (a*g)/6) + x^7*((b*e)/7 + (a*h)/7
) + (b*h*x^10)/10 + (a*c*x^2)/2 + (a*d*x^3)/3 + (a*e*x^4)/4 + (b*f*x^8)/8
+ (b*g*x^9)/9
```

3.377 $\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

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3.377.1 Optimal result

Integrand size = 33, antiderivative size = 92

$$\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}(bc+af)x^4+\frac{1}{5}(bd+ag)x^5 + \frac{1}{6}(be+ah)x^6+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

output
```
a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*(a*f+b*c)*x^4+1/5*(a*g+b*d)*x^5+1/6*(a*h
+b*e)*x^6+1/7*b*f*x^7+1/8*b*g*x^8+1/9*b*h*x^9
```

3.377.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}(bc+af)x^4+\frac{1}{5}(bd+ag)x^5 + \frac{1}{6}(be+ah)x^6+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

input
```
Integrate[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5
)/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9
```

3.377.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right) \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(x^3(af+bc)+x^4(ag+bd)+x^5(ah+be)+ac+adx+aex^2+bfx^6+bgx^7+bhx^8\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(af+bc)+\frac{1}{5}x^5(ag+bd)+\frac{1}{6}x^6(ah+be)+acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{7}bfx^7+\frac{1}{8}bgx^8+\frac{1}{9}bhx^9$$

input
```
Int[(a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + ((b*c + a*f)*x^4)/4 + ((b*d + a*g)*x^5
)/5 + ((b*e + a*h)*x^6)/6 + (b*f*x^7)/7 + (b*g*x^8)/8 + (b*h*x^9)/9
```

3.377.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.377.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result
default	$acx+\frac{ad\,x^{2}}{2}+\frac{ae\,x^{3}}{3}+\frac{(af+bc)x^{4}}{4}+\frac{(ag+bd)x^{5}}{5}+\frac{(ah+be)x^{6}}{6}+\frac{bf\,x^{7}}{7}+\frac{bg\,x^{8}}{8}+\frac{bh\,x^{9}}{9}$
norman	$\frac{bh\,x^{9}}{9}+\frac{bg\,x^{8}}{8}+\frac{bf\,x^{7}}{7}+\left(\frac{ah}{6}+\frac{be}{6}\right)x^{6}+\left(\frac{ag}{5}+\frac{bd}{5}\right)x^{5}+\left(\frac{af}{4}+\frac{bc}{4}\right)x^{4}+\frac{ae\,x^{3}}{3}+\frac{ad\,x^{2}}{2}+acx$
gosper	$\frac{1}{9}bh\,x^{9}+\frac{1}{8}bg\,x^{8}+\frac{1}{7}bf\,x^{7}+\frac{1}{6}x^{6}ah+\frac{1}{6}be\,x^{6}+\frac{1}{5}x^{5}ag+\frac{1}{5}bd\,x^{5}+\frac{1}{4}af\,x^{4}+\frac{1}{4}bc\,x^{4}+\frac{1}{3}ae\,x^{3}+$
risch	$\frac{1}{9}bh\,x^{9}+\frac{1}{8}bg\,x^{8}+\frac{1}{7}bf\,x^{7}+\frac{1}{6}x^{6}ah+\frac{1}{6}be\,x^{6}+\frac{1}{5}x^{5}ag+\frac{1}{5}bd\,x^{5}+\frac{1}{4}af\,x^{4}+\frac{1}{4}bc\,x^{4}+\frac{1}{3}ae\,x^{3}+$
parallelrisch	$\frac{1}{9}bh\,x^{9}+\frac{1}{8}bg\,x^{8}+\frac{1}{7}bf\,x^{7}+\frac{1}{6}x^{6}ah+\frac{1}{6}be\,x^{6}+\frac{1}{5}x^{5}ag+\frac{1}{5}bd\,x^{5}+\frac{1}{4}af\,x^{4}+\frac{1}{4}bc\,x^{4}+\frac{1}{3}ae\,x^{3}+$

input
```
int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*(a*f+b*c)*x^4+1/5*(a*g+b*d)*x^5+1/6*(a*h
+b*e)*x^6+1/7*b*f*x^7+1/8*b*g*x^8+1/9*b*h*x^9
```

3.377.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int (a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{9}\,bhx^9+\frac{1}{8}\,bgx^8+\frac{1}{7}\,bfx^7+\frac{1}{6}\,(be+ah)x^6+\frac{1}{5}\,(bd+ag)x^5+\frac{1}{3}\,aex^3+\frac{1}{4}\,(bc+af)x^4+\frac{1}{2}\,adx^2+acx$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

output
```
1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*(b*e + a*h)*x^6 + 1/5*(b*d +
 a*g)*x^5 + 1/3*a*e*x^3 + 1/4*(b*c + a*f)*x^4 + 1/2*a*d*x^2 + a*c*x
```

3.377.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{bfx^7}{7}+\frac{bgx^8}{8}+\frac{bhx^9}{9}+x^6\left(\frac{ah}{6}+\frac{be}{6}\right)+x^5\left(\frac{ag}{5}+\frac{bd}{5}\right)+x^4\left(\frac{af}{4}+\frac{bc}{4}\right)$$

input
```
integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a*c*x + a*d*x**2/2 + a*e*x**3/3 + b*f*x**7/7 + b*g*x**8/8 + b*h*x**9/9 + x
**6*(a*h/6 + b*e/6) + x**5*(a*g/5 + b*d/5) + x**4*(a*f/4 + b*c/4)
```

3.377.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{9}bhx^9+\frac{1}{8}bgx^8+\frac{1}{7}bfx^7+\frac{1}{6}(be+ah)x^6 + \frac{1}{5}(bd+ag)x^5+\frac{1}{3}aex^3+\frac{1}{4}(bc+af)x^4+\frac{1}{2}adx^2+acx$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
```

output
```
1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*(b*e + a*h)*x^6 + 1/5*(b*d +
 a*g)*x^5 + 1/3*a*e*x^3 + 1/4*(b*c + a*f)*x^4 + 1/2*a*d*x^2 + a*c*x
```

3.377.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int (a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{9}bhx^9+\frac{1}{8}bgx^8+\frac{1}{7}bfx^7+\frac{1}{6}bex^6+\frac{1}{6}ahx^6+\frac{1}{5}bdx^5 + \frac{1}{5}agx^5+\frac{1}{4}bcx^4+\frac{1}{4}afx^4+\frac{1}{3}aex^3+\frac{1}{2}adx^2+acx$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

output
```
1/9*b*h*x^9 + 1/8*b*g*x^8 + 1/7*b*f*x^7 + 1/6*b*e*x^6 + 1/6*a*h*x^6 + 1/5*
b*d*x^5 + 1/5*a*g*x^5 + 1/4*b*c*x^4 + 1/4*a*f*x^4 + 1/3*a*e*x^3 + 1/2*a*d*
x^2 + a*c*x
```

3.377.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$\begin{aligned}\int (a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\ &=\frac{b\,h\,x^9}{9}+\frac{b\,g\,x^8}{8}+\frac{b\,f\,x^7}{7}+\left(\frac{b\,e}{6}+\frac{a\,h}{6}\right)\,x^6+\left(\frac{b\,d}{5}+\frac{a\,g}{5}\right)\,x^5\\ &\quad+\left(\frac{b\,c}{4}+\frac{a\,f}{4}\right)\,x^4+\frac{a\,e\,x^3}{3}+\frac{a\,d\,x^2}{2}+a\,c\,x\end{aligned}$$

input
```
int((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^4*((b*c)/4 + (a*f)/4) + x^5*((b*d)/5 + (a*g)/5) + x^6*((b*e)/6 + (a*h)/6
) + (b*h*x^9)/9 + a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*f*x^7)/7 + (b*g*x
^8)/8
```

3.378 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

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3.378.1 Optimal result

Integrand size = 36, antiderivative size = 88

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = adx+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+\frac{1}{4}(bd+ag)x^4+\frac{1}{5}(be+ah)x^5+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8+ac\log(x)$$

output
```
a*d*x+1/2*a*e*x^2+1/3*(a*f+b*c)*x^3+1/4*(a*g+b*d)*x^4+1/5*(a*h+b*e)*x^5+1/
6*b*f*x^6+1/7*b*g*x^7+1/8*b*h*x^8+a*c*ln(x)
```

3.378.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = adx+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+\frac{1}{4}(bd+ag)x^4+\frac{1}{5}(be+ah)x^5+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8+ac\log(x)$$

input
```
Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

output
```
a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e +
a*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]
```

3.378.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}dx$$

$$\Big\downarrow \text{2360}$$

$$\int \left(x^2(af+bc)+x^3(ag+bd)+x^4(ah+be)+\frac{ac}{x}+ad+aex+bfx^5+bgx^6+bhx^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(af+bc)+\frac{1}{4}x^4(ag+bd)+\frac{1}{5}x^5(ah+be)+ac\log(x)+adx+\frac{1}{2}aex^2+\frac{1}{6}bfx^6+\frac{1}{7}bgx^7+\frac{1}{8}bhx^8$$

input
```
Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

output
```
a*d*x + (a*e*x^2)/2 + ((b*c + a*f)*x^3)/3 + ((b*d + a*g)*x^4)/4 + ((b*e +
a*h)*x^5)/5 + (b*f*x^6)/6 + (b*g*x^7)/7 + (b*h*x^8)/8 + a*c*Log[x]
```

3.378.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.378.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

method	result
norman	$\left(\frac{af}{3}+\frac{bc}{3}\right)x^{3}+\left(\frac{ag}{4}+\frac{bd}{4}\right)x^{4}+\left(\frac{ah}{5}+\frac{be}{5}\right)x^{5}+adx+\frac{ae\,x^{2}}{2}+\frac{bf\,x^{6}}{6}+\frac{bg\,x^{7}}{7}+\frac{bh\,x^{8}}{8}+ac\ln\left(x\right)$
default	$\frac{bh\,x^{8}}{8}+\frac{bg\,x^{7}}{7}+\frac{bf\,x^{6}}{6}+\frac{ah\,x^{5}}{5}+\frac{be\,x^{5}}{5}+\frac{ag\,x^{4}}{4}+\frac{bd\,x^{4}}{4}+\frac{af\,x^{3}}{3}+\frac{bc\,x^{3}}{3}+\frac{ae\,x^{2}}{2}+adx+ac\ln\left(x\right)$
risch	$\frac{bh\,x^{8}}{8}+\frac{bg\,x^{7}}{7}+\frac{bf\,x^{6}}{6}+\frac{ah\,x^{5}}{5}+\frac{be\,x^{5}}{5}+\frac{ag\,x^{4}}{4}+\frac{bd\,x^{4}}{4}+\frac{af\,x^{3}}{3}+\frac{bc\,x^{3}}{3}+\frac{ae\,x^{2}}{2}+adx+ac\ln\left(x\right)$
parallelrisch	$\frac{bh\,x^{8}}{8}+\frac{bg\,x^{7}}{7}+\frac{bf\,x^{6}}{6}+\frac{ah\,x^{5}}{5}+\frac{be\,x^{5}}{5}+\frac{ag\,x^{4}}{4}+\frac{bd\,x^{4}}{4}+\frac{af\,x^{3}}{3}+\frac{bc\,x^{3}}{3}+\frac{ae\,x^{2}}{2}+adx+ac\ln\left(x\right)$

input
```
int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/3*a*f+1/3*b*c)*x^3+(1/4*a*g+1/4*b*d)*x^4+(1/5*a*h+1/5*b*e)*x^5+a*d*x+1/
2*a*e*x^2+1/6*b*f*x^6+1/7*b*g*x^7+1/8*b*h*x^8+a*c*ln(x)
```

3.378.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\begin{aligned}\int\frac{(a+bx^3)\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx &= \frac{1}{8}\,bhx^8+\frac{1}{7}\,bgx^7+\frac{1}{6}\,bfx^6+\frac{1}{5}\,(be+ah)x^5+\frac{1}{4}\,(bd+ag)x^4\\&\quad+\frac{1}{2}\,aex^2+\frac{1}{3}\,(bc+af)x^3+adx+ac\log\left(x\right)\end{aligned}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas
")
```

output
```
1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(b*e + a*h)*x^5 + 1/4*(b*d +
 a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*c + a*f)*x^3 + a*d*x + a*c*log(x)
```

3.378.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$$
$$= ac\log(x)+adx+\frac{aex^2}{2}+\frac{bfx^6}{6}+\frac{bgx^7}{7}+\frac{bhx^8}{8}+x^5\left(\frac{ah}{5}+\frac{be}{5}\right)+x^4\left(\frac{ag}{4}+\frac{bd}{4}\right)+x^3\left(\frac{af}{3}+\frac{bc}{3}\right)$$

input
```
integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

output
```
a*c*log(x) + a*d*x + a*e*x**2/2 + b*f*x**6/6 + b*g*x**7/7 + b*h*x**8/8 + x
**5*(a*h/5 + b*e/5) + x**4*(a*g/4 + b*d/4) + x**3*(a*f/3 + b*c/3)
```

3.378.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$$
$$= \frac{1}{8}bhx^8+\frac{1}{7}bgx^7+\frac{1}{6}bfx^6+\frac{1}{5}(be+ah)x^5+\frac{1}{4}(bd+ag)x^4+\frac{1}{2}aex^2+\frac{1}{3}(bc+af)x^3+adx+ac\log(x)$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima
")
```

output
```
1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*(b*e + a*h)*x^5 + 1/4*(b*d +
 a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*c + a*f)*x^3 + a*d*x + a*c*log(x)
```

3.378.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{1}{8}bhx^8+\frac{1}{7}bgx^7+\frac{1}{6}bfx^6+\frac{1}{5}bex^5+\frac{1}{5}ahx^5+\frac{1}{4}bdx^4 + \frac{1}{4}agx^4+\frac{1}{3}bcx^3+\frac{1}{3}afx^3+\frac{1}{2}aex^2+adx+ac\log(|x|)$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")
```

output
```
1/8*b*h*x^8 + 1/7*b*g*x^7 + 1/6*b*f*x^6 + 1/5*b*e*x^5 + 1/5*a*h*x^5 + 1/4*
b*d*x^4 + 1/4*a*g*x^4 + 1/3*b*c*x^3 + 1/3*a*f*x^3 + 1/2*a*e*x^2 + a*d*x +
a*c*log(abs(x))
```

3.378.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = x^3\left(\frac{b\,c}{3}+\frac{a\,f}{3}\right)+x^4\left(\frac{b\,d}{4}+\frac{a\,g}{4}\right)+x^5\left(\frac{b\,e}{5}+\frac{a\,h}{5}\right) + \frac{b\,h\,x^8}{8}+a\,c\,\ln(x)+a\,d\,x+\frac{a\,e\,x^2}{2}+\frac{b\,f\,x^6}{6}+\frac{b\,g\,x^7}{7}$$

input
```
int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x)
```

output
```
x^3*((b*c)/3 + (a*f)/3) + x^4*((b*d)/4 + (a*g)/4) + x^5*((b*e)/5 + (a*h)/5
) + (b*h*x^8)/8 + a*c*log(x) + a*d*x + (a*e*x^2)/2 + (b*f*x^6)/6 + (b*g*x^
7)/7
```

3.379 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

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3.379.2 Mathematica [A] (verified) 2831
3.379.3 Rubi [A] (verified) 2832
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3.379.5 Fricas [A] (verification not implemented) 2833
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3.379.8 Giac [A] (verification not implemented) 2835
3.379.9 Mupad [B] (verification not implemented) 2835

3.379.1 Optimal result

Integrand size = 36, antiderivative size = 86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = -\frac{ac}{x}+aex+\frac{1}{2}(bc+af)x^2+\frac{1}{3}(bd+ag)x^3 + \frac{1}{4}(be+ah)x^4+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7+ad\log(x)$$

output
```
-a*c/x+a*e*x+1/2*(a*f+b*c)*x^2+1/3*(a*g+b*d)*x^3+1/4*(a*h+b*e)*x^4+1/5*b*f
*x^5+1/6*b*g*x^6+1/7*b*h*x^7+a*d*ln(x)
```

3.379.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = -\frac{ac}{x}+aex+\frac{1}{2}(bc+af)x^2+\frac{1}{3}(bd+ag)x^3 + \frac{1}{4}(be+ah)x^4+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7+ad\log(x)$$

input
```
Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

output
```
-((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 + ((b*e + a
*h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a*d*Log[x]
```

3.379.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$

$$\Big\downarrow \text{2360}$$

$$\int \left(x(af+bc)+x^2(ag+bd)+x^3(ah+be)+\frac{ac}{x^2}+\frac{ad}{x}+ae+bfx^4+bgx^5+bhx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(af+bc)+\frac{1}{3}x^3(ag+bd)+\frac{1}{4}x^4(ah+be)-\frac{ac}{x}+ad\log(x)+aex+\frac{1}{5}bfx^5+\frac{1}{6}bgx^6+\frac{1}{7}bhx^7$$

input
```
Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

output
```
-((a*c)/x) + a*e*x + ((b*c + a*f)*x^2)/2 + ((b*d + a*g)*x^3)/3 + ((b*e + a
*h)*x^4)/4 + (b*f*x^5)/5 + (b*g*x^6)/6 + (b*h*x^7)/7 + a*d*Log[x]
```

3.379.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.379.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

method	result
default	$\frac{bh\,x^{7}}{7}+\frac{bg\,x^{6}}{6}+\frac{bf\,x^{5}}{5}+\frac{ah\,x^{4}}{4}+\frac{be\,x^{4}}{4}+\frac{ag\,x^{3}}{3}+\frac{bd\,x^{3}}{3}+\frac{x^{2}af}{2}+\frac{cb\,x^{2}}{2}+aex+ad\ln\left(x\right)-\frac{ac}{x}$
risch	$\frac{bh\,x^{7}}{7}+\frac{bg\,x^{6}}{6}+\frac{bf\,x^{5}}{5}+\frac{ah\,x^{4}}{4}+\frac{be\,x^{4}}{4}+\frac{ag\,x^{3}}{3}+\frac{bd\,x^{3}}{3}+\frac{x^{2}af}{2}+\frac{cb\,x^{2}}{2}+aex+ad\ln\left(x\right)-\frac{ac}{x}$
norman	$\frac{\left(\frac{af}{2}+\frac{bc}{2}\right)x^{3}+\left(\frac{ag}{3}+\frac{bd}{3}\right)x^{4}+\left(\frac{ah}{4}+\frac{be}{4}\right)x^{5}+ae\,x^{2}-ac+\frac{bf\,x^{6}}{5}+\frac{bg\,x^{7}}{6}+\frac{bh\,x^{8}}{7}}{x}+ad\ln\left(x\right)$
parallelrisch	$\frac{60bh\,x^{8}+70bg\,x^{7}+84bf\,x^{6}+105ah\,x^{5}+105be\,x^{5}+140ag\,x^{4}+140bd\,x^{4}+210af\,x^{3}+210bc\,x^{3}+420ad\ln(x)x+420ae\,x^{2}-420ac}{420x}$

input
```
int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*b*h*x^7+1/6*b*g*x^6+1/5*b*f*x^5+1/4*a*h*x^4+1/4*b*e*x^4+1/3*a*g*x^3+1/
3*b*d*x^3+1/2*x^2*a*f+1/2*c*b*x^2+a*e*x+a*d*ln(x)-a*c/x
```

3.379.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^2}\,dx$$
$$= \frac{60\,bhx^8+70\,bgx^7+84\,bfx^6+105\,(be+ah)x^5+140\,(bd+ag)x^4+420\,aex^2+210\,(bc+af)x^3+420\,ad}{420\,x}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fric
as")
```

output
```
1/420*(60*b*h*x^8 + 70*b*g*x^7 + 84*b*f*x^6 + 105*(b*e + a*h)*x^5 + 140*(b
*d + a*g)*x^4 + 420*a*e*x^2 + 210*(b*c + a*f)*x^3 + 420*a*d*x*log(x) - 420
*a*c)/x
```

3.379.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = -\frac{ac}{x}+ad\log{(x)}+aex+\frac{bfx^5}{5}+\frac{bgx^6}{6}+\frac{bhx^7}{7}+x^4\left(\frac{ah}{4}+\frac{be}{4}\right)+x^3\left(\frac{ag}{3}+\frac{bd}{3}\right)+x^2\left(\frac{af}{2}+\frac{bc}{2}\right)$$

input
```
integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

output
```
-a*c/x + a*d*log(x) + a*e*x + b*f*x**5/5 + b*g*x**6/6 + b*h*x**7/7 + x**4*
(a*h/4 + b*e/4) + x**3*(a*g/3 + b*d/3) + x**2*(a*f/2 + b*c/2)
```

3.379.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx = \frac{1}{7}\,bhx^7+\frac{1}{6}\,bgx^6+\frac{1}{5}\,bfx^5+\frac{1}{4}\,(be+ah)x^4+\frac{1}{3}\,(bd+ag)x^3+aex+\frac{1}{2}\,(bc+af)x^2+ad\log{(x)}-\frac{ac}{x}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxi
ma")
```

output
```
1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*(b*e + a*h)*x^4 + 1/3*(b*d +
 a*g)*x^3 + a*e*x + 1/2*(b*c + a*f)*x^2 + a*d*log(x) - a*c/x
```

3.379.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= \frac{1}{7}bhx^7 + \frac{1}{6}bgx^6 + \frac{1}{5}bfx^5 + \frac{1}{4}bex^4 + \frac{1}{4}ahx^4 + \frac{1}{3}bdx^3 + \frac{1}{3}agx^3 + \frac{1}{2}bcx^2 + \frac{1}{2}afx^2 + aex + ad\log(|x|) - \frac{ac}{x}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac
")
```

output
```
1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/5*b*f*x^5 + 1/4*b*e*x^4 + 1/4*a*h*x^4 + 1/3*
b*d*x^3 + 1/3*a*g*x^3 + 1/2*b*c*x^2 + 1/2*a*f*x^2 + a*e*x + a*d*log(abs(x)
) - a*c/x
```

3.379.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= x^2\left(\frac{b\,c}{2}+\frac{a\,f}{2}\right) + x^3\left(\frac{b\,d}{3}+\frac{a\,g}{3}\right) + x^4\left(\frac{b\,e}{4}+\frac{a\,h}{4}\right) + \frac{b\,h\,x^7}{7} + a\,d\,\ln(x) + a\,e\,x - \frac{a\,c}{x} + \frac{b\,f\,x^5}{5} + \frac{b\,g\,x^6}{6}$$

input
```
int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x)
```

output
```
x^2*((b*c)/2 + (a*f)/2) + x^3*((b*d)/3 + (a*g)/3) + x^4*((b*e)/4 + (a*h)/4
) + (b*h*x^7)/7 + a*d*log(x) + a*e*x - (a*c)/x + (b*f*x^5)/5 + (b*g*x^6)/6
```

3.380 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

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3.380.2 Mathematica [A] (verified) 2836
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3.380.1 Optimal result

Integrand size = 36, antiderivative size = 86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = -\frac{ac}{2x^2}-\frac{ad}{x}+(bc+af)x+\frac{1}{2}(bd+ag)x^2+\frac{1}{3}(be+ah)x^3+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6+ae\log(x)$$

output
```
-1/2*a*c/x^2-a*d/x+(a*f+b*c)*x+1/2*(a*g+b*d)*x^2+1/3*(a*h+b*e)*x^3+1/4*b*f
*x^4+1/5*b*g*x^5+1/6*b*h*x^6+a*e*ln(x)
```

3.380.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = bcx+\frac{a(-3c-6dx+6fx^3+3gx^4+2hx^5)}{6x^2} + \frac{1}{60}bx^2\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)+ae\log(x)$$

input
```
Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

output
```
b*c*x + (a*(-3*c - 6*d*x + 6*f*x^3 + 3*g*x^4 + 2*h*x^5))/(6*x^2) + (b*x^2*
(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3)))/60 + a*e*Log[x]
```

3.380.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$

$$\Big\downarrow \text{2360}$$

$$\int \left(bc\left(\frac{af}{bc}+1\right)+x(ag+bd)+x^2(ah+be)+\frac{ac}{x^3}+\frac{ad}{x^2}+\frac{ae}{x}+bfx^3+bgx^4+bhx^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x(af+bc)+\frac{1}{2}x^2(ag+bd)+\frac{1}{3}x^3(ah+be)-\frac{ac}{2x^2}-\frac{ad}{x}+ae\log(x)+\frac{1}{4}bfx^4+\frac{1}{5}bgx^5+\frac{1}{6}bhx^6$$

input
```
Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

output
```
-1/2*(a*c)/x^2 - (a*d)/x + (b*c + a*f)*x + ((b*d + a*g)*x^2)/2 + ((b*e + a
*h)*x^3)/3 + (b*f*x^4)/4 + (b*g*x^5)/5 + (b*h*x^6)/6 + a*e*Log[x]
```

3.380.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.380.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{bh\,x^{6}}{6}+\frac{bg\,x^{5}}{5}+\frac{bf\,x^{4}}{4}+\frac{ah\,x^{3}}{3}+\frac{be\,x^{3}}{3}+\frac{ag\,x^{2}}{2}+\frac{bd\,x^{2}}{2}+afx+bcx+ae\ln\left(x\right)-\frac{ad}{x}-\frac{ac}{2x^{2}}$	78
risch	$\frac{bh\,x^{6}}{6}+\frac{bg\,x^{5}}{5}+\frac{bf\,x^{4}}{4}+\frac{ah\,x^{3}}{3}+\frac{be\,x^{3}}{3}+\frac{ag\,x^{2}}{2}+\frac{bd\,x^{2}}{2}+afx+bcx+\frac{-adx-\frac{1}{2}ac}{x^{2}}+ae\ln\left(x\right)$	78
norman	$\frac{\left(\frac{ag}{2}+\frac{bd}{2}\right)x^{4}+\left(\frac{ah}{3}+\frac{be}{3}\right)x^{5}+(af+bc)x^{3}-\frac{ac}{2}-adx+\frac{bf\,x^{6}}{4}+\frac{bg\,x^{7}}{5}+\frac{bh\,x^{8}}{6}}{x^{2}}+ae\ln\left(x\right)$	79
parallelrisch	$\frac{10bh\,x^{8}+12bg\,x^{7}+15bf\,x^{6}+20ah\,x^{5}+20be\,x^{5}+30ag\,x^{4}+30bd\,x^{4}+60ae\ln(x)x^{2}+60af\,x^{3}+60bc\,x^{3}-60adx-30ac}{60x^{2}}$	88

input
```
int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*b*h*x^6+1/5*b*g*x^5+1/4*b*f*x^4+1/3*a*h*x^3+1/3*b*e*x^3+1/2*a*g*x^2+1/
2*b*d*x^2+a*f*x+b*c*x+a*e*ln(x)-a*d/x-1/2*a*c/x^2
```

3.380.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$=\frac{10\,bhx^8+12\,bgx^7+15\,bfx^6+20\,(be+ah)x^5+30\,(bd+ag)x^4+60\,aex^2\log\left(x\right)+60\,(bc+af)x^3-60\,ac}{60\,x^2}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fric
as")
```

output
```
1/60*(10*b*h*x^8 + 12*b*g*x^7 + 15*b*f*x^6 + 20*(b*e + a*h)*x^5 + 30*(b*d
+ a*g)*x^4 + 60*a*e*x^2*log(x) + 60*(b*c + a*f)*x^3 - 60*a*d*x - 30*a*c)/x
^2
```

3.380.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = ae\log(x)+\frac{bfx^4}{4}+\frac{bgx^5}{5}+\frac{bhx^6}{6}+x^3\left(\frac{ah}{3}+\frac{be}{3}\right)+x^2\left(\frac{ag}{2}+\frac{bd}{2}\right)+x(af+bc)+\frac{-ac-2adx}{2x^2}$$

input
```
integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

output
```
a*e*log(x) + b*f*x**4/4 + b*g*x**5/5 + b*h*x**6/6 + x**3*(a*h/3 + b*e/3) +
 x**2*(a*g/2 + b*d/2) + x*(a*f + b*c) + (-a*c - 2*a*d*x)/(2*x**2)
```

3.380.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = \frac{1}{6}\,bhx^6+\frac{1}{5}\,bgx^5+\frac{1}{4}\,bfx^4+\frac{1}{3}\,(be+ah)x^3+\frac{1}{2}\,(bd+ag)x^2+ae\log(x)+(bc+af)x-\frac{2\,adx+ac}{2\,x^2}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxi
ma")
```

output
```
1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*(b*e + a*h)*x^3 + 1/2*(b*d +
 a*g)*x^2 + a*e*log(x) + (b*c + a*f)*x - 1/2*(2*a*d*x + a*c)/x^2
```

3.380.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = \frac{1}{6}\,bhx^6+\frac{1}{5}\,bgx^5+\frac{1}{4}\,bfx^4+\frac{1}{3}\,bex^3+\frac{1}{3}\,ahx^3+\frac{1}{2}\,bdx^2 + \frac{1}{2}\,agx^2+bcx+afx+ae\log(|x|)-\frac{2\,adx+ac}{2\,x^2}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac
")
```

output
```
1/6*b*h*x^6 + 1/5*b*g*x^5 + 1/4*b*f*x^4 + 1/3*b*e*x^3 + 1/3*a*h*x^3 + 1/2*
b*d*x^2 + 1/2*a*g*x^2 + b*c*x + a*f*x + a*e*log(abs(x)) - 1/2*(2*a*d*x + a
*c)/x^2
```

3.380.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx = x\,(b\,c+a\,f)-\frac{\frac{a\,c}{2}+a\,d\,x}{x^2}+x^2\left(\frac{b\,d}{2}+\frac{a\,g}{2}\right) + x^3\left(\frac{b\,e}{3}+\frac{a\,h}{3}\right)+\frac{b\,h\,x^6}{6}+a\,e\,\ln(x)+\frac{b\,f\,x^4}{4}+\frac{b\,g\,x^5}{5}$$

input
```
int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x)
```

output
```
x*(b*c + a*f) - ((a*c)/2 + a*d*x)/x^2 + x^2*((b*d)/2 + (a*g)/2) + x^3*((b*
e)/3 + (a*h)/3) + (b*h*x^6)/6 + a*e*log(x) + (b*f*x^4)/4 + (b*g*x^5)/5
```

3.381 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx$

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3.381.2 Mathematica [A] (verified) 2841
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3.381.7 Maxima [A] (verification not implemented) 2844
3.381.8 Giac [A] (verification not implemented) 2845
3.381.9 Mupad [B] (verification not implemented) 2845

3.381.1 Optimal result

Integrand size = 36, antiderivative size = 86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx = -\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+(bd+ag)x+\frac{1}{2}(be+ah)x^2+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5+(bc+af)\log(x)$$

output
```
-1/3*a*c/x^3-1/2*a*d/x^2-a*e/x+(a*g+b*d)*x+1/2*(a*h+b*e)*x^2+1/3*b*f*x^3+1
/4*b*g*x^4+1/5*b*h*x^5+(a*f+b*c)*ln(x)
```

3.381.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4} dx = -\frac{a(2c+3x(d+2ex-x^3(2g+hx)))}{6x^3} + \frac{1}{60}bx(60d+x(30e+x(20f+15gx+12hx^2)))+(bc+af)\log(x)$$

input
```
Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

output
```
-1/6*(a*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/x^3 + (b*x*(60*d + x*(3
0*e + x*(20*f + 15*g*x + 12*h*x^2))))/60 + (b*c + a*f)*Log[x]
```

3.381.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}dx$$

$$\Big\downarrow \text{2360}$$

$$\int \left(\frac{af+bc}{x}+bd\left(\frac{ag}{bd}+1\right)+x(ah+be)+\frac{ac}{x^4}+\frac{ad}{x^3}+\frac{ae}{x^2}+bfx^2+bgx^3+bhx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x)(af+bc)+x(ag+bd)+\frac{1}{2}x^2(ah+be)-\frac{ac}{3x^3}-\frac{ad}{2x^2}-\frac{ae}{x}+\frac{1}{3}bfx^3+\frac{1}{4}bgx^4+\frac{1}{5}bhx^5$$

input
```
Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

output
```
-1/3*(a*c)/x^3 - (a*d)/(2*x^2) - (a*e)/x + (b*d + a*g)*x + ((b*e + a*h)*x^
2)/2 + (b*f*x^3)/3 + (b*g*x^4)/4 + (b*h*x^5)/5 + (b*c + a*f)*Log[x]
```

3.381.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.381.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

method	result	siz
default	$\frac{bh\,x^5}{5}+\frac{bg\,x^4}{4}+\frac{f\,x^3 b}{3}+\frac{ah\,x^2}{2}+\frac{be\,x^2}{2}+agx+bdx+(af+bc)\ln(x)-\frac{ac}{3x^3}-\frac{ae}{x}-\frac{ad}{2x^2}$	76
risch	$\frac{bh\,x^5}{5}+\frac{bg\,x^4}{4}+\frac{f\,x^3 b}{3}+\frac{ah\,x^2}{2}+\frac{be\,x^2}{2}+agx+bdx+\frac{-ae\,x^2-\frac{1}{2}adx-\frac{1}{3}ac}{x^3}+\ln(x)\,af+\ln(x)\,bc$	76
norman	$\frac{\left(\frac{ah}{2}+\frac{be}{2}\right)x^5+(ag+bd)x^4-\frac{ac}{3}-\frac{adx}{2}-ae\,x^2+\frac{bf\,x^6}{3}+\frac{bg\,x^7}{4}+\frac{bh\,x^8}{5}}{x^3}+(af+bc)\ln(x)$	78
parallelrisch	$\frac{12bh\,x^8+15bg\,x^7+20bf\,x^6+30ah\,x^5+30be\,x^5+60\ln(x)x^3af+60\ln(x)x^3bc+60ag\,x^4+60bd\,x^4-60ae\,x^2-30adx-20ac}{60x^3}$	90

input
```
int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*b*h*x^5+1/4*b*g*x^4+1/3*f*x^3*b+1/2*a*h*x^2+1/2*b*e*x^2+a*g*x+b*d*x+(a
*f+b*c)*ln(x)-1/3*a*c/x^3-a*e/x-1/2*a*d/x^2
```

3.381.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = \frac{12\,bhx^8+15\,bgx^7+20\,bfx^6+30\,(be+ah)x^5+60\,(bd+ag)x^4+60\,(bc+af)x^3\log(x)-60\,aex^2-30\,ac}{60\,x^3}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fric
as")
```

output
```
1/60*(12*b*h*x^8 + 15*b*g*x^7 + 20*b*f*x^6 + 30*(b*e + a*h)*x^5 + 60*(b*d
+ a*g)*x^4 + 60*(b*c + a*f)*x^3*log(x) - 60*a*e*x^2 - 30*a*d*x - 20*a*c)/x
^3
```

3.381.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$=\frac{bfx^3}{3}+\frac{bgx^4}{4}+\frac{bhx^5}{5}+x^2\left(\frac{ah}{2}+\frac{be}{2}\right)+x(ag+bd)$$
$$+(af+bc)\log(x)+\frac{-2ac-3adx-6aex^2}{6x^3}$$

input
```
integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

output
```
b*f*x**3/3 + b*g*x**4/4 + b*h*x**5/5 + x**2*(a*h/2 + b*e/2) + x*(a*g + b*d
) + (a*f + b*c)*log(x) + (-2*a*c - 3*a*d*x - 6*a*e*x**2)/(6*x**3)
```

3.381.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$=\frac{1}{5}bhx^5+\frac{1}{4}bgx^4+\frac{1}{3}bfx^3+\frac{1}{2}(be+ah)x^2+(bd+ag)x$$
$$+(bc+af)\log(x)-\frac{6\,aex^2+3\,adx+2\,ac}{6\,x^3}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxi
ma")
```

output
```
1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*(b*e + a*h)*x^2 + (b*d + a*g
)*x + (b*c + a*f)*log(x) - 1/6*(6*a*e*x^2 + 3*a*d*x + 2*a*c)/x^3
```

3.381.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = \frac{1}{5}\,bhx^5+\frac{1}{4}\,bgx^4+\frac{1}{3}\,bfx^3+\frac{1}{2}\,bex^2+\frac{1}{2}\,ahx^2+bdx + agx+(bc+af)\log(|x|)-\frac{6\,aex^2+3\,adx+2\,ac}{6\,x^3}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="giac
")
```

output
```
1/5*b*h*x^5 + 1/4*b*g*x^4 + 1/3*b*f*x^3 + 1/2*b*e*x^2 + 1/2*a*h*x^2 + b*d*
x + a*g*x + (b*c + a*f)*log(abs(x)) - 1/6*(6*a*e*x^2 + 3*a*d*x + 2*a*c)/x^
3
```

3.381.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = x\,(b\,d+a\,g)-\frac{a\,e\,x^2+\frac{a\,d\,x}{2}+\frac{a\,c}{3}}{x^3}+x^2\left(\frac{b\,e}{2}+\frac{a\,h}{2}\right) + \ln(x)\,\left(b\,c+a\,f\right)+\frac{b\,h\,x^5}{5}+\frac{b\,f\,x^3}{3}+\frac{b\,g\,x^4}{4}$$

input
```
int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x)
```

output
```
x*(b*d + a*g) - ((a*c)/3 + (a*d*x)/2 + a*e*x^2)/x^3 + x^2*((b*e)/2 + (a*h)
/2) + log(x)*(b*c + a*f) + (b*h*x^5)/5 + (b*f*x^3)/3 + (b*g*x^4)/4
```

3.382 $\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx$

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3.382.1 Optimal result

Integrand size = 36, antiderivative size = 86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx = -\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}-\frac{bc+af}{x}+(be+ah)x+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4+(bd+ag)\log(x)$$

output
```
-1/4*a*c/x^4-1/3*a*d/x^3-1/2*a*e/x^2+(-a*f-b*c)/x+(a*h+b*e)*x+1/2*b*f*x^2+
1/3*b*g*x^3+1/4*b*h*x^4+(a*g+b*d)*ln(x)
```

3.382.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} dx = b\left(-\frac{c}{x}+ex+\frac{1}{12}x^2(6f+4gx+3hx^2)\right) - \frac{a(3c+4dx+6x^2(e+2fx-2hx^3))}{12x^4}+(bd+ag)\log(x)$$

input
```
Integrate[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

output
```
b*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) - (a*(3*c + 4*d*x + 6*
x^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4) + (b*d + a*g)*Log[x]
```

3.382.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} \, dx$$

$$\Big\downarrow \text{ 2360}$$

$$\int \left(\frac{af+bc}{x^2}+\frac{ag+bd}{x}+be\left(\frac{ah}{be}+1\right)+\frac{ac}{x^5}+\frac{ad}{x^4}+\frac{ae}{x^3}+bfx+bgx^2+bhx^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{af+bc}{x}+\log(x)(ag+bd)+x(ah+be)-\frac{ac}{4x^4}-\frac{ad}{3x^3}-\frac{ae}{2x^2}+\frac{1}{2}bfx^2+\frac{1}{3}bgx^3+\frac{1}{4}bhx^4$$

input
```
Int[((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

output
```
-1/4*(a*c)/x^4 - (a*d)/(3*x^3) - (a*e)/(2*x^2) - (b*c + a*f)/x + (b*e + a*
h)*x + (b*f*x^2)/2 + (b*g*x^3)/3 + (b*h*x^4)/4 + (b*d + a*g)*Log[x]
```

3.382.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.382.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{bh\,x^{4}}{4}+\frac{bg\,x^{3}}{3}+\frac{bf\,x^{2}}{2}+ahx+bex+\left(ag+bd\right)\ln\left(x\right)-\frac{ad}{3x^{3}}-\frac{af+bc}{x}-\frac{ae}{2x^{2}}-\frac{ac}{4x^{4}}$	74
risch	$\frac{bh\,x^{4}}{4}+\frac{bg\,x^{3}}{3}+\frac{bf\,x^{2}}{2}+ahx+bex+\frac{\left(-af-bc\right)x^{3}-\frac{ae\,x^{2}}{2}-\frac{adx}{3}-\frac{ac}{4}}{x^{4}}+\ln\left(x\right)ag+\ln\left(x\right)bd$	75
norman	$\frac{\left(-af-bc\right)x^{3}+\left(ah+be\right)x^{5}-\frac{ac}{4}-\frac{adx}{3}-\frac{ae\,x^{2}}{2}+\frac{bf\,x^{6}}{2}+\frac{bg\,x^{7}}{3}+\frac{bh\,x^{8}}{4}}{x^{4}}+\left(ag+bd\right)\ln\left(x\right)$	78
parallelrisch	$\frac{3bh\,x^{8}+4bg\,x^{7}+6bf\,x^{6}+12\ln\left(x\right)x^{4}ag+12\ln\left(x\right)x^{4}bd+12ah\,x^{5}+12be\,x^{5}-12af\,x^{3}-12bc\,x^{3}-6ae\,x^{2}-4adx-3ac}{12x^{4}}$	90

input
```
int((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*b*h*x^4+1/3*b*g*x^3+1/2*b*f*x^2+a*h*x+b*e*x+(a*g+b*d)*ln(x)-1/3*a*d/x^
3-(a*f+b*c)/x-1/2*a*e/x^2-1/4*a*c/x^4
```

3.382.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^5}\,dx$$
$$=\frac{3\,bhx^{8}+4\,bgx^{7}+6\,bfx^{6}+12\,(be+ah)x^{5}+12\,(bd+ag)x^{4}\log\left(x\right)-6\,aex^{2}-12\,(bc+af)x^{3}-4\,adx-3}{12\,x^{4}}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fric
as")
```

output
```
1/12*(3*b*h*x^8 + 4*b*g*x^7 + 6*b*f*x^6 + 12*(b*e + a*h)*x^5 + 12*(b*d + a
*g)*x^4*log(x) - 6*a*e*x^2 - 12*(b*c + a*f)*x^3 - 4*a*d*x - 3*a*c)/x^4
```

3.382.6 Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = \frac{bfx^2}{2}+\frac{bgx^3}{3}+\frac{bhx^4}{4}+x(ah+be)+(ag+bd)\log(x) + \frac{-3ac-4adx-6aex^2+x^3(-12af-12bc)}{12x^4}$$

input
```
integrate((b*x**3+a)*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

output
```
b*f*x**2/2 + b*g*x**3/3 + b*h*x**4/4 + x*(a*h + b*e) + (a*g + b*d)*log(x)
+ (-3*a*c - 4*a*d*x - 6*a*e*x**2 + x**3*(-12*a*f - 12*b*c))/(12*x**4)
```

3.382.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = \frac{1}{4}\,bhx^4+\frac{1}{3}\,bgx^3+\frac{1}{2}\,bfx^2+(be+ah)x+(bd+ag)\log(x) - \frac{6\,aex^2+12\,(bc+af)x^3+4\,adx+3\,ac}{12\,x^4}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxi
ma")
```

output
```
1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + (b*e + a*h)*x + (b*d + a*g)*log(
x) - 1/12*(6*a*e*x^2 + 12*(b*c + a*f)*x^3 + 4*a*d*x + 3*a*c)/x^4
```

3.382.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$=\frac{1}{4}\,bhx^4+\frac{1}{3}\,bgx^3+\frac{1}{2}\,bfx^2+bex+ahx+(bd+ag)\log(|x|)-\frac{6\,aex^2+12\,(bc+af)x^3+4\,adx+3\,ac}{12\,x^4}$$

input
```
integrate((b*x^3+a)*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="giac
")
```

output
```
1/4*b*h*x^4 + 1/3*b*g*x^3 + 1/2*b*f*x^2 + b*e*x + a*h*x + (b*d + a*g)*log(
abs(x)) - 1/12*(6*a*e*x^2 + 12*(b*c + a*f)*x^3 + 4*a*d*x + 3*a*c)/x^4
```

3.382.9 Mupad [B] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$=x\,(b\,e+a\,h)-\frac{(b\,c+a\,f)\;x^3+\frac{a\,e\,x^2}{2}+\frac{a\,d\,x}{3}+\frac{a\,c}{4}}{x^4}+\ln(x)\;(b\,d+a\,g)+\frac{b\,h\,x^4}{4}+\frac{b\,f\,x^2}{2}+\frac{b\,g\,x^3}{3}$$

input
```
int(((a + b*x^3)*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x)
```

output
```
x*(b*e + a*h) - ((a*c)/4 + x^3*(b*c + a*f) + (a*d*x)/3 + (a*e*x^2)/2)/x^4
+ log(x)*(b*d + a*g) + (b*h*x^4)/4 + (b*f*x^2)/2 + (b*g*x^3)/3
```

3.383 $\int x^4(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

3.383.1 Optimal result 2851
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3.383.9 Mupad [B] (verification not implemented) 2856

3.383.1 Optimal result

Integrand size = 38, antiderivative size = 163

$$\begin{aligned}\int x^4(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{8}a(2bc+af)x^8+\frac{1}{9}a(2bd+ag)x^9\\ &\quad+\frac{1}{10}a(2be+ah)x^{10}+\frac{1}{11}b(bc+2af)x^{11}+\frac{1}{12}b(bd+2ag)x^{12}\\ &\quad+\frac{1}{13}b(be+2ah)x^{13}+\frac{1}{14}b^2fx^{14}+\frac{1}{15}b^2gx^{15}+\frac{1}{16}b^2hx^{16}\end{aligned}$$

output
```
1/5*a^2*c*x^5+1/6*a^2*d*x^6+1/7*a^2*e*x^7+1/8*a*(a*f+2*b*c)*x^8+1/9*a*(a*g
+2*b*d)*x^9+1/10*a*(a*h+2*b*e)*x^10+1/11*b*(2*a*f+b*c)*x^11+1/12*b*(2*a*g+
b*d)*x^12+1/13*b*(2*a*h+b*e)*x^13+1/14*b^2*f*x^14+1/15*b^2*g*x^15+1/16*b^2
*h*x^16
```

3.383.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^4(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{8}a(2bc+af)x^8+\frac{1}{9}a(2bd+ag)x^9\\ &\quad+\frac{1}{10}a(2be+ah)x^{10}+\frac{1}{11}b(bc+2af)x^{11}+\frac{1}{12}b(bd+2ag)x^{12}\\ &\quad+\frac{1}{13}b(be+2ah)x^{13}+\frac{1}{14}b^2fx^{14}+\frac{1}{15}b^2gx^{15}+\frac{1}{16}b^2hx^{16}\end{aligned}$$

input
```
Integrate[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 +
(a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^1
1)/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^1
4)/14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

3.383.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx$$

$$\Big\downarrow \text{2360}$$

$$\int (a^2cx^4+a^2dx^5+a^2ex^6+bx^{10}(2af+bc)+ax^7(af+2bc)+bx^{11}(2ag+bd)+ax^8(ag+2bd)+bx^{12}(2ah+be)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^2cx^5+\frac{1}{6}a^2dx^6+\frac{1}{7}a^2ex^7+\frac{1}{11}bx^{11}(2af+bc)+\frac{1}{8}ax^8(af+2bc)+\frac{1}{12}bx^{12}(2ag+bd)+\frac{1}{9}ax^9(ag+2bd)+\frac{1}{13}bx^{13}(2ah+be)+\frac{1}{10}ax^{10}(ah+2be)+\frac{1}{14}b^2fx^{14}+\frac{1}{15}b^2gx^{15}+\frac{1}{16}b^2hx^{16}$$

input
```
Int[x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^2*c*x^5)/5 + (a^2*d*x^6)/6 + (a^2*e*x^7)/7 + (a*(2*b*c + a*f)*x^8)/8 +
(a*(2*b*d + a*g)*x^9)/9 + (a*(2*b*e + a*h)*x^10)/10 + (b*(b*c + 2*a*f)*x^1
1)/11 + (b*(b*d + 2*a*g)*x^12)/12 + (b*(b*e + 2*a*h)*x^13)/13 + (b^2*f*x^1
4)/14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

3.383.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.383.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.93

method	result
default	$\frac{b^2 h\,x^{16}}{16}+\frac{b^2 g\,x^{15}}{15}+\frac{b^2 f\,x^{14}}{14}+\frac{(2abh+b^2e)x^{13}}{13}+\frac{(2abg+b^2d)x^{12}}{12}+\frac{(2afb+b^2c)x^{11}}{11}+\frac{(a^2h+2aeb)x^{10}}{10}+$
norman	$\frac{a^2 c\,x^5}{5}+\frac{a^2 d\,x^6}{6}+\frac{a^2 e\,x^7}{7}+\left(\frac{1}{8}a^2 f+\frac{1}{4}abc\right)x^8+\left(\frac{1}{9}a^2 g+\frac{2}{9}abd\right)x^9+\left(\frac{1}{10}a^2 h+\frac{1}{5}aeb\right)x^{10}+$
gosper	$\frac{1}{5}a^2c\,x^5+\frac{1}{6}a^2d\,x^6+\frac{1}{7}a^2e\,x^7+\frac{1}{8}x^8a^2f+\frac{1}{4}x^8abc+\frac{1}{9}x^9a^2g+\frac{2}{9}abd\,x^9+\frac{1}{10}x^{10}a^2h+\frac{1}{5}abe\,x^{10}$
risch	$\frac{1}{5}a^2c\,x^5+\frac{1}{6}a^2d\,x^6+\frac{1}{7}a^2e\,x^7+\frac{1}{8}x^8a^2f+\frac{1}{4}x^8abc+\frac{1}{9}x^9a^2g+\frac{2}{9}abd\,x^9+\frac{1}{10}x^{10}a^2h+\frac{1}{5}abe\,x^{10}$
parallelrisch	$\frac{1}{5}a^2c\,x^5+\frac{1}{6}a^2d\,x^6+\frac{1}{7}a^2e\,x^7+\frac{1}{8}x^8a^2f+\frac{1}{4}x^8abc+\frac{1}{9}x^9a^2g+\frac{2}{9}abd\,x^9+\frac{1}{10}x^{10}a^2h+\frac{1}{5}abe\,x^{10}$

input
```
int(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOS
E)
```

output
```
1/16*b^2*h*x^16+1/15*b^2*g*x^15+1/14*b^2*f*x^14+1/13*(2*a*b*h+b^2*e)*x^13+
1/12*(2*a*b*g+b^2*d)*x^12+1/11*(2*a*b*f+b^2*c)*x^11+1/10*(a^2*h+2*a*b*e)*x
^10+1/9*(a^2*g+2*a*b*d)*x^9+1/8*(a^2*f+2*a*b*c)*x^8+1/7*a^2*e*x^7+1/6*a^2*
d*x^6+1/5*a^2*c*x^5
```

3.383.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int x^4(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{16}\,b^2hx^{16}+\frac{1}{15}\,b^2gx^{15}+\frac{1}{14}\,b^2fx^{14}+\frac{1}{13}\,(b^2e+2\,abh)x^{13} + \frac{1}{12}\,(b^2d+2\,abg)x^{12}+\frac{1}{11}\,(b^2c+2\,abf)x^{11}+\frac{1}{10}\,(2\,abe+a^2h)x^{10} + \frac{1}{7}\,a^2ex^7+\frac{1}{9}\,(2\,abd+a^2g)x^9+\frac{1}{6}\,a^2dx^6+\frac{1}{8}\,(2\,abc+a^2f)x^8+\frac{1}{5}\,a^2cx^5$$

input
```
integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fr
icas")
```

output
```
1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 1/13*(b^2*e + 2*a*b*
h)*x^13 + 1/12*(b^2*d + 2*a*b*g)*x^12 + 1/11*(b^2*c + 2*a*b*f)*x^11 + 1/10
*(2*a*b*e + a^2*h)*x^10 + 1/7*a^2*e*x^7 + 1/9*(2*a*b*d + a^2*g)*x^9 + 1/6*
a^2*d*x^6 + 1/8*(2*a*b*c + a^2*f)*x^8 + 1/5*a^2*c*x^5
```

3.383.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int x^4(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{a^2cx^5}{5}+\frac{a^2dx^6}{6}+\frac{a^2ex^7}{7}+\frac{b^2fx^{14}}{14}+\frac{b^2gx^{15}}{15}+\frac{b^2hx^{16}}{16}+x^{13}\cdot\left(\frac{2abh}{13}+\frac{b^2e}{13}\right)+x^{12}\left(\frac{abg}{6}+\frac{b^2d}{12}\right)+x^{11}\cdot\left(\frac{2abf}{11}+\frac{b^2c}{11}\right) + x^{10}\left(\frac{a^2h}{10}+\frac{abe}{5}\right)+x^9\left(\frac{a^2g}{9}+\frac{2abd}{9}\right)+x^8\left(\frac{a^2f}{8}+\frac{abc}{4}\right)$$

input
```
integrate(x**4*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**2*c*x**5/5 + a**2*d*x**6/6 + a**2*e*x**7/7 + b**2*f*x**14/14 + b**2*g*x
**15/15 + b**2*h*x**16/16 + x**13*(2*a*b*h/13 + b**2*e/13) + x**12*(a*b*g/
6 + b**2*d/12) + x**11*(2*a*b*f/11 + b**2*c/11) + x**10*(a**2*h/10 + a*b*e
/5) + x**9*(a**2*g/9 + 2*a*b*d/9) + x**8*(a**2*f/8 + a*b*c/4)
```

3.383.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int x^4(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{16}\,b^2hx^{16}+\frac{1}{15}\,b^2gx^{15}+\frac{1}{14}\,b^2fx^{14}+\frac{1}{13}\,(b^2e+2\,abh)x^{13}+\frac{1}{12}\,(b^2d+2\,abg)x^{12}+\frac{1}{11}\,(b^2c+2\,abf)x^{11}+\frac{1}{10}\,(2\,abe+a^2h)x^{10}+\frac{1}{7}\,a^2ex^7+\frac{1}{9}\,(2\,abd+a^2g)x^9+\frac{1}{6}\,a^2dx^6+\frac{1}{8}\,(2\,abc+a^2f)x^8+\frac{1}{5}\,a^2cx^5$$

input
```
integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="ma
xima")
```

output
```
1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 1/13*(b^2*e + 2*a*b*
h)*x^13 + 1/12*(b^2*d + 2*a*b*g)*x^12 + 1/11*(b^2*c + 2*a*b*f)*x^11 + 1/10
*(2*a*b*e + a^2*h)*x^10 + 1/7*a^2*e*x^7 + 1/9*(2*a*b*d + a^2*g)*x^9 + 1/6*
a^2*d*x^6 + 1/8*(2*a*b*c + a^2*f)*x^8 + 1/5*a^2*c*x^5
```

3.383.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

$$\int x^4(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{16}\,b^2hx^{16}+\frac{1}{15}\,b^2gx^{15}+\frac{1}{14}\,b^2fx^{14}+\frac{1}{13}\,b^2ex^{13}+\frac{2}{13}\,abhx^{13}+\frac{1}{12}\,b^2dx^{12}+\frac{1}{6}\,abgx^{12}+\frac{1}{11}\,b^2cx^{11}+\frac{2}{11}\,abfx^{11}+\frac{1}{5}\,abex^{10}+\frac{1}{10}\,a^2hx^{10}+\frac{2}{9}\,abdx^9+\frac{1}{9}\,a^2gx^9+\frac{1}{4}\,abcx^8+\frac{1}{8}\,a^2fx^8+\frac{1}{7}\,a^2ex^7+\frac{1}{6}\,a^2dx^6+\frac{1}{5}\,a^2cx^5$$

input
```
integrate(x^4*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gi
ac")
```

output
```
1/16*b^2*h*x^16 + 1/15*b^2*g*x^15 + 1/14*b^2*f*x^14 + 1/13*b^2*e*x^13 + 2/
13*a*b*h*x^13 + 1/12*b^2*d*x^12 + 1/6*a*b*g*x^12 + 1/11*b^2*c*x^11 + 2/11*
a*b*f*x^11 + 1/5*a*b*e*x^10 + 1/10*a^2*h*x^10 + 2/9*a*b*d*x^9 + 1/9*a^2*g*
x^9 + 1/4*a*b*c*x^8 + 1/8*a^2*f*x^8 + 1/7*a^2*e*x^7 + 1/6*a^2*d*x^6 + 1/5*
a^2*c*x^5
```

3.383.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int x^4 \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$= x^8\left(\frac{f\,a^2}{8}+\frac{b\,c\,a}{4}\right)+x^{11}\left(\frac{c\,b^2}{11}+\frac{2\,a\,f\,b}{11}\right)+x^9\left(\frac{g\,a^2}{9}+\frac{2\,b\,d\,a}{9}\right)$$
$$+x^{12}\left(\frac{d\,b^2}{12}+\frac{a\,g\,b}{6}\right)+x^{10}\left(\frac{h\,a^2}{10}+\frac{b\,e\,a}{5}\right)+x^{13}\left(\frac{e\,b^2}{13}+\frac{2\,a\,h\,b}{13}\right)$$
$$+\frac{a^2\,c\,x^5}{5}+\frac{a^2\,d\,x^6}{6}+\frac{a^2\,e\,x^7}{7}+\frac{b^2\,f\,x^{14}}{14}+\frac{b^2\,g\,x^{15}}{15}+\frac{b^2\,h\,x^{16}}{16}$$

input
```
int(x^4*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^8*((a^2*f)/8 + (a*b*c)/4) + x^11*((b^2*c)/11 + (2*a*b*f)/11) + x^9*((a^2
*g)/9 + (2*a*b*d)/9) + x^12*((b^2*d)/12 + (a*b*g)/6) + x^10*((a^2*h)/10 +
(a*b*e)/5) + x^13*((b^2*e)/13 + (2*a*b*h)/13) + (a^2*c*x^5)/5 + (a^2*d*x^6
)/6 + (a^2*e*x^7)/7 + (b^2*f*x^14)/14 + (b^2*g*x^15)/15 + (b^2*h*x^16)/16
```

3.384 $\int x^3(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

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3.384.1 Optimal result

Integrand size = 38, antiderivative size = 163

$$\begin{aligned}\int x^3(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a(2bc+af)x^7+\frac{1}{8}a(2bd+ag)x^8\\&+\frac{1}{9}a(2be+ah)x^9+\frac{1}{10}b(bc+2af)x^{10}+\frac{1}{11}b(bd+2ag)x^{11}\\&+\frac{1}{12}b(be+2ah)x^{12}+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}\end{aligned}$$

output
```
1/4*a^2*c*x^4+1/5*a^2*d*x^5+1/6*a^2*e*x^6+1/7*a*(a*f+2*b*c)*x^7+1/8*a*(a*g
+2*b*d)*x^8+1/9*a*(a*h+2*b*e)*x^9+1/10*b*(2*a*f+b*c)*x^10+1/11*b*(2*a*g+b*
d)*x^11+1/12*b*(2*a*h+b*e)*x^12+1/13*b^2*f*x^13+1/14*b^2*g*x^14+1/15*b^2*h
*x^15
```

3.384.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^3(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a(2bc+af)x^7+\frac{1}{8}a(2bd+ag)x^8\\&+\frac{1}{9}a(2be+ah)x^9+\frac{1}{10}b(bc+2af)x^{10}+\frac{1}{11}b(bd+2ag)x^{11}\\&+\frac{1}{12}b(be+2ah)x^{12}+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}\end{aligned}$$

input
```
Integrate[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 +
(a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)
/10 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)
/13 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

3.384.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx$$

$$\Big\downarrow \text{2360}$$

$$\int (a^2cx^3+a^2dx^4+a^2ex^5+bx^9(2af+bc)+ax^6(af+2bc)+bx^{10}(2ag+bd)+ax^7(ag+2bd)+bx^{11}(2ah+be)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{10}bx^{10}(2af+bc)+\frac{1}{7}ax^7(af+2bc)+\frac{1}{11}bx^{11}(2ag+bd)+\frac{1}{8}ax^8(ag+2bd)+\frac{1}{12}bx^{12}(2ah+be)+\frac{1}{9}ax^9(ah+2be)+\frac{1}{13}b^2fx^{13}+\frac{1}{14}b^2gx^{14}+\frac{1}{15}b^2hx^{15}$$

input
```
Int[x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a*(2*b*c + a*f)*x^7)/7 +
(a*(2*b*d + a*g)*x^8)/8 + (a*(2*b*e + a*h)*x^9)/9 + (b*(b*c + 2*a*f)*x^10)
/10 + (b*(b*d + 2*a*g)*x^11)/11 + (b*(b*e + 2*a*h)*x^12)/12 + (b^2*f*x^13)
/13 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

3.384.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.384.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.93

method	result
default	$\frac{b^2 h\,x^{15}}{15}+\frac{b^2 g\,x^{14}}{14}+\frac{b^2 f\,x^{13}}{13}+\frac{(2abh+b^2 e)x^{12}}{12}+\frac{(2abg+b^2 d)x^{11}}{11}+\frac{(2afb+b^2 c)x^{10}}{10}+\frac{(a^2 h+2aeb)x^9}{9}+\frac{(a^2 g+2ab}{8}$
norman	$\frac{a^2 c\,x^4}{4}+\frac{a^2 d\,x^5}{5}+\frac{a^2 e\,x^6}{6}+\left(\frac{1}{7}a^2 f+\frac{2}{7}abc\right)x^7+\left(\frac{1}{8}a^2 g+\frac{1}{4}abd\right)x^8+\left(\frac{1}{9}a^2 h+\frac{2}{9}aeb\right)x^9+\left(\frac{1}{5}afb\right.$
gosper	$\frac{1}{4}a^2 c\,x^4+\frac{1}{5}a^2 d\,x^5+\frac{1}{6}a^2 e\,x^6+\frac{1}{7}a^2 f\,x^7+\frac{2}{7}x^7 abc+\frac{1}{8}x^8 a^2 g+\frac{1}{4}x^8 abd+\frac{1}{9}x^9 a^2 h+\frac{2}{9}x^9 aeb+$
risch	$\frac{1}{4}a^2 c\,x^4+\frac{1}{5}a^2 d\,x^5+\frac{1}{6}a^2 e\,x^6+\frac{1}{7}a^2 f\,x^7+\frac{2}{7}x^7 abc+\frac{1}{8}x^8 a^2 g+\frac{1}{4}x^8 abd+\frac{1}{9}x^9 a^2 h+\frac{2}{9}x^9 aeb+$
parallelrisch	$\frac{1}{4}a^2 c\,x^4+\frac{1}{5}a^2 d\,x^5+\frac{1}{6}a^2 e\,x^6+\frac{1}{7}a^2 f\,x^7+\frac{2}{7}x^7 abc+\frac{1}{8}x^8 a^2 g+\frac{1}{4}x^8 abd+\frac{1}{9}x^9 a^2 h+\frac{2}{9}x^9 aeb+$

input
```
int(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOS
E)
```

output
```
1/15*b^2*h*x^15+1/14*b^2*g*x^14+1/13*b^2*f*x^13+1/12*(2*a*b*h+b^2*e)*x^12+
1/11*(2*a*b*g+b^2*d)*x^11+1/10*(2*a*b*f+b^2*c)*x^10+1/9*(a^2*h+2*a*b*e)*x^
9+1/8*(a^2*g+2*a*b*d)*x^8+1/7*(a^2*f+2*a*b*c)*x^7+1/6*a^2*e*x^6+1/5*a^2*d*
x^5+1/4*a^2*c*x^4
```

3.384.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$
$$=\frac{1}{15}b^2hx^{15}+\frac{1}{14}b^2gx^{14}+\frac{1}{13}b^2fx^{13}+\frac{1}{12}(b^2e+2abh)x^{12}+\frac{1}{11}(b^2d+2abg)x^{11}+\frac{1}{10}(b^2c+2abf)x^{10}+\frac{1}{9}(2abe+a^2h)x^9+\frac{1}{6}a^2ex^6+\frac{1}{8}(2abd+a^2g)x^8+\frac{1}{5}a^2dx^5+\frac{1}{7}(2abc+a^2f)x^7+\frac{1}{4}a^2cx^4$$

input
```
integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fr
icas")
```

output
```
1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/12*(b^2*e + 2*a*b*
h)*x^12 + 1/11*(b^2*d + 2*a*b*g)*x^11 + 1/10*(b^2*c + 2*a*b*f)*x^10 + 1/9*
(2*a*b*e + a^2*h)*x^9 + 1/6*a^2*e*x^6 + 1/8*(2*a*b*d + a^2*g)*x^8 + 1/5*a^
2*d*x^5 + 1/7*(2*a*b*c + a^2*f)*x^7 + 1/4*a^2*c*x^4
```

3.384.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int x^3(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$
$$=\frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{b^2fx^{13}}{13}+\frac{b^2gx^{14}}{14}+\frac{b^2hx^{15}}{15}+x^{12}\left(\frac{abh}{6}+\frac{b^2e}{12}\right)+x^{11}\cdot\left(\frac{2abg}{11}+\frac{b^2d}{11}\right)+x^{10}\left(\frac{abf}{5}+\frac{b^2c}{10}\right)+x^9\left(\frac{a^2h}{9}+\frac{2abe}{9}\right)+x^8\left(\frac{a^2g}{8}+\frac{abd}{4}\right)+x^7\left(\frac{a^2f}{7}+\frac{2abc}{7}\right)$$

input
```
integrate(x**3*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + b**2*f*x**13/13 + b**2*g*x
**14/14 + b**2*h*x**15/15 + x**12*(a*b*h/6 + b**2*e/12) + x**11*(2*a*b*g/1
1 + b**2*d/11) + x**10*(a*b*f/5 + b**2*c/10) + x**9*(a**2*h/9 + 2*a*b*e/9)
 + x**8*(a**2*g/8 + a*b*d/4) + x**7*(a**2*f/7 + 2*a*b*c/7)
```

3.384.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{15}\,b^2hx^{15}+\frac{1}{14}\,b^2gx^{14}+\frac{1}{13}\,b^2fx^{13}+\frac{1}{12}\,(b^2e+2\,abh)x^{12}+\frac{1}{11}\,(b^2d+2\,abg)x^{11}+\frac{1}{10}\,(b^2c+2\,abf)x^{10}+\frac{1}{9}\,(2\,abe+a^2h)x^9+\frac{1}{6}\,a^2ex^6+\frac{1}{8}\,(2\,abd+a^2g)x^8+\frac{1}{5}\,a^2dx^5+\frac{1}{7}\,(2\,abc+a^2f)x^7+\frac{1}{4}\,a^2cx^4$$

input
```
integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="ma
xima")
```

output
```
1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/12*(b^2*e + 2*a*b*
h)*x^12 + 1/11*(b^2*d + 2*a*b*g)*x^11 + 1/10*(b^2*c + 2*a*b*f)*x^10 + 1/9*
(2*a*b*e + a^2*h)*x^9 + 1/6*a^2*e*x^6 + 1/8*(2*a*b*d + a^2*g)*x^8 + 1/5*a^
2*d*x^5 + 1/7*(2*a*b*c + a^2*f)*x^7 + 1/4*a^2*c*x^4
```

3.384.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

$$\int x^3(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{15}\,b^2hx^{15}+\frac{1}{14}\,b^2gx^{14}+\frac{1}{13}\,b^2fx^{13}+\frac{1}{12}\,b^2ex^{12}+\frac{1}{6}\,abhx^{12}+\frac{1}{11}\,b^2dx^{11}+\frac{2}{11}\,abgx^{11}+\frac{1}{10}\,b^2cx^{10}+\frac{1}{5}\,abfx^{10}+\frac{2}{9}\,abex^9+\frac{1}{9}\,a^2hx^9+\frac{1}{4}\,abdx^8+\frac{1}{8}\,a^2gx^8+\frac{2}{7}\,abcx^7+\frac{1}{7}\,a^2fx^7+\frac{1}{6}\,a^2ex^6+\frac{1}{5}\,a^2dx^5+\frac{1}{4}\,a^2cx^4$$

input
```
integrate(x^3*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gi
ac")
```

output
```
1/15*b^2*h*x^15 + 1/14*b^2*g*x^14 + 1/13*b^2*f*x^13 + 1/12*b^2*e*x^12 + 1/
6*a*b*h*x^12 + 1/11*b^2*d*x^11 + 2/11*a*b*g*x^11 + 1/10*b^2*c*x^10 + 1/5*a
*b*f*x^10 + 2/9*a*b*e*x^9 + 1/9*a^2*h*x^9 + 1/4*a*b*d*x^8 + 1/8*a^2*g*x^8
+ 2/7*a*b*c*x^7 + 1/7*a^2*f*x^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*
c*x^4
```

3.384.9 Mupad [B] (verification not implemented)

Time = 10.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int x^3 (a + bx^3)^2 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) \, dx$$
$$= x^7 \left(\frac{f\,a^2}{7} + \frac{2\,b\,c\,a}{7} \right) + x^{10} \left(\frac{c\,b^2}{10} + \frac{a\,f\,b}{5} \right) + x^8 \left(\frac{g\,a^2}{8} + \frac{b\,d\,a}{4} \right)$$
$$+ x^{11} \left(\frac{d\,b^2}{11} + \frac{2\,a\,g\,b}{11} \right) + x^9 \left(\frac{h\,a^2}{9} + \frac{2\,b\,e\,a}{9} \right) + x^{12} \left(\frac{e\,b^2}{12} + \frac{a\,h\,b}{6} \right)$$
$$+ \frac{a^2\,c\,x^4}{4} + \frac{a^2\,d\,x^5}{5} + \frac{a^2\,e\,x^6}{6} + \frac{b^2\,f\,x^{13}}{13} + \frac{b^2\,g\,x^{14}}{14} + \frac{b^2\,h\,x^{15}}{15}$$

input
```
int(x^3*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^7*((a^2*f)/7 + (2*a*b*c)/7) + x^10*((b^2*c)/10 + (a*b*f)/5) + x^8*((a^2*
g)/8 + (a*b*d)/4) + x^11*((b^2*d)/11 + (2*a*b*g)/11) + x^9*((a^2*h)/9 + (2
*a*b*e)/9) + x^12*((b^2*e)/12 + (a*b*h)/6) + (a^2*c*x^4)/4 + (a^2*d*x^5)/5
 + (a^2*e*x^6)/6 + (b^2*f*x^13)/13 + (b^2*g*x^14)/14 + (b^2*h*x^15)/15
```

3.385 $\int x^2(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

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3.385.1 Optimal result

Integrand size = 38, antiderivative size = 158

$$\int x^2(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{1}{7}a(2bd+ag)x^7+\frac{1}{8}a(2be+ah)x^8+\frac{2}{9}abfx^9+\frac{1}{10}b(bd+2ag)x^{10}+\frac{1}{11}b(be+2ah)x^{11}+\frac{1}{12}b^2fx^{12}+\frac{1}{13}b^2gx^{13}+\frac{1}{14}b^2hx^{14}+\frac{c(a+bx^3)^3}{9b}$$

output
```
1/4*a^2*d*x^4+1/5*a^2*e*x^5+1/6*a^2*f*x^6+1/7*a*(a*g+2*b*d)*x^7+1/8*a*(a*h
+2*b*e)*x^8+2/9*a*b*f*x^9+1/10*b*(2*a*g+b*d)*x^10+1/11*b*(2*a*h+b*e)*x^11+
1/12*b^2*f*x^12+1/13*b^2*g*x^13+1/14*b^2*h*x^14+1/9*c*(b*x^3+a)^3/b
```

3.385.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.95

$$\int x^2(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = a^2\left(\frac{cx^3}{3}+\frac{dx^4}{4}+\frac{ex^5}{5}+\frac{fx^6}{6}+\frac{gx^7}{7}+\frac{hx^8}{8}\right)+ab\left(\frac{cx^6}{3}+\frac{2dx^7}{7}+\frac{ex^8}{4}+\frac{2fx^9}{9}+\frac{gx^{10}}{5}+\frac{2hx^{11}}{11}\right)+\frac{b^2x^9(20020c+3x(6006d+5460ex+55x^2(91f+84gx+78hx^2)))}{180180}$$

input
```
Integrate[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
a^2*((c*x^3)/3 + (d*x^4)/4 + (e*x^5)/5 + (f*x^6)/6 + (g*x^7)/7 + (h*x^8)/8
) + a*b*((c*x^6)/3 + (2*d*x^7)/7 + (e*x^8)/4 + (2*f*x^9)/9 + (g*x^10)/5 +
(2*h*x^11)/11) + (b^2*x^9*(20020*c + 3*x*(6006*d + 5460*e*x + 55*x^2*(91*f
 + 84*g*x + 78*h*x^2))))/180180
```

3.385.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^3+a\right)^2\left(x^2\left(hx^5+gx^4+fx^3+ex^2+dx+c\right)-cx^2\right)dx+\frac{c\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^2hx^{13}+b^2gx^{12}+b^2fx^{11}+b(be+2ah)x^{10}+b(bd+2ag)x^9+2abfx^8+a(2be+ah)x^7+a(2bd+ag)x^6+a^2\right.$$
$$\frac{c\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^2dx^4+\frac{1}{5}a^2ex^5+\frac{1}{6}a^2fx^6+\frac{c\left(a+bx^3\right)^3}{9b}+\frac{1}{10}bx^{10}(2ag+bd)+\frac{1}{7}ax^7(ag+2bd)+\frac{1}{11}bx^{11}(2ah+be)+\frac{1}{8}ax^8(ah+2be)+\frac{2}{9}abfx^9+\frac{1}{12}b^2fx^{12}+\frac{1}{13}b^2gx^{13}+\frac{1}{14}b^2hx^{14}$$

input
```
Int[x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^2*d*x^4)/4 + (a^2*e*x^5)/5 + (a^2*f*x^6)/6 + (a*(2*b*d + a*g)*x^7)/7 +
(a*(2*b*e + a*h)*x^8)/8 + (2*a*b*f*x^9)/9 + (b*(b*d + 2*a*g)*x^10)/10 + (b
*(b*e + 2*a*h)*x^11)/11 + (b^2*f*x^12)/12 + (b^2*g*x^13)/13 + (b^2*h*x^14)
/14 + (c*(a + b*x^3)^3)/(9*b)
```

3.385.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.385.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

method	result
default	$\frac{b^2 h\,x^{14}}{14}+\frac{b^2 g\,x^{13}}{13}+\frac{b^2 f\,x^{12}}{12}+\frac{(2abh+b^2e)x^{11}}{11}+\frac{(2abg+b^2d)x^{10}}{10}+\frac{(2afb+b^2c)x^9}{9}+\frac{(a^2h+2aeb)x^8}{8}+\frac{(a^2g+2abc}{7}$
norman	$\frac{a^2c\,x^3}{3}+\frac{a^2d\,x^4}{4}+\frac{a^2e\,x^5}{5}+\left(\frac{1}{6}a^2f+\frac{1}{3}abc\right)x^6+\left(\frac{1}{7}a^2g+\frac{2}{7}abd\right)x^7+\left(\frac{1}{8}a^2h+\frac{1}{4}aeb\right)x^8+\left(\frac{2}{9}afb\right.$
gosper	$\frac{1}{3}a^2c\,x^3+\frac{1}{4}a^2d\,x^4+\frac{1}{5}a^2e\,x^5+\frac{1}{6}a^2f\,x^6+\frac{1}{3}abc\,x^6+\frac{1}{7}x^7a^2g+\frac{2}{7}ad\,x^7b+\frac{1}{8}x^8a^2h+\frac{1}{4}abe\,x^8+$
risch	$\frac{1}{3}a^2c\,x^3+\frac{1}{4}a^2d\,x^4+\frac{1}{5}a^2e\,x^5+\frac{1}{6}a^2f\,x^6+\frac{1}{3}abc\,x^6+\frac{1}{7}x^7a^2g+\frac{2}{7}ad\,x^7b+\frac{1}{8}x^8a^2h+\frac{1}{4}abe\,x^8+$
parallelrisch	$\frac{1}{3}a^2c\,x^3+\frac{1}{4}a^2d\,x^4+\frac{1}{5}a^2e\,x^5+\frac{1}{6}a^2f\,x^6+\frac{1}{3}abc\,x^6+\frac{1}{7}x^7a^2g+\frac{2}{7}ad\,x^7b+\frac{1}{8}x^8a^2h+\frac{1}{4}abe\,x^8+$

input
```
int(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOS
E)
```

output
```
1/14*b^2*h*x^14+1/13*b^2*g*x^13+1/12*b^2*f*x^12+1/11*(2*a*b*h+b^2*e)*x^11+
1/10*(2*a*b*g+b^2*d)*x^10+1/9*(2*a*b*f+b^2*c)*x^9+1/8*(a^2*h+2*a*b*e)*x^8+
1/7*(a^2*g+2*a*b*d)*x^7+1/6*(a^2*f+2*a*b*c)*x^6+1/5*a^2*e*x^5+1/4*a^2*d*x^
4+1/3*a^2*c*x^3
```

3.385.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\begin{aligned}&\int x^2(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx\\&=\frac{1}{14}b^2hx^{14}+\frac{1}{13}b^2gx^{13}+\frac{1}{12}b^2fx^{12}+\frac{1}{11}(b^2e+2\,abh)x^{11}\\&\quad+\frac{1}{10}(b^2d+2\,abg)x^{10}+\frac{1}{9}(b^2c+2\,abf)x^9+\frac{1}{8}(2\,abe+a^2h)x^8\\&\quad+\frac{1}{5}a^2ex^5+\frac{1}{7}(2\,abd+a^2g)x^7+\frac{1}{4}a^2dx^4+\frac{1}{6}(2\,abc+a^2f)x^6+\frac{1}{3}a^2cx^3\end{aligned}$$

input
```
integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fr
icas")
```

output
```
1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*(b^2*e + 2*a*b*
h)*x^11 + 1/10*(b^2*d + 2*a*b*g)*x^10 + 1/9*(b^2*c + 2*a*b*f)*x^9 + 1/8*(2
*a*b*e + a^2*h)*x^8 + 1/5*a^2*e*x^5 + 1/7*(2*a*b*d + a^2*g)*x^7 + 1/4*a^2*
d*x^4 + 1/6*(2*a*b*c + a^2*f)*x^6 + 1/3*a^2*c*x^3
```

3.385.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

$$\begin{aligned}&\int x^2(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx\\&=\frac{a^2cx^3}{3}+\frac{a^2dx^4}{4}+\frac{a^2ex^5}{5}+\frac{b^2fx^{12}}{12}+\frac{b^2gx^{13}}{13}+\frac{b^2hx^{14}}{14}+x^{11}\\&\quad\cdot\left(\frac{2abh}{11}+\frac{b^2e}{11}\right)+x^{10}\left(\frac{abg}{5}+\frac{b^2d}{10}\right)+x^9\cdot\left(\frac{2abf}{9}+\frac{b^2c}{9}\right)\\&\quad+x^8\left(\frac{a^2h}{8}+\frac{abe}{4}\right)+x^7\left(\frac{a^2g}{7}+\frac{2abd}{7}\right)+x^6\left(\frac{a^2f}{6}+\frac{abc}{3}\right)\end{aligned}$$

input
```
integrate(x**2*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**2*c*x**3/3 + a**2*d*x**4/4 + a**2*e*x**5/5 + b**2*f*x**12/12 + b**2*g*x
**13/13 + b**2*h*x**14/14 + x**11*(2*a*b*h/11 + b**2*e/11) + x**10*(a*b*g/
5 + b**2*d/10) + x**9*(2*a*b*f/9 + b**2*c/9) + x**8*(a**2*h/8 + a*b*e/4) +
 x**7*(a**2*g/7 + 2*a*b*d/7) + x**6*(a**2*f/6 + a*b*c/3)
```

3.385.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int x^2(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{14}b^2hx^{14}+\frac{1}{13}b^2gx^{13}+\frac{1}{12}b^2fx^{12}+\frac{1}{11}(b^2e+2abh)x^{11} + \frac{1}{10}(b^2d+2abg)x^{10}+\frac{1}{9}(b^2c+2abf)x^9+\frac{1}{8}(2abe+a^2h)x^8 + \frac{1}{5}a^2ex^5+\frac{1}{7}(2abd+a^2g)x^7+\frac{1}{4}a^2dx^4+\frac{1}{6}(2abc+a^2f)x^6+\frac{1}{3}a^2cx^3$$

input
```
integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="ma
xima")
```

output
```
1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*(b^2*e + 2*a*b*
h)*x^11 + 1/10*(b^2*d + 2*a*b*g)*x^10 + 1/9*(b^2*c + 2*a*b*f)*x^9 + 1/8*(2
*a*b*e + a^2*h)*x^8 + 1/5*a^2*e*x^5 + 1/7*(2*a*b*d + a^2*g)*x^7 + 1/4*a^2*
d*x^4 + 1/6*(2*a*b*c + a^2*f)*x^6 + 1/3*a^2*c*x^3
```

3.385.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int x^2(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{14}b^2hx^{14}+\frac{1}{13}b^2gx^{13}+\frac{1}{12}b^2fx^{12}+\frac{1}{11}b^2ex^{11}+\frac{2}{11}abhx^{11}+\frac{1}{10}b^2dx^{10} + \frac{1}{5}abgx^{10}+\frac{1}{9}b^2cx^9+\frac{2}{9}abfx^9+\frac{1}{4}abex^8+\frac{1}{8}a^2hx^8+\frac{2}{7}abdx^7 + \frac{1}{7}a^2gx^7+\frac{1}{3}abcx^6+\frac{1}{6}a^2fx^6+\frac{1}{5}a^2ex^5+\frac{1}{4}a^2dx^4+\frac{1}{3}a^2cx^3$$

input
```
integrate(x^2*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gi
ac")
```

output
```
1/14*b^2*h*x^14 + 1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 2/
11*a*b*h*x^11 + 1/10*b^2*d*x^10 + 1/5*a*b*g*x^10 + 1/9*b^2*c*x^9 + 2/9*a*b
*f*x^9 + 1/4*a*b*e*x^8 + 1/8*a^2*h*x^8 + 2/7*a*b*d*x^7 + 1/7*a^2*g*x^7 + 1
/3*a*b*c*x^6 + 1/6*a^2*f*x^6 + 1/5*a^2*e*x^5 + 1/4*a^2*d*x^4 + 1/3*a^2*c*x
^3
```

3.385.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int x^2\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=x^6\left(\frac{f\,a^2}{6}+\frac{b\,c\,a}{3}\right)+x^9\left(\frac{c\,b^2}{9}+\frac{2\,a\,f\,b}{9}\right)+x^7\left(\frac{g\,a^2}{7}+\frac{2\,b\,d\,a}{7}\right)+x^{10}\left(\frac{d\,b^2}{10}+\frac{a\,g\,b}{5}\right)+x^8\left(\frac{h\,a^2}{8}+\frac{b\,e\,a}{4}\right)+x^{11}\left(\frac{e\,b^2}{11}+\frac{2\,a\,h\,b}{11}\right)+\frac{a^2\,c\,x^3}{3}+\frac{a^2\,d\,x^4}{4}+\frac{a^2\,e\,x^5}{5}+\frac{b^2\,f\,x^{12}}{12}+\frac{b^2\,g\,x^{13}}{13}+\frac{b^2\,h\,x^{14}}{14}$$

input
```
int(x^2*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^6*((a^2*f)/6 + (a*b*c)/3) + x^9*((b^2*c)/9 + (2*a*b*f)/9) + x^7*((a^2*g)
/7 + (2*a*b*d)/7) + x^10*((b^2*d)/10 + (a*b*g)/5) + x^8*((a^2*h)/8 + (a*b*
e)/4) + x^11*((b^2*e)/11 + (2*a*b*h)/11) + (a^2*c*x^3)/3 + (a^2*d*x^4)/4 +
 (a^2*e*x^5)/5 + (b^2*f*x^12)/12 + (b^2*g*x^13)/13 + (b^2*h*x^14)/14
```

3.386 $\int x(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

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3.386.1 Optimal result

Integrand size = 36, antiderivative size = 158

$$\int x(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{5}a(2bc+af)x^5+\frac{1}{6}a^2gx^6+\frac{1}{7}a(2be+ah)x^7+\frac{1}{8}b(bc+2af)x^8+\frac{2}{9}abgx^9+\frac{1}{10}b(be+2ah)x^{10}+\frac{1}{11}b^2fx^{11}+\frac{1}{12}b^2gx^{12}+\frac{1}{13}b^2hx^{13}+\frac{d(a+bx^3)^3}{9b}$$

output
```
1/2*a^2*c*x^2+1/4*a^2*e*x^4+1/5*a*(a*f+2*b*c)*x^5+1/6*a^2*g*x^6+1/7*a*(a*h
+2*b*e)*x^7+1/8*b*(2*a*f+b*c)*x^8+2/9*a*b*g*x^9+1/10*b*(2*a*h+b*e)*x^10+1/
11*b^2*f*x^11+1/12*b^2*g*x^12+1/13*b^2*h*x^13+1/9*d*(b*x^3+a)^3/b
```

3.386.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\int x(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{2}a^2cx^2+\frac{1}{3}a^2dx^3+\frac{1}{4}a^2ex^4+\frac{1}{5}a(2bc+af)x^5+\frac{1}{6}a(2bd+ag)x^6+\frac{1}{7}a(2be+ah)x^7+\frac{1}{8}b(bc+2af)x^8+\frac{1}{9}b(bd+2ag)x^9+\frac{1}{10}b(be+2ah)x^{10}+\frac{1}{11}b^2fx^{11}+\frac{1}{12}b^2gx^{12}+\frac{1}{13}b^2hx^{13}$$

input
```
Integrate[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 +
(a*(2*b*d + a*g)*x^6)/6 + (a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/
8 + (b*(b*d + 2*a*g)*x^9)/9 + (b*(b*e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11
+ (b^2*g*x^12)/12 + (b^2*h*x^13)/13
```

3.386.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^3+a\right)^2\left(x\left(hx^5+gx^4+fx^3+ex^2+dx+c\right)-dx^2\right)dx+\frac{d\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^2hx^{12}+b^2gx^{11}+b^2fx^{10}+b(be+2ah)x^9+2abgx^8+b(bc+2af)x^7+a(2be+ah)x^6+a^2gx^5+a(2bc+af)$$
$$\frac{d\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^2cx^2+\frac{1}{4}a^2ex^4+\frac{1}{6}a^2gx^6+\frac{1}{8}bx^8(2af+bc)+\frac{1}{5}ax^5(af+2bc)+\frac{d\left(a+bx^3\right)^3}{9b}+\frac{1}{10}bx^{10}(2ah+be)+\frac{1}{7}ax^7(ah+2be)+\frac{2}{9}abgx^9+\frac{1}{11}b^2fx^{11}+\frac{1}{12}b^2gx^{12}+\frac{1}{13}b^2hx^{13}$$

input
```
Int[x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^2*c*x^2)/2 + (a^2*e*x^4)/4 + (a*(2*b*c + a*f)*x^5)/5 + (a^2*g*x^6)/6 +
(a*(2*b*e + a*h)*x^7)/7 + (b*(b*c + 2*a*f)*x^8)/8 + (2*a*b*g*x^9)/9 + (b*(
b*e + 2*a*h)*x^10)/10 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/1
3 + (d*(a + b*x^3)^3)/(9*b)
```

3.386.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.386.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

method	result
default	$\frac{b^2 h\,x^{13}}{13}+\frac{b^2 g\,x^{12}}{12}+\frac{b^2 f\,x^{11}}{11}+\frac{(2abh+b^2e)x^{10}}{10}+\frac{(2abg+b^2d)x^9}{9}+\frac{(2afb+b^2c)x^8}{8}+\frac{(a^2h+2aeb)x^7}{7}+\frac{(a^2g+2abd}{6}$
norman	$\frac{b^2 h\,x^{13}}{13}+\frac{b^2 g\,x^{12}}{12}+\frac{b^2 f\,x^{11}}{11}+\left(\frac{1}{5}abh+\frac{1}{10}b^2e\right)x^{10}+\left(\frac{2}{9}abg+\frac{1}{9}b^2d\right)x^9+\left(\frac{1}{4}afb+\frac{1}{8}b^2c\right)x^8+\left(\frac{1}{7}a\right.$
gosper	$\frac{1}{13}b^2h\,x^{13}+\frac{1}{12}b^2g\,x^{12}+\frac{1}{11}b^2f\,x^{11}+\frac{1}{5}x^{10}abh+\frac{1}{10}b^2e\,x^{10}+\frac{2}{9}abg\,x^9+\frac{1}{9}b^2d\,x^9+\frac{1}{4}abf\,x^8+\frac{1}{8}$
risch	$\frac{1}{13}b^2h\,x^{13}+\frac{1}{12}b^2g\,x^{12}+\frac{1}{11}b^2f\,x^{11}+\frac{1}{5}x^{10}abh+\frac{1}{10}b^2e\,x^{10}+\frac{2}{9}abg\,x^9+\frac{1}{9}b^2d\,x^9+\frac{1}{4}abf\,x^8+\frac{1}{8}$
parallelrisch	$\frac{1}{13}b^2h\,x^{13}+\frac{1}{12}b^2g\,x^{12}+\frac{1}{11}b^2f\,x^{11}+\frac{1}{5}x^{10}abh+\frac{1}{10}b^2e\,x^{10}+\frac{2}{9}abg\,x^9+\frac{1}{9}b^2d\,x^9+\frac{1}{4}abf\,x^8+\frac{1}{8}$

input
```
int(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^2*h*x^13+1/12*b^2*g*x^12+1/11*b^2*f*x^11+1/10*(2*a*b*h+b^2*e)*x^10+
1/9*(2*a*b*g+b^2*d)*x^9+1/8*(2*a*b*f+b^2*c)*x^8+1/7*(a^2*h+2*a*b*e)*x^7+1/
6*(a^2*g+2*a*b*d)*x^6+1/5*(a^2*f+2*a*b*c)*x^5+1/4*a^2*e*x^4+1/3*a^2*d*x^3+
1/2*a^2*c*x^2
```

3.386.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int x(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{1}{13}\,b^2hx^{13}+\frac{1}{12}\,b^2gx^{12}+\frac{1}{11}\,b^2fx^{11}+\frac{1}{10}\,(b^2e+2\,abh)x^{10}+\frac{1}{9}\,(b^2d+2\,abg)x^9+\frac{1}{8}\,(b^2c+2\,abf)x^8+\frac{1}{7}\,(2\,abe+a^2h)x^7+\frac{1}{4}\,a^2ex^4+\frac{1}{6}\,(2\,abd+a^2g)x^6+\frac{1}{3}\,a^2dx^3+\frac{1}{5}\,(2\,abc+a^2f)x^5+\frac{1}{2}\,a^2cx^2$$

input
```
integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fric
as")
```

output
```
1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/10*(b^2*e + 2*a*b*
h)*x^10 + 1/9*(b^2*d + 2*a*b*g)*x^9 + 1/8*(b^2*c + 2*a*b*f)*x^8 + 1/7*(2*a
*b*e + a^2*h)*x^7 + 1/4*a^2*e*x^4 + 1/6*(2*a*b*d + a^2*g)*x^6 + 1/3*a^2*d*
x^3 + 1/5*(2*a*b*c + a^2*f)*x^5 + 1/2*a^2*c*x^2
```

3.386.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

$$\int x(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = \frac{a^2cx^2}{2}+\frac{a^2dx^3}{3}+\frac{a^2ex^4}{4}+\frac{b^2fx^{11}}{11}+\frac{b^2gx^{12}}{12}+\frac{b^2hx^{13}}{13}+x^{10}\left(\frac{abh}{5}+\frac{b^2e}{10}\right)+x^9\cdot\left(\frac{2abg}{9}+\frac{b^2d}{9}\right)+x^8\left(\frac{abf}{4}+\frac{b^2c}{8}\right)+x^7\left(\frac{a^2h}{7}+\frac{2abe}{7}\right)+x^6\left(\frac{a^2g}{6}+\frac{abd}{3}\right)+x^5\left(\frac{a^2f}{5}+\frac{2abc}{5}\right)$$

input
```
integrate(x*(b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**2*c*x**2/2 + a**2*d*x**3/3 + a**2*e*x**4/4 + b**2*f*x**11/11 + b**2*g*x
**12/12 + b**2*h*x**13/13 + x**10*(a*b*h/5 + b**2*e/10) + x**9*(2*a*b*g/9
+ b**2*d/9) + x**8*(a*b*f/4 + b**2*c/8) + x**7*(a**2*h/7 + 2*a*b*e/7) + x*
*6*(a**2*g/6 + a*b*d/3) + x**5*(a**2*f/5 + 2*a*b*c/5)
```

3.386.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int x(a+bx^3)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \frac{1}{13}b^2hx^{13} + \frac{1}{12}b^2gx^{12} + \frac{1}{11}b^2fx^{11} + \frac{1}{10}\left(b^2e+2\,abh\right)x^{10} + \frac{1}{9}\left(b^2d+2\,abg\right)x^9 + \frac{1}{8}\left(b^2c+2\,abf\right)x^8 + \frac{1}{7}\left(2\,abe+a^2h\right)x^7 + \frac{1}{4}a^2ex^4 + \frac{1}{6}\left(2\,abd+a^2g\right)x^6 + \frac{1}{3}a^2dx^3 + \frac{1}{5}\left(2\,abc+a^2f\right)x^5 + \frac{1}{2}a^2cx^2$$

input
```
integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxi
ma")
```

output
```
1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/10*(b^2*e + 2*a*b*
h)*x^10 + 1/9*(b^2*d + 2*a*b*g)*x^9 + 1/8*(b^2*c + 2*a*b*f)*x^8 + 1/7*(2*a
*b*e + a^2*h)*x^7 + 1/4*a^2*e*x^4 + 1/6*(2*a*b*d + a^2*g)*x^6 + 1/3*a^2*d*
x^3 + 1/5*(2*a*b*c + a^2*f)*x^5 + 1/2*a^2*c*x^2
```

3.386.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int x(a+bx^3)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx = \frac{1}{13}b^2hx^{13} + \frac{1}{12}b^2gx^{12} + \frac{1}{11}b^2fx^{11} + \frac{1}{10}b^2ex^{10} + \frac{1}{5}abhx^{10} + \frac{1}{9}b^2dx^9 + \frac{2}{9}abgx^9 + \frac{1}{8}b^2cx^8 + \frac{1}{4}abfx^8 + \frac{2}{7}abex^7 + \frac{1}{7}a^2hx^7 + \frac{1}{3}abdx^6 + \frac{1}{6}a^2gx^6 + \frac{2}{5}abcx^5 + \frac{1}{5}a^2fx^5 + \frac{1}{4}a^2ex^4 + \frac{1}{3}a^2dx^3 + \frac{1}{2}a^2cx^2$$

input
```
integrate(x*(b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac
")
```

output
```
1/13*b^2*h*x^13 + 1/12*b^2*g*x^12 + 1/11*b^2*f*x^11 + 1/10*b^2*e*x^10 + 1/
5*a*b*h*x^10 + 1/9*b^2*d*x^9 + 2/9*a*b*g*x^9 + 1/8*b^2*c*x^8 + 1/4*a*b*f*x
^8 + 2/7*a*b*e*x^7 + 1/7*a^2*h*x^7 + 1/3*a*b*d*x^6 + 1/6*a^2*g*x^6 + 2/5*a
*b*c*x^5 + 1/5*a^2*f*x^5 + 1/4*a^2*e*x^4 + 1/3*a^2*d*x^3 + 1/2*a^2*c*x^2
```

3.386.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int x(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=x^5\left(\frac{f\,a^2}{5}+\frac{2\,b\,c\,a}{5}\right)+x^8\left(\frac{c\,b^2}{8}+\frac{a\,f\,b}{4}\right)+x^6\left(\frac{g\,a^2}{6}+\frac{b\,d\,a}{3}\right)+x^9\left(\frac{d\,b^2}{9}+\frac{2\,a\,g\,b}{9}\right)+x^7\left(\frac{h\,a^2}{7}+\frac{2\,b\,e\,a}{7}\right)+x^{10}\left(\frac{e\,b^2}{10}+\frac{a\,h\,b}{5}\right)+\frac{a^2\,c\,x^2}{2}+\frac{a^2\,d\,x^3}{3}+\frac{a^2\,e\,x^4}{4}+\frac{b^2\,f\,x^{11}}{11}+\frac{b^2\,g\,x^{12}}{12}+\frac{b^2\,h\,x^{13}}{13}$$

input
```
int(x*(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^5*((a^2*f)/5 + (2*a*b*c)/5) + x^8*((b^2*c)/8 + (a*b*f)/4) + x^6*((a^2*g)
/6 + (a*b*d)/3) + x^9*((b^2*d)/9 + (2*a*b*g)/9) + x^7*((a^2*h)/7 + (2*a*b*
e)/7) + x^10*((b^2*e)/10 + (a*b*h)/5) + (a^2*c*x^2)/2 + (a^2*d*x^3)/3 + (a
^2*e*x^4)/4 + (b^2*f*x^11)/11 + (b^2*g*x^12)/12 + (b^2*h*x^13)/13
```

3.387 $\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx$

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3.387.1 Optimal result

Integrand size = 35, antiderivative size = 153

$$\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{4}a(2bc+af)x^4+\frac{1}{5}a(2bd+ag)x^5+\frac{1}{6}a^2hx^6+\frac{1}{7}b(bc+2af)x^7 + \frac{1}{8}b(bd+2ag)x^8+\frac{2}{9}abhx^9+\frac{1}{10}b^2fx^{10}+\frac{1}{11}b^2gx^{11}+\frac{1}{12}b^2hx^{12}+\frac{e(a+bx^3)^3}{9b}$$

output
```
a^2*c*x+1/2*a^2*d*x^2+1/4*a*(a*f+2*b*c)*x^4+1/5*a*(a*g+2*b*d)*x^5+1/6*a^2*
h*x^6+1/7*b*(2*a*f+b*c)*x^7+1/8*b*(2*a*g+b*d)*x^8+2/9*a*b*h*x^9+1/10*b^2*f
*x^10+1/11*b^2*g*x^11+1/12*b^2*h*x^12+1/9*e*(b*x^3+a)^3/b
```

3.387.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

$$\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = \frac{b^2x^7(3960c+7x(495d+440ex+6x^2(66f+60gx+55hx^2)))+462a^2x(60c+x(30d+x(20e+15fx+1}{27720}$$

input
```
Integrate[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(b^2*x^7*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h*x^2)
)) + 462*a^2*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3)))
 + 22*a*b*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x))
))))/27720
```

3.387.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx$$

$$\Big\downarrow \; 2017$$

$$\int \left(bx^3+a\right)^2 \left(hx^5+gx^4+fx^3+dx+c\right) dx+\frac{e\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \; 2389$$

$$\int \left(b^2hx^{11}+b^2gx^{10}+b^2fx^9+2abhx^8+b(bd+2ag)x^7+b(bc+2af)x^6+a^2hx^5+a(2bd+ag)x^4+a(2bc+af)x$$

$$\frac{e\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \; 2009$$

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{6}a^2hx^6+\frac{1}{7}bx^7(2af+bc)+\frac{1}{4}ax^4(af+2bc)+\frac{1}{8}bx^8(2ag+bd)+\frac{1}{5}ax^5(ag+2bd)+\frac{e\left(a+bx^3\right)^3}{9b}+\frac{2}{9}abhx^9+\frac{1}{10}b^2fx^{10}+\frac{1}{11}b^2gx^{11}+\frac{1}{12}b^2hx^{12}$$

input
```
Int[(a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a*(2*b*c + a*f)*x^4)/4 + (a*(2*b*d + a*g)*x^5)/
5 + (a^2*h*x^6)/6 + (b*(b*c + 2*a*f)*x^7)/7 + (b*(b*d + 2*a*g)*x^8)/8 + (2
*a*b*h*x^9)/9 + (b^2*f*x^10)/10 + (b^2*g*x^11)/11 + (b^2*h*x^12)/12 + (e*(
a + b*x^3)^3)/(9*b)
```

3.387.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.387.4 Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^2 h\,x^{12}}{12}+\frac{b^2 g\,x^{11}}{11}+\frac{b^2 f\,x^{10}}{10}+\frac{(2abh+b^2e)x^9}{9}+\frac{(2abg+b^2d)x^8}{8}+\frac{(2afb+b^2c)x^7}{7}+\frac{(a^2h+2aeb)x^6}{6}+\frac{(a^2g+2abd)}{5}$
norman	$\frac{b^2 h\,x^{12}}{12}+\frac{b^2 g\,x^{11}}{11}+\frac{b^2 f\,x^{10}}{10}+\left(\frac{2}{9}abh+\frac{1}{9}b^2e\right)x^9+\left(\frac{1}{4}abg+\frac{1}{8}b^2d\right)x^8+\left(\frac{2}{7}afb+\frac{1}{7}b^2c\right)x^7+\left(\frac{1}{6}a^2\right.$
gosper	$\frac{1}{12}b^2h\,x^{12}+\frac{1}{11}b^2g\,x^{11}+\frac{1}{10}b^2f\,x^{10}+\frac{2}{9}abh\,x^9+\frac{1}{9}b^2e\,x^9+\frac{1}{4}x^8abg+\frac{1}{8}b^2d\,x^8+\frac{2}{7}x^7afb+\frac{1}{7}b^2c$
risch	$\frac{1}{12}b^2h\,x^{12}+\frac{1}{11}b^2g\,x^{11}+\frac{1}{10}b^2f\,x^{10}+\frac{2}{9}abh\,x^9+\frac{1}{9}b^2e\,x^9+\frac{1}{4}x^8abg+\frac{1}{8}b^2d\,x^8+\frac{2}{7}x^7afb+\frac{1}{7}b^2c$
parallelrisch	$\frac{1}{12}b^2h\,x^{12}+\frac{1}{11}b^2g\,x^{11}+\frac{1}{10}b^2f\,x^{10}+\frac{2}{9}abh\,x^9+\frac{1}{9}b^2e\,x^9+\frac{1}{4}x^8abg+\frac{1}{8}b^2d\,x^8+\frac{2}{7}x^7afb+\frac{1}{7}b^2c$

input
```
int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/12*b^2*h*x^12+1/11*b^2*g*x^11+1/10*b^2*f*x^10+1/9*(2*a*b*h+b^2*e)*x^9+1/
8*(2*a*b*g+b^2*d)*x^8+1/7*(2*a*b*f+b^2*c)*x^7+1/6*(a^2*h+2*a*b*e)*x^6+1/5*
(a^2*g+2*a*b*d)*x^5+1/4*(a^2*f+2*a*b*c)*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^
2*c*x
```

3.387.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = \frac{1}{12} b^2hx^{12} + \frac{1}{11} b^2gx^{11} + \frac{1}{10} b^2fx^{10} + \frac{1}{9} (b^2e + 2\,abh)x^9 + \frac{1}{8} (b^2d + 2\,abg)x^8 + \frac{1}{7} (b^2c + 2\,abf)x^7 + \frac{1}{6} (2\,abe + a^2h)x^6 + \frac{1}{3} a^2ex^3 + \frac{1}{5} (2\,abd + a^2g)x^5 + \frac{1}{2} a^2dx^2 + \frac{1}{4} (2\,abc + a^2f)x^4 + a^2cx$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas
")
```

output
```
1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 1/9*(b^2*e + 2*a*b*h
)*x^9 + 1/8*(b^2*d + 2*a*b*g)*x^8 + 1/7*(b^2*c + 2*a*b*f)*x^7 + 1/6*(2*a*b
*e + a^2*h)*x^6 + 1/3*a^2*e*x^3 + 1/5*(2*a*b*d + a^2*g)*x^5 + 1/2*a^2*d*x^
2 + 1/4*(2*a*b*c + a^2*f)*x^4 + a^2*c*x
```

3.387.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07

$$\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = a^2cx + \frac{a^2dx^2}{2} + \frac{a^2ex^3}{3} + \frac{b^2fx^{10}}{10} + \frac{b^2gx^{11}}{11} + \frac{b^2hx^{12}}{12} + x^9 \cdot \left(\frac{2abh}{9} + \frac{b^2e}{9}\right) + x^8\left(\frac{abg}{4} + \frac{b^2d}{8}\right) + x^7 \cdot \left(\frac{2abf}{7} + \frac{b^2c}{7}\right) + x^6\left(\frac{a^2h}{6} + \frac{abe}{3}\right) + x^5\left(\frac{a^2g}{5} + \frac{2abd}{5}\right) + x^4\left(\frac{a^2f}{4} + \frac{abc}{2}\right)$$

input
```
integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + b**2*f*x**10/10 + b**2*g*x**11/
11 + b**2*h*x**12/12 + x**9*(2*a*b*h/9 + b**2*e/9) + x**8*(a*b*g/4 + b**2*
d/8) + x**7*(2*a*b*f/7 + b**2*c/7) + x**6*(a**2*h/6 + a*b*e/3) + x**5*(a**
2*g/5 + 2*a*b*d/5) + x**4*(a**2*f/4 + a*b*c/2)
```

3.387.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = \frac{1}{12} b^2hx^{12} + \frac{1}{11} b^2gx^{11} + \frac{1}{10} b^2fx^{10} + \frac{1}{9} (b^2e+2\,abh)x^9 + \frac{1}{8} (b^2d+2\,abg)x^8 + \frac{1}{7} (b^2c+2\,abf)x^7 + \frac{1}{6} (2\,abe+a^2h)x^6 + \frac{1}{3} a^2ex^3 + \frac{1}{5} (2\,abd+a^2g)x^5 + \frac{1}{2} a^2dx^2 + \frac{1}{4} (2\,abc+a^2f)x^4 + a^2cx$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima
")
```

output
```
1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 1/9*(b^2*e + 2*a*b*h
)*x^9 + 1/8*(b^2*d + 2*a*b*g)*x^8 + 1/7*(b^2*c + 2*a*b*f)*x^7 + 1/6*(2*a*b
*e + a^2*h)*x^6 + 1/3*a^2*e*x^3 + 1/5*(2*a*b*d + a^2*g)*x^5 + 1/2*a^2*d*x^
2 + 1/4*(2*a*b*c + a^2*f)*x^4 + a^2*c*x
```

3.387.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

$$\int (a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = \frac{1}{12} b^2hx^{12} + \frac{1}{11} b^2gx^{11} + \frac{1}{10} b^2fx^{10} + \frac{1}{9} b^2ex^9 + \frac{2}{9} abhx^9 + \frac{1}{8} b^2dx^8 + \frac{1}{4} abgx^8 + \frac{1}{7} b^2cx^7 + \frac{2}{7} abfx^7 + \frac{1}{3} abex^6 + \frac{1}{6} a^2hx^6 + \frac{2}{5} abdx^5 + \frac{1}{5} a^2gx^5 + \frac{1}{2} abcx^4 + \frac{1}{4} a^2fx^4 + \frac{1}{3} a^2ex^3 + \frac{1}{2} a^2dx^2 + a^2cx$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

output
```
1/12*b^2*h*x^12 + 1/11*b^2*g*x^11 + 1/10*b^2*f*x^10 + 1/9*b^2*e*x^9 + 2/9*
a*b*h*x^9 + 1/8*b^2*d*x^8 + 1/4*a*b*g*x^8 + 1/7*b^2*c*x^7 + 2/7*a*b*f*x^7
+ 1/3*a*b*e*x^6 + 1/6*a^2*h*x^6 + 2/5*a*b*d*x^5 + 1/5*a^2*g*x^5 + 1/2*a*b*
c*x^4 + 1/4*a^2*f*x^4 + 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

3.387.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx = x^4\left(\frac{f\,a^2}{4}+\frac{b\,c\,a}{2}\right)+x^7\left(\frac{c\,b^2}{7}+\frac{2\,a\,f\,b}{7}\right)+x^5\left(\frac{g\,a^2}{5}+\frac{2\,b\,d\,a}{5}\right)+x^8\left(\frac{d\,b^2}{8}+\frac{a\,g\,b}{4}\right)+x^6\left(\frac{h\,a^2}{6}+\frac{b\,e\,a}{3}\right)+x^9\left(\frac{e\,b^2}{9}+\frac{2\,a\,h\,b}{9}\right)+\frac{a^2\,d\,x^2}{2}+\frac{a^2\,e\,x^3}{3}+\frac{b^2\,f\,x^{10}}{10}+\frac{b^2\,g\,x^{11}}{11}+\frac{b^2\,h\,x^{12}}{12}+a^2\,c\,x$$

input
```
int((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^4*((a^2*f)/4 + (a*b*c)/2) + x^7*((b^2*c)/7 + (2*a*b*f)/7) + x^5*((a^2*g)
/5 + (2*a*b*d)/5) + x^8*((b^2*d)/8 + (a*b*g)/4) + x^6*((a^2*h)/6 + (a*b*e)
/3) + x^9*((b^2*e)/9 + (2*a*b*h)/9) + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (b^2
*f*x^10)/10 + (b^2*g*x^11)/11 + (b^2*h*x^12)/12 + a^2*c*x
```

3.388 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} dx$

3.388.1 Optimal result 2881
3.388.2 Mathematica [A] (verified) 2881
3.388.3 Rubi [A] (verified) 2882
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3.388.1 Optimal result

Integrand size = 38, antiderivative size = 149

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} dx$$
$$= a^2dx + \frac{1}{2}a^2ex^2 + \frac{2}{3}abcx^3 + \frac{1}{4}a(2bd+ag)x^4 + \frac{1}{5}a(2be+ah)x^5 + \frac{1}{6}b^2cx^6$$
$$+ \frac{1}{7}b(bd+2ag)x^7 + \frac{1}{8}b(be+2ah)x^8 + \frac{1}{10}b^2gx^{10} + \frac{1}{11}b^2hx^{11} + \frac{f(a+bx^3)^3}{9b} + a^2c\log(x)$$

output
```
a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*c*x^3+1/4*a*(a*g+2*b*d)*x^4+1/5*a*(a*h+2*b*e
)*x^5+1/6*b^2*c*x^6+1/7*b*(2*a*g+b*d)*x^7+1/8*b*(2*a*h+b*e)*x^8+1/10*b^2*g
*x^10+1/11*b^2*h*x^11+1/9*f*(b*x^3+a)^3/b+a^2*c*ln(x)
```

3.388.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x} dx$$
$$= a^2dx + \frac{1}{2}a^2ex^2 + \frac{1}{3}a(2bc+af)x^3 + \frac{1}{4}a(2bd+ag)x^4 + \frac{1}{5}a(2be+ah)x^5 + \frac{1}{6}b(bc+2af)x^6$$
$$+ \frac{1}{7}b(bd+2ag)x^7 + \frac{1}{8}b(be+2ah)x^8 + \frac{1}{9}b^2fx^9 + \frac{1}{10}b^2gx^{10} + \frac{1}{11}b^2hx^{11} + a^2c\log(x)$$

input
```
Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

output
```
a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*c + a*f)*x^3)/3 + (a*(2*b*d + a*g)*x^4)/
4 + (a*(2*b*e + a*h)*x^5)/5 + (b*(b*c + 2*a*f)*x^6)/6 + (b*(b*d + 2*a*g)*x
^7)/7 + (b*(b*e + 2*a*h)*x^8)/8 + (b^2*f*x^9)/9 + (b^2*g*x^10)/10 + (b^2*h
*x^11)/11 + a^2*c*Log[x]
```

3.388.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2018, 2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x} \, dx$$

$$\Big\downarrow \text{2018}$$

$$\int \frac{\left(bx^3+a\right)^2 \left(hx^5+gx^4+ex^2+dx+c\right)}{x}dx+\frac{f\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \text{2360}$$

$$\int \Big(b^2hx^{10}+b^2gx^9+b(be+2ah)x^7+b(bd+2ag)x^6+b^2cx^5+a(2be+ah)x^4+a(2bd+ag)x^3+2abcx^2+a^2ex$$

$$\frac{f\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \text{2009}$$

$$a^2c\log(x)+a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abcx^3+\frac{1}{7}bx^7(2ag+bd)+\frac{1}{4}ax^4(ag+2bd)+\frac{1}{8}bx^8(2ah+be)+\frac{1}{5}ax^5(ah+2be)+\frac{f\left(a+bx^3\right)^3}{9b}+\frac{1}{6}b^2cx^6+\frac{1}{10}b^2gx^{10}+\frac{1}{11}b^2hx^{11}$$

input
```
Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

output
```
a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*c*x^3)/3 + (a*(2*b*d + a*g)*x^4)/4 + (a*(
2*b*e + a*h)*x^5)/5 + (b^2*c*x^6)/6 + (b*(b*d + 2*a*g)*x^7)/7 + (b*(b*e +
2*a*h)*x^8)/8 + (b^2*g*x^10)/10 + (b^2*h*x^11)/11 + (f*(a + b*x^3)^3)/(9*b
) + a^2*c*Log[x]
```

3.388.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.388.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99

method	result
norman	$\left(\frac{1}{3}a^2f+\frac{2}{3}abc\right)x^3+\left(\frac{1}{4}a^2g+\frac{1}{2}abd\right)x^4+\left(\frac{1}{5}a^2h+\frac{2}{5}aeb\right)x^5+\left(\frac{2}{7}abg+\frac{1}{7}b^2d\right)x^7+\left(\frac{1}{4}abh+\frac{1}{8}\right.$
default	$\frac{b^2h\,x^{11}}{11}+\frac{b^2g\,x^{10}}{10}+\frac{b^2f\,x^9}{9}+\frac{abh\,x^8}{4}+\frac{b^2e\,x^8}{8}+\frac{2abg\,x^7}{7}+\frac{b^2d\,x^7}{7}+\frac{abf\,x^6}{3}+\frac{b^2c\,x^6}{6}+\frac{a^2h\,x^5}{5}+\frac{2abe\,x^5}{5}+$
risch	$\frac{b^2h\,x^{11}}{11}+\frac{b^2g\,x^{10}}{10}+\frac{b^2f\,x^9}{9}+\frac{abh\,x^8}{4}+\frac{b^2e\,x^8}{8}+\frac{2abg\,x^7}{7}+\frac{b^2d\,x^7}{7}+\frac{abf\,x^6}{3}+\frac{b^2c\,x^6}{6}+\frac{a^2h\,x^5}{5}+\frac{2abe\,x^5}{5}+$
parallelrisch	$\frac{b^2h\,x^{11}}{11}+\frac{b^2g\,x^{10}}{10}+\frac{b^2f\,x^9}{9}+\frac{abh\,x^8}{4}+\frac{b^2e\,x^8}{8}+\frac{2abg\,x^7}{7}+\frac{b^2d\,x^7}{7}+\frac{abf\,x^6}{3}+\frac{b^2c\,x^6}{6}+\frac{a^2h\,x^5}{5}+\frac{2abe\,x^5}{5}+$

input
```
int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/3*a^2*f+2/3*a*b*c)*x^3+(1/4*a^2*g+1/2*a*b*d)*x^4+(1/5*a^2*h+2/5*a*e*b)*
x^5+(2/7*a*b*g+1/7*b^2*d)*x^7+(1/4*a*b*h+1/8*b^2*e)*x^8+(1/3*a*f*b+1/6*b^2
*c)*x^6+a^2*d*x+1/2*a^2*e*x^2+1/9*b^2*f*x^9+1/10*b^2*g*x^10+1/11*b^2*h*x^1
1+a^2*c*ln(x)
```

3.388.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$$
$$= \frac{1}{11}b^2hx^{11} + \frac{1}{10}b^2gx^{10} + \frac{1}{9}b^2fx^9 + \frac{1}{8}(b^2e+2\,abh)x^8 + \frac{1}{7}(b^2d+2\,abg)x^7 + \frac{1}{6}(b^2c+2\,abf)x^6 + \frac{1}{5}(2\,abe+a^2h)x^5 + \frac{1}{2}a^2ex^2 + \frac{1}{4}(2\,abd+a^2g)x^4 + a^2dx + \frac{1}{3}(2\,abc+a^2f)x^3 + a^2c\log(x)$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fric
as")
```

output
```
1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e + 2*a*b*h)*
x^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*x^6 + 1/5*(2*a*b*e
 + a^2*h)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*d + a^2*g)*x^4 + a^2*d*x + 1/3*
(2*a*b*c + a^2*f)*x^3 + a^2*c*log(x)
```

3.388.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$$
$$= a^2c\log(x) + a^2dx + \frac{a^2ex^2}{2} + \frac{b^2fx^9}{9} + \frac{b^2gx^{10}}{10} + \frac{b^2hx^{11}}{11} + x^8\left(\frac{abh}{4}+\frac{b^2e}{8}\right) + x^7\cdot\left(\frac{2abg}{7}+\frac{b^2d}{7}\right) + x^6\left(\frac{abf}{3}+\frac{b^2c}{6}\right) + x^5\left(\frac{a^2h}{5}+\frac{2abe}{5}\right) + x^4\left(\frac{a^2g}{4}+\frac{abd}{2}\right) + x^3\left(\frac{a^2f}{3}+\frac{2abc}{3}\right)$$

input
```
integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

output
```
a**2*c*log(x) + a**2*d*x + a**2*e*x**2/2 + b**2*f*x**9/9 + b**2*g*x**10/10
 + b**2*h*x**11/11 + x**8*(a*b*h/4 + b**2*e/8) + x**7*(2*a*b*g/7 + b**2*d/
7) + x**6*(a*b*f/3 + b**2*c/6) + x**5*(a**2*h/5 + 2*a*b*e/5) + x**4*(a**2*
g/4 + a*b*d/2) + x**3*(a**2*f/3 + 2*a*b*c/3)
```

3.388.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{1}{11}b^2hx^{11} + \frac{1}{10}b^2gx^{10} + \frac{1}{9}b^2fx^9 + \frac{1}{8}\left(b^2e+2\,abh\right)x^8 + \frac{1}{7}\left(b^2d+2\,abg\right)x^7 + \frac{1}{6}\left(b^2c+2\,abf\right)x^6 + \frac{1}{5}\left(2\,abe+a^2h\right)x^5 + \frac{1}{2}a^2ex^2 + \frac{1}{4}\left(2\,abd+a^2g\right)x^4 + a^2dx + \frac{1}{3}\left(2\,abc+a^2f\right)x^3 + a^2c\log(x)$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxi
ma")
```

output
```
1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*(b^2*e + 2*a*b*h)*
x^8 + 1/7*(b^2*d + 2*a*b*g)*x^7 + 1/6*(b^2*c + 2*a*b*f)*x^6 + 1/5*(2*a*b*e
 + a^2*h)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*d + a^2*g)*x^4 + a^2*d*x + 1/3*
(2*a*b*c + a^2*f)*x^3 + a^2*c*log(x)
```

3.388.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{1}{11}b^2hx^{11} + \frac{1}{10}b^2gx^{10} + \frac{1}{9}b^2fx^9 + \frac{1}{8}b^2ex^8 + \frac{1}{4}abhx^8 + \frac{1}{7}b^2dx^7 + \frac{2}{7}abgx^7 + \frac{1}{6}b^2cx^6 + \frac{1}{3}abfx^6 + \frac{2}{5}abex^5 + \frac{1}{5}a^2hx^5 + \frac{1}{2}abdx^4 + \frac{1}{4}a^2gx^4 + \frac{2}{3}abcx^3 + \frac{1}{3}a^2fx^3 + \frac{1}{2}a^2ex^2 + a^2dx + a^2c\log(|x|)$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac
")
```

output
```
1/11*b^2*h*x^11 + 1/10*b^2*g*x^10 + 1/9*b^2*f*x^9 + 1/8*b^2*e*x^8 + 1/4*a*
b*h*x^8 + 1/7*b^2*d*x^7 + 2/7*a*b*g*x^7 + 1/6*b^2*c*x^6 + 1/3*a*b*f*x^6 +
2/5*a*b*e*x^5 + 1/5*a^2*h*x^5 + 1/2*a*b*d*x^4 + 1/4*a^2*g*x^4 + 2/3*a*b*c*
x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*e*x^2 + a^2*d*x + a^2*c*log(abs(x))
```

3.388.9 Mupad [B] (verification not implemented)

Time = 10.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx &= x^3\left(\frac{f\,a^2}{3}+\frac{2\,b\,c\,a}{3}\right)+x^6\left(\frac{c\,b^2}{6}+\frac{a\,f\,b}{3}\right)+x^4\left(\frac{g\,a^2}{4}+\frac{b\,d\,a}{2}\right)\\&\quad+x^7\left(\frac{d\,b^2}{7}+\frac{2\,a\,g\,b}{7}\right)+x^5\left(\frac{h\,a^2}{5}+\frac{2\,b\,e\,a}{5}\right)+x^8\left(\frac{e\,b^2}{8}+\frac{a\,h\,b}{4}\right)\\&\quad+\frac{a^2\,e\,x^2}{2}+\frac{b^2\,f\,x^9}{9}+\frac{b^2\,g\,x^{10}}{10}+\frac{b^2\,h\,x^{11}}{11}+a^2\,c\,\ln(x)+a^2\,d\,x\end{aligned}$$

input
```
int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x)
```

output
```
x^3*((a^2*f)/3 + (2*a*b*c)/3) + x^6*((b^2*c)/6 + (a*b*f)/3) + x^4*((a^2*g)
/4 + (a*b*d)/2) + x^7*((b^2*d)/7 + (2*a*b*g)/7) + x^5*((a^2*h)/5 + (2*a*b*
e)/5) + x^8*((b^2*e)/8 + (a*b*h)/4) + (a^2*e*x^2)/2 + (b^2*f*x^9)/9 + (b^2
*g*x^10)/10 + (b^2*h*x^11)/11 + a^2*c*log(x) + a^2*d*x
```

3.389 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

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3.389.1 Optimal result

Integrand size = 38, antiderivative size = 147

$$\begin{aligned}&\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx\\&=-\frac{a^2c}{x}+a^2ex+\frac{1}{2}a(2bc+af)x^2+\frac{2}{3}abdx^3+\frac{1}{4}a(2be+ah)x^4+\frac{1}{5}b(bc+2af)x^5\\&\quad+\frac{1}{6}b^2dx^6+\frac{1}{7}b(be+2ah)x^7+\frac{1}{8}b^2fx^8+\frac{1}{10}b^2hx^{10}+\frac{g(a+bx^3)^3}{9b}+a^2d\log(x)\end{aligned}$$

output
```
-a^2*c/x+a^2*e*x+1/2*a*(a*f+2*b*c)*x^2+2/3*a*b*d*x^3+1/4*a*(a*h+2*b*e)*x^4
+1/5*b*(2*a*f+b*c)*x^5+1/6*b^2*d*x^6+1/7*b*(2*a*h+b*e)*x^7+1/8*b^2*f*x^8+1
/10*b^2*h*x^10+1/9*g*(b*x^3+a)^3/b+a^2*d*ln(x)
```

3.389.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03

$$\begin{aligned}&\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx\\&=-\frac{a^2c}{x}+a^2ex+\frac{1}{2}a(2bc+af)x^2+\frac{1}{3}a(2bd+ag)x^3+\frac{1}{4}a(2be+ah)x^4+\frac{1}{5}b(bc+2af)x^5\\&\quad+\frac{1}{6}b(bd+2ag)x^6+\frac{1}{7}b(be+2ah)x^7+\frac{1}{8}b^2fx^8+\frac{1}{9}b^2gx^9+\frac{1}{10}b^2hx^{10}+a^2d\log(x)\end{aligned}$$

input
```
Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

output
```
-((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (a*(2*b*d + a*g)*x^3)/3
 + (a*(2*b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b*(b*d + 2*a*g)*x^
6)/6 + (b*(b*e + 2*a*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*g*x^9)/9 + (b^2*h*x^
10)/10 + a^2*d*Log[x]
```

3.389.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2018, 2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^2} \, dx$$

$$\Big\downarrow \text{2018}$$

$$\int \frac{\left(bx^3+a\right)^2 \left(hx^5+fx^3+ex^2+dx+c\right)}{x^2}dx+\frac{g\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \text{2360}$$

$$\int \Big(b^2hx^9+b^2fx^7+b(be+2ah)x^6+b^2dx^5+b(bc+2af)x^4+a(2be+ah)x^3+2abdx^2+a(2bc+af)x+a^2e+$$
$$\frac{g\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{1}{5}bx^5(2af+bc)+\frac{1}{2}ax^2(af+2bc)+\frac{2}{3}abdx^3+\frac{1}{7}bx^7(2ah+be)+$$
$$\frac{1}{4}ax^4(ah+2be)+\frac{g\left(a+bx^3\right)^3}{9b}+\frac{1}{6}b^2dx^6+\frac{1}{8}b^2fx^8+\frac{1}{10}b^2hx^{10}$$

input
```
Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

output
```
-((a^2*c)/x) + a^2*e*x + (a*(2*b*c + a*f)*x^2)/2 + (2*a*b*d*x^3)/3 + (a*(2
*b*e + a*h)*x^4)/4 + (b*(b*c + 2*a*f)*x^5)/5 + (b^2*d*x^6)/6 + (b*(b*e + 2
*a*h)*x^7)/7 + (b^2*f*x^8)/8 + (b^2*h*x^10)/10 + (g*(a + b*x^3)^3)/(9*b) +
 a^2*d*Log[x]
```

3.389.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.389.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02

method	result
norman	$\frac{\left(\frac{1}{2}a^2f+abc\right)x^3+\left(\frac{1}{3}a^2g+\frac{2}{3}abd\right)x^4+\left(\frac{1}{4}a^2h+\frac{1}{2}aeb\right)x^5+\left(\frac{1}{3}abg+\frac{1}{6}b^2d\right)x^7+\left(\frac{2}{7}abh+\frac{1}{7}b^2e\right)x^8+\left(\frac{2}{5}afb+\frac{1}{5}b^2c\right)x^6+a^2e\,x^2-a^2c+}{x}$
default	$\frac{b^2h\,x^{10}}{10}+\frac{b^2g\,x^9}{9}+\frac{b^2f\,x^8}{8}+\frac{2abh\,x^7}{7}+\frac{b^2e\,x^7}{7}+\frac{abg\,x^6}{3}+\frac{b^2d\,x^6}{6}+\frac{2abf\,x^5}{5}+\frac{b^2c\,x^5}{5}+\frac{a^2h\,x^4}{4}+\frac{abe\,x^4}{2}+$
risch	$\frac{b^2h\,x^{10}}{10}+\frac{b^2g\,x^9}{9}+\frac{b^2f\,x^8}{8}+\frac{2abh\,x^7}{7}+\frac{b^2e\,x^7}{7}+\frac{abg\,x^6}{3}+\frac{b^2d\,x^6}{6}+\frac{2abf\,x^5}{5}+\frac{b^2c\,x^5}{5}+\frac{a^2h\,x^4}{4}+\frac{abe\,x^4}{2}+$
parallelrisch	$\frac{252b^2h\,x^{11}+280b^2g\,x^{10}+315b^2f\,x^9+720abh\,x^8+360b^2e\,x^8+840abg\,x^7+420b^2d\,x^7+1008abf\,x^6+504b^2c\,x^6+630a^2h\,x^5+1260}{2520x}$

input
```
int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x,method=_RETURNVERBOS
E)
```

output
```
((1/2*a^2*f+a*b*c)*x^3+(1/3*a^2*g+2/3*a*b*d)*x^4+(1/4*a^2*h+1/2*a*e*b)*x^5
+(1/3*a*b*g+1/6*b^2*d)*x^7+(2/7*a*b*h+1/7*b^2*e)*x^8+(2/5*a*f*b+1/5*b^2*c)
*x^6+a^2*e*x^2-a^2*c+1/8*b^2*f*x^9+1/9*b^2*g*x^10+1/10*b^2*h*x^11)/x+a^2*d
*ln(x)
```

3.389.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= \frac{252\,b^2hx^{11}+280\,b^2gx^{10}+315\,b^2fx^9+360\,(b^2e+2\,abh)x^8+420\,(b^2d+2\,abg)x^7+504\,(b^2c+2\,abf)x^6+}{2}$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fr
icas")
```

output
```
1/2520*(252*b^2*h*x^11 + 280*b^2*g*x^10 + 315*b^2*f*x^9 + 360*(b^2*e + 2*a
*b*h)*x^8 + 420*(b^2*d + 2*a*b*g)*x^7 + 504*(b^2*c + 2*a*b*f)*x^6 + 630*(2
*a*b*e + a^2*h)*x^5 + 2520*a^2*e*x^2 + 840*(2*a*b*d + a^2*g)*x^4 + 2520*a^
2*d*x*log(x) + 1260*(2*a*b*c + a^2*f)*x^3 - 2520*a^2*c)/x
```

3.389.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= -\frac{a^2c}{x}+a^2d\log(x)+a^2ex+\frac{b^2fx^8}{8}+\frac{b^2gx^9}{9}+\frac{b^2hx^{10}}{10}+x^7\cdot\left(\frac{2abh}{7}+\frac{b^2e}{7}\right)+x^6\left(\frac{abg}{3}+\frac{b^2d}{6}\right)$$
$$+x^5\cdot\left(\frac{2abf}{5}+\frac{b^2c}{5}\right)+x^4\left(\frac{a^2h}{4}+\frac{abe}{2}\right)+x^3\left(\frac{a^2g}{3}+\frac{2abd}{3}\right)+x^2\left(\frac{a^2f}{2}+abc\right)$$

input
```
integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

output
```
-a**2*c/x + a**2*d*log(x) + a**2*e*x + b**2*f*x**8/8 + b**2*g*x**9/9 + b**
2*h*x**10/10 + x**7*(2*a*b*h/7 + b**2*e/7) + x**6*(a*b*g/3 + b**2*d/6) + x
**5*(2*a*b*f/5 + b**2*c/5) + x**4*(a**2*h/4 + a*b*e/2) + x**3*(a**2*g/3 +
2*a*b*d/3) + x**2*(a**2*f/2 + a*b*c)
```

3.389.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^2\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$=\frac{1}{10}\,b^2hx^{10}+\frac{1}{9}\,b^2gx^9+\frac{1}{8}\,b^2fx^8+\frac{1}{7}\,(b^2e+2\,abh)x^7+\frac{1}{6}\,(b^2d+2\,abg)x^6+\frac{1}{5}\,(b^2c+2\,abf)x^5+\frac{1}{4}\,(2\,abe+a^2h)x^4+a^2ex+\frac{1}{3}\,(2\,abd+a^2g)x^3+a^2d\log(x)+\frac{1}{2}\,(2\,abc+a^2f)x^2-\frac{a^2c}{x}$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="ma
xima")
```

output
```
1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 1/7*(b^2*e + 2*a*b*h)*x^
7 + 1/6*(b^2*d + 2*a*b*g)*x^6 + 1/5*(b^2*c + 2*a*b*f)*x^5 + 1/4*(2*a*b*e +
 a^2*h)*x^4 + a^2*e*x + 1/3*(2*a*b*d + a^2*g)*x^3 + a^2*d*log(x) + 1/2*(2*
a*b*c + a^2*f)*x^2 - a^2*c/x
```

3.389.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^2\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$=\frac{1}{10}\,b^2hx^{10}+\frac{1}{9}\,b^2gx^9+\frac{1}{8}\,b^2fx^8+\frac{1}{7}\,b^2ex^7+\frac{2}{7}\,abhx^7+\frac{1}{6}\,b^2dx^6+\frac{1}{3}\,abgx^6+\frac{1}{5}\,b^2cx^5+\frac{2}{5}\,abfx^5+\frac{1}{2}\,abex^4+\frac{1}{4}\,a^2hx^4+\frac{2}{3}\,abdx^3+\frac{1}{3}\,a^2gx^3+abcx^2+\frac{1}{2}\,a^2fx^2+a^2ex+a^2d\log(|x|)-\frac{a^2c}{x}$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="gi
ac")
```

output
```
1/10*b^2*h*x^10 + 1/9*b^2*g*x^9 + 1/8*b^2*f*x^8 + 1/7*b^2*e*x^7 + 2/7*a*b*
h*x^7 + 1/6*b^2*d*x^6 + 1/3*a*b*g*x^6 + 1/5*b^2*c*x^5 + 2/5*a*b*f*x^5 + 1/
2*a*b*e*x^4 + 1/4*a^2*h*x^4 + 2/3*a*b*d*x^3 + 1/3*a^2*g*x^3 + a*b*c*x^2 +
1/2*a^2*f*x^2 + a^2*e*x + a^2*d*log(abs(x)) - a^2*c/x
```

3.389.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= x^2\left(\frac{f\,a^2}{2}+b\,c\,a\right)+x^5\left(\frac{c\,b^2}{5}+\frac{2\,a\,f\,b}{5}\right)+x^3\left(\frac{g\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right)$$
$$+x^6\left(\frac{d\,b^2}{6}+\frac{a\,g\,b}{3}\right)+x^4\left(\frac{h\,a^2}{4}+\frac{b\,e\,a}{2}\right)+x^7\left(\frac{e\,b^2}{7}+\frac{2\,a\,h\,b}{7}\right)$$
$$-\frac{a^2\,c}{x}+\frac{b^2\,f\,x^8}{8}+\frac{b^2\,g\,x^9}{9}+\frac{b^2\,h\,x^{10}}{10}+a^2\,d\,\ln(x)+a^2\,e\,x$$

input
```
int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x)
```

output
```
x^2*((a^2*f)/2 + a*b*c) + x^5*((b^2*c)/5 + (2*a*b*f)/5) + x^3*((a^2*g)/3 +
 (2*a*b*d)/3) + x^6*((b^2*d)/6 + (a*b*g)/3) + x^4*((a^2*h)/4 + (a*b*e)/2)
+ x^7*((b^2*e)/7 + (2*a*b*h)/7) - (a^2*c)/x + (b^2*f*x^8)/8 + (b^2*g*x^9)/
9 + (b^2*h*x^10)/10 + a^2*d*log(x) + a^2*e*x
```

3.390 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

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3.390.1 Optimal result

Integrand size = 38, antiderivative size = 147

$$\begin{aligned}\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx &= -\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a(2bc+af)x+\frac{1}{2}a(2bd+ag)x^2+\frac{2}{3}abex^3+\frac{1}{4}b(bc+2af)x^4\\&\quad+\frac{1}{5}b(bd+2ag)x^5+\frac{1}{6}b^2ex^6+\frac{1}{7}b^2fx^7+\frac{1}{8}b^2gx^8+\frac{h(a+bx^3)^3}{9b}+a^2e\log(x)\end{aligned}$$

output
```
-1/2*a^2*c/x^2-a^2*d/x+a*(a*f+2*b*c)*x+1/2*a*(a*g+2*b*d)*x^2+2/3*a*b*e*x^3
+1/4*b*(2*a*f+b*c)*x^4+1/5*b*(2*a*g+b*d)*x^5+1/6*b^2*e*x^6+1/7*b^2*f*x^7+1
/8*b^2*g*x^8+1/9*h*(b*x^3+a)^3/b+a^2*e*ln(x)
```

3.390.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx &= \frac{a^2(-3c-6dx+x^3(6f+3gx+2hx^2))}{6x^2}\\&\quad+\frac{1}{30}abx(60c+x(30d+x(20e+15fx+12gx^2+10hx^3)))\\&\quad+\frac{b^2x^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx)))))}{2520}+a^2e\log(x)\end{aligned}$$

input
```
Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

output
```
(a^2*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (a*b*x*(60*c
+ x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))))/30 + (b^2*x^4*(630*
c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x))))))/2520 + a^2*e*L
og[x]
```

3.390.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2018, 2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^3}dx$$

$$\Big\downarrow \text{2018}$$

$$\int \frac{\left(bx^3+a\right)^2 \left(gx^4+fx^3+ex^2+dx+c\right)}{x^3}dx+\frac{h\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \text{2360}$$

$$\int \left(b^2gx^7+b^2fx^6+b^2ex^5+b(bd+2ag)x^4+b(bc+2af)x^3+2abex^2+a(2bd+ag)x+a(2bc+af)+\frac{a^2e}{x}+\frac{a^2}{x^2}\right.$$
$$\frac{h\left(a+bx^3\right)^3}{9b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2c}{2x^2}-\frac{a^2d}{x}+a^2e\log(x)+\frac{1}{4}bx^4(2af+bc)+ax(af+2bc)+\frac{1}{5}bx^5(2ag+bd)+\frac{1}{2}ax^2(ag+2bd)+\frac{2}{3}abex^3+\frac{h\left(a+bx^3\right)^3}{9b}+\frac{1}{6}b^2ex^6+\frac{1}{7}b^2fx^7+\frac{1}{8}b^2gx^8$$

input
```
Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

output
```
-1/2*(a^2*c)/x^2 - (a^2*d)/x + a*(2*b*c + a*f)*x + (a*(2*b*d + a*g)*x^2)/2
 + (2*a*b*e*x^3)/3 + (b*(b*c + 2*a*f)*x^4)/4 + (b*(b*d + 2*a*g)*x^5)/5 + (
b^2*e*x^6)/6 + (b^2*f*x^7)/7 + (b^2*g*x^8)/8 + (h*(a + b*x^3)^3)/(9*b) + a
^2*e*Log[x]
```

3.390.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.390.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

method	result
norman	$\frac{\left(\frac{1}{2}a^{2}g+abd\right)x^{4}+\left(\frac{1}{3}a^{2}h+\frac{2}{3}aeb\right)x^{5}+\left(\frac{2}{5}abg+\frac{1}{5}b^{2}d\right)x^{7}+\left(\frac{1}{3}abh+\frac{1}{6}b^{2}e\right)x^{8}+\left(\frac{1}{2}afb+\frac{1}{4}b^{2}c\right)x^{6}+\left(a^{2}f+2abc\right)x^{3}-\frac{a^{2}c}{2}-a^{2}dx+\frac{b^{2}fx}{7}}{x^{2}}$
default	$\frac{b^{2}h\,x^{9}}{9}+\frac{b^{2}g\,x^{8}}{8}+\frac{b^{2}f\,x^{7}}{7}+\frac{abh\,x^{6}}{3}+\frac{b^{2}e\,x^{6}}{6}+\frac{2abg\,x^{5}}{5}+\frac{b^{2}d\,x^{5}}{5}+\frac{abf\,x^{4}}{2}+\frac{b^{2}c\,x^{4}}{4}+\frac{a^{2}h\,x^{3}}{3}+\frac{2abe\,x^{3}}{3}+a^{2}$
risch	$\frac{b^{2}h\,x^{9}}{9}+\frac{b^{2}g\,x^{8}}{8}+\frac{b^{2}f\,x^{7}}{7}+\frac{abh\,x^{6}}{3}+\frac{b^{2}e\,x^{6}}{6}+\frac{2abg\,x^{5}}{5}+\frac{b^{2}d\,x^{5}}{5}+\frac{abf\,x^{4}}{2}+\frac{b^{2}c\,x^{4}}{4}+\frac{a^{2}h\,x^{3}}{3}+\frac{2abe\,x^{3}}{3}+a^{2}$
parallelrisch	$\frac{280b^{2}h\,x^{11}+315b^{2}g\,x^{10}+360b^{2}f\,x^{9}+840abh\,x^{8}+420b^{2}e\,x^{8}+1008abg\,x^{7}+504b^{2}d\,x^{7}+1260abf\,x^{6}+630b^{2}c\,x^{6}+840a^{2}h\,x^{5}+1680}{2520x^{2}}$

input
```
int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x,method=_RETURNVERBOS
E)
```

output
```
((1/2*a^2*g+a*b*d)*x^4+(1/3*a^2*h+2/3*a*e*b)*x^5+(2/5*a*b*g+1/5*b^2*d)*x^7
+(1/3*a*b*h+1/6*b^2*e)*x^8+(1/2*a*f*b+1/4*b^2*c)*x^6+(a^2*f+2*a*b*c)*x^3-1
/2*a^2*c-a^2*d*x+1/7*b^2*f*x^9+1/8*b^2*g*x^10+1/9*b^2*h*x^11)/x^2+a^2*e*ln
(x)
```

3.390.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= \frac{280\,b^2hx^{11}+315\,b^2gx^{10}+360\,b^2fx^9+420\,(b^2e+2\,abh)x^8+504\,(b^2d+2\,abg)x^7+630\,(b^2c+2\,abf)x^6+}{2}$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fr
icas")
```

output
```
1/2520*(280*b^2*h*x^11 + 315*b^2*g*x^10 + 360*b^2*f*x^9 + 420*(b^2*e + 2*a
*b*h)*x^8 + 504*(b^2*d + 2*a*b*g)*x^7 + 630*(b^2*c + 2*a*b*f)*x^6 + 840*(2
*a*b*e + a^2*h)*x^5 + 2520*a^2*e*x^2*log(x) + 1260*(2*a*b*d + a^2*g)*x^4 -
 2520*a^2*d*x + 2520*(2*a*b*c + a^2*f)*x^3 - 1260*a^2*c)/x^2
```

3.390.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= a^2e\log(x)+\frac{b^2fx^7}{7}+\frac{b^2gx^8}{8}+\frac{b^2hx^9}{9}+x^6\left(\frac{abh}{3}+\frac{b^2e}{6}\right)+x^5\cdot\left(\frac{2abg}{5}+\frac{b^2d}{5}\right)$$
$$+x^4\left(\frac{abf}{2}+\frac{b^2c}{4}\right)+x^3\left(\frac{a^2h}{3}+\frac{2abe}{3}\right)+x^2\left(\frac{a^2g}{2}+abd\right)+x(a^2f+2abc)+\frac{-a^2c-2a^2dx}{2x^2}$$

input
```
integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

output
```
a**2*e*log(x) + b**2*f*x**7/7 + b**2*g*x**8/8 + b**2*h*x**9/9 + x**6*(a*b*
h/3 + b**2*e/6) + x**5*(2*a*b*g/5 + b**2*d/5) + x**4*(a*b*f/2 + b**2*c/4)
+ x**3*(a**2*h/3 + 2*a*b*e/3) + x**2*(a**2*g/2 + a*b*d) + x*(a**2*f + 2*a*
b*c) + (-a**2*c - 2*a**2*d*x)/(2*x**2)
```

3.390.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$=\frac{1}{9}\,b^2hx^9+\frac{1}{8}\,b^2gx^8+\frac{1}{7}\,b^2fx^7+\frac{1}{6}\,(b^2e+2\,abh)x^6+\frac{1}{5}\,(b^2d+2\,abg)x^5+\frac{1}{4}\,(b^2c+2\,abf)x^4+\frac{1}{3}\,(2\,abe+a^2h)x^3+a^2e\log(x)+\frac{1}{2}\,(2\,abd+a^2g)x^2+(2\,abc+a^2f)x-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="ma
xima")
```

output
```
1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/6*(b^2*e + 2*a*b*h)*x^6
+ 1/5*(b^2*d + 2*a*b*g)*x^5 + 1/4*(b^2*c + 2*a*b*f)*x^4 + 1/3*(2*a*b*e + a
^2*h)*x^3 + a^2*e*log(x) + 1/2*(2*a*b*d + a^2*g)*x^2 + (2*a*b*c + a^2*f)*x
 - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

3.390.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$=\frac{1}{9}\,b^2hx^9+\frac{1}{8}\,b^2gx^8+\frac{1}{7}\,b^2fx^7+\frac{1}{6}\,b^2ex^6+\frac{1}{3}\,abhx^6+\frac{1}{5}\,b^2dx^5+\frac{2}{5}\,abgx^5+\frac{1}{4}\,b^2cx^4+\frac{1}{2}\,abfx^4+\frac{2}{3}\,abex^3+\frac{1}{3}\,a^2hx^3+abdx^2+\frac{1}{2}\,a^2gx^2+2\,abcx+a^2fx+a^2e\log(|x|)-\frac{2\,a^2dx+a^2c}{2\,x^2}$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="gi
ac")
```

output
```
1/9*b^2*h*x^9 + 1/8*b^2*g*x^8 + 1/7*b^2*f*x^7 + 1/6*b^2*e*x^6 + 1/3*a*b*h*
x^6 + 1/5*b^2*d*x^5 + 2/5*a*b*g*x^5 + 1/4*b^2*c*x^4 + 1/2*a*b*f*x^4 + 2/3*
a*b*e*x^3 + 1/3*a^2*h*x^3 + a*b*d*x^2 + 1/2*a^2*g*x^2 + 2*a*b*c*x + a^2*f*
x + a^2*e*log(abs(x)) - 1/2*(2*a^2*d*x + a^2*c)/x^2
```

3.390.9 Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^2 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= x\left(f\,a^2+2\,b\,c\,a\right)-\frac{\frac{a^2\,c}{2}+a^2\,d\,x}{x^2}+x^4\left(\frac{c\,b^2}{4}+\frac{a\,f\,b}{2}\right)$$
$$+x^2\left(\frac{g\,a^2}{2}+b\,d\,a\right)+x^5\left(\frac{d\,b^2}{5}+\frac{2\,a\,g\,b}{5}\right)+x^3\left(\frac{h\,a^2}{3}+\frac{2\,b\,e\,a}{3}\right)$$
$$+x^6\left(\frac{e\,b^2}{6}+\frac{a\,h\,b}{3}\right)+\frac{b^2\,f\,x^7}{7}+\frac{b^2\,g\,x^8}{8}+\frac{b^2\,h\,x^9}{9}+a^2\,e\,\ln\left(x\right)$$

input
```
int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x)
```

output
```
x*(a^2*f + 2*a*b*c) - ((a^2*c)/2 + a^2*d*x)/x^2 + x^4*((b^2*c)/4 + (a*b*f)
/2) + x^2*((a^2*g)/2 + a*b*d) + x^5*((b^2*d)/5 + (2*a*b*g)/5) + x^3*((a^2*
h)/3 + (2*a*b*e)/3) + x^6*((b^2*e)/6 + (a*b*h)/3) + (b^2*f*x^7)/7 + (b^2*g
*x^8)/8 + (b^2*h*x^9)/9 + a^2*e*log(x)
```

3.391 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$

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3.391.1 Optimal result

Integrand size = 38, antiderivative size = 152

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = -\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+a(2bd+ag)x+\frac{1}{2}a(2be+ah)x^2+\frac{1}{3}b(bc+2af)x^3+\frac{1}{4}b(bd+2ag)x^4+\frac{1}{5}b(be+2ah)x^5+\frac{1}{6}b^2fx^6+\frac{1}{7}b^2gx^7+\frac{1}{8}b^2hx^8+a(2bc+af)\log(x)$$

output
```
-1/3*a^2*c/x^3-1/2*a^2*d/x^2-a^2*e/x+a*(a*g+2*b*d)*x+1/2*a*(a*h+2*b*e)*x^2
+1/3*b*(2*a*f+b*c)*x^3+1/4*b*(2*a*g+b*d)*x^4+1/5*b*(2*a*h+b*e)*x^5+1/6*b^2
*f*x^6+1/7*b^2*g*x^7+1/8*b^2*h*x^8+a*(a*f+2*b*c)*ln(x)
```

3.391.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = -\frac{a^2(2c+3x(d+2ex-x^3(2g+hx)))}{6x^3}+\frac{1}{30}abx(60d+x(30e+x(20f+15gx+12hx^2)))+\frac{1}{840}b^2x^3(280c+x(210d+x(168e+140fx+120gx^2+105hx^3)))+a(2bc+af)\log(x)$$

input
```
Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

output
```
-1/6*(a^2*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/x^3 + (a*b*x*(60*d +
x*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/30 + (b^2*x^3*(280*c + x*(210*d
+ x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3))))/840 + a*(2*b*c + a*f)*Log
[x]
```

3.391.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$

$$\Big\downarrow \text{2360}$$

$$\int \left(\frac{a^2c}{x^4}+\frac{a^2d}{x^3}+\frac{a^2e}{x^2}+bx^2(2af+bc)+\frac{a(af+2bc)}{x}+bx^3(2ag+bd)+a(ag+2bd)+bx^4(2ah+be)+ax(ah+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2c}{3x^3}-\frac{a^2d}{2x^2}-\frac{a^2e}{x}+\frac{1}{3}bx^3(2af+bc)+a\log(x)(af+2bc)+\frac{1}{4}bx^4(2ag+bd)+ax(ag+2bd)+\frac{1}{5}bx^5(2ah+be)+\frac{1}{2}ax^2(ah+2be)+\frac{1}{6}b^2fx^6+\frac{1}{7}b^2gx^7+\frac{1}{8}b^2hx^8$$

input
```
Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

output
```
-1/3*(a^2*c)/x^3 - (a^2*d)/(2*x^2) - (a^2*e)/x + a*(2*b*d + a*g)*x + (a*(2
*b*e + a*h)*x^2)/2 + (b*(b*c + 2*a*f)*x^3)/3 + (b*(b*d + 2*a*g)*x^4)/4 + (
b*(b*e + 2*a*h)*x^5)/5 + (b^2*f*x^6)/6 + (b^2*g*x^7)/7 + (b^2*h*x^8)/8 + a
*(2*b*c + a*f)*Log[x]
```

3.391.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.391.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^2 h\,x^8}{8}+\frac{b^2 g\,x^7}{7}+\frac{b^2 f\,x^6}{6}+\frac{2abh\,x^5}{5}+\frac{b^2 e\,x^5}{5}+\frac{abg\,x^4}{2}+\frac{b^2 d\,x^4}{4}+\frac{2abf\,x^3}{3}+\frac{b^2 c\,x^3}{3}+\frac{a^2 h\,x^2}{2}+abe\,x^2+a$
norman	$\frac{\left(\frac{1}{2}a^2 h+aeb\right)x^5+\left(\frac{1}{2}abg+\frac{1}{4}b^2 d\right)x^7+\left(\frac{2}{5}abh+\frac{1}{5}b^2 e\right)x^8+\left(\frac{2}{3}afb+\frac{1}{3}b^2 c\right)x^6+\left(a^2 g+2abd\right)x^4-\frac{a^2 c}{3}-\frac{a^2 dx}{2}-a^2 e\,x^2+\frac{b^2 f\,x^9}{6}+\frac{b^2 g\,x^{10}}{7}}{x^3}$
risch	$\frac{b^2 h\,x^8}{8}+\frac{b^2 g\,x^7}{7}+\frac{b^2 f\,x^6}{6}+\frac{2abh\,x^5}{5}+\frac{b^2 e\,x^5}{5}+\frac{abg\,x^4}{2}+\frac{b^2 d\,x^4}{4}+\frac{2abf\,x^3}{3}+\frac{b^2 c\,x^3}{3}+\frac{a^2 h\,x^2}{2}+abe\,x^2+a$
parallelrisch	$\frac{105b^2 h\,x^{11}+120b^2 g\,x^{10}+140b^2 f\,x^9+336abh\,x^8+168b^2 e\,x^8+420abg\,x^7+210b^2 d\,x^7+560abf\,x^6+280b^2 c\,x^6+420a^2 h\,x^5+840abe}{840x^3}$

input
```
int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x,method=_RETURNVERBOS
E)
```

output
```
1/8*b^2*h*x^8+1/7*b^2*g*x^7+1/6*b^2*f*x^6+2/5*a*b*h*x^5+1/5*b^2*e*x^5+1/2*
a*b*g*x^4+1/4*b^2*d*x^4+2/3*a*b*f*x^3+1/3*b^2*c*x^3+1/2*a^2*h*x^2+a*b*e*x^
2+a^2*g*x+2*a*x*b*d+a*(a*f+2*b*c)*ln(x)-1/3*a^2*c/x^3-a^2*e/x-1/2/x^2*a^2*
d
```

3.391.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^2\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$=\frac{105\,b^2hx^{11}+120\,b^2gx^{10}+140\,b^2fx^9+168\,(b^2e+2\,abh)x^8+210\,(b^2d+2\,abg)x^7+280\,(b^2c+2\,abf)x^6+}{840}$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fr
icas")
```

output
```
1/840*(105*b^2*h*x^11 + 120*b^2*g*x^10 + 140*b^2*f*x^9 + 168*(b^2*e + 2*a*
b*h)*x^8 + 210*(b^2*d + 2*a*b*g)*x^7 + 280*(b^2*c + 2*a*b*f)*x^6 + 420*(2*
a*b*e + a^2*h)*x^5 - 840*a^2*e*x^2 + 840*(2*a*b*d + a^2*g)*x^4 + 840*(2*a*
b*c + a^2*f)*x^3*log(x) - 420*a^2*d*x - 280*a^2*c)/x^3
```

3.391.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = a(af+2bc)\log(x)+\frac{b^2fx^6}{6}+\frac{b^2gx^7}{7}+\frac{b^2hx^8}{8}+x^5\cdot\left(\frac{2abh}{5}+\frac{b^2e}{5}\right)+x^4\left(\frac{abg}{2}+\frac{b^2d}{4}\right)+x^3\cdot\left(\frac{2abf}{3}+\frac{b^2c}{3}\right)+x^2\left(\frac{a^2h}{2}+abe\right)+x\left(a^2g+2abd\right)+\frac{-2a^2c-3a^2dx-6a^2ex^2}{6x^3}$$

input
```
integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

output
```
a*(a*f + 2*b*c)*log(x) + b**2*f*x**6/6 + b**2*g*x**7/7 + b**2*h*x**8/8 + x
**5*(2*a*b*h/5 + b**2*e/5) + x**4*(a*b*g/2 + b**2*d/4) + x**3*(2*a*b*f/3 +
 b**2*c/3) + x**2*(a**2*h/2 + a*b*e) + x*(a**2*g + 2*a*b*d) + (-2*a**2*c -
 3*a**2*d*x - 6*a**2*e*x**2)/(6*x**3)
```

3.391.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = \frac{1}{8}b^2hx^8+\frac{1}{7}b^2gx^7+\frac{1}{6}b^2fx^6+\frac{1}{5}\left(b^2e+2\,abh\right)x^5+\frac{1}{4}\left(b^2d+2\,abg\right)x^4+\frac{1}{3}\left(b^2c+2\,abf\right)x^3+\frac{1}{2}\left(2\,abe+a^2h\right)x^2+\left(2\,abd+a^2g\right)x+\left(2\,abc+a^2f\right)\log(x)-\frac{6\,a^2ex^2+3\,a^2dx+2\,a^2c}{6\,x^3}$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="ma
xima")
```

output
```
1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 1/5*(b^2*e + 2*a*b*h)*x^5
+ 1/4*(b^2*d + 2*a*b*g)*x^4 + 1/3*(b^2*c + 2*a*b*f)*x^3 + 1/2*(2*a*b*e + a
^2*h)*x^2 + (2*a*b*d + a^2*g)*x + (2*a*b*c + a^2*f)*log(x) - 1/6*(6*a^2*e*
x^2 + 3*a^2*d*x + 2*a^2*c)/x^3
```

3.391.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= \frac{1}{8}b^2hx^8 + \frac{1}{7}b^2gx^7 + \frac{1}{6}b^2fx^6 + \frac{1}{5}b^2ex^5 + \frac{2}{5}abhx^5 + \frac{1}{4}b^2dx^4 + \frac{1}{2}abgx^4 + \frac{1}{3}b^2cx^3 + \frac{2}{3}abfx^3 + abex^2 + \frac{1}{2}a^2hx^2 + 2abdx + a^2gx + (2abc + a^2f)\log(|x|) - \frac{6a^2ex^2 + 3a^2dx + 2a^2c}{6x^3}$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="gi
ac")
```

output
```
1/8*b^2*h*x^8 + 1/7*b^2*g*x^7 + 1/6*b^2*f*x^6 + 1/5*b^2*e*x^5 + 2/5*a*b*h*
x^5 + 1/4*b^2*d*x^4 + 1/2*a*b*g*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*f*x^3 + a*b*
e*x^2 + 1/2*a^2*h*x^2 + 2*a*b*d*x + a^2*g*x + (2*a*b*c + a^2*f)*log(abs(x)
) - 1/6*(6*a^2*e*x^2 + 3*a^2*d*x + 2*a^2*c)/x^3
```

3.391.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= x\,(g\,a^2 + 2\,b\,d\,a) - \frac{e\,a^2\,x^2 + \frac{d\,a^2\,x}{2} + \frac{c\,a^2}{3}}{x^3} + x^3\left(\frac{c\,b^2}{3} + \frac{2\,a\,f\,b}{3}\right) + x^4\left(\frac{d\,b^2}{4} + \frac{a\,g\,b}{2}\right) + x^2\left(\frac{h\,a^2}{2} + b\,e\,a\right) + x^5\left(\frac{e\,b^2}{5} + \frac{2\,a\,h\,b}{5}\right) + \ln(x)\,\left(f\,a^2 + 2\,b\,c\,a\right) + \frac{b^2\,f\,x^6}{6} + \frac{b^2\,g\,x^7}{7} + \frac{b^2\,h\,x^8}{8}$$

input
```
int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x)
```

output
```
x*(a^2*g + 2*a*b*d) - ((a^2*c)/3 + a^2*e*x^2 + (a^2*d*x)/2)/x^3 + x^3*((b^
2*c)/3 + (2*a*b*f)/3) + x^4*((b^2*d)/4 + (a*b*g)/2) + x^2*((a^2*h)/2 + a*b
*e) + x^5*((b^2*e)/5 + (2*a*b*h)/5) + log(x)*(a^2*f + 2*a*b*c) + (b^2*f*x^
6)/6 + (b^2*g*x^7)/7 + (b^2*h*x^8)/8
```

3.391. $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$

3.392 $\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$

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3.392.2 Mathematica [A] (verified) 2905
3.392.3 Rubi [A] (verified) 2906
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3.392.8 Giac [A] (verification not implemented) 2909
3.392.9 Mupad [B] (verification not implemented) 2909

3.392.1 Optimal result

Integrand size = 38, antiderivative size = 152

$$\begin{aligned}\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx &= -\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}-\frac{a(2bc+af)}{x}+a(2be+ah)x+\frac{1}{2}b(bc+2af)x^2+\frac{1}{3}b(bd+2ag)x^3\\&\quad+\frac{1}{4}b(be+2ah)x^4+\frac{1}{5}b^2fx^5+\frac{1}{6}b^2gx^6+\frac{1}{7}b^2hx^7+a(2bd+ag)\log(x)\end{aligned}$$

output
```
-1/4*a^2*c/x^4-1/3*a^2*d/x^3-1/2*a^2*e/x^2-a*(a*f+2*b*c)/x+a*(a*h+2*b*e)*x
+1/2*b*(2*a*f+b*c)*x^2+1/3*b*(2*a*g+b*d)*x^3+1/4*b*(2*a*h+b*e)*x^4+1/5*b^2
*f*x^5+1/6*b^2*g*x^6+1/7*b^2*h*x^7+a*(a*g+2*b*d)*ln(x)
```

3.392.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

$$\begin{aligned}\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx &= -\frac{2abc}{x}-\frac{a^2(3c+4dx+6x^2(e+2fx-2hx^3))}{12x^4}+\frac{1}{6}abx(12e+x(6f+x(4g+3hx)))\\&\quad+\frac{1}{420}b^2x^2\left(210c+x\left(140d+x\left(105e+84fx+70gx^2+60hx^3\right)\right)\right)+a(2bd+ag)\log(x)\end{aligned}$$

input
```
Integrate[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

output
```
(-2*a*b*c)/x - (a^2*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)))/(12*x^4)
+ (a*b*x*(12*e + x*(6*f + x*(4*g + 3*h*x))))/6 + (b^2*x^2*(210*c + x*(140*
d + x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3))))/420 + a*(2*b*d + a*g)*Log[
x]
```

3.392.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^5}dx$$

$$\Big\downarrow \text{2360}$$

$$\int \left(\frac{a^2c}{x^5}+\frac{a^2d}{x^4}+\frac{a^2e}{x^3}+\frac{a(af+2bc)}{x^2}+bx(2af+bc)+bx^2(2ag+bd)+\frac{a(ag+2bd)}{x}+bx^3(2ah+be)+a(ah+2$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2c}{4x^4}-\frac{a^2d}{3x^3}-\frac{a^2e}{2x^2}+\frac{1}{2}bx^2(2af+bc)-\frac{a(af+2bc)}{x}+\frac{1}{3}bx^3(2ag+bd)+a\log(x)(ag+2bd)+\frac{1}{4}bx^4(2ah+be)+ax(ah+2be)+\frac{1}{5}b^2fx^5+\frac{1}{6}b^2gx^6+\frac{1}{7}b^2hx^7$$

input
```
Int[((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

output
```
-1/4*(a^2*c)/x^4 - (a^2*d)/(3*x^3) - (a^2*e)/(2*x^2) - (a*(2*b*c + a*f))/x
 + a*(2*b*e + a*h)*x + (b*(b*c + 2*a*f)*x^2)/2 + (b*(b*d + 2*a*g)*x^3)/3 +
 (b*(b*e + 2*a*h)*x^4)/4 + (b^2*f*x^5)/5 + (b^2*g*x^6)/6 + (b^2*h*x^7)/7 +
 a*(2*b*d + a*g)*Log[x]
```

3.392.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.392.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95

method	result
default	$\frac{b^2 h\,x^7}{7}+\frac{b^2 g\,x^6}{6}+\frac{f\,x^5 b^2}{5}+\frac{abh\,x^4}{2}+\frac{b^2 e\,x^4}{4}+\frac{2abg\,x^3}{3}+\frac{d\,x^3 b^2}{3}+abf\,x^2+\frac{b^2 c\,x^2}{2}+a^2hx+2abex+a$
norman	$\frac{\left(\frac{2}{3}abg+\frac{1}{3}b^2d\right)x^7+\left(\frac{1}{2}abh+\frac{1}{4}b^2e\right)x^8+\left(afb+\frac{1}{2}b^2c\right)x^6+\left(-a^2f-2abc\right)x^3+\left(a^2h+2aeb\right)x^5-\frac{a^2c}{4}-\frac{a^2dx}{3}-\frac{a^2e\,x^2}{2}+\frac{b^2f\,x^9}{5}+\frac{b^2g\,x^{10}}{6}}{x^4}$
risch	$\frac{b^2 h\,x^7}{7}+\frac{b^2 g\,x^6}{6}+\frac{f\,x^5 b^2}{5}+\frac{abh\,x^4}{2}+\frac{b^2 e\,x^4}{4}+\frac{2abg\,x^3}{3}+\frac{d\,x^3 b^2}{3}+abf\,x^2+\frac{b^2 c\,x^2}{2}+a^2hx+2abex+\frac{(}{}$
parallelrisch	$\frac{60b^2h\,x^{11}+70b^2g\,x^{10}+84b^2f\,x^9+210abh\,x^8+105b^2e\,x^8+280abg\,x^7+140b^2d\,x^7+420abf\,x^6+210b^2c\,x^6+420\ln(x)x^4a^2g+840\ln}{420x^4}$

input
```
int((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x,method=_RETURNVERBOS
E)
```

output
```
1/7*b^2*h*x^7+1/6*b^2*g*x^6+1/5*f*x^5*b^2+1/2*a*b*h*x^4+1/4*b^2*e*x^4+2/3*
a*b*g*x^3+1/3*d*x^3*b^2+a*b*f*x^2+1/2*b^2*c*x^2+a^2*h*x+2*a*b*e*x+a*(a*g+2
*b*d)*ln(x)-1/3*a^2*d/x^3-a*(a*f+2*b*c)/x-1/2*a^2*e/x^2-1/4*a^2*c/x^4
```

3.392.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^2\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$=\frac{60\,b^2hx^{11}+70\,b^2gx^{10}+84\,b^2fx^9+105\,(b^2e+2\,abh)x^8+140\,(b^2d+2\,abg)x^7+210\,(b^2c+2\,abf)x^6+420}{420\,x}$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fr
icas")
```

output
```
1/420*(60*b^2*h*x^11 + 70*b^2*g*x^10 + 84*b^2*f*x^9 + 105*(b^2*e + 2*a*b*h
)*x^8 + 140*(b^2*d + 2*a*b*g)*x^7 + 210*(b^2*c + 2*a*b*f)*x^6 + 420*(2*a*b
*e + a^2*h)*x^5 + 420*(2*a*b*d + a^2*g)*x^4*log(x) - 210*a^2*e*x^2 - 140*a
^2*d*x - 420*(2*a*b*c + a^2*f)*x^3 - 105*a^2*c)/x^4
```

3.392.6 Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = a(ag+2bd)\log(x)+\frac{b^2fx^5}{5}+\frac{b^2gx^6}{6}+\frac{b^2hx^7}{7}+x^4\left(\frac{abh}{2}+\frac{b^2e}{4}\right)+x^3\cdot\left(\frac{2abg}{3}+\frac{b^2d}{3}\right)+x^2\left(abf+\frac{b^2c}{2}\right)+x\left(a^2h+2abe\right)+\frac{-3a^2c-4a^2dx-6a^2ex^2+x^3(-12a^2f-24abc)}{12x^4}$$

input
```
integrate((b*x**3+a)**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

output
```
a*(a*g + 2*b*d)*log(x) + b**2*f*x**5/5 + b**2*g*x**6/6 + b**2*h*x**7/7 + x
**4*(a*b*h/2 + b**2*e/4) + x**3*(2*a*b*g/3 + b**2*d/3) + x**2*(a*b*f + b**
2*c/2) + x*(a**2*h + 2*a*b*e) + (-3*a**2*c - 4*a**2*d*x - 6*a**2*e*x**2 +
x**3*(-12*a**2*f - 24*a*b*c))/(12*x**4)
```

3.392.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx = \frac{1}{7}b^2hx^7+\frac{1}{6}b^2gx^6+\frac{1}{5}b^2fx^5+\frac{1}{4}\left(b^2e+2\,abh\right)x^4+\frac{1}{3}\left(b^2d+2\,abg\right)x^3+\frac{1}{2}\left(b^2c+2\,abf\right)x^2+\left(2\,abe+a^2h\right)x+\left(2\,abd+a^2g\right)\log(x)-\frac{6\,a^2ex^2+4\,a^2dx+12\,(2\,abc+a^2f)x^3+3\,a^2c}{12\,x^4}$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="ma
xima")
```

output
```
1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/4*(b^2*e + 2*a*b*h)*x^4
+ 1/3*(b^2*d + 2*a*b*g)*x^3 + 1/2*(b^2*c + 2*a*b*f)*x^2 + (2*a*b*e + a^2*h
)*x + (2*a*b*d + a^2*g)*log(x) - 1/12*(6*a^2*e*x^2 + 4*a^2*d*x + 12*(2*a*b
*c + a^2*f)*x^3 + 3*a^2*c)/x^4
```

3.392.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^5}\,dx$$
$$= \frac{1}{7}b^2hx^7 + \frac{1}{6}b^2gx^6 + \frac{1}{5}b^2fx^5 + \frac{1}{4}b^2ex^4 + \frac{1}{2}abhx^4 + \frac{1}{3}b^2dx^3 + \frac{2}{3}abgx^3 + \frac{1}{2}b^2cx^2 + abfx^2 + 2abex + a^2hx + \left(2abd + a^2g\right)\log\left(|x|\right) - \frac{6a^2ex^2 + 4a^2dx + 12\left(2abc + a^2f\right)x^3 + 3a^2c}{12x^4}$$

input
```
integrate((b*x^3+a)^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="gi
ac")
```

output
```
1/7*b^2*h*x^7 + 1/6*b^2*g*x^6 + 1/5*b^2*f*x^5 + 1/4*b^2*e*x^4 + 1/2*a*b*h*
x^4 + 1/3*b^2*d*x^3 + 2/3*a*b*g*x^3 + 1/2*b^2*c*x^2 + a*b*f*x^2 + 2*a*b*e*
x + a^2*h*x + (2*a*b*d + a^2*g)*log(abs(x)) - 1/12*(6*a^2*e*x^2 + 4*a^2*d*
x + 12*(2*a*b*c + a^2*f)*x^3 + 3*a^2*c)/x^4
```

3.392.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^5}\,dx$$
$$= x\left(h\,a^2 + 2\,b\,e\,a\right) - \frac{\frac{a^2\,c}{4} + x^3\left(f\,a^2 + 2\,b\,c\,a\right) + \frac{a^2\,e\,x^2}{2} + \frac{a^2\,d\,x}{3}}{x^4} + x^2\left(\frac{c\,b^2}{2} + a\,f\,b\right) + x^3\left(\frac{d\,b^2}{3} + \frac{2\,a\,g\,b}{3}\right) + x^4\left(\frac{e\,b^2}{4} + \frac{a\,h\,b}{2}\right) + \ln\left(x\right)\,\left(g\,a^2 + 2\,b\,d\,a\right) + \frac{b^2\,f\,x^5}{5} + \frac{b^2\,g\,x^6}{6} + \frac{b^2\,h\,x^7}{7}$$

input
```
int(((a + b*x^3)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x)
```

output
```
x*(a^2*h + 2*a*b*e) - ((a^2*c)/4 + x^3*(a^2*f + 2*a*b*c) + (a^2*e*x^2)/2 +
 (a^2*d*x)/3)/x^4 + x^2*((b^2*c)/2 + a*b*f) + x^3*((b^2*d)/3 + (2*a*b*g)/3
) + x^4*((b^2*e)/4 + (a*b*h)/2) + log(x)*(a^2*g + 2*a*b*d) + (b^2*f*x^5)/5
 + (b^2*g*x^6)/6 + (b^2*h*x^7)/7
```

3.392. $\int \frac{\left(a+bx^3\right)^2 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^5} \, dx$

3.393 $\int x^4(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

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3.393.1 Optimal result

Integrand size = 38, antiderivative size = 223

$$\begin{aligned}\int x^4(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2(3bc+af)x^8+\frac{1}{9}a^2(3bd+ag)x^9+\frac{1}{10}a^2(3be+ah)x^{10}\\ &+\frac{3}{11}ab(bc+af)x^{11}+\frac{1}{4}ab(bd+ag)x^{12}+\frac{3}{13}ab(be+ah)x^{13}+\frac{1}{14}b^2(bc+3af)x^{14}\\ &+\frac{1}{15}b^2(bd+3ag)x^{15}+\frac{1}{16}b^2(be+3ah)x^{16}+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}\end{aligned}$$

output
```
1/5*a^3*c*x^5+1/6*a^3*d*x^6+1/7*a^3*e*x^7+1/8*a^2*(a*f+3*b*c)*x^8+1/9*a^2*
(a*g+3*b*d)*x^9+1/10*a^2*(a*h+3*b*e)*x^10+3/11*a*b*(a*f+b*c)*x^11+1/4*a*b*
(a*g+b*d)*x^12+3/13*a*b*(a*h+b*e)*x^13+1/14*b^2*(3*a*f+b*c)*x^14+1/15*b^2*
(3*a*g+b*d)*x^15+1/16*b^2*(3*a*h+b*e)*x^16+1/17*b^3*f*x^17+1/18*b^3*g*x^18
+1/19*b^3*h*x^19
```

3.393.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2(3bc+af)x^8+\frac{1}{9}a^2(3bd+ag)x^9+\frac{1}{10}a^2(3be+ah)x^{10} + \frac{3}{11}ab(bc+af)x^{11}+\frac{1}{4}ab(bd+ag)x^{12}+\frac{3}{13}ab(be+ah)x^{13}+\frac{1}{14}b^2(bc+3af)x^{14} + \frac{1}{15}b^2(bd+3ag)x^{15}+\frac{1}{16}b^2(be+3ah)x^{16}+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}$$

input
```
Integrate[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8
+ (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/10 + (3*a*b*(b*c +
a*f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a*b*(b*e + a*h)*x^13)/13 + (
b^2*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a
*h)*x^16)/16 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

3.393.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$

$$\Big\downarrow \text{2360}$$

$$\int (a^3cx^4+a^3dx^5+a^3ex^6+a^2x^7(af+3bc)+a^2x^8(ag+3bd)+a^2x^9(ah+3be)+b^2x^{13}(3af+bc)+b^2x^{14}(3ag$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^3cx^5+\frac{1}{6}a^3dx^6+\frac{1}{7}a^3ex^7+\frac{1}{8}a^2x^8(af+3bc)+\frac{1}{9}a^2x^9(ag+3bd)+\frac{1}{10}a^2x^{10}(ah+3be)+\frac{1}{14}b^2x^{14}(3af+bc)+\frac{1}{15}b^2x^{15}(3ag+bd)+\frac{1}{16}b^2x^{16}(3ah+be)+\frac{3}{11}abx^{11}(af+bc)+\frac{1}{4}abx^{12}(ag+bd)+\frac{3}{13}abx^{13}(ah+be)+\frac{1}{17}b^3fx^{17}+\frac{1}{18}b^3gx^{18}+\frac{1}{19}b^3hx^{19}$$

input
```
Int[x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^3*c*x^5)/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (a^2*(3*b*c + a*f)*x^8)/8
+ (a^2*(3*b*d + a*g)*x^9)/9 + (a^2*(3*b*e + a*h)*x^10)/10 + (3*a*b*(b*c +
a*f)*x^11)/11 + (a*b*(b*d + a*g)*x^12)/4 + (3*a*b*(b*e + a*h)*x^13)/13 + (
b^2*(b*c + 3*a*f)*x^14)/14 + (b^2*(b*d + 3*a*g)*x^15)/15 + (b^2*(b*e + 3*a
*h)*x^16)/16 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (b^3*h*x^19)/19
```

3.393.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.393.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.99

method	result
norman	$\frac{a^3c\,x^5}{5}+\frac{a^3d\,x^6}{6}+\frac{a^3e\,x^7}{7}+\left(\frac{1}{8}f\,a^3+\frac{3}{8}a^2bc\right)x^8+\left(\frac{1}{9}a^3g+\frac{1}{3}d\,a^2b\right)x^9+\left(\frac{1}{10}a^3h+\frac{3}{10}a^2be\right)x^{10}+$
default	$\frac{b^3h\,x^{19}}{19}+\frac{b^3g\,x^{18}}{18}+\frac{b^3f\,x^{17}}{17}+\frac{(3a\,b^2h+b^3e)x^{16}}{16}+\frac{(3a\,b^2g+b^3d)x^{15}}{15}+\frac{(3a\,b^2f+b^3c)x^{14}}{14}+\frac{(3a^2bh+3a\,b^2e)x^{13}}{13}+$
gosper	$\frac{1}{5}a^3c\,x^5+\frac{1}{6}a^3d\,x^6+\frac{1}{7}a^3e\,x^7+\frac{1}{8}x^8f\,a^3+\frac{3}{8}x^8a^2bc+\frac{1}{9}x^9a^3g+\frac{1}{3}a^2bd\,x^9+\frac{1}{10}x^{10}a^3h+\frac{3}{10}a^2be$
risch	$\frac{1}{5}a^3c\,x^5+\frac{1}{6}a^3d\,x^6+\frac{1}{7}a^3e\,x^7+\frac{1}{8}x^8f\,a^3+\frac{3}{8}x^8a^2bc+\frac{1}{9}x^9a^3g+\frac{1}{3}a^2bd\,x^9+\frac{1}{10}x^{10}a^3h+\frac{3}{10}a^2be$
parallelrisch	$\frac{1}{5}a^3c\,x^5+\frac{1}{6}a^3d\,x^6+\frac{1}{7}a^3e\,x^7+\frac{1}{8}x^8f\,a^3+\frac{3}{8}x^8a^2bc+\frac{1}{9}x^9a^3g+\frac{1}{3}a^2bd\,x^9+\frac{1}{10}x^{10}a^3h+\frac{3}{10}a^2be$

input
```
int(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOS
E)
```

output
```
1/5*a^3*c*x^5+1/6*a^3*d*x^6+1/7*a^3*e*x^7+(1/8*f*a^3+3/8*a^2*b*c)*x^8+(1/9
*a^3*g+1/3*d*a^2*b)*x^9+(1/10*a^3*h+3/10*a^2*b*e)*x^10+(3/11*f*a^2*b+3/11*
a*b^2*c)*x^11+(1/4*a^2*b*g+1/4*a*b^2*d)*x^12+(3/13*a^2*b*h+3/13*a*b^2*e)*x
^13+(3/14*a*b^2*f+1/14*b^3*c)*x^14+(1/5*a*b^2*g+1/15*b^3*d)*x^15+(3/16*a*b
^2*h+1/16*b^3*e)*x^16+1/17*b^3*f*x^17+1/18*b^3*g*x^18+1/19*b^3*h*x^19
```

3.393.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$
\begin{aligned}
&\int x^4(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx \\
&= \frac{1}{19}b^3hx^{19} + \frac{1}{18}b^3gx^{18} + \frac{1}{17}b^3fx^{17} + \frac{1}{16}(b^3e+3ab^2h)x^{16} \\
&+ \frac{1}{15}(b^3d+3ab^2g)x^{15} + \frac{1}{14}(b^3c+3ab^2f)x^{14} + \frac{3}{13}(ab^2e+a^2bh)x^{13} \\
&+ \frac{1}{4}(ab^2d+a^2bg)x^{12} + \frac{3}{11}(ab^2c+a^2bf)x^{11} + \frac{1}{7}a^3ex^7 + \frac{1}{10}(3a^2be+a^3h)x^{10} \\
&+ \frac{1}{6}a^3dx^6 + \frac{1}{9}(3a^2bd+a^3g)x^9 + \frac{1}{5}a^3cx^5 + \frac{1}{8}(3a^2bc+a^3f)x^8
\end{aligned}
$$

input
```
integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fr
icas")
```

output
```
1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 1/16*(b^3*e + 3*a*b^
2*h)*x^16 + 1/15*(b^3*d + 3*a*b^2*g)*x^15 + 1/14*(b^3*c + 3*a*b^2*f)*x^14
+ 3/13*(a*b^2*e + a^2*b*h)*x^13 + 1/4*(a*b^2*d + a^2*b*g)*x^12 + 3/11*(a*b
^2*c + a^2*b*f)*x^11 + 1/7*a^3*e*x^7 + 1/10*(3*a^2*b*e + a^3*h)*x^10 + 1/6
*a^3*d*x^6 + 1/9*(3*a^2*b*d + a^3*g)*x^9 + 1/5*a^3*c*x^5 + 1/8*(3*a^2*b*c
+ a^3*f)*x^8
```

3.393.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.10

$$\int x^4(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{a^3cx^5}{5}+\frac{a^3dx^6}{6}+\frac{a^3ex^7}{7}+\frac{b^3fx^{17}}{17}+\frac{b^3gx^{18}}{18}+\frac{b^3hx^{19}}{19}+x^{16}\cdot\left(\frac{3ab^2h}{16}+\frac{b^3e}{16}\right)+x^{15}\left(\frac{ab^2g}{5}+\frac{b^3d}{15}\right)+x^{14}\cdot\left(\frac{3ab^2f}{14}+\frac{b^3c}{14}\right)+x^{13}\cdot\left(\frac{3a^2bh}{13}+\frac{3ab^2e}{13}\right)+x^{12}\left(\frac{a^2bg}{4}+\frac{ab^2d}{4}\right)+x^{11}\cdot\left(\frac{3a^2bf}{11}+\frac{3ab^2c}{11}\right)+x^{10}\left(\frac{a^3h}{10}+\frac{3a^2be}{10}\right)+x^9\left(\frac{a^3g}{9}+\frac{a^2bd}{3}\right)+x^8\left(\frac{a^3f}{8}+\frac{3a^2bc}{8}\right)$$

input
```
integrate(x**4*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**3*c*x**5/5 + a**3*d*x**6/6 + a**3*e*x**7/7 + b**3*f*x**17/17 + b**3*g*x
**18/18 + b**3*h*x**19/19 + x**16*(3*a*b**2*h/16 + b**3*e/16) + x**15*(a*b
**2*g/5 + b**3*d/15) + x**14*(3*a*b**2*f/14 + b**3*c/14) + x**13*(3*a**2*b
*h/13 + 3*a*b**2*e/13) + x**12*(a**2*b*g/4 + a*b**2*d/4) + x**11*(3*a**2*b
*f/11 + 3*a*b**2*c/11) + x**10*(a**3*h/10 + 3*a**2*b*e/10) + x**9*(a**3*g/
9 + a**2*b*d/3) + x**8*(a**3*f/8 + 3*a**2*b*c/8)
```

3.393.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\int x^4(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{19}b^3hx^{19}+\frac{1}{18}b^3gx^{18}+\frac{1}{17}b^3fx^{17}+\frac{1}{16}(b^3e+3ab^2h)x^{16}+\frac{1}{15}(b^3d+3ab^2g)x^{15}+\frac{1}{14}(b^3c+3ab^2f)x^{14}+\frac{3}{13}(ab^2e+a^2bh)x^{13}+\frac{1}{4}(ab^2d+a^2bg)x^{12}+\frac{3}{11}(ab^2c+a^2bf)x^{11}+\frac{1}{7}a^3ex^7+\frac{1}{10}(3a^2be+a^3h)x^{10}+\frac{1}{6}a^3dx^6+\frac{1}{9}(3a^2bd+a^3g)x^9+\frac{1}{5}a^3cx^5+\frac{1}{8}(3a^2bc+a^3f)x^8$$

input
```
integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="ma
xima")
```

output
```
1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 1/16*(b^3*e + 3*a*b^
2*h)*x^16 + 1/15*(b^3*d + 3*a*b^2*g)*x^15 + 1/14*(b^3*c + 3*a*b^2*f)*x^14
+ 3/13*(a*b^2*e + a^2*b*h)*x^13 + 1/4*(a*b^2*d + a^2*b*g)*x^12 + 3/11*(a*b
^2*c + a^2*b*f)*x^11 + 1/7*a^3*e*x^7 + 1/10*(3*a^2*b*e + a^3*h)*x^10 + 1/6
*a^3*d*x^6 + 1/9*(3*a^2*b*d + a^3*g)*x^9 + 1/5*a^3*c*x^5 + 1/8*(3*a^2*b*c
+ a^3*f)*x^8
```

3.393.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.03

$$\begin{aligned}\int x^4(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx\\ =\ &\frac{1}{19}\,b^3hx^{19}+\frac{1}{18}\,b^3gx^{18}+\frac{1}{17}\,b^3fx^{17}+\frac{1}{16}\,b^3ex^{16}+\frac{3}{16}\,ab^2hx^{16}+\frac{1}{15}\,b^3dx^{15}\\ &+\frac{1}{5}\,ab^2gx^{15}+\frac{1}{14}\,b^3cx^{14}+\frac{3}{14}\,ab^2fx^{14}+\frac{3}{13}\,ab^2ex^{13}+\frac{3}{13}\,a^2bhx^{13}+\frac{1}{4}\,ab^2dx^{12}\\ &+\frac{1}{4}\,a^2bgx^{12}+\frac{3}{11}\,ab^2cx^{11}+\frac{3}{11}\,a^2bfx^{11}+\frac{3}{10}\,a^2bex^{10}+\frac{1}{10}\,a^3hx^{10}\\ &+\frac{1}{3}\,a^2bdx^9+\frac{1}{9}\,a^3gx^9+\frac{3}{8}\,a^2bcx^8+\frac{1}{8}\,a^3fx^8+\frac{1}{7}\,a^3ex^7+\frac{1}{6}\,a^3dx^6+\frac{1}{5}\,a^3cx^5\end{aligned}$$

input
```
integrate(x^4*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gi
ac")
```

output
```
1/19*b^3*h*x^19 + 1/18*b^3*g*x^18 + 1/17*b^3*f*x^17 + 1/16*b^3*e*x^16 + 3/
16*a*b^2*h*x^16 + 1/15*b^3*d*x^15 + 1/5*a*b^2*g*x^15 + 1/14*b^3*c*x^14 + 3
/14*a*b^2*f*x^14 + 3/13*a*b^2*e*x^13 + 3/13*a^2*b*h*x^13 + 1/4*a*b^2*d*x^1
2 + 1/4*a^2*b*g*x^12 + 3/11*a*b^2*c*x^11 + 3/11*a^2*b*f*x^11 + 3/10*a^2*b*
e*x^10 + 1/10*a^3*h*x^10 + 1/3*a^2*b*d*x^9 + 1/9*a^3*g*x^9 + 3/8*a^2*b*c*x
^8 + 1/8*a^3*f*x^8 + 1/7*a^3*e*x^7 + 1/6*a^3*d*x^6 + 1/5*a^3*c*x^5
```

3.393.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.92

$$\int x^4(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=x^8\left(\frac{f\,a^3}{8}+\frac{3\,b\,c\,a^2}{8}\right)+x^{14}\left(\frac{c\,b^3}{14}+\frac{3\,a\,f\,b^2}{14}\right)+x^9\left(\frac{g\,a^3}{9}+\frac{b\,d\,a^2}{3}\right)+x^{15}\left(\frac{d\,b^3}{15}+\frac{a\,g\,b^2}{5}\right)+x^{10}\left(\frac{h\,a^3}{10}+\frac{3\,b\,e\,a^2}{10}\right)+x^{16}\left(\frac{e\,b^3}{16}+\frac{3\,a\,h\,b^2}{16}\right)+\frac{a^3\,c\,x^5}{5}+\frac{a^3\,d\,x^6}{6}+\frac{a^3\,e\,x^7}{7}+\frac{b^3\,f\,x^{17}}{17}+\frac{b^3\,g\,x^{18}}{18}+\frac{b^3\,h\,x^{19}}{19}+\frac{3\,a\,b\,x^{11}\,(b\,c+a\,f)}{11}+\frac{a\,b\,x^{12}\,(b\,d+a\,g)}{4}+\frac{3\,a\,b\,x^{13}\,(b\,e+a\,h)}{13}$$

input
```
int(x^4*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^8*((a^3*f)/8 + (3*a^2*b*c)/8) + x^14*((b^3*c)/14 + (3*a*b^2*f)/14) + x^9
*((a^3*g)/9 + (a^2*b*d)/3) + x^15*((b^3*d)/15 + (a*b^2*g)/5) + x^10*((a^3*
h)/10 + (3*a^2*b*e)/10) + x^16*((b^3*e)/16 + (3*a*b^2*h)/16) + (a^3*c*x^5)
/5 + (a^3*d*x^6)/6 + (a^3*e*x^7)/7 + (b^3*f*x^17)/17 + (b^3*g*x^18)/18 + (
b^3*h*x^19)/19 + (3*a*b*x^11*(b*c + a*f))/11 + (a*b*x^12*(b*d + a*g))/4 +
(3*a*b*x^13*(b*e + a*h))/13
```

3.394 $\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

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3.394.1 Optimal result

Integrand size = 38, antiderivative size = 223

$$\begin{aligned}\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2(3bc+af)x^7+\frac{1}{8}a^2(3bd+ag)x^8+\frac{1}{9}a^2(3be+ah)x^9\\&\quad+\frac{3}{10}ab(bc+af)x^{10}+\frac{3}{11}ab(bd+ag)x^{11}+\frac{1}{4}ab(be+ah)x^{12}+\frac{1}{13}b^2(bc+3af)x^{13}\\&\quad+\frac{1}{14}b^2(bd+3ag)x^{14}+\frac{1}{15}b^2(be+3ah)x^{15}+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}\end{aligned}$$

output
```
1/4*a^3*c*x^4+1/5*a^3*d*x^5+1/6*a^3*e*x^6+1/7*a^2*(a*f+3*b*c)*x^7+1/8*a^2*
(a*g+3*b*d)*x^8+1/9*a^2*(a*h+3*b*e)*x^9+3/10*a*b*(a*f+b*c)*x^10+3/11*a*b*(
a*g+b*d)*x^11+1/4*a*b*(a*h+b*e)*x^12+1/13*b^2*(3*a*f+b*c)*x^13+1/14*b^2*(3
*a*g+b*d)*x^14+1/15*b^2*(3*a*h+b*e)*x^15+1/16*b^3*f*x^16+1/17*b^3*g*x^17+1
/18*b^3*h*x^18
```

3.394.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2(3bc+af)x^7+\frac{1}{8}a^2(3bd+ag)x^8+\frac{1}{9}a^2(3be+ah)x^9+\frac{3}{10}ab(bc+af)x^{10}+\frac{3}{11}ab(bd+ag)x^{11}+\frac{1}{4}ab(be+ah)x^{12}+\frac{1}{13}b^2(bc+3af)x^{13}+\frac{1}{14}b^2(bd+3ag)x^{14}+\frac{1}{15}b^2(be+3ah)x^{15}+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}$$

input
```
Integrate[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f)*x^7)/7
+ (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9 + (3*a*b*(b*c + a*
f)*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*b*(b*e + a*h)*x^12)/4 + (b^
2*(b*c + 3*a*f)*x^13)/13 + (b^2*(b*d + 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h
)*x^15)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

3.394.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^3\right)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$

$$\Big\downarrow \text{2360}$$

$$\int \left(a^3cx^3+a^3dx^4+a^3ex^5+a^2x^6(af+3bc)+a^2x^7(ag+3bd)+a^2x^8(ah+3be)+b^2x^{12}(3af+bc)+b^2x^{13}(3ag\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^2x^7(af+3bc)+\frac{1}{8}a^2x^8(ag+3bd)+\frac{1}{9}a^2x^9(ah+3be)+\frac{1}{13}b^2x^{13}(3af+bc)+\frac{1}{14}b^2x^{14}(3ag+bd)+\frac{1}{15}b^2x^{15}(3ah+be)+\frac{3}{10}abx^{10}(af+bc)+\frac{3}{11}abx^{11}(ag+bd)+\frac{1}{4}abx^{12}(ah+be)+\frac{1}{16}b^3fx^{16}+\frac{1}{17}b^3gx^{17}+\frac{1}{18}b^3hx^{18}$$

input
```
Int[x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^2*(3*b*c + a*f)*x^7)/7
+ (a^2*(3*b*d + a*g)*x^8)/8 + (a^2*(3*b*e + a*h)*x^9)/9 + (3*a*b*(b*c + a*
f)*x^10)/10 + (3*a*b*(b*d + a*g)*x^11)/11 + (a*b*(b*e + a*h)*x^12)/4 + (b^
2*(b*c + 3*a*f)*x^13)/13 + (b^2*(b*d + 3*a*g)*x^14)/14 + (b^2*(b*e + 3*a*h
)*x^15)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3*h*x^18)/18
```

3.394.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.394.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.99

method	result
norman	$\frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\left(\frac{1}{7}fa^3+\frac{3}{7}a^2bc\right)x^7+\left(\frac{1}{8}a^3g+\frac{3}{8}da^2b\right)x^8+\left(\frac{1}{9}a^3h+\frac{1}{3}a^2be\right)x^9+\left(\frac{3}{10}\right.$
default	$\frac{b^3hx^{18}}{18}+\frac{b^3gx^{17}}{17}+\frac{b^3fx^{16}}{16}+\frac{(3ab^2h+b^3e)x^{15}}{15}+\frac{(3ab^2g+b^3d)x^{14}}{14}+\frac{(3ab^2f+b^3c)x^{13}}{13}+\frac{(3a^2bh+3ab^2e)x^{12}}{12}+$
gosper	$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{3}{7}x^7a^2bc+\frac{1}{8}x^8a^3g+\frac{3}{8}x^8da^2b+\frac{1}{9}x^9a^3h+\frac{1}{3}x^9a^2be$
risch	$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{3}{7}x^7a^2bc+\frac{1}{8}x^8a^3g+\frac{3}{8}x^8da^2b+\frac{1}{9}x^9a^3h+\frac{1}{3}x^9a^2be$
parallelrisch	$\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{3}{7}x^7a^2bc+\frac{1}{8}x^8a^3g+\frac{3}{8}x^8da^2b+\frac{1}{9}x^9a^3h+\frac{1}{3}x^9a^2be$

input
```
int(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOS
E)
```

output
```
1/4*a^3*c*x^4+1/5*a^3*d*x^5+1/6*a^3*e*x^6+(1/7*f*a^3+3/7*a^2*b*c)*x^7+(1/8
*a^3*g+3/8*d*a^2*b)*x^8+(1/9*a^3*h+1/3*a^2*b*e)*x^9+(3/10*f*a^2*b+3/10*a*b
^2*c)*x^10+(3/11*a^2*b*g+3/11*a*b^2*d)*x^11+(1/4*a^2*b*h+1/4*a*b^2*e)*x^12
+(3/13*a*b^2*f+1/13*b^3*c)*x^13+(3/14*a*b^2*g+1/14*b^3*d)*x^14+(1/5*a*b^2*
h+1/15*b^3*e)*x^15+1/16*b^3*f*x^16+1/17*b^3*g*x^17+1/18*b^3*h*x^18
```

3.394.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$
\begin{aligned}
&\int x^3(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx \\
&= \frac{1}{18}\,b^3hx^{18}+\frac{1}{17}\,b^3gx^{17}+\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,\left(b^3e+3\,ab^2h\right)x^{15} \\
&\quad+\frac{1}{14}\,\left(b^3d+3\,ab^2g\right)x^{14}+\frac{1}{13}\,\left(b^3c+3\,ab^2f\right)x^{13}+\frac{1}{4}\,\left(ab^2e+a^2bh\right)x^{12} \\
&\quad+\frac{3}{11}\,\left(ab^2d+a^2bg\right)x^{11}+\frac{3}{10}\,\left(ab^2c+a^2bf\right)x^{10}+\frac{1}{6}\,a^3ex^6+\frac{1}{9}\,\left(3\,a^2be+a^3h\right)x^9 \\
&\quad+\frac{1}{5}\,a^3dx^5+\frac{1}{8}\,\left(3\,a^2bd+a^3g\right)x^8+\frac{1}{4}\,a^3cx^4+\frac{1}{7}\,\left(3\,a^2bc+a^3f\right)x^7
\end{aligned}
$$

input
```
integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fr
icas")
```

output
```
1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*(b^3*e + 3*a*b^
2*h)*x^15 + 1/14*(b^3*d + 3*a*b^2*g)*x^14 + 1/13*(b^3*c + 3*a*b^2*f)*x^13
+ 1/4*(a*b^2*e + a^2*b*h)*x^12 + 3/11*(a*b^2*d + a^2*b*g)*x^11 + 3/10*(a*b
^2*c + a^2*b*f)*x^10 + 1/6*a^3*e*x^6 + 1/9*(3*a^2*b*e + a^3*h)*x^9 + 1/5*a
^3*d*x^5 + 1/8*(3*a^2*b*d + a^3*g)*x^8 + 1/4*a^3*c*x^4 + 1/7*(3*a^2*b*c +
a^3*f)*x^7
```

3.394.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.10

$$\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{b^3fx^{16}}{16}+\frac{b^3gx^{17}}{17}+\frac{b^3hx^{18}}{18}+x^{15}\left(\frac{ab^2h}{5}+\frac{b^3e}{15}\right)+x^{14}\cdot\left(\frac{3ab^2g}{14}+\frac{b^3d}{14}\right)+x^{13}\cdot\left(\frac{3ab^2f}{13}+\frac{b^3c}{13}\right)+x^{12}\left(\frac{a^2bh}{4}+\frac{ab^2e}{4}\right)+x^{11}\cdot\left(\frac{3a^2bg}{11}+\frac{3ab^2d}{11}\right)+x^{10}\cdot\left(\frac{3a^2bf}{10}+\frac{3ab^2c}{10}\right)+x^9\left(\frac{a^3h}{9}+\frac{a^2be}{3}\right)+x^8\left(\frac{a^3g}{8}+\frac{3a^2bd}{8}\right)+x^7\left(\frac{a^3f}{7}+\frac{3a^2bc}{7}\right)$$

input
```
integrate(x**3*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + b**3*f*x**16/16 + b**3*g*x
**17/17 + b**3*h*x**18/18 + x**15*(a*b**2*h/5 + b**3*e/15) + x**14*(3*a*b*
*2*g/14 + b**3*d/14) + x**13*(3*a*b**2*f/13 + b**3*c/13) + x**12*(a**2*b*h
/4 + a*b**2*e/4) + x**11*(3*a**2*b*g/11 + 3*a*b**2*d/11) + x**10*(3*a**2*b
*f/10 + 3*a*b**2*c/10) + x**9*(a**3*h/9 + a**2*b*e/3) + x**8*(a**3*g/8 + 3
*a**2*b*d/8) + x**7*(a**3*f/7 + 3*a**2*b*c/7)
```

3.394.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{18}b^3hx^{18}+\frac{1}{17}b^3gx^{17}+\frac{1}{16}b^3fx^{16}+\frac{1}{15}(b^3e+3ab^2h)x^{15}+\frac{1}{14}(b^3d+3ab^2g)x^{14}+\frac{1}{13}(b^3c+3ab^2f)x^{13}+\frac{1}{4}(ab^2e+a^2bh)x^{12}+\frac{3}{11}(ab^2d+a^2bg)x^{11}+\frac{3}{10}(ab^2c+a^2bf)x^{10}+\frac{1}{6}a^3ex^6+\frac{1}{9}(3a^2be+a^3h)x^9+\frac{1}{5}a^3dx^5+\frac{1}{8}(3a^2bd+a^3g)x^8+\frac{1}{4}a^3cx^4+\frac{1}{7}(3a^2bc+a^3f)x^7$$

input
```
integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="ma
xima")
```

output
```
1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*(b^3*e + 3*a*b^
2*h)*x^15 + 1/14*(b^3*d + 3*a*b^2*g)*x^14 + 1/13*(b^3*c + 3*a*b^2*f)*x^13
+ 1/4*(a*b^2*e + a^2*b*h)*x^12 + 3/11*(a*b^2*d + a^2*b*g)*x^11 + 3/10*(a*b
^2*c + a^2*b*f)*x^10 + 1/6*a^3*e*x^6 + 1/9*(3*a^2*b*e + a^3*h)*x^9 + 1/5*a
^3*d*x^5 + 1/8*(3*a^2*b*d + a^3*g)*x^8 + 1/4*a^3*c*x^4 + 1/7*(3*a^2*b*c +
a^3*f)*x^7
```

3.394.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.03

$$\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$
$$= \frac{1}{18}b^3hx^{18} + \frac{1}{17}b^3gx^{17} + \frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3ex^{15} + \frac{1}{5}ab^2hx^{15} + \frac{1}{14}b^3dx^{14} + \frac{3}{14}ab^2gx^{14} + \frac{1}{13}b^3cx^{13} + \frac{3}{13}ab^2fx^{13} + \frac{1}{4}ab^2ex^{12} + \frac{1}{4}a^2bhx^{12} + \frac{3}{11}ab^2dx^{11} + \frac{3}{11}a^2bgx^{11} + \frac{3}{10}ab^2cx^{10} + \frac{3}{10}a^2bfx^{10} + \frac{1}{3}a^2bex^9 + \frac{1}{9}a^3hx^9 + \frac{3}{8}a^2bdx^8 + \frac{1}{8}a^3gx^8 + \frac{3}{7}a^2bcx^7 + \frac{1}{7}a^3fx^7 + \frac{1}{6}a^3ex^6 + \frac{1}{5}a^3dx^5 + \frac{1}{4}a^3cx^4$$

input
```
integrate(x^3*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gi
ac")
```

output
```
1/18*b^3*h*x^18 + 1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/
5*a*b^2*h*x^15 + 1/14*b^3*d*x^14 + 3/14*a*b^2*g*x^14 + 1/13*b^3*c*x^13 + 3
/13*a*b^2*f*x^13 + 1/4*a*b^2*e*x^12 + 1/4*a^2*b*h*x^12 + 3/11*a*b^2*d*x^11
 + 3/11*a^2*b*g*x^11 + 3/10*a*b^2*c*x^10 + 3/10*a^2*b*f*x^10 + 1/3*a^2*b*e
*x^9 + 1/9*a^3*h*x^9 + 3/8*a^2*b*d*x^8 + 1/8*a^3*g*x^8 + 3/7*a^2*b*c*x^7 +
 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

3.394.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.92

$$\int x^3(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$
$$=x^7\left(\frac{f\,a^3}{7}+\frac{3\,b\,c\,a^2}{7}\right)+x^{13}\left(\frac{c\,b^3}{13}+\frac{3\,a\,f\,b^2}{13}\right)+x^8\left(\frac{g\,a^3}{8}+\frac{3\,b\,d\,a^2}{8}\right)+x^{14}\left(\frac{d\,b^3}{14}+\frac{3\,a\,g\,b^2}{14}\right)+x^9\left(\frac{h\,a^3}{9}+\frac{b\,e\,a^2}{3}\right)+x^{15}\left(\frac{e\,b^3}{15}+\frac{a\,h\,b^2}{5}\right)+\frac{a^3\,c\,x^4}{4}+\frac{a^3\,d\,x^5}{5}+\frac{a^3\,e\,x^6}{6}+\frac{b^3\,f\,x^{16}}{16}+\frac{b^3\,g\,x^{17}}{17}+\frac{b^3\,h\,x^{18}}{18}+\frac{3\,a\,b\,x^{10}\,(b\,c+a\,f)}{10}+\frac{3\,a\,b\,x^{11}\,(b\,d+a\,g)}{11}+\frac{a\,b\,x^{12}\,(b\,e+a\,h)}{4}$$

input
```
int(x^3*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^7*((a^3*f)/7 + (3*a^2*b*c)/7) + x^13*((b^3*c)/13 + (3*a*b^2*f)/13) + x^8
*((a^3*g)/8 + (3*a^2*b*d)/8) + x^14*((b^3*d)/14 + (3*a*b^2*g)/14) + x^9*((
a^3*h)/9 + (a^2*b*e)/3) + x^15*((b^3*e)/15 + (a*b^2*h)/5) + (a^3*c*x^4)/4
+ (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + (b^3
*h*x^18)/18 + (3*a*b*x^10*(b*c + a*f))/10 + (3*a*b*x^11*(b*d + a*g))/11 +
(a*b*x^12*(b*e + a*h))/4
```

3.395 $\int x^2(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

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3.395.1 Optimal result

Integrand size = 38, antiderivative size = 212

$$\begin{aligned}\int x^2(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{7}a^2(3bd+ag)x^7+\frac{1}{8}a^2(3be+ah)x^8+\frac{1}{3}a^2bfx^9\\&+\frac{3}{10}ab(bd+ag)x^{10}+\frac{3}{11}ab(be+ah)x^{11}+\frac{1}{4}ab^2fx^{12}+\frac{1}{13}b^2(bd+3ag)x^{13}\\&+\frac{1}{14}b^2(be+3ah)x^{14}+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}+\frac{c(a+bx^3)^4}{12b}\end{aligned}$$

output
```
1/4*a^3*d*x^4+1/5*a^3*e*x^5+1/6*a^3*f*x^6+1/7*a^2*(a*g+3*b*d)*x^7+1/8*a^2*
(a*h+3*b*e)*x^8+1/3*a^2*b*f*x^9+3/10*a*b*(a*g+b*d)*x^10+3/11*a*b*(a*h+b*e)
*x^11+1/4*a*b^2*f*x^12+1/13*b^2*(3*a*g+b*d)*x^13+1/14*b^2*(3*a*h+b*e)*x^14
+1/15*b^3*f*x^15+1/16*b^3*g*x^16+1/17*b^3*h*x^17+1/12*c*(b*x^3+a)^4/b
```

3.395.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.05

$$\begin{aligned}\int x^2(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx &= \frac{1}{3}a^3cx^3+\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^2(3bc+af)x^6+\frac{1}{7}a^2(3bd+ag)x^7+\frac{1}{8}a^2(3be+ah)x^8\\&+\frac{1}{3}ab(bc+af)x^9+\frac{3}{10}ab(bd+ag)x^{10}+\frac{3}{11}ab(be+ah)x^{11}+\frac{1}{12}b^2(bc+3af)x^{12}\\&+\frac{1}{13}b^2(bd+3ag)x^{13}+\frac{1}{14}b^2(be+3ah)x^{14}+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}\end{aligned}$$

input
```
Integrate[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^3*c*x^3)/3 + (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^2*(3*b*c + a*f)*x^6)/6
+ (a^2*(3*b*d + a*g)*x^7)/7 + (a^2*(3*b*e + a*h)*x^8)/8 + (a*b*(b*c + a*f)
*x^9)/3 + (3*a*b*(b*d + a*g)*x^10)/10 + (3*a*b*(b*e + a*h)*x^11)/11 + (b^2
*(b*c + 3*a*f)*x^12)/12 + (b^2*(b*d + 3*a*g)*x^13)/13 + (b^2*(b*e + 3*a*h)
*x^14)/14 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^3*h*x^17)/17
```

3.395.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx^3)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) dx$$

$$\Big\downarrow \text{2017}$$

$$\int (bx^3+a)^3 \left(x^2 \left(hx^5+gx^4+fx^3+ex^2+dx+c\right)-cx^2\right) dx+\frac{c(a+bx^3)^4}{12b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^3hx^{16}+b^3gx^{15}+b^3fx^{14}+b^2(be+3ah)x^{13}+b^2(bd+3ag)x^{12}+3ab^2fx^{11}+3ab(be+ah)x^{10}+3ab(bd+ag)\right.$$
$$\frac{c(a+bx^3)^4}{12b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^3dx^4+\frac{1}{5}a^3ex^5+\frac{1}{6}a^3fx^6+\frac{1}{7}a^2x^7(ag+3bd)+\frac{1}{8}a^2x^8(ah+3be)+\frac{1}{3}a^2bfx^9+\frac{1}{13}b^2x^{13}(3ag+bd)+\frac{1}{14}b^2x^{14}(3ah+be)+\frac{1}{4}ab^2fx^{12}+\frac{c(a+bx^3)^4}{12b}+\frac{3}{10}abx^{10}(ag+bd)+\frac{3}{11}abx^{11}(ah+be)+\frac{1}{15}b^3fx^{15}+\frac{1}{16}b^3gx^{16}+\frac{1}{17}b^3hx^{17}$$

input
```
Int[x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (a^3*f*x^6)/6 + (a^2*(3*b*d + a*g)*x^7)/7
+ (a^2*(3*b*e + a*h)*x^8)/8 + (a^2*b*f*x^9)/3 + (3*a*b*(b*d + a*g)*x^10)/1
0 + (3*a*b*(b*e + a*h)*x^11)/11 + (a*b^2*f*x^12)/4 + (b^2*(b*d + 3*a*g)*x^
13)/13 + (b^2*(b*e + 3*a*h)*x^14)/14 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 +
 (b^3*h*x^17)/17 + (c*(a + b*x^3)^4)/(12*b)
```

3.395.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.395.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.04

method	result
norman	$\frac{c\,a^{3} x^{3}}{3}+\frac{a^{3} d\,x^{4}}{4}+\frac{a^{3} e\,x^{5}}{5}+\left(\frac{1}{6} f\,a^{3}+\frac{1}{2} a^{2} b c \right) x^{6}+\left(\frac{1}{7} a^{3} g +\frac{3}{7} d\,a^{2} b \right) x^{7}+\left(\frac{1}{8} a^{3} h +\frac{3}{8} a^{2} b e \right) x^{8}+\left(\frac{1}{3}\right.$
default	$\frac{b^{3} h\,x^{17}}{17}+\frac{b^{3} g\,x^{16}}{16}+\frac{b^{3} f\,x^{15}}{15}+\frac{\left(3 a\,b^{2} h +b^{3} e \right) x^{14}}{14}+\frac{\left(3 a\,b^{2} g +b^{3} d \right) x^{13}}{13}+\frac{\left(3 a\,b^{2} f +b^{3} c \right) x^{12}}{12}+\frac{\left(3 a^{2} b h +3 a\,b^{2} e \right) x^{11}}{11}+$
gosper	$\frac{1}{3} c\,a^{3} x^{3}+\frac{1}{4} a^{3} d\,x^{4}+\frac{1}{5} a^{3} e\,x^{5}+\frac{1}{6} a^{3} f\,x^{6}+\frac{1}{2} a^{2} b c\,x^{6}+\frac{1}{7} x^{7} a^{3} g +\frac{3}{7} a^{2} b d\,x^{7}+\frac{1}{8} x^{8} a^{3} h +\frac{3}{8} a^{2} b e\,x$
risch	$\frac{1}{3} c\,a^{3} x^{3}+\frac{1}{4} a^{3} d\,x^{4}+\frac{1}{5} a^{3} e\,x^{5}+\frac{1}{6} a^{3} f\,x^{6}+\frac{1}{2} a^{2} b c\,x^{6}+\frac{1}{7} x^{7} a^{3} g +\frac{3}{7} a^{2} b d\,x^{7}+\frac{1}{8} x^{8} a^{3} h +\frac{3}{8} a^{2} b e\,x$
parallelrisch	$\frac{1}{3} c\,a^{3} x^{3}+\frac{1}{4} a^{3} d\,x^{4}+\frac{1}{5} a^{3} e\,x^{5}+\frac{1}{6} a^{3} f\,x^{6}+\frac{1}{2} a^{2} b c\,x^{6}+\frac{1}{7} x^{7} a^{3} g +\frac{3}{7} a^{2} b d\,x^{7}+\frac{1}{8} x^{8} a^{3} h +\frac{3}{8} a^{2} b e\,x$

input
```
int(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOS
E)
```

output
```
1/3*c*a^3*x^3+1/4*a^3*d*x^4+1/5*a^3*e*x^5+(1/6*f*a^3+1/2*a^2*b*c)*x^6+(1/7
*a^3*g+3/7*d*a^2*b)*x^7+(1/8*a^3*h+3/8*a^2*b*e)*x^8+(1/3*f*a^2*b+1/3*a*b^2
*c)*x^9+(3/10*a^2*b*g+3/10*a*b^2*d)*x^10+(3/11*a^2*b*h+3/11*a*b^2*e)*x^11+
(1/4*a*b^2*f+1/12*b^3*c)*x^12+(3/13*a*b^2*g+1/13*b^3*d)*x^13+(3/14*a*b^2*h
+1/14*b^3*e)*x^14+1/15*b^3*f*x^15+1/16*b^3*g*x^16+1/17*b^3*h*x^17
```

3.395.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

$$\begin{aligned}
&\int x^2(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx\\
&=\frac{1}{17}b^3hx^{17}+\frac{1}{16}b^3gx^{16}+\frac{1}{15}b^3fx^{15}+\frac{1}{14}(b^3e+3ab^2h)x^{14}\\
&+\frac{1}{13}(b^3d+3ab^2g)x^{13}+\frac{1}{12}(b^3c+3ab^2f)x^{12}+\frac{3}{11}(ab^2e+a^2bh)x^{11}\\
&+\frac{3}{10}(ab^2d+a^2bg)x^{10}+\frac{1}{3}(ab^2c+a^2bf)x^9+\frac{1}{5}a^3ex^5+\frac{1}{8}(3a^2be+a^3h)x^8\\
&+\frac{1}{4}a^3dx^4+\frac{1}{7}(3a^2bd+a^3g)x^7+\frac{1}{3}a^3cx^3+\frac{1}{6}(3a^2bc+a^3f)x^6
\end{aligned}$$

input
```
integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fr
icas")
```

output
```
1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 1/14*(b^3*e + 3*a*b^
2*h)*x^14 + 1/13*(b^3*d + 3*a*b^2*g)*x^13 + 1/12*(b^3*c + 3*a*b^2*f)*x^12
+ 3/11*(a*b^2*e + a^2*b*h)*x^11 + 3/10*(a*b^2*d + a^2*b*g)*x^10 + 1/3*(a*b
^2*c + a^2*b*f)*x^9 + 1/5*a^3*e*x^5 + 1/8*(3*a^2*b*e + a^3*h)*x^8 + 1/4*a^
3*d*x^4 + 1/7*(3*a^2*b*d + a^3*g)*x^7 + 1/3*a^3*c*x^3 + 1/6*(3*a^2*b*c + a
^3*f)*x^6
```

3.395.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.16

$$\int x^2 (a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) \, dx$$
$$= \frac{a^3cx^3}{3} + \frac{a^3dx^4}{4} + \frac{a^3ex^5}{5} + \frac{b^3fx^{15}}{15} + \frac{b^3gx^{16}}{16} + \frac{b^3hx^{17}}{17} + x^{14} \cdot \left(\frac{3ab^2h}{14} + \frac{b^3e}{14}\right) + x^{13} \cdot \left(\frac{3ab^2g}{13} + \frac{b^3d}{13}\right) + x^{12}\left(\frac{ab^2f}{4} + \frac{b^3c}{12}\right) + x^{11} \cdot \left(\frac{3a^2bh}{11} + \frac{3ab^2e}{11}\right) + x^{10} \cdot \left(\frac{3a^2bg}{10} + \frac{3ab^2d}{10}\right) + x^9\left(\frac{a^2bf}{3} + \frac{ab^2c}{3}\right) + x^8\left(\frac{a^3h}{8} + \frac{3a^2be}{8}\right) + x^7\left(\frac{a^3g}{7} + \frac{3a^2bd}{7}\right) + x^6\left(\frac{a^3f}{6} + \frac{a^2bc}{2}\right)$$

input
```
integrate(x**2*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**3*c*x**3/3 + a**3*d*x**4/4 + a**3*e*x**5/5 + b**3*f*x**15/15 + b**3*g*x
**16/16 + b**3*h*x**17/17 + x**14*(3*a*b**2*h/14 + b**3*e/14) + x**13*(3*a
*b**2*g/13 + b**3*d/13) + x**12*(a*b**2*f/4 + b**3*c/12) + x**11*(3*a**2*b
*h/11 + 3*a*b**2*e/11) + x**10*(3*a**2*b*g/10 + 3*a*b**2*d/10) + x**9*(a**
2*b*f/3 + a*b**2*c/3) + x**8*(a**3*h/8 + 3*a**2*b*e/8) + x**7*(a**3*g/7 +
3*a**2*b*d/7) + x**6*(a**3*f/6 + a**2*b*c/2)
```

3.395.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

$$\int x^2 (a + bx^3)^3 (c + dx + ex^2 + fx^3 + gx^4 + hx^5) \, dx$$
$$= \frac{1}{17} b^3hx^{17} + \frac{1}{16} b^3gx^{16} + \frac{1}{15} b^3fx^{15} + \frac{1}{14} (b^3e + 3\,ab^2h)x^{14} + \frac{1}{13} (b^3d + 3\,ab^2g)x^{13} + \frac{1}{12} (b^3c + 3\,ab^2f)x^{12} + \frac{3}{11} (ab^2e + a^2bh)x^{11} + \frac{3}{10} (ab^2d + a^2bg)x^{10} + \frac{1}{3} (ab^2c + a^2bf)x^9 + \frac{1}{5} a^3ex^5 + \frac{1}{8} (3\,a^2be + a^3h)x^8 + \frac{1}{4} a^3dx^4 + \frac{1}{7} (3\,a^2bd + a^3g)x^7 + \frac{1}{3} a^3cx^3 + \frac{1}{6} (3\,a^2bc + a^3f)x^6$$

input
```
integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="ma
xima")
```

output
```
1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 1/14*(b^3*e + 3*a*b^
2*h)*x^14 + 1/13*(b^3*d + 3*a*b^2*g)*x^13 + 1/12*(b^3*c + 3*a*b^2*f)*x^12
+ 3/11*(a*b^2*e + a^2*b*h)*x^11 + 3/10*(a*b^2*d + a^2*b*g)*x^10 + 1/3*(a*b
^2*c + a^2*b*f)*x^9 + 1/5*a^3*e*x^5 + 1/8*(3*a^2*b*e + a^3*h)*x^8 + 1/4*a^
3*d*x^4 + 1/7*(3*a^2*b*d + a^3*g)*x^7 + 1/3*a^3*c*x^3 + 1/6*(3*a^2*b*c + a
^3*f)*x^6
```

3.395.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.08

$$\begin{aligned}\int x^2(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{17}b^3hx^{17}+\frac{1}{16}b^3gx^{16}+\frac{1}{15}b^3fx^{15}+\frac{1}{14}b^3ex^{14}+\frac{3}{14}ab^2hx^{14}+\frac{1}{13}b^3dx^{13}\\ &+\frac{3}{13}ab^2gx^{13}+\frac{1}{12}b^3cx^{12}+\frac{1}{4}ab^2fx^{12}+\frac{3}{11}ab^2ex^{11}+\frac{3}{11}a^2bhx^{11}\\ &+\frac{3}{10}ab^2dx^{10}+\frac{3}{10}a^2bgx^{10}+\frac{1}{3}ab^2cx^9+\frac{1}{3}a^2bfx^9+\frac{3}{8}a^2bex^8+\frac{1}{8}a^3hx^8\\ &+\frac{3}{7}a^2bdx^7+\frac{1}{7}a^3gx^7+\frac{1}{2}a^2bcx^6+\frac{1}{6}a^3fx^6+\frac{1}{5}a^3ex^5+\frac{1}{4}a^3dx^4+\frac{1}{3}a^3cx^3\end{aligned}$$

input
```
integrate(x^2*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="gi
ac")
```

output
```
1/17*b^3*h*x^17 + 1/16*b^3*g*x^16 + 1/15*b^3*f*x^15 + 1/14*b^3*e*x^14 + 3/
14*a*b^2*h*x^14 + 1/13*b^3*d*x^13 + 3/13*a*b^2*g*x^13 + 1/12*b^3*c*x^12 +
1/4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/11*a^2*b*h*x^11 + 3/10*a*b^2*d*x^
10 + 3/10*a^2*b*g*x^10 + 1/3*a*b^2*c*x^9 + 1/3*a^2*b*f*x^9 + 3/8*a^2*b*e*x
^8 + 1/8*a^3*h*x^8 + 3/7*a^2*b*d*x^7 + 1/7*a^3*g*x^7 + 1/2*a^2*b*c*x^6 + 1
/6*a^3*f*x^6 + 1/5*a^3*e*x^5 + 1/4*a^3*d*x^4 + 1/3*a^3*c*x^3
```

3.395.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97

$$\int x^2(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=x^6\left(\frac{f\,a^3}{6}+\frac{b\,c\,a^2}{2}\right)+x^{12}\left(\frac{c\,b^3}{12}+\frac{a\,f\,b^2}{4}\right)+x^7\left(\frac{g\,a^3}{7}+\frac{3\,b\,d\,a^2}{7}\right)+x^{13}\left(\frac{d\,b^3}{13}+\frac{3\,a\,g\,b^2}{13}\right)$$
$$+x^8\left(\frac{h\,a^3}{8}+\frac{3\,b\,e\,a^2}{8}\right)+x^{14}\left(\frac{e\,b^3}{14}+\frac{3\,a\,h\,b^2}{14}\right)+\frac{a^3\,c\,x^3}{3}+\frac{a^3\,d\,x^4}{4}+\frac{a^3\,e\,x^5}{5}+\frac{b^3\,f\,x^{15}}{15}$$
$$+\frac{b^3\,g\,x^{16}}{16}+\frac{b^3\,h\,x^{17}}{17}+\frac{a\,b\,x^9\,(b\,c+a\,f)}{3}+\frac{3\,a\,b\,x^{10}\,(b\,d+a\,g)}{10}+\frac{3\,a\,b\,x^{11}\,(b\,e+a\,h)}{11}$$

input
```
int(x^2*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^6*((a^3*f)/6 + (a^2*b*c)/2) + x^12*((b^3*c)/12 + (a*b^2*f)/4) + x^7*((a^
3*g)/7 + (3*a^2*b*d)/7) + x^13*((b^3*d)/13 + (3*a*b^2*g)/13) + x^8*((a^3*h
)/8 + (3*a^2*b*e)/8) + x^14*((b^3*e)/14 + (3*a*b^2*h)/14) + (a^3*c*x^3)/3
+ (a^3*d*x^4)/4 + (a^3*e*x^5)/5 + (b^3*f*x^15)/15 + (b^3*g*x^16)/16 + (b^3
*h*x^17)/17 + (a*b*x^9*(b*c + a*f))/3 + (3*a*b*x^10*(b*d + a*g))/10 + (3*a
*b*x^11*(b*e + a*h))/11
```

3.396 $\int x(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$

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3.396.1 Optimal result

Integrand size = 36, antiderivative size = 212

$$\begin{aligned}
&\int x(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx \\
&= \frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{5}a^2(3bc+af)x^5+\frac{1}{6}a^3gx^6+\frac{1}{7}a^2(3be+ah)x^7 \\
&\quad+\frac{3}{8}ab(bc+af)x^8+\frac{1}{3}a^2bgx^9+\frac{3}{10}ab(be+ah)x^{10}+\frac{1}{11}b^2(bc+3af)x^{11}+\frac{1}{4}ab^2gx^{12} \\
&\quad+\frac{1}{13}b^2(be+3ah)x^{13}+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}+\frac{d(a+bx^3)^4}{12b}
\end{aligned}$$

output
```
1/2*a^3*c*x^2+1/4*a^3*e*x^4+1/5*a^2*(a*f+3*b*c)*x^5+1/6*a^3*g*x^6+1/7*a^2*
(a*h+3*b*e)*x^7+3/8*a*b*(a*f+b*c)*x^8+1/3*a^2*b*g*x^9+3/10*a*b*(a*h+b*e)*x
^10+1/11*b^2*(3*a*f+b*c)*x^11+1/4*a*b^2*g*x^12+1/13*b^2*(3*a*h+b*e)*x^13+1
/14*b^3*f*x^14+1/15*b^3*g*x^15+1/16*b^3*h*x^16+1/12*d*(b*x^3+a)^4/b
```

3.396.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.05

$$\begin{aligned}
&\int x(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx \\
&= \frac{1}{2}a^3cx^2+\frac{1}{3}a^3dx^3+\frac{1}{4}a^3ex^4+\frac{1}{5}a^2(3bc+af)x^5+\frac{1}{6}a^2(3bd+ag)x^6+\frac{1}{7}a^2(3be+ah)x^7 \\
&\quad+\frac{3}{8}ab(bc+af)x^8+\frac{1}{3}ab(bd+ag)x^9+\frac{3}{10}ab(be+ah)x^{10}+\frac{1}{11}b^2(bc+3af)x^{11} \\
&\quad+\frac{1}{12}b^2(bd+3ag)x^{12}+\frac{1}{13}b^2(be+3ah)x^{13}+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}
\end{aligned}$$

input
```
Integrate[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^3*c*x^2)/2 + (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5
+ (a^2*(3*b*d + a*g)*x^6)/6 + (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*
f)*x^8)/8 + (a*b*(b*d + a*g)*x^9)/3 + (3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(
b*c + 3*a*f)*x^11)/11 + (b^2*(b*d + 3*a*g)*x^12)/12 + (b^2*(b*e + 3*a*h)*x
^13)/13 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3*h*x^16)/16
```

3.396.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^3+a\right)^3\left(x\left(hx^5+gx^4+fx^3+ex^2+dx+c\right)-dx^2\right)dx+\frac{d(a+bx^3)^4}{12b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^3hx^{15}+b^3gx^{14}+b^3fx^{13}+b^2(be+3ah)x^{12}+3ab^2gx^{11}+b^2(bc+3af)x^{10}+3ab(be+ah)x^9+3a^2bgx^8+3ab\right.$$
$$\frac{d(a+bx^3)^4}{12b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^3cx^2+\frac{1}{4}a^3ex^4+\frac{1}{6}a^3gx^6+\frac{1}{5}a^2x^5(af+3bc)+\frac{1}{7}a^2x^7(ah+3be)+\frac{1}{3}a^2bgx^9+\frac{1}{11}b^2x^{11}(3af+bc)+\frac{1}{13}b^2x^{13}(3ah+be)+\frac{1}{4}ab^2gx^{12}+\frac{3}{8}abx^8(af+bc)+\frac{d(a+bx^3)^4}{12b}+\frac{3}{10}abx^{10}(ah+be)+\frac{1}{14}b^3fx^{14}+\frac{1}{15}b^3gx^{15}+\frac{1}{16}b^3hx^{16}$$

input
```
Int[x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(a^3*c*x^2)/2 + (a^3*e*x^4)/4 + (a^2*(3*b*c + a*f)*x^5)/5 + (a^3*g*x^6)/6
+ (a^2*(3*b*e + a*h)*x^7)/7 + (3*a*b*(b*c + a*f)*x^8)/8 + (a^2*b*g*x^9)/3
+ (3*a*b*(b*e + a*h)*x^10)/10 + (b^2*(b*c + 3*a*f)*x^11)/11 + (a*b^2*g*x^1
2)/4 + (b^2*(b*e + 3*a*h)*x^13)/13 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (
b^3*h*x^16)/16 + (d*(a + b*x^3)^4)/(12*b)
```

3.396.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.396.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.04

method	result
norman	$\frac{a^{3} c \,x^{2}}{2}+\frac{a^{3} d \,x^{3}}{3}+\frac{a^{3} e \,x^{4}}{4}+\left(\frac{1}{5} f \,a^{3}+\frac{3}{5} a^{2} b c \right) x^{5}+\left(\frac{1}{6} a^{3} g +\frac{1}{2} d \,a^{2} b \right) x^{6}+\left(\frac{1}{7} a^{3} h +\frac{3}{7} a^{2} b e \right) x^{7}+\left(\frac{3}{8}\right.$
default	$\frac{b^{3} h \,x^{16}}{16}+\frac{b^{3} g \,x^{15}}{15}+\frac{b^{3} f \,x^{14}}{14}+\frac{\left(3 a \,b^{2} h +b^{3} e \right) x^{13}}{13}+\frac{\left(3 a \,b^{2} g +b^{3} d \right) x^{12}}{12}+\frac{\left(3 a \,b^{2} f +b^{3} c \right) x^{11}}{11}+\frac{\left(3 a^{2} b h +3 a \,b^{2} e \right) x^{10}}{10}+$
gosper	$\frac{1}{2} a^{3} c \,x^{2}+\frac{1}{3} a^{3} d \,x^{3}+\frac{1}{4} a^{3} e \,x^{4}+\frac{1}{5} x^{5} f \,a^{3}+\frac{3}{5} a^{2} b c \,x^{5}+\frac{1}{6} a^{3} g \,x^{6}+\frac{1}{2} a^{2} b d \,x^{6}+\frac{1}{7} x^{7} a^{3} h +\frac{3}{7} a^{2} b e \,x$
risch	$\frac{1}{2} a^{3} c \,x^{2}+\frac{1}{3} a^{3} d \,x^{3}+\frac{1}{4} a^{3} e \,x^{4}+\frac{1}{5} x^{5} f \,a^{3}+\frac{3}{5} a^{2} b c \,x^{5}+\frac{1}{6} a^{3} g \,x^{6}+\frac{1}{2} a^{2} b d \,x^{6}+\frac{1}{7} x^{7} a^{3} h +\frac{3}{7} a^{2} b e \,x$
parallelrisch	$\frac{1}{2} a^{3} c \,x^{2}+\frac{1}{3} a^{3} d \,x^{3}+\frac{1}{4} a^{3} e \,x^{4}+\frac{1}{5} x^{5} f \,a^{3}+\frac{3}{5} a^{2} b c \,x^{5}+\frac{1}{6} a^{3} g \,x^{6}+\frac{1}{2} a^{2} b d \,x^{6}+\frac{1}{7} x^{7} a^{3} h +\frac{3}{7} a^{2} b e \,x$

input
```
int(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*c*x^2+1/3*a^3*d*x^3+1/4*a^3*e*x^4+(1/5*f*a^3+3/5*a^2*b*c)*x^5+(1/6
*a^3*g+1/2*d*a^2*b)*x^6+(1/7*a^3*h+3/7*a^2*b*e)*x^7+(3/8*f*a^2*b+3/8*a*b^2
*c)*x^8+(1/3*a^2*b*g+1/3*a*b^2*d)*x^9+(3/10*a^2*b*h+3/10*a*b^2*e)*x^10+(3/
11*a*b^2*f+1/11*b^3*c)*x^11+(1/4*a*b^2*g+1/12*b^3*d)*x^12+(3/13*a*b^2*h+1/
13*b^3*e)*x^13+1/14*b^3*f*x^14+1/15*b^3*g*x^15+1/16*b^3*h*x^16
```

3.396.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

$$\int x(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$=\frac{1}{16}b^3hx^{16}+\frac{1}{15}b^3gx^{15}+\frac{1}{14}b^3fx^{14}+\frac{1}{13}\left(b^3e+3ab^2h\right)x^{13}+\frac{1}{12}\left(b^3d+3ab^2g\right)x^{12}+\frac{1}{11}\left(b^3c+3ab^2f\right)x^{11}+\frac{3}{10}\left(ab^2e+a^2bh\right)x^{10}+\frac{1}{3}\left(ab^2d+a^2bg\right)x^9+\frac{3}{8}\left(ab^2c+a^2bf\right)x^8+\frac{1}{4}a^3ex^4+\frac{1}{7}\left(3a^2be+a^3h\right)x^7+\frac{1}{3}a^3dx^3+\frac{1}{6}\left(3a^2bd+a^3g\right)x^6+\frac{1}{2}a^3cx^2+\frac{1}{5}\left(3a^2bc+a^3f\right)x^5$$

input
```
integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fric
as")
```

output
```
1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 1/13*(b^3*e + 3*a*b^
2*h)*x^13 + 1/12*(b^3*d + 3*a*b^2*g)*x^12 + 1/11*(b^3*c + 3*a*b^2*f)*x^11
+ 3/10*(a*b^2*e + a^2*b*h)*x^10 + 1/3*(a*b^2*d + a^2*b*g)*x^9 + 3/8*(a*b^2
*c + a^2*b*f)*x^8 + 1/4*a^3*e*x^4 + 1/7*(3*a^2*b*e + a^3*h)*x^7 + 1/3*a^3*
d*x^3 + 1/6*(3*a^2*b*d + a^3*g)*x^6 + 1/2*a^3*c*x^2 + 1/5*(3*a^2*b*c + a^3
*f)*x^5
```

3.396.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.16

$$\int x(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx$$
$$=\frac{a^3cx^2}{2}+\frac{a^3dx^3}{3}+\frac{a^3ex^4}{4}+\frac{b^3fx^{14}}{14}+\frac{b^3gx^{15}}{15}+\frac{b^3hx^{16}}{16}+x^{13}\cdot\left(\frac{3ab^2h}{13}+\frac{b^3e}{13}\right)+x^{12}\left(\frac{ab^2g}{4}+\frac{b^3d}{12}\right)+x^{11}\cdot\left(\frac{3ab^2f}{11}+\frac{b^3c}{11}\right)+x^{10}\cdot\left(\frac{3a^2bh}{10}+\frac{3ab^2e}{10}\right)+x^9\left(\frac{a^2bg}{3}+\frac{ab^2d}{3}\right)+x^8\cdot\left(\frac{3a^2bf}{8}+\frac{3ab^2c}{8}\right)+x^7\left(\frac{a^3h}{7}+\frac{3a^2be}{7}\right)+x^6\left(\frac{a^3g}{6}+\frac{a^2bd}{2}\right)+x^5\left(\frac{a^3f}{5}+\frac{3a^2bc}{5}\right)$$

input
```
integrate(x*(b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**3*c*x**2/2 + a**3*d*x**3/3 + a**3*e*x**4/4 + b**3*f*x**14/14 + b**3*g*x
**15/15 + b**3*h*x**16/16 + x**13*(3*a*b**2*h/13 + b**3*e/13) + x**12*(a*b
**2*g/4 + b**3*d/12) + x**11*(3*a*b**2*f/11 + b**3*c/11) + x**10*(3*a**2*b
*h/10 + 3*a*b**2*e/10) + x**9*(a**2*b*g/3 + a*b**2*d/3) + x**8*(3*a**2*b*f
/8 + 3*a*b**2*c/8) + x**7*(a**3*h/7 + 3*a**2*b*e/7) + x**6*(a**3*g/6 + a**
2*b*d/2) + x**5*(a**3*f/5 + 3*a**2*b*c/5)
```

3.396.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

$$\int x(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)dx$$
$$=\frac{1}{16}b^3hx^{16}+\frac{1}{15}b^3gx^{15}+\frac{1}{14}b^3fx^{14}+\frac{1}{13}\left(b^3e+3ab^2h\right)x^{13}+\frac{1}{12}\left(b^3d+3ab^2g\right)x^{12}+\frac{1}{11}\left(b^3c+3ab^2f\right)x^{11}+\frac{3}{10}\left(ab^2e+a^2bh\right)x^{10}+\frac{1}{3}\left(ab^2d+a^2bg\right)x^9+\frac{3}{8}\left(ab^2c+a^2bf\right)x^8+\frac{1}{4}a^3ex^4+\frac{1}{7}\left(3a^2be+a^3h\right)x^7+\frac{1}{3}a^3dx^3+\frac{1}{6}\left(3a^2bd+a^3g\right)x^6+\frac{1}{2}a^3cx^2+\frac{1}{5}\left(3a^2bc+a^3f\right)x^5$$

input
```
integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxi
ma")
```

output
```
1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 1/13*(b^3*e + 3*a*b^
2*h)*x^13 + 1/12*(b^3*d + 3*a*b^2*g)*x^12 + 1/11*(b^3*c + 3*a*b^2*f)*x^11
+ 3/10*(a*b^2*e + a^2*b*h)*x^10 + 1/3*(a*b^2*d + a^2*b*g)*x^9 + 3/8*(a*b^2
*c + a^2*b*f)*x^8 + 1/4*a^3*e*x^4 + 1/7*(3*a^2*b*e + a^3*h)*x^7 + 1/3*a^3*
d*x^3 + 1/6*(3*a^2*b*d + a^3*g)*x^6 + 1/2*a^3*c*x^2 + 1/5*(3*a^2*b*c + a^3
*f)*x^5
```

3.396.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.08

$$\begin{aligned}\int x(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx &= \frac{1}{16}b^3hx^{16}+\frac{1}{15}b^3gx^{15}+\frac{1}{14}b^3fx^{14}+\frac{1}{13}b^3ex^{13}+\frac{3}{13}ab^2hx^{13}+\frac{1}{12}b^3dx^{12}\\&+\frac{1}{4}ab^2gx^{12}+\frac{1}{11}b^3cx^{11}+\frac{3}{11}ab^2fx^{11}+\frac{3}{10}ab^2ex^{10}+\frac{3}{10}a^2bhx^{10}\\&+\frac{1}{3}ab^2dx^9+\frac{1}{3}a^2bgx^9+\frac{3}{8}ab^2cx^8+\frac{3}{8}a^2bfx^8+\frac{3}{7}a^2bex^7+\frac{1}{7}a^3hx^7\\&+\frac{1}{2}a^2bdx^6+\frac{1}{6}a^3gx^6+\frac{3}{5}a^2bcx^5+\frac{1}{5}a^3fx^5+\frac{1}{4}a^3ex^4+\frac{1}{3}a^3dx^3+\frac{1}{2}a^3cx^2\end{aligned}$$

input
```
integrate(x*(b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac
")
```

output
```
1/16*b^3*h*x^16 + 1/15*b^3*g*x^15 + 1/14*b^3*f*x^14 + 1/13*b^3*e*x^13 + 3/
13*a*b^2*h*x^13 + 1/12*b^3*d*x^12 + 1/4*a*b^2*g*x^12 + 1/11*b^3*c*x^11 + 3
/11*a*b^2*f*x^11 + 3/10*a*b^2*e*x^10 + 3/10*a^2*b*h*x^10 + 1/3*a*b^2*d*x^9
 + 1/3*a^2*b*g*x^9 + 3/8*a*b^2*c*x^8 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 +
 1/7*a^3*h*x^7 + 1/2*a^2*b*d*x^6 + 1/6*a^3*g*x^6 + 3/5*a^2*b*c*x^5 + 1/5*a
^3*f*x^5 + 1/4*a^3*e*x^4 + 1/3*a^3*d*x^3 + 1/2*a^3*c*x^2
```

3.396.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97

$$\int x(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$
$$= x^5\left(\frac{f\,a^3}{5}+\frac{3\,b\,c\,a^2}{5}\right)+x^{11}\left(\frac{c\,b^3}{11}+\frac{3\,a\,f\,b^2}{11}\right)+x^6\left(\frac{g\,a^3}{6}+\frac{b\,d\,a^2}{2}\right)+x^{12}\left(\frac{d\,b^3}{12}+\frac{a\,g\,b^2}{4}\right)+x^7\left(\frac{h\,a^3}{7}+\frac{3\,b\,e\,a^2}{7}\right)+x^{13}\left(\frac{e\,b^3}{13}+\frac{3\,a\,h\,b^2}{13}\right)+\frac{a^3\,c\,x^2}{2}+\frac{a^3\,d\,x^3}{3}+\frac{a^3\,e\,x^4}{4}+\frac{b^3\,f\,x^{14}}{14}+\frac{b^3\,g\,x^{15}}{15}+\frac{b^3\,h\,x^{16}}{16}+\frac{3\,a\,b\,x^8\,(b\,c+a\,f)}{8}+\frac{a\,b\,x^9\,(b\,d+a\,g)}{3}+\frac{3\,a\,b\,x^{10}\,(b\,e+a\,h)}{10}$$

input
```
int(x*(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^5*((a^3*f)/5 + (3*a^2*b*c)/5) + x^11*((b^3*c)/11 + (3*a*b^2*f)/11) + x^6
*((a^3*g)/6 + (a^2*b*d)/2) + x^12*((b^3*d)/12 + (a*b^2*g)/4) + x^7*((a^3*h
)/7 + (3*a^2*b*e)/7) + x^13*((b^3*e)/13 + (3*a*b^2*h)/13) + (a^3*c*x^2)/2
+ (a^3*d*x^3)/3 + (a^3*e*x^4)/4 + (b^3*f*x^14)/14 + (b^3*g*x^15)/15 + (b^3
*h*x^16)/16 + (3*a*b*x^8*(b*c + a*f))/8 + (a*b*x^9*(b*d + a*g))/3 + (3*a*b
*x^10*(b*e + a*h))/10
```

3.397 $\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx$

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3.397.1 Optimal result

Integrand size = 35, antiderivative size = 207

$$\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{4}a^2(3bc+af)x^4+\frac{1}{5}a^2(3bd+ag)x^5+\frac{1}{6}a^3hx^6 +\frac{3}{7}ab(bc+af)x^7+\frac{3}{8}ab(bd+ag)x^8+\frac{1}{3}a^2bhx^9+\frac{1}{10}b^2(bc+3af)x^{10} +\frac{1}{11}b^2(bd+3ag)x^{11}+\frac{1}{4}ab^2hx^{12}+\frac{1}{13}b^3fx^{13}+\frac{1}{14}b^3gx^{14}+\frac{1}{15}b^3hx^{15}+\frac{e(a+bx^3)^4}{12b}$$

output
```
a^3*c*x+1/2*a^3*d*x^2+1/4*a^2*(a*f+3*b*c)*x^4+1/5*a^2*(a*g+3*b*d)*x^5+1/6*
a^3*h*x^6+3/7*a*b*(a*f+b*c)*x^7+3/8*a*b*(a*g+b*d)*x^8+1/3*a^2*b*h*x^9+1/10
*b^2*(3*a*f+b*c)*x^10+1/11*b^2*(3*a*g+b*d)*x^11+1/4*a*b^2*h*x^12+1/13*b^3*
f*x^13+1/14*b^3*g*x^14+1/15*b^3*h*x^15+1/12*e*(b*x^3+a)^4/b
```

3.397.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.82

$$\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5) \, dx = \frac{x(13ab^2x^6(3960c+7x(495d+440ex+6x^2(66f+60gx+55hx^2)))+2002a^3(60c+x(30d+x(20e+15f}{}$$

input
```
Integrate[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
(x*(13*a*b^2*x^6*(3960*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 5
5*h*x^2))) + 2002*a^3*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h
*x^3))) + 2*b^3*x^9*(6006*c + x*(5460*d + 11*x*(455*e + 420*f*x + 390*g*x^
2 + 364*h*x^3))) + 143*a^2*b*x^3*(630*c + x*(504*d + 5*x*(84*e + x*(72*f +
 7*x*(9*g + 8*h*x)))))))/120120
```

3.397.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^3+a\right)^3 \left(hx^5+gx^4+fx^3+dx+c\right)dx+\frac{e\left(a+bx^3\right)^4}{12b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^3hx^{14}+b^3gx^{13}+b^3fx^{12}+3ab^2hx^{11}+b^2(bd+3ag)x^{10}+b^2(bc+3af)x^9+3a^2bhx^8+3ab(bd+ag)x^7+3ab\right.$$
$$\frac{e\left(a+bx^3\right)^4}{12b}$$

$$\Big\downarrow \text{2009}$$

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{6}a^3hx^6+\frac{1}{4}a^2x^4(af+3bc)+\frac{1}{5}a^2x^5(ag+3bd)+\frac{1}{3}a^2bhx^9+\frac{1}{10}b^2x^{10}(3af+bc)+\frac{1}{11}b^2x^{11}(3ag+bd)+\frac{1}{4}ab^2hx^{12}+\frac{3}{7}abx^7(af+bc)+\frac{3}{8}abx^8(ag+bd)+\frac{e\left(a+bx^3\right)^4}{12b}+\frac{1}{13}b^3fx^{13}+\frac{1}{14}b^3gx^{14}+\frac{1}{15}b^3hx^{15}$$

input
```
Int[(a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (a^2*(3*b*c + a*f)*x^4)/4 + (a^2*(3*b*d + a*g)*x
^5)/5 + (a^3*h*x^6)/6 + (3*a*b*(b*c + a*f)*x^7)/7 + (3*a*b*(b*d + a*g)*x^8
)/8 + (a^2*b*h*x^9)/3 + (b^2*(b*c + 3*a*f)*x^10)/10 + (b^2*(b*d + 3*a*g)*x
^11)/11 + (a*b^2*h*x^12)/4 + (b^3*f*x^13)/13 + (b^3*g*x^14)/14 + (b^3*h*x^
15)/15 + (e*(a + b*x^3)^4)/(12*b)
```

3.397.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.397.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.05

method	result
norman	$a^3cx+\frac{a^3d\,x^2}{2}+\frac{a^3e\,x^3}{3}+\left(\frac{1}{4}f\,a^3+\frac{3}{4}a^2bc\right)x^4+\left(\frac{1}{5}a^3g+\frac{3}{5}d\,a^2b\right)x^5+\left(\frac{1}{6}a^3h+\frac{1}{2}a^2be\right)x^6+\left(\frac{3}{7}f\right.$
default	$\frac{b^3h\,x^{15}}{15}+\frac{b^3g\,x^{14}}{14}+\frac{b^3f\,x^{13}}{13}+\frac{(3a\,b^2h+b^3e)x^{12}}{12}+\frac{(3a\,b^2g+b^3d)x^{11}}{11}+\frac{(3a\,b^2f+b^3c)x^{10}}{10}+\frac{(3a^2bh+3a\,b^2e)x^9}{9}+($
gosper	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{4}a^2bc\,x^4+\frac{1}{5}x^5a^3g+\frac{3}{5}x^5bd\,a^2+\frac{1}{6}a^3h\,x^6+\frac{1}{2}a^2be\,x^6-$
risch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{4}a^2bc\,x^4+\frac{1}{5}x^5a^3g+\frac{3}{5}x^5bd\,a^2+\frac{1}{6}a^3h\,x^6+\frac{1}{2}a^2be\,x^6-$
parallelrisch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{4}a^2bc\,x^4+\frac{1}{5}x^5a^3g+\frac{3}{5}x^5bd\,a^2+\frac{1}{6}a^3h\,x^6+\frac{1}{2}a^2be\,x^6-$

input
```
int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+(1/4*f*a^3+3/4*a^2*b*c)*x^4+(1/5*a^3*g
+3/5*d*a^2*b)*x^5+(1/6*a^3*h+1/2*a^2*b*e)*x^6+(3/7*f*a^2*b+3/7*a*b^2*c)*x^
7+(3/8*a^2*b*g+3/8*a*b^2*d)*x^8+(1/3*a^2*b*h+1/3*a*b^2*e)*x^9+(3/10*a*b^2*
f+1/10*b^3*c)*x^10+(3/11*a*b^2*g+1/11*b^3*d)*x^11+(1/4*a*b^2*h+1/12*b^3*e)
*x^12+1/13*b^3*f*x^13+1/14*b^3*g*x^14+1/15*b^3*h*x^15
```

3.397.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.03

$$\int \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$= \frac{1}{15}\,b^3hx^{15} + \frac{1}{14}\,b^3gx^{14} + \frac{1}{13}\,b^3fx^{13} + \frac{1}{12}\,\left(b^3e + 3\,ab^2h\right)x^{12} + \frac{1}{11}\,\left(b^3d + 3\,ab^2g\right)x^{11} + \frac{1}{10}\,\left(b^3c + 3\,ab^2f\right)x^{10} + \frac{1}{3}\,\left(ab^2e + a^2bh\right)x^9 + \frac{3}{8}\,\left(ab^2d + a^2bg\right)x^8 + \frac{3}{7}\,\left(ab^2c + a^2bf\right)x^7 + \frac{1}{3}\,a^3ex^3 + \frac{1}{6}\,\left(3\,a^2be + a^3h\right)x^6 + \frac{1}{2}\,a^3dx^2 + \frac{1}{5}\,\left(3\,a^2bd + a^3g\right)x^5 + a^3cx + \frac{1}{4}\,\left(3\,a^2bc + a^3f\right)x^4$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas
")
```

output
```
1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/12*(b^3*e + 3*a*b^
2*h)*x^12 + 1/11*(b^3*d + 3*a*b^2*g)*x^11 + 1/10*(b^3*c + 3*a*b^2*f)*x^10
+ 1/3*(a*b^2*e + a^2*b*h)*x^9 + 3/8*(a*b^2*d + a^2*b*g)*x^8 + 3/7*(a*b^2*c
 + a^2*b*f)*x^7 + 1/3*a^3*e*x^3 + 1/6*(3*a^2*b*e + a^3*h)*x^6 + 1/2*a^3*d*
x^2 + 1/5*(3*a^2*b*d + a^3*g)*x^5 + a^3*c*x + 1/4*(3*a^2*b*c + a^3*f)*x^4
```

3.397.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.17

$$\int \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right)\,dx$$
$$= a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{b^3fx^{13}}{13} + \frac{b^3gx^{14}}{14} + \frac{b^3hx^{15}}{15} + x^{12}\left(\frac{ab^2h}{4} + \frac{b^3e}{12}\right) + x^{11} \cdot \left(\frac{3ab^2g}{11} + \frac{b^3d}{11}\right) + x^{10} \cdot \left(\frac{3ab^2f}{10} + \frac{b^3c}{10}\right) + x^9\left(\frac{a^2bh}{3} + \frac{ab^2e}{3}\right) + x^8 \cdot \left(\frac{3a^2bg}{8} + \frac{3ab^2d}{8}\right) + x^7 \cdot \left(\frac{3a^2bf}{7} + \frac{3ab^2c}{7}\right) + x^6\left(\frac{a^3h}{6} + \frac{a^2be}{2}\right) + x^5\left(\frac{a^3g}{5} + \frac{3a^2bd}{5}\right) + x^4\left(\frac{a^3f}{4} + \frac{3a^2bc}{4}\right)$$

input
```
integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + b**3*f*x**13/13 + b**3*g*x**14/
14 + b**3*h*x**15/15 + x**12*(a*b**2*h/4 + b**3*e/12) + x**11*(3*a*b**2*g/
11 + b**3*d/11) + x**10*(3*a*b**2*f/10 + b**3*c/10) + x**9*(a**2*b*h/3 + a
*b**2*e/3) + x**8*(3*a**2*b*g/8 + 3*a*b**2*d/8) + x**7*(3*a**2*b*f/7 + 3*a
*b**2*c/7) + x**6*(a**3*h/6 + a**2*b*e/2) + x**5*(a**3*g/5 + 3*a**2*b*d/5)
 + x**4*(a**3*f/4 + 3*a**2*b*c/4)
```

3.397.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.03

$$\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx$$
$$= \frac{1}{15}b^3hx^{15} + \frac{1}{14}b^3gx^{14} + \frac{1}{13}b^3fx^{13} + \frac{1}{12}(b^3e+3ab^2h)x^{12} + \frac{1}{11}(b^3d+3ab^2g)x^{11} + \frac{1}{10}(b^3c+3ab^2f)x^{10} + \frac{1}{3}(ab^2e+a^2bh)x^9 + \frac{3}{8}(ab^2d+a^2bg)x^8 + \frac{3}{7}(ab^2c+a^2bf)x^7 + \frac{1}{3}a^3ex^3 + \frac{1}{6}(3a^2be+a^3h)x^6 + \frac{1}{2}a^3dx^2 + \frac{1}{5}(3a^2bd+a^3g)x^5 + a^3cx + \frac{1}{4}(3a^2bc+a^3f)x^4$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima
")
```

output
```
1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/12*(b^3*e + 3*a*b^
2*h)*x^12 + 1/11*(b^3*d + 3*a*b^2*g)*x^11 + 1/10*(b^3*c + 3*a*b^2*f)*x^10
+ 1/3*(a*b^2*e + a^2*b*h)*x^9 + 3/8*(a*b^2*d + a^2*b*g)*x^8 + 3/7*(a*b^2*c
 + a^2*b*f)*x^7 + 1/3*a^3*e*x^3 + 1/6*(3*a^2*b*e + a^3*h)*x^6 + 1/2*a^3*d*
x^2 + 1/5*(3*a^2*b*d + a^3*g)*x^5 + a^3*c*x + 1/4*(3*a^2*b*c + a^3*f)*x^4
```

3.397.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.09

$$\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = \frac{1}{15}b^3hx^{15}+\frac{1}{14}b^3gx^{14}+\frac{1}{13}b^3fx^{13}+\frac{1}{12}b^3ex^{12}+\frac{1}{4}ab^2hx^{12}+\frac{1}{11}b^3dx^{11}+\frac{3}{11}ab^2gx^{11}+\frac{1}{10}b^3cx^{10}+\frac{3}{10}ab^2fx^{10}+\frac{1}{3}ab^2ex^9+\frac{1}{3}a^2bhx^9+\frac{3}{8}ab^2dx^8+\frac{3}{8}a^2bgx^8+\frac{3}{7}ab^2cx^7+\frac{3}{7}a^2bfx^7+\frac{1}{2}a^2bex^6+\frac{1}{6}a^3hx^6+\frac{3}{5}a^2bdx^5+\frac{1}{5}a^3gx^5+\frac{3}{4}a^2bcx^4+\frac{1}{4}a^3fx^4+\frac{1}{3}a^3ex^3+\frac{1}{2}a^3dx^2+a^3cx$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

output
```
1/15*b^3*h*x^15 + 1/14*b^3*g*x^14 + 1/13*b^3*f*x^13 + 1/12*b^3*e*x^12 + 1/
4*a*b^2*h*x^12 + 1/11*b^3*d*x^11 + 3/11*a*b^2*g*x^11 + 1/10*b^3*c*x^10 + 3
/10*a*b^2*f*x^10 + 1/3*a*b^2*e*x^9 + 1/3*a^2*b*h*x^9 + 3/8*a*b^2*d*x^8 + 3
/8*a^2*b*g*x^8 + 3/7*a*b^2*c*x^7 + 3/7*a^2*b*f*x^7 + 1/2*a^2*b*e*x^6 + 1/6
*a^3*h*x^6 + 3/5*a^2*b*d*x^5 + 1/5*a^3*g*x^5 + 3/4*a^2*b*c*x^4 + 1/4*a^3*f
*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.397.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.98

$$\int (a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)\,dx = x^4\left(\frac{f\,a^3}{4}+\frac{3\,b\,c\,a^2}{4}\right)+x^{10}\left(\frac{c\,b^3}{10}+\frac{3\,a\,f\,b^2}{10}\right)+x^5\left(\frac{g\,a^3}{5}+\frac{3\,b\,d\,a^2}{5}\right)+x^{11}\left(\frac{d\,b^3}{11}+\frac{3\,a\,g\,b^2}{11}\right)+x^6\left(\frac{h\,a^3}{6}+\frac{b\,e\,a^2}{2}\right)+x^{12}\left(\frac{e\,b^3}{12}+\frac{a\,h\,b^2}{4}\right)+\frac{a^3\,d\,x^2}{2}+\frac{a^3\,e\,x^3}{3}+\frac{b^3\,f\,x^{13}}{13}+\frac{b^3\,g\,x^{14}}{14}+\frac{b^3\,h\,x^{15}}{15}+a^3\,c\,x+\frac{3\,a\,b\,x^7\,(b\,c+a\,f)}{7}+\frac{3\,a\,b\,x^8\,(b\,d+a\,g)}{8}+\frac{a\,b\,x^9\,(b\,e+a\,h)}{3}$$

input
```
int((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5),x)
```

output
```
x^4*((a^3*f)/4 + (3*a^2*b*c)/4) + x^10*((b^3*c)/10 + (3*a*b^2*f)/10) + x^5
*((a^3*g)/5 + (3*a^2*b*d)/5) + x^11*((b^3*d)/11 + (3*a*b^2*g)/11) + x^6*((
a^3*h)/6 + (a^2*b*e)/2) + x^12*((b^3*e)/12 + (a*b^2*h)/4) + (a^3*d*x^2)/2
+ (a^3*e*x^3)/3 + (b^3*f*x^13)/13 + (b^3*g*x^14)/14 + (b^3*h*x^15)/15 + a^
3*c*x + (3*a*b*x^7*(b*c + a*f))/7 + (3*a*b*x^8*(b*d + a*g))/8 + (a*b*x^9*(
b*e + a*h))/3
```

3.397. $\int \left(a+bx^3\right)^3 \left(c+dx+ex^2+fx^3+gx^4+hx^5\right) \, dx$

3.398 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

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3.398.1 Optimal result

Integrand size = 38, antiderivative size = 200

$$\begin{aligned}\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx &= a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{1}{4}a^2(3bd+ag)x^4+\frac{1}{5}a^2(3be+ah)x^5+\frac{1}{2}ab^2cx^6\\&\quad+\frac{3}{7}ab(bd+ag)x^7+\frac{3}{8}ab(be+ah)x^8+\frac{1}{9}b^3cx^9+\frac{1}{10}b^2(bd+3ag)x^{10}\\&\quad+\frac{1}{11}b^2(be+3ah)x^{11}+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}+\frac{f(a+bx^3)^4}{12b}+a^3c\log(x)\end{aligned}$$

output
```
a^3*d*x+1/2*a^3*e*x^2+a^2*b*c*x^3+1/4*a^2*(a*g+3*b*d)*x^4+1/5*a^2*(a*h+3*b
*e)*x^5+1/2*a*b^2*c*x^6+3/7*a*b*(a*g+b*d)*x^7+3/8*a*b*(a*h+b*e)*x^8+1/9*b^
3*c*x^9+1/10*b^2*(3*a*g+b*d)*x^10+1/11*b^2*(3*a*h+b*e)*x^11+1/13*b^3*g*x^1
3+1/14*b^3*h*x^14+1/12*f*(b*x^3+a)^4/b+a^3*c*ln(x)
```

3.398.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$$
$$= a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{3}a^2(3bc+af)x^3+\frac{1}{4}a^2(3bd+ag)x^4+\frac{1}{5}a^2(3be+ah)x^5$$
$$+\frac{1}{2}ab(bc+af)x^6+\frac{3}{7}ab(bd+ag)x^7+\frac{3}{8}ab(be+ah)x^8+\frac{1}{9}b^2(bc+3af)x^9$$
$$+\frac{1}{10}b^2(bd+3ag)x^{10}+\frac{1}{11}b^2(be+3ah)x^{11}+\frac{1}{12}b^3fx^{12}+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}+a^3c\log(x)$$

input
```
Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

output
```
a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*c + a*f)*x^3)/3 + (a^2*(3*b*d + a*g)*x
^4)/4 + (a^2*(3*b*e + a*h)*x^5)/5 + (a*b*(b*c + a*f)*x^6)/2 + (3*a*b*(b*d
+ a*g)*x^7)/7 + (3*a*b*(b*e + a*h)*x^8)/8 + (b^2*(b*c + 3*a*f)*x^9)/9 + (b
^2*(b*d + 3*a*g)*x^10)/10 + (b^2*(b*e + 3*a*h)*x^11)/11 + (b^3*f*x^12)/12
+ (b^3*g*x^13)/13 + (b^3*h*x^14)/14 + a^3*c*Log[x]
```

3.398.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2018, 2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$$
$$\Big\downarrow \text{2018}$$
$$\int \frac{(bx^3+a)^3(hx^5+gx^4+ex^2+dx+c)}{x}dx+\frac{f(a+bx^3)^4}{12b}$$
$$\Big\downarrow \text{2360}$$

$$\int \Big(b^3hx^{13}+b^3gx^{12}+b^2(be+3ah)x^{10}+b^2(bd+3ag)x^9+b^3cx^8+3ab(be+ah)x^7+3ab(bd+ag)x^6+3ab^2cx^5$$

$$\frac{f\left(a+bx^3\right)^4}{12b}$$

$$\Big\downarrow \text{ 2009}$$

$$a^3c\log(x)+a^3dx+\frac{1}{2}a^3ex^2+a^2bcx^3+\frac{1}{4}a^2x^4(ag+3bd)+\frac{1}{5}a^2x^5(ah+3be)+\frac{1}{2}ab^2cx^6+\frac{1}{10}b^2x^{10}(3ag+bd)+\frac{1}{11}b^2x^{11}(3ah+be)+\frac{3}{7}abx^7(ag+bd)+\frac{3}{8}abx^8(ah+be)+\frac{f\left(a+bx^3\right)^4}{12b}+\frac{1}{9}b^3cx^9+\frac{1}{13}b^3gx^{13}+\frac{1}{14}b^3hx^{14}$$

input
```
Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x]
```

output
```
a^3*d*x + (a^3*e*x^2)/2 + a^2*b*c*x^3 + (a^2*(3*b*d + a*g)*x^4)/4 + (a^2*(
3*b*e + a*h)*x^5)/5 + (a*b^2*c*x^6)/2 + (3*a*b*(b*d + a*g)*x^7)/7 + (3*a*b
*(b*e + a*h)*x^8)/8 + (b^3*c*x^9)/9 + (b^2*(b*d + 3*a*g)*x^10)/10 + (b^2*(
b*e + 3*a*h)*x^11)/11 + (b^3*g*x^13)/13 + (b^3*h*x^14)/14 + (f*(a + b*x^3)
^4)/(12*b) + a^3*c*Log[x]
```

3.398.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.398. $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$

3.398.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.08

method	result
norman	$\left(\frac{1}{4}a^{3}g+\frac{3}{4}d\,a^{2}b\right)x^{4}+\left(\frac{1}{5}a^{3}h+\frac{3}{5}a^{2}be\right)x^{5}+\left(\frac{1}{3}f\,a^{3}+a^{2}bc\right)x^{3}+\left(\frac{1}{3}a\,b^{2}f+\frac{1}{9}b^{3}c\right)x^{9}+\left(\frac{3}{10}a\,b^{2}\right.$
default	$\frac{b^{3}h\,x^{14}}{14}+\frac{b^{3}g\,x^{13}}{13}+\frac{b^{3}f\,x^{12}}{12}+\frac{3a\,b^{2}h\,x^{11}}{11}+\frac{b^{3}e\,x^{11}}{11}+\frac{3a\,b^{2}g\,x^{10}}{10}+\frac{b^{3}d\,x^{10}}{10}+\frac{a\,b^{2}f\,x^{9}}{3}+\frac{b^{3}c\,x^{9}}{9}+\frac{3a^{2}bh\,x^{8}}{8}+$
risch	$\frac{b^{3}h\,x^{14}}{14}+\frac{b^{3}g\,x^{13}}{13}+\frac{b^{3}f\,x^{12}}{12}+\frac{3a\,b^{2}h\,x^{11}}{11}+\frac{b^{3}e\,x^{11}}{11}+\frac{3a\,b^{2}g\,x^{10}}{10}+\frac{b^{3}d\,x^{10}}{10}+\frac{a\,b^{2}f\,x^{9}}{3}+\frac{b^{3}c\,x^{9}}{9}+\frac{3a^{2}bh\,x^{8}}{8}+$
parallelrisch	$\frac{b^{3}h\,x^{14}}{14}+\frac{b^{3}g\,x^{13}}{13}+\frac{b^{3}f\,x^{12}}{12}+\frac{3a\,b^{2}h\,x^{11}}{11}+\frac{b^{3}e\,x^{11}}{11}+\frac{3a\,b^{2}g\,x^{10}}{10}+\frac{b^{3}d\,x^{10}}{10}+\frac{a\,b^{2}f\,x^{9}}{3}+\frac{b^{3}c\,x^{9}}{9}+\frac{3a^{2}bh\,x^{8}}{8}+$

input
```
int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/4*a^3*g+3/4*d*a^2*b)*x^4+(1/5*a^3*h+3/5*a^2*b*e)*x^5+(1/3*f*a^3+a^2*b*c
)*x^3+(1/3*a*b^2*f+1/9*b^3*c)*x^9+(3/10*a*b^2*g+1/10*b^3*d)*x^10+(3/11*a*b
^2*h+1/11*b^3*e)*x^11+(3/7*a^2*b*g+3/7*a*b^2*d)*x^7+(3/8*a^2*b*h+3/8*a*b^2
*e)*x^8+(1/2*f*a^2*b+1/2*a*b^2*c)*x^6+a^3*d*x+1/2*a^3*e*x^2+1/12*b^3*f*x^1
2+1/13*b^3*g*x^13+1/14*b^3*h*x^14+a^3*c*ln(x)
```

3.398.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx &= \frac{1}{14}\,b^3hx^{14}+\frac{1}{13}\,b^3gx^{13}+\frac{1}{12}\,b^3fx^{12}+\frac{1}{11}\,(b^3e+3\,ab^2h)x^{11}\\&\quad+\frac{1}{10}\,(b^3d+3\,ab^2g)x^{10}+\frac{1}{9}\,(b^3c+3\,ab^2f)x^9+\frac{3}{8}\,(ab^2e+a^2bh)x^8\\&\quad+\frac{3}{7}\,(ab^2d+a^2bg)x^7+\frac{1}{2}\,(ab^2c+a^2bf)x^6+\frac{1}{2}\,a^3ex^2+\frac{1}{5}\,(3\,a^2be+a^3h)x^5\\&\quad+a^3dx+\frac{1}{4}\,(3\,a^2bd+a^3g)x^4+a^3c\log{(x)}+\frac{1}{3}\,(3\,a^2bc+a^3f)x^3\end{aligned}$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fric
as")
```

output
```
1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*(b^3*e + 3*a*b^
2*h)*x^11 + 1/10*(b^3*d + 3*a*b^2*g)*x^10 + 1/9*(b^3*c + 3*a*b^2*f)*x^9 +
3/8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^2*b*g)*x^7 + 1/2*(a*b^2*c +
 a^2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/
4*(3*a^2*b*d + a^3*g)*x^4 + a^3*c*log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3
```

3.398.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$$
$$= a^3c\log(x) + a^3dx + \frac{a^3ex^2}{2} + \frac{b^3fx^{12}}{12} + \frac{b^3gx^{13}}{13} + \frac{b^3hx^{14}}{14} + x^{11}\cdot\left(\frac{3ab^2h}{11} + \frac{b^3e}{11}\right) + x^{10}\cdot\left(\frac{3ab^2g}{10} + \frac{b^3d}{10}\right) + x^9\left(\frac{ab^2f}{3} + \frac{b^3c}{9}\right) + x^8\cdot\left(\frac{3a^2bh}{8} + \frac{3ab^2e}{8}\right) + x^7\cdot\left(\frac{3a^2bg}{7} + \frac{3ab^2d}{7}\right) + x^6\left(\frac{a^2bf}{2} + \frac{ab^2c}{2}\right) + x^5\left(\frac{a^3h}{5} + \frac{3a^2be}{5}\right) + x^4\left(\frac{a^3g}{4} + \frac{3a^2bd}{4}\right) + x^3\left(\frac{a^3f}{3} + a^2bc\right)$$

input
```
integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

output
```
a**3*c*log(x) + a**3*d*x + a**3*e*x**2/2 + b**3*f*x**12/12 + b**3*g*x**13/
13 + b**3*h*x**14/14 + x**11*(3*a*b**2*h/11 + b**3*e/11) + x**10*(3*a*b**2
*g/10 + b**3*d/10) + x**9*(a*b**2*f/3 + b**3*c/9) + x**8*(3*a**2*b*h/8 + 3
*a*b**2*e/8) + x**7*(3*a**2*b*g/7 + 3*a*b**2*d/7) + x**6*(a**2*b*f/2 + a*b
**2*c/2) + x**5*(a**3*h/5 + 3*a**2*b*e/5) + x**4*(a**3*g/4 + 3*a**2*b*d/4)
 + x**3*(a**3*f/3 + a**2*b*c)
```

3.398.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{1}{14}\,b^3hx^{14}+\frac{1}{13}\,b^3gx^{13}+\frac{1}{12}\,b^3fx^{12}+\frac{1}{11}\,(b^3e+3\,ab^2h)x^{11} + \frac{1}{10}\,(b^3d+3\,ab^2g)x^{10}+\frac{1}{9}\,(b^3c+3\,ab^2f)x^9+\frac{3}{8}\,(ab^2e+a^2bh)x^8 + \frac{3}{7}\,(ab^2d+a^2bg)x^7+\frac{1}{2}\,(ab^2c+a^2bf)x^6+\frac{1}{2}\,a^3ex^2+\frac{1}{5}\,(3\,a^2be+a^3h)x^5 + a^3dx+\frac{1}{4}\,(3\,a^2bd+a^3g)x^4+a^3c\log(x)+\frac{1}{3}\,(3\,a^2bc+a^3f)x^3$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxi
ma")
```

output
```
1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*(b^3*e + 3*a*b^
2*h)*x^11 + 1/10*(b^3*d + 3*a*b^2*g)*x^10 + 1/9*(b^3*c + 3*a*b^2*f)*x^9 +
3/8*(a*b^2*e + a^2*b*h)*x^8 + 3/7*(a*b^2*d + a^2*b*g)*x^7 + 1/2*(a*b^2*c +
 a^2*b*f)*x^6 + 1/2*a^3*e*x^2 + 1/5*(3*a^2*b*e + a^3*h)*x^5 + a^3*d*x + 1/
4*(3*a^2*b*d + a^3*g)*x^4 + a^3*c*log(x) + 1/3*(3*a^2*b*c + a^3*f)*x^3
```

3.398.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx = \frac{1}{14}\,b^3hx^{14}+\frac{1}{13}\,b^3gx^{13}+\frac{1}{12}\,b^3fx^{12}+\frac{1}{11}\,b^3ex^{11}+\frac{3}{11}\,ab^2hx^{11}+\frac{1}{10}\,b^3dx^{10} + \frac{3}{10}\,ab^2gx^{10}+\frac{1}{9}\,b^3cx^9+\frac{1}{3}\,ab^2fx^9+\frac{3}{8}\,ab^2ex^8+\frac{3}{8}\,a^2bhx^8+\frac{3}{7}\,ab^2dx^7 + \frac{3}{7}\,a^2bgx^7+\frac{1}{2}\,ab^2cx^6+\frac{1}{2}\,a^2bfx^6+\frac{3}{5}\,a^2bex^5+\frac{1}{5}\,a^3hx^5+\frac{3}{4}\,a^2bdx^4 + \frac{1}{4}\,a^3gx^4+a^2bcx^3+\frac{1}{3}\,a^3fx^3+\frac{1}{2}\,a^3ex^2+a^3dx+a^3c\log(|x|)$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac
")
```

output
```
1/14*b^3*h*x^14 + 1/13*b^3*g*x^13 + 1/12*b^3*f*x^12 + 1/11*b^3*e*x^11 + 3/
11*a*b^2*h*x^11 + 1/10*b^3*d*x^10 + 3/10*a*b^2*g*x^10 + 1/9*b^3*c*x^9 + 1/
3*a*b^2*f*x^9 + 3/8*a*b^2*e*x^8 + 3/8*a^2*b*h*x^8 + 3/7*a*b^2*d*x^7 + 3/7*
a^2*b*g*x^7 + 1/2*a*b^2*c*x^6 + 1/2*a^2*b*f*x^6 + 3/5*a^2*b*e*x^5 + 1/5*a^
3*h*x^5 + 3/4*a^2*b*d*x^4 + 1/4*a^3*g*x^4 + a^2*b*c*x^3 + 1/3*a^3*f*x^3 +
1/2*a^3*e*x^2 + a^3*d*x + a^3*c*log(abs(x))
```

3.398.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x}\,dx$$
$$= x^3\left(\frac{f\,a^3}{3}+b\,c\,a^2\right)+x^9\left(\frac{c\,b^3}{9}+\frac{a\,f\,b^2}{3}\right)+x^4\left(\frac{g\,a^3}{4}+\frac{3\,b\,d\,a^2}{4}\right)+x^{10}\left(\frac{d\,b^3}{10}+\frac{3\,a\,g\,b^2}{10}\right)$$
$$+x^5\left(\frac{h\,a^3}{5}+\frac{3\,b\,e\,a^2}{5}\right)+x^{11}\left(\frac{e\,b^3}{11}+\frac{3\,a\,h\,b^2}{11}\right)+\frac{a^3\,e\,x^2}{2}+\frac{b^3\,f\,x^{12}}{12}+\frac{b^3\,g\,x^{13}}{13}+\frac{b^3\,h\,x^{14}}{14}$$
$$+a^3\,c\,\ln(x)+a^3\,d\,x+\frac{a\,b\,x^6\,(b\,c+a\,f)}{2}+\frac{3\,a\,b\,x^7\,(b\,d+a\,g)}{7}+\frac{3\,a\,b\,x^8\,(b\,e+a\,h)}{8}$$

input
```
int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x,x)
```

output
```
x^3*((a^3*f)/3 + a^2*b*c) + x^9*((b^3*c)/9 + (a*b^2*f)/3) + x^4*((a^3*g)/4
 + (3*a^2*b*d)/4) + x^10*((b^3*d)/10 + (3*a*b^2*g)/10) + x^5*((a^3*h)/5 +
(3*a^2*b*e)/5) + x^11*((b^3*e)/11 + (3*a*b^2*h)/11) + (a^3*e*x^2)/2 + (b^3
*f*x^12)/12 + (b^3*g*x^13)/13 + (b^3*h*x^14)/14 + a^3*c*log(x) + a^3*d*x +
 (a*b*x^6*(b*c + a*f))/2 + (3*a*b*x^7*(b*d + a*g))/7 + (3*a*b*x^8*(b*e + a
*h))/8
```

3.399 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

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3.399.1 Optimal result

Integrand size = 38, antiderivative size = 198

$$\begin{aligned}
&\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx \\
&= -\frac{a^3c}{x}+a^3ex+\frac{1}{2}a^2(3bc+af)x^2+a^2bdx^3+\frac{1}{4}a^2(3be+ah)x^4 \\
&\quad +\frac{3}{5}ab(bc+af)x^5+\frac{1}{2}ab^2dx^6+\frac{3}{7}ab(be+ah)x^7+\frac{1}{8}b^2(bc+3af)x^8+\frac{1}{9}b^3dx^9 \\
&\quad +\frac{1}{10}b^2(be+3ah)x^{10}+\frac{1}{11}b^3fx^{11}+\frac{1}{13}b^3hx^{13}+\frac{g(a+bx^3)^4}{12b}+a^3d\log(x)
\end{aligned}$$

output
```
-a^3*c/x+a^3*e*x+1/2*a^2*(a*f+3*b*c)*x^2+a^2*b*d*x^3+1/4*a^2*(a*h+3*b*e)*x
^4+3/5*a*b*(a*f+b*c)*x^5+1/2*a*b^2*d*x^6+3/7*a*b*(a*h+b*e)*x^7+1/8*b^2*(3*
a*f+b*c)*x^8+1/9*b^3*d*x^9+1/10*b^2*(3*a*h+b*e)*x^10+1/11*b^3*f*x^11+1/13*
b^3*h*x^13+1/12*g*(b*x^3+a)^4/b+a^3*d*ln(x)
```

3.399.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= a^3\left(-\frac{c}{x}+ex+\frac{1}{12}x^2(6f+4gx+3hx^2)\right)$$
$$+\frac{b^3x^8(6435c+5720dx+6x^2(858e+780fx+715gx^2+660hx^3))}{51480}$$
$$+\frac{1}{140}a^2bx^2(210c+x(140d+x(105e+84fx+70gx^2+60hx^3)))$$
$$+\frac{1}{840}ab^2x^5(504c+x(420d+x(360e+315fx+280gx^2+252hx^3)))+a^3d\log(x)$$

input
```
Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

output
```
a^3*(-(c/x) + e*x + (x^2*(6*f + 4*g*x + 3*h*x^2))/12) + (b^3*x^8*(6435*c +
 5720*d*x + 6*x^2*(858*e + 780*f*x + 715*g*x^2 + 660*h*x^3)))/51480 + (a^2
*b*x^2*(210*c + x*(140*d + x*(105*e + 84*f*x + 70*g*x^2 + 60*h*x^3))))/140
 + (a*b^2*x^5*(504*c + x*(420*d + x*(360*e + 315*f*x + 280*g*x^2 + 252*h*x
^3))))/840 + a^3*d*Log[x]
```

3.399.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2018, 2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$\Big\downarrow \text{2018}$$
$$\int \frac{(bx^3+a)^3(hx^5+fx^3+ex^2+dx+c)}{x^2}dx+\frac{g(a+bx^3)^4}{12b}$$
$$\Big\downarrow \text{2360}$$

$$\int \Big(b^3hx^{12} + b^3fx^{10} + b^2(be+3ah)x^9 + b^3dx^8 + b^2(bc+3af)x^7 + 3ab(be+ah)x^6 + 3ab^2dx^5 + 3ab(bc+af)x^4 +$$

$$\frac{g(a+bx^3)^4}{12b}$$

$$\Big\downarrow 2009$$

$$-\frac{a^3c}{x}+a^3d\log(x)+a^3ex+\frac{1}{2}a^2x^2(af+3bc)+a^2bdx^3+\frac{1}{4}a^2x^4(ah+3be)+\frac{1}{8}b^2x^8(3af+bc)+\frac{1}{2}ab^2dx^6+\frac{1}{10}b^2x^{10}(3ah+be)+\frac{3}{5}abx^5(af+bc)+\frac{3}{7}abx^7(ah+be)+\frac{g(a+bx^3)^4}{12b}+\frac{1}{9}b^3dx^9+\frac{1}{11}b^3fx^{11}+\frac{1}{13}b^3hx^{13}$$

input
```
Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x]
```

output
```
-((a^3*c)/x) + a^3*e*x + (a^2*(3*b*c + a*f)*x^2)/2 + a^2*b*d*x^3 + (a^2*(3
*b*e + a*h)*x^4)/4 + (3*a*b*(b*c + a*f)*x^5)/5 + (a*b^2*d*x^6)/2 + (3*a*b*
(b*e + a*h)*x^7)/7 + (b^2*(b*c + 3*a*f)*x^8)/8 + (b^3*d*x^9)/9 + (b^2*(b*e
 + 3*a*h)*x^10)/10 + (b^3*f*x^11)/11 + (b^3*h*x^13)/13 + (g*(a + b*x^3)^4)
/(12*b) + a^3*d*Log[x]
```

3.399.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.399. $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

3.399.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11

method	result
norman	$\frac{\left(\frac{1}{3}a^3g+d\,a^2b\right)x^4+\left(\frac{1}{4}a^3h+\frac{3}{4}a^2be\right)x^5+\left(\frac{1}{2}f\,a^3+\frac{3}{2}a^2bc\right)x^3+\left(\frac{3}{8}a\,b^2f+\frac{1}{8}b^3c\right)x^9+\left(\frac{1}{3}a\,b^2g+\frac{1}{9}b^3d\right)x^{10}+\left(\frac{3}{10}a\,b^2h+\frac{1}{10}b^3e\right)x^{11}+\left(\frac{1}{2}\right.}{x}$
default	$\frac{b^3h\,x^{13}}{13}+\frac{b^3g\,x^{12}}{12}+\frac{b^3f\,x^{11}}{11}+\frac{3a\,b^2h\,x^{10}}{10}+\frac{b^3e\,x^{10}}{10}+\frac{a\,b^2g\,x^9}{3}+\frac{b^3d\,x^9}{9}+\frac{3x^8a\,b^2f}{8}+\frac{b^3c\,x^8}{8}+\frac{3a^2bh\,x^7}{7}+3$
risch	$\frac{b^3h\,x^{13}}{13}+\frac{b^3g\,x^{12}}{12}+\frac{b^3f\,x^{11}}{11}+\frac{3a\,b^2h\,x^{10}}{10}+\frac{b^3e\,x^{10}}{10}+\frac{a\,b^2g\,x^9}{3}+\frac{b^3d\,x^9}{9}+\frac{3x^8a\,b^2f}{8}+\frac{b^3c\,x^8}{8}+\frac{3a^2bh\,x^7}{7}+3$
parallelrisch	$32760b^3f\,x^{12}+154440a^2bh\,x^8+360360a^2bd\,x^4+120120a\,b^2g\,x^{10}+180180f\,a^3x^3+45045b^3c\,x^9+120120a^3g\,x^4+90090a^3h\,x^5-3$

input
```
int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x,method=_RETURNVERBOS
E)
```

output
```
((1/3*a^3*g+d*a^2*b)*x^4+(1/4*a^3*h+3/4*a^2*b*e)*x^5+(1/2*f*a^3+3/2*a^2*b*
c)*x^3+(3/8*a*b^2*f+1/8*b^3*c)*x^9+(1/3*a*b^2*g+1/9*b^3*d)*x^10+(3/10*a*b^
2*h+1/10*b^3*e)*x^11+(1/2*a^2*b*g+1/2*a*b^2*d)*x^7+(3/7*a^2*b*h+3/7*a*b^2*
e)*x^8+(3/5*f*a^2*b+3/5*a*b^2*c)*x^6+a^3*e*x^2-c*a^3+1/11*b^3*f*x^12+1/12*
b^3*g*x^13+1/13*b^3*h*x^14)/x+a^3*d*ln(x)
```

3.399.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$=\frac{27720\,b^3hx^{14}+30030\,b^3gx^{13}+32760\,b^3fx^{12}+36036\,(b^3e+3\,ab^2h)x^{11}+40040\,(b^3d+3\,ab^2g)x^{10}+45045}{}$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fr
icas")
```

output
```
1/360360*(27720*b^3*h*x^14 + 30030*b^3*g*x^13 + 32760*b^3*f*x^12 + 36036*(
b^3*e + 3*a*b^2*h)*x^11 + 40040*(b^3*d + 3*a*b^2*g)*x^10 + 45045*(b^3*c +
3*a*b^2*f)*x^9 + 154440*(a*b^2*e + a^2*b*h)*x^8 + 180180*(a*b^2*d + a^2*b*
g)*x^7 + 216216*(a*b^2*c + a^2*b*f)*x^6 + 360360*a^3*e*x^2 + 90090*(3*a^2*
b*e + a^3*h)*x^5 + 360360*a^3*d*x*log(x) + 120120*(3*a^2*b*d + a^3*g)*x^4
- 360360*a^3*c + 180180*(3*a^2*b*c + a^3*f)*x^3)/x
```

3.399.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= -\frac{a^3c}{x} + a^3d\log(x) + a^3ex + \frac{b^3fx^{11}}{11} + \frac{b^3gx^{12}}{12} + \frac{b^3hx^{13}}{13} + x^{10}\cdot\left(\frac{3ab^2h}{10} + \frac{b^3e}{10}\right) + x^9\left(\frac{ab^2g}{3} + \frac{b^3d}{9}\right) + x^8\cdot\left(\frac{3ab^2f}{8} + \frac{b^3c}{8}\right) + x^7\cdot\left(\frac{3a^2bh}{7} + \frac{3ab^2e}{7}\right) + x^6\left(\frac{a^2bg}{2} + \frac{ab^2d}{2}\right) + x^5\cdot\left(\frac{3a^2bf}{5} + \frac{3ab^2c}{5}\right) + x^4\left(\frac{a^3h}{4} + \frac{3a^2be}{4}\right) + x^3\left(\frac{a^3g}{3} + a^2bd\right) + x^2\left(\frac{a^3f}{2} + \frac{3a^2bc}{2}\right)$$

input
```
integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

output
```
-a**3*c/x + a**3*d*log(x) + a**3*e*x + b**3*f*x**11/11 + b**3*g*x**12/12 +
 b**3*h*x**13/13 + x**10*(3*a*b**2*h/10 + b**3*e/10) + x**9*(a*b**2*g/3 +
b**3*d/9) + x**8*(3*a*b**2*f/8 + b**3*c/8) + x**7*(3*a**2*b*h/7 + 3*a*b**2
*e/7) + x**6*(a**2*b*g/2 + a*b**2*d/2) + x**5*(3*a**2*b*f/5 + 3*a*b**2*c/5
) + x**4*(a**3*h/4 + 3*a**2*b*e/4) + x**3*(a**3*g/3 + a**2*b*d) + x**2*(a*
*3*f/2 + 3*a**2*b*c/2)
```

3.399.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= \frac{1}{13}b^3hx^{13} + \frac{1}{12}b^3gx^{12} + \frac{1}{11}b^3fx^{11} + \frac{1}{10}\left(b^3e + 3\,ab^2h\right)x^{10} + \frac{1}{9}\left(b^3d + 3\,ab^2g\right)x^9 + \frac{1}{8}\left(b^3c + 3\,ab^2f\right)x^8 + \frac{3}{7}\left(ab^2e + a^2bh\right)x^7 + \frac{1}{2}\left(ab^2d + a^2bg\right)x^6 + \frac{3}{5}\left(ab^2c + a^2bf\right)x^5 + a^3ex + \frac{1}{4}\left(3\,a^2be + a^3h\right)x^4 + a^3d\log(x) + \frac{1}{3}\left(3\,a^2bd + a^3g\right)x^3 - \frac{a^3c}{x} + \frac{1}{2}\left(3\,a^2bc + a^3f\right)x^2$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="ma
xima")
```

output
```
1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 1/10*(b^3*e + 3*a*b^
2*h)*x^10 + 1/9*(b^3*d + 3*a*b^2*g)*x^9 + 1/8*(b^3*c + 3*a*b^2*f)*x^8 + 3/
7*(a*b^2*e + a^2*b*h)*x^7 + 1/2*(a*b^2*d + a^2*b*g)*x^6 + 3/5*(a*b^2*c + a
^2*b*f)*x^5 + a^3*e*x + 1/4*(3*a^2*b*e + a^3*h)*x^4 + a^3*d*log(x) + 1/3*(
3*a^2*b*d + a^3*g)*x^3 - a^3*c/x + 1/2*(3*a^2*b*c + a^3*f)*x^2
```

3.399.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= \frac{1}{13}\,b^3hx^{13} + \frac{1}{12}\,b^3gx^{12} + \frac{1}{11}\,b^3fx^{11} + \frac{1}{10}\,b^3ex^{10} + \frac{3}{10}\,ab^2hx^{10} + \frac{1}{9}\,b^3dx^9 + \frac{1}{3}\,ab^2gx^9 + \frac{1}{8}\,b^3cx^8 + \frac{3}{8}\,ab^2fx^8 + \frac{3}{7}\,ab^2ex^7 + \frac{3}{7}\,a^2bhx^7 + \frac{1}{2}\,ab^2dx^6 + \frac{1}{2}\,a^2bgx^6 + \frac{3}{5}\,ab^2cx^5 + \frac{3}{5}\,a^2bfx^5 + \frac{3}{4}\,a^2bex^4 + \frac{1}{4}\,a^3hx^4 + a^2bdx^3 + \frac{1}{3}\,a^3gx^3 + \frac{3}{2}\,a^2bcx^2 + \frac{1}{2}\,a^3fx^2 + a^3ex + a^3d\log(|x|) - \frac{a^3c}{x}$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="gi
ac")
```

output
```
1/13*b^3*h*x^13 + 1/12*b^3*g*x^12 + 1/11*b^3*f*x^11 + 1/10*b^3*e*x^10 + 3/
10*a*b^2*h*x^10 + 1/9*b^3*d*x^9 + 1/3*a*b^2*g*x^9 + 1/8*b^3*c*x^8 + 3/8*a*
b^2*f*x^8 + 3/7*a*b^2*e*x^7 + 3/7*a^2*b*h*x^7 + 1/2*a*b^2*d*x^6 + 1/2*a^2*
b*g*x^6 + 3/5*a*b^2*c*x^5 + 3/5*a^2*b*f*x^5 + 3/4*a^2*b*e*x^4 + 1/4*a^3*h*
x^4 + a^2*b*d*x^3 + 1/3*a^3*g*x^3 + 3/2*a^2*b*c*x^2 + 1/2*a^3*f*x^2 + a^3*
e*x + a^3*d*log(abs(x)) - a^3*c/x
```

3.399.9 Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$$
$$= x^2\left(\frac{f\,a^3}{2}+\frac{3\,b\,c\,a^2}{2}\right) + x^8\left(\frac{c\,b^3}{8}+\frac{3\,a\,f\,b^2}{8}\right) + x^3\left(\frac{g\,a^3}{3}+b\,d\,a^2\right) + x^9\left(\frac{d\,b^3}{9}+\frac{a\,g\,b^2}{3}\right) + x^4\left(\frac{h\,a^3}{4}+\frac{3\,b\,e\,a^2}{4}\right) + x^{10}\left(\frac{e\,b^3}{10}+\frac{3\,a\,h\,b^2}{10}\right) - \frac{a^3\,c}{x} + \frac{b^3\,f\,x^{11}}{11} + \frac{b^3\,g\,x^{12}}{12} + \frac{b^3\,h\,x^{13}}{13} + a^3\,d\,\ln(x) + a^3\,e\,x + \frac{3\,a\,b\,x^5\,(b\,c+a\,f)}{5} + \frac{a\,b\,x^6\,(b\,d+a\,g)}{2} + \frac{3\,a\,b\,x^7\,(b\,e+a\,h)}{7}$$

input
```
int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^2,x)
```

output
```
x^2*((a^3*f)/2 + (3*a^2*b*c)/2) + x^8*((b^3*c)/8 + (3*a*b^2*f)/8) + x^3*((
a^3*g)/3 + a^2*b*d) + x^9*((b^3*d)/9 + (a*b^2*g)/3) + x^4*((a^3*h)/4 + (3*
a^2*b*e)/4) + x^10*((b^3*e)/10 + (3*a*b^2*h)/10) - (a^3*c)/x + (b^3*f*x^11
)/11 + (b^3*g*x^12)/12 + (b^3*h*x^13)/13 + a^3*d*log(x) + a^3*e*x + (3*a*b
*x^5*(b*c + a*f))/5 + (a*b*x^6*(b*d + a*g))/2 + (3*a*b*x^7*(b*e + a*h))/7
```

3.399. $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^2}\,dx$

3.400 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

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3.400.1 Optimal result

Integrand size = 38, antiderivative size = 198

$$\begin{aligned}\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx &= -\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^2(3bc+af)x+\frac{1}{2}a^2(3bd+ag)x^2+a^2bex^3+\frac{3}{4}ab(bc+af)x^4\\ &\quad+\frac{3}{5}ab(bd+ag)x^5+\frac{1}{2}ab^2ex^6+\frac{1}{7}b^2(bc+3af)x^7+\frac{1}{8}b^2(bd+3ag)x^8\\ &\quad+\frac{1}{9}b^3ex^9+\frac{1}{10}b^3fx^{10}+\frac{1}{11}b^3gx^{11}+\frac{h(a+bx^3)^4}{12b}+a^3e\log(x)\end{aligned}$$

output
```
-1/2*a^3*c/x^2-a^3*d/x+a^2*(a*f+3*b*c)*x+1/2*a^2*(a*g+3*b*d)*x^2+a^2*b*e*x
^3+3/4*a*b*(a*f+b*c)*x^4+3/5*a*b*(a*g+b*d)*x^5+1/2*a*b^2*e*x^6+1/7*b^2*(3*
a*f+b*c)*x^7+1/8*b^2*(3*a*g+b*d)*x^8+1/9*b^3*e*x^9+1/10*b^3*f*x^10+1/11*b^
3*g*x^11+1/12*h*(b*x^3+a)^4/b+a^3*e*ln(x)
```

3.400.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= \frac{a^3(-3c-6dx+x^3(6f+3gx+2hx^2))}{6x^2} + \frac{b^3x^7(3960c+7x(495d+440ex+6x^2(66f+60gx+55hx^2)))}{27720} + \frac{1}{20}a^2bx\left(60c+x\left(30d+x\left(20e+15fx+12gx^2+10hx^3\right)\right)\right) + \frac{1}{840}ab^2x^4(630c+x(504d+5x(84e+x(72f+7x(9g+8hx))))) + a^3e\log(x)$$

input
```
Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

output
```
(a^3*(-3*c - 6*d*x + x^3*(6*f + 3*g*x + 2*h*x^2)))/(6*x^2) + (b^3*x^7*(396
0*c + 7*x*(495*d + 440*e*x + 6*x^2*(66*f + 60*g*x + 55*h*x^2))))/27720 + (
a^2*b*x*(60*c + x*(30*d + x*(20*e + 15*f*x + 12*g*x^2 + 10*h*x^3))))/20 +
(a*b^2*x^4*(630*c + x*(504*d + 5*x*(84*e + x*(72*f + 7*x*(9*g + 8*h*x)))))
)/840 + a^3*e*Log[x]
```

3.400.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2018, 2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$

$$\Big\downarrow \; 2018$$

$$\int \frac{(bx^3+a)^3\,(gx^4+fx^3+ex^2+dx+c)}{x^3}dx + \frac{h(a+bx^3)^4}{12b}$$

$$\Big\downarrow \; 2360$$

$$\int \Big(b^3gx^{10}+b^3fx^9+b^3ex^8+b^2(bd+3ag)x^7+b^2(bc+3af)x^6+3ab^2ex^5+3ab(bd+ag)x^4+3ab(bc+af)x^3+$$

$$\frac{h(a+bx^3)^4}{12b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3c}{2x^2}-\frac{a^3d}{x}+a^3e\log(x)+a^2x(af+3bc)+\frac{1}{2}a^2x^2(ag+3bd)+a^2bex^3+\frac{1}{7}b^2x^7(3af+bc)+\frac{1}{8}b^2x^8(3ag+bd)+\frac{1}{2}ab^2ex^6+\frac{3}{4}abx^4(af+bc)+\frac{3}{5}abx^5(ag+bd)+\frac{h(a+bx^3)^4}{12b}+\frac{1}{9}b^3ex^9+\frac{1}{10}b^3fx^{10}+\frac{1}{11}b^3gx^{11}$$

input
```
Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x]
```

output
```
-1/2*(a^3*c)/x^2 - (a^3*d)/x + a^2*(3*b*c + a*f)*x + (a^2*(3*b*d + a*g)*x^
2)/2 + a^2*b*e*x^3 + (3*a*b*(b*c + a*f)*x^4)/4 + (3*a*b*(b*d + a*g)*x^5)/5
 + (a*b^2*e*x^6)/2 + (b^2*(b*c + 3*a*f)*x^7)/7 + (b^2*(b*d + 3*a*g)*x^8)/8
 + (b^3*e*x^9)/9 + (b^3*f*x^10)/10 + (b^3*g*x^11)/11 + (h*(a + b*x^3)^4)/(
12*b) + a^3*e*Log[x]
```

3.400.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.400.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.10

method	result
norman	$\frac{\left(\frac{1}{2}a^3g+\frac{3}{2}d\,a^2b\right)x^4+\left(\frac{1}{3}a^3h+a^2be\right)x^5+\left(\frac{3}{7}a\,b^2f+\frac{1}{7}b^3c\right)x^9+\left(\frac{3}{8}a\,b^2g+\frac{1}{8}b^3d\right)x^{10}+\left(\frac{1}{3}a\,b^2h+\frac{1}{9}b^3e\right)x^{11}+\left(\frac{3}{5}a^2bg+\frac{3}{5}a\,b^2d\right)x^7+\left(\frac{1}{2}a\right.}{x^2}$
default	$\frac{b^3h\,x^{12}}{12}+\frac{b^3g\,x^{11}}{11}+\frac{b^3f\,x^{10}}{10}+\frac{a\,b^2h\,x^9}{3}+\frac{b^3e\,x^9}{9}+\frac{3a\,b^2g\,x^8}{8}+\frac{b^3d\,x^8}{8}+\frac{3x^7a\,b^2f}{7}+\frac{b^3c\,x^7}{7}+\frac{a^2bh\,x^6}{2}+\frac{a\,b^2e}{2}$
risch	$\frac{b^3h\,x^{12}}{12}+\frac{b^3g\,x^{11}}{11}+\frac{b^3f\,x^{10}}{10}+\frac{a\,b^2h\,x^9}{3}+\frac{b^3e\,x^9}{9}+\frac{3a\,b^2g\,x^8}{8}+\frac{b^3d\,x^8}{8}+\frac{3x^7a\,b^2f}{7}+\frac{b^3c\,x^7}{7}+\frac{a^2bh\,x^6}{2}+\frac{a\,b^2e}{2}$
parallelrisch	$2772b^3f\,x^{12}+13860a^2bh\,x^8-27720a^3dx+41580a^2bd\,x^4+10395a\,b^2g\,x^{10}+27720f\,a^3x^3+3960b^3c\,x^9+13860a^3g\,x^4+9240a^3h\,x^5$

input
```
int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x,method=_RETURNVERBOS
E)
```

output
```
((1/2*a^3*g+3/2*d*a^2*b)*x^4+(1/3*a^3*h+a^2*b*e)*x^5+(3/7*a*b^2*f+1/7*b^3*
c)*x^9+(3/8*a*b^2*g+1/8*b^3*d)*x^10+(1/3*a*b^2*h+1/9*b^3*e)*x^11+(3/5*a^2*
b*g+3/5*a*b^2*d)*x^7+(1/2*a^2*b*h+1/2*a*b^2*e)*x^8+(3/4*f*a^2*b+3/4*a*b^2*
c)*x^6+(a^3*f+3*a^2*b*c)*x^3-1/2*c*a^3-a^3*d*x+1/10*b^3*f*x^12+1/11*b^3*g*
x^13+1/12*b^3*h*x^14)/x^2+a^3*e*ln(x)
```

3.400.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$=\frac{2310\,b^3hx^{14}+2520\,b^3gx^{13}+2772\,b^3fx^{12}+3080\,(b^3e+3\,ab^2h)x^{11}+3465\,(b^3d+3\,ab^2g)x^{10}+3960\,(b^3c+}{}$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fr
icas")
```

output
```
1/27720*(2310*b^3*h*x^14 + 2520*b^3*g*x^13 + 2772*b^3*f*x^12 + 3080*(b^3*e
 + 3*a*b^2*h)*x^11 + 3465*(b^3*d + 3*a*b^2*g)*x^10 + 3960*(b^3*c + 3*a*b^2
*f)*x^9 + 13860*(a*b^2*e + a^2*b*h)*x^8 + 16632*(a*b^2*d + a^2*b*g)*x^7 +
20790*(a*b^2*c + a^2*b*f)*x^6 + 27720*a^3*e*x^2*log(x) + 9240*(3*a^2*b*e +
 a^3*h)*x^5 - 27720*a^3*d*x + 13860*(3*a^2*b*d + a^3*g)*x^4 - 13860*a^3*c
+ 27720*(3*a^2*b*c + a^3*f)*x^3)/x^2
```

3.400.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= a^3e\log(x) + \frac{b^3fx^{10}}{10} + \frac{b^3gx^{11}}{11} + \frac{b^3hx^{12}}{12} + x^9\left(\frac{ab^2h}{3} + \frac{b^3e}{9}\right) + x^8\cdot\left(\frac{3ab^2g}{8} + \frac{b^3d}{8}\right) + x^7 \cdot\left(\frac{3ab^2f}{7} + \frac{b^3c}{7}\right) + x^6\left(\frac{a^2bh}{2} + \frac{ab^2e}{2}\right) + x^5\cdot\left(\frac{3a^2bg}{5} + \frac{3ab^2d}{5}\right) + x^4\cdot\left(\frac{3a^2bf}{4} + \frac{3ab^2c}{4}\right) + x^3\left(\frac{a^3h}{3} + a^2be\right) + x^2\left(\frac{a^3g}{2} + \frac{3a^2bd}{2}\right) + x(a^3f + 3a^2bc) + \frac{-a^3c - 2a^3dx}{2x^2}$$

input
```
integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

output
```
a**3*e*log(x) + b**3*f*x**10/10 + b**3*g*x**11/11 + b**3*h*x**12/12 + x**9
*(a*b**2*h/3 + b**3*e/9) + x**8*(3*a*b**2*g/8 + b**3*d/8) + x**7*(3*a*b**2
*f/7 + b**3*c/7) + x**6*(a**2*b*h/2 + a*b**2*e/2) + x**5*(3*a**2*b*g/5 + 3
*a*b**2*d/5) + x**4*(3*a**2*b*f/4 + 3*a*b**2*c/4) + x**3*(a**3*h/3 + a**2*
b*e) + x**2*(a**3*g/2 + 3*a**2*b*d/2) + x*(a**3*f + 3*a**2*b*c) + (-a**3*c
 - 2*a**3*d*x)/(2*x**2)
```

3.400.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= \frac{1}{12}b^3hx^{12} + \frac{1}{11}b^3gx^{11} + \frac{1}{10}b^3fx^{10} + \frac{1}{9}(b^3e + 3ab^2h)x^9 + \frac{1}{8}(b^3d + 3ab^2g)x^8 + \frac{1}{7}(b^3c + 3ab^2f)x^7 + \frac{1}{2}(ab^2e + a^2bh)x^6 + \frac{3}{5}(ab^2d + a^2bg)x^5 + \frac{3}{4}(ab^2c + a^2bf)x^4 + a^3e\log(x) + \frac{1}{3}(3a^2be + a^3h)x^3 + \frac{1}{2}(3a^2bd + a^3g)x^2 + (3a^2bc + a^3f)x - \frac{2a^3dx + a^3c}{2x^2}$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="ma
xima")
```

output
```
1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/9*(b^3*e + 3*a*b^2
*h)*x^9 + 1/8*(b^3*d + 3*a*b^2*g)*x^8 + 1/7*(b^3*c + 3*a*b^2*f)*x^7 + 1/2*
(a*b^2*e + a^2*b*h)*x^6 + 3/5*(a*b^2*d + a^2*b*g)*x^5 + 3/4*(a*b^2*c + a^2
*b*f)*x^4 + a^3*e*log(x) + 1/3*(3*a^2*b*e + a^3*h)*x^3 + 1/2*(3*a^2*b*d +
a^3*g)*x^2 + (3*a^2*b*c + a^3*f)*x - 1/2*(2*a^3*d*x + a^3*c)/x^2
```

3.400.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= \frac{1}{12}b^3hx^{12}+\frac{1}{11}b^3gx^{11}+\frac{1}{10}b^3fx^{10}+\frac{1}{9}b^3ex^9+\frac{1}{3}ab^2hx^9+\frac{1}{8}b^3dx^8+\frac{3}{8}ab^2gx^8+\frac{1}{7}b^3cx^7+\frac{3}{7}ab^2fx^7+\frac{1}{2}ab^2ex^6+\frac{1}{2}a^2bhx^6+\frac{3}{5}ab^2dx^5+\frac{3}{5}a^2bgx^5+\frac{3}{4}ab^2cx^4+\frac{3}{4}a^2bfx^4+a^2bex^3+\frac{1}{3}a^3hx^3+\frac{3}{2}a^2bdx^2+\frac{1}{2}a^3gx^2+3a^2bcx+a^3fx+a^3e\log(|x|)-\frac{2a^3dx+a^3c}{2x^2}$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="gi
ac")
```

output
```
1/12*b^3*h*x^12 + 1/11*b^3*g*x^11 + 1/10*b^3*f*x^10 + 1/9*b^3*e*x^9 + 1/3*
a*b^2*h*x^9 + 1/8*b^3*d*x^8 + 3/8*a*b^2*g*x^8 + 1/7*b^3*c*x^7 + 3/7*a*b^2*
f*x^7 + 1/2*a*b^2*e*x^6 + 1/2*a^2*b*h*x^6 + 3/5*a*b^2*d*x^5 + 3/5*a^2*b*g*
x^5 + 3/4*a*b^2*c*x^4 + 3/4*a^2*b*f*x^4 + a^2*b*e*x^3 + 1/3*a^3*h*x^3 + 3/
2*a^2*b*d*x^2 + 1/2*a^3*g*x^2 + 3*a^2*b*c*x + a^3*f*x + a^3*e*log(abs(x))
- 1/2*(2*a^3*d*x + a^3*c)/x^2
```

3.400.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$$
$$= x^7\left(\frac{c\,b^3}{7}+\frac{3\,a\,f\,b^2}{7}\right)+x^2\left(\frac{g\,a^3}{2}+\frac{3\,b\,d\,a^2}{2}\right)+x^8\left(\frac{d\,b^3}{8}+\frac{3\,a\,g\,b^2}{8}\right)+x^3\left(\frac{h\,a^3}{3}+b\,e\,a^2\right)+x^9\left(\frac{e\,b^3}{9}+\frac{a\,h\,b^2}{3}\right)-\frac{\frac{a^3\,c}{2}+a^3\,d\,x}{x^2}+x\left(f\,a^3+3\,b\,c\,a^2\right)+\frac{b^3\,f\,x^{10}}{10}+\frac{b^3\,g\,x^{11}}{11}+\frac{b^3\,h\,x^{12}}{12}+a^3\,e\,\ln(x)+\frac{3\,a\,b\,x^4\,(b\,c+a\,f)}{4}+\frac{3\,a\,b\,x^5\,(b\,d+a\,g)}{5}+\frac{a\,b\,x^6\,(b\,e+a\,h)}{2}$$

input
```
int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^3,x)
```

output
```
x^7*((b^3*c)/7 + (3*a*b^2*f)/7) + x^2*((a^3*g)/2 + (3*a^2*b*d)/2) + x^8*((
b^3*d)/8 + (3*a*b^2*g)/8) + x^3*((a^3*h)/3 + a^2*b*e) + x^9*((b^3*e)/9 + (
a*b^2*h)/3) - ((a^3*c)/2 + a^3*d*x)/x^2 + x*(a^3*f + 3*a^2*b*c) + (b^3*f*x
^10)/10 + (b^3*g*x^11)/11 + (b^3*h*x^12)/12 + a^3*e*log(x) + (3*a*b*x^4*(b
*c + a*f))/4 + (3*a*b*x^5*(b*d + a*g))/5 + (a*b*x^6*(b*e + a*h))/2
```

3.400. $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^3}\,dx$

3.401 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$

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3.401.1 Optimal result

Integrand size = 38, antiderivative size = 209

$$\begin{aligned}\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx &= -\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2(3bd+ag)x+\frac{1}{2}a^2(3be+ah)x^2+ab(bc+af)x^3\\ &\quad+\frac{3}{4}ab(bd+ag)x^4+\frac{3}{5}ab(be+ah)x^5+\frac{1}{6}b^2(bc+3af)x^6+\frac{1}{7}b^2(bd+3ag)x^7\\ &\quad+\frac{1}{8}b^2(be+3ah)x^8+\frac{1}{9}b^3fx^9+\frac{1}{10}b^3gx^{10}+\frac{1}{11}b^3hx^{11}+a^2(3bc+af)\log(x)\end{aligned}$$

output
```
-1/3*a^3*c/x^3-1/2*a^3*d/x^2-a^3*e/x+a^2*(a*g+3*b*d)*x+1/2*a^2*(a*h+3*b*e)
*x^2+a*b*(a*f+b*c)*x^3+3/4*a*b*(a*g+b*d)*x^4+3/5*a*b*(a*h+b*e)*x^5+1/6*b^2
*(3*a*f+b*c)*x^6+1/7*b^2*(3*a*g+b*d)*x^7+1/8*b^2*(3*a*h+b*e)*x^8+1/9*b^3*f
*x^9+1/10*b^3*g*x^10+1/11*b^3*h*x^11+a^2*(a*f+3*b*c)*ln(x)
```

3.401.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= -\frac{a^3(2c+3x(d+2ex-x^3(2g+hx)))}{6x^3}+\frac{1}{20}a^2bx\big(60d+x\big(30e+x\big(20f+15gx+12hx^2\big)\big)\big)$$
$$+\frac{1}{280}ab^2x^3\big(280c+x\big(210d+x\big(168e+140fx+120gx^2+105hx^3\big)\big)\big)$$
$$+\frac{b^3x^6(4620c+x(3960d+7x(495e+4x(110f+99gx+90hx^2))))}{27720}+a^2(3bc+af)\log(x)$$

input
```
Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

output
```
-1/6*(a^3*(2*c + 3*x*(d + 2*e*x - x^3*(2*g + h*x))))/x^3 + (a^2*b*x*(60*d
+ x*(30*e + x*(20*f + 15*g*x + 12*h*x^2))))/20 + (a*b^2*x^3*(280*c + x*(21
0*d + x*(168*e + 140*f*x + 120*g*x^2 + 105*h*x^3))))/280 + (b^3*x^6*(4620*
c + x*(3960*d + 7*x*(495*e + 4*x*(110*f + 99*g*x + 90*h*x^2)))))/27720 + a
^2*(3*b*c + a*f)*Log[x]
```

3.401.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$\Big\downarrow \; 2360$$
$$\int \left(\frac{a^3c}{x^4}+\frac{a^3d}{x^3}+\frac{a^3e}{x^2}+\frac{a^2(af+3bc)}{x}+a^2(ag+3bd)+a^2x(ah+3be)+b^2x^5(3af+bc)+b^2x^6(3ag+bd)+b^2x^7\right.$$
$$\Big\downarrow \; 2009$$

$$-\frac{a^3c}{3x^3}-\frac{a^3d}{2x^2}-\frac{a^3e}{x}+a^2\log(x)(af+3bc)+a^2x(ag+3bd)+\frac{1}{2}a^2x^2(ah+3be)+\frac{1}{6}b^2x^6(3af+bc)+\frac{1}{7}b^2x^7(3ag+bd)+\frac{1}{8}b^2x^8(3ah+be)+abx^3(af+bc)+\frac{3}{4}abx^4(ag+bd)+\frac{3}{5}abx^5(ah+be)+\frac{1}{9}b^3fx^9+\frac{1}{10}b^3gx^{10}+\frac{1}{11}b^3hx^{11}$$

input
```
Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x]
```

output
```
-1/3*(a^3*c)/x^3 - (a^3*d)/(2*x^2) - (a^3*e)/x + a^2*(3*b*d + a*g)*x + (a^
2*(3*b*e + a*h)*x^2)/2 + a*b*(b*c + a*f)*x^3 + (3*a*b*(b*d + a*g)*x^4)/4 +
 (3*a*b*(b*e + a*h)*x^5)/5 + (b^2*(b*c + 3*a*f)*x^6)/6 + (b^2*(b*d + 3*a*g
)*x^7)/7 + (b^2*(b*e + 3*a*h)*x^8)/8 + (b^3*f*x^9)/9 + (b^3*g*x^10)/10 + (
b^3*h*x^11)/11 + a^2*(3*b*c + a*f)*Log[x]
```

3.401.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.401.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.03

method	result
norman	$\frac{\left(\frac{1}{2}a^3h+\frac{3}{2}a^2be\right)x^5+\left(\frac{1}{2}a\,b^2f+\frac{1}{6}b^3c\right)x^9+\left(\frac{3}{7}a\,b^2g+\frac{1}{7}b^3d\right)x^{10}+\left(\frac{3}{8}a\,b^2h+\frac{1}{8}b^3e\right)x^{11}+\left(\frac{3}{4}a^2bg+\frac{3}{4}a\,b^2d\right)x^7+\left(\frac{3}{5}a^2bh+\frac{3}{5}a\,b^2e\right)x^8+(}{x^3}$
default	$\frac{b^3h\,x^{11}}{11}+\frac{b^3g\,x^{10}}{10}+\frac{b^3f\,x^9}{9}+\frac{3a\,b^2h\,x^8}{8}+\frac{x^8b^3e}{8}+\frac{3a\,b^2g\,x^7}{7}+\frac{b^3d\,x^7}{7}+\frac{a\,b^2f\,x^6}{2}+\frac{b^3c\,x^6}{6}+\frac{3a^2bh\,x^5}{5}+\frac{3a\,b^2}{5}$
risch	$\frac{b^3h\,x^{11}}{11}+\frac{b^3g\,x^{10}}{10}+\frac{b^3f\,x^9}{9}+\frac{3a\,b^2h\,x^8}{8}+\frac{x^8b^3e}{8}+\frac{3a\,b^2g\,x^7}{7}+\frac{b^3d\,x^7}{7}+\frac{a\,b^2f\,x^6}{2}+\frac{b^3c\,x^6}{6}+\frac{3a^2bh\,x^5}{5}+\frac{3a\,b^2}{5}$
parallelrisch	$\frac{3080b^3f\,x^{12}+16632a^2bh\,x^8-13860a^3dx+83160a^2bd\,x^4+11880a\,b^2g\,x^{10}+4620b^3c\,x^9+27720a^3g\,x^4+13860a^3h\,x^5-9240c\,a^3+}{}$

input
```
int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x,method=_RETURNVERBOS
E)
```

output
```
((1/2*a^3*h+3/2*a^2*b*e)*x^5+(1/2*a*b^2*f+1/6*b^3*c)*x^9+(3/7*a*b^2*g+1/7*
b^3*d)*x^10+(3/8*a*b^2*h+1/8*b^3*e)*x^11+(3/4*a^2*b*g+3/4*a*b^2*d)*x^7+(3/
5*a^2*b*h+3/5*a*b^2*e)*x^8+(a^2*b*f+a*b^2*c)*x^6+(a^3*g+3*a^2*b*d)*x^4-1/3
*c*a^3-1/2*a^3*d*x-a^3*e*x^2+1/9*b^3*f*x^12+1/10*b^3*g*x^13+1/11*b^3*h*x^1
4)/x^3+(a^3*f+3*a^2*b*c)*ln(x)
```

3.401.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = \frac{2520\,b^3hx^{14}+2772\,b^3gx^{13}+3080\,b^3fx^{12}+3465\,(b^3e+3\,ab^2h)x^{11}+3960\,(b^3d+3\,ab^2g)x^{10}+4620\,(b^3c+$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fr
icas")
```

output
```
1/27720*(2520*b^3*h*x^14 + 2772*b^3*g*x^13 + 3080*b^3*f*x^12 + 3465*(b^3*e
 + 3*a*b^2*h)*x^11 + 3960*(b^3*d + 3*a*b^2*g)*x^10 + 4620*(b^3*c + 3*a*b^2
*f)*x^9 + 16632*(a*b^2*e + a^2*b*h)*x^8 + 20790*(a*b^2*d + a^2*b*g)*x^7 +
27720*(a*b^2*c + a^2*b*f)*x^6 - 27720*a^3*e*x^2 + 13860*(3*a^2*b*e + a^3*h
)*x^5 - 13860*a^3*d*x + 27720*(3*a^2*b*d + a^3*g)*x^4 + 27720*(3*a^2*b*c +
 a^3*f)*x^3*log(x) - 9240*a^3*c)/x^3
```

3.401.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx = a^2(af+3bc)\log(x)+\frac{b^3fx^9}{9}+\frac{b^3gx^{10}}{10}+\frac{b^3hx^{11}}{11}+x^8\cdot\left(\frac{3ab^2h}{8}+\frac{b^3e}{8}\right)+x^7\cdot\left(\frac{3ab^2g}{7}+\frac{b^3d}{7}\right)+x^6\left(\frac{ab^2f}{2}+\frac{b^3c}{6}\right)+x^5\cdot\left(\frac{3a^2bh}{5}+\frac{3ab^2e}{5}\right)+x^4\cdot\left(\frac{3a^2bg}{4}+\frac{3ab^2d}{4}\right)+x^3(a^2bf+ab^2c)+x^2\left(\frac{a^3h}{2}+\frac{3a^2be}{2}\right)+x(a^3g+3a^2bd)+\frac{-2a^3c-3a^3dx-6a^3ex^2}{6x^3}$$

input
```
integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

output
```
a**2*(a*f + 3*b*c)*log(x) + b**3*f*x**9/9 + b**3*g*x**10/10 + b**3*h*x**11
/11 + x**8*(3*a*b**2*h/8 + b**3*e/8) + x**7*(3*a*b**2*g/7 + b**3*d/7) + x*
*6*(a*b**2*f/2 + b**3*c/6) + x**5*(3*a**2*b*h/5 + 3*a*b**2*e/5) + x**4*(3*
a**2*b*g/4 + 3*a*b**2*d/4) + x**3*(a**2*b*f + a*b**2*c) + x**2*(a**3*h/2 +
 3*a**2*b*e/2) + x*(a**3*g + 3*a**2*b*d) + (-2*a**3*c - 3*a**3*d*x - 6*a**
3*e*x**2)/(6*x**3)
```

3.401.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx &= \frac{1}{11}\,b^3hx^{11}+\frac{1}{10}\,b^3gx^{10}+\frac{1}{9}\,b^3fx^9+\frac{1}{8}\,\left(b^3e+3\,ab^2h\right)x^8 \\ &\quad +\frac{1}{7}\,\left(b^3d+3\,ab^2g\right)x^7+\frac{1}{6}\,\left(b^3c+3\,ab^2f\right)x^6+\frac{3}{5}\,\left(ab^2e+a^2bh\right)x^5 \\ &\quad +\frac{3}{4}\,\left(ab^2d+a^2bg\right)x^4+\left(ab^2c+a^2bf\right)x^3+\frac{1}{2}\,\left(3\,a^2be+a^3h\right)x^2 \\ &\quad +\left(3\,a^2bd+a^3g\right)x+\left(3\,a^2bc+a^3f\right)\log\left(x\right)-\frac{6\,a^3ex^2+3\,a^3dx+2\,a^3c}{6\,x^3}\end{aligned}$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="ma
xima")
```

output
```
1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 1/8*(b^3*e + 3*a*b^2*h
)*x^8 + 1/7*(b^3*d + 3*a*b^2*g)*x^7 + 1/6*(b^3*c + 3*a*b^2*f)*x^6 + 3/5*(a
*b^2*e + a^2*b*h)*x^5 + 3/4*(a*b^2*d + a^2*b*g)*x^4 + (a*b^2*c + a^2*b*f)*
x^3 + 1/2*(3*a^2*b*e + a^3*h)*x^2 + (3*a^2*b*d + a^3*g)*x + (3*a^2*b*c + a
^3*f)*log(x) - 1/6*(6*a^3*e*x^2 + 3*a^3*d*x + 2*a^3*c)/x^3
```

3.401.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= \frac{1}{11}b^3hx^{11} + \frac{1}{10}b^3gx^{10} + \frac{1}{9}b^3fx^9 + \frac{1}{8}b^3ex^8 + \frac{3}{8}ab^2hx^8 + \frac{1}{7}b^3dx^7 + \frac{3}{7}ab^2gx^7 + \frac{1}{6}b^3cx^6 + \frac{1}{2}ab^2fx^6 + \frac{3}{5}ab^2ex^5 + \frac{3}{5}a^2bhx^5 + \frac{3}{4}ab^2dx^4 + \frac{3}{4}a^2bgx^4 + ab^2cx^3 + a^2bfx^3 + \frac{3}{2}a^2bex^2 + \frac{1}{2}a^3hx^2 + 3\,a^2bdx + a^3gx + \left(3\,a^2bc + a^3f\right)\log\left(|x|\right) - \frac{6\,a^3ex^2 + 3\,a^3dx + 2\,a^3c}{6\,x^3}$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="gi
ac")
```

output
```
1/11*b^3*h*x^11 + 1/10*b^3*g*x^10 + 1/9*b^3*f*x^9 + 1/8*b^3*e*x^8 + 3/8*a*
b^2*h*x^8 + 1/7*b^3*d*x^7 + 3/7*a*b^2*g*x^7 + 1/6*b^3*c*x^6 + 1/2*a*b^2*f*
x^6 + 3/5*a*b^2*e*x^5 + 3/5*a^2*b*h*x^5 + 3/4*a*b^2*d*x^4 + 3/4*a^2*b*g*x^
4 + a*b^2*c*x^3 + a^2*b*f*x^3 + 3/2*a^2*b*e*x^2 + 1/2*a^3*h*x^2 + 3*a^2*b*
d*x + a^3*g*x + (3*a^2*b*c + a^3*f)*log(abs(x)) - 1/6*(6*a^3*e*x^2 + 3*a^3
*d*x + 2*a^3*c)/x^3
```

3.401.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$$
$$= x^6\left(\frac{c\,b^3}{6} + \frac{a\,f\,b^2}{2}\right) + x^7\left(\frac{d\,b^3}{7} + \frac{3\,a\,g\,b^2}{7}\right) + x^2\left(\frac{h\,a^3}{2} + \frac{3\,b\,e\,a^2}{2}\right) + x^8\left(\frac{e\,b^3}{8} + \frac{3\,a\,h\,b^2}{8}\right) + \ln\left(x\right)\,\left(f\,a^3 + 3\,b\,c\,a^2\right) - \frac{e\,a^3\,x^2 + \frac{d\,a^3\,x}{2} + \frac{c\,a^3}{3}}{x^3} + x\,\left(g\,a^3 + 3\,b\,d\,a^2\right) + \frac{b^3\,f\,x^9}{9} + \frac{b^3\,g\,x^{10}}{10} + \frac{b^3\,h\,x^{11}}{11} + a\,b\,x^3\,\left(b\,c + a\,f\right) + \frac{3\,a\,b\,x^4\,\left(b\,d + a\,g\right)}{4} + \frac{3\,a\,b\,x^5\,\left(b\,e + a\,h\right)}{5}$$

input
```
int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^4,x)
```

output
```
x^6*((b^3*c)/6 + (a*b^2*f)/2) + x^7*((b^3*d)/7 + (3*a*b^2*g)/7) + x^2*((a^
3*h)/2 + (3*a^2*b*e)/2) + x^8*((b^3*e)/8 + (3*a*b^2*h)/8) + log(x)*(a^3*f
+ 3*a^2*b*c) - ((a^3*c)/3 + a^3*e*x^2 + (a^3*d*x)/2)/x^3 + x*(a^3*g + 3*a^
2*b*d) + (b^3*f*x^9)/9 + (b^3*g*x^10)/10 + (b^3*h*x^11)/11 + a*b*x^3*(b*c
+ a*f) + (3*a*b*x^4*(b*d + a*g))/4 + (3*a*b*x^5*(b*e + a*h))/5
```

3.401. $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^4}\,dx$

3.402 $\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$

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3.402.1 Optimal result

Integrand size = 38, antiderivative size = 209

$$\begin{aligned}\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx &= -\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(3bc+af)}{x}+a^2(3be+ah)x+\frac{3}{2}ab(bc+af)x^2\\&\quad+ab(bd+ag)x^3+\frac{3}{4}ab(be+ah)x^4+\frac{1}{5}b^2(bc+3af)x^5+\frac{1}{6}b^2(bd+3ag)x^6\\&\quad+\frac{1}{7}b^2(be+3ah)x^7+\frac{1}{8}b^3fx^8+\frac{1}{9}b^3gx^9+\frac{1}{10}b^3hx^{10}+a^2(3bd+ag)\log(x)\end{aligned}$$

output
```
-1/4*a^3*c/x^4-1/3*a^3*d/x^3-1/2*a^3*e/x^2-a^2*(a*f+3*b*c)/x+a^2*(a*h+3*b*
e)*x+3/2*a*b*(a*f+b*c)*x^2+a*b*(a*g+b*d)*x^3+3/4*a*b*(a*h+b*e)*x^4+1/5*b^2
*(3*a*f+b*c)*x^5+1/6*b^2*(3*a*g+b*d)*x^6+1/7*b^2*(3*a*h+b*e)*x^7+1/8*b^3*f
*x^8+1/9*b^3*g*x^9+1/10*b^3*h*x^10+a^2*(a*g+3*b*d)*ln(x)
```

3.402.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx\\&=\frac{-210a^3(3c+4dx+6x^2(e+2fx-2hx^3))+630a^2bx^3(-12c+x^2(12e+6fx+4gx^2+3hx^3))+18ab^2x^6(}{}\\&\quad+a^2(3bd+ag)\log(x)\end{aligned}$$

input
```
Integrate[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

output
```
(-210*a^3*(3*c + 4*d*x + 6*x^2*(e + 2*f*x - 2*h*x^3)) + 630*a^2*b*x^3*(-12
*c + x^2*(12*e + 6*f*x + 4*g*x^2 + 3*h*x^3)) + 18*a*b^2*x^6*(210*c + x*(14
0*d + 105*e*x + 84*f*x^2 + 70*g*x^3 + 60*h*x^4)) + b^3*x^9*(504*c + x*(420
*d + 360*e*x + 315*f*x^2 + 280*g*x^3 + 252*h*x^4)))/(2520*x^4) + a^2*(3*b*
d + a*g)*Log[x]
```

3.402.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5} \, dx$$

$$\Big\downarrow \text{2360}$$

$$\int \left(\frac{a^3c}{x^5}+\frac{a^3d}{x^4}+\frac{a^3e}{x^3}+\frac{a^2(af+3bc)}{x^2}+\frac{a^2(ag+3bd)}{x}+a^2(ah+3be)+b^2x^4(3af+bc)+b^2x^5(3ag+bd)+b^2x^6(\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3c}{4x^4}-\frac{a^3d}{3x^3}-\frac{a^3e}{2x^2}-\frac{a^2(af+3bc)}{x}+a^2\log(x)(ag+3bd)+a^2x(ah+3be)+\frac{1}{5}b^2x^5(3af+bc)+\frac{1}{6}b^2x^6(3ag+bd)+\frac{1}{7}b^2x^7(3ah+be)+\frac{3}{2}abx^2(af+bc)+abx^3(ag+bd)+\frac{3}{4}abx^4(ah+be)+\frac{1}{8}b^3fx^8+\frac{1}{9}b^3gx^9+\frac{1}{10}b^3hx^{10}$$

input
```
Int[((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x]
```

output
```
-1/4*(a^3*c)/x^4 - (a^3*d)/(3*x^3) - (a^3*e)/(2*x^2) - (a^2*(3*b*c + a*f))
/x + a^2*(3*b*e + a*h)*x + (3*a*b*(b*c + a*f)*x^2)/2 + a*b*(b*d + a*g)*x^3
 + (3*a*b*(b*e + a*h)*x^4)/4 + (b^2*(b*c + 3*a*f)*x^5)/5 + (b^2*(b*d + 3*a
*g)*x^6)/6 + (b^2*(b*e + 3*a*h)*x^7)/7 + (b^3*f*x^8)/8 + (b^3*g*x^9)/9 + (
b^3*h*x^10)/10 + a^2*(3*b*d + a*g)*Log[x]
```

3.402.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.402.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.03

method	result
default	$\frac{b^3 h\,x^{10}}{10}+\frac{b^3 g\,x^9}{9}+\frac{b^3 f\,x^8}{8}+\frac{3a\,b^2 h\,x^7}{7}+\frac{x^7 b^3 e}{7}+\frac{a\,b^2 g\,x^6}{2}+\frac{b^3 d\,x^6}{6}+\frac{3a\,b^2 f\,x^5}{5}+\frac{b^3 c\,x^5}{5}+\frac{3a^2 bh\,x^4}{4}+\frac{3a\,b^2 e}{4}$
norman	$\frac{\left(\frac{3}{5}a\,b^2 f+\frac{1}{5}b^3 c\right)x^9+\left(\frac{1}{2}a\,b^2 g+\frac{1}{6}b^3 d\right)x^{10}+\left(\frac{3}{7}a\,b^2 h+\frac{1}{7}b^3 e\right)x^{11}+\left(\frac{3}{4}a^2 bh+\frac{3}{4}a\,b^2 e\right)x^8+\left(\frac{3}{2}f\,a^2 b+\frac{3}{2}a\,b^2 c\right)x^6+\left(-f\,a^3-3a^2 bc\right)x^3+}{x^4}$
risch	$\frac{b^3 h\,x^{10}}{10}+\frac{b^3 g\,x^9}{9}+\frac{b^3 f\,x^8}{8}+\frac{3a\,b^2 h\,x^7}{7}+\frac{x^7 b^3 e}{7}+\frac{a\,b^2 g\,x^6}{2}+\frac{b^3 d\,x^6}{6}+\frac{3a\,b^2 f\,x^5}{5}+\frac{b^3 c\,x^5}{5}+\frac{3a^2 bh\,x^4}{4}+\frac{3a\,b^2 e}{4}$
parallelrisch	$\frac{315b^3 f\,x^{12}+1890a^2 bh\,x^8-840a^3 dx+1260a\,b^2 g\,x^{10}+7560\ln(x)x^4 a^2 bd-2520f\,a^3 x^3+504b^3 c\,x^9+2520a^3 h\,x^5-630c\,a^3+1080a}{}$

input
```
int((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x,method=_RETURNVERBOS
E)
```

output
```
1/10*b^3*h*x^10+1/9*b^3*g*x^9+1/8*b^3*f*x^8+3/7*a*b^2*h*x^7+1/7*x^7*b^3*e+
1/2*a*b^2*g*x^6+1/6*b^3*d*x^6+3/5*a*b^2*f*x^5+1/5*b^3*c*x^5+3/4*a^2*b*h*x^
4+3/4*a*b^2*e*x^4+a^2*b*g*x^3+a*b^2*d*x^3+3/2*x^2*f*a^2*b+3/2*a*b^2*c*x^2+
a^3*h*x+3*a^2*b*e*x+a^2*(a*g+3*b*d)*ln(x)-1/3*a^3*d/x^3-a^2*(a*f+3*b*c)/x-
1/2*a^3*e/x^2-1/4*a^3*c/x^4
```

3.402.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^3\,(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$=\frac{252\,b^3hx^{14}+280\,b^3gx^{13}+315\,b^3fx^{12}+360\,(b^3e+3\,ab^2h)x^{11}+420\,(b^3d+3\,ab^2g)x^{10}+504\,(b^3c+3\,ab^2f)}{}$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fr
icas")
```

output
```
1/2520*(252*b^3*h*x^14 + 280*b^3*g*x^13 + 315*b^3*f*x^12 + 360*(b^3*e + 3*
a*b^2*h)*x^11 + 420*(b^3*d + 3*a*b^2*g)*x^10 + 504*(b^3*c + 3*a*b^2*f)*x^9
 + 1890*(a*b^2*e + a^2*b*h)*x^8 + 2520*(a*b^2*d + a^2*b*g)*x^7 + 3780*(a*b
^2*c + a^2*b*f)*x^6 - 1260*a^3*e*x^2 + 2520*(3*a^2*b*e + a^3*h)*x^5 + 2520
*(3*a^2*b*d + a^3*g)*x^4*log(x) - 840*a^3*d*x - 630*a^3*c - 2520*(3*a^2*b*
c + a^3*f)*x^3)/x^4
```

3.402.6 Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{(a+bx^3)^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{x^5}\,dx &= a^2(ag+3bd)\log(x)+\frac{b^3fx^8}{8}+\frac{b^3gx^9}{9}+\frac{b^3hx^{10}}{10}+x^7\cdot\left(\frac{3ab^2h}{7}+\frac{b^3e}{7}\right)+x^6\left(\frac{ab^2g}{2}+\frac{b^3d}{6}\right)\\&\quad+x^5\cdot\left(\frac{3ab^2f}{5}+\frac{b^3c}{5}\right)+x^4\cdot\left(\frac{3a^2bh}{4}+\frac{3ab^2e}{4}\right)+x^3\left(a^2bg+ab^2d\right)+x^2\cdot\left(\frac{3a^2bf}{2}+\frac{3ab^2c}{2}\right)\\&\quad+x\left(a^3h+3a^2be\right)+\frac{-3a^3c-4a^3dx-6a^3ex^2+x^3(-12a^3f-36a^2bc)}{12x^4}\end{aligned}$$

input
```
integrate((b*x**3+a)**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

output
```
a**2*(a*g + 3*b*d)*log(x) + b**3*f*x**8/8 + b**3*g*x**9/9 + b**3*h*x**10/1
0 + x**7*(3*a*b**2*h/7 + b**3*e/7) + x**6*(a*b**2*g/2 + b**3*d/6) + x**5*(
3*a*b**2*f/5 + b**3*c/5) + x**4*(3*a**2*b*h/4 + 3*a*b**2*e/4) + x**3*(a**2
*b*g + a*b**2*d) + x**2*(3*a**2*b*f/2 + 3*a*b**2*c/2) + x*(a**3*h + 3*a**2
*b*e) + (-3*a**3*c - 4*a**3*d*x - 6*a**3*e*x**2 + x**3*(-12*a**3*f - 36*a*
*2*b*c))/(12*x**4)
```

3.402.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$= \frac{1}{10}\,b^3hx^{10} + \frac{1}{9}\,b^3gx^9 + \frac{1}{8}\,b^3fx^8 + \frac{1}{7}\,(b^3e+3\,ab^2h)x^7 + \frac{1}{6}\,(b^3d+3\,ab^2g)x^6 + \frac{1}{5}\,(b^3c+3\,ab^2f)x^5 + \frac{3}{4}\,(ab^2e+a^2bh)x^4 + (ab^2d+a^2bg)x^3 + \frac{3}{2}\,(ab^2c+a^2bf)x^2 + (3\,a^2be+a^3h)x + (3\,a^2bd+a^3g)\log(x) - \frac{6\,a^3ex^2+4\,a^3dx+3\,a^3c+12\,(3\,a^2bc+a^3f)x^3}{12\,x^4}$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="ma
xima")
```

output
```
1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 1/7*(b^3*e + 3*a*b^2*h)*
x^7 + 1/6*(b^3*d + 3*a*b^2*g)*x^6 + 1/5*(b^3*c + 3*a*b^2*f)*x^5 + 3/4*(a*b
^2*e + a^2*b*h)*x^4 + (a*b^2*d + a^2*b*g)*x^3 + 3/2*(a*b^2*c + a^2*b*f)*x^
2 + (3*a^2*b*e + a^3*h)*x + (3*a^2*b*d + a^3*g)*log(x) - 1/12*(6*a^3*e*x^2
 + 4*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^3*f)*x^3)/x^4
```

3.402.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^3 (c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$= \frac{1}{10}\,b^3hx^{10} + \frac{1}{9}\,b^3gx^9 + \frac{1}{8}\,b^3fx^8 + \frac{1}{7}\,b^3ex^7 + \frac{3}{7}\,ab^2hx^7 + \frac{1}{6}\,b^3dx^6 + \frac{1}{2}\,ab^2gx^6 + \frac{1}{5}\,b^3cx^5 + \frac{3}{5}\,ab^2fx^5 + \frac{3}{4}\,ab^2ex^4 + \frac{3}{4}\,a^2bhx^4 + ab^2dx^3 + a^2bgx^3 + \frac{3}{2}\,ab^2cx^2 + \frac{3}{2}\,a^2bfx^2 + 3\,a^2bex + a^3hx + (3\,a^2bd+a^3g)\log(|x|) - \frac{6\,a^3ex^2+4\,a^3dx+3\,a^3c+12\,(3\,a^2bc+a^3f)x^3}{12\,x^4}$$

input
```
integrate((b*x^3+a)^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="gi
ac")
```

output
```
1/10*b^3*h*x^10 + 1/9*b^3*g*x^9 + 1/8*b^3*f*x^8 + 1/7*b^3*e*x^7 + 3/7*a*b^
2*h*x^7 + 1/6*b^3*d*x^6 + 1/2*a*b^2*g*x^6 + 1/5*b^3*c*x^5 + 3/5*a*b^2*f*x^
5 + 3/4*a*b^2*e*x^4 + 3/4*a^2*b*h*x^4 + a*b^2*d*x^3 + a^2*b*g*x^3 + 3/2*a*
b^2*c*x^2 + 3/2*a^2*b*f*x^2 + 3*a^2*b*e*x + a^3*h*x + (3*a^2*b*d + a^3*g)*
log(abs(x)) - 1/12*(6*a^3*e*x^2 + 4*a^3*d*x + 3*a^3*c + 12*(3*a^2*b*c + a^
3*f)*x^3)/x^4
```

3.402.9 Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{x^5}\,dx$$
$$= x^5\left(\frac{c\,b^3}{5}+\frac{3\,a\,f\,b^2}{5}\right)+x^6\left(\frac{d\,b^3}{6}+\frac{a\,g\,b^2}{2}\right)+x^7\left(\frac{e\,b^3}{7}+\frac{3\,a\,h\,b^2}{7}\right)+\ln(x)\,\left(g\,a^3+3\,b\,d\,a^2\right)$$
$$-\frac{x^3\,(f\,a^3+3\,b\,c\,a^2)+\frac{a^3\,c}{4}+\frac{a^3\,e\,x^2}{2}+\frac{a^3\,d\,x}{3}}{x^4}+x\,\left(h\,a^3+3\,b\,e\,a^2\right)+\frac{b^3\,f\,x^8}{8}$$
$$+\frac{b^3\,g\,x^9}{9}+\frac{b^3\,h\,x^{10}}{10}+\frac{3\,a\,b\,x^2\,(b\,c+a\,f)}{2}+a\,b\,x^3\,(b\,d+a\,g)+\frac{3\,a\,b\,x^4\,(b\,e+a\,h)}{4}$$

input
```
int(((a + b*x^3)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/x^5,x)
```

output
```
x^5*((b^3*c)/5 + (3*a*b^2*f)/5) + x^6*((b^3*d)/6 + (a*b^2*g)/2) + x^7*((b^
3*e)/7 + (3*a*b^2*h)/7) + log(x)*(a^3*g + 3*a^2*b*d) - (x^3*(a^3*f + 3*a^2
*b*c) + (a^3*c)/4 + (a^3*e*x^2)/2 + (a^3*d*x)/3)/x^4 + x*(a^3*h + 3*a^2*b*
e) + (b^3*f*x^8)/8 + (b^3*g*x^9)/9 + (b^3*h*x^10)/10 + (3*a*b*x^2*(b*c + a
*f))/2 + a*b*x^3*(b*d + a*g) + (3*a*b*x^4*(b*e + a*h))/4
```

3.403 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$

3.403.1 Optimal result . . . 2980
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3.403.1 Optimal result

Integrand size = 38, antiderivative size = 331

$$\begin{aligned}
&\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx \\
&= -\frac{a(be-ah)x}{b^3} + \frac{(bc-af)x^2}{2b^2} + \frac{(bd-ag)x^3}{3b^2} + \frac{(be-ah)x^4}{4b^2} + \frac{fx^5}{5b} + \frac{gx^6}{6b} \\
&\quad + \frac{hx^7}{7b} + \frac{a^{2/3}\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}} \\
&\quad + \frac{a^{2/3}\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} \\
&\quad - \frac{a^{2/3}\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}} \\
&\quad - \frac{a(bd-ag)\log\left(a+bx^3\right)}{3b^3}
\end{aligned}$$

output
```
-a*(-a*h+b*e)*x/b^3+1/2*(-a*f+b*c)*x^2/b^2+1/3*(-a*g+b*d)*x^3/b^2+1/4*(-a*
h+b*e)*x^4/b^2+1/5*f*x^5/b+1/6*g*x^6/b+1/7*h*x^7/b+1/3*a^(2/3)*(b^(2/3)*(-
a*f+b*c)+a^(2/3)*(-a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/b^(10/3)-1/6*a^(2/3)*(b
^(2/3)*(-a*f+b*c)+a^(2/3)*(-a*h+b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/b^(10/3)-1/3*a*(-a*g+b*d)*ln(b*x^3+a)/b^3+1/3*a^(2/3)*(b^(5/3)*c-a^(
2/3)*b*e-a*b^(2/3)*f+a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3
^(1/2))/b^(10/3)*3^(1/2)
```

3.403.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.01

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= \frac{a(-be+ah)x}{b^3}+\frac{(bc-af)x^2}{2b^2}+\frac{(bd-ag)x^3}{3b^2}+\frac{(be-ah)x^4}{4b^2}+\frac{fx^5}{5b}+\frac{gx^6}{6b}$$
$$+\frac{hx^7}{7b}+\frac{a^{2/3}\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^{10/3}}$$
$$+\frac{a^{2/3}\left(b^{5/3}c+a^{2/3}be-ab^{2/3}f-a^{5/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}}$$
$$+\frac{a^{2/3}\left(-b^{5/3}c-a^{2/3}be+ab^{2/3}f+a^{5/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}$$
$$+\frac{a(-bd+ag)\log(a+bx^3)}{3b^3}$$

input
```
Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

output
```
(a*(-(b*e) + a*h)*x)/b^3 + ((b*c - a*f)*x^2)/(2*b^2) + ((b*d - a*g)*x^3)/(
3*b^2) + ((b*e - a*h)*x^4)/(4*b^2) + (f*x^5)/(5*b) + (g*x^6)/(6*b) + (h*x^
7)/(7*b) + (a^(2/3)*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(10/3)) + (a^(2/3)*(
b^(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x
])/(3*b^(10/3)) + (a^(2/3)*(-(b^(5/3)*c) - a^(2/3)*b*e + a*b^(2/3)*f + a^(
5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(10/3)) + (a*
(-(b*d) + a*g)*Log[a + b*x^3])/(3*b^3)
```

3.403.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2375, 27, 2375, 27, 2375, 27, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}dx$$

$$\Big\downarrow \ 2375$$

$$\frac{\int \frac{7x^4\left(bgx^4+bfx^3+(be-ah)x^2+bdx+bc\right)}{bx^3+a}dx}{7b}+\frac{hx^7}{7b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{x^4\left(bgx^4+bfx^3+(be-ah)x^2+bdx+bc\right)}{bx^3+a}dx}{b}+\frac{hx^7}{7b}$$

$$\Big\downarrow \ 2375$$

$$\frac{\frac{\int \frac{6x^4\left(b^2fx^3+b(be-ah)x^2+b(bd-ag)x+b^2c\right)}{bx^3+a}dx}{6b}+\frac{gx^6}{6}}{b}+\frac{hx^7}{7b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{x^4\left(b^2fx^3+b(be-ah)x^2+b(bd-ag)x+b^2c\right)}{bx^3+a}dx}{b}+\frac{gx^6}{6}}{b}+\frac{hx^7}{7b}$$

$$\Big\downarrow \ 2375$$

$$\frac{\frac{\frac{\int \frac{5x^4\left((be-ah)x^2b^2+(bc-af)b^2+(bd-ag)xb^2\right)}{bx^3+a}dx}{5b}+\frac{1}{5}bfx^5}{b}+\frac{gx^6}{6}}{b}+\frac{hx^7}{7b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{x^4\left((be-ah)x^2b^2+(bc-af)b^2+(bd-ag)xb^2\right)}{bx^3+a}dx}{b}+\frac{1}{5}bfx^5}{b}+\frac{gx^6}{6}}{b}+\frac{hx^7}{7b}$$

$$\Big\downarrow \ 2426$$

$$\frac{\frac{\frac{\int \left(b(be-ah)x^3+b(bd-ag)x^2+b(bc-af)x-a(be-ah)+\frac{(be-ah)a^2-b(bd-ag)x^2a-b(bc-af)xa}{bx^3+a}\right)dx}{b}+\frac{1}{5}bfx^5}{b}+\frac{gx^6}{6}}{b}+\frac{hx^7}{7b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\frac{\frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6\sqrt[3]{b}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{}}{b}}{b}}{b}$$

$$\frac{hx^7}{7b}$$

3.403. $\int \frac{x^4\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}dx$

input
```
Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

output
```
(h*x^7)/(7*b) + ((g*x^6)/6 + ((b*f*x^5)/5 + (-(a*(b*e - a*h)*x) + (b*(b*c
- a*f)*x^2)/2 + (b*(b*d - a*g)*x^3)/3 + (b*(b*e - a*h)*x^4)/4 + (a^(2/3)*(
b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3)) + (a^(2/3)*(b^(2/3)*(b*c - a
*f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*b^(1/3)) - (a^(2/3
)*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(6*b^(1/3)) - (a*(b*d - a*g)*Log[a + b*x^3])/3)/b)/b)/
b
```

3.403.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.403.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.50

method	result
risch	$\frac{h\,x^{7}}{7b}+\frac{g\,x^{6}}{6b}+\frac{f\,x^{5}}{5b}-\frac{ah\,x^{4}}{4b^{2}}+\frac{e\,x^{4}}{4b}-\frac{ag\,x^{3}}{3b^{2}}+\frac{d\,x^{3}}{3b}-\frac{af\,x^{2}}{2b^{2}}+\frac{c\,x^{2}}{2b}+\frac{a^{2}hx}{b^{3}}-\frac{aex}{b^{2}}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(b(a\right.}{}\right.}{}$
default	$\frac{\frac{1}{7}b^{2}h\,x^{7}+\frac{1}{6}b^{2}g\,x^{6}+\frac{1}{5}f\,x^{5}b^{2}-\frac{1}{4}abh\,x^{4}+\frac{1}{4}b^{2}e\,x^{4}-\frac{1}{3}abg\,x^{3}+\frac{1}{3}d\,x^{3}b^{2}-\frac{1}{2}abf\,x^{2}+\frac{1}{2}b^{2}c\,x^{2}+a^{2}hx-abex}{b^{3}}-\frac{\left(\left(a^{2}h-aeb\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.\right.}{}$

input
```
int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/7*h*x^7/b+1/6*g*x^6/b+1/5*f*x^5/b-1/4/b^2*a*h*x^4+1/4/b*e*x^4-1/3/b^2*a*
g*x^3+1/3*d*x^3/b-1/2/b^2*a*f*x^2+1/2*c*x^2/b+1/b^3*a^2*h*x-1/b^2*a*e*x+1/
3/b^4*a*sum((b*(a*g-b*d)*_R^2+b*(a*f-b*c)*_R-a^2*h+a*e*b)/_R^2*ln(x-_R),_R
=RootOf(_Z^3*b+a))
```

3.403.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 15635, normalized size of antiderivative = 47.24

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fric
as")
```

output
```
Too large to include
```

3.403.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

3.403.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.14

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= -\frac{\sqrt{3}\left(ab^2c\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^2bf\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^2be\left(\frac{a}{b}\right)^{\frac{1}{3}} + a^3h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}$$
$$+ \frac{60\,b^2hx^7 + 70\,b^2gx^6 + 84\,b^2fx^5 + 105\,(b^2e-abh)x^4 + 140\,(b^2d-abg)x^3 + 210\,(b^2c-abf)x^2 - 420\,(ab}{420\,b^3}$$
$$- \frac{\left(2\,ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,a^2bg\left(\frac{a}{b}\right)^{\frac{2}{3}} + ab^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2bf\left(\frac{a}{b}\right)^{\frac{1}{3}} + a^2be - a^3h\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{\left(ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^2bg\left(\frac{a}{b}\right)^{\frac{2}{3}} - ab^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} + a^2bf\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2be + a^3h\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxi
ma")
```

output
```
-1/3*sqrt(3)*(a*b^2*c*(a/b)^(2/3) - a^2*b*f*(a/b)^(2/3) - a^2*b*e*(a/b)^(1
/3) + a^3*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a*b^3) + 1/420*(60*b^2*h*x^7 + 70*b^2*g*x^6 + 84*b^2*f*x^5 + 105*(b^2*
e - a*b*h)*x^4 + 140*(b^2*d - a*b*g)*x^3 + 210*(b^2*c - a*b*f)*x^2 - 420*(
a*b*e - a^2*h)*x)/b^3 - 1/6*(2*a*b^2*d*(a/b)^(2/3) - 2*a^2*b*g*(a/b)^(2/3)
 + a*b^2*c*(a/b)^(1/3) - a^2*b*f*(a/b)^(1/3) + a^2*b*e - a^3*h)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) - 1/3*(a*b^2*d*(a/b)^(2/3)
- a^2*b*g*(a/b)^(2/3) - a*b^2*c*(a/b)^(1/3) + a^2*b*f*(a/b)^(1/3) - a^2*b*
e + a^3*h)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

3.403.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.13

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = -\frac{(abd-a^2g)\log(|bx^3+a|)}{3\,b^3} + \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}\,abe-(-ab^2)^{\frac{1}{3}}\,a^2h+(-ab^2)^{\frac{2}{3}}\,bc-(-ab^2)^{\frac{2}{3}}\,af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} + \frac{\left((-ab^2)^{\frac{1}{3}}\,abe-(-ab^2)^{\frac{1}{3}}\,a^2h-(-ab^2)^{\frac{2}{3}}\,bc+(-ab^2)^{\frac{2}{3}}\,af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4} + \frac{60\,b^6hx^7+70\,b^6gx^6+84\,b^6fx^5+105\,b^6ex^4-105\,ab^5hx^4+140\,b^6dx^3-140\,ab^5gx^3+210\,b^6cx^2-210\,a}{420\,b^7} + \frac{\left(ab^{14}c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^{13}f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^{13}e+a^3b^{12}h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{15}}$$

input
```
integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac
")
```

output
```
-1/3*(a*b*d - a^2*g)*log(abs(b*x^3 + a))/b^3 + 1/3*sqrt(3)*((-a*b^2)^(1/3)
*a*b*e - (-a*b^2)^(1/3)*a^2*h + (-a*b^2)^(2/3)*b*c - (-a*b^2)^(2/3)*a*f)*a
rctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/6*((-a*b^2)^(
1/3)*a*b*e - (-a*b^2)^(1/3)*a^2*h - (-a*b^2)^(2/3)*b*c + (-a*b^2)^(2/3)*a*
f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/420*(60*b^6*h*x^7 + 70
*b^6*g*x^6 + 84*b^6*f*x^5 + 105*b^6*e*x^4 - 105*a*b^5*h*x^4 + 140*b^6*d*x^
3 - 140*a*b^5*g*x^3 + 210*b^6*c*x^2 - 210*a*b^5*f*x^2 - 420*a*b^5*e*x + 42
0*a^2*b^4*h*x)/b^7 + 1/3*(a*b^14*c*(-a/b)^(1/3) - a^2*b^13*f*(-a/b)^(1/3)
- a^2*b^13*e + a^3*b^12*h)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^15
)
```

3.403.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 1271, normalized size of antiderivative = 3.84

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= x^2\left(\frac{c}{2\,b}-\frac{a\,f}{2\,b^2}\right)+x^3\left(\frac{d}{3\,b}-\frac{a\,g}{3\,b^2}\right)+x^4\left(\frac{e}{4\,b}-\frac{a\,h}{4\,b^2}\right)$$
$$+\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\big(27\,b^{10}\,z^3+27\,a\,b^8\,d\,z^2-27\,a^2\,b^7\,g\,z^2-9\,a^4\,b^4\,f\,h\,z-18\,a^3\,b^5\,d\,g\,z+9\,a^3\,b^5\,e\,f\,z+9\,a^3\,b^5\,c\,h\,z\right.\right.$$
$$+\frac{a^5\,g^2+a^3\,b^2\,d^2-a^5\,f\,h+a^4\,b\,c\,h-2\,a^4\,b\,d\,g+a^4\,b\,e\,f-a^3\,b^2\,c\,e}{b^4}$$
$$\left.+\frac{x\,(a^4\,b\,f^2+a^2\,b^3\,c^2+a^5\,g\,h-a^4\,b\,d\,h-a^4\,b\,e\,g-2\,a^3\,b^2\,c\,f+a^3\,b^2\,d\,e)}{b^4}\right)\mathrm{root}\big(27\,b^{10}\,z^3$$
$$+27\,a\,b^8\,d\,z^2-27\,a^2\,b^7\,g\,z^2-9\,a^4\,b^4\,f\,h\,z-18\,a^3\,b^5\,d\,g\,z+9\,a^3\,b^5\,e\,f\,z+9\,a^3\,b^5\,c\,h\,z$$
$$-9\,a^2\,b^6\,c\,e\,z+9\,a^4\,b^4\,g^2\,z+9\,a^2\,b^6\,d^2\,z+3\,a^6\,b\,f\,g\,h-3\,a^5\,b^2\,e\,f\,g-3\,a^5\,b^2\,d\,f\,h$$
$$-3\,a^5\,b^2\,c\,g\,h+3\,a^4\,b^3\,d\,e\,f+3\,a^4\,b^3\,c\,e\,g+3\,a^4\,b^3\,c\,d\,h-3\,a^3\,b^4\,c\,d\,e-3\,a^6\,b\,e\,h^2$$
$$+3\,a^5\,b^2\,e^2\,h+3\,a^5\,b^2\,d\,g^2-3\,a^4\,b^3\,d^2\,g-3\,a^4\,b^3\,c\,f^2+3\,a^3\,b^4\,c^2\,f+a^5\,b^2\,f^3+a^3\,b^4\,d^3$$
$$\left.+a^7\,h^3-a^4\,b^3\,e^3-a^2\,b^5\,c^3-a^6\,b\,g^3,z,k\big)\right)+\frac{f\,x^5}{5\,b}+\frac{g\,x^6}{6\,b}+\frac{h\,x^7}{7\,b}-\frac{a\,x\left(\frac{e}{b}-\frac{a\,h}{b^2}\right)}{b}$$

input
```
int((x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

output
```
x^2*(c/(2*b) - (a*f)/(2*b^2)) + x^3*(d/(3*b) - (a*g)/(3*b^2)) + x^4*(e/(4*
b) - (a*h)/(4*b^2)) + symsum(log(root(27*b^10*z^3 + 27*a*b^8*d*z^2 - 27*a^
2*b^7*g*z^2 - 9*a^4*b^4*f*h*z - 18*a^3*b^5*d*g*z + 9*a^3*b^5*e*f*z + 9*a^3
*b^5*c*h*z - 9*a^2*b^6*c*e*z + 9*a^4*b^4*g^2*z + 9*a^2*b^6*d^2*z + 3*a^6*b
*f*g*h - 3*a^5*b^2*e*f*g - 3*a^5*b^2*d*f*h - 3*a^5*b^2*c*g*h + 3*a^4*b^3*d
*e*f + 3*a^4*b^3*c*e*g + 3*a^4*b^3*c*d*h - 3*a^3*b^4*c*d*e - 3*a^6*b*e*h^2
 + 3*a^5*b^2*e^2*h + 3*a^5*b^2*d*g^2 - 3*a^4*b^3*d^2*g - 3*a^4*b^3*c*f^2 +
 3*a^3*b^4*c^2*f + a^5*b^2*f^3 + a^3*b^4*d^3 + a^7*h^3 - a^4*b^3*e^3 - a^2
*b^5*c^3 - a^6*b*g^3, z, k)*((6*a^2*b^4*d - 6*a^3*b^3*g)/b^4 + (x*(3*a^2*b
^4*e - 3*a^3*b^3*h))/b^4 + 9*root(27*b^10*z^3 + 27*a*b^8*d*z^2 - 27*a^2*b^
7*g*z^2 - 9*a^4*b^4*f*h*z - 18*a^3*b^5*d*g*z + 9*a^3*b^5*e*f*z + 9*a^3*b^5
*c*h*z - 9*a^2*b^6*c*e*z + 9*a^4*b^4*g^2*z + 9*a^2*b^6*d^2*z + 3*a^6*b*f*g
*h - 3*a^5*b^2*e*f*g - 3*a^5*b^2*d*f*h - 3*a^5*b^2*c*g*h + 3*a^4*b^3*d*e*f
 + 3*a^4*b^3*c*e*g + 3*a^4*b^3*c*d*h - 3*a^3*b^4*c*d*e - 3*a^6*b*e*h^2 + 3
*a^5*b^2*e^2*h + 3*a^5*b^2*d*g^2 - 3*a^4*b^3*d^2*g - 3*a^4*b^3*c*f^2 + 3*a
^3*b^4*c^2*f + a^5*b^2*f^3 + a^3*b^4*d^3 + a^7*h^3 - a^4*b^3*e^3 - a^2*b^5
*c^3 - a^6*b*g^3, z, k)*a*b^2) + (a^5*g^2 + a^3*b^2*d^2 - a^5*f*h + a^4*b*
c*h - 2*a^4*b*d*g + a^4*b*e*f - a^3*b^2*c*e)/b^4 + (x*(a^4*b*f^2 + a^2*b^3
*c^2 + a^5*g*h - a^4*b*d*h - a^4*b*e*g - 2*a^3*b^2*c*f + a^3*b^2*d*e))/b^4
)*root(27*b^10*z^3 + 27*a*b^8*d*z^2 - 27*a^2*b^7*g*z^2 - 9*a^4*b^4*f*h*...
```

3.403. $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

3.404 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

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3.404.1 Optimal result

Integrand size = 38, antiderivative size = 313

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx &= \frac{(bc-af)x}{b^2}+\frac{(bd-ag)x^2}{2b^2}+\frac{(be-ah)x^3}{3b^2}+\frac{fx^4}{4b}+\frac{gx^5}{5b}+\frac{hx^6}{6b}\\&+\frac{\sqrt[3]{a}\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}\\&-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}\\&+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}\\&-\frac{a(be-ah)\log(a+bx^3)}{3b^3}\end{aligned}$$

output
```
(-a*f+b*c)*x/b^2+1/2*(-a*g+b*d)*x^2/b^2+1/3*(-a*h+b*e)*x^3/b^2+1/4*f*x^4/b
+1/5*g*x^5/b+1/6*h*x^6/b-1/3*a^(1/3)*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+b*d
))*ln(a^(1/3)+b^(1/3)*x)/b^(8/3)+1/6*a^(1/3)*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(
-a*g+b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(8/3)-1/3*a*(-a*h+b
*e)*ln(b*x^3+a)/b^3+1/3*a^(1/3)*(b^(4/3)*c+a^(1/3)*b*d-a*b^(1/3)*f-a^(4/3)
*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(8/3)*3^(1/2)
```

3.404.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$

$$= \frac{60b(bc-af)x+30b(bd-ag)x^2+20b(be-ah)x^3+15b^2fx^4+12b^2gx^5+10b^2hx^6-20\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(-b^{4/3}\right.}{}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

output
```
(60*b*(b*c - a*f)*x + 30*b*(b*d - a*g)*x^2 + 20*b*(b*e - a*h)*x^3 + 15*b^2
*f*x^4 + 12*b^2*g*x^5 + 10*b^2*h*x^6 - 20*Sqrt[3]*a^(1/3)*b^(1/3)*(-(b^(4/
3)*c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] - 20*a^(1/3)*b^(1/3)*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)
*f + a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x] + 10*a^(1/3)*b^(1/3)*(b^(4/3)*c -
 a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2] + 20*a*(-(b*e) + a*h)*Log[a + b*x^3])/(60*b^3)
```

3.404.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2375, 27, 2375, 27, 2375, 27, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}\,dx$$

$$\Big\downarrow \ 2375$$

$$\frac{\int \frac{6x^3\left(bgx^4+bfx^3+(be-ah)x^2+bdx+bc\right)}{bx^3+a}dx}{6b}+\frac{hx^6}{6b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{x^3\left(bgx^4+bfx^3+(be-ah)x^2+bdx+bc\right)}{bx^3+a}dx}{b}+\frac{hx^6}{6b}$$

$$\Big\downarrow \text{2375}$$

$$\frac{\frac{\frac{\int \frac{5x^3\left(b^2fx^3+b(be-ah)x^2+b(bd-ag)x+b^2c\right)}{bx^3+a}dx}{5b}+\frac{gx^5}{5}}{b}}{b}+\frac{hx^6}{6b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{x^3\left(b^2fx^3+b(be-ah)x^2+b(bd-ag)x+b^2c\right)}{bx^3+a}dx}{b}+\frac{gx^5}{5}}{b}}{b}+\frac{hx^6}{6b}$$

$$\Big\downarrow \text{2375}$$

$$\frac{\frac{\frac{\frac{\int \frac{4x^3\left((be-ah)x^2b^2+(bc-af)b^2+(bd-ag)xb^2\right)}{bx^3+a}dx}{4b}+\frac{1}{4}bfx^4}{b}+\frac{gx^5}{5}}{b}}{b}+\frac{hx^6}{6b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\frac{\int \frac{x^3\left((be-ah)x^2b^2+(bc-af)b^2+(bd-ag)xb^2\right)}{bx^3+a}dx}{b}+\frac{1}{4}bfx^4}{b}+\frac{gx^5}{5}}{b}}{b}+\frac{hx^6}{6b}$$

$$\Big\downarrow \text{2426}$$

$$\frac{\frac{\frac{\frac{\int \left(b(be-ah)x^2+b(bd-ag)x+b(bc-af)-\frac{ab(be-ah)x^2+ab(bd-ag)x+ab(bc-af)}{bx^3+a}\right)dx}{b}+\frac{1}{4}bfx^4}{b}+\frac{gx^5}{5}}{b}}{b}+\frac{hx^6}{6b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\frac{\frac{\sqrt[3]{a}\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}}+\frac{1}{6}\sqrt[3]{a}\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)-\frac{1}{3}\sqrt[3]{a}\ \cdots}{b}}{b}}{b}$$

$$\frac{hx^6}{6b}$$

input

```
Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

3.404. $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

output
```
(h*x^6)/(6*b) + ((g*x^5)/5 + ((b*f*x^4)/4 + (b*(b*c - a*f)*x + (b*(b*d - a
*g)*x^2)/2 + (b*(b*e - a*h)*x^3)/3 + (a^(1/3)*b^(1/3)*(b^(4/3)*c + a^(1/3)
*b*d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/Sqrt[3] - (a^(1/3)*b^(1/3)*(b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d -
a*g))*Log[a^(1/3) + b^(1/3)*x])/3 + (a^(1/3)*b^(1/3)*(b^(1/3)*(b*c - a*f)
- a^(1/3)*(b*d - a*g))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/6 -
 (a*(b*e - a*h)*Log[a + b*x^3])/3)/b)/b)/b
```

3.404.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.404. $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

3.404.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.44

method	result
risch	$\frac{h\,x^{6}}{6b}+\frac{g\,x^{5}}{5b}+\frac{f\,x^{4}}{4b}-\frac{ah\,x^{3}}{3b^{2}}+\frac{e\,x^{3}}{3b}-\frac{ag\,x^{2}}{2b^{2}}+\frac{d\,x^{2}}{2b}-\frac{afx}{b^{2}}+\frac{cx}{b}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left((ah-be)\_R^{2}+(ag-bd)\_\right.}{\_I}\right.}{3b^{3}}$
default	$-\frac{-\frac{1}{6}bh\,x^{6}-\frac{1}{5}bg\,x^{5}-\frac{1}{4}bf\,x^{4}+\frac{1}{3}ah\,x^{3}-\frac{1}{3}be\,x^{3}+\frac{1}{2}ag\,x^{2}-\frac{1}{2}bd\,x^{2}+afx-bcx}{b^{2}}+\frac{\left((af-bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.\right.}{}$

input
```
int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*h*x^6/b+1/5*g*x^5/b+1/4*f*x^4/b-1/3/b^2*a*h*x^3+1/3*e*x^3/b-1/2/b^2*a*
g*x^2+1/2*d*x^2/b-1/b^2*a*f*x+c*x/b+1/3/b^3*a*sum(((a*h-b*e)*_R^2+(a*g-b*d
)*_R+a*f-b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.404.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 15451, normalized size of antiderivative = 49.36

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fric
as")
```

output
```
Too large to include
```

3.404.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

3.404.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.06

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$=\frac{10\,bhx^6+12\,bgx^5+15\,bfx^4+20\,(be-ah)x^3+30\,(bd-ag)x^2+60\,(bc-af)x}{60\,b^2}$$
$$-\frac{\sqrt{3}\left(ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2bg\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2bf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}$$
$$-\frac{\left(2\,abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}-abc+a^2f\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}+abc-a^2f\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxi
ma")
```

output
```
1/60*(10*b*h*x^6 + 12*b*g*x^5 + 15*b*f*x^4 + 20*(b*e - a*h)*x^3 + 30*(b*d
- a*g)*x^2 + 60*(b*c - a*f)*x)/b^2 - 1/3*sqrt(3)*(a*b^2*d*(a/b)^(2/3) - a^
2*b*g*(a/b)^(2/3) + a*b^2*c*(a/b)^(1/3) - a^2*b*f*(a/b)^(1/3))*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) - 1/6*(2*a*b*e*(a/b)^(2/3
) - 2*a^2*h*(a/b)^(2/3) + a*b*d*(a/b)^(1/3) - a^2*g*(a/b)^(1/3) - a*b*c +
a^2*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 1/3*(a*b
*e*(a/b)^(2/3) - a^2*h*(a/b)^(2/3) - a*b*d*(a/b)^(1/3) + a^2*g*(a/b)^(1/3)
 + a*b*c - a^2*f)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

3.404.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.12

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = -\frac{(abe-a^2h)\log(|bx^3+a|)}{3\,b^3} - \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}\,b^2c-(-ab^2)^{\frac{1}{3}}\,abf-(-ab^2)^{\frac{2}{3}}\,bd+(-ab^2)^{\frac{2}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} - \frac{\left((-ab^2)^{\frac{1}{3}}\,b^2c-(-ab^2)^{\frac{1}{3}}\,abf+(-ab^2)^{\frac{2}{3}}\,bd-(-ab^2)^{\frac{2}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4} + \frac{10\,b^5hx^6+12\,b^5gx^5+15\,b^5fx^4+20\,b^5ex^3-20\,ab^4hx^3+30\,b^5dx^2-30\,ab^4gx^2+60\,b^5cx-60\,ab^4fx}{60\,b^6} + \frac{\left(ab^{12}d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^{11}g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^{12}c-a^2b^{11}f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{13}}$$

input
```
integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac
")
```

output
```
-1/3*(a*b*e - a^2*h)*log(abs(b*x^3 + a))/b^3 - 1/3*sqrt(3)*((-a*b^2)^(1/3)
*b^2*c - (-a*b^2)^(1/3)*a*b*f - (-a*b^2)^(2/3)*b*d + (-a*b^2)^(2/3)*a*g)*a
rctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(
1/3)*b^2*c - (-a*b^2)^(1/3)*a*b*f + (-a*b^2)^(2/3)*b*d - (-a*b^2)^(2/3)*a*
g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/60*(10*b^5*h*x^6 + 12*
b^5*g*x^5 + 15*b^5*f*x^4 + 20*b^5*e*x^3 - 20*a*b^4*h*x^3 + 30*b^5*d*x^2 -
30*a*b^4*g*x^2 + 60*b^5*c*x - 60*a*b^4*f*x)/b^6 + 1/3*(a*b^12*d*(-a/b)^(1/
3) - a^2*b^11*g*(-a/b)^(1/3) + a*b^12*c - a^2*b^11*f)*(-a/b)^(1/3)*log(abs
(x - (-a/b)^(1/3)))/(a*b^13)
```

3.404.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1236, normalized size of antiderivative = 3.95

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = x^2\left(\frac{d}{2\,b}-\frac{a\,g}{2\,b^2}\right)+x^3\left(\frac{e}{3\,b}-\frac{a\,h}{3\,b^2}\right) + \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,b^9\,z^3+27\,a\,b^7\,e\,z^2-27\,a^2\,b^6\,h\,z^2+9\,a\,b^6\,c\,d\,z-18\,a^3\,b^4\,e\,h\,z+9\,a^3\,b^4\,f\,g\,z-9\,a^2\,b^5\,d\,f\,z-\right.\right.\right.$$
$$+\frac{a^5\,h^2+a^3\,b^2\,e^2-2\,a^4\,b\,e\,h+a^4\,b\,f\,g+a^2\,b^3\,c\,d-a^3\,b^2\,c\,g-a^3\,b^2\,d\,f}{b^4}$$
$$\left.+\frac{x\,(a^4\,g^2+a^2\,b^2\,d^2-a^4\,f\,h+a^3\,b\,c\,h-2\,a^3\,b\,d\,g+a^3\,b\,e\,f-a^2\,b^2\,c\,e)}{b^3}\right)\mathrm{root}\left(27\,b^9\,z^3\right.$$
$$+27\,a\,b^7\,e\,z^2-27\,a^2\,b^6\,h\,z^2+9\,a\,b^6\,c\,d\,z-18\,a^3\,b^4\,e\,h\,z+9\,a^3\,b^4\,f\,g\,z-9\,a^2\,b^5\,d\,f\,z$$
$$-9\,a^2\,b^5\,c\,g\,z+9\,a^4\,b^3\,h^2\,z+9\,a^2\,b^5\,e^2\,z-3\,a^5\,b\,f\,g\,h+3\,a^4\,b^2\,e\,f\,g+3\,a^4\,b^2\,d\,f\,h$$
$$+3\,a^4\,b^2\,c\,g\,h-3\,a^3\,b^3\,d\,e\,f-3\,a^3\,b^3\,c\,e\,g-3\,a^3\,b^3\,c\,d\,h+3\,a^2\,b^4\,c\,d\,e+3\,a^5\,b\,e\,h^2$$
$$-3\,a^4\,b^2\,e^2\,h-3\,a^4\,b^2\,d\,g^2+3\,a^3\,b^3\,d^2\,g+3\,a^3\,b^3\,c\,f^2-3\,a^2\,b^4\,c^2\,f+a^3\,b^3\,e^3+a^5\,b\,g^3$$
$$\left.\left.+a\,b^5\,c^3-a^4\,b^2\,f^3-a^2\,b^4\,d^3-a^6\,h^3,z,k\right)\right)+x\left(\frac{c}{b}-\frac{a\,f}{b^2}\right)+\frac{f\,x^4}{4\,b}+\frac{g\,x^5}{5\,b}+\frac{h\,x^6}{6\,b}$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

output
```
x^2*(d/(2*b) - (a*g)/(2*b^2)) + x^3*(e/(3*b) - (a*h)/(3*b^2)) + symsum(log
(root(27*b^9*z^3 + 27*a*b^7*e*z^2 - 27*a^2*b^6*h*z^2 + 9*a*b^6*c*d*z - 18*
a^3*b^4*e*h*z + 9*a^3*b^4*f*g*z - 9*a^2*b^5*d*f*z - 9*a^2*b^5*c*g*z + 9*a^
4*b^3*h^2*z + 9*a^2*b^5*e^2*z - 3*a^5*b*f*g*h + 3*a^4*b^2*e*f*g + 3*a^4*b^
2*d*f*h + 3*a^4*b^2*c*g*h - 3*a^3*b^3*d*e*f - 3*a^3*b^3*c*e*g - 3*a^3*b^3*
c*d*h + 3*a^2*b^4*c*d*e + 3*a^5*b*e*h^2 - 3*a^4*b^2*e^2*h - 3*a^4*b^2*d*g^
2 + 3*a^3*b^3*d^2*g + 3*a^3*b^3*c*f^2 - 3*a^2*b^4*c^2*f + a^3*b^3*e^3 + a^
5*b*g^3 + a*b^5*c^3 - a^4*b^2*f^3 - a^2*b^4*d^3 - a^6*h^3, z, k)*((6*a^2*b
^4*e - 6*a^3*b^3*h)/b^4 + (x*(3*a^2*b^3*f - 3*a*b^4*c))/b^3 + 9*root(27*b^
9*z^3 + 27*a*b^7*e*z^2 - 27*a^2*b^6*h*z^2 + 9*a*b^6*c*d*z - 18*a^3*b^4*e*h
*z + 9*a^3*b^4*f*g*z - 9*a^2*b^5*d*f*z - 9*a^2*b^5*c*g*z + 9*a^4*b^3*h^2*z
 + 9*a^2*b^5*e^2*z - 3*a^5*b*f*g*h + 3*a^4*b^2*e*f*g + 3*a^4*b^2*d*f*h + 3
*a^4*b^2*c*g*h - 3*a^3*b^3*d*e*f - 3*a^3*b^3*c*e*g - 3*a^3*b^3*c*d*h + 3*a
^2*b^4*c*d*e + 3*a^5*b*e*h^2 - 3*a^4*b^2*e^2*h - 3*a^4*b^2*d*g^2 + 3*a^3*b
^3*d^2*g + 3*a^3*b^3*c*f^2 - 3*a^2*b^4*c^2*f + a^3*b^3*e^3 + a^5*b*g^3 + a
*b^5*c^3 - a^4*b^2*f^3 - a^2*b^4*d^3 - a^6*h^3, z, k)*a*b^2) + (a^5*h^2 +
a^3*b^2*e^2 - 2*a^4*b*e*h + a^4*b*f*g + a^2*b^3*c*d - a^3*b^2*c*g - a^3*b^
2*d*f)/b^4 + (x*(a^4*g^2 + a^2*b^2*d^2 - a^4*f*h + a^3*b*c*h - 2*a^3*b*d*g
 + a^3*b*e*f - a^2*b^2*c*e))/b^3)*root(27*b^9*z^3 + 27*a*b^7*e*z^2 - 27*a^
2*b^6*h*z^2 + 9*a*b^6*c*d*z - 18*a^3*b^4*e*h*z + 9*a^3*b^4*f*g*z - 9*a^...
```

3.404. $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} \, dx$

3.405 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

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3.405.1 Optimal result

Integrand size = 38, antiderivative size = 294

$$\begin{aligned}
&\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx \\
&= \frac{(bd-ag)x}{b^2}+\frac{(be-ah)x^2}{2b^2}+\frac{fx^3}{3b}+\frac{gx^4}{4b}+\frac{hx^5}{5b} \\
&\quad+\frac{\sqrt[3]{a}\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}} \\
&\quad-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} \\
&\quad+\frac{\sqrt[3]{a}\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}} \\
&\quad+\frac{(bc-af)\log(a+bx^3)}{3b^2}
\end{aligned}$$

output
```
(-a*g+b*d)*x/b^2+1/2*(-a*h+b*e)*x^2/b^2+1/3*f*x^3/b+1/4*g*x^4/b+1/5*h*x^5/
b-1/3*a^(1/3)*(b^(1/3)*(-a*g+b*d)-a^(1/3)*(-a*h+b*e))*ln(a^(1/3)+b^(1/3)*x
)/b^(8/3)+1/6*a^(1/3)*(b^(1/3)*(-a*g+b*d)-a^(1/3)*(-a*h+b*e))*ln(a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(8/3)+1/3*(-a*f+b*c)*ln(b*x^3+a)/b^2+1/3*a
^(1/3)*(b^(4/3)*d+a^(1/3)*b*e-a*b^(1/3)*g-a^(4/3)*h)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(8/3)*3^(1/2)
```

3.405.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.99

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$

$$= \frac{60b^{2/3}(bd-ag)x+30b^{2/3}(be-ah)x^2+20b^{5/3}fx^3+15b^{5/3}gx^4+12b^{5/3}hx^5-20\sqrt{3}\sqrt[3]{a}\left(-b^{4/3}d-\sqrt[3]{a}be\right.}{}$$

input
```
Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

output
```
(60*b^(2/3)*(b*d - a*g)*x + 30*b^(2/3)*(b*e - a*h)*x^2 + 20*b^(5/3)*f*x^3
+ 15*b^(5/3)*g*x^4 + 12*b^(5/3)*h*x^5 - 20*Sqrt[3]*a^(1/3)*(-(b^(4/3)*d) -
 a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]] + 20*a^(1/3)*(-(b^(4/3)*d) + a^(1/3)*b*e + a*b^(1/3)*g - a^(4/3)
*h)*Log[a^(1/3) + b^(1/3)*x] + 10*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(
1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 20*b^
(2/3)*(b*c - a*f)*Log[a + b*x^3])/(60*b^(8/3))
```

3.405.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2375, 27, 2375, 27, 2375, 27, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}\,dx$$

$$\Big\downarrow \; 2375$$

$$\frac{\int \frac{5x^2\left(bgx^4+bfx^3+(be-ah)x^2+bdx+bc\right)}{bx^3+a}dx}{5b}+\frac{hx^5}{5b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x^2\left(bgx^4+bfx^3+(be-ah)x^2+bdx+bc\right)}{bx^3+a}dx}{b}+\frac{hx^5}{5b}$$

$\Big\downarrow$ 2375

$$\frac{\frac{\int \frac{4x^2\left(b^2fx^3+b(be-ah)x^2+b(bd-ag)x+b^2c\right)}{bx^3+a}dx}{4b}+\frac{gx^4}{4}}{b}+\frac{hx^5}{5b}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{x^2\left(b^2fx^3+b(be-ah)x^2+b(bd-ag)x+b^2c\right)}{bx^3+a}dx}{b}+\frac{gx^4}{4}}{b}+\frac{hx^5}{5b}$$

$\Big\downarrow$ 2375

$$\frac{\frac{\frac{\int \frac{3x^2\left((be-ah)x^2b^2+(bc-af)b^2+(bd-ag)xb^2\right)}{bx^3+a}dx}{3b}+\frac{1}{3}bfx^3}{b}+\frac{gx^4}{4}}{b}+\frac{hx^5}{5b}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{x^2\left((be-ah)x^2b^2+(bc-af)b^2+(bd-ag)xb^2\right)}{bx^3+a}dx}{b}+\frac{1}{3}bfx^3}{b}+\frac{gx^4}{4}}{b}+\frac{hx^5}{5b}$$

$\Big\downarrow$ 2426

$$\frac{\frac{\frac{\int \left(b(bd-ag)+b(be-ah)x-\frac{-b^2(bc-af)x^2+ab(be-ah)x+ab(bd-ag)}{bx^3+a}\right)dx}{b}+\frac{1}{3}bfx^3}{b}+\frac{gx^4}{4}}{b}+\frac{hx^5}{5b}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\frac{\frac{\sqrt[3]{a}\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}}+\frac{1}{6}\sqrt[3]{a}\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)+\frac{1}{3}b(bc-}{b}}{b}}{b}$$

$$\frac{hx^5}{5b}$$

input
```
Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

3.405. $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

output
```
(h*x^5)/(5*b) + ((g*x^4)/4 + ((b*f*x^3)/3 + (b*(b*d - a*g)*x + (b*(b*e - a
*h)*x^2)/2 + (a^(1/3)*b^(1/3)*(b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^(
4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (a^(1
/3)*b^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1
/3)*x])/3 + (a^(1/3)*b^(1/3)*(b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/6 + (b*(b*c - a*f)*Log[a +
b*x^3])/3)/b)/b)/b
```

3.405.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.405. $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} \, dx$

3.405.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.42

method	result
risch	$\frac{h\,x^5}{5b}+\frac{g\,x^4}{4b}+\frac{f\,x^3}{3b}-\frac{ah\,x^2}{2b^2}+\frac{e\,x^2}{2b}-\frac{agx}{b^2}+\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(b(-af+bc)\_R^2+a(ah-be)\_R+a^2g-abd\right)\ln\left(x\right.}{\_R^2}}{3b^3}$
default	$-\frac{-\frac{1}{5}bh\,x^5-\frac{1}{4}bg\,x^4-\frac{1}{3}f\,x^3b+\frac{1}{2}ah\,x^2-\frac{1}{2}be\,x^2+agx-bdx}{b^2}+\frac{(a^2g-abd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(\ldots}{\ldots}\right.}{3b\left(\frac{a}{b}\right.}\right.}{\ldots}$

input
```
int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*h*x^5/b+1/4*g*x^4/b+1/3*f*x^3/b-1/2/b^2*a*h*x^2+1/2*e*x^2/b-1/b^2*a*g*
x+d*x/b+1/3/b^3*sum((b*(-a*f+b*c)*_R^2+a*(a*h-b*e)*_R+a^2*g-a*b*d)/_R^2*ln
(x-_R),_R=RootOf(_Z^3*b+a))
```

3.405.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 14746, normalized size of antiderivative = 50.16

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fric
as")
```

output
```
Too large to include
```

3.405.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

3.405.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.06

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= -\frac{\sqrt{3}\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}} + abd\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{12\,bhx^5+15\,bgx^4+20\,bfx^3+30\,(be-ah)x^2+60\,(bd-ag)x}{60\,b^2} + \frac{\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}} - abe\left(\frac{a}{b}\right)^{\frac{1}{3}} + a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}} + abd - a^2g\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}} - abf\left(\frac{a}{b}\right)^{\frac{2}{3}} + abe\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}} - abd + a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxi
ma")
```

output
```
-1/3*sqrt(3)*(a*b*e*(a/b)^(2/3) - a^2*h*(a/b)^(2/3) + a*b*d*(a/b)^(1/3) -
a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*
b^2) + 1/60*(12*b*h*x^5 + 15*b*g*x^4 + 20*b*f*x^3 + 30*(b*e - a*h)*x^2 + 6
0*(b*d - a*g)*x)/b^2 + 1/6*(2*b^2*c*(a/b)^(2/3) - 2*a*b*f*(a/b)^(2/3) - a*
b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3) + a*b*d - a^2*g)*log(x^2 - x*(a/b)^(1/
3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(b^2*c*(a/b)^(2/3) - a*b*f*(a/b)
^(2/3) + a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) - a*b*d + a^2*g)*log(x + (a
/b)^(1/3))/(b^3*(a/b)^(2/3))
```

3.405.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.12

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \frac{(bc-af)\log(|bx^3+a|)}{3\,b^2} - \frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}\,b^2d-(-ab^2)^{\frac{1}{3}}\,abg-(-ab^2)^{\frac{2}{3}}\,be+(-ab^2)^{\frac{2}{3}}\,ah\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} - \frac{\left((-ab^2)^{\frac{1}{3}}\,b^2d-(-ab^2)^{\frac{1}{3}}\,abg+(-ab^2)^{\frac{2}{3}}\,be-(-ab^2)^{\frac{2}{3}}\,ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4} + \frac{12\,b^4hx^5+15\,b^4gx^4+20\,b^4fx^3+30\,b^4ex^2-30\,ab^3hx^2+60\,b^4dx-60\,ab^3gx}{60\,b^5} + \frac{\left(ab^{10}e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^9h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^{10}d-a^2b^9g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{11}}$$

input
```
integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac
")
```

output
```
1/3*(b*c - a*f)*log(abs(b*x^3 + a))/b^2 - 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*
d - (-a*b^2)^(1/3)*a*b*g - (-a*b^2)^(2/3)*b*e + (-a*b^2)^(2/3)*a*h)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*
b^2*d - (-a*b^2)^(1/3)*a*b*g + (-a*b^2)^(2/3)*b*e - (-a*b^2)^(2/3)*a*h)*lo
g(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/60*(12*b^4*h*x^5 + 15*b^4*g
*x^4 + 20*b^4*f*x^3 + 30*b^4*e*x^2 - 30*a*b^3*h*x^2 + 60*b^4*d*x - 60*a*b^
3*g*x)/b^5 + 1/3*(a*b^10*e*(-a/b)^(1/3) - a^2*b^9*h*(-a/b)^(1/3) + a*b^10*
d - a^2*b^9*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^11)
```

3.405.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 1170, normalized size of antiderivative = 3.98

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = x^2\left(\frac{e}{2\,b}-\frac{a\,h}{2\,b^2}\right)$$
$$+\left(\sum_{k=1}^{3}\ln\left(\operatorname{root}\left(27\,b^8\,z^3+27\,a\,b^6\,f\,z^2-27\,b^7\,c\,z^2-18\,a\,b^5\,c\,f\,z+9\,a\,b^5\,d\,e\,z+9\,a^3\,b^3\,g\,h\,z-9\,a^2\,b^4\,e\,g\,z-9\,a^2\right.\right.\right.$$
$$+\frac{a\,b^3\,c^2+a^3\,b\,f^2+a^4\,g\,h-a^3\,b\,d\,h-a^3\,b\,e\,g-2\,a^2\,b^2\,c\,f+a^2\,b^2\,d\,e}{b^3}$$
$$\left.+\frac{x\,(a^4\,h^2+a^2\,b^2\,e^2+a\,b^3\,c\,d-2\,a^3\,b\,e\,h+a^3\,b\,f\,g-a^2\,b^2\,c\,g-a^2\,b^2\,d\,f)}{b^3}\right)\operatorname{root}\left(27\,b^8\,z^3\right.$$
$$+27\,a\,b^6\,f\,z^2-27\,b^7\,c\,z^2-18\,a\,b^5\,c\,f\,z+9\,a\,b^5\,d\,e\,z+9\,a^3\,b^3\,g\,h\,z-9\,a^2\,b^4\,e\,g\,z-9\,a^2\,b^4\,d\,h\,z$$
$$+9\,a^2\,b^4\,f^2\,z+9\,b^6\,c^2\,z+3\,a^4\,b\,f\,g\,h-3\,a\,b^4\,c\,d\,e-3\,a^3\,b^2\,e\,f\,g-3\,a^3\,b^2\,d\,f\,h-3\,a^3\,b^2\,c\,g\,h$$
$$+3\,a^2\,b^3\,d\,e\,f+3\,a^2\,b^3\,c\,e\,g+3\,a^2\,b^3\,c\,d\,h-3\,a^4\,b\,e\,h^2+3\,a\,b^4\,c^2\,f+3\,a^3\,b^2\,e^2\,h+3\,a^3\,b^2\,d\,g^2$$
$$\left.\left.-3\,a^2\,b^3\,d^2\,g-3\,a^2\,b^3\,c\,f^2+a^3\,b^2\,f^3+a\,b^4\,d^3+a^5\,h^3-a^2\,b^3\,e^3-a^4\,b\,g^3-b^5\,c^3,z,k\right)\right)$$
$$+x\left(\frac{d}{b}-\frac{a\,g}{b^2}\right)+\frac{f\,x^3}{3\,b}+\frac{g\,x^4}{4\,b}+\frac{h\,x^5}{5\,b}$$

input
```
int((x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

output
```
x^2*(e/(2*b) - (a*h)/(2*b^2)) + symsum(log(root(27*b^8*z^3 + 27*a*b^6*f*z^
2 - 27*b^7*c*z^2 - 18*a*b^5*c*f*z + 9*a*b^5*d*e*z + 9*a^3*b^3*g*h*z - 9*a^
2*b^4*e*g*z - 9*a^2*b^4*d*h*z + 9*a^2*b^4*f^2*z + 9*b^6*c^2*z + 3*a^4*b*f*
g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h
+ 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*
a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*
b^3*c*f^2 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 -
b^5*c^3, z, k)*((6*a^2*b^3*f - 6*a*b^4*c)/b^3 + (x*(3*a^2*b^3*g - 3*a*b^4*
d))/b^3 + 9*root(27*b^8*z^3 + 27*a*b^6*f*z^2 - 27*b^7*c*z^2 - 18*a*b^5*c*f
*z + 9*a*b^5*d*e*z + 9*a^3*b^3*g*h*z - 9*a^2*b^4*e*g*z - 9*a^2*b^4*d*h*z +
 9*a^2*b^4*f^2*z + 9*b^6*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2
*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c
*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h +
 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3 + a*b^4
*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k)*a*b^2) + (a*b^3*
c^2 + a^3*b*f^2 + a^4*g*h - a^3*b*d*h - a^3*b*e*g - 2*a^2*b^2*c*f + a^2*b^
2*d*e)/b^3 + (x*(a^4*h^2 + a^2*b^2*e^2 + a*b^3*c*d - 2*a^3*b*e*h + a^3*b*f
*g - a^2*b^2*c*g - a^2*b^2*d*f))/b^3)*root(27*b^8*z^3 + 27*a*b^6*f*z^2 - 2
7*b^7*c*z^2 - 18*a*b^5*c*f*z + 9*a*b^5*d*e*z + 9*a^3*b^3*g*h*z - 9*a^2*b^4
*e*g*z - 9*a^2*b^4*d*h*z + 9*a^2*b^4*f^2*z + 9*b^6*c^2*z + 3*a^4*b*f*g*...
```

3.405. $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

3.406 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$

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3.406.1 Optimal result

Integrand size = 36, antiderivative size = 275

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3} dx$$
$$= \frac{(be-ah)x}{b^2}+\frac{fx^2}{2b}+\frac{gx^3}{3b}+\frac{hx^4}{4b}-\frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{7/3}}$$
$$-\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{7/3}}$$
$$+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{7/3}}$$
$$+\frac{(bd-ag)\log\left(a+bx^3\right)}{3b^2}$$

output
```
(-a*h+b*e)*x/b^2+1/2*f*x^2/b+1/3*g*x^3/b+1/4*h*x^4/b-1/3*(b^(2/3)*(-a*f+b*
c)+a^(2/3)*(-a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(7/3)+1/6*(b^(2/3)*
(-a*f+b*c)+a^(2/3)*(-a*h+b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a
^(1/3)/b^(7/3)+1/3*(-a*g+b*d)*ln(b*x^3+a)/b^2-1/3*(b^(5/3)*c-a^(2/3)*b*e-a
*b^(2/3)*f+a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^
(1/3)/b^(7/3)*3^(1/2)
```

3.406.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.99

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$

$$= \frac{12\sqrt[3]{b}(be-ah)x+6b^{4/3}fx^2+4b^{4/3}gx^3+3b^{4/3}hx^4-\frac{4\sqrt{3}(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{4(-b^{5/3}}{}}{}$$

input
```
Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

output
```
(12*b^(1/3)*(b*e - a*h)*x + 6*b^(4/3)*f*x^2 + 4*b^(4/3)*g*x^3 + 3*b^(4/3)*
h*x^4 - (4*Sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*Arc
Tan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (4*(-(b^(5/3)*c) - a^(
2/3)*b*e + a*b^(2/3)*f + a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (2
*(b^(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/3)*h)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3) + 4*b^(1/3)*(b*d - a*g)*Log[a + b*x^3])
/(12*b^(7/3))
```

3.406.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2375, 27, 2375, 27, 2375, 27, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{a+bx^3}\,dx$$

$$\Big\downarrow \text{2375}$$

$$\frac{\int \frac{4x(bgx^4+bfx^3+(be-ah)x^2+bdx+bc)}{bx^3+a}dx}{4b}+\frac{hx^4}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x(bgx^4+bfx^3+(be-ah)x^2+bdx+bc)}{bx^3+a}dx}{b}+\frac{hx^4}{4b}$$

$\downarrow$ 2375

$$\frac{\frac{\int \frac{3x\left(b^2fx^3+b(be-ah)x^2+b(bd-ag)x+b^2c\right)}{bx^3+a}dx}{3b}+\frac{gx^3}{3}}{b}+\frac{hx^4}{4b}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{x\left(b^2fx^3+b(be-ah)x^2+b(bd-ag)x+b^2c\right)}{bx^3+a}dx}{b}+\frac{gx^3}{3}}{b}+\frac{hx^4}{4b}$$

$\downarrow$ 2375

$$\frac{\frac{\frac{\int \frac{2x\left((be-ah)x^2b^2+(bc-af)b^2+(bd-ag)xb^2\right)}{bx^3+a}dx}{2b}+\frac{1}{2}bfx^2}{b}+\frac{gx^3}{3}}{b}+\frac{hx^4}{4b}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{x\left((be-ah)x^2b^2+(bc-af)b^2+(bd-ag)xb^2\right)}{bx^3+a}dx}{b}+\frac{1}{2}bfx^2}{b}+\frac{gx^3}{3}}{b}+\frac{hx^4}{4b}$$

$\downarrow$ 2426

$$\frac{\frac{\frac{\int \left(b(be-ah)-\frac{-\left((bd-ag)x^2b^2\right)-(bc-af)xb^2+a(be-ah)b}{bx^3+a}\right)dx}{b}+\frac{1}{2}bfx^2}{b}+\frac{gx^3}{3}}{b}+\frac{hx^4}{4b}$$

$\downarrow$ 2009

$$\frac{\frac{\frac{-\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}\sqrt[3]{a}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6\sqrt[3]{a}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{\cdots}}{b}}{b}}{b}$$

$$\frac{hx^4}{4b}$$

input
```
Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x]
```

3.406. $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

output
```
(h*x^4)/(4*b) + ((g*x^3)/3 + ((b*f*x^2)/2 + (b*(b*e - a*h)*x - (b^(2/3)*(b
^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)) - (b^(2/3)*(b^(2/3)*(b*c - a*
f) + a^(2/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)) + (b^(2/3)
*(b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(6*a^(1/3)) + (b*(b*d - a*g)*Log[a + b*x^3])/3)/b)/b)/b
```

3.406.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.406. $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

3.406.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.38

method	result
risch	$\frac{h\,x^4}{4b}+\frac{g\,x^3}{3b}+\frac{f\,x^2}{2b}-\frac{ahx}{b^2}+\frac{ex}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^3+a\right)}\frac{\left(b(-ag+bd)\_R^2+b(-af+bc)\_R+a^2h-aeb\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^3}$
default	$-\frac{-\frac{1}{4}bh\,x^4-\frac{1}{3}bg\,x^3-\frac{1}{2}bf\,x^2+ahx-bex}{b^2}+\frac{\left(a^2h-aeb\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+}{}$

input
```
int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*h*x^4/b+1/3*g*x^3/b+1/2*f*x^2/b-1/b^2*a*h*x+e*x/b+1/3/b^3*sum((b*(-a*g
+b*d)*_R^2+b*(-a*f+b*c)*_R+a^2*h-a*e*b)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.406.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 14875, normalized size of antiderivative = 54.09

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas
")
```

output
```
Too large to include
```

3.406.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

3.406.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.09

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= \frac{\sqrt{3}\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}} - abf\left(\frac{a}{b}\right)^{\frac{2}{3}} - abe\left(\frac{a}{b}\right)^{\frac{1}{3}} + a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{3\,bhx^4 + 4\,bgx^3 + 6\,bfx^2 + 12\,(be-ah)x}{12\,b^2} + \frac{\left(2\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}} + b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} - abf\left(\frac{a}{b}\right)^{\frac{1}{3}} + abe - a^2h\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}} - abg\left(\frac{a}{b}\right)^{\frac{2}{3}} - b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} + abf\left(\frac{a}{b}\right)^{\frac{1}{3}} - abe + a^2h\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima
")
```

output
```
1/3*sqrt(3)*(b^2*c*(a/b)^(2/3) - a*b*f*(a/b)^(2/3) - a*b*e*(a/b)^(1/3) + a
^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b
^2) + 1/12*(3*b*h*x^4 + 4*b*g*x^3 + 6*b*f*x^2 + 12*(b*e - a*h)*x)/b^2 + 1/
6*(2*b^2*d*(a/b)^(2/3) - 2*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) - a*b*f*(
a/b)^(1/3) + a*b*e - a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a
/b)^(2/3)) + 1/3*(b^2*d*(a/b)^(2/3) - a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3
) + a*b*f*(a/b)^(1/3) - a*b*e + a^2*h)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/
3))
```

3.406.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.06

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= \frac{\sqrt{3}\left(abe-a^2h+(-ab^2)^{\frac{1}{3}}bc-(-ab^2)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}\,b}$$
$$+\frac{\left(abe-a^2h-(-ab^2)^{\frac{1}{3}}bc+(-ab^2)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}\,b}$$
$$+\frac{(bd-ag)\log(|bx^3+a|)}{3\,b^2}+\frac{3\,b^3hx^4+4\,b^3gx^3+6\,b^3fx^2+12\,b^3ex-12\,ab^2hx}{12\,b^4}$$
$$-\frac{\left(b^9c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^8f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^8e+a^2b^7h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^9}$$

input
```
integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(a*b*e - a^2*h + (-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*f)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) + 1/6
*(a*b*e - a^2*h - (-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a
/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) + 1/3*(b*d - a*g)*log(abs(b*x
^3 + a))/b^2 + 1/12*(3*b^3*h*x^4 + 4*b^3*g*x^3 + 6*b^3*f*x^2 + 12*b^3*e*x
- 12*a*b^2*h*x)/b^4 - 1/3*(b^9*c*(-a/b)^(1/3) - a*b^8*f*(-a/b)^(1/3) - a*b
^8*e + a^2*b^7*h)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^9)
```

3.406.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 1161, normalized size of antiderivative = 4.22

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,a\,b^7\,z^3-27\,a\,b^6\,d\,z^2+27\,a^2\,b^5\,g\,z^2-9\,a\,b^5\,c\,e\,z-9\,a^3\,b^3\,f\,h\,z-18\,a^2\,b^4\,d\,g\,z+9\,a^2\,b^4\,e\,f\,z+\right.\right.\right.$$
$$+\frac{a^3\,g^2+a\,b^2\,d^2-a^3\,f\,h-a\,b^2\,c\,e+a^2\,b\,c\,h-2\,a^2\,b\,d\,g+a^2\,b\,e\,f}{b^2}$$
$$\left.+\frac{x\,(b^3\,c^2+a^2\,b\,f^2+a^3\,g\,h-2\,a\,b^2\,c\,f+a\,b^2\,d\,e-a^2\,b\,d\,h-a^2\,b\,e\,g)}{b^2}\right)\mathrm{root}\left(27\,a\,b^7\,z^3\right.$$
$$-27\,a\,b^6\,d\,z^2+27\,a^2\,b^5\,g\,z^2-9\,a\,b^5\,c\,e\,z-9\,a^3\,b^3\,f\,h\,z-18\,a^2\,b^4\,d\,g\,z+9\,a^2\,b^4\,e\,f\,z$$
$$+9\,a^2\,b^4\,c\,h\,z+9\,a\,b^5\,d^2\,z+9\,a^3\,b^3\,g^2\,z-3\,a^4\,b\,f\,g\,h+3\,a\,b^4\,c\,d\,e+3\,a^3\,b^2\,e\,f\,g$$
$$+3\,a^3\,b^2\,d\,f\,h+3\,a^3\,b^2\,c\,g\,h-3\,a^2\,b^3\,d\,e\,f-3\,a^2\,b^3\,c\,e\,g-3\,a^2\,b^3\,c\,d\,h+3\,a^4\,b\,e\,h^2$$
$$-3\,a\,b^4\,c^2\,f-3\,a^3\,b^2\,e^2\,h-3\,a^3\,b^2\,d\,g^2+3\,a^2\,b^3\,d^2\,g+3\,a^2\,b^3\,c\,f^2+a^2\,b^3\,e^3+a^4\,b\,g^3$$
$$\left.\left.+b^5\,c^3-a^3\,b^2\,f^3-a\,b^4\,d^3-a^5\,h^3,z,k\right)\right)+x\left(\frac{e}{b}-\frac{a\,h}{b^2}\right)+\frac{f\,x^2}{2\,b}+\frac{g\,x^3}{3\,b}+\frac{h\,x^4}{4\,b}$$

input
```
int((x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3),x)
```

output
```
symsum(log(root(27*a*b^7*z^3 - 27*a*b^6*d*z^2 + 27*a^2*b^5*g*z^2 - 9*a*b^5
*c*e*z - 9*a^3*b^3*f*h*z - 18*a^2*b^4*d*g*z + 9*a^2*b^4*e*f*z + 9*a^2*b^4*
c*h*z + 9*a*b^5*d^2*z + 9*a^3*b^3*g^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e +
3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*
a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^
2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^
3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k)*((6*a^2
*b^2*g - 6*a*b^3*d)/b^2 + (x*(3*a^2*b^2*h - 3*a*b^3*e))/b^2 + 9*root(27*a*
b^7*z^3 - 27*a*b^6*d*z^2 + 27*a^2*b^5*g*z^2 - 9*a*b^5*c*e*z - 9*a^3*b^3*f*
h*z - 18*a^2*b^4*d*g*z + 9*a^2*b^4*e*f*z + 9*a^2*b^4*c*h*z + 9*a*b^5*d^2*z
 + 9*a^3*b^3*g^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a
^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2
*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d
*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c
^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3, z, k)*a*b^2) + (a^3*g^2 + a*b^2*d^
2 - a^3*f*h - a*b^2*c*e + a^2*b*c*h - 2*a^2*b*d*g + a^2*b*e*f)/b^2 + (x*(b
^3*c^2 + a^2*b*f^2 + a^3*g*h - 2*a*b^2*c*f + a*b^2*d*e - a^2*b*d*h - a^2*b
*e*g))/b^2)*root(27*a*b^7*z^3 - 27*a*b^6*d*z^2 + 27*a^2*b^5*g*z^2 - 9*a*b^
5*c*e*z - 9*a^3*b^3*f*h*z - 18*a^2*b^4*d*g*z + 9*a^2*b^4*e*f*z + 9*a^2*b^4
*c*h*z + 9*a*b^5*d^2*z + 9*a^3*b^3*g^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*...
```

3.406. $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{a+bx^3}\,dx$

3.407 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3} \, dx$

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3.407.1 Optimal result

Integrand size = 35, antiderivative size = 259

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3} \, dx &= \frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}-\frac{\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{5/3}} \\ &\quad+\frac{\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{5/3}} \\ &\quad-\frac{\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{5/3}} \\ &\quad+\frac{(be-ah)\log\left(a+bx^3\right)}{3b^2}\end{aligned}$$

output
```
f*x/b+1/2*g*x^2/b+1/3*h*x^3/b+1/3*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+b*d))*
ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(5/3)-1/6*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*
g+b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(5/3)+1/3*(-a*
h+b*e)*ln(b*x^3+a)/b^2-1/3*(b^(4/3)*c+a^(1/3)*b*d-a*b^(1/3)*f-a^(4/3)*g)*a
rctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(5/3)*3^(1/2)
```

3.407.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx$$

$$= \frac{6b^{2/3}fx+3b^{2/3}gx^2+2b^{2/3}hx^3+\frac{2\sqrt{3}\left(-b^{4/3}c-\sqrt[3]{a}bd+a\sqrt[3]{b}f+a^{4/3}g\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{2\left(b^{4/3}c-\sqrt[3]{a}bd-a\sqrt[3]{b}f+a^{4/3}\right.}{a^{2/3}}}{6b^{5/3}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

output
```
(6*b^(2/3)*f*x + 3*b^(2/3)*g*x^2 + 2*b^(2/3)*h*x^3 + (2*Sqrt[3]*(-(b^(4/3)
*c) - a^(1/3)*b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]])/a^(2/3) + (2*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f + a^(4
/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - ((b^(4/3)*c - a^(1/3)*b*d - a*b
^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2
/3) + (2*(b*e - a*h)*Log[a + b*x^3])/b^(1/3))/(6*b^(5/3))
```

3.407.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx$$

$$\Big\downarrow \text{ 2426}$$

$$\int \left(\frac{x(bd-ag)+x^2(be-ah)-af+bc}{b\left(a+bx^3\right)}+\frac{f}{b}+\frac{gx}{b}+\frac{hx^2}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}a^{2/3}b^{5/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}-af+bc\right)}{6a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{2/3}b^{5/3}}+\frac{(be-ah)\log\left(a+bx^3\right)}{3b^2}+\frac{fx}{b}+\frac{gx^2}{2b}+\frac{hx^3}{3b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x]
```

output
```
(f*x)/b + (g*x^2)/(2*b) + (h*x^3)/(3*b) - ((b^(4/3)*c + a^(1/3)*b*d - a*b^
(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(S
qrt[3]*a^(2/3)*b^(5/3)) + ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log
[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b*c - a*f - (a^(1/3)*(b*d -
 a*g))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)
*b^(4/3)) + ((b*e - a*h)*Log[a + b*x^3])/(3*b^2)
```

3.407.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.407.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.32

method	result
risch	$\frac{h\,x^{3}}{3b}+\frac{g\,x^{2}}{2b}+\frac{fx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(bc-af+\left(-ag+bd\right)\_R+\left(-ah+be\right)\_R^{2}\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3b^{2}}$
default	$\frac{\frac{1}{3}h\,x^{3}+\frac{1}{2}g\,x^{2}+fx}{b}+\frac{\left(-af+bc\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\left(-ag+bd\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\cdots\right.\right.}{3b\left(\frac{a}{b}\cdots\right.}\cdots\right.}{b}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*h*x^3/b+1/2*g*x^2/b+f*x/b+1/3/b^2*sum((b*c-a*f+(-a*g+b*d)*_R+(-a*h+b*e
)*_R^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.407.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 15235, normalized size of antiderivative = 58.82

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.407.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

3.407.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.03

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx \\
&= \frac{2\,hx^3+3\,gx^2+6\,fx}{6\,b} \\
&\quad + \frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} \\
&\quad + \frac{\left(2\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc+af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&\quad + \frac{\left(be\left(\frac{a}{b}\right)^{\frac{2}{3}}-ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc-af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}
\end{aligned}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/6*(2*h*x^3 + 3*g*x^2 + 6*f*x)/b + 1/3*sqrt(3)*(b^2*d*(a/b)^(2/3) - a*b*g
*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) - a*b*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) + 1/6*(2*b*e*(a/b)^(2/3) - 2*a*h*(
a/b)^(2/3) + b*d*(a/b)^(1/3) - a*g*(a/b)^(1/3) - b*c + a*f)*log(x^2 - x*(a
/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*e*(a/b)^(2/3) - a*h*(a
/b)^(2/3) - b*d*(a/b)^(1/3) + a*g*(a/b)^(1/3) + b*c - a*f)*log(x + (a/b)^(
1/3))/(b^2*(a/b)^(2/3))
```

3.407.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx$$
$$= -\frac{\sqrt{3}\left(b^2c-abf-\left(-ab^2\right)^{\frac{1}{3}}bd+\left(-ab^2\right)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b} - \frac{\left(b^2c-abf+\left(-ab^2\right)^{\frac{1}{3}}bd-\left(-ab^2\right)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b} + \frac{(be-ah)\log\left(|bx^3+a|\right)}{3b^2} + \frac{2b^2hx^3+3b^2gx^2+6b^2fx}{6b^3} - \frac{\left(b^7d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^6g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b^7c-ab^6f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^7}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b^2*c - a*b*f - (-a*b^2)^(1/3)*b*d + (-a*b^2)^(1/3)*a*g)*arc
tan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/
6*(b^2*c - a*b*f + (-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) + 1/3*(b*e - a*h)*log(abs(b*
x^3 + a))/b^2 + 1/6*(2*b^2*h*x^3 + 3*b^2*g*x^2 + 6*b^2*f*x)/b^3 - 1/3*(b^7
*d*(-a/b)^(1/3) - a*b^6*g*(-a/b)^(1/3) + b^7*c - a*b^6*f)*(-a/b)^(1/3)*log
(abs(x - (-a/b)^(1/3)))/(a*b^7)
```

3.407.9 Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 1150, normalized size of antiderivative = 4.44

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\frac{a^3\,h^2+a\,b^2\,e^2+b^3\,c\,d-a\,b^2\,c\,g-a\,b^2\,d\,f-2\,a^2\,b\,e\,h+a^2\,b\,f\,g}{b^2}\right.\right.$$
$$+\mathrm{root}\big(27\,a^2\,b^6\,z^3+27\,a^3\,b^4\,h\,z^2-27\,a^2\,b^5\,e\,z^2+9\,a\,b^5\,c\,d\,z-18\,a^3\,b^3\,e\,h\,z+9\,a^3\,b^3\,f\,g\,z-9\,a^2\,b^4\,d\,f\,z-9\,a^2\,b$$
$$\left.+\frac{x\,(b^2\,d^2+a^2\,g^2-b^2\,c\,e-a^2\,f\,h+a\,b\,c\,h-2\,a\,b\,d\,g+a\,b\,e\,f)}{b}\right)\mathrm{root}\big(27\,a^2\,b^6\,z^3$$
$$+27\,a^3\,b^4\,h\,z^2-27\,a^2\,b^5\,e\,z^2+9\,a\,b^5\,c\,d\,z-18\,a^3\,b^3\,e\,h\,z+9\,a^3\,b^3\,f\,g\,z-9\,a^2\,b^4\,d\,f\,z$$
$$-9\,a^2\,b^4\,c\,g\,z+9\,a^4\,b^2\,h^2\,z+9\,a^2\,b^4\,e^2\,z+3\,a^4\,b\,f\,g\,h-3\,a\,b^4\,c\,d\,e-3\,a^3\,b^2\,e\,f\,g$$
$$-3\,a^3\,b^2\,d\,f\,h-3\,a^3\,b^2\,c\,g\,h+3\,a^2\,b^3\,d\,e\,f+3\,a^2\,b^3\,c\,e\,g+3\,a^2\,b^3\,c\,d\,h-3\,a^4\,b\,e\,h^2$$
$$+3\,a\,b^4\,c^2\,f+3\,a^3\,b^2\,e^2\,h+3\,a^3\,b^2\,d\,g^2-3\,a^2\,b^3\,d^2\,g-3\,a^2\,b^3\,c\,f^2+a^3\,b^2\,f^3+a\,b^4\,d^3$$
$$\left.+a^5\,h^3-a^2\,b^3\,e^3-a^4\,b\,g^3-b^5\,c^3,z,k\big)\right)+\frac{g\,x^2}{2\,b}+\frac{h\,x^3}{3\,b}+\frac{f\,x}{b}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3),x)
```

output
```
symsum(log((a^3*h^2 + a*b^2*e^2 + b^3*c*d - a*b^2*c*g - a*b^2*d*f - 2*a^2*
b*e*h + a^2*b*f*g)/b^2 + root(27*a^2*b^6*z^3 + 27*a^3*b^4*h*z^2 - 27*a^2*b
^5*e*z^2 + 9*a*b^5*c*d*z - 18*a^3*b^3*e*h*z + 9*a^3*b^3*f*g*z - 9*a^2*b^4*
d*f*z - 9*a^2*b^4*c*g*z + 9*a^4*b^2*h^2*z + 9*a^2*b^4*e^2*z + 3*a^4*b*f*g*
h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h +
3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*
b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^
3*c*f^2 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^
5*c^3, z, k)*((6*a^2*b^2*h - 6*a*b^3*e)/b^2 + (x*(3*b^3*c - 3*a*b^2*f))/b
+ 9*root(27*a^2*b^6*z^3 + 27*a^3*b^4*h*z^2 - 27*a^2*b^5*e*z^2 + 9*a*b^5*c*
d*z - 18*a^3*b^3*e*h*z + 9*a^3*b^3*f*g*z - 9*a^2*b^4*d*f*z - 9*a^2*b^4*c*g
*z + 9*a^4*b^2*h^2*z + 9*a^2*b^4*e^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3
*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a
^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2
*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 + a^3*b^2*f^3
 + a*b^4*d^3 + a^5*h^3 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3, z, k)*a*b^2) +
 (x*(b^2*d^2 + a^2*g^2 - b^2*c*e - a^2*f*h + a*b*c*h - 2*a*b*d*g + a*b*e*f
))/b)*root(27*a^2*b^6*z^3 + 27*a^3*b^4*h*z^2 - 27*a^2*b^5*e*z^2 + 9*a*b^5*
c*d*z - 18*a^3*b^3*e*h*z + 9*a^3*b^3*f*g*z - 9*a^2*b^4*d*f*z - 9*a^2*b^4*c
*g*z + 9*a^4*b^2*h^2*z + 9*a^2*b^4*e^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*...
```

3.407. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{a+bx^3}\,dx$

3.408 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx$

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3.408.1 Optimal result

Integrand size = 38, antiderivative size = 258

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx = \frac{gx}{b}+\frac{hx^2}{2b}-\frac{\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{5/3}}+\frac{c\log(x)}{a}+\frac{\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{5/3}}-\frac{\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{5/3}}-\frac{(bc-af)\log(a+bx^3)}{3ab}$$

output
```
g*x/b+1/2*h*x^2/b+c*ln(x)/a+1/3*(b^(1/3)*(-a*g+b*d)-a^(1/3)*(-a*h+b*e))*ln
(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(5/3)-1/6*(b^(1/3)*(-a*g+b*d)-a^(1/3)*(-a*h+
b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(5/3)-1/3*(-a*f+
b*c)*ln(b*x^3+a)/a/b-1/3*(b^(4/3)*d+a^(1/3)*b*e-a*b^(1/3)*g-a^(4/3)*h)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(5/3)*3^(1/2)
```

3.408.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx$$

$$= \frac{6ab^{2/3}gx+3ab^{2/3}hx^2+2\sqrt{3}\sqrt[3]{a}\left(-b^{4/3}d-\sqrt[3]{a}be+a\sqrt[3]{b}g+a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6b^{5/3}c\log(x)+2}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x]
```

output
```
(6*a*b^(2/3)*g*x + 3*a*b^(2/3)*h*x^2 + 2*Sqrt[3]*a^(1/3)*(-(b^(4/3)*d) - a
^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]] + 6*b^(5/3)*c*Log[x] + 2*a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1
/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*(b^(4/3)*d - a^(1/3)
*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2] - 2*b^(2/3)*(b*c - a*f)*Log[a + b*x^3])/(6*a*b^(5/3))
```

3.408.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx$$

$$\Big\downarrow \; 2373$$

$$\int \left(\frac{-bx^2(bc-af)+a(bd-ag)+ax(be-ah)}{ab(a+bx^3)}+\frac{c}{ax}+\frac{g}{b}+\frac{hx}{b}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}a^{2/3}b^{5/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}-ag+bd\right)}{6a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{2/3}b^{5/3}}-\frac{(bc-af)\log\left(a+bx^3\right)}{3ab}+\frac{c\log(x)}{a}+\frac{gx}{b}+\frac{hx^2}{2b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x]
```

output
```
(g*x)/b + (h*x^2)/(2*b) - ((b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g - a^(4/3
)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b
^(5/3)) + (c*Log[x])/a + ((b^(1/3)*(b*d - a*g) - a^(1/3)*(b*e - a*h))*Log[
a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b*d - a*g - (a^(1/3)*(b*e -
a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*
b^(4/3)) - ((b*c - a*f)*Log[a + b*x^3])/(3*a*b)
```

3.408.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.408.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00

method	result
default	$\frac{\frac{1}{2}h\,x^2+gx}{b}+\frac{c\ln(x)}{a}+\frac{(-a^2g+abd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(-a^2h+aeb)\left(-\right.}{ba}$
risch	$\frac{h\,x^2}{2b}+\frac{gx}{b}+\frac{\_R=\text{RootOf}\left(a^3b^2\_Z^3+\left(-3a^3b^2f+3a^2c\,b^3\right)\_Z^2+\left(3a^4bgh-3a^3b^2dh-3a^3b^2eg+3a^3b^2f^2-6a^2b^3cf+3a^2b^3de+3c^2a\,b^4\right)\_Z-a}{}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*h*x^2+g*x)+c*ln(x)/a+((-a^2*g+a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b
)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(-a^2*h+a*b*e)*(-1/3
/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
)))+1/3*(a*b*f-b^2*c)*ln(b*x^3+a)/b)/b/a
```

3.408.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 84.14 (sec) , antiderivative size = 15327, normalized size of antiderivative = 59.41

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\,(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="fricas
")
```

output
```
Too large to include
```

3.408.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx=\text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a),x)
```

output
```
Timed out
```

3.408.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx &= \frac{c\log(x)}{a}+\frac{hx^2+2\,gx}{2\,b}\\
&+\frac{\sqrt{3}\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b}\\
&-\frac{\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+abd-a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\
&-\frac{\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd+a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="maxima
")
```

output
```
c*log(x)/a + 1/2*(h*x^2 + 2*g*x)/b + 1/3*sqrt(3)*(a*b*e*(a/b)^(2/3) - a^2*
h*(a/b)^(2/3) + a*b*d*(a/b)^(1/3) - a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b) - 1/6*(2*b^2*c*(a/b)^(2/3) - 2*a*
b*f*(a/b)^(2/3) - a*b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3) + a*b*d - a^2*g)*l
og(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) - 1/3*(b^2*c*(a/
b)^(2/3) - a*b*f*(a/b)^(2/3) + a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) - a*b
*d + a^2*g)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

3.408.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx$$
$$= \frac{c\log(|x|)}{a} - \frac{\sqrt{3}\left(b^2d-abg-(-ab^2)^{\frac{1}{3}}be+(-ab^2)^{\frac{1}{3}}ah\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(-ab^2)^{\frac{2}{3}}b}$$
$$- \frac{\left(b^2d-abg+(-ab^2)^{\frac{1}{3}}be-(-ab^2)^{\frac{1}{3}}ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(-ab^2)^{\frac{2}{3}}b}$$
$$- \frac{(bc-af)\log(|bx^3+a|)}{3ab} + \frac{bhx^2+2bgx}{2b^2}$$
$$- \frac{\left(a^2b^3e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^3d-a^3b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b^3}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

output
```
c*log(abs(x))/a - 1/3*sqrt(3)*(b^2*d - a*b*g - (-a*b^2)^(1/3)*b*e + (-a*b^
2)^(1/3)*a*h)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b
^2)^(2/3)*b) - 1/6*(b^2*d - a*b*g + (-a*b^2)^(1/3)*b*e - (-a*b^2)^(1/3)*a*
h)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/3*(b*c
- a*f)*log(abs(b*x^3 + a))/(a*b) + 1/2*(b*h*x^2 + 2*b*g*x)/b^2 - 1/3*(a^2*
b^3*e*(-a/b)^(1/3) - a^3*b^2*h*(-a/b)^(1/3) + a^2*b^3*d - a^3*b^2*g)*(-a/b
)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3)
```

3.408.9 Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 1731, normalized size of antiderivative = 6.71

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)),x)
```

output
```
symsum(log(b^2*c*d^2 - root(27*a^3*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5
*c*z^2 + 9*a^4*b^2*g*h*z - 9*a^3*b^3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*
c*f*z + 9*a^2*b^4*d*e*z + 9*a*b^5*c^2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h
+ 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*
a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^
4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*
c*f^2 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*
h^3, z, k)*(a^3*g^2 - root(27*a^3*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5*
c*z^2 + 9*a^4*b^2*g*h*z - 9*a^3*b^3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c
*f*z + 9*a^2*b^4*d*e*z + 9*a*b^5*c^2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h +
 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a
^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4
*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c
*f^2 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h
^3, z, k)*((x*(33*a^2*b^4*f - 24*a*b^5*c))/b^2 + 3*a^2*b^2*e - 3*a^3*b*h -
 36*root(27*a^3*b^5*z^3 - 27*a^3*b^4*f*z^2 + 27*a^2*b^5*c*z^2 + 9*a^4*b^2*
g*h*z - 9*a^3*b^3*e*g*z - 9*a^3*b^3*d*h*z - 18*a^2*b^4*c*f*z + 9*a^2*b^4*d
*e*z + 9*a*b^5*c^2*z + 9*a^3*b^3*f^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3
*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a
^2*b^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*...
```

3.408. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)}\,dx$

3.409 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx$

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3.409.1 Optimal result

Integrand size = 38, antiderivative size = 253

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx = -\frac{c}{ax}+\frac{hx}{b}+\frac{\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{4/3}} + \frac{d\log(x)}{a}+\frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{4/3}} - \frac{\left(b^{2/3}(bc-af)+a^{2/3}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{4/3}} - \frac{(bd-ag)\log\left(a+bx^3\right)}{3ab}$$

output
```
-c/a/x+h*x/b+d*ln(x)/a+1/3*(b^(2/3)*(-a*f+b*c)+a^(2/3)*(-a*h+b*e))*ln(a^(1
/3)+b^(1/3)*x)/a^(4/3)/b^(4/3)-1/6*(b^(2/3)*(-a*f+b*c)+a^(2/3)*(-a*h+b*e))
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(4/3)-1/3*(-a*g+b*d)*
ln(b*x^3+a)/a/b+1/3*(b^(5/3)*c-a^(2/3)*b*e-a*b^(2/3)*f+a^(5/3)*h)*arctan(1
/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(4/3)*3^(1/2)
```

3.409.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx = \frac{1}{6}\left(-\frac{6c}{ax}+\frac{6hx}{b}+\frac{2\sqrt{3}\left(b^{5/3}c-a^{2/3}be-ab^{2/3}f+a^{5/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}b^{4/3}} + \frac{6d\log(x)}{a}+\frac{2\left(b^{5/3}c+a^{2/3}be-ab^{2/3}f-a^{5/3}h\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}b^{4/3}} + \frac{\left(-b^{5/3}c-a^{2/3}be+ab^{2/3}f+a^{5/3}h\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{4/3}b^{4/3}} + \frac{2(-bd+ag)\log\left(a+bx^3\right)}{ab}\right)$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]
```

output
```
((-6*c)/(a*x) + (6*h*x)/b + (2*Sqrt[3]*(b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3
)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(4/3)*b^(
4/3)) + (6*d*Log[x])/a + (2*(b^(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f - a^(5/
3)*h)*Log[a^(1/3) + b^(1/3)*x])/(a^(4/3)*b^(4/3)) + ((-(b^(5/3)*c) - a^(2/
3)*b*e + a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(a^(4/3)*b^(4/3)) + (2*(-(b*d) + a*g)*Log[a + b*x^3])/(a*b))/6
```

3.409.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}dx$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{-bx(bc-af)-bx^2(bd-ag)+a(be-ah)}{ab(a+bx^3)}+\frac{c}{ax^2}+\frac{d}{ax}+\frac{h}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-a^{2/3}be+a^{5/3}h-ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}a^{4/3}b^{4/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{6a^{4/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(a^{2/3}(be-ah)+b^{2/3}(bc-af)\right)}{3a^{4/3}b^{4/3}}-\frac{(bd-ag)\log\left(a+bx^3\right)}{3ab}-\frac{c}{ax}+\frac{d\log(x)}{a}+\frac{hx}{b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x]
```

output
```
-(c/(a*x)) + (h*x)/b + ((b^(5/3)*c - a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*b^(4
/3)) + (d*Log[x])/a + ((b^(2/3)*(b*c - a*f) + a^(2/3)*(b*e - a*h))*Log[a^(
1/3) + b^(1/3)*x])/(3*a^(4/3)*b^(4/3)) - ((b^(2/3)*(b*c - a*f) + a^(2/3)*(
b*e - a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(
4/3)) - ((b*d - a*g)*Log[a + b*x^3])/(3*a*b)
```

3.409.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.409.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.03

method	result
default	$\frac{hx}{b}-\frac{c}{ax}+\frac{d\ln(x)}{a}+\frac{(-a^2h+aeb)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(afb-b^2c)\left(-\ldots\right.}{ba\ldots}$
risch	$\frac{hx}{b}-\frac{c}{ax}+\frac{d\ln(-x)}{a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a^4b\_Z^3+\left(-3a^4bg+3a^3d\,b^2\right)\_Z^2+\left(-3a^4bfh+3a^4b\,g^2+3a^3b^2ch-6a^3b^2dg+3a^3b^2ef-3a^2b^3ce+3a^2\ldots\right.\right.}}{\ldots}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
h*x/b-c/a/x+d*ln(x)/a+((-a^2*h+a*b*e)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))
-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(a*b*f-b^2*c)*(-1/3/b/(a/b)^
(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(
a*b*g-b^2*d)*ln(b*x^3+a)/b)/b/a
```

3.409.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 85.94 (sec) , antiderivative size = 15238, normalized size of antiderivative = 60.23

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="fric
as")
```

output
```
Too large to include
```

3.409.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a),x)
```

output
```
Timed out
```

3.409.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx &= \frac{hx}{b} + \frac{d\log(x)}{a} \\ &\quad - \frac{\sqrt{3}\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}} - abf\left(\frac{a}{b}\right)^{\frac{2}{3}} - abe\left(\frac{a}{b}\right)^{\frac{1}{3}} + a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b} - \frac{c}{ax} \\ &\quad - \frac{\left(2b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2abg\left(\frac{a}{b}\right)^{\frac{2}{3}} + b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} - abf\left(\frac{a}{b}\right)^{\frac{1}{3}} + abe - a^2h\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ &\quad - \frac{\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}} - abg\left(\frac{a}{b}\right)^{\frac{2}{3}} - b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} + abf\left(\frac{a}{b}\right)^{\frac{1}{3}} - abe + a^2h\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="maxi
ma")
```

output
```
h*x/b + d*log(x)/a - 1/3*sqrt(3)*(b^2*c*(a/b)^(2/3) - a*b*f*(a/b)^(2/3) -
a*b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/
3))/(a/b)^(1/3))/(a^2*b) - c/(a*x) - 1/6*(2*b^2*d*(a/b)^(2/3) - 2*a*b*g*(a
/b)^(2/3) + b^2*c*(a/b)^(1/3) - a*b*f*(a/b)^(1/3) + a*b*e - a^2*h)*log(x^2
 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) - 1/3*(b^2*d*(a/b)^(2/
3) - a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3) - a*b*e + a
^2*h)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

3.409.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)}\,dx$$

$$= \frac{hx}{b}+\frac{d\log(|x|)}{a}-\frac{\sqrt{3}\left(abe-a^2h+(-ab^2)^{\frac{1}{3}}bc-(-ab^2)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}a}$$
$$-\frac{\left(abe-a^2h-(-ab^2)^{\frac{1}{3}}bc+(-ab^2)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}a}$$
$$-\frac{(bd-ag)\log(|bx^3+a|)}{3\,ab}-\frac{c}{ax}$$
$$+\frac{\left(ab^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^3e+a^3b^2h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3b^3}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a),x, algorithm="giac
")
```

output
```
h*x/b + d*log(abs(x))/a - 1/3*sqrt(3)*(a*b*e - a^2*h + (-a*b^2)^(1/3)*b*c
- (-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3)
)/((-a*b^2)^(2/3)*a) - 1/6*(a*b*e - a^2*h - (-a*b^2)^(1/3)*b*c + (-a*b^2)^
(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1
/3*(b*d - a*g)*log(abs(b*x^3 + a))/(a*b) - c/(a*x) + 1/3*(a*b^4*c*(-a/b)^(
1/3) - a^2*b^3*f*(-a/b)^(1/3) - a^2*b^3*e + a^3*b^2*h)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a^3*b^3)
```

3.409.9 Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 1802, normalized size of antiderivative = 7.12

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\,(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)),x)
```

output
```
symsum(log((b^3*c*d^2 + a^3*d*h^2 + a*b^2*d*e^2 - a*b^2*d^2*f - a*b^2*c*d*
g - 2*a^2*b*d*e*h + a^2*b*d*f*g)/a - root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^
2 + 27*a^3*b^4*d*z^2 - 9*a^4*b^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*
z + 9*a^3*b^3*c*h*z - 9*a^2*b^4*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z
+ 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^
3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*
b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^
2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 +
a*b^4*d^3 + a^5*h^3, z, k)*(root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2 + 27*a^
3*b^4*d*z^2 - 9*a^4*b^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z + 9*a^3
*b^3*c*h*z - 9*a^2*b^4*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*a^4*b
*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g
*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 +
 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a
^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3
 + a^5*h^3, z, k)*((3*a^2*b^3*c - 3*a^3*b^2*f)/a + (x*(24*a^3*b^4*d - 33*a
^4*b^3*g))/(a^2*b) + 36*root(27*a^4*b^4*z^3 - 27*a^4*b^3*g*z^2 + 27*a^3*b^
4*d*z^2 - 9*a^4*b^2*f*h*z - 18*a^3*b^3*d*g*z + 9*a^3*b^3*e*f*z + 9*a^3*b^3
*c*h*z - 9*a^2*b^4*c*e*z + 9*a^4*b^2*g^2*z + 9*a^2*b^4*d^2*z + 3*a^4*b*f*g
*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*...
```

3.410 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx$

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3.410.1 Optimal result

Integrand size = 38, antiderivative size = 260

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx = -\frac{c}{2ax^2}-\frac{d}{ax}+\frac{\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{2/3}} + \frac{e\log(x)}{a}-\frac{\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{2/3}} + \frac{\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}b^{2/3}} - \frac{(be-ah)\log(a+bx^3)}{3ab}$$

output
```
-1/2*c/a/x^2-d/a/x+e*ln(x)/a-1/3*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g+b*d))*l
n(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(2/3)+1/6*(b^(1/3)*(-a*f+b*c)-a^(1/3)*(-a*g
+b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)-1/3*(-a*h
+b*e)*ln(b*x^3+a)/a/b+1/3*(b^(4/3)*c+a^(1/3)*b*d-a*b^(1/3)*f-a^(4/3)*g)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(2/3)*3^(1/2)
```

3.410.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx$$

$$= \frac{-\frac{3a^{2/3}c}{x^2}-\frac{6a^{2/3}d}{x}+\frac{2\sqrt{3}\left(b^{4/3}c+\sqrt[3]{a}bd-a\sqrt[3]{b}f-a^{4/3}g\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+6a^{2/3}e\log(x)-\frac{2\left(b^{4/3}c-\sqrt[3]{a}bd-a\sqrt[3]{b}f+a^{4/3}\right.}{b^{2/3}}}{6a^{5/3}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x]
```

output
```
((-3*a^(2/3)*c)/x^2 - (6*a^(2/3)*d)/x + (2*Sqrt[3]*(b^(4/3)*c + a^(1/3)*b*
d - a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/
b^(2/3) + 6*a^(2/3)*e*Log[x] - (2*(b^(4/3)*c - a^(1/3)*b*d - a*b^(1/3)*f +
 a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((b^(4/3)*c - a^(1/3)*b*d
- a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/b^(2/3) + (2*a^(2/3)*(-(b*e) + a*h)*Log[a + b*x^3])/b)/(6*a^(5/3))
```

3.410.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{-x(bd-ag)-\left(x^2(be-ah)\right)+af-bc}{a(a+bx^3)}+\frac{c}{ax^3}+\frac{d}{ax^2}+\frac{e}{ax}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+\sqrt[3]{a}bd-a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}a^{5/3}b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(bd-ag)}{\sqrt[3]{b}}-af+bc\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bc-af)-\sqrt[3]{a}(bd-ag)\right)}{3a^{5/3}b^{2/3}}-\frac{(be-ah)\log\left(a+bx^3\right)}{3ab}-\frac{c}{2ax^2}-\frac{d}{ax}+\frac{e\log(x)}{a}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*c/(a*x^2) - d/(a*x) + ((b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f - a^(4/
3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*
b^(2/3)) + (e*Log[x])/a - ((b^(1/3)*(b*c - a*f) - a^(1/3)*(b*d - a*g))*Log
[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b*c - a*f - (a^(1/3)*(b*d -
 a*g))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)
*b^(1/3)) - ((b*e - a*h)*Log[a + b*x^3])/(3*a*b)
```

3.410.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.410.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.97

method	result
default	$-\frac{c}{2a\,x^{2}}-\frac{d}{ax}+\frac{e\ln(x)}{a}+\frac{(af-bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(ag-bd)\left(-\frac{\ln(\ldots}{3\ldots}\right.}{a}$
risch	$\frac{-\frac{xd}{a}-\frac{c}{2a}}{x^{2}}+\frac{e\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{5}b^{3}\_Z^{3}+\left(-3a^{5}b^{2}h+3a^{4}b^{3}e\right)\_Z^{2}+\left(3a^{5}b\,h^{2}-6a^{4}b^{2}eh+3a^{4}b^{2}fg-3a^{3}b^{3}cg-3a^{3}b^{3}df+3a^{3}b^{3}e^{2}+3a\ldots\right.\right.}\right.}{}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/a/x^2-d/a/x+e*ln(x)/a+((a*f-b*c)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(a*g-b*d)*(-1/3/b/(a/b)^(1
/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+
1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(a*
h-b*e)*ln(b*x^3+a)/b)/a
```

3.410.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 45.13 (sec) , antiderivative size = 15424, normalized size of antiderivative = 59.32

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)}\,dx=\text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="fric
as")
```

output
```
Too large to include
```

3.410.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a),x)
```

output
```
Timed out
```

3.410.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)}\,dx = \frac{e\log(x)}{a} - \frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}} - abg\left(\frac{a}{b}\right)^{\frac{2}{3}} + b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} - abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b} - \frac{\left(2be\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2ah\left(\frac{a}{b}\right)^{\frac{2}{3}} + bd\left(\frac{a}{b}\right)^{\frac{1}{3}} - ag\left(\frac{a}{b}\right)^{\frac{1}{3}} - bc + af\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(be\left(\frac{a}{b}\right)^{\frac{2}{3}} - ah\left(\frac{a}{b}\right)^{\frac{2}{3}} - bd\left(\frac{a}{b}\right)^{\frac{1}{3}} + ag\left(\frac{a}{b}\right)^{\frac{1}{3}} + bc - af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{2dx+c}{2ax^2}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="maxi
ma")
```

output
```
e*log(x)/a - 1/3*sqrt(3)*(b^2*d*(a/b)^(2/3) - a*b*g*(a/b)^(2/3) + b^2*c*(a
/b)^(1/3) - a*b*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(a^2*b) - 1/6*(2*b*e*(a/b)^(2/3) - 2*a*h*(a/b)^(2/3) + b*d*(a/b)^
(1/3) - a*g*(a/b)^(1/3) - b*c + a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(a*b*(a/b)^(2/3)) - 1/3*(b*e*(a/b)^(2/3) - a*h*(a/b)^(2/3) - b*d*(a/b)^(
1/3) + a*g*(a/b)^(1/3) + b*c - a*f)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
 - 1/2*(2*d*x + c)/(a*x^2)
```

3.410.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx$$
$$= \frac{e\log(|x|)}{a} + \frac{\sqrt{3}\left(b^2c-abf-(-ab^2)^{\frac{1}{3}}bd+(-ab^2)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(-ab^2)^{\frac{2}{3}}a}$$
$$+ \frac{\left(b^2c-abf+(-ab^2)^{\frac{1}{3}}bd-(-ab^2)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(-ab^2)^{\frac{2}{3}}a}$$
$$- \frac{(be-ah)\log(|bx^3+a|)}{3ab}$$
$$+ \frac{\left(ab^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^2c-a^2bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3b} - \frac{2dx+c}{2ax^2}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a),x, algorithm="giac
")
```

output
```
e*log(abs(x))/a + 1/3*sqrt(3)*(b^2*c - a*b*f - (-a*b^2)^(1/3)*b*d + (-a*b^
2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b
^2)^(2/3)*a) + 1/6*(b^2*c - a*b*f + (-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*
g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/3*(b*e
- a*h)*log(abs(b*x^3 + a))/(a*b) + 1/3*(a*b^2*d*(-a/b)^(1/3) - a^2*b*g*(-a
/b)^(1/3) + a*b^2*c - a^2*b*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^
3*b) - 1/2*(2*d*x + c)/(a*x^2)
```

3.410.9 Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 6948, normalized size of antiderivative = 26.72

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)),x)
```

output
```
symsum(log(-(b^5*c^3*x - a^5*h^3*x - a^2*b^3*d*e^2 + 36*root(27*a^5*b^3*z^
3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h*z + 9*a^4*b^2*f*g
*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z + 9*a^5*b*h^2*z +
 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3*b^2*e*f*g + 3*a^3
*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b^3*c*e*g - 3*a^2*b
^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2*h - 3*a^3*b^2*d*g
^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a*b^4*d^3 - a^5*h^3
 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)^3*a^5*b^3*x - a^3*b^2*e*f^2 +
a^3*b^2*e^2*g - a^3*b^2*f^3*x - a*b^4*c^2*e - a*b^4*d^3*x + a^4*b*g^3*x +
root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2*e*h
*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c*d*z
 + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3*a^3
*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - 3*a^2*b
^3*c*e*g - 3*a^2*b^3*c*d*h + 3*a^4*b*e*h^2 - 3*a*b^4*c^2*f - 3*a^3*b^2*e^2
*h - 3*a^3*b^2*d*g^2 + 3*a^2*b^3*d^2*g + 3*a^2*b^3*c*f^2 - a^3*b^2*f^3 - a
*b^4*d^3 - a^5*h^3 + a^2*b^3*e^3 + a^4*b*g^3 + b^5*c^3, z, k)*a^2*b^4*c^2
+ 3*root(27*a^5*b^3*z^3 - 27*a^5*b^2*h*z^2 + 27*a^4*b^3*e*z^2 - 18*a^4*b^2
*e*h*z + 9*a^4*b^2*f*g*z - 9*a^3*b^3*d*f*z - 9*a^3*b^3*c*g*z + 9*a^2*b^4*c
*d*z + 9*a^5*b*h^2*z + 9*a^3*b^3*e^2*z - 3*a^4*b*f*g*h + 3*a*b^4*c*d*e + 3
*a^3*b^2*e*f*g + 3*a^3*b^2*d*f*h + 3*a^3*b^2*c*g*h - 3*a^2*b^3*d*e*f - ...
```

3.410. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)} \, dx$

3.411 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx$

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3.411.1 Optimal result

Integrand size = 38, antiderivative size = 276

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx \\
&= -\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}+\frac{\left(b^{4/3}d+\sqrt[3]{a}be-a\sqrt[3]{b}g-a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}b^{2/3}} \\
&\quad-\frac{(bc-af)\log(x)}{a^2}-\frac{\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}b^{2/3}} \\
&\quad+\frac{\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}b^{2/3}} \\
&\quad+\frac{(bc-af)\log(a+bx^3)}{3a^2}
\end{aligned}$$

output
```
-1/3*c/a/x^3-1/2*d/a/x^2-e/a/x-(-a*f+b*c)*ln(x)/a^2-1/3*(b^(1/3)*(-a*g+b*d
)-a^(1/3)*(-a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(2/3)+1/6*(b^(1/3)*(
-a*g+b*d)-a^(1/3)*(-a*h+b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^
(5/3)/b^(2/3)+1/3*(-a*f+b*c)*ln(b*x^3+a)/a^2+1/3*(b^(4/3)*d+a^(1/3)*b*e-a*
b^(1/3)*g-a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
5/3)/b^(2/3)*3^(1/2)
```

3.411.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)}\,dx =$$

$$-\frac{\frac{2ac}{x^3}+\frac{3ad}{x^2}+\frac{6ae}{x}+\frac{2\sqrt{3}\sqrt[3]{a}\left(-b^{4/3}d-\sqrt[3]{a}be+a\sqrt[3]{b}g+a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+6(bc-af)\log(x)+\frac{2\sqrt[3]{a}\left(b^{4/3}d-\sqrt[3]{\ldots}\right.}{\ldots}}{\ldots}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x]
```

output
```
-1/6*((2*a*c)/x^3 + (3*a*d)/x^2 + (6*a*e)/x + (2*Sqrt[3]*a^(1/3)*(-(b^(4/3
)*d) - a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]])/b^(2/3) + 6*(b*c - a*f)*Log[x] + (2*a^(1/3)*(b^(4/3)*d -
a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) -
 (a^(1/3)*(b^(4/3)*d - a^(1/3)*b*e - a*b^(1/3)*g + a^(4/3)*h)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) - 2*(b*c - a*f)*Log[a + b*x^3]
)/a^2
```

3.411.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{bx^2(bc-af)-a(bd-ag)-ax(be-ah)}{a^2\left(a+bx^3\right)}+\frac{af-bc}{a^2x}+\frac{c}{ax^4}+\frac{d}{ax^3}+\frac{e}{ax^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+\sqrt[3]{a}be-a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}a^{5/3}b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(be-ah)}{\sqrt[3]{b}}-ag+bd\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(bd-ag)-\sqrt[3]{a}(be-ah)\right)}{3a^{5/3}b^{2/3}}+\frac{(bc-af)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(bc-af)}{a^2}-\frac{c}{3ax^3}-\frac{d}{2ax^2}-\frac{e}{ax}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x]
```

output
```
-1/3*c/(a*x^3) - d/(2*a*x^2) - e/(a*x) + ((b^(4/3)*d + a^(1/3)*b*e - a*b^(
1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(5/3)*b^(2/3)) - ((b*c - a*f)*Log[x])/a^2 - ((b^(1/3)*(b*d - a*g)
- a^(1/3)*(b*e - a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)*b^(2/3)) + ((b
*d - a*g - (a^(1/3)*(b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(6*a^(5/3)*b^(1/3)) + ((b*c - a*f)*Log[a + b*x^3])/(3*a^2)
```

3.411.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.411. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx$

3.411.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00

method	result
default	$-\frac{e}{ax}-\frac{c}{3a\,x^{3}}-\frac{d}{2a\,x^{2}}+\frac{(af-bc)\ln(x)}{a^{2}}+\frac{(a^{2}g-abd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$
risch	$\frac{-\frac{e\,x^{2}}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{6}b^{2}\_Z^{3}+\left(3a^{5}b^{2}f-3a^{4}b^{3}c\right)\_Z^{2}+\left(3a^{5}bgh-3a^{4}b^{2}dh-3a^{4}b^{2}eg+3a^{4}b^{2}f^{2}-6a^{3}b^{3}cf+3a^{3}b^{3}de+3a^{2}b^{4}c^{2}\right)\_\right.}\right.}{}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-e/a/x-1/3*c/a/x^3-1/2*d/a/x^2+(a*f-b*c)/a^2*ln(x)+((a^2*g-a*b*d)*(1/3/b/(
a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+
(a^2*h-a*b*e)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1)))+1/3*(-a*b*f+b^2*c)*ln(b*x^3+a)/b)/a^2
```

3.411.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 74.15 (sec) , antiderivative size = 15204, normalized size of antiderivative = 55.09

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\,(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="fric
as")
```

output
```
Too large to include
```

3.411.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx=\text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a),x)
```

output
```
Timed out
```

3.411.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.09

$$\begin{aligned}&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx\\&=-\frac{(bc-af)\log(x)}{a^2}\\&-\frac{\sqrt{3}\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3}\\&+\frac{\left(2b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+abd-a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&+\frac{\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd+a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\\&-\frac{6ex^2+3dx+2c}{6ax^3}\end{aligned}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="maxi
ma")
```

output
```
-(b*c - a*f)*log(x)/a^2 - 1/3*sqrt(3)*(a*b*e*(a/b)^(2/3) - a^2*h*(a/b)^(2/
3) + a*b*d*(a/b)^(1/3) - a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b
)^(1/3))/(a/b)^(1/3))/a^3 + 1/6*(2*b^2*c*(a/b)^(2/3) - 2*a*b*f*(a/b)^(2/3)
 - a*b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3) + a*b*d - a^2*g)*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) + 1/3*(b^2*c*(a/b)^(2/3) - a*b*
f*(a/b)^(2/3) + a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) - a*b*d + a^2*g)*log
(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) - 1/6*(6*e*x^2 + 3*d*x + 2*c)/(a*x^3
)
```

3.411.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx = \frac{\sqrt{3}\left(b^2d-abg-(-ab^2)^{\frac{1}{3}}be+(-ab^2)^{\frac{1}{3}}ah\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}a} + \frac{\left(b^2d-abg+(-ab^2)^{\frac{1}{3}}be-(-ab^2)^{\frac{1}{3}}ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}a} + \frac{(bc-af)\log\left(|bx^3+a|\right)}{3a^2} - \frac{(bc-af)\log\left(|x|\right)}{a^2} + \frac{\left(a^3b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^4bh\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^3b^2d-a^4bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^5b} - \frac{6aex^2+3adx+2ac}{6a^2x^3}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a),x, algorithm="giac
")
```

output
```
1/3*sqrt(3)*(b^2*d - a*b*g - (-a*b^2)^(1/3)*b*e + (-a*b^2)^(1/3)*a*h)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) + 1/6
*(b^2*d - a*b*g + (-a*b^2)^(1/3)*b*e - (-a*b^2)^(1/3)*a*h)*log(x^2 + x*(-a
/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) + 1/3*(b*c - a*f)*log(abs(b*x
^3 + a))/a^2 - (b*c - a*f)*log(abs(x))/a^2 + 1/3*(a^3*b^2*e*(-a/b)^(1/3) -
 a^4*b*h*(-a/b)^(1/3) + a^3*b^2*d - a^4*b*g)*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/(a^5*b) - 1/6*(6*a*e*x^2 + 3*a*d*x + 2*a*c)/(a^2*x^3)
```

3.411.9 Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 1842, normalized size of antiderivative = 6.67

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)),x)
```

output
```
symsum(log(- (b^5*c*d^2 - b^5*c^2*e + a^2*b^3*c*g^2 - a^2*b^3*e*f^2 - a^3*
b^2*f*g^2 + a^3*b^2*f^2*h - a*b^4*d^2*f + a*b^4*c^2*h - 2*a^2*b^3*c*f*h +
2*a^2*b^3*d*f*g - 2*a*b^4*c*d*g + 2*a*b^4*c*e*f)/a^3 - root(27*a^6*b^2*z^3
 + 27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g*h*z - 9*a^4*b^2*e*g*z -
 9*a^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b^3*d*e*z + 9*a^4*b^2*f^2*z +
9*a^2*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d*e - 3*a^3*b^2*e*f*g - 3*a^3*
b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f + 3*a^2*b^3*c*e*g + 3*a^2*b^
3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a^3*b^2*e^2*h + 3*a^3*b^2*d*g^
2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b^3*e^3 - a^4*b*g^3 - b^5*c^3
+ a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*((a^2*b^4*d^2 + a^4*b^2*g^2 + 2
*a^2*b^4*c*e - 2*a^3*b^3*c*h - 2*a^3*b^3*d*g - 2*a^3*b^3*e*f + 2*a^4*b^2*f
*h)/a^3 + root(27*a^6*b^2*z^3 + 27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^
5*b*g*h*z - 9*a^4*b^2*e*g*z - 9*a^4*b^2*d*h*z - 18*a^3*b^3*c*f*z + 9*a^3*b
^3*d*e*z + 9*a^4*b^2*f^2*z + 9*a^2*b^4*c^2*z + 3*a^4*b*f*g*h - 3*a*b^4*c*d
*e - 3*a^3*b^2*e*f*g - 3*a^3*b^2*d*f*h - 3*a^3*b^2*c*g*h + 3*a^2*b^3*d*e*f
 + 3*a^2*b^3*c*e*g + 3*a^2*b^3*c*d*h - 3*a^4*b*e*h^2 + 3*a*b^4*c^2*f + 3*a
^3*b^2*e^2*h + 3*a^3*b^2*d*g^2 - 3*a^2*b^3*d^2*g - 3*a^2*b^3*c*f^2 - a^2*b
^3*e^3 - a^4*b*g^3 - b^5*c^3 + a^3*b^2*f^3 + a*b^4*d^3 + a^5*h^3, z, k)*((
3*a^4*b^3*e - 3*a^5*b^2*h)/a^3 - (x*(24*a^3*b^4*c - 24*a^4*b^3*f))/a^3 + 3
6*root(27*a^6*b^2*z^3 + 27*a^5*b^2*f*z^2 - 27*a^4*b^3*c*z^2 + 9*a^5*b*g...
```

3.412 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx$

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3.412.1 Optimal result

Integrand size = 38, antiderivative size = 337

$$\begin{aligned}
&\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2} dx \\
&= \frac{(be-2ah)x}{b^3} + \frac{fx^2}{2b^2} + \frac{gx^3}{3b^2} + \frac{hx^4}{4b^2} + \frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3b^3(a+bx^3)} \\
&\quad - \frac{(2b^{5/3}c-4a^{2/3}be-5ab^{2/3}f+7a^{5/3}h)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{10/3}} \\
&\quad - \frac{(b^{2/3}(2bc-5af)+a^{2/3}(4be-7ah))\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{10/3}} \\
&\quad + \frac{(b^{2/3}(2bc-5af)+a^{2/3}(4be-7ah))\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{10/3}} \\
&\quad + \frac{(bd-2ag)\log(a+bx^3)}{3b^3}
\end{aligned}$$

output
```
(-2*a*h+b*e)*x/b^3+1/2*f*x^2/b^2+1/3*g*x^3/b^2+1/4*h*x^4/b^2+1/3*x*(a*(-a*
h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/b^3/(b*x^3+a)-1/9*(b^(2/3)*(-5*a*f
+2*b*c)+a^(2/3)*(-7*a*h+4*b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(10/3)+1/1
8*(b^(2/3)*(-5*a*f+2*b*c)+a^(2/3)*(-7*a*h+4*b*e))*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(1/3)/b^(10/3)+1/3*(-2*a*g+b*d)*ln(b*x^3+a)/b^3-1/9*(2
*b^(5/3)*c-4*a^(2/3)*b*e-5*a*b^(2/3)*f+7*a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*
b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(10/3)*3^(1/2)
```

3.412.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.99

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$

$$= \frac{36b^{2/3}(be-2ah)x+18b^{5/3}fx^2+12b^{5/3}gx^3+9b^{5/3}hx^4-\frac{12b^{2/3}(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{a+bx^3}-\frac{4\sqrt{3}\left(2b^2c-4a^{2/}\right.}{\ldots}}{\ldots}$$

input
```
Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

output
```
(36*b^(2/3)*(b*e - 2*a*h)*x + 18*b^(5/3)*f*x^2 + 12*b^(5/3)*g*x^3 + 9*b^(5
/3)*h*x^4 - (12*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x))
))/(a + b*x^3) - (4*Sqrt[3]*(2*b^2*c - 4*a^(2/3)*b^(4/3)*e - 5*a*b*f + 7*a
^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (
4*(-2*b^2*c - 4*a^(2/3)*b^(4/3)*e + 5*a*b*f + 7*a^(5/3)*b^(1/3)*h)*Log[a^(
1/3) + b^(1/3)*x])/a^(1/3) + (2*(2*b^2*c + 4*a^(2/3)*b^(4/3)*e - 5*a*b*f -
 7*a^(5/3)*b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1
/3) + 12*b^(2/3)*(b*d - 2*a*g)*Log[a + b*x^3])/(36*b^(11/3))
```

3.412.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2367, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 2367$$

$$\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3b^3(a+bx^3)}-\frac{\int \frac{-3ab^2hx^6-3ab^2gx^5-3ab^2fx^4-3ab(be-ah)x^3-3ab(bd-ag)x^2-2ab(bc-af)x+a^2(be-ah)}{bx^3+a}dx}{3ab^3}$$

$$\Big\downarrow \text{2426}$$

$$\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3b^3\left(a+bx^3\right)}-\frac{\int\left(-3abhx^3-3abgx^2-3abfx-3a(be-2ah)+\frac{(4be-7ah)a^2-3b(bd-2ag)x^2a-b(2bc-5af)xa}{bx^3+a}\right)dx}{3ab^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3b^3\left(a+bx^3\right)}-\frac{\frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-4a^{2/3}be+7a^{5/3}h-5ab^{2/3}f+2b^{5/3}c\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(4be-7ah)+b^{2/3}(2bc-5af)\right)}{6\sqrt[3]{b}}+}{}$$

input
```
Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

output
```
(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*b^3*(a + b*x^
3)) - (-3*a*(b*e - 2*a*h)*x - (3*a*b*f*x^2)/2 - a*b*g*x^3 - (3*a*b*h*x^4)/
4 + (a^(2/3)*(2*b^(5/3)*c - 4*a^(2/3)*b*e - 5*a*b^(2/3)*f + 7*a^(5/3)*h)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3)) + (a^(
2/3)*(b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e - 7*a*h))*Log[a^(1/3) + b^(
1/3)*x])/(3*b^(1/3)) - (a^(2/3)*(b^(2/3)*(2*b*c - 5*a*f) + a^(2/3)*(4*b*e
- 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(1/3)) - a*
(b*d - 2*a*g)*Log[a + b*x^3])/(3*a*b^3)
```

3.412.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

3.412. $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.412.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.48

method	result
risch	$\frac{h\,x^4}{4b^2}+\frac{g\,x^3}{3b^2}+\frac{f\,x^2}{2b^2}-\frac{2ahx}{b^3}+\frac{ex}{b^2}+\frac{(\frac{1}{3}afb-\frac{1}{3}b^2c)x^2+(-\frac{1}{3}a^2h+\frac{1}{3}aeb)x-\frac{a(ag-bd)}{3}}{b^3(b\,x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(3b(-2ag+bd)\_\cdots\right.}{\cdots}}{\cdots}$
default	$-\frac{-\frac{1}{4}bh\,x^4-\frac{1}{3}bg\,x^3-\frac{1}{2}bf\,x^2+2ahx-bex}{b^3}+\frac{\left(\frac{1}{3}afb-\frac{1}{3}b^2c\right)x^2+\left(-\frac{1}{3}a^2h+\frac{1}{3}aeb\right)x-\frac{a(ag-bd)}{3}}{b\,x^3+a}+\frac{\left(7a^2h-4aeb\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\cdots\right.\right.}{6\cdots}\cdots\right)}{\cdots}$

input
```
int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOS
E)
```

output
```
1/4*h*x^4/b^2+1/3*g*x^3/b^2+1/2*f*x^2/b^2-2/b^3*a*h*x+1/b^2*e*x+((1/3*a*f*
b-1/3*b^2*c)*x^2+(-1/3*a^2*h+1/3*a*e*b)*x-1/3*a*(a*g-b*d))/b^3/(b*x^3+a)+1
/9/b^4*sum((3*b*(-2*a*g+b*d)*_R^2+b*(-5*a*f+2*b*c)*_R+7*a^2*h-4*a*e*b)/_R^
2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.412.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 16147, normalized size of antiderivative = 47.91

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.412.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.412.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.08

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \frac{abd-a^2g-(b^2c-abf)x^2+(abe-a^2h)x}{3\,(b^4x^3+ab^3)}$$
$$+\frac{\sqrt{3}\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+7\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3}$$
$$+\frac{3\,bhx^4+4\,bgx^3+6\,bfx^2+12\,(be-2\,ah)x}{12\,b^3}$$
$$+\frac{\left(6\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-12\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+4\,abe-7\,a^2h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(3\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-6\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,abe+7\,a^2h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="ma
xima")
```

output
```
1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 + (a*b*e - a^2*h)*x)/(b^4*x^3 + a
*b^3) + 1/9*sqrt(3)*(2*b^2*c*(a/b)^(2/3) - 5*a*b*f*(a/b)^(2/3) - 4*a*b*e*(
a/b)^(1/3) + 7*a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(
a/b)^(1/3))/(a*b^3) + 1/12*(3*b*h*x^4 + 4*b*g*x^3 + 6*b*f*x^2 + 12*(b*e -
2*a*h)*x)/b^3 + 1/18*(6*b^2*d*(a/b)^(2/3) - 12*a*b*g*(a/b)^(2/3) + 2*b^2*c
*(a/b)^(1/3) - 5*a*b*f*(a/b)^(1/3) + 4*a*b*e - 7*a^2*h)*log(x^2 - x*(a/b)^
(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/9*(3*b^2*d*(a/b)^(2/3) - 6*a*b*
g*(a/b)^(2/3) - 2*b^2*c*(a/b)^(1/3) + 5*a*b*f*(a/b)^(1/3) - 4*a*b*e + 7*a^
2*h)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

3.412.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.04

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= \frac{\sqrt{3}\left(4\,abe-7\,a^2h+2\,(-ab^2)^{\frac{1}{3}}\,bc-5\,(-ab^2)^{\frac{1}{3}}\,af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,b^2}$$
$$+ \frac{\left(4\,abe-7\,a^2h-2\,(-ab^2)^{\frac{1}{3}}\,bc+5\,(-ab^2)^{\frac{1}{3}}\,af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,b^2}$$
$$+ \frac{(bd-2\,ag)\log\left(|bx^3+a|\right)}{3\,b^3} + \frac{abd-a^2g-(b^2c-abf)x^2+(abe-a^2h)x}{3\,(bx^3+a)b^3}$$
$$- \frac{\left(2\,b^6c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^5f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,ab^5e+7\,a^2b^4h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^7}$$
$$+ \frac{3\,b^6hx^4+4\,b^6gx^3+6\,b^6fx^2+12\,b^6ex-24\,ab^5hx}{12\,b^8}$$

input
```
integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gi
ac")
```

output
```
1/9*sqrt(3)*(4*a*b*e - 7*a^2*h + 2*(-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a
*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*
b^2) + 1/18*(4*a*b*e - 7*a^2*h - 2*(-a*b^2)^(1/3)*b*c + 5*(-a*b^2)^(1/3)*a
*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) + 1/3*(b
*d - 2*a*g)*log(abs(b*x^3 + a))/b^3 + 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)
*x^2 + (a*b*e - a^2*h)*x)/((b*x^3 + a)*b^3) - 1/9*(2*b^6*c*(-a/b)^(1/3) -
5*a*b^5*f*(-a/b)^(1/3) - 4*a*b^5*e + 7*a^2*b^4*h)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a*b^7) + 1/12*(3*b^6*h*x^4 + 4*b^6*g*x^3 + 6*b^6*f*x^2 +
12*b^6*e*x - 24*a*b^5*h*x)/b^8
```

3.412.9 Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 1241, normalized size of antiderivative = 3.68

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$

$$= \left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\left(729\,a\,b^{10}\,z^3-729\,a\,b^8\,d\,z^2+1458\,a^2\,b^7\,g\,z^2-216\,a\,b^6\,c\,e\,z-945\,a^3\,b^4\,f\,h\,z-972\,a^2\,b^5\,d\,g\,z+54\right.\right.\right.$$
$$+\frac{36\,a^3\,g^2+9\,a\,b^2\,d^2-35\,a^3\,f\,h-8\,a\,b^2\,c\,e+14\,a^2\,b\,c\,h-36\,a^2\,b\,d\,g+20\,a^2\,b\,e\,f}{9\,b^4}$$
$$+\frac{x\,(4\,b^3\,c^2+25\,a^2\,b\,f^2+42\,a^3\,g\,h-20\,a\,b^2\,c\,f+12\,a\,b^2\,d\,e-21\,a^2\,b\,d\,h-24\,a^2\,b\,e\,g)}{9\,b^4}\Bigg)\,\mathrm{root}\big(729\,a\,b^{10}\,z^3$$
$$-729\,a\,b^8\,d\,z^2+1458\,a^2\,b^7\,g\,z^2-216\,a\,b^6\,c\,e\,z-945\,a^3\,b^4\,f\,h\,z-972\,a^2\,b^5\,d\,g\,z$$
$$+540\,a^2\,b^5\,e\,f\,z+378\,a^2\,b^5\,c\,h\,z+243\,a\,b^6\,d^2\,z+972\,a^3\,b^4\,g^2\,z-630\,a^4\,b\,f\,g\,h$$
$$+72\,a\,b^4\,c\,d\,e+360\,a^3\,b^2\,e\,f\,g+315\,a^3\,b^2\,d\,f\,h+252\,a^3\,b^2\,c\,g\,h-180\,a^2\,b^3\,d\,e\,f$$
$$-144\,a^2\,b^3\,c\,e\,g-126\,a^2\,b^3\,c\,d\,h+588\,a^4\,b\,e\,h^2-60\,a\,b^4\,c^2\,f-336\,a^3\,b^2\,e^2\,h$$
$$-324\,a^3\,b^2\,d\,g^2+162\,a^2\,b^3\,d^2\,g+150\,a^2\,b^3\,c\,f^2-125\,a^3\,b^2\,f^3+64\,a^2\,b^3\,e^3+216\,a^4\,b\,g^3$$
$$\left.\left.-27\,a\,b^4\,d^3-343\,a^5\,h^3+8\,b^5\,c^3,z,k\right)\right)$$
$$+x\left(\frac{e}{b^2}-\frac{2\,a\,h}{b^3}\right)-\frac{x\left(\frac{a^2\,h}{3}-\frac{a\,b\,e}{3}\right)+\frac{a^2\,g}{3}+x^2\left(\frac{b^2\,c}{3}-\frac{a\,b\,f}{3}\right)-\frac{a\,b\,d}{3}}{b^4\,x^3+a\,b^3}+\frac{f\,x^2}{2\,b^2}+\frac{g\,x^3}{3\,b^2}+\frac{h\,x^4}{4\,b^2}$$

input
```
int((x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

output
```
symsum(log(root(729*a*b^10*z^3 - 729*a*b^8*d*z^2 + 1458*a^2*b^7*g*z^2 - 21
6*a*b^6*c*e*z - 945*a^3*b^4*f*h*z - 972*a^2*b^5*d*g*z + 540*a^2*b^5*e*f*z
+ 378*a^2*b^5*c*h*z + 243*a*b^6*d^2*z + 972*a^3*b^4*g^2*z - 630*a^4*b*f*g*
h + 72*a*b^4*c*d*e + 360*a^3*b^2*e*f*g + 315*a^3*b^2*d*f*h + 252*a^3*b^2*c
*g*h - 180*a^2*b^3*d*e*f - 144*a^2*b^3*c*e*g - 126*a^2*b^3*c*d*h + 588*a^4
*b*e*h^2 - 60*a*b^4*c^2*f - 336*a^3*b^2*e^2*h - 324*a^3*b^2*d*g^2 + 162*a^
2*b^3*d^2*g + 150*a^2*b^3*c*f^2 - 125*a^3*b^2*f^3 + 64*a^2*b^3*e^3 + 216*a
^4*b*g^3 - 27*a*b^4*d^3 - 343*a^5*h^3 + 8*b^5*c^3, z, k)*((108*a^2*b^3*g -
 54*a*b^4*d)/(9*b^4) + (x*(63*a^2*b^3*h - 36*a*b^4*e))/(9*b^4) + 9*root(72
9*a*b^10*z^3 - 729*a*b^8*d*z^2 + 1458*a^2*b^7*g*z^2 - 216*a*b^6*c*e*z - 94
5*a^3*b^4*f*h*z - 972*a^2*b^5*d*g*z + 540*a^2*b^5*e*f*z + 378*a^2*b^5*c*h*
z + 243*a*b^6*d^2*z + 972*a^3*b^4*g^2*z - 630*a^4*b*f*g*h + 72*a*b^4*c*d*e
 + 360*a^3*b^2*e*f*g + 315*a^3*b^2*d*f*h + 252*a^3*b^2*c*g*h - 180*a^2*b^3
*d*e*f - 144*a^2*b^3*c*e*g - 126*a^2*b^3*c*d*h + 588*a^4*b*e*h^2 - 60*a*b^
4*c^2*f - 336*a^3*b^2*e^2*h - 324*a^3*b^2*d*g^2 + 162*a^2*b^3*d^2*g + 150*
a^2*b^3*c*f^2 - 125*a^3*b^2*f^3 + 64*a^2*b^3*e^3 + 216*a^4*b*g^3 - 27*a*b^
4*d^3 - 343*a^5*h^3 + 8*b^5*c^3, z, k)*a*b^2) + (36*a^3*g^2 + 9*a*b^2*d^2
- 35*a^3*f*h - 8*a*b^2*c*e + 14*a^2*b*c*h - 36*a^2*b*d*g + 20*a^2*b*e*f)/(
9*b^4) + (x*(4*b^3*c^2 + 25*a^2*b*f^2 + 42*a^3*g*h - 20*a*b^2*c*f + 12*a*b
^2*d*e - 21*a^2*b*d*h - 24*a^2*b*e*g))/(9*b^4))*root(729*a*b^10*z^3 - 7...
```

3.412. $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

3.413 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

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3.413.1 Optimal result

Integrand size = 38, antiderivative size = 311

$$\begin{aligned}
&\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx \\
&= \frac{fx}{b^2}+\frac{gx^2}{2b^2}+\frac{hx^3}{3b^2}-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3b^2(a+bx^3)} \\
&\quad -\frac{\left(b^{4/3}c+2\sqrt[3]{a}bd-4a\sqrt[3]{b}f-5a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{8/3}} \\
&\quad +\frac{\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{8/3}} \\
&\quad -\frac{\left(\sqrt[3]{b}(bc-4af)-\sqrt[3]{a}(2bd-5ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{8/3}} \\
&\quad +\frac{(be-2ah)\log(a+bx^3)}{3b^3}
\end{aligned}$$

output
```
f*x/b^2+1/2*g*x^2/b^2+1/3*h*x^3/b^2-1/3*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)
*x^2)/b^2/(b*x^3+a)+1/9*(b^(1/3)*(-4*a*f+b*c)-a^(1/3)*(-5*a*g+2*b*d))*ln(a
^(1/3)+b^(1/3)*x)/a^(2/3)/b^(8/3)-1/18*(b^(1/3)*(-4*a*f+b*c)-a^(1/3)*(-5*a
*g+2*b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(8/3)+1/3*(
-2*a*h+b*e)*ln(b*x^3+a)/b^3-1/9*(b^(4/3)*c+2*a^(1/3)*b*d-4*a*b^(1/3)*f-5*a
^(4/3)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(8/3
)*3^(1/2)
```

3.413.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$

$$= \frac{18bfx+9bgx^2+6bhx^3-\frac{6(a^2h+b^2x(c+dx)-ab(e+x(f+gx)))}{a+bx^3}+\frac{2\sqrt{3}\sqrt[3]{b}\left(-b^{4/3}c-2\sqrt[3]{a}bd+4a\sqrt[3]{b}f+5a^{4/3}g\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{a^{2/3}}}{}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

output
```
(18*b*f*x + 9*b*g*x^2 + 6*b*h*x^3 - (6*(a^2*h + b^2*x*(c + d*x) - a*b*(e +
 x*(f + g*x))))/(a + b*x^3) + (2*Sqrt[3]*b^(1/3)*(-(b^(4/3)*c) - 2*a^(1/3)
*b*d + 4*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/a^(2/3) + (2*b^(1/3)*(b^(4/3)*c - 2*a^(1/3)*b*d - 4*a*b^(1/3)*f + 5
*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - (b^(1/3)*(b^(4/3)*c - 2*a^
(1/3)*b*d - 4*a*b^(1/3)*f + 5*a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/a^(2/3) + 6*(b*e - 2*a*h)*Log[a + b*x^3])/(18*b^3)
```

3.413.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2367, 25, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 2367$$

$$-\frac{\int -\frac{3ab^2hx^5+3ab^2gx^4+3ab^2fx^3+3ab(be-ah)x^2+2ab(bd-ag)x+ab(bc-af)}{bx^3+a}dx}{3ab^3}-\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{3b^2(a+bx^3)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3ab^2hx^5+3ab^2gx^4+3ab^2fx^3+3ab(be-ah)x^2+2ab(bd-ag)x+ab(bc-af)}{bx^3+a}dx}{3ab^3} - \frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{3b^2(a+bx^3)}$$

$\Big\downarrow$ 2426

$$\frac{\int \left(3abhx^2+3abgx+3abf+\frac{3ab(be-2ah)x^2+ab(2bd-5ag)x+ab(bc-4af)}{bx^3+a}\right)dx}{3ab^3} - \frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{3b^2(a+bx^3)}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{\sqrt[3]{a}\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{4/3}g+2\sqrt[3]{a}bd-4a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt{3}} - \frac{1}{6}\sqrt[3]{a}\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(\sqrt[3]{b}(bc-4af\right.}{} - \frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{3b^2(a+bx^3)}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

output
```
-1/3*(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(b^2*(a + b*x^3)) +
 (3*a*b*f*x + (3*a*b*g*x^2)/2 + a*b*h*x^3 - (a^(1/3)*b^(1/3)*(b^(4/3)*c +
2*a^(1/3)*b*d - 4*a*b^(1/3)*f - 5*a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/Sqrt[3] + (a^(1/3)*b^(1/3)*(b^(1/3)*(b*c - 4*a*f) -
a^(1/3)*(2*b*d - 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/3 - (a^(1/3)*b^(1/3)*(b
^(1/3)*(b*c - 4*a*f) - a^(1/3)*(2*b*d - 5*a*g))*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/6 + a*(b*e - 2*a*h)*Log[a + b*x^3])/(3*a*b^3)
```

3.413.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.413. $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.413.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.44

method	result
risch	$\frac{h x^3}{3b^2} + \frac{g x^2}{2b^2} + \frac{f x}{b^2} + \frac{\left(\frac{ag}{3}-\frac{bd}{3}\right)x^2+\left(\frac{af}{3}-\frac{bc}{3}\right)x-\frac{a(ah-be)}{3b}}{b^2(b x^3+a)} + \frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)} \frac{\left(3(-2ah+be)\_R^2+(-5ag+2bd)\_R-4af+\cdots\right.}{\_R^2}}{9b^3}$
default	$\frac{\frac{1}{3}h x^3+\frac{1}{2}g x^2+f x}{b^2} - \frac{\left(-\frac{ag}{3}+\frac{bd}{3}\right)x^2+\left(-\frac{af}{3}+\frac{bc}{3}\right)x+\frac{a(ah-be)}{3b}}{b x^3+a} + \frac{(4af-bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\cdots\right)}{\cdots}\right)}{3b\left(\frac{a}{b}\right)^{\cdots}}\cdots\right.}{3}\cdots$

input
```
int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOS
E)
```

output
```
1/3*h*x^3/b^2+1/2*g*x^2/b^2+f*x/b^2+((1/3*a*g-1/3*b*d)*x^2+(1/3*a*f-1/3*b*
c)*x-1/3*a*(a*h-b*e)/b)/b^2/(b*x^3+a)+1/9/b^3*sum((3*(-2*a*h+b*e)*_R^2+(-5
*a*g+2*b*d)*_R-4*a*f+b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.413.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 16285, normalized size of antiderivative = 52.36

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.413.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.413.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.06

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= \frac{abe-a^2h-(b^2d-abg)x^2-(b^2c-abf)x}{3\,(b^4x^3+ab^3)} + \frac{2\,hx^3+3\,gx^2+6\,fx}{6\,b^2}$$
$$+ \frac{\sqrt{3}\left(2\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}} - 5\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}} + b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} - 4\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3}$$
$$+ \frac{\left(6\,be\left(\frac{a}{b}\right)^{\frac{2}{3}} - 12\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}} - 5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}} - bc + 4\,af\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+ \frac{\left(3\,be\left(\frac{a}{b}\right)^{\frac{2}{3}} - 6\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}} + 5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}} + bc - 4\,af\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="ma
xima")
```

output
```
1/3*(a*b*e - a^2*h - (b^2*d - a*b*g)*x^2 - (b^2*c - a*b*f)*x)/(b^4*x^3 + a
*b^3) + 1/6*(2*h*x^3 + 3*g*x^2 + 6*f*x)/b^2 + 1/9*sqrt(3)*(2*b^2*d*(a/b)^(
2/3) - 5*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) - 4*a*b*f*(a/b)^(1/3))*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^3) + 1/18*(6*b*e*(a/b
)^(2/3) - 12*a*h*(a/b)^(2/3) + 2*b*d*(a/b)^(1/3) - 5*a*g*(a/b)^(1/3) - b*c
 + 4*a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/9*(
3*b*e*(a/b)^(2/3) - 6*a*h*(a/b)^(2/3) - 2*b*d*(a/b)^(1/3) + 5*a*g*(a/b)^(1
/3) + b*c - 4*a*f)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

3.413.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.06

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= -\frac{\sqrt{3}\left(b^2c-4\,abf-2\,(-ab^2)^{\frac{1}{3}}\,bd+5\,(-ab^2)^{\frac{1}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,b^2}$$
$$-\frac{\left(b^2c-4\,abf+2\,(-ab^2)^{\frac{1}{3}}\,bd-5\,(-ab^2)^{\frac{1}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,b^2}$$
$$+\frac{(be-2\,ah)\log\left(|bx^3+a|\right)}{3\,b^3}+\frac{abe-a^2h-(b^2d-abg)x^2-(b^2c-abf)x}{3\,(bx^3+a)b^3}$$
$$-\frac{\left(2\,b^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^3g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b^4c-4\,ab^3f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}$$
$$+\frac{2\,b^4hx^3+3\,b^4gx^2+6\,b^4fx}{6\,b^6}$$

input
```
integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gi
ac")
```

output
```
-1/9*sqrt(3)*(b^2*c - 4*a*b*f - 2*(-a*b^2)^(1/3)*b*d + 5*(-a*b^2)^(1/3)*a*
g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b
^2) - 1/18*(b^2*c - 4*a*b*f + 2*(-a*b^2)^(1/3)*b*d - 5*(-a*b^2)^(1/3)*a*g)
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) + 1/3*(b*e
- 2*a*h)*log(abs(b*x^3 + a))/b^3 + 1/3*(a*b*e - a^2*h - (b^2*d - a*b*g)*x^
2 - (b^2*c - a*b*f)*x)/((b*x^3 + a)*b^3) - 1/9*(2*b^4*d*(-a/b)^(1/3) - 5*a
*b^3*g*(-a/b)^(1/3) + b^4*c - 4*a*b^3*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(
1/3)))/(a*b^5) + 1/6*(2*b^4*h*x^3 + 3*b^4*g*x^2 + 6*b^4*f*x)/b^6
```

3.413.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1229, normalized size of antiderivative = 3.95

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$

$$= \left(\sum_{k=1}^{3} \ln\left(\frac{36\,a^3\,h^2+9\,a\,b^2\,e^2+2\,b^3\,c\,d-5\,a\,b^2\,c\,g-8\,a\,b^2\,d\,f-36\,a^2\,b\,e\,h+20\,a^2\,b\,f\,g}{9\,b^4}\right.\right.$$

$$+\mathrm{root}\big(729\,a^2\,b^9\,z^3+1458\,a^3\,b^6\,h\,z^2-729\,a^2\,b^7\,e\,z^2+54\,a\,b^6\,c\,d\,z-972\,a^3\,b^4\,e\,h\,z+540\,a^3\,b^4\,f\,g\,z-216\,a^2\,b^5$$

$$\left.+\frac{x\,(4\,b^2\,d^2+25\,a^2\,g^2-3\,b^2\,c\,e-24\,a^2\,f\,h+6\,a\,b\,c\,h-20\,a\,b\,d\,g+12\,a\,b\,e\,f)}{9\,b^3}\right)\mathrm{root}\big(729\,a^2\,b^9\,z^3$$

$$+1458\,a^3\,b^6\,h\,z^2-729\,a^2\,b^7\,e\,z^2+54\,a\,b^6\,c\,d\,z-972\,a^3\,b^4\,e\,h\,z+540\,a^3\,b^4\,f\,g\,z$$

$$-216\,a^2\,b^5\,d\,f\,z-135\,a^2\,b^5\,c\,g\,z+972\,a^4\,b^3\,h^2\,z+243\,a^2\,b^5\,e^2\,z+360\,a^4\,b\,f\,g\,h$$

$$-18\,a\,b^4\,c\,d\,e-180\,a^3\,b^2\,e\,f\,g-144\,a^3\,b^2\,d\,f\,h-90\,a^3\,b^2\,c\,g\,h+72\,a^2\,b^3\,d\,e\,f+45\,a^2\,b^3\,c\,e\,g$$

$$+36\,a^2\,b^3\,c\,d\,h-324\,a^4\,b\,e\,h^2+12\,a\,b^4\,c^2\,f+162\,a^3\,b^2\,e^2\,h+150\,a^3\,b^2\,d\,g^2-60\,a^2\,b^3\,d^2\,g$$

$$\left.-48\,a^2\,b^3\,c\,f^2+64\,a^3\,b^2\,f^3-27\,a^2\,b^3\,e^3-125\,a^4\,b\,g^3+8\,a\,b^4\,d^3+216\,a^5\,h^3-b^5\,c^3,z,k\big)\right)$$

$$-\frac{x\left(\frac{b\,c}{3}-\frac{a\,f}{3}\right)+\frac{a^2\,h-a\,b\,e}{3\,b}+x^2\left(\frac{b\,d}{3}-\frac{a\,g}{3}\right)}{b^3\,x^3+a\,b^2}+\frac{g\,x^2}{2\,b^2}+\frac{h\,x^3}{3\,b^2}+\frac{f\,x}{b^2}$$

input

```
int((x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

output
```
symsum(log((36*a^3*h^2 + 9*a*b^2*e^2 + 2*b^3*c*d - 5*a*b^2*c*g - 8*a*b^2*d
*f - 36*a^2*b*e*h + 20*a^2*b*f*g)/(9*b^4) + root(729*a^2*b^9*z^3 + 1458*a^
3*b^6*h*z^2 - 729*a^2*b^7*e*z^2 + 54*a*b^6*c*d*z - 972*a^3*b^4*e*h*z + 540
*a^3*b^4*f*g*z - 216*a^2*b^5*d*f*z - 135*a^2*b^5*c*g*z + 972*a^4*b^3*h^2*z
 + 243*a^2*b^5*e^2*z + 360*a^4*b*f*g*h - 18*a*b^4*c*d*e - 180*a^3*b^2*e*f*
g - 144*a^3*b^2*d*f*h - 90*a^3*b^2*c*g*h + 72*a^2*b^3*d*e*f + 45*a^2*b^3*c
*e*g + 36*a^2*b^3*c*d*h - 324*a^4*b*e*h^2 + 12*a*b^4*c^2*f + 162*a^3*b^2*e
^2*h + 150*a^3*b^2*d*g^2 - 60*a^2*b^3*d^2*g - 48*a^2*b^3*c*f^2 + 64*a^3*b^
2*f^3 - 27*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8*a*b^4*d^3 + 216*a^5*h^3 - b^5*c
^3, z, k)*((108*a^2*b^3*h - 54*a*b^4*e)/(9*b^4) + (x*(9*b^4*c - 36*a*b^3*f
))/(9*b^3) + 9*root(729*a^2*b^9*z^3 + 1458*a^3*b^6*h*z^2 - 729*a^2*b^7*e*z
^2 + 54*a*b^6*c*d*z - 972*a^3*b^4*e*h*z + 540*a^3*b^4*f*g*z - 216*a^2*b^5*
d*f*z - 135*a^2*b^5*c*g*z + 972*a^4*b^3*h^2*z + 243*a^2*b^5*e^2*z + 360*a^
4*b*f*g*h - 18*a*b^4*c*d*e - 180*a^3*b^2*e*f*g - 144*a^3*b^2*d*f*h - 90*a^
3*b^2*c*g*h + 72*a^2*b^3*d*e*f + 45*a^2*b^3*c*e*g + 36*a^2*b^3*c*d*h - 324
*a^4*b*e*h^2 + 12*a*b^4*c^2*f + 162*a^3*b^2*e^2*h + 150*a^3*b^2*d*g^2 - 60
*a^2*b^3*d^2*g - 48*a^2*b^3*c*f^2 + 64*a^3*b^2*f^3 - 27*a^2*b^3*e^3 - 125*
a^4*b*g^3 + 8*a*b^4*d^3 + 216*a^5*h^3 - b^5*c^3, z, k)*a*b^2) + (x*(4*b^2*
d^2 + 25*a^2*g^2 - 3*b^2*c*e - 24*a^2*f*h + 6*a*b*c*h - 20*a*b*d*g + 12*a*
b*e*f))/(9*b^3))*root(729*a^2*b^9*z^3 + 1458*a^3*b^6*h*z^2 - 729*a^2*b^...
```

3.413. $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

3.414 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

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3.414.1 Optimal result

Integrand size = 38, antiderivative size = 290

$$\begin{aligned}\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx &= \frac{4gx}{3b^2}+\frac{5hx^2}{6b^2}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b(a+bx^3)}\\ &-\frac{\left(b^{4/3}d+2\sqrt[3]{a}be-4a\sqrt[3]{b}g-5a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{8/3}}\\ &+\frac{\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{8/3}}\\ &-\frac{\left(\sqrt[3]{b}(bd-4ag)-\sqrt[3]{a}(2be-5ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{8/3}}+\frac{f\log(a+bx^3)}{3b^2}\end{aligned}$$

output
```
4/3*g*x/b^2+5/6*h*x^2/b^2+1/3*(-h*x^5-g*x^4-f*x^3-e*x^2-d*x-c)/b/(b*x^3+a)
+1/9*(b^(1/3)*(-4*a*g+b*d)-a^(1/3)*(-5*a*h+2*b*e))*ln(a^(1/3)+b^(1/3)*x)/a
^(2/3)/b^(8/3)-1/18*(b^(1/3)*(-4*a*g+b*d)-a^(1/3)*(-5*a*h+2*b*e))*ln(a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(8/3)+1/3*f*ln(b*x^3+a)/b^2-1/
9*(b^(4/3)*d+2*a^(1/3)*b*e-4*a*b^(1/3)*g-5*a^(4/3)*h)*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(8/3)*3^(1/2)
```

3.414.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.97

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$

$$= \frac{18b^{2/3}gx+9b^{2/3}hx^2-\frac{6b^{2/3}(b(c+x(d+ex))-a(f+x(g+hx)))}{a+bx^3}+\frac{2\sqrt{3}\left(-b^{4/3}d-2\sqrt[3]{a}be+4a\sqrt[3]{b}g+5a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+}{}$$

input
```
Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

output
```
(18*b^(2/3)*g*x + 9*b^(2/3)*h*x^2 - (6*b^(2/3)*(b*(c + x*(d + e*x)) - a*(f
 + x*(g + h*x))))/(a + b*x^3) + (2*Sqrt[3]*(-(b^(4/3)*d) - 2*a^(1/3)*b*e +
 4*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])
/a^(2/3) + (2*(b^(4/3)*d - 2*a^(1/3)*b*e - 4*a*b^(1/3)*g + 5*a^(4/3)*h)*Lo
g[a^(1/3) + b^(1/3)*x])/a^(2/3) - ((b^(4/3)*d - 2*a^(1/3)*b*e - 4*a*b^(1/3
)*g + 5*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3)
 + 6*b^(2/3)*f*Log[a + b*x^3])/(18*b^(8/3))
```

3.414.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2363, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{ 2363}$$

$$\frac{\int \frac{5hx^4+4gx^3+3fx^2+2ex+d}{bx^3+a}dx}{3b}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b(a+bx^3)}$$

$$\Big\downarrow \text{ 2426}$$

$$\frac{\int \left(\frac{4g}{b}+\frac{5hx}{b}+\frac{3bfx^2+(2be-5ah)x+bd-4ag}{b(bx^3+a)}\right)dx}{3b}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{4/3}h+2\sqrt[3]{a}be-4a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt{3}a^{2/3}b^{5/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(2be-5ah)}{\sqrt[3]{b}}-4ag+bd\right)}{6a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\right.}{}}{3b}$$

$$\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{3b\left(a+bx^3\right)}$$

input
```
Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

output
```
-1/3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(b*(a + b*x^3)) + ((4*g*x)/
b + (5*h*x^2)/(2*b) - ((b^(4/3)*d + 2*a^(1/3)*b*e - 4*a*b^(1/3)*g - 5*a^(4
/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)
*b^(5/3)) + ((b^(1/3)*(b*d - 4*a*g) - a^(1/3)*(2*b*e - 5*a*h))*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(2/3)*b^(5/3)) - ((b*d - 4*a*g - (a^(1/3)*(2*b*e - 5*a
*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b
^(4/3)) + (f*Log[a + b*x^3])/b)/(3*b)
```

3.414.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.414. $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

3.414.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.40

method	result
risch	$\frac{h\,x^{2}}{2b^{2}}+\frac{gx}{b^{2}}+\frac{\left(\frac{ah}{3}-\frac{be}{3}\right)x^{2}+\left(\frac{ag}{3}-\frac{bd}{3}\right)x+\frac{af}{3}-\frac{bc}{3}}{b^{2}\left(b\,x^{3}+a\right)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(3bf\_R^{2}+\left(-5ah+2be\right)\_R-4ag+bd\right)\ln\left(x-\_R\right)}{\_R^{2}}}{9b^{3}}$
default	$\frac{\frac{1}{2}h\,x^{2}+gx}{b^{2}}-\frac{\frac{\left(-\frac{ah}{3}+\frac{be}{3}\right)x^{2}+\left(-\frac{ag}{3}+\frac{bd}{3}\right)x-\frac{af}{3}+\frac{bc}{3}}{b\,x^{3}+a}+\frac{\left(4ag-bd\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{3}}{b^{2}}$

input
```
int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOS
E)
```

output
```
1/2*h*x^2/b^2+g*x/b^2+((1/3*a*h-1/3*b*e)*x^2+(1/3*a*g-1/3*b*d)*x+1/3*a*f-1
/3*b*c)/b^2/(b*x^3+a)+1/9/b^3*sum((3*b*f*_R^2+(-5*a*h+2*b*e)*_R-4*a*g+b*d)
/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.414.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 12153, normalized size of antiderivative = 41.91

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.414.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.414.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= -\frac{(be-ah)x^2+bc-af+(bd-ag)x}{3\,(b^3x^3+ab^2)}$$
$$+\frac{\sqrt{3}\left(2\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2}+\frac{hx^2+2\,gx}{2\,b^2}$$
$$+\frac{\left(6\,bf\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,be\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}-bd+4\,ag\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(3\,bf\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,be\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}+bd-4\,ag\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="ma
xima")
```

output
```
-1/3*((b*e - a*h)*x^2 + b*c - a*f + (b*d - a*g)*x)/(b^3*x^3 + a*b^2) + 1/9
*sqrt(3)*(2*b*e*(a/b)^(2/3) - 5*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/3) - 4*a*g*
(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) +
 1/2*(h*x^2 + 2*g*x)/b^2 + 1/18*(6*b*f*(a/b)^(2/3) + 2*b*e*(a/b)^(1/3) - 5
*a*h*(a/b)^(1/3) - b*d + 4*a*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^
3*(a/b)^(2/3)) + 1/9*(3*b*f*(a/b)^(2/3) - 2*b*e*(a/b)^(1/3) + 5*a*h*(a/b)^
(1/3) + b*d - 4*a*g)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

3.414.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.04

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= \frac{f\log\left(|bx^3+a|\right)}{3\,b^2} - \frac{\sqrt{3}\left(b^2d-4\,abg-2\,(-ab^2)^{\frac{1}{3}}\,be+5\,(-ab^2)^{\frac{1}{3}}\,ah\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,b^2} - \frac{\left(b^2d-4\,abg+2\,(-ab^2)^{\frac{1}{3}}\,be-5\,(-ab^2)^{\frac{1}{3}}\,ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,b^2} - \frac{(be-ah)x^2+bc-af+(bd-ag)x}{3\,(bx^3+a)b^2} + \frac{b^2hx^2+2\,b^2gx}{2\,b^4} - \frac{\left(2\,b^4e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,ab^3h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b^4d-4\,ab^3g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^5}$$

input
```
integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="gi
ac")
```

output
```
1/3*f*log(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(b^2*d - 4*a*b*g - 2*(-a*b^2)^
(1/3)*b*e + 5*(-a*b^2)^(1/3)*a*h)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/
(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/18*(b^2*d - 4*a*b*g + 2*(-a*b^2)^(1
/3)*b*e - 5*(-a*b^2)^(1/3)*a*h)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(
(-a*b^2)^(2/3)*b^2) - 1/3*((b*e - a*h)*x^2 + b*c - a*f + (b*d - a*g)*x)/((
b*x^3 + a)*b^2) + 1/2*(b^2*h*x^2 + 2*b^2*g*x)/b^4 - 1/9*(2*b^4*e*(-a/b)^(1
/3) - 5*a*b^3*h*(-a/b)^(1/3) + b^4*d - 4*a*b^3*g)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a*b^5)
```

3.414.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.81

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$

$$= \left(\sum_{k=1}^{3} \ln\left(\frac{9\,a\,b\,f^2+2\,b^2\,d\,e+20\,a^2\,g\,h-5\,a\,b\,d\,h-8\,a\,b\,e\,g}{9\,b^3} + \mathrm{root}\left(729\,a^2\,b^8\,z^3-729\,a^2\,b^6\,f\,z^2+54\,a\,b^5\,d\,e\,z+540\,a^3\,b^3\,g\,h\,z-216\,a^2\,b^4\,e\,g\,z-135\,a^2\,b^4\,d\,h\,z+243\,a^2\,b^4\,\ldots\right.\right.\right.$$

$$\left.+\frac{x\,(25\,a^2\,h^2-20\,a\,b\,e\,h+12\,f\,g\,a\,b+4\,b^2\,e^2-3\,d\,f\,b^2)}{9\,b^3}\right)\mathrm{root}\left(729\,a^2\,b^8\,z^3\right.$$

$$-729\,a^2\,b^6\,f\,z^2+54\,a\,b^5\,d\,e\,z+540\,a^3\,b^3\,g\,h\,z-216\,a^2\,b^4\,e\,g\,z-135\,a^2\,b^4\,d\,h\,z+243\,a^2\,b^4\,f^2\,z$$

$$-180\,a^3\,b\,f\,g\,h-18\,a\,b^3\,d\,e\,f+72\,a^2\,b^2\,e\,f\,g+45\,a^2\,b^2\,d\,f\,h+150\,a^3\,b\,e\,h^2+12\,a\,b^3\,d^2\,g$$

$$\left.\left.-60\,a^2\,b^2\,e^2\,h-48\,a^2\,b^2\,d\,g^2-27\,a^2\,b^2\,f^3+64\,a^3\,b\,g^3+8\,a\,b^3\,e^3-125\,a^4\,h^3-b^4\,d^3,z,k\right)\right)$$

$$-\frac{\left(\frac{b\,e}{3}-\frac{a\,h}{3}\right)\,x^2+\left(\frac{b\,d}{3}-\frac{a\,g}{3}\right)\,x+\frac{b\,c}{3}-\frac{a\,f}{3}}{b^3\,x^3+a\,b^2}+\frac{h\,x^2}{2\,b^2}+\frac{g\,x}{b^2}$$

input

```
int((x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

output
```
symsum(log((9*a*b*f^2 + 2*b^2*d*e + 20*a^2*g*h - 5*a*b*d*h - 8*a*b*e*g)/(9
*b^3) + root(729*a^2*b^8*z^3 - 729*a^2*b^6*f*z^2 + 54*a*b^5*d*e*z + 540*a^
3*b^3*g*h*z - 216*a^2*b^4*e*g*z - 135*a^2*b^4*d*h*z + 243*a^2*b^4*f^2*z -
180*a^3*b*f*g*h - 18*a*b^3*d*e*f + 72*a^2*b^2*e*f*g + 45*a^2*b^2*d*f*h + 1
50*a^3*b*e*h^2 + 12*a*b^3*d^2*g - 60*a^2*b^2*e^2*h - 48*a^2*b^2*d*g^2 - 27
*a^2*b^2*f^3 + 64*a^3*b*g^3 + 8*a*b^3*e^3 - 125*a^4*h^3 - b^4*d^3, z, k)*(
(x*(9*b^4*d - 36*a*b^3*g))/(9*b^3) - 6*a*f + 9*root(729*a^2*b^8*z^3 - 729*
a^2*b^6*f*z^2 + 54*a*b^5*d*e*z + 540*a^3*b^3*g*h*z - 216*a^2*b^4*e*g*z - 1
35*a^2*b^4*d*h*z + 243*a^2*b^4*f^2*z - 180*a^3*b*f*g*h - 18*a*b^3*d*e*f +
72*a^2*b^2*e*f*g + 45*a^2*b^2*d*f*h + 150*a^3*b*e*h^2 + 12*a*b^3*d^2*g - 6
0*a^2*b^2*e^2*h - 48*a^2*b^2*d*g^2 - 27*a^2*b^2*f^3 + 64*a^3*b*g^3 + 8*a*b
^3*e^3 - 125*a^4*h^3 - b^4*d^3, z, k)*a*b^2) + (x*(4*b^2*e^2 + 25*a^2*h^2
- 3*b^2*d*f - 20*a*b*e*h + 12*a*b*f*g))/(9*b^3))*root(729*a^2*b^8*z^3 - 72
9*a^2*b^6*f*z^2 + 54*a*b^5*d*e*z + 540*a^3*b^3*g*h*z - 216*a^2*b^4*e*g*z -
 135*a^2*b^4*d*h*z + 243*a^2*b^4*f^2*z - 180*a^3*b*f*g*h - 18*a*b^3*d*e*f
+ 72*a^2*b^2*e*f*g + 45*a^2*b^2*d*f*h + 150*a^3*b*e*h^2 + 12*a*b^3*d^2*g -
 60*a^2*b^2*e^2*h - 48*a^2*b^2*d*g^2 - 27*a^2*b^2*f^3 + 64*a^3*b*g^3 + 8*a
*b^3*e^3 - 125*a^4*h^3 - b^4*d^3, z, k), k, 1, 3) - ((b*c)/3 - (a*f)/3 + x
*((b*d)/3 - (a*g)/3) + x^2*((b*e)/3 - (a*h)/3))/(a*b^2 + b^3*x^3) + (h*x^2
)/(2*b^2) + (g*x)/b^2
```

3.414. $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

3.415 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

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3.415.1 Optimal result

Integrand size = 36, antiderivative size = 289

$$\begin{aligned}&\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx\\&=\frac{hx}{b^2}-\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3ab^2(a+bx^3)}\\&\quad-\frac{\left(b^{5/3}c+a^{2/3}be+2ab^{2/3}f-4a^{5/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{7/3}}\\&\quad-\frac{\left(b^{2/3}(bc+2af)-a^{2/3}(be-4ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{7/3}}\\&\quad+\frac{\left(b^{2/3}(bc+2af)-a^{2/3}(be-4ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{7/3}}+\frac{g\log(a+bx^3)}{3b^2}\end{aligned}$$

output
```
h*x/b^2-1/3*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/a/b^2/(b*x^3+
a)-1/9*(b^(2/3)*(2*a*f+b*c)-a^(2/3)*(-4*a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^
(4/3)/b^(7/3)+1/18*(b^(2/3)*(2*a*f+b*c)-a^(2/3)*(-4*a*h+b*e))*ln(a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(7/3)+1/3*g*ln(b*x^3+a)/b^2-1/9*(b
^(5/3)*c+a^(2/3)*b*e+2*a*b^(2/3)*f-4*a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(7/3)*3^(1/2)
```

3.415.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.99

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$

$$= \frac{18b^{2/3}hx + \frac{6b^{2/3}(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{a(a+bx^3)} - \frac{2\sqrt{3}\left(b^2c+a^{2/3}b^{4/3}e+2abf-4a^{5/3}\sqrt[3]{b}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} - \frac{2\left(b^2c-a^{2/3}\right.}{}}{}$$

input
```
Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

output
```
(18*b^(2/3)*h*x + (6*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + 
f*x))))/(a*(a + b*x^3)) - (2*Sqrt[3]*(b^2*c + a^(2/3)*b^(4/3)*e + 2*a*b*f 
- 4*a^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4/3
) - (2*(b^2*c - a^(2/3)*b^(4/3)*e + 2*a*b*f + 4*a^(5/3)*b^(1/3)*h)*Log[a^(
1/3) + b^(1/3)*x])/a^(4/3) + ((b^2*c - a^(2/3)*b^(4/3)*e + 2*a*b*f + 4*a^(
5/3)*b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) + 
6*b^(2/3)*g*Log[a + b*x^3])/(18*b^(8/3))
```

3.415.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2367, 25, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 2367$$

$$-\frac{\int -\frac{3abhx^3+3abgx^2+b(bc+2af)x+a(be-ah)}{bx^3+a}dx}{3ab^2} - \frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3ab^2\,(a+bx^3)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3abhx^3+3abgx^2+b(bc+2af)x+a(be-ah)}{bx^3+a}dx}{3ab^2}-\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{3ab^2(a+bx^3)}$$

$$\Big\downarrow \text{ 2426}$$

$$\frac{\int \left(3ah+\frac{3abgx^2+b(bc+2af)x+a(be-4ah)}{bx^3+a}\right)dx}{3ab^2}-\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{3ab^2(a+bx^3)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(a^{2/3}be-4a^{5/3}h+2ab^{2/3}f+b^{5/3}c)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)(b^{2/3}(2af+bc)-a^{2/3}(be-4ah))}{6\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{\cdots}}{3ab^2}-\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{3ab^2(a+bx^3)}$$

input
```
Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x]
```

output
```
-1/3*(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(a*b^2*(a +
 b*x^3)) + (3*a*h*x - ((b^(5/3)*c + a^(2/3)*b*e + 2*a*b^(2/3)*f - 4*a^(5/3
)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b
^(1/3)) - ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e - 4*a*h))*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(1/3)*b^(1/3)) + ((b^(2/3)*(b*c + 2*a*f) - a^(2/3)*(b*e -
4*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(1/3)
) + a*g*Log[a + b*x^3])/(3*a*b^2)
```

3.415.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.415. $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.415.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.39

method	result
risch	$\frac{hx}{b^2}+\frac{-\frac{b(af-bc)x^2}{3a}+\left(\frac{ah}{3}-\frac{be}{3}\right)x+\frac{ag}{3}-\frac{bd}{3}}{b^2(bx^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(3gb\_R^2+\frac{b(2af+bc)\_R}{a}-4ah+be\right)\ln\left(x-\_R\right)}{\_R^2}}{9b^3}$
default	$\frac{hx}{b^2}-\frac{\frac{\frac{b(af-bc)x^2}{3a}+\left(-\frac{ah}{3}+\frac{be}{3}\right)x-\frac{ag}{3}+\frac{bd}{3}}{bx^3+a}+\frac{\left(4a^2h-aeb\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(\cdots}{\cdots}}{b^2}$

input
```
int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
h*x/b^2+(-1/3*b*(a*f-b*c)/a*x^2+(1/3*a*h-1/3*b*e)*x+1/3*a*g-1/3*b*d)/b^2/(
b*x^3+a)+1/9/b^3*sum((3*g*b*_R^2+b*(2*a*f+b*c)/a*_R-4*a*h+b*e)/_R^2*ln(x-_
R),_R=RootOf(_Z^3*b+a))
```

3.415.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 12617, normalized size of antiderivative = 43.66

$$\int \frac{x(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^2} dx = \text{Too large to display}$$

input
```
integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fric
as")
```

output
```
Too large to include
```

3.415.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^2} dx = \text{Timed out}$$

input
```
integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.415.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.08

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= -\frac{abd-a^2g-(b^2c-abf)x^2+(abe-a^2h)x}{3\,(ab^3x^3+a^2b^2)}+\frac{hx}{b^2}$$
$$+\frac{\sqrt{3}\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}$$
$$+\frac{\left(6\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-abe+4\,a^2h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(3\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+abe-4\,a^2h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxi
ma")
```

output
```
-1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 + (a*b*e - a^2*h)*x)/(a*b^3*x^3
+ a^2*b^2) + h*x/b^2 + 1/9*sqrt(3)*(b^2*c*(a/b)^(2/3) + 2*a*b*f*(a/b)^(2/3
) + a*b*e*(a/b)^(1/3) - 4*a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/(a^2*b^2) + 1/18*(6*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)
^(1/3) + 2*a*b*f*(a/b)^(1/3) - a*b*e + 4*a^2*h)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/9*(3*a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^
(1/3) - 2*a*b*f*(a/b)^(1/3) + a*b*e - 4*a^2*h)*log(x + (a/b)^(1/3))/(a*b^3
*(a/b)^(2/3))
```

3.415.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.08

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= \frac{hx}{b^2} + \frac{g\log(|bx^3+a|)}{3\,b^2} - \frac{\sqrt{3}\left(abe-4\,a^2h-(-ab^2)^{\frac{1}{3}}\,bc-2\,(-ab^2)^{\frac{1}{3}}\,af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab} - \frac{\left(abe-4\,a^2h+(-ab^2)^{\frac{1}{3}}\,bc+2\,(-ab^2)^{\frac{1}{3}}\,af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,ab} - \frac{abd-a^2g-(b^2c-abf)x^2+(abe-a^2h)x}{3\,(bx^3+a)ab^2} - \frac{\left(ab^5c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2b^4f\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^4e-4\,a^3b^3h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^5}$$

input
```
integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac
")
```

output
```
h*x/b^2 + 1/3*g*log(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(a*b*e - 4*a^2*h - (
-a*b^2)^(1/3)*b*c - 2*(-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/18*(a*b*e - 4*a^2*h + (-a*b
^2)^(1/3)*b*c + 2*(-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/((-a*b^2)^(2/3)*a*b) - 1/3*(a*b*d - a^2*g - (b^2*c - a*b*f)*x^2 + (a*
b*e - a^2*h)*x)/((b*x^3 + a)*a*b^2) - 1/9*(a*b^5*c*(-a/b)^(1/3) + 2*a^2*b^
4*f*(-a/b)^(1/3) + a^2*b^4*e - 4*a^3*b^3*h)*(-a/b)^(1/3)*log(abs(x - (-a/b
)^(1/3)))/(a^3*b^5)
```

3.415.9 Mupad [B] (verification not implemented)

Time = 10.61 (sec) , antiderivative size = 827, normalized size of antiderivative = 2.86

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^2}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\frac{9\,a^2\,g^2+b^2\,c\,e-8\,a^2\,f\,h-4\,a\,b\,c\,h+2\,a\,b\,e\,f}{9\,a\,b^2} -\mathrm{root}\left(729\,a^4\,b^7\,z^3-729\,a^4\,b^5\,g\,z^2-216\,a^4\,b^3\,f\,h\,z-108\,a^3\,b^4\,c\,h\,z+54\,a^3\,b^4\,e\,f\,z+27\,a^2\,b^5\,c\,e\,z+243\,a^4\,b^3\,g \right.\right.\right.$$
$$\left.\left. + \frac{x\left(12\,g\,h\,a^3+4\,a^2\,b\,f^2-3\,e\,g\,a^2\,b+4\,a\,b^2\,c\,f+b^3\,c^2\right)}{9\,a^2\,b^2}\right)\mathrm{root}\left(729\,a^4\,b^7\,z^3-729\,a^4\,b^5\,g\,z^2 - 216\,a^4\,b^3\,f\,h\,z-108\,a^3\,b^4\,c\,h\,z+54\,a^3\,b^4\,e\,f\,z+27\,a^2\,b^5\,c\,e\,z+243\,a^4\,b^3\,g^2\,z + 72\,a^4\,b\,f\,g\,h+36\,a^3\,b^2\,c\,g\,h-18\,a^3\,b^2\,e\,f\,g-9\,a^2\,b^3\,c\,e\,g-48\,a^4\,b\,e\,h^2+6\,a\,b^4\,c^2\,f + 12\,a^3\,b^2\,e^2\,h+12\,a^2\,b^3\,c\,f^2+8\,a^3\,b^2\,f^3-27\,a^4\,b\,g^3+64\,a^5\,h^3+b^5\,c^3-a^2\,b^3\,e^3,z,k\right)\right)$$
$$-\frac{\frac{b\,d}{3}-\frac{a\,g}{3}+x\left(\frac{b\,e}{3}-\frac{a\,h}{3}\right)-\frac{b\,x^2\,(b\,c-a\,f)}{3\,a}}{b^3\,x^3+a\,b^2}+\frac{h\,x}{b^2}$$

input

```
int((x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^2,x)
```

output

```
symsum(log((9*a^2*g^2 + b^2*c*e - 8*a^2*f*h - 4*a*b*c*h + 2*a*b*e*f)/(9*a*
b^2) - root(729*a^4*b^7*z^3 - 729*a^4*b^5*g*z^2 - 216*a^4*b^3*f*h*z - 108*
a^3*b^4*c*h*z + 54*a^3*b^4*e*f*z + 27*a^2*b^5*c*e*z + 243*a^4*b^3*g^2*z +
72*a^4*b*f*g*h + 36*a^3*b^2*c*g*h - 18*a^3*b^2*e*f*g - 9*a^2*b^3*c*e*g - 4
8*a^4*b*e*h^2 + 6*a*b^4*c^2*f + 12*a^3*b^2*e^2*h + 12*a^2*b^3*c*f^2 + 8*a^
3*b^2*f^3 - 27*a^4*b*g^3 + 64*a^5*h^3 + b^5*c^3 - a^2*b^3*e^3, z, k)*(6*a*
g - b*e*x + 4*a*h*x - 9*root(729*a^4*b^7*z^3 - 729*a^4*b^5*g*z^2 - 216*a^4
*b^3*f*h*z - 108*a^3*b^4*c*h*z + 54*a^3*b^4*e*f*z + 27*a^2*b^5*c*e*z + 243
*a^4*b^3*g^2*z + 72*a^4*b*f*g*h + 36*a^3*b^2*c*g*h - 18*a^3*b^2*e*f*g - 9*
a^2*b^3*c*e*g - 48*a^4*b*e*h^2 + 6*a*b^4*c^2*f + 12*a^3*b^2*e^2*h + 12*a^2
*b^3*c*f^2 + 8*a^3*b^2*f^3 - 27*a^4*b*g^3 + 64*a^5*h^3 + b^5*c^3 - a^2*b^3
*e^3, z, k)*a*b^2) + (x*(b^3*c^2 + 4*a^2*b*f^2 + 12*a^3*g*h + 4*a*b^2*c*f
- 3*a^2*b*e*g))/(9*a^2*b^2))*root(729*a^4*b^7*z^3 - 729*a^4*b^5*g*z^2 - 21
6*a^4*b^3*f*h*z - 108*a^3*b^4*c*h*z + 54*a^3*b^4*e*f*z + 27*a^2*b^5*c*e*z
+ 243*a^4*b^3*g^2*z + 72*a^4*b*f*g*h + 36*a^3*b^2*c*g*h - 18*a^3*b^2*e*f*g
 - 9*a^2*b^3*c*e*g - 48*a^4*b*e*h^2 + 6*a*b^4*c^2*f + 12*a^3*b^2*e^2*h + 1
2*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 27*a^4*b*g^3 + 64*a^5*h^3 + b^5*c^3 - a^
2*b^3*e^3, z, k), k, 1, 3) - ((b*d)/3 - (a*g)/3 + x*((b*e)/3 - (a*h)/3) -
(b*x^2*(b*c - a*f))/(3*a))/(a*b^2 + b^3*x^3) + (h*x)/b^2
```

3.416 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2} dx$

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3.416.2 Mathematica [A] (verified) . . . 3086
3.416.3 Rubi [A] (verified) . . . 3086
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3.416.9 Mupad [B] (verification not implemented) . . . 3095

3.416.1 Optimal result

Integrand size = 35, antiderivative size = 276

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2} dx = \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3ab(a+bx^3)} - \frac{\left(2b^{4/3}c+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{5/3}} + \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{5/3}} - \frac{\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{5/3}} + \frac{h\log(a+bx^3)}{3b^2}$$

output
```
1/3*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/a/b/(b*x^3+a)+1/9*(b^(1/3)*(a*
f+2*b*c)-a^(1/3)*(2*a*g+b*d))*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(5/3)-1/18*(
b^(1/3)*(a*f+2*b*c)-a^(1/3)*(2*a*g+b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/a^(5/3)/b^(5/3)+1/3*h*ln(b*x^3+a)/b^2-1/9*(2*b^(4/3)*c+a^(1/3)*b*
d+a*b^(1/3)*f+2*a^(4/3)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(5/3)/b^(5/3)*3^(1/2)
```

3.416.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$$

$$= \frac{\frac{6(a^2h+b^2x(c+dx)-ab(e+x(f+gx)))}{a(a+bx^3)} - \frac{2\sqrt{3}\sqrt[3]{b}\left(2b^{4/3}c+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2a^{4/3}g\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2\sqrt[3]{b}\left(2b^{4/3}c-\sqrt[3]{a}bd+a\sqrt[3]{b}\right.}{a^{5}}}{18b^2}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```

output
```
((6*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a*(a + b*x^3)) - (
2*Sqrt[3]*b^(1/3)*(2*b^(4/3)*c + a^(1/3)*b*d + a*b^(1/3)*f + 2*a^(4/3)*g)*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*b^(1/3)*(2*b^(4/
3)*c - a^(1/3)*b*d + a*b^(1/3)*f - 2*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/
a^(5/3) + (b^(1/3)*(-2*b^(4/3)*c + a^(1/3)*b*d - a*b^(1/3)*f + 2*a^(4/3)*g
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 6*h*Log[a + b*
x^3])/(18*b^2)
```

3.416.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2397, 25, 2410, 792, 2399, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{ 2397}$$

$$\frac{x(x(bd-ag)+x^2(be-ah)-af+bc)}{3ab(a+bx^3)} - \frac{\int -\frac{3abhx^2+b(bd+2ag)x+b(2bc+af)}{bx^3+a}dx}{3ab^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3abhx^2+b(bd+2ag)x+b(2bc+af)}{bx^3+a}dx}{3ab^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 2410

$$\frac{\int \frac{b(2bc+af)+b(bd+2ag)x}{bx^3+a}dx+3abh\int \frac{x^2}{bx^3+a}dx}{3ab^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 792

$$\frac{\int \frac{b(2bc+af)+b(bd+2ag)x}{bx^3+a}dx+ah\log\left(a+bx^3\right)}{3ab^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 2399

$$\frac{\frac{\int \frac{b\left(\sqrt[3]{a}\left(2\sqrt[3]{b}(2bc+af)+\sqrt[3]{a}(bd+2ag)\right)-\sqrt[3]{b}\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{b^{2/3}\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}+a}{3ab^2}$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 16

$$\frac{\frac{\int \frac{b\left(\sqrt[3]{a}\left(2\sqrt[3]{b}(2bc+af)+\sqrt[3]{a}(bd+2ag)\right)-\sqrt[3]{b}\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{3a^{2/3}}+}{3ab^2}$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 27

$$\frac{\frac{b^{2/3}\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(2bc+af)+\sqrt[3]{a}(bd+2ag)\right)-\sqrt[3]{b}\left(\sqrt[3]{b}(2bc+af)-\sqrt[3]{a}(bd+2ag)\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}(af+2bc)-\sqrt[3]{a}(2ag+bd)\right)}{3a^{2/3}}+}{3ab^2}$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

$\big\downarrow$ 1142

3.416. $\displaystyle\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^3\right)^2}\,dx$

$$\frac{\frac{b^{2/3}\left(\frac{3}{2}\sqrt[3]{a}\left(2a^{4/3}g+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(-\frac{\sqrt[3]{a}(2ag+bd)}{\sqrt[3]{b}}+af+2bc\right)\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx\right)}{3a^{2/3}}+}{3ab^2}$$
$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 25

$$\frac{\frac{b^{2/3}\left(\frac{3}{2}\sqrt[3]{a}\left(2a^{4/3}g+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(-\frac{\sqrt[3]{a}(2ag+bd)}{\sqrt[3]{b}}+af+2bc\right)\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx\right)}{3a^{2/3}}+}{3ab^2}$$
$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 27

$$\frac{\frac{b^{2/3}\left(\frac{3}{2}\sqrt[3]{a}\left(2a^{4/3}g+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(-\frac{\sqrt[3]{a}(2ag+bd)}{\sqrt[3]{b}}+af+2bc\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx\right)}{3a^{2/3}}}{3ab^2}$$
$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 1082

$$\frac{\frac{b^{2/3}\left(\frac{1}{2}\sqrt[3]{b}\left(-\frac{\sqrt[3]{a}(2ag+bd)}{\sqrt[3]{b}}+af+2bc\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(2a^{4/3}g+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}\right)}{3a^{2/3}}}{3ab^2}$$
$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 217

3.416. $\int\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^3\right)^2}\,dx$

$$\frac{\frac{b^{2/3}\left(\frac{1}{2}\sqrt[3]{b}\left(-\frac{\sqrt[3]{a}(2ag+bd)}{\sqrt[3]{b}}+af+2bc\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2a^{4/3}g+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)}{\sqrt[3]{b}}\right)}{3a^{2/3}}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{\cdots}\right.}{\cdots}}{3ab^2}$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{b^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2a^{4/3}g+\sqrt[3]{a}bd+a\sqrt[3]{b}f+2b^{4/3}c\right)}{\sqrt[3]{b}}-\frac{1}{2}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(2ag+bd)}{\sqrt[3]{b}}+af+2bc\right)\right)}{3a^{2/3}}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{\cdots}\right.}{\cdots}}{3ab^2}$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3ab\left(a+bx^3\right)}$$

input

```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x]
```

output

```
(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(3*a*b*(a + b*x^3)) + ((
b^(1/3)*(b^(1/3)*(2*b*c + a*f) - a^(1/3)*(b*d + 2*a*g))*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)) + (b^(2/3)*(-((Sqrt[3]*(2*b^(4/3)*c + a^(1/3)*b*d + a*
b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1
/3)) - ((2*b*c + a*f - (a^(1/3)*(b*d + 2*a*g))/b^(1/3))*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/2))/(3*a^(2/3)) + a*h*Log[a + b*x^3])/(3*a*b
^2)
```

3.416. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$

3.416.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.416. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2} dx$

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.416.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.43

method	result
risch	$\frac{-\frac{(ag-bd)x^2}{3ab}-\frac{(af-bc)x}{3ab}+\frac{ah-be}{3b^2}}{b\,x^3+a}+\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)}\frac{\left(3h\_R^2+\frac{(2ag+bd)\_R}{a}+\frac{af+2bc}{a}\right)\ln\left(x-\_R\right)}{\_R^2}}{9b^2}$
default	$\frac{-\frac{(ag-bd)x^2}{3ab}-\frac{(af-bc)x}{3ab}+\frac{ah-be}{3b^2}}{b\,x^3+a}+\frac{(af+2bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(2ag+bd)}{3ba}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*(a*g-b*d)/a/b*x^2-1/3*(a*f-b*c)/a/b*x+1/3*(a*h-b*e)/b^2)/(b*x^3+a)+1
/9/b^2*sum((3*h*_R^2+1/a*(2*a*g+b*d)*_R+(a*f+2*b*c)/a)/_R^2*ln(x-_R),_R=Ro
otOf(_Z^3*b+a))
```

3.416.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 12636, normalized size of antiderivative = 45.78

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

3.416.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.416.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.06

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$$
$$= -\frac{abe-a^2h-(b^2d-abg)x^2-(b^2c-abf)x}{3\,(ab^3x^3+a^2b^2)} + \frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2} + \frac{\left(6\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,bc-af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(3\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,bc+af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="maxima
")
```

output
```
-1/3*(a*b*e - a^2*h - (b^2*d - a*b*g)*x^2 - (b^2*c - a*b*f)*x)/(a*b^3*x^3
+ a^2*b^2) + 1/9*sqrt(3)*(b^2*d*(a/b)^(2/3) + 2*a*b*g*(a/b)^(2/3) + 2*b^2*
c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a^2*b^2) + 1/18*(6*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/3) + 2*a*g
*(a/b)^(1/3) - 2*b*c - a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*
(a/b)^(2/3)) + 1/9*(3*a*h*(a/b)^(2/3) - b*d*(a/b)^(1/3) - 2*a*g*(a/b)^(1/3
) + 2*b*c + a*f)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

3.416.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.09

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$$
$$= \frac{h\log\left(|bx^3+a|\right)}{3\,b^2} - \frac{\sqrt{3}\left(2\,b^2c+abf-(-ab^2)^{\frac{1}{3}}\,bd-2\,(-ab^2)^{\frac{1}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab} - \frac{\left(2\,b^2c+abf+(-ab^2)^{\frac{1}{3}}\,bd+2\,(-ab^2)^{\frac{1}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,ab} + \frac{(bd-ag)x^2+(bc-af)x-\frac{abe-a^2h}{b}}{3\,(bx^3+a)ab} - \frac{\left(ab^3d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2b^2g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^3c+a^2b^2f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3b^3}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*h*log(abs(b*x^3 + a))/b^2 - 1/9*sqrt(3)*(2*b^2*c + a*b*f - (-a*b^2)^(1
/3)*b*d - 2*(-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/18*(2*b^2*c + a*b*f + (-a*b^2)^(1/3)*
b*d + 2*(-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*
b^2)^(2/3)*a*b) + 1/3*((b*d - a*g)*x^2 + (b*c - a*f)*x - (a*b*e - a^2*h)/b
)/((b*x^3 + a)*a*b) - 1/9*(a*b^3*d*(-a/b)^(1/3) + 2*a^2*b^2*g*(-a/b)^(1/3)
 + 2*a*b^3*c + a^2*b^2*f)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^3
)
```

3.416.9 Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 835, normalized size of antiderivative = 3.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^2}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\frac{\mathrm{root}(729\,a^5\,b^6\,z^3-729\,a^5\,b^4\,h\,z^2+54\,a^4\,b^3\,f\,g\,z+108\,a^3\,b^4\,c\,g\,z+27\,a^3\,b^4\,d\,f\,z+54\,a^2\,b^5\,c\,d\,z+\right.\right.\right.$$
$$+\frac{9\,a^3\,h^2+2\,b^3\,c\,d+4\,a\,b^2\,c\,g+a\,b^2\,d\,f+2\,a^2\,b\,f\,g}{9\,a^2\,b^2}$$
$$\left.+\frac{x\,(4\,a^2\,g^2-3\,f\,h\,a^2+4\,a\,b\,d\,g-6\,c\,h\,a\,b+b^2\,d^2)}{9\,a^2\,b}\right)\mathrm{root}\big(729\,a^5\,b^6\,z^3-729\,a^5\,b^4\,h\,z^2$$
$$+54\,a^4\,b^3\,f\,g\,z+108\,a^3\,b^4\,c\,g\,z+27\,a^3\,b^4\,d\,f\,z+54\,a^2\,b^5\,c\,d\,z+243\,a^5\,b^2\,h^2\,z$$
$$-18\,a^4\,b\,f\,g\,h-36\,a^3\,b^2\,c\,g\,h-9\,a^3\,b^2\,d\,f\,h-18\,a^2\,b^3\,c\,d\,h-12\,a\,b^4\,c^2\,f+12\,a^3\,b^2\,d\,g^2$$
$$\left.+6\,a^2\,b^3\,d^2\,g-6\,a^2\,b^3\,c\,f^2+8\,a^4\,b\,g^3+a\,b^4\,d^3-27\,a^5\,h^3-8\,b^5\,c^3-a^3\,b^2\,f^3,z,k\big)\right)$$
$$+\frac{\frac{x\,(b\,c-a\,f)}{3\,a\,b}-\frac{b\,e-a\,h}{3\,b^2}+\frac{x^2\,(b\,d-a\,g)}{3\,a\,b}}{b\,x^3+a}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^2,x)
```

output
```
symsum(log((root(729*a^5*b^6*z^3 - 729*a^5*b^4*h*z^2 + 54*a^4*b^3*f*g*z +
108*a^3*b^4*c*g*z + 27*a^3*b^4*d*f*z + 54*a^2*b^5*c*d*z + 243*a^5*b^2*h^2*
z - 18*a^4*b*f*g*h - 36*a^3*b^2*c*g*h - 9*a^3*b^2*d*f*h - 18*a^2*b^3*c*d*h
 - 12*a*b^4*c^2*f + 12*a^3*b^2*d*g^2 + 6*a^2*b^3*d^2*g - 6*a^2*b^3*c*f^2 +
 8*a^4*b*g^3 + a*b^4*d^3 - 27*a^5*h^3 - 8*b^5*c^3 - a^3*b^2*f^3, z, k)*(9*
root(729*a^5*b^6*z^3 - 729*a^5*b^4*h*z^2 + 54*a^4*b^3*f*g*z + 108*a^3*b^4*
c*g*z + 27*a^3*b^4*d*f*z + 54*a^2*b^5*c*d*z + 243*a^5*b^2*h^2*z - 18*a^4*b
*f*g*h - 36*a^3*b^2*c*g*h - 9*a^3*b^2*d*f*h - 18*a^2*b^3*c*d*h - 12*a*b^4*
c^2*f + 12*a^3*b^2*d*g^2 + 6*a^2*b^3*d^2*g - 6*a^2*b^3*c*f^2 + 8*a^4*b*g^3
 + a*b^4*d^3 - 27*a^5*h^3 - 8*b^5*c^3 - a^3*b^2*f^3, z, k)*a^2*b^2 - 6*a^2
*h + 2*b^2*c*x + a*b*f*x))/a + (9*a^3*h^2 + 2*b^3*c*d + 4*a*b^2*c*g + a*b^
2*d*f + 2*a^2*b*f*g)/(9*a^2*b^2) + (x*(b^2*d^2 + 4*a^2*g^2 - 3*a^2*f*h - 6
*a*b*c*h + 4*a*b*d*g))/(9*a^2*b))*root(729*a^5*b^6*z^3 - 729*a^5*b^4*h*z^2
 + 54*a^4*b^3*f*g*z + 108*a^3*b^4*c*g*z + 27*a^3*b^4*d*f*z + 54*a^2*b^5*c*
d*z + 243*a^5*b^2*h^2*z - 18*a^4*b*f*g*h - 36*a^3*b^2*c*g*h - 9*a^3*b^2*d*
f*h - 18*a^2*b^3*c*d*h - 12*a*b^4*c^2*f + 12*a^3*b^2*d*g^2 + 6*a^2*b^3*d^2
*g - 6*a^2*b^3*c*f^2 + 8*a^4*b*g^3 + a*b^4*d^3 - 27*a^5*h^3 - 8*b^5*c^3 -
a^3*b^2*f^3, z, k), k, 1, 3) + ((x*(b*c - a*f))/(3*a*b) - (b*e - a*h)/(3*b
^2) + (x^2*(b*d - a*g))/(3*a*b))/(a + b*x^3)
```

3.417 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx$

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3.417.1 Optimal result

Integrand size = 38, antiderivative size = 289

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx &= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{3a^2b(a+bx^3)} \\ &- \frac{\left(2b^{4/3}d+\sqrt[3]{a}be+a\sqrt[3]{b}g+2a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{5/3}} \\ &+ \frac{c\log(x)}{a^2}+\frac{\left(\sqrt[3]{b}(2bd+ag)-\sqrt[3]{a}(be+2ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{5/3}} \\ &- \frac{\left(\sqrt[3]{b}(2bd+ag)-\sqrt[3]{a}(be+2ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{5/3}}-\frac{c\log(a+bx^3)}{3a^2}\end{aligned}$$

output
```
1/3*x*(a*(-a*g+b*d)+a*(-a*h+b*e)*x-b*(-a*f+b*c)*x^2)/a^2/b/(b*x^3+a)+c*ln(
x)/a^2+1/9*(b^(1/3)*(a*g+2*b*d)-a^(1/3)*(2*a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)
/a^(5/3)/b^(5/3)-1/18*(b^(1/3)*(a*g+2*b*d)-a^(1/3)*(2*a*h+b*e))*ln(a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(5/3)-1/3*c*ln(b*x^3+a)/a^2-1/9*
(2*b^(4/3)*d+a^(1/3)*b*e+a*b^(1/3)*g+2*a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(5/3)*3^(1/2)
```

3.417.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{6a(-b(c+x(d+ex))+a(f+x(g+hx)))}{b(a+bx^3)} - \frac{2\sqrt{3}\sqrt[3]{a}\left(2b^{4/3}d+\sqrt[3]{a}be+a\sqrt[3]{b}g+2a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} + 18c\log(x) + \frac{2\sqrt[3]{a}\left(2b\right.}{}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x]
```

output
```
((-6*a*(-(b*(c + x*(d + e*x)) + a*(f + x*(g + h*x))))/(b*(a + b*x^3)) - (
2*Sqrt[3]*a^(1/3)*(2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h)*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) + 18*c*Log[x] + (2*a^
(1/3)*(2*b^(4/3)*d - a^(1/3)*b*e + a*b^(1/3)*g - 2*a^(4/3)*h)*Log[a^(1/3)
+ b^(1/3)*x])/b^(5/3) + (a^(1/3)*(-2*b^(4/3)*d + a^(1/3)*b*e - a*b^(1/3)*g
 + 2*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) -
6*c*Log[a + b*x^3])/(18*a^2)
```

3.417.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \text{ 2368}$$

$$\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{3a^2b\left(a+bx^3\right)}-\frac{\int -\frac{3cb^2+(be+2ah)x^2b+(2bd+ag)xb}{x(bx^3+a)}dx}{3ab^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3cb^2+(be+2ah)x^2b+(2bd+ag)xb}{x(bx^3+a)}dx}{3ab^2}+\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{3a^2b\left(a+bx^3\right)}$$

$\downarrow$ 2373

$$\frac{\int \left(\frac{3cb^2}{ax}+\frac{(-3b^2cx^2+a(be+2ah)x+a(2bd+ag))b}{a(bx^3+a)}\right)dx}{3ab^2}+\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{3a^2b\left(a+bx^3\right)}$$

$\downarrow$ 2009

$$\frac{-\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{4/3}h+\sqrt[3]{a}be+a\sqrt[3]{b}g+2b^{4/3}d\right)}{\sqrt{3}a^{2/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(2ah+be)}{\sqrt[3]{b}}+ag+2bd\right)}{6a^{2/3}}+\frac{\sqrt[3]{b}\mathrm{lo}}{}}{3ab^2}$$
$$\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{3a^2b\left(a+bx^3\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x]
```

output
```
(x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(3*a^2*b*(a + b*
x^3)) + (-((b^(1/3)*(2*b^(4/3)*d + a^(1/3)*b*e + a*b^(1/3)*g + 2*a^(4/3)*h
)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3))) +
(3*b^2*c*Log[x])/a + (b^(1/3)*(b^(1/3)*(2*b*d + a*g) - a^(1/3)*(b*e + 2*a*
h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)) - (b^(2/3)*(2*b*d + a*g - (a^(1/
3)*(b*e + 2*a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(6*a^(2/3)) - (b^2*c*Log[a + b*x^3])/a)/(3*a*b^2)
```

3.417.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.417.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.01

method	result
default	$\frac{c\ln(x)}{a^2}+\frac{\frac{-\frac{a(ah-be)x^2}{3b}-\frac{a(ag-bd)x}{3b}-\frac{a(af-bc)}{3b}}{b\,x^3+a}+(a^2g+2abd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{a^2}$
risch	$\frac{-\frac{(ah-be)x^2}{3ab}-\frac{(ag-bd)x}{3ab}-\frac{af-bc}{3ab}}{b\,x^3+a}+\frac{c\ln(-x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^6b^5\_Z^3+9a^4b^5c\_Z^2+\left(6a^5b^2gh+12a^4b^3dh+3a^4b^3eg+6a^3b^4de+27a^2b^5\right.\right.}\right.}{}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c*ln(x)/a^2+1/a^2*((-1/3*a*(a*h-b*e)/b*x^2-1/3*a*(a*g-b*d)/b*x-1/3*a*(a*f-
b*c)/b)/(b*x^3+a)+1/3/b*((a^2*g+2*a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/
3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(2*a^2*h+a*b*e)*(-1/3/b/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-
b*c*ln(b*x^3+a)))
```

3.417.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.33 (sec) , antiderivative size = 12541, normalized size of antiderivative = 43.39

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="fric
as")
```

output
```
Too large to include
```

3.417.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.417.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx$$
$$= \frac{(be-ah)x^2+bc-af+(bd-ag)x}{3(ab^2x^3+a^2b)} + \frac{c\log(x)}{a^2} + \frac{\sqrt{3}\left(abe\left(\frac{a}{b}\right)^{\frac{2}{3}}+2a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+2abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^3b} - \frac{\left(6b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-2a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+2abd+a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(3b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+2a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-2abd-a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxi
ma")
```

output
```
1/3*((b*e - a*h)*x^2 + b*c - a*f + (b*d - a*g)*x)/(a*b^2*x^3 + a^2*b) + c*
log(x)/a^2 + 1/9*sqrt(3)*(a*b*e*(a/b)^(2/3) + 2*a^2*h*(a/b)^(2/3) + 2*a*b*
d*(a/b)^(1/3) + a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a^3*b) - 1/18*(6*b^2*c*(a/b)^(2/3) - a*b*e*(a/b)^(1/3) - 2*a
^2*h*(a/b)^(1/3) + 2*a*b*d + a^2*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a^2*b^2*(a/b)^(2/3)) - 1/9*(3*b^2*c*(a/b)^(2/3) + a*b*e*(a/b)^(1/3) + 2*
a^2*h*(a/b)^(1/3) - 2*a*b*d - a^2*g)*log(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(
2/3))
```

3.417.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.09

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^2}\,dx$$
$$= -\frac{c\log(|bx^3+a|)}{3a^2} + \frac{c\log(|x|)}{a^2} - \frac{\sqrt{3}\left(2b^2d+abg-(-ab^2)^{\frac{1}{3}}be-2(-ab^2)^{\frac{1}{3}}ah\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9(-ab^2)^{\frac{2}{3}}ab} - \frac{\left(2b^2d+abg+(-ab^2)^{\frac{1}{3}}be+2(-ab^2)^{\frac{1}{3}}ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18(-ab^2)^{\frac{2}{3}}ab} + \frac{abc-a^2f+(abe-a^2h)x^2+(abd-a^2g)x}{3(bx^3+a)a^2b} - \frac{\left(a^3b^3e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2a^4b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2a^3b^3d+a^4b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^5b^3}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac
")
```

output
```
-1/3*c*log(abs(b*x^3 + a))/a^2 + c*log(abs(x))/a^2 - 1/9*sqrt(3)*(2*b^2*d
+ a*b*g - (-a*b^2)^(1/3)*b*e - 2*(-a*b^2)^(1/3)*a*h)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/18*(2*b^2*d + a*
b*g + (-a*b^2)^(1/3)*b*e + 2*(-a*b^2)^(1/3)*a*h)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) + 1/3*(a*b*c - a^2*f + (a*b*e - a^2*h
)*x^2 + (a*b*d - a^2*g)*x)/((b*x^3 + a)*a^2*b) - 1/9*(a^3*b^3*e*(-a/b)^(1/
3) + 2*a^4*b^2*h*(-a/b)^(1/3) + 2*a^3*b^3*d + a^4*b^2*g)*(-a/b)^(1/3)*log(
abs(x - (-a/b)^(1/3)))/(a^5*b^3)
```

3.417.9 Mupad [B] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 1660, normalized size of antiderivative = 5.74

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^2}\,dx=\text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^2),x)
```

output
```
((b*c - a*f)/(3*a*b) + (x*(b*d - a*g))/(3*a*b) + (x^2*(b*e - a*h))/(3*a*b)
)/(a + b*x^3) + symsum(log((c*(4*b^2*d^2 + a^2*g^2 - 3*b^2*c*e - 6*a*b*c*h
 + 4*a*b*d*g))/(9*a^3) - (root(729*a^6*b^5*z^3 + 729*a^4*b^5*c*z^2 + 54*a^
5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3*b^4*d*e*z + 24
3*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2*c*g*h + 36*a^2*b^3*c*d*h + 9
*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e^2*h - 12*a^2*b^3*d^2*g - 6*a
^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^5*h^3 + 27*b^5*c^3 + a^2*b^3*
e^3, z, k)*(a^3*g^2 + 4*a*b^2*d^2 + 36*b^3*c^2*x + 324*root(729*a^6*b^5*z^
3 + 729*a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*
e*g*z + 54*a^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b^2
*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2*e
^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*a^
5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z, k)^2*a^4*b^3*x - 18*root(729*a^6*b^5*
z^3 + 729*a^4*b^5*c*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^
3*e*g*z + 54*a^3*b^4*d*e*z + 243*a^2*b^5*c^2*z + 18*a*b^4*c*d*e + 18*a^3*b
^2*c*g*h + 36*a^2*b^3*c*d*h + 9*a^2*b^3*c*e*g + 12*a^4*b*e*h^2 + 6*a^3*b^2
*e^2*h - 12*a^2*b^3*d^2*g - 6*a^3*b^2*d*g^2 - a^4*b*g^3 - 8*a*b^4*d^3 + 8*
a^5*h^3 + 27*b^5*c^3 + a^2*b^3*e^3, z, k)*a^4*b*h + 6*a*b^2*c*e + 12*a^2*b
*c*h + 4*a^2*b*d*g + 20*a^3*g*h*x - 9*root(729*a^6*b^5*z^3 + 729*a^4*b^5*c
*z^2 + 54*a^5*b^2*g*h*z + 108*a^4*b^3*d*h*z + 27*a^4*b^3*e*g*z + 54*a^3...
```

3.418 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2}\,dx$

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3.418.1 Optimal result

Integrand size = 38, antiderivative size = 301

$$\begin{aligned}&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2}\,dx\\&=-\frac{c}{a^2x}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{3a^2b(a+bx^3)}\\&\quad+\frac{\left(4b^{5/3}c-2a^{2/3}be-ab^{2/3}f-a^{5/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{4/3}}\\&\quad+\frac{d\log(x)}{a^2}+\frac{\left(b^{2/3}(4bc-af)+a^{2/3}(2be+ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{4/3}}\\&\quad-\frac{\left(b^{2/3}(4bc-af)+a^{2/3}(2be+ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{4/3}}-\frac{d\log(a+bx^3)}{3a^2}\end{aligned}$$

output
```
-c/a^2/x+1/3*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/a^2/b/(b*x^3
+a)+d*ln(x)/a^2+1/9*(b^(2/3)*(-a*f+4*b*c)+a^(2/3)*(a*h+2*b*e))*ln(a^(1/3)+
b^(1/3)*x)/a^(7/3)/b^(4/3)-1/18*(b^(2/3)*(-a*f+4*b*c)+a^(2/3)*(a*h+2*b*e))
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(4/3)-1/3*d*ln(b*x^3+
a)/a^2+1/9*(4*b^(5/3)*c-2*a^(2/3)*b*e-a*b^(2/3)*f-a^(5/3)*h)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(4/3)*3^(1/2)
```

3.418.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2}\,dx =$$

$$-\frac{18ac}{x}+\frac{6a(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{b(a+bx^3)}+\frac{2\sqrt{3}a^{2/3}(-4b^{5/3}c+2a^{2/3}be+ab^{2/3}f+a^{5/3}h)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}}-18ad\log(x)$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x]
```

output
```
-1/18*((18*a*c)/x + (6*a*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*x)
)))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-4*b^(5/3)*c + 2*a^(2/3)*b*e + a
*b^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(4/
3) - 18*a*d*Log[x] - (2*a^(2/3)*(4*b^(5/3)*c + 2*a^(2/3)*b*e - a*b^(2/3)*f
 + a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(2/3)*(4*b^(5/3)*c +
2*a^(2/3)*b*e - a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/b^(4/3) + 6*a*d*Log[a + b*x^3])/a^3
```

3.418.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{3a^2b(a+bx^3)}-\frac{\int -\frac{-b^2\left(\frac{bc}{a}-f\right)x^3+b(2be+ah)x^2+3b^2dx+3b^2c}{x^2(bx^3+a)}dx}{3ab^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-b^2\left(\frac{bc}{a}-f\right)x^3+b(2be+ah)x^2+3b^2dx+3b^2c}{x^2(bx^3+a)}dx}{3ab^2}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3a^2b\,(a+bx^3)}$$

$\Big\downarrow$ 2373

$$\frac{\int \left(\frac{3db^2}{ax}+\frac{3cb^2}{ax^2}+\frac{\left(-3b^2dx^2-b(4bc-af)x+a(2be+ah)\right)b}{a(bx^3+a)}\right)dx}{3ab^2}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3a^2b\,(a+bx^3)}$$

$\Big\downarrow$ 2009

$$\frac{\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{2/3}be+a^{5/3}(-h)-ab^{2/3}f+4b^{5/3}c\right)}{\sqrt{3}a^{4/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+2be)+b^{2/3}(4bc-af)\right)}{6a^{4/3}}+\ldots}{3ab^2}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{3a^2b\,(a+bx^3)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x]
```

output
```
(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(3*a^2*b*(a + b*
x^3)) + ((-3*b^2*c)/(a*x) + (b^(2/3)*(4*b^(5/3)*c - 2*a^(2/3)*b*e - a*b^(2
/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqr
t[3]*a^(4/3)) + (3*b^2*d*Log[x])/a + (b^(2/3)*(b^(2/3)*(4*b*c - a*f) + a^(
2/3)*(2*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)) - (b^(2/3)*(b^(2
/3)*(4*b*c - a*f) + a^(2/3)*(2*b*e + a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2])/(6*a^(4/3)) - (b^2*d*Log[a + b*x^3])/a)/(3*a*b^2)
```

3.418.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.418.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.99

method	result
default	$-\frac{c}{a^2 x}+\frac{d\ln(x)}{a^2}+\frac{\frac{\left(\frac{af}{3}-\frac{bc}{3}\right)x^2-\frac{a(ah-be)x}{3b}-\frac{a(ag-bd)}{3b}}{b\,x^3+a}+\frac{\left(a^2h+2aeb\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\sqrt{3}\left(\cdots\right.\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}\cdots}\right.}{\cdots}}{\cdots}$
risch	$\frac{\frac{(af-4bc)x^3}{3a^2}-\frac{(ah-be)x^2}{3ab}-\frac{(ag-bd)x}{3ab}-\frac{c}{a}}{x(b\,x^3+a)}+\frac{d\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^7b^4\_Z^3+9a^5b^4d\_Z^2+\left(3a^5b^2fh-12a^4b^3ch+6a^4b^3ef-24a^3b^4ce+\cdots\right.\right.}\cdots\right.}{\cdots}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOS
E)
```

output
```
-c/a^2/x+d*ln(x)/a^2+1/a^2*(((1/3*a*f-1/3*b*c)*x^2-1/3*a*(a*h-b*e)/b*x-1/3
*a*(a*g-b*d)/b)/(b*x^3+a)+1/3/b*((a^2*h+2*a*b*e)*(1/3/b/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(a*b*f-4*b^2*c)*(
-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))-b*d*ln(b*x^3+a)))
```

3.418.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.13 (sec) , antiderivative size = 12556, normalized size of antiderivative = 41.71

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.418.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.418.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.09

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2}\,dx$$
$$= -\frac{(4\,b^2c-abf)x^3+3\,abc-(abe-a^2h)x^2-(abd-a^2g)x}{3\,(a^2b^2x^4+a^3bx)}+\frac{d\log(x)}{a^2}$$
$$-\frac{\sqrt{3}\left(4\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b}$$
$$-\frac{\left(6\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abe+a^2h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(3\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abe-a^2h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="ma
xima")
```

output
```
-1/3*((4*b^2*c - a*b*f)*x^3 + 3*a*b*c - (a*b*e - a^2*h)*x^2 - (a*b*d - a^2
*g)*x)/(a^2*b^2*x^4 + a^3*b*x) + d*log(x)/a^2 - 1/9*sqrt(3)*(4*b^2*c*(a/b)
^(2/3) - a*b*f*(a/b)^(2/3) - 2*a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3))*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b) - 1/18*(6*b^2*d*(a
/b)^(2/3) + 4*b^2*c*(a/b)^(1/3) - a*b*f*(a/b)^(1/3) + 2*a*b*e + a^2*h)*log
(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) - 1/9*(3*b^2*d*(
a/b)^(2/3) - 4*b^2*c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3) - 2*a*b*e - a^2*h)*lo
g(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

3.418.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^2}\,dx$$
$$= -\frac{d\log(|bx^3+a|)}{3a^2}+\frac{d\log(|x|)}{a^2}$$
$$-\frac{\sqrt{3}\left(2abe+a^2h+4(-ab^2)^{\frac{1}{3}}bc-(-ab^2)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9(-ab^2)^{\frac{2}{3}}a^2}$$
$$-\frac{\left(2abe+a^2h-4(-ab^2)^{\frac{1}{3}}bc+(-ab^2)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18(-ab^2)^{\frac{2}{3}}a^2}$$
$$-\frac{4b^2cx^3-abfx^3-abex^2+a^2hx^2-abdx+a^2gx+3abc}{3(bx^4+ax)a^2b}$$
$$+\frac{\left(4a^2b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2a^3b^3e-a^4b^2h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^5b^3}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="gi
ac")
```

output
```
-1/3*d*log(abs(b*x^3 + a))/a^2 + d*log(abs(x))/a^2 - 1/9*sqrt(3)*(2*a*b*e
+ a^2*h + 4*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/18*(2*a*b*e + a^
2*h - 4*(-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/3*(4*b^2*c*x^3 - a*b*f*x^3 - a*b*
e*x^2 + a^2*h*x^2 - a*b*d*x + a^2*g*x + 3*a*b*c)/((b*x^4 + a*x)*a^2*b) + 1
/9*(4*a^2*b^4*c*(-a/b)^(1/3) - a^3*b^3*f*(-a/b)^(1/3) - 2*a^3*b^3*e - a^4*
b^2*h)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*b^3)
```

3.418.9 Mupad [B] (verification not implemented)

Time = 10.01 (sec) , antiderivative size = 1684, normalized size of antiderivative = 5.59

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^2),x)
```

output
```
symsum(log((d*(a^3*h^2 + 4*a*b^2*e^2 + 12*b^3*c*d - 3*a*b^2*d*f + 4*a^2*b*
e*h))/(9*a^4) - (root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f*h
*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b^
4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3
*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*
f^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z
, k)*(a^3*h^2 + 4*a*b^2*e^2 + 36*b^3*d^2*x - 24*b^3*c*d + 324*root(729*a^7
*b^4*z^3 + 729*a^5*b^4*d*z^2 + 27*a^5*b^2*f*h*z - 108*a^4*b^3*c*h*z + 54*a
^4*b^3*e*f*z - 216*a^3*b^4*c*e*z + 243*a^3*b^4*d^2*z - 72*a*b^4*c*d*e + 9*
a^3*b^2*d*f*h - 36*a^2*b^3*c*d*h + 18*a^2*b^3*d*e*f - 6*a^4*b*e*h^2 + 48*a
*b^4*c^2*f - 12*a^3*b^2*e^2*h - 12*a^2*b^3*c*f^2 - 8*a^2*b^3*e^3 + 27*a*b^
4*d^3 - a^5*h^3 - 64*b^5*c^3 + a^3*b^2*f^3, z, k)^2*a^4*b^3*x + 6*a*b^2*d*
f + 4*a^2*b*e*h - 80*b^3*c*e*x + 36*root(729*a^7*b^4*z^3 + 729*a^5*b^4*d*z
^2 + 27*a^5*b^2*f*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b^4
*c*e*z + 243*a^3*b^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^2*b^3
*c*d*h + 18*a^2*b^3*d*e*f - 6*a^4*b*e*h^2 + 48*a*b^4*c^2*f - 12*a^3*b^2*e^
2*h - 12*a^2*b^3*c*f^2 - 8*a^2*b^3*e^3 + 27*a*b^4*d^3 - a^5*h^3 - 64*b^5*c
^3 + a^3*b^2*f^3, z, k)*a^2*b^3*c - 9*root(729*a^7*b^4*z^3 + 729*a^5*b^4*d
*z^2 + 27*a^5*b^2*f*h*z - 108*a^4*b^3*c*h*z + 54*a^4*b^3*e*f*z - 216*a^3*b
^4*c*e*z + 243*a^3*b^4*d^2*z - 72*a*b^4*c*d*e + 9*a^3*b^2*d*f*h - 36*a^...
```

3.419 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2} dx$

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3.419.1 Optimal result

Integrand size = 38, antiderivative size = 306

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2} dx \\
&= -\frac{c}{2a^2x^2} - \frac{d}{a^2x} - \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{3a^2(a+bx^3)} \\
&\quad + \frac{\left(5b^{4/3}c+4\sqrt[3]{a}bd-2a\sqrt[3]{b}f-a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}b^{2/3}} \\
&\quad + \frac{e\log(x)}{a^2} - \frac{\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}b^{2/3}} \\
&\quad + \frac{\left(\sqrt[3]{b}(5bc-2af)-\sqrt[3]{a}(4bd-ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}b^{2/3}} - \frac{e\log(a+bx^3)}{3a^2}
\end{aligned}$$

output
```
-1/2*c/a^2/x^2-d/a^2/x-1/3*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/a^2/(b*
x^3+a)+e*ln(x)/a^2-1/9*(b^(1/3)*(-2*a*f+5*b*c)-a^(1/3)*(-a*g+4*b*d))*ln(a^
(1/3)+b^(1/3)*x)/a^(8/3)/b^(2/3)+1/18*(b^(1/3)*(-2*a*f+5*b*c)-a^(1/3)*(-a*
g+4*b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(2/3)-1/3*e*
ln(b*x^3+a)/a^2+1/9*(5*b^(4/3)*c+4*a^(1/3)*b*d-2*a*b^(1/3)*f-a^(4/3)*g)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(2/3)*3^(1/2)
```

3.419.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^2}\,dx =$$

$$-\frac{\frac{9ac}{x^2}+\frac{18ad}{x}+\frac{6a\left(a^2h+b^2x(c+dx)-ab(e+x(f+gx))\right)}{b(a+bx^3)}+\frac{2\sqrt{3}\sqrt[3]{a}\left(-5b^{4/3}c-4\sqrt[3]{a}bd+2a\sqrt[3]{b}f+a^{4/3}g\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}-18a}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x]
```

output
```
-1/18*((9*a*c)/x^2 + (18*a*d)/x + (6*a*(a^2*h + b^2*x*(c + d*x) - a*b*(e +
 x*(f + g*x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(1/3)*(-5*b^(4/3)*c - 4*a^(
1/3)*b*d + 2*a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]])/b^(2/3) - 18*a*e*Log[x] + (2*a^(1/3)*(5*b^(4/3)*c - 4*a^(1/3)*b*d
 - 2*a*b^(1/3)*f + a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (a^(1/3)
*(5*b^(4/3)*c - 4*a^(1/3)*b*d - 2*a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 6*a*e*Log[a + b*x^3])/a^3
```

3.419.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \text{ 2368}$$

$$-\frac{\int -\frac{-b^2\left(\frac{bd}{a}-g\right)x^4-2b^2\left(\frac{bc}{a}-f\right)x^3+3b^2ex^2+3b^2dx+3b^2c}{x^3(bx^3+a)}dx}{3ab^2}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3a^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-b^2\left(\frac{bd}{a}-g\right)x^4-2b^2\left(\frac{bc}{a}-f\right)x^3+3b^2ex^2+3b^2dx+3b^2c}{x^3(bx^3+a)}dx}{3ab^2}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3a^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 2373$$

$$\frac{\int \left(\frac{3eb^2}{ax}+\frac{\left(-3bex^2-(4bd-ag)x-5bc+2af\right)b^2}{a(bx^3+a)}+\frac{3db^2}{ax^2}+\frac{3cb^2}{ax^3}\right)dx}{3ab^2}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3a^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-g)+4\sqrt[3]{a}bd-2a\sqrt[3]{b}f+5b^{4/3}c\right)}{\sqrt{3}a^{5/3}}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(4bd-ag)}{\sqrt[3]{b}}-2af+5bc\right)}{6a^{5/3}}-\frac{b^4}{\cdots}}{3ab^2}-$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{3a^2\left(a+bx^3\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x]
```

output
```
-1/3*(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(a^2*(a + b*x^3)) +
 ((-3*b^2*c)/(2*a*x^2) - (3*b^2*d)/(a*x) + (b^(4/3)*(5*b^(4/3)*c + 4*a^(1/
3)*b*d - 2*a*b^(1/3)*f - a^(4/3)*g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (3*b^2*e*Log[x])/a - (b^(4/3)*(b^(1/3)*(5
*b*c - 2*a*f) - a^(1/3)*(4*b*d - a*g))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3
)) + (b^(5/3)*(5*b*c - 2*a*f - (a^(1/3)*(4*b*d - a*g))/b^(1/3))*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)) - (b^2*e*Log[a + b*x^3])
/a)/(3*a*b^2)
```

3.419.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.419.4 Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.96

method	result
default	$-\frac{c}{2a^2x^2}-\frac{d}{a^2x}+\frac{e\ln(x)}{a^2}+\frac{\left(\frac{ag}{3}-\frac{bd}{3}\right)x^2+\left(\frac{af}{3}-\frac{bc}{3}\right)x-\frac{a(ah-be)}{3b}}{b\,x^3+a}+\frac{(2af-5bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan}{}\right.}{3}$
risch	$\frac{\frac{(ag-4bd)x^4}{3a^2}+\frac{(2af-5bc)x^3}{6a^2}-\frac{(ah-be)x^2}{3ab}-\frac{xd}{a}-\frac{c}{2a}}{x^2(b\,x^3+a)}+\frac{\left(\_R=\text{RootOf}\left(a^8b^2\_Z^3+9a^6b^2e\_Z^2+\left(6a^5bfg-15a^4b^2cg-24a^4b^2df+27a^4b^2e^2+60a\right.\right.\right.}{}$

input

```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/2*c/a^2/x^2-d/a^2/x+e*ln(x)/a^2+1/a^2*(((1/3*a*g-1/3*b*d)*x^2+(1/3*a*f-
1/3*b*c)*x-1/3*a*(a*h-b*e)/b)/(b*x^3+a)+1/3*(2*a*f-5*b*c)*(1/3/b/(a/b)^(2/
3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1
/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(a*g
-4*b*d)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1)))-1/3*e*ln(b*x^3+a))
```

3.419.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.92 (sec) , antiderivative size = 12231, normalized size of antiderivative = 39.97

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^3 (a + bx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.419.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^3 (a + bx^3)^2} \, dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.419.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.03

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx = -\frac{2\,(4\,b^2d-abg)x^4+6\,abdx+(5\,b^2c-2\,abf)x^3+3\,abc-2\,(abe-a^2h)x^2}{6\,(a^2b^2x^5+a^3bx^2)}+\frac{e\log(x)}{a^2} - \frac{\sqrt{3}\left(4\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}}-ag\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,af\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3} - \frac{\left(6\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,bc+2\,af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(3\,be\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,bc-2\,af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="ma
xima")
```

output
```
-1/6*(2*(4*b^2*d - a*b*g)*x^4 + 6*a*b*d*x + (5*b^2*c - 2*a*b*f)*x^3 + 3*a*
b*c - 2*(a*b*e - a^2*h)*x^2)/(a^2*b^2*x^5 + a^3*b*x^2) + e*log(x)/a^2 - 1/
9*sqrt(3)*(4*b*d*(a/b)^(2/3) - a*g*(a/b)^(2/3) + 5*b*c*(a/b)^(1/3) - 2*a*f
*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^3 - 1/
18*(6*b*e*(a/b)^(2/3) + 4*b*d*(a/b)^(1/3) - a*g*(a/b)^(1/3) - 5*b*c + 2*a*
f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/9*(3*b*e
*(a/b)^(2/3) - 4*b*d*(a/b)^(1/3) + a*g*(a/b)^(1/3) + 5*b*c - 2*a*f)*log(x
+ (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

3.419.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.09

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx$$
$$= -\frac{e\log(|bx^3+a|)}{3a^2} + \frac{e\log(|x|)}{a^2} + \frac{\sqrt{3}\left(5b^2c-2abf-4(-ab^2)^{\frac{1}{3}}bd+(-ab^2)^{\frac{1}{3}}ag\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9(-ab^2)^{\frac{2}{3}}a^2} + \frac{\left(5b^2c-2abf+4(-ab^2)^{\frac{1}{3}}bd-(-ab^2)^{\frac{1}{3}}ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18(-ab^2)^{\frac{2}{3}}a^2} + \frac{\left(4a^2b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^3bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5a^2b^2c-2a^3bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^5b} - \frac{2(4b^2d-abg)x^4+6abdx+(5b^2c-2abf)x^3+3abc-2(abe-a^2h)x^2}{6(bx^3+a)a^2bx^2}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="gi
ac")
```

output
```
-1/3*e*log(abs(b*x^3 + a))/a^2 + e*log(abs(x))/a^2 + 1/9*sqrt(3)*(5*b^2*c
- 2*a*b*f - 4*(-a*b^2)^(1/3)*b*d + (-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) + 1/18*(5*b^2*c -
2*a*b*f + 4*(-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) + 1/9*(4*a^2*b^2*d*(-a/b)^(1/3) -
 a^3*b*g*(-a/b)^(1/3) + 5*a^2*b^2*c - 2*a^3*b*f)*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/(a^5*b) - 1/6*(2*(4*b^2*d - a*b*g)*x^4 + 6*a*b*d*x + (5*b^2
*c - 2*a*b*f)*x^3 + 3*a*b*c - 2*(a*b*e - a^2*h)*x^2)/((b*x^3 + a)*a^2*b*x^
2)
```

3.419.9 Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 1632, normalized size of antiderivative = 5.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^2),x)
```

output
```
symsum(log((b^2*e*(25*b^2*c^2 + 4*a^2*f^2 - 3*a^2*e*g - 20*a*b*c*f + 12*a*
b*d*e))/(9*a^5) - (root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g
*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b
^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^
2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2
*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g
^3, z, k)*b^2*(25*b^2*c^2 + 4*a^2*f^2 - 9*root(729*a^8*b^2*z^3 + 729*a^6*b
^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^
3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^
2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*c^2*f + 48*a^2
*b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3 - 64*a*b^3*d^
3 + 125*b^4*c^3 + a^4*g^3, z, k)*a^4*g + 6*a^2*e*g + 36*root(729*a^8*b^2*z
^3 + 729*a^6*b^2*e*z^2 + 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*
c*g*z + 540*a^3*b^3*c*d*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3
*c*d*e - 72*a^2*b^2*d*e*f - 45*a^2*b^2*c*e*g - 12*a^3*b*d*g^2 - 150*a*b^3*
c^2*f + 48*a^2*b^2*d^2*g + 60*a^2*b^2*c*f^2 + 27*a^2*b^2*e^3 - 8*a^3*b*f^3
 - 64*a*b^3*d^3 + 125*b^4*c^3 + a^4*g^3, z, k)*a^3*b*d + 36*a*b*e^2*x + 20
0*b^2*c*d*x + 20*a^2*f*g*x + 324*root(729*a^8*b^2*z^3 + 729*a^6*b^2*e*z^2
+ 54*a^5*b*f*g*z - 216*a^4*b^2*d*f*z - 135*a^4*b^2*c*g*z + 540*a^3*b^3*c*d
*z + 243*a^4*b^2*e^2*z + 18*a^3*b*e*f*g + 180*a*b^3*c*d*e - 72*a^2*b^2*...
```

3.420 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx$

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3.420.1 Optimal result

Integrand size = 38, antiderivative size = 338

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx &= -\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}-\frac{e}{a^2x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{3a^2(a+bx^3)}\\ &+\frac{\left(5b^{4/3}d+4\sqrt[3]{a}be-2a\sqrt[3]{b}g-a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}b^{2/3}}\\ &-\frac{(2bc-af)\log(x)}{a^3}-\frac{\left(\sqrt[3]{b}(5bd-2ag)-\sqrt[3]{a}(4be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}b^{2/3}}\\ &+\frac{\left(\sqrt[3]{b}(5bd-2ag)-\sqrt[3]{a}(4be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}b^{2/3}}\\ &+\frac{(2bc-af)\log(a+bx^3)}{3a^3}\end{aligned}$$

output
```
-1/3*c/a^2/x^3-1/2*d/a^2/x^2-e/a^2/x-1/3*x*(b*d-a*g+(-a*h+b*e)*x-b*(b*c/a-
f)*x^2)/a^2/(b*x^3+a)-(-a*f+2*b*c)*ln(x)/a^3-1/9*(b^(1/3)*(-2*a*g+5*b*d)-a
^(1/3)*(-a*h+4*b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(2/3)+1/18*(b^(1/3)*(
-2*a*g+5*b*d)-a^(1/3)*(-a*h+4*b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/a^(8/3)/b^(2/3)+1/3*(-a*f+2*b*c)*ln(b*x^3+a)/a^3+1/9*(5*b^(4/3)*d+4*a^
(1/3)*b*e-2*a*b^(1/3)*g-a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3
)*3^(1/2))/a^(8/3)/b^(2/3)*3^(1/2)
```

3.420.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{6ac}{x^3}-\frac{9ad}{x^2}-\frac{18ae}{x}+\frac{a(-6b(c+x(d+ex))+6a(f+x(g+hx)))}{a+bx^3}-\frac{2\sqrt{3}\sqrt[3]{a}\left(-5b^{4/3}d-4\sqrt[3]{a}be+2a\sqrt[3]{b}g+a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}-}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x]
```

output
```
((-6*a*c)/x^3 - (9*a*d)/x^2 - (18*a*e)/x + (a*(-6*b*(c + x*(d + e*x)) + 6*
a*(f + x*(g + h*x))))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(-5*b^(4/3)*d - 4*a
^(1/3)*b*e + 2*a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]])/b^(2/3) + 18*(-2*b*c + a*f)*Log[x] - (2*a^(1/3)*(5*b^(4/3)*d -
4*a^(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(2/
3) + (a^(1/3)*(5*b^(4/3)*d - 4*a^(1/3)*b*e - 2*a*b^(1/3)*g + a^(4/3)*h)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 6*(2*b*c - a*f)*Lo
g[a + b*x^3])/(18*a^3)
```

3.420.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \ 2368$$

$$-\frac{\int -\frac{-b^2\left(\frac{be}{a}-h\right)x^5-2b^2\left(\frac{bd}{a}-g\right)x^4-3b^2\left(\frac{bc}{a}-f\right)x^3+3b^2ex^2+3b^2dx+3b^2c}{x^4\left(bx^3+a\right)}dx}{3ab^2}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{3a^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-b^2\left(\frac{be}{a}-h\right)x^5-2b^2\left(\frac{bd}{a}-g\right)x^4-3b^2\left(\frac{bc}{a}-f\right)x^3+3b^2ex^2+3b^2dx+3b^2c}{x^4(bx^3+a)}dx}{3ab^2}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{3a^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2373}$$

$$\frac{\int \left(\frac{3(af-2bc)b^2}{a^2x}+\frac{(3b(2bc-af)x^2-a(4be-ah)x-a(5bd-2ag))b^2}{a^2(bx^3+a)}+\frac{3eb^2}{ax^2}+\frac{3db^2}{ax^3}+\frac{3cb^2}{ax^4}\right)dx}{3ab^2}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{3a^2\left(a+bx^3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}(-h)+4\sqrt[3]{a}be-2a\sqrt[3]{b}g+5b^{4/3}d\right)}{\sqrt{3}a^{5/3}}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(4be-ah)}{\sqrt[3]{b}}-2ag+5bd\right)}{6a^{5/3}}-\frac{b^4}{}}{3ab^2}$$

$$\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{3a^2\left(a+bx^3\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x]
```

output
```
-1/3*(x*(b*d - a*g + (b*e - a*h)*x - b*((b*c)/a - f)*x^2))/(a^2*(a + b*x^3
)) + (-((b^2*c)/(a*x^3)) - (3*b^2*d)/(2*a*x^2) - (3*b^2*e)/(a*x) + (b^(4/3
)*(5*b^(4/3)*d + 4*a^(1/3)*b*e - 2*a*b^(1/3)*g - a^(4/3)*h)*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) - (3*b^2*(2*b*c - a
*f)*Log[x])/a^2 - (b^(4/3)*(b^(1/3)*(5*b*d - 2*a*g) - a^(1/3)*(4*b*e - a*h
))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (b^(5/3)*(5*b*d - 2*a*g - (a^(1
/3)*(4*b*e - a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*a^(5/3)) + (b^2*(2*b*c - a*f)*Log[a + b*x^3])/a^2)/(3*a*b^2)
```

3.420. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx$

3.420.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.420.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.98

method	result
default	$-\frac{e}{a^2 x}-\frac{c}{3a^2x^3}-\frac{d}{2a^2x^2}+\frac{(af-2bc)\ln(x)}{a^3}+\frac{\frac{\left(\frac{1}{3}a^2h-\frac{1}{3}aeb\right)x^2+\left(\frac{1}{3}a^2g-\frac{1}{3}abd\right)x+\frac{a(af-bc)}{3}}{bx^3+a}+\frac{\left(2a^2g-5abd\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln(\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
risch	$\frac{\frac{(ah-4be)x^5}{3a^2}+\frac{(2ag-5bd)x^4}{6a^2}+\frac{(af-2bc)x^3}{3a^2}-\frac{e\,x^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3\left(b\,x^3+a\right)}+\frac{\left(\_R=\text{RootOf}\left(a^9b^2\_Z^3+\left(9a^7b^2f-18a^6b^3c\right)\_Z^2+\left(6a^6bgh-15a^5b^2dh-24a^5\cdots\right.\right.\right.}{\cdots}$

3.420. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOS
E)
```

output
```
-e/a^2/x-1/3*c/a^2/x^3-1/2*d/a^2/x^2+(a*f-2*b*c)/a^3*ln(x)+1/a^3*(((1/3*a^
2*h-1/3*a*e*b)*x^2+(1/3*a^2*g-1/3*a*b*d)*x+1/3*a*(a*f-b*c))/(b*x^3+a)+1/3*
(2*a^2*g-5*a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(a^2*h-4*a*b*e)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/
b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/
b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/9*(-3*a*b*f+6*b^2
*c)*ln(b*x^3+a)/b)
```

3.420.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 71.98 (sec) , antiderivative size = 16568, normalized size of antiderivative = 49.02

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.420.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**2,x)
```

output
```
Timed out
```

3.420.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx$$
$$= -\frac{2\,(4\,be-ah)x^5+(5\,bd-2\,ag)x^4+6\,aex^2+2\,(2\,bc-af)x^3+3\,adx+2\,ac}{6\,(a^2bx^6+a^3x^3)}$$
$$-\frac{(2\,bc-af)\log(x)}{a^3}$$
$$-\frac{\sqrt{3}\left(4\,abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4}$$
$$+\frac{\left(12\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-6\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abd-2\,a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(6\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-3\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abd+2\,a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="ma
xima")
```

output
```
-1/6*(2*(4*b*e - a*h)*x^5 + (5*b*d - 2*a*g)*x^4 + 6*a*e*x^2 + 2*(2*b*c - a
*f)*x^3 + 3*a*d*x + 2*a*c)/(a^2*b*x^6 + a^3*x^3) - (2*b*c - a*f)*log(x)/a^
3 - 1/9*sqrt(3)*(4*a*b*e*(a/b)^(2/3) - a^2*h*(a/b)^(2/3) + 5*a*b*d*(a/b)^(
1/3) - 2*a^2*g*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/a^4 + 1/18*(12*b^2*c*(a/b)^(2/3) - 6*a*b*f*(a/b)^(2/3) - 4*a*b*e*(a/
b)^(1/3) + a^2*h*(a/b)^(1/3) + 5*a*b*d - 2*a^2*g)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) + 1/9*(6*b^2*c*(a/b)^(2/3) - 3*a*b*f*(a
/b)^(2/3) + 4*a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) - 5*a*b*d + 2*a^2*g)*l
og(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

3.420.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.06

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx$$
$$= \frac{\sqrt{3}\left(5\,b^2d-2\,abg-4\,(-ab^2)^{\frac{1}{3}}\,be+(-ab^2)^{\frac{1}{3}}\,ah\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a^2}$$
$$+\frac{\left(5\,b^2d-2\,abg+4\,(-ab^2)^{\frac{1}{3}}\,be-(-ab^2)^{\frac{1}{3}}\,ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a^2}$$
$$+\frac{(2\,bc-af)\log\left(|bx^3+a|\right)}{3\,a^3}-\frac{(2\,bc-af)\log\left(|x|\right)}{a^3}$$
$$+\frac{\left(4\,a^4b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^5bh\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^4b^2d-2\,a^5bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^7b}$$
$$-\frac{2\,(4\,abe-a^2h)x^5+6\,a^2ex^2+(5\,abd-2\,a^2g)x^4+3\,a^2dx+2\,(2\,abc-a^2f)x^3+2\,a^2c}{6\,(bx^3+a)a^3x^3}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^2,x, algorithm="gi
ac")
```

output
```
1/9*sqrt(3)*(5*b^2*d - 2*a*b*g - 4*(-a*b^2)^(1/3)*b*e + (-a*b^2)^(1/3)*a*h
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^
2) + 1/18*(5*b^2*d - 2*a*b*g + 4*(-a*b^2)^(1/3)*b*e - (-a*b^2)^(1/3)*a*h)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) + 1/3*(2*b*c
 - a*f)*log(abs(b*x^3 + a))/a^3 - (2*b*c - a*f)*log(abs(x))/a^3 + 1/9*(4*a
^4*b^2*e*(-a/b)^(1/3) - a^5*b*h*(-a/b)^(1/3) + 5*a^4*b^2*d - 2*a^5*b*g)*(-
a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b) - 1/6*(2*(4*a*b*e - a^2*h)*x
^5 + 6*a^2*e*x^2 + (5*a*b*d - 2*a^2*g)*x^4 + 3*a^2*d*x + 2*(2*a*b*c - a^2*
f)*x^3 + 2*a^2*c)/((b*x^3 + a)*a^3*x^3)
```

3.420.9 Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 1924, normalized size of antiderivative = 5.69

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^2),x)
```

output
```
symsum(log(- (50*b^5*c*d^2 - 48*b^5*c^2*e + 8*a^2*b^3*c*g^2 - 12*a^2*b^3*e
*f^2 - 4*a^3*b^2*f*g^2 + 3*a^3*b^2*f^2*h - 25*a*b^4*d^2*f + 12*a*b^4*c^2*h
 - 12*a^2*b^3*c*f*h + 20*a^2*b^3*d*f*g - 40*a*b^4*c*d*g + 48*a*b^4*c*e*f)/
(9*a^6) - root(729*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1458*a^6*b^3*c*z^2 +
54*a^6*b*g*h*z - 216*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z - 972*a^4*b^3*c*f*z
 + 540*a^4*b^3*d*e*z + 243*a^5*b^2*f^2*z + 972*a^3*b^4*c^2*z + 18*a^4*b*f*
g*h - 360*a*b^4*c*d*e - 72*a^3*b^2*e*f*g - 45*a^3*b^2*d*f*h - 36*a^3*b^2*c
*g*h + 180*a^2*b^3*d*e*f + 144*a^2*b^3*c*e*g + 90*a^2*b^3*c*d*h - 12*a^4*b
*e*h^2 + 324*a*b^4*c^2*f + 48*a^3*b^2*e^2*h - 150*a^2*b^3*d^2*g + 60*a^3*b
^2*d*g^2 - 162*a^2*b^3*c*f^2 + 27*a^3*b^2*f^3 - 64*a^2*b^3*e^3 - 8*a^4*b*g
^3 + 125*a*b^4*d^3 - 216*b^5*c^3 + a^5*h^3, z, k)*((25*a^3*b^4*d^2 + 4*a^5
*b^2*g^2 + 48*a^3*b^4*c*e - 12*a^4*b^3*c*h - 20*a^4*b^3*d*g - 24*a^4*b^3*e
*f + 6*a^5*b^2*f*h)/(9*a^6) + root(729*a^9*b^2*z^3 + 729*a^7*b^2*f*z^2 - 1
458*a^6*b^3*c*z^2 + 54*a^6*b*g*h*z - 216*a^5*b^2*e*g*z - 135*a^5*b^2*d*h*z
 - 972*a^4*b^3*c*f*z + 540*a^4*b^3*d*e*z + 243*a^5*b^2*f^2*z + 972*a^3*b^4
*c^2*z + 18*a^4*b*f*g*h - 360*a*b^4*c*d*e - 72*a^3*b^2*e*f*g - 45*a^3*b^2*
d*f*h - 36*a^3*b^2*c*g*h + 180*a^2*b^3*d*e*f + 144*a^2*b^3*c*e*g + 90*a^2*
b^3*c*d*h - 12*a^4*b*e*h^2 + 324*a*b^4*c^2*f + 48*a^3*b^2*e^2*h - 150*a^2*
b^3*d^2*g + 60*a^3*b^2*d*g^2 - 162*a^2*b^3*c*f^2 + 27*a^3*b^2*f^3 - 64*a^2
*b^3*e^3 - 8*a^4*b*g^3 + 125*a*b^4*d^3 - 216*b^5*c^3 + a^5*h^3, z, k)*(...
```

3.421 $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

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3.421.1 Optimal result

Integrand size = 38, antiderivative size = 345

$$\begin{aligned}
&\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx \\
&= \frac{hx}{b^3}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6b^3(a+bx^3)^2} \\
&\quad -\frac{x(a(7be-13ah)-2b(bc-4af)x-3b(bd-3ag)x^2)}{18ab^3(a+bx^3)} \\
&\quad -\frac{(b^{5/3}c+2a^{2/3}be+5ab^{2/3}f-14a^{5/3}h)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{10/3}} \\
&\quad -\frac{(b^{2/3}(bc+5af)-2a^{2/3}(be-7ah))\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{10/3}} \\
&\quad +\frac{(b^{2/3}(bc+5af)-2a^{2/3}(be-7ah))\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{10/3}}+\frac{g\log(a+bx^3)}{3b^3}
\end{aligned}$$

output
```
h*x/b^3+1/6*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/b^3/(b*x^3+a)
^2-1/18*x*(a*(-13*a*h+7*b*e)-2*b*(-4*a*f+b*c)*x-3*b*(-3*a*g+b*d)*x^2)/a/b^
3/(b*x^3+a)-1/27*(b^(2/3)*(5*a*f+b*c)-2*a^(2/3)*(-7*a*h+b*e))*ln(a^(1/3)+b
^(1/3)*x)/a^(4/3)/b^(10/3)+1/54*(b^(2/3)*(5*a*f+b*c)-2*a^(2/3)*(-7*a*h+b*e
))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(10/3)+1/3*g*ln(b*x
^3+a)/b^3-1/27*(b^(5/3)*c+2*a^(2/3)*b*e+5*a*b^(2/3)*f-14*a^(5/3)*h)*arctan
(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(10/3)*3^(1/2)
```

3.421.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.99

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$= \frac{54b^{2/3}hx - \frac{9b^{2/3}(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{(a+bx^3)^2} + \frac{3b^{2/3}(2b^2cx^2+a^2(12g+13hx)-ab(6d+x(7e+8fx)))}{a(a+bx^3)} - \frac{2\sqrt{3}\left(b^2c+2a^{2/3}b^{4/3}e+\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

output
```
(54*b^(2/3)*h*x - (9*b^(2/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e +
f*x))))/(a + b*x^3)^2 + (3*b^(2/3)*(2*b^2*c*x^2 + a^2*(12*g + 13*h*x) - a*
b*(6*d + x*(7*e + 8*f*x))))/(a*(a + b*x^3)) - (2*Sqrt[3]*(b^2*c + 2*a^(2/3
)*b^(4/3)*e + 5*a*b*f - 14*a^(5/3)*b^(1/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]])/a^(4/3) - (2*(b^2*c - 2*a^(2/3)*b^(4/3)*e + 5*a*b*f + 14*
a^(5/3)*b^(1/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + ((b^2*c - 2*a^(2/3)
*b^(4/3)*e + 5*a*b*f + 14*a^(5/3)*b^(1/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/a^(4/3) + 18*b^(2/3)*g*Log[a + b*x^3])/(54*b^(11/3))
```

3.421.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2367, 2397, 27, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \; 2367$$

$$\frac{x(-bx(bc-af)-bx^2(bd-ag)+a(be-ah))}{6b^3(a+bx^3)^2} - \frac{\int \frac{-6ab^2hx^6-6ab^2gx^5-6ab^2fx^4-6ab(be-ah)x^3-3ab(bd-ag)x^2-2ab(bc-af)x+a^2(be-ah)}{(bx^3+a)^2}dx}{6ab^3}$$

$$\Big\downarrow \text{2397}$$

$$\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6b^3\left(a+bx^3\right)^2}-\frac{\frac{x(-2bx(bc-4af)-3bx^2(bd-3ag)+a(7be-13ah))}{3(a+bx^3)}-\frac{\int \frac{2\left(9a^2hx^3b^3+9a^2gx^2b^3+a(bc+5af)xb^3+a^2(2be-5ah)b^2\right)}{bx^3+a}dx}{3ab^2}}{6ab^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6b^3\left(a+bx^3\right)^2}-\frac{\frac{x(-2bx(bc-4af)-3bx^2(bd-3ag)+a(7be-13ah))}{3(a+bx^3)}-\frac{2\int \frac{9a^2hx^3b^3+9a^2gx^2b^3+a(bc+5af)xb^3+a^2(2be-5ah)b^2}{bx^3+a}dx}{3ab^2}}{6ab^3}$$

$$\Big\downarrow \text{2426}$$

$$\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6b^3\left(a+bx^3\right)^2}-\frac{\frac{x(-2bx(bc-4af)-3bx^2(bd-3ag)+a(7be-13ah))}{3(a+bx^3)}-\frac{2\int \left(9a^2hb^2+\frac{9a^2gx^2b^3+a(bc+5af)xb^3+2a^2(be-7ah)b^2}{bx^3+a}\right)dx}{3ab^2}}{6ab^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6b^3\left(a+bx^3\right)^2}-\frac{\frac{x(-2bx(bc-4af)-3bx^2(bd-3ag)+a(7be-13ah))}{3(a+bx^3)}-\frac{2\left(-\frac{a^{2/3}b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(2a^{2/3}be-14a^{5/3}h+5ab^{2/3}f+b^{5/3}c\right)}{\sqrt{3}}+\frac{1}{6}a^{2/3}b^{5/3}\log\left(a^{2/3}\right.\right.}{}}{}$$

input
```
Int[(x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

output
```
(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*b^3*(a + b*x^
3)^2) - ((x*(a*(7*b*e - 13*a*h) - 2*b*(b*c - 4*a*f)*x - 3*b*(b*d - 3*a*g)*
x^2))/(3*(a + b*x^3)) - (2*(9*a^2*b^2*h*x - (a^(2/3)*b^(5/3)*(b^(5/3)*c +
2*a^(2/3)*b*e + 5*a*b^(2/3)*f - 14*a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*
x)/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (a^(2/3)*b^(5/3)*(b^(2/3)*(b*c + 5*a*f) -
 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(1/3) + b^(1/3)*x])/3 + (a^(2/3)*b^(5/3)*(
b^(2/3)*(b*c + 5*a*f) - 2*a^(2/3)*(b*e - 7*a*h))*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/6 + 3*a^2*b^2*g*Log[a + b*x^3]))/(3*a*b^2))/(6*a*b^
3)
```

3.421. $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

3.421.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.421. $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

3.421.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.49

method	result
risch	$\frac{hx}{b^3} + \frac{-\frac{b^2(4af-bc)x^5}{9a} + \left(\frac{13}{18}abh - \frac{7}{18}b^2 e\right)x^4 + \left(\frac{2}{3}abg - \frac{1}{3}b^2 d\right)x^3 - \frac{b(5af+bc)x^2}{18} + \frac{a(5ah-2be)x}{9} + \frac{a^2 g}{2} - \frac{abd}{6}}{b^3(bx^3+a)^2} + \sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{\left(9 \cdots\right.}{\cdots}$
default	$\frac{hx}{b^3} - \frac{\frac{\frac{b^2(4af-bc)x^5}{9a} + \left(-\frac{13}{18}abh + \frac{7}{18}b^2 e\right)x^4 + \left(-\frac{2}{3}abg + \frac{1}{3}b^2 d\right)x^3 + \frac{b(5af+bc)x^2}{18} - \frac{a(5ah-2be)x}{9} - \frac{a^2 g}{2} + \frac{abd}{6}}{(bx^3+a)^2} + \left(14a^2h-2aeb\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} \cdots\right.}{\cdots}$

input
```
int(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOS
E)
```

output
```
h*x/b^3+(-1/9*b^2*(4*a*f-b*c)/a*x^5+(13/18*a*b*h-7/18*b^2*e)*x^4+(2/3*a*b*
g-1/3*b^2*d)*x^3-1/18*b*(5*a*f+b*c)*x^2+1/9*a*(5*a*h-2*b*e)*x+1/2*a^2*g-1/
6*a*b*d)/b^3/(b*x^3+a)^2+1/27/b^4*sum((9*g*b*_R^2+b*(5*a*f+b*c)/a*_R-14*a*
h+2*b*e)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.421.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 12967, normalized size of antiderivative = 37.59

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.421.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.421.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.13

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= \frac{2\,(b^3c-4\,ab^2f)x^5-(7\,ab^2e-13\,a^2bh)x^4-3\,a^2bd+9\,a^3g-6\,(ab^2d-2\,a^2bg)x^3-(ab^2c+5\,a^2bf)x^2-2}{18\,(ab^5x^6+2\,a^2b^4x^3+a^3b^3)}$$
$$+\frac{hx}{b^3}+\frac{\sqrt{3}\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-14\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3}$$
$$+\frac{\left(18\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abe+14\,a^2h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(9\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abe-14\,a^2h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="ma
xima")
```

output
```
1/18*(2*(b^3*c - 4*a*b^2*f)*x^5 - (7*a*b^2*e - 13*a^2*b*h)*x^4 - 3*a^2*b*d
 + 9*a^3*g - 6*(a*b^2*d - 2*a^2*b*g)*x^3 - (a*b^2*c + 5*a^2*b*f)*x^2 - 2*(
2*a^2*b*e - 5*a^3*h)*x)/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3) + h*x/b^3 +
1/27*sqrt(3)*(b^2*c*(a/b)^(2/3) + 5*a*b*f*(a/b)^(2/3) + 2*a*b*e*(a/b)^(1/3
) - 14*a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(a^2*b^3) + 1/54*(18*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) + 5*a*b*f*(
a/b)^(1/3) - 2*a*b*e + 14*a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a
*b^4*(a/b)^(2/3)) + 1/27*(9*a*b*g*(a/b)^(2/3) - b^2*c*(a/b)^(1/3) - 5*a*b*
f*(a/b)^(1/3) + 2*a*b*e - 14*a^2*h)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3
))
```

3.421.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.10

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \frac{hx}{b^3} + \frac{g\log\left(|bx^3+a|\right)}{3\,b^3} - \frac{\sqrt{3}\left(2\,abe - 14\,a^2h - \left(-ab^2\right)^{\frac{1}{3}}bc - 5\left(-ab^2\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab^2} - \frac{\left(2\,abe - 14\,a^2h + \left(-ab^2\right)^{\frac{1}{3}}bc + 5\left(-ab^2\right)^{\frac{1}{3}}af\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab^2} + \frac{2\left(b^3c - 4\,ab^2f\right)x^5 - \left(7\,ab^2e - 13\,a^2bh\right)x^4 - 3\,a^2bd + 9\,a^3g - 6\left(ab^2d - 2\,a^2bg\right)x^3 - \left(ab^2c + 5\,a^2bf\right)x^2 -}{18\left(bx^3+a\right)^2ab^3} - \frac{\left(ab^6c\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 5\,a^2b^5f\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\,a^2b^5e - 14\,a^3b^4h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^7}$$

input
```
integrate(x^4*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gi
ac")
```

output
```
h*x/b^3 + 1/3*g*log(abs(b*x^3 + a))/b^3 - 1/27*sqrt(3)*(2*a*b*e - 14*a^2*h
 - (-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3)*(2*x + (-
a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1/54*(2*a*b*e - 14*a^2*
h + (-a*b^2)^(1/3)*b*c + 5*(-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) + 1/18*(2*(b^3*c - 4*a*b^2*f)*x^5 - (
7*a*b^2*e - 13*a^2*b*h)*x^4 - 3*a^2*b*d + 9*a^3*g - 6*(a*b^2*d - 2*a^2*b*g
)*x^3 - (a*b^2*c + 5*a^2*b*f)*x^2 - 2*(2*a^2*b*e - 5*a^3*h)*x)/((b*x^3 + a
)^2*a*b^3) - 1/27*(a*b^6*c*(-a/b)^(1/3) + 5*a^2*b^5*f*(-a/b)^(1/3) + 2*a^2
*b^5*e - 14*a^3*b^4*h)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^7)
```

3.421.9 Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 916, normalized size of antiderivative = 2.66

$$\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$= \left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\big(19683\,a^4\,b^{10}\,z^3 - 19683\,a^4\,b^7\,g\,z^2 - 5670\,a^4\,b^4\,f\,h\,z - 1134\,a^3\,b^5\,c\,h\,z + 810\,a^3\,b^5\,e\,f\,z + 162\,a^2\,b\right.\right.$$

$$+ \frac{81\,a^2\,g^2 + 2\,b^2\,c\,e - 70\,a^2\,f\,h - 14\,a\,b\,c\,h + 10\,a\,b\,e\,f}{81\,a\,b^4}$$

$$\left. + \frac{x\,(126\,g\,h\,a^3 + 25\,a^2\,b\,f^2 - 18\,e\,g\,a^2\,b + 10\,a\,b^2\,c\,f + b^3\,c^2)}{81\,a^2\,b^4}\right)\mathrm{root}\big(19683\,a^4\,b^{10}\,z^3$$

$$- 19683\,a^4\,b^7\,g\,z^2 - 5670\,a^4\,b^4\,f\,h\,z - 1134\,a^3\,b^5\,c\,h\,z + 810\,a^3\,b^5\,e\,f\,z + 162\,a^2\,b^6\,c\,e\,z$$

$$+ 6561\,a^4\,b^4\,g^2\,z + 1890\,a^4\,b\,f\,g\,h + 378\,a^3\,b^2\,c\,g\,h - 270\,a^3\,b^2\,e\,f\,g - 54\,a^2\,b^3\,c\,e\,g$$

$$- 1176\,a^4\,b\,e\,h^2 + 15\,a\,b^4\,c^2\,f + 168\,a^3\,b^2\,e^2\,h + 75\,a^2\,b^3\,c\,f^2 + 125\,a^3\,b^2\,f^3 - 8\,a^2\,b^3\,e^3$$

$$\left. - 729\,a^4\,b\,g^3 + 2744\,a^5\,h^3 + b^5\,c^3, z, k\big)\right)$$

$$- \frac{x^2\left(\frac{c\,b^2}{18} + \frac{5\,a\,f\,b}{18}\right) - \frac{a^2\,g}{2} - x\left(\frac{5\,a^2\,h}{9} - \frac{2\,a\,b\,e}{9}\right) + x^3\left(\frac{b^2\,d}{3} - \frac{2\,a\,b\,g}{3}\right) + \frac{b\,x^4\,(7\,b\,e - 13\,a\,h)}{18} + \frac{a\,b\,d}{6} - \frac{b\,x^5\,(b^2\,c - 4\,a\,b\,f)}{9\,a}}{a^2\,b^3 + 2\,a\,b^4\,x^3 + b^5\,x^6}$$

$$+ \frac{h\,x}{b^3}$$

input
```
int((x^4*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

output
```
symsum(log(root(19683*a^4*b^10*z^3 - 19683*a^4*b^7*g*z^2 - 5670*a^4*b^4*f*
h*z - 1134*a^3*b^5*c*h*z + 810*a^3*b^5*e*f*z + 162*a^2*b^6*c*e*z + 6561*a^
4*b^4*g^2*z + 1890*a^4*b*f*g*h + 378*a^3*b^2*c*g*h - 270*a^3*b^2*e*f*g - 5
4*a^2*b^3*c*e*g - 1176*a^4*b*e*h^2 + 15*a*b^4*c^2*f + 168*a^3*b^2*e^2*h +
75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 8*a^2*b^3*e^3 - 729*a^4*b*g^3 + 2744*
a^5*h^3 + b^5*c^3, z, k)*(9*root(19683*a^4*b^10*z^3 - 19683*a^4*b^7*g*z^2
- 5670*a^4*b^4*f*h*z - 1134*a^3*b^5*c*h*z + 810*a^3*b^5*e*f*z + 162*a^2*b^
6*c*e*z + 6561*a^4*b^4*g^2*z + 1890*a^4*b*f*g*h + 378*a^3*b^2*c*g*h - 270*
a^3*b^2*e*f*g - 54*a^2*b^3*c*e*g - 1176*a^4*b*e*h^2 + 15*a*b^4*c^2*f + 168
*a^3*b^2*e^2*h + 75*a^2*b^3*c*f^2 + 125*a^3*b^2*f^3 - 8*a^2*b^3*e^3 - 729*
a^4*b*g^3 + 2744*a^5*h^3 + b^5*c^3, z, k)*a*b^2 - (6*a*g)/b + (x*(54*a^2*b
^4*e - 378*a^3*b^3*h))/(81*a^2*b^4)) + (81*a^2*g^2 + 2*b^2*c*e - 70*a^2*f*
h - 14*a*b*c*h + 10*a*b*e*f)/(81*a*b^4) + (x*(b^3*c^2 + 25*a^2*b*f^2 + 126
*a^3*g*h + 10*a*b^2*c*f - 18*a^2*b*e*g))/(81*a^2*b^4))*root(19683*a^4*b^10
*z^3 - 19683*a^4*b^7*g*z^2 - 5670*a^4*b^4*f*h*z - 1134*a^3*b^5*c*h*z + 810
*a^3*b^5*e*f*z + 162*a^2*b^6*c*e*z + 6561*a^4*b^4*g^2*z + 1890*a^4*b*f*g*h
 + 378*a^3*b^2*c*g*h - 270*a^3*b^2*e*f*g - 54*a^2*b^3*c*e*g - 1176*a^4*b*e
*h^2 + 15*a*b^4*c^2*f + 168*a^3*b^2*e^2*h + 75*a^2*b^3*c*f^2 + 125*a^3*b^2
*f^3 - 8*a^2*b^3*e^3 - 729*a^4*b*g^3 + 2744*a^5*h^3 + b^5*c^3, z, k), k, 1
, 3) - (x^2*((b^2*c)/18 + (5*a*b*f)/18) - (a^2*g)/2 - x*((5*a^2*h)/9 - ...
```

3.421. $\int \frac{x^4(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

3.422 $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

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3.422.1 Optimal result

Integrand size = 38, antiderivative size = 325

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx &= -\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6b^2(a+bx^3)^2} \\ &\quad+\frac{x(bc-7af+2(bd-4ag)x+3(be-3ah)x^2)}{18ab^2(a+bx^3)} \\ &\quad-\frac{\left(b^{4/3}c+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+5a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{8/3}} \\ &\quad+\frac{\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{8/3}} \\ &\quad-\frac{\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{8/3}}+\frac{h\log(a+bx^3)}{3b^3}\end{aligned}$$

output
```
-1/6*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/b^2/(b*x^3+a)^2+1/18*x*(b*c-7
*a*f+2*(-4*a*g+b*d)*x+3*(-3*a*h+b*e)*x^2)/a/b^2/(b*x^3+a)+1/27*(b^(1/3)*(2
*a*f+b*c)-a^(1/3)*(5*a*g+b*d))*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(8/3)-1/54*
(b^(1/3)*(2*a*f+b*c)-a^(1/3)*(5*a*g+b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/a^(5/3)/b^(8/3)+1/3*h*ln(b*x^3+a)/b^3-1/27*(b^(4/3)*c+a^(1/3)*b*
d+2*a*b^(1/3)*f+5*a^(4/3)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1
/2))/a^(5/3)/b^(8/3)*3^(1/2)
```

3.422.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{9(a^2h+b^2x(c+dx)-ab(e+x(f+gx)))}{(a+bx^3)^2}+\frac{36a^2h+3b^2x(c+2dx)-3ab(6e+x(7f+8gx))}{a(a+bx^3)}-\frac{2\sqrt{3}\sqrt[3]{b}\left(b^{4/3}c+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+5a^{4/3}g\right)\arctan}{a^{5/3}}}{}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

output
```
((-9*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a + b*x^3)^2 + (3
6*a^2*h + 3*b^2*x*(c + 2*d*x) - 3*a*b*(6*e + x*(7*f + 8*g*x)))/(a*(a + b*x
^3)) - (2*Sqrt[3]*b^(1/3)*(b^(4/3)*c + a^(1/3)*b*d + 2*a*b^(1/3)*f + 5*a^(
4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*b^(1/3)*
(b^(4/3)*c - a^(1/3)*b*d + 2*a*b^(1/3)*f - 5*a^(4/3)*g)*Log[a^(1/3) + b^(1
/3)*x])/a^(5/3) + (b^(1/3)*(-(b^(4/3)*c) + a^(1/3)*b*d - 2*a*b^(1/3)*f + 5
*a^(4/3)*g)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + 18*h
*Log[a + b*x^3])/(54*b^3)
```

3.422.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.395, Rules used = {2367, 25, 2397, 27, 2410, 792, 2399, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \; 2367$$

$$-\frac{\int -\frac{6ab^2hx^5+6ab^2gx^4+6ab^2fx^3+3ab(be-ah)x^2+2ab(bd-ag)x+ab(bc-af)}{(bx^3+a)^2}dx}{6ab^3}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\frac{\int \frac{6ab^2hx^5+6ab^2gx^4+6ab^2fx^3+3ab(be-ah)x^2+2ab(bd-ag)x+ab(bc-af)}{(bx^3+a)^2}dx}{6ab^3}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textcolor{blue}{2397}$$

$$\frac{\frac{x\left(b(bc-7af)+2bx(bd-4ag)+3bx^2(be-3ah)\right)}{3(a+bx^3)}-\frac{\int -\frac{2\left(9a^2hx^2b^3+a(bc+2af)b^3+a(bd+5ag)xb^3\right)}{bx^3+a}dx}{3ab^2}}{6ab^3}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\frac{2\int \frac{9a^2hx^2b^3+a(bc+2af)b^3+a(bd+5ag)xb^3}{bx^3+a}dx}{3ab^2}+\frac{x\left(b(bc-7af)+2bx(bd-4ag)+3bx^2(be-3ah)\right)}{3(a+bx^3)}}{6ab^3}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textcolor{blue}{2410}$$

$$\frac{\frac{2\left(9a^2b^3h\int \frac{x^2}{bx^3+a}dx+\int \frac{a(bc+2af)b^3+a(bd+5ag)xb^3}{bx^3+a}dx\right)}{3ab^2}+\frac{x\left(b(bc-7af)+2bx(bd-4ag)+3bx^2(be-3ah)\right)}{3(a+bx^3)}}{6ab^3}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textcolor{blue}{792}$$

$$\frac{\frac{2\left(\int \frac{a(bc+2af)b^3+a(bd+5ag)xb^3}{bx^3+a}dx+3a^2b^2h\log\left(a+bx^3\right)\right)}{3ab^2}+\frac{x\left(b(bc-7af)+2bx(bd-4ag)+3bx^2(be-3ah)\right)}{3(a+bx^3)}}{6ab^3}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \textcolor{blue}{2399}$$

3.422. $\displaystyle\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{\frac{2\left(\frac{\int \frac{ab^3\left(\sqrt[3]{a}\left(2\sqrt[3]{b}(bc+2af)+\sqrt[3]{a}(bd+5ag)\right)-\sqrt[3]{b}\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{1}{3}\sqrt[3]{a}b^{8/3}\left(\sqrt[3]{b}(2af+bc)-\sqrt[3]{a}(5ag+bd)\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}\cdots\right)}{3ab^2}\cdots}{6ab^3}\cdots$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{2\left(\frac{\int \frac{ab^3\left(\sqrt[3]{a}\left(2\sqrt[3]{b}(bc+2af)+\sqrt[3]{a}(bd+5ag)\right)-\sqrt[3]{b}\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+3a^2b^2h\log\left(a+bx^3\right)+\frac{1}{3}\sqrt[3]{a}b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\sqrt[3]{b}\cdots\right.\right.}{3ab^2}\cdots}{6ab^3}\cdots$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(\frac{1}{3}\sqrt[3]{a}b^{8/3}\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(bc+2af)+\sqrt[3]{a}(bd+5ag)\right)-\sqrt[3]{b}\left(\sqrt[3]{b}(bc+2af)-\sqrt[3]{a}(bd+5ag)\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+3a^2b^2h\log\left(a+bx^3\right)+\frac{1}{3}\sqrt[3]{a}b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\cdots\right.\right.}{3ab^2}\cdots}{6ab^3}\cdots$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{2\left(\frac{1}{3}\sqrt[3]{a}b^{8/3}\left(\frac{3}{2}\sqrt[3]{a}\left(5a^{4/3}g+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(-\frac{\sqrt[3]{a}(5ag+bd)}{\sqrt[3]{b}}+2af+bc\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\cdots\right.\right.}{3ab^2}\cdots}{6ab^3}\cdots$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

3.422. $\int \frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{2\left(\frac{1}{3}\sqrt[3]{a}b^{8/3}\left(\frac{3}{2}\sqrt[3]{a}\left(5a^{4/3}g+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(-\frac{\sqrt[3]{a}(5ag+bd)}{\sqrt[3]{b}}+2af+bc\right)\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}\right.\right.}{3ab^2}}{6ab^3}$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{3}\sqrt[3]{a}b^{8/3}\left(\frac{3}{2}\sqrt[3]{a}\left(5a^{4/3}g+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(-\frac{\sqrt[3]{a}(5ag+bd)}{\sqrt[3]{b}}+2af+bc\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+\ldots}\right.\right.}{3ab^2}}{6a\ldots}$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$\downarrow$ 1082

$$\frac{2\left(\frac{1}{3}\sqrt[3]{a}b^{8/3}\left(\frac{1}{2}\sqrt[3]{b}\left(-\frac{\sqrt[3]{a}(5ag+bd)}{\sqrt[3]{b}}+2af+bc\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(5a^{4/3}g+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\ldots\right)}{\sqrt[3]{b}}\right.\right.}{3ab^2}}{6ab\ldots}$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$\downarrow$ 217

$$\frac{2\left(\frac{1}{3}\sqrt[3]{a}b^{8/3}\left(\frac{1}{2}\sqrt[3]{b}\left(-\frac{\sqrt[3]{a}(5ag+bd)}{\sqrt[3]{b}}+2af+bc\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{4/3}g+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt[3]{b}}\right)+3a^2b\ldots\right.}{3ab^2}}{6ab^3}$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

$\downarrow$ 1103

3.422. $\int\frac{x^3\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{2\left(\frac{1}{3}\sqrt[3]{a}b^{8/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{4/3}g+\sqrt[3]{a}bd+2a\sqrt[3]{b}f+b^{4/3}c\right)}{\sqrt[3]{b}}-\frac{1}{2}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(5ag+bd)}{\sqrt[3]{b}}+2af+bc\right)\right)+3a^2b\cdots}{3ab^2}}{6ab^3}$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6b^2\left(a+bx^3\right)^2}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

output
```
-1/6*(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(b^2*(a + b*x^3)^2)
 + ((x*(b*(b*c - 7*a*f) + 2*b*(b*d - 4*a*g)*x + 3*b*(b*e - 3*a*h)*x^2))/(3
*(a + b*x^3)) + (2*((a^(1/3)*b^(7/3)*(b^(1/3)*(b*c + 2*a*f) - a^(1/3)*(b*d
 + 5*a*g))*Log[a^(1/3) + b^(1/3)*x])/3 + (a^(1/3)*b^(8/3)*(-((Sqrt[3]*(b^(
4/3)*c + a^(1/3)*b*d + 2*a*b^(1/3)*f + 5*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - ((b*c + 2*a*f - (a^(1/3)*(b*d + 5*a*g))/
b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/2))/3 + 3*a^2*b^2
*h*Log[a + b*x^3]))/(3*a*b^2))/(6*a*b^3)
```

3.422.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

3.422. $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.422.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.53

method	result
risch	$\frac{-\frac{(4ag-bd)x^5}{9ab}-\frac{(7af-bc)x^4}{18ab}+\frac{(2ah-be)x^3}{3b^2}-\frac{(5ag+bd)x^2}{18b^2}-\frac{(2af+bc)x}{9b^2}+\frac{a(3ah-be)}{6b^3}}{(bx^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\left(9h\_R^2+\frac{(5ag+bd)\_R}{a}+\right.}{\_R^2}}{27b^3}$
default	$\frac{-\frac{(4ag-bd)x^5}{9ab}-\frac{(7af-bc)x^4}{18ab}+\frac{(2ah-be)x^3}{3b^2}-\frac{(5ag+bd)x^2}{18b^2}-\frac{(2af+bc)x}{9b^2}+\frac{a(3ah-be)}{6b^3}}{(bx^3+a)^2}+\frac{(2af+bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$

input
```
int(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOS
E)
```

output
```
(-1/9*(4*a*g-b*d)/a/b*x^5-1/18*(7*a*f-b*c)/a/b*x^4+1/3*(2*a*h-b*e)/b^2*x^3
-1/18*(5*a*g+b*d)/b^2*x^2-1/9*(2*a*f+b*c)/b^2*x+1/6*a*(3*a*h-b*e)/b^3)/(b*
x^3+a)^2+1/27/b^3*sum((9*h*_R^2+1/a*(5*a*g+b*d)*_R+(2*a*f+b*c)/a)/_R^2*ln(
x-_R),_R=RootOf(_Z^3*b+a))
```

3.422.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 12939, normalized size of antiderivative = 39.81

$$\int \frac{x^3(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^3} dx = \text{Too large to display}$$

input
```
integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.422.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^3} dx = \text{Timed out}$$

input
```
integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.422.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.13

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$= \frac{2\,(b^3d-4\,ab^2g)x^5+(b^3c-7\,ab^2f)x^4-3\,a^2be+9\,a^3h-6\,(ab^2e-2\,a^2bh)x^3-(ab^2d+5\,a^2bg)x^2-2\,(ab^2}{18\,(ab^5x^6+2\,a^2b^4x^3+a^3b^3)}$$
$$+\frac{\sqrt{3}\left(b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,abg\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3}$$
$$+\frac{\left(18\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}+bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-bc-2\,af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(9\,ah\left(\frac{a}{b}\right)^{\frac{2}{3}}-bd\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+bc+2\,af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="ma
xima")
```

output
```
1/18*(2*(b^3*d - 4*a*b^2*g)*x^5 + (b^3*c - 7*a*b^2*f)*x^4 - 3*a^2*b*e + 9*
a^3*h - 6*(a*b^2*e - 2*a^2*b*h)*x^3 - (a*b^2*d + 5*a^2*b*g)*x^2 - 2*(a*b^2
*c + 2*a^2*b*f)*x)/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3) + 1/27*sqrt(3)*(b
^2*d*(a/b)^(2/3) + 5*a*b*g*(a/b)^(2/3) + b^2*c*(a/b)^(1/3) + 2*a*b*f*(a/b)
^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3) + 1/
54*(18*a*h*(a/b)^(2/3) + b*d*(a/b)^(1/3) + 5*a*g*(a/b)^(1/3) - b*c - 2*a*f
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/27*(9*a*h
*(a/b)^(2/3) - b*d*(a/b)^(1/3) - 5*a*g*(a/b)^(1/3) + b*c + 2*a*f)*log(x +
(a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

3.422.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.10

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \frac{h\log\left(|bx^3+a|\right)}{3\,b^3}$$
$$-\frac{\sqrt{3}\left(b^2c+2\,abf-(-ab^2)^{\frac{1}{3}}\,bd-5\,(-ab^2)^{\frac{1}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,ab^2}$$
$$-\frac{\left(b^2c+2\,abf+(-ab^2)^{\frac{1}{3}}\,bd+5\,(-ab^2)^{\frac{1}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,ab^2}$$
$$+\frac{2\,(b^2d-4\,abg)x^5+(b^2c-7\,abf)x^4-6\,(abe-2\,a^2h)x^3-(abd+5\,a^2g)x^2-2\,(abc+2\,a^2f)x-\frac{3\,(a^2be-3}{b}}{18\,(bx^3+a)^2ab^2}$$
$$-\frac{\left(ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^2b^3g\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^4c+2\,a^2b^3f\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^5}$$

input
```
integrate(x^3*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gi
ac")
```

output
```
1/3*h*log(abs(b*x^3 + a))/b^3 - 1/27*sqrt(3)*(b^2*c + 2*a*b*f - (-a*b^2)^(
1/3)*b*d - 5*(-a*b^2)^(1/3)*a*g)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1/54*(b^2*c + 2*a*b*f + (-a*b^2)^(1/
3)*b*d + 5*(-a*b^2)^(1/3)*a*g)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((
-a*b^2)^(2/3)*a*b^2) + 1/18*(2*(b^2*d - 4*a*b*g)*x^5 + (b^2*c - 7*a*b*f)*x
^4 - 6*(a*b*e - 2*a^2*h)*x^3 - (a*b*d + 5*a^2*g)*x^2 - 2*(a*b*c + 2*a^2*f)
*x - 3*(a^2*b*e - 3*a^3*h)/b)/((b*x^3 + a)^2*a*b^2) - 1/27*(a*b^4*d*(-a/b)
^(1/3) + 5*a^2*b^3*g*(-a/b)^(1/3) + a*b^4*c + 2*a^2*b^3*f)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a^3*b^5)
```

3.422.9 Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 908, normalized size of antiderivative = 2.79

$$\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= \frac{\frac{3a^2h-abe}{6b^3}-\frac{x(bc+2af)}{9b^2}-\frac{x^2(bd+5ag)}{18b^2}-\frac{x^3(be-2ah)}{3b^2}+\frac{x^4(bc-7af)}{18ab}+\frac{x^5(bd-4ag)}{9ab}}{a^2+2abx^3+b^2x^6}$$
$$+\left(\sum_{k=1}^{3}\ln\Bigg(\mathrm{root}\big(19683a^5b^9z^3-19683a^5b^6hz^2+810a^4b^4fgz+405a^3b^5cgz+162a^3b^5dfz+81a^2b^6c$$
$$+\frac{81a^3h^2+b^3cd+5ab^2cg+2ab^2df+10a^2bfg}{81a^2b^4}$$
$$+\frac{x(25a^2g^2-18fha^2+10abdg-9chab+b^2d^2)}{81a^2b^3}\Bigg)\,\mathrm{root}\big(19683a^5b^9z^3$$
$$-19683a^5b^6hz^2+810a^4b^4fgz+405a^3b^5cgz+162a^3b^5dfz+81a^2b^6cdz$$
$$+6561a^5b^3h^2z-270a^4bfgh-135a^3b^2cgh-54a^3b^2dfh-27a^2b^3cdh$$
$$-6ab^4c^2f+75a^3b^2dg^2+15a^2b^3d^2g-12a^2b^3cf^2-8a^3b^2f^3+125a^4bg^3+ab^4d^3$$
$$-729a^5h^3-b^5c^3,z,k\big)\right)$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

output
```
((3*a^2*h - a*b*e)/(6*b^3) - (x*(b*c + 2*a*f))/(9*b^2) - (x^2*(b*d + 5*a*g
))/(18*b^2) - (x^3*(b*e - 2*a*h))/(3*b^2) + (x^4*(b*c - 7*a*f))/(18*a*b) +
 (x^5*(b*d - 4*a*g))/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log(roo
t(19683*a^5*b^9*z^3 - 19683*a^5*b^6*h*z^2 + 810*a^4*b^4*f*g*z + 405*a^3*b^
5*c*g*z + 162*a^3*b^5*d*f*z + 81*a^2*b^6*c*d*z + 6561*a^5*b^3*h^2*z - 270*
a^4*b*f*g*h - 135*a^3*b^2*c*g*h - 54*a^3*b^2*d*f*h - 27*a^2*b^3*c*d*h - 6*
a*b^4*c^2*f + 75*a^3*b^2*d*g^2 + 15*a^2*b^3*d^2*g - 12*a^2*b^3*c*f^2 - 8*a
^3*b^2*f^3 + 125*a^4*b*g^3 + a*b^4*d^3 - 729*a^5*h^3 - b^5*c^3, z, k)*(9*r
oot(19683*a^5*b^9*z^3 - 19683*a^5*b^6*h*z^2 + 810*a^4*b^4*f*g*z + 405*a^3*
b^5*c*g*z + 162*a^3*b^5*d*f*z + 81*a^2*b^6*c*d*z + 6561*a^5*b^3*h^2*z - 27
0*a^4*b*f*g*h - 135*a^3*b^2*c*g*h - 54*a^3*b^2*d*f*h - 27*a^2*b^3*c*d*h -
6*a*b^4*c^2*f + 75*a^3*b^2*d*g^2 + 15*a^2*b^3*d^2*g - 12*a^2*b^3*c*f^2 - 8
*a^3*b^2*f^3 + 125*a^4*b*g^3 + a*b^4*d^3 - 729*a^5*h^3 - b^5*c^3, z, k)*a*
b^2 - (6*a*h)/b + (x*(54*a^2*b^3*f + 27*a*b^4*c))/(81*a^2*b^3)) + (81*a^3*
h^2 + b^3*c*d + 5*a*b^2*c*g + 2*a*b^2*d*f + 10*a^2*b*f*g)/(81*a^2*b^4) + (
x*(b^2*d^2 + 25*a^2*g^2 - 18*a^2*f*h - 9*a*b*c*h + 10*a*b*d*g))/(81*a^2*b^
3))*root(19683*a^5*b^9*z^3 - 19683*a^5*b^6*h*z^2 + 810*a^4*b^4*f*g*z + 405
*a^3*b^5*c*g*z + 162*a^3*b^5*d*f*z + 81*a^2*b^6*c*d*z + 6561*a^5*b^3*h^2*z
 - 270*a^4*b*f*g*h - 135*a^3*b^2*c*g*h - 54*a^3*b^2*d*f*h - 27*a^2*b^3*c*d
*h - 6*a*b^4*c^2*f + 75*a^3*b^2*d*g^2 + 15*a^2*b^3*d^2*g - 12*a^2*b^3*c...
```

3.422. $\int \frac{x^3(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

3.423 $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

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3.423.1 Optimal result

Integrand size = 38, antiderivative size = 297

$$\begin{aligned}
&\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx \\
&= \frac{x(bd-4ag+(2be-5ah)x+3bfx^2)}{18ab^2(a+bx^3)} - \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b(a+bx^3)^2} \\
&- \frac{\left(b^{4/3}d+\sqrt[3]{a}be+2a\sqrt[3]{b}g+5a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{8/3}} \\
&+ \frac{\left(\sqrt[3]{b}(bd+2ag)-\sqrt[3]{a}(be+5ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{8/3}} \\
&- \frac{\left(\sqrt[3]{b}(bd+2ag)-\sqrt[3]{a}(be+5ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{8/3}}
\end{aligned}$$

output
```
1/18*x*(b*d-4*a*g+(-5*a*h+2*b*e)*x+3*b*f*x^2)/a/b^2/(b*x^3+a)+1/6*(-h*x^5-
g*x^4-f*x^3-e*x^2-d*x-c)/b/(b*x^3+a)^2+1/27*(b^(1/3)*(2*a*g+b*d)-a^(1/3)*(
5*a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(8/3)-1/54*(b^(1/3)*(2*a*g+b*d
)-a^(1/3)*(5*a*h+b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b
^(8/3)-1/27*(b^(4/3)*d+a^(1/3)*b*e+2*a*b^(1/3)*g+5*a^(4/3)*h)*arctan(1/3*(
a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(8/3)*3^(1/2)
```

3.423.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.97

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{9b^{2/3}(b(c+x(d+ex))-a(f+x(g+hx)))}{(a+bx^3)^2}+\frac{3b^{2/3}(bx(d+2ex)-a(6f+x(7g+8hx)))}{a(a+bx^3)}-\frac{2\sqrt{3}\left(b^{4/3}d+\sqrt[3]{a}be+2a\sqrt[3]{b}g+5a^{4/3}h\right)\arctan\left(\frac{1-2\ldots}{\sqrt{\ldots}}\right)}{a^{5/3}}\ldots}{54b\ldots}$$

input
```
Integrate[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

output
```
((-9*b^(2/3)*(b*(c + x*(d + e*x)) - a*(f + x*(g + h*x))))/(a + b*x^3)^2 +
(3*b^(2/3)*(b*x*(d + 2*e*x) - a*(6*f + x*(7*g + 8*h*x))))/(a*(a + b*x^3))
- (2*Sqrt[3]*(b^(4/3)*d + a^(1/3)*b*e + 2*a*b^(1/3)*g + 5*a^(4/3)*h)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*(b^(4/3)*d - a^(1/3)*
b*e + 2*a*b^(1/3)*g - 5*a^(4/3)*h)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) + ((-
(b^(4/3)*d) + a^(1/3)*b*e - 2*a*b^(1/3)*g + 5*a^(4/3)*h)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(54*b^(8/3))
```

3.423.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {2363, 2397, 27, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \; 2363$$

$$\frac{\int \frac{5hx^4+4gx^3+3fx^2+2ex+d}{(bx^3+a)^2}dx}{6b}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\,(a+bx^3)^2}$$

$$\Big\downarrow \; 2397$$

$$\frac{\frac{x(x(2be-5ah)-4ag+bd+3bfx^2)}{3ab(a+bx^3)}-\frac{\int -\frac{2b(bd+2ag+(be+5ah)x)}{bx^3+a}dx}{3ab^2}}{6b}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

$\big\downarrow$ 27

$$\frac{\frac{2\int \frac{bd+2ag+(be+5ah)x}{bx^3+a}dx}{3ab}+\frac{x(x(2be-5ah)-4ag+bd+3bfx^2)}{3ab(a+bx^3)}}{6b}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

$\big\downarrow$ 2399

$$\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(bd+2ag)+\sqrt[3]{a}(be+5ah)\right)-\sqrt[3]{b}\left(\sqrt[3]{b}(bd+2ag)-\sqrt[3]{a}(be+5ah)\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(-\frac{\sqrt[3]{a}(5ah+be)}{\sqrt[3]{b}}+2ag+bd\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3ab}+\frac{x(x(2be-5a}{3a}}{6b}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

$\big\downarrow$ 16

$$\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(bd+2ag)+\sqrt[3]{a}(be+5ah)\right)-\sqrt[3]{b}\left(\sqrt[3]{b}(bd+2ag)-\sqrt[3]{a}(be+5ah)\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\frac{\sqrt[3]{a}(5ah+be)}{\sqrt[3]{b}}+2ag+bd\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(x(2be-5a}{3}}{6b}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

$\big\downarrow$ 1142

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(5a^{4/3}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(-\frac{\sqrt[3]{a}(5ah+be)}{\sqrt[3]{b}}+2ag+bd\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\right.}{}\right.}{3ab}}{6b}-\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

$\big\downarrow$ 25

3.423. $\displaystyle\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx$

$$2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(5a^{4/3}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(-\frac{\sqrt[3]{a}(5ah+be)}{\sqrt[3]{b}}+2ag+bd\right)\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{\cdots}}{3ab}\right.$$
$$\overline{\qquad 6b \qquad}$$
$$\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

$\big\downarrow$ 27

$$2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(5a^{4/3}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(-\frac{\sqrt[3]{a}(5ah+be)}{\sqrt[3]{b}}+2ag+bd\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}\cdots\right.}{\cdots}}{3ab}\right.$$
$$\overline{\qquad 6b \qquad}$$
$$\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

$\big\downarrow$ 1082

$$2\left(\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(-\frac{\sqrt[3]{a}(5ah+be)}{\sqrt[3]{b}}+2ag+bd\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(5a^{4/3}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{\cdots}\right.}{\cdots}}{3ab}\right.$$
$$\overline{\qquad 6b \qquad}$$
$$\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

$\big\downarrow$ 217

3.423. $\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx$

$$\frac{2\left(\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(-\frac{\sqrt[3]{a}(5ah+be)}{\sqrt[3]{b}}+2ag+bd\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{4/3}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\cdots\right.}{3a^{2\cdots}}\right.}{3ab}}{6b}$$

$$\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 1103

$$\frac{2\left(\frac{\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(5a^{4/3}h+\sqrt[3]{a}be+2a\sqrt[3]{b}g+b^{4/3}d\right)}{\sqrt[3]{b}}-\frac{1}{2}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(5ah+be)}{\sqrt[3]{b}}+2ag+bd\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\cdots\right.}{3a^{2/\cdots}}\right.}{3ab}}{6b}$$

$$\frac{c+dx+ex^2+fx^3+gx^4+hx^5}{6b\left(a+bx^3\right)^2}$$

input
```
Int[(x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

output
```
-1/6*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(b*(a + b*x^3)^2) + ((x*(b*
d - 4*a*g + (2*b*e - 5*a*h)*x + 3*b*f*x^2))/(3*a*b*(a + b*x^3)) + (2*(((b*
d + 2*a*g - (a^(1/3)*(b*e + 5*a*h))/b^(1/3))*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*b^(1/3)) + (-((Sqrt[3]*(b^(4/3)*d + a^(1/3)*b*e + 2*a*b^(1/3)*g +
5*a^(4/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - ((b*d
 + 2*a*g - (a^(1/3)*(b*e + 5*a*h))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/2)/(3*a^(2/3)*b^(1/3))))/(3*a*b))/(6*b)
```

3.423. $\int \frac{x^2\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx$

3.423.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

3.423. $\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.423.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.51

method	result
risch	$\frac{-\frac{(4ah-be)x^5}{9ab}-\frac{(7ag-bd)x^4}{18ab}-\frac{f x^3}{3b}-\frac{(5ah+be)x^2}{18b^2}-\frac{(2ag+bd)x}{9b^2}-\frac{af+bc}{6b^2}}{(b x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left((5ah+be)\_R+2ag+bd\right)\ln\left(x-\_R\right)}{\_R^2}}{27a b^3}$
default	$\frac{-\frac{(4ah-be)x^5}{9ab}-\frac{(7ag-bd)x^4}{18ab}-\frac{f x^3}{3b}-\frac{(5ah+be)x^2}{18b^2}-\frac{(2ag+bd)x}{9b^2}-\frac{af+bc}{6b^2}}{(b x^3+a)^2}+\frac{(2ag+bd)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\mathrm{arcta}}{}\right.}{}$

input
```
int(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOS
E)
```

output
```
(-1/9*(4*a*h-b*e)/a/b*x^5-1/18*(7*a*g-b*d)/a/b*x^4-1/3*f*x^3/b-1/18*(5*a*h
+b*e)/b^2*x^2-1/9*(2*a*g+b*d)/b^2*x-1/6*(a*f+b*c)/b^2)/(b*x^3+a)^2+1/27/a/
b^3*sum(((5*a*h+b*e)*_R+2*a*g+b*d)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.423.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 6926, normalized size of antiderivative = 23.32

$$\int \frac{x^2(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.423.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{(a + bx^3)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.423.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.04

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = -\frac{6\,abfx^3-2\,(b^2e-4\,abh)x^5-(b^2d-7\,abg)x^4+3\,abc+3\,a^2f+(abe+5\,a^2h)x^2+2\,(abd+2\,a^2g)x}{18\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)} + \frac{\sqrt{3}\left(be\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}+bd+2\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(be\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}-bd-2\,ag\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(be\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,ah\left(\frac{a}{b}\right)^{\frac{1}{3}}-bd-2\,ag\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="ma
xima")
```

output
```
-1/18*(6*a*b*f*x^3 - 2*(b^2*e - 4*a*b*h)*x^5 - (b^2*d - 7*a*b*g)*x^4 + 3*a
*b*c + 3*a^2*f + (a*b*e + 5*a^2*h)*x^2 + 2*(a*b*d + 2*a^2*g)*x)/(a*b^4*x^6
 + 2*a^2*b^3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(b*e*(a/b)^(1/3) + 5*a*h*(a/b)^
(1/3) + b*d + 2*a*g)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(
a*b^3*(a/b)^(2/3)) + 1/54*(b*e*(a/b)^(1/3) + 5*a*h*(a/b)^(1/3) - b*d - 2*a
*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) - 1/27*(b*e
*(a/b)^(1/3) + 5*a*h*(a/b)^(1/3) - b*d - 2*a*g)*log(x + (a/b)^(1/3))/(a*b^
3*(a/b)^(2/3))
```

3.423.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.06

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= -\frac{\sqrt{3}\left(b^2d+2\,abg-(-ab^2)^{\frac{1}{3}}\,be-5\,(-ab^2)^{\frac{1}{3}}\,ah\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,ab^2}$$
$$-\frac{\left(b^2d+2\,abg+(-ab^2)^{\frac{1}{3}}\,be+5\,(-ab^2)^{\frac{1}{3}}\,ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,ab^2}$$
$$-\frac{\left(be\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,ah\left(-\frac{a}{b}\right)^{\frac{1}{3}}+bd+2\,ag\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^2}$$
$$+\frac{2\,b^2ex^5-8\,abhx^5+b^2dx^4-7\,abgx^4-6\,abfx^3-abex^2-5\,a^2hx^2-2\,abdx-4\,a^2gx-3\,abc-3\,a^2f}{18\,(bx^3+a)^2ab^2}$$

input
```
integrate(x^2*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="gi
ac")
```

output
```
-1/27*sqrt(3)*(b^2*d + 2*a*b*g - (-a*b^2)^(1/3)*b*e - 5*(-a*b^2)^(1/3)*a*h
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*
b^2) - 1/54*(b^2*d + 2*a*b*g + (-a*b^2)^(1/3)*b*e + 5*(-a*b^2)^(1/3)*a*h)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) - 1/27*(b*
e*(-a/b)^(1/3) + 5*a*h*(-a/b)^(1/3) + b*d + 2*a*g)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^2*b^2) + 1/18*(2*b^2*e*x^5 - 8*a*b*h*x^5 + b^2*d*x^4 -
 7*a*b*g*x^4 - 6*a*b*f*x^3 - a*b*e*x^2 - 5*a^2*h*x^2 - 2*a*b*d*x - 4*a^2*g
*x - 3*a*b*c - 3*a^2*f)/((b*x^3 + a)^2*a*b^2)
```

3.423.9 Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.11

$$\int \frac{x^2(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$= \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(19683\,a^5\,b^8\,z^3+810\,a^4\,b^3\,g\,h\,z+405\,a^3\,b^4\,d\,h\,z+162\,a^3\,b^4\,e\,g\,z+81\,a^2\,b^5\,d\,e\,z+75\,a^3\,b\,e\,h^2-6 + \frac{b^2\,d\,e+10\,a^2\,g\,h+5\,a\,b\,d\,h+2\,a\,b\,e\,g}{81\,a^2\,b^3} + \frac{x\,(25\,a^2\,h^2+10\,a\,b\,e\,h+b^2\,e^2)}{81\,a^2\,b^3}\right)\mathrm{root}\left(19683\,a^5\,b^8\,z^3+810\,a^4\,b^3\,g\,h\,z+405\,a^3\,b^4\,d\,h\,z + 162\,a^3\,b^4\,e\,g\,z+81\,a^2\,b^5\,d\,e\,z+75\,a^3\,b\,e\,h^2-6\,a\,b^3\,d^2\,g+15\,a^2\,b^2\,e^2\,h-12\,a^2\,b^2\,d\,g^2 - 8\,a^3\,b\,g^3+a\,b^3\,e^3+125\,a^4\,h^3-b^4\,d^3,z,k\right)\right) - \frac{\frac{b\,c+a\,f}{6\,b^2}+\frac{x\,(b\,d+2\,a\,g)}{9\,b^2}+\frac{f\,x^3}{3\,b}+\frac{x^2\,(b\,e+5\,a\,h)}{18\,b^2}-\frac{x^4\,(b\,d-7\,a\,g)}{18\,a\,b}-\frac{x^5\,(b\,e-4\,a\,h)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

input
```
int((x^2*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

output
```
symsum(log(root(19683*a^5*b^8*z^3 + 810*a^4*b^3*g*h*z + 405*a^3*b^4*d*h*z
+ 162*a^3*b^4*e*g*z + 81*a^2*b^5*d*e*z + 75*a^3*b*e*h^2 - 6*a*b^3*d^2*g +
15*a^2*b^2*e^2*h - 12*a^2*b^2*d*g^2 - 8*a^3*b*g^3 + a*b^3*e^3 + 125*a^4*h^
3 - b^4*d^3, z, k)*(9*root(19683*a^5*b^8*z^3 + 810*a^4*b^3*g*h*z + 405*a^3
*b^4*d*h*z + 162*a^3*b^4*e*g*z + 81*a^2*b^5*d*e*z + 75*a^3*b*e*h^2 - 6*a*b
^3*d^2*g + 15*a^2*b^2*e^2*h - 12*a^2*b^2*d*g^2 - 8*a^3*b*g^3 + a*b^3*e^3 +
 125*a^4*h^3 - b^4*d^3, z, k)*a*b^2 + (x*(54*a^2*b^3*g + 27*a*b^4*d))/(81*
a^2*b^3)) + (b^2*d*e + 10*a^2*g*h + 5*a*b*d*h + 2*a*b*e*g)/(81*a^2*b^3) +
(x*(b^2*e^2 + 25*a^2*h^2 + 10*a*b*e*h))/(81*a^2*b^3))*root(19683*a^5*b^8*z
^3 + 810*a^4*b^3*g*h*z + 405*a^3*b^4*d*h*z + 162*a^3*b^4*e*g*z + 81*a^2*b^
5*d*e*z + 75*a^3*b*e*h^2 - 6*a*b^3*d^2*g + 15*a^2*b^2*e^2*h - 12*a^2*b^2*d
*g^2 - 8*a^3*b*g^3 + a*b^3*e^3 + 125*a^4*h^3 - b^4*d^3, z, k), k, 1, 3) -
((b*c + a*f)/(6*b^2) + (x*(b*d + 2*a*g))/(9*b^2) + (f*x^3)/(3*b) + (x^2*(b
*e + 5*a*h))/(18*b^2) - (x^4*(b*d - 7*a*g))/(18*a*b) - (x^5*(b*e - 4*a*h))
/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

3.424 $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$

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3.424.1 Optimal result

Integrand size = 36, antiderivative size = 323

$$\begin{aligned}\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx &= -\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6ab^2(a+bx^3)^2} \\ &\quad+\frac{x(a(be-7ah)+2b(2bc+af)x+3b(bd+ag)x^2)}{18a^2b^2(a+bx^3)} \\ &\quad-\frac{(2b^{5/3}c+a^{2/3}be+ab^{2/3}f+2a^{5/3}h)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{7/3}} \\ &\quad-\frac{(b^{2/3}(2bc+af)-a^{2/3}(be+2ah))\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{7/3}} \\ &\quad+\frac{(b^{2/3}(2bc+af)-a^{2/3}(be+2ah))\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{7/3}}\end{aligned}$$

output
```
-1/6*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/a/b^2/(b*x^3+a)^2+1/
18*x*(a*(-7*a*h+b*e)+2*b*(a*f+2*b*c)*x+3*b*(a*g+b*d)*x^2)/a^2/b^2/(b*x^3+a
)-1/27*(b^(2/3)*(a*f+2*b*c)-a^(2/3)*(2*a*h+b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(
7/3)/b^(7/3)+1/54*(b^(2/3)*(a*f+2*b*c)-a^(2/3)*(2*a*h+b*e))*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(7/3)-1/27*(2*b^(5/3)*c+a^(2/3)*b*e+
a*b^(2/3)*f+2*a^(5/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))
/a^(7/3)/b^(7/3)*3^(1/2)
```

3.424.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.92

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{3\sqrt[3]{a}\sqrt[3]{b}(-4b^2cx^2-abx(e+2fx)+a^2(6g+7hx))}{a+bx^3}+\frac{9a^{4/3}\sqrt[3]{b}(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{(a+bx^3)^2}-2\sqrt{3}\left(2b^{5/3}c+a^{2/3}be+ab^2\right.}{}$$

input
```
Integrate[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

output
```
((-3*a^(1/3)*b^(1/3)*(-4*b^2*c*x^2 - a*b*x*(e + 2*f*x) + a^2*(6*g + 7*h*x)
))/(a + b*x^3) + (9*a^(4/3)*b^(1/3)*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d +
x*(e + f*x))))/(a + b*x^3)^2 - 2*Sqrt[3]*(2*b^(5/3)*c + a^(2/3)*b*e + a*b^
(2/3)*f + 2*a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(-2
*b^(5/3)*c + a^(2/3)*b*e - a*b^(2/3)*f + 2*a^(5/3)*h)*Log[a^(1/3) + b^(1/3
)*x] + (2*b^(5/3)*c - a^(2/3)*b*e + a*b^(2/3)*f - 2*a^(5/3)*h)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(7/3))
```

3.424.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {2367, 25, 2397, 27, 2399, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \; 2367$$

$$-\frac{\int -\frac{6abhx^3+3b(bd+ag)x^2+2b(2bc+af)x+a(be-ah)}{(bx^3+a)^2}dx}{6ab^2}-\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{6abhx^3+3b(bd+ag)x^2+2b(2bc+af)x+a(be-ah)}{(bx^3+a)^2}dx}{6ab^2}-\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 2397}$$

$$\frac{\frac{x\left(2bx(af+2bc)+3bx^2(ag+bd)+a(be-7ah)\right)}{3a(a+bx^3)}-\frac{\int -\frac{2b(a(be+2ah)+b(2bc+af)x)}{bx^3+a}dx}{3ab}}{6ab^2}-\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2\int \frac{a(be+2ah)+b(2bc+af)x}{bx^3+a}dx}{3a}+\frac{x\left(2bx(af+2bc)+3bx^2(ag+bd)+a(be-7ah)\right)}{3a(a+bx^3)}}{6ab^2}-\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 2399}$$

$$\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}\left(b^{2/3}(2bc+af)+2a^{2/3}(be+2ah)\right)+\sqrt[3]{b}\left(b^{2/3}(2bc+af)-a^{2/3}(be+2ah)\right)x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}}\right)}{3a}+}{6ab^2}$$

$$-\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}\left(b^{2/3}(2bc+af)+2a^{2/3}(be+2ah)\right)+\sqrt[3]{b}\left(b^{2/3}(2bc+af)-a^{2/3}(be+2ah)\right)x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}+}{6ab^2}$$

$$-\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 27}$$

3.424. $\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{(a+bx^3)^3}\,dx$

$$\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\left(b^{2/3}(2bc+af)+2a^{2/3}(be+2ah)\right)+\sqrt[3]{b}\left(b^{2/3}(2bc+af)-a^{2/3}(be+2ah)\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}+\frac{x(2bx(af+}{}}{6ab^2}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{2/3}be+2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\log\left(\sqrt[3]{a}+\right.}{}\right.}{3a}}{6ab^2}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{2/3}be+2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{}\right.}{3a}}{6ab^2}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{2/3}be+2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{}\right.}{3a}}{6ab^2}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

3.424. $\displaystyle\int \frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx$

$\Big\downarrow$ 1082

$$2\left(\frac{\frac{\frac{3\left(a^{2/3}be+2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}}-\frac{\log\left(\sqrt[3]{a}-\right.}{}}{3a}\right.}{6ab^2}$$

$$\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 217

$$2\left(\frac{\frac{-\frac{1}{2}\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{2/3}be+2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}\right.}{3\sqrt[3]{}}}{3a}\right.}{6ab^2}$$

$$\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1103

$$2\left(\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(b^{2/3}(af+2bc)-a^{2/3}(2ah+be)\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{2/3}be+2a^{5/3}h+ab^{2/3}f+2b^{5/3}c\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(b^{2/3}\right.}{3\sqrt[3]{a}}}{3a}\right.}{6ab^2}$$

$$\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6ab^2\left(a+bx^3\right)^2}$$

3.424. $\displaystyle\int\frac{x\left(c+dx+ex^2+fx^3+gx^4+hx^5\right)}{\left(a+bx^3\right)^3}\,dx$

input
```
Int[(x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x]
```

output
```
-1/6*(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(a*b^2*(a +
 b*x^3)^2) + ((x*(a*(b*e - 7*a*h) + 2*b*(2*b*c + a*f)*x + 3*b*(b*d + a*g)*
x^2))/(3*a*(a + b*x^3)) + (2*(-1/3*((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e
+ 2*a*h))*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*b^(1/3)) + (-((Sqrt[3]*(2*b^(
5/3)*c + a^(2/3)*b*e + a*b^(2/3)*f + 2*a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]])/b^(1/3)) + ((b^(2/3)*(2*b*c + a*f) - a^(2/3)*(b*e + 2
*a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)))/(3*a^(
1/3))))/(3*a))/(6*a*b^2)
```

3.424.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.424. $\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3} dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.424.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.67 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.49

method	result
risch	$\frac{\frac{(af+2bc)x^5}{9a^2}-\frac{(7ah-be)x^4}{18ab}-\frac{g\,x^3}{3b}-\frac{(af-7bc)x^2}{18ab}-\frac{(2ah+be)x}{9b^2}-\frac{ag+bd}{6b^2}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\left(\frac{(af+2bc)\_R}{a}+\frac{2ah+be}{b}\right)\ln(x-\_R)}{\_R^2}}{27a\,b^2}$
default	$\frac{\frac{(af+2bc)x^5}{9a^2}-\frac{(7ah-be)x^4}{18ab}-\frac{g\,x^3}{3b}-\frac{(af-7bc)x^2}{18ab}-\frac{(2ah+be)x}{9b^2}-\frac{ag+bd}{6b^2}}{(b\,x^3+a)^2}+\frac{(2a^2h+aeb)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\ \mathrm{arct}}{}\right.}{}$

input
```
int(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/9*(a*f+2*b*c)/a^2*x^5-1/18*(7*a*h-b*e)/a/b*x^4-1/3*g*x^3/b-1/18*(a*f-7*
b*c)/a/b*x^2-1/9*(2*a*h+b*e)/b^2*x-1/6*(a*g+b*d)/b^2)/(b*x^3+a)^2+1/27/a/b
^2*sum((1/a*(a*f+2*b*c)*_R+1/b*(2*a*h+b*e))/_R^2*ln(x-_R),_R=RootOf(_Z^3*b
+a))
```

3.424.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 7190, normalized size of antiderivative = 22.26

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fric
as")
```

output
```
Too large to include
```

3.424.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.424.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.07

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx =$$

$$-\frac{6\,a^2bgx^3 - 2\,(2\,b^3c + ab^2f)x^5 - (ab^2e - 7\,a^2bh)x^4 + 3\,a^2bd + 3\,a^3g - (7\,ab^2c - a^2bf)x^2 + 2\,(a^2be + 2\,a^3}{18\,(a^2b^4x^6 + 2\,a^3b^3x^3 + a^4b^2)}$$
$$+\frac{\sqrt{3}\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} + abf\left(\frac{a}{b}\right)^{\frac{1}{3}} + abe + 2\,a^2h\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} + abf\left(\frac{a}{b}\right)^{\frac{1}{3}} - abe - 2\,a^2h\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(2\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}} + abf\left(\frac{a}{b}\right)^{\frac{1}{3}} - abe - 2\,a^2h\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxi
ma")
```

output
```
-1/18*(6*a^2*b*g*x^3 - 2*(2*b^3*c + a*b^2*f)*x^5 - (a*b^2*e - 7*a^2*b*h)*x
^4 + 3*a^2*b*d + 3*a^3*g - (7*a*b^2*c - a^2*b*f)*x^2 + 2*(a^2*b*e + 2*a^3*
h)*x)/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2) + 1/27*sqrt(3)*(2*b^2*c*(a/b
)^(1/3) + a*b*f*(a/b)^(1/3) + a*b*e + 2*a^2*h)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3)) + 1/54*(2*b^2*c*(a/b)^(1/3)
 + a*b*f*(a/b)^(1/3) - a*b*e - 2*a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2
/3))/(a^2*b^3*(a/b)^(2/3)) - 1/27*(2*b^2*c*(a/b)^(1/3) + a*b*f*(a/b)^(1/3)
 - a*b*e - 2*a^2*h)*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))
```

3.424.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.04

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$
$$= -\frac{\sqrt{3}\left(abe+2\,a^2h-2\,(-ab^2)^{\frac{1}{3}}\,bc-(-ab^2)^{\frac{1}{3}}\,af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^2b} - \frac{\left(abe+2\,a^2h+2\,(-ab^2)^{\frac{1}{3}}\,bc+(-ab^2)^{\frac{1}{3}}\,af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^2b} - \frac{\left(2\,b^2c\left(-\frac{a}{b}\right)^{\frac{1}{3}}+abf\left(-\frac{a}{b}\right)^{\frac{1}{3}}+abe+2\,a^2h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b^2} + \frac{4\,b^3cx^5+2\,ab^2fx^5+ab^2ex^4-7\,a^2bhx^4-6\,a^2bgx^3+7\,ab^2cx^2-a^2bfx^2-2\,a^2bex-4\,a^3hx-3\,a^2bd-}{18\,(bx^3+a)^2a^2b^2}$$

input
```
integrate(x*(h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac
")
```

output
```
-1/27*sqrt(3)*(a*b*e + 2*a^2*h - 2*(-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*f
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^
2*b) - 1/54*(a*b*e + 2*a^2*h + 2*(-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*f)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) - 1/27*(2*
b^2*c*(-a/b)^(1/3) + a*b*f*(-a/b)^(1/3) + a*b*e + 2*a^2*h)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a^3*b^2) + 1/18*(4*b^3*c*x^5 + 2*a*b^2*f*x^5 + a
*b^2*e*x^4 - 7*a^2*b*h*x^4 - 6*a^2*b*g*x^3 + 7*a*b^2*c*x^2 - a^2*b*f*x^2 -
 2*a^2*b*e*x - 4*a^3*h*x - 3*a^2*b*d - 3*a^3*g)/((b*x^3 + a)^2*a^2*b^2)
```

3.424.9 Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.98

$$\int \frac{x(c+dx+ex^2+fx^3+gx^4+hx^5)}{(a+bx^3)^3}\,dx$$

$$= \left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\left(19683\,a^7\,b^7\,z^3+162\,a^5\,b^3\,f\,h\,z+324\,a^4\,b^4\,c\,h\,z+81\,a^4\,b^4\,e\,f\,z+162\,a^3\,b^5\,c\,e\,z-12\,a^4\,b\,e\,h^2+1\right.\right.\right.$$

$$+\frac{2\,b^2\,c\,e+2\,a^2\,f\,h+4\,a\,b\,c\,h+a\,b\,e\,f}{81\,a^3\,b^2}$$

$$\left.+\frac{x\,(a^2\,f^2+4\,a\,b\,c\,f+4\,b^2\,c^2)}{81\,a^4\,b}\right)\mathrm{root}\left(19683\,a^7\,b^7\,z^3+162\,a^5\,b^3\,f\,h\,z+324\,a^4\,b^4\,c\,h\,z\right.$$

$$+81\,a^4\,b^4\,e\,f\,z+162\,a^3\,b^5\,c\,e\,z-12\,a^4\,b\,e\,h^2+12\,a\,b^4\,c^2\,f-6\,a^3\,b^2\,e^2\,h+6\,a^2\,b^3\,c\,f^2$$

$$\left.\left.+a^3\,b^2\,f^3-8\,a^5\,h^3+8\,b^5\,c^3-a^2\,b^3\,e^3,z,k\right)\right)$$

$$-\frac{\frac{b\,d+a\,g}{6\,b^2}+\frac{x\,(b\,e+2\,a\,h)}{9\,b^2}+\frac{g\,x^3}{3\,b}-\frac{x^5\,(2\,b\,c+a\,f)}{9\,a^2}-\frac{x^2\,(7\,b\,c-a\,f)}{18\,a\,b}-\frac{x^4\,(b\,e-7\,a\,h)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

input
```
int((x*(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5))/(a + b*x^3)^3,x)
```

output
```
symsum(log(root(19683*a^7*b^7*z^3 + 162*a^5*b^3*f*h*z + 324*a^4*b^4*c*h*z
+ 81*a^4*b^4*e*f*z + 162*a^3*b^5*c*e*z - 12*a^4*b*e*h^2 + 12*a*b^4*c^2*f -
 6*a^3*b^2*e^2*h + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 - 8*a^5*h^3 + 8*b^5*c^3 -
 a^2*b^3*e^3, z, k)*(9*root(19683*a^7*b^7*z^3 + 162*a^5*b^3*f*h*z + 324*a^
4*b^4*c*h*z + 81*a^4*b^4*e*f*z + 162*a^3*b^5*c*e*z - 12*a^4*b*e*h^2 + 12*a
*b^4*c^2*f - 6*a^3*b^2*e^2*h + 6*a^2*b^3*c*f^2 + a^3*b^2*f^3 - 8*a^5*h^3 +
 8*b^5*c^3 - a^2*b^3*e^3, z, k)*a*b^2 + (x*(27*a^3*b^2*e + 54*a^4*b*h))/(8
1*a^4*b)) + (2*b^2*c*e + 2*a^2*f*h + 4*a*b*c*h + a*b*e*f)/(81*a^3*b^2) + (
x*(4*b^2*c^2 + a^2*f^2 + 4*a*b*c*f))/(81*a^4*b))*root(19683*a^7*b^7*z^3 +
162*a^5*b^3*f*h*z + 324*a^4*b^4*c*h*z + 81*a^4*b^4*e*f*z + 162*a^3*b^5*c*e
*z - 12*a^4*b*e*h^2 + 12*a*b^4*c^2*f - 6*a^3*b^2*e^2*h + 6*a^2*b^3*c*f^2 +
 a^3*b^2*f^3 - 8*a^5*h^3 + 8*b^5*c^3 - a^2*b^3*e^3, z, k), k, 1, 3) - ((b*
d + a*g)/(6*b^2) + (x*(b*e + 2*a*h))/(9*b^2) + (g*x^3)/(3*b) - (x^5*(2*b*c
 + a*f))/(9*a^2) - (x^2*(7*b*c - a*f))/(18*a*b) - (x^4*(b*e - 7*a*h))/(18*
a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

3.425 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$

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3.425.1 Optimal result

Integrand size = 35, antiderivative size = 313

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx = \frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6ab(a+bx^3)^2} - \frac{3a(be+ah)-bx(5bc+af+2(2bd+ag)x)}{18a^2b^2(a+bx^3)} - \frac{\left(5b^{4/3}c+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{5/3}} + \frac{\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{5/3}} - \frac{\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{5/3}}$$

output
```
1/6*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/a/b/(b*x^3+a)^2+1/18*(-3*a*(a*
h+b*e)+b*x*(5*b*c+a*f+2*(a*g+2*b*d)*x))/a^2/b^2/(b*x^3+a)+1/27*(b^(1/3)*(a
*f+5*b*c)-a^(1/3)*(a*g+2*b*d))*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(5/3)-1/54*
(b^(1/3)*(a*f+5*b*c)-a^(1/3)*(a*g+2*b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/a^(8/3)/b^(5/3)-1/27*(5*b^(4/3)*c+2*a^(1/3)*b*d+a*b^(1/3)*f+a^(4
/3)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(5/3)*3
^(1/2)
```

3.425.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.94

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$$

$$= \frac{\frac{3a^{2/3}(-6a^2h+b^2x(5c+4dx)+abx(f+2gx))}{a+bx^3}+\frac{9a^{5/3}(a^2h+b^2x(c+dx)-ab(e+x(f+gx)))}{(a+bx^3)^2}-2\sqrt{3}\sqrt[3]{b}\left(5b^{4/3}c+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]
```

output
```
((3*a^(2/3)*(-6*a^2*h + b^2*x*(5*c + 4*d*x) + a*b*x*(f + 2*g*x)))/(a + b*x
^3) + (9*a^(5/3)*(a^2*h + b^2*x*(c + d*x) - a*b*(e + x*(f + g*x))))/(a + b
*x^3)^2 - 2*Sqrt[3]*b^(1/3)*(5*b^(4/3)*c + 2*a^(1/3)*b*d + a*b^(1/3)*f + a
^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*(5*b^(4/
3)*c - 2*a^(1/3)*b*d + a*b^(1/3)*f - a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x] +
 b^(1/3)*(-5*b^(4/3)*c + 2*a^(1/3)*b*d - a*b^(1/3)*f + a^(4/3)*g)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^2)
```

3.425.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {2397, 25, 2393, 27, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \; 2397$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\,(a+bx^3)^2}-\frac{\int -\frac{3b(be+ah)x^2+2b(2bd+ag)x+b(5bc+af)}{(bx^3+a)^2}dx}{6ab^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3b(be+ah)x^2+2b(2bd+ag)x+b(5bc+af)}{(bx^3+a)^2}dx}{6ab^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 2393

$$\frac{-\frac{\int -\frac{2b(5bc+af+(2bd+ag)x)}{bx^3+a}dx}{3a}-\frac{3a(ah+be)-bx(2x(ag+2bd)+af+5bc)}{3a(a+bx^3)}}{6ab^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{2b\int \frac{5bc+af+(2bd+ag)x}{bx^3+a}dx}{3a}-\frac{3a(ah+be)-bx(2x(ag+2bd)+af+5bc)}{3a(a+bx^3)}}{6ab^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 2399

$$\frac{\frac{2b\left(\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(5bc+af)+\sqrt[3]{a}(2bd+ag)\right)-\sqrt[3]{b}\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(-\frac{\sqrt[3]{a}(ag+2bd)}{\sqrt[3]{b}}+af+5bc\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}-\frac{3a(ah+be)-}{}}{6ab^2}$$
$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 16

$$\frac{\frac{2b\left(\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}(5bc+af)+\sqrt[3]{a}(2bd+ag)\right)-\sqrt[3]{b}\left(\sqrt[3]{b}(5bc+af)-\sqrt[3]{a}(2bd+ag)\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\frac{\sqrt[3]{a}(ag+2bd)}{\sqrt[3]{b}}+af+5bc\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}-\frac{3a(ah+be)}{}}{6ab^2}$$
$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\left(a+bx^3\right)^2}$$

$\downarrow$ 1142

3.425. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$

$$\frac{2b\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{4/3}g+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{4/3}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(-\frac{\sqrt[3]{a}(ag+2bd)}{\sqrt[3]{b}}+af+5bc\right)\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\cdots\right.}{\cdots}\cdots\right.}{3a}\cdots}{6ab^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{2b\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{4/3}g+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{4/3}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(-\frac{\sqrt[3]{a}(ag+2bd)}{\sqrt[3]{b}}+af+5bc\right)\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\cdots\right.}{\cdots}\cdots\right.}{3a}\cdots}{6ab^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{4/3}g+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{4/3}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(-\frac{\sqrt[3]{a}(ag+2bd)}{\sqrt[3]{b}}+af+5bc\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\cdots\right.}{\cdots}\cdots\right.}{3a}\cdots}{6ab^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{2b\left(\frac{\frac{1}{2}\sqrt[3]{b}\left(-\frac{\sqrt[3]{a}(ag+2bd)}{\sqrt[3]{b}}+af+5bc\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(a^{4/3}g+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{4/3}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\cdots\right.}{\cdots}\cdots\right.}{3a}\cdots}{6ab^2}+\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\left(a+bx^3\right)^2}$$

3.425. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{\left(a+bx^3\right)^3}\,dx$

$$\downarrow 217$$

$$\frac{\frac{2b\left(\frac{\frac{1}{2}\sqrt[3]{b}\left(-\frac{\sqrt[3]{a}(ag+2bd)}{\sqrt[3]{b}}+af+5bc\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}g+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{4/3}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\cdots\right.}{3a\cdots}\right.}{3a}}{6ab^2}\cdots$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\left(a+bx^3\right)^2}$$

$$\downarrow 1103$$

$$\frac{\frac{2b\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(a^{4/3}g+2\sqrt[3]{a}bd+a\sqrt[3]{b}f+5b^{4/3}c\right)}{\sqrt[3]{b}}-\frac{1}{2}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(ag+2bd)}{\sqrt[3]{b}}+af+5bc\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(-\cdots\right.}{3a^{2/\cdots}}\right.}{3a}}{6ab^2}\cdots$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6ab\left(a+bx^3\right)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x]
```

output
```
(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(6*a*b*(a + b*x^3)^2) +
(-1/3*(3*a*(b*e + a*h) - b*x*(5*b*c + a*f + 2*(2*b*d + a*g)*x))/(a*(a + b*
x^3)) + (2*b*(((5*b*c + a*f - (a^(1/3)*(2*b*d + a*g))/b^(1/3))*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*(5*b^(4/3)*c + 2*a^(1/3)*
b*d + a*b^(1/3)*f + a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
)/b^(1/3)) - ((5*b*c + a*f - (a^(1/3)*(2*b*d + a*g))/b^(1/3))*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)*b^(1/3))))/(3*a))/(6*a*b
^2)
```

3.425. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$

3.425.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

3.425. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.425.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.71 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.48

method	result
risch	$\frac{\frac{(ag+2bd)x^5}{9a^2}+\frac{(af+5bc)x^4}{18a^2}-\frac{h x^3}{3b}-\frac{(ag-7bd)x^2}{18ab}-\frac{(af-4bc)x}{9ab}-\frac{ah+be}{6b^2}}{(b x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left((ag+2bd)\_R+af+5bc\right)\ln\left(x-\_R\right)}{\_R^2}}{27a^2b^2}$
default	$\frac{\frac{(ag+2bd)x^5}{9a^2}+\frac{(af+5bc)x^4}{18a^2}-\frac{h x^3}{3b}-\frac{(ag-7bd)x^2}{18ab}-\frac{(af-4bc)x}{9ab}-\frac{ah+be}{6b^2}}{(b x^3+a)^2}+\frac{(af+5bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan}{}\right.}{}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/9*(a*g+2*b*d)/a^2*x^5+1/18*(a*f+5*b*c)/a^2*x^4-1/3*h*x^3/b-1/18*(a*g-7*
b*d)/a/b*x^2-1/9*(a*f-4*b*c)/a/b*x-1/6*(a*h+b*e)/b^2)/(b*x^3+a)^2+1/27/a^2
/b^2*sum(((a*g+2*b*d)*_R+a*f+5*b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

3.425. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$

3.425.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 6984, normalized size of antiderivative = 22.31

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a + bx^3)^3} \, dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="fricas
")
```

output
```
Too large to include
```

3.425.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{(a + bx^3)^3} \, dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.425.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx = -\frac{6\,a^2bhx^3-2\,(2\,b^3d+ab^2g)x^5-(5\,b^3c+ab^2f)x^4+3\,a^2be+3\,a^3h-(7\,ab^2d-a^2bg)x^2-2\,(4\,ab^2c-a^2b}{18\,(a^2b^4x^6+2\,a^3b^3x^3+a^4b^2)} + \frac{\sqrt{3}\left(2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,bc+af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,bc-af\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(2\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,bc-af\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="maxima
")
```

output
```
-1/18*(6*a^2*b*h*x^3 - 2*(2*b^3*d + a*b^2*g)*x^5 - (5*b^3*c + a*b^2*f)*x^4
 + 3*a^2*b*e + 3*a^3*h - (7*a*b^2*d - a^2*b*g)*x^2 - 2*(4*a*b^2*c - a^2*b*
f)*x)/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2) + 1/27*sqrt(3)*(2*b*d*(a/b)^
(1/3) + a*g*(a/b)^(1/3) + 5*b*c + a*f)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/
3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) + 1/54*(2*b*d*(a/b)^(1/3) + a*g*(a/
b)^(1/3) - 5*b*c - a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a
/b)^(2/3)) - 1/27*(2*b*d*(a/b)^(1/3) + a*g*(a/b)^(1/3) - 5*b*c - a*f)*log(
x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

3.425.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$$

$$= -\frac{\sqrt{3}\left(5\,b^2c+abf-2\,(-ab^2)^{\frac{1}{3}}\,bd-(-ab^2)^{\frac{1}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^2b} - \frac{\left(5\,b^2c+abf+2\,(-ab^2)^{\frac{1}{3}}\,bd+(-ab^2)^{\frac{1}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^2b} - \frac{\left(2\,bd\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ag\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,bc+af\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b} + \frac{4\,b^3dx^5+2\,ab^2gx^5+5\,b^3cx^4+ab^2fx^4-6\,a^2bhx^3+7\,ab^2dx^2-a^2bgx^2+8\,ab^2cx-2\,a^2bfx-3\,a^2be-}{18\,(bx^3+a)^2a^2b^2}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(5*b^2*c + a*b*f - 2*(-a*b^2)^(1/3)*b*d - (-a*b^2)^(1/3)*a*g
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^
2*b) - 1/54*(5*b^2*c + a*b*f + 2*(-a*b^2)^(1/3)*b*d + (-a*b^2)^(1/3)*a*g)*
log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) - 1/27*(2*
b*d*(-a/b)^(1/3) + a*g*(-a/b)^(1/3) + 5*b*c + a*f)*(-a/b)^(1/3)*log(abs(x
- (-a/b)^(1/3)))/(a^3*b) + 1/18*(4*b^3*d*x^5 + 2*a*b^2*g*x^5 + 5*b^3*c*x^4
 + a*b^2*f*x^4 - 6*a^2*b*h*x^3 + 7*a*b^2*d*x^2 - a^2*b*g*x^2 + 8*a*b^2*c*x
 - 2*a^2*b*f*x - 3*a^2*b*e - 3*a^3*h)/((b*x^3 + a)^2*a^2*b^2)
```

3.425.9 Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.01

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{(a+bx^3)^3}\,dx$$
$$=\frac{\frac{x^4\,(5\,b\,c+a\,f)}{18\,a^2}-\frac{h\,x^3}{3\,b}-\frac{b\,e+a\,h}{6\,b^2}+\frac{x^5\,(2\,b\,d+a\,g)}{9\,a^2}+\frac{x\,(4\,b\,c-a\,f)}{9\,a\,b}+\frac{x^2\,(7\,b\,d-a\,g)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$
$$+\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\big(19683\,a^8\,b^5\,z^3+81\,a^5\,b^2\,f\,g\,z+405\,a^4\,b^3\,c\,g\,z+162\,a^4\,b^3\,d\,f\,z+810\,a^3\,b^4\,c\,d\,z+6\,a^3\,b\,d\,g^2-\right.\right.$$
$$+\frac{10\,b^2\,c\,d+a^2\,f\,g+5\,a\,b\,c\,g+2\,a\,b\,d\,f}{81\,a^4\,b}$$
$$\left.+\frac{x\,(a^2\,g^2+4\,a\,b\,d\,g+4\,b^2\,d^2)}{81\,a^4\,b}\right)\mathrm{root}\big(19683\,a^8\,b^5\,z^3+81\,a^5\,b^2\,f\,g\,z+405\,a^4\,b^3\,c\,g\,z$$
$$+162\,a^4\,b^3\,d\,f\,z+810\,a^3\,b^4\,c\,d\,z+6\,a^3\,b\,d\,g^2-75\,a\,b^3\,c^2\,f+12\,a^2\,b^2\,d^2\,g-15\,a^2\,b^2\,c\,f^2$$
$$\left.+8\,a\,b^3\,d^3+a^4\,g^3-125\,b^4\,c^3-a^3\,b\,f^3,z,k\big)\right)$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(a + b*x^3)^3,x)
```

output
```
((x^4*(5*b*c + a*f))/(18*a^2) - (h*x^3)/(3*b) - (b*e + a*h)/(6*b^2) + (x^5
*(2*b*d + a*g))/(9*a^2) + (x*(4*b*c - a*f))/(9*a*b) + (x^2*(7*b*d - a*g))/
(18*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log(root(19683*a^8*b^5*z^3
+ 81*a^5*b^2*f*g*z + 405*a^4*b^3*c*g*z + 162*a^4*b^3*d*f*z + 810*a^3*b^4*c
*d*z + 6*a^3*b*d*g^2 - 75*a*b^3*c^2*f + 12*a^2*b^2*d^2*g - 15*a^2*b^2*c*f^
2 + 8*a*b^3*d^3 + a^4*g^3 - 125*b^4*c^3 - a^3*b*f^3, z, k)*(9*root(19683*a
^8*b^5*z^3 + 81*a^5*b^2*f*g*z + 405*a^4*b^3*c*g*z + 162*a^4*b^3*d*f*z + 81
0*a^3*b^4*c*d*z + 6*a^3*b*d*g^2 - 75*a*b^3*c^2*f + 12*a^2*b^2*d^2*g - 15*a
^2*b^2*c*f^2 + 8*a*b^3*d^3 + a^4*g^3 - 125*b^4*c^3 - a^3*b*f^3, z, k)*a*b^
2 + (x*(135*a^2*b^3*c + 27*a^3*b^2*f))/(81*a^4*b)) + (10*b^2*c*d + a^2*f*g
 + 5*a*b*c*g + 2*a*b*d*f)/(81*a^4*b) + (x*(4*b^2*d^2 + a^2*g^2 + 4*a*b*d*g
))/(81*a^4*b))*root(19683*a^8*b^5*z^3 + 81*a^5*b^2*f*g*z + 405*a^4*b^3*c*g
*z + 162*a^4*b^3*d*f*z + 810*a^3*b^4*c*d*z + 6*a^3*b*d*g^2 - 75*a*b^3*c^2*
f + 12*a^2*b^2*d^2*g - 15*a^2*b^2*c*f^2 + 8*a*b^3*d^3 + a^4*g^3 - 125*b^4*
c^3 - a^3*b*f^3, z, k), k, 1, 3)
```

3.426 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^3}\,dx$

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3.426.1 Optimal result

Integrand size = 38, antiderivative size = 347

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^3}\,dx &= \frac{x(a(bd-ag)+a(be-ah)x-b(bc-af)x^2)}{6a^2b(a+bx^3)^2} \\ &+ \frac{x(a(5bd+ag)+2a(2be+ah)x-3b(3bc-af)x^2)}{18a^3b(a+bx^3)} \\ &- \frac{\left(5b^{4/3}d+2\sqrt[3]{a}be+a\sqrt[3]{b}g+a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{5/3}} \\ &+ \frac{c\log(x)}{a^3} + \frac{\left(\sqrt[3]{b}(5bd+ag)-\sqrt[3]{a}(2be+ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{5/3}} \\ &- \frac{\left(\sqrt[3]{b}(5bd+ag)-\sqrt[3]{a}(2be+ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{5/3}} - \frac{c\log(a+bx^3)}{3a^3}\end{aligned}$$

output
```
1/6*x*(a*(-a*g+b*d)+a*(-a*h+b*e)*x-b*(-a*f+b*c)*x^2)/a^2/b/(b*x^3+a)^2+1/1
8*x*(a*(a*g+5*b*d)+2*a*(a*h+2*b*e)*x-3*b*(-a*f+3*b*c)*x^2)/a^3/b/(b*x^3+a)
+c*ln(x)/a^3+1/27*(b^(1/3)*(a*g+5*b*d)-a^(1/3)*(a*h+2*b*e))*ln(a^(1/3)+b^(
1/3)*x)/a^(8/3)/b^(5/3)-1/54*(b^(1/3)*(a*g+5*b*d)-a^(1/3)*(a*h+2*b*e))*ln(
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(5/3)-1/3*c*ln(b*x^3+a)/a
^3-1/27*(5*b^(4/3)*d+2*a^(1/3)*b*e+a*b^(1/3)*g+a^(4/3)*h)*arctan(1/3*(a^(1
/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(5/3)*3^(1/2)
```

3.426.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.90

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\,(a+bx^3)^3}\,dx$$

$$= \frac{\frac{3a(6bc+bx(5d+4ex)+ax(g+2hx))}{b(a+bx^3)} - \frac{9a^2(-b(c+x(d+ex))+a(f+x(g+hx)))}{b(a+bx^3)^2} - \frac{2\sqrt{3}\sqrt[3]{a}\left(5b^{4/3}d+2\sqrt[3]{a}be+a\sqrt[3]{b}g+a^{4/3}h\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}}{\sqrt{3}}\right.}{b^{5/3}}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]
```

output
```
((3*a*(6*b*c + b*x*(5*d + 4*e*x) + a*x*(g + 2*h*x)))/(b*(a + b*x^3)) - (9*
a^2*(-(b*(c + x*(d + e*x))) + a*(f + x*(g + h*x))))/(b*(a + b*x^3)^2) - (2
*Sqrt[3]*a^(1/3)*(5*b^(4/3)*d + 2*a^(1/3)*b*e + a*b^(1/3)*g + a^(4/3)*h)*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(5/3) + 54*c*Log[x] + (2*a^(
1/3)*(5*b^(4/3)*d - 2*a^(1/3)*b*e + a*b^(1/3)*g - a^(4/3)*h)*Log[a^(1/3) +
 b^(1/3)*x])/b^(5/3) + (a^(1/3)*(-5*b^(4/3)*d + 2*a^(1/3)*b*e - a*b^(1/3)*
g + a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - 1
8*c*Log[a + b*x^3])/(54*a^3)
```

3.426.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\,(a+bx^3)^3}\,dx$$

$$\Big\downarrow \; 2368$$

$$\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{6a^2b\,(a+bx^3)^2} - \frac{\int -\frac{-3b^2\left(\frac{bc}{a}-f\right)x^3+2b(2be+ah)x^2+b(5bd+ag)x+6b^2c}{x(bx^3+a)^2}dx}{6ab^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-3b^2\left(\frac{bc}{a}-f\right)x^3+2b(2be+ah)x^2+b(5bd+ag)x+6b^2c}{x(bx^3+a)^2}dx}{6ab^2}+\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{2368}$$

$$\frac{\frac{x(-3b^2x^2(3bc-af)+ab(ag+5bd)+2abx(ah+2be))}{3a^2(a+bx^3)}-\frac{\int -\frac{2\left(9cb^3+(2be+ah)x^2b^2+(5bd+ag)xb^2\right)}{x\left(bx^3+a\right)}dx}{3ab}}{6ab^2}+\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{9cb^3+(2be+ah)x^2b^2+(5bd+ag)xb^2}{x\left(bx^3+a\right)}dx}{3ab}+\frac{x(-3b^2x^2(3bc-af)+ab(ag+5bd)+2abx(ah+2be))}{3a^2(a+bx^3)}}{6ab^2}+\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{2373}$$

$$\frac{\frac{2\int \left(\frac{9cb^3}{ax}+\frac{\left(-9b^2cx^2+a(2be+ah)x+a(5bd+ag)\right)b^2}{a\left(bx^3+a\right)}\right)dx}{3ab}+\frac{x(-3b^2x^2(3bc-af)+ab(ag+5bd)+2abx(ah+2be))}{3a^2(a+bx^3)}}{6ab^2}+\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-bx^2(bc-af)+a(bd-ag)+ax(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}+$$

$$\frac{\frac{2\left(-\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(a^{4/3}h+2\sqrt[3]{a}be+a\sqrt[3]{b}g+5b^{4/3}d\right)}{\sqrt{3}a^{2/3}}-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{\sqrt[3]{a}(ah+2be)}{\sqrt[3]{b}}+ag+5bd\right)}{6a^{2/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\right.}{}\right.}{3ab}}{6ab^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x]
```

3.426. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^3}\,dx$

output
```
(x*(a*(b*d - a*g) + a*(b*e - a*h)*x - b*(b*c - a*f)*x^2))/(6*a^2*b*(a + b*
x^3)^2) + ((x*(a*b*(5*b*d + a*g) + 2*a*b*(2*b*e + a*h)*x - 3*b^2*(3*b*c -
a*f)*x^2))/(3*a^2*(a + b*x^3)) + (2*(-((b^(4/3)*(5*b^(4/3)*d + 2*a^(1/3)*b
*e + a*b^(1/3)*g + a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(2/3))) + (9*b^3*c*Log[x])/a + (b^(4/3)*(b^(1/3)*(5*b*d
+ a*g) - a^(1/3)*(2*b*e + a*h))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)) - (b
^(5/3)*(5*b*d + a*g - (a^(1/3)*(2*b*e + a*h))/b^(1/3))*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)) - (3*b^3*c*Log[a + b*x^3])/a))/(3
*a*b))/(6*a*b^2)
```

3.426.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.426. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x(a+bx^3)^3}\,dx$

3.426.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.98

method	result
default	$\frac{c\ln(x)}{a^3}+\frac{\frac{\left(\frac{1}{9}a^2h+\frac{2}{9}aeb\right)x^5+\left(\frac{1}{18}a^2g+\frac{5}{18}abd\right)x^4+\frac{abc\,x^3}{3}-\frac{a^2(ah-7be)x^2}{18b}-\frac{a^2(ag-4bd)x}{9b}-\frac{a^2(af-3bc)}{6b}}{\left(b\,x^3+a\right)^2}+\frac{\left(a^2g+5abd\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\right.}{}\right.}{}}{}$
risch	$\frac{\frac{(ah+2be)x^5}{9a^2}+\frac{(ag+5bd)x^4}{18a^2}+\frac{bc\,x^3}{3a^2}-\frac{(ah-7be)x^2}{18ab}-\frac{(ag-4bd)x}{9ab}-\frac{af-3bc}{6ab}}{(b\,x^3+a)^2}+\frac{c\ln(-x)}{a^3}+\frac{\left(\_R=\text{RootOf}\left(a^9b^5\_Z^3+27a^6b^5c\_Z^2+\left(3a^6b^2gh-\right.\right.\right.}{}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
c*ln(x)/a^3+1/a^3*(((1/9*a^2*h+2/9*a*e*b)*x^5+(1/18*a^2*g+5/18*a*b*d)*x^4+
1/3*a*b*c*x^3-1/18*a^2*(a*h-7*b*e)/b*x^2-1/9*a^2*(a*g-4*b*d)/b*x-1/6*a^2*(
a*f-3*b*c)/b)/(b*x^3+a)^2+1/9/b*((a^2*g+5*a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b
)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(a^2*h+2*a*b*e)*(
-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))-3*b*c*ln(b*x^3+a)))
```

3.426.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.99 (sec) , antiderivative size = 12815, normalized size of antiderivative = 36.93

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="fric
as")
```

output
```
Too large to include
```

3.426.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.426.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx \\
&= \frac{6\,b^2cx^3+2\,(2\,b^2e+abh)x^5+(5\,b^2d+abg)x^4+9\,abc-3\,a^2f+(7\,abe-a^2h)x^2+2\,(4\,abd-a^2g)x}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} \\
&\quad+\frac{c\log(x)}{a^3}+\frac{\sqrt{3}\left(2\,abe\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+5\,abd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b} \\
&\quad-\frac{\left(18\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abd+a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&\quad-\frac{\left(9\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abd-a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}
\end{aligned}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxi
ma")
```

output
```
1/18*(6*b^2*c*x^3 + 2*(2*b^2*e + a*b*h)*x^5 + (5*b^2*d + a*b*g)*x^4 + 9*a*
b*c - 3*a^2*f + (7*a*b*e - a^2*h)*x^2 + 2*(4*a*b*d - a^2*g)*x)/(a^2*b^3*x^
6 + 2*a^3*b^2*x^3 + a^4*b) + c*log(x)/a^3 + 1/27*sqrt(3)*(2*a*b*e*(a/b)^(2
/3) + a^2*h*(a/b)^(2/3) + 5*a*b*d*(a/b)^(1/3) + a^2*g*(a/b)^(1/3))*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b) - 1/54*(18*b^2*c*(a/b
)^(2/3) - 2*a*b*e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3) + 5*a*b*d + a^2*g)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) - 1/27*(9*b^2*c*(a
/b)^(2/3) + 2*a*b*e*(a/b)^(1/3) + a^2*h*(a/b)^(1/3) - 5*a*b*d - a^2*g)*log
(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3))
```

3.426.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.07

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx = -\frac{c\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{c\log\left(\left|x\right|\right)}{a^3}$$
$$-\frac{\sqrt{3}\left(5\,b^2d+abg-2\left(-ab^2\right)^{\frac{1}{3}}be-\left(-ab^2\right)^{\frac{1}{3}}ah\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2b}$$
$$-\frac{\left(5\,b^2d+abg+2\left(-ab^2\right)^{\frac{1}{3}}be+\left(-ab^2\right)^{\frac{1}{3}}ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2b}$$
$$+\frac{6\,ab^2cx^3+2\left(2\,ab^2e+a^2bh\right)x^5+\left(5\,ab^2d+a^2bg\right)x^4+9\,a^2bc-3\,a^3f+\left(7\,a^2be-a^3h\right)x^2+2\left(4\,a^2bd-a^3\right.}{18\left(bx^3+a\right)^2a^3b}$$
$$-\frac{\left(2\,a^4b^3e\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^5b^2h\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,a^4b^3d+a^5b^2g\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b^3}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac
")
```

output
```
-1/3*c*log(abs(b*x^3 + a))/a^3 + c*log(abs(x))/a^3 - 1/27*sqrt(3)*(5*b^2*d
 + a*b*g - 2*(-a*b^2)^(1/3)*b*e - (-a*b^2)^(1/3)*a*h)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2*b) - 1/54*(5*b^2*d +
 a*b*g + 2*(-a*b^2)^(1/3)*b*e + (-a*b^2)^(1/3)*a*h)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b) + 1/18*(6*a*b^2*c*x^3 + 2*(2*a*b
^2*e + a^2*b*h)*x^5 + (5*a*b^2*d + a^2*b*g)*x^4 + 9*a^2*b*c - 3*a^3*f + (7
*a^2*b*e - a^3*h)*x^2 + 2*(4*a^2*b*d - a^3*g)*x)/((b*x^3 + a)^2*a^3*b) - 1
/27*(2*a^4*b^3*e*(-a/b)^(1/3) + a^5*b^2*h*(-a/b)^(1/3) + 5*a^4*b^3*d + a^5
*b^2*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b^3)
```

3.426.9 Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 1716, normalized size of antiderivative = 4.95

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x*(a + b*x^3)^3),x)
```

output
```
((3*b*c - a*f)/(6*a*b) + (x^4*(5*b*d + a*g))/(18*a^2) + (x^5*(2*b*e + a*h)
)/(9*a^2) + (x*(4*b*d - a*g))/(9*a*b) + (x^2*(7*b*e - a*h))/(18*a*b) + (b*
c*x^3)/(3*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((c*(25*b^2*d^2 +
a^2*g^2 - 18*b^2*c*e - 9*a*b*c*h + 10*a*b*d*g))/(81*a^6) - (root(19683*a^9
*b^5*z^3 + 19683*a^6*b^5*c*z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 16
2*a^5*b^3*e*g*z + 810*a^4*b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e
 + 27*a^3*b^2*c*g*h + 135*a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2
 + 12*a^3*b^2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3
- a^4*b*g^3 - 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*h^3, z, k)*(a^3*g^2 + 25*a
*b^2*d^2 + 324*b^3*c^2*x + 2916*root(19683*a^9*b^5*z^3 + 19683*a^6*b^5*c*z
^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*g*z + 810*a^4*b^
4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^2*c*g*h + 135*a^
2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^2*e^2*h - 75*a^2
*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3 - 125*a*b^4*d^3
+ 729*b^5*c^3 + a^5*h^3, z, k)^2*a^6*b^3*x - 27*root(19683*a^9*b^5*z^3 + 1
9683*a^6*b^5*c*z^2 + 81*a^6*b^2*g*h*z + 405*a^5*b^3*d*h*z + 162*a^5*b^3*e*
g*z + 810*a^4*b^4*d*e*z + 6561*a^3*b^5*c^2*z + 270*a*b^4*c*d*e + 27*a^3*b^
2*c*g*h + 135*a^2*b^3*c*d*h + 54*a^2*b^3*c*e*g + 6*a^4*b*e*h^2 + 12*a^3*b^
2*e^2*h - 75*a^2*b^3*d^2*g - 15*a^3*b^2*d*g^2 + 8*a^2*b^3*e^3 - a^4*b*g^3
- 125*a*b^4*d^3 + 729*b^5*c^3 + a^5*h^3, z, k)*a^5*b*h + 36*a*b^2*c*e +...
```

3.427 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^3}\,dx$

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3.427.1 Optimal result

Integrand size = 38, antiderivative size = 362

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^3}\,dx &= -\frac{c}{a^3x}+\frac{x(a(be-ah)-b(bc-af)x-b(bd-ag)x^2)}{6a^2b(a+bx^3)^2}\\ &+\frac{x(a(5be+ah)-2b(5bc-2af)x-3b(3bd-ag)x^2)}{18a^3b(a+bx^3)}\\ &+\frac{(14b^{5/3}c-5a^{2/3}be-2ab^{2/3}f-a^{5/3}h)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{4/3}}\\ &+\frac{d\log(x)}{a^3}+\frac{(2b^{2/3}(7bc-af)+a^{2/3}(5be+ah))\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{4/3}}\\ &-\frac{(2b^{2/3}(7bc-af)+a^{2/3}(5be+ah))\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{10/3}b^{4/3}}-\frac{d\log(a+bx^3)}{3a^3}\end{aligned}$$

output
```
-c/a^3/x+1/6*x*(a*(-a*h+b*e)-b*(-a*f+b*c)*x-b*(-a*g+b*d)*x^2)/a^2/b/(b*x^3
+a)^2+1/18*x*(a*(a*h+5*b*e)-2*b*(-2*a*f+5*b*c)*x-3*b*(-a*g+3*b*d)*x^2)/a^3
/b/(b*x^3+a)+d*ln(x)/a^3+1/27*(2*b^(2/3)*(-a*f+7*b*c)+a^(2/3)*(a*h+5*b*e))
*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(4/3)-1/54*(2*b^(2/3)*(-a*f+7*b*c)+a^(2/
3)*(a*h+5*b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(4/3)
-1/3*d*ln(b*x^3+a)/a^3+1/27*(14*b^(5/3)*c-5*a^(2/3)*b*e-2*a*b^(2/3)*f-a^(5
/3)*h)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(4/3)*
3^(1/2)
```

3.427.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.93

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{\frac{54ac}{x}+\frac{9a^2(b^2cx^2+a^2(g+hx)-ab(d+x(e+fx)))}{b(a+bx^3)^2}-\frac{3a(a^2hx-10b^2cx^2+ab(6d+x(5e+4fx)))}{b(a+bx^3)}+\frac{2\sqrt{3}a^{2/3}(-14b^{5/3}c+5a^{2/3}be+2ab^{2/3}f+}{b^{4/3}}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]
```

output
```
-1/54*((54*a*c)/x + (9*a^2*(b^2*c*x^2 + a^2*(g + h*x) - a*b*(d + x*(e + f*
x))))/(b*(a + b*x^3)^2) - (3*a*(a^2*h*x - 10*b^2*c*x^2 + a*b*(6*d + x*(5*e
 + 4*f*x))))/(b*(a + b*x^3)) + (2*Sqrt[3]*a^(2/3)*(-14*b^(5/3)*c + 5*a^(2/
3)*b*e + 2*a*b^(2/3)*f + a^(5/3)*h)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/b^(4/3) - 54*a*d*Log[x] - (2*a^(2/3)*(14*b^(5/3)*c + 5*a^(2/3)*b*e
- 2*a*b^(2/3)*f + a^(5/3)*h)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(2/3)*
(14*b^(5/3)*c + 5*a^(2/3)*b*e - 2*a*b^(2/3)*f + a^(5/3)*h)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + 18*a*d*Log[a + b*x^3])/a^4
```

3.427.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}dx$$

$$\Big\downarrow \; 2368$$

$$\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}-\frac{\int -\frac{-3b^2\left(\frac{bd}{a}-g\right)x^4-4b^2\left(\frac{bc}{a}-f\right)x^3+b(5be+ah)x^2+6b^2dx+6b^2c}{x^2(bx^3+a)^2}dx}{6ab^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-3b^2\left(\frac{bd}{a}-g\right)x^4-4b^2\left(\frac{bc}{a}-f\right)x^3+b(5be+ah)x^2+6b^2dx+6b^2c}{x^2(bx^3+a)^2}dx}{6ab^2}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2368$$

$$\frac{\frac{x\left(-2b^2x(5bc-2af)-3b^2x^2(3bd-ag)+ab(ah+5be)\right)}{3a^2(a+bx^3)}-\frac{\int -\frac{2\left(-\left(\left(\frac{5bc}{a}-2f\right)x^3b^4\right)+9cb^4+9dxb^4+(5be+ah)x^2b^3\right)}{x^2\left(bx^3+a\right)}dx}{3ab^2}}{6ab^2}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{-\left(\left(\frac{5bc}{a}-2f\right)x^3b^4\right)+9cb^4+9dxb^4+(5be+ah)x^2b^3}{x^2\left(bx^3+a\right)}dx}{3ab^2}+\frac{x\left(-2b^2x(5bc-2af)-3b^2x^2(3bd-ag)+ab(ah+5be)\right)}{3a^2(a+bx^3)}}{6ab^2}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2373$$

$$\frac{\frac{2\int \left(\frac{9db^4}{ax}+\frac{9cb^4}{ax^2}+\frac{\left(-9b^2dx^2-2b(7bc-af)x+a(5be+ah)\right)b^3}{a\left(bx^3+a\right)}\right)dx}{3ab^2}+\frac{x\left(-2b^2x(5bc-2af)-3b^2x^2(3bd-ag)+ab(ah+5be)\right)}{3a^2(a+bx^3)}}{6ab^2}+\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(-bx(bc-af)-bx^2(bd-ag)+a(be-ah)\right)}{6a^2b\left(a+bx^3\right)^2}+$$

$$\frac{\frac{2\left(\frac{b^{8/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-5a^{2/3}be+a^{5/3}(-h)-2ab^{2/3}f+14b^{5/3}c\right)}{\sqrt{3}a^{4/3}}-\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(a^{2/3}(ah+5be)+2b^{2/3}(7bc-af)\right)}{6a^{4/3}}+\frac{b^{8/3}\log \cdots}{\cdots}\right.}{3ab^2}}{6ab^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x]
```

3.427. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^3}\,dx$

output
```
(x*(a*(b*e - a*h) - b*(b*c - a*f)*x - b*(b*d - a*g)*x^2))/(6*a^2*b*(a + b*
x^3)^2) + ((x*(a*b*(5*b*e + a*h) - 2*b^2*(5*b*c - 2*a*f)*x - 3*b^2*(3*b*d
- a*g)*x^2))/(3*a^2*(a + b*x^3)) + (2*((-9*b^4*c)/(a*x) + (b^(8/3)*(14*b^(
5/3)*c - 5*a^(2/3)*b*e - 2*a*b^(2/3)*f - a^(5/3)*h)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)) + (9*b^4*d*Log[x])/a + (b^(
8/3)*(2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e + a*h))*Log[a^(1/3) + b^(1/
3)*x])/(3*a^(4/3)) - (b^(8/3)*(2*b^(2/3)*(7*b*c - a*f) + a^(2/3)*(5*b*e +
a*h))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)) - (3*b^4
*d*Log[a + b*x^3])/a))/(3*a*b^2))/(6*a*b^2)
```

3.427.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.427. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2(a+bx^3)^3} \, dx$

3.427.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.95

method	result
default	$-\frac{c}{a^{3}x}+\frac{d\ln(x)}{a^{3}}+\frac{\frac{\left(\frac{2}{9}afb-\frac{5}{9}b^{2}c\right)x^{5}+\left(\frac{1}{18}a^{2}h+\frac{5}{18}aeb\right)x^{4}+\frac{x^{3}abd}{3}+\frac{a(7af-13bc)x^{2}}{18}-\frac{a^{2}(ah-4be)x}{9b}-\frac{a^{2}(ag-3bd)}{6b}}{\left(bx^{3}+a\right)^{2}}+\frac{\left(a^{2}h+5aeb\right)\left(\frac{\ln\left(x+\left(\cdots\right.\right.}{3b\left(\frac{a}{b}\cdots\right.}\cdots\right.}{\cdots}$
risch	$\frac{\frac{2b(af-7bc)x^{6}}{9a^{3}}+\frac{(ah+5be)x^{5}}{18a^{2}}+\frac{bdx^{4}}{3a^{2}}+\frac{7(af-7bc)x^{3}}{18a^{2}}-\frac{(ah-4be)x^{2}}{9ab}-\frac{(ag-3bd)x}{6ab}-\frac{c}{a}}{x\left(bx^{3}+a\right)^{2}}+\frac{d\ln(x)}{a^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}b^{4}\_Z^{3}+27a^{7}b^{4}d\_Z^{2}-\cdots\right.}\cdots\right.}{\cdots}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOS
E)
```

output
```
-c/a^3/x+d*ln(x)/a^3+1/a^3*(((2/9*a*f*b-5/9*b^2*c)*x^5+(1/18*a^2*h+5/18*a*
e*b)*x^4+1/3*x^3*a*b*d+1/18*a*(7*a*f-13*b*c)*x^2-1/9*a^2*(a*h-4*b*e)/b*x-1
/6*a^2*(a*g-3*b*d)/b)/(b*x^3+a)^2+1/9/b*((a^2*h+5*a*b*e)*(1/3/b/(a/b)^(2/3
)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/
3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+(2*a*b*f-
14*b^2*c)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1)))-3*b*d*ln(b*x^3+a)))
```

3.427.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.72 (sec) , antiderivative size = 12951, normalized size of antiderivative = 35.78

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.427.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**2/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.427.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx$$
$$=\frac{6\,ab^2dx^4-4\,(7\,b^3c-ab^2f)x^6+(5\,ab^2e+a^2bh)x^5-18\,a^2bc-7\,(7\,ab^2c-a^2bf)x^3+2\,(4\,a^2be-a^3h)x^2+}{18\,(a^3b^3x^7+2\,a^4b^2x^4+a^5bx)}$$
$$+\frac{d\log(x)}{a^3}$$
$$-\frac{\sqrt{3}\left(14\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-5\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b}$$
$$-\frac{\left(18\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}+5\,abe+a^2h\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{\left(9\,b^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}-14\,b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,abf\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,abe-a^2h\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="ma
xima")
```

output
```
1/18*(6*a*b^2*d*x^4 - 4*(7*b^3*c - a*b^2*f)*x^6 + (5*a*b^2*e + a^2*b*h)*x^
5 - 18*a^2*b*c - 7*(7*a*b^2*c - a^2*b*f)*x^3 + 2*(4*a^2*b*e - a^3*h)*x^2 +
 3*(3*a^2*b*d - a^3*g)*x)/(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*x) + d*log(
x)/a^3 - 1/27*sqrt(3)*(14*b^2*c*(a/b)^(2/3) - 2*a*b*f*(a/b)^(2/3) - 5*a*b*
e*(a/b)^(1/3) - a^2*h*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a^4*b) - 1/54*(18*b^2*d*(a/b)^(2/3) + 14*b^2*c*(a/b)^(1/3) -
 2*a*b*f*(a/b)^(1/3) + 5*a*b*e + a^2*h)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2
/3))/(a^3*b^2*(a/b)^(2/3)) - 1/27*(9*b^2*d*(a/b)^(2/3) - 14*b^2*c*(a/b)^(1
/3) + 2*a*b*f*(a/b)^(1/3) - 5*a*b*e - a^2*h)*log(x + (a/b)^(1/3))/(a^3*b^2
*(a/b)^(2/3))
```

3.427.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.07

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx = -\frac{d\log\left(|bx^3+a|\right)}{3\,a^3}+\frac{d\log\left(|x|\right)}{a^3}$$
$$-\frac{\sqrt{3}\left(5\,abe+a^2h+14\left(-ab^2\right)^{\frac{1}{3}}bc-2\left(-ab^2\right)^{\frac{1}{3}}af\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^3}$$
$$-\frac{\left(5\,abe+a^2h-14\left(-ab^2\right)^{\frac{1}{3}}bc+2\left(-ab^2\right)^{\frac{1}{3}}af\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^3}$$
$$+\frac{6\,ab^2dx^4-4\left(7\,b^3c-ab^2f\right)x^6+\left(5\,ab^2e+a^2bh\right)x^5-18\,a^2bc-7\left(7\,ab^2c-a^2bf\right)x^3+2\left(4\,a^2be-a^3h\right)x^2}{18\left(bx^3+a\right)^2a^3bx}$$
$$+\frac{\left(14\,a^3b^4c\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^4b^3f\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^4b^3e-a^5b^2h\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b^3}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^2/(b*x^3+a)^3,x, algorithm="gi
ac")
```

output
```
-1/3*d*log(abs(b*x^3 + a))/a^3 + d*log(abs(x))/a^3 - 1/27*sqrt(3)*(5*a*b*e
 + a^2*h + 14*(-a*b^2)^(1/3)*b*c - 2*(-a*b^2)^(1/3)*a*f)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/54*(5*a*b*e
+ a^2*h - 14*(-a*b^2)^(1/3)*b*c + 2*(-a*b^2)^(1/3)*a*f)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) + 1/18*(6*a*b^2*d*x^4 - 4*(7*b
^3*c - a*b^2*f)*x^6 + (5*a*b^2*e + a^2*b*h)*x^5 - 18*a^2*b*c - 7*(7*a*b^2*
c - a^2*b*f)*x^3 + 2*(4*a^2*b*e - a^3*h)*x^2 + 3*(3*a^2*b*d - a^3*g)*x)/((
b*x^3 + a)^2*a^3*b*x) + 1/27*(14*a^3*b^4*c*(-a/b)^(1/3) - 2*a^4*b^3*f*(-a/
b)^(1/3) - 5*a^4*b^3*e - a^5*b^2*h)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3))
)/(a^7*b^3)
```

3.427.9 Mupad [B] (verification not implemented)

Time = 9.87 (sec) , antiderivative size = 1747, normalized size of antiderivative = 4.83

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^2\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^2*(a + b*x^3)^3),x)
```

output
```
symsum(log((d*(a^3*h^2 + 25*a*b^2*e^2 + 126*b^3*c*d - 18*a*b^2*d*f + 10*a^
2*b*e*h))/(81*a^7) - (root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162*
a^6*b^2*f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e*
z + 6561*a^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^3
*c*d*h + 270*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b^
2*e^2*h - 168*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4*
d^3 - a^5*h^3 - 2744*b^5*c^3, z, k)*(a^3*h^2 + 25*a*b^2*e^2 + 324*b^3*d^2*
x - 252*b^3*c*d + 2916*root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 + 162
*a^6*b^2*f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4*c*e
*z + 6561*a^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^2*b^
3*c*d*h + 270*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a^3*b
^2*e^2*h - 168*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a*b^4
*d^3 - a^5*h^3 - 2744*b^5*c^3, z, k)^2*a^6*b^3*x + 36*a*b^2*d*f + 10*a^2*b
*e*h - 700*b^3*c*e*x + 378*root(19683*a^10*b^4*z^3 + 19683*a^7*b^4*d*z^2 +
 162*a^6*b^2*f*h*z - 1134*a^5*b^3*c*h*z + 810*a^5*b^3*e*f*z - 5670*a^4*b^4
*c*e*z + 6561*a^4*b^4*d^2*z - 1890*a*b^4*c*d*e + 54*a^3*b^2*d*f*h - 378*a^
2*b^3*c*d*h + 270*a^2*b^3*d*e*f - 15*a^4*b*e*h^2 + 1176*a*b^4*c^2*f - 75*a
^3*b^2*e^2*h - 168*a^2*b^3*c*f^2 + 8*a^3*b^2*f^3 - 125*a^2*b^3*e^3 + 729*a
*b^4*d^3 - a^5*h^3 - 2744*b^5*c^3, z, k)*a^3*b^3*c - 54*root(19683*a^10*b^
4*z^3 + 19683*a^7*b^4*d*z^2 + 162*a^6*b^2*f*h*z - 1134*a^5*b^3*c*h*z + ...
```

3.428 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^3}\,dx$

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3.428.1 Optimal result

Integrand size = 38, antiderivative size = 360

$$\begin{aligned}&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^3}\,dx\\
&=-\frac{c}{2a^3x^2}-\frac{d}{a^3x}-\frac{x(bc-af+(bd-ag)x+(be-ah)x^2)}{6a^2(a+bx^3)^2}\\
&\quad-\frac{x(11bc-5af+2(5bd-2ag)x+3(3be-ah)x^2)}{18a^3(a+bx^3)}\\
&\quad+\frac{\left(20b^{4/3}c+14\sqrt[3]{a}bd-5a\sqrt[3]{b}f-2a^{4/3}g\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}b^{2/3}}\\
&\quad+\frac{e\log(x)}{a^3}-\frac{\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}b^{2/3}}\\
&\quad+\frac{\left(5\sqrt[3]{b}(4bc-af)-2\sqrt[3]{a}(7bd-ag)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}b^{2/3}}-\frac{e\log(a+bx^3)}{3a^3}\end{aligned}$$

output
```
-1/2*c/a^3/x^2-d/a^3/x-1/6*x*(b*c-a*f+(-a*g+b*d)*x+(-a*h+b*e)*x^2)/a^2/(b*
x^3+a)^2-1/18*x*(11*b*c-5*a*f+2*(-2*a*g+5*b*d)*x+3*(-a*h+3*b*e)*x^2)/a^3/(
b*x^3+a)+e*ln(x)/a^3-1/27*(5*b^(1/3)*(-a*f+4*b*c)-2*a^(1/3)*(-a*g+7*b*d))*
ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(2/3)+1/54*(5*b^(1/3)*(-a*f+4*b*c)-2*a^(1
/3)*(-a*g+7*b*d))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(2/
3)-1/3*e*ln(b*x^3+a)/a^3+1/27*(20*b^(4/3)*c+14*a^(1/3)*b*d-5*a*b^(1/3)*f-2
*a^(4/3)*g)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(
2/3)*3^(1/2)
```

3.428.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.94

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{\frac{27ac}{x^2}+\frac{54ad}{x}-\frac{3a(6ae-bx(11c+10dx)+ax(5f+4gx))}{a+bx^3}+\frac{9a^2\left(a^2h+b^2x(c+dx)-ab(e+x(f+gx))\right)}{b\left(a+bx^3\right)^2}+\frac{2\sqrt{3}\sqrt[3]{a}\left(-20b^{4/3}c-14\sqrt[3]{a}bd+5\right.}{}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]
```

output
```
-1/54*((27*a*c)/x^2 + (54*a*d)/x - (3*a*(6*a*e - b*x*(11*c + 10*d*x) + a*x
*(5*f + 4*g*x)))/(a + b*x^3) + (9*a^2*(a^2*h + b^2*x*(c + d*x) - a*b*(e +
x*(f + g*x))))/(b*(a + b*x^3)^2) + (2*Sqrt[3]*a^(1/3)*(-20*b^(4/3)*c - 14*
a^(1/3)*b*d + 5*a*b^(1/3)*f + 2*a^(4/3)*g)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/b^(2/3) - 54*a*e*Log[x] + (2*a^(1/3)*(20*b^(4/3)*c - 14*a^(1
/3)*b*d - 5*a*b^(1/3)*f + 2*a^(4/3)*g)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) -
 (a^(1/3)*(20*b^(4/3)*c - 14*a^(1/3)*b*d - 5*a*b^(1/3)*f + 2*a^(4/3)*g)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3) + 18*a*e*Log[a + b*x
^3])/a^4
```

3.428.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx$$

$$\Big\downarrow \text{ 2368}$$

$$-\frac{\int -\frac{-3b^2\left(\frac{be}{a}-h\right)x^5-4b^2\left(\frac{bd}{a}-g\right)x^4-5b^2\left(\frac{bc}{a}-f\right)x^3+6b^2ex^2+6b^2dx+6b^2c}{x^3(bx^3+a)^2}dx}{6ab^2}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6a^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-3b^2\left(\frac{be}{a}-h\right)x^5-4b^2\left(\frac{bd}{a}-g\right)x^4-5b^2\left(\frac{bc}{a}-f\right)x^3+6b^2ex^2+6b^2dx+6b^2c}{x^3(bx^3+a)^2}dx}{6ab^2}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6a^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2368$$

$$-\frac{-\frac{\int -\frac{2\left(-\left(\left(\frac{5bd}{a}-2g\right)x^4b^4\right)-\left(\frac{11bc}{a}-5f\right)x^3b^4+9ex^2b^4+9cb^4+9dxb^4\right)}{x^3\left(bx^3+a\right)}dx}{3ab^2}-\frac{x\left(b^2(11bc-5af)+2b^2x(5bd-2ag)+3b^2x^2(3be-ah)\right)}{3a^2(a+bx^3)}}{6ab^2}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6a^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{-\left(\left(\frac{5bd}{a}-2g\right)x^4b^4\right)-\left(\frac{11bc}{a}-5f\right)x^3b^4+9ex^2b^4+9cb^4+9dxb^4}{x^3\left(bx^3+a\right)}dx}{3ab^2}-\frac{x\left(b^2(11bc-5af)+2b^2x(5bd-2ag)+3b^2x^2(3be-ah)\right)}{3a^2(a+bx^3)}}{6ab^2}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6a^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2373$$

$$\frac{\frac{2\int \left(\frac{9eb^4}{ax}+\frac{\left(-9bex^2-2(7bd-ag)x-5(4bc-af)\right)b^4}{a\left(bx^3+a\right)}+\frac{9db^4}{ax^2}+\frac{9cb^4}{ax^3}\right)dx}{3ab^2}-\frac{x\left(b^2(11bc-5af)+2b^2x(5bd-2ag)+3b^2x^2(3be-ah)\right)}{3a^2(a+bx^3)}}{6ab^2}-\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6a^2\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(\frac{b^{10/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{4/3}g+14\sqrt[3]{a}bd-5a\sqrt[3]{b}f+20b^{4/3}c\right)}{\sqrt{3}a^{5/3}}+\frac{b^{11/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7bd-ag)}{\sqrt[3]{b}}-5af+20bc\right)}{6a^{5/3}}-\frac{b^{10/3}\cdots}{\cdots}\right.}{3ab^2}$$

$$\frac{x\left(x(bd-ag)+x^2(be-ah)-af+bc\right)}{6a^2\left(a+bx^3\right)^2}$$

3.428. $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^3}\,dx$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x]
```

output
```
-1/6*(x*(b*c - a*f + (b*d - a*g)*x + (b*e - a*h)*x^2))/(a^2*(a + b*x^3)^2)
 + (-1/3*(x*(b^2*(11*b*c - 5*a*f) + 2*b^2*(5*b*d - 2*a*g)*x + 3*b^2*(3*b*e
 - a*h)*x^2))/(a^2*(a + b*x^3)) + (2*((-9*b^4*c)/(2*a*x^2) - (9*b^4*d)/(a*
x) + (b^(10/3)*(20*b^(4/3)*c + 14*a^(1/3)*b*d - 5*a*b^(1/3)*f - 2*a^(4/3)*
g)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) +
(9*b^4*e*Log[x])/a - (b^(10/3)*(5*b^(1/3)*(4*b*c - a*f) - 2*a^(1/3)*(7*b*d
 - a*g))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (b^(11/3)*(20*b*c - 5*a*f
 - (2*a^(1/3)*(7*b*d - a*g))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(6*a^(5/3)) - (3*b^4*e*Log[a + b*x^3])/a))/(3*a*b^2))/(6*a*b^2
)
```

3.428.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.428.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.94

method	result
default	$(5af-20bc)$ … $-\frac{c}{2a^3x^2}-\frac{d}{a^3x}+\frac{e\ln(x)}{a^3}+\frac{\frac{\left(\frac{2}{9}abg-\frac{5}{9}b^2d\right)x^5+\left(\frac{5}{18}afb-\frac{11}{18}b^2c\right)x^4+\frac{abe\,x^3}{3}+\frac{a(7ag-13bd)x^2}{18}+\frac{a(4af-7bc)x}{9}-\frac{a^2(ah-3be)}{6b}}{\left(b\,x^3+a\right)^2}+ \cdots}{\cdots}$
risch	$\frac{\frac{2b(ag-7bd)x^7}{9a^3}+\frac{5b(af-4bc)x^6}{18a^3}+\frac{be\,x^5}{3a^2}+\frac{7(ag-7bd)x^4}{18a^2}+\frac{4(af-4bc)x^3}{9a^2}-\frac{(ah-3be)x^2}{6ab}-\frac{xd}{a}-\frac{c}{2a}}{x^2\left(b\,x^3+a\right)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}b^2\_Z^3+27a^8b^2e\_Z^2+\left(\cdots\right.\right.}\cdots\right.}{\cdots}$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOS
E)
```

output
```
-1/2*c/a^3/x^2-d/a^3/x+e*ln(x)/a^3+1/a^3*(((2/9*a*b*g-5/9*b^2*d)*x^5+(5/18
*a*f*b-11/18*b^2*c)*x^4+1/3*a*b*e*x^3+1/18*a*(7*a*g-13*b*d)*x^2+1/9*a*(4*a
*f-7*b*c)*x-1/6*a^2*(a*h-3*b*e)/b)/(b*x^3+a)^2+1/9*(5*a*f-20*b*c)*(1/3/b/(
a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+
1/9*(2*a*g-14*b*d)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1)))-1/3*e*ln(b*x^3+a))
```

3.428.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.28 (sec) , antiderivative size = 12435, normalized size of antiderivative = 34.54

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.428.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**3/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.428.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^3}\,dx$$
$$= \frac{6\,ab^2ex^5 - 4\,(7\,b^3d - ab^2g)x^7 - 5\,(4\,b^3c - ab^2f)x^6 - 18\,a^2bdx - 7\,(7\,ab^2d - a^2bg)x^4 - 9\,a^2bc - 8\,(4\,ab^2c -}{18\,(a^3b^3x^8 + 2\,a^4b^2x^5 + a^5bx^2)}$$
$$+ \frac{e\log(x)}{a^3} - \frac{\sqrt{3}\left(14\,bd\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,ag\left(\frac{a}{b}\right)^{\frac{2}{3}} + 20\,bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - 5\,af\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4}$$
$$- \frac{\left(18\,be\left(\frac{a}{b}\right)^{\frac{2}{3}} + 14\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}} - 2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}} - 20\,bc + 5\,af\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{\left(9\,be\left(\frac{a}{b}\right)^{\frac{2}{3}} - 14\,bd\left(\frac{a}{b}\right)^{\frac{1}{3}} + 2\,ag\left(\frac{a}{b}\right)^{\frac{1}{3}} + 20\,bc - 5\,af\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="ma
xima")
```

output
```
1/18*(6*a*b^2*e*x^5 - 4*(7*b^3*d - a*b^2*g)*x^7 - 5*(4*b^3*c - a*b^2*f)*x^
6 - 18*a^2*b*d*x - 7*(7*a*b^2*d - a^2*b*g)*x^4 - 9*a^2*b*c - 8*(4*a*b^2*c
- a^2*b*f)*x^3 + 3*(3*a^2*b*e - a^3*h)*x^2)/(a^3*b^3*x^8 + 2*a^4*b^2*x^5 +
 a^5*b*x^2) + e*log(x)/a^3 - 1/27*sqrt(3)*(14*b*d*(a/b)^(2/3) - 2*a*g*(a/b
)^(2/3) + 20*b*c*(a/b)^(1/3) - 5*a*f*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/a^4 - 1/54*(18*b*e*(a/b)^(2/3) + 14*b*d*(a/b)^
(1/3) - 2*a*g*(a/b)^(1/3) - 20*b*c + 5*a*f)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*b*(a/b)^(2/3)) - 1/27*(9*b*e*(a/b)^(2/3) - 14*b*d*(a/b)^(1/3
) + 2*a*g*(a/b)^(1/3) + 20*b*c - 5*a*f)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^
(2/3))
```

3.428.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3(a+bx^3)^3}\,dx = -\frac{e\log(|bx^3+a|)}{3\,a^3} + \frac{e\log(|x|)}{a^3} + \frac{\sqrt{3}\left(20\,b^2c-5\,abf-14\,(-ab^2)^{\frac{1}{3}}\,bd+2\,(-ab^2)^{\frac{1}{3}}\,ag\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^3} + \frac{\left(20\,b^2c-5\,abf+14\,(-ab^2)^{\frac{1}{3}}\,bd-2\,(-ab^2)^{\frac{1}{3}}\,ag\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{28\,b^3dx^7-4\,ab^2gx^7+20\,b^3cx^6-5\,ab^2fx^6-6\,ab^2ex^5+49\,ab^2dx^4-7\,a^2bgx^4+32\,ab^2cx^3-8\,a^2bfx^3-}{18\,(bx^4+ax)^2a^3b} + \frac{\left(14\,a^3b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^4bg\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,a^3b^2c-5\,a^4bf\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^7b}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^3/(b*x^3+a)^3,x, algorithm="gi
ac")
```

output
```
-1/3*e*log(abs(b*x^3 + a))/a^3 + e*log(abs(x))/a^3 + 1/27*sqrt(3)*(20*b^2*
c - 5*a*b*f - 14*(-a*b^2)^(1/3)*b*d + 2*(-a*b^2)^(1/3)*a*g)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) + 1/54*(20*b^
2*c - 5*a*b*f + 14*(-a*b^2)^(1/3)*b*d - 2*(-a*b^2)^(1/3)*a*g)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) - 1/18*(28*b^3*d*x^7 - 4
*a*b^2*g*x^7 + 20*b^3*c*x^6 - 5*a*b^2*f*x^6 - 6*a*b^2*e*x^5 + 49*a*b^2*d*x
^4 - 7*a^2*b*g*x^4 + 32*a*b^2*c*x^3 - 8*a^2*b*f*x^3 - 9*a^2*b*e*x^2 + 3*a^
3*h*x^2 + 18*a^2*b*d*x + 9*a^2*b*c)/((b*x^4 + a*x)^2*a^3*b) + 1/27*(14*a^3
*b^2*d*(-a/b)^(1/3) - 2*a^4*b*g*(-a/b)^(1/3) + 20*a^3*b^2*c - 5*a^4*b*f)*(
-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^7*b)
```

3.428.9 Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 1697, normalized size of antiderivative = 4.71

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^3\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^3*(a + b*x^3)^3),x)
```

output
```
symsum(log((b^2*e*(400*b^2*c^2 + 25*a^2*f^2 - 18*a^2*e*g - 200*a*b*c*f + 1
26*a*b*d*e))/(81*a^8) - (root(19683*a^11*b^2*z^3 + 19683*a^8*b^2*e*z^2 + 8
10*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c*g*z + 22680*a^4*b^3*c
*d*z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*b^3*c*d*e - 1890*a^2*
b^2*d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 6000*a*b^3*c^2*f + 1176
*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 - 125*a^3*b*f^3 - 27
44*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z, k)*b^2*(400*b^2*c^2 + 25*a^2*f
^2 - 54*root(19683*a^11*b^2*z^3 + 19683*a^8*b^2*e*z^2 + 810*a^6*b*f*g*z -
5670*a^5*b^2*d*f*z - 3240*a^5*b^2*c*g*z + 22680*a^4*b^3*c*d*z + 6561*a^5*b
^2*e^2*z + 270*a^3*b*e*f*g + 7560*a*b^3*c*d*e - 1890*a^2*b^2*d*e*f - 1080*
a^2*b^2*c*e*g - 168*a^3*b*d*g^2 - 6000*a*b^3*c^2*f + 1176*a^2*b^2*d^2*g +
1500*a^2*b^2*c*f^2 + 729*a^2*b^2*e^3 - 125*a^3*b*f^3 - 2744*a*b^3*d^3 + 8*
a^4*g^3 + 8000*b^4*c^3, z, k)*a^5*g + 36*a^2*e*g + 378*root(19683*a^11*b^2
*z^3 + 19683*a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2*d*f*z - 3240*a
^5*b^2*c*g*z + 22680*a^4*b^3*c*d*z + 6561*a^5*b^2*e^2*z + 270*a^3*b*e*f*g
+ 7560*a*b^3*c*d*e - 1890*a^2*b^2*d*e*f - 1080*a^2*b^2*c*e*g - 168*a^3*b*d
*g^2 - 6000*a*b^3*c^2*f + 1176*a^2*b^2*d^2*g + 1500*a^2*b^2*c*f^2 + 729*a^
2*b^2*e^3 - 125*a^3*b*f^3 - 2744*a*b^3*d^3 + 8*a^4*g^3 + 8000*b^4*c^3, z,
k)*a^4*b*d + 324*a*b*e^2*x + 2800*b^2*c*d*x + 100*a^2*f*g*x + 2916*root(19
683*a^11*b^2*z^3 + 19683*a^8*b^2*e*z^2 + 810*a^6*b*f*g*z - 5670*a^5*b^2...
```

3.429 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^3}\,dx$

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3.429.1 Optimal result

Integrand size = 38, antiderivative size = 395

$$\begin{aligned}
&\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4(a+bx^3)^3}\,dx \\
&= -\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}-\frac{x\left(bd-ag+(be-ah)x-b\left(\frac{bc}{a}-f\right)x^2\right)}{6a^2(a+bx^3)^2} \\
&\quad-\frac{x\left(11bd-5ag+2(5be-2ah)x-3b\left(\frac{5bc}{a}-3f\right)x^2\right)}{18a^3(a+bx^3)} \\
&\quad+\frac{\left(20b^{4/3}d+14\sqrt[3]{a}be-5a\sqrt[3]{b}g-2a^{4/3}h\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}b^{2/3}} \\
&\quad-\frac{(3bc-af)\log(x)}{a^4}-\frac{\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}b^{2/3}} \\
&\quad+\frac{\left(5\sqrt[3]{b}(4bd-ag)-2\sqrt[3]{a}(7be-ah)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}b^{2/3}} \\
&\quad+\frac{(3bc-af)\log(a+bx^3)}{3a^4}
\end{aligned}$$

output
```
-1/3*c/a^3/x^3-1/2*d/a^3/x^2-e/a^3/x-1/6*x*(b*d-a*g+(-a*h+b*e)*x-b*(b*c/a-
f)*x^2)/a^2/(b*x^3+a)^2-1/18*x*(11*b*d-5*a*g+2*(-2*a*h+5*b*e)*x-3*b*(5*b*c
/a-3*f)*x^2)/a^3/(b*x^3+a)-(-a*f+3*b*c)*ln(x)/a^4-1/27*(5*b^(1/3)*(-a*g+4*
b*d)-2*a^(1/3)*(-a*h+7*b*e))*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(2/3)+1/54*(
5*b^(1/3)*(-a*g+4*b*d)-2*a^(1/3)*(-a*h+7*b*e))*ln(a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/a^(11/3)/b^(2/3)+1/3*(-a*f+3*b*c)*ln(b*x^3+a)/a^4+1/27*(20*
b^(4/3)*d+14*a^(1/3)*b*e-5*a*b^(1/3)*g-2*a^(4/3)*h)*arctan(1/3*(a^(1/3)-2*
b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(2/3)*3^(1/2)
```

3.429.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.89

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{18ac}{x^3}-\frac{27ad}{x^2}-\frac{54ae}{x}+\frac{3a(-12bc+6af-bx(11d+10ex)+ax(5g+4hx))}{a+bx^3}+\frac{a^2(-9b(c+x(d+ex))+9a(f+x(g+hx)))}{(a+bx^3)^2}+\frac{2\sqrt{3}\sqrt[3]{a}\left(20b^{4/3}\right.}{}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]
```

output
```
((-18*a*c)/x^3 - (27*a*d)/x^2 - (54*a*e)/x + (3*a*(-12*b*c + 6*a*f - b*x*(
11*d + 10*e*x) + a*x*(5*g + 4*h*x)))/(a + b*x^3) + (a^2*(-9*b*(c + x*(d +
e*x)) + 9*a*(f + x*(g + h*x))))/(a + b*x^3)^2 + (2*Sqrt[3]*a^(1/3)*(20*b^(
4/3)*d + 14*a^(1/3)*b*e - 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(1 - (2*b^(1
/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + 54*(-3*b*c + a*f)*Log[x] - (2*a^(1/3)*
(20*b^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1/3)*g + 2*a^(4/3)*h)*Log[a^(1/3)
+ b^(1/3)*x])/b^(2/3) + (a^(1/3)*(20*b^(4/3)*d - 14*a^(1/3)*b*e - 5*a*b^(1
/3)*g + 2*a^(4/3)*h)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/
3) + 18*(3*b*c - a*f)*Log[a + b*x^3])/(54*a^4)
```

3.429.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2368, 25, 2368, 27, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4 (a+bx^3)^3} \, dx$$

$$\Big\downarrow \textbf{2368}$$

$$-\frac{\int -\frac{\frac{3b^3(bc-af)x^6}{a^2}-4b^2\left(\frac{be}{a}-h\right)x^5-5b^2\left(\frac{bd}{a}-g\right)x^4-6b^2\left(\frac{bc}{a}-f\right)x^3+6b^2ex^2+6b^2dx+6b^2c}{x^4(bx^3+a)^2}dx}{6ab^2}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{6a^2 (a+bx^3)^2}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{\frac{3b^3(bc-af)x^6}{a^2}-4b^2\left(\frac{be}{a}-h\right)x^5-5b^2\left(\frac{bd}{a}-g\right)x^4-6b^2\left(\frac{bc}{a}-f\right)x^3+6b^2ex^2+6b^2dx+6b^2c}{x^4(bx^3+a)^2}dx}{6ab^2}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{6a^2 (a+bx^3)^2}$$

$$\Big\downarrow \textbf{2368}$$

$$\frac{-\frac{\int -\frac{2\left(-b^4\left(\frac{5be}{a}-2h\right)x^5-b^4\left(\frac{11bd}{a}-5g\right)x^4-9b^4\left(\frac{2bc}{a}-f\right)x^3+9b^4ex^2+9b^4dx+9b^4c\right)}{x^4\left(bx^3+a\right)}dx}{3ab^2}-\frac{x\left(-3b^3x^2\left(\frac{5bc}{a}-3f\right)+b^2(11bd-5ag)+2b^2x(5be-2ah)\right)}{3a^2(a+bx^3)}}{6ab^2}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{6a^2 (a+bx^3)^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{2\int \frac{-b^4\left(\frac{5be}{a}-2h\right)x^5-b^4\left(\frac{11bd}{a}-5g\right)x^4-9b^4\left(\frac{2bc}{a}-f\right)x^3+9b^4ex^2+9b^4dx+9b^4c}{x^4\left(bx^3+a\right)}dx}{3ab^2}-\frac{x\left(-3b^3x^2\left(\frac{5bc}{a}-3f\right)+b^2(11bd-5ag)+2b^2x(5be-2ah)\right)}{3a^2(a+bx^3)}}{6ab^2}-\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{6a^2 (a+bx^3)^2}$$

$$\Big\downarrow \textbf{2373}$$

$$\frac{\frac{2\int \left(\frac{9(af-3bc)b^4}{a^2x}+\frac{\left(9b(3bc-af)x^2-2a(7be-ah)x-5a(4bd-ag)\right)b^4}{a^2\left(bx^3+a\right)}+\frac{9eb^4}{ax^2}+\frac{9db^4}{ax^3}+\frac{9cb^4}{ax^4}\right)dx}{3ab^2}-\frac{x\left(-3b^3x^2\left(\frac{5bc}{a}-3f\right)+b^2(11bd-5ag)+2b^2x(5be-2ah)\right)}{3a^2(a+bx^3)}}{6ab^2}$$

$$\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{6a^2\left(a+bx^3\right)^2}$$

$\downarrow$ 2009

$$\frac{2\left(\frac{b^{10/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-2a^{4/3}h+14\sqrt[3]{a}be-5a\sqrt[3]{b}g+20b^{4/3}d\right)}{\sqrt{3}a^{5/3}}+\frac{b^{11/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\left(-\frac{2\sqrt[3]{a}(7be-ah)}{\sqrt[3]{b}}-5ag+20bd\right)}{6a^{5/3}}-\frac{b^{10/3}}{}\right.}{3ab^2}$$

$$\frac{x\left(-bx^2\left(\frac{bc}{a}-f\right)+x(be-ah)-ag+bd\right)}{6a^2\left(a+bx^3\right)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x]
```

output
```
-1/6*(x*(b*d - a*g + (b*e - a*h)*x - b*((b*c)/a - f)*x^2))/(a^2*(a + b*x^3
)^2) + (-1/3*(x*(b^2*(11*b*d - 5*a*g) + 2*b^2*(5*b*e - 2*a*h)*x - 3*b^3*((
5*b*c)/a - 3*f)*x^2))/(a^2*(a + b*x^3)) + (2*((-3*b^4*c)/(a*x^3) - (9*b^4*
d)/(2*a*x^2) - (9*b^4*e)/(a*x) + (b^(10/3)*(20*b^(4/3)*d + 14*a^(1/3)*b*e
- 5*a*b^(1/3)*g - 2*a^(4/3)*h)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(5/3)) - (9*b^4*(3*b*c - a*f)*Log[x])/a^2 - (b^(10/3)*(
5*b^(1/3)*(4*b*d - a*g) - 2*a^(1/3)*(7*b*e - a*h))*Log[a^(1/3) + b^(1/3)*x
])/(3*a^(5/3)) + (b^(11/3)*(20*b*d - 5*a*g - (2*a^(1/3)*(7*b*e - a*h))/b^(
1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)) + (3*b^4
*(3*b*c - a*f)*Log[a + b*x^3])/a^2))/(3*a*b^2))/(6*a*b^2)
```

3.429.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

3.429.4 Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00

method	result
default	$-\frac{c}{3a^3x^3}-\frac{d}{2a^3x^2}-\frac{e}{a^3x}+\frac{(af-3bc)\ln(x)}{a^4}+\frac{\left(\frac{2}{9}a^2bh-\frac{5}{9}a\,b^2e\right)x^5+\left(\frac{5}{18}a^2bg-\frac{11}{18}a\,b^2d\right)x^4+\left(\frac{1}{3}f\,a^2b-\frac{2}{3}a\,b^2c\right)x^3+\frac{a^2(7ah-13be)x^2}{18}+}{\left(b\,x^3+a\right)^2}$
risch	$\frac{\frac{2b(ah-7be)x^8}{9a^3}+\frac{5b(ag-4bd)x^7}{18a^3}+\frac{b(af-3bc)x^6}{3a^3}+\frac{7(ah-7be)x^5}{18a^2}+\frac{4(ag-4bd)x^4}{9a^2}+\frac{(af-3bc)x^3}{2a^2}-\frac{e\,x^2}{a}-\frac{xd}{2a}-\frac{c}{3a}}{x^3(b\,x^3+a)^2}+\frac{\ln(-x)f}{a^3}-\frac{3\ln(-x)bc}{a^4}+$

input
```
int((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOS
E)
```

output
```
-1/3*c/a^3/x^3-1/2*d/a^3/x^2-e/a^3/x+(a*f-3*b*c)/a^4*ln(x)+1/a^4*(((2/9*a^
2*b*h-5/9*a*b^2*e)*x^5+(5/18*a^2*b*g-11/18*a*b^2*d)*x^4+(1/3*f*a^2*b-2/3*a
*b^2*c)*x^3+1/18*a^2*(7*a*h-13*b*e)*x^2+1/9*a^2*(4*a*g-7*b*d)*x+1/2*f*a^3-
5/6*a^2*b*c)/(b*x^3+a)^2+1/9*(5*a^2*g-20*a*b*d)*(1/3/b/(a/b)^(2/3)*ln(x+(a
/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/9*(2*a^2*h-14*a*
b*e)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^
(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1)))+1/27*(-9*a*b*f+27*b^2*c)*ln(b*x^3+a)/b)
```

3.429.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 101.30 (sec) , antiderivative size = 16697, normalized size of antiderivative = 42.27

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="fr
icas")
```

output
```
Too large to include
```

3.429.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/x**4/(b*x**3+a)**3,x)
```

output
```
Timed out
```

3.429.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.12

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{4\,(7\,b^2e-abh)x^8+5\,(4\,b^2d-abg)x^7+6\,(3\,b^2c-abf)x^6+7\,(7\,abe-a^2h)x^5+18\,a^2ex^2+8\,(4\,abd-a}{18\,(a^3b^2x^9+2\,a^4bx^6+a^5x^3)}$$

$$-\frac{(3\,bc-af)\log{(x)}}{a^4}$$

$$-\frac{\sqrt{3}\left(14\,abe\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2h\left(\frac{a}{b}\right)^{\frac{2}{3}}+20\,abd\left(\frac{a}{b}\right)^{\frac{1}{3}}-5\,a^2g\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5}$$

$$+\frac{\left(54\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-18\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}-14\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}+20\,abd-5\,a^2g\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$+\frac{\left(27\,b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-9\,abf\left(\frac{a}{b}\right)^{\frac{2}{3}}+14\,abe\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2h\left(\frac{a}{b}\right)^{\frac{1}{3}}-20\,abd+5\,a^2g\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="ma
xima")
```

output
```
-1/18*(4*(7*b^2*e - a*b*h)*x^8 + 5*(4*b^2*d - a*b*g)*x^7 + 6*(3*b^2*c - a*
b*f)*x^6 + 7*(7*a*b*e - a^2*h)*x^5 + 18*a^2*e*x^2 + 8*(4*a*b*d - a^2*g)*x^
4 + 9*a^2*d*x + 9*(3*a*b*c - a^2*f)*x^3 + 6*a^2*c)/(a^3*b^2*x^9 + 2*a^4*b*
x^6 + a^5*x^3) - (3*b*c - a*f)*log(x)/a^4 - 1/27*sqrt(3)*(14*a*b*e*(a/b)^(
2/3) - 2*a^2*h*(a/b)^(2/3) + 20*a*b*d*(a/b)^(1/3) - 5*a^2*g*(a/b)^(1/3))*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/a^5 + 1/54*(54*b^2*c*(a
/b)^(2/3) - 18*a*b*f*(a/b)^(2/3) - 14*a*b*e*(a/b)^(1/3) + 2*a^2*h*(a/b)^(1
/3) + 20*a*b*d - 5*a^2*g)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a
/b)^(2/3)) + 1/27*(27*b^2*c*(a/b)^(2/3) - 9*a*b*f*(a/b)^(2/3) + 14*a*b*e*(
a/b)^(1/3) - 2*a^2*h*(a/b)^(1/3) - 20*a*b*d + 5*a^2*g)*log(x + (a/b)^(1/3)
)/(a^4*b*(a/b)^(2/3))
```

3.429.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.08

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5}{x^4\left(a+bx^3\right)^3}\,dx$$

$$= \frac{\sqrt{3}\left(20\,b^2d-5\,abg-14\,(-ab^2)^{\frac{1}{3}}\,be+2\,(-ab^2)^{\frac{1}{3}}\,ah\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^3} + \frac{\left(20\,b^2d-5\,abg+14\,(-ab^2)^{\frac{1}{3}}\,be-2\,(-ab^2)^{\frac{1}{3}}\,ah\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^3} + \frac{(3\,bc-af)\log\left(|bx^3+a|\right)}{3\,a^4} - \frac{(3\,bc-af)\log\left(|x|\right)}{a^4} + \frac{\left(14\,a^5b^2e\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^6bh\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,a^5b^2d-5\,a^6bg\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^9b} - \frac{4\,(7\,ab^2e-a^2bh)x^8+5\,(4\,ab^2d-a^2bg)x^7+6\,(3\,ab^2c-a^2bf)x^6+18\,a^3ex^2+7\,(7\,a^2be-a^3h)x^5+9\,a^3}{18\,(bx^3+a)^2a^4x^3}$$

input
```
integrate((h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/x^4/(b*x^3+a)^3,x, algorithm="gi
ac")
```

output
```
1/27*sqrt(3)*(20*b^2*d - 5*a*b*g - 14*(-a*b^2)^(1/3)*b*e + 2*(-a*b^2)^(1/3
)*a*h)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/
3)*a^3) + 1/54*(20*b^2*d - 5*a*b*g + 14*(-a*b^2)^(1/3)*b*e - 2*(-a*b^2)^(1
/3)*a*h)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) + 1
/3*(3*b*c - a*f)*log(abs(b*x^3 + a))/a^4 - (3*b*c - a*f)*log(abs(x))/a^4 +
 1/27*(14*a^5*b^2*e*(-a/b)^(1/3) - 2*a^6*b*h*(-a/b)^(1/3) + 20*a^5*b^2*d -
 5*a^6*b*g)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^9*b) - 1/18*(4*(7*a
*b^2*e - a^2*b*h)*x^8 + 5*(4*a*b^2*d - a^2*b*g)*x^7 + 6*(3*a*b^2*c - a^2*b
*f)*x^6 + 18*a^3*e*x^2 + 7*(7*a^2*b*e - a^3*h)*x^5 + 9*a^3*d*x + 8*(4*a^2*
b*d - a^3*g)*x^4 + 6*a^3*c + 9*(3*a^2*b*c - a^3*f)*x^3)/((b*x^3 + a)^2*a^4
*x^3)
```

3.429.9 Mupad [B] (verification not implemented)

Time = 10.35 (sec) , antiderivative size = 1994, normalized size of antiderivative = 5.05

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{x^4 (a + bx^3)^3} \, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5)/(x^4*(a + b*x^3)^3),x)
```

output
```
symsum(log(- (1200*b^5*c*d^2 - 1134*b^5*c^2*e + 75*a^2*b^3*c*g^2 - 126*a^2
*b^3*e*f^2 - 25*a^3*b^2*f*g^2 + 18*a^3*b^2*f^2*h - 400*a*b^4*d^2*f + 162*a
*b^4*c^2*h - 108*a^2*b^3*c*f*h + 200*a^2*b^3*d*f*g - 600*a*b^4*c*d*g + 756
*a*b^4*c*e*f)/(81*a^9) - root(19683*a^12*b^2*z^3 + 19683*a^9*b^2*f*z^2 - 5
9049*a^8*b^3*c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z - 3240*a^6*b^2*d
*h*z - 39366*a^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b^2*f^2*z + 59
049*a^4*b^4*c^2*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890*a^3*b^2*e*f
*g - 1080*a^3*b^2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*f + 5670*a^
2*b^3*c*e*g + 3240*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4*c^2*f + 1
176*a^3*b^2*e^2*h - 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 6561*a^2*b^3
*c*f^2 + 729*a^3*b^2*f^3 - 2744*a^2*b^3*e^3 - 125*a^4*b*g^3 + 8000*a*b^4*d
^3 + 8*a^5*h^3 - 19683*b^5*c^3, z, k)*((400*a^4*b^4*d^2 + 25*a^6*b^2*g^2 +
 756*a^4*b^4*c*e - 108*a^5*b^3*c*h - 200*a^5*b^3*d*g - 252*a^5*b^3*e*f + 3
6*a^6*b^2*f*h)/(81*a^9) + root(19683*a^12*b^2*z^3 + 19683*a^9*b^2*f*z^2 -
59049*a^8*b^3*c*z^2 + 810*a^7*b*g*h*z - 5670*a^6*b^2*e*g*z - 3240*a^6*b^2*
d*h*z - 39366*a^5*b^3*c*f*z + 22680*a^5*b^3*d*e*z + 6561*a^6*b^2*f^2*z + 5
9049*a^4*b^4*c^2*z + 270*a^4*b*f*g*h - 22680*a*b^4*c*d*e - 1890*a^3*b^2*e*
f*g - 1080*a^3*b^2*d*f*h - 810*a^3*b^2*c*g*h + 7560*a^2*b^3*d*e*f + 5670*a
^2*b^3*c*e*g + 3240*a^2*b^3*c*d*h - 168*a^4*b*e*h^2 + 19683*a*b^4*c^2*f +
1176*a^3*b^2*e^2*h - 6000*a^2*b^3*d^2*g + 1500*a^3*b^2*d*g^2 - 6561*a^2...
```

3.430 $\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}} \, dx$

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3.430.1 Optimal result

Integrand size = 25, antiderivative size = 583

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}} \, dx = -\frac{4ae\sqrt{a+bx^3}}{9b^2} + \frac{2cx\sqrt{a+bx^3}}{5b} + \frac{2dx^2\sqrt{a+bx^3}}{7b} + \frac{2ex^3\sqrt{a+bx^3}}{9b} - \frac{8ad\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$- \frac{4\sqrt{2+\sqrt{3}}a\left(7\sqrt[3]{b}c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-4/9*a*e*(b*x^3+a)^(1/2)/b^2+2/5*c*x*(b*x^3+a)^(1/2)/b+2/7*d*x^2*(b*x^3+a)
^(1/2)/b+2/9*e*x^3*(b*x^3+a)^(1/2)/b-8/7*a*d*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))+4/7*3^(1/4)*a^(4/3)*d*(a^(1/3)+b^(1/3)*x)*Ellip
ticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-4/105*a*(a
^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*c-10*a^(1/3)*d*(1-3^(1/2)))*(1
/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.430.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.23

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{-2(a+bx^3)\left(70ae-bx(63c+5x(9d+7ex))\right)-126abcx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)-}{315b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

output
```
(-2*(a + b*x^3)*(70*a*e - b*x*(63*c + 5*x*(9*d + 7*e*x))) - 126*a*b*c*x*Sq
rt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 90*a*b*
d*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/
(315*b^2*Sqrt[a + b*x^3])
```

3.430.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2427, 27, 2028, 2427, 27, 2028, 2427, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (c+dx+ex^2)}{\sqrt{a+bx^3}} \, dx$$

$$\Big\downarrow \text{2427}$$

$$\frac{2\int -\frac{3(-3bdx^4-3bcx^3+2aex^2)}{2\sqrt{bx^3+a}}dx}{9b}+\frac{2ex^3\sqrt{a+bx^3}}{9b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{\int \frac{-3bdx^4-3bcx^3+2aex^2}{\sqrt{bx^3+a}}dx}{3b}$$

$$\Big\downarrow \text{2028}$$

$$\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{\int \frac{x^2(-3bdx^2-3bcx+2ae)}{\sqrt{bx^3+a}}dx}{3b}$$

$$\Big\downarrow \text{2427}$$

$$\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{\frac{2\int \frac{-21b^2cx^3+14abex^2+12abdx}{2\sqrt{bx^3+a}}dx}{7b}-\frac{6}{7}dx^2\sqrt{a+bx^3}}{3b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{\frac{\int \frac{-21b^2cx^3+14abex^2+12abdx}{\sqrt{bx^3+a}}dx}{7b}-\frac{6}{7}dx^2\sqrt{a+bx^3}}{3b}$$

$$\Big\downarrow \text{2028}$$

$$\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{\frac{\int \frac{x(-21b^2cx^2+14abex+12abd)}{\sqrt{bx^3+a}}dx}{7b}-\frac{6}{7}dx^2\sqrt{a+bx^3}}{3b}$$

$$\Big\downarrow \text{2427}$$

$$\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{\frac{\frac{2\int \frac{35aex^2b^2+21acb^2+30adxb^2}{\sqrt{bx^3+a}}dx}{5b}-\frac{42}{5}bcx\sqrt{a+bx^3}}{7b}-\frac{6}{7}dx^2\sqrt{a+bx^3}}{3b}$$

$$\Big\downarrow \text{2425}$$

$$\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{\frac{\frac{2\left(\int\frac{21acb^2+30adxb^2}{\sqrt{bx^3+a}}dx+35ab^2e\int\frac{x^2}{\sqrt{bx^3+a}}dx\right)}{5b}-\frac{42}{5}bcx\sqrt{a+bx^3}}{7b}-\frac{6}{7}dx^2\sqrt{a+bx^3}}{3b}$$

$\downarrow$ 793

$$\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{\frac{\frac{2\left(\int\frac{21acb^2+30adxb^2}{\sqrt{bx^3+a}}dx+\frac{70}{3}abe\sqrt{a+bx^3}\right)}{5b}-\frac{42}{5}bcx\sqrt{a+bx^3}}{7b}-\frac{6}{7}dx^2\sqrt{a+bx^3}}{3b}$$

$\downarrow$ 2417

$$\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{\frac{\frac{2\left(3ab^{5/3}\left(7\sqrt[3]{b}c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\int\frac{1}{\sqrt{bx^3+a}}dx+30ab^{5/3}d\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{70}{3}abe\sqrt{a+bx^3}\right)}{5b}-\frac{42}{5}bcx\sqrt{a+bx^3}}{7b}-\frac{6}{7}dx^2\sqrt{a+bx^3}}{3b}$$

$\downarrow$ 759

$$\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{\frac{2\left(30ab^{5/3}d\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}ab^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x\ldots}{\sqrt[3]{b}x\ldots}\right.\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{5b}\ldots}{7b}\ldots}{3b}$$

$\downarrow$ 2416

$$\frac{2ex^3\sqrt{a+bx^3}}{9b}-\frac{\frac{2\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}ab^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}c-10\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+30ab^{5/\ldots}\right.}{\ldots}}{\ldots}$$

input
```
Int[(x^3*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

3.430. $\int\frac{x^3\left(c+dx+ex^2\right)}{\sqrt{a+bx^3}}dx$

output
```
(2*e*x^3*Sqrt[a + b*x^3])/(9*b) - ((-6*d*x^2*Sqrt[a + b*x^3])/7 + ((-42*b*
c*x*Sqrt[a + b*x^3])/5 + (2*((70*a*b*e*Sqrt[a + b*x^3])/3 + 30*a*b^(5/3)*d
*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(
1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (2*3^(3/4)*Sqrt[
2 + Sqrt[3]]*a*b^(4/3)*(7*b^(1/3)*c - 10*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])))/(5*b))/(7*b))/(3*b)
```

3.430.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

3.430. $\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.430.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{2(-35be\,x^3-45bd\,x^2-63bcx+70ae)\sqrt{b\,x^3+a}}{315b^2}-\frac{2a\left(-\frac{14ic\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b\right.}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{}}{}}}}{}\right.}{}$ ... $4iac\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{}$
elliptic	$\frac{2e\,x^3\sqrt{b\,x^3+a}}{9b}+\frac{2d\,x^2\sqrt{b\,x^3+a}}{7b}+\frac{2cx\sqrt{b\,x^3+a}}{5b}-\frac{4ae\sqrt{b\,x^3+a}}{9b^2}+\frac{}{}$
default	Expression too large to display

3.430. $\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

input
```
int(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(-35*b*e*x^3-45*b*d*x^2-63*b*c*x+70*a*e)/b^2*(b*x^3+a)^(1/2)-2/35*a
/b*(-14/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))-20/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b
^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(
-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b
*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.430.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.14

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(126\,a\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-180\,a\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0\right.\right.\right.}{315\,b^2}$$

input
```
integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(126*a*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 180*a*sqrt(b)*
d*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (35*b*e*
x^3 + 45*b*d*x^2 + 63*b*c*x - 70*a*e)*sqrt(b*x^3 + a))/b^2
```

3.430.6 Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.22

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = e\left(\begin{cases} -\frac{4a\sqrt{a+bx^3}}{9b^2}+\frac{2x^3\sqrt{a+bx^3}}{9b} & \text{for } b\neq 0 \\ \frac{x^6}{6\sqrt{a}} & \text{otherwise} \end{cases}\right) + \frac{cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

output
```
e*Piecewise((-4*a*sqrt(a + b*x**3)/(9*b**2) + 2*x**3*sqrt(a + b*x**3)/(9*b
), Ne(b, 0)), (x**6/(6*sqrt(a)), True)) + c*x**4*gamma(4/3)*hyper((1/2, 4/
3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + d*x**5*gamm
a(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamm
a(8/3))
```

3.430.7 Maxima [F]

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{(ex^2+dx+c)x^3}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a), x)
```

3.430.8 Giac [F]

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx=\int \frac{(ex^2+dx+c)x^3}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)*x^3/sqrt(b*x^3 + a), x)
```

3.430.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx=\int \frac{x^3\left(e\,x^2+d\,x+c\right)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((x^3*(c + d*x + e*x^2))/(a + b*x^3)^(1/2),x)
```

output
```
int((x^3*(c + d*x + e*x^2))/(a + b*x^3)^(1/2), x)
```

3.431 $\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

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3.431.1 Optimal result

Integrand size = 25, antiderivative size = 560

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \frac{2c\sqrt{a+bx^3}}{3b}+\frac{2dx\sqrt{a+bx^3}}{5b}+\frac{2ex^2\sqrt{a+bx^3}}{7b}-\frac{8ae\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{4\sqrt{2+\sqrt{3}}a\left(7\sqrt[3]{b}d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\ldots}{\left(1+\sqrt{3}\right)\ldots}\right.\right.}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*c*(b*x^3+a)^(1/2)/b+2/5*d*x*(b*x^3+a)^(1/2)/b+2/7*e*x^2*(b*x^3+a)^(1/2
)/b-8/7*a*e*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+4/7*3^
(1/4)*a^(4/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2
))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-4/105*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(7*b^(1/3)*d-10*a^(1/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3
^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.431.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.22

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{2(a+bx^3)\left(35c+3x(7d+5ex)\right)-42adx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)-30aex^2\sqrt{1+\frac{bx^3}{a}}\cdots}{105b\sqrt{a+bx^3}}$$

input
```
Integrate[(x^2*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

output
```
(2*(a + b*x^3)*(35*c + 3*x*(7*d + 5*e*x)) - 42*a*d*x*Sqrt[1 + (b*x^3)/a]*H
ypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 30*a*e*x^2*Sqrt[1 + (b*x^3
)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(105*b*Sqrt[a + b*x^3
])
```

3.431.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2427, 27, 2028, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx+ex^2)}{\sqrt{a+bx^3}} \, dx$$

$$\Big\downarrow \text{2427}$$

$$\frac{2\int -\frac{-7bdx^3-7bcx^2+4aex}{2\sqrt{bx^3+a}}dx}{7b}+\frac{2ex^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2ex^2\sqrt{a+bx^3}}{7b}-\frac{\int \frac{-7bdx^3-7bcx^2+4aex}{\sqrt{bx^3+a}}dx}{7b}$$

$$\Big\downarrow \text{2028}$$

$$\frac{2ex^2\sqrt{a+bx^3}}{7b}-\frac{\int \frac{x(-7bdx^2-7bcx+4ae)}{\sqrt{bx^3+a}}dx}{7b}$$

$$\Big\downarrow \text{2427}$$

$$\frac{2ex^2\sqrt{a+bx^3}}{7b}-\frac{\frac{2\int \frac{-35b^2cx^2+20abex+14abd}{2\sqrt{bx^3+a}}dx}{5b}-\frac{14}{5}dx\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2ex^2\sqrt{a+bx^3}}{7b}-\frac{\frac{\int \frac{-35b^2cx^2+20abex+14abd}{\sqrt{bx^3+a}}dx}{5b}-\frac{14}{5}dx\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{2425}$$

$$\frac{2ex^2\sqrt{a+bx^3}}{7b}-\frac{\frac{\int \frac{14abd+20abex}{\sqrt{bx^3+a}}dx-35b^2c\int \frac{x^2}{\sqrt{bx^3+a}}dx}{5b}-\frac{14}{5}dx\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{793}$$

$$\frac{2ex^2\sqrt{a+bx^3}}{7b}-\frac{\frac{\int \frac{14abd+20abex}{\sqrt{bx^3+a}}dx-\frac{70}{3}bc\sqrt{a+bx^3}}{5b}-\frac{14}{5}dx\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{2417}$$

$$\frac{2ex^2\sqrt{a+bx^3}}{7b}-\frac{\frac{2ab^{2/3}\left(7\sqrt[3]{b}d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\int\frac{1}{\sqrt{bx^3+a}}dx+20ab^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{70}{3}bc\sqrt{a+bx^3}}{5b}-\frac{14}{5}dx\sqrt{a+bx^3}}{7b}$$

$\Big\downarrow$ 759

$$\frac{2ex^2\sqrt{a+bx^3}}{7b}-\frac{\frac{20ab^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{4\sqrt{2+\sqrt{3}}a\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{}\right.}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{5b}}{7b}$$

$\Big\downarrow$ 2416

$$\frac{2ex^2\sqrt{a+bx^3}}{7b}-\frac{\frac{4\sqrt{2+\sqrt{3}}a\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7\sqrt[3]{b}d-10\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+20ab^{2/3}e\left(\frac{\sqrt[3]{}}{}\right.}{5b}}{}$$

input
```
Int[(x^2*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

3.431. $\int\frac{x^2\left(c+dx+ex^2\right)}{\sqrt{a+bx^3}}\,dx$

output
```
(2*e*x^2*Sqrt[a + b*x^3])/(7*b) - ((-14*d*x*Sqrt[a + b*x^3])/5 + ((-70*b*c
*Sqrt[a + b*x^3])/3 + 20*a*b^(2/3)*e*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])) + (4*Sqrt[2 + Sqrt[3]]*a*b^(1/3)*(7*b^(1/3)*d - 10*(1 -
Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(5*b))/(7*b)
```

3.431.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

3.431. $\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}} \, dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.431.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.35

method	result
risch	$\frac{2(15e\,x^2+21dx+35c)\sqrt{b\,x^3+a}}{105b} - \frac{2a\left(-\frac{14id\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-}}{}\right.}{}$
elliptic	$\frac{2e\,x^2\sqrt{b\,x^3+a}}{7b}+\frac{2dx\sqrt{b\,x^3+a}}{5b}+\frac{2c\sqrt{b\,x^3+a}}{3b}+\frac{4iad\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-\right.}{}}}}{}$

3.431. $\int \frac{x^2\left(c+dx+ex^2\right)}{\sqrt{a+bx^3}}\,dx$

input
```
int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(15*e*x^2+21*d*x+35*c)/b*(b*x^3+a)^(1/2)-2/35*a/b*(-14/3*I*d*3^(1/2)
/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-20/3*I*e*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.431.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.13

$$\int \frac{x^2(c + dx + ex^2)}{\sqrt{a + bx^3}}\, dx =$$
$$-\frac{2\left(42\, a\sqrt{b}d\text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - 60\, a\sqrt{b}e\text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\right.\right.\right.}{105\, b^2}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(42*a*sqrt(b)*d*weierstrassPInverse(0, -4*a/b, x) - 60*a*sqrt(b)*e*
weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (15*b*e*x^
2 + 21*b*d*x + 35*b*c)*sqrt(b*x^3 + a))/b^2
```

3.431.6 Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.19

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = c\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right) + \frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{ex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

output
```
c*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True)
) + d*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/
(3*sqrt(a)*gamma(7/3)) + e*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**
3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

3.431.7 Maxima [F]

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{(ex^2+dx+c)x^2}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(b*x^3 + a)*c/b + integrate((e*x^4 + d*x^3)/sqrt(b*x^3 + a), x)
```

3.431.8 Giac [F]

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{(ex^2+dx+c)x^2}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)*x^2/sqrt(b*x^3 + a), x)
```

3.431.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{x^2\,(e\,x^2+d\,x+c)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^(1/2),x)
```

output
```
int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^(1/2), x)
```

3.432 $\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$

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3.432.1 Optimal result

Integrand size = 23, antiderivative size = 537

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \frac{2d\sqrt{a+bx^3}}{3b}+\frac{2ex\sqrt{a+bx^3}}{5b}+\frac{2c\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(5\left(1-\sqrt{3}\right)b^{2/3}c+2a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{(1-\sqrt{\ }}{(1+\sqrt{\ }}\right.\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*d*(b*x^3+a)^(1/2)/b+2/5*e*x*(b*x^3+a)^(1/2)/b+2*c*(b*x^3+a)^(1/2)/b^(2
/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)*c*(a^(1/3)+b^(1/3)*x)*
EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(
a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/15*
3^(3/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*a^(2/3)*e+5*b^(2/3)*
c*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(4/3)/(b*x^3+a)^(1/2)/(
a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.432.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.21

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{4(5d+3ex)\left(a+bx^3\right)-12aex\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+15bcx^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeo}}{30b\sqrt{a+bx^3}}$$

input
```
Integrate[(x*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

output
```
(4*(5*d + 3*e*x)*(a + b*x^3) - 12*a*e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 15*b*c*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeo
metric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(30*b*Sqrt[a + b*x^3])
```

3.432.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx+ex^2\right)}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \ 2427$$

$$\frac{2\int -\frac{-5bdx^2-5bcx+2ae}{2\sqrt{bx^3+a}}dx}{5b}+\frac{2ex\sqrt{a+bx^3}}{5b}$$

$$\Big\downarrow \ 27$$

$$\frac{2ex\sqrt{a+bx^3}}{5b}-\frac{\int \frac{-5bdx^2-5bcx+2ae}{\sqrt{bx^3+a}}dx}{5b}$$

$$\Big\downarrow \ 2425$$

$$\frac{2ex\sqrt{a+bx^3}}{5b}-\frac{\int \frac{2ae-5bcx}{\sqrt{bx^3+a}}dx-5bd\int \frac{x^2}{\sqrt{bx^3+a}}dx}{5b}$$

$$\Big\downarrow \ 793$$

$$\frac{2ex\sqrt{a+bx^3}}{5b}-\frac{\int \frac{2ae-5bcx}{\sqrt{bx^3+a}}dx-\frac{10}{3}d\sqrt{a+bx^3}}{5b}$$

$$\Big\downarrow \ 2417$$

$$\frac{2ex\sqrt{a+bx^3}}{5b}-\frac{\sqrt[3]{a}\left(2a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)\int \frac{1}{\sqrt{bx^3+a}}dx-5b^{2/3}c\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{10}{3}d\sqrt{a+bx^3}}{5b}$$

$$\Big\downarrow \ 759$$

$$\frac{2ex\sqrt{a+bx^3}}{5b}-\frac{-5b^{2/3}c\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{5b}$$

$$\Big\downarrow \ 2416$$

3.432. $\int \frac{x\left(c+dx+ex^2\right)}{\sqrt{a+bx^3}}\,dx$

$$\frac{2ex\sqrt{a+bx^3}}{5b} - \frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - 5b^{2/}$$

input
```
Int[(x*(c + d*x + e*x^2))/Sqrt[a + b*x^3],x]
```

output
```
(2*e*x*Sqrt[a + b*x^3])/(5*b) - ((-10*d*Sqrt[a + b*x^3])/3 - 5*b^(2/3)*c*(
(2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (2*Sqrt[2 + Sqrt[3
]]*a^(1/3)*(5*(1 - Sqrt[3])*b^(2/3)*c + 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]))/(5*b)
```

3.432.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.432. $\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}} dx$

3.432.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.39

method	result
risch	$\frac{2(3ex+5d)\sqrt{b\,x^3+a}}{15b}-\frac{-\frac{4iae\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\cdots\right.}{\left(-\cdots\right.}}\cdots}{\cdots}}{3b\sqrt{b\,x\cdots}}$
elliptic	$\frac{2ex\sqrt{b\,x^3+a}}{5b}+\frac{2d\sqrt{b\,x^3+a}}{3b}+\frac{4iae\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\cdots\right.}{\cdots}}\cdots}{\cdots}$
default	$e\left(\frac{2x\sqrt{b\,x^3+a}}{5b}+\frac{4ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{\cdots}}{\cdots}\right.}{\left(-a\,b^2\cdots\right.}}\cdots}{15b^2\sqrt{b\,x^3+\cdots}}\right.$

3.432. $\int \frac{x\left(c+dx+ex^2\right)}{\sqrt{a+bx^3}}\,dx$

input
```
int(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*e*x+5*d)/b*(b*x^3+a)^(1/2)-1/5/b*(-4/3*I*a*e*3^(1/2)/b*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+10/3*I*c*3^(1/2)*(-a*b^2)
^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.432.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.12

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(6\,a\sqrt{b}e\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+15\,b^{\frac{3}{2}}c\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b}\right.\right.\right.}{15\,b^2}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(6*a*sqrt(b)*e*weierstrassPInverse(0, -4*a/b, x) + 15*b^(3/2)*c*weie
rstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - sqrt(b*x^3 + a
)*(3*b*e*x + 5*b*d))/b^2
```

3.432.6 Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.20

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = d\left(\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{for } b=0 \\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise} \end{cases}\right) + \frac{cx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

output
```
d*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True)
) + c*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/
(3*sqrt(a)*gamma(5/3)) + e*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**
3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3))
```

3.432.7 Maxima [F]

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{(ex^2+dx+c)x}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a), x)
```

3.432.8 Giac [F]

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{(ex^2+dx+c)x}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)*x/sqrt(b*x^3 + a), x)
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2)}{\sqrt{a+bx^3}}\,dx = \int \frac{x\,(e\,x^2+d\,x+c)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((x*(c + d*x + e*x^2))/(a + b*x^3)^(1/2),x)
```

output
```
int((x*(c + d*x + e*x^2))/(a + b*x^3)^(1/2), x)
```

3.433 $\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx$

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3.433.1 Optimal result

Integrand size = 22, antiderivative size = 509

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx = \frac{2e\sqrt{a+bx^3}}{3b} + \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*e*(b*x^3+a)^(1/2)/b+2*d*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))-3^(1/4)*a^(1/3)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2
)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*(a^(1/3)+b^(1/3)*x)*Ellipti
cF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/
2)+2*I)*(b^(1/3)*c-a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.433.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.21

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx = \frac{4e(a+bx^3)+6bcx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+3bdx^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\right.}{6b\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^3],x]
```

output
```
(4*e*(a + b*x^3) + 6*b*c*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2,
 4/3, -((b*x^3)/a)] + 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2,
 2/3, 5/3, -((b*x^3)/a)])/(6*b*Sqrt[a + b*x^3])
```

3.433.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx$$

$\big\downarrow$ 2425

$$\int \frac{c+dx}{\sqrt{bx^3+a}}dx+e\int \frac{x^2}{\sqrt{bx^3+a}}dx$$

$\big\downarrow$ 793

$$\int \frac{c+dx}{\sqrt{bx^3+a}}dx+\frac{2e\sqrt{a+bx^3}}{3b}$$

$\big\downarrow$ 2417

$$\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2e\sqrt{a+bx^3}}{3b}$$

$\big\downarrow$ 759

$$\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2e\sqrt{a+bx^3}}{3b}$$

$\big\downarrow$ 2416

3.433. $\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{d\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$\frac{2e\sqrt{a+bx^3}}{3b}$$

input
```
Int[(c + d*x + e*x^2)/Sqrt[a + b*x^3],x]
```

output
```
(2*e*Sqrt[a + b*x^3])/(3*b) + (d*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3])))/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(c - ((1 - Sqrt[3])*a^(1/3)*
d)/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.433.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

3.433. $\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx$

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

3.433.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.44

method	result
default	$-\frac{2ic\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{b\,x^3+a}}$
risch	$-\frac{2ic\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{b\,x^3+a}}$
elliptic	$-\frac{2ic\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{b\,x^3+a}}$

3.433. $\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx$

input
```
int((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+2/3*e*(b*x^3+a)^(1/2)/b-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)))
```

3.433.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx = \frac{2\left(3\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-3\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right.}{3b}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(3*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 3*sqrt(b)*d*weierstra
ssZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + sqrt(b*x^3 + a)*e)/
b
```

3.433.6 Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.21

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx = e\left(\begin{cases}\frac{x^3}{3\sqrt{a}} & \text{for } b=0\\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise}\end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

output
```
e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True)
) + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

3.433.7 Maxima [F]

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

3.433.8 Giac [F]

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

3.433.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^3}}\,dx = \int \frac{e\,x^2+d\,x+c}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c + d*x + e*x^2)/(a + b*x^3)^(1/2),x)
```

output
```
int((c + d*x + e*x^2)/(a + b*x^3)^(1/2), x)
```

3.434 $\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx$

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3.434.1 Optimal result

Integrand size = 25, antiderivative size = 518

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = \frac{2e\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2c\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}d-\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2*e*(b*x^3+a)^(1/2)/b^(2/3
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3^(1/4)*a^(1/3)*e*(a^(1/3)+b^(1/3)*x)*El
lipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*(a^
(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*d-a^(1/3)*e*(1-3^(1/2)))*(1/2*6^(
1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.434.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.25

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = -\frac{2c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} + \frac{dx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{a+bx^3}} + \frac{ex^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x + e*x^2)/(x*Sqrt[a + b*x^3]),x]
```

output
```
(-2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) + (d*x*Sqrt[1 + (b*x^3
)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/Sqrt[a + b*x^3] + (e*
x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(2
*Sqrt[a + b*x^3])
```

3.434.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{2371}$$

$$c\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{d+ex}{\sqrt{bx^3+a}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}c\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int \frac{d+ex}{\sqrt{bx^3+a}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{2c\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int \frac{d+ex}{\sqrt{bx^3+a}}dx$$

$$\Big\downarrow \text{221}$$

$$\int \frac{d+ex}{\sqrt{bx^3+a}}dx-\frac{2\text{carctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$\Big\downarrow \text{2417}$$

$$\left(d-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{e\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\text{carctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$\Big\downarrow \text{759}$$

$$\frac{e\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(d-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{2c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$\big\downarrow$ 2416

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(d-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{e\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$\frac{2c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Int[(c + d*x + e*x^2)/(x*Sqrt[a + b*x^3]),x]
```

3.434. $\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx$

output
```
(-2*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) + (e*((2*Sqrt[a + b*x^
3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt
[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(d -
 ((1 - Sqrt[3])*a^(1/3)*e)/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.434.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.434.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.43

method	result
default	$-\frac{2 i d \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{3b\sqrt{b\,x^{3}+a}}$
elliptic	$-\frac{2 i d \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{3b\sqrt{b\,x^{3}+a}}$

input
```
int((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

3.434. $\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx$

output
```
-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))-2/3*I*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*c*a
rctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

3.434.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.37

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = \left[\frac{\sqrt{a}bc\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+12\,a\sqrt{b}d\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-12\,a\sqrt{b}e\mathrm{wei}}{6\,ab}\right.$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(a)*b*c*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 +
 a)*sqrt(a) + 8*a^2)/x^6) + 12*a*sqrt(b)*d*weierstrassPInverse(0, -4*a/b,
x) - 12*a*sqrt(b)*e*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a
/b, x)))/(a*b), 1/3*(sqrt(-a)*b*c*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3
 + 2*a)) + 6*a*sqrt(b)*d*weierstrassPInverse(0, -4*a/b, x) - 6*a*sqrt(b)*e
*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/(a*b)]
```

3.434.6 Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.20

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = -\frac{2c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}} + \frac{dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(1/2),x)
```

output
```
-2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)) + d*x*gamma(1/3)*hyper(
(1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + e*x
**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*gamma(5/3))
```

3.434.7 Maxima [F]

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+a}\,x}\,dx$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x), x)
```

3.434.8 Giac [F]

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx=\int \frac{ex^2+dx+c}{\sqrt{bx^3+ax}}\,dx$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x), x)
```

3.434.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x\sqrt{a+bx^3}}\,dx=\int \frac{e\,x^2+d\,x+c}{x\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c + d*x + e*x^2)/(x*(a + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x + e*x^2)/(x*(a + b*x^3)^(1/2)), x)
```

3.435 $\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx$

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3.435.1 Optimal result

Integrand size = 25, antiderivative size = 547

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = -\frac{c\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}c\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2d\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt{2+\sqrt{3}}\left(\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots}\right.\right.}{\sqrt[4]{3}a^{2/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-c*(b*x^3+a)^(1/2)/a/x+b^(1
/3)*c*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-1/2*3^(1/4)*b^(1/3
)*c*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a
^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)-1/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(-2*a^(2/3)*e+b^
(2/3)*c*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(
1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)
```

3.435.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.23

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = -\frac{2d\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{c\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt{a+bx^3}} + \frac{ex\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^2*Sqrt[a + b*x^3]),x]
```

output
```
(-2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (c*Sqrt[1 + (b*x^3)/
a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(x*Sqrt[a + b*x^3]) +
(e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/S
qrt[a + b*x^3]
```

3.435.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2374, 25, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \ 2374$$

$$-\frac{\int -\frac{bcx^2+2aex+2ad}{x\sqrt{bx^3+a}}dx}{2a}-\frac{c\sqrt{a+bx^3}}{ax}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{bcx^2+2aex+2ad}{x\sqrt{bx^3+a}}dx}{2a}-\frac{c\sqrt{a+bx^3}}{ax}$$

$$\Big\downarrow \ 2371$$

$$\frac{\int \frac{2ae+bcx}{\sqrt{bx^3+a}}dx+2ad\int \frac{1}{x\sqrt{bx^3+a}}dx}{2a}-\frac{c\sqrt{a+bx^3}}{ax}$$

$$\Big\downarrow \ 798$$

$$\frac{\int \frac{2ae+bcx}{\sqrt{bx^3+a}}dx+\frac{2}{3}ad\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{2a}-\frac{c\sqrt{a+bx^3}}{ax}$$

$$\Big\downarrow \ 73$$

$$\frac{\int \frac{2ae+bcx}{\sqrt{bx^3+a}}dx+\frac{4ad\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}}{2a}-\frac{c\sqrt{a+bx^3}}{ax}$$

$$\Big\downarrow \ 221$$

$$\frac{\int \frac{2ae+bcx}{\sqrt{bx^3+a}}dx-\frac{4}{3}\sqrt{a}d\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{2a}-\frac{c\sqrt{a+bx^3}}{ax}$$

$$\Big\downarrow \ 2417$$

$$\frac{-\sqrt[3]{a}\left(\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\int \frac{1}{\sqrt{bx^3+a}}dx+b^{2/3}c\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{4}{3}\sqrt{a}d\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{2a}-\frac{c\sqrt{a+bx^3}}{ax}$$

$\downarrow$ 759

$$\frac{b^{2/3}c\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x-}{\sqrt[3]{b}x-}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2a}$$

$$\frac{c\sqrt{a+bx^3}}{ax}$$

$\downarrow$ 2416

$$\frac{-\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+b^{2/3}}{}$$

$$\frac{c\sqrt{a+bx^3}}{ax}$$

input
```
Int[(c + d*x + e*x^2)/(x^2*Sqrt[a + b*x^3]),x]
```

output
```
-((c*Sqrt[a + b*x^3])/(a*x)) + ((-4*Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/3 + b^(2/3)*c*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))
 - (2*Sqrt[2 + Sqrt[3]]*a^(1/3)*((1 - Sqrt[3])*b^(2/3)*c - 2*a^(2/3)*e)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*a)
```

3.435. $\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx$

3.435.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

3.435. $\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.435.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 758, normalized size of antiderivative = 1.39

method	result
elliptic	$-\frac{c\sqrt{b\,x^{3}+a}}{a x}-\frac{2 i e \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-\right.}{2}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{3 b\sqrt{b\,x^{3}+a}}$
default	$-\frac{2 i e \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F}{3 b\sqrt{b\,x^{3}+a}}$

3.435. $\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx$

input
```
int((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-c*(b*x^3+a)^(1/2)/a/x-2/3*I*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))-1/3*I*c/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)))-2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

3.435.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.43

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx$$
$$= \left[\frac{\sqrt{a}bdx\log\left(\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+12\,a\sqrt{b}ex\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-6\,b^{\frac{3}{2}}cx\text{weier}}{6\,abx}\right.$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(a)*b*d*x*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3
+ a)*sqrt(a) + 8*a^2)/x^6) + 12*a*sqrt(b)*e*x*weierstrassPInverse(0, -4*a/
b, x) - 6*b^(3/2)*c*x*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, x)) - 6*sqrt(b*x^3 + a)*b*c)/(a*b*x), 1/3*(sqrt(-a)*b*d*x*arctan(1/2
*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) + 6*a*sqrt(b)*e*x
*weierstrassPInverse(0, -4*a/b, x) - 3*b^(3/2)*c*x*weierstrassZeta(0, -4*a
/b, weierstrassPInverse(0, -4*a/b, x)) - 3*sqrt(b*x^3 + a)*b*c)/(a*b*x)]
```

3.435.6 Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.20

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = \frac{c\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)} - \frac{2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}} + \frac{ex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(1/2),x)
```

output
```
c*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x*gamma(2/3)) - 2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)) + e*
x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a
)*gamma(4/3))
```

3.435.7 Maxima [F]

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)
```

3.435.8 Giac [F]

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^2), x)
```

3.435.9 Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.22

$$\int \frac{c+dx+ex^2}{x^2\sqrt{a+bx^3}}\,dx = \frac{d\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\sqrt{a}} - \frac{2\,c\,\sqrt{\frac{a}{b\,x^3}+1}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\ \frac{11}{6};\ -\frac{a}{b\,x^3}\right)}{5\,x\,\sqrt{b\,x^3+a}} + \frac{e\,x\,\sqrt{\frac{b\,x^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\sqrt{b\,x^3+a}}$$

input
```
int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^(1/2)),x)
```

output
```
(d*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6
))/(3*a^(1/2)) - (2*c*(a/(b*x^3) + 1)^(1/2)*hypergeom([1/2, 5/6], 11/6, -a
/(b*x^3)))/(5*x*(a + b*x^3)^(1/2)) + (e*x*((b*x^3)/a + 1)^(1/2)*hypergeom(
[1/3, 1/2], 4/3, -(b*x^3)/a))/(a + b*x^3)^(1/2)
```

3.436 $\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx$

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3.436.1 Optimal result

Integrand size = 25, antiderivative size = 569

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx$$

$$= -\frac{c\sqrt{a+bx^3}}{2ax^2} - \frac{d\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}d\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{b}c+2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}}{\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}}\right.\right.}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/2*c*(b*x^3+a)^(1/2)/a/x^
2-d*(b*x^3+a)^(1/2)/a/x+b^(1/3)*d*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))-1/2*3^(1/4)*b^(1/3)*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^
(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/6*3^(3/4)*b^(1/3)*(a^(1/3
)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c+2*a^(1/3)*d*(1-3^(1/2)))*(1/2*6^(1/
2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.436.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.23

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = -\frac{2e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{c\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2\sqrt{a+bx^3}} - \frac{d\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^3*Sqrt[a + b*x^3]),x]
```

output
```
(-2*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) - (c*Sqrt[1 + (b*x^3)/
a]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/(2*x^2*Sqrt[a + b*x^3]
) - (d*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)]
)/(x*Sqrt[a + b*x^3])
```

3.436.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2374, 25, 2374, 27, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{2374}$$

$$-\frac{\int -\frac{-bcx^2+4aex+4ad}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{c\sqrt{a+bx^3}}{2ax^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-bcx^2+4aex+4ad}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{c\sqrt{a+bx^3}}{2ax^2}$$

$$\Big\downarrow \text{2374}$$

$$\frac{-\frac{\int -\frac{2\left(4ea^2+2bdx^2a-bcxa\right)}{x\sqrt{bx^3+a}}dx}{2a}-\frac{4d\sqrt{a+bx^3}}{x}}{4a}-\frac{c\sqrt{a+bx^3}}{2ax^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{4ea^2+2bdx^2a-bcxa}{x\sqrt{bx^3+a}}dx}{a}-\frac{4d\sqrt{a+bx^3}}{x}}{4a}-\frac{c\sqrt{a+bx^3}}{2ax^2}$$

$$\Big\downarrow \text{2371}$$

$$\frac{\frac{4a^2e\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{2abdx-abc}{\sqrt{bx^3+a}}dx}{a}-\frac{4d\sqrt{a+bx^3}}{x}}{4a}-\frac{c\sqrt{a+bx^3}}{2ax^2}$$

$$\Big\downarrow \text{798}$$

$$\frac{\frac{\frac{4}{3}a^2e\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int \frac{2abdx-abc}{\sqrt{bx^3+a}}dx}{a}-\frac{4d\sqrt{a+bx^3}}{x}}{4a}-\frac{c\sqrt{a+bx^3}}{2ax^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{8a^2e\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int \frac{2abdx-abc}{\sqrt{bx^3+a}}dx}{a}-\frac{4d\sqrt{a+bx^3}}{x}}{4a}-\frac{c\sqrt{a+bx^3}}{2ax^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\int \frac{2abdx-abc}{\sqrt{bx^3+a}}dx-\frac{8}{3}a^{3/2}e\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a}-\frac{4d\sqrt{a+bx^3}}{x}}{4a}-\frac{c\sqrt{a+bx^3}}{2ax^2}$$

$\downarrow$ 2417

$$\frac{\frac{-ab^{2/3}\left(2\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{\sqrt{bx^3+a}}dx+2ab^{2/3}d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{8}{3}a^{3/2}e\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a}-\frac{4d\sqrt{a+bx^3}}{x}}{4a}-\frac{c\sqrt{a+bx^3}}{2ax^2}$$

$\downarrow$ 759

$$\frac{\frac{2ab^{2/3}d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{2\sqrt{2+\sqrt{3}}a\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{a}}{4a}$$

$$\frac{c\sqrt{a+bx^3}}{2ax^2}$$

$\downarrow$ 2416

$$-\frac{2\sqrt{2+\sqrt{3}}a\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2\left(1-\sqrt{3}\right)\sqrt[3]{a}d+\sqrt[3]{b}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+2ab^{2/3}d\left(\sqrt[3]{b}\right.$$

$$\frac{c\sqrt{a+bx^3}}{2ax^2}$$

input
```
Int[(c + d*x + e*x^2)/(x^3*Sqrt[a + b*x^3]),x]
```

3.436. $\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx$

output
```
-1/2*(c*Sqrt[a + b*x^3])/(a*x^2) + ((-4*d*Sqrt[a + b*x^3])/x + ((-8*a^(3/2
)*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 + 2*a*b^(2/3)*d*((2*Sqrt[a + b*x^3
])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) - (2*Sqrt[2 + Sqrt[3]]*a*b^(1/3)*(b^(
1/3)*c + 2*(1 - Sqrt[3])*a^(1/3)*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/a)/(4*a)
```

3.436.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.436. $\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx$

3.436.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 763, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,(2dx+c)}{2a\,x^{2}}+\frac{2ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\cdots\right.}{\left(-a\,b^{2}\right.}}\cdots}{3\sqrt{b\,x^{3}+\cdots}}$
elliptic	$-\frac{c\sqrt{b\,x^{3}+a}}{2a\,x^{2}}-\frac{d\sqrt{b\,x^{3}+a}}{ax}+\frac{ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\right.}{\cdots}\right.}{\cdots}}\cdots}{\cdots}$
default	Expression too large to display

input
```
int((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^3+a)^(1/2)*(2*d*x+c)/a/x^2+1/4/a*(2/3*I*c*3^(1/2)*(-a*b^2)^(1/3)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-4/3*I*d*3^(1/2)*(-a*b^2)^(1/
3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-8/3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(
1/2))
```

3.436.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.43

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx$$
$$= \left[\frac{\sqrt{a}ex^2\log\left(\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)-3\,\sqrt{b}cx^2\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-6\,\sqrt{b}dx^2\text{weie}}{6\,ax^2}\right.$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(a)*e*x^2*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3
+ a)*sqrt(a) + 8*a^2)/x^6) - 3*sqrt(b)*c*x^2*weierstrassPInverse(0, -4*a/b
, x) - 6*sqrt(b)*d*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -
4*a/b, x)) - 3*sqrt(b*x^3 + a)*(2*d*x + c))/(a*x^2), 1/6*(2*sqrt(-a)*e*x^2
*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) - 3*sq
rt(b)*c*x^2*weierstrassPInverse(0, -4*a/b, x) - 6*sqrt(b)*d*x^2*weierstras
sZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - 3*sqrt(b*x^3 + a)*(2
*d*x + c))/(a*x^2)]
```

3.436.6 Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.20

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = \frac{c\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{1}{3}\right)} + \frac{d\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)} - \frac{2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

input
```
integrate((e*x**2+d*x+c)/x**3/(b*x**3+a)**(1/2),x)
```

output
```
c*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**2*gamma(1/3)) + d*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3)) - 2*e*asinh(sqrt(a)/(sqrt(b)*x**(
3/2)))/(3*sqrt(a))
```

3.436.7 Maxima [F]

$$\int \frac{c+dx+ex^2}{x^3\sqrt{a+bx^3}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^3+a}\,x^3}\,dx$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)
```

3.436.8 Giac [F]

$$\int \frac{c + dx + ex^2}{x^3\sqrt{a + bx^3}}\,dx = \int \frac{ex^2 + dx + c}{\sqrt{bx^3 + a}x^3}\,dx$$

input
```
integrate((e*x^2+d*x+c)/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)/(sqrt(b*x^3 + a)*x^3), x)
```

3.436.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2}{x^3\sqrt{a + bx^3}}\,dx = \int \frac{e\,x^2 + d\,x + c}{x^3\,\sqrt{b\,x^3 + a}}\,dx$$

input
```
int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x + e*x^2)/(x^3*(a + b*x^3)^(1/2)), x)
```

3.437 $\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

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3.437.1 Optimal result

Integrand size = 25, antiderivative size = 594

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2x(ad+aex-bcx^2)}{3b^2\sqrt{a+bx^3}}+\frac{4c\sqrt{a+bx^3}}{3b^2}$$
$$+\frac{2dx\sqrt{a+bx^3}}{5b^2}+\frac{2ex^2\sqrt{a+bx^3}}{7b^2}-\frac{80ae\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{40\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7\ 3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{16\sqrt{2+\sqrt{3}}a\left(14\sqrt[3]{b}d-25\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{105\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*x*(-b*c*x^2+a*e*x+a*d)/b^2/(b*x^3+a)^(1/2)+4/3*c*(b*x^3+a)^(1/2)/b^2+2
/5*d*x*(b*x^3+a)^(1/2)/b^2+2/7*e*x^2*(b*x^3+a)^(1/2)/b^2-80/21*a*e*(b*x^3+
a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+40/21*a^(4/3)*e*(a^(1/3)+
b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(8/3
)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)-16/315*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(14*b^(1/3)*d-25
*a^(1/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(8/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)
```

3.437.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.23

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(70ac+56adx-150aex^2+35bcx^3+21bdx^4+15bex^5-56adx\sqrt{1+\frac{bx^3}{a}}\,\text{Hyper}\cdots\right.}{105b\cdots}$$

input
```
Integrate[(x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(70*a*c + 56*a*d*x - 150*a*e*x^2 + 35*b*c*x^3 + 21*b*d*x^4 + 15*b*e*x^5
 - 56*a*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)
/a)] + 150*a*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -(
(b*x^3)/a)]))/(105*b^2*Sqrt[a + b*x^3])
```

3.437.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2367, 27, 2427, 27, 2427, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}} \, dx$$

$$\Big\downarrow \; 2367$$

$$\frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}}-\frac{2\int \frac{-3ab^2ex^4-3ab^2dx^3-6ab^2cx^2+4a^2bex+2a^2bd}{2\sqrt{bx^3+a}}dx}{3ab^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}}-\frac{\int \frac{-3ab^2ex^4-3ab^2dx^3-6ab^2cx^2+4a^2bex+2a^2bd}{\sqrt{bx^3+a}}dx}{3ab^3}$$

$$\Big\downarrow \; 2427$$

$$\frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}}-\frac{\frac{2\int \frac{-21adx^3b^3-42acx^2b^3+14a^2db^2+40a^2exb^2}{2\sqrt{bx^3+a}}dx}{7b}-\frac{6}{7}abex^2\sqrt{a+bx^3}}{3ab^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}}-\frac{\frac{\int \frac{-21adx^3b^3-42acx^2b^3+14a^2db^2+40a^2exb^2}{\sqrt{bx^3+a}}dx}{7b}-\frac{6}{7}abex^2\sqrt{a+bx^3}}{3ab^3}$$

$$\Big\downarrow \; 2427$$

$$\frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}}-\frac{\frac{\frac{2\int \frac{-105acx^2b^4+56a^2db^3+100a^2exb^3}{\sqrt{bx^3+a}}dx}{5b}-\frac{42}{5}ab^2dx\sqrt{a+bx^3}}{7b}-\frac{6}{7}abex^2\sqrt{a+bx^3}}{3ab^3}$$

$$\Big\downarrow \; 2425$$

$$\frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}}-\frac{\frac{\frac{2\left(\int \frac{56a^2db^3+100a^2exb^3}{\sqrt{bx^3+a}}dx-105ab^4c\int \frac{x^2}{\sqrt{bx^3+a}}dx\right)}{5b}-\frac{42}{5}ab^2dx\sqrt{a+bx^3}}{7b}-\frac{6}{7}abex^2\sqrt{a+bx^3}}{3ab^3}$$

$$\Big\downarrow \; 793$$

$$\frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}}-\frac{\frac{\frac{2\left(\int \frac{56a^2db^3+100a^2exb^3}{\sqrt{bx^3+a}}dx-70ab^3c\sqrt{a+bx^3}\right)}{5b}-\frac{42}{5}ab^2dx\sqrt{a+bx^3}}{7b}-\frac{6}{7}abex^2\sqrt{a+bx^3}}{3ab^3}$$

$$\Big\downarrow \; 2417$$

$$\frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}}-\frac{\frac{\frac{2\left(4a^2b^{8/3}\left(14\sqrt[3]{b}d-25\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\int \frac{1}{\sqrt{bx^3+a}}dx+100a^2b^{8/3}e\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-70ab^3c\sqrt{a+bx^3}\right)}{5b}-\frac{42}{5}ab^2dx\sqrt{a+bx^3}}{7b}-\frac{6}{7}abex^2\sqrt{a+bx^3}}{3ab^3}$$

3.437. $\int \frac{x^5\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

$\Big\downarrow$ 759

$$\frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}}-\frac{\frac{2\left(100a^2b^{8/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{8\sqrt{2+\sqrt{3}}a^2b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(14\sqrt[3]{b}d-25\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+}{\sqrt[3]{b}x+}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{5b}}{7b}}{3ab^3}$$

$\Big\downarrow$ 2416

$$\frac{2x\left(ad+aex-bcx^2\right)}{3b^2\sqrt{a+bx^3}}-\frac{\frac{\frac{2\left(\frac{8\sqrt{2+\sqrt{3}}a^2b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(14\sqrt[3]{b}d-25\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+100a^2b^{8/3}\right.}{}}{}}{}$$

input
```
Int[(x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

output
```
(2*x*(a*d + a*e*x - b*c*x^2))/(3*b^2*Sqrt[a + b*x^3]) - ((-6*a*b*e*x^2*Sqr
t[a + b*x^3])/7 + ((-42*a*b^2*d*x*Sqrt[a + b*x^3])/5 + (2*(-70*a*b^3*c*Sqr
t[a + b*x^3] + 100*a^2*b^(8/3)*e*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3])) + (8*Sqrt[2 + Sqrt[3]]*a^2*b^(7/3)*(14*b^(1/3)*d - 25*(1 - S
qrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(5*b))/(7*b))/(3*a*b^3)
```

3.437. $\int \frac{x^5\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

3.437.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.437. $\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.437.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 813, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	813
default	Expression too large to display	836
risch	Expression too large to display	1587

input
```
int(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/3/b^3*a*e*x^2-1/3*a*d/b^3*x-1/3*a*c/b^3)/((x^3+a/b)*b)^(1/2)+2/7*
e*x^2*(b*x^3+a)^(1/2)/b^2+2/5*d*x*(b*x^3+a)^(1/2)/b^2+2/3*c*(b*x^3+a)^(1/2
)/b^2+32/45*I*a*d/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*
b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2))+80/63*I*a*e/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/...
```

3.437.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.23

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx =$$

$$-\frac{2\left(112\left(abdx^3+a^2d\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-200\left(abex^3+a^2e\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{we}\right.\right.}{105\,(b^4x}$$

input
```
integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/105*(112*(a*b*d*x^3 + a^2*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x)
- 200*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPI
nverse(0, -4*a/b, x)) - (15*b^2*e*x^5 + 21*b^2*d*x^4 + 35*b^2*c*x^3 + 50*a
*b*e*x^2 + 56*a*b*d*x + 70*a*b*c)*sqrt(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

3.437.6 Sympy [A] (verification not implemented)

Time = 6.88 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.22

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = c\left(\begin{cases} \frac{4a}{3b^2\sqrt{a+bx^3}} + \frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{dx^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)} + \frac{ex^8\Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**5*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

output
```
c*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3))
, Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + d*x**7*gamma(7/3)*hyper((3/2, 7/
3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3)) + e*x**8*g
amma(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)
*gamma(11/3))
```

3.437.7 Maxima [F]

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^5}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/3*c*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) + integrate((e*x^7 +
 d*x^6)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

3.437.8 Giac [F]

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^5}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)*x^5/(b*x^3 + a)^(3/2), x)
```

3.437.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^5\,(e\,x^2+d\,x+c)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
```

output
```
int((x^5*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.438 $\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

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3.438.2 Mathematica [C] (verified) . . . 3295
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3.438.8 Giac [F] . . . 3302
3.438.9 Mupad [F(-1)] . . . 3302

3.438.1 Optimal result

Integrand size = 25, antiderivative size = 574

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}}+\frac{4d\sqrt{a+bx^3}}{3b^2} + \frac{2ex\sqrt{a+bx^3}}{5b^2}+\frac{8c\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{8\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(5\left(1-\sqrt{3}\right)b^{2/3}c+4a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*x*(-b*d*x^2-b*c*x+a*e)/b^2/(b*x^3+a)^(1/2)+4/3*d*(b*x^3+a)^(1/2)/b^2+2
/5*e*x*(b*x^3+a)^(1/2)/b^2+8/3*c*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))-4/3*a^(1/3)*c*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/
2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-8/45*a^(1/3)*(a^(1/3)+
b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(4*a^(2/3)*e+5*b^(2/3)*c*(1-3^(1/2)))*(1/2*6^(1/
2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.438.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.22

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(10ad+8aex+15bcx^2+5bdx^3+3bex^4-8aex\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\right.\right.}{15b^2\sqrt{a+bx}}$$

input
```
Integrate[(x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(10*a*d + 8*a*e*x + 15*b*c*x^2 + 5*b*d*x^3 + 3*b*e*x^4 - 8*a*e*x*Sqrt[1
 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 15*b*c*x^2*
Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(15*b
^2*Sqrt[a + b*x^3])
```

3.438.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2367, 27, 2427, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (c + dx + ex^2)}{(a + bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \; 2367$$

$$\frac{2x(ae - bcx - bdx^2)}{3b^2\sqrt{a + bx^3}} - \frac{2\int \frac{-3abex^3 - 6abdx^2 - 4abcx + 2a^2e}{2\sqrt{bx^3+a}}dx}{3ab^2}$$

$$\Big\downarrow \; 27$$

$$\frac{2x(ae - bcx - bdx^2)}{3b^2\sqrt{a + bx^3}} - \frac{\int \frac{-3abex^3 - 6abdx^2 - 4abcx + 2a^2e}{\sqrt{bx^3+a}}dx}{3ab^2}$$

$$\Big\downarrow \; 2427$$

$$\frac{2x(ae - bcx - bdx^2)}{3b^2\sqrt{a + bx^3}} - \frac{\frac{2\int \frac{8bea^2 - 15b^2dx^2a - 10b^2cxa}{\sqrt{bx^3+a}}dx}{5b} - \frac{6}{5}aex\sqrt{a + bx^3}}{3ab^2}$$

$$\Big\downarrow \; 2425$$

$$\frac{2x(ae - bcx - bdx^2)}{3b^2\sqrt{a + bx^3}} - \frac{\frac{2\left(\int \frac{8a^2be - 10ab^2cx}{\sqrt{bx^3+a}}dx - 15ab^2d\int \frac{x^2}{\sqrt{bx^3+a}}dx\right)}{5b} - \frac{6}{5}aex\sqrt{a + bx^3}}{3ab^2}$$

$$\Big\downarrow \; 793$$

$$\frac{2x(ae - bcx - bdx^2)}{3b^2\sqrt{a + bx^3}} - \frac{\frac{2\left(\int \frac{8a^2be - 10ab^2cx}{\sqrt{bx^3+a}}dx - 10abd\sqrt{a+bx^3}\right)}{5b} - \frac{6}{5}aex\sqrt{a + bx^3}}{3ab^2}$$

$$\Big\downarrow \; 2417$$

$$\frac{2x(ae - bcx - bdx^2)}{3b^2\sqrt{a + bx^3}} - \frac{\frac{2\left(2a^{4/3}b\left(4a^{2/3}e + 5\left(1-\sqrt{3}\right)b^{2/3}c\right)\int \frac{1}{\sqrt{bx^3+a}}dx - 10ab^{5/3}c\int \frac{\sqrt[3]{b}x + \left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx - 10abd\sqrt{a+bx^3}\right)}{5b} - \frac{6}{5}aex\sqrt{a + bx^3}}{3ab^2}$$

$$\Big\downarrow \; 759$$

3.438. $\int \frac{x^4 (c+dx+ex^2)}{(a+bx^3)^{3/2}} \, dx$

$$\frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} - \frac{\frac{2\left(-10ab^{5/3}c\int \frac{\sqrt[3]{b}x+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt{bx^3+a}}dx + \frac{4\sqrt{2+\sqrt{3}}a^{4/3}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(4a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+}{\sqrt[3]{b}x+}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{5b}}{3ab^2}$$

$\downarrow$ 2416

$$\frac{2x(ae-bcx-bdx^2)}{3b^2\sqrt{a+bx^3}} - \frac{2\left(\frac{4\sqrt{2+\sqrt{3}}a^{4/3}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(4a^{2/3}e+5\left(1-\sqrt{3}\right)b^{2/3}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - 10ab^{5/3}c\left(\right.\right.}{}$$

input
```
Int[(x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

output
```
(2*x*(a*e - b*c*x - b*d*x^2))/(3*b^2*Sqrt[a + b*x^3]) - ((-6*a*e*x*Sqrt[a
+ b*x^3])/5 + (2*(-10*a*b*d*Sqrt[a + b*x^3] - 10*a*b^(5/3)*c*((2*Sqrt[a +
b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 -
Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (4*Sqrt[2 + Sqrt[3]]*a^(4/3)*b
^(2/3)*(5*(1 - Sqrt[3])*b^(2/3)*c + 4*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))
/(5*b))/(3*a*b^2)
```

3.438. $\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

3.438.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.438. $\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.438.4 Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 793, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	793
default	Expression too large to display	817
risch	Expression too large to display	1123

input
```
int(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/3*c/b^2*x^2-1/3/b^3*a*e*x-1/3*a*d/b^3)/((x^3+a/b)*b)^(1/2)+2/5*e*x
*(b*x^3+a)^(1/2)/b^2+2/3*d*(b*x^3+a)^(1/2)/b^2+32/45*I*a*e/b^3*3^(1/2)*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-8/9*I*c/b^2*3^(1/
2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.438.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.23

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx =$$
$$-\frac{2\left(16\,(abex^3+a^2e)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+20\,(b^2cx^3+abc)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weier}\right.\right.}{15\,(b^4x^3+ab^3)}$$

input
```
integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(16*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) +
20*(b^2*c*x^3 + a*b*c)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInve
rse(0, -4*a/b, x)) - (3*b^2*e*x^4 + 5*b^2*d*x^3 - 5*b^2*c*x^2 + 8*a*b*e*x
+ 10*a*b*d)*sqrt(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

3.438.6 Sympy [A] (verification not implemented)

Time = 5.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.22

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = d\left(\begin{cases} \frac{4a}{3b^2\sqrt{a+bx^3}} + \frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)} + \frac{ex^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**4*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

output
```
d*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3))
, Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + c*x**5*gamma(5/3)*hyper((3/2, 5/
3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3)) + e*x**7*gam
ma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*g
amma(10/3))
```

3.438.7 Maxima [F]

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^(3/2), x)
```

3.438.8 Giac [F]

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx=\int \frac{(ex^2+dx+c)x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)*x^4/(b*x^3 + a)^(3/2), x)
```

3.438.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx=\int \frac{x^4\,(e\,x^2+d\,x+c)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
```

output
```
int((x^4*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.439 $\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

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3.439.2 Mathematica [C] (verified) . . . 3304
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3.439.1 Optimal result

Integrand size = 25, antiderivative size = 542

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{2x(c+dx+ex^2)}{3b\sqrt{a+bx^3}}+\frac{4e\sqrt{a+bx^3}}{3b^2}+\frac{8d\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*x*(e*x^2+d*x+c)/b/(b*x^3+a)^(1/2)+4/3*e*(b*x^3+a)^(1/2)/b^2+8/3*d*(b*
x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-4/3*a^(1/3)*d*(a^(1/3
)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5
/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)+4/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c-2*a^(1/3
)*d*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+
a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

3.439.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.22

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(2ae-bcx+3bdx^2+bex^3+bcx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\cdots\right.}{3b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

output
```
(2*(2*a*e - b*c*x + 3*b*d*x^2 + b*e*x^3 + b*c*x*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] - 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*H
ypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(3*b^2*Sqrt[a + b*x^3])
```

3.439.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2367, 25, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 2367$$

$$-\frac{2\int -\frac{3abex^2+2abdx+abc}{\sqrt{bx^3+a}}dx}{3ab^2}-\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 25$$

$$\frac{2\int \frac{3abex^2+2abdx+abc}{\sqrt{bx^3+a}}dx}{3ab^2}-\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 2425$$

$$\frac{2\left(\int \frac{abc+2abdx}{\sqrt{bx^3+a}}dx+3abe\int \frac{x^2}{\sqrt{bx^3+a}}dx\right)}{3ab^2}-\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 793$$

$$\frac{2\left(\int \frac{abc+2abdx}{\sqrt{bx^3+a}}dx+2ae\sqrt{a+bx^3}\right)}{3ab^2}-\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 2417$$

$$\frac{2\left(ab^{2/3}\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\int \frac{1}{\sqrt{bx^3+a}}dx+2ab^{2/3}d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+2ae\sqrt{a+bx^3}\right)}{3ab^2}-\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 759$$

$$\frac{2\left(2ab^{2/3}d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{2\sqrt{2+\sqrt{3}}a\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\text{arcsi}\right.}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{3ab^2}$$

$$\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 2416$$

3.439. $\int \frac{x^3\left(c+dx+ex^2\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

$$2\left(\frac{2\sqrt{2+\sqrt{3}}a\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}c-2\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+2\cdots\right.$$

$$\frac{2x\left(c+dx+ex^2\right)}{3b\sqrt{a+bx^3}}$$

input
```
Int[(x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

output
```
(-2*x*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (2*(2*a*e*Sqrt[a + b*x^3]
 + 2*a*b^(2/3)*d*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) +
(2*Sqrt[2 + Sqrt[3]]*a*b^(1/3)*(b^(1/3)*c - 2*(1 - Sqrt[3])*a^(1/3)*d)*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])))/(3*a*b^2)
```

3.439.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

3.439. $\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

3.439. $\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

3.439.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.43

method	result
elliptic	$-\frac{2b\left(\frac{d\,x^{2}}{3b^{2}}+\frac{cx}{3b^{2}}-\frac{ae}{3b^{3}}\right)}{\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2e\sqrt{b\,x^{3}+a}}{3b^{2}}-\frac{4ic\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\ \cdots}{\cdots}$
default	Expression too large to display
risch	Expression too large to display

input
```
int(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/3*d*x^2/b^2+1/3*c/b^2*x-1/3*a*e/b^3)/((x^3+a/b)*b)^(1/2)+2/3*e*(b*
x^3+a)^(1/2)/b^2-4/9*I*c/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))-8/9*I*d/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)))
```

3.439.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.20

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(2\,(bcx^3+ac)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-4\,(bdx^3+ad)\sqrt{b}\text{weierstrassZ}\right.}{3\,(b^3}$$

input
```
integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(2*(b*c*x^3 + a*c)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - 4*(b*d*
x^3 + a*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/
b, x)) + (b*e*x^3 - b*d*x^2 - b*c*x + 2*a*e)*sqrt(b*x^3 + a))/(b^3*x^3 + a
*b^2)
```

3.439.6 Sympy [A] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.24

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = e\left(\begin{cases} \frac{4a}{3b^2\sqrt{a+bx^3}}+\frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**3*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

output
```
e*Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3))
, Ne(b, 0)), (x**6/(6*a**(3/2)), True)) + c*x**4*gamma(4/3)*hyper((4/3, 3/
2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + d*x**5*gam
ma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*ga
mma(8/3))
```

3.439.7 Maxima [F]

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2), x)
```

3.439.8 Giac [F]

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)*x^3/(b*x^3 + a)^(3/2), x)
```

3.439.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^3\,(e\,x^2+d\,x+c)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
```

output
```
int((x^3*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.440 $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

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3.440.1 Optimal result

Integrand size = 25, antiderivative size = 522

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}} + \frac{8e\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}d-2\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}\right.\right.}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*(e*x^2+d*x+c)/b/(b*x^3+a)^(1/2)+8/3*e*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))-4/3*a^(1/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/9*(a^(1/3)
+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*d-2*a^(1/3)*e*(1-3^(1/2)))*(1/2*6^(1/2
)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.440.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.20

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2dx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)-2\left(c+x(d-3ex)+3ex^2\sqrt{1\right.}{3b\sqrt{a+bx^3}}$$

input
```
Integrate[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

output
```
(2*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
- 2*(c + x*(d - 3*e*x) + 3*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3
, 3/2, 5/3, -((b*x^3)/a)]))/(3*b*Sqrt[a + b*x^3])
```

3.440.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2363, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{2363}$$

$$\frac{2\int \frac{d+2ex}{\sqrt{bx^3+a}}dx}{3b}-\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2417}$$

$$\frac{2\left(\left(d-\frac{2\left(1-\sqrt{3}\right)\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{2e\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{3b}-\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

$$\frac{2\left(\frac{2e\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(d-\frac{2\left(1-\sqrt{3}\right)\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{3b}-\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{2\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(d-\frac{2\left(1-\sqrt{3}\right)\sqrt[3]{a}e}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2e\left(\cdots\right.}{\sqrt[3]{b}(\cdots}\right)}{3b}-\frac{2(c+dx+ex^2)}{3b\sqrt{a+bx^3}}$$

input
```
Int[(x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

3.440. $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

output
```
(-2*(c + d*x + e*x^2))/(3*b*Sqrt[a + b*x^3]) + (2*((2*e*((2*Sqrt[a + b*x^3
])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(d -
(2*(1 - Sqrt[3])*a^(1/3)*e)/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) +
 b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(3*
b)
```

3.440.3.1 Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.440. $\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.440.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 759, normalized size of antiderivative = 1.45

method	result
elliptic	$-\frac{2b\left(\frac{e\,x^2}{3b^2}+\frac{dx}{3b^2}+\frac{c}{3b^2}\right)}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{4id\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)}{2b}\right.}{}}}{9b^2}$
default	Expression too large to display

input
```
int(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/3*e/b^2*x^2+1/3*d*x/b^2+1/3*c/b^2)/((x^3+a/b)*b)^(1/2)-4/9*I*d/b^2
*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-8/9*I
/b^2*e*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.440.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.19

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(2\,(bdx^3+ad)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-4\,(bex^3+ae)\sqrt{b}\text{weierstrassZ}\right.}{3\,(b^3x^3+}$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(2*(b*d*x^3 + a*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - 4*(b*e*
x^3 + a*e)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/
b, x)) - (b*e*x^2 + b*d*x + b*c)*sqrt(b*x^3 + a))/(b^3*x^3 + a*b^2)
```

3.440.6 Sympy [A] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.21

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = c\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{dx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{ex^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**2*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

output
```
c*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), Tru
e)) + d*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a
)/(3*a**(3/2)*gamma(7/3)) + e*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))
```

3.440.7 Maxima [F]

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^2}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
-2/3*c/(sqrt(b*x^3 + a)*b) + integrate((e*x^4 + d*x^3)*sqrt(b*x^3 + a)/(b^
2*x^6 + 2*a*b*x^3 + a^2), x)
```

3.440.8 Giac [F]

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x^2}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)*x^2/(b*x^3 + a)^(3/2), x)
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^2\,(e\,x^2+d\,x+c)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
```

output
```
int((x^2*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.441 $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

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3.441.1 Optimal result

Integrand size = 23, antiderivative size = 561

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = -\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab} - \frac{2c\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{\sqrt{2-\sqrt{3}}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{2\sqrt{2+\sqrt{3}}\left(b^{2/3}(c-\sqrt{3}c)+2a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}\right.\right.}{3\sqrt[4]{3}a^{2/3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*x*(-b*d*x^2-b*c*x+a*e)/a/b/(b*x^3+a)^(1/2)-2/3*d*(b*x^3+a)^(1/2)/a/b-
2/3*c*(b*x^3+a)^(1/2)/a/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/3*c*(a^(
1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a
^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^
(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*a^(2/
3)*e+b^(2/3)*(c-c*3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2
/3)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

3.441.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.19

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{-4a(d+ex)+4aex\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+3bcx^2\sqrt{1+\frac{bx^3}{a}}}{6ab\sqrt{a+bx^3}}$$

input
```
Integrate[(x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

output
```
(-4*a*(d + e*x) + 4*a*e*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2,
4/3, -((b*x^3)/a)] + 3*b*c*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3,
3/2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqrt[a + b*x^3])
```

3.441.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2367, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2367}$$

$$-\frac{2\int -\frac{-3bdx^2-bcx+2ae}{2\sqrt{bx^3+a}}dx}{3ab}-\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-3bdx^2-bcx+2ae}{\sqrt{bx^3+a}}dx}{3ab}-\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 2425}$$

$$\frac{\int \frac{2ae-bcx}{\sqrt{bx^3+a}}dx-3bd\int \frac{x^2}{\sqrt{bx^3+a}}dx}{3ab}-\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\int \frac{2ae-bcx}{\sqrt{bx^3+a}}dx-2d\sqrt{a+bx^3}}{3ab}-\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 2417}$$

$$\frac{\sqrt[3]{a}\left(2a^{2/3}e+\left(1-\sqrt{3}\right)b^{2/3}c\right)\int \frac{1}{\sqrt{bx^3+a}}dx-b^{2/3}c\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-2d\sqrt{a+bx^3}}{3ab}-\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{-b^{2/3}c\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+\left(1-\sqrt{3}\right)b^{2/3}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\cdots}{\sqrt[3]{b}\cdots}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{3ab}-\frac{2x(ae-bcx-bdx^2)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 2416}$$

3.441. $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

$$\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(2a^{2/3}e+\left(1-\sqrt{3}\right)b^{2/3}c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - b^{2/3}c$$

$$\frac{2x\left(ae-bcx-bdx^2\right)}{3ab\sqrt{a+bx^3}}$$

input
```
Int[(x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x]
```

output
```
(-2*x*(a*e - b*c*x - b*d*x^2))/(3*a*b*Sqrt[a + b*x^3]) + (-2*d*Sqrt[a + b*
x^3] - b^(2/3)*c*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) +
(2*Sqrt[2 + Sqrt[3]]*a^(1/3)*((1 - Sqrt[3])*b^(2/3)*c + 2*a^(2/3)*e)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a*b)
```

3.441.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

3.441. $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

3.441. $\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx$

3.441.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.36

method	result
elliptic	$-\frac{2b\left(-\frac{c\,x^{2}}{3ba}+\frac{ex}{3b^{2}}+\frac{d}{3b^{2}}\right)}{\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{4ie\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b\cdots\right.}{2b\cdots}\right.\cdots}{\cdots}}\cdots}{9b\cdots}$
default	Expression too large to display

input
```
int(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/3/b/a*c*x^2+1/3*e/b^2*x+1/3*d/b^2)/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*
e*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/9*
I*c/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.441.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.20

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(2\left(abex^3+a^2e\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\left(b^2cx^3+abc\right)\sqrt{b}\text{weierstrass}\right.}{3\left(ab^3x^3\right.}$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(2*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (b^
2*c*x^3 + a*b*c)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0,
 -4*a/b, x)) + (b^2*c*x^2 - a*b*e*x - a*b*d)*sqrt(b*x^3 + a))/(a*b^3*x^3 +
 a^2*b^2)
```

3.441.6 Sympy [A] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.19

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = d\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x*(e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

output
```
d*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), Tru
e)) + c*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*a**(3/2)*gamma(5/3)) + e*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3))
```

3.441.7 Maxima [F]

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2), x)
```

3.441.8 Giac [F]

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(ex^2+dx+c)x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)*x/(b*x^3 + a)^(3/2), x)
```

3.441.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x\,(e\,x^2+d\,x+c)}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2),x)
```

output
```
int((x*(c + d*x + e*x^2))/(a + b*x^3)^(3/2), x)
```

3.442 $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^{3/2}}\,dx$

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3.442.1 Optimal result

Integrand size = 22, antiderivative size = 532

$$\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c+\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}\right.\right.}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*(a*e-b*x*(d*x+c))/a/b/(b*x^3+a)^(1/2)-2/3*d*(b*x^3+a)^(1/2)/a/b^(2/3)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/3*d*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/9*
(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*c+a^(1/3)*d*(1-3^(1/2)))*(1/2*
6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.442.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.20

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = \frac{-4ae+4bcx+2bcx\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+3bdx^2\sqrt{1+\frac{bx^3}{a}}\,\text{H}}{6ab\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^3)^(3/2),x]
```

output
```
(-4*a*e + 4*b*c*x + 2*b*c*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2
, 4/3, -((b*x^3)/a)] + 3*b*d*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3
, 3/2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqrt[a + b*x^3])
```

3.442.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2393, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2393}$$

$$-\frac{2\int -\frac{c-dx}{2\sqrt{bx^3+a}}dx}{3a}-\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{c-dx}{\sqrt{bx^3+a}}dx}{3a}-\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 2417}$$

$$\frac{\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}-\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{\right.}}{\sqrt{bx^3-}}}{\sqrt[3]{b}}}{3a}$$

$$\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{d\left(\frac{2\sqrt{}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right.\right.}\right.}{}}{3a}$$

$$\frac{2(ae-bx(c+dx))}{3ab\sqrt{a+bx^3}}$$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^3)^(3/2),x]
```

3.442. $\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx$

output
```
(-2*(a*e - b*x*(c + d*x)))/(3*a*b*Sqrt[a + b*x^3]) + (-((d*((2*Sqrt[a + b*
x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sq
rt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3)) + (2*Sqrt[2 + Sqrt[3]]*(
c + ((1 - Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3
*a)
```

3.442.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.442.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.45

method	result
elliptic	$-\frac{2b\left(-\frac{d\,x^2}{3ab}-\frac{cx}{3ba}+\frac{e}{3b^2}\right)}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2ic\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b\right.}{2b}\right.}{}}}{9a}$
default	$c\left(\frac{2x}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i}{}\right.}{\left(-a\,b\right.}}}{9ab\sqrt{b\,x^3}}\right.$

3.442. $\int \frac{c+dx+ex^2}{\left(a+bx^3\right)^{3/2}}\,dx$

input
```
int((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/3/a/b*d*x^2-1/3/b/a*c*x+1/3*e/b^2)/((x^3+a/b)*b)^(1/2)-2/9*I*c/a*
3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/9*
I*d/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

3.442.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.18

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = \frac{2\left((bcx^3+ac)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+(bdx^3+ad)\sqrt{b}\text{weierstrassZeta}\left(0,\right.\right.}{3\left(ab^2x^3+a^2b\right)}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*((b*c*x^3 + a*c)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (b*d*x^3
+ a*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x
)) + (b*d*x^2 + b*c*x - a*e)*sqrt(b*x^3 + a))/(a*b^2*x^3 + a^2*b)
```

3.442.6 Sympy [A] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.20

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = e\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{3}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{3}{2} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

output
```
e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), Tru
e)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**
3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

3.442.7 Maxima [F]

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = \int \frac{ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

3.442.8 Giac [F]

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx = \int \frac{ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

3.442.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{(a+bx^3)^{3/2}}\,dx=\int \frac{e\,x^2+d\,x+c}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

input
```
int((c + d*x + e*x^2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c + d*x + e*x^2)/(a + b*x^3)^(3/2), x)
```

3.443 $\int \frac{c+dx+ex^2}{x(a+bx^3)^{3/2}}\,dx$

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3.443.1 Optimal result

Integrand size = 25, antiderivative size = 579

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{2x(ad+aex-bcx^2)}{3a^2\sqrt{a+bx^3}}+\frac{2c\sqrt{a+bx^3}}{3a^2}$$
$$-\frac{2e\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$
$$+\frac{\sqrt{2-\sqrt{3}}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}d+\left(1-\sqrt{3}\right)\sqrt[3]{a}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}\right.\right.}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)+2/3*x*(-b*c*x^2+a*e*x+a*d)
/a^2/(b*x^3+a)^(1/2)+2/3*c*(b*x^3+a)^(1/2)/a^2-2/3*e*(b*x^3+a)^(1/2)/a/b^(
2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/3*e*(a^(1/3)+b^(1/3)*x)*EllipticE((
b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1
/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+
2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*d+a^(1/3)*e*(1-3^(1/2)))*(
1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.443.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.21

$$\int \frac{c+dx+ex^2}{x(a+bx^3)^{3/2}}\,dx = \frac{4c\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^3}{a}\right)+x\left(4d+2d\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F}\right.}{6a\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x]
```

output
```
(4*c*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/a] + x*(4*d + 2*d*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*e*x*Sqrt[1
 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(6*a*Sqrt[a
 + b*x^3])
```

3.443.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2368, 27, 2371, 798, 73, 221, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}-\frac{2\int -\frac{\frac{3b^2cx^3}{a}-bex^2+bdx+3bc}{2x\sqrt{bx^3+a}}dx}{3ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\frac{3b^2cx^3}{a}-bex^2+bdx+3bc}{x\sqrt{bx^3+a}}dx}{3ab}+\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2371}$$

$$\frac{\int \frac{\frac{3b^2cx^2}{a}-bex+bd}{\sqrt{bx^3+a}}dx+3bc\int \frac{1}{x\sqrt{bx^3+a}}dx}{3ab}+\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{\frac{3b^2cx^2}{a}-bex+bd}{\sqrt{bx^3+a}}dx+bc\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{3ab}+\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{\frac{3b^2cx^2}{a}-bex+bd}{\sqrt{bx^3+a}}dx+2c\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3ab}+\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\int \frac{\frac{3b^2cx^2}{a}-bex+bd}{\sqrt{bx^3+a}}dx-\frac{2bc\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}}{3ab}+\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2425}$$

$$\frac{\frac{3b^2c\int \frac{x^2}{\sqrt{bx^3+a}}dx}{a}+\int \frac{bd-bex}{\sqrt{bx^3+a}}dx-\frac{2bc\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}}{3ab}+\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{793}$$

$$\frac{\int \frac{bd-bex}{\sqrt{bx^3+a}}dx-\frac{2bc\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2bc\sqrt{a+bx^3}}{a}}{3ab}+\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2417}$$

3.443. $\int \frac{c+dx+ex^2}{x(a+bx^3)^{3/2}}\,dx$

$$\frac{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}e+\sqrt[3]{b}d\right)\int \frac{1}{\sqrt{bx^3+a}}dx-b^{2/3}e\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{2bc\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2bc\sqrt{a+bx^3}}{a}}{3ab}+\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$\big\downarrow$ 759

$$\frac{-b^{2/3}e\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}e+\sqrt[3]{b}d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\ldots}{\sqrt[3]{b}\ldots}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{3ab}$$

$$\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$\big\downarrow$ 2416

$$\frac{\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}e+\sqrt[3]{b}d\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-b^{2/3}e\ldots}{}$$

$$\frac{2x\left(ad+aex-bcx^2\right)}{3a^2\sqrt{a+bx^3}}$$

input
```
Int[(c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x]
```

3.443. $\int \frac{c+dx+ex^2}{x(a+bx^3)^{3/2}}\,dx$

output
```
(2*x*(a*d + a*e*x - b*c*x^2))/(3*a^2*Sqrt[a + b*x^3]) + ((2*b*c*Sqrt[a + b
*x^3])/a - (2*b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/Sqrt[a] - b^(2/3)*e*((
2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (2*Sqrt[2 + Sqrt[3]
]*b^(1/3)*(b^(1/3)*d + (1 - Sqrt[3])*a^(1/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(
3*a*b)
```

3.443.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.443. $\int \frac{c+dx+ex^2}{x(a+bx^3)^{3/2}}\,dx$

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

3.443.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	794
default	Expression too large to display	810

input
```
int((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/3/a/b*x^2*e-1/3/a/b*d*x-1/3/b/a*c)/((x^3+a/b)*b)^(1/2)-2/9*I*d/a*
3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/9*
I/a*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*c*arctanh(
(b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

3.443.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.60

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx = \left[\frac{(b^2cx^3+abc)\sqrt{a}\log\left(\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+4\,(abdx^3+a^2d)\sqrt{b}\text{weiers}}{}\right.$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*((b^2*c*x^3 + a*b*c)*sqrt(a)*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*
a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 4*(a*b*d*x^3 + a^2*d)*sqrt(b)*w
eierstrassPInverse(0, -4*a/b, x) + 4*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstra
ssZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 4*(a*b*e*x^2 + a*b*
d*x + a*b*c)*sqrt(b*x^3 + a))/(a^2*b^2*x^3 + a^3*b), 1/3*((b^2*c*x^3 + a*b
*c)*sqrt(-a)*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 +
a^2)) + 2*(a*b*d*x^3 + a^2*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) +
2*(a*b*e*x^3 + a^2*e)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInver
se(0, -4*a/b, x)) + 2*(a*b*e*x^2 + a*b*d*x + a*b*c)*sqrt(b*x^3 + a))/(a^2*
b^2*x^3 + a^3*b)]
```

3.443.6 Sympy [A] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx = c\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right) + \frac{dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((e*x**2+d*x+c)/x/(b*x**3+a)**(3/2),x)
```

output
```
c*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b
*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a)
+ 1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9
/2) + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a*
*(9/2) + 3*a**(7/2)*b*x**3)) + d*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3)) + e*x**2*gamma(2/3)*hyper(
(2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

3.443.7 Maxima [F]

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{ex^2+dx+c}{\left(bx^3+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x), x)
```

3.443.8 Giac [F]

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{ex^2+dx+c}{\left(bx^3+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((e*x^2+d*x+c)/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x), x)
```

3.443.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{x\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{e\,x^2+d\,x+c}{x\,\left(b\,x^3+a\right)^{3/2}}\,dx$$

input
```
int((c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)),x)
```

output
```
int((c + d*x + e*x^2)/(x*(a + b*x^3)^(3/2)), x)
```

3.444 $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^{3/2}}\,dx$

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3.444.1 Optimal result

Integrand size = 25, antiderivative size = 607

$$\int \frac{c+dx+ex^2}{x^2(a+bx^3)^{3/2}}\,dx=\frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}}+\frac{2d\sqrt{a+bx^3}}{3a^2}-\frac{c\sqrt{a+bx^3}}{a^2x}+\frac{5\sqrt[3]{b}c\sqrt{a+bx^3}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2d\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{2+\sqrt{3}}\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{3\sqrt[4]{3}a^{5/3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)+2/3*x*(-b*d*x^2-b*c*x+a*e)
/a^2/(b*x^3+a)^(1/2)+2/3*d*(b*x^3+a)^(1/2)/a^2-c*(b*x^3+a)^(1/2)/a^2/x+5/3
*b^(1/3)*c*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-5/6*b^(1/3)
*c*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^
(1/4)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)-1/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(
1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(-2*a^(2/
3)*e+5*b^(2/3)*c*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^
(5/3)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)
```

3.444.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.20

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{2dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^3}{a}\right)-3c\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3}\right.}{3ax\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x]
```

output
```
(2*d*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/a] - 3*c*Sqrt[1 + (b*x^
3)/a]*Hypergeometric2F1[-1/3, 3/2, 2/3, -((b*x^3)/a)] + e*x^2*(2 + Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(3*a*x*Sqrt[
a + b*x^3])
```

3.444.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2368, 27, 2374, 25, 2371, 798, 73, 221, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \; 2368$$

$$\frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}}-\frac{2\int -\frac{\frac{3b^2dx^4}{a}+\frac{b^2cx^3}{a}+bex^2+3bdx+3bc}{2x^2\sqrt{bx^3+a}}dx}{3ab}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\frac{3b^2dx^4}{a}+\frac{b^2cx^3}{a}+bex^2+3bdx+3bc}{x^2\sqrt{bx^3+a}}dx}{3ab}+\frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 2374$$

$$\frac{-\frac{\int -\frac{6b^2dx^3+5b^2cx^2+2abex+6abd}{x\sqrt{bx^3+a}}dx}{2a}-\frac{3bc\sqrt{a+bx^3}}{ax}}{3ab}+\frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{6b^2dx^3+5b^2cx^2+2abex+6abd}{x\sqrt{bx^3+a}}dx}{2a}-\frac{3bc\sqrt{a+bx^3}}{ax}}{3ab}+\frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 2371$$

$$\frac{\frac{\int \frac{6dx^2b^2+5cxb^2+2aeb}{\sqrt{bx^3+a}}dx+6abd\int \frac{1}{x\sqrt{bx^3+a}}dx}{2a}-\frac{3bc\sqrt{a+bx^3}}{ax}}{3ab}+\frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 798$$

$$\frac{\frac{\int \frac{6dx^2b^2+5cxb^2+2aeb}{\sqrt{bx^3+a}}dx+2abd\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{2a}-\frac{3bc\sqrt{a+bx^3}}{ax}}{3ab}+\frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\int \frac{6dx^2b^2+5cxb^2+2aeb}{\sqrt{bx^3+a}}dx+4ad\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{2a}-\frac{3bc\sqrt{a+bx^3}}{ax}}{3ab}+\frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\int \frac{6dx^2b^2+5cxb^2+2aeb}{\sqrt{bx^3+a}}dx-4\sqrt{a}bd\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{2a}-\frac{3bc\sqrt{a+bx^3}}{ax}}{3ab}+\frac{2x\left(ae-bcx-bdx^2\right)}{3a^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 2425$$

3.444. $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^{3/2}}\,dx$

$$\frac{\frac{\int \frac{5cxb^2+2aeb}{\sqrt{bx^3+a}}dx+6b^2d\int \frac{x^2}{\sqrt{bx^3+a}}dx-4\sqrt{a}bd\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{2a}-\frac{3bc\sqrt{a+bx^3}}{ax}}{3ab}+\frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}}$$

$\downarrow$ 793

$$\frac{\frac{\int \frac{5cxb^2+2aeb}{\sqrt{bx^3+a}}dx-4\sqrt{a}bd\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+4bd\sqrt{a+bx^3}}{2a}-\frac{3bc\sqrt{a+bx^3}}{ax}}{3ab}+\frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}}$$

$\downarrow$ 2417

$$\frac{\frac{-\sqrt[3]{a}b\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\int \frac{1}{\sqrt{bx^3+a}}dx+5b^{5/3}c\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-4\sqrt{a}bd\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+4bd\sqrt{a+bx^3}}{2a}-\frac{3bc\sqrt{a+bx^3}}{ax}}{3ab}+\frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}}$$

$\downarrow$ 759

$$\frac{\frac{5b^{5/3}c\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right.}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2a}}{3ab}$$

$$\frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}}$$

$\downarrow$ 2416

$$\frac{-\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{a}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(5\left(1-\sqrt{3}\right)b^{2/3}c-2a^{2/3}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+5b^{5/3}c\left(\frac{\sqrt[3]{\,}}{\,}\right.}{\,}$$

$$\frac{2x(ae-bcx-bdx^2)}{3a^2\sqrt{a+bx^3}}$$

input
```
Int[(c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x]
```

3.444. $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^{3/2}}\,dx$

output
```
(2*x*(a*e - b*c*x - b*d*x^2))/(3*a^2*Sqrt[a + b*x^3]) + ((-3*b*c*Sqrt[a +
b*x^3])/(a*x) + (4*b*d*Sqrt[a + b*x^3] - 4*Sqrt[a]*b*d*ArcTanh[Sqrt[a + b*
x^3]/Sqrt[a]] + 5*b^(5/3)*c*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])) - (2*Sqrt[2 + Sqrt[3]]*a^(1/3)*b^(2/3)*(5*(1 - Sqrt[3])*b^(2/3)*c
 - 2*a^(2/3)*e)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(3^(1/4)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*a))/(3*a*b)
```

3.444.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.444. $\int \frac{c+dx+ex^2}{x^2(a+bx^3)^{3/2}}\,dx$

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

3.444.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	806
default	Expression too large to display	825
risch	Expression too large to display	1306

input
```
int((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-c*(b*x^3+a)^(1/2)/a^2/x-2*b*(1/3/a^2*c*x^2-1/3/a/b*x*e-1/3/a/b*d)/((x^3+a
/b)*b)^(1/2)-2/9*I/a*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2))-5/9*I*c/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)))-2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

3.444.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.62

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \left[\frac{(b^2dx^4+abdx)\sqrt{a}\log\left(\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+4\,(abex^4+a^2ex)\sqrt{b}\mathrm{we}}{}\right.$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*((b^2*d*x^4 + a*b*d*x)*sqrt(a)*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 +
2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 4*(a*b*e*x^4 + a^2*e*x)*sqrt(
b)*weierstrassPInverse(0, -4*a/b, x) - 10*(b^2*c*x^4 + a*b*c*x)*sqrt(b)*we
ierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - 2*(5*b^2*c*x
^3 - 2*a*b*e*x^2 - 2*a*b*d*x + 3*a*b*c)*sqrt(b*x^3 + a))/(a^2*b^2*x^4 + a^
3*b*x), 1/3*((b^2*d*x^4 + a*b*d*x)*sqrt(-a)*arctan(1/2*(b*x^3 + 2*a)*sqrt(
b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) + 2*(a*b*e*x^4 + a^2*e*x)*sqrt(b)*wei
erstrassPInverse(0, -4*a/b, x) - 5*(b^2*c*x^4 + a*b*c*x)*sqrt(b)*weierstra
ssZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (5*b^2*c*x^3 - 2*a*
b*e*x^2 - 2*a*b*d*x + 3*a*b*c)*sqrt(b*x^3 + a))/(a^2*b^2*x^4 + a^3*b*x)]
```

3.444.6 Sympy [A] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.44

$$\int \frac{c+dx+ex^2}{x^2\left(a+bx^3\right)^{3/2}}\,dx = d\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right)+\frac{c\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{3}{2}\\\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}+\frac{ex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((e*x**2+d*x+c)/x**2/(b*x**3+a)**(3/2),x)
```

output
```
d*(2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b
*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a)
+ 1)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9
/2) + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a*
*(9/2) + 3*a**(7/2)*b*x**3)) + c*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*x*gamma(2/3)) + e*x*gamma(1/3)*hyper((
1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3))
```

3.444.7 Maxima [F]

$$\int \frac{c+dx+ex^2}{x^2 (a+bx^3)^{3/2}}\,dx = \int \frac{ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2), x)
```

3.444.8 Giac [F]

$$\int \frac{c+dx+ex^2}{x^2 (a+bx^3)^{3/2}}\,dx = \int \frac{ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((e*x^2+d*x+c)/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)/((b*x^3 + a)^(3/2)*x^2), x)
```

3.444.9 Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.22

$$\int \frac{c+dx+ex^2}{x^2 (a+bx^3)^{3/2}}\,dx = \frac{2\,d}{3\,a\,\sqrt{b\,x^3+a}} + \frac{d\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{3/2}} - \frac{2\,c\,\left(\frac{a}{b\,x^3}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{11}{6};\ \frac{17}{6};\ -\frac{a}{b\,x^3}\right)}{11\,x\,(b\,x^3+a)^{3/2}} + \frac{e\,x\,\left(\frac{b\,x^3}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{3}{2};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{(b\,x^3+a)^{3/2}}$$

input
```
int((c + d*x + e*x^2)/(x^2*(a + b*x^3)^(3/2)),x)
```

output
```
(2*d)/(3*a*(a + b*x^3)^(1/2)) + (d*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((
a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(3/2)) - (2*c*(a/(b*x^3) + 1)^(3/2
)*hypergeom([3/2, 11/6], 17/6, -a/(b*x^3)))/(11*x*(a + b*x^3)^(3/2)) + (e*
x*((b*x^3)/a + 1)^(3/2)*hypergeom([1/3, 3/2], 4/3, -(b*x^3)/a))/(a + b*x^3
)^(3/2)
```

3.445 $\int x^3\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

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3.445.1 Optimal result

Integrand size = 35, antiderivative size = 733

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$= -\frac{4a^2e\sqrt{a+bx^3}}{45b^2}+\frac{6a(17bc-8af)x\sqrt{a+bx^3}}{935b^2}+\frac{6a(19bd-10ag)x^2\sqrt{a+bx^3}}{1729b^2}$$
$$+\frac{2aex^3\sqrt{a+bx^3}}{45b}+\frac{6afx^4\sqrt{a+bx^3}}{187b}+\frac{6agx^5\sqrt{a+bx^3}}{247b}-\frac{24a^2(19bd-10ag)\sqrt{a+bx^3}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{2x^3\sqrt{a+bx^3}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19bd-10ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(1729\sqrt[3]{b}(17bc-8af)-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}(19bd-10ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{\cdots}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}{1616615b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx\cdots}}$$

output
```
-4/45*a^2*e*(b*x^3+a)^(1/2)/b^2+6/935*a*(-8*a*f+17*b*c)*x*(b*x^3+a)^(1/2)/
b^2+6/1729*a*(-10*a*g+19*b*d)*x^2*(b*x^3+a)^(1/2)/b^2+2/45*a*e*x^3*(b*x^3+
a)^(1/2)/b+6/187*a*f*x^4*(b*x^3+a)^(1/2)/b+6/247*a*g*x^5*(b*x^3+a)^(1/2)/b
+2/692835*x^3*(36465*g*x^5+40755*f*x^4+46189*e*x^3+53295*d*x^2+62985*c*x)*
(b*x^3+a)^(1/2)-24/1729*a^2*(-10*a*g+19*b*d)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))+12/1729*3^(1/4)*a^(7/3)*(-10*a*g+19*b*d)*(a^(1/
3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*
x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)-4/1616615*3^(3/4)*a^2*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^
(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1729*b^
(1/3)*(-8*a*f+17*b*c)-1870*a^(1/3)*(-10*a*g+19*b*d)*(1-3^(1/2)))*(1/2*6^(1
/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)
*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.445.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.91 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.23

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\frac{2\sqrt{a+bx^3}\left(-\left(\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\left(a(92378e+90x(988f+935gx))-3bx(62985c+11x(4845d+13x(3\right.\right.\right.}{}$$

input
```
Integrate[x^3*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
(2*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(a*(92378*e + 90*x*(
988*f + 935*g*x)) - 3*b*x*(62985*c + 11*x*(4845*d + 13*x*(323*e + 285*f*x
+ 255*g*x^2))))) + 11115*a*(-17*b*c + 8*a*f)*x*Hypergeometric2F1[-1/2, 1/3
, 4/3, -((b*x^3)/a)] + 8415*a*(-19*b*d + 10*a*g)*x^2*Hypergeometric2F1[-1/
2, 2/3, 5/3, -((b*x^3)/a)]))/(2078505*b^2*Sqrt[1 + (b*x^3)/a])
```

3.445.3 Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.543, Rules used = {2365, 27, 2375, 27, 2375, 27, 2427, 27, 2028, 2427, 27, 2028, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$

$$\Big\downarrow \text{2365}$$

$$\frac{3}{2}a\int \frac{2x^3\left(36465gx^4+40755fx^3+46189ex^2+53295dx+62985c\right)}{692835\sqrt{bx^3+a}}dx+\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{x^3\left(36465gx^4+40755fx^3+46189ex^2+53295dx+62985c\right)}{\sqrt{bx^3+a}}dx}{230945}+\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$\Big\downarrow \text{2375}$$

$$\frac{a\left(\frac{2\int \frac{13x^3\left(40755bfx^3+46189bex^2+2805(19bd-10ag)x+62985bc\right)}{2\sqrt{bx^3+a}}dx}{13b}+\frac{5610gx^5\sqrt{a+bx^3}}{b}\right)}{230945}+\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\left(\frac{\int \frac{x^3\left(40755bfx^3+46189bex^2+2805(19bd-10ag)x+62985bc\right)}{\sqrt{bx^3+a}}dx}{b}+\frac{5610gx^5\sqrt{a+bx^3}}{b}\right)}{230945}+\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$\Big\downarrow \text{2375}$$

$$\frac{a\left(\frac{\frac{2\int \frac{11x^3\left(46189b^2ex^2+2805b(19bd-10ag)x+3705b(17bc-8af)\right)}{2\sqrt{bx^3+a}}dx}{11b}+7410fx^4\sqrt{a+bx^3}}{b}+\frac{5610gx^5\sqrt{a+bx^3}}{b}\right)}{230945}+\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$\Big\downarrow \; 27$$

$$\frac{a\left(\frac{\frac{\int \frac{x^3\left(46189b^2ex^2+2805b(19bd-10ag)x+3705b(17bc-8af)\right)}{\sqrt{bx^3+a}}dx}{b}+7410fx^4\sqrt{a+bx^3}}{b}+\frac{5610gx^5\sqrt{a+bx^3}}{b}\right)}{230945}+\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$\Big\downarrow \; 2427$$

$$\frac{a\left(\frac{\frac{\frac{2\int -\frac{3\left(-8415b^2(19bd-10ag)x^4-11115b^2(17bc-8af)x^3+92378ab^2ex^2\right)}{2\sqrt{bx^3+a}}dx}{9b}+\frac{92378}{9}bex^3\sqrt{a+bx^3}}{b}+7410fx^4\sqrt{a+bx^3}}{b}+\frac{5610gx^5\sqrt{a+bx^3}}{b}\right)}{230945}+\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$\Big\downarrow \; 27$$

$$\frac{a\left(\frac{\frac{\frac{92378}{9}bex^3\sqrt{a+bx^3}-\frac{\int \frac{-8415b^2(19bd-10ag)x^4-11115b^2(17bc-8af)x^3+92378ab^2ex^2}{\sqrt{bx^3+a}}dx}{3b}}{b}+7410fx^4\sqrt{a+bx^3}}{b}+\frac{5610gx^5\sqrt{a+bx^3}}{b}\right)}{230945}+\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$\Big\downarrow \; 2028$$

$$\frac{a\left(\frac{\frac{\frac{92378}{9}bex^3\sqrt{a+bx^3}-\frac{\int \frac{x^2\left(-8415(19bd-10ag)x^2b^2+92378aeb^2-11115(17bc-8af)xb^2\right)}{\sqrt{bx^3+a}}dx}{3b}}{b}+7410fx^4\sqrt{a+bx^3}}{b}+\frac{5610gx^5\sqrt{a+bx^3}}{b}\right)}{230945}+\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

3.445. $\int x^3\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

$\downarrow$ 2427

$$\frac{a\left(\frac{\frac{\frac{92378}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{2\int\frac{-77805(17bc-8af)x^3b^3+646646aex^2b^3+33660a(19bd-10ag)xb^2}{2\sqrt{bx^3+a}}dx}{7b}-\frac{16830}{7}bx^2\sqrt{a+bx^3}(19bd-10ag)}{3b}}{b}}{b}+7410fx^4\sqrt{a+bx^3}+\frac{561}{}\right.}{230945}$$

$$\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 27

$$\frac{a\left(\frac{\frac{\frac{92378}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\int\frac{-77805(17bc-8af)x^3b^3+646646aex^2b^3+33660a(19bd-10ag)xb^2}{\sqrt{bx^3+a}}dx}{7b}-\frac{16830}{7}bx^2\sqrt{a+bx^3}(19bd-10ag)}{3b}}{b}}{b}+7410fx^4\sqrt{a+bx^3}+\frac{5610}{}\right.}{230945}$$

$$\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 2028

$$\frac{a\left(\frac{\frac{\frac{92378}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\int\frac{x\left(-77805(17bc-8af)x^2b^3+646646aexb^3+33660a(19bd-10ag)b^2\right)}{\sqrt{bx^3+a}}dx}{7b}-\frac{16830}{7}bx^2\sqrt{a+bx^3}(19bd-10ag)}{3b}}{b}}{b}+7410fx^4\sqrt{a+bx^3}+\frac{561}{}\right.}{230945}$$

$$\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 2427

$$\frac{a\left(\frac{\frac{\frac{92378}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\frac{2\int\frac{5\left(323323aex^2b^4+15561a(17bc-8af)b^3+16830a(19bd-10ag)xb^3\right)}{\sqrt{bx^3+a}}dx}{5b}-31122b^2x\sqrt{a+bx^3}(17bc-8af)}{7b}-\frac{16830}{7}bx^2\sqrt{a+bx^3}(19bd-10a}{3b}}{b}}{b}\right.}{230945}$$

$$\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 27

3.445. $\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$\frac{a\left(\frac{\frac{\frac{92378}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\frac{2\int\frac{323323aex^2b^4+15561a(17bc-8af)b^3+16830a(19bd-10ag)xb^3}{\sqrt{bx^3+a}}dx}{b}-31122b^2x\sqrt{a+bx^3}(17bc-8af)}{7b}-\frac{16830}{7}bx^2\sqrt{a+bx^3}(19bd-10ag)}{3b}}{b}-\cdots}{b}\right.}{230945}$$

$$\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 2425

$$\frac{a\left(\frac{\frac{\frac{92378}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\frac{2\left(323323ab^4e\int\frac{x^2}{\sqrt{bx^3+a}}dx+\int\frac{15561a(17bc-8af)b^3+16830a(19bd-10ag)xb^3}{\sqrt{bx^3+a}}dx\right)}{b}-31122b^2x\sqrt{a+bx^3}(17bc-8af)}{7b}-\frac{16830}{7}bx^2\sqrt{a+bx^3}\cdots}{3b}}{b}\cdots}{b}\right.}{230945}$$

$$\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 793

$$\frac{a\left(\frac{\frac{\frac{92378}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\frac{2\left(\int\frac{15561a(17bc-8af)b^3+16830a(19bd-10ag)xb^3}{\sqrt{bx^3+a}}dx+\frac{646646}{3}ab^3e\sqrt{a+bx^3}\right)}{b}-31122b^2x\sqrt{a+bx^3}(17bc-8af)}{7b}-\frac{16830}{7}bx^2\sqrt{a+bx^3}(19b\cdots}{3b}}{b}\cdots}{b}\right.}{230945}$$

$$\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 2417

$$\frac{a\left(\frac{\frac{\frac{92378}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\frac{2\left(9ab^{8/3}\left(1729\sqrt[3]{b}(17bc-8af)-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}(19bd-10ag)\right)\int\frac{1}{\sqrt{bx^3+a}}dx+16830ab^{8/3}(19bd-10ag)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\cdots\right.}{b}\cdots}{7b}\cdots}{3b}}{b}\cdots}{b}\right.}{230945}$$

$$\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

3.445. $\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$\big\downarrow$ 759

$$a\left(\frac{\frac{92378}{9}bex^3\sqrt{a+bx^3}-\frac{2\left(16830ab^{8/3}(19bd-10ag)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}ab^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(a\cdots\right.}{\sqrt{\frac{\sqrt[3]{a}\cdots}{\left(\left(1+\cdots\right.\right.}}\cdots}\right)}{b}\cdots}{\cdots}\right.$$

$$\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\big\downarrow$ 2416

$$a\left(\frac{\frac{92378}{9}bex^3\sqrt{a+bx^3}-\frac{2\left(\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}ab^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}\cdots\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right)}{\cdots}}{\cdots}\right.$$

$$\frac{2x^3\sqrt{a+bx^3}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

input
```
Int[x^3*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

3.445. $\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

output
```
(2*x^3*Sqrt[a + b*x^3]*(62985*c*x + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^
4 + 36465*g*x^5))/692835 + (a*((5610*g*x^5*Sqrt[a + b*x^3])/b + (7410*f*x^
4*Sqrt[a + b*x^3] + ((92378*b*e*x^3*Sqrt[a + b*x^3])/9 - ((-16830*b*(19*b*
d - 10*a*g)*x^2*Sqrt[a + b*x^3])/7 + (-31122*b^2*(17*b*c - 8*a*f)*x*Sqrt[a
 + b*x^3] + (2*((646646*a*b^3*e*Sqrt[a + b*x^3])/3 + 16830*a*b^(8/3)*(19*b
*d - 10*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (6*3
^(3/4)*Sqrt[2 + Sqrt[3]]*a*b^(7/3)*(1729*b^(1/3)*(17*b*c - 8*a*f) - 1870*(
1 - Sqrt[3])*a^(1/3)*(19*b*d - 10*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b)/(7*b))/(3*b))
/b)/b))/230945
```

3.445.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.445. $\int x^3\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.445.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 956, normalized size of antiderivative = 1.30

method	result	size
elliptic	Expression too large to display	956
risch	Expression too large to display	1138
default	Expression too large to display	1674

input
```
int(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/19*g*x^8*(b*x^3+a)^(1/2)+2/17*f*x^7*(b*x^3+a)^(1/2)+2/15*e*x^6*(b*x^3+a)
^(1/2)+2/13*(3/19*a*g+b*d)/b*x^5*(b*x^3+a)^(1/2)+2/11*(3/17*a*f+b*c)/b*x^4
*(b*x^3+a)^(1/2)+2/45*a*e*x^3*(b*x^3+a)^(1/2)/b+2/7*(a*d-10/13*a/b*(3/19*a
*g+b*d))/b*x^2*(b*x^3+a)^(1/2)+2/5*(a*c-8/11*a/b*(3/17*a*f+b*c))/b*x*(b*x^
3+a)^(1/2)-4/45*a^2*e*(b*x^3+a)^(1/2)/b^2+4/15*I*a/b^2*(a*c-8/11*a/b*(3/17
*a*f+b*c))*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+8/21*I*a/b^2*(a*d-10/13*a/b*(3/19*a*g+b*d))*3^(1/2)*(-a*b^2)^(1/3)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2...
```

3.445.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.28

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx =$$

$$-\frac{2\left(93366\left(17\,a^2bc-8\,a^3f\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-100980\left(19\,a^2bd-10\,a^3g\right)\sqrt{b}\text{weierstrass}\right.}{}$$

input
```
integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fric
as")
```

output
```
-2/14549535*(93366*(17*a^2*b*c - 8*a^3*f)*sqrt(b)*weierstrassPInverse(0, -
4*a/b, x) - 100980*(19*a^2*b*d - 10*a^3*g)*sqrt(b)*weierstrassZeta(0, -4*a
/b, weierstrassPInverse(0, -4*a/b, x)) - (765765*b^3*g*x^8 + 855855*b^3*f*
x^7 + 969969*b^3*e*x^6 + 323323*a*b^2*e*x^3 + 58905*(19*b^3*d + 3*a*b^2*g)
*x^5 + 77805*(17*b^3*c + 3*a*b^2*f)*x^4 - 646646*a^2*b*e + 25245*(19*a*b^2
*d - 10*a^2*b*g)*x^2 + 46683*(17*a*b^2*c - 8*a^2*b*f)*x)*sqrt(b*x^3 + a))/
b^3
```

3.445.6 Sympy [A] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.32

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\frac{\sqrt{a}cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$
$$+\frac{\sqrt{a}fx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt{a}gx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$
$$+e\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**3*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + sqrt(a)*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*f*x**7*gamma(7/3)*hype
r((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a
)*g*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(11/3)) + e*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x*
*3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(
a)*x**6/6, True))
```

3.445.7 Maxima [F]

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3, x)
```

3.445.8 Giac [F]

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^3, x)
```

3.445.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\int x^3\,\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^3*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

output
```
int(x^3*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.446 $\int x^2\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

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3.446.1 Optimal result

Integrand size = 35, antiderivative size = 681

$$
\begin{aligned}
&\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx\\
&=\frac{2a(5bc-2af)\sqrt{a+bx^3}}{45b^2}+\frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2}+\frac{6aex^2\sqrt{a+bx^3}}{91b}\\
&\quad+\frac{2afx^3\sqrt{a+bx^3}}{45b}+\frac{6agx^4\sqrt{a+bx^3}}{187b}-\frac{24a^2e\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}\\
&\quad+\frac{2x^2\sqrt{a+bx^3}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395}\\
&\quad+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\\
&\quad-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(1547bd-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-728ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{Ellip}}{85085b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

output
```
2/45*a*(-2*a*f+5*b*c)*(b*x^3+a)^(1/2)/b^2+6/935*a*(-8*a*g+17*b*d)*x*(b*x^3
+a)^(1/2)/b^2+6/91*a*e*x^2*(b*x^3+a)^(1/2)/b+2/45*a*f*x^3*(b*x^3+a)^(1/2)/
b+6/187*a*g*x^4*(b*x^3+a)^(1/2)/b+2/109395*x^2*(6435*g*x^5+7293*f*x^4+8415
*e*x^3+9945*d*x^2+12155*c*x)*(b*x^3+a)^(1/2)-24/91*a^2*e*(b*x^3+a)^(1/2)/b
^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+12/91*3^(1/4)*a^(7/3)*e*(a^(1/3)+b^
(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a
)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)-4/85085*3^(3/4)*a^2*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1547*b*d-728*a
*g-1870*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b
^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

3.446.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.23

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$= \frac{2\sqrt{a+bx^3}\left(-\left(\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\left(26a(187f+180gx)-b(12155c+9945dx+33x^2(255e+13x(17f+\right.\right.\right.}{}$$

input
```
Integrate[x^2*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
(2*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(26*a*(187*f + 180*g
*x) - b*(12155*c + 9945*d*x + 33*x^2*(255*e + 13*x*(17*f + 15*g*x))))) + 5
85*a*(-17*b*d + 8*a*g)*x*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)] -
 8415*a*b*e*x^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(109395*
b^2*Sqrt[1 + (b*x^3)/a])
```

3.446.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.457, Rules used = {2365, 27, 2375, 27, 2375, 27, 2427, 27, 2028, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$

$$\Big\downarrow \text{ 2365}$$

$$\frac{3}{2}a\int \frac{2x^2\left(6435gx^4+7293fx^3+8415ex^2+9945dx+12155c\right)}{109395\sqrt{bx^3+a}}dx+\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\int \frac{x^2\left(6435gx^4+7293fx^3+8415ex^2+9945dx+12155c\right)}{\sqrt{bx^3+a}}dx}{36465}+\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$$\Big\downarrow \text{ 2375}$$

$$\frac{a\left(\frac{2\int \frac{11x^2\left(7293bfx^3+8415bex^2+585(17bd-8ag)x+12155bc\right)}{2\sqrt{bx^3+a}}dx}{11b}+\frac{1170gx^4\sqrt{a+bx^3}}{b}\right)}{36465}+\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\left(\frac{\int \frac{x^2\left(7293bfx^3+8415bex^2+585(17bd-8ag)x+12155bc\right)}{\sqrt{bx^3+a}}dx}{b}+\frac{1170gx^4\sqrt{a+bx^3}}{b}\right)}{36465}+\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$$\Big\downarrow \text{ 2375}$$

$$\frac{a\left(\frac{\frac{2\int \frac{9x^2\left(8415b^2ex^2+585b(17bd-8ag)x+2431b(5bc-2af)\right)}{2\sqrt{bx^3+a}}dx}{9b}+\frac{4862}{3}fx^3\sqrt{a+bx^3}}{b}+\frac{1170gx^4\sqrt{a+bx^3}}{b}\right)}{36465}+\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$\Big\downarrow$ 27

$$\frac{a\left(\frac{\frac{\int \frac{x^2\left(8415b^2ex^2+585b(17bd-8ag)x+2431b(5bc-2af)\right)}{\sqrt{bx^3+a}}dx}{b}+\frac{4862}{3}fx^3\sqrt{a+bx^3}}{b}+\frac{1170gx^4\sqrt{a+bx^3}}{b}\right)}{36465}+\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$\Big\downarrow$ 2427

$$\frac{a\left(\frac{\frac{\frac{2\int -\frac{-4095b^2(17bd-8ag)x^3-17017b^2(5bc-2af)x^2+33660ab^2ex}{2\sqrt{bx^3+a}}dx}{7b}+\frac{16830}{7}bex^2\sqrt{a+bx^3}}{b}+\frac{4862}{3}fx^3\sqrt{a+bx^3}}{b}+\frac{1170gx^4\sqrt{a+bx^3}}{b}\right)}{36465}+\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$\Big\downarrow$ 27

$$\frac{a\left(\frac{\frac{\frac{16830}{7}bex^2\sqrt{a+bx^3}-\frac{\int \frac{-4095b^2(17bd-8ag)x^3-17017b^2(5bc-2af)x^2+33660ab^2ex}{\sqrt{bx^3+a}}dx}{7b}}{b}+\frac{4862}{3}fx^3\sqrt{a+bx^3}}{b}+\frac{1170gx^4\sqrt{a+bx^3}}{b}\right)}{36465}+\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$\Big\downarrow$ 2028

$$\frac{a\left(\frac{\frac{\frac{16830}{7}bex^2\sqrt{a+bx^3}-\frac{\int \frac{x\left(-4095(17bd-8ag)x^2b^2+33660aeb^2-17017(5bc-2af)xb^2\right)}{\sqrt{bx^3+a}}dx}{7b}}{b}+\frac{4862}{3}fx^3\sqrt{a+bx^3}}{b}+\frac{1170gx^4\sqrt{a+bx^3}}{b}\right)}{36465}+\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

3.446. $\int x^2\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

$$\Big\downarrow \text{2427}$$

$$\frac{a\left(\frac{\frac{\frac{16830}{7}bex^2\sqrt{a+bx^3}-\frac{\frac{2\int \frac{5\left(-17017(5bc-2af)x^2b^3+33660aexb^3+1638a(17bd-8ag)b^2\right)}{2\sqrt{bx^3+a}}dx}{5b}-1638bx\sqrt{a+bx^3}(17bd-8ag)}{7b}}{b}+\frac{4862}{3}fx^3\sqrt{a+bx^3}}{b}+\frac{1170gx^4\sqrt{\cdots}}{b}\cdots\right)}{36465}$$

$$\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\left(\frac{\frac{\frac{16830}{7}bex^2\sqrt{a+bx^3}-\frac{\frac{\int \frac{-17017(5bc-2af)x^2b^3+33660aexb^3+1638a(17bd-8ag)b^2}{\sqrt{bx^3+a}}dx}{b}-1638bx\sqrt{a+bx^3}(17bd-8ag)}{7b}}{b}+\frac{4862}{3}fx^3\sqrt{a+bx^3}}{b}+\frac{1170gx^4\sqrt{a+b\cdots}}{b}\cdots\right)}{36465}$$

$$\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$$\Big\downarrow \text{2425}$$

$$\frac{a\left(\frac{\frac{\frac{16830}{7}bex^2\sqrt{a+bx^3}-\frac{\frac{\int \frac{33660aexb^3+1638a(17bd-8ag)b^2}{\sqrt{bx^3+a}}dx-17017b^3(5bc-2af)\int \frac{x^2}{\sqrt{bx^3+a}}dx}{b}-1638bx\sqrt{a+bx^3}(17bd-8ag)}{7b}}{b}+\frac{4862}{3}fx^3\sqrt{a+bx^3}}{b}+\frac{1170g\cdots}{}\cdots\right)}{36465}$$

$$\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$$\Big\downarrow \text{793}$$

$$\frac{a\left(\frac{\frac{\frac{16830}{7}bex^2\sqrt{a+bx^3}-\frac{\frac{\int \frac{33660aexb^3+1638a(17bd-8ag)b^2}{\sqrt{bx^3+a}}dx-\frac{34034}{3}b^2\sqrt{a+bx^3}(5bc-2af)}{b}-1638bx\sqrt{a+bx^3}(17bd-8ag)}{7b}}{b}+\frac{4862}{3}fx^3\sqrt{a+bx^3}}{b}+\frac{1170gx^4\sqrt{\cdots}}{b}\cdots\right)}{36465}$$

$$\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$$\Big\downarrow \text{2417}$$

3.446. $\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$\frac{a\left(\frac{\frac{16830}{7}bex^2\sqrt{a+bx^3}-\frac{\frac{33660ab^{8/3}e\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+18ab^2\left(-1870\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-728ag+1547bd\right)\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{34034}{3}b^2\sqrt{a+bx^3}(5bc-2af)\cdots}{b}}{7b}}{b}\cdots\right.}{36465}$$

$$\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$\downarrow$ 759

$$\frac{a\left(\frac{\frac{16830}{7}bex^2\sqrt{a+bx^3}-\frac{\frac{33660ab^{8/3}e\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{12\ 3^{3/4}\sqrt{2+\sqrt{3}}ab^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\cdots}{\sqrt[3]{b}x+\cdots}\right.\right.\cdots}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\cdots}\cdots}{b}\cdots}{7b}}{b}\cdots\right.}{\cdots}$$

$$\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

$\downarrow$ 2416

3.446. $\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$a\left(\frac{\frac{16830}{7}bex^2\sqrt{a+bx^3}-\frac{12\ 3^{3/4}\sqrt{2+\sqrt{3}}ab^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(-1870\left(1-\sqrt{\right.\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}\right.$$

$$\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}$$

input
```
Int[x^2*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
(2*x^2*Sqrt[a + b*x^3]*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 +
 6435*g*x^5))/109395 + (a*((1170*g*x^4*Sqrt[a + b*x^3])/b + ((4862*f*x^3*S
qrt[a + b*x^3])/3 + ((16830*b*e*x^2*Sqrt[a + b*x^3])/7 - (-1638*b*(17*b*d
- 8*a*g)*x*Sqrt[a + b*x^3] + ((-34034*b^2*(5*b*c - 2*a*f)*Sqrt[a + b*x^3])
/3 + 33660*a*b^(8/3)*e*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])) + (12*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*b^(5/3)*(1547*b*d - 1870*(1 - Sqrt[3
])*a^(1/3)*b^(2/3)*e - 728*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b)/(7*b))/b)/b))/36465
```

3.446. $\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

3.446.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.446. $\int x^2\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.446.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	920
risch	Expression too large to display	1115
default	Expression too large to display	1197

input
```
int(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/17*g*x^7*(b*x^3+a)^(1/2)+2/15*f*x^6*(b*x^3+a)^(1/2)+2/13*e*x^5*(b*x^3+a)
^(1/2)+2/11*(3/17*a*g+b*d)/b*x^4*(b*x^3+a)^(1/2)+2/9*(1/5*a*f+b*c)/b*x^3*(
b*x^3+a)^(1/2)+6/91*a*e*x^2*(b*x^3+a)^(1/2)/b+2/5*(a*d-8/11*a/b*(3/17*a*g+
b*d))/b*x*(b*x^3+a)^(1/2)+2/3*(a*c-2/3*a/b*(1/5*a*f+b*c))/b*(b*x^3+a)^(1/2
)+4/15*I*a/b^2*(a*d-8/11*a/b*(3/17*a*g+b*d))*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+8/91*I/b^2*a^2*e*3^(1/2)*(-a*b^2)^(
1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2...
```

3.446.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.26

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$

$$= \frac{2\left(100980\,a^2b^{\frac{3}{2}}e\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-4914\left(17\,a^2bd-8\,a^3g\right)\sqrt{b}\text{weie}\right.}{}$$

input
```
integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fric
as")
```

output
```
2/765765*(100980*a^2*b^(3/2)*e*weierstrassZeta(0, -4*a/b, weierstrassPInve
rse(0, -4*a/b, x)) - 4914*(17*a^2*b*d - 8*a^3*g)*sqrt(b)*weierstrassPInver
se(0, -4*a/b, x) + (45045*b^3*g*x^7 + 51051*b^3*f*x^6 + 58905*b^3*e*x^5 +
25245*a*b^2*e*x^2 + 4095*(17*b^3*d + 3*a*b^2*g)*x^4 + 85085*a*b^2*c - 3403
4*a^2*b*f + 17017*(5*b^3*c + a*b^2*f)*x^3 + 2457*(17*a*b^2*d - 8*a^2*b*g)*
x)*sqrt(b*x^3 + a))/b^3
```

3.446.6 Sympy [A] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.33

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{\sqrt{a}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}ex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}gx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + c\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + f\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + sqrt(a)*e*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*g*x**7*gamma(7/3)*hype
r((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + c*Piec
ewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + f
*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)
/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

3.446.7 Maxima [F]

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}x^2\,dx$$

input
```
integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
2/9*(b*x^3 + a)^(3/2)*c/b + integrate((g*x^6 + f*x^5 + e*x^4 + d*x^3)*sqrt
(b*x^3 + a), x)
```

3.446.8 Giac [F]

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}x^2\,dx$$

input
```
integrate(x^2*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x^2, x)
```

3.446.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\int x^2\,\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^2*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

output
```
int(x^2*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.447 $\int x\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

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3.447.1 Optimal result

Integrand size = 33, antiderivative size = 667

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{2a(5bd-2ag)\sqrt{a+bx^3}}{45b^2}+\frac{6aex\sqrt{a+bx^3}}{55b}$$
$$+\frac{6afx^2\sqrt{a+bx^3}}{91b}+\frac{2agx^3\sqrt{a+bx^3}}{45b}+\frac{6a(13bc-4af)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{2x\sqrt{a+bx^3}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bc-4af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^{4/3}\left(182a^{2/3}\sqrt[3]{b}e+55\left(1-\sqrt{3}\right)(13bc-4af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Elli}}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/45*a*(-2*a*g+5*b*d)*(b*x^3+a)^(1/2)/b^2+6/55*a*e*x*(b*x^3+a)^(1/2)/b+6/9
1*a*f*x^2*(b*x^3+a)^(1/2)/b+2/45*a*g*x^3*(b*x^3+a)^(1/2)/b+2/45045*x*(3003
*g*x^5+3465*f*x^4+4095*e*x^3+5005*d*x^2+6435*c*x)*(b*x^3+a)^(1/2)+6/91*a*(
-4*a*f+13*b*c)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3/9
1*3^(1/4)*a^(4/3)*(-4*a*f+13*b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/5005*3^(3/4)*a^(4/3)*(a
^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(182*a^(2/3)*b^(1/3)*e+55*(-4*a*f+13*b*c)
*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.447.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.21

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\frac{\sqrt{a+bx^3}\left(-4(a+bx^3)\sqrt{1+\frac{bx^3}{a}}(286ag-b(715d+585ex+495fx^2+429gx^3))-2340abex\,\text{Hypergeom}\right.}{12870b^2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[x*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
(Sqrt[a + b*x^3]*(-4*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(286*a*g - b*(715*d +
 585*e*x + 495*f*x^2 + 429*g*x^3)) - 2340*a*b*e*x*Hypergeometric2F1[-1/2,
1/3, 4/3, -((b*x^3)/a)] + 495*b*(13*b*c - 4*a*f)*x^2*Hypergeometric2F1[-1/
2, 2/3, 5/3, -((b*x^3)/a)]))/(12870*b^2*Sqrt[1 + (b*x^3)/a])
```

3.447.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {2365, 27, 2375, 27, 2375, 27, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$

$$\Big\downarrow \ 2365$$

$$\frac{3}{2}a\int \frac{2x\left(3003gx^4+3465fx^3+4095ex^2+5005dx+6435c\right)}{45045\sqrt{bx^3+a}}dx+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$$\Big\downarrow \ 27$$

$$\frac{a\int \frac{x\left(3003gx^4+3465fx^3+4095ex^2+5005dx+6435c\right)}{\sqrt{bx^3+a}}dx}{15015}+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$$\Big\downarrow \ 2375$$

$$\frac{a\left(\frac{2\int \frac{9x\left(3465bfx^3+4095bex^2+1001(5bd-2ag)x+6435bc\right)}{2\sqrt{bx^3+a}}dx}{9b}+\frac{2002gx^3\sqrt{a+bx^3}}{3b}\right)}{15015}+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$$\Big\downarrow \ 27$$

$$\frac{a\left(\frac{\int \frac{x\left(3465bfx^3+4095bex^2+1001(5bd-2ag)x+6435bc\right)}{\sqrt{bx^3+a}}dx}{b}+\frac{2002gx^3\sqrt{a+bx^3}}{3b}\right)}{15015}+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$$\Big\downarrow \ 2375$$

$$\frac{a\left(\frac{\frac{2\int \frac{7x\left(4095b^2ex^2+1001b(5bd-2ag)x+495b(13bc-4af)\right)}{2\sqrt{bx^3+a}}dx}{7b}+990fx^2\sqrt{a+bx^3}}{b}+\frac{2002gx^3\sqrt{a+bx^3}}{3b}\right)}{15015}+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$\Big\downarrow$ 27

$$\frac{a\left(\frac{\frac{\int \frac{x\left(4095b^2ex^2+1001b(5bd-2ag)x+495b(13bc-4af)\right)}{\sqrt{bx^3+a}}dx}{b}+990fx^2\sqrt{a+bx^3}}{b}+\frac{2002gx^3\sqrt{a+bx^3}}{3b}\right)}{15015}+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$\Big\downarrow$ 2427

$$\frac{a\left(\frac{\frac{\frac{2\int -\frac{5\left(-1001(5bd-2ag)x^2b^2+1638aeb^2-495(13bc-4af)xb^2\right)}{2\sqrt{bx^3+a}}dx}{5b}+1638bex\sqrt{a+bx^3}}{b}+990fx^2\sqrt{a+bx^3}}{b}+\frac{2002gx^3\sqrt{a+bx^3}}{3b}\right)}{15015}+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$\Big\downarrow$ 27

$$\frac{a\left(\frac{\frac{1638bex\sqrt{a+bx^3}-\frac{\int \frac{-1001(5bd-2ag)x^2b^2+1638aeb^2-495(13bc-4af)xb^2}{\sqrt{bx^3+a}}dx}{b}}{b}+990fx^2\sqrt{a+bx^3}}{b}+\frac{2002gx^3\sqrt{a+bx^3}}{3b}\right)}{15015}+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$\Big\downarrow$ 2425

$$\frac{a\left(\frac{\frac{1638bex\sqrt{a+bx^3}-\frac{\int \frac{1638ab^2e-495b^2(13bc-4af)x}{\sqrt{bx^3+a}}dx-1001b^2(5bd-2ag)\int \frac{x^2}{\sqrt{bx^3+a}}dx}{b}}{b}+990fx^2\sqrt{a+bx^3}}{b}+\frac{2002gx^3\sqrt{a+bx^3}}{3b}\right)}{15015}+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$\Big\downarrow$ 793

3.447. $\int x\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\ dx$

$$\frac{a\left(\frac{\frac{1638bex\sqrt{a+bx^3}-\frac{\int\frac{1638ab^2e-495b^2(13bc-4af)x}{\sqrt{bx^3+a}}dx-\frac{2002}{3}b\sqrt{a+bx^3}(5bd-2ag)}{b}}{b}}{b}+990fx^2\sqrt{a+bx^3}+\frac{2002gx^3\sqrt{a+bx^3}}{3b}\right)}{15015}+\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$\downarrow$ 2417

$$\frac{a\left(\frac{\frac{1638bex\sqrt{a+bx^3}-\frac{9\sqrt[3]{a}b^{5/3}\left(182a^{2/3}\sqrt[3]{b}e+55\left(1-\sqrt{3}\right)(13bc-4af)\right)\int\frac{1}{\sqrt{bx^3+a}}dx-495b^{5/3}(13bc-4af)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{2002}{3}b\sqrt{a+bx^3}(5bd-2a}{b}}{b}}{b}}{15015}$$

$$\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$\downarrow$ 759

$$\frac{a\left(\frac{\frac{1638bex\sqrt{a+bx^3}-\frac{-495b^{5/3}(13bc-4af)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}}}}{b}}{b}}{b}}{15015}$$

$$\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

$\downarrow$ 2416

3.447. $\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$a\left(\frac{1638bex\sqrt{a+bx^3}-\frac{6\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(182a^{2/3}\sqrt[3]{b}\cdots\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\cdots}\right.$$

$$\frac{2x\sqrt{a+bx^3}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045}$$

input
```
Int[x*Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(6435*c*x + 5005*d*x^2 + 4095*e*x^3 + 3465*f*x^4 + 30
03*g*x^5))/45045 + (a*((2002*g*x^3*Sqrt[a + b*x^3])/(3*b) + (990*f*x^2*Sqr
t[a + b*x^3] + (1638*b*e*x*Sqrt[a + b*x^3] - ((-2002*b*(5*b*d - 2*a*g)*Sqr
t[a + b*x^3])/3 - 495*b^(5/3)*(13*b*c - 4*a*f)*((2*Sqrt[a + b*x^3])/(b^(1/
3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/
3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3])) + (6*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*b^(4/3)
*(182*a^(2/3)*b^(1/3)*e + 55*(1 - Sqrt[3])*(13*b*c - 4*a*f))*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
]))/b)/b)/b))/15015
```

3.447. $\int x\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

3.447.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.447. $\int x\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.447.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.24

method	result	size
risch	Expression too large to display	829
elliptic	Expression too large to display	884
default	Expression too large to display	1311

input
```
int(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45045*(-3003*b^2*g*x^6-3465*b^2*f*x^5-4095*b^2*e*x^4-1001*a*b*g*x^3-500
5*b^2*d*x^3-1485*a*b*f*x^2-6435*b^2*c*x^2-2457*a*b*e*x+2002*a^2*g-5005*a*b
*d)/b^2*(b*x^3+a)^(1/2)-3/5005*a/b*(-364/3*I*a*e*3^(1/2)/b*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(220*a*f-715*b*c)*3^(1/
2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*...
```

3.447.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.22

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx =$$
$$-\frac{2\left(4914\,a^2\sqrt{b}e\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+1485\left(13\,abc-4\,a^2f\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weier}\right.\right.}{}$$

input
```
integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas
")
```

output
```
-2/45045*(4914*a^2*sqrt(b)*e*weierstrassPInverse(0, -4*a/b, x) + 1485*(13*
a*b*c - 4*a^2*f)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0,
 -4*a/b, x)) - (3003*b^2*g*x^6 + 3465*b^2*f*x^5 + 4095*b^2*e*x^4 + 2457*a*
b*e*x + 1001*(5*b^2*d + a*b*g)*x^3 + 5005*a*b*d - 2002*a^2*g + 495*(13*b^2
*c + 3*a*b*f)*x^2)*sqrt(b*x^3 + a))/b^2
```

3.447.6 Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.33

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\frac{\sqrt{a}cx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt{a}ex^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$
$$+\frac{\sqrt{a}fx^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+d\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$
$$+g\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x*(g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(5/3)) + sqrt(a)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*f*x**5*gamma(5/3)*hype
r((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + d*Piecew
ise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + g*P
iecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(
45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

3.447.7 Maxima [F]

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}x\,dx$$

input
```
integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x, x)
```

3.447.8 Giac [F]

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int\left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}x\,dx$$

input
```
integrate(x*(g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)*x, x)
```

3.447.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int x\,\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

output
```
int(x*(a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.448 $\int \sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

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3.448.1 Optimal result

Integrand size = 32, antiderivative size = 639

$$\int \sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$$

$$= \frac{2ae\sqrt{a+bx^3}}{9b} + \frac{6afx\sqrt{a+bx^3}}{55b} + \frac{6agx^2\sqrt{a+bx^3}}{91b} + \frac{6a(13bd-4ag)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+ \frac{2\sqrt{a+bx^3}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7 ...}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(91\sqrt[3]{b}(11bc-2af)-55\left(1-\sqrt{3}\right)\sqrt[3]{a}(13bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/ ...}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b ...}\right)}}}{5005b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*a*e*(b*x^3+a)^(1/2)/b+6/55*a*f*x*(b*x^3+a)^(1/2)/b+6/91*a*g*x^2*(b*x^3
+a)^(1/2)/b+2/45045*(3465*g*x^5+4095*f*x^4+5005*e*x^3+6435*d*x^2+9009*c*x)
*(b*x^3+a)^(1/2)+6/91*a*(-4*a*g+13*b*d)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))-3/91*3^(1/4)*a^(4/3)*(-4*a*g+13*b*d)*(a^(1/3)+b^(1/3
)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1
/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+
2/5005*3^(3/4)*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(91*b^(1/3)*(-2*a*f+11
*b*c)-55*a^(1/3)*(-4*a*g+13*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)
```

3.448.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.21

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$

$$= \frac{\sqrt{a+bx^3}\left(4(a+bx^3)\sqrt{1+\frac{bx^3}{a}}(143e+9x(13f+11gx))+234(11bc-2af)x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\right.\right.}{2574b\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
(Sqrt[a + b*x^3]*(4*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(143*e + 9*x*(13*f + 1
1*g*x)) + 234*(11*b*c - 2*a*f)*x*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^
3)/a)] + 99*(13*b*d - 4*a*g)*x^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^
3)/a)]))/(2574*b*Sqrt[1 + (b*x^3)/a])
```

3.448.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 634, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2392, 27, 2427, 27, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$

$$\Big\downarrow \text{ 2392}$$

$$\frac{3}{2}a\int \frac{2\left(3465gx^4+4095fx^3+5005ex^2+6435dx+9009c\right)}{45045\sqrt{bx^3+a}}dx+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\int \frac{3465gx^4+4095fx^3+5005ex^2+6435dx+9009c}{\sqrt{bx^3+a}}dx}{15015}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

$$\Big\downarrow \text{ 2427}$$

$$\frac{a\left(\frac{2\int \frac{7\left(4095bfx^3+5005bex^2+495(13bd-4ag)x+9009bc\right)}{2\sqrt{bx^3+a}}dx}{7b}+\frac{990gx^2\sqrt{a+bx^3}}{b}\right)}{15015}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\left(\frac{\int \frac{4095bfx^3+5005bex^2+495(13bd-4ag)x+9009bc}{\sqrt{bx^3+a}}dx}{b}+\frac{990gx^2\sqrt{a+bx^3}}{b}\right)}{15015}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

$$\Big\downarrow \text{ 2427}$$

$$\frac{a\left(\frac{\frac{2\int \frac{5\left(5005b^2ex^2+495b(13bd-4ag)x+819b(11bc-2af)\right)}{2\sqrt{bx^3+a}}dx}{5b}+1638fx\sqrt{a+bx^3}}{b}+\frac{990gx^2\sqrt{a+bx^3}}{b}\right)}{15015}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

$$\Big\downarrow \; 27$$

$$\frac{a\left(\frac{\frac{\int \frac{5005b^2ex^2+495b(13bd-4ag)x+819b(11bc-2af)}{\sqrt{bx^3+a}}dx}{b}+1638fx\sqrt{a+bx^3}}{b}+\frac{990gx^2\sqrt{a+bx^3}}{b}\right)}{15015}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

$$\Big\downarrow \; 2425$$

$$\frac{a\left(\frac{\frac{5005b^2e\int \frac{x^2}{\sqrt{bx^3+a}}dx+\int \frac{819b(11bc-2af)+495b(13bd-4ag)x}{\sqrt{bx^3+a}}dx}{b}+1638fx\sqrt{a+bx^3}}{b}+\frac{990gx^2\sqrt{a+bx^3}}{b}\right)}{15015}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

$$\Big\downarrow \; 793$$

$$\frac{a\left(\frac{\frac{\int \frac{819b(11bc-2af)+495b(13bd-4ag)x}{\sqrt{bx^3+a}}dx+\frac{10010}{3}be\sqrt{a+bx^3}}{b}+1638fx\sqrt{a+bx^3}}{b}+\frac{990gx^2\sqrt{a+bx^3}}{b}\right)}{15015}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

$$\Big\downarrow \; 2417$$

$$\frac{a\left(\frac{\frac{9b^{2/3}\left(91\sqrt[3]{b}(11bc-2af)-55\left(1-\sqrt{3}\right)\sqrt[3]{a}(13bd-4ag)\right)\int \frac{1}{\sqrt{bx^3+a}}dx+495b^{2/3}(13bd-4ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{10010}{3}be\sqrt{a+bx^3}}{b}+1638fx\sqrt{a+bx^3}}{b}\right.}{15015}+\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

$$\Big\downarrow \; 759$$

3.448. $\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$a\left(\frac{\frac{495b^{2/3}(13bd-4ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{b}}{b}\right.$$

$$15015$$

$$\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

$$\Big\downarrow \ 2416$$

$$a\left(\frac{\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(91\sqrt[3]{b}(11bc-2af)-55\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}\right.$$

$$\frac{2\sqrt{a+bx^3}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045}$$

input
```
Int[Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

3.448. $\int \sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)\,dx$

output
```
(2*Sqrt[a + b*x^3]*(9009*c*x + 6435*d*x^2 + 5005*e*x^3 + 4095*f*x^4 + 3465
*g*x^5))/45045 + (a*((990*g*x^2*Sqrt[a + b*x^3])/b + (1638*f*x*Sqrt[a + b*
x^3] + ((10010*b*e*Sqrt[a + b*x^3])/3 + 495*b^(2/3)*(13*b*d - 4*a*g)*((2*S
qrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*S
qrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (6*3^(3/4)*Sqrt[2 + Sq
rt[3]]*b^(1/3)*(91*b^(1/3)*(11*b*c - 2*a*f) - 55*(1 - Sqrt[3])*a^(1/3)*(13
*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b)/b))/15015
```

3.448.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.448.4 Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	863
risch	Expression too large to display	1080
default	Expression too large to display	1557

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/13*g*x^5*(b*x^3+a)^(1/2)+2/11*f*x^4*(b*x^3+a)^(1/2)+2/9*e*x^3*(b*x^3+a)^
(1/2)+2/7*(3/13*a*g+b*d)/b*x^2*(b*x^3+a)^(1/2)+2/5*(3/11*a*f+b*c)/b*x*(b*x
^3+a)^(1/2)+2/9*a*e*(b*x^3+a)^(1/2)/b-2/3*I*(a*c-2/5*a/b*(3/11*a*f+b*c))*3
^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I
*(a*d-4/7*a/b*(3/13*a*g+b*d))*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^...
```

3.448.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.22

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$= \frac{2\left(2457\,(11\,abc-2\,a^2f)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-1485\,(13\,abd-4\,a^2g)\sqrt{b}\text{weierstrassZeta}\left(0,-\right.\right.}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45045*(2457*(11*a*b*c - 2*a^2*f)*sqrt(b)*weierstrassPInverse(0, -4*a/b,
x) - 1485*(13*a*b*d - 4*a^2*g)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstr
assPInverse(0, -4*a/b, x)) + (3465*b^2*g*x^5 + 4095*b^2*f*x^4 + 5005*b^2*e
*x^3 + 5005*a*b*e + 495*(13*b^2*d + 3*a*b*g)*x^2 + 819*(11*b^2*c + 3*a*b*f
)*x)*sqrt(b*x^3 + a))/b^2
```

3.448.6 Sympy [A] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.30

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{\sqrt{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + e\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*f*x**4*gamma(4/3)*hyper((
-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*g*x
**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(8/3)) + e*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/
(9*b), True))
```

3.448.7 Maxima [F]

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)
```

3.448.8 Giac [F]

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\sqrt{bx^3+a}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a), x)
```

3.448.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

output
```
int((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.449 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$

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3.449.2 Mathematica [C] (verified) . . . 3405
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3.449.5 Fricas [C] (verification not implemented) . . . 3412
3.449.6 Sympy [A] (verification not implemented) . . . 3413
3.449.7 Maxima [F] . . . 3414
3.449.8 Giac [F] . . . 3414
3.449.9 Mupad [F(-1)] . . . 3415

3.449.1 Optimal result

Integrand size = 35, antiderivative size = 620

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \frac{2af\sqrt{a+bx^3}}{9b}+\frac{6agx\sqrt{a+bx^3}}{55b}+\frac{6ae\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x}$$
$$-\frac{2}{3}\sqrt{a}\,\mathrm{carctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(77bd-55\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-14ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\Big(}{385b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/9*a*f*(b*x^3+a)^(1/2)/b+
6/55*a*g*x*(b*x^3+a)^(1/2)/b+2/3465*(315*g*x^5+385*f*x^4+495*e*x^3+693*d*x
^2+1155*c*x)*(b*x^3+a)^(1/2)/x+6/7*a*e*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))-3/7*3^(1/4)*a^(4/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((
b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/385*3^(3/4)*a*
(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(77*b*d-14*a*g-55*a^(1/3)*b^(2/3)*e*(1-
3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.449.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$$
$$= \frac{4\sqrt{1+\frac{bx^3}{a}}\left(\sqrt{a+bx^3}(33bc+11af+9agx+11bfx^3+9bgx^4)-33\sqrt{a}bc\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+18(11bd-}{198b}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

output
```
(4*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3]*(33*b*c + 11*a*f + 9*a*g*x + 11*b*
f*x^3 + 9*b*g*x^4) - 33*Sqrt[a]*b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) + 18
*(11*b*d - 2*a*g)*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b
*x^3)/a)] + 99*b*e*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 2/3, 5/3, -
((b*x^3)/a)])/(198*b*Sqrt[1 + (b*x^3)/a])
```

3.449.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2365, 27, 2371, 798, 73, 221, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$$

$$\Big\downarrow \ 2365$$

$$\frac{3}{2}a\int \frac{2(315gx^4+385fx^3+495ex^2+693dx+1155c)}{3465x\sqrt{bx^3+a}}dx+\frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x}$$

$$\Big\downarrow \ 27$$

$$\frac{a\int \frac{315gx^4+385fx^3+495ex^2+693dx+1155c}{x\sqrt{bx^3+a}}dx}{1155}+\frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x}$$

$$\Big\downarrow \ 2371$$

$$\frac{a\left(1155c\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{315gx^3+385fx^2+495ex+693d}{\sqrt{bx^3+a}}dx\right)}{1155}+\frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x}$$

$$\Big\downarrow \ 798$$

$$\frac{a\left(385c\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int \frac{315gx^3+385fx^2+495ex+693d}{\sqrt{bx^3+a}}dx\right)}{1155}+\frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x}$$

$$\Big\downarrow \ 73$$

$$\frac{a\left(\frac{770c\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+\int \frac{315gx^3+385fx^2+495ex+693d}{\sqrt{bx^3+a}}dx\right)}{1155}+\frac{2\sqrt{a+bx^3}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x}$$

$$\Big\downarrow \ 221$$

$$\frac{a\left(\int \frac{315gx^3+385fx^2+495ex+693d}{\sqrt{bx^3+a}}dx-\frac{770c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{1155}+\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x}$$

$\Big\downarrow$ 2427

$$\frac{a\left(\frac{2\int \frac{5\left(385bfx^2+495bex+63(11bd-2ag)\right)}{2\sqrt{bx^3+a}}dx}{5b}-\frac{770c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{126gx\sqrt{a+bx^3}}{b}\right)}{1155}+\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x}$$

$\Big\downarrow$ 27

$$\frac{a\left(\frac{\int \frac{385bfx^2+495bex+63(11bd-2ag)}{\sqrt{bx^3+a}}dx}{b}-\frac{770c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{126gx\sqrt{a+bx^3}}{b}\right)}{1155}+\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x}$$

$\Big\downarrow$ 2425

$$\frac{a\left(\frac{\int \frac{63(11bd-2ag)+495bex}{\sqrt{bx^3+a}}dx+385bf\int \frac{x^2}{\sqrt{bx^3+a}}dx}{b}-\frac{770c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{126gx\sqrt{a+bx^3}}{b}\right)}{1155}+\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x}$$

$\Big\downarrow$ 793

$$\frac{a\left(\frac{\int \frac{63(11bd-2ag)+495bex}{\sqrt{bx^3+a}}dx+\frac{770}{3}f\sqrt{a+bx^3}}{b}-\frac{770c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{126gx\sqrt{a+bx^3}}{b}\right)}{1155}+\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x}$$

$\Big\downarrow$ 2417

3.449. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}\,dx$

$$a\left(\frac{\frac{9\left(-55\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-14ag+77bd\right)\int\frac{1}{\sqrt{bx^3+a}}dx+495b^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{770}{3}f\sqrt{a+bx^3}}{b}-\frac{770c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}+12}{1155}\right.$$

$$\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x}$$

$\downarrow$ 759

$$a\left(\frac{\frac{495b^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(-\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{b}}{1155}\right.$$

$$\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x}$$

$\downarrow$ 2416

$$a\left(\frac{\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(-55\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-14ag+77bd\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+4}{}\right.$$

$$\frac{2\sqrt{a+bx^3}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x}$$

3.449. $\int\frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}\,dx$

input
```
Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

output
```
(2*Sqrt[a + b*x^3]*(1155*c*x + 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g*x
^5))/(3465*x) + (a*((126*g*x*Sqrt[a + b*x^3])/b - (770*c*ArcTanh[Sqrt[a +
b*x^3]/Sqrt[a]])/Sqrt[a] + ((770*f*Sqrt[a + b*x^3])/3 + 495*b^(2/3)*e*((2*
Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*
Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (6*3^(3/4)*Sqrt[2 + S
qrt[3]]*(77*b*d - 55*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 14*a*g)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]))/b))/1155
```

3.449.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.449. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.449. $\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.449.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	848
default	Expression too large to display	1118

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/11*g*x^4*(b*x^3+a)^(1/2)+2/9*f*x^3*(b*x^3+a)^(1/2)+2/7*e*x^2*(b*x^3+a)^(
1/2)+2/5*(3/11*a*g+b*d)/b*x*(b*x^3+a)^(1/2)+2/3*(1/3*a*f+b*c)/b*(b*x^3+a)^
(1/2)-2/3*I*(a*d-2/5*(3/11*a*g+b*d)/b*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/7*I*a*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/...
```

3.449.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$$
$$= \left[\frac{1155\sqrt{a}b^2c\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right) - 5940\,ab^{\frac{3}{2}}e\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPI}\right.}{}\right.$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas
")
```

output
```
[1/6930*(1155*sqrt(a)*b^2*c*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sq
rt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 5940*a*b^(3/2)*e*weierstrassZeta(0,
-4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 756*(11*a*b*d - 2*a^2*g)*sqrt
(b)*weierstrassPInverse(0, -4*a/b, x) + 4*(315*b^2*g*x^4 + 385*b^2*f*x^3 +
 495*b^2*e*x^2 + 1155*b^2*c + 385*a*b*f + 63*(11*b^2*d + 3*a*b*g)*x)*sqrt(
b*x^3 + a))/b^2, 1/3465*(1155*sqrt(-a)*b^2*c*arctan(2*sqrt(b*x^3 + a)*sqrt
(-a)/(b*x^3 + 2*a)) - 2970*a*b^(3/2)*e*weierstrassZeta(0, -4*a/b, weierstr
assPInverse(0, -4*a/b, x)) + 378*(11*a*b*d - 2*a^2*g)*sqrt(b)*weierstrassP
Inverse(0, -4*a/b, x) + 2*(315*b^2*g*x^4 + 385*b^2*f*x^3 + 495*b^2*e*x^2 +
 1155*b^2*c + 385*a*b*f + 63*(11*b^2*d + 3*a*b*g)*x)*sqrt(b*x^3 + a))/b^2]
```

3.449.6 Sympy [A] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = -\frac{2\sqrt{a}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}gx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2ac}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2\sqrt{b}cx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + f\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x,x)
```

output
```
-2*sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*d*x*gamma(1/3)*
hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt
(a)*e*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(5/3)) + sqrt(a)*g*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a*c/(3*sqrt(b)*x**(3/2)*sqrt(a/(
b*x**3) + 1)) + 2*sqrt(b)*c*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + f*Piecewis
e((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

3.449.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="maxima
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)
```

3.449.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x, x)
```

3.449.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx=\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x}\,dx$$

input
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x)
```

output
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x, x)
```

3.450 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

3.450.1 Optimal result . . . 3416
3.450.2 Mathematica [C] (verified) . . . 3417
3.450.3 Rubi [A] (verified) . . . 3418
3.450.4 Maple [A] (verified) . . . 3424
3.450.5 Fricas [C] (verification not implemented) . . . 3425
3.450.6 Sympy [A] (verification not implemented) . . . 3426
3.450.7 Maxima [F] . . . 3427
3.450.8 Giac [F] . . . 3427
3.450.9 Mupad [F(-1)] . . . 3427

3.450.1 Optimal result

Integrand size = 35, antiderivative size = 638

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$$
$$= \frac{2ag\sqrt{a+bx^3}}{9b} - \frac{3c\sqrt{a+bx^3}}{x} + \frac{3(7bc+2af)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{2\sqrt{a+bx^3}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^2} - \frac{2}{3}\sqrt{a}d\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$
$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bc+2af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(14a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(7bc+2af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/9*a*g*(b*x^3+a)^(1/2)/b-
3*c*(b*x^3+a)^(1/2)/x+2/315*(35*g*x^5+45*f*x^4+63*e*x^3+105*d*x^2+315*c*x)
*(b*x^3+a)^(1/2)/x^2+3/7*(2*a*f+7*b*c)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))-3/14*3^(1/4)*a^(1/3)*(2*a*f+7*b*c)*(a^(1/3)+b^(1/3)*x
)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)
/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/3
5*3^(3/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(14*a^(2/3)*b^(1/3)*e
-5*(2*a*f+7*b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b
*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)
```

3.450.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.33

$$\begin{aligned}&\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx\\&=\frac{2}{3}d\sqrt{a+bx^3}+\frac{2g(a+bx^3)^{3/2}}{9b}\\&\quad-\frac{2}{3}\sqrt{a}d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{c\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt{1+\frac{bx^3}{a}}}\\&\quad+\frac{ex\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\\&\quad+\frac{fx^2\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\end{aligned}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

output
```
(2*d*Sqrt[a + b*x^3])/3 + (2*g*(a + b*x^3)^(3/2))/(9*b) - (2*Sqrt[a]*d*Arc
Tanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - (c*Sqrt[a + b*x^3]*Hypergeometric2F1[-1
/2, -1/3, 2/3, -((b*x^3)/a)])/(x*Sqrt[1 + (b*x^3)/a]) + (e*x*Sqrt[a + b*x^
3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a] +
(f*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(2
*Sqrt[1 + (b*x^3)/a])
```

3.450.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2365, 27, 2374, 25, 2371, 798, 73, 221, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^2}\,dx$$

$$\Big\downarrow \text{2365}$$

$$\frac{3}{2}a\int \frac{2\left(35gx^4+45fx^3+63ex^2+105dx+315c\right)}{315x^2\sqrt{bx^3+a}}dx+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105}a\int \frac{35gx^4+45fx^3+63ex^2+105dx+315c}{x^2\sqrt{bx^3+a}}dx+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$$\Big\downarrow \text{2374}$$

$$\frac{1}{105}a\left(-\frac{\int -\frac{70agx^3+45(7bc+2af)x^2+126aex+210ad}{x\sqrt{bx^3+a}}dx}{2a}-\frac{315c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{105}a\left(\frac{\int \frac{70agx^3+45(7bc+2af)x^2+126aex+210ad}{x\sqrt{bx^3+a}}dx}{2a}-\frac{315c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$$\Big\downarrow \ 2371$$

$$\frac{1}{105}a\left(\frac{\int \frac{70agx^2+45(7bc+2af)x+126ae}{\sqrt{bx^3+a}}dx+210ad\int \frac{1}{x\sqrt{bx^3+a}}dx}{2a}-\frac{315c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$$\Big\downarrow \ 798$$

$$\frac{1}{105}a\left(\frac{\int \frac{70agx^2+45(7bc+2af)x+126ae}{\sqrt{bx^3+a}}dx+70ad\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{2a}-\frac{315c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$$\Big\downarrow \ 73$$

$$\frac{1}{105}a\left(\frac{\int \frac{70agx^2+45(7bc+2af)x+126ae}{\sqrt{bx^3+a}}dx+\frac{140ad\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}}{2a}-\frac{315c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$$\Big\downarrow \ 221$$

$$\frac{1}{105}a\left(\frac{\int \frac{70agx^2+45(7bc+2af)x+126ae}{\sqrt{bx^3+a}}dx-140\sqrt{a}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{2a}-\frac{315c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$$\Big\downarrow \ 2425$$

$$\frac{1}{105}a\left(\frac{\int \frac{126ae+45(7bc+2af)x}{\sqrt{bx^3+a}}dx+70ag\int \frac{x^2}{\sqrt{bx^3+a}}dx-140\sqrt{a}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{2a}-\frac{315c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$$\Big\downarrow \ 793$$

3.450. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^2}\,dx$

$$\frac{1}{105}a\left(\frac{\int \frac{126ae+45(7bc+2af)x}{\sqrt{bx^3+a}}dx-140\sqrt{a}d\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{140ag\sqrt{a+bx^3}}{3b}}{2a}-\frac{315c\sqrt{a+bx^3}}{ax}\right)+$$
$$\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$\downarrow$ 2417

$$\frac{1}{105}a\left(\frac{9\sqrt[3]{a}\left(14a^{2/3}e-\frac{5\left(1-\sqrt{3}\right)(2af+7bc)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{45(2af+7bc)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-140\sqrt{a}d\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right.}{2a}\right.$$
$$\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$\downarrow$ 759

$$\frac{1}{105}a\left(\frac{\frac{45(2af+7bc)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\right.}}{\sqrt[3]{b}x+\left(1+\sqrt{\right.}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2a}\right.$$
$$\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

$\downarrow$ 2416

3.450. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^2}\,dx$

$$\frac{1}{105}a\Bigg(\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\Bigg(14a^{2/3}e-\frac{5\left(1-\sqrt{3}\right)(\cdots}{\sqrt[3]{\cdots}}}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots$$

$$\frac{2\sqrt{a+bx^3}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^2}$$

input

```
Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

output

```
(2*Sqrt[a + b*x^3]*(315*c*x + 105*d*x^2 + 63*e*x^3 + 45*f*x^4 + 35*g*x^5))
/(315*x^2) + (a*((-315*c*Sqrt[a + b*x^3])/(a*x) + ((140*a*g*Sqrt[a + b*x^3
])/(3*b) - 140*Sqrt[a]*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + (45*(7*b*c + 2
*a*f)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
 (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (6
*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(14*a^(2/3)*e - (5*(1 - Sqrt[3])*(7*b*c
 + 2*a*f))/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*a)))/105
```

3.450. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^2}\,dx$

3.450.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.450. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2} \, dx$

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.450. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

3.450.4 Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.30

method	result	size
elliptic	Expression too large to display	829
default	Expression too large to display	1248
risch	Expression too large to display	2011

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-c*(b*x^3+a)^(1/2)/x+2/9*g*x^3*(b*x^3+a)^(1/2)+2/7*f*x^2*(b*x^3+a)^(1/2)+2
/5*e*x*(b*x^3+a)^(1/2)+2/3*(1/3*a*g+b*d)/b*(b*x^3+a)^(1/2)-2/5*I*a*e*3^(1/
2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(3/
7*a*f+3/2*b*c)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a
*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/...
```

3.450.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$$
$$= \left[\frac{105\sqrt{a}bdx\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+756\,a\sqrt{b}ex\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-270\right.$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="fric
as")
```

output
```
[1/630*(105*sqrt(a)*b*d*x*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt
(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 756*a*sqrt(b)*e*x*weierstrassPInverse(
0, -4*a/b, x) - 270*(7*b*c + 2*a*f)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, w
eierstrassPInverse(0, -4*a/b, x)) + 2*(70*b*g*x^4 + 90*b*f*x^3 + 126*b*e*x
^2 - 315*b*c + 70*(3*b*d + a*g)*x)*sqrt(b*x^3 + a))/(b*x), 1/315*(105*sqrt
(-a)*b*d*x*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 378*a*sqrt(b
)*e*x*weierstrassPInverse(0, -4*a/b, x) - 135*(7*b*c + 2*a*f)*sqrt(b)*x*we
ierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (70*b*g*x^4
+ 90*b*f*x^3 + 126*b*e*x^2 - 315*b*c + 70*(3*b*d + a*g)*x)*sqrt(b*x^3 + a)
)/(b*x)]
```

3.450.6 Sympy [A] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}ex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}fx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{2ad}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2\sqrt{b}dx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + g\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**2,x)
```

output
```
sqrt(a)*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x*gamma(2/3)) - 2*sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqr
t(a)*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) + sqrt(a)*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**
3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a*d/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*
x**3) + 1)) + 2*sqrt(b)*d*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + g*Piecewise(
(sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

3.450.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^2}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2, x)
```

3.450.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^2}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^2,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^2, x)
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \int \frac{\sqrt{b\,x^3+a}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^2}\,dx$$

input
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x)
```

output
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2, x)
```

3.451 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

3.451.1 Optimal result . . . 3428
3.451.2 Mathematica [C] (verified) . . . 3429
3.451.3 Rubi [A] (verified) . . . 3430
3.451.4 Maple [A] (verified) . . . 3435
3.451.5 Fricas [C] (verification not implemented) . . . 3436
3.451.6 Sympy [A] (verification not implemented) . . . 3437
3.451.7 Maxima [F] . . . 3438
3.451.8 Giac [F] . . . 3438
3.451.9 Mupad [F(-1)] . . . 3438

3.451.1 Optimal result

Integrand size = 35, antiderivative size = 640

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$$
$$= \frac{3c\sqrt{a+bx^3}}{2x^2} - \frac{3d\sqrt{a+bx^3}}{x} + \frac{3(7bd+2ag)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$- \frac{2\sqrt{a+bx^3}(105cx-105dx^2-35ex^3-21fx^4-15gx^5)}{105x^3} - \frac{2}{3}\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$
$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bd+2ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(5bc+4af)-10\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd+2ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{Elli}}{70b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+3/2*c*(b*x^3+a)^(1/2)/x^2-
3*d*(b*x^3+a)^(1/2)/x-2/105*(-15*g*x^5-21*f*x^4-35*e*x^3-105*d*x^2+105*c*x
)*(b*x^3+a)^(1/2)/x^3+3/7*(2*a*g+7*b*d)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))-3/14*3^(1/4)*a^(1/3)*(2*a*g+7*b*d)*(a^(1/3)+b^(1/3)*
x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2
)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/
70*3^(3/4)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*(4*a*f+5*b*c)-10*
a^(1/3)*(2*a*g+7*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

3.451.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.78 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$$
$$= \frac{-3c\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)+x\left(-6d\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3}\right.\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

output
```
(-3*c*Sqrt[a + b*x^3]*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + x
*(-6*d*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)] +
x*(4*e*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x
^3]/Sqrt[a]]) + 6*f*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -(
(b*x^3)/a)] + 3*g*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 2/3, 5/3, -(
(b*x^3)/a)])))/(6*x^2*Sqrt[1 + (b*x^3)/a])
```

3.451.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {2365, 27, 2374, 2374, 27, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx$$

$$\Big\downarrow \text{2365}$$

$$\frac{3}{2}a\int -\frac{2\left(-15gx^4-21fx^3-35ex^2-105dx+105c\right)}{105x^3\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{35}a\int \frac{-15gx^4-21fx^3-35ex^2-105dx+105c}{x^3\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$$\Big\downarrow \text{2374}$$

$$-\frac{1}{35}a\left(-\frac{\int \frac{60agx^3+21(5bc+4af)x^2+140aex+420ad}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{105c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$$\Big\downarrow \text{2374}$$

$$-\frac{1}{35}a\left(-\frac{-\frac{\int -\frac{2\left(140ea^2+30(7bd+2ag)x^2a+21(5bc+4af)xa\right)}{x\sqrt{bx^3+a}}dx}{2a}-\frac{420d\sqrt{a+bx^3}}{x}}{4a}-\frac{105c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{35}a\left(-\frac{\frac{\int \frac{140ea^2+30(7bd+2ag)x^2a+21(5bc+4af)xa}{x\sqrt{bx^3+a}}dx}{a}-\frac{420d\sqrt{a+bx^3}}{x}}{4a}-\frac{105c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$$\Big\downarrow \; 2371$$

$$-\frac{1}{35}a\left(-\frac{\frac{140a^2e\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{21a(5bc+4af)+30a(7bd+2ag)x}{\sqrt{bx^3+a}}dx}{a}-\frac{420d\sqrt{a+bx^3}}{x}}{4a}-\frac{105c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$$\Big\downarrow \; 798$$

$$-\frac{1}{35}a\left(-\frac{\frac{\frac{140}{3}a^2e\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int \frac{21a(5bc+4af)+30a(7bd+2ag)x}{\sqrt{bx^3+a}}dx}{a}-\frac{420d\sqrt{a+bx^3}}{x}}{4a}-\frac{105c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$$\Big\downarrow \; 73$$

$$-\frac{1}{35}a\left(-\frac{\frac{\frac{280a^2e\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int \frac{21a(5bc+4af)+30a(7bd+2ag)x}{\sqrt{bx^3+a}}dx}{a}-\frac{420d\sqrt{a+bx^3}}{x}}{4a}-\frac{105c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{1}{35}a\left(-\frac{\frac{\int \frac{21a(5bc+4af)+30a(7bd+2ag)x}{\sqrt{bx^3+a}}dx-\frac{280}{3}a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a}-\frac{420d\sqrt{a+bx^3}}{x}}{4a}-\frac{105c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$$\Big\downarrow \; 2417$$

$$-\frac{1}{35}a\left(-\frac{\frac{3a\left(7(4af+5bc)-\frac{10\left(1-\sqrt{3}\right)\sqrt[3]{a}(2ag+7bd)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{30a(2ag+7bd)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{280}{3}a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a}-}{4a}\right.$$

$$\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$$\Big\downarrow \; 759$$

3.451. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx$

$$-\frac{1}{35}a\left(-\frac{\frac{\frac{30a(2ag+7bd)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{a}}{4a}\right.$$

$$\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

$\downarrow$ 2416

$$-\frac{1}{35}a\left(-\frac{\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(7(4af+5bc)-\frac{10\left(1-\sqrt{3}\right)\sqrt[3]{a}(}{\sqrt[3]{b}}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}\right.$$

$$\frac{2\sqrt{a+bx^3}\left(105cx-105dx^2-35ex^3-21fx^4-15gx^5\right)}{105x^3}$$

input
```
Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

3.451. $\int\frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx$

output
```
(-2*Sqrt[a + b*x^3]*(105*c*x - 105*d*x^2 - 35*e*x^3 - 21*f*x^4 - 15*g*x^5)
)/(105*x^3) - (a*((-105*c*Sqrt[a + b*x^3])/(2*a*x^2) - ((-420*d*Sqrt[a + b
*x^3])/x + ((-280*a^(3/2)*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 + (30*a*(7
*b*d + 2*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1
/3) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(7*(5*b*c + 4*a*f) - (10*(1 - Sqrt[3]
)*a^(1/3)*(7*b*d + 2*a*g))/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/a)/(4*a)))/35
```

3.451.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

3.451. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.451. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

3.451.4 Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.29

method	result	size
elliptic	Expression too large to display	826
default	Expression too large to display	1529
risch	Expression too large to display	2244

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*(b*x^3+a)^(1/2)/x^2-d*(b*x^3+a)^(1/2)/x+2/7*g*x^2*(b*x^3+a)^(1/2)+2
/5*f*x*(b*x^3+a)^(1/2)+2/3*e*(b*x^3+a)^(1/2)-2/3*I*(3/5*a*f+3/4*b*c)*3^(1/
2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(3/
7*a*g+3/2*b*d)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a
*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-2/...
```

3.451.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$$
$$= \left[\frac{35\sqrt{a}bex^2\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+63\,(5\,bc+4\,af)\sqrt{b}x^2\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\right.}{}\right.$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="fric
as")
```

output
```
[1/210*(35*sqrt(a)*b*e*x^2*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqr
t(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 63*(5*b*c + 4*a*f)*sqrt(b)*x^2*weiers
trassPInverse(0, -4*a/b, x) - 90*(7*b*d + 2*a*g)*sqrt(b)*x^2*weierstrassZe
ta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (60*b*g*x^4 + 84*b*f*x^
3 + 140*b*e*x^2 - 210*b*d*x - 105*b*c)*sqrt(b*x^3 + a))/(b*x^2), 1/210*(70
*sqrt(-a)*b*e*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 63*(5
*b*c + 4*a*f)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) - 90*(7*b*d +
2*a*g)*sqrt(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/
b, x)) + (60*b*g*x^4 + 84*b*f*x^3 + 140*b*e*x^2 - 210*b*d*x - 105*b*c)*sqr
t(b*x^3 + a))/(b*x^2)]
```

3.451.6 Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}d\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}e\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}gx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{2ae}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2\sqrt{b}ex^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**3,x)
```

output
```
sqrt(a)*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + sqrt(a)*d*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*e*asinh(sqrt(a)/(sq
rt(b)*x**(3/2)))/3 + sqrt(a)*f*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*g*x**2*gamma(2/3)*hyper((-
1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a*e/(3*sqr
t(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*sqrt(b)*e*x**(3/2)/(3*sqrt(a/(b*x*
*3) + 1))
```

3.451.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^3}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3, x)
```

3.451.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^3}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^3,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^3, x)
```

3.451.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \int \frac{\sqrt{b\,x^3+a}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^3}\,dx$$

input
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x)
```

output
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3, x)
```

3.452 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

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3.452.1 Optimal result

Integrand size = 35, antiderivative size = 637

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \frac{c\sqrt{a+bx^3}}{3x^3}+\frac{3d\sqrt{a+bx^3}}{2x^2}-\frac{3e\sqrt{a+bx^3}}{x}+\frac{3\sqrt[3]{b}e\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}-\frac{2\sqrt{a+bx^3}(5cx+15dx^2-15ex^3-5fx^4-3gx^5)}{15x^4}-\frac{(bc+2af)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(5bd-10\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arcsin}\right.}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/3*(2*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+1/3*c*(b*x^3+a)^
(1/2)/x^3+3/2*d*(b*x^3+a)^(1/2)/x^2-3*e*(b*x^3+a)^(1/2)/x-2/15*(-3*g*x^5-5
*f*x^4-15*e*x^3+15*d*x^2+5*c*x)*(b*x^3+a)^(1/2)/x^4+3*b^(1/3)*e*(b*x^3+a)^
(1/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3/2*3^(1/4)*a^(1/3)*b^(1/3)*e*(a^(1/
3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
+1/10*3^(3/4)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(5*b*d+4*a*g-10*a^(1/3)*b
^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a)^
(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)
```

3.452.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$$
$$= \frac{-2ac-2bcx^3+4afx^3+4bfx^6-4\sqrt{a}fx^3\sqrt{a+bx^3}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-2bcx^3\sqrt{1+\frac{bx^3}{a}}\text{arctanh}\left(\sqrt{1+\frac{bx^3}{a}}\right)\cdots}{\cdots}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

output
```
(-2*a*c - 2*b*c*x^3 + 4*a*f*x^3 + 4*b*f*x^6 - 4*Sqrt[a]*f*x^3*Sqrt[a + b*x
^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] - 2*b*c*x^3*Sqrt[1 + (b*x^3)/a]*ArcTa
nh[Sqrt[1 + (b*x^3)/a]] - 3*a*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2
/3, -1/2, 1/3, -((b*x^3)/a)] - 6*a*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometri
c2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)] + 6*a*g*x^4*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/(6*x^3*Sqrt[a + b*x^3])
```

3.452.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2365, 27, 2374, 27, 2374, 27, 2374, 27, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^4}dx$$

$$\Big\downarrow \text{2365}$$

$$\frac{3}{2}a\int -\frac{2\left(-3gx^4-5fx^3-15ex^2+15dx+5c\right)}{15x^4\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}a\int \frac{-3gx^4-5fx^3-15ex^2+15dx+5c}{x^4\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$$\Big\downarrow \text{2374}$$

$$-\frac{1}{5}a\left(-\frac{\int -\frac{3\left(-6agx^3-5(bc+2af)x^2-30aex+30ad\right)}{x^3\sqrt{bx^3+a}}dx}{6a}-\frac{5c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}a\left(\frac{\int \frac{-6agx^3-5(bc+2af)x^2-30aex+30ad}{x^3\sqrt{bx^3+a}}dx}{2a}-\frac{5c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$$\Big\downarrow \text{2374}$$

$$-\frac{1}{5}a\left(\frac{-\frac{\int \frac{2\left(60ea^2+3(5bd+4ag)x^2a+10(bc+2af)xa\right)}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{15d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{5c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}a\left(\frac{-\frac{\int \frac{60ea^2+3(5bd+4ag)x^2a+10(bc+2af)xa}{x^2\sqrt{bx^3+a}}dx}{2a}-\frac{15d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{5c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$\Big\downarrow$ 2374

$$-\frac{1}{5}a\left(\frac{-\frac{-\frac{\int -\frac{2\left(30bex^2a^2+10(bc+2af)a^2+3(5bd+4ag)xa^2\right)}{x\sqrt{bx^3+a}}dx}{2a}-\frac{60ae\sqrt{a+bx^3}}{x}}{2a}-\frac{15d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{5c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$\Big\downarrow$ 27

$$-\frac{1}{5}a\left(\frac{-\frac{\frac{\int \frac{30bex^2a^2+10(bc+2af)a^2+3(5bd+4ag)xa^2}{x\sqrt{bx^3+a}}dx}{a}-\frac{60ae\sqrt{a+bx^3}}{x}}{2a}-\frac{15d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{5c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$\Big\downarrow$ 2371

$$-\frac{1}{5}a\left(\frac{-\frac{\frac{10a^2(2af+bc)\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{3(5bd+4ag)a^2+30bexa^2}{\sqrt{bx^3+a}}dx}{a}-\frac{60ae\sqrt{a+bx^3}}{x}}{2a}-\frac{15d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{5c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$\Big\downarrow$ 798

$$-\frac{1}{5}a\left(\frac{-\frac{\frac{\frac{10}{3}a^2(2af+bc)\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int \frac{3(5bd+4ag)a^2+30bexa^2}{\sqrt{bx^3+a}}dx}{a}-\frac{60ae\sqrt{a+bx^3}}{x}}{2a}-\frac{15d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{5c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$\Big\downarrow$ 73

3.452. $\displaystyle\int \frac{\sqrt{a+bx^3}\,(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

$$-\frac{1}{5}a\left(-\frac{-\frac{\frac{\frac{20a^2(2af+bc)\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int \frac{3(5bd+4ag)a^2+30bexa^2}{\sqrt{bx^3+a}}dx}{a}-\frac{60ae\sqrt{a+bx^3}}{x}}{2a}-\frac{15d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{5c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$\big\downarrow$ 221

$$-\frac{1}{5}a\left(-\frac{-\frac{\frac{\int \frac{3(5bd+4ag)a^2+30bexa^2}{\sqrt{bx^3+a}}dx-\frac{20}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2af+bc)}{a}-\frac{60ae\sqrt{a+bx^3}}{x}}{2a}-\frac{15d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{5c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$\big\downarrow$ 2417

$$-\frac{1}{5}a\left(-\frac{-\frac{\frac{3a^2\left(-10\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag+5bd\right)\int \frac{1}{\sqrt{bx^3+a}}dx+30a^2b^{2/3}e\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{20}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2af+bc)}{a}-\frac{60ae\sqrt{a+b}}{x}}{2a}}{2a}\right.$$

$$-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

$\big\downarrow$ 759

$$-\frac{1}{5}a\left(-\frac{-\frac{\frac{30a^2b^{2/3}e\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{20}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2af+bc)+\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{Ellip}}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{\ }}{\left(\left(1\right.\right.}}}}{a}}{2a}}{2a}\right.$$

$$-\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

3.452. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^4}\,dx$

$\Big\downarrow$ 2416

$$-\frac{1}{5}a\left(-\frac{-\frac{20}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2af+bc)+\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}\right.$$

$$\frac{2\sqrt{a+bx^3}\left(5cx+15dx^2-15ex^3-5fx^4-3gx^5\right)}{15x^4}$$

input
```
Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

output
```
(-2*Sqrt[a + b*x^3]*(5*c*x + 15*d*x^2 - 15*e*x^3 - 5*f*x^4 - 3*g*x^5))/(15
*x^4) - (a*((-5*c*Sqrt[a + b*x^3])/(3*a*x^3) + ((-15*d*Sqrt[a + b*x^3])/x^
2 - ((-60*a*e*Sqrt[a + b*x^3])/x + ((-20*a^(3/2)*(b*c + 2*a*f)*ArcTanh[Sqr
t[a + b*x^3]/Sqrt[a]])/3 + 30*a^2*b^(2/3)*e*((2*Sqrt[a + b*x^3])/(b^(1/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(5*b*d - 10*(1
 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/a)/(2*a))/
(2*a)))/5
```

3.452. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^4}\,dx$

3.452.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.452. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^4}\,dx$

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.452.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 822, normalized size of antiderivative = 1.29

method	result	size
elliptic	Expression too large to display	822
default	Expression too large to display	1114
risch	Expression too large to display	1368

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c*(b*x^3+a)^(1/2)/x^3-1/2*d*(b*x^3+a)^(1/2)/x^2-e*(b*x^3+a)^(1/2)/x+2
/5*g*x*(b*x^3+a)^(1/2)+2/3*f*(b*x^3+a)^(1/2)-2/3*I*(3/5*a*g+3/4*b*d)*3^(1/
2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-I*e*3^(1/
2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*(a*f+1/2*b*c)*arctan...
```

3.452.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$$

$$= \left[-\frac{180\,ab^{\frac{3}{2}}ex^3\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-5\left(b^2c+2\,abf\right)\sqrt{a}x^3\log\left(-\frac{b^2x^6}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{90\,ab^{\frac{3}{2}}ex^3\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-5\left(b^2c+2\,abf\right)\sqrt{-a}x^3\arctan\left(\frac{2\,\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="fric
as")
```

output
```
[-1/60*(180*a*b^(3/2)*e*x^3*weierstrassZeta(0, -4*a/b, weierstrassPInverse
(0, -4*a/b, x)) - 5*(b^2*c + 2*a*b*f)*sqrt(a)*x^3*log(-(b^2*x^6 + 8*a*b*x^
3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 18*(5*a*b*d +
4*a^2*g)*sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) - 2*(12*a*b*g*x^4 +
 20*a*b*f*x^3 - 30*a*b*e*x^2 - 15*a*b*d*x - 10*a*b*c)*sqrt(b*x^3 + a))/(a*
b*x^3), -1/30*(90*a*b^(3/2)*e*x^3*weierstrassZeta(0, -4*a/b, weierstrassPI
nverse(0, -4*a/b, x)) - 5*(b^2*c + 2*a*b*f)*sqrt(-a)*x^3*arctan(2*sqrt(b*x
^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 9*(5*a*b*d + 4*a^2*g)*sqrt(b)*x^3*weiers
trassPInverse(0, -4*a/b, x) - (12*a*b*g*x^4 + 20*a*b*f*x^3 - 30*a*b*e*x^2
- 15*a*b*d*x - 10*a*b*c)*sqrt(b*x^3 + a))/(a*b*x^3)]
```

3.452.6 Sympy [A] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}gx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2af}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}c\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2\sqrt{b}fx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} - \frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**4,x)
```

output
```
sqrt(a)*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + sqrt(a)*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*sqrt(a)*f*asinh(sqrt(a)/(sq
rt(b)*x**(3/2)))/3 + sqrt(a)*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + 2*a*f/(3*sqrt(b)*x**(3/2)*sqrt(a/(
b*x**3) + 1)) - sqrt(b)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*sqrt(b)*f*
x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - b*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/
(3*sqrt(a))
```

3.452.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^4}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4, x)
```

3.452.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^4}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^4,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^4, x)
```

3.452.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx=\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^4}\,dx$$

input
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x)
```

output
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4, x)
```

3.453 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx$

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3.453.1 Optimal result

Integrand size = 35, antiderivative size = 694

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}$$

$$+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3(bc+8af)\sqrt{a+bx^3}}{8ax}+\frac{3\sqrt[3]{b}(bc+8af)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{2\sqrt{a+bx^3}(3cx+5dx^2+15ex^3-15fx^4-5gx^5)}{15x^5}-\frac{(bd+2ag)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(bc+8af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(4a^{2/3}\sqrt[3]{b}e-\left(1-\sqrt{3}\right)(bc+8af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arc}\right.}{8a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/3*(2*a*g+b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+3/20*c*(b*x^3+a)
^(1/2)/x^4+1/3*d*(b*x^3+a)^(1/2)/x^3+3/2*e*(b*x^3+a)^(1/2)/x^2-3/8*(8*a*f+
b*c)*(b*x^3+a)^(1/2)/a/x-2/15*(-5*g*x^5-15*f*x^4+15*e*x^3+5*d*x^2+3*c*x)*(
b*x^3+a)^(1/2)/x^5+3/8*b^(1/3)*(8*a*f+b*c)*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))-3/16*3^(1/4)*b^(1/3)*(8*a*f+b*c)*(a^(1/3)+b^(1/3)*x)*El
lipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/8*3^(
3/4)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(4*a^(2/3)*b^(1/3)*e-(8*a*
f+b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1
/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.453.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.45 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx =$$

$$-\frac{4adx+4bdx^4-8agx^4-8bgx^7+8\sqrt{a}gx^4\sqrt{a+bx^3}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+4bdx^4\sqrt{1+\frac{bx^3}{a}}\text{arctanh}\left(\sqrt{1+\right.}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

output
```
-1/12*(4*a*d*x + 4*b*d*x^4 - 8*a*g*x^4 - 8*b*g*x^7 + 8*Sqrt[a]*g*x^4*Sqrt[
a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + 4*b*d*x^4*Sqrt[1 + (b*x^3)/a
]*ArcTanh[Sqrt[1 + (b*x^3)/a]] + 3*a*c*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 6*a*e*x^2*Sqrt[1 + (b*x^3)/a]*Hyperge
ometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + 12*a*f*x^3*Sqrt[1 + (b*x^3)/a]
*Hypergeometric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)])/(x^4*Sqrt[a + b*x^3])
```

3.453.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {2365, 27, 2374, 27, 2374, 27, 2374, 27, 2374, 25, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}dx$$

$$\Big\downarrow \text{2365}$$

$$\frac{3}{2}a\int -\frac{2\left(-5gx^4-15fx^3+15ex^2+5dx+3c\right)}{15x^5\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}a\int \frac{-5gx^4-15fx^3+15ex^2+5dx+3c}{x^5\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$$\Big\downarrow \text{2374}$$

$$-\frac{1}{5}a\left(-\frac{\int -\frac{5\left(-8agx^3-3(bc+8af)x^2+24aex+8ad\right)}{x^4\sqrt{bx^3+a}}dx}{8a}-\frac{3c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}a\left(\frac{5\int \frac{-8agx^3-3(bc+8af)x^2+24aex+8ad}{x^4\sqrt{bx^3+a}}dx}{8a}-\frac{3c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$$\Big\downarrow \text{2374}$$

$$-\frac{1}{5}a\left(\frac{5\left(-\frac{\int -\frac{6\left(24ea^2-4(bd+2ag)x^2a-3(bc+8af)xa\right)}{x^3\sqrt{bx^3+a}}dx}{6a}-\frac{8d\sqrt{a+bx^3}}{3x^3}\right)}{8a}-\frac{3c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{1}{5}a\left(\frac{5\left(\frac{\int \frac{24ea^2-4(bd+2ag)x^2a-3(bc+8af)xa}{x^3\sqrt{bx^3+a}}dx}{a}-\frac{8d\sqrt{a+bx^3}}{3x^3}\right)}{8a}-\frac{3c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$$\Big\downarrow \textbf{2374}$$

$$-\frac{1}{5}a\left(\frac{5\left(\frac{-\frac{\int \frac{4\left(6bex^2a^2+3(bc+8af)a^2+4(bd+2ag)xa^2\right)}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{12ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{8d\sqrt{a+bx^3}}{3x^3}\right)}{8a}-\frac{3c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{1}{5}a\left(\frac{5\left(\frac{-\frac{\int \frac{6bex^2a^2+3(bc+8af)a^2+4(bd+2ag)xa^2}{x^2\sqrt{bx^3+a}}dx}{a}-\frac{12ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{8d\sqrt{a+bx^3}}{3x^3}\right)}{8a}-\frac{3c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$$\Big\downarrow \textbf{2374}$$

$$-\frac{1}{5}a\left(\frac{5\left(\frac{-\frac{-\frac{\int -\frac{8(bd+2ag)a^3+12bexa^3+3b(bc+8af)x^2a^2}{x\sqrt{bx^3+a}}dx}{2a}-\frac{3a\sqrt{a+bx^3}(8af+bc)}{x}}{a}-\frac{12ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{8d\sqrt{a+bx^3}}{3x^3}\right)}{8a}-\frac{3c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

3.453. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx$

$$\Big\downarrow \; 25$$

$$-\frac{1}{5}a\left(\frac{5\left(\frac{-\frac{\frac{\int \frac{8(bd+2ag)a^3+12bexa^3+3b(bc+8af)x^2a^2}{x\sqrt{bx^3+a}}dx}{2a}-\frac{3a\sqrt{a+bx^3}(8af+bc)}{x}}{a}-\frac{12ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{8d\sqrt{a+bx^3}}{3x^3}\right)}{8a}-\frac{3c\sqrt{a+bx^3}}{4ax^4}\right)-$$

$$\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$$\Big\downarrow \; 2371$$

$$-\frac{1}{5}a\left(\frac{5\left(\frac{-\frac{\frac{8a^3(2ag+bd)\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{12bea^3+3b(bc+8af)xa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{3a\sqrt{a+bx^3}(8af+bc)}{x}}{a}-\frac{12ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{8d\sqrt{a+bx^3}}{3x^3}\right)}{8a}-\frac{3c\sqrt{a+bx^3}}{4ax^4}\right)-$$

$$\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$$\Big\downarrow \; 798$$

$$-\frac{1}{5}a\left(\frac{5\left(\frac{-\frac{\frac{\frac{8}{3}a^3(2ag+bd)\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int \frac{12bea^3+3b(bc+8af)xa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{3a\sqrt{a+bx^3}(8af+bc)}{x}}{a}-\frac{12ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{8d\sqrt{a+bx^3}}{3x^3}\right)}{8a}-\frac{3c\sqrt{a+bx^3}}{4ax^4}\right)$$

$$\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$$\Big\downarrow \; 73$$

3.453. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx$

$$-\frac{1}{5}a\left(\frac{5\left(-\frac{\frac{\frac{16a^3(2ag+bd)\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int\frac{12bea^3+3b(bc+8af)xa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{3a\sqrt{a+bx^3}(8af+bc)}{x}}{a}-\frac{12ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{8d\sqrt{a+bx^3}}{3x^3}\right)}{8a}-\frac{3c\sqrt{a+bx}}{4ax^4}\right.$$

$$\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$\downarrow$ 221

$$-\frac{1}{5}a\left(\frac{5\left(-\frac{\frac{\frac{\int\frac{12bea^3+3b(bc+8af)xa^2}{\sqrt{bx^3+a}}dx-\frac{16}{3}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2ag+bd)}{2a}-\frac{3a\sqrt{a+bx^3}(8af+bc)}{x}}{a}-\frac{12ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{8d\sqrt{a+bx^3}}{3x^3}\right)}{8a}-\frac{3c\sqrt{a}}{4a}\right.$$

$$\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$\downarrow$ 2417

$$-\frac{1}{5}a\left(\frac{5\left(-\frac{\frac{\frac{3a^{7/3}b^{2/3}\left(4a^{2/3}\sqrt[3]{b}e-\left(1-\sqrt{3}\right)(8af+bc)\right)\int\frac{1}{\sqrt{bx^3+a}}dx+3a^2b^{2/3}(8af+bc)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{16}{3}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2ag+}{2a}}{a}}{a}\right.}{8a}\right.$$

$$\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$\downarrow$ 759

3.453. $\int\frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx$

$$-\frac{1}{5}a\left(\frac{5\left(-\frac{\frac{\frac{3a^2b^{2/3}(8af+bc)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(\cdots}{\sqrt[3]{b}x+(\cdots}\right.\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}\cdots}}{2a}}{a}}{a}\right.}{\cdots}\right.$$

$$\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

$\downarrow$ 2416

3.453. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx$

$$-\frac{1}{5}a\left(5\left(-\frac{\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(4a^{2/3}\sqrt[3]{b}e-\left(1-\sqrt{\right.\right.}}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}\right.\right.$$

$$\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}$$

input
```
Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

3.453. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx$

output
```
(-2*Sqrt[a + b*x^3]*(3*c*x + 5*d*x^2 + 15*e*x^3 - 15*f*x^4 - 5*g*x^5))/(15
*x^5) - (a*((-3*c*Sqrt[a + b*x^3])/(4*a*x^4) + (5*((-8*d*Sqrt[a + b*x^3])/
(3*x^3) + ((-12*a*e*Sqrt[a + b*x^3])/x^2 - ((-3*a*(b*c + 8*a*f)*Sqrt[a + b
*x^3])/x + ((-16*a^(5/2)*(b*d + 2*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
 + 3*a^2*b^(2/3)*(b*c + 8*a*f)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3])) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*b^(1/3)*(4*a^(2/3)*b^(1
/3)*e - (1 - Sqrt[3])*(b*c + 8*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*a))/a)/a))/(8*a))
)/5
```

3.453.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.453. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5} \, dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

3.453. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5} dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.453.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 845, normalized size of antiderivative = 1.22

method	result	size
elliptic	Expression too large to display	845
risch	Expression too large to display	1243
default	Expression too large to display	1286

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*c*(b*x^3+a)^(1/2)/x^4-1/3*d*(b*x^3+a)^(1/2)/x^3-1/2*e*(b*x^3+a)^(1/2)
/x^2-1/8*(8*a*f+3*b*c)/a*(b*x^3+a)^(1/2)/x+2/3*g*(b*x^3+a)^(1/2)-1/2*I*e*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^
3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(
b*f+1/16*b*(8*a*f+3*b*c)/a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-
1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)...
```

3.453.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx$$
$$= \left[\frac{36\,a\sqrt{b}ex^4\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+2\,(bd+2\,ag)\sqrt{a}x^4\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)}{}\right.$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="fric
as")
```

output
```
[1/24*(36*a*sqrt(b)*e*x^4*weierstrassPInverse(0, -4*a/b, x) + 2*(b*d + 2*a
*g)*sqrt(a)*x^4*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a
)*sqrt(a) + 8*a^2)/x^6) - 9*(b*c + 8*a*f)*sqrt(b)*x^4*weierstrassZeta(0, -
4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (16*a*g*x^4 - 12*a*e*x^2 - 3*(
3*b*c + 8*a*f)*x^3 - 8*a*d*x - 6*a*c)*sqrt(b*x^3 + a))/(a*x^4), 1/24*(36*a
*sqrt(b)*e*x^4*weierstrassPInverse(0, -4*a/b, x) + 4*(b*d + 2*a*g)*sqrt(-a
)*x^4*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 9*(b*c + 8*a*f)*s
qrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) +
 (16*a*g*x^4 - 12*a*e*x^2 - 3*(3*b*c + 8*a*f)*x^3 - 8*a*d*x - 6*a*c)*sqrt(
b*x^3 + a))/(a*x^4)]
```

3.453.6 Sympy [A] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\ -\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2\sqrt{a}g\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{2ag}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2\sqrt{b}gx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} - \frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**5,x)
```

output
```
sqrt(a)*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**4*gamma(-1/3)) + sqrt(a)*e*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*f*gamma(-1/3)*hy
per((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*s
qrt(a)*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2*a*g/(3*sqrt(b)*x**(3/2)*s
qrt(a/(b*x**3) + 1)) - sqrt(b)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*sqr
t(b)*g*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - b*d*asinh(sqrt(a)/(sqrt(b)*x**(
3/2)))/(3*sqrt(a))
```

3.453.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^5}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5, x)
```

3.453.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^5}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^5,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^5, x)
```

3.453.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \int \frac{\sqrt{b\,x^3+a}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^5}\,dx$$

input
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x)
```

output
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5, x)
```

3.454 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$

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3.454.1 Optimal result

Integrand size = 35, antiderivative size = 652

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\sqrt{a+bx^3}$$

$$-\frac{3bc\sqrt{a+bx^3}}{20ax^2}-\frac{3bd\sqrt{a+bx^3}}{8ax}+\frac{3\sqrt[3]{b}(bd+8ag)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{be\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(bd+8ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(2\sqrt[3]{b}(bc-10af)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd+8ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{El}}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/3*b*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/60*(12*c/x^5+15*d/x^4+
20*e/x^3+30*f/x^2+60*g/x)*(b*x^3+a)^(1/2)-3/20*b*c*(b*x^3+a)^(1/2)/a/x^2-3
/8*b*d*(b*x^3+a)^(1/2)/a/x+3/8*b^(1/3)*(8*a*g+b*d)*(b*x^3+a)^(1/2)/a/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))-3/16*3^(1/4)*b^(1/3)*(8*a*g+b*d)*(a^(1/3)+b^(1/
3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
-1/40*3^(3/4)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(2*b^(1/3)*(-10*a
*f+b*c)+5*a^(1/3)*(8*a*g+b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/
2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)
```

3.454.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx =$$

$$-\frac{\sqrt{a+bx^3}\left(12ac\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)+5x\left(3ad\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{2},-\frac{1}{3}\right.\right.\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

output
```
-1/60*(Sqrt[a + b*x^3]*(12*a*c*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^
3)/a)] + 5*x*(3*a*d*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 2*
x*(2*a*e*Sqrt[1 + (b*x^3)/a] + 2*b*e*x^3*ArcTanh[Sqrt[1 + (b*x^3)/a]] + 3*
a*f*x*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + 6*a*g*x^2*Hyperge
ometric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)]))))/(a*x^5*Sqrt[1 + (b*x^3)/a])
```

3.454.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2364, 27, 2374, 27, 2374, 25, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}dx$$

$$\Big\downarrow \text{2364}$$

$$-\frac{3}{2}b\int -\frac{60gx^4+30fx^3+20ex^2+15dx+12c}{60x^3\sqrt{bx^3+a}}dx-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{40}b\int \frac{60gx^4+30fx^3+20ex^2+15dx+12c}{x^3\sqrt{bx^3+a}}dx-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{2374}$$

$$\frac{1}{40}b\left(-\frac{\int -\frac{4\left(60agx^3-3(bc-10af)x^2+20aex+15ad\right)}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{6c\sqrt{a+bx^3}}{ax^2}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{40}b\left(\frac{\int \frac{60agx^3-3(bc-10af)x^2+20aex+15ad}{x^2\sqrt{bx^3+a}}dx}{a}-\frac{6c\sqrt{a+bx^3}}{ax^2}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{2374}$$

$$\frac{1}{40}b\left(\frac{-\frac{\int -\frac{40ea^2+15(bd+8ag)x^2a-6(bc-10af)xa}{x\sqrt{bx^3+a}}dx}{2a}-\frac{15d\sqrt{a+bx^3}}{x}}{a}-\frac{6c\sqrt{a+bx^3}}{ax^2}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{40}b\left(\frac{\frac{\int\frac{40ea^2+15(bd+8ag)x^2a-6(bc-10af)xa}{x\sqrt{bx^3+a}}dx}{2a}-\frac{15d\sqrt{a+bx^3}}{x}}{a}-\frac{6c\sqrt{a+bx^3}}{ax^2}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{2371}$$

$$\frac{1}{40}b\left(\frac{\frac{40a^2e\int\frac{1}{x\sqrt{bx^3+a}}dx+\int\frac{15a(bd+8ag)x-6a(bc-10af)}{\sqrt{bx^3+a}}dx}{2a}-\frac{15d\sqrt{a+bx^3}}{x}}{a}-\frac{6c\sqrt{a+bx^3}}{ax^2}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{40}b\left(\frac{\frac{\frac{40}{3}a^2e\int\frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int\frac{15a(bd+8ag)x-6a(bc-10af)}{\sqrt{bx^3+a}}dx}{2a}-\frac{15d\sqrt{a+bx^3}}{x}}{a}-\frac{6c\sqrt{a+bx^3}}{ax^2}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{40}b\left(\frac{\frac{\frac{80a^2e\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int\frac{15a(bd+8ag)x-6a(bc-10af)}{\sqrt{bx^3+a}}dx}{2a}-\frac{15d\sqrt{a+bx^3}}{x}}{a}-\frac{6c\sqrt{a+bx^3}}{ax^2}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{40}b\left(\frac{\frac{\int\frac{15a(bd+8ag)x-6a(bc-10af)}{\sqrt{bx^3+a}}dx-\frac{80}{3}a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{2a}-\frac{15d\sqrt{a+bx^3}}{x}}{a}-\frac{6c\sqrt{a+bx^3}}{ax^2}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{2417}$$

3.454. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}\,dx$

$$\frac{1}{40}b\left(\frac{\frac{-3a\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(8ag+bd)}{\sqrt[3]{b}}-20af+2bc\right)\int\frac{1}{\sqrt{bx^3+a}}dx+\frac{15a(8ag+bd)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{80}{3}a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{2a}}{a}-\frac{15d\sqrt{a+\cdots}}{x}\right.$$

$$\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \; 759$$

$$\frac{1}{40}b\left(\frac{\frac{\frac{15a(8ag+bd)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\cdots\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2a}}{a}\right.$$

$$\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \; 2416$$

3.454. $\displaystyle\int\frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}\,dx$

$$\frac{1}{40}b\left(-\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(8ag+bd)}{\sqrt[3]{b}}-20af+2\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.$$

$$\frac{1}{60}\sqrt{a+bx^3}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

input
```
Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

output
```
-1/60*(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*Sqrt
[a + b*x^3]) + (b*((-6*c*Sqrt[a + b*x^3])/(a*x^2) + ((-15*d*Sqrt[a + b*x^3
])/x + ((-80*a^(3/2)*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 + (15*a*(b*d +
8*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
- (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) - (
2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(2*b*c - 20*a*f + (5*(1 - Sqrt[3])*a^(1/3)*(
b*d + 8*a*g))/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(2*a))/a))/40
```

3.454. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$

3.454.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

3.454. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.454.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	874
risch	Expression too large to display	1523
default	Expression too large to display	1571

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

3.454. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$

output
```
-1/5*c*(b*x^3+a)^(1/2)/x^5-1/4*d*(b*x^3+a)^(1/2)/x^4-1/3*e*(b*x^3+a)^(1/2)
/x^3-1/20*(10*a*f+3*b*c)/a*(b*x^3+a)^(1/2)/x^2-1/8*(8*a*g+3*b*d)/a*(b*x^3+
a)^(1/2)/x-2/3*I*(b*f-1/40*b*(10*a*f+3*b*c)/a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(g*b+1/16*b/a*(8*a*g+3*b*
d))*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3...
```

3.454.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$$
$$= \left[\frac{10\sqrt{a}bex^5\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)-18\,(bc-10\,af)\sqrt{b}x^5\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\right.}{}\right.$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="fric
as")
```

output
```
[1/120*(10*sqrt(a)*b*e*x^5*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqr
t(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 18*(b*c - 10*a*f)*sqrt(b)*x^5*weierst
rassPInverse(0, -4*a/b, x) - 45*(b*d + 8*a*g)*sqrt(b)*x^5*weierstrassZeta(
0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (15*(3*b*d + 8*a*g)*x^4 +
40*a*e*x^2 + 6*(3*b*c + 10*a*f)*x^3 + 30*a*d*x + 24*a*c)*sqrt(b*x^3 + a))/
(a*x^5), 1/120*(20*sqrt(-a)*b*e*x^5*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x
^3 + 2*a)) - 18*(b*c - 10*a*f)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b,
x) - 45*(b*d + 8*a*g)*sqrt(b)*x^5*weierstrassZeta(0, -4*a/b, weierstrassPI
nverse(0, -4*a/b, x)) - (15*(3*b*d + 8*a*g)*x^4 + 40*a*e*x^2 + 6*(3*b*c +
10*a*f)*x^3 + 30*a*d*x + 24*a*c)*sqrt(b*x^3 + a))/(a*x^5)]
```

3.454.6 Sympy [A] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}d\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}g\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**6,x)
```

output
```
sqrt(a)*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + sqrt(a)*d*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*f*gamma(-2/3)*
hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3))
+ sqrt(a)*g*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x*gamma(2/3)) - sqrt(b)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b*e*a
sinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

3.454.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^6}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6, x)
```

3.454.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^6}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^6,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^6, x)
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^6}\,dx$$

input
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x)
```

output
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6, x)
```

3.455 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$

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3.455.1 Optimal result

Integrand size = 35, antiderivative size = 659

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$$
$$= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)\sqrt{a+bx^3}-\frac{bc\sqrt{a+bx^3}}{12ax^3}-\frac{3bd\sqrt{a+bx^3}}{20ax^2}$$
$$-\frac{3be\sqrt{a+bx^3}}{8ax}+\frac{3b^{4/3}e\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{b(bc-4af)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(2bd+5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-20ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\mathrm{ar}\right.}{40a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/12*b*(-4*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/60*(10*c/x^
6+12*d/x^5+15*e/x^4+20*f/x^3+30*g/x^2)*(b*x^3+a)^(1/2)-1/12*b*c*(b*x^3+a)^
(1/2)/a/x^3-3/20*b*d*(b*x^3+a)^(1/2)/a/x^2-3/8*b*e*(b*x^3+a)^(1/2)/a/x+3/8
*b^(4/3)*e*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-3/16*3^(1/4)*
b^(4/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)-1/40*3^(3/4)*b^(2/3)*(a^(1/3)+b^(1/3)*x)*Elliptic
F((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(2*b*d-20*a*g+5*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(
1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.455.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.43 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx =$$

$$-\frac{\sqrt{a+bx^3}\left(36a^3d\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)+5x\left(9a^3e\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{2},-\right.\right.\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

output
```
-1/180*(Sqrt[a + b*x^3]*(36*a^3*d*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b
*x^3)/a)] + 5*x*(9*a^3*e*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)]
 + 2*x*(6*a^2*f*(a*Sqrt[1 + (b*x^3)/a] + b*x^3*ArcTanh[Sqrt[1 + (b*x^3)/a]
]) + 9*a^3*g*x*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)] + 4*b^2*c*
x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*
x^3)/a]))))/(a^3*x^5*Sqrt[1 + (b*x^3)/a])
```

3.455.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2364, 27, 2374, 27, 2374, 27, 2374, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx$$

$$\Big\downarrow \ 2364$$

$$-\frac{3}{2}b\int -\frac{30gx^4+20fx^3+15ex^2+12dx+10c}{60x^4\sqrt{bx^3+a}}dx-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{40}b\int \frac{30gx^4+20fx^3+15ex^2+12dx+10c}{x^4\sqrt{bx^3+a}}dx-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \ 2374$$

$$\frac{1}{40}b\left(-\frac{\int -\frac{6\left(30agx^3-5(bc-4af)x^2+15aex+12ad\right)}{x^3\sqrt{bx^3+a}}dx}{6a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{40}b\left(\frac{\int \frac{30agx^3-5(bc-4af)x^2+15aex+12ad}{x^3\sqrt{bx^3+a}}dx}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \ 2374$$

$$\frac{1}{40}b\left(\frac{-\frac{\int -\frac{4\left(15ea^2-3(bd-10ag)x^2a-5(bc-4af)xa\right)}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{6d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{40}b\left(\frac{\frac{\int \frac{15ea^2-3(bd-10ag)x^2a-5(bc-4af)xa}{x^2\sqrt{bx^3+a}}dx}{a}-\frac{6d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \text{2374}$$

$$\frac{1}{40}b\left(\frac{\frac{-\frac{\int \frac{-15bex^2a^2+10(bc-4af)a^2+6(bd-10ag)xa^2}{x\sqrt{bx^3+a}}dx}{2a}-\frac{15ae\sqrt{a+bx^3}}{x}}{a}-\frac{6d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \text{2371}$$

$$\frac{1}{40}b\left(\frac{\frac{-\frac{10a^2(bc-4af)\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{6a^2(bd-10ag)-15a^2bex}{\sqrt{bx^3+a}}dx}{2a}-\frac{15ae\sqrt{a+bx^3}}{x}}{a}-\frac{6d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{40}b\left(\frac{\frac{-\frac{\frac{10}{3}a^2(bc-4af)\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int \frac{6a^2(bd-10ag)-15a^2bex}{\sqrt{bx^3+a}}dx}{2a}-\frac{15ae\sqrt{a+bx^3}}{x}}{a}-\frac{6d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{40}b\left(\frac{\frac{-\frac{\frac{20a^2(bc-4af)\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int \frac{6a^2(bd-10ag)-15a^2bex}{\sqrt{bx^3+a}}dx}{2a}-\frac{15ae\sqrt{a+bx^3}}{x}}{a}-\frac{6d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \text{221}$$

3.455. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}dx$

$$\frac{1}{40}b\left(\frac{\frac{-\frac{\int \frac{6a^2(bd-10ag)-15a^2bex}{\sqrt{bx^3+a}}dx-\frac{20}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bc-4af)}{2a}-\frac{15ae\sqrt{a+bx^3}}{x}}{a}-\frac{6d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$\downarrow$ 2417

$$\frac{1}{40}b\left(\frac{\frac{-\frac{3a^2\left(5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-20ag+2bd\right)\int\frac{1}{\sqrt{bx^3+a}}dx-15a^2b^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{20}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bc-4af)}{2a}-\frac{15ae\sqrt{a+bx^3}}{x}\cdots}{a}\cdots}{a}\cdots\right.$$

$$\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$\downarrow$ 759

$$\frac{1}{40}b\left(\frac{\frac{-\frac{-15a^2b^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{20}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bc-4af)+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{Ellip}\cdots}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{\cdots}}{\left(\left(1+\cdots\right.\right.}}\cdots}}{2a}\cdots}{a}\cdots}{a}\cdots\right.$$

$$\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$\downarrow$ 2416

3.455. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx$

$$\frac{1}{40}b\left(-\frac{-\frac{20}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bc-4af)+\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}\right.$$

$$\frac{1}{60}\sqrt{a+bx^3}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

input
```
Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

output
```
-1/60*(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2)*Sq
rt[a + b*x^3]) + (b*((-10*c*Sqrt[a + b*x^3])/(3*a*x^3) + ((-6*d*Sqrt[a + b
*x^3])/x^2 + ((-15*a*e*Sqrt[a + b*x^3])/x - ((-20*a^(3/2)*(b*c - 4*a*f)*Ar
cTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 - 15*a^2*b^(2/3)*e*((2*Sqrt[a + b*x^3])/
(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]
*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(2*b*
d + 5*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 20*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(
2*a))/a)/a))/40
```

3.455. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$

3.455.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.455. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.455.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	883
risch	Expression too large to display	1102
default	Expression too large to display	1180

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*c*(b*x^3+a)^(1/2)/x^6-1/5*d*(b*x^3+a)^(1/2)/x^5-1/4*e*(b*x^3+a)^(1/2)
/x^4-1/12*(4*a*f+b*c)/a*(b*x^3+a)^(1/2)/x^3-1/20/a*(10*a*g+3*b*d)*(b*x^3+a
)^(1/2)/x^2-3/8*b*e*(b*x^3+a)^(1/2)/a/x-2/3*I*(g*b-1/40*b/a*(10*a*g+3*b*d)
)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-1/
8*I/a*b*e*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)...
```

3.455.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$$

$$= \left[-\frac{90\,ab^{\frac{3}{2}}ex^6\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+5\left(b^2c-4\,abf\right)\sqrt{a}x^6\log\left(\frac{b^2x^6+8a}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{45\,ab^{\frac{3}{2}}ex^6\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+5\left(b^2c-4\,abf\right)\sqrt{-a}x^6\arctan\left(\frac{(b}{\cdots}\right.}{\cdots}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="fric
as")
```

output
```
[-1/240*(90*a*b^(3/2)*e*x^6*weierstrassZeta(0, -4*a/b, weierstrassPInverse
(0, -4*a/b, x)) + 5*(b^2*c - 4*a*b*f)*sqrt(a)*x^6*log((b^2*x^6 + 8*a*b*x^3
 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 36*(a*b*d - 10*
a^2*g)*sqrt(b)*x^6*weierstrassPInverse(0, -4*a/b, x) + 2*(45*a*b*e*x^5 + 3
0*a^2*e*x^2 + 6*(3*a*b*d + 10*a^2*g)*x^4 + 24*a^2*d*x + 10*(a*b*c + 4*a^2*
f)*x^3 + 20*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^6), -1/120*(45*a*b^(3/2)*e*x^6*
weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 5*(b^2*c -
 4*a*b*f)*sqrt(-a)*x^6*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(
a*b*x^3 + a^2)) + 18*(a*b*d - 10*a^2*g)*sqrt(b)*x^6*weierstrassPInverse(0,
 -4*a/b, x) + (45*a*b*e*x^5 + 30*a^2*e*x^2 + 6*(3*a*b*d + 10*a^2*g)*x^4 +
24*a^2*d*x + 10*(a*b*c + 4*a^2*f)*x^3 + 20*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^
6)]
```

3.455.6 Sympy [A] (verification not implemented)

Time = 4.76 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\ -\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\ -\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}g\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} - \frac{ac}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}c}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}c}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}} + \frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**7,x)
```

output
```
sqrt(a)*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + sqrt(a)*e*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*g*gamma(-2/3)*
hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3))
- a*c/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)*c/(4*x**(9/2)*s
qrt(a/(b*x**3) + 1)) - sqrt(b)*f*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3
/2)*c/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b*f*asinh(sqrt(a)/(sqrt(b)*x*
*(3/2)))/(3*sqrt(a)) + b**2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/
2))
```

3.455.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^7}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="maxi
ma")
```

output
```
-1/24*(b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^
(3/2) + 2*((b*x^3 + a)^(3/2)*b^2 + sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a
 - 2*(b*x^3 + a)*a^2 + a^3))*c + integrate(sqrt(b*x^3 + a)*(g*x^3 + f*x^2
+ e*x + d)/x^6, x)
```

3.455.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^7}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^7,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^7, x)
```

3.455.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx=\int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^7}\,dx$$

input
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x)
```

output
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7, x)
```

3.456 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

3.456.1 Optimal result 3490
3.456.2 Mathematica [C] (verified) 3491
3.456.3 Rubi [A] (verified) 3492
3.456.4 Maple [A] (verified) 3498
3.456.5 Fricas [C] (verification not implemented) 3499
3.456.6 Sympy [A] (verification not implemented) 3500
3.456.7 Maxima [F] 3501
3.456.8 Giac [F] 3501
3.456.9 Mupad [F(-1)] 3502

3.456.1 Optimal result

Integrand size = 35, antiderivative size = 711

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)\sqrt{a+bx^3} - \frac{3bc\sqrt{a+bx^3}}{56ax^4} - \frac{bd\sqrt{a+bx^3}}{12ax^3} - \frac{3be\sqrt{a+bx^3}}{20ax^2} + \frac{3b(5bc-14af)\sqrt{a+bx^3}}{112a^2x} - \frac{3b^{4/3}(5bc-14af)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{b(bd-4ag)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}} + \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}(5bc-14af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\cdots\right)}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{4/3}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(5bc-14af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\cdots}{560a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/12*b*(-4*a*g+b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/420*(60*c/x
^7+70*d/x^6+84*e/x^5+105*f/x^4+140*g/x^3)*(b*x^3+a)^(1/2)-3/56*b*c*(b*x^3+
a)^(1/2)/a/x^4-1/12*b*d*(b*x^3+a)^(1/2)/a/x^3-3/20*b*e*(b*x^3+a)^(1/2)/a/x
^2+3/112*b*(-14*a*f+5*b*c)*(b*x^3+a)^(1/2)/a^2/x-3/112*b^(4/3)*(-14*a*f+5*
b*c)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+3/224*3^(1/4)*b^(
4/3)*(-14*a*f+5*b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/560*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)
*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*(28*a^(2/3)*b^(1/3)*e-5*(-14*a*f+5*b*c)*(1-3^(1/2)))*(1
/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.456.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.49 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx =$$

$$-\frac{\sqrt{a+bx^3}\left(180a^3c\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{1}{2},-\frac{4}{3},-\frac{bx^3}{a}\right)+7x^2\left(36a^3e\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{2}\right.\right.\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

output
```
-1/1260*(Sqrt[a + b*x^3]*(180*a^3*c*Hypergeometric2F1[-7/3, -1/2, -4/3, -(
(b*x^3)/a)] + 7*x^2*(36*a^3*e*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3
)/a)] + 5*x*(12*a^2*g*x*(a*Sqrt[1 + (b*x^3)/a] + b*x^3*ArcTanh[Sqrt[1 + (b
*x^3)/a]]) + 9*a^3*f*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 8
*b^2*d*x^4*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, 3, 5/2,
1 + (b*x^3)/a])))/(a^3*x^7*Sqrt[1 + (b*x^3)/a])
```

3.456.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.01, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {2364, 27, 2374, 27, 2374, 27, 2374, 27, 2374, 25, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^8}dx$$

$$\Big\downarrow \text{2364}$$

$$-\frac{3}{2}b\int -\frac{140gx^4+105fx^3+84ex^2+70dx+60c}{420x^5\sqrt{bx^3+a}}dx-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}b\int \frac{140gx^4+105fx^3+84ex^2+70dx+60c}{x^5\sqrt{bx^3+a}}dx-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{2374}$$

$$\frac{1}{280}b\left(-\frac{\int -\frac{4(280agx^3-15(5bc-14af)x^2+168aex+140ad)}{x^4\sqrt{bx^3+a}}dx}{8a}-\frac{15c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}b\left(\frac{\int \frac{280agx^3-15(5bc-14af)x^2+168aex+140ad}{x^4\sqrt{bx^3+a}}dx}{2a}-\frac{15c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{2374}$$

$$\frac{1}{280}b\left(\frac{-\frac{\int -\frac{6\left(168ea^2-70(bd-4ag)x^2a-15(5bc-14af)xa\right)}{x^3\sqrt{bx^3+a}}dx}{6a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{15c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}b\left(\frac{\frac{\int \frac{168ea^2-70(bd-4ag)x^2a-15(5bc-14af)xa}{x^3\sqrt{bx^3+a}}dx}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{15c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{2374}$$

$$\frac{1}{280}b\left(\frac{\frac{-\frac{\int \frac{4\left(42bex^2a^2+15(5bc-14af)a^2+70(bd-4ag)xa^2\right)}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{84ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{15c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}b\left(\frac{\frac{-\frac{\int \frac{42bex^2a^2+15(5bc-14af)a^2+70(bd-4ag)xa^2}{x^2\sqrt{bx^3+a}}dx}{a}-\frac{84ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{15c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{2374}$$

$$\frac{1}{280}b\left(\frac{\frac{-\frac{-\frac{\int -\frac{140(bd-4ag)a^3+84bexa^3+15b(5bc-14af)x^2a^2}{x\sqrt{bx^3+a}}dx}{2a}-\frac{15a\sqrt{a+bx^3}(5bc-14af)}{x}}{a}-\frac{84ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{15c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{280}b\left(\frac{\frac{-\frac{\frac{\int \frac{140(bd-4ag)a^3+84bexa^3+15b(5bc-14af)x^2a^2}{x\sqrt{bx^3+a}}dx}{2a}-\frac{15a\sqrt{a+bx^3}(5bc-14af)}{x}}{a}-\frac{84ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{15c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

3.456. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^8}\,dx$

$$\Big\downarrow \text{2371}$$

$$\frac{1}{280}b\left(\frac{-\frac{-\frac{\frac{140a^3(bd-4ag)\int\frac{1}{x\sqrt{bx^3+a}}dx+\int\frac{84bea^3+15b(5bc-14af)xa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{15a\sqrt{a+bx^3}(5bc-14af)}{x}}{a}-\frac{84ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{15c\sqrt{a+bx}}{ax^4}\right.$$

$$\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{280}b\left(\frac{-\frac{-\frac{\frac{\frac{140}{3}a^3(bd-4ag)\int\frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int\frac{84bea^3+15b(5bc-14af)xa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{15a\sqrt{a+bx^3}(5bc-14af)}{x}}{a}-\frac{84ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{15c\sqrt{a+}}{ax^4}\right.$$

$$\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{280}b\left(\frac{-\frac{-\frac{\frac{\frac{280a^3(bd-4ag)\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int\frac{84bea^3+15b(5bc-14af)xa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{15a\sqrt{a+bx^3}(5bc-14af)}{x}}{a}-\frac{84ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{15c\sqrt{a+}}{ax^4}\right.$$

$$\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{280}b\left(\frac{-\frac{-\frac{\frac{\int\frac{84bea^3+15b(5bc-14af)xa^2}{\sqrt{bx^3+a}}dx-\frac{280}{3}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bd-4ag)}{2a}-\frac{15a\sqrt{a+bx^3}(5bc-14af)}{x}}{a}-\frac{84ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-15c\right.$$

$$\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{2417}$$

3.456. $\int\frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

$$\frac{1}{280}b\left(\frac{\frac{\frac{-\frac{3a^{7/3}b^{2/3}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(5bc-14af)\right)\int\frac{1}{\sqrt{bx^3+a}}dx+15a^2b^{2/3}(5bc-14af)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{280}{3}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(b}{2a}}{a}}{a}}{2a}\right.$$

$$\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$\downarrow$ 759

$$\frac{1}{280}b\left(\frac{\frac{-\frac{15a^2b^{2/3}(5bc-14af)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(1}{\sqrt[3]{b}x+(1}\right.\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2a}}{a}}{a}\right.$$

$$\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$\downarrow$ 2416

3.456. $\int\frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^8}\,dx$

$$\frac{1}{280}b\left(-\frac{\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right.\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\ldots}\right.$$

$$\frac{1}{420}\sqrt{a+bx^3}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

input
```
Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

output
```
-1/420*(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x^3)
*Sqrt[a + b*x^3]) + (b*((-15*c*Sqrt[a + b*x^3])/(a*x^4) + ((-140*d*Sqrt[a
+ b*x^3])/(3*x^3) + ((-84*a*e*Sqrt[a + b*x^3])/x^2 - ((-15*a*(5*b*c - 14*a
*f)*Sqrt[a + b*x^3])/x + ((-280*a^(5/2)*(b*d - 4*a*g)*ArcTanh[Sqrt[a + b*x
^3]/Sqrt[a]])/3 + 15*a^2*b^(2/3)*(5*b*c - 14*a*f)*((2*Sqrt[a + b*x^3])/(b^
(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^
(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*b^(1
/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(5*b*c - 14*a*f))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3]))/(2*a))/a)/a)/(2*a)))/280
```

3.456. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^8}\,dx$

3.456.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

3.456. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.456.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.27

method	result	size
elliptic	Expression too large to display	901
risch	Expression too large to display	1283
default	Expression too large to display	1376

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*c*(b*x^3+a)^(1/2)/x^7-1/6*d*(b*x^3+a)^(1/2)/x^6-1/5*e*(b*x^3+a)^(1/2)
/x^5-1/56*(14*a*f+3*b*c)/a*(b*x^3+a)^(1/2)/x^4-1/12/a*(4*a*g+b*d)*(b*x^3+a
)^(1/2)/x^3-3/20*b*e*(b*x^3+a)^(1/2)/a/x^2-3/112*(14*a*f-5*b*c)*b/a^2*(b*x
^3+a)^(1/2)/x+1/20*I/a*b*e*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))-1/112*I*(14*a*f-5*b*c)*b/a^2*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^...
```

3.456.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$$

$$= \left[-\frac{252\,ab^{\frac{3}{2}}ex^7\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+35\,(b^2d-4\,abg)\sqrt{a}x^7\log\left(\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8}{x^6}\right.}{} \right.$$

$$-\frac{252\,ab^{\frac{3}{2}}ex^7\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+70\,(b^2d-4\,abg)\sqrt{-a}x^7\arctan\left(\frac{(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{-a}}{2\,(abx^3+a^2)}\right)-45\,(}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="fric
as")
```

output
```
[-1/1680*(252*a*b^(3/2)*e*x^7*weierstrassPInverse(0, -4*a/b, x) + 35*(b^2*
d - 4*a*b*g)*sqrt(a)*x^7*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b
*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 45*(5*b^2*c - 14*a*b*f)*sqrt(b)*x^7*weie
rstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (252*a*b*e*x^5
 - 45*(5*b^2*c - 14*a*b*f)*x^6 + 336*a^2*e*x^2 + 140*(a*b*d + 4*a^2*g)*x^4
 + 280*a^2*d*x + 30*(3*a*b*c + 14*a^2*f)*x^3 + 240*a^2*c)*sqrt(b*x^3 + a))
/(a^2*x^7), -1/1680*(252*a*b^(3/2)*e*x^7*weierstrassPInverse(0, -4*a/b, x)
 + 70*(b^2*d - 4*a*b*g)*sqrt(-a)*x^7*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 +
 a)*sqrt(-a)/(a*b*x^3 + a^2)) - 45*(5*b^2*c - 14*a*b*f)*sqrt(b)*x^7*weiers
trassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (252*a*b*e*x^5 -
 45*(5*b^2*c - 14*a*b*f)*x^6 + 336*a^2*e*x^2 + 140*(a*b*d + 4*a^2*g)*x^4 +
 280*a^2*d*x + 30*(3*a*b*c + 14*a^2*f)*x^3 + 240*a^2*c)*sqrt(b*x^3 + a))/(
a^2*x^7)]
```

3.456.6 Sympy [A] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} - \frac{ad}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}d}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}g\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}d}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{bg\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}} + \frac{b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**8,x)
```

output
```
sqrt(a)*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**7*gamma(-4/3)) + sqrt(a)*e*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*f*gamma(-4/3)*
hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)
) - a*d/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)*d/(4*x**(9/2)
*sqrt(a/(b*x**3) + 1)) - sqrt(b)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**
(3/2)*d/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b*g*asinh(sqrt(a)/(sqrt(b)*
x**(3/2)))/(3*sqrt(a)) + b**2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(
3/2))
```

3.456.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^8}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8, x)
```

3.456.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^8}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^8,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^8, x)
```

3.456.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \int \frac{\sqrt{b\,x^3+a}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^8}\,dx$$

input
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x)
```

output
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8, x)
```

3.457 $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$

3.457.1 Optimal result . . . 3503
3.457.2 Mathematica [C] (verified) . . . 3504
3.457.3 Rubi [A] (verified) . . . 3505
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3.457.5 Fricas [C] (verification not implemented) . . . 3513
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3.457.9 Mupad [F(-1)] . . . 3515

3.457.1 Optimal result

Integrand size = 35, antiderivative size = 743

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$$
$$= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\sqrt{a+bx^3}$$
$$-\frac{3bc\sqrt{a+bx^3}}{80ax^5}-\frac{3bd\sqrt{a+bx^3}}{56ax^4}-\frac{be\sqrt{a+bx^3}}{12ax^3}+\frac{3b(7bc-16af)\sqrt{a+bx^3}}{320a^2x^2}$$
$$+\frac{3b(5bd-14ag)\sqrt{a+bx^3}}{112a^2x}-\frac{3b^{4/3}(5bd-14ag)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{b^2e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$
$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(5bd-14ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(7\sqrt[3]{b}(7bc-16af)+20\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-14ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right.}}}{2240a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/12*b^2*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/840*(105*c/x^8+120*d
/x^7+140*e/x^6+168*f/x^5+210*g/x^4)*(b*x^3+a)^(1/2)-3/80*b*c*(b*x^3+a)^(1/
2)/a/x^5-3/56*b*d*(b*x^3+a)^(1/2)/a/x^4-1/12*b*e*(b*x^3+a)^(1/2)/a/x^3+3/3
20*b*(-16*a*f+7*b*c)*(b*x^3+a)^(1/2)/a^2/x^2+3/112*b*(-14*a*g+5*b*d)*(b*x^
3+a)^(1/2)/a^2/x-3/112*b^(4/3)*(-14*a*g+5*b*d)*(b*x^3+a)^(1/2)/a^2/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))+3/224*3^(1/4)*b^(4/3)*(-14*a*g+5*b*d)*(a^(1/3)+b^
(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a
)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)+1/2240*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b^(1/3)*(
-16*a*f+7*b*c)+20*a^(1/3)*(-14*a*g+5*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)/a^2/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.457.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx =$$

$$-\frac{\sqrt{a+bx^3}\left(315a^3c\,\text{Hypergeometric2F1}\left(-\frac{8}{3},-\frac{1}{2},-\frac{5}{3},-\frac{bx^3}{a}\right)+360a^3dx\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{1}{2},-\right.\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

output
```
-1/2520*(Sqrt[a + b*x^3]*(315*a^3*c*Hypergeometric2F1[-8/3, -1/2, -5/3, -(
(b*x^3)/a)] + 360*a^3*d*x*Hypergeometric2F1[-7/3, -1/2, -4/3, -((b*x^3)/a)
] + 14*x^3*(36*a^3*f*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)] + 4
5*a^3*g*x*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)] + 40*b^2*e*x^5
*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^3
)/a])))/(a^3*x^8*Sqrt[1 + (b*x^3)/a])
```

3.457.3 Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.01, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {2364, 27, 2374, 27, 2374, 27, 2374, 27, 2374, 25, 2374, 27, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$$

$$\Big\downarrow \text{2364}$$

$$-\frac{3}{2}b\int -\frac{210gx^4+168fx^3+140ex^2+120dx+105c}{840x^6\sqrt{bx^3+a}}dx-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{560}b\int \frac{210gx^4+168fx^3+140ex^2+120dx+105c}{x^6\sqrt{bx^3+a}}dx-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$$\Big\downarrow \text{2374}$$

$$\frac{1}{560}b\left(-\frac{\int -\frac{5(420agx^3-21(7bc-16af)x^2+280aex+240ad)}{x^5\sqrt{bx^3+a}}dx}{10a}-\frac{21c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{560}b\left(\frac{\int \frac{420agx^3-21(7bc-16af)x^2+280aex+240ad}{x^5\sqrt{bx^3+a}}dx}{2a}-\frac{21c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$$\Big\downarrow \text{2374}$$

$$\frac{1}{560}b\left(\frac{-\frac{\int -\frac{8\left(280ea^2-30(5bd-14ag)x^2a-21(7bc-16af)xa\right)}{x^4\sqrt{bx^3+a}}dx}{8a}-\frac{60d\sqrt{a+bx^3}}{x^4}}{2a}-\frac{21c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 27

$$\frac{1}{560}b\left(\frac{\frac{\int \frac{280ea^2-30(5bd-14ag)x^2a-21(7bc-16af)xa}{x^4\sqrt{bx^3+a}}dx}{a}-\frac{60d\sqrt{a+bx^3}}{x^4}}{2a}-\frac{21c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 2374

$$\frac{1}{560}b\left(\frac{\frac{-\frac{\int \frac{6\left(140bex^2a^2+21(7bc-16af)a^2+30(5bd-14ag)xa^2\right)}{x^3\sqrt{bx^3+a}}dx}{6a}-\frac{280ae\sqrt{a+bx^3}}{3x^3}}{a}-\frac{60d\sqrt{a+bx^3}}{x^4}}{2a}-\frac{21c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 27

$$\frac{1}{560}b\left(\frac{\frac{-\frac{\int \frac{140bex^2a^2+21(7bc-16af)a^2+30(5bd-14ag)xa^2}{x^3\sqrt{bx^3+a}}dx}{a}-\frac{280ae\sqrt{a+bx^3}}{3x^3}}{a}-\frac{60d\sqrt{a+bx^3}}{x^4}}{2a}-\frac{21c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 2374

$$\frac{1}{560}b\left(\frac{\frac{-\frac{-\frac{\int -\frac{120(5bd-14ag)a^3+560bexa^3-21b(7bc-16af)x^2a^2}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{21a\sqrt{a+bx^3}(7bc-16af)}{2x^2}}{a}-\frac{280ae\sqrt{a+bx^3}}{3x^3}}{a}-\frac{60d\sqrt{a+bx^3}}{x^4}}{2a}-\frac{21c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 25

3.457. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx$

$$\frac{1}{560}b\left(\frac{\frac{-\frac{\frac{\int \frac{120(5bd-14ag)a^3+560bexa^3-21b(7bc-16af)x^2a^2}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{21a\sqrt{a+bx^3}(7bc-16af)}{2x^2}}{a}-\frac{280ae\sqrt{a+bx^3}}{3x^3}}{a}-\frac{60d\sqrt{a+bx^3}}{x^4}}{2a}-\frac{21c\sqrt{a+bx^3}}{ax^5}\right)-$$

$$\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\Big\downarrow$ 2374

$$\frac{1}{560}b\left(\frac{\frac{-\frac{-\frac{-\frac{\int -\frac{2\left(560bea^4+60b(5bd-14ag)x^2a^3-21b(7bc-16af)xa^3\right)}{x\sqrt{bx^3+a}}dx}{2a}-\frac{120a^2\sqrt{a+bx^3}(5bd-14ag)}{x}}{4a}-\frac{21a\sqrt{a+bx^3}(7bc-16af)}{2x^2}}{a}-\frac{280ae\sqrt{a+bx^3}}{3x^3}}{a}-\frac{60d\sqrt{a-\cdots}}{x^4}}{2a}\right.$$

$$\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{560}b\left(\frac{\frac{-\frac{-\frac{\frac{\int \frac{560bea^4+60b(5bd-14ag)x^2a^3-21b(7bc-16af)xa^3}{x\sqrt{bx^3+a}}dx}{a}-\frac{120a^2\sqrt{a+bx^3}(5bd-14ag)}{x}}{4a}-\frac{21a\sqrt{a+bx^3}(7bc-16af)}{2x^2}}{a}-\frac{280ae\sqrt{a+bx^3}}{3x^3}}{a}-\frac{60d\sqrt{a+bx^3}}{x^4}}{2a}\right.$$

$$\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\Big\downarrow$ 2371

$$\frac{1}{560}b\left(\frac{\frac{-\frac{-\frac{\frac{560a^4be\int\frac{1}{x\sqrt{bx^3+a}}dx+\int\frac{60a^3b(5bd-14ag)x-21a^3b(7bc-16af)}{\sqrt{bx^3+a}}dx}{a}-\frac{120a^2\sqrt{a+bx^3}(5bd-14ag)}{x}}{4a}-\frac{21a\sqrt{a+bx^3}(7bc-16af)}{2x^2}}{a}-\frac{280ae\sqrt{a+bx^3}}{3x^3}}{a}-60\cdots}{2a}\right.$$

$$\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\Big\downarrow$ 798

3.457. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx$

$$\frac{1}{560}b\left(-\frac{-\frac{-\frac{\frac{\frac{560}{3}a^4be\int\frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int\frac{60a^3b(5bd-14ag)x-21a^3b(7bc-16af)}{\sqrt{bx^3+a}}dx}{a}-\frac{120a^2\sqrt{a+bx^3}(5bd-14ag)}{x}}{4a}-\frac{21a\sqrt{a+bx^3}(7bc-16af)}{2x^2}}{a}-\frac{280ae\sqrt{a+bx^3}}{3x^3}}{a}-}{2a}\right.$$

$$\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{560}b\left(-\frac{-\frac{-\frac{\frac{\frac{1120}{3}a^4e\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}+\int\frac{60a^3b(5bd-14ag)x-21a^3b(7bc-16af)}{\sqrt{bx^3+a}}dx}{a}-\frac{120a^2\sqrt{a+bx^3}(5bd-14ag)}{x}}{4a}-\frac{21a\sqrt{a+bx^3}(7bc-16af)}{2x^2}}{a}-\frac{280ae\sqrt{a+bx^3}}{3x^3}}{a}-}{2a}\right.$$

$$\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{560}b\left(-\frac{-\frac{-\frac{\frac{\int\frac{60a^3b(5bd-14ag)x-21a^3b(7bc-16af)}{\sqrt{bx^3+a}}dx-\frac{1120}{3}a^{7/2}be\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a}-\frac{120a^2\sqrt{a+bx^3}(5bd-14ag)}{x}}{4a}-\frac{21a\sqrt{a+bx^3}(7bc-16af)}{2x^2}}{a}-\frac{280ae\sqrt{a-}}{3x^3}}{a}}{2a}\right.$$

$$\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$$\Big\downarrow \; 2417$$

$$\frac{1}{560}b\left(-\frac{-\frac{-\frac{\frac{-3a^3b^{2/3}\left(7\sqrt[3]{b}(7bc-16af)+20\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-14ag)\right)\int\frac{1}{\sqrt{bx^3+a}}dx+60a^3b^{2/3}(5bd-14ag)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{1120}{3}a^{7/2}be\,\text{arctanh}}{a}}{4a}}{a}}{a}}{2a}\right.$$

$$\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$$\Big\downarrow \; 759$$

3.457. $\int\frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx$

$$\frac{1}{560}b\left(60a^3b^{2/3}(5bd-14ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{1120}{3}a^{7/2}be\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\ldots}\right.}}\ldots}{\ldots}\ldots\right)$$

$$\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 2416

$$\frac{1}{560}b\left(-\frac{1120}{3}a^{7/2}be\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),\ldots\right)}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\ldots\right)$$

$$\frac{1}{840}\sqrt{a+bx^3}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

input
```
Int[(Sqrt[a + b*x^3]*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

3.457. $\int \frac{\sqrt{a+bx^3}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx$

output
```
-1/840*(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210*g)/x
^4)*Sqrt[a + b*x^3]) + (b*((-21*c*Sqrt[a + b*x^3])/(a*x^5) + ((-60*d*Sqrt[
a + b*x^3])/x^4 + ((-280*a*e*Sqrt[a + b*x^3])/(3*x^3) - ((-21*a*(7*b*c - 1
6*a*f)*Sqrt[a + b*x^3])/(2*x^2) + ((-120*a^2*(5*b*d - 14*a*g)*Sqrt[a + b*x
^3])/x + ((-1120*a^(7/2)*b*e*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 + 60*a^3*
b^(2/3)*(5*b*d - 14*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3])) - (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*b^(1/3)*(7*b^(1/3)*(7*b*c - 16*a*
f) + 20*(1 - Sqrt[3])*a^(1/3)*(5*b*d - 14*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt
[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/a)/(4*a))
/a)/a)/(2*a)))/560
```

3.457.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.457. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.457. $\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.457.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 931, normalized size of antiderivative = 1.25

method	result	size
elliptic	Expression too large to display	931
risch	Expression too large to display	1579
default	Expression too large to display	1679

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*c*(b*x^3+a)^(1/2)/x^8-1/7*d*(b*x^3+a)^(1/2)/x^7-1/6*e*(b*x^3+a)^(1/2)
/x^6-1/80*(16*a*f+3*b*c)/a*(b*x^3+a)^(1/2)/x^5-1/56/a*(14*a*g+3*b*d)*(b*x^
3+a)^(1/2)/x^4-1/12*b*e*(b*x^3+a)^(1/2)/a/x^3-3/320*b*(16*a*f-7*b*c)/a^2*(
b*x^3+a)^(1/2)/x^2-3/112*(14*a*g-5*b*d)*b/a^2*(b*x^3+a)^(1/2)/x+1/320*I*(1
6*a*f-7*b*c)*b/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2
)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))-1/112*I*(14*a*g-5*b*d)*b/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*...
```

3.457.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$$

$$= \left[\frac{140\sqrt{a}b^2ex^8\log\left(\frac{b^2x^6+8\,abx^3+4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+63\,(7\,b^2c-16\,abf)\sqrt{b}x^8\text{weierstrassPInverse}(0,-\right.$$

$$-\frac{280\sqrt{-a}b^2ex^8\arctan\left(\frac{(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{-a}}{2\,(abx^3+a^2)}\right)-63\,(7\,b^2c-16\,abf)\sqrt{b}x^8\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="fric
as")
```

output
```
[1/6720*(140*sqrt(a)*b^2*e*x^8*log((b^2*x^6 + 8*a*b*x^3 + 4*(b*x^3 + 2*a)*
sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 63*(7*b^2*c - 16*a*b*f)*sqrt(b)*x^
8*weierstrassPInverse(0, -4*a/b, x) + 180*(5*b^2*d - 14*a*b*g)*sqrt(b)*x^8
*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (560*a*b*
e*x^5 - 180*(5*b^2*d - 14*a*b*g)*x^7 - 63*(7*b^2*c - 16*a*b*f)*x^6 + 1120*
a^2*e*x^2 + 120*(3*a*b*d + 14*a^2*g)*x^4 + 960*a^2*d*x + 84*(3*a*b*c + 16*
a^2*f)*x^3 + 840*a^2*c)*sqrt(b*x^3 + a))/(a^2*x^8), -1/6720*(280*sqrt(-a)*
b^2*e*x^8*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2
)) - 63*(7*b^2*c - 16*a*b*f)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b, x)
 - 180*(5*b^2*d - 14*a*b*g)*sqrt(b)*x^8*weierstrassZeta(0, -4*a/b, weierst
rassPInverse(0, -4*a/b, x)) + (560*a*b*e*x^5 - 180*(5*b^2*d - 14*a*b*g)*x^
7 - 63*(7*b^2*c - 16*a*b*f)*x^6 + 1120*a^2*e*x^2 + 120*(3*a*b*d + 14*a^2*g
)*x^4 + 960*a^2*d*x + 84*(3*a*b*c + 16*a^2*f)*x^3 + 840*a^2*c)*sqrt(b*x^3
+ a))/(a^2*x^8)]
```

3.457.6 Sympy [A] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{\sqrt{a}d\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}g\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} - \frac{ae}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}e}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}e}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b^2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)*(b*x**3+a)**(1/2)/x**9,x)
```

output
```
sqrt(a)*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**8*gamma(-5/3)) + sqrt(a)*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*f*gamma(-5/3)*
hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)
) + sqrt(a)*g*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x**4*gamma(-1/3)) - a*e/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1
)) - sqrt(b)*e/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*e/(12*a*x**(3/
2)*sqrt(a/(b*x**3) + 1)) + b**2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a*
*(3/2))
```

3.457.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^9}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9, x)
```

3.457.8 Giac [F]

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)\sqrt{bx^3+a}}{x^9}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)*(b*x^3+a)^(1/2)/x^9,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*sqrt(b*x^3 + a)/x^9, x)
```

3.457.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx=\int \frac{\sqrt{b\,x^3+a}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^9}\,dx$$

input
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x)
```

output
```
int(((a + b*x^3)^(1/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9, x)
```

3.458 $\int x^3(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)\,dx$

3.458.1 Optimal result 3516
3.458.2 Mathematica [C] (verified) 3517
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3.458.1 Optimal result

Integrand size = 35, antiderivative size = 791

$$\int x^3(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)\,dx = -\frac{4a^3e\sqrt{a+bx^3}}{105b^2}$$
$$+\frac{54a^2(23bc-8af)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2(5bd-2ag)x^2\sqrt{a+bx^3}}{8645b^2}+\frac{2a^2ex^3\sqrt{a+bx^3}}{105b}$$
$$+\frac{54a^2fx^4\sqrt{a+bx^3}}{4301b}+\frac{54a^2gx^5\sqrt{a+bx^3}}{6175b}-\frac{216a^3(5bd-2ag)\sqrt{a+bx^3}}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225}$$
$$+\frac{2ax^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5311735ex^3+4279275fx^4+3522519gx^5)}{185910725}$$
$$+\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}(5bd-2ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(1729\sqrt[3]{b}(23bc-8af)-8602\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-2ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}}}{37182145b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3900225*x^3*(b*x^3+a)^(3/2)*(156009*g*x^5+169575*f*x^4+185725*e*x^3+2052
75*d*x^2+229425*c*x)-4/105*a^3*e*(b*x^3+a)^(1/2)/b^2+54/21505*a^2*(-8*a*f+
23*b*c)*x*(b*x^3+a)^(1/2)/b^2+54/8645*a^2*(-2*a*g+5*b*d)*x^2*(b*x^3+a)^(1/
2)/b^2+2/105*a^2*e*x^3*(b*x^3+a)^(1/2)/b+54/4301*a^2*f*x^4*(b*x^3+a)^(1/2)
/b+54/6175*a^2*g*x^5*(b*x^3+a)^(1/2)/b+2/185910725*a*x^3*(3522519*g*x^5+42
79275*f*x^4+5311735*e*x^3+6774075*d*x^2+8947575*c*x)*(b*x^3+a)^(1/2)-216/8
645*a^3*(-2*a*g+5*b*d)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))+108/8645*3^(1/4)*a^(10/3)*(-2*a*g+5*b*d)*(a^(1/3)+b^(1/3)*x)*Elliptic
E((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-36/37182145*3
^(3/4)*a^3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1729*b^(1/3)*(-8*a*f+23*b*c
)-8602*a^(1/3)*(-2*a*g+5*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/
2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)
```

3.458.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.56 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.23

$$\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{2\sqrt{a+bx^3}\left(-(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}(10a(7429e+21x(380f+391gx))-bx(229425c+17x(1207\right.}{}$$

input
```
Integrate[x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
(2*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(10*a*(7429*e + 21
*x*(380*f + 391*g*x)) - b*x*(229425*c + 17*x*(12075*d + 19*x*(575*e + 525*
f*x + 483*g*x^2))))) + 9975*a^2*(-23*b*c + 8*a*f)*x*Hypergeometric2F1[-3/2
, 1/3, 4/3, -((b*x^3)/a)] + 41055*a^2*(-5*b*d + 2*a*g)*x^2*Hypergeometric2
F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(3900225*b^2*Sqrt[1 + (b*x^3)/a])
```

3.458.3 Rubi [A] (verified)

Time = 2.33 (sec) , antiderivative size = 789, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2365, 27, 2365, 27, 2375, 27, 2375, 27, 2427, 27, 2028, 2427, 27, 2028, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^3)^{3/2} (c+dx+ex^2+fx^3+gx^4)\, dx$$

$$\Big\downarrow \text{ 2365}$$

$$\frac{9}{2}a\int \frac{2x^3\sqrt{bx^3+a}(156009gx^4+169575fx^3+185725ex^2+205275dx+229425c)}{3900225}dx+\frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3a\int x^3\sqrt{bx^3+a}(156009gx^4+169575fx^3+185725ex^2+205275dx+229425c)\,dx}{1300075}+\frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225}$$

$$\Big\downarrow \text{ 2365}$$

$$\frac{3a\left(\frac{3}{2}a\int \frac{2x^3(3522519gx^4+4279275fx^3+5311735ex^2+6774075dx+8947575c)}{429\sqrt{bx^3+a}}dx+\frac{2}{429}x^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+5\right.}{1300075}$$
$$\frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3a\left(\frac{1}{143}a\int \frac{x^3(3522519gx^4+4279275fx^3+5311735ex^2+6774075dx+8947575c)}{\sqrt{bx^3+a}}dx+\frac{2}{429}x^3\sqrt{a+bx^3}(8947575cx+6774075dx^2+\right.}{1300075}$$
$$\frac{2x^3(a+bx^3)^{3/2}(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5)}{3900225}$$

$$\Big\downarrow \text{ 2375}$$

$$\frac{3a\left(\frac{1}{143}a\left(\frac{2\int \frac{65x^3\left(855855bfx^3+1062347bex^2+270963(5bd-2ag)x+1789515bc\right)}{2\sqrt{bx^3+a}}dx}{13b}+\frac{541926gx^5\sqrt{a+bx^3}}{b}\right)+\frac{2}{429}x^3\sqrt{a+bx^3}\left(8947575cx+\right.}{1300075}$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\downarrow$ 27

$$\frac{3a\left(\frac{1}{143}a\left(\frac{5\int \frac{x^3\left(855855bfx^3+1062347bex^2+270963(5bd-2ag)x+1789515bc\right)}{\sqrt{bx^3+a}}dx}{b}+\frac{541926gx^5\sqrt{a+bx^3}}{b}\right)+\frac{2}{429}x^3\sqrt{a+bx^3}\left(8947575cx+\right.}{1300075}$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\downarrow$ 2375

$$\frac{3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{2\int \frac{11x^3\left(1062347b^2ex^2+270963b(5bd-2ag)x+77805b(23bc-8af)\right)}{2\sqrt{bx^3+a}}dx}{11b}+155610fx^4\sqrt{a+bx^3}\right)}{b}+\frac{541926gx^5\sqrt{a+bx^3}}{b}\right)+\frac{2}{429}x^3\sqrt{a\right.}{1300075}$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\downarrow$ 27

$$\frac{3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{\int \frac{x^3\left(1062347b^2ex^2+270963b(5bd-2ag)x+77805b(23bc-8af)\right)}{\sqrt{bx^3+a}}dx}{b}+155610fx^4\sqrt{a+bx^3}\right)}{b}+\frac{541926gx^5\sqrt{a+bx^3}}{b}\right)+\frac{2}{429}x^3\sqrt{a+\right.}{1300075}$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\downarrow$ 2427

3.458. $\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$\frac{3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{\frac{2\int -\frac{3\left(-812889b^2(5bd-2ag)x^4-233415b^2(23bc-8af)x^3+2124694ab^2ex^2\right)}{2\sqrt{bx^3+a}}dx}{9b}+\frac{2124694}{9}bex^3\sqrt{a+bx^3}}{b}+155610fx^4\sqrt{a+bx^3}\right)}{b}+\frac{541926g\ldots}{}\right.\right.}{1300075}$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\big\downarrow$ 27

$$\frac{3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{\frac{2124694}{9}bex^3\sqrt{a+bx^3}-\frac{\int \frac{-812889b^2(5bd-2ag)x^4-233415b^2(23bc-8af)x^3+2124694ab^2ex^2}{\sqrt{bx^3+a}}dx}{3b}}{b}+155610fx^4\sqrt{a+bx^3}\right)}{b}+\frac{541926gx^5\sqrt{a\ldots}}{b}\right.\right.}{1300075}$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\big\downarrow$ 2028

$$\frac{3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{\frac{2124694}{9}bex^3\sqrt{a+bx^3}-\frac{\int \frac{x^2\left(-812889(5bd-2ag)x^2b^2+2124694aeb^2-233415(23bc-8af)xb^2\right)}{\sqrt{bx^3+a}}dx}{3b}}{b}+155610fx^4\sqrt{a+bx^3}\right)}{b}+\frac{541926gx^5\sqrt{\ldots}}{b}\right.\right.}{1300075}$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\big\downarrow$ 2427

3.458. $\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{\frac{2124694}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{2\int\frac{7\left(-233415(23bc-8af)x^3b^3+2124694aex^2b^3+464508a(5bd-2ag)xb^2\right)}{2\sqrt{bx^3+a}}dx}{7b}-232254bx^2\sqrt{a+bx^3}(5bd-2ag)}{3b}}{b}+155610\right.}{b}\right.\right.$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\downarrow$ 27

$$3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{\frac{2124694}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\int\frac{-233415(23bc-8af)x^3b^3+2124694aex^2b^3+464508a(5bd-2ag)xb^2}{\sqrt{bx^3+a}}dx}{b}-232254bx^2\sqrt{a+bx^3}(5bd-2ag)}{3b}}{b}+155610fx^4\right.}{b}\right.\right.$$

1

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\downarrow$ 2028

$$3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{\frac{2124694}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\int\frac{x\left(-233415(23bc-8af)x^2b^3+2124694aexb^3+464508a(5bd-2ag)b^2\right)}{\sqrt{bx^3+a}}dx}{b}-232254bx^2\sqrt{a+bx^3}(5bd-2ag)}{3b}}{b}+155610fx^4\right.}{b}\right.\right.$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\downarrow$ 2427

3.458. $\int x^3\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{\frac{2124694}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\frac{\frac{2\int\frac{5\left(1062347aex^2b^4+46683a(23bc-8af)b^3+232254a(5bd-2ag)xb^3\right)}{\sqrt{bx^3+a}}dx}{5b}-93366b^2x\sqrt{a+bx^3}(23bc-8af)}{b}-232254bx^2\sqrt{\cdots}}{3b}}{b}\cdots}{b}\cdots\right.\right.\right.$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\downarrow$ 27

$$3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{\frac{2124694}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\frac{\frac{2\int\frac{1062347aex^2b^4+46683a(23bc-8af)b^3+232254a(5bd-2ag)xb^3}{\sqrt{bx^3+a}}dx}{b}-93366b^2x\sqrt{a+bx^3}(23bc-8af)}{b}-232254bx^2\sqrt{a+b\cdots}}{3b}}{b}\cdots}{b}\cdots\right.\right.\right.$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\downarrow$ 2425

$$3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{\frac{2124694}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\frac{2\left(1062347ab^4e\int\frac{x^2}{\sqrt{bx^3+a}}dx+\int\frac{46683a(23bc-8af)b^3+232254a(5bd-2ag)xb^3}{\sqrt{bx^3+a}}dx\right)}{b}-93366b^2x\sqrt{a+bx^3}(23bc-8af)}{b}-2\cdots}{3b}}{b}\cdots}{b}\cdots\right.\right.\right.$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\downarrow$ 793

3.458. $\int x^3\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{2124694}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\frac{2\left(\int\frac{46683a(23bc-8af)b^3+232254a(5bd-2ag)xb^3}{\sqrt{bx^3+a}}dx+\frac{2124694}{3}ab^3e\sqrt{a+bx^3}\right)}{b}-93366b^2x\sqrt{a+bx^3}(23bc-8af)-23225\cdots}{b}}{3b}}{b}\right)}{b}\cdots\right.\right.$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\Big\downarrow$ 2417

$$3a\left(\frac{1}{143}a\left(\frac{5\left(\frac{2124694}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{\frac{2\left(27ab^{8/3}\left(1729\sqrt[3]{b}(23bc-8af)-8602\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd-2ag)\right)\int\frac{1}{\sqrt{bx^3+a}}dx+232254ab^{8/3}(5bd-2ag)\int\frac{\sqrt[3]{b}x+\left(1\cdots\right.}{\sqrt{bx\cdots}}\cdots\right.}{b}}{b}}{3b}}{b}\right)}{b}\cdots\right.\right.$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$\Big\downarrow$ 759

3.458. $\int x^3\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$2\left(232254ab^{8/3}(5bd-2ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}ab^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/\cdots}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}}}{\cdots}\right.$$

$$5\left(\frac{2124694}{9}bex^3\sqrt{a+bx^3}-\frac{\cdots}{\cdots}\right.$$

$$3a\left(\frac{1}{143}a\left(\frac{\cdots}{\cdots}\right.\right.$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

$$\downarrow 2416$$

3.458. $\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$3a\left(\frac{1}{143}a\,\frac{5\left(\frac{2124694}{9}bex^3\sqrt{a+bx^3}-\frac{\frac{2\left(\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}ab^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{}}{}\right)}{}\right)}{}$$

$$\frac{2x^3\left(a+bx^3\right)^{3/2}\left(229425cx+205275dx^2+185725ex^3+169575fx^4+156009gx^5\right)}{3900225}$$

input
```
Int[x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

3.458. $\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

output
```
(2*x^3*(a + b*x^3)^(3/2)*(229425*c*x + 205275*d*x^2 + 185725*e*x^3 + 16957
5*f*x^4 + 156009*g*x^5))/3900225 + (3*a*((2*x^3*Sqrt[a + b*x^3]*(8947575*c
*x + 6774075*d*x^2 + 5311735*e*x^3 + 4279275*f*x^4 + 3522519*g*x^5))/429 +
 (a*((541926*g*x^5*Sqrt[a + b*x^3])/b + (5*(155610*f*x^4*Sqrt[a + b*x^3] +
 ((2124694*b*e*x^3*Sqrt[a + b*x^3])/9 - (-232254*b*(5*b*d - 2*a*g)*x^2*Sqr
t[a + b*x^3] + (-93366*b^2*(23*b*c - 8*a*f)*x*Sqrt[a + b*x^3] + (2*((21246
94*a*b^3*e*Sqrt[a + b*x^3])/3 + 232254*a*b^(8/3)*(5*b*d - 2*a*g)*((2*Sqrt[
a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[
2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (18*3^(3/4)*Sqrt[2 + Sqrt[
3]]*a*b^(7/3)*(1729*b^(1/3)*(23*b*c - 8*a*f) - 8602*(1 - Sqrt[3])*a^(1/3)*
(5*b*d - 2*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b)/b)/(3*b))/b))/b))/143))/1300075
```

3.458.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.458. $\int x^3 (a+bx^3)^{3/2} (c+dx+ex^2+fx^3+gx^4)\,dx$

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.458. $\int x^3(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)\,dx$

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.458.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 1161, normalized size of antiderivative = 1.47

method	result	size
elliptic	Expression too large to display	1161
risch	Expression too large to display	1198
default	Expression too large to display	1764

input
```
int(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2/25*g*b*x^11*(b*x^3+a)^(1/2)+2/23*b*f*x^10*(b*x^3+a)^(1/2)+2/21*b*e*x^9*(
b*x^3+a)^(1/2)+2/19*(28/25*a*b*g+b^2*d)/b*x^8*(b*x^3+a)^(1/2)+2/17*(26/23*
a*f*b+b^2*c)/b*x^7*(b*x^3+a)^(1/2)+16/105*a*e*x^6*(b*x^3+a)^(1/2)+2/13*(a^
2*g+2*a*b*d-16/19*a/b*(28/25*a*b*g+b^2*d))/b*x^5*(b*x^3+a)^(1/2)+2/11*(a^2
*f+2*a*b*c-14/17*a/b*(26/23*a*f*b+b^2*c))/b*x^4*(b*x^3+a)^(1/2)+2/105*a^2*
e*x^3*(b*x^3+a)^(1/2)/b+2/7*(a^2*d-10/13*a/b*(a^2*g+2*a*b*d-16/19*a/b*(28/
25*a*b*g+b^2*d)))/b*x^2*(b*x^3+a)^(1/2)+2/5*(a^2*c-8/11*a/b*(a^2*f+2*a*b*c
-14/17*a/b*(26/23*a*f*b+b^2*c)))/b*x*(b*x^3+a)^(1/2)-4/105*a^3*e*(b*x^3+a)
^(1/2)/b^2+4/15*I*a/b^2*(a^2*c-8/11*a/b*(a^2*f+2*a*b*c-14/17*a/b*(26/23*a*
f*b+b^2*c)))*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))+8/21*I*a/b^2*(a^2*d-10/13*a/b*(a^2*g+2*a*b*d-16/19*a/b*(28/25*a*b*g
+b^2*d)))*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/...
```

3.458.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.33

$$\int x^3 (a + bx^3)^{3/2} (c + dx + ex^2 + fx^3 + gx^4) \, dx =$$

$$-\frac{2 \left(1400490 \, (23 \, a^3 bc - 8 \, a^4 f) \sqrt{b} \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, x\right) - 6967620 \, (5 \, a^3 bd - 2 \, a^4 g) \sqrt{b} \text{weierstrassZ}\right.}{}$$

input
```
integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fric
as")
```

output
```
-2/557732175*(1400490*(23*a^3*b*c - 8*a^4*f)*sqrt(b)*weierstrassPInverse(0
, -4*a/b, x) - 6967620*(5*a^3*b*d - 2*a^4*g)*sqrt(b)*weierstrassZeta(0, -4
*a/b, weierstrassPInverse(0, -4*a/b, x)) - (22309287*b^4*g*x^11 + 24249225
*b^4*f*x^10 + 26558675*b^4*e*x^9 + 42493880*a*b^3*e*x^6 + 1174173*(25*b^4*
d + 28*a*b^3*g)*x^8 + 5311735*a^2*b^2*e*x^3 + 1426425*(23*b^4*c + 26*a*b^3
*f)*x^7 + 90321*(550*a*b^3*d + 27*a^2*b^2*g)*x^5 - 10623470*a^3*b*e + 1296
75*(460*a*b^3*c + 27*a^2*b^2*f)*x^4 + 1741905*(5*a^2*b^2*d - 2*a^3*b*g)*x^
2 + 700245*(23*a^2*b^2*c - 8*a^3*b*f)*x)*sqrt(b*x^3 + a))/b^3
```

3.458.6 Sympy [A] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 512, normalized size of antiderivative = 0.65

$$\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{a^{\frac{3}{2}}cx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{3}{2}}dx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{a^{\frac{3}{2}}fx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{a^{\frac{3}{2}}gx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + \frac{\sqrt{a}bcx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}bdx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + \frac{\sqrt{a}bfx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)} + \frac{\sqrt{a}bgx^{11}\Gamma\left(\frac{11}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{3}\\ \frac{14}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{14}{3}\right)} + ae\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right) + be\left(\begin{cases}\frac{16a^3\sqrt{a+bx^3}}{315b^3}-\frac{8a^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2ax^6\sqrt{a+bx^3}}{105b}+\frac{2x^9\sqrt{a+bx^3}}{21} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^9}{9} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**3*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**(3/2)*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(7/3)) + a**(3/2)*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + a**(3/2)*f*x**7*gamma(7/3)*h
yper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**
(3/2)*g*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(11/3)) + sqrt(a)*b*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*d*x**8*gamma(8/3
)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) +
sqrt(a)*b*f*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*gamma(13/3)) + sqrt(a)*b*g*x**11*gamma(11/3)*hyper((-1/2, 11
/3), (14/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(14/3)) + a*e*Piecewise((-
4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x
**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + b*e*Piecewis
e((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*x**3)/(315
*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x**3)/21, N
e(b, 0)), (sqrt(a)*x**9/9, True))
```

3.458.7 Maxima [F]

$$\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3, x)
```

3.458.8 Giac [F]

$$\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^3, x)
```

3.458.9 Mupad [F(-1)]

Timed out.

$$\int x^3(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int x^3\,\left(b\,x^3+a\right)^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

output
```
int(x^3*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.459 $\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

3.459.1 Optimal result . . . 3533
3.459.2 Mathematica [C] (verified) . . . 3534
3.459.3 Rubi [A] (verified) . . . 3535
3.459.4 Maple [A] (verified) . . . 3542
3.459.5 Fricas [C] (verification not implemented) . . . 3543
3.459.6 Sympy [A] (verification not implemented) . . . 3544
3.459.7 Maxima [F] . . . 3545
3.459.8 Giac [F] . . . 3546
3.459.9 Mupad [F(-1)] . . . 3546

3.459.1 Optimal result

Integrand size = 35, antiderivative size = 742

$$\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\frac{2a^2(7bc-2af)\sqrt{a+bx^3}}{105b^2}$$
$$+\frac{54a^2(23bd-8ag)x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2ex^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2fx^3\sqrt{a+bx^3}}{105b}$$
$$+\frac{54a^2gx^4\sqrt{a+bx^3}}{4301b}-\frac{216a^3e\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{2x^2(a+bx^3)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$
$$+\frac{2ax^2\sqrt{a+bx^3}(7436429cx+5368545dx^2+4064445ex^3+3187041fx^4+2567565gx^5)}{111546435}$$
$$+\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(43010\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-1729(23bd-8ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{Elli}}{37182145b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/780045*x^2*(b*x^3+a)^(3/2)*(33915*g*x^5+37145*f*x^4+41055*e*x^3+45885*d*
x^2+52003*c*x)+2/105*a^2*(-2*a*f+7*b*c)*(b*x^3+a)^(1/2)/b^2+54/21505*a^2*(
-8*a*g+23*b*d)*x*(b*x^3+a)^(1/2)/b^2+54/1729*a^2*e*x^2*(b*x^3+a)^(1/2)/b+2
/105*a^2*f*x^3*(b*x^3+a)^(1/2)/b+54/4301*a^2*g*x^4*(b*x^3+a)^(1/2)/b+2/111
546435*a*x^2*(2567565*g*x^5+3187041*f*x^4+4064445*e*x^3+5368545*d*x^2+7436
429*c*x)*(b*x^3+a)^(1/2)-216/1729*a^3*e*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))+108/1729*3^(1/4)*a^(10/3)*e*(a^(1/3)+b^(1/3)*x)*Elli
pticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+36/371821
45*3^(3/4)*a^3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(13832*a*g-39767*b*d+430
10*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3
)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)
```

3.459.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.22

$$\int x^2(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)\,dx = \frac{2\Big((a+bx^3)^3(52003bc-38a(391f+420gx)+5bx(9177d+17x(483e+19x(23f+21gx))))+\cdots}{\cdots}$$

input
```
Integrate[x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
(2*((a + b*x^3)^3*(52003*b*c - 38*a*(391*f + 420*g*x) + 5*b*x*(9177*d + 17
*x*(483*e + 19*x*(23*f + 21*g*x)))) + 1995*a^3*(-23*b*d + 8*a*g)*x*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] - 41055*a^3*b
*e*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]
))/(780045*b^2*Sqrt[a + b*x^3])
```

3.459.3 Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.514, Rules used = {2365, 27, 2365, 27, 2375, 27, 2375, 27, 2427, 27, 2028, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right) \, dx$$

$$\Big\downarrow \text{2365}$$

$$\frac{9}{2} a \int \frac{2x^2 \sqrt{bx^3+a} \left(33915 g x^4+37145 f x^3+41055 e x^2+45885 d x+52003 c\right)}{780045}dx+\frac{2x^2 \left(a+bx^3\right)^{3/2} \left(52003 c x+45885 d x^2+41055 e x^3+37145 f x^4+33915 g x^5\right)}{780045}$$

$$\Big\downarrow \text{27}$$

$$\frac{3 a \int x^2 \sqrt{bx^3+a} \left(33915 g x^4+37145 f x^3+41055 e x^2+45885 d x+52003 c\right) \, dx}{260015}+\frac{2x^2 \left(a+bx^3\right)^{3/2} \left(52003 c x+45885 d x^2+41055 e x^3+37145 f x^4+33915 g x^5\right)}{780045}$$

$$\Big\downarrow \text{2365}$$

$$\frac{3 a \left(\frac{3}{2} a \int \frac{2 x^2 \left(2567565 g x^4+3187041 f x^3+4064445 e x^2+5368545 d x+7436429 c\right)}{1287 \sqrt{bx^3+a}}dx+\frac{2 x^2 \sqrt{a+bx^3} (7436429 c x+5368545 d x^2+4064445 e x^3+3187}{1287}\right.}{260015}$$
$$\frac{2x^2 \left(a+bx^3\right)^{3/2} \left(52003 c x+45885 d x^2+41055 e x^3+37145 f x^4+33915 g x^5\right)}{780045}$$

$$\Big\downarrow \text{27}$$

$$\frac{3 a \left(\frac{1}{429} a \int \frac{x^2 \left(2567565 g x^4+3187041 f x^3+4064445 e x^2+5368545 d x+7436429 c\right)}{\sqrt{bx^3+a}}dx+\frac{2 x^2 \sqrt{a+bx^3} (7436429 c x+5368545 d x^2+4064445 e x^3+318}{1287}\right.}{260015}$$
$$\frac{2x^2 \left(a+bx^3\right)^{3/2} \left(52003 c x+45885 d x^2+41055 e x^3+37145 f x^4+33915 g x^5\right)}{780045}$$

$$\Big\downarrow \text{2375}$$

$$\frac{3a\left(\frac{1}{429}a\left(\frac{2\int \frac{11x^2\left(3187041bfx^3+4064445bex^2+233415(23bd-8ag)x+7436429bc\right)}{2\sqrt{bx^3+a}}dx}{11b}+\frac{466830gx^4\sqrt{a+bx^3}}{b}\right)+\frac{2x^2\sqrt{a+bx^3}(7436429cx+5368545}{}\right.}{260015}$$
$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$\Big\downarrow$ 27

$$\frac{3a\left(\frac{1}{429}a\left(\frac{\int \frac{x^2\left(3187041bfx^3+4064445bex^2+233415(23bd-8ag)x+7436429bc\right)}{\sqrt{bx^3+a}}dx}{b}+\frac{466830gx^4\sqrt{a+bx^3}}{b}\right)+\frac{2x^2\sqrt{a+bx^3}(7436429cx+5368545dx^2}{}\right.}{260015}$$
$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$\Big\downarrow$ 2375

$$\frac{3a\left(\frac{1}{429}a\left(\frac{\frac{2\int \frac{9x^2\left(4064445b^2ex^2+233415b(23bd-8ag)x+1062347b(7bc-2af)\right)}{2\sqrt{bx^3+a}}dx}{9b}+\frac{2124694}{3}fx^3\sqrt{a+bx^3}}{b}+\frac{466830gx^4\sqrt{a+bx^3}}{b}\right)+\frac{2x^2\sqrt{a+bx^3}(74}{}\right.}{260015}$$
$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$\Big\downarrow$ 27

$$\frac{3a\left(\frac{1}{429}a\left(\frac{\frac{\int \frac{x^2\left(4064445b^2ex^2+233415b(23bd-8ag)x+1062347b(7bc-2af)\right)}{\sqrt{bx^3+a}}dx}{b}+\frac{2124694}{3}fx^3\sqrt{a+bx^3}}{b}+\frac{466830gx^4\sqrt{a+bx^3}}{b}\right)+\frac{2x^2\sqrt{a+bx^3}(7436}{}\right.}{260015}$$
$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$\Big\downarrow$ 2427

$$\frac{3a\left(\frac{1}{429}a\left(\frac{\frac{\frac{2\int -\frac{7\left(-233415b^2(23bd-8ag)x^3-1062347b^2(7bc-2af)x^2+2322540ab^2ex\right)}{2\sqrt{bx^3+a}}dx}{7b}+1161270bex^2\sqrt{a+bx^3}}{b}+\frac{2124694}{3}fx^3\sqrt{a+bx^3}}{b}+\frac{466830gx^4\sqrt{a}}{b}\right.\right.}{260015}$$
$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

3.459. $\int x^2\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$\Big\downarrow 27$$

$$\frac{3a\left(\frac{1}{429}a\left(\frac{\frac{1161270bex^2\sqrt{a+bx^3}-\frac{\int\frac{-233415b^2(23bd-8ag)x^3-1062347b^2(7bc-2af)x^2+2322540ab^2ex}{\sqrt{bx^3+a}}dx}{b}}{b}}{b}+\frac{2124694}{3}fx^3\sqrt{a+bx^3}+\frac{466830gx^4\sqrt{a+bx^3}}{b}\right.\right.}{260015}$$

$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$$\Big\downarrow 2028$$

$$\frac{3a\left(\frac{1}{429}a\left(\frac{\frac{1161270bex^2\sqrt{a+bx^3}-\frac{\int\frac{x\left(-233415(23bd-8ag)x^2b^2+2322540aeb^2-1062347(7bc-2af)xb^2\right)}{\sqrt{bx^3+a}}dx}{b}}{b}}{b}+\frac{2124694}{3}fx^3\sqrt{a+bx^3}+\frac{466830gx^4\sqrt{a+bx^3}}{b}\right.\right.}{260015}$$

$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$$\Big\downarrow 2427$$

$$\frac{3a\left(\frac{1}{429}a\left(\frac{\frac{1161270bex^2\sqrt{a+bx^3}-\frac{\frac{2\int\frac{5\left(-1062347(7bc-2af)x^2b^3+2322540aexb^3+93366a(23bd-8ag)b^2\right)}{2\sqrt{bx^3+a}}dx}{5b}-93366bx\sqrt{a+bx^3}(23bd-8ag)}{b}}{b}}{b}+\frac{2124694}{3}fx^3\sqrt{a+bx^3}\right.\right.}{260015}$$

$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$$\Big\downarrow 27$$

$$\frac{3a\left(\frac{1}{429}a\left(\frac{\frac{1161270bex^2\sqrt{a+bx^3}-\frac{\frac{\int\frac{-1062347(7bc-2af)x^2b^3+2322540aexb^3+93366a(23bd-8ag)b^2}{\sqrt{bx^3+a}}dx}{b}-93366bx\sqrt{a+bx^3}(23bd-8ag)}{b}}{b}}{b}+\frac{2124694}{3}fx^3\sqrt{a+bx^3}\right.\right.}{260015}$$

$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$$\Big\downarrow 2425$$

3.459. $\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$3a\left(\frac{1}{429}a\left(\frac{\frac{1161270bex^2\sqrt{a+bx^3}-\frac{\frac{\int\frac{2322540aexb^3+93366a(23bd-8ag)b^2}{\sqrt{bx^3+a}}dx-1062347b^3(7bc-2af)\int\frac{x^2}{\sqrt{bx^3+a}}dx}{b}-93366bx\sqrt{a+bx^3}(23bd-8ag)}{b}}{b}}{b}+\frac{2124694}{3}fx\ldots\right.\right.$$

$$\ldots 260015$$

$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$\downarrow$ 793

$$3a\left(\frac{1}{429}a\left(\frac{\frac{1161270bex^2\sqrt{a+bx^3}-\frac{\frac{\int\frac{2322540aexb^3+93366a(23bd-8ag)b^2}{\sqrt{bx^3+a}}dx-\frac{2124694}{3}b^2\sqrt{a+bx^3}(7bc-2af)}{b}-93366bx\sqrt{a+bx^3}(23bd-8ag)}{b}}{b}}{b}+\frac{2124694}{3}fx^3\sqrt{a\ldots}\right.\right.$$

$$\ldots 260015$$

$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$\downarrow$ 2417

$$3a\left(\frac{1}{429}a\left(\frac{\frac{1161270bex^2\sqrt{a+bx^3}-\frac{\frac{2322540ab^{8/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-54ab^2\left(43010\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-1729(23bd-8ag)\right)\int\frac{1}{\sqrt{bx^3+a}}dx-\frac{2124694}{3}b^2\ldots}{b}\ldots}{b}}{b}}{b}\ldots\right.\right.$$

$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$\downarrow$ 759

3.459. $\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$3a\left(\frac{1}{429}a\left(\frac{1161270bex^2\sqrt{a+bx^3}-\frac{2322540ab^{8/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}ab^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\ldots\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\ldots\right.}{\left(\left(1+\sqrt{3}\right)\ldots\right.}}\ldots}}{b\ldots}}{\ldots}\right.\right.$$

$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

$\downarrow$ 2416

$$3a\left(\frac{1}{429}a\left(\frac{1161270bex^2\sqrt{a+bx^3}-\frac{-\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}ab^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\ldots}{\ldots}}{\ldots}\right.\right.$$

$$\frac{2x^2\left(a+bx^3\right)^{3/2}\left(52003cx+45885dx^2+41055ex^3+37145fx^4+33915gx^5\right)}{780045}$$

input
```
Int[x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

3.459. $\int x^2\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

output
```
(2*x^2*(a + b*x^3)^(3/2)*(52003*c*x + 45885*d*x^2 + 41055*e*x^3 + 37145*f*
x^4 + 33915*g*x^5))/780045 + (3*a*((2*x^2*Sqrt[a + b*x^3]*(7436429*c*x + 5
368545*d*x^2 + 4064445*e*x^3 + 3187041*f*x^4 + 2567565*g*x^5))/1287 + (a*(
(466830*g*x^4*Sqrt[a + b*x^3])/b + ((2124694*f*x^3*Sqrt[a + b*x^3])/3 + (1
161270*b*e*x^2*Sqrt[a + b*x^3] - (-93366*b*(23*b*d - 8*a*g)*x*Sqrt[a + b*x
^3] + ((-2124694*b^2*(7*b*c - 2*a*f)*Sqrt[a + b*x^3])/3 + 2322540*a*b^(8/3
)*e*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (
3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) - (36*3^(3/4)*S
qrt[2 + Sqrt[3]]*a*b^(5/3)*(43010*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 1729*(
23*b*d - 8*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b)/b)/b)/b))/429))/260015
```

3.459.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] && !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.459. $\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)dx$

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.459.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 1103, normalized size of antiderivative = 1.49

method	result	size
elliptic	Expression too large to display	1103
risch	Expression too large to display	1175
default	Expression too large to display	1269

input
```
int(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2/23*g*b*x^10*(b*x^3+a)^(1/2)+2/21*b*f*x^9*(b*x^3+a)^(1/2)+2/19*b*e*x^8*(b
*x^3+a)^(1/2)+2/17*(26/23*a*b*g+b^2*d)/b*x^7*(b*x^3+a)^(1/2)+2/15*(8/7*a*f
*b+b^2*c)/b*x^6*(b*x^3+a)^(1/2)+44/247*a*e*x^5*(b*x^3+a)^(1/2)+2/11*(a^2*g
+2*a*b*d-14/17*a/b*(26/23*a*b*g+b^2*d))/b*x^4*(b*x^3+a)^(1/2)+2/9*(a^2*f+2
*a*b*c-4/5*a/b*(8/7*a*f*b+b^2*c))/b*x^3*(b*x^3+a)^(1/2)+54/1729*a^2*e*x^2*
(b*x^3+a)^(1/2)/b+2/5*(a^2*d-8/11*a/b*(a^2*g+2*a*b*d-14/17*a/b*(26/23*a*b*
g+b^2*d)))/b*x*(b*x^3+a)^(1/2)+2/3*(a^2*c-2/3*a/b*(a^2*f+2*a*b*c-4/5*a/b*(
8/7*a*f*b+b^2*c)))/b*(b*x^3+a)^(1/2)+4/15*I*a/b^2*(a^2*d-8/11*a/b*(a^2*g+2
*a*b*d-14/17*a/b*(26/23*a*b*g+b^2*d)))*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+72/1729*I*e*a^3/b^2*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2...
```

3.459.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.32

$$\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{2\left(6967620\,a^3b^{\frac{3}{2}}e\,\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-280098\,(23\,a^3bd-\right.}{}$$

input
```
integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fric
as")
```

output
```
2/111546435*(6967620*a^3*b^(3/2)*e*weierstrassZeta(0, -4*a/b, weierstrassP
Inverse(0, -4*a/b, x)) - 280098*(23*a^3*b*d - 8*a^4*g)*sqrt(b)*weierstrass
PInverse(0, -4*a/b, x) + (4849845*b^4*g*x^10 + 5311735*b^4*f*x^9 + 5870865
*b^4*e*x^8 + 9935310*a*b^3*e*x^5 + 285285*(23*b^4*d + 26*a*b^3*g)*x^7 + 17
41905*a^2*b^2*e*x^2 + 1062347*(7*b^4*c + 8*a*b^3*f)*x^6 + 7436429*a^2*b^2*
c - 2124694*a^3*b*f + 25935*(460*a*b^3*d + 27*a^2*b^2*g)*x^4 + 1062347*(14
*a*b^3*c + a^2*b^2*f)*x^3 + 140049*(23*a^2*b^2*d - 8*a^3*b*g)*x)*sqrt(b*x^
3 + a))/b^3
```

3.459.6 Sympy [A] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.71

$$\int x^2\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{a^{\frac{3}{2}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{3}{2}}ex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{a^{\frac{3}{2}}gx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}bex^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + \frac{\sqrt{a}bgx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)} + ac\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for}\ b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + af\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right) + bc\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right) + bf\left(\begin{cases}\frac{16a^3\sqrt{a+bx^3}}{315b^3}-\frac{8a^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2ax^6\sqrt{a+bx^3}}{105b}+\frac{2x^9\sqrt{a+bx^3}}{21} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^9}{9} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**(3/2)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(7/3)) + a**(3/2)*e*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + a**(3/2)*g*x**7*gamma(7/3)*h
yper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqr
t(a)*b*d*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(10/3)) + sqrt(a)*b*e*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + sqrt(a)*b*g*x**10*gamma(1
0/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/3)
) + a*c*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b)
, True)) + a*f*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sq
rt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x*
*6/6, True)) + b*c*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**
3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a
)*x**6/6, True)) + b*f*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*
a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b)
+ 2*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

3.459.7 Maxima [F]

$$\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int\left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxi
ma")
```

output
```
2/15*(b*x^3 + a)^(5/2)*c/b + integrate((b*g*x^9 + b*f*x^8 + b*e*x^7 + a*f*
x^5 + (b*d + a*g)*x^6 + a*e*x^4 + a*d*x^3)*sqrt(b*x^3 + a), x)
```

3.459.8 Giac [F]

$$\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x^2, x)
```

3.459.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int x^2\,\left(b\,x^3+a\right)^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

output
```
int(x^2*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.460 $\int x(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

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3.460.1 Optimal result

Integrand size = 33, antiderivative size = 723

$$\int x\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\frac{2a^2(7bd-2ag)\sqrt{a+bx^3}}{105b^2}+\frac{54a^2ex\sqrt{a+bx^3}}{935b}$$
$$+\frac{54a^2fx^2\sqrt{a+bx^3}}{1729b}+\frac{2a^2gx^3\sqrt{a+bx^3}}{105b}+\frac{54a^2(19bc-4af)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$
$$+\frac{2ax\sqrt{a+bx^3}\left(479655cx+323323dx^2+233415ex^3+176715fx^4+138567gx^5\right)}{4849845}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19bc-4af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\left(3458a^{2/3}\sqrt[3]{b}e+935\left(1-\sqrt{3}\right)(19bc-4af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{Ell}}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/440895*x*(b*x^3+a)^(3/2)*(20995*g*x^5+23205*f*x^4+25935*e*x^3+29393*d*x^
2+33915*c*x)+2/105*a^2*(-2*a*g+7*b*d)*(b*x^3+a)^(1/2)/b^2+54/935*a^2*e*x*(
b*x^3+a)^(1/2)/b+54/1729*a^2*f*x^2*(b*x^3+a)^(1/2)/b+2/105*a^2*g*x^3*(b*x^
3+a)^(1/2)/b+2/4849845*a*x*(138567*g*x^5+176715*f*x^4+233415*e*x^3+323323*
d*x^2+479655*c*x)*(b*x^3+a)^(1/2)+54/1729*a^2*(-4*a*f+19*b*c)*(b*x^3+a)^(1
/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/1729*3^(1/4)*a^(7/3)*(-4*a*
f+19*b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)-18/1616615*3^(3/4)*a^(7/3)*(a^(1/3)+b^(1/3)*x)*El
lipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*(3458*a^(2/3)*b^(1/3)*e+935*(-4*a*f+19*b*c)*(1-3^(1/2)))*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.460.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.20

$$\int x\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx =$$

$$-\frac{\sqrt{a+bx^3}\left(4\left(a+bx^3\right)^2\sqrt{1+\frac{bx^3}{a}}(-2261bd+646ag-5bx(399e+17x(21f+19gx)))+7980a^2bex\,\text{Hyper}\right.}{67830b^2\sqrt{1-}}$$

input
```
Integrate[x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
-1/67830*(Sqrt[a + b*x^3]*(4*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(-2261*b*d
+ 646*a*g - 5*b*x*(399*e + 17*x*(21*f + 19*g*x))) + 7980*a^2*b*e*x*Hyperge
ometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 1785*a*b*(-19*b*c + 4*a*f)*x^2*
Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(b^2*Sqrt[1 + (b*x^3)/a]
)
```

3.460.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 717, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {2365, 27, 2365, 27, 2375, 27, 2375, 27, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$

$$\Big\downarrow \text{2365}$$

$$\frac{9}{2}a\int \frac{2x\sqrt{bx^3+a}(20995gx^4+23205fx^3+25935ex^2+29393dx+33915c)}{440895}dx+\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$$\Big\downarrow \text{27}$$

$$\frac{3a\int x\sqrt{bx^3+a}(20995gx^4+23205fx^3+25935ex^2+29393dx+33915c)\,dx}{146965}+\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$$\Big\downarrow \text{2365}$$

$$\frac{3a\left(\frac{3}{2}a\int \frac{2x(138567gx^4+176715fx^3+233415ex^2+323323dx+479655c)}{99\sqrt{bx^3+a}}dx+\frac{2}{99}x\sqrt{a+bx^3}(479655cx+323323dx^2+233415ex^3\right.}{146965}$$
$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$$\Big\downarrow \text{27}$$

$$\frac{3a\left(\frac{1}{33}a\int \frac{x(138567gx^4+176715fx^3+233415ex^2+323323dx+479655c)}{\sqrt{bx^3+a}}dx+\frac{2}{99}x\sqrt{a+bx^3}(479655cx+323323dx^2+233415ex^3\right.}{146965}$$
$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$$\Big\downarrow \text{2375}$$

$$\frac{3a\left(\frac{1}{33}a\left(\frac{2\int \frac{9x\left(176715bfx^3+233415bex^2+46189(7bd-2ag)x+479655bc\right)}{2\sqrt{bx^3+a}}dx}{9b}+\frac{92378gx^3\sqrt{a+bx^3}}{3b}\right)+\frac{2}{99}x\sqrt{a+bx^3}\left(479655cx+323323\right.}{146965}$$

$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$\downarrow$ 27

$$\frac{3a\left(\frac{1}{33}a\left(\frac{\int \frac{x\left(176715bfx^3+233415bex^2+46189(7bd-2ag)x+479655bc\right)}{\sqrt{bx^3+a}}dx}{b}+\frac{92378gx^3\sqrt{a+bx^3}}{3b}\right)+\frac{2}{99}x\sqrt{a+bx^3}\left(479655cx+323323dx\right.}{146965}$$

$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$\downarrow$ 2375

$$\frac{3a\left(\frac{1}{33}a\left(\frac{\frac{2\int \frac{7x\left(233415b^2ex^2+46189b(7bd-2ag)x+25245b(19bc-4af)\right)}{2\sqrt{bx^3+a}}dx}{7b}+50490fx^2\sqrt{a+bx^3}}{b}+\frac{92378gx^3\sqrt{a+bx^3}}{3b}\right)+\frac{2}{99}x\sqrt{a+bx^3}\left(47965\right.}{146965}$$

$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$\downarrow$ 27

$$\frac{3a\left(\frac{1}{33}a\left(\frac{\frac{\int \frac{x\left(233415b^2ex^2+46189b(7bd-2ag)x+25245b(19bc-4af)\right)}{\sqrt{bx^3+a}}dx}{b}+50490fx^2\sqrt{a+bx^3}}{b}+\frac{92378gx^3\sqrt{a+bx^3}}{3b}\right)+\frac{2}{99}x\sqrt{a+bx^3}\left(479655\right.}{146965}$$

$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$\downarrow$ 2427

$$\frac{3a\left(\frac{1}{33}a\left(\frac{\frac{\frac{2\int -\frac{5\left(-46189(7bd-2ag)x^2b^2+93366aeb^2-25245(19bc-4af)xb^2\right)}{2\sqrt{bx^3+a}}dx}{5b}+93366bex\sqrt{a+bx^3}}{b}+50490fx^2\sqrt{a+bx^3}}{b}+\frac{92378gx^3\sqrt{a+bx^3}}{3b}\right)+\frac{2}{99}\right.}{146965}$$

$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

3.460. $\int x\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$\Big\downarrow 27$$

$$\frac{3a\left(\frac{1}{33}a\left(\frac{\frac{93366bex\sqrt{a+bx^3}-\frac{\int \frac{-46189(7bd-2ag)x^2b^2+93366aeb^2-25245(19bc-4af)xb^2}{\sqrt{bx^3+a}}dx}{b}}{b}+50490fx^2\sqrt{a+bx^3}}{b}+\frac{92378gx^3\sqrt{a+bx^3}}{3b}\right)+\frac{2}{99}x\sqrt{a\ldots}}{146965}$$

$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$$\Big\downarrow 2425$$

$$\frac{3a\left(\frac{1}{33}a\left(\frac{\frac{93366bex\sqrt{a+bx^3}-\frac{\frac{\int \frac{93366ab^2e-25245b^2(19bc-4af)x}{\sqrt{bx^3+a}}dx-46189b^2(7bd-2ag)\int \frac{x^2}{\sqrt{bx^3+a}}dx}{b}}{b}}{b}+50490fx^2\sqrt{a+bx^3}}{b}+\frac{92378gx^3\sqrt{a+bx^3}}{3b}\right)+\ldots}{146965}$$

$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$$\Big\downarrow 793$$

$$\frac{3a\left(\frac{1}{33}a\left(\frac{\frac{93366bex\sqrt{a+bx^3}-\frac{\frac{\int \frac{93366ab^2e-25245b^2(19bc-4af)x}{\sqrt{bx^3+a}}dx-\frac{92378}{3}b\sqrt{a+bx^3}(7bd-2ag)}{b}}{b}}{b}+50490fx^2\sqrt{a+bx^3}}{b}+\frac{92378gx^3\sqrt{a+bx^3}}{3b}\right)+\frac{2}{99}x\ldots}{146965}$$

$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$$\Big\downarrow 2417$$

$$3a\left(\frac{1}{33}a\left(\frac{\frac{93366bex\sqrt{a+bx^3}-\frac{\frac{27\sqrt[3]{a}b^{5/3}\left(3458a^{2/3}\sqrt[3]{b}e+935\left(1-\sqrt{3}\right)(19bc-4af)\right)\int \frac{1}{\sqrt{bx^3+a}}dx-25245b^{5/3}(19bc-4af)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{92378}{3}\ldots}{b}}{b}}{b}\ldots\right.\right.$$

$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$$\Big\downarrow 759$$

3.460. $\int x\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$3a\left(\frac{1}{33}a\left(\frac{93366bex\sqrt{a+bx^3}-\frac{-25245b^{5/3}(19bc-4af)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{Elli}\cdots}{\sqrt{\frac{\sqrt[3]{a}\cdots}{\left(\left(1+\cdots\right)\cdots\right)}}\cdots}}{b}\cdots}{b}\cdots\right.\right.$$

$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

$\downarrow$ 2416

$$3a\left(\frac{1}{33}a\left(\frac{93366bex\sqrt{a+bx^3}-\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\cdots\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots}{\cdots}\right.\right.$$

$$\frac{2x\left(a+bx^3\right)^{3/2}\left(33915cx+29393dx^2+25935ex^3+23205fx^4+20995gx^5\right)}{440895}$$

input
```
Int[x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

3.460. $\int x(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

output
```
(2*x*(a + b*x^3)^(3/2)*(33915*c*x + 29393*d*x^2 + 25935*e*x^3 + 23205*f*x^
4 + 20995*g*x^5))/440895 + (3*a*((2*x*Sqrt[a + b*x^3]*(479655*c*x + 323323
*d*x^2 + 233415*e*x^3 + 176715*f*x^4 + 138567*g*x^5))/99 + (a*((92378*g*x^
3*Sqrt[a + b*x^3])/(3*b) + (50490*f*x^2*Sqrt[a + b*x^3] + (93366*b*e*x*Sqr
t[a + b*x^3] - ((-92378*b*(7*b*d - 2*a*g)*Sqrt[a + b*x^3])/3 - 25245*b^(5/
3)*(19*b*c - 4*a*f)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))
 + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*b^(4/3)*(3458*a^(2/3)*b^(1/3)*e +
 935*(1 - Sqrt[3])*(19*b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b)/b)/b))/33))/14696
5
```

3.460.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.460. $\int x(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

3.460. $\int x(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)\,dx$

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.460.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 889, normalized size of antiderivative = 1.23

method	result	size
risch	Expression too large to display	889
elliptic	Expression too large to display	1045
default	Expression too large to display	1383

input
```
int(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/4849845/b^2*(-230945*b^3*g*x^9-255255*b^3*f*x^8-285285*b^3*e*x^7-369512
*a*b^2*g*x^6-323323*b^3*d*x^6-431970*a*b^2*f*x^5-373065*b^3*c*x^5-518700*a
*b^2*e*x^4-46189*a^2*b*g*x^3-646646*a*b^2*d*x^3-75735*a^2*b*f*x^2-852720*a
*b^2*c*x^2-140049*a^2*b*e*x+92378*a^3*g-323323*a^2*b*d)*(b*x^3+a)^(1/2)-27
/1616615*a^2/b*(-6916/3*I*a*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(3740*a*f-17765*b*c)*3^(1/2)/b*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)...
```

3.460.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.29

$$\int x\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx =$$

$$-\frac{2\left(280098\,a^3\sqrt{b}e\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+75735\left(19\,a^2bc-4\,a^3f\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{wei}\right.\right.}{}$$

input
```
integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas
")
```

output
```
-2/4849845*(280098*a^3*sqrt(b)*e*weierstrassPInverse(0, -4*a/b, x) + 75735
*(19*a^2*b*c - 4*a^3*f)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInv
erse(0, -4*a/b, x)) - (230945*b^3*g*x^9 + 255255*b^3*f*x^8 + 285285*b^3*e*
x^7 + 518700*a*b^2*e*x^4 + 46189*(7*b^3*d + 8*a*b^2*g)*x^6 + 19635*(19*b^3
*c + 22*a*b^2*f)*x^5 + 140049*a^2*b*e*x + 323323*a^2*b*d - 92378*a^3*g + 4
6189*(14*a*b^2*d + a^2*b*g)*x^3 + 2805*(304*a*b^2*c + 27*a^2*b*f)*x^2)*sqr
t(b*x^3 + a))/b^2
```

3.460.6 Sympy [A] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.73

$$\int x(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{a^{\frac{3}{2}}cx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}}ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{3}{2}}fx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}bcx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}bex^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}bfx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + ad\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + ag\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right) + bd\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right) + bg\left(\begin{cases}\frac{16a^3\sqrt{a+bx^3}}{315b^3}-\frac{8a^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2ax^6\sqrt{a+bx^3}}{105b}+\frac{2x^9\sqrt{a+bx^3}}{21} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^9}{9} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x*(b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**(3/2)*c*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(5/3)) + a**(3/2)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)*f*x**5*gamma(5/3)*h
yper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(
a)*b*c*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(8/3)) + sqrt(a)*b*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b*f*x**8*gamma(8/3)*h
yper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + a*d
*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)
) + a*g*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a +
b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, T
rue)) + b*d*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(
a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/
6, True)) + b*g*Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x*
*3*sqrt(a + b*x**3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**
9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

3.460.7 Maxima [F]

$$\int x(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x, x)
```

3.460.8 Giac [F]

$$\int x(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)*x, x)
```

3.460.9 Mupad [F(-1)]

Timed out.

$$\int x(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int x\,\left(b\,x^3+a\right)^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

output
```
int(x*(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.461 $\int (a+bx^3)^{3/2} (c+dx+ex^2+fx^3+gx^4)\, dx$

3.461.1 Optimal result . . . 3560
3.461.2 Mathematica [C] (verified) . . . 3561
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3.461.5 Fricas [C] (verification not implemented) . . . 3568
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3.461.9 Mupad [F(-1)] . . . 3571

3.461.1 Optimal result

Integrand size = 32, antiderivative size = 694

$$\int (a+bx^3)^{3/2} (c+dx+ex^2+fx^3+gx^4)\, dx = \frac{2a^2e\sqrt{a+bx^3}}{15b} + \frac{54a^2fx\sqrt{a+bx^3}}{935b} + \frac{54a^2gx^2\sqrt{a+bx^3}}{1729b} + \frac{54a^2(19bd-4ag)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2(a+bx^3)^{3/2}(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5)}{692835} + \frac{2a\sqrt{a+bx^3}(793611cx+479655dx^2+323323ex^3+233415fx^4+176715gx^5)}{4849845} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(1729\sqrt[3]{b}(17bc-2af)-935\left(1-\sqrt{3}\right)\sqrt[3]{a}(19bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{1616615b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/692835*(b*x^3+a)^(3/2)*(36465*g*x^5+40755*f*x^4+46189*e*x^3+53295*d*x^2+
62985*c*x)+2/15*a^2*e*(b*x^3+a)^(1/2)/b+54/935*a^2*f*x*(b*x^3+a)^(1/2)/b+5
4/1729*a^2*g*x^2*(b*x^3+a)^(1/2)/b+2/4849845*a*(176715*g*x^5+233415*f*x^4+
323323*e*x^3+479655*d*x^2+793611*c*x)*(b*x^3+a)^(1/2)+54/1729*a^2*(-4*a*g+
19*b*d)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/1729*3^
(1/4)*a^(7/3)*(-4*a*g+19*b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(
1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1
/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)
*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+18/1616615*3^(3/4)*a^2*(a^(1/
3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*(1729*b^(1/3)*(-2*a*f+17*b*c)-935*a^(1/3)*(-4
*a*g+19*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

3.461.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.20

$$\int \left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{\sqrt{a+bx^3}\left(4\left(a+bx^3\right)^2\sqrt{1+\frac{bx^3}{a}}(323e+15x(19f+17gx))-570a(-17bc+2af)x\,\text{Hypergeom}\cdots\right.}{9690b\sqrt{1\cdots}}$$

input
```
Integrate[(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
(Sqrt[a + b*x^3]*(4*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(323*e + 15*x*(19*f
+ 17*g*x)) - 570*a*(-17*b*c + 2*a*f)*x*Hypergeometric2F1[-3/2, 1/3, 4/3, -
((b*x^3)/a)] - 255*a*(-19*b*d + 4*a*g)*x^2*Hypergeometric2F1[-3/2, 2/3, 5/
3, -((b*x^3)/a)]))/(9690*b*Sqrt[1 + (b*x^3)/a])
```

3.461.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 684, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {2392, 27, 2392, 27, 2427, 27, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right) \, dx$$

$$\Big\downarrow \; 2392$$

$$\frac{9}{2}a\int \frac{2\sqrt{bx^3+a}\left(36465gx^4+40755fx^3+46189ex^2+53295dx+62985c\right)}{692835}dx+\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$\Big\downarrow \; 27$$

$$\frac{3a\int \sqrt{bx^3+a}\left(36465gx^4+40755fx^3+46189ex^2+53295dx+62985c\right)dx}{230945}+\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$\Big\downarrow \; 2392$$

$$\frac{3a\left(\frac{3}{2}a\int \frac{2\left(176715gx^4+233415fx^3+323323ex^2+479655dx+793611c\right)}{63\sqrt{bx^3+a}}dx+\frac{2}{63}\sqrt{a+bx^3}\left(793611cx+479655dx^2+323323ex^3+\right.}{230945}$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$\Big\downarrow \; 27$$

$$\frac{3a\left(\frac{1}{21}a\int \frac{176715gx^4+233415fx^3+323323ex^2+479655dx+793611c}{\sqrt{bx^3+a}}dx+\frac{2}{63}\sqrt{a+bx^3}\left(793611cx+479655dx^2+323323ex^3+2\right.}{230945}$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$$\Big\downarrow \; 2427$$

$$\frac{3a\left(\frac{1}{21}a\left(\frac{2\int \frac{7\left(233415bfx^3+323323bex^2+25245(19bd-4ag)x+793611bc\right)}{2\sqrt{bx^3+a}}dx}{7b}+\frac{50490gx^2\sqrt{a+bx^3}}{b}\right)+\frac{2}{63}\sqrt{a+bx^3}\left(793611cx+479655d\right.}{230945}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 27

$$\frac{3a\left(\frac{1}{21}a\left(\frac{\int \frac{233415bfx^3+323323bex^2+25245(19bd-4ag)x+793611bc}{\sqrt{bx^3+a}}dx}{b}+\frac{50490gx^2\sqrt{a+bx^3}}{b}\right)+\frac{2}{63}\sqrt{a+bx^3}\left(793611cx+479655dx^2+\right.}{230945}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 2427

$$\frac{3a\left(\frac{1}{21}a\left(\frac{\frac{2\int \frac{5\left(323323b^2ex^2+25245b(19bd-4ag)x+46683b(17bc-2af)\right)}{2\sqrt{bx^3+a}}dx}{5b}+93366fx\sqrt{a+bx^3}}{b}+\frac{50490gx^2\sqrt{a+bx^3}}{b}\right)+\frac{2}{63}\sqrt{a+bx^3}\left(793611c\right.}{230945}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 27

$$\frac{3a\left(\frac{1}{21}a\left(\frac{\frac{\int \frac{323323b^2ex^2+25245b(19bd-4ag)x+46683b(17bc-2af)}{\sqrt{bx^3+a}}dx}{b}+93366fx\sqrt{a+bx^3}}{b}+\frac{50490gx^2\sqrt{a+bx^3}}{b}\right)+\frac{2}{63}\sqrt{a+bx^3}\left(793611cx+\right.}{230945}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 2425

$$\frac{3a\left(\frac{1}{21}a\left(\frac{\frac{323323b^2e\int \frac{x^2}{\sqrt{bx^3+a}}dx+\int \frac{46683b(17bc-2af)+25245b(19bd-4ag)x}{\sqrt{bx^3+a}}dx}{b}+93366fx\sqrt{a+bx^3}}{b}+\frac{50490gx^2\sqrt{a+bx^3}}{b}\right)+\frac{2}{63}\sqrt{a+bx^3}\left(793\right.}{230945}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 793

3.461. $\int \left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$\frac{3a\left(\frac{1}{21}a\left(\frac{\frac{\int \frac{46683b(17bc-2af)+25245b(19bd-4ag)x}{\sqrt{bx^3+a}}dx+\frac{646646}{3}be\sqrt{a+bx^3}}{b}+93366fx\sqrt{a+bx^3}}{b}+\frac{50490gx^2\sqrt{a+bx^3}}{b}\right)+\frac{2}{63}\sqrt{a+bx^3}\left(793611\right.}{230945}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 2417

$$\frac{3a\left(\frac{1}{21}a\left(\frac{\frac{27b^{2/3}\left(1729\sqrt[3]{b}(17bc-2af)-935\left(1-\sqrt{3}\right)\sqrt[3]{a}(19bd-4ag)\right)\int \frac{1}{\sqrt{bx^3+a}}dx+25245b^{2/3}(19bd-4ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{646646}{3}be\sqrt{a+bx^3}}{b}+}{b}\right.\right.}{23}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 759

$$3a\left(\frac{1}{21}a\left(\frac{\frac{25245b^{2/3}(19bd-4ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\right.}{\sqrt[3]{b}x+\left(1+\right.}\right.\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}}{b}}{b}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

$\downarrow$ 2416

3.461. $\int \left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

$$3a\left(\frac{1}{21}a\left(\frac{\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(1729\sqrt[3]{b}(17bc-2af)-93\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(62985cx+53295dx^2+46189ex^3+40755fx^4+36465gx^5\right)}{692835}$$

input
```
Int[(a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
(2*(a + b*x^3)^(3/2)*(62985*c*x + 53295*d*x^2 + 46189*e*x^3 + 40755*f*x^4
+ 36465*g*x^5))/692835 + (3*a*((2*Sqrt[a + b*x^3]*(793611*c*x + 479655*d*x
^2 + 323323*e*x^3 + 233415*f*x^4 + 176715*g*x^5))/63 + (a*((50490*g*x^2*Sq
rt[a + b*x^3])/b + (93366*f*x*Sqrt[a + b*x^3] + ((646646*b*e*Sqrt[a + b*x^
3])/3 + 25245*b^(2/3)*(19*b*d - 4*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3])) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(1/3)*(1729*b^(1/3)*
(17*b*c - 2*a*f) - 935*(1 - Sqrt[3])*a^(1/3)*(19*b*d - 4*a*g))*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3]))/b)/b))/21))/230945
```

3.461. $\int \left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

3.461.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.461. $\int (a+bx^3)^{3/2} (c+dx+ex^2+fx^3+gx^4)\, dx$

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.461.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 1024, normalized size of antiderivative = 1.48

method	result	size
elliptic	Expression too large to display	1024
risch	Expression too large to display	1138
default	Expression too large to display	1629

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2/19*g*b*x^8*(b*x^3+a)^(1/2)+2/17*b*f*x^7*(b*x^3+a)^(1/2)+2/15*b*e*x^6*(b*
x^3+a)^(1/2)+2/13*(22/19*a*b*g+b^2*d)/b*x^5*(b*x^3+a)^(1/2)+2/11*(20/17*a*
f*b+b^2*c)/b*x^4*(b*x^3+a)^(1/2)+4/15*a*e*x^3*(b*x^3+a)^(1/2)+2/7*(a^2*g+2
*a*b*d-10/13*a/b*(22/19*a*b*g+b^2*d))/b*x^2*(b*x^3+a)^(1/2)+2/5*(a^2*f+2*a
*b*c-8/11*a/b*(20/17*a*f*b+b^2*c))/b*x*(b*x^3+a)^(1/2)+2/15*a^2*e*(b*x^3+a
)^(1/2)/b-2/3*I*(a^2*c-2/5*a/b*(a^2*f+2*a*b*c-8/11*a/b*(20/17*a*f*b+b^2*c)
))*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2
/3*I*(a^2*d-4/7*a/b*(a^2*g+2*a*b*d-10/13*a/b*(22/19*a*b*g+b^2*d)))*3^(1/2)
/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3...
```

3.461.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.29

$$\int (a+bx^3)^{3/2} (c+dx+ex^2+fx^3+gx^4)\ dx = \frac{2\left(140049\,(17\,a^2bc-2\,a^3f)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-75735\,(19\,a^2bd-4\,a^3g)\sqrt{b}\text{weie}\right.}{}$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

output
```
2/4849845*(140049*(17*a^2*b*c - 2*a^3*f)*sqrt(b)*weierstrassPInverse(0, -4
*a/b, x) - 75735*(19*a^2*b*d - 4*a^3*g)*sqrt(b)*weierstrassZeta(0, -4*a/b,
 weierstrassPInverse(0, -4*a/b, x)) + (255255*b^3*g*x^8 + 285285*b^3*f*x^7
 + 323323*b^3*e*x^6 + 646646*a*b^2*e*x^3 + 19635*(19*b^3*d + 22*a*b^2*g)*x
^5 + 25935*(17*b^3*c + 20*a*b^2*f)*x^4 + 323323*a^2*b*e + 2805*(304*a*b^2*
d + 27*a^2*b*g)*x^2 + 5187*(238*a*b^2*c + 27*a^2*b*f)*x)*sqrt(b*x^3 + a))/
b^2
```

3.461.6 Sympy [A] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.64

$$\int \left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx = \frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}}fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{3}{2}}gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{\sqrt{a}bfx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt{a}bgx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + ae\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + be\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

input

```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + a**(3/2)*f*x**4*gamma(4/3)*hype
r((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(3/2)
*g*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(8/3)) + sqrt(a)*b*c*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d*x**5*gamma(5/3)*hyper((
-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + sqrt(a)*b*f
*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(10/3)) + sqrt(a)*b*g*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + a*e*Piecewise((sqrt(a)*x**3/3, Eq
(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True)) + b*e*Piecewise((-4*a**2*sqr
t(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a
 + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

3.461.7 Maxima [F]

$$\int \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2), x)
```

3.461.8 Giac [F]

$$\int \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(gx^4+fx^3+ex^2+dx+c\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2), x)
```

3.461. $\int \left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right)\,dx$

3.461.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)\,dx=\int \left(b\,x^3+a\right)^{3/2}\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

output
```
int((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4), x)
```

3.462 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$

3.462.1 Optimal result 3572
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3.462.1 Optimal result

Integrand size = 35, antiderivative size = 676

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \frac{2a^2f\sqrt{a+bx^3}}{15b}$$
$$+ \frac{54a^2gx\sqrt{a+bx^3}}{935b} + \frac{54a^2e\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{2(a+bx^3)^{3/2}(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5)}{109395x}$$
$$+ \frac{2a\sqrt{a+bx^3}(85085cx+41769dx^2+25245ex^3+17017fx^4+12285gx^5)}{255255x}$$
$$-\frac{2}{3}a^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/109395*(b*x^3+a)^(3/2)*(6435*g*x^5+7293*f*x^4+8415*e*x^3+9945*d*x^2+1215
5*c*x)/x-2/3*a^(3/2)*c*arctanh((b*x^3+a)^(1/2)/a^(1/2))+2/15*a^2*f*(b*x^3+
a)^(1/2)/b+54/935*a^2*g*x*(b*x^3+a)^(1/2)/b+2/255255*a*(12285*g*x^5+17017*
f*x^4+25245*e*x^3+41769*d*x^2+85085*c*x)*(b*x^3+a)^(1/2)/x+54/91*a^2*e*(b*
x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/91*3^(1/4)*a^(7/3)
*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^
(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)+18/85085*3^(3/4)*a^2*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(1547*b*d-182*a*g-935*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1
/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.462.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.45 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \frac{4\sqrt{1+\frac{bx^3}{a}}\left(\sqrt{a+bx^3}(a^2(51f+45gx)+b^2x^3(85c+51fx^3\right.}{}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

output
```
(4*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3]*(a^2*(51*f + 45*g*x) + b^2*x^3*(85
*c + 51*f*x^3 + 45*g*x^4) + 2*a*b*(170*c + 51*f*x^3 + 45*g*x^4)) - 255*a^(
3/2)*b*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) - 90*a*(-17*b*d + 2*a*g)*x*Sqrt
[a + b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 765*a*b*e*x^
2*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(1530*b
*Sqrt[1 + (b*x^3)/a])
```

3.462.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2365, 27, 2365, 27, 2371, 798, 73, 221, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}dx$$

$$\Big\downarrow \ 2365$$

$$\frac{9}{2}a\int \frac{2\sqrt{bx^3+a}\left(6435gx^4+7293fx^3+8415ex^2+9945dx+12155c\right)}{109395x}dx+\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$$\Big\downarrow \ 27$$

$$\frac{a\int \frac{\sqrt{bx^3+a}\left(6435gx^4+7293fx^3+8415ex^2+9945dx+12155c\right)}{x}dx}{12155}+\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$$\Big\downarrow \ 2365$$

$$\frac{a\left(\frac{3}{2}a\int \frac{2\left(12285gx^4+17017fx^3+25245ex^2+41769dx+85085c\right)}{21x\sqrt{bx^3+a}}dx+\frac{2\sqrt{a+bx^3}\left(85085cx+41769dx^2+25245ex^3+17017fx^4+12285gx^5\right)}{21x}\right)}{12155}+\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$$\Big\downarrow \ 27$$

$$\frac{a\left(\frac{1}{7}a\int \frac{12285gx^4+17017fx^3+25245ex^2+41769dx+85085c}{x\sqrt{bx^3+a}}dx+\frac{2\sqrt{a+bx^3}\left(85085cx+41769dx^2+25245ex^3+17017fx^4+12285gx^5\right)}{21x}\right)}{12155}+\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$$\Big\downarrow \ 2371$$

$$\frac{a\left(\frac{1}{7}a\left(85085c\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{12285gx^3+17017fx^2+25245ex+41769d}{\sqrt{bx^3+a}}dx\right)+\frac{2\sqrt{a+bx^3}\left(85085cx+41769dx^2+25245ex^3+17017fx^4+\right.}{21x}}{12155}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$\Big\downarrow$ 798

$$\frac{a\left(\frac{1}{7}a\left(\frac{85085}{3}c\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int \frac{12285gx^3+17017fx^2+25245ex+41769d}{\sqrt{bx^3+a}}dx\right)+\frac{2\sqrt{a+bx^3}\left(85085cx+41769dx^2+25245ex^3+17017fx^4\right.}{21x}}{12155}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$\Big\downarrow$ 73

$$\frac{a\left(\frac{1}{7}a\left(\frac{170170c\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int \frac{12285gx^3+17017fx^2+25245ex+41769d}{\sqrt{bx^3+a}}dx\right)+\frac{2\sqrt{a+bx^3}\left(85085cx+41769dx^2+25245ex^3+17017f\right.}{21x}}{12155}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$\Big\downarrow$ 221

$$\frac{a\left(\frac{1}{7}a\left(\int \frac{12285gx^3+17017fx^2+25245ex+41769d}{\sqrt{bx^3+a}}dx-\frac{170170c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}\right)+\frac{2\sqrt{a+bx^3}\left(85085cx+41769dx^2+25245ex^3+1701\right.}{21x}}{12155}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$\Big\downarrow$ 2427

$$\frac{a\left(\frac{1}{7}a\left(\frac{2\int \frac{5\left(17017bfx^2+25245bex+2457(17bd-2ag)\right)}{2\sqrt{bx^3+a}}dx}{5b}-\frac{170170c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{4914gx\sqrt{a+bx^3}}{b}\right)+\frac{2\sqrt{a+bx^3}\left(85085cx+41769d\right.}{}}{12155}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$\Big\downarrow$ 27

3.462. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}\,dx$

$$\frac{a\left(\frac{1}{7}a\left(\frac{\int \frac{17017bfx^2+25245bex+2457(17bd-2ag)}{\sqrt{bx^3+a}}dx}{b}-\frac{170170c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{4914gx\sqrt{a+bx^3}}{b}\right)+\frac{2\sqrt{a+bx^3}(85085cx+41769dx^2+2}{21}\right.}{12155}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$\downarrow$ 2425

$$\frac{a\left(\frac{1}{7}a\left(\frac{\int \frac{2457(17bd-2ag)+25245bex}{\sqrt{bx^3+a}}dx+17017bf\int \frac{x^2}{\sqrt{bx^3+a}}dx}{b}-\frac{170170c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{4914gx\sqrt{a+bx^3}}{b}\right)+\frac{2\sqrt{a+bx^3}(85085cx+}{}\right.}{12155}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$\downarrow$ 793

$$\frac{a\left(\frac{1}{7}a\left(\frac{\int \frac{2457(17bd-2ag)+25245bex}{\sqrt{bx^3+a}}dx+\frac{34034}{3}f\sqrt{a+bx^3}}{b}-\frac{170170c\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{4914gx\sqrt{a+bx^3}}{b}\right)+\frac{2\sqrt{a+bx^3}(85085cx+41769}{}\right.}{12155}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$\downarrow$ 2417

$$\frac{a\left(\frac{1}{7}a\left(\frac{27\left(-935\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-182ag+1547bd\right)\int \frac{1}{\sqrt{bx^3+a}}dx+25245b^{2/3}e\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{34034}{3}f\sqrt{a+bx^3}}{b}-\frac{170170c\,\text{arctan}}{3\sqrt{a}}\right.\right.}{12155}$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$\downarrow$ 759

3.462. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}\,dx$

$$a\left(\frac{1}{7}a\left(\frac{25245b^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{b}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

$\downarrow$ 2416

$$a\left(\frac{1}{7}a\left(\frac{\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(-935\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-182ag+\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395x}$$

input

```
Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x]
```

3.462. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x}\,dx$

output
```
(2*(a + b*x^3)^(3/2)*(12155*c*x + 9945*d*x^2 + 8415*e*x^3 + 7293*f*x^4 + 6
435*g*x^5))/(109395*x) + (a*((2*Sqrt[a + b*x^3]*(85085*c*x + 41769*d*x^2 +
 25245*e*x^3 + 17017*f*x^4 + 12285*g*x^5))/(21*x) + (a*((4914*g*x*Sqrt[a +
 b*x^3])/b - (170170*c*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]) + ((3
4034*f*Sqrt[a + b*x^3])/3 + 25245*b^(2/3)*e*((2*Sqrt[a + b*x^3])/(b^(1/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1547*b*d - 935*(
1 - Sqrt[3])*a^(1/3)*b^(2/3)*e - 182*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b))/7))
/12155
```

3.462.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

3.462. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.462. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.462.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 987, normalized size of antiderivative = 1.46

method	result	size
elliptic	Expression too large to display	987
default	Expression too large to display	1188

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
2/17*g*b*x^7*(b*x^3+a)^(1/2)+2/15*b*f*x^6*(b*x^3+a)^(1/2)+2/13*b*e*x^5*(b*
x^3+a)^(1/2)+2/11*(20/17*a*b*g+b^2*d)/b*x^4*(b*x^3+a)^(1/2)+2/9*(6/5*a*f*b
+b^2*c)/b*x^3*(b*x^3+a)^(1/2)+32/91*a*e*x^2*(b*x^3+a)^(1/2)+2/5*(a^2*g+2*a
*b*d-8/11*(20/17*a*b*g+b^2*d)/b*a)/b*x*(b*x^3+a)^(1/2)+2/3*(a^2*f+2*a*b*c-
2/3*(6/5*a*f*b+b^2*c)/b*a)/b*(b*x^3+a)^(1/2)-2/3*I*(a^2*d-2/5*(a^2*g+2*a*b
*d-8/11*(20/17*a*b*g+b^2*d)/b*a)/b*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-18/91*I*a^2*e*3^(1/2)/b*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/...
```

3.462.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \left[\frac{255255\,a^{\frac{3}{2}}b^2c\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)-}{}\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="fricas
")
```

output
```
[1/1531530*(255255*a^(3/2)*b^2*c*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*
a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 908820*a^2*b^(3/2)*e*weierstras
sZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 88452*(17*a^2*b*d -
2*a^3*g)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + 4*(45045*b^3*g*x^7 +
51051*b^3*f*x^6 + 58905*b^3*e*x^5 + 134640*a*b^2*e*x^2 + 4095*(17*b^3*d +
20*a*b^2*g)*x^4 + 340340*a*b^2*c + 51051*a^2*b*f + 17017*(5*b^3*c + 6*a*b^
2*f)*x^3 + 819*(238*a*b^2*d + 27*a^2*b*g)*x)*sqrt(b*x^3 + a))/b^2, 1/76576
5*(255255*sqrt(-a)*a*b^2*c*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)
) - 454410*a^2*b^(3/2)*e*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0,
 -4*a/b, x)) + 44226*(17*a^2*b*d - 2*a^3*g)*sqrt(b)*weierstrassPInverse(0,
 -4*a/b, x) + 2*(45045*b^3*g*x^7 + 51051*b^3*f*x^6 + 58905*b^3*e*x^5 + 134
640*a*b^2*e*x^2 + 4095*(17*b^3*d + 20*a*b^2*g)*x^4 + 340340*a*b^2*c + 5105
1*a^2*b*f + 17017*(5*b^3*c + 6*a*b^2*f)*x^3 + 819*(238*a*b^2*d + 27*a^2*b*
g)*x)*sqrt(b*x^3 + a))/b^2]
```

3.462. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx$

3.462.6 Sympy [A] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx =$$

$$-\frac{2a^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3}+\frac{a^{\frac{3}{2}}dx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$
$$+\frac{a^{\frac{3}{2}}ex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{a^{\frac{3}{2}}gx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$
$$+\frac{\sqrt{a}bdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}bex^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$
$$+\frac{\sqrt{a}bgx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{2a^2c}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2a\sqrt{b}cx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$
$$+af\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)+bc\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$
$$+bf\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x,x)
```

output
```
-2*a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a**(3/2)*d*x*gamma(1/3
)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a*
*(3/2)*e*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(5/3)) + a**(3/2)*g*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*d*x**4*gamma(4/3)*hy
per((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a
)*b*e*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(8/3)) + sqrt(a)*b*g*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**2*c/(3*sqrt(b)*x**(3/2)*s
qrt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*c*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + a
*f*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), Tru
e)) + b*c*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*
b), True)) + b*f*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*
sqrt(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*
x**6/6, True))
```

3.462.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="maxima
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x, x)
```

3.462.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x,x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x, x)
```

3.462.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x}\,dx$$

input
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x,x)
```

output
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x, x)
```

3.463 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

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3.463.8 Giac [F] . . . 3598
3.463.9 Mupad [F(-1)] . . . 3599

3.463.1 Optimal result

Integrand size = 35, antiderivative size = 692

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{2a^2g\sqrt{a+bx^3}}{15b} - \frac{27ac\sqrt{a+bx^3}}{7x} + \frac{27a(13bc+2af)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2a\sqrt{a+bx^3}(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5)}{15015x^2} + \frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2} - \frac{2}{3}a^{3/2}d\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bc+2af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\mathrm{ar}\right.}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/45045*(b*x^3+a)^(3/2)*(3003*g*x^5+3465*f*x^4+4095*e*x^3+5005*d*x^2+6435*
c*x)/x^2-2/3*a^(3/2)*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))+2/15*a^2*g*(b*x^3+
a)^(1/2)/b-27/7*a*c*(b*x^3+a)^(1/2)/x+2/15015*a*(1001*g*x^5+1485*f*x^4+245
7*e*x^3+5005*d*x^2+19305*c*x)*(b*x^3+a)^(1/2)/x^2+27/91*a*(2*a*f+13*b*c)*(
b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/182*3^(1/4)*a^(4
/3)*(2*a*f+13*b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(
1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+9/5005*3^(3/4)*a^(4/3)*(a^(1/3)+b^(1/3)*
x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(182*a^(2/3)*b^(1/3)*e-55*(2*a*f+13*b*c)*(1-3^(1/2)))*(1
/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.463.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.37 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{2g(a+bx^3)^{5/2}}{15b} + \frac{2}{9}d\left(\sqrt{a+bx^3}(4a+bx^3)-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right) - \frac{ac\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

output
```
(2*g*(a + b*x^3)^(5/2))/(15*b) + (2*d*(Sqrt[a + b*x^3]*(4*a + b*x^3) - 3*a
^(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))/9 - (a*c*Sqrt[a + b*x^3]*Hyperge
ometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)])/(x*Sqrt[1 + (b*x^3)/a]) + (a*e*
x*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1
+ (b*x^3)/a] + (a*f*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 2/3, 5/3,
-((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

3.463.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2365, 27, 2365, 27, 2374, 25, 2371, 798, 73, 221, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^2}dx$$

$$\Big\downarrow \text{2365}$$

$$\frac{9}{2}a\int \frac{2\sqrt{bx^3+a}\left(3003gx^4+3465fx^3+4095ex^2+5005dx+6435c\right)}{45045x^2}dx+\frac{2\left(a+bx^3\right)^{3/2}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{\sqrt{bx^3+a}\left(3003gx^4+3465fx^3+4095ex^2+5005dx+6435c\right)}{x^2}dx}{5005}+\frac{2\left(a+bx^3\right)^{3/2}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045x^2}$$

$$\Big\downarrow \text{2365}$$

$$\frac{a\left(\frac{3}{2}a\int \frac{2\left(1001gx^4+1485fx^3+2457ex^2+5005dx+19305c\right)}{3x^2\sqrt{bx^3+a}}dx+\frac{2\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5\right)}{3x^2}\right)}{5005}+\frac{2\left(a+bx^3\right)^{3/2}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\left(a\int \frac{1001gx^4+1485fx^3+2457ex^2+5005dx+19305c}{x^2\sqrt{bx^3+a}}dx+\frac{2\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5\right)}{3x^2}\right)}{5005}+\frac{2\left(a+bx^3\right)^{3/2}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045x^2}$$

$$\Big\downarrow \text{2374}$$

$$\frac{a\left(a\left(-\frac{\int -\frac{2002agx^3+1485(13bc+2af)x^2+4914aex+10010ad}{x\sqrt{bx^3+a}}dx}{2a}-\frac{19305c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+1485fx^4+1001g\right.}{3x^2}}{5005}$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045x^2}$$

$\Big\downarrow$ 25

$$\frac{a\left(a\left(\frac{\int \frac{2002agx^3+1485(13bc+2af)x^2+4914aex+10010ad}{x\sqrt{bx^3+a}}dx}{2a}-\frac{19305c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+1485fx^4+1001gx^5\right)}{3x^2}}{5005}$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045x^2}$$

$\Big\downarrow$ 2371

$$\frac{a\left(a\left(\frac{\int \frac{2002agx^2+1485(13bc+2af)x+4914ae}{\sqrt{bx^3+a}}dx+10010ad\int \frac{1}{x\sqrt{bx^3+a}}dx}{2a}-\frac{19305c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+1485f\right.}{3x^2}}{5005}$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045x^2}$$

$\Big\downarrow$ 798

$$\frac{a\left(a\left(\frac{\int \frac{2002agx^2+1485(13bc+2af)x+4914ae}{\sqrt{bx^3+a}}dx+\frac{10010}{3}ad\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{2a}-\frac{19305c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+1485\right.}{3x^2}}{5005}$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045x^2}$$

$\Big\downarrow$ 73

$$\frac{a\left(a\left(\frac{\int \frac{2002agx^2+1485(13bc+2af)x+4914ae}{\sqrt{bx^3+a}}dx+\frac{20020ad\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}}{2a}-\frac{19305c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}\left(19305cx+5005dx^2+2457ex^3+148\right.}{3x^2}}{5005}$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045x^2}$$

$\Big\downarrow$ 221

3.463. $\displaystyle \int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^2}\,dx$

$$\frac{a\left(a\left(\frac{\int \frac{2002agx^2+1485(13bc+2af)x+4914ae}{\sqrt{bx^3+a}}dx-\frac{20020}{3}\sqrt{a}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{2a}-\frac{19305c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}(19305cx+5005dx^2+2457ex}{3x^2}}{5005}$$

$$\frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2}$$

$\downarrow$ 2425

$$\frac{a\left(a\left(\frac{\int \frac{4914ae+1485(13bc+2af)x}{\sqrt{bx^3+a}}dx+2002ag\int \frac{x^2}{\sqrt{bx^3+a}}dx-\frac{20020}{3}\sqrt{a}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{2a}-\frac{19305c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}(19305cx+5005}{}}{5005}$$

$$\frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2}$$

$\downarrow$ 793

$$\frac{a\left(a\left(\frac{\int \frac{4914ae+1485(13bc+2af)x}{\sqrt{bx^3+a}}dx-\frac{20020}{3}\sqrt{a}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{4004ag\sqrt{a+bx^3}}{3b}}{2a}-\frac{19305c\sqrt{a+bx^3}}{ax}\right)+\frac{2\sqrt{a+bx^3}(19305cx+5005dx^2+}{3x}}{5005}$$

$$\frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2}$$

$\downarrow$ 2417

$$\frac{a\left(a\left(\frac{\frac{27\sqrt[3]{a}\left(182a^{2/3}e-\frac{55(1-\sqrt{3})(2af+13bc)}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+1485(2af+13bc)\int \frac{\sqrt[3]{b}x+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{20020}{3}\sqrt{a}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{4004a}{}}{2a}\right.\right.}{5005}$$

$$\frac{2(a+bx^3)^{3/2}(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5)}{45045x^2}$$

$\downarrow$ 759

3.463. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

$$a\left(a\left(\frac{\frac{1485(2af+13bc)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{2a}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045x^2}$$

$\downarrow$ 2416

$$a\left(a\left(\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(182a^{2/3}e-\frac{55\left(1-\sqrt{3}\right)(2af+1\right.}{\sqrt[3]{b}}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(6435cx+5005dx^2+4095ex^3+3465fx^4+3003gx^5\right)}{45045x^2}$$

input
```
Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x]
```

3.463. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

output
```
(2*(a + b*x^3)^(3/2)*(6435*c*x + 5005*d*x^2 + 4095*e*x^3 + 3465*f*x^4 + 30
03*g*x^5))/(45045*x^2) + (a*((2*Sqrt[a + b*x^3]*(19305*c*x + 5005*d*x^2 +
2457*e*x^3 + 1485*f*x^4 + 1001*g*x^5))/(3*x^2) + a*((-19305*c*Sqrt[a + b*x
^3])/(a*x) + ((4004*a*g*Sqrt[a + b*x^3])/(3*b) - (20020*Sqrt[a]*d*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]])/3 + (1485*(13*b*c + 2*a*f)*((2*Sqrt[a + b*x^3])/
(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]
*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[
3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]
*a^(1/3)*(182*a^(2/3)*e - (55*(1 - Sqrt[3])*(13*b*c + 2*a*f))/b^(1/3))*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3]))/(2*a))))/5005
```

3.463.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.463. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

3.463. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

3.463.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 946, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	946
default	Expression too large to display	1317
risch	Expression too large to display	3382

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a*c*(b*x^3+a)^(1/2)/x+2/15*g*b*x^6*(b*x^3+a)^(1/2)+2/13*b*f*x^5*(b*x^3+a)
^(1/2)+2/11*b*e*x^4*(b*x^3+a)^(1/2)+2/9*(6/5*a*b*g+b^2*d)/b*x^3*(b*x^3+a)^
(1/2)+2/7*(16/13*a*f*b+b^2*c)/b*x^2*(b*x^3+a)^(1/2)+28/55*a*e*x*(b*x^3+a)^
(1/2)+2/3*(a^2*g+2*a*b*d-2/3*(6/5*a*b*g+b^2*d)/b*a)/b*(b*x^3+a)^(1/2)-18/5
5*I*a^2*e*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))-2/3*I*(a^2*f+5/2*a*b*c-4/7*(16/13*a*f*b+b^2*c)/b*a)*3^(1/2)/b*(-a*b^
2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^...
```

3.463.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \left[\frac{15015\,a^{\frac{3}{2}}bdx\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+8\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="fric
as")
```

output
```
[1/90090*(15015*a^(3/2)*b*d*x*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*
sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 88452*a^2*sqrt(b)*e*x*weierstrassP
Inverse(0, -4*a/b, x) - 26730*(13*a*b*c + 2*a^2*f)*sqrt(b)*x*weierstrassZe
ta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + 2*(6006*b^2*g*x^7 + 693
0*b^2*f*x^6 + 8190*b^2*e*x^5 + 22932*a*b*e*x^2 + 2002*(5*b^2*d + 6*a*b*g)*
x^4 + 990*(13*b^2*c + 16*a*b*f)*x^3 - 45045*a*b*c + 2002*(20*a*b*d + 3*a^2
*g)*x)*sqrt(b*x^3 + a))/(b*x), 1/45045*(15015*sqrt(-a)*a*b*d*x*arctan(2*sq
rt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 44226*a^2*sqrt(b)*e*x*weierstrassP
Inverse(0, -4*a/b, x) - 13365*(13*a*b*c + 2*a^2*f)*sqrt(b)*x*weierstrassZe
ta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (6006*b^2*g*x^7 + 6930*
b^2*f*x^6 + 8190*b^2*e*x^5 + 22932*a*b*e*x^2 + 2002*(5*b^2*d + 6*a*b*g)*x^
4 + 990*(13*b^2*c + 16*a*b*f)*x^3 - 45045*a*b*c + 2002*(20*a*b*d + 3*a^2*g
)*x)*sqrt(b*x^3 + a))/(b*x)]
```

3.463. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx$

3.463.6 Sympy [A] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 474, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2a^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{a^{\frac{3}{2}}ex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}fx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bcx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}bfx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{2a^2d}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2a\sqrt{b}dx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + ag\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + bd\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right) + bg\left(\begin{cases}-\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^6}{6} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**2,x)
```

output
```
a**(3/2)*c*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x*gamma(2/3)) - 2*a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + a
**(3/2)*e*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(3/2)*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x**2*gamma(2/3)*hype
r((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*
b*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(7/3)) + sqrt(a)*b*f*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + 2*a**2*d/(3*sqrt(b)*x**(3/2)*sqrt(
a/(b*x**3) + 1)) + 2*a*sqrt(b)*d*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + a*g*P
iecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
 + b*d*Piecewise((sqrt(a)*x**3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b),
True)) + b*g*Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt
(a + b*x**3)/(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6
/6, True))
```

3.463.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2, x)
```

3.463.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^2,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^2, x)
```

3.463.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^2}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^2}\,dx$$

input
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2,x)
```

output
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^2, x)
```

3.464 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

3.464.1 Optimal result . . . 3600
3.464.2 Mathematica [C] (verified) . . . 3601
3.464.3 Rubi [A] (verified) . . . 3602
3.464.4 Maple [A] (verified) . . . 3608
3.464.5 Fricas [C] (verification not implemented) . . . 3608
3.464.6 Sympy [A] (verification not implemented) . . . 3610
3.464.7 Maxima [F] . . . 3611
3.464.8 Giac [F] . . . 3611
3.464.9 Mupad [F(-1)] . . . 3612

3.464.1 Optimal result

Integrand size = 35, antiderivative size = 694

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \frac{27ac\sqrt{a+bx^3}}{10x^2} - \frac{27ad\sqrt{a+bx^3}}{7x} + \frac{27a(13bd+2ag)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2a\sqrt{a+bx^3}(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5)}{15015x^3} + \frac{2(a+bx^3)^{3/2}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045x^3} - \frac{2}{3}a^{3/2}\text{earctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13bd+2ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\text{arc}\right.}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/45045*(b*x^3+a)^(3/2)*(3465*g*x^5+4095*f*x^4+5005*e*x^3+6435*d*x^2+9009*
c*x)/x^3-2/3*a^(3/2)*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))+27/10*a*c*(b*x^3+a
)^(1/2)/x^2-27/7*a*d*(b*x^3+a)^(1/2)/x-2/15015*a*(-1485*g*x^5-2457*f*x^4-5
005*e*x^3-19305*d*x^2+27027*c*x)*(b*x^3+a)^(1/2)/x^3+27/91*a*(2*a*g+13*b*d
)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/182*3^(1/4)*a
^(4/3)*(2*a*g+13*b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*
2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+9/10010*3^(3/4)*a*(a^(1/3)+b^(1/3)*x)
*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(91*b^(1/3)*(4*a*f+11*b*c)-110*a^(1/3)*(2*a*g+13*b*d)*(1-3
^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.464.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \frac{4ex^2\sqrt{1+\frac{bx^3}{a}}\left(\sqrt{a+bx^3}(4a+bx^3)-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right.\right.}{}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

output
```
(4*e*x^2*Sqrt[1 + (b*x^3)/a]*(Sqrt[a + b*x^3]*(4*a + b*x^3) - 3*a^(3/2)*Ar
cTanh[Sqrt[a + b*x^3]/Sqrt[a]]) - 9*a*c*Sqrt[a + b*x^3]*Hypergeometric2F1[
-3/2, -2/3, 1/3, -((b*x^3)/a)] - 18*a*d*x*Sqrt[a + b*x^3]*Hypergeometric2F
1[-3/2, -1/3, 2/3, -((b*x^3)/a)] + 18*a*f*x^3*Sqrt[a + b*x^3]*Hypergeometr
ic2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)] + 9*a*g*x^4*Sqrt[a + b*x^3]*Hypergeome
tric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(18*x^2*Sqrt[1 + (b*x^3)/a])
```

3.464.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2365, 27, 2365, 27, 2374, 2374, 27, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}dx$$

$$\Big\downarrow \text{2365}$$

$$\frac{9}{2}a\int \frac{2\sqrt{bx^3+a}\left(3465gx^4+4095fx^3+5005ex^2+6435dx+9009c\right)}{45045x^3}dx+\frac{2\left(a+bx^3\right)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{\sqrt{bx^3+a}\left(3465gx^4+4095fx^3+5005ex^2+6435dx+9009c\right)}{x^3}dx}{5005}+\frac{2\left(a+bx^3\right)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3}$$

$$\Big\downarrow \text{2365}$$

$$\frac{a\left(\frac{3}{2}a\int -\frac{2\left(-1485gx^4-2457fx^3-5005ex^2-19305dx+27027c\right)}{3x^3\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}\left(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5\right)}{3x^3}\right)}{5005}+\frac{2\left(a+bx^3\right)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\left(-a\int \frac{-1485gx^4-2457fx^3-5005ex^2-19305dx+27027c}{x^3\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}\left(27027cx-19305dx^2-5005ex^3-2457fx^4-1485gx^5\right)}{3x^3}\right)}{5005}+\frac{2\left(a+bx^3\right)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3}$$

$$\Big\downarrow \text{2374}$$

$$\frac{a\left(-a\left(-\frac{\int \frac{5940agx^3+2457(11bc+4af)x^2+20020aex+77220ad}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{27027c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}(27027cx-19305dx^2-5005ex^3-2457fx^4-148}{3x^3}}{5005}$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3}$$

$\Big\downarrow$ 2374

$$\frac{a\left(-a\left(-\frac{-\frac{\int -\frac{2\left(20020ea^2+2970(13bd+2ag)x^2a+2457(11bc+4af)xa\right)}{x\sqrt{bx^3+a}}dx}{2a}-\frac{77220d\sqrt{a+bx^3}}{x}}{4a}-\frac{27027c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}(27027cx-19305dx^2-}{3a}}{5005}$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3}$$

$\Big\downarrow$ 27

$$\frac{a\left(-a\left(-\frac{\frac{\int \frac{20020ea^2+2970(13bd+2ag)x^2a+2457(11bc+4af)xa}{x\sqrt{bx^3+a}}dx}{a}-\frac{77220d\sqrt{a+bx^3}}{x}}{4a}-\frac{27027c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}(27027cx-19305dx^2-5005e}{3x^3}}{5005}$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3}$$

$\Big\downarrow$ 2371

$$\frac{a\left(-a\left(-\frac{\frac{20020a^2e\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{2457a(11bc+4af)+2970a(13bd+2ag)x}{\sqrt{bx^3+a}}dx}{a}-\frac{77220d\sqrt{a+bx^3}}{x}}{4a}-\frac{27027c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}(27027cx-19305}{}}{5005}$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3}$$

$\Big\downarrow$ 798

$$\frac{a\left(-a\left(-\frac{\frac{\frac{20020}{3}a^2e\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int \frac{2457a(11bc+4af)+2970a(13bd+2ag)x}{\sqrt{bx^3+a}}dx}{a}-\frac{77220d\sqrt{a+bx^3}}{x}}{4a}-\frac{27027c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}(27027cx-193}{}}{5005}$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3}$$

$\Big\downarrow$ 73

3.464. $\displaystyle\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx$

$$a\left(-a\left(-\frac{\frac{\frac{40040a^2e\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int\frac{2457a(11bc+4af)+2970a(13bd+2ag)x}{\sqrt{bx^3+a}}dx}{a}-\frac{77220d\sqrt{a+bx^3}}{x}}{4a}-\frac{27027c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}(27027cx-1}{}\right.$$

$$5005$$

$$\frac{2(a+bx^3)^{3/2}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045x^3}$$

$\downarrow$ 221

$$a\left(-a\left(-\frac{\frac{\int\frac{2457a(11bc+4af)+2970a(13bd+2ag)x}{\sqrt{bx^3+a}}dx-\frac{40040}{3}a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a}-\frac{77220d\sqrt{a+bx^3}}{x}}{4a}-\frac{27027c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{2\sqrt{a+bx^3}(27027}{}\right.$$

$$5005$$

$$\frac{2(a+bx^3)^{3/2}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045x^3}$$

$\downarrow$ 2417

$$a\left(-a\left(-\frac{\frac{27a\left(91(4af+11bc)-\frac{110(1-\sqrt{3})\sqrt[3]{a}(2ag+13bd)}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{bx^3+a}}dx+\frac{2970a(2ag+13bd)\int\frac{\sqrt[3]{b}x+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{40040}{3}a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right.}{a}}{4a}\right.\right.$$

$$5005$$

$$\frac{2(a+bx^3)^{3/2}(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5)}{45045x^3}$$

$\downarrow$ 759

3.464. $\int\frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

$$a\left(-a\left(-\frac{\frac{2970a(2ag+13bd)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+}}}{\frac{a}{4a}}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3}$$

↓ 2416

$$a\left(-a\left(-\frac{\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(91(4af+11bc)-\frac{110\left(1-\sqrt{3}\right)}{}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(9009cx+6435dx^2+5005ex^3+4095fx^4+3465gx^5\right)}{45045x^3}$$

input

```
Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x]
```

3.464. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^3}\,dx$

output
```
(2*(a + b*x^3)^(3/2)*(9009*c*x + 6435*d*x^2 + 5005*e*x^3 + 4095*f*x^4 + 34
65*g*x^5))/(45045*x^3) + (a*((-2*Sqrt[a + b*x^3]*(27027*c*x - 19305*d*x^2
- 5005*e*x^3 - 2457*f*x^4 - 1485*g*x^5))/(3*x^3) - a*((-27027*c*Sqrt[a + b
*x^3])/(2*a*x^2) - ((-77220*d*Sqrt[a + b*x^3])/x + ((-40040*a^(3/2)*e*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/3 + (2970*a*(13*b*d + 2*a*g)*((2*Sqrt[a + b*
x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sq
rt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (18*3^(3/4)*Sqrt[2 + Sq
rt[3]]*a*(91*(11*b*c + 4*a*f) - (110*(1 - Sqrt[3])*a^(1/3)*(13*b*d + 2*a*g
))/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/a)/(4*a))))/5005
```

3.464.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

3.464. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.464. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

3.464.4 Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 941, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	941
default	Expression too large to display	1613
risch	Expression too large to display	3858

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*c*(b*x^3+a)^(1/2)/x^2-a*d*(b*x^3+a)^(1/2)/x+2/13*g*b*x^5*(b*x^3+a)^
(1/2)+2/11*b*f*x^4*(b*x^3+a)^(1/2)+2/9*b*e*x^3*(b*x^3+a)^(1/2)+2/7*(16/13*
a*b*g+b^2*d)/b*x^2*(b*x^3+a)^(1/2)+2/5*(14/11*a*f*b+b^2*c)/b*x*(b*x^3+a)^(
1/2)+8/9*a*e*(b*x^3+a)^(1/2)-2/3*I*(a^2*f+7/4*a*b*c-2/5*(14/11*a*f*b+b^2*c
)/b*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))-2/3*I*(a^2*g+5/2*a*b*d-4/7*(16/13*a*b*g+b^2*d)/b*a)*3^(1/2)/b*(-a*b^2)^
(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/...
```

3.464.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \left[\frac{15015\,a^{\frac{3}{2}}bex^2\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+ \right.$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="fric
as")
```

output
```
[1/90090*(15015*a^(3/2)*b*e*x^2*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a
)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 22113*(11*a*b*c + 4*a^2*f)*sqrt(
b)*x^2*weierstrassPInverse(0, -4*a/b, x) - 26730*(13*a*b*d + 2*a^2*g)*sqrt
(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (1
3860*b^2*g*x^7 + 16380*b^2*f*x^6 + 20020*b^2*e*x^5 + 80080*a*b*e*x^2 + 198
0*(13*b^2*d + 16*a*b*g)*x^4 - 90090*a*b*d*x + 3276*(11*b^2*c + 14*a*b*f)*x
^3 - 45045*a*b*c)*sqrt(b*x^3 + a))/(b*x^2), 1/90090*(30030*sqrt(-a)*a*b*e*
x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 22113*(11*a*b*c + 4
*a^2*f)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) - 26730*(13*a*b*d +
2*a^2*g)*sqrt(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*
a/b, x)) + (13860*b^2*g*x^7 + 16380*b^2*f*x^6 + 20020*b^2*e*x^5 + 80080*a*
b*e*x^2 + 1980*(13*b^2*d + 16*a*b*g)*x^4 - 90090*a*b*d*x + 3276*(11*b^2*c
+ 14*a*b*f)*x^3 - 45045*a*b*c)*sqrt(b*x^3 + a))/(b*x^2)]
```

3.464. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx$

3.464.6 Sympy [A] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 462, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2a^{\frac{3}{2}}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{a^{\frac{3}{2}}fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}gx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bcx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}bdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bfx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}bgx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{2a^2e}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2a\sqrt{b}ex^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + be\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**3,x)
```

output
```
a**(3/2)*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**2*gamma(1/3)) + a**(3/2)*d*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*e*asinh(sqrt(a)/
(sqrt(b)*x**(3/2)))/3 + a**(3/2)*f*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**(3/2)*g*x**2*gamma(2/3)*hyp
er((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)
*b*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(4/3)) + sqrt(a)*b*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**
3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*f*x**4*gamma(4/3)*hyper((-
1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*b*g*
x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*ga
mma(8/3)) + 2*a**2*e/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*a*sqrt(
b)*e*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + b*e*Piecewise((sqrt(a)*x**3/3, Eq
(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

3.464.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3, x)
```

3.464.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^3,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^3, x)
```

3.464.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^3}\,dx=\int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^3}\,dx$$

input
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3,x)
```

output
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^3, x)
```

3.465 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

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3.465.1 Optimal result

Integrand size = 35, antiderivative size = 692

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \frac{ac\sqrt{a+bx^3}}{x^3} + \frac{27ad\sqrt{a+bx^3}}{10x^2} - \frac{27ae\sqrt{a+bx^3}}{7x} + \frac{27a\sqrt[3]{b}e\sqrt{a+bx^3}}{7\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2a\sqrt{a+bx^3}(1155cx+2079dx^2-1485ex^3-385fx^4-189gx^5)}{1155x^4} + \frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4} - \frac{1}{3}\sqrt{a}(3bc+2af)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\sqrt[3]{b}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{14\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(77bd-110\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+28ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\text{ar}\right.}{770\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3465*(b*x^3+a)^(3/2)*(315*g*x^5+385*f*x^4+495*e*x^3+693*d*x^2+1155*c*x)/
x^4-1/3*(2*a*f+3*b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+a*c*(b*x^3+
a)^(1/2)/x^3+27/10*a*d*(b*x^3+a)^(1/2)/x^2-27/7*a*e*(b*x^3+a)^(1/2)/x-2/11
55*a*(-189*g*x^5-385*f*x^4-1485*e*x^3+2079*d*x^2+1155*c*x)*(b*x^3+a)^(1/2)
/x^4+27/7*a*b^(1/3)*e*(b*x^3+a)^(1/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/1
4*3^(1/4)*a^(4/3)*b^(1/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2
)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+9/770*3^(3/4)*a*(a^(1/3)+b^(1/3)*x)*Ell
ipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(77*b*d+28*a*g-110*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)
+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.465.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.60 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.35

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \frac{-45a^3d\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)-$$

input
```
Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

output
```
(-45*a^3*d*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)
] - 90*a^3*e*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3
)/a)] + 90*a^3*g*x^3*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((
b*x^3)/a)] + 4*x^2*Sqrt[1 + (b*x^3)/a]*(5*a^2*f*(Sqrt[a + b*x^3]*(4*a + b*
x^3) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) + 3*b*c*(a + b*x^3)^(5/
2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a]))/(90*a^2*x^2*Sqrt[1 + (b
*x^3)/a])
```

3.465.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {2365, 27, 2365, 27, 2374, 27, 2374, 27, 2374, 27, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^4}dx$$

$$\Big\downarrow \; 2365$$

$$\frac{9}{2}a\int \frac{2\sqrt{bx^3+a}\left(315gx^4+385fx^3+495ex^2+693dx+1155c\right)}{3465x^4}dx+\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{385}a\int \frac{\sqrt{bx^3+a}\left(315gx^4+385fx^3+495ex^2+693dx+1155c\right)}{x^4}dx+\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$$\Big\downarrow \; 2365$$

$$\frac{1}{385}a\left(\frac{3}{2}a\int -\frac{2\left(-189gx^4-385fx^3-1485ex^2+2079dx+1155c\right)}{3x^4\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}\left(1155cx+2079dx^2-1485ex\right.}{3x^4}\right.$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{385}a\left(-a\int \frac{-189gx^4-385fx^3-1485ex^2+2079dx+1155c}{x^4\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}\left(1155cx+2079dx^2-1485ex^3-3\right.}{3x^4}\right.$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$$\Big\downarrow \; 2374$$

$$\frac{1}{385}a\left(-a\left(-\frac{\int -\frac{3(-378agx^3-385(3bc+2af)x^2-2970aex+4158ad)}{x^3\sqrt{bx^3+a}}dx}{6a}-\frac{385c\sqrt{a+bx^3}}{ax^3}\right)-\frac{2\sqrt{a+bx^3}(1155cx+2079dx^2}{}\right.$$

$$\frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4}$$

$\downarrow$ 27

$$\frac{1}{385}a\left(-a\left(\frac{\int \frac{-378agx^3-385(3bc+2af)x^2-2970aex+4158ad}{x^3\sqrt{bx^3+a}}dx}{2a}-\frac{385c\sqrt{a+bx^3}}{ax^3}\right)-\frac{2\sqrt{a+bx^3}(1155cx+2079dx^2-148}{3x^4}\right.$$

$$\frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4}$$

$\downarrow$ 2374

$$\frac{1}{385}a\left(-a\left(\frac{-\frac{\int \frac{2(5940ea^2+189(11bd+4ag)x^2a+770(3bc+2af)xa)}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{2079d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{385c\sqrt{a+bx^3}}{ax^3}\right)-\frac{2\sqrt{a+bx^3}(1155cx+}{}\right.$$

$$\frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4}$$

$\downarrow$ 27

$$\frac{1}{385}a\left(-a\left(\frac{-\frac{\int \frac{5940ea^2+189(11bd+4ag)x^2a+770(3bc+2af)xa}{x^2\sqrt{bx^3+a}}dx}{2a}-\frac{2079d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{385c\sqrt{a+bx^3}}{ax^3}\right)-\frac{2\sqrt{a+bx^3}(1155cx+207}{}\right.$$

$$\frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4}$$

$\downarrow$ 2374

$$\frac{1}{385}a\left(-a\left(\frac{-\frac{-\frac{\int -\frac{2(2970bex^2a^2+770(3bc+2af)a^2+189(11bd+4ag)xa^2)}{x\sqrt{bx^3+a}}dx}{2a}-\frac{5940ae\sqrt{a+bx^3}}{x}}{2a}-\frac{2079d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{385c\sqrt{a+bx^3}}{ax^3}\right)-\frac{2\sqrt{a}}{}\right.$$

$$\frac{2(a+bx^3)^{3/2}(1155cx+693dx^2+495ex^3+385fx^4+315gx^5)}{3465x^4}$$

$\downarrow$ 27

3.465. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

$$\frac{1}{385}a\left(-a\left(\frac{-\frac{\frac{\int\frac{2970bex^2a^2+770(3bc+2af)a^2+189(11bd+4ag)xa^2}{x\sqrt{bx^3+a}}dx}{a}-\frac{5940ae\sqrt{a+bx^3}}{x}}{2a}-\frac{2079d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{385c\sqrt{a+bx^3}}{ax^3}\right)-\frac{2\sqrt{a+bx}}{}\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$\downarrow$ 2371

$$\frac{1}{385}a\left(-a\left(\frac{-\frac{\frac{770a^2(2af+3bc)\int\frac{1}{x\sqrt{bx^3+a}}dx+\int\frac{189(11bd+4ag)a^2+2970bexa^2}{\sqrt{bx^3+a}}dx}{a}-\frac{5940ae\sqrt{a+bx^3}}{x}}{2a}-\frac{2079d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{385c\sqrt{a+bx^3}}{ax^3}\right)-\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$\downarrow$ 798

$$\frac{1}{385}a\left(-a\left(\frac{-\frac{\frac{\frac{770}{3}a^2(2af+3bc)\int\frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int\frac{189(11bd+4ag)a^2+2970bexa^2}{\sqrt{bx^3+a}}dx}{a}-\frac{5940ae\sqrt{a+bx^3}}{x}}{2a}-\frac{2079d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{385c\sqrt{a+bx^3}}{ax^3}\right)\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$\downarrow$ 73

$$\frac{1}{385}a\left(-a\left(\frac{-\frac{\frac{\frac{1540a^2(2af+3bc)\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int\frac{189(11bd+4ag)a^2+2970bexa^2}{\sqrt{bx^3+a}}dx}{a}-\frac{5940ae\sqrt{a+bx^3}}{x}}{2a}-\frac{2079d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{385c\sqrt{a+bx^3}}{ax^3}\right)\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$\downarrow$ 221

3.465. $\int\frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^4}\,dx$

$$\frac{1}{385}a\left(-a\left(-\frac{\frac{\int\frac{189(11bd+4ag)a^2+2970bexa^2}{\sqrt{bx^3+a}}dx-\frac{1540}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2af+3bc)}{a}-\frac{5940ae\sqrt{a+bx^3}}{x}}{2a}-\frac{2079d\sqrt{a+bx^3}}{x^2}}{2a}-\frac{385c\sqrt{a+b}}{ax^3}\right)\right)$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$\downarrow$ 2417

$$\frac{1}{385}a\left(-a\left(-\frac{\frac{27a^2\left(-110\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+28ag+77bd\right)\int\frac{1}{\sqrt{bx^3+a}}dx+2970a^2b^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{1540}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2af+}{a}}{2a}}{2a}\right)\right)$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$\downarrow$ 759

$$\frac{1}{385}a\left(-a\left(-\frac{\frac{2970a^2b^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{1540}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2af+3bc)+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{}}{a}}{2a}}{2}\right)\right)$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

$\downarrow$ 2416

3.465. $\int\frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^4}\,dx$

$$\frac{1}{385}a\left(-a\left(-\frac{-\frac{1540}{3}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2af+3bc)+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(1}{\sqrt[3]{b}x+(1}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{}}}{}\right.\right.$$

$$\frac{2(a+bx^3)^{3/2}\left(1155cx+693dx^2+495ex^3+385fx^4+315gx^5\right)}{3465x^4}$$

input
```
Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x]
```

output
```
(2*(a + b*x^3)^(3/2)*(1155*c*x + 693*d*x^2 + 495*e*x^3 + 385*f*x^4 + 315*g
*x^5))/(3465*x^4) + (a*((-2*Sqrt[a + b*x^3]*(1155*c*x + 2079*d*x^2 - 1485*
e*x^3 - 385*f*x^4 - 189*g*x^5))/(3*x^4) - a*((-385*c*Sqrt[a + b*x^3])/(a*x
^3) + ((-2079*d*Sqrt[a + b*x^3])/x^2 - ((-5940*a*e*Sqrt[a + b*x^3])/x + ((
-1540*a^(3/2)*(3*b*c + 2*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 + 2970*a
^2*b^(2/3)*e*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (18*
3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(77*b*d - 110*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*
e + 28*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/a)/(2*a))/(2*a))))/385
```

3.465. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

3.465.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.465. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.465.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	920
default	Expression too large to display	1193
risch	Expression too large to display	2513

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*c*(b*x^3+a)^(1/2)/x^3-1/2*a*d*(b*x^3+a)^(1/2)/x^2-a*e*(b*x^3+a)^(1/
2)/x+2/11*g*b*x^4*(b*x^3+a)^(1/2)+2/9*b*f*x^3*(b*x^3+a)^(1/2)+2/7*b*e*x^2*
(b*x^3+a)^(1/2)+2/5*(14/11*a*b*g+b^2*d)/b*x*(b*x^3+a)^(1/2)+2/3*(4/3*a*f*b
+b^2*c)/b*(b*x^3+a)^(1/2)-2/3*I*(a^2*g+7/4*a*b*d-2/5*(14/11*a*b*g+b^2*d)/b
*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-
9/7*I*a*e*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(...
```

3.465.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.67 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \left[-\frac{53460\,ab^{\frac{3}{2}}ex^3\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInve}\right.}{}\right.$$

$$-\frac{26730\,ab^{\frac{3}{2}}ex^3\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-1155\left(3\,b^2c+2\,abf\right)\sqrt{-a}x^3\,\text{arcta}}{}$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="fric
as")
```

output
```
[-1/13860*(53460*a*b^(3/2)*e*x^3*weierstrassZeta(0, -4*a/b, weierstrassPIn
verse(0, -4*a/b, x)) - 1155*(3*b^2*c + 2*a*b*f)*sqrt(a)*x^3*log(-(b^2*x^6
+ 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 3402
*(11*a*b*d + 4*a^2*g)*sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) - 2*(1
260*b^2*g*x^7 + 1540*b^2*f*x^6 + 1980*b^2*e*x^5 - 6930*a*b*e*x^2 + 252*(11
*b^2*d + 14*a*b*g)*x^4 - 3465*a*b*d*x + 1540*(3*b^2*c + 4*a*b*f)*x^3 - 231
0*a*b*c)*sqrt(b*x^3 + a))/(b*x^3), -1/6930*(26730*a*b^(3/2)*e*x^3*weierstr
assZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - 1155*(3*b^2*c + 2*
a*b*f)*sqrt(-a)*x^3*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 170
1*(11*a*b*d + 4*a^2*g)*sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) - (12
60*b^2*g*x^7 + 1540*b^2*f*x^6 + 1980*b^2*e*x^5 - 6930*a*b*e*x^2 + 252*(11*
b^2*d + 14*a*b*g)*x^4 - 3465*a*b*d*x + 1540*(3*b^2*c + 4*a*b*f)*x^3 - 2310
*a*b*c)*sqrt(b*x^3 + a))/(b*x^3)]
```

3.465. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

3.465.6 Sympy [A] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2a^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{a^{\frac{3}{2}}gx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \sqrt{a}bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) + \frac{\sqrt{a}bdx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}bex^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt{a}bgx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^2f}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2a\sqrt{b}c}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2a\sqrt{b}fx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + \frac{2b^{\frac{3}{2}}cx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + bf\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**4,x)
```

output
```
a**(3/2)*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**2*gamma(1/3)) + a**(3/2)*e*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - 2*a**(3/2)*f*asinh(sqrt(a)/
(sqrt(b)*x**(3/2)))/3 + a**(3/2)*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - sqrt(a)*b*c*asinh(sqrt(a)/(sqr
t(b)*x**(3/2))) + sqrt(a)*b*d*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(a)*b*e*x**2*gamma(2/3)*hyper((
-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*g
*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(7/3)) + 2*a**2*f/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b
)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*c/(3*x**(3/2)*sqrt(a/(
b*x**3) + 1)) + 2*a*sqrt(b)*f*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + 2*b**(3/
2)*c*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + b*f*Piecewise((sqrt(a)*x**3/3, Eq
(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

3.465.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4, x)
```

3.465.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^4,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^4, x)
```

3.465. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx$

3.465.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^4}\,dx=\int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^4}\,dx$$

input
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4,x)
```

output
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^4, x)
```

3.466 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx$

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3.466.1 Optimal result

Integrand size = 35, antiderivative size = 741

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \frac{27ac\sqrt{a+bx^3}}{20x^4}+\frac{ad\sqrt{a+bx^3}}{x^3}$$
$$+\frac{27ae\sqrt{a+bx^3}}{10x^2}-\frac{27(7bc+8af)\sqrt{a+bx^3}}{56x}+\frac{27\sqrt[3]{b}(7bc+8af)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{2a\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-35gx^5)}{105x^5}$$
$$+\frac{2(a+bx^3)^{3/2}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^5}$$
$$-\frac{1}{3}\sqrt{a}(3bd+2ag)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}(7bc+8af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\cdots\right.}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(7bc+8af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\cdots}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/315*(b*x^3+a)^(3/2)*(35*g*x^5+45*f*x^4+63*e*x^3+105*d*x^2+315*c*x)/x^5-1
/3*(2*a*g+3*b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+27/20*a*c*(b*x^3
+a)^(1/2)/x^4+a*d*(b*x^3+a)^(1/2)/x^3+27/10*a*e*(b*x^3+a)^(1/2)/x^2-27/56*
(8*a*f+7*b*c)*(b*x^3+a)^(1/2)/x-2/105*a*(-35*g*x^5-135*f*x^4+189*e*x^3+105
*d*x^2+189*c*x)*(b*x^3+a)^(1/2)/x^5+27/56*b^(1/3)*(8*a*f+7*b*c)*(b*x^3+a)^
(1/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/112*3^(1/4)*a^(1/3)*b^(1/3)*(8*a*
f+7*b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)+9/280*3^(3/4)*a^(1/3)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*Elliptic
F((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(28*a^(2/3)*b^(1/3)*e-5*(8*a*f+7*b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2
*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.466.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \frac{-45a^3c\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

output
```
(-45*a^3*c*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a
)] - 90*a^3*e*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*
x^3)/a)] + 4*x^3*(-45*a^3*f*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -1/3,
2/3, -((b*x^3)/a)] + 2*x*Sqrt[1 + (b*x^3)/a]*(5*a^2*g*(Sqrt[a + b*x^3]*(4*
a + b*x^3) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]) + 3*b*d*(a + b*x^
3)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a])))/(180*a^2*x^4*Sqr
t[1 + (b*x^3)/a])
```

3.466.3 Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.04, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.543, Rules used = {2365, 27, 2365, 27, 2374, 25, 2374, 27, 2374, 27, 2374, 25, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}dx$$

$$\Big\downarrow \text{ 2365}$$

$$\frac{9}{2}a\int \frac{2\sqrt{bx^3+a}(35gx^4+45fx^3+63ex^2+105dx+315c)}{315x^5}dx+\frac{2(a+bx^3)^{3/2}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{35}a\int \frac{\sqrt{bx^3+a}(35gx^4+45fx^3+63ex^2+105dx+315c)}{x^5}dx+\frac{2(a+bx^3)^{3/2}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^5}$$

$$\Big\downarrow \text{ 2365}$$

$$\frac{1}{35}a\left(\frac{3}{2}a\int -\frac{2(-35gx^4-135fx^3+189ex^2+105dx+189c)}{3x^5\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx\right.$$
$$\frac{2(a+bx^3)^{3/2}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{35}a\left(-a\int \frac{-35gx^4-135fx^3+189ex^2+105dx+189c}{x^5\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}(189cx+105dx^2+189ex^3-135fx^4-3\right.$$
$$\frac{2(a+bx^3)^{3/2}(315cx+105dx^2+63ex^3+45fx^4+35gx^5)}{315x^5}$$

$$\Big\downarrow \text{ 2374}$$

$$\frac{1}{35}a\left(-a\left(-\frac{\int -\frac{-280agx^3-135(7bc+8af)x^2+1512aex+840ad}{x^4\sqrt{bx^3+a}}dx}{8a}-\frac{189c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}\left(189cx+105dx^2+189\right.}{3x^5}\right.$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$\downarrow$ 25

$$\frac{1}{35}a\left(-a\left(\frac{\int \frac{-280agx^3-135(7bc+8af)x^2+1512aex+840ad}{x^4\sqrt{bx^3+a}}dx}{8a}-\frac{189c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}\left(189cx+105dx^2+189ex^3\right.}{3x^5}\right.$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$\downarrow$ 2374

$$\frac{1}{35}a\left(-a\left(\frac{-\frac{\int -\frac{6\left(1512ea^2-140(3bd+2ag)x^2a-135(7bc+8af)xa\right)}{x^3\sqrt{bx^3+a}}dx}{6a}-\frac{280d\sqrt{a+bx^3}}{x^3}}{8a}-\frac{189c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}\left(189cx+105\right.}{}\right.$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$\downarrow$ 27

$$\frac{1}{35}a\left(-a\left(\frac{\frac{\int \frac{1512ea^2-140(3bd+2ag)x^2a-135(7bc+8af)xa}{x^3\sqrt{bx^3+a}}dx}{a}-\frac{280d\sqrt{a+bx^3}}{x^3}}{8a}-\frac{189c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}\left(189cx+105dx^2+\right.}{3x}\right.$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$\downarrow$ 2374

$$\frac{1}{35}a\left(-a\left(\frac{\frac{-\frac{\int \frac{4\left(378bex^2a^2+135(7bc+8af)a^2+140(3bd+2ag)xa^2\right)}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{756ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{x^3}}{8a}-\frac{189c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}\left(\right.}{}\right.$$
$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$\downarrow$ 27

3.466. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx$

$$\frac{1}{35}a\left(-a\left(\frac{\frac{-\frac{\int \frac{378bex^2a^2+135(7bc+8af)a^2+140(3bd+2ag)xa^2}{x^2\sqrt{bx^3+a}}dx}{a}-\frac{756ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{x^3}}{8a}-\frac{189c\sqrt{a+bx^3}}{4ax^4}\right)-\frac{2\sqrt{a+bx^3}(18\cdots}{}\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$$\Big\downarrow \; 2374$$

$$\frac{1}{35}a\left(-a\left(\frac{\frac{-\frac{\frac{\int -\frac{280(3bd+2ag)a^3+756bexa^3+135b(7bc+8af)x^2a^2}{x\sqrt{bx^3+a}}dx}{2a}-\frac{135a\sqrt{a+bx^3}(8af+7bc)}{x}}{a}-\frac{756ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{x^3}}{8a}-\frac{189c\sqrt{a+b\cdots}}{4ax^4}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{35}a\left(-a\left(\frac{\frac{-\frac{\frac{\int \frac{280(3bd+2ag)a^3+756bexa^3+135b(7bc+8af)x^2a^2}{x\sqrt{bx^3+a}}dx}{2a}-\frac{135a\sqrt{a+bx^3}(8af+7bc)}{x}}{a}-\frac{756ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{x^3}}{8a}-\frac{189c\sqrt{a+bx^3}}{4ax^4}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$$\Big\downarrow \; 2371$$

$$\frac{1}{35}a\left(-a\left(\frac{\frac{-\frac{\frac{280a^3(2ag+3bd)\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{756bea^3+135b(7bc+8af)xa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{135a\sqrt{a+bx^3}(8af+7bc)}{x}}{a}-\frac{756ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{x^3}}{8a}-\frac{189c\cdots}{4\cdots}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$$\Big\downarrow \; 798$$

3.466. $\displaystyle\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx$

$$\frac{1}{35}a\left(-a\left(\frac{-\frac{\frac{\frac{280}{3}a^3(2ag+3bd)\int\frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int\frac{756bea^3+135b(7bc+8af)xa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{135a\sqrt{a+bx^3}(8af+7bc)}{x}}{a}-\frac{756ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{x^3}}{8a}-\frac{189\cdots}{\cdots}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$\downarrow$ 73

$$\frac{1}{35}a\left(-a\left(\frac{-\frac{\frac{\frac{560a^3(2ag+3bd)\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int\frac{756bea^3+135b(7bc+8af)xa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{135a\sqrt{a+bx^3}(8af+7bc)}{x}}{a}-\frac{756ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{x^3}}{8a}-\frac{18\cdots}{\cdots}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$\downarrow$ 221

$$\frac{1}{35}a\left(-a\left(\frac{-\frac{\frac{\int\frac{756bea^3+135b(7bc+8af)xa^2}{\sqrt{bx^3+a}}dx-\frac{560}{3}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(2ag+3bd)}{2a}-\frac{135a\sqrt{a+bx^3}(8af+7bc)}{x}}{a}-\frac{756ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{x^3}}{8a}\cdots\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$\downarrow$ 2417

$$\frac{1}{35}a\left(-a\left(\frac{\frac{-\frac{27a^{7/3}b^{2/3}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(8af+7bc)\right)\int\frac{1}{\sqrt{bx^3+a}}dx+135a^2b^{2/3}(8af+7bc)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{560}{3}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b\cdots}}{\sqrt{a}}\right.\cdots}{2a}\cdots}{a}\cdots}{8a}\cdots\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$\downarrow$ 759

3.466. $\displaystyle\int\frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx$

$$\frac{1}{35}a\left(-a\left(\frac{-\frac{\frac{135a^2b^{2/3}(8af+7bc)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{}}{\sqrt[3]{}}\right.\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{}}}{2a}}{a}}{}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

$\Big\downarrow$ 2416

$$\frac{1}{35}a\left(-a\left(\frac{-\frac{\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(28a^{2/3}\sqrt[3]{b}e-5\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}}{}\right.\right.$$

$$\frac{2\left(a+bx^3\right)^{3/2}\left(315cx+105dx^2+63ex^3+45fx^4+35gx^5\right)}{315x^5}$$

input
```
Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x]
```

3.466. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx$

output
```
(2*(a + b*x^3)^(3/2)*(315*c*x + 105*d*x^2 + 63*e*x^3 + 45*f*x^4 + 35*g*x^5
))/(315*x^5) + (a*((-2*Sqrt[a + b*x^3]*(189*c*x + 105*d*x^2 + 189*e*x^3 -
135*f*x^4 - 35*g*x^5))/(3*x^5) - a*((-189*c*Sqrt[a + b*x^3])/(4*a*x^4) + (
(-280*d*Sqrt[a + b*x^3])/x^3 + ((-756*a*e*Sqrt[a + b*x^3])/x^2 - ((-135*a*
(7*b*c + 8*a*f)*Sqrt[a + b*x^3])/x + ((-560*a^(5/2)*(3*b*d + 2*a*g)*ArcTan
h[Sqrt[a + b*x^3]/Sqrt[a]])/3 + 135*a^2*b^(2/3)*(7*b*c + 8*a*f)*((2*Sqrt[a
 + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2
 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (18*3^(3/4)*Sqrt[2 + Sqrt[3
]]*a^(7/3)*b^(1/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(7*b*c + 8*a*f)
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3]))/(2*a))/a)/a)/(8*a))))/35
```

3.466.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.466. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

3.466. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.466.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.21

method	result	size
elliptic	Expression too large to display	900
default	Expression too large to display	1342
risch	Expression too large to display	2048

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*c*(b*x^3+a)^(1/2)/x^4-1/3*a*d*(b*x^3+a)^(1/2)/x^3-1/2*a*e*(b*x^3+a)
^(1/2)/x^2-(a*f+11/8*b*c)*(b*x^3+a)^(1/2)/x+2/9*g*b*x^3*(b*x^3+a)^(1/2)+2/
7*b*f*x^2*(b*x^3+a)^(1/2)+2/5*b*e*x*(b*x^3+a)^(1/2)+2/3*(4/3*a*b*g+b^2*d)/
b*(b*x^3+a)^(1/2)-9/10*I*a*e*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))-2/3*I*(27/14*a*f*b+27/16*b^2*c)*3^(1/2)/b*(-a*b^2)^
(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-...
```

3.466.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.82 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \left[\frac{6804\,a\sqrt{b}ex^4\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+210\,(3\,bd\ \cdots}{\cdots}\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="fric
as")
```

output
```
[1/2520*(6804*a*sqrt(b)*e*x^4*weierstrassPInverse(0, -4*a/b, x) + 210*(3*b
*d + 2*a*g)*sqrt(a)*x^4*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b
*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 1215*(7*b*c + 8*a*f)*sqrt(b)*x^4*weierst
rassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (560*b*g*x^7 + 72
0*b*f*x^6 + 1008*b*e*x^5 + 560*(3*b*d + 4*a*g)*x^4 - 1260*a*e*x^2 - 315*(1
1*b*c + 8*a*f)*x^3 - 840*a*d*x - 630*a*c)*sqrt(b*x^3 + a))/x^4, 1/2520*(68
04*a*sqrt(b)*e*x^4*weierstrassPInverse(0, -4*a/b, x) + 420*(3*b*d + 2*a*g)
*sqrt(-a)*x^4*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 1215*(7*b
*c + 8*a*f)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0,
-4*a/b, x)) + (560*b*g*x^7 + 720*b*f*x^6 + 1008*b*e*x^5 + 560*(3*b*d + 4*a
*g)*x^4 - 1260*a*e*x^2 - 315*(11*b*c + 8*a*f)*x^3 - 840*a*d*x - 630*a*c)*s
qrt(b*x^3 + a))/x^4]
```

3.466.6 Sympy [A] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 495, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{2a^{\frac{3}{2}}g\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{a}bc\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \sqrt{a}bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) + \frac{\sqrt{a}bex\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}bfx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{2a^2g}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}d\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2a\sqrt{b}d}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2a\sqrt{b}gx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + \frac{2b^{\frac{3}{2}}dx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + bg\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for}\ b=0\\\frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**5,x)
```

output
```
a**(3/2)*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*e*gamma(-2/3)*hyper((-2/3, -1/2), (1/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + a**(3/2)*f*gamma(-1/3)
*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) -
2*a**(3/2)*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + sqrt(a)*b*c*gamma(-1/3)
*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) -
sqrt(a)*b*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*e*x*gamma(1/3)*h
yper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqrt(
a)*b*f*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(5/3)) + 2*a**2*g/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) - a*
sqrt(b)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*d/(3*x**(3/2)*sq
rt(a/(b*x**3) + 1)) + 2*a*sqrt(b)*g*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + 2*
b**(3/2)*d*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + b*g*Piecewise((sqrt(a)*x**3
/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

3.466.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5, x)
```

3.466.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^5}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^5,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^5, x)
```

3.466.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^5}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{3/2}\,\left(g\,x^4+f\,x^3+e\,x^2+d\,x+c\right)}{x^5}\,dx$$

input
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5,x)
```

output
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^5, x)
```

3.467 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$

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3.467.1 Optimal result

Integrand size = 35, antiderivative size = 689

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \frac{27bc\sqrt{a+bx^3}}{20x^2} - \frac{27bd\sqrt{a+bx^3}}{8x}$$
$$+ \frac{27\sqrt[3]{b}(7bd+8ag)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)\left(a+bx^3\right)^{3/2}$$
$$- \frac{b\sqrt{a+bx^3}(252cx-315dx^2-140ex^3-126fx^4-180gx^5)}{140x^3} - \sqrt{a}be\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$
$$- \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}(7bd+8ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(14\sqrt[3]{b}(bc+2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd+8ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{El}}{280\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/60*(12*c/x^5+15*d/x^4+20*e/x^3+30*f/x^2+60*g/x)*(b*x^3+a)^(3/2)-b*e*arc
tanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+27/20*b*c*(b*x^3+a)^(1/2)/x^2-27/8*b
*d*(b*x^3+a)^(1/2)/x-1/140*b*(-180*g*x^5-126*f*x^4-140*e*x^3-315*d*x^2+252
*c*x)*(b*x^3+a)^(1/2)/x^3+27/56*b^(1/3)*(8*a*g+7*b*d)*(b*x^3+a)^(1/2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))-27/112*3^(1/4)*a^(1/3)*b^(1/3)*(8*a*g+7*b*d)*(
a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)+9/280*3^(3/4)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(14*b^(1/3)
*(2*a*f+b*c)-5*a^(1/3)*(8*a*g+7*b*d)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2)
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)
```

3.467.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.28

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \frac{\sqrt{a+bx^3}\Big(-12a^3c\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},-\frac{bx^3}{a}\right.$$

input
```
Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

output
```
(Sqrt[a + b*x^3]*(-12*a^3*c*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/
a)] - 15*a^3*d*x*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)] - 30*a^
3*f*x^3*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)] - 60*a^3*g*x^4*Hy
pergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)] + 8*b*e*x^5*(a + b*x^3)^2*S
qrt[1 + (b*x^3)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a]))/(60*a^2
*x^5*Sqrt[1 + (b*x^3)/a])
```

3.467.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2364, 27, 2365, 27, 2374, 27, 2374, 25, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2} \left(c+dx+ex^2+fx^3+gx^4\right)}{x^6} \, dx$$

$$\Big\downarrow \text{2364}$$

$$-\frac{9}{2}b\int -\frac{\sqrt{bx^3+a}(60gx^4+30fx^3+20ex^2+15dx+12c)}{60x^3}dx - \frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{40}b\int \frac{\sqrt{bx^3+a}(60gx^4+30fx^3+20ex^2+15dx+12c)}{x^3}dx - \frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{2365}$$

$$\frac{3}{40}b\left(\frac{3}{2}a\int -\frac{2(-180gx^4-126fx^3-140ex^2-315dx+252c)}{21x^3\sqrt{bx^3+a}}dx - \frac{2\sqrt{a+bx^3}(252cx-315dx^2-140ex^3-126f}{21x^3}\right. \\ \left.- \frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{40}b\left(-\frac{1}{7}a\int \frac{-180gx^4-126fx^3-140ex^2-315dx+252c}{x^3\sqrt{bx^3+a}}dx - \frac{2\sqrt{a+bx^3}(252cx-315dx^2-140ex^3-126fx^4}{21x^3}\right. \\ \left.- \frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{2374}$$

$$\frac{3}{40}b\left(-\frac{1}{7}a\left(-\frac{\int \frac{4(180agx^3+63(bc+2af)x^2+140aex+315ad)}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{126c\sqrt{a+bx^3}}{ax^2}\right)-\frac{2\sqrt{a+bx^3}(252cx-315dx^2-140ex}{21x^3}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$\downarrow$ 27

$$\frac{3}{40}b\left(-\frac{1}{7}a\left(-\frac{\int \frac{180agx^3+63(bc+2af)x^2+140aex+315ad}{x^2\sqrt{bx^3+a}}dx}{a}-\frac{126c\sqrt{a+bx^3}}{ax^2}\right)-\frac{2\sqrt{a+bx^3}(252cx-315dx^2-140ex^3}{21x^3}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$\downarrow$ 2374

$$\frac{3}{40}b\left(-\frac{1}{7}a\left(-\frac{-\frac{\int -\frac{280ea^2+45(7bd+8ag)x^2a+126(bc+2af)xa}{x\sqrt{bx^3+a}}dx}{2a}-\frac{315d\sqrt{a+bx^3}}{x}}{a}-\frac{126c\sqrt{a+bx^3}}{ax^2}\right)-\frac{2\sqrt{a+bx^3}(252cx-315d}{}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$\downarrow$ 25

$$\frac{3}{40}b\left(-\frac{1}{7}a\left(-\frac{\frac{\int \frac{280ea^2+45(7bd+8ag)x^2a+126(bc+2af)xa}{x\sqrt{bx^3+a}}dx}{2a}-\frac{315d\sqrt{a+bx^3}}{x}}{a}-\frac{126c\sqrt{a+bx^3}}{ax^2}\right)-\frac{2\sqrt{a+bx^3}(252cx-315dx^2}{2}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$\downarrow$ 2371

$$\frac{3}{40}b\left(-\frac{1}{7}a\left(-\frac{\frac{280a^2e\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{126a(bc+2af)+45a(7bd+8ag)x}{\sqrt{bx^3+a}}dx}{2a}-\frac{315d\sqrt{a+bx^3}}{x}}{a}-\frac{126c\sqrt{a+bx^3}}{ax^2}\right)-\frac{2\sqrt{a+bx^3}(252cx}{}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$\downarrow$ 798

3.467. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$

$$\frac{3}{40}b\left(-\frac{1}{7}a\left(-\frac{\frac{\frac{280}{3}a^2e\int\frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int\frac{126a(bc+2af)+45a(7bd+8ag)x}{\sqrt{bx^3+a}}dx}{2a}-\frac{315d\sqrt{a+bx^3}}{x}}{a}-\frac{126c\sqrt{a+bx^3}}{ax^2}\right)-\frac{2\sqrt{a+bx^3}(252c}{}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$\downarrow$ 73

$$\frac{3}{40}b\left(-\frac{1}{7}a\left(-\frac{\frac{\frac{560a^2e\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int\frac{126a(bc+2af)+45a(7bd+8ag)x}{\sqrt{bx^3+a}}dx}{2a}-\frac{315d\sqrt{a+bx^3}}{x}}{a}-\frac{126c\sqrt{a+bx^3}}{ax^2}\right)-\frac{2\sqrt{a+bx^3}(252c}{}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$\downarrow$ 221

$$\frac{3}{40}b\left(-\frac{1}{7}a\left(-\frac{\frac{\int\frac{126a(bc+2af)+45a(7bd+8ag)x}{\sqrt{bx^3+a}}dx-\frac{560}{3}a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{2a}-\frac{315d\sqrt{a+bx^3}}{x}}{a}-\frac{126c\sqrt{a+bx^3}}{ax^2}\right)-\frac{2\sqrt{a+bx^3}}{}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$\downarrow$ 2417

$$\frac{3}{40}b\left(-\frac{1}{7}a\left(-\frac{\frac{9a\left(14(2af+bc)-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(8ag+7bd)}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{bx^3+a}}dx+\frac{45a(8ag+7bd)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{560}{3}a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right.}{2a}}{a}\right.\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$\downarrow$ 759

3.467. $\int\frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}\,dx$

$$\frac{3}{40}b\left(-\frac{1}{7}a\left(-\frac{\frac{\frac{45a(8ag+7bd)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\cdots}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\cdots}\right)\cdots\right)\cdots}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+b\cdots}}}{2a}\cdots}{a}\cdots\right)\cdots\right)$$

$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

$$\Big\downarrow \text{2416}$$

$$\frac{3}{40}b\left(-\frac{1}{7}a\left(-\frac{\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(14(2af+bc)-\frac{5\left(1-\sqrt{3}\right)\cdots}{\cdots}\cdots\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots}{\cdots}\cdots\right)\cdots\right)$$

$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}+\frac{60g}{x}\right)$$

input
```
Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x]
```

3.467. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^6}\,dx$

output
```
-1/60*(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2 + (60*g)/x)*(a +
 b*x^3)^(3/2)) + (3*b*((-2*Sqrt[a + b*x^3]*(252*c*x - 315*d*x^2 - 140*e*x^
3 - 126*f*x^4 - 180*g*x^5))/(21*x^3) - (a*((-126*c*Sqrt[a + b*x^3])/(a*x^2
) - ((-315*d*Sqrt[a + b*x^3])/x + ((-560*a^(3/2)*e*ArcTanh[Sqrt[a + b*x^3]
/Sqrt[a]])/3 + (45*a*(7*b*d + 8*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])))/b^(1/3) + (6*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(14*(b*c + 2*a
*f) - (5*(1 - Sqrt[3])*a^(1/3)*(7*b*d + 8*a*g))/b^(1/3))*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b
*x^3]))/(2*a))/a))/7))/40
```

3.467.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.467. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

3.467. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.467.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	920
default	Expression too large to display	1606
risch	Expression too large to display	2289

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a*c*(b*x^3+a)^(1/2)/x^5-1/4*a*d*(b*x^3+a)^(1/2)/x^4-1/3*a*e*(b*x^3+a)
^(1/2)/x^3-1/2*(a*f+13/10*b*c)*(b*x^3+a)^(1/2)/x^2-(a*g+11/8*b*d)*(b*x^3+a
)^(1/2)/x+2/7*g*b*x^2*(b*x^3+a)^(1/2)+2/5*b*f*x*(b*x^3+a)^(1/2)+2/3*b*e*(b
*x^3+a)^(1/2)-2/3*I*(8/5*a*f*b+b^2*c-1/40*b*(10*a*f+13*b*c))*3^(1/2)/b*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(10/7*a*b*g
+b^2*d+1/16*b*(8*a*g+11*b*d))*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(...
```

3.467.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \left[\frac{210\sqrt{a}bex^5\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+1\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="fric
as")
```

output
```
[1/840*(210*sqrt(a)*b*e*x^5*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sq
rt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 1134*(b*c + 2*a*f)*sqrt(b)*x^5*weier
strassPInverse(0, -4*a/b, x) - 405*(7*b*d + 8*a*g)*sqrt(b)*x^5*weierstrass
Zeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (240*b*g*x^7 + 336*b*
f*x^6 + 560*b*e*x^5 - 105*(11*b*d + 8*a*g)*x^4 - 280*a*e*x^2 - 42*(13*b*c
+ 10*a*f)*x^3 - 210*a*d*x - 168*a*c)*sqrt(b*x^3 + a))/x^5, 1/840*(420*sqrt
(-a)*b*e*x^5*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) + 1134*(b*c
+ 2*a*f)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) - 405*(7*b*d + 8*a*
g)*sqrt(b)*x^5*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x
)) + (240*b*g*x^7 + 336*b*f*x^6 + 560*b*e*x^5 - 105*(11*b*d + 8*a*g)*x^4 -
 280*a*e*x^2 - 42*(13*b*c + 10*a*f)*x^3 - 210*a*d*x - 168*a*c)*sqrt(b*x^3
+ a))/x^5]
```

3.467.6 Sympy [A] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 476, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}g\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \sqrt{a}be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) + \frac{\sqrt{a}bfx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}bgx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{a\sqrt{b}e\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2a\sqrt{b}e}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2b^{\frac{3}{2}}ex^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

input
```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**6,x)
```

output
```
a**(3/2)*c*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**5*gamma(-2/3)) + a**(3/2)*d*gamma(-4/3)*hyper((-4/3, -1/2), (-1/
3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*f*gamma(-2/
3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3
)) + a**(3/2)*g*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I
*pi)/a)/(3*x*gamma(2/3)) + sqrt(a)*b*c*gamma(-2/3)*hyper((-2/3, -1/2), (1/
3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*d*gamma(-1/
3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3))
- sqrt(a)*b*e*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*f*x*gamma(1/3)
*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + sqr
t(a)*b*g*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(5/3)) - a*sqrt(b)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*s
qrt(b)*e/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*e*x**(3/2)/(3*sqrt
(a/(b*x**3) + 1))
```

3.467.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6, x)
```

3.467.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^6,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^6, x)
```

3.467.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^6}\,dx = \int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^6}\,dx$$

input
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6,x)
```

output
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^6, x)
```

3.468 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$

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3.468.1 Optimal result

Integrand size = 35, antiderivative size = 692

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \frac{bc\sqrt{a+bx^3}}{4x^3} + \frac{27bd\sqrt{a+bx^3}}{20x^2} - \frac{27be\sqrt{a+bx^3}}{8x} + \frac{27b^{4/3}e\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)(a+bx^3)^{3/2} - \frac{b\sqrt{a+bx^3}(10cx+36dx^2-45ex^3-20fx^4-18gx^5)}{20x^4} - \frac{b(bc+4af)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}b^{4/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(2bd-5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arcs}\right.}{40\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/60*(10*c/x^6+12*d/x^5+15*e/x^4+20*f/x^3+30*g/x^2)*(b*x^3+a)^(3/2)-1/4*b
*(4*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+1/4*b*c*(b*x^3+a)^(1
/2)/x^3+27/20*b*d*(b*x^3+a)^(1/2)/x^2-27/8*b*e*(b*x^3+a)^(1/2)/x-1/20*b*(-
18*g*x^5-20*f*x^4-45*e*x^3+36*d*x^2+10*c*x)*(b*x^3+a)^(1/2)/x^4+27/8*b^(4/
3)*e*(b*x^3+a)^(1/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/16*3^(1/4)*a^(1/3)
*b^(4/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)+9/40*3^(3/4)*b^(2/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(2*b*d+4*a*g-5*a^(1/3)*b^(2/3)*e*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

3.468.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.56 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.35

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \frac{-\frac{15bc(a+bx^3)}{x^3} - \frac{10c(a+bx^3)^2}{x^6} - 15b^2c\sqrt{1+\frac{bx^3}{a}}\operatorname{arctanh}\left(\sqrt{1+\right.}}{}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

output
```
((-15*b*c*(a + b*x^3))/x^3 - (10*c*(a + b*x^3)^2)/x^6 - 15*b^2*c*Sqrt[1 +
(b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]] - (12*a^2*d*Sqrt[1 + (b*x^3)/a]*Hy
pergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/x^5 - (15*a^2*e*Sqrt[1 +
(b*x^3)/a]*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])/x^4 - (30*a^
2*g*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)])/
x^2 + (8*b*f*(a + b*x^3)^3*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a])/
a^2)/(60*Sqrt[a + b*x^3])
```

3.468.3 Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {2364, 27, 2365, 27, 2374, 27, 2374, 27, 2374, 25, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}dx$$

$$\Big\downarrow \text{ 2364}$$

$$-\frac{9}{2}b\int -\frac{\sqrt{bx^3+a}\left(30gx^4+20fx^3+15ex^2+12dx+10c\right)}{60x^4}dx-\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{40}b\int \frac{\sqrt{bx^3+a}\left(30gx^4+20fx^3+15ex^2+12dx+10c\right)}{x^4}dx-\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \text{ 2365}$$

$$\frac{3}{40}b\left(\frac{3}{2}a\int -\frac{2\left(-18gx^4-20fx^3-45ex^2+36dx+10c\right)}{3x^4\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}\left(10cx+36dx^2-45ex^3-20fx^4-18gx^5\right)}{3x^4}\right)-\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{40}b\left(-a\int \frac{-18gx^4-20fx^3-45ex^2+36dx+10c}{x^4\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}\left(10cx+36dx^2-45ex^3-20fx^4-18gx^5\right)}{3x^4}\right)-\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \text{ 2374}$$

$$\frac{3}{40}b\left(-a\left(-\frac{\int -\frac{6(-18agx^3-5(bc+4af)x^2-45aex+36ad)}{x^3\sqrt{bx^3+a}}dx}{6a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}(10cx+36dx^2-45ex^3-20}{3x^4}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$\downarrow$ 27

$$\frac{3}{40}b\left(-a\left(\frac{\int \frac{-18agx^3-5(bc+4af)x^2-45aex+36ad}{x^3\sqrt{bx^3+a}}dx}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}(10cx+36dx^2-45ex^3-20fx^4-}{3x^4}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$\downarrow$ 2374

$$\frac{3}{40}b\left(-a\left(\frac{-\frac{\int \frac{4\left(45ea^2+9(bd+2ag)x^2a+5(bc+4af)xa\right)}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{18d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}(10cx+36dx^2-45ex^3}{3x^4}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$\downarrow$ 27

$$\frac{3}{40}b\left(-a\left(\frac{-\frac{\int \frac{45ea^2+9(bd+2ag)x^2a+5(bc+4af)xa}{x^2\sqrt{bx^3+a}}dx}{a}-\frac{18d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}(10cx+36dx^2-45ex^3-}{3x^4}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$\downarrow$ 2374

$$\frac{3}{40}b\left(-a\left(\frac{-\frac{-\frac{\int -\frac{45bex^2a^2+10(bc+4af)a^2+18(bd+2ag)xa^2}{x\sqrt{bx^3+a}}dx}{2a}-\frac{45ae\sqrt{a+bx^3}}{x}}{a}-\frac{18d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}(10cx+}{}\right.$$
$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$\downarrow$ 25

3.468. $\displaystyle\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx$

$$\frac{3}{40}b\left(-a\left(-\frac{\frac{\frac{\int \frac{45bex^2a^2+10(bc+4af)a^2+18(bd+2ag)xa^2}{x\sqrt{bx^3+a}}dx}{2a}-\frac{45ae\sqrt{a+bx^3}}{x}}{a}-\frac{18d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}\left(10cx+30\right.}{}\right.$$

$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \; 2371$$

$$\frac{3}{40}b\left(-a\left(-\frac{\frac{\frac{10a^2(4af+bc)\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{18(bd+2ag)a^2+45bexa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{45ae\sqrt{a+bx^3}}{x}}{a}-\frac{18d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx^3}\left(\right.}{}\right.$$

$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \; 798$$

$$\frac{3}{40}b\left(-a\left(-\frac{\frac{\frac{\frac{10}{3}a^2(4af+bc)\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int \frac{18(bd+2ag)a^2+45bexa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{45ae\sqrt{a+bx^3}}{x}}{a}-\frac{18d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+bx}}{}\right.$$

$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{3}{40}b\left(-a\left(-\frac{\frac{\frac{\frac{20a^2(4af+bc)\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int \frac{18(bd+2ag)a^2+45bexa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{45ae\sqrt{a+bx^3}}{x}}{a}-\frac{18d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a+b}}{}\right.$$

$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{3}{40}b\left(-a\left(-\frac{\frac{\frac{\int \frac{18(bd+2ag)a^2+45bexa^2}{\sqrt{bx^3+a}}dx-\frac{20}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(4af+bc)}{2a}-\frac{45ae\sqrt{a+bx^3}}{x}}{a}-\frac{18d\sqrt{a+bx^3}}{x^2}}{a}-\frac{10c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{2\sqrt{a}}{}\right.$$

$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

3.468. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx$

$\downarrow$ 2417

$$\frac{3}{40}b\left(-a\left(\frac{-\frac{\frac{9a^2\left(-5\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e+4ag+2bd\right)\int\frac{1}{\sqrt{bx^3+a}}dx+45a^2b^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{20}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(4af+bc)}{2a}-\frac{45ae\sqrt{}}{}}{a}}{a}\right.\right.$$

$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$\downarrow$ 759

$$\frac{3}{40}b\left(-a\left(\frac{-\frac{\frac{45a^2b^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{20}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(4af+bc)+\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}}}{\sqrt[3]{b}}}{2a}}{a}}{a}\right.\right.$$

$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

$\downarrow$ 2416

3.468. $\int\frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx$

$$\frac{3}{40}b\left(-a\left(\frac{-\frac{-\frac{20}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(4af+bc)+\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\cdots}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\cdots}\right)\cdots\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots}{\cdots}}{\cdots}\cdots\right)\cdots\right)$$

$$\frac{1}{60}\left(a+bx^3\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}+\frac{30g}{x^2}\right)$$

input
```
Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x]
```

output
```
-1/60*(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3 + (30*g)/x^2)*(a
 + b*x^3)^(3/2)) + (3*b*((-2*Sqrt[a + b*x^3]*(10*c*x + 36*d*x^2 - 45*e*x^3
 - 20*f*x^4 - 18*g*x^5))/(3*x^4) - a*((-10*c*Sqrt[a + b*x^3])/(3*a*x^3) +
((-18*d*Sqrt[a + b*x^3])/x^2 - ((-45*a*e*Sqrt[a + b*x^3])/x + ((-20*a^(3/2
)*(b*c + 4*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 + 45*a^2*b^(2/3)*e*((2
*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)
*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (6*3^(3/4)*Sqrt[2 +
Sqrt[3]]*a^2*(2*b*d - 5*(1 - Sqrt[3])*a^(1/3)*b^(2/3)*e + 4*a*g)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3]))/(2*a))/a)/a)))/40
```

3.468. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^7}\,dx$

3.468.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

3.468. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.468. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx$

3.468.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.30

method	result	size
elliptic	Expression too large to display	903
default	Expression too large to display	1196
risch	Expression too large to display	1411

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*c*(b*x^3+a)^(1/2)/x^6-1/5*a*d*(b*x^3+a)^(1/2)/x^5-1/4*a*e*(b*x^3+a)
^(1/2)/x^4-1/3*(a*f+5/4*b*c)*(b*x^3+a)^(1/2)/x^3-1/2*(a*g+13/10*b*d)*(b*x^
3+a)^(1/2)/x^2-11/8*b*e*(b*x^3+a)^(1/2)/x+2/5*g*b*x*(b*x^3+a)^(1/2)+2/3*b*
f*(b*x^3+a)^(1/2)-2/3*I*(8/5*a*b*g+b^2*d-1/40*b*(10*a*g+13*b*d))*3^(1/2)/b
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-9/8*I*b*e*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)...
```

3.468.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \left[-\frac{810\,ab^{\frac{3}{2}}ex^6\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\right.}{}\right.$$

$$-\frac{405\,ab^{\frac{3}{2}}ex^6\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)-15\,(b^2c+4\,abf)\sqrt{-a}x^6\arctan\left(\frac{2\sqrt{}}{}\right.}{}$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="fric
as")
```

output
```
[-1/240*(810*a*b^(3/2)*e*x^6*weierstrassZeta(0, -4*a/b, weierstrassPInvers
e(0, -4*a/b, x)) - 15*(b^2*c + 4*a*b*f)*sqrt(a)*x^6*log(-(b^2*x^6 + 8*a*b*
x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 324*(a*b*d +
 2*a^2*g)*sqrt(b)*x^6*weierstrassPInverse(0, -4*a/b, x) - 2*(48*a*b*g*x^7
+ 80*a*b*f*x^6 - 165*a*b*e*x^5 - 30*a^2*e*x^2 - 6*(13*a*b*d + 10*a^2*g)*x^
4 - 24*a^2*d*x - 10*(5*a*b*c + 4*a^2*f)*x^3 - 20*a^2*c)*sqrt(b*x^3 + a))/(
a*x^6), -1/120*(405*a*b^(3/2)*e*x^6*weierstrassZeta(0, -4*a/b, weierstrass
PInverse(0, -4*a/b, x)) - 15*(b^2*c + 4*a*b*f)*sqrt(-a)*x^6*arctan(2*sqrt(
b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 162*(a*b*d + 2*a^2*g)*sqrt(b)*x^6*wei
erstrassPInverse(0, -4*a/b, x) - (48*a*b*g*x^7 + 80*a*b*f*x^6 - 165*a*b*e*
x^5 - 30*a^2*e*x^2 - 6*(13*a*b*d + 10*a^2*g)*x^4 - 24*a^2*d*x - 10*(5*a*b*
c + 4*a^2*f)*x^3 - 20*a^2*c)*sqrt(b*x^3 + a))/(a*x^6)]
```

3.468.6 Sympy [A] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{a^{\frac{3}{2}}g\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \sqrt{a}bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) + \frac{\sqrt{a}bgx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{a^2c}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}c}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}f\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2a\sqrt{b}f}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}c}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2b^{\frac{3}{2}}fx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} - \frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}}$$

input
```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**7,x)
```

output
```
a**(3/2)*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**5*gamma(-2/3)) + a**(3/2)*e*gamma(-4/3)*hyper((-4/3, -1/2), (-1/
3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + a**(3/2)*g*gamma(-2/
3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3
)) + sqrt(a)*b*d*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(
I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*e*gamma(-1/3)*hyper((-1/2, -1/3),
 (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - sqrt(a)*b*f*asinh(sq
rt(a)/(sqrt(b)*x**(3/2))) + sqrt(a)*b*g*x*gamma(1/3)*hyper((-1/2, 1/3), (4
/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - a**2*c/(6*sqrt(b)*x**(15/
2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*c/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) -
 a*sqrt(b)*f*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*f/(3*x**(3/2)
*sqrt(a/(b*x**3) + 1)) - b**(3/2)*c*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b*
*(3/2)*c/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*f*x**(3/2)/(3*sqr
t(a/(b*x**3) + 1)) - b**2*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))
```

3.468.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="maxi
ma")
```

output
```
1/24*(3*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/s
qrt(a) - 2*(5*(b*x^3 + a)^(3/2)*b^2 - 3*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a
)^2 - 2*(b*x^3 + a)*a + a^2))*c + integrate((b*g*x^6 + b*f*x^5 + b*e*x^4 +
 a*f*x^2 + (b*d + a*g)*x^3 + a*e*x + a*d)*sqrt(b*x^3 + a)/x^6, x)
```

3.468.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^7,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^7, x)
```

3.468.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^7}\,dx = \int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^7}\,dx$$

input
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7,x)
```

output
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^7, x)
```

3.469 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8} dx$

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3.469.1 Optimal result

Integrand size = 35, antiderivative size = 746

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8} dx = \frac{27bc\sqrt{a+bx^3}}{280x^4} + \frac{bd\sqrt{a+bx^3}}{4x^3} + \frac{27be\sqrt{a+bx^3}}{20x^2} - \frac{27b(bc+14af)\sqrt{a+bx^3}}{112ax} + \frac{27b^{4/3}(bc+14af)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)(a+bx^3)^{3/2} - \frac{b\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140gx^5)}{140x^5} - \frac{b(bd+4ag)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(bc+14af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^{4/3}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc+14af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\right.}{560a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/420*(60*c/x^7+70*d/x^6+84*e/x^5+105*f/x^4+140*g/x^3)*(b*x^3+a)^(3/2)-1/
4*b*(4*a*g+b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+27/280*b*c*(b*x^3
+a)^(1/2)/x^4+1/4*b*d*(b*x^3+a)^(1/2)/x^3+27/20*b*e*(b*x^3+a)^(1/2)/x^2-27
/112*b*(14*a*f+b*c)*(b*x^3+a)^(1/2)/a/x-1/140*b*(-140*g*x^5-315*f*x^4+252*
e*x^3+70*d*x^2+36*c*x)*(b*x^3+a)^(1/2)/x^5+27/112*b^(4/3)*(14*a*f+b*c)*(b*
x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/224*3^(1/4)*b^(4/3)*(14*
a*f+b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)+9/560*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*Elliptic
F((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(28*a^(2/3)*b^(1/3)*e-5*(14*a*f+b*c)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*
2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.469.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.88 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \frac{-\frac{105bd(a+bx^3)}{x^3}-\frac{70d(a+bx^3)^2}{x^6}-105b^2d\sqrt{1+\frac{bx^3}{a}}\text{arctanh}\left(\sqrt{1\right.}}{}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

output
```
((-105*b*d*(a + b*x^3))/x^3 - (70*d*(a + b*x^3)^2)/x^6 - 105*b^2*d*Sqrt[1
+ (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]] - (60*a^2*c*Sqrt[1 + (b*x^3)/a]*
Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)])/x^7 - (84*a^2*e*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/x^5 - (105
*a^2*f*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a
)])/x^4 + (56*b*g*(a + b*x^3)^3*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)
/a])/a^2)/(420*Sqrt[a + b*x^3])
```

3.469.3 Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 767, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.543, Rules used = {2364, 27, 2365, 27, 2374, 27, 2374, 27, 2374, 27, 2374, 25, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$$

$$\Big\downarrow \text{2364}$$

$$-\frac{9}{2}b\int -\frac{\sqrt{bx^3+a}(140gx^4+105fx^3+84ex^2+70dx+60c)}{420x^5}dx-\frac{1}{420}(a+bx^3)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{280}b\int \frac{\sqrt{bx^3+a}(140gx^4+105fx^3+84ex^2+70dx+60c)}{x^5}dx-\frac{1}{420}(a+bx^3)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{2365}$$

$$\frac{3}{280}b\left(\frac{3}{2}a\int -\frac{2(-140gx^4-315fx^3+252ex^2+70dx+36c)}{3x^5\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4\right.}{3x^5}$$
$$\frac{1}{420}(a+bx^3)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{280}b\left(-a\int \frac{-140gx^4-315fx^3+252ex^2+70dx+36c}{x^5\sqrt{bx^3+a}}dx-\frac{2\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315fx^4-140\right.}{3x^5}$$
$$\frac{1}{420}(a+bx^3)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{2374}$$

$$\frac{3}{280}b\left(-a\left(-\frac{\int -\frac{4(-280agx^3-45(bc+14af)x^2+504aex+140ad)}{x^4\sqrt{bx^3+a}}dx}{8a}-\frac{9c\sqrt{a+bx^3}}{ax^4}\right)-\frac{2\sqrt{a+bx^3}(36cx+70dx^2+252ex^3}{3x^5}\right.$$
$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$\big\downarrow$ 27

$$\frac{3}{280}b\left(-a\left(\frac{\int \frac{-280agx^3-45(bc+14af)x^2+504aex+140ad}{x^4\sqrt{bx^3+a}}dx}{2a}-\frac{9c\sqrt{a+bx^3}}{ax^4}\right)-\frac{2\sqrt{a+bx^3}(36cx+70dx^2+252ex^3-315}{3x^5}\right.$$
$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$\big\downarrow$ 2374

$$\frac{3}{280}b\left(-a\left(\frac{-\frac{\int -\frac{6(504ea^2-70(bd+4ag)x^2a-45(bc+14af)xa)}{x^3\sqrt{bx^3+a}}dx}{6a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{9c\sqrt{a+bx^3}}{ax^4}\right)-\frac{2\sqrt{a+bx^3}(36cx+70dx^2+}{3}\right.$$
$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$\big\downarrow$ 27

$$\frac{3}{280}b\left(-a\left(\frac{\frac{\int \frac{504ea^2-70(bd+4ag)x^2a-45(bc+14af)xa}{x^3\sqrt{bx^3+a}}dx}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{9c\sqrt{a+bx^3}}{ax^4}\right)-\frac{2\sqrt{a+bx^3}(36cx+70dx^2+252ex}{3x^5}\right.$$
$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$\big\downarrow$ 2374

$$\frac{3}{280}b\left(-a\left(\frac{\frac{-\frac{\int \frac{4(126bex^2a^2+45(bc+14af)a^2+70(bd+4ag)xa^2)}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{252ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{9c\sqrt{a+bx^3}}{ax^4}\right)-\frac{2\sqrt{a+bx^3}(36c}{}\right.$$
$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$\big\downarrow$ 27

3.469. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

$$\frac{3}{280}b\left(-a\left(\frac{-\frac{\int\frac{126bex^2a^2+45(bc+14af)a^2+70(bd+4ag)xa^2}{x^2\sqrt{bx^3+a}}dx}{a}-\frac{252ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{9c\sqrt{a+bx^3}}{ax^4}\right)-\frac{2\sqrt{a+bx^3}(36cx+\right.$$

$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{2374}$$

$$\frac{3}{280}b\left(-a\left(\frac{-\frac{-\frac{\int-\frac{140(bd+4ag)a^3+252bexa^3+45b(bc+14af)x^2a^2}{x\sqrt{bx^3+a}}dx}{2a}-\frac{45a\sqrt{a+bx^3}(14af+bc)}{x}}{a}-\frac{252ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{9c\sqrt{a+bx^3}}{ax^4}\right)\right.$$

$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{280}b\left(-a\left(\frac{-\frac{\frac{\int\frac{140(bd+4ag)a^3+252bexa^3+45b(bc+14af)x^2a^2}{x\sqrt{bx^3+a}}dx}{2a}-\frac{45a\sqrt{a+bx^3}(14af+bc)}{x}}{a}-\frac{252ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{9c\sqrt{a+bx^3}}{ax^4}\right)-\right.$$

$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{2371}$$

$$\frac{3}{280}b\left(-a\left(\frac{-\frac{\frac{140a^3(4ag+bd)\int\frac{1}{x\sqrt{bx^3+a}}dx+\int\frac{252bea^3+45b(bc+14af)xa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{45a\sqrt{a+bx^3}(14af+bc)}{x}}{a}-\frac{252ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{9c\sqrt{a-}}{ax}\right.\right.$$

$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \text{798}$$

3.469. $\int\frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

$$\frac{3}{280}b\left(-a\left(\frac{-\frac{-\frac{\frac{\frac{140}{3}a^3(4ag+bd)\int\frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int\frac{252bea^3+45b(bc+14af)xa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{45a\sqrt{a+bx^3}(14af+bc)}{x}}{a}-\frac{252ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}-\frac{9c\sqrt{}}{}}{2a}\right.\right.$$

$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{3}{280}b\left(-a\left(\frac{-\frac{-\frac{\frac{\frac{280a^3(4ag+bd)\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int\frac{252bea^3+45b(bc+14af)xa^2}{\sqrt{bx^3+a}}dx}{2a}-\frac{45a\sqrt{a+bx^3}(14af+bc)}{x}}{a}-\frac{252ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}-\frac{9c\sqrt{}}{}}{2a}\right.\right.$$

$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{3}{280}b\left(-a\left(\frac{-\frac{-\frac{\frac{\int\frac{252bea^3+45b(bc+14af)xa^2}{\sqrt{bx^3+a}}dx-\frac{280}{3}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(4ag+bd)}{2a}-\frac{45a\sqrt{a+bx^3}(14af+bc)}{x}}{a}-\frac{252ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{140d\sqrt{a+bx^3}}{3x^3}-}{2a}\right.\right.$$

$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \; 2417$$

$$\frac{3}{280}b\left(-a\left(\frac{-\frac{-\frac{\frac{9a^{7/3}b^{2/3}\left(28a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(14af+bc)\right)\int\frac{1}{\sqrt{bx^3+a}}dx+45a^2b^{2/3}(14af+bc)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{280}{3}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right.}{2a}}{a}}{a}}{2a}\right.\right.$$

$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$$\Big\downarrow \; 759$$

3.469. $\int\frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^8}dx$

$$\frac{3}{280}b\left(-a\left(-\frac{\frac{45a^2b^{2/3}(14af+bc)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}\ldots}{\sqrt[3]{b}\ldots}\right.\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{\ldots}}}{2a}}{a}\ldots\right.\right.$$

$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

$\downarrow$ 2416

$$\frac{3}{280}b\left(-a\left(-\frac{\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(28a^{2/3}\sqrt[3]{b}e-5\ldots\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\ldots}\ldots\right.\right.$$

$$\frac{1}{420}\left(a+bx^3\right)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}+\frac{140g}{x^3}\right)$$

input
```
Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x]
```

3.469. $\int\frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^8}\,dx$

output
```
-1/420*(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4 + (140*g)/x^3)
*(a + b*x^3)^(3/2)) + (3*b*((-2*Sqrt[a + b*x^3]*(36*c*x + 70*d*x^2 + 252*e
*x^3 - 315*f*x^4 - 140*g*x^5))/(3*x^5) - a*((-9*c*Sqrt[a + b*x^3])/(a*x^4)
 + ((-140*d*Sqrt[a + b*x^3])/(3*x^3) + ((-252*a*e*Sqrt[a + b*x^3])/x^2 - (
(-45*a*(b*c + 14*a*f)*Sqrt[a + b*x^3])/x + ((-280*a^(5/2)*(b*d + 4*a*g)*Ar
cTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 + 45*a^2*b^(2/3)*(b*c + 14*a*f)*((2*Sqrt
[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt
[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (6*3^(3/4)*Sqrt[2 + Sqrt[
3]]*a^(7/3)*b^(1/3)*(28*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(b*c + 14*a*f)
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3]))/(2*a))/a)/a)/(2*a))))/280
```

3.469.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.469. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2365
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[Coeff[Pq, x, i]
*(x^(i + 1)/(m + n*p + i + 1)), {i, 0, q}], x] + Simp[a*n*p   Int[(c*x)^m*(
a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(x^i/(m + n*p + i + 1)), {i, 0, q}],
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
 GtQ[p, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

3.469. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx$

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.469.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 916, normalized size of antiderivative = 1.23

method	result	size
elliptic	Expression too large to display	916
risch	Expression too large to display	1295
default	Expression too large to display	1375

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a*c*(b*x^3+a)^(1/2)/x^7-1/6*a*d*(b*x^3+a)^(1/2)/x^6-1/5*a*e*(b*x^3+a)
^(1/2)/x^5-1/4*(a*f+17/14*b*c)*(b*x^3+a)^(1/2)/x^4-1/3*(a*g+5/4*b*d)*(b*x^
3+a)^(1/2)/x^3-13/20*b*e*(b*x^3+a)^(1/2)/x^2-1/112*b*(154*a*f+27*b*c)/a*(b
*x^3+a)^(1/2)/x+2/3*g*b*(b*x^3+a)^(1/2)-9/20*I*b*e*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(b^2*f+1/224*b^2*(154*a
*f+27*b*c)/a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*
b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/...
```

3.469.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \left[\frac{2268\,ab^{\frac{3}{2}}ex^7\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+105\,(b^2d+\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="fric
as")
```

output
```
[1/1680*(2268*a*b^(3/2)*e*x^7*weierstrassPInverse(0, -4*a/b, x) + 105*(b^2
*d + 4*a*b*g)*sqrt(a)*x^7*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt
(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 405*(b^2*c + 14*a*b*f)*sqrt(b)*x^7*wei
erstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (1120*a*b*g*x
^7 - 1092*a*b*e*x^5 - 15*(27*b^2*c + 154*a*b*f)*x^6 - 336*a^2*e*x^2 - 140*
(5*a*b*d + 4*a^2*g)*x^4 - 280*a^2*d*x - 30*(17*a*b*c + 14*a^2*f)*x^3 - 240
*a^2*c)*sqrt(b*x^3 + a))/(a*x^7), 1/1680*(2268*a*b^(3/2)*e*x^7*weierstrass
PInverse(0, -4*a/b, x) + 210*(b^2*d + 4*a*b*g)*sqrt(-a)*x^7*arctan(2*sqrt(
b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 405*(b^2*c + 14*a*b*f)*sqrt(b)*x^7*we
ierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (1120*a*b*g*
x^7 - 1092*a*b*e*x^5 - 15*(27*b^2*c + 154*a*b*f)*x^6 - 336*a^2*e*x^2 - 140
*(5*a*b*d + 4*a^2*g)*x^4 - 280*a^2*d*x - 30*(17*a*b*c + 14*a^2*f)*x^3 - 24
0*a^2*c)*sqrt(b*x^3 + a))/(a*x^7)]
```

3.469.6 Sympy [A] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \sqrt{a}bg\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) - \frac{a^2d}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}d}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}g\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2a\sqrt{b}g}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}d}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2b^{\frac{3}{2}}gx^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} - \frac{b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}}$$

input
```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**8,x)
```

output
```
a**(3/2)*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*e*gamma(-5/3)*hyper((-5/3, -1/2), (-2/
3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + a**(3/2)*f*gamma(-4/
3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1
/3)) + sqrt(a)*b*c*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*e*gamma(-2/3)*hyper((-2/3, -1
/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*f*g
amma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamm
a(2/3)) - sqrt(a)*b*g*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - a**2*d/(6*sqrt(b
)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*d/(4*x**(9/2)*sqrt(a/(b*x**3
) + 1)) - a*sqrt(b)*g*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*a*sqrt(b)*g/(3
*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*d*sqrt(a/(b*x**3) + 1)/(3*x**(3
/2)) - b**(3/2)*d/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/2)*g*x**(3/
2)/(3*sqrt(a/(b*x**3) + 1)) - b**2*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*
sqrt(a))
```

3.469.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8, x)
```

3.469.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^8,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^8, x)
```

3.469.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^8}\,dx = \int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^8}\,dx$$

input
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8,x)
```

output
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^8, x)
```

3.470 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$

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3.470.1 Optimal result

Integrand size = 35, antiderivative size = 705

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx =$$

$$-\frac{1}{560}b\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\sqrt{a+bx^3}$$

$$-\frac{27b^2c\sqrt{a+bx^3}}{320ax^2}-\frac{27b^2d\sqrt{a+bx^3}}{112ax}+\frac{27b^{4/3}(bd+14ag)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)\left(a+bx^3\right)^{3/2}-\frac{b^2e\,\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}(bd+14ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}\,b^{4/3}\left(7\sqrt[3]{b}(bc-16af)+20\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd+14ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/840*(105*c/x^8+120*d/x^7+140*e/x^6+168*f/x^5+210*g/x^4)*(b*x^3+a)^(3/2)
-1/4*b^2*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/560*b*(63*c/x^5+90*d
/x^4+140*e/x^3+252*f/x^2+630*g/x)*(b*x^3+a)^(1/2)-27/320*b^2*c*(b*x^3+a)^(
1/2)/a/x^2-27/112*b^2*d*(b*x^3+a)^(1/2)/a/x+27/112*b^(4/3)*(14*a*g+b*d)*(b
*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-27/224*3^(1/4)*b^(4/3)*(14
*a*g+b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)-9/2240*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*Ellipt
icF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1
/2)+2*I)*(7*b^(1/3)*(-16*a*f+b*c)+20*a^(1/3)*(14*a*g+b*d)*(1-3^(1/2)))*(1/
2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)
*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.470.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.44 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.29

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx =$$

$$-\frac{\sqrt{a+bx^3}\left(105a^2c\,\text{Hypergeometric2F1}\left(-\frac{8}{3},-\frac{3}{2},-\frac{5}{3},-\frac{bx^3}{a}\right)+2x\left(60a^2d\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{3}{2},-\right.\right.\right.}{}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

output
```
-1/840*(Sqrt[a + b*x^3]*(105*a^2*c*Hypergeometric2F1[-8/3, -3/2, -5/3, -((
b*x^3)/a)] + 2*x*(60*a^2*d*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a
)] + 7*x*(12*a^2*f*x*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)] + 5
*(a*e*(2*a + 5*b*x^3)*Sqrt[1 + (b*x^3)/a] + 3*b^2*e*x^6*ArcTanh[Sqrt[1 + (
b*x^3)/a]] + 3*a^2*g*x^2*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)]
)))))/(a*x^8*Sqrt[1 + (b*x^3)/a])
```

3.470.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2364, 27, 2364, 27, 2374, 25, 2374, 27, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}dx$$

$$\Big\downarrow \text{ 2364}$$

$$-\frac{9}{2}b\int -\frac{\sqrt{bx^3+a}(210gx^4+168fx^3+140ex^2+120dx+105c)}{840x^6}dx-\frac{1}{840}(a+bx^3)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{560}b\int \frac{\sqrt{bx^3+a}(210gx^4+168fx^3+140ex^2+120dx+105c)}{x^6}dx-\frac{1}{840}(a+bx^3)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$$\Big\downarrow \text{ 2364}$$

$$\frac{3}{560}b\left(-\frac{3}{2}b\int -\frac{630gx^4+252fx^3+140ex^2+90dx+63c}{3x^3\sqrt{bx^3+a}}dx-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\right.$$
$$\frac{1}{840}(a+bx^3)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{560}b\left(\frac{1}{2}b\int \frac{630gx^4+252fx^3+140ex^2+90dx+63c}{x^3\sqrt{bx^3+a}}dx-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}+\frac{630g}{x}\right)\right)-$$
$$\frac{1}{840}(a+bx^3)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$$\Big\downarrow \text{ 2374}$$

$$\frac{3}{560}b\left(\frac{1}{2}b\left(-\frac{\int -\frac{2520agx^3-63(bc-16af)x^2+560aex+360ad}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{63c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{25}{}\right.\right.$$
$$\frac{1}{840}(a+bx^3)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 25

$$\frac{3}{560}b\left(\frac{1}{2}b\left(\frac{\int \frac{2520agx^3-63(bc-16af)x^2+560aex+360ad}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{63c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}+\frac{252f}{x^2}\right.\right.$$
$$\frac{1}{840}\left(a+bx^3\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 2374

$$\frac{3}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{\int -\frac{2\left(560ea^2+180(bd+14ag)x^2a-63(bc-16af)xa\right)}{x\sqrt{bx^3+a}}dx}{2a}-\frac{360d\sqrt{a+bx^3}}{x}}{4a}-\frac{63c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\right.\right.$$
$$\frac{1}{840}\left(a+bx^3\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 27

$$\frac{3}{560}b\left(\frac{1}{2}b\left(\frac{\frac{\int \frac{560ea^2+180(bd+14ag)x^2a-63(bc-16af)xa}{x\sqrt{bx^3+a}}dx}{a}-\frac{360d\sqrt{a+bx^3}}{x}}{4a}-\frac{63c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{63c}{x^5}+\frac{90d}{x^4}+\frac{140e}{x^3}\right.\right.$$
$$\frac{1}{840}\left(a+bx^3\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 2371

$$\frac{3}{560}b\left(\frac{1}{2}b\left(\frac{\frac{560a^2e\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{180a(bd+14ag)x-63a(bc-16af)}{\sqrt{bx^3+a}}dx}{a}-\frac{360d\sqrt{a+bx^3}}{x}}{4a}-\frac{63c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{63c}{x^5}+\right.\right.$$
$$\frac{1}{840}\left(a+bx^3\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 798

$$\frac{3}{560}b\left(\frac{1}{2}b\left(\frac{\frac{\frac{560}{3}a^2e\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int \frac{180a(bd+14ag)x-63a(bc-16af)}{\sqrt{bx^3+a}}dx}{a}-\frac{360d\sqrt{a+bx^3}}{x}}{4a}-\frac{63c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{63c}{x^5}+\right.\right.$$
$$\frac{1}{840}\left(a+bx^3\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 73

3.470. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx$

$$\frac{3}{560}b\left(\frac{1}{2}b\left(\frac{\frac{\frac{1120a^2e\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int\frac{180a(bd+14ag)x-63a(bc-16af)}{\sqrt{bx^3+a}}dx}{a}-\frac{360d\sqrt{a+bx^3}}{x}}{4a}-\frac{63c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{63c}{x^5}\right.\right.$$

$$\frac{1}{840}\left(a+bx^3\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 221

$$\frac{3}{560}b\left(\frac{1}{2}b\left(\frac{\frac{\int\frac{180a(bd+14ag)x-63a(bc-16af)}{\sqrt{bx^3+a}}dx-\frac{1120}{3}a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a}-\frac{360d\sqrt{a+bx^3}}{x}}{4a}-\frac{63c\sqrt{a+bx^3}}{2ax^2}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\right.\right.$$

$$\frac{1}{840}\left(a+bx^3\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 2417

$$\frac{3}{560}b\left(\frac{1}{2}b\left(\frac{\frac{-9a\left(7(bc-16af)+\frac{20\left(1-\sqrt{3}\right)\sqrt[3]{a}(14ag+bd)}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{bx^3+a}}dx+\frac{180a(14ag+bd)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{1120}{3}a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right.}{a}}{4a}\right.\right.$$

$$\frac{1}{840}\left(a+bx^3\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

$\downarrow$ 759

$$\frac{3}{560}b\left(\frac{1}{2}b\left(\frac{\frac{\frac{180a(14ag+bd)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{6\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{a}}{4a}\right.\right.$$

$$\frac{1}{840}\left(a+bx^3\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

3.470. $\int\frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx$

$$\Big\downarrow \text{2416}$$

$$\frac{3}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{6\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(7(bc-16af)+\frac{20\left(1-\sqrt{3}\right)}{}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}\right.\right.$$

$$\frac{1}{840}\left(a+bx^3\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}+\frac{210g}{x^4}\right)$$

input
```
Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x]
```

3.470. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^9}\,dx$

output
```
-1/840*(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5 + (210*g)/x
^4)*(a + b*x^3)^(3/2)) + (3*b*(-1/3*(((63*c)/x^5 + (90*d)/x^4 + (140*e)/x^
3 + (252*f)/x^2 + (630*g)/x)*Sqrt[a + b*x^3]) + (b*((-63*c*Sqrt[a + b*x^3]
)/(2*a*x^2) + ((-360*d*Sqrt[a + b*x^3])/x + ((-1120*a^(3/2)*e*ArcTanh[Sqrt
[a + b*x^3]/Sqrt[a]])/3 + (180*a*(b*d + 14*a*g)*((2*Sqrt[a + b*x^3])/(b^(1
/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1
/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/
(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) - (6*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(7*
(b*c - 16*a*f) + (20*(1 - Sqrt[3])*a^(1/3)*(b*d + 14*a*g))/b^(1/3))*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3]))/a)/(4*a)))/2))/560
```

3.470.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.470. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.470. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx$

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.470.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 949, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	949
risch	Expression too large to display	1579
default	Expression too large to display	1663

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a*c*(b*x^3+a)^(1/2)/x^8-1/7*a*d*(b*x^3+a)^(1/2)/x^7-1/6*a*e*(b*x^3+a)
^(1/2)/x^6-1/5*(a*f+19/16*b*c)*(b*x^3+a)^(1/2)/x^5-1/4*(a*g+17/14*b*d)*(b*
x^3+a)^(1/2)/x^4-5/12*b*e*(b*x^3+a)^(1/2)/x^3-1/320*b*(208*a*f+27*b*c)/a*(
b*x^3+a)^(1/2)/x^2-1/112*(154*a*g+27*b*d)*b/a*(b*x^3+a)^(1/2)/x-2/3*I*(b^2
*f-1/640*b^2*(208*a*f+27*b*c)/a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(b^2*g+1/224*b^2/a*(154*a*g+27*b*d))*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*El...
```

3.470.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = \left[\frac{420\sqrt{a}b^2ex^8\log\left(-\frac{b^2x^6+8\,abx^3-4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)-\right.$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="fric
as")
```

output
```
[1/6720*(420*sqrt(a)*b^2*e*x^8*log(-(b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)
*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 567*(b^2*c - 16*a*b*f)*sqrt(b)*x^
8*weierstrassPInverse(0, -4*a/b, x) - 1620*(b^2*d + 14*a*b*g)*sqrt(b)*x^8*
weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (2800*a*b*
e*x^5 + 60*(27*b^2*d + 154*a*b*g)*x^7 + 21*(27*b^2*c + 208*a*b*f)*x^6 + 11
20*a^2*e*x^2 + 120*(17*a*b*d + 14*a^2*g)*x^4 + 960*a^2*d*x + 84*(19*a*b*c
+ 16*a^2*f)*x^3 + 840*a^2*c)*sqrt(b*x^3 + a))/(a*x^8), 1/6720*(840*sqrt(-a
)*b^2*e*x^8*arctan(2*sqrt(b*x^3 + a)*sqrt(-a)/(b*x^3 + 2*a)) - 567*(b^2*c
- 16*a*b*f)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b, x) - 1620*(b^2*d +
14*a*b*g)*sqrt(b)*x^8*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, x)) - (2800*a*b*e*x^5 + 60*(27*b^2*d + 154*a*b*g)*x^7 + 21*(27*b^2*c
 + 208*a*b*f)*x^6 + 1120*a^2*e*x^2 + 120*(17*a*b*d + 14*a^2*g)*x^4 + 960*a
^2*d*x + 84*(19*a*b*c + 16*a^2*f)*x^3 + 840*a^2*c)*sqrt(b*x^3 + a))/(a*x^8
)]
```

3.470.6 Sympy [A] (verification not implemented)

Time = 7.32 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{a^{\frac{3}{2}}g\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{\sqrt{a}bg\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{a^2e}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}e}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}e}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}}$$

input
```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**9,x)
```

output
```
a**(3/2)*c*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/
3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*f*gamma(-5/
3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2
/3)) + a**(3/2)*g*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*c*gamma(-5/3)*hyper((-5/3, -1/
2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*d*
gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4
*gamma(-1/3)) + sqrt(a)*b*f*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3
*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + sqrt(a)*b*g*gamma(-1/3)*hyper((-
1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) - a**2*e/(6
*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*e/(4*x**(9/2)*sqrt(a/
(b*x**3) + 1)) - b**(3/2)*e*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*e
/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*e*asinh(sqrt(a)/(sqrt(b)*x**(3/
2)))/(4*sqrt(a))
```

3.470.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="maxi
ma")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^9, x)
```

3.470.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^9,x, algorithm="giac
")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^9, x)
```

3.470.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^9}\,dx=\int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^9}\,dx$$

input
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9,x)
```

output
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^9, x)
```

3.471 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx$

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3.471.1 Optimal result

Integrand size = 35, antiderivative size = 714

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx = -\frac{b\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\sqrt{a+bx^3}}{1680} - \frac{b^2c\sqrt{a+bx^3}}{24ax^3} - \frac{27b^2d\sqrt{a+bx^3}}{320ax^2} - \frac{27b^2e\sqrt{a+bx^3}}{112ax} + \frac{27b^{7/3}e\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)(a+bx^3)^{3/2}}{2520} + \frac{b^2(bc-6af)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(7bd+20\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-112ag\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\right.}{2240a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/2520*(280*c/x^9+315*d/x^8+360*e/x^7+420*f/x^6+504*g/x^5)*(b*x^3+a)^(3/2
)+1/24*b^2*(-6*a*f+b*c)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/1680*b*
(140*c/x^6+189*d/x^5+270*e/x^4+420*f/x^3+756*g/x^2)*(b*x^3+a)^(1/2)-1/24*b
^2*c*(b*x^3+a)^(1/2)/a/x^3-27/320*b^2*d*(b*x^3+a)^(1/2)/a/x^2-27/112*b^2*e
*(b*x^3+a)^(1/2)/a/x+27/112*b^(7/3)*e*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))-27/224*3^(1/4)*b^(7/3)*e*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-9/2240*3^(3/4)*b^(5/
3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*b*d-112*a*g+20*a^(1/3)*b^(2/3)*e*
(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.471.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.70 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx =$$

$$-\frac{\sqrt{a+bx^3}\left(105a^5d\,\text{Hypergeometric2F1}\left(-\frac{8}{3},-\frac{3}{2},-\frac{5}{3},-\frac{bx^3}{a}\right)+2x\left(60a^5e\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{3}{2},-\right.\right.\right.}{}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x]
```

output
```
-1/840*(Sqrt[a + b*x^3]*(105*a^5*d*Hypergeometric2F1[-8/3, -3/2, -5/3, -((
b*x^3)/a)] + 2*x*(60*a^5*e*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a
)] + 7*x*(5*a^3*f*(a*(2*a + 5*b*x^3)*Sqrt[1 + (b*x^3)/a] + 3*b^2*x^6*ArcTa
nh[Sqrt[1 + (b*x^3)/a]]) + 12*a^5*g*x*Hypergeometric2F1[-5/3, -3/2, -2/3,
-((b*x^3)/a)] - 8*b^3*c*x^6*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometr
ic2F1[5/2, 4, 7/2, 1 + (b*x^3)/a])))/(a^4*x^8*Sqrt[1 + (b*x^3)/a])
```

3.471.3 Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.02, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {2364, 27, 2364, 27, 2374, 27, 2374, 25, 2374, 27, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{10}}dx$$

$$\Big\downarrow \text{2364}$$

$$-\frac{9}{2}b\int -\frac{\sqrt{bx^3+a}\left(504gx^4+420fx^3+360ex^2+315dx+280c\right)}{2520x^7}dx-\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{560}b\int \frac{\sqrt{bx^3+a}\left(504gx^4+420fx^3+360ex^2+315dx+280c\right)}{x^7}dx-\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$$\Big\downarrow \text{2364}$$

$$\frac{1}{560}b\left(-\frac{3}{2}b\int -\frac{756gx^4+420fx^3+270ex^2+189dx+140c}{3x^4\sqrt{bx^3+a}}dx-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{75}{x}\right.\right.$$
$$-\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{560}b\left(\frac{1}{2}b\int \frac{756gx^4+420fx^3+270ex^2+189dx+140c}{x^4\sqrt{bx^3+a}}dx-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}+\frac{756g}{x^2}\right)\right.$$
$$-\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$$\Big\downarrow \text{2374}$$

$$\frac{1}{560}b\left(\frac{1}{2}b\left(-\frac{\int -\frac{6\left(756agx^3-70(bc-6af)x^2+270aex+189ad\right)}{x^3\sqrt{bx^3+a}}dx}{6a}-\frac{140c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\right.\right.$$
$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$\downarrow$ 27

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{\int \frac{756agx^3-70(bc-6af)x^2+270aex+189ad}{x^3\sqrt{bx^3+a}}dx}{a}-\frac{140c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\frac{270e}{x^4}+\frac{420f}{x^3}\right.\right.$$
$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$\downarrow$ 2374

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{\int -\frac{1080ea^2-189(bd-16ag)x^2a-280(bc-6af)xa}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{189d\sqrt{a+bx^3}}{2x^2}}{a}-\frac{140c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{140c}{x^6}+\frac{189d}{x^5}\right.\right.$$
$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$\downarrow$ 25

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{\frac{\int \frac{1080ea^2-189(bd-16ag)x^2a-280(bc-6af)xa}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{189d\sqrt{a+bx^3}}{2x^2}}{a}-\frac{140c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{140c}{x^6}+\frac{189d}{x^5}+\right.\right.$$
$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$\downarrow$ 2374

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{\frac{-\frac{\int \frac{2\left(-540bex^2a^2+280(bc-6af)a^2+189(bd-16ag)xa^2\right)}{x\sqrt{bx^3+a}}dx}{2a}-\frac{1080ae\sqrt{a+bx^3}}{x}}{4a}-\frac{189d\sqrt{a+bx^3}}{2x^2}}{a}-\frac{140c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{3}\sqrt{a+bx^3}\right.$$
$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$\downarrow$ 27

3.471. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{10}}\,dx$

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{-\frac{\int \frac{-540bex^2a^2+280(bc-6af)a^2+189(bd-16ag)xa^2}{x\sqrt{bx^3+a}}dx}{a}-\frac{1080ae\sqrt{a+bx^3}}{x}}{4a}-\frac{189d\sqrt{a+bx^3}}{2x^2}}{a}-\frac{140c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$\downarrow$ 2371

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{-\frac{280a^2(bc-6af)\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{189a^2(bd-16ag)-540a^2bex}{\sqrt{bx^3+a}}dx}{a}-\frac{1080ae\sqrt{a+bx^3}}{x}}{4a}-\frac{189d\sqrt{a+bx^3}}{2x^2}}{a}-\frac{140c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{3}\sqrt{a\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$\downarrow$ 798

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{-\frac{\frac{280}{3}a^2(bc-6af)\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int \frac{189a^2(bd-16ag)-540a^2bex}{\sqrt{bx^3+a}}dx}{a}-\frac{1080ae\sqrt{a+bx^3}}{x}}{4a}-\frac{189d\sqrt{a+bx^3}}{2x^2}}{a}-\frac{140c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{3}\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$\downarrow$ 73

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{-\frac{\frac{560a^2(bc-6af)\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int \frac{189a^2(bd-16ag)-540a^2bex}{\sqrt{bx^3+a}}dx}{a}-\frac{1080ae\sqrt{a+bx^3}}{x}}{4a}-\frac{189d\sqrt{a+bx^3}}{2x^2}}{a}-\frac{140c\sqrt{a+bx^3}}{3ax^3}\right)-\frac{1}{3}\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$\downarrow$ 221

3.471. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{10}}\,dx$

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{\frac{\int \frac{189a^2(bd-16ag)-540a^2bex}{\sqrt{bx^3+a}}dx-\frac{560}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bc-6af)}{a}-\frac{1080ae\sqrt{a+bx^3}}{x}}{4a}-\frac{189d\sqrt{a+bx^3}}{2x^2}}{a}-\frac{140c\sqrt{a+bx^3}}{3ax^3}\right)\right.$$

$$\frac{(a+bx^3)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$\downarrow$ 2417

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{\frac{27a^2\left(20\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}e-112ag+7bd\right)\int\frac{1}{\sqrt{bx^3+a}}dx-540a^2b^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{560}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bc-6af)}{a}-}{4a}}{a}\right.\right.$$

$$\frac{(a+bx^3)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$\downarrow$ 759

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{\frac{-540a^2b^{2/3}e\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{560}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bc-6af)+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}\right.}}}{}}{a}}{4a}}{a}\right.\right.$$

$$\frac{(a+bx^3)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

$\downarrow$ 2416

3.471. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx$

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{-\frac{560}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bc-6af)+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\cdots}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{\cdots}\right.}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx\cdots}}}{\cdots}}{\cdots}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{280c}{x^9}+\frac{315d}{x^8}+\frac{360e}{x^7}+\frac{420f}{x^6}+\frac{504g}{x^5}\right)}{2520}$$

input
```
Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x]
```

output
```
-1/2520*(((280*c)/x^9 + (315*d)/x^8 + (360*e)/x^7 + (420*f)/x^6 + (504*g)/
x^5)*(a + b*x^3)^(3/2)) + (b*(-1/3*(((140*c)/x^6 + (189*d)/x^5 + (270*e)/x
^4 + (420*f)/x^3 + (756*g)/x^2)*Sqrt[a + b*x^3]) + (b*((-140*c*Sqrt[a + b*
x^3])/(3*a*x^3) + ((-189*d*Sqrt[a + b*x^3])/(2*x^2) + ((-1080*a*e*Sqrt[a +
 b*x^3])/x - ((-560*a^(3/2)*(b*c - 6*a*f)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]
)/3 - 540*a^2*b^(2/3)*e*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(7*b*d + 20*(1 - Sqrt[3])*a^(1/3)
*b^(2/3)*e - 112*a*g)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/a)/(4*a))/a))/2))/560
```

3.471. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{10}}\,dx$

3.471.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

3.471. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx$

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.471.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 958, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	958
risch	Expression too large to display	1160
default	Expression too large to display	1273

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x,method=_RETURNVERBOSE
)
```

output
```
-1/9*a*c*(b*x^3+a)^(1/2)/x^9-1/8*a*d*(b*x^3+a)^(1/2)/x^8-1/7*a*e*(b*x^3+a)
^(1/2)/x^7-1/6*(a*f+7/6*b*c)*(b*x^3+a)^(1/2)/x^6-1/5*(a*g+19/16*b*d)*(b*x^
3+a)^(1/2)/x^5-17/56*b*e*(b*x^3+a)^(1/2)/x^4-1/24*b*(10*a*f+b*c)/a*(b*x^3+
a)^(1/2)/x^3-1/320*b/a*(208*a*g+27*b*d)*(b*x^3+a)^(1/2)/x^2-27/112*b^2*e*(
b*x^3+a)^(1/2)/a/x-2/3*I*(b^2*g-1/640*b^2/a*(208*a*g+27*b*d))*3^(1/2)/b*(-
a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-9/112*I/a*b^2*e*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*El...
```

3.471.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx = \left[-\frac{4860\,ab^{\frac{5}{2}}ex^9\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInvers}\right.}{}\right.$$

$$-\frac{4860\,ab^{\frac{5}{2}}ex^9\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+420\left(b^3c-6\,ab^2f\right)\sqrt{-a}x^9\arctan\left(\right.}{}$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="fri
cas")
```

output
```
[-1/20160*(4860*a*b^(5/2)*e*x^9*weierstrassZeta(0, -4*a/b, weierstrassPInv
erse(0, -4*a/b, x)) + 210*(b^3*c - 6*a*b^2*f)*sqrt(a)*x^9*log((b^2*x^6 + 8
*a*b*x^3 - 4*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 1701*(a
*b^2*d - 16*a^2*b*g)*sqrt(b)*x^9*weierstrassPInverse(0, -4*a/b, x) + (4860
*a*b^2*e*x^8 + 6120*a^2*b*e*x^5 + 63*(27*a*b^2*d + 208*a^2*b*g)*x^7 + 840*
(a*b^2*c + 10*a^2*b*f)*x^6 + 2880*a^3*e*x^2 + 2520*a^3*d*x + 252*(19*a^2*b
*d + 16*a^3*g)*x^4 + 2240*a^3*c + 560*(7*a^2*b*c + 6*a^3*f)*x^3)*sqrt(b*x^
3 + a))/(a^2*x^9), -1/20160*(4860*a*b^(5/2)*e*x^9*weierstrassZeta(0, -4*a/
b, weierstrassPInverse(0, -4*a/b, x)) + 420*(b^3*c - 6*a*b^2*f)*sqrt(-a)*x
^9*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) + 17
01*(a*b^2*d - 16*a^2*b*g)*sqrt(b)*x^9*weierstrassPInverse(0, -4*a/b, x) +
(4860*a*b^2*e*x^8 + 6120*a^2*b*e*x^5 + 63*(27*a*b^2*d + 208*a^2*b*g)*x^7 +
 840*(a*b^2*c + 10*a^2*b*f)*x^6 + 2880*a^3*e*x^2 + 2520*a^3*d*x + 252*(19*
a^2*b*d + 16*a^3*g)*x^4 + 2240*a^3*c + 560*(7*a^2*b*c + 6*a^3*f)*x^3)*sqrt
(b*x^3 + a))/(a^2*x^9)]
```

3.471. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx$

3.471.6 Sympy [A] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{a^{\frac{3}{2}}g\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt{a}bg\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} - \frac{a^2c}{9\sqrt{b}x^{\frac{21}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a^2f}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{11a\sqrt{b}c}{36x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}f}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{17b^{\frac{3}{2}}c}{72x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}f}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{5}{2}}c}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^2f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}} + \frac{b^3c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{24a^{\frac{3}{2}}}$$

input
```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**10,x)
```

output
```
a**(3/2)*d*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*e*gamma(-7/3)*hyper((-7/3, -1/2), (-4/
3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + a**(3/2)*g*gamma(-5/
3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2
/3)) + sqrt(a)*b*d*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*e*gamma(-4/3)*hyper((-4/3, -1
/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + sqrt(a)*b*g
*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2
*gamma(1/3)) - a**2*c/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - a**2*f/
(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*c/(36*x**(15/2)*
sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*f/(4*x**(9/2)*sqrt(a/(b*x**3) + 1)) - 17
*b**(3/2)*c/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*f*sqrt(a/(b*x**3
) + 1)/(3*x**(3/2)) - b**(3/2)*f/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**(
5/2)*c/(24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*f*asinh(sqrt(a)/(sqrt(b
)*x**(3/2)))/(4*sqrt(a)) + b**3*c*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(24*a*
*(3/2))
```

3.471.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="max
ima")
```

output
```
-1/144*(3*b^3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))
/a^(3/2) + 2*(3*(b*x^3 + a)^(5/2)*b^3 + 8*(b*x^3 + a)^(3/2)*a*b^3 - 3*sqrt
(b*x^3 + a)*a^2*b^3)/((b*x^3 + a)^3*a - 3*(b*x^3 + a)^2*a^2 + 3*(b*x^3 + a
)*a^3 - a^4))*c + integrate((b*g*x^6 + b*f*x^5 + b*e*x^4 + a*f*x^2 + (b*d
+ a*g)*x^3 + a*e*x + a*d)*sqrt(b*x^3 + a)/x^9, x)
```

3.471.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx=\int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^10,x, algorithm="gia
c")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^10, x)
```

3.471.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{10}}\,dx=\int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^{10}}\,dx$$

input
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10,x)
```

output
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^10, x)
```

3.472 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx$

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3.472.1 Optimal result

Integrand size = 35, antiderivative size = 764

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx = -\frac{b\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\sqrt{a+bx^3}}{1680} - \frac{27b^2c\sqrt{a+bx^3}}{1120ax^4} - \frac{b^2d\sqrt{a+bx^3}}{24ax^3}$$
$$- \frac{27b^2e\sqrt{a+bx^3}}{320ax^2} + \frac{27b^2(bc-4af)\sqrt{a+bx^3}}{448a^2x} - \frac{27b^{7/3}(bc-4af)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$- \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)(a+bx^3)^{3/2}}{2520} + \frac{b^2(bd-6ag)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}$$
$$+ \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}(bc-4af)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{\ldots}\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$- \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{7/3}\left(7a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc-4af)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\operatorname{EllipticF}\left(a\ldots\right.}{2240a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/2520*(252*c/x^10+280*d/x^9+315*e/x^8+360*f/x^7+420*g/x^6)*(b*x^3+a)^(3/
2)+1/24*b^2*(-6*a*g+b*d)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/1680*b
*(108*c/x^7+140*d/x^6+189*e/x^5+270*f/x^4+420*g/x^3)*(b*x^3+a)^(1/2)-27/11
20*b^2*c*(b*x^3+a)^(1/2)/a/x^4-1/24*b^2*d*(b*x^3+a)^(1/2)/a/x^3-27/320*b^2
*e*(b*x^3+a)^(1/2)/a/x^2+27/448*b^2*(-4*a*f+b*c)*(b*x^3+a)^(1/2)/a^2/x-27/
448*b^(7/3)*(-4*a*f+b*c)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))+27/896*3^(1/4)*b^(7/3)*(-4*a*f+b*c)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-9/2240*3^(3/4)*b^(7/
3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(7*a^(2/3)*b^(1/3)*e-5*(-4*a*f+b*c)*
(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.472.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.54 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx =$$

$$-\frac{\sqrt{a+bx^3}\Big(84a^5c\,\text{Hypergeometric2F1}\left(-\frac{10}{3},-\frac{3}{2},-\frac{7}{3},-\frac{bx^3}{a}\right)+105a^5ex^2\,\text{Hypergeometric2F1}\left(-\frac{8}{3},-\frac{3}{2},-\frac{5}{3}\right.}{}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x]
```

output
```
-1/840*(Sqrt[a + b*x^3]*(84*a^5*c*Hypergeometric2F1[-10/3, -3/2, -7/3, -((
b*x^3)/a)] + 105*a^5*e*x^2*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b*x^3)/a
)] + 2*x^3*(35*a^3*g*x*(a*(2*a + 5*b*x^3)*Sqrt[1 + (b*x^3)/a] + 3*b^2*x^6*
ArcTanh[Sqrt[1 + (b*x^3)/a]]) + 60*a^5*f*Hypergeometric2F1[-7/3, -3/2, -4/
3, -((b*x^3)/a)] - 56*b^3*d*x^7*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeo
metric2F1[5/2, 4, 7/2, 1 + (b*x^3)/a]))/(a^4*x^10*Sqrt[1 + (b*x^3)/a])
```

3.472.3 Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 767, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.543, Rules used = {2364, 27, 2364, 27, 2374, 27, 2374, 27, 2374, 27, 2374, 27, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}dx$$

$$\Big\downarrow \text{2364}$$

$$-\frac{9}{2}b\int -\frac{\sqrt{bx^3+a}(420gx^4+360fx^3+315ex^2+280dx+252c)}{2520x^8}dx-\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{560}b\int \frac{\sqrt{bx^3+a}(420gx^4+360fx^3+315ex^2+280dx+252c)}{x^8}dx-\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$$\Big\downarrow \text{2364}$$

$$\frac{1}{560}b\left(-\frac{3}{2}b\int -\frac{420gx^4+270fx^3+189ex^2+140dx+108c}{3x^5\sqrt{bx^3+a}}dx-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{42}{x}\right.\right.$$
$$-\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{560}b\left(\frac{1}{2}b\int \frac{420gx^4+270fx^3+189ex^2+140dx+108c}{x^5\sqrt{bx^3+a}}dx-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}+\frac{420g}{x^3}\right)\right.$$
$$-\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$$\Big\downarrow \text{2374}$$

$$\frac{1}{560}b\left(\frac{1}{2}b\left(-\frac{\int -\frac{4(840agx^3-135(bc-4af)x^2+378aex+280ad)}{x^4\sqrt{bx^3+a}}dx}{8a}-\frac{27c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}\right.\right.$$
$$\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$\downarrow$ 27

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{\int \frac{840agx^3-135(bc-4af)x^2+378aex+280ad}{x^4\sqrt{bx^3+a}}dx}{2a}-\frac{27c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\frac{189e}{x^5}+\frac{270f}{x^4}\right.\right.$$
$$\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$\downarrow$ 2374

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{\int -\frac{6\left(378ea^2-140(bd-6ag)x^2a-135(bc-4af)xa\right)}{x^3\sqrt{bx^3+a}}dx}{6a}-\frac{280d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{27c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{108c}{x^7}+\frac{140d}{x^6}\right.\right.$$
$$\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$\downarrow$ 27

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{\frac{\int \frac{378ea^2-140(bd-6ag)x^2a-135(bc-4af)xa}{x^3\sqrt{bx^3+a}}dx}{a}-\frac{280d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{27c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{108c}{x^7}+\frac{140d}{x^6}+\right.\right.$$
$$\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$\downarrow$ 2374

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{\frac{-\frac{\int \frac{2\left(189bex^2a^2+270(bc-4af)a^2+280(bd-6ag)xa^2\right)}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{189ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{27c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\right.\right.$$
$$\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$\downarrow$ 27

3.472. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx$

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{\frac{-\frac{\int \frac{189bex^2a^2+270(bc-4af)a^2+280(bd-6ag)xa^2}{x^2\sqrt{bx^3+a}}dx}{2a}-\frac{189ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{27c\sqrt{a+bx^3}}{ax^4}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{108}{x^7}\right.\right.$$

$$\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$\downarrow$ 2374

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{\frac{-\frac{-\frac{\int -\frac{2\left(280(bd-6ag)a^3+189bexa^3+135b(bc-4af)x^2a^2\right)}{x\sqrt{bx^3+a}}dx}{2a}-\frac{270a\sqrt{a+bx^3}(bc-4af)}{x}}{2a}-\frac{189ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{27c\sqrt{a+bx^3}}{ax^4}\right.\right.$$

$$\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$\downarrow$ 27

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{\frac{-\frac{\frac{\int \frac{280(bd-6ag)a^3+189bexa^3+135b(bc-4af)x^2a^2}{x\sqrt{bx^3+a}}dx}{a}-\frac{270a\sqrt{a+bx^3}(bc-4af)}{x}}{2a}-\frac{189ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{27c\sqrt{a+bx^3}}{ax^4}\right)\right.$$

$$\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$\downarrow$ 2371

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{\frac{-\frac{\frac{280a^3(bd-6ag)\int \frac{1}{x\sqrt{bx^3+a}}dx+\int \frac{189bea^3+135b(bc-4af)xa^2}{\sqrt{bx^3+a}}dx}{a}-\frac{270a\sqrt{a+bx^3}(bc-4af)}{x}}{2a}-\frac{189ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{3x^3}}{2a}-\frac{27c\sqrt{a}}{ax}\right.\right.$$

$$\frac{(a+bx^3)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$\downarrow$ 798

3.472. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx$

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{\frac{\frac{280}{3}a^3(bd-6ag)\int\frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int\frac{189bea^3+135b(bc-4af)xa^2}{\sqrt{bx^3+a}}dx}{a}}{2a}-\frac{270a\sqrt{a+bx^3}(bc-4af)}{x}}{2a}-\frac{189ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{3x^3}-\frac{27c\sqrt{}}{}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$$\downarrow 73$$

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{\frac{\frac{560a^3(bd-6ag)\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}+\int\frac{189bea^3+135b(bc-4af)xa^2}{\sqrt{bx^3+a}}dx}{a}}{2a}-\frac{270a\sqrt{a+bx^3}(bc-4af)}{x}}{2a}-\frac{189ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{3x^3}-\frac{27c}{}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$$\downarrow 221$$

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{\frac{\int\frac{189bea^3+135b(bc-4af)xa^2}{\sqrt{bx^3+a}}dx-\frac{560}{3}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)(bd-6ag)}{a}}{2a}-\frac{270a\sqrt{a+bx^3}(bc-4af)}{x}}{2a}-\frac{189ae\sqrt{a+bx^3}}{x^2}}{a}-\frac{280d\sqrt{a+bx^3}}{3x^3}-\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$$\downarrow 2417$$

$$\frac{1}{560}b\left(\frac{1}{2}b\left(\frac{-\frac{\frac{27a^{7/3}b^{2/3}\left(7a^{2/3}\sqrt[3]{b}e-5\left(1-\sqrt{3}\right)(bc-4af)\right)\int\frac{1}{\sqrt{bx^3+a}}dx+135a^2b^{2/3}(bc-4af)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{560}{3}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right.}{a}}{2a}}{a}}{2a}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$$\downarrow 759$$

3.472. $\int\frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{11}}\,dx$

$$\frac{1}{560}b\left(\frac{1}{2}b\left(-\frac{135a^2b^{2/3}(bc-4af)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{}}{\sqrt[3]{}}\right.\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a}}}{a}}{2a}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

$\downarrow$ 2416

$$\frac{1}{560}b\left(\frac{1}{2}b\left(-\frac{\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^{7/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(7a^{2/3}\sqrt[3]{b}e-5\right.}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}+\frac{420g}{x^6}\right)}{2520}$$

input

```
Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x]
```

3.472. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{11}}\,dx$

output
```
-1/2520*(((252*c)/x^10 + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7 + (420*g)
/x^6)*(a + b*x^3)^(3/2)) + (b*(-1/3*(((108*c)/x^7 + (140*d)/x^6 + (189*e)/
x^5 + (270*f)/x^4 + (420*g)/x^3)*Sqrt[a + b*x^3]) + (b*((-27*c*Sqrt[a + b*
x^3])/(a*x^4) + ((-280*d*Sqrt[a + b*x^3])/(3*x^3) + ((-189*a*e*Sqrt[a + b*
x^3])/x^2 - ((-270*a*(b*c - 4*a*f)*Sqrt[a + b*x^3])/x + ((-560*a^(5/2)*(b*
d - 6*a*g)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 + 135*a^2*b^(2/3)*(b*c - 4*
a*f)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
(3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) + (18*3^(3/4)*
Sqrt[2 + Sqrt[3]]*a^(7/3)*b^(1/3)*(7*a^(2/3)*b^(1/3)*e - 5*(1 - Sqrt[3])*(
b*c - 4*a*f))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3]))/a)/(2*a))/a)/(2*a)))/2))/560
```

3.472.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.472. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.472. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx$

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.472.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.28

method	result	size
elliptic	Expression too large to display	976
risch	Expression too large to display	1343
default	Expression too large to display	1470

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x,method=_RETURNVERBOSE
)
```

output
```
-1/10*a*c*(b*x^3+a)^(1/2)/x^10-1/9*a*d*(b*x^3+a)^(1/2)/x^9-1/8*a*e*(b*x^3+
a)^(1/2)/x^8-1/7*(a*f+23/20*b*c)*(b*x^3+a)^(1/2)/x^7-1/6*(a*g+7/6*b*d)*(b*
x^3+a)^(1/2)/x^6-19/80*b*e*(b*x^3+a)^(1/2)/x^5-1/1120*b*(340*a*f+27*b*c)/a
*(b*x^3+a)^(1/2)/x^4-1/24*b/a*(10*a*g+b*d)*(b*x^3+a)^(1/2)/x^3-27/320*b^2*
e*(b*x^3+a)^(1/2)/a/x^2-27/448*(4*a*f-b*c)*b^2/a^2*(b*x^3+a)^(1/2)/x+9/320
*I/a*b^2*e*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))-9/448*I*b^2*(4*a*f-b*c)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/...
```

3.472.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 559, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx = \left[-\frac{1701\,ab^{\frac{5}{2}}ex^{10}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+210\left(b^3d\right.}{}\right.$$

$$-\frac{1701\,ab^{\frac{5}{2}}ex^{10}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+420\left(b^3d-6ab^2g\right)\sqrt{-a}x^{10}\arctan\left(\frac{(bx^3+2a)\sqrt{bx^3+a}\sqrt{-a}}{2(abx^3+a^2)}\right)-1}{}$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="fri
cas")
```

output
```
[-1/20160*(1701*a*b^(5/2)*e*x^10*weierstrassPInverse(0, -4*a/b, x) + 210*(
b^3*d - 6*a*b^2*g)*sqrt(a)*x^10*log((b^2*x^6 + 8*a*b*x^3 - 4*(b*x^3 + 2*a)
*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) - 1215*(b^3*c - 4*a*b^2*f)*sqrt(b)*
x^10*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (1701
*a*b^2*e*x^8 - 1215*(b^3*c - 4*a*b^2*f)*x^9 + 4788*a^2*b*e*x^5 + 840*(a*b^
2*d + 10*a^2*b*g)*x^7 + 18*(27*a*b^2*c + 340*a^2*b*f)*x^6 + 2520*a^3*e*x^2
 + 2240*a^3*d*x + 560*(7*a^2*b*d + 6*a^3*g)*x^4 + 2016*a^3*c + 144*(23*a^2
*b*c + 20*a^3*f)*x^3)*sqrt(b*x^3 + a))/(a^2*x^10), -1/20160*(1701*a*b^(5/2
)*e*x^10*weierstrassPInverse(0, -4*a/b, x) + 420*(b^3*d - 6*a*b^2*g)*sqrt(
-a)*x^10*arctan(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)
) - 1215*(b^3*c - 4*a*b^2*f)*sqrt(b)*x^10*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)) + (1701*a*b^2*e*x^8 - 1215*(b^3*c - 4*a*b^2*
f)*x^9 + 4788*a^2*b*e*x^5 + 840*(a*b^2*d + 10*a^2*b*g)*x^7 + 18*(27*a*b^2*
c + 340*a^2*b*f)*x^6 + 2520*a^3*e*x^2 + 2240*a^3*d*x + 560*(7*a^2*b*d + 6*
a^3*g)*x^4 + 2016*a^3*c + 144*(23*a^2*b*c + 20*a^3*f)*x^3)*sqrt(b*x^3 + a)
)/(a^2*x^10)]
```

3.472.6 Sympy [A] (verification not implemented)

Time = 14.17 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{10}{3},-\frac{1}{2}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{8}\Gamma\left(-\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{5}\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)} - \frac{a^2d}{9\sqrt{b}x^{\frac{21}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a^2g}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{11a\sqrt{b}d}{36x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{a\sqrt{b}g}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{17b^{\frac{3}{2}}d}{72x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}g\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b^{\frac{3}{2}}g}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{5}{2}}d}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^2g\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}} + \frac{b^3d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{24a^{\frac{3}{2}}}$$

input
```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**11,x)
```

output
```
a**(3/2)*c*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*p
i)/a)/(3*x**10*gamma(-7/3)) + a**(3/2)*e*gamma(-8/3)*hyper((-8/3, -1/2), (
-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + a**(3/2)*f*gamma(
-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma
(-4/3)) + sqrt(a)*b*c*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_
polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*e*gamma(-5/3)*hyper((-5/3,
 -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*
b*f*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*
x**4*gamma(-1/3)) - a**2*d/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - a*
*2*g/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*d/(36*x**(1
5/2)*sqrt(a/(b*x**3) + 1)) - a*sqrt(b)*g/(4*x**(9/2)*sqrt(a/(b*x**3) + 1))
 - 17*b**(3/2)*d/(72*x**(9/2)*sqrt(a/(b*x**3) + 1)) - b**(3/2)*g*sqrt(a/(b
*x**3) + 1)/(3*x**(3/2)) - b**(3/2)*g/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) -
 b**(5/2)*d/(24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*g*asinh(sqrt(a)/(s
qrt(b)*x**(3/2)))/(4*sqrt(a)) + b**3*d*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(
24*a**(3/2))
```

3.472.7 Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="max
ima")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^11, x)
```

3.472.8 Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^11,x, algorithm="gia
c")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^11, x)
```

3.472.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^{11}}\,dx = \int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^{11}}\,dx$$

input
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11,x)
```

output
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^11, x)
```

3.473 $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx$

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3.473.1 Optimal result

Integrand size = 35, antiderivative size = 796

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = -\frac{b\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\sqrt{a+bx^3}}{18480}-\frac{27b^2c\sqrt{a+bx^3}}{1760ax^5}-\frac{27b^2d\sqrt{a+bx^3}}{1120ax^4}-\frac{b^2e\sqrt{a+bx^3}}{24ax^3}+\frac{27b^2(7bc-22af)\sqrt{a+bx^3}}{7040a^2x^2}+\frac{27b^2(bd-4ag)\sqrt{a+bx^3}}{448a^2x}-\frac{27b^{7/3}(bd-4ag)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)(a+bx^3)^{3/2}}{27720}+\frac{b^3e\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{3/2}}+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}(bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{\ }\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{7/3}\left(7\sqrt[3]{b}(7bc-22af)+110\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}{49280a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/27720*(2520*c/x^11+2772*d/x^10+3080*e/x^9+3465*f/x^8+3960*g/x^7)*(b*x^3
+a)^(3/2)+1/24*b^3*e*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/18480*b*(9
45*c/x^8+1188*d/x^7+1540*e/x^6+2079*f/x^5+2970*g/x^4)*(b*x^3+a)^(1/2)-27/1
760*b^2*c*(b*x^3+a)^(1/2)/a/x^5-27/1120*b^2*d*(b*x^3+a)^(1/2)/a/x^4-1/24*b
^2*e*(b*x^3+a)^(1/2)/a/x^3+27/7040*b^2*(-22*a*f+7*b*c)*(b*x^3+a)^(1/2)/a^2
/x^2+27/448*b^2*(-4*a*g+b*d)*(b*x^3+a)^(1/2)/a^2/x-27/448*b^(7/3)*(-4*a*g+
b*d)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+27/896*3^(1/4)*b^
(7/3)*(-4*a*g+b*d)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(
1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+9/49280*3^(3/4)*b^(7/3)*(a^(1/3)+b^(1/3)
*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*(7*b^(1/3)*(-22*a*f+7*b*c)+110*a^(1/3)*(-4*a*g+b*d)*(1-
3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^2/(b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.473.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.43 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.24

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = \frac{\sqrt{a+bx^3}\left(-840a^5c\,\text{Hypergeometric2F1}\left(-\frac{11}{3},-\frac{3}{2},-\frac{8}{3},-\right.\right.}{}$$

input
```
Integrate[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x]
```

output
```
(Sqrt[a + b*x^3]*(-840*a^5*c*Hypergeometric2F1[-11/3, -3/2, -8/3, -((b*x^3
)/a)] - 924*a^5*d*x*Hypergeometric2F1[-10/3, -3/2, -7/3, -((b*x^3)/a)] + 1
1*x^3*(-105*a^5*f*Hypergeometric2F1[-8/3, -3/2, -5/3, -((b*x^3)/a)] - 120*
a^5*g*x*Hypergeometric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)] + 112*b^3*e*x^8*
(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x^
3)/a])))/(9240*a^4*x^11*Sqrt[1 + (b*x^3)/a])
```

3.473.3 Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.01, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2364, 27, 2364, 27, 2374, 27, 2374, 27, 2374, 27, 2374, 25, 2374, 27, 2371, 798, 73, 221, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}dx$$

$$\Big\downarrow \ 2364$$

$$-\frac{9}{2}b\int -\frac{\sqrt{bx^3+a}\left(3960gx^4+3465fx^3+3080ex^2+2772dx+2520c\right)}{27720x^9}dx-\frac{(a+bx^3)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$$\Big\downarrow \ 27$$

$$\frac{b\int \frac{\sqrt{bx^3+a}(3960gx^4+3465fx^3+3080ex^2+2772dx+2520c)}{x^9}dx}{6160}-\frac{(a+bx^3)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$$\Big\downarrow \ 2364$$

$$\frac{b\left(-\frac{3}{2}b\int -\frac{2970gx^4+2079fx^3+1540ex^2+1188dx+945c}{3x^6\sqrt{bx^3+a}}dx-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\right)}{6160}-\frac{(a+bx^3)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{1}{2}b\int \frac{2970gx^4+2079fx^3+1540ex^2+1188dx+945c}{x^6\sqrt{bx^3+a}}dx-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\right)}{6160}-\frac{(a+bx^3)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$$\Big\downarrow \ 2374$$

$$\frac{b\left(\frac{1}{2}b\left(-\frac{\int -\frac{5\left(5940agx^3-189(7bc-22af)x^2+3080aex+2376ad\right)}{x^5\sqrt{bx^3+a}}dx}{10a}-\frac{189c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\cdots\right.}{6160}$$
$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$\downarrow$ 27

$$\frac{b\left(\frac{1}{2}b\left(\frac{\int \frac{5940agx^3-189(7bc-22af)x^2+3080aex+2376ad}{x^5\sqrt{bx^3+a}}dx}{2a}-\frac{189c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079f}{x^5}+\frac{2970g}{x^4}\right)\cdots}{6160}$$
$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$\downarrow$ 2374

$$\frac{b\left(\frac{1}{2}b\left(\frac{-\frac{\int -\frac{8\left(3080ea^2-1485(bd-4ag)x^2a-189(7bc-22af)xa\right)}{x^4\sqrt{bx^3+a}}dx}{8a}-\frac{594d\sqrt{a+bx^3}}{x^4}}{2a}-\frac{189c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\cdots\right.}{6160}$$
$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$\downarrow$ 27

$$\frac{b\left(\frac{1}{2}b\left(\frac{\frac{\int \frac{3080ea^2-1485(bd-4ag)x^2a-189(7bc-22af)xa}{x^4\sqrt{bx^3+a}}dx}{a}-\frac{594d\sqrt{a+bx^3}}{x^4}}{2a}-\frac{189c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{945c}{x^8}+\frac{1188d}{x^7}+\frac{1540e}{x^6}+\frac{2079\cdots}{x^5}\right.}{6160}$$
$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$\downarrow$ 2374

3.473. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{12}}\,dx$

$$\frac{b\left(\frac{1}{2}b\left(\frac{\frac{-\frac{\int \frac{6\left(1540bex^2a^2+189(7bc-22af)a^2+1485(bd-4ag)xa^2\right)}{x^3\sqrt{bx^3+a}}dx}{6a}-\frac{3080ae\sqrt{a+bx^3}}{3x^3}}{a}-\frac{594d\sqrt{a+bx^3}}{x^4}}{2a}-\frac{189c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{945c}{x^8}+\cdots\right.\right.}{6160}$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{1}{2}b\left(\frac{\frac{-\frac{\int \frac{1540bex^2a^2+189(7bc-22af)a^2+1485(bd-4ag)xa^2}{x^3\sqrt{bx^3+a}}dx}{a}-\frac{3080ae\sqrt{a+bx^3}}{3x^3}}{a}-\frac{594d\sqrt{a+bx^3}}{x^4}}{2a}-\frac{189c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{3}\sqrt{a+bx^3}\left(\frac{945c}{x^8}+\frac{118\cdots}{x^{\cdots}}\right.\right.}{6160}$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$$\Big\downarrow \; 2374$$

$$\frac{b\left(\frac{1}{2}b\left(\frac{\frac{-\frac{-\frac{\int -\frac{5940(bd-4ag)a^3+6160bexa^3-189b(7bc-22af)x^2a^2}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{189a\sqrt{a+bx^3}(7bc-22af)}{2x^2}}{a}-\frac{3080ae\sqrt{a+bx^3}}{3x^3}}{a}-\frac{594d\sqrt{a+bx^3}}{x^4}}{2a}-\frac{189c\sqrt{a+bx^3}}{ax^5}\right)-\cdots\right.}{6160}$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{1}{2}b\left(\frac{\frac{-\frac{\frac{\int \frac{5940(bd-4ag)a^3+6160bexa^3-189b(7bc-22af)x^2a^2}{x^2\sqrt{bx^3+a}}dx}{4a}-\frac{189a\sqrt{a+bx^3}(7bc-22af)}{2x^2}}{a}-\frac{3080ae\sqrt{a+bx^3}}{3x^3}}{a}-\frac{594d\sqrt{a+bx^3}}{x^4}}{2a}-\frac{189c\sqrt{a+bx^3}}{ax^5}\right)-\frac{1}{3}\cdots\right.}{6160}$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$$\Big\downarrow \; 2374$$

3.473. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{12}}\,dx$

$$b\left(\frac{1}{2}b\left(\frac{-\frac{-\frac{-\frac{-\frac{\int -\frac{2\left(6160bea^4+2970b(bd-4ag)x^2a^3-189b(7bc-22af)xa^3\right)}{x\sqrt{bx^3+a}}dx}{2a}-\frac{5940a^2\sqrt{a+bx^3}(bd-4ag)}{x}}{4a}-\frac{189a\sqrt{a+bx^3}(7bc-22af)}{2x^2}}{a}-\frac{3080ae\sqrt{a+bx^3}}{3x^3}}{a}-\frac{594d\sqrt{\cdots}}{x\cdots}}{2a}\cdots\right.\right.$$

$$\overline{\qquad\qquad 6160 \qquad\qquad}$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$$\Big\downarrow \; 27$$

$$b\left(\frac{1}{2}b\left(\frac{-\frac{-\frac{-\frac{\frac{\int \frac{6160bea^4+2970b(bd-4ag)x^2a^3-189b(7bc-22af)xa^3}{x\sqrt{bx^3+a}}dx}{a}-\frac{5940a^2\sqrt{a+bx^3}(bd-4ag)}{x}}{4a}-\frac{189a\sqrt{a+bx^3}(7bc-22af)}{2x^2}}{a}-\frac{3080ae\sqrt{a+bx^3}}{3x^3}}{a}-\frac{594d\sqrt{a+bx^3}}{x^4}\cdots}{2a}\cdots\right.\right.$$

$$\overline{\qquad\qquad 6160 \qquad\qquad}$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$$\Big\downarrow \; 2371$$

$$b\left(\frac{1}{2}b\left(\frac{-\frac{-\frac{-\frac{\frac{6160a^4be\int\frac{1}{x\sqrt{bx^3+a}}dx+\int\frac{2970a^3b(bd-4ag)x-189a^3b(7bc-22af)}{\sqrt{bx^3+a}}dx}{a}-\frac{5940a^2\sqrt{a+bx^3}(bd-4ag)}{x}}{4a}-\frac{189a\sqrt{a+bx^3}(7bc-22af)}{2x^2}}{a}-\frac{3080ae\sqrt{a+bx^3}}{3x^3}}{a}-5\cdots}{2a}\cdots\right.\right.$$

$$\overline{\qquad\qquad 6160 \qquad\qquad}$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$$\Big\downarrow \; 798$$

3.473. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{12}}\,dx$

$$\frac{b\left(\frac{1}{2}b\left(-\frac{\frac{\frac{\frac{\frac{6160}{3}a^4be\int\frac{1}{x^3\sqrt{bx^3+a}}dx^3+\int\frac{2970a^3b(bd-4ag)x-189a^3b(7bc-22af)}{\sqrt{bx^3+a}}dx}{a}-\frac{5940a^2\sqrt{a+bx^3}(bd-4ag)}{x}}{4a}-\frac{189a\sqrt{a+bx^3}(7bc-22af)}{2x^2}}{a}-\frac{3080ae\sqrt{a+bx^3}}{3x^3}\cdots}{a}\cdots}{2a}\cdots\right.\right.}{6160}\cdots$$

$$\frac{(a+bx^3)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$\big\downarrow$ 73

$$\frac{b\left(\frac{1}{2}b\left(-\frac{\frac{\frac{\frac{\frac{12320}{3}a^4e\int\frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}+\int\frac{2970a^3b(bd-4ag)x-189a^3b(7bc-22af)}{\sqrt{bx^3+a}}dx}{a}-\frac{5940a^2\sqrt{a+bx^3}(bd-4ag)}{x}}{4a}-\frac{189a\sqrt{a+bx^3}(7bc-22af)}{2x^2}}{a}-\frac{3080ae\sqrt{a+bx^3}}{3x^3}\cdots}{a}\cdots}{2a}\cdots\right.\right.}{6160}\cdots$$

$$\frac{(a+bx^3)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$\big\downarrow$ 221

$$\frac{b\left(\frac{1}{2}b\left(-\frac{\frac{\frac{\frac{\int\frac{2970a^3b(bd-4ag)x-189a^3b(7bc-22af)}{\sqrt{bx^3+a}}dx-\frac{12320}{3}a^{7/2}be\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a}-\frac{5940a^2\sqrt{a+bx^3}(bd-4ag)}{x}}{4a}-\frac{189a\sqrt{a+bx^3}(7bc-22af)}{2x^2}}{a}-\frac{3080ae\cdots}{3\cdots}}{a}\cdots}{2a}\cdots\right.\right.}{6160}\cdots$$

$$\frac{(a+bx^3)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$\big\downarrow$ 2417

3.473. $\int\frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx$

$$b\left(\frac{1}{2}b\left(-\frac{\frac{\frac{\frac{-27a^3b^{2/3}\left(7\sqrt[3]{b}(7bc-22af)+110\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\int\frac{1}{\sqrt{bx^3+a}}dx+2970a^3b^{2/3}(bd-4ag)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{12320}{3}a^{7/2}be\,\text{arctanh}}{a}}{4a}}{a}}{a}}{2a}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$\Big\downarrow$ 759

$$b\left(\frac{1}{2}b\left(-\frac{\frac{\frac{2970a^3b^{2/3}(bd-4ag)\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-\frac{12320}{3}a^{7/2}be\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{}}{a}}{4a}}{}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}\left(\frac{2520c}{x^{11}}+\frac{2772d}{x^{10}}+\frac{3080e}{x^9}+\frac{3465f}{x^8}+\frac{3960g}{x^7}\right)}{27720}$$

$\Big\downarrow$ 2416

3.473. $\displaystyle\int\frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{12}}\,dx$

$$b\left(\frac{1}{2}b\left(\frac{-\frac{3080a\sqrt{bx^3+a}e}{3x^3}-\frac{-\frac{12320}{3}be\,\text{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)a^{7/2}+2970b^{2/3}(bd-4ag)\left(\frac{2\sqrt{bx^3+a}}{\sqrt[3]{b}\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)\sqrt{\cdots}}{\sqrt[3]{\cdots}}\right.}{\cdots}}{\cdots}\right.\right.$$

$$\frac{\left(\frac{2520c}{x^{11}}+\frac{3960g}{x^7}+\frac{3465f}{x^8}+\frac{3080e}{x^9}+\frac{2772d}{x^{10}}\right)\left(bx^3+a\right)^{3/2}}{27720}$$

input
```
Int[((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x]
```

output
```
-1/27720*(((2520*c)/x^11 + (2772*d)/x^10 + (3080*e)/x^9 + (3465*f)/x^8 + (
3960*g)/x^7)*(a + b*x^3)^(3/2)) + (b*(-1/3*(((945*c)/x^8 + (1188*d)/x^7 +
(1540*e)/x^6 + (2079*f)/x^5 + (2970*g)/x^4)*Sqrt[a + b*x^3]) + (b*((-189*c
*Sqrt[a + b*x^3])/(a*x^5) + ((-594*d*Sqrt[a + b*x^3])/x^4 + ((-3080*a*e*Sq
rt[a + b*x^3])/(3*x^3) - ((-189*a*(7*b*c - 22*a*f)*Sqrt[a + b*x^3])/(2*x^2
) + ((-5940*a^2*(b*d - 4*a*g)*Sqrt[a + b*x^3])/x + ((-12320*a^(7/2)*b*e*Ar
cTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3 + 2970*a^3*b^(2/3)*(b*d - 4*a*g)*((2*Sqr
t[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqr
t[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])) - (18*3^(3/4)*Sqrt[2 + Sqr
t[3]]*a^3*b^(1/3)*(7*b^(1/3)*(7*b*c - 22*a*f) + 110*(1 - Sqrt[3])*a^(1/3)*
(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/a)/(4*a))/a)/a)/(2*a)))/2))/6160
```

3.473. $\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{12}}\,dx$

3.473.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

3.473. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx$

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.473.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 1006, normalized size of antiderivative = 1.26

method	result	size
elliptic	Expression too large to display	1006
risch	Expression too large to display	1639
default	Expression too large to display	1773

3.473. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx$

input
```
int((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x,method=_RETURNVERBOSE
)
```

output
```
-1/11*a*c*(b*x^3+a)^(1/2)/x^11-1/10*a*d*(b*x^3+a)^(1/2)/x^10-1/9*a*e*(b*x^
3+a)^(1/2)/x^9-1/8*(a*f+25/22*b*c)*(b*x^3+a)^(1/2)/x^8-1/7*(a*g+23/20*b*d)
*(b*x^3+a)^(1/2)/x^7-7/36*b*e*(b*x^3+a)^(1/2)/x^6-1/1760*b*(418*a*f+27*b*c
)/a*(b*x^3+a)^(1/2)/x^5-1/1120*b/a*(340*a*g+27*b*d)*(b*x^3+a)^(1/2)/x^4-1/
24*b^2*e*(b*x^3+a)^(1/2)/a/x^3-27/7040*b^2*(22*a*f-7*b*c)/a^2*(b*x^3+a)^(1
/2)/x^2-27/448*(4*a*g-b*d)*b^2/a^2*(b*x^3+a)^(1/2)/x+9/7040*I*(22*a*f-7*b*
c)*b^2/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))-9/448*I*(4*a*g-b*d)*b^2/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^...
```

3.473.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 606, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = \left[\frac{4620\sqrt{a}b^3ex^{11}\log\left(\frac{b^2x^6+8\,abx^3+4\,(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{a}+8\,a^2}{x^6}\right)+}{}\right.$$

$$-\frac{9240\sqrt{-a}b^3ex^{11}\arctan\left(\frac{(bx^3+2\,a)\sqrt{bx^3+a}\sqrt{-a}}{2\,(abx^3+a^2)}\right)-1701\,(7\,b^3c-22\,ab^2f)\sqrt{b}x^{11}\text{weierstrassPInverse}(0,-\frac{4\,a}{b},x}{}$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="fri
cas")
```

output
```
[1/443520*(4620*sqrt(a)*b^3*e*x^11*log((b^2*x^6 + 8*a*b*x^3 + 4*(b*x^3 + 2
*a)*sqrt(b*x^3 + a)*sqrt(a) + 8*a^2)/x^6) + 1701*(7*b^3*c - 22*a*b^2*f)*sq
rt(b)*x^11*weierstrassPInverse(0, -4*a/b, x) + 26730*(b^3*d - 4*a*b^2*g)*s
qrt(b)*x^11*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
 - (18480*a*b^2*e*x^8 - 26730*(b^3*d - 4*a*b^2*g)*x^10 - 1701*(7*b^3*c - 22
*a*b^2*f)*x^9 + 86240*a^2*b*e*x^5 + 396*(27*a*b^2*d + 340*a^2*b*g)*x^7 + 2
52*(27*a*b^2*c + 418*a^2*b*f)*x^6 + 49280*a^3*e*x^2 + 44352*a^3*d*x + 3168
*(23*a^2*b*d + 20*a^3*g)*x^4 + 40320*a^3*c + 2520*(25*a^2*b*c + 22*a^3*f)*
x^3)*sqrt(b*x^3 + a))/(a^2*x^11), -1/443520*(9240*sqrt(-a)*b^3*e*x^11*arct
an(1/2*(b*x^3 + 2*a)*sqrt(b*x^3 + a)*sqrt(-a)/(a*b*x^3 + a^2)) - 1701*(7*b
^3*c - 22*a*b^2*f)*sqrt(b)*x^11*weierstrassPInverse(0, -4*a/b, x) - 26730*
(b^3*d - 4*a*b^2*g)*sqrt(b)*x^11*weierstrassZeta(0, -4*a/b, weierstrassPIn
verse(0, -4*a/b, x)) + (18480*a*b^2*e*x^8 - 26730*(b^3*d - 4*a*b^2*g)*x^10
 - 1701*(7*b^3*c - 22*a*b^2*f)*x^9 + 86240*a^2*b*e*x^5 + 396*(27*a*b^2*d +
 340*a^2*b*g)*x^7 + 252*(27*a*b^2*c + 418*a^2*b*f)*x^6 + 49280*a^3*e*x^2 +
 44352*a^3*d*x + 3168*(23*a^2*b*d + 20*a^3*g)*x^4 + 40320*a^3*c + 2520*(25
*a^2*b*c + 22*a^3*f)*x^3)*sqrt(b*x^3 + a))/(a^2*x^11)]
```

3.473. $\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx$

3.473.6 Sympy [A] (verification not implemented)

Time = 12.15 (sec) , antiderivative size = 541, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^3)^{3/2}(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{11}{3}\right){}_2F_1\left(\begin{matrix}-\frac{11}{3},-\frac{1}{2}\\-\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{11}\Gamma\left(-\frac{8}{3}\right)} + \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{10}{3},-\frac{1}{2}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{8}\Gamma\left(-\frac{5}{3}\right)} + \frac{a^{\frac{3}{2}}g\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{8}\Gamma\left(-\frac{5}{3}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{5}\Gamma\left(-\frac{2}{3}\right)} + \frac{\sqrt{a}bg\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)} - \frac{a^2e}{9\sqrt{b}x^{\frac{21}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{11a\sqrt{b}e}{36x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{17b^{\frac{3}{2}}e}{72x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{5}{2}}e}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b^3e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{24a^{\frac{3}{2}}}$$

input
```
integrate((b*x**3+a)**(3/2)*(g*x**4+f*x**3+e*x**2+d*x+c)/x**12,x)
```

output
```
a**(3/2)*c*gamma(-11/3)*hyper((-11/3, -1/2), (-8/3,), b*x**3*exp_polar(I*p
i)/a)/(3*x**11*gamma(-8/3)) + a**(3/2)*d*gamma(-10/3)*hyper((-10/3, -1/2),
 (-7/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**10*gamma(-7/3)) + a**(3/2)*f*gam
ma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*ga
mma(-5/3)) + a**(3/2)*g*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + sqrt(a)*b*c*gamma(-8/3)*hyper((-8/
3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**8*gamma(-5/3)) + sqrt(a
)*b*d*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*x**7*gamma(-4/3)) + sqrt(a)*b*f*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + sqrt(a)*b*g*gamma(-4/3)*
hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)
) - a**2*e/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - 11*a*sqrt(b)*e/(36
*x**(15/2)*sqrt(a/(b*x**3) + 1)) - 17*b**(3/2)*e/(72*x**(9/2)*sqrt(a/(b*x*
*3) + 1)) - b**(5/2)*e/(24*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + b**3*e*asinh
(sqrt(a)/(sqrt(b)*x**(3/2)))/(24*a**(3/2))
```

3.473.7 Maxima [F]

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{12}}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="max
ima")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^12, x)
```

3.473.8 Giac [F]

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(c+dx+ex^2+fx^3+gx^4\right)}{x^{12}}\,dx = \int \frac{(gx^4+fx^3+ex^2+dx+c)(bx^3+a)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)*(g*x^4+f*x^3+e*x^2+d*x+c)/x^12,x, algorithm="gia
c")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)*(b*x^3 + a)^(3/2)/x^12, x)
```

3.473.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(c+dx+ex^2+fx^3+gx^4)}{x^{12}}\,dx=\int \frac{(b\,x^3+a)^{3/2}\,(g\,x^4+f\,x^3+e\,x^2+d\,x+c)}{x^{12}}\,dx$$

input
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12,x)
```

output
```
int(((a + b*x^3)^(3/2)*(c + d*x + e*x^2 + f*x^3 + g*x^4))/x^12, x)
```

3.474 $\int (c+dx+ex^2)(a+bx^3)^p\,dx$

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3.474.1 Optimal result

Integrand size = 20, antiderivative size = 102

$$\int (c+dx+ex^2)(a+bx^3)^p\,dx = \frac{e(a+bx^3)^{1+p}}{3b(1+p)} + \frac{cx(a+bx^3)^{1+p}\text{Hypergeometric2F1}\left(1,\frac{4}{3}+p,\frac{4}{3},-\frac{bx^3}{a}\right)}{a} + \frac{dx^2(a+bx^3)^{1+p}\text{Hypergeometric2F1}\left(1,\frac{5}{3}+p,\frac{5}{3},-\frac{bx^3}{a}\right)}{2a}$$

output
```
1/3*e*(b*x^3+a)^(p+1)/b/(p+1)+c*x*(b*x^3+a)^(p+1)*hypergeom([1, 4/3+p],[4/
3],-b*x^3/a)/a+1/2*d*x^2*(b*x^3+a)^(p+1)*hypergeom([1, 5/3+p],[5/3],-b*x^3
/a)/a
```

3.474.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int (c+dx+ex^2)(a+bx^3)^p\,dx$$
$$= \frac{(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(2e(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p + 6bc(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\frac{bx^3}{a}\right)+3b\right.}{6b(1+p)}$$

input
```
Integrate[(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

output
```
((a + b*x^3)^p*(2*e*(a + b*x^3)*(1 + (b*x^3)/a)^p + 6*b*c*(1 + p)*x*Hyperg
eometric2F1[1/3, -p, 4/3, -((b*x^3)/a)] + 3*b*d*(1 + p)*x^2*Hypergeometric
2F1[2/3, -p, 5/3, -((b*x^3)/a)]))/(6*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

3.474.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^p \left(c+dx+ex^2\right) \, dx$$

$$\Big\downarrow \text{2425}$$

$$\int (c+dx)\left(bx^3+a\right)^p dx+e\int x^2\left(bx^3+a\right)^p dx$$

$$\Big\downarrow \text{793}$$

$$\int (c+dx)\left(bx^3+a\right)^p dx+\frac{e\left(a+bx^3\right)^{p+1}}{3b(p+1)}$$

$$\Big\downarrow \text{2432}$$

$$\int \left(c\left(bx^3+a\right)^p+dx\left(bx^3+a\right)^p\right)dx+\frac{e\left(a+bx^3\right)^{p+1}}{3b(p+1)}$$

$$\Big\downarrow \text{2009}$$

$$cx\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\frac{bx^3}{a}\right)+\frac{1}{2}dx^2\left(a+bx^3\right)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)+\frac{e\left(a+bx^3\right)^{p+1}}{3b(p+1)}$$

input
```
Int[(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

output
```
(e*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x*(a + b*x^3)^p*Hypergeometric2
F1[1/3, -p, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^p + (d*x^2*(a + b*x^3)^p*H
ypergeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(2*(1 + (b*x^3)/a)^p)
```

3.474.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.474.4 Maple [F]

$$\int \left(e\,x^{2}+dx+c\right)\left(b\,x^{3}+a\right)^{p}dx$$

input
```
int((e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

output
```
int((e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

3.474.5 Fricas [F]

$$\int (c + dx + ex^2) (a + bx^3)^p \, dx = \int (ex^2 + dx + c) (bx^3 + a)^p \, dx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

3.474.6 Sympy [A] (verification not implemented)

Time = 28.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

$$\int (c + dx + ex^2) (a + bx^3)^p \, dx = \frac{a^p c x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -p \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^p d x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, -p \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + e\left(\begin{cases} \frac{a^p x^3}{3} & \text{for } b = 0 \\ \frac{\begin{cases} \frac{(a+bx^3)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a + bx^3) & \text{otherwise} \end{cases}}{3b} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

output
```
a**p*c*x*gamma(1/3)*hyper((1/3, -p), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(4/3)) + a**p*d*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*gamma(5/3)) + e*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piece
wise(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True))/
(3*b), True))
```

3.474.7 Maxima [F]

$$\int (c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int (ex^2+dx+c)\left(bx^3+a\right)^p\,dx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

3.474.8 Giac [F]

$$\int (c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int (ex^2+dx+c)\left(bx^3+a\right)^p\,dx$$

input
```
integrate((e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p, x)
```

3.474.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int \left(b\,x^3+a\right)^p\,\left(e\,x^2+d\,x+c\right)\,dx$$

input
```
int((a + b*x^3)^p*(c + d*x + e*x^2),x)
```

output
```
int((a + b*x^3)^p*(c + d*x + e*x^2), x)
```

3.475 $\int x(c+dx+ex^2)(a+bx^3)^p\,dx$

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3.475.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx = \frac{d(a+bx^3)^{1+p}}{3b(1+p)}+\frac{cx^2(a+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{5}{3}+p,\frac{5}{3},-\frac{bx^3}{a}\right)}{2a} + \frac{ex^4(a+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{7}{3}+p,\frac{7}{3},-\frac{bx^3}{a}\right)}{4a}$$

output
```
1/3*d*(b*x^3+a)^(p+1)/b/(p+1)+1/2*c*x^2*(b*x^3+a)^(p+1)*hypergeom([1, 5/3+
p],[5/3],-b*x^3/a)/a+1/4*e*x^4*(b*x^3+a)^(p+1)*hypergeom([1, 7/3+p],[7/3],
-b*x^3/a)/a
```

3.475.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx = \frac{(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(4d(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p+6bc(1+p)x^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)+3\right.}{12b(1+p)}$$

input
```
Integrate[x*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

output
```
((a + b*x^3)^p*(4*d*(a + b*x^3)*(1 + (b*x^3)/a)^p + 6*b*c*(1 + p)*x^2*Hype
rgeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)] + 3*b*e*(1 + p)*x^4*Hypergeometr
ic2F1[4/3, -p, 7/3, -((b*x^3)/a)]))/(12*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

3.475.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^3)^p\left(c+dx+ex^2\right)\,dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(cx(a+bx^3)^p+dx^2(a+bx^3)^p+ex^3(a+bx^3)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}cx^2(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)+\frac{d(a+bx^3)^{p+1}}{3b(p+1)}+\frac{1}{4}ex^4(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

input
```
Int[x*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

output
```
(d*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (c*x^2*(a + b*x^3)^p*Hypergeometri
c2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(2*(1 + (b*x^3)/a)^p) + (e*x^4*(a + b*x^
3)^p*Hypergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(4*(1 + (b*x^3)/a)^p)
```

3.475.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.475.4 Maple [F]

$$\int x\left(e\,x^{2}+dx+c\right)\left(b\,x^{3}+a\right)^{p}dx$$

input
```
int(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

output
```
int(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

3.475.5 Fricas [F]

$$\int x\left(c+dx+ex^{2}\right)\left(a+bx^{3}\right)^{p}\,dx=\int\left(ex^{2}+dx+c\right)\left(bx^{3}+a\right)^{p}x\,dx$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^3 + d*x^2 + c*x)*(b*x^3 + a)^p, x)
```

3.475.6 Sympy [A] (verification not implemented)

Time = 43.56 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\frac{a^pcx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},-p\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{a^pex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},-p\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+d\left(\begin{cases}\frac{a^px^3}{3} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^3)^{p+1}}{p+1} & \text{for } p\neq-1\\ \log\left(a+bx^3\right) & \text{otherwise}\end{cases}}{3b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

output
```
a**p*c*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(5/3)) + a**p*e*x**4*gamma(4/3)*hyper((4/3, -p), (7/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(7/3)) + d*Piecewise((a**p*x**3/3, Eq(b, 0)), (Pi
ecewise(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True
))/(3*b), True))
```

3.475.7 Maxima [F]

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int\left(ex^2+dx+c\right)\left(bx^3+a\right)^px\,dx$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x, x)
```

3.475.8 Giac [F]

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int (ex^2+dx+c)\left(bx^3+a\right)^p x\,dx$$

input
```
integrate(x*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x, x)
```

3.475.9 Mupad [F(-1)]

Timed out.

$$\int x(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int x\left(b\,x^3+a\right)^p\left(e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x*(a + b*x^3)^p*(c + d*x + e*x^2),x)
```

output
```
int(x*(a + b*x^3)^p*(c + d*x + e*x^2), x)
```

3.476 $\int x^2(c+dx+ex^2)(a+bx^3)^p\,dx$

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3.476.7 Maxima [F] . . . 3752
3.476.8 Giac [F] . . . 3753
3.476.9 Mupad [F(-1)] . . . 3753

3.476.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int x^2(c+dx+ex^2)(a+bx^3)^p\,dx = \frac{c(a+bx^3)^{1+p}}{3b(1+p)} + \frac{dx^4(a+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{7}{3}+p,\frac{7}{3},-\frac{bx^3}{a}\right)}{4a} + \frac{ex^5(a+bx^3)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{8}{3}+p,\frac{8}{3},-\frac{bx^3}{a}\right)}{5a}$$

output
```
1/3*c*(b*x^3+a)^(p+1)/b/(p+1)+1/4*d*x^4*(b*x^3+a)^(p+1)*hypergeom([1, 7/3+
p],[7/3],-b*x^3/a)/a+1/5*e*x^5*(b*x^3+a)^(p+1)*hypergeom([1, 8/3+p],[8/3],
-b*x^3/a)/a
```

3.476.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int x^2(c+dx+ex^2)(a+bx^3)^p\,dx = \frac{(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\left(20c(a+bx^3)\left(1+\frac{bx^3}{a}\right)^p+15bd(1+p)x^4\,\text{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{bx^3}{a}\right)+\right.}{60b(1+p)}$$

input
```
Integrate[x^2*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

output
```
((a + b*x^3)^p*(20*c*(a + b*x^3)*(1 + (b*x^3)/a)^p + 15*b*d*(1 + p)*x^4*Hy
pergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)] + 12*b*e*(1 + p)*x^5*Hypergeom
etric2F1[5/3, -p, 8/3, -((b*x^3)/a)]))/(60*b*(1 + p)*(1 + (b*x^3)/a)^p)
```

3.476.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right)^p \left(c+dx+ex^2\right) dx$$

$$\Big\downarrow \text{ 2432}$$

$$\int \left(cx^2 \left(a+bx^3\right)^p+dx^3 \left(a+bx^3\right)^p+ex^4 \left(a+bx^3\right)^p\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\left(a+bx^3\right)^{p+1}}{3b(p+1)}+\frac{1}{4}dx^4 \left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{bx^3}{a}\right)+\frac{1}{5}ex^5 \left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{3},-p,\frac{8}{3},-\frac{bx^3}{a}\right)$$

input
```
Int[x^2*(c + d*x + e*x^2)*(a + b*x^3)^p,x]
```

output
```
(c*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (d*x^4*(a + b*x^3)^p*Hypergeometri
c2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(4*(1 + (b*x^3)/a)^p) + (e*x^5*(a + b*x^
3)^p*Hypergeometric2F1[5/3, -p, 8/3, -((b*x^3)/a)])/(5*(1 + (b*x^3)/a)^p)
```

3.476.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.476.4 Maple [F]

$$\int x^{2} \left(e\,x^{2}+dx+c\right) \left(b\,x^{3}+a\right)^{p}dx$$

input
```
int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

output
```
int(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x)
```

3.476.5 Fricas [F]

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int \left(ex^2+dx+c\right)\left(bx^3+a\right)^px^2\,dx$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^4 + d*x^3 + c*x^2)*(b*x^3 + a)^p, x)
```

3.476.6 Sympy [A] (verification not implemented)

Time = 62.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^p\,dx = \frac{a^p d x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, -p \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^p e x^5 \Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} \frac{5}{3}, -p \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + c\left(\begin{cases} \frac{a^p x^3}{3} & \text{for}\ b=0 \\ \frac{\begin{cases} \frac{(a+bx^3)^{p+1}}{p+1} & \text{for}\ p\neq -1 \\ \log\left(a+bx^3\right) & \text{otherwise} \end{cases}}{3b} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(x**2*(e*x**2+d*x+c)*(b*x**3+a)**p,x)
```

output
```
a**p*d*x**4*gamma(4/3)*hyper((4/3, -p), (7/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(7/3)) + a**p*e*x**5*gamma(5/3)*hyper((5/3, -p), (8/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(8/3)) + c*Piecewise((a**p*x**3/3, Eq(b, 0)), (Pi
ecewise(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True
))/(3*b), True))
```

3.476.7 Maxima [F]

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^p\,dx = \int \left(ex^2+dx+c\right)\left(bx^3+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="maxima")
```

output
```
1/3*(b*x^3 + a)^(p + 1)*c/(b*(p + 1)) + integrate((e*x^4 + d*x^3)*(b*x^3 +
 a)^p, x)
```

3.476.8 Giac [F]

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int \left(ex^2+dx+c\right)\left(bx^3+a\right)^px^2\,dx$$

input
```
integrate(x^2*(e*x^2+d*x+c)*(b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)*(b*x^3 + a)^p*x^2, x)
```

3.476.9 Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx+ex^2)\left(a+bx^3\right)^p\,dx=\int x^2\,\left(b\,x^3+a\right)^p\,\left(e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^2*(a + b*x^3)^p*(c + d*x + e*x^2),x)
```

output
```
int(x^2*(a + b*x^3)^p*(c + d*x + e*x^2), x)
```

3.477 $\int (c+dx+ex^2+fx^3)(a+bx^4)\,dx$

3.477.1 Optimal result 3754
3.477.2 Mathematica [A] (verified) 3754
3.477.3 Rubi [A] (verified) 3755
3.477.4 Maple [A] (verified) 3756
3.477.5 Fricas [A] (verification not implemented) 3756
3.477.6 Sympy [A] (verification not implemented) 3757
3.477.7 Maxima [A] (verification not implemented) 3757
3.477.8 Giac [A] (verification not implemented) 3757
3.477.9 Mupad [B] (verification not implemented) 3758

3.477.1 Optimal result

Integrand size = 23, antiderivative size = 68

$$\int (c+dx+ex^2+fx^3)(a+bx^4)\,dx = acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

output
```
a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*a*f*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*
x^7+1/8*b*f*x^8
```

3.477.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int (c+dx+ex^2+fx^3)(a+bx^4)\,dx = acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/
6 + (b*e*x^7)/7 + (b*f*x^8)/8
```

3.477.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)\left(c+dx+ex^2+fx^3\right)dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(ac+adx+aex^2+afx^3+bcx^4+bdx^5+bex^6+bfx^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/
6 + (b*e*x^7)/7 + (b*f*x^8)/8
```

3.477.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.477.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

method	result	size
gosper	$acx + \frac{1}{2}ad\,x^2 + \frac{1}{3}ae\,x^3 + \frac{1}{4}af\,x^4 + \frac{1}{5}bc\,x^5 + \frac{1}{6}bd\,x^6 + \frac{1}{7}be\,x^7 + \frac{1}{8}bf\,x^8$	55
default	$acx + \frac{1}{2}ad\,x^2 + \frac{1}{3}ae\,x^3 + \frac{1}{4}af\,x^4 + \frac{1}{5}bc\,x^5 + \frac{1}{6}bd\,x^6 + \frac{1}{7}be\,x^7 + \frac{1}{8}bf\,x^8$	55
norman	$acx + \frac{1}{2}ad\,x^2 + \frac{1}{3}ae\,x^3 + \frac{1}{4}af\,x^4 + \frac{1}{5}bc\,x^5 + \frac{1}{6}bd\,x^6 + \frac{1}{7}be\,x^7 + \frac{1}{8}bf\,x^8$	55
risch	$acx + \frac{1}{2}ad\,x^2 + \frac{1}{3}ae\,x^3 + \frac{1}{4}af\,x^4 + \frac{1}{5}bc\,x^5 + \frac{1}{6}bd\,x^6 + \frac{1}{7}be\,x^7 + \frac{1}{8}bf\,x^8$	55
parallelrisch	$acx + \frac{1}{2}ad\,x^2 + \frac{1}{3}ae\,x^3 + \frac{1}{4}af\,x^4 + \frac{1}{5}bc\,x^5 + \frac{1}{6}bd\,x^6 + \frac{1}{7}be\,x^7 + \frac{1}{8}bf\,x^8$	55

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*a*f*x^4+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*
x^7+1/8*b*f*x^8
```

3.477.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (c + dx + ex^2 + fx^3)(a + bx^4)\,dx = \frac{1}{8}bfx^8 + \frac{1}{7}bex^7 + \frac{1}{6}bdx^6 + \frac{1}{5}bcx^5 + \frac{1}{4}afx^4 + \frac{1}{3}aex^3 + \frac{1}{2}adx^2 + acx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")
```

output
```
1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*
a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

3.477.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx=acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{afx^4}{4}+\frac{bcx^5}{5}+\frac{bdx^6}{6}+\frac{bex^7}{7}+\frac{bfx^8}{8}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

output
```
a*c*x + a*d*x**2/2 + a*e*x**3/3 + a*f*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b
*e*x**7/7 + b*f*x**8/8
```

3.477.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx=\frac{1}{8}\,bfx^8+\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,afx^4+\frac{1}{3}\,aex^3+\frac{1}{2}\,adx^2+acx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*
a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

3.477.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)\,dx=\frac{1}{8}\,bfx^8+\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{5}\,bcx^5+\frac{1}{4}\,afx^4+\frac{1}{3}\,aex^3+\frac{1}{2}\,adx^2+acx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")
```

output
```
1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*
a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

3.477.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (c+dx+ex^2+fx^3)\,(a+bx^4)\;dx = \frac{b\,f\,x^8}{8}+\frac{b\,e\,x^7}{7}+\frac{b\,d\,x^6}{6}+\frac{b\,c\,x^5}{5}+\frac{a\,f\,x^4}{4}+\frac{a\,e\,x^3}{3}+\frac{a\,d\,x^2}{2}+a\,c\,x$$

input
```
int((a + b*x^4)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
a*c*x + (a*d*x^2)/2 + (b*c*x^5)/5 + (a*e*x^3)/3 + (b*d*x^6)/6 + (a*f*x^4)/
4 + (b*e*x^7)/7 + (b*f*x^8)/8
```

3.478 $\int x^3(c+dx+ex^2+fx^3)(a+bx^4)\,dx$

3.478.1 Optimal result 3759
3.478.2 Mathematica [A] (verified) 3759
3.478.3 Rubi [A] (verified) 3760
3.478.4 Maple [A] (verified) 3761
3.478.5 Fricas [A] (verification not implemented) 3761
3.478.6 Sympy [A] (verification not implemented) 3762
3.478.7 Maxima [A] (verification not implemented) 3762
3.478.8 Giac [A] (verification not implemented) 3762
3.478.9 Mupad [B] (verification not implemented) 3763

3.478.1 Optimal result

Integrand size = 26, antiderivative size = 73

$$\int x^3(c+dx+ex^2+fx^3)(a+bx^4)\,dx = \frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}afx^7+\frac{1}{8}bcx^8+\frac{1}{9}bdx^9+\frac{1}{10}bex^{10}+\frac{1}{11}bfx^{11}$$

output
```
1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*a*f*x^7+1/8*b*c*x^8+1/9*b*d*x^9+1/
10*b*e*x^10+1/11*b*f*x^11
```

3.478.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^3(c+dx+ex^2+fx^3)(a+bx^4)\,dx = \frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}afx^7+\frac{1}{8}bcx^8+\frac{1}{9}bdx^9+\frac{1}{10}bex^{10}+\frac{1}{11}bfx^{11}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

output
```
(a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d
*x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

3.478.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^4) (c+dx+ex^2+fx^3) \, dx$$

$$\Big\downarrow \text{2360}$$

$$\int \left(acx^3+adx^4+aex^5+afx^6+bcx^7+bdx^8+bex^9+bfx^{10}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}afx^7+\frac{1}{8}bcx^8+\frac{1}{9}bdx^9+\frac{1}{10}bex^{10}+\frac{1}{11}bfx^{11}$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

output
```
(a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d
*x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

3.478.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

3.478.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{1}{4}a\,x^4c+\frac{1}{5}ad\,x^5+\frac{1}{6}ae\,x^6+\frac{1}{7}af\,x^7+\frac{1}{8}bc\,x^8+\frac{1}{9}bd\,x^9+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
default	$\frac{1}{4}a\,x^4c+\frac{1}{5}ad\,x^5+\frac{1}{6}ae\,x^6+\frac{1}{7}af\,x^7+\frac{1}{8}bc\,x^8+\frac{1}{9}bd\,x^9+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
norman	$\frac{1}{4}a\,x^4c+\frac{1}{5}ad\,x^5+\frac{1}{6}ae\,x^6+\frac{1}{7}af\,x^7+\frac{1}{8}bc\,x^8+\frac{1}{9}bd\,x^9+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
risch	$\frac{1}{4}a\,x^4c+\frac{1}{5}ad\,x^5+\frac{1}{6}ae\,x^6+\frac{1}{7}af\,x^7+\frac{1}{8}bc\,x^8+\frac{1}{9}bd\,x^9+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
parallelrisch	$\frac{1}{4}a\,x^4c+\frac{1}{5}ad\,x^5+\frac{1}{6}ae\,x^6+\frac{1}{7}af\,x^7+\frac{1}{8}bc\,x^8+\frac{1}{9}bd\,x^9+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4*c+1/5*a*d*x^5+1/6*a*e*x^6+1/7*a*f*x^7+1/8*b*c*x^8+1/9*b*d*x^9+1/
10*b*e*x^10+1/11*b*f*x^11
```

3.478.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx=\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bex^{10}+\frac{1}{9}\,bdx^9+\frac{1}{8}\,bcx^8+\frac{1}{7}\,afx^7+\frac{1}{6}\,aex^6+\frac{1}{5}\,adx^5+\frac{1}{4}\,acx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")
```

output
```
1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 +
1/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

3.478.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx = \frac{acx^4}{4}+\frac{adx^5}{5}+\frac{aex^6}{6}+\frac{afx^7}{7}+\frac{bcx^8}{8}+\frac{bdx^9}{9}+\frac{bex^{10}}{10}+\frac{bfx^{11}}{11}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

output
```
a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + a*f*x**7/7 + b*c*x**8/8 + b*d*x**9/
9 + b*e*x**10/10 + b*f*x**11/11
```

3.478.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx = \frac{1}{11}bfx^{11}+\frac{1}{10}bex^{10}+\frac{1}{9}bdx^9+\frac{1}{8}bcx^8+\frac{1}{7}afx^7+\frac{1}{6}aex^6+\frac{1}{5}adx^5+\frac{1}{4}acx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")
```

output
```
1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 +
1/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

3.478.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx = \frac{1}{11}bfx^{11}+\frac{1}{10}bex^{10}+\frac{1}{9}bdx^9+\frac{1}{8}bcx^8+\frac{1}{7}afx^7+\frac{1}{6}aex^6+\frac{1}{5}adx^5+\frac{1}{4}acx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")
```

output
```
1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 +
1/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

3.478.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3(c+dx+ex^2+fx^3)\,(a+bx^4)\,dx=\frac{b\,f\,x^{11}}{11}+\frac{b\,e\,x^{10}}{10}+\frac{b\,d\,x^9}{9}+\frac{b\,c\,x^8}{8}+\frac{a\,f\,x^7}{7}+\frac{a\,e\,x^6}{6}+\frac{a\,d\,x^5}{5}+\frac{a\,c\,x^4}{4}$$

input
```
int(x^3*(a + b*x^4)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a*c*x^4)/4 + (a*d*x^5)/5 + (b*c*x^8)/8 + (a*e*x^6)/6 + (b*d*x^9)/9 + (a*f
*x^7)/7 + (b*e*x^10)/10 + (b*f*x^11)/11
```

3.479 $\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx$

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3.479.1 Optimal result

Integrand size = 25, antiderivative size = 109

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{f(a+bx^4)^3}{12b}$$

output
```
a^2*c*x+1/2*a^2*d*x^2+1/3*a^2*e*x^3+2/5*a*b*c*x^5+1/3*a*b*d*x^6+2/7*a*b*e*
x^7+1/9*b^2*c*x^9+1/10*b^2*d*x^10+1/11*b^2*e*x^11+1/12*f*(b*x^4+a)^3/b
```

3.479.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx^{12}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5
+ (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d
*x^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

3.479.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^2 \left(c+dx+ex^2+fx^3\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(ex^2+dx+c\right) \left(bx^4+a\right)^2dx+\frac{f \left(a+bx^4\right)^3}{12 b}$$

$$\Big\downarrow \text{2188}$$

$$\int \left(b^2 e x^{10}+b^2 d x^9+b^2 c x^8+2 a b e x^6+2 a b d x^5+2 a b c x^4+a^2 e x^2+a^2 d x+a^2 c\right)dx+\frac{f \left(a+bx^4\right)^3}{12 b}$$

$$\Big\downarrow \text{2009}$$

$$a^2 c x+\frac{1}{2} a^2 d x^2+\frac{1}{3} a^2 e x^3+\frac{2}{5} a b c x^5+\frac{1}{3} a b d x^6+\frac{2}{7} a b e x^7+\frac{f \left(a+bx^4\right)^3}{12 b}+\frac{1}{9} b^2 c x^9+\frac{1}{10} b^2 d x^{10}+\frac{1}{11} b^2 e x^{11}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3
+ (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f
*(a + b*x^4)^3)/(12*b)
```

3.479.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.479.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2$
default	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2$
norman	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2$
risch	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2$
parallelrisch	$\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^2$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*a*b*f*x^
8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*a^2*f*x^4+1/3*a^2*e*x^3+1/
2*a^2*d*x^2+a^2*c*x
```

3.479.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (c + dx + ex^2 + fx^3)(a + bx^4)^2 \, dx = \frac{1}{12}b^2fx^{12} + \frac{1}{11}b^2ex^{11} + \frac{1}{10}b^2dx^{10} + \frac{1}{9}b^2cx^9 + \frac{1}{4}abfx^8 + \frac{2}{7}abex^7 + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2ex^3 + \frac{1}{2}a^2dx^2 + a^2cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*
a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4
+ 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

3.479.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11

$$\int (c + dx + ex^2 + fx^3)(a + bx^4)^2 \, dx = a^2cx + \frac{a^2dx^2}{2} + \frac{a^2ex^3}{3} + \frac{a^2fx^4}{4} + \frac{2abcx^5}{5} + \frac{abdx^6}{3} + \frac{2abex^7}{7} + \frac{abfx^8}{4} + \frac{b^2cx^9}{9} + \frac{b^2dx^{10}}{10} + \frac{b^2ex^{11}}{11} + \frac{b^2fx^{12}}{12}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

output
```
a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5
+ a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x**
*10/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

3.479.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2 \, dx = \frac{1}{12}b^2fx^{12} + \frac{1}{11}b^2ex^{11} + \frac{1}{10}b^2dx^{10} + \frac{1}{9}b^2cx^9 + \frac{1}{4}abfx^8 + \frac{2}{7}abex^7 + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2ex^3 + \frac{1}{2}a^2dx^2 + a^2cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*
a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4
+ 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

3.479.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2 \, dx = \frac{1}{12}b^2fx^{12} + \frac{1}{11}b^2ex^{11} + \frac{1}{10}b^2dx^{10} + \frac{1}{9}b^2cx^9 + \frac{1}{4}abfx^8 + \frac{2}{7}abex^7 + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2ex^3 + \frac{1}{2}a^2dx^2 + a^2cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*
a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4
+ 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

3.479.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (c+dx+ex^2+fx^3)\,(a+bx^4)^2\,dx=\frac{f\,a^2\,x^4}{4}+\frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{f\,a\,b\,x^8}{4}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{f\,b^2\,x^{12}}{12}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

input
```
int((a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^2*d*x^2)/2 + (b^2*c*x^9)/9 + (a^2*e*x^3)/3 + (b^2*d*x^10)/10 + (a^2*f*x
^4)/4 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12 + a^2*c*x + (2*a*b*c*x^5)/5 + (a
*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4
```

3.480 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx$

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3.480.1 Optimal result

Integrand size = 28, antiderivative size = 114

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx=\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}+\frac{c(a+bx^4)^3}{12b}$$

output
```
1/5*a^2*d*x^5+1/6*a^2*e*x^6+1/7*a^2*f*x^7+2/9*a*b*d*x^9+1/5*a*b*e*x^10+2/1
1*a*b*f*x^11+1/13*b^2*d*x^13+1/14*b^2*e*x^14+1/15*b^2*f*x^15+1/12*c*(b*x^4
+a)^3/b
```

3.480.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx=\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{1}{4}abcx^8+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{12}b^2cx^{12}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

output
```
(a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (a*b*c*x^8
)/4 + (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11 + (b^2*c*x^12)/
12 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15
```

3.480.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^4\right)^2 \left(c+dx+ex^2+fx^3\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^4+a\right)^2 \left(x^3 \left(fx^3+ex^2+dx+c\right)-cx^3\right) dx+\frac{c \left(a+bx^4\right)^3}{12 b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^2 f x^{14}+b^2 e x^{13}+b^2 d x^{12}+2 a b f x^{10}+2 a b e x^9+2 a b d x^8+a^2 f x^6+a^2 e x^5+a^2 d x^4\right) dx+\frac{c \left(a+bx^4\right)^3}{12 b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5} a^2 d x^5+\frac{1}{6} a^2 e x^6+\frac{1}{7} a^2 f x^7+\frac{c \left(a+bx^4\right)^3}{12 b}+\frac{2}{9} a b d x^9+\frac{1}{5} a b e x^{10}+\frac{2}{11} a b f x^{11}+\frac{1}{13} b^2 d x^{13}+\frac{1}{14} b^2 e x^{14}+\frac{1}{15} b^2 f x^{15}$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

output
```
(a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (2*a*b*d*x^9)/9 + (a*b*e*x
^10)/5 + (2*a*b*f*x^11)/11 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^
15)/15 + (c*(a + b*x^4)^3)/(12*b)
```

3.480.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.480.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c$
default	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c$
norman	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c$
risch	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c$
parallelrisch	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*c*x^4+1/5*a^2*d*x^5+1/6*a^2*e*x^6+1/7*a^2*f*x^7+1/4*a*b*c*x^8+2/9*
a*b*d*x^9+1/5*a*b*e*x^10+2/11*a*b*f*x^11+1/12*b^2*c*x^12+1/13*b^2*d*x^13+1
/14*b^2*e*x^14+1/15*b^2*f*x^15
```

3.480.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx = \frac{1}{15}\,b^2fx^{15}+\frac{1}{14}\,b^2ex^{14}+\frac{1}{13}\,b^2dx^{13}+\frac{1}{12}\,b^2cx^{12}+\frac{2}{11}\,abfx^{11}+\frac{1}{5}\,abex^{10}+\frac{2}{9}\,abdx^9+\frac{1}{4}\,abcx^8+\frac{1}{7}\,a^2fx^7+\frac{1}{6}\,a^2ex^6+\frac{1}{5}\,a^2dx^5+\frac{1}{4}\,a^2cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/
11*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f
*x^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

3.480.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx = \frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{a^2fx^7}{7}+\frac{abcx^8}{4}+\frac{2abdx^9}{9}+\frac{abex^{10}}{5}+\frac{2abfx^{11}}{11}+\frac{b^2cx^{12}}{12}+\frac{b^2dx^{13}}{13}+\frac{b^2ex^{14}}{14}+\frac{b^2fx^{15}}{15}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

output
```
a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + a**2*f*x**7/7 + a*b*c*x**8
/4 + 2*a*b*d*x**9/9 + a*b*e*x**10/5 + 2*a*b*f*x**11/11 + b**2*c*x**12/12 +
 b**2*d*x**13/13 + b**2*e*x**14/14 + b**2*f*x**15/15
```

3.480.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx=\frac{1}{15}\,b^2fx^{15}+\frac{1}{14}\,b^2ex^{14}+\frac{1}{13}\,b^2dx^{13}+\frac{1}{12}\,b^2cx^{12}+\frac{2}{11}\,abfx^{11}+\frac{1}{5}\,abex^{10}+\frac{2}{9}\,abdx^9+\frac{1}{4}\,abcx^8+\frac{1}{7}\,a^2fx^7+\frac{1}{6}\,a^2ex^6+\frac{1}{5}\,a^2dx^5+\frac{1}{4}\,a^2cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/
11*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f
*x^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

3.480.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx=\frac{1}{15}\,b^2fx^{15}+\frac{1}{14}\,b^2ex^{14}+\frac{1}{13}\,b^2dx^{13}+\frac{1}{12}\,b^2cx^{12}+\frac{2}{11}\,abfx^{11}+\frac{1}{5}\,abex^{10}+\frac{2}{9}\,abdx^9+\frac{1}{4}\,abcx^8+\frac{1}{7}\,a^2fx^7+\frac{1}{6}\,a^2ex^6+\frac{1}{5}\,a^2dx^5+\frac{1}{4}\,a^2cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/
11*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f
*x^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

3.480.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx = \frac{f\,a^2\,x^7}{7}+\frac{e\,a^2\,x^6}{6}+\frac{d\,a^2\,x^5}{5}+\frac{c\,a^2\,x^4}{4}+\frac{2\,f\,a\,b\,x^{11}}{11}+\frac{e\,a\,b\,x^{10}}{5}+\frac{2\,d\,a\,b\,x^9}{9}+\frac{c\,a\,b\,x^8}{4}+\frac{f\,b^2\,x^{15}}{15}+\frac{e\,b^2\,x^{14}}{14}+\frac{d\,b^2\,x^{13}}{13}+\frac{c\,b^2\,x^{12}}{12}$$

input
```
int(x^3*(a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (b^2*c*x^12)/12 + (a^2*e*x^6)/6 + (b^2*d*x
^13)/13 + (a^2*f*x^7)/7 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15 + (a*b*c*x^8)/
4 + (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11
```

3.481 $\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx$

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3.481.8 Giac [A] (verification not implemented) 3780
3.481.9 Mupad [B] (verification not implemented) 3781

3.481.1 Optimal result

Integrand size = 25, antiderivative size = 151

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{f(a+bx^4)^4}{16b}$$

output
```
a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+3/5*a^2*b*c*x^5+1/2*a^2*b*d*x^6+3/7*a^
2*b*e*x^7+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^11+1/13*b^3*c*x
^13+1/14*b^3*d*x^14+1/15*b^3*e*x^15+1/16*f*(b*x^4+a)^4/b
```

3.481.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{4}ab^2fx^{12}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{16}b^3fx^{16}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/
5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9
)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*
c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

3.481.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^3 \left(c+dx+ex^2+fx^3\right) dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(ex^2+dx+c\right) \left(bx^4+a\right)^3 dx+\frac{f\left(a+bx^4\right)^4}{16b}$$

$$\Big\downarrow \text{2188}$$

$$\int \left(b^3ex^{14}+b^3dx^{13}+b^3cx^{12}+3ab^2ex^{10}+3ab^2dx^9+3ab^2cx^8+3a^2bex^6+3a^2bdx^5+3a^2bcx^4+a^3ex^2+a^3dx+\right.$$
$$\frac{f\left(a+bx^4\right)^4}{16b}$$

$$\Big\downarrow \text{2009}$$

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6
)/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2
*e*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a
+ b*x^4)^4)/(16*b)
```

3.481.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.481.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

method	result
gosper	$a^3cx + \frac{1}{2}a^3d\,x^2 + \frac{1}{3}a^3e\,x^3 + \frac{1}{4}f\,a^3x^4 + \frac{3}{5}a^2bc\,x^5 + \frac{1}{2}a^2bd\,x^6 + \frac{3}{7}a^2be\,x^7 + \frac{3}{8}f\,a^2b\,x^8 + \frac{1}{3}a\,b^2c\,x$
default	$a^3cx + \frac{1}{2}a^3d\,x^2 + \frac{1}{3}a^3e\,x^3 + \frac{1}{4}f\,a^3x^4 + \frac{3}{5}a^2bc\,x^5 + \frac{1}{2}a^2bd\,x^6 + \frac{3}{7}a^2be\,x^7 + \frac{3}{8}f\,a^2b\,x^8 + \frac{1}{3}a\,b^2c\,x$
norman	$a^3cx + \frac{1}{2}a^3d\,x^2 + \frac{1}{3}a^3e\,x^3 + \frac{1}{4}f\,a^3x^4 + \frac{3}{5}a^2bc\,x^5 + \frac{1}{2}a^2bd\,x^6 + \frac{3}{7}a^2be\,x^7 + \frac{3}{8}f\,a^2b\,x^8 + \frac{1}{3}a\,b^2c\,x$
risch	$a^3cx + \frac{1}{2}a^3d\,x^2 + \frac{1}{3}a^3e\,x^3 + \frac{1}{4}f\,a^3x^4 + \frac{3}{5}a^2bc\,x^5 + \frac{1}{2}a^2bd\,x^6 + \frac{3}{7}a^2be\,x^7 + \frac{3}{8}f\,a^2b\,x^8 + \frac{1}{3}a\,b^2c\,x$
parallelrisch	$a^3cx + \frac{1}{2}a^3d\,x^2 + \frac{1}{3}a^3e\,x^3 + \frac{1}{4}f\,a^3x^4 + \frac{3}{5}a^2bc\,x^5 + \frac{1}{2}a^2bd\,x^6 + \frac{3}{7}a^2be\,x^7 + \frac{3}{8}f\,a^2b\,x^8 + \frac{1}{3}a\,b^2c\,x$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+1/4*f*a^3*x^4+3/5*a^2*b*c*x^5+1/2*a^2*
b*d*x^6+3/7*a^2*b*e*x^7+3/8*f*a^2*b*x^8+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+
3/11*a*b^2*e*x^11+1/4*a*b^2*f*x^12+1/13*b^3*c*x^13+1/14*b^3*d*x^14+1/15*b^
3*e*x^15+1/16*b^3*f*x^16
```

3.481.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx = \frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,b^3ex^{15}+\frac{1}{14}\,b^3dx^{14}+\frac{1}{13}\,b^3cx^{13}+\frac{1}{4}\,ab^2fx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3fx^4+\frac{1}{3}\,a^3ex^3+\frac{1}{2}\,a^3dx^2+a^3cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 +
 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1
/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.481.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx = a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{a^3fx^4}{4}+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3a^2bfx^8}{8}+\frac{ab^2cx^9}{3}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{ab^2fx^{12}}{4}+\frac{b^3cx^{13}}{13}+\frac{b^3dx^{14}}{14}+\frac{b^3ex^{15}}{15}+\frac{b^3fx^{16}}{16}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

output
```
a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5
/5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x*
*9/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3
*c*x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16
```

3.481.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c + dx + ex^2 + fx^3) (a + bx^4)^3 \, dx = \frac{1}{16} b^3 f x^{16} + \frac{1}{15} b^3 e x^{15} + \frac{1}{14} b^3 d x^{14} + \frac{1}{13} b^3 c x^{13} + \frac{1}{4} a b^2 f x^{12} + \frac{3}{11} a b^2 e x^{11} + \frac{3}{10} a b^2 d x^{10} + \frac{1}{3} a b^2 c x^9 + \frac{3}{8} a^2 b f x^8 + \frac{3}{7} a^2 b e x^7 + \frac{1}{2} a^2 b d x^6 + \frac{3}{5} a^2 b c x^5 + \frac{1}{4} a^3 f x^4 + \frac{1}{3} a^3 e x^3 + \frac{1}{2} a^3 d x^2 + a^3 c x$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 +
 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1
/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.481.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c + dx + ex^2 + fx^3) (a + bx^4)^3 \, dx = \frac{1}{16} b^3 f x^{16} + \frac{1}{15} b^3 e x^{15} + \frac{1}{14} b^3 d x^{14} + \frac{1}{13} b^3 c x^{13} + \frac{1}{4} a b^2 f x^{12} + \frac{3}{11} a b^2 e x^{11} + \frac{3}{10} a b^2 d x^{10} + \frac{1}{3} a b^2 c x^9 + \frac{3}{8} a^2 b f x^8 + \frac{3}{7} a^2 b e x^7 + \frac{1}{2} a^2 b d x^6 + \frac{3}{5} a^2 b c x^5 + \frac{1}{4} a^3 f x^4 + \frac{1}{3} a^3 e x^3 + \frac{1}{2} a^3 d x^2 + a^3 c x$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 +
 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1
/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

3.481.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3) (a+bx^4)^3 \, dx = \frac{f\,a^3\,x^4}{4}+\frac{e\,a^3\,x^3}{3}+\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{3\,f\,a^2\,b\,x^8}{8}+\frac{3\,e\,a^2\,b\,x^7}{7}+\frac{d\,a^2\,b\,x^6}{2}+\frac{3\,c\,a^2\,b\,x^5}{5}+\frac{f\,a\,b^2\,x^{12}}{4}+\frac{3\,e\,a\,b^2\,x^{11}}{11}+\frac{3\,d\,a\,b^2\,x^{10}}{10}+\frac{c\,a\,b^2\,x^9}{3}+\frac{f\,b^3\,x^{16}}{16}+\frac{e\,b^3\,x^{15}}{15}+\frac{d\,b^3\,x^{14}}{14}+\frac{c\,b^3\,x^{13}}{13}$$

input
```
int((a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^3*d*x^2)/2 + (b^3*c*x^13)/13 + (a^3*e*x^3)/3 + (b^3*d*x^14)/14 + (a^3*f
*x^4)/4 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16 + a^3*c*x + (3*a^2*b*c*x^5)/5
+ (a*b^2*c*x^9)/3 + (a^2*b*d*x^6)/2 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^7
)/7 + (3*a*b^2*e*x^11)/11 + (3*a^2*b*f*x^8)/8 + (a*b^2*f*x^12)/4
```

3.482 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx$

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3.482.1 Optimal result

Integrand size = 28, antiderivative size = 156

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}+\frac{c(a+bx^4)^4}{16b}$$

output
```
1/5*a^3*d*x^5+1/6*a^3*e*x^6+1/7*a^3*f*x^7+1/3*a^2*b*d*x^9+3/10*a^2*b*e*x^1
0+3/11*a^2*b*f*x^11+3/13*a*b^2*d*x^13+3/14*a*b^2*e*x^14+1/5*a*b^2*f*x^15+1
/17*b^3*d*x^17+1/18*b^3*e*x^18+1/19*b^3*f*x^19+1/16*c*(b*x^4+a)^4/b
```

3.482.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{3}{8}a^2bcx^8+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{1}{4}ab^2cx^{12}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{1}{16}b^3cx^{16}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

output
```
(a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (3*a^2*b*c
*x^8)/8 + (a^2*b*d*x^9)/3 + (3*a^2*b*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (a
*b^2*c*x^12)/4 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/14 + (a*b^2*f*x^15
)/5 + (b^3*c*x^16)/16 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/1
9
```

3.482.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^4\right)^3 \left(c+dx+ex^2+fx^3\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^4+a\right)^3 \left(x^3\left(fx^3+ex^2+dx+c\right)-cx^3\right)dx+\frac{c\left(a+bx^4\right)^4}{16b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^3fx^{18}+b^3ex^{17}+b^3dx^{16}+3ab^2fx^{14}+3ab^2ex^{13}+3ab^2dx^{12}+3a^2bfx^{10}+3a^2bex^9+3a^2bdx^8+a^3fx^6+a^3e$$
$$\frac{c\left(a+bx^4\right)^4}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{c\left(a+bx^4\right)^4}{16b}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

output
```
(a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (a^2*b*d*x^9)/3 + (3*a^2*b
*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)
/14 + (a*b^2*f*x^15)/5 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/
19 + (c*(a + b*x^4)^4)/(16*b)
```

3.482.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.482.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

method	result
gosper	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}a^2bc\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}$
default	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}a^2bc\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}$
norman	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}a^2bc\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}$
risch	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}a^2bc\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}$
parallelrisch	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}a^2bc\,x^8+\frac{1}{3}a^2bd\,x^9+\frac{3}{10}a^2be\,x^{10}+\frac{3}{11}a^2bf\,x^{11}+\frac{1}{4}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^3*c*x^4+1/5*a^3*d*x^5+1/6*a^3*e*x^6+1/7*a^3*f*x^7+3/8*a^2*b*c*x^8+1/
3*a^2*b*d*x^9+3/10*a^2*b*e*x^10+3/11*a^2*b*f*x^11+1/4*a*b^2*c*x^12+3/13*a*
b^2*d*x^13+3/14*a*b^2*e*x^14+1/5*a*b^2*f*x^15+1/16*b^3*c*x^16+1/17*b^3*d*x
^17+1/18*b^3*e*x^18+1/19*b^3*f*x^19
```

3.482.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{1}{19}\,b^3fx^{19}+\frac{1}{18}\,b^3ex^{18}+\frac{1}{17}\,b^3dx^{17}+\frac{1}{16}\,b^3cx^{16}+\frac{1}{5}\,ab^2fx^{15}+\frac{3}{14}\,ab^2ex^{14}+\frac{3}{13}\,ab^2dx^{13}+\frac{1}{4}\,ab^2cx^{12}+\frac{3}{11}\,a^2bfx^{11}+\frac{3}{10}\,a^2bex^{10}+\frac{1}{3}\,a^2bdx^9+\frac{3}{8}\,a^2bcx^8+\frac{1}{7}\,a^3fx^7+\frac{1}{6}\,a^3ex^6+\frac{1}{5}\,a^3dx^5+\frac{1}{4}\,a^3cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/
5*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12
+ 3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^
8 + 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

3.482.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.18

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{a^3fx^7}{7}+\frac{3a^2bcx^8}{8}+\frac{a^2bdx^9}{3}+\frac{3a^2bex^{10}}{10}+\frac{3a^2bfx^{11}}{11}+\frac{ab^2cx^{12}}{4}+\frac{3ab^2dx^{13}}{13}+\frac{3ab^2ex^{14}}{14}+\frac{ab^2fx^{15}}{5}+\frac{b^3cx^{16}}{16}+\frac{b^3dx^{17}}{17}+\frac{b^3ex^{18}}{18}+\frac{b^3fx^{19}}{19}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

output
```
a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + a**3*f*x**7/7 + 3*a**2*b*c
*x**8/8 + a**2*b*d*x**9/3 + 3*a**2*b*e*x**10/10 + 3*a**2*b*f*x**11/11 + a*
b**2*c*x**12/4 + 3*a*b**2*d*x**13/13 + 3*a*b**2*e*x**14/14 + a*b**2*f*x**1
5/5 + b**3*c*x**16/16 + b**3*d*x**17/17 + b**3*e*x**18/18 + b**3*f*x**19/1
9
```

3.482.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx = \frac{1}{19}\,b^3fx^{19}+\frac{1}{18}\,b^3ex^{18}+\frac{1}{17}\,b^3dx^{17}+\frac{1}{16}\,b^3cx^{16}+\frac{1}{5}\,ab^2fx^{15}+\frac{3}{14}\,ab^2ex^{14}+\frac{3}{13}\,ab^2dx^{13}+\frac{1}{4}\,ab^2cx^{12}+\frac{3}{11}\,a^2bfx^{11}+\frac{3}{10}\,a^2bex^{10}+\frac{1}{3}\,a^2bdx^9+\frac{3}{8}\,a^2bcx^8+\frac{1}{7}\,a^3fx^7+\frac{1}{6}\,a^3ex^6+\frac{1}{5}\,a^3dx^5+\frac{1}{4}\,a^3cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/
5*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12
+ 3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^
8 + 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

3.482.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx = \frac{1}{19}b^3fx^{19}+\frac{1}{18}b^3ex^{18}+\frac{1}{17}b^3dx^{17}+\frac{1}{16}b^3cx^{16}+\frac{1}{5}ab^2fx^{15}+\frac{3}{14}ab^2ex^{14}+\frac{3}{13}ab^2dx^{13}+\frac{1}{4}ab^2cx^{12}+\frac{3}{11}a^2bfx^{11}+\frac{3}{10}a^2bex^{10}+\frac{1}{3}a^2bdx^9+\frac{3}{8}a^2bcx^8+\frac{1}{7}a^3fx^7+\frac{1}{6}a^3ex^6+\frac{1}{5}a^3dx^5+\frac{1}{4}a^3cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/
5*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12
+ 3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^
8 + 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

3.482.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx = \frac{f\,a^3\,x^7}{7}+\frac{e\,a^3\,x^6}{6}+\frac{d\,a^3\,x^5}{5}+\frac{c\,a^3\,x^4}{4}+\frac{3\,f\,a^2\,b\,x^{11}}{11}+\frac{3\,e\,a^2\,b\,x^{10}}{10}+\frac{d\,a^2\,b\,x^9}{3}+\frac{3\,c\,a^2\,b\,x^8}{8}+\frac{f\,a\,b^2\,x^{15}}{5}+\frac{3\,e\,a\,b^2\,x^{14}}{14}+\frac{3\,d\,a\,b^2\,x^{13}}{13}+\frac{c\,a\,b^2\,x^{12}}{4}+\frac{f\,b^3\,x^{19}}{19}+\frac{e\,b^3\,x^{18}}{18}+\frac{d\,b^3\,x^{17}}{17}+\frac{c\,b^3\,x^{16}}{16}$$

input
```
int(x^3*(a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (b^3*c*x^16)/16 + (a^3*e*x^6)/6 + (b^3*d*x
^17)/17 + (a^3*f*x^7)/7 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19 + (3*a^2*b*c*x
^8)/8 + (a*b^2*c*x^12)/4 + (a^2*b*d*x^9)/3 + (3*a*b^2*d*x^13)/13 + (3*a^2*
b*e*x^10)/10 + (3*a*b^2*e*x^14)/14 + (3*a^2*b*f*x^11)/11 + (a*b^2*f*x^15)/
5
```

3.483 $\int (c+dx+ex^2+fx^3)(a+bx^4)^4\,dx$

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3.483.1 Optimal result

Integrand size = 25, antiderivative size = 193

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^4\,dx = a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6 +\frac{4}{7}a^3bex^7+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11} +\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{1}{17}b^4cx^{17} +\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}+\frac{f(a+bx^4)^5}{20b}$$

output
```
a^4*c*x+1/2*a^4*d*x^2+1/3*a^4*e*x^3+4/5*a^3*b*c*x^5+2/3*a^3*b*d*x^6+4/7*a^
3*b*e*x^7+2/3*a^2*b^2*c*x^9+3/5*a^2*b^2*d*x^10+6/11*a^2*b^2*e*x^11+4/13*a*
b^3*c*x^13+2/7*a*b^3*d*x^14+4/15*a*b^3*e*x^15+1/17*b^4*c*x^17+1/18*b^4*d*x
^18+1/19*b^4*e*x^19+1/20*f*(b*x^4+a)^5/b
```

3.483.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.22

$$\int \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^4 \, dx = a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{1}{4}a^4fx^4+\frac{4}{5}a^3bcx^5 +\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{1}{2}a^3bfx^8+\frac{2}{3}a^2b^2cx^9 +\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{1}{2}a^2b^2fx^{12} +\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{1}{4}ab^3fx^{16} +\frac{1}{17}b^4cx^{17}+\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}+\frac{1}{20}b^4fx^{20}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

output
```
a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (a^4*f*x^4)/4 + (4*a^3*b*c*x^5)/
5 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (a^3*b*f*x^8)/2 + (2*a^2*b^2*c
*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a^2*b^2*f*x^12)/
2 + (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3*e*x^15)/15 + (a*b^
3*f*x^16)/4 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (b^4*f
*x^20)/20
```

3.483.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^4 \left(c+dx+ex^2+fx^3\right) \, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int \left(ex^2+dx+c\right) \left(bx^4+a\right)^4 dx+\frac{f\left(a+bx^4\right)^5}{20b}$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(b^4ex^{18}+b^4dx^{17}+b^4cx^{16}+4ab^3ex^{14}+4ab^3dx^{13}+4ab^3cx^{12}+6a^2b^2ex^{10}+6a^2b^2dx^9+6a^2b^2cx^8+4a^3bex^6+\right.$$

$$\frac{f(a+bx^4)^5}{20b}$$

$$\Big\downarrow \text{2009}$$

$$a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{f(a+bx^4)^5}{20b}+\frac{1}{17}b^4cx^{17}+\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

output
```
a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (4*a^3*b*c*x^5)/5 + (2*a^3*b*d*x
^6)/3 + (4*a^3*b*e*x^7)/7 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (
6*a^2*b^2*e*x^11)/11 + (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3
*e*x^15)/15 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (f*(a
+ b*x^4)^5)/(20*b)
```

3.483.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.483.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03

method	result
gosper	$a^4cx + \frac{1}{2}a^4d\,x^2 + \frac{1}{3}a^4e\,x^3 + \frac{1}{4}a^4f\,x^4 + \frac{4}{5}a^3bc\,x^5 + \frac{2}{3}a^3bd\,x^6 + \frac{4}{7}a^3be\,x^7 + \frac{1}{2}a^3bf\,x^8 + \frac{2}{3}a^2b^2c$
default	$a^4cx + \frac{1}{2}a^4d\,x^2 + \frac{1}{3}a^4e\,x^3 + \frac{1}{4}a^4f\,x^4 + \frac{4}{5}a^3bc\,x^5 + \frac{2}{3}a^3bd\,x^6 + \frac{4}{7}a^3be\,x^7 + \frac{1}{2}a^3bf\,x^8 + \frac{2}{3}a^2b^2c$
norman	$a^4cx + \frac{1}{2}a^4d\,x^2 + \frac{1}{3}a^4e\,x^3 + \frac{1}{4}a^4f\,x^4 + \frac{4}{5}a^3bc\,x^5 + \frac{2}{3}a^3bd\,x^6 + \frac{4}{7}a^3be\,x^7 + \frac{1}{2}a^3bf\,x^8 + \frac{2}{3}a^2b^2c$
risch	$a^4cx + \frac{1}{2}a^4d\,x^2 + \frac{1}{3}a^4e\,x^3 + \frac{1}{4}a^4f\,x^4 + \frac{4}{5}a^3bc\,x^5 + \frac{2}{3}a^3bd\,x^6 + \frac{4}{7}a^3be\,x^7 + \frac{1}{2}a^3bf\,x^8 + \frac{2}{3}a^2b^2c$
parallelrisch	$a^4cx + \frac{1}{2}a^4d\,x^2 + \frac{1}{3}a^4e\,x^3 + \frac{1}{4}a^4f\,x^4 + \frac{4}{5}a^3bc\,x^5 + \frac{2}{3}a^3bd\,x^6 + \frac{4}{7}a^3be\,x^7 + \frac{1}{2}a^3bf\,x^8 + \frac{2}{3}a^2b^2c$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*c*x+1/2*a^4*d*x^2+1/3*a^4*e*x^3+1/4*a^4*f*x^4+4/5*a^3*b*c*x^5+2/3*a^3*
b*d*x^6+4/7*a^3*b*e*x^7+1/2*a^3*b*f*x^8+2/3*a^2*b^2*c*x^9+3/5*a^2*b^2*d*x^
10+6/11*a^2*b^2*e*x^11+1/2*f*a^2*b^2*x^12+4/13*a*b^3*c*x^13+2/7*a*b^3*d*x^
14+4/15*a*b^3*e*x^15+1/4*f*a*b^3*x^16+1/17*b^4*c*x^17+1/18*b^4*d*x^18+1/19
*b^4*e*x^19+1/20*f*b^4*x^20
```

3.483.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\begin{aligned}\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx &= \frac{1}{20}\,b^4fx^{20}+\frac{1}{19}\,b^4ex^{19}+\frac{1}{18}\,b^4dx^{18}+\frac{1}{17}\,b^4cx^{17}\\ &\quad+\frac{1}{4}\,ab^3fx^{16}+\frac{4}{15}\,ab^3ex^{15}+\frac{2}{7}\,ab^3dx^{14}+\frac{4}{13}\,ab^3cx^{13}\\ &\quad+\frac{1}{2}\,a^2b^2fx^{12}+\frac{6}{11}\,a^2b^2ex^{11}+\frac{3}{5}\,a^2b^2dx^{10}\\ &\quad+\frac{2}{3}\,a^2b^2cx^9+\frac{1}{2}\,a^3bfx^8+\frac{4}{7}\,a^3bex^7+\frac{2}{3}\,a^3bdx^6\\ &\quad+\frac{4}{5}\,a^3bcx^5+\frac{1}{4}\,a^4fx^4+\frac{1}{3}\,a^4ex^3+\frac{1}{2}\,a^4dx^2+a^4cx\end{aligned}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/
4*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13
+ 1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*
b^2*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*
b*c*x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

3.483.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.25

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^4\,dx = a^4cx + \frac{a^4dx^2}{2} + \frac{a^4ex^3}{3} + \frac{a^4fx^4}{4} + \frac{4a^3bcx^5}{5} + \frac{2a^3bdx^6}{3} + \frac{4a^3bex^7}{7} + \frac{a^3bfx^8}{2} + \frac{2a^2b^2cx^9}{3} + \frac{3a^2b^2dx^{10}}{5} + \frac{6a^2b^2ex^{11}}{11} + \frac{a^2b^2fx^{12}}{2} + \frac{4ab^3cx^{13}}{13} + \frac{2ab^3dx^{14}}{7} + \frac{4ab^3ex^{15}}{15} + \frac{ab^3fx^{16}}{4} + \frac{b^4cx^{17}}{17} + \frac{b^4dx^{18}}{18} + \frac{b^4ex^{19}}{19} + \frac{b^4fx^{20}}{20}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

output
```
a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**4*f*x**4/4 + 4*a**3*b*c*x**5
/5 + 2*a**3*b*d*x**6/3 + 4*a**3*b*e*x**7/7 + a**3*b*f*x**8/2 + 2*a**2*b**2
*c*x**9/3 + 3*a**2*b**2*d*x**10/5 + 6*a**2*b**2*e*x**11/11 + a**2*b**2*f*x
**12/2 + 4*a*b**3*c*x**13/13 + 2*a*b**3*d*x**14/7 + 4*a*b**3*e*x**15/15 +
a*b**3*f*x**16/4 + b**4*c*x**17/17 + b**4*d*x**18/18 + b**4*e*x**19/19 + b
**4*f*x**20/20
```

3.483.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx = \frac{1}{20}b^4fx^{20}+\frac{1}{19}b^4ex^{19}+\frac{1}{18}b^4dx^{18}+\frac{1}{17}b^4cx^{17}+\frac{1}{4}ab^3fx^{16}+\frac{4}{15}ab^3ex^{15}+\frac{2}{7}ab^3dx^{14}+\frac{4}{13}ab^3cx^{13}+\frac{1}{2}a^2b^2fx^{12}+\frac{6}{11}a^2b^2ex^{11}+\frac{3}{5}a^2b^2dx^{10}+\frac{2}{3}a^2b^2cx^9+\frac{1}{2}a^3bfx^8+\frac{4}{7}a^3bex^7+\frac{2}{3}a^3bdx^6+\frac{4}{5}a^3bcx^5+\frac{1}{4}a^4fx^4+\frac{1}{3}a^4ex^3+\frac{1}{2}a^4dx^2+a^4cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/
4*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13
+ 1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*
b^2*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*
b*c*x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

3.483.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx = \frac{1}{20}b^4fx^{20}+\frac{1}{19}b^4ex^{19}+\frac{1}{18}b^4dx^{18}+\frac{1}{17}b^4cx^{17}+\frac{1}{4}ab^3fx^{16}+\frac{4}{15}ab^3ex^{15}+\frac{2}{7}ab^3dx^{14}+\frac{4}{13}ab^3cx^{13}+\frac{1}{2}a^2b^2fx^{12}+\frac{6}{11}a^2b^2ex^{11}+\frac{3}{5}a^2b^2dx^{10}+\frac{2}{3}a^2b^2cx^9+\frac{1}{2}a^3bfx^8+\frac{4}{7}a^3bex^7+\frac{2}{3}a^3bdx^6+\frac{4}{5}a^3bcx^5+\frac{1}{4}a^4fx^4+\frac{1}{3}a^4ex^3+\frac{1}{2}a^4dx^2+a^4cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/
4*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13
+ 1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*
b^2*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*
b*c*x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

3.483.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^4\,dx = \frac{f\,a^4\,x^4}{4}+\frac{e\,a^4\,x^3}{3}+\frac{d\,a^4\,x^2}{2}+c\,a^4\,x+\frac{f\,a^3\,b\,x^8}{2}+\frac{4\,e\,a^3\,b\,x^7}{7}+\frac{2\,d\,a^3\,b\,x^6}{3}+\frac{4\,c\,a^3\,b\,x^5}{5}+\frac{f\,a^2\,b^2\,x^{12}}{2}+\frac{6\,e\,a^2\,b^2\,x^{11}}{11}+\frac{3\,d\,a^2\,b^2\,x^{10}}{5}+\frac{2\,c\,a^2\,b^2\,x^9}{3}+\frac{f\,a\,b^3\,x^{16}}{4}+\frac{4\,e\,a\,b^3\,x^{15}}{15}+\frac{2\,d\,a\,b^3\,x^{14}}{7}+\frac{4\,c\,a\,b^3\,x^{13}}{13}+\frac{f\,b^4\,x^{20}}{20}+\frac{e\,b^4\,x^{19}}{19}+\frac{d\,b^4\,x^{18}}{18}+\frac{c\,b^4\,x^{17}}{17}$$

input
```
int((a + b*x^4)^4*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^4*d*x^2)/2 + (b^4*c*x^17)/17 + (a^4*e*x^3)/3 + (b^4*d*x^18)/18 + (a^4*f
*x^4)/4 + (b^4*e*x^19)/19 + (b^4*f*x^20)/20 + a^4*c*x + (2*a^2*b^2*c*x^9)/
3 + (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a^2*b^2*f*x^12)/2 + (4
*a^3*b*c*x^5)/5 + (4*a*b^3*c*x^13)/13 + (2*a^3*b*d*x^6)/3 + (2*a*b^3*d*x^1
4)/7 + (4*a^3*b*e*x^7)/7 + (4*a*b^3*e*x^15)/15 + (a^3*b*f*x^8)/2 + (a*b^3*
f*x^16)/4
```

3.484 $\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^4\,dx$

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3.484.1 Optimal result

Integrand size = 28, antiderivative size = 198

$$\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^4\,dx=\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{1}{21}b^4dx^{21}+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}+\frac{c(a+bx^4)^5}{20b}$$

output
```
1/5*a^4*d*x^5+1/6*a^4*e*x^6+1/7*a^4*f*x^7+4/9*a^3*b*d*x^9+2/5*a^3*b*e*x^10
+4/11*a^3*b*f*x^11+6/13*a^2*b^2*d*x^13+3/7*a^2*b^2*e*x^14+2/5*a^2*b^2*f*x^
15+4/17*a*b^3*d*x^17+2/9*a*b^3*e*x^18+4/19*a*b^3*f*x^19+1/21*b^4*d*x^21+1/
22*b^4*e*x^22+1/23*b^4*f*x^23+1/20*c*(b*x^4+a)^5/b
```

3.484.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.22

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx=\frac{1}{4}a^4cx^4+\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{1}{2}a^3bcx^8+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{1}{2}a^2b^2cx^{12}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{1}{4}ab^3cx^{16}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{1}{20}b^4cx^{20}+\frac{1}{21}b^4dx^{21}+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

output
```
(a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (a^3*b*c*x
^8)/2 + (4*a^3*b*d*x^9)/9 + (2*a^3*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (a^
2*b^2*c*x^12)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^
2*f*x^15)/5 + (a*b^3*c*x^16)/4 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9
+ (4*a*b^3*f*x^19)/19 + (b^4*c*x^20)/20 + (b^4*d*x^21)/21 + (b^4*e*x^22)/2
2 + (b^4*f*x^23)/23
```

3.484.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^4\right)^4\left(c+dx+ex^2+fx^3\right)dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^4+a\right)^4\left(x^3\left(fx^3+ex^2+dx+c\right)-cx^3\right)dx+\frac{c\left(a+bx^4\right)^5}{20b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^4fx^{22}+b^4ex^{21}+b^4dx^{20}+4ab^3fx^{18}+4ab^3ex^{17}+4ab^3dx^{16}+6a^2b^2fx^{14}+6a^2b^2ex^{13}+6a^2b^2dx^{12}+4a^3bfx\right.$$

$$\frac{c(a+bx^4)^5}{20b}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{c(a+bx^4)^5}{20b}+\frac{1}{21}b^4dx^{21}+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

output
```
(a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (4*a^3*b*d*x^9)/9 + (2*a^3
*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*
x^14)/7 + (2*a^2*b^2*f*x^15)/5 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9
+ (4*a*b^3*f*x^19)/19 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (b^4*f*x^23)/2
3 + (c*(a + b*x^4)^5)/(20*b)
```

3.484.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.484.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.02

method	result
gosper	$\frac{1}{4}a^4c\,x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}a^3bc\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}a$
default	$\frac{1}{4}a^4c\,x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}a^3bc\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}a$
norman	$\frac{1}{4}a^4c\,x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}a^3bc\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}a$
risch	$\frac{1}{4}a^4c\,x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}a^3bc\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}a$
parallelrisch	$\frac{1}{4}a^4c\,x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}a^3bc\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+\frac{1}{2}a$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^4*c*x^4+1/5*a^4*d*x^5+1/6*a^4*e*x^6+1/7*a^4*f*x^7+1/2*a^3*b*c*x^8+4/
9*a^3*b*d*x^9+2/5*a^3*b*e*x^10+4/11*a^3*b*f*x^11+1/2*a^2*b^2*c*x^12+6/13*a
^2*b^2*d*x^13+3/7*a^2*b^2*e*x^14+2/5*a^2*b^2*f*x^15+1/4*a*b^3*c*x^16+4/17*
a*b^3*d*x^17+2/9*a*b^3*e*x^18+4/19*a*b^3*f*x^19+1/20*b^4*c*x^20+1/21*b^4*d
*x^21+1/22*b^4*e*x^22+1/23*b^4*f*x^23
```

3.484.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx=&\ \frac{1}{23}\,b^4fx^{23}+\frac{1}{22}\,b^4ex^{22}+\frac{1}{21}\,b^4dx^{21}+\frac{1}{20}\,b^4cx^{20}\\&+\frac{4}{19}\,ab^3fx^{19}+\frac{2}{9}\,ab^3ex^{18}+\frac{4}{17}\,ab^3dx^{17}\\&+\frac{1}{4}\,ab^3cx^{16}+\frac{2}{5}\,a^2b^2fx^{15}+\frac{3}{7}\,a^2b^2ex^{14}\\&+\frac{6}{13}\,a^2b^2dx^{13}+\frac{1}{2}\,a^2b^2cx^{12}+\frac{4}{11}\,a^3bfx^{11}\\&+\frac{2}{5}\,a^3bex^{10}+\frac{4}{9}\,a^3bdx^9+\frac{1}{2}\,a^3bcx^8\\&+\frac{1}{7}\,a^4fx^7+\frac{1}{6}\,a^4ex^6+\frac{1}{5}\,a^4dx^5+\frac{1}{4}\,a^4cx^4\end{aligned}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/
19*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16
+ 2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*
b^2*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*
a^3*b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^
4
```

3.484.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.24

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{a^4cx^4}{4}+\frac{a^4dx^5}{5}+\frac{a^4ex^6}{6}+\frac{a^4fx^7}{7}+\frac{a^3bcx^8}{2} + \frac{4a^3bdx^9}{9}+\frac{2a^3bex^{10}}{5}+\frac{4a^3bfx^{11}}{11}+\frac{a^2b^2cx^{12}}{2} + \frac{6a^2b^2dx^{13}}{13}+\frac{3a^2b^2ex^{14}}{7}+\frac{2a^2b^2fx^{15}}{5} + \frac{ab^3cx^{16}}{4}+\frac{4ab^3dx^{17}}{17}+\frac{2ab^3ex^{18}}{9}+\frac{4ab^3fx^{19}}{19} + \frac{b^4cx^{20}}{20}+\frac{b^4dx^{21}}{21}+\frac{b^4ex^{22}}{22}+\frac{b^4fx^{23}}{23}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

output
```
a**4*c*x**4/4 + a**4*d*x**5/5 + a**4*e*x**6/6 + a**4*f*x**7/7 + a**3*b*c*x
**8/2 + 4*a**3*b*d*x**9/9 + 2*a**3*b*e*x**10/5 + 4*a**3*b*f*x**11/11 + a**
2*b**2*c*x**12/2 + 6*a**2*b**2*d*x**13/13 + 3*a**2*b**2*e*x**14/7 + 2*a**2
*b**2*f*x**15/5 + a*b**3*c*x**16/4 + 4*a*b**3*d*x**17/17 + 2*a*b**3*e*x**1
8/9 + 4*a*b**3*f*x**19/19 + b**4*c*x**20/20 + b**4*d*x**21/21 + b**4*e*x**
22/22 + b**4*f*x**23/23
```

3.484.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{1}{23}\,b^4fx^{23}+\frac{1}{22}\,b^4ex^{22}+\frac{1}{21}\,b^4dx^{21}+\frac{1}{20}\,b^4cx^{20}+\frac{4}{19}\,ab^3fx^{19}+\frac{2}{9}\,ab^3ex^{18}+\frac{4}{17}\,ab^3dx^{17}+\frac{1}{4}\,ab^3cx^{16}+\frac{2}{5}\,a^2b^2fx^{15}+\frac{3}{7}\,a^2b^2ex^{14}+\frac{6}{13}\,a^2b^2dx^{13}+\frac{1}{2}\,a^2b^2cx^{12}+\frac{4}{11}\,a^3bfx^{11}+\frac{2}{5}\,a^3bex^{10}+\frac{4}{9}\,a^3bdx^9+\frac{1}{2}\,a^3bcx^8+\frac{1}{7}\,a^4fx^7+\frac{1}{6}\,a^4ex^6+\frac{1}{5}\,a^4dx^5+\frac{1}{4}\,a^4cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/
19*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16
+ 2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*
b^2*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*
a^3*b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^
4
```

3.484.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{1}{23}\,b^4fx^{23}+\frac{1}{22}\,b^4ex^{22}+\frac{1}{21}\,b^4dx^{21}+\frac{1}{20}\,b^4cx^{20}+\frac{4}{19}\,ab^3fx^{19}+\frac{2}{9}\,ab^3ex^{18}+\frac{4}{17}\,ab^3dx^{17}+\frac{1}{4}\,ab^3cx^{16}+\frac{2}{5}\,a^2b^2fx^{15}+\frac{3}{7}\,a^2b^2ex^{14}+\frac{6}{13}\,a^2b^2dx^{13}+\frac{1}{2}\,a^2b^2cx^{12}+\frac{4}{11}\,a^3bfx^{11}+\frac{2}{5}\,a^3bex^{10}+\frac{4}{9}\,a^3bdx^9+\frac{1}{2}\,a^3bcx^8+\frac{1}{7}\,a^4fx^7+\frac{1}{6}\,a^4ex^6+\frac{1}{5}\,a^4dx^5+\frac{1}{4}\,a^4cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/
19*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16
+ 2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*
b^2*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*
a^3*b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^
4
```

3.484.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{f\,a^4\,x^7}{7}+\frac{e\,a^4\,x^6}{6}+\frac{d\,a^4\,x^5}{5}+\frac{c\,a^4\,x^4}{4}+\frac{4\,f\,a^3\,b\,x^{11}}{11}+\frac{2\,e\,a^3\,b\,x^{10}}{5}+\frac{4\,d\,a^3\,b\,x^9}{9}+\frac{c\,a^3\,b\,x^8}{2}+\frac{2\,f\,a^2\,b^2\,x^{15}}{5}+\frac{3\,e\,a^2\,b^2\,x^{14}}{7}+\frac{6\,d\,a^2\,b^2\,x^{13}}{13}+\frac{c\,a^2\,b^2\,x^{12}}{2}+\frac{4\,f\,a\,b^3\,x^{19}}{19}+\frac{2\,e\,a\,b^3\,x^{18}}{9}+\frac{4\,d\,a\,b^3\,x^{17}}{17}+\frac{c\,a\,b^3\,x^{16}}{4}+\frac{f\,b^4\,x^{23}}{23}+\frac{e\,b^4\,x^{22}}{22}+\frac{d\,b^4\,x^{21}}{21}+\frac{c\,b^4\,x^{20}}{20}$$

input
```
int(x^3*(a + b*x^4)^4*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (b^4*c*x^20)/20 + (a^4*e*x^6)/6 + (b^4*d*x
^21)/21 + (a^4*f*x^7)/7 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 + (a^2*b^2*c*x
^12)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f*x^15)
/5 + (a^3*b*c*x^8)/2 + (a*b^3*c*x^16)/4 + (4*a^3*b*d*x^9)/9 + (4*a*b^3*d*x
^17)/17 + (2*a^3*b*e*x^10)/5 + (2*a*b^3*e*x^18)/9 + (4*a^3*b*f*x^11)/11 +
(4*a*b^3*f*x^19)/19
```

3.485 $\int \frac{c+dx+ex^2+fx^3}{a-bx^4} dx$

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3.485.1 Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4} dx = \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{f\log\left(a-bx^4\right)}{4b}$$

output
```
-1/4*f*ln(-b*x^4+a)/b+1/2*d*arctanh(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)+1
/2*arctan(b^(1/4)*x/a^(1/4))*(-e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*ar
ctanh(b^(1/4)*x/a^(1/4))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

3.485.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.61

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \frac{\left(\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2ab^{3/4}} - \frac{\left(\sqrt[4]{a}\sqrt{b}c+\sqrt{a}\sqrt[4]{b}d+a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{4ab^{3/4}} - \frac{\left(-\sqrt[4]{a}\sqrt{b}c+\sqrt{a}\sqrt[4]{b}d-a^{3/4}e\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{4ab^{3/4}} + \frac{d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{a}\sqrt{b}} - \frac{f\log\left(a-bx^4\right)}{4b}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

output
```
((a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b^(3/4)
) - ((a^(1/4)*Sqrt[b]*c + Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(
1/4)*x])/(4*a*b^(3/4)) - ((-(a^(1/4)*Sqrt[b]*c) + Sqrt[a]*b^(1/4)*d - a^(3
/4)*e)*Log[a^(1/4) + b^(1/4)*x])/(4*a*b^(3/4)) + (d*Log[Sqrt[a] + Sqrt[b]*
x^2])/(4*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

3.485.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx$$

$$\Big\downarrow \text{ 2415}$$

$$\int \left(\frac{c+ex^2}{a-bx^4}+\frac{x\left(d+fx^2\right)}{a-bx^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}c-\sqrt{a}e\right)}{2a^{3/4}b^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2a^{3/4}b^{3/4}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

output
```
((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4
)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b
*x^4])/(4*b)
```

3.485.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.485.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.33

method	result
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(\_R^3 f+\_R^2 e+\_R d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{f\ln(-b\,x^4+a}{4b}$

input
```
int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*sum((_R^3*f+_R^2*e+_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.485.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.08 (sec) , antiderivative size = 241149, normalized size of antiderivative = 1813.15

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.485.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \text{Timed out}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

output
```
Timed out
```

3.485.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.31

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}b} - \frac{\left(\sqrt{b}d+\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}b} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*(sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/4*(sqrt(b)*d - sqrt(a)*f)*log(sqrt(b
)*x^2 + sqrt(a))/(sqrt(a)*b) - 1/4*(sqrt(b)*d + sqrt(a)*f)*log(sqrt(b)*x^2
 - sqrt(a))/(sqrt(a)*b) - 1/4*(sqrt(b)*c + sqrt(a)*e)*log((sqrt(b)*x - sqr
t(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))*sqrt(b))
```

3.485.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(93) = 186$.

Time = 0.28 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.08

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx$$
$$= -\frac{\sqrt{2}\left(b^2c-\sqrt{2}(-ab^3)^{\frac{1}{4}}bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}\left(b^2c+\sqrt{2}(-ab^3)^{\frac{1}{4}}bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,(-ab^3)^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}}$$
$$+\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,(-ab^3)^{\frac{3}{4}}}-\frac{f\log\left(|bx^4-a|\right)}{4\,b}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(b^2*c - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/
4*sqrt(2)*(b^2*c + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*s
qrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a
/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 - sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*f*log(abs(b*x^4 - a))
/b
```

3.485.9 Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 1970, normalized size of antiderivative = 14.81

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x)
```

output
```
symsum(log(b^2*c^2*e - b^2*c*d^2 - b^2*d^3*x - a*b*e^3 - a*b*c*f^2 - 16*ro
ot(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f
^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16
*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4
*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a
*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)^2*a*b^3*c
- 4*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*
z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2
*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2
 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)*b^3*
c^2*x - b^2*c^2*f*x + 16*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2
*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*
z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*
f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^
2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4
- b^3*c^4, z, k)^2*a*b^3*d*x - 4*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3
- 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^
2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 1
6*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4...
```

3.486 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$

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3.486.1 Optimal result

Integrand size = 29, antiderivative size = 162

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}-\frac{c\log\left(a-bx^4\right)}{4b}$$

output
```
-d*x/b-1/2*e*x^2/b-1/3*f*x^3/b-1/4*c*ln(-b*x^4+a)/b+1/2*e*arctanh(x^2*b^(1
/2)/a^(1/2))*a^(1/2)/b^(3/2)+1/2*a^(1/4)*arctan(b^(1/4)*x/a^(1/4))*(-f*a^(
1/2)+d*b^(1/2))/b^(7/4)+1/2*a^(1/4)*arctanh(b^(1/4)*x/a^(1/4))*(f*a^(1/2)+
d*b^(1/2))/b^(7/4)
```

3.486.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.36

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$$
$$= \frac{-12b^{3/4}dx-6b^{3/4}ex^2-4b^{3/4}fx^3+6\left(\sqrt[4]{a}\sqrt{b}d-a^{3/4}f\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-3\left(\sqrt[4]{a}\sqrt{b}d+\sqrt{a}\sqrt[4]{b}e+a^{3/4}f\right)}{}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

output
```
(-12*b^(3/4)*d*x - 6*b^(3/4)*e*x^2 - 4*b^(3/4)*f*x^3 + 6*(a^(1/4)*Sqrt[b]*
d - a^(3/4)*f)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(a^(1/4)*Sqrt[b]*d + Sqrt[a
]*b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) - b^(1/4)*x] + 3*(a^(1/4)*Sqrt[b]*d -
 Sqrt[a]*b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) + b^(1/4)*x] + 3*Sqrt[a]*b^(1/
4)*e*Log[Sqrt[a] + Sqrt[b]*x^2] - 3*b^(3/4)*c*Log[a - b*x^4])/(12*b^(7/4))
```

3.486.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{a-bx^4}\,dx$$
$$\Big\downarrow \text{2370}$$
$$\int \left(\frac{x^3\left(c+ex^2\right)}{a-bx^4}+\frac{x^4\left(d+fx^2\right)}{a-bx^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}d-\sqrt{a}f\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2b^{7/4}}+\frac{\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}-\frac{c\log\left(a-bx^4\right)}{4b}-\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

output
```
-((d*x)/b) - (e*x^2)/(2*b) - (f*x^3)/(3*b) + (a^(1/4)*(Sqrt[b]*d - Sqrt[a]
*f)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[
a]*f)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (Sqrt[a]*e*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a]])/(2*b^(3/2)) - (c*Log[a - b*x^4])/(4*b)
```

3.486.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

3.486.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.49

method	result
risch	$-\frac{f\,x^{3}}{3b}-\frac{e\,x^{2}}{2b}-\frac{dx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}b-a\right)}\frac{\left(-\_R^{3}bc-\_R^{2}af-\_Rae-ad\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$-\frac{\frac{1}{3}f\,x^{3}+\frac{1}{2}e\,x^{2}+dx}{b}+\frac{\frac{d\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4}+\frac{ae\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{af\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-c\ldots}{b}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*f*x^3/b-1/2*e*x^2/b-d*x/b+1/4/b^2*sum((-_R^3*b*c-_R^2*a*f-_R*a*e-a*d)
/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.486.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.17 (sec) , antiderivative size = 220680, normalized size of antiderivative = 1362.22

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.486.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

output
```
Timed out
```

3.486.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.28

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = -\frac{2\,fx^3+3\,ex^2+6\,dx}{6\,b} + \frac{\frac{2\left(a\sqrt{b}d-a^{\frac{3}{2}}f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{a}bc-a\sqrt{b}e\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(\sqrt{a}bc+a\sqrt{b}e\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(a\sqrt{b}d+a^{\frac{3}{2}}f\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{4\,b}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
-1/6*(2*f*x^3 + 3*e*x^2 + 6*d*x)/b + 1/4*(2*(a*sqrt(b)*d - a^(3/2)*f)*arct
an(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)
) - (sqrt(a)*b*c - a*sqrt(b)*e)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (
sqrt(a)*b*c + a*sqrt(b)*e)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (a*sqr
t(b)*d + a^(3/2)*f)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + s
qrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

3.486.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(118) = 236$.

Time = 0.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.01

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$$
$$= -\frac{c\log\left(|bx^4-a|\right)}{4\,b} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2e-\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2e-\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4} + \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,b^4} - \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2d-\left(-ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,b^4} - \frac{2\,b^2fx^3+3\,b^2ex^2+6\,b^2dx}{6\,b^3}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*c*log(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-a
*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/b^4 - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-
a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(
-a/b)^(1/4))/(-a/b)^(1/4))/b^4 + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*d - (-a*b
^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 - 1/8*sqrt
(2)*((-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(-a/b)^(
1/4) + sqrt(-a/b))/b^4 - 1/6*(2*b^2*f*x^3 + 3*b^2*e*x^2 + 6*b^2*d*x)/b^3
```

3.486.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 846, normalized size of antiderivative = 5.22

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$$

$$= \left(\sum_{k=1}^{4}\ln\left(-\frac{a^4\,f^3-2\,a^3\,b\,c\,e\,f-a^3\,b\,d^2\,f+a^3\,b\,d\,e^2+a^2\,b^2\,c^2\,d}{b^2}\right.\right.$$
$$-\mathrm{root}\big(256\,b^7\,z^4+256\,b^6\,c\,z^3-64\,a\,b^4\,d\,f\,z^2-32\,a\,b^4\,e^2\,z^2+96\,b^5\,c^2\,z^2-32\,a\,b^3\,c\,d\,f\,z+16\,a^2\,b^2\,e\,f^2\,z+16\,a\,b$$
$$\left.-\frac{x\,(a^3\,c\,f^2-2\,a^3\,d\,e\,f+a^3\,e^3-b\,a^2\,c^2\,e+b\,a^2\,c\,d^2)}{b}\right)\mathrm{root}\big(256\,b^7\,z^4+256\,b^6\,c\,z^3$$
$$-64\,a\,b^4\,d\,f\,z^2-32\,a\,b^4\,e^2\,z^2+96\,b^5\,c^2\,z^2-32\,a\,b^3\,c\,d\,f\,z+16\,a^2\,b^2\,e\,f^2\,z+16\,a\,b^3\,d^2\,e\,z$$
$$-16\,a\,b^3\,c\,e^2\,z+16\,b^4\,c^3\,z-4\,a^2\,b\,d\,e^2\,f+4\,a^2\,b\,c\,e\,f^2-4\,a\,b^2\,c^2\,d\,f+4\,a\,b^2\,c\,d^2\,e$$
$$\left.+2\,a^2\,b\,d^2\,f^2-2\,a\,b^2\,c^2\,e^2+a^2\,b\,e^4+b^3\,c^4-a\,b^2\,d^4-a^3\,f^4,z,k\big)\right)-\frac{e\,x^2}{2\,b}-\frac{f\,x^3}{3\,b}-\frac{d\,x}{b}$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x)
```

output
```
symsum(log(- (a^4*f^3 + a^2*b^2*c^2*d + a^3*b*d*e^2 - a^3*b*d^2*f - 2*a^3*
b*c*e*f)/b^2 - root(256*b^7*z^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*
b^4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*
a*b^3*d^2*e*z - 16*a*b^3*c*e^2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*
b*c*e*f^2 - 4*a*b^2*c^2*d*f + 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*
c^2*e^2 + a^2*b*e^4 + b^3*c^4 - a*b^2*d^4 - a^3*f^4, z, k)*(root(256*b^7*z
^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^4*e^2*z^2 + 96*b^5*c^2*z^2
- 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c*e^
2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 - 4*a*b^2*c^2*d*f +
 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e^2 + a^2*b*e^4 + b^3*c^4
 - a*b^2*d^4 - a^3*f^4, z, k)*(16*a^2*b^2*d - 16*a^2*b^2*e*x) + (8*a^2*b^3
*c*d - 8*a^3*b^2*e*f)/b^2 + (x*(4*a^3*b*f^2 + 4*a^2*b^2*d^2 - 8*a^2*b^2*c*
e))/b) - (x*(a^3*e^3 + a^3*c*f^2 - 2*a^3*d*e*f + a^2*b*c*d^2 - a^2*b*c^2*e
))/b)*root(256*b^7*z^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^4*e^2*z
^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2
*e*z - 16*a*b^3*c*e^2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2
 - 4*a*b^2*c^2*d*f + 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e^2 +
 a^2*b*e^4 + b^3*c^4 - a*b^2*d^4 - a^3*f^4, z, k), k, 1, 4) - (e*x^2)/(2*b
) - (f*x^3)/(3*b) - (d*x)/b
```

3.486. $\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$

3.487 $\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx$

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3.487.1 Optimal result

Integrand size = 25, antiderivative size = 293

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{f\log\left(a+bx^4\right)}{4b}$$

output
```
1/4*f*ln(b*x^4+a)/b+1/2*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)-1/8*
ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+c*b^(1/2))/
a^(3/4)/b^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))*(-e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)*x
*2^(1/2)/a^(1/4))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan
(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2
)
```

3.487.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx$$
$$= \frac{-2\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{2}\sqrt{b}c+2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{2}\sqrt{b}c-2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

output
```
(-2*a^(1/4)*b^(1/4)*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqr
t[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(Sqrt[
2]*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*b^(1/4)*(a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(a^
(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] + 2*a*f*Log[a + b*x^4])/(8*a*b)
```

3.487.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx$$
$$\Big\downarrow \text{ 2415}$$
$$\int \left(\frac{c+ex^2}{a+bx^4}+\frac{x(d+fx^2)}{a+bx^4}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt
[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)
) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c
 - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*S
qrt[2]*a^(3/4)*b^(3/4)) + (f*Log[a + b*x^4])/(4*b)
```

3.487.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.487.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.14

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)} \frac{\left(\_R^3 f+\_R^2 e+\_R d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\dots\right.}{x^2+\left(\dots\right.}\right.\right.}{\dots}$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum((_R^3*f+_R^2*e+_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.487.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.41 (sec) , antiderivative size = 254687, normalized size of antiderivative = 869.24

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.487.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.487.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.95

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f+bc-\sqrt{a}\sqrt{b}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f-bc+\sqrt{a}\sqrt{b}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{5}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{3}{4}}e-2\,\sqrt{a}bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{5}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{5}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{3}{4}}e+2\,\sqrt{a}bd\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{5}{4}}}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(sqrt(2)*a^(3/4)*b^(1/4)*f + b*c - sqrt(a)*sqrt(b)*e)*log(sqrt
(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/8*sqr
t(2)*(sqrt(2)*a^(3/4)*b^(1/4)*f - b*c + sqrt(a)*sqrt(b)*e)*log(sqrt(b)*x^2
 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/4*(sqrt(2)*a
^(1/4)*b^(5/4)*c + sqrt(2)*a^(3/4)*b^(3/4)*e - 2*sqrt(a)*b*d)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(5/4)*c + sqr
t(2)*a^(3/4)*b^(3/4)*e + 2*sqrt(a)*b*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x -
sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt
(b))*b^(5/4))
```

3.487.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.98

$$\int \frac{c + dx + ex^2 + fx^3}{a + bx^4} \, dx$$
$$= \frac{f \log(|bx^4 + a|)}{4b} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d - (ab^3)^{\frac{1}{4}}b^2c - (ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}$$
$$- \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d - (ab^3)^{\frac{1}{4}}b^2c - (ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c - (ab^3)^{\frac{3}{4}}e\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8ab^3}$$
$$- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c - (ab^3)^{\frac{3}{4}}e\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8ab^3}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*f*log(abs(b*x^4 + a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^
3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^
3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)
*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a*b^3)
```

3.487.9 Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 1952, normalized size of antiderivative = 6.66

$$\int \frac{c + dx + ex^2 + fx^3}{a + bx^4} \, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x)
```

output
```
symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - a*b*e^3 - a*b*c*f^2 - 16*ro
ot(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f
^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16
*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4
*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a
*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)^2*a*b^3*c
- 4*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*
z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2
*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2
 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*b^3*
c^2*x + b^2*c^2*f*x + 16*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2
*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*
z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*
f^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^
2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4
+ b^3*c^4, z, k)^2*a*b^3*d*x + 4*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3
+ 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^
2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 1
6*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4...
```

3.487. $\int \frac{c+dx+ex^2+fx^3}{a+bx^4} \, dx$

3.488 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$

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3.488.1 Optimal result

Integrand size = 28, antiderivative size = 321

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx =& \frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}-\frac{\sqrt{a}e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}\\
&+\frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}}\\
&-\frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}}\\
&+\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}\\
&-\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}\\
&+\frac{c\log\left(a+bx^4\right)}{4b}\end{aligned}$$

output
```
d*x/b+1/2*e*x^2/b+1/3*f*x^3/b+1/4*c*ln(b*x^4+a)/b-1/2*e*arctan(x^2*b^(1/2)
/a^(1/2))*a^(1/2)/b^(3/2)+1/8*a^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*b^(1/2))*(-f*a^(1/2)+d*b^(1/2))/b^(7/4)*2^(1/2)-1/8*a^(1/4)*ln(a^(1/
4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-f*a^(1/2)+d*b^(1/2))/b^(7/4)*2
^(1/2)-1/4*a^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(f*a^(1/2)+d*b^(1/
2))/b^(7/4)*2^(1/2)-1/4*a^(1/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(f*a^(
1/2)+d*b^(1/2))/b^(7/4)*2^(1/2)
```

3.488.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$$
$$= \frac{24b^{3/4}dx+12b^{3/4}ex^2+8b^{3/4}fx^3+6\sqrt[4]{a}\left(\sqrt{2}\sqrt{b}d+2\sqrt[4]{a}\sqrt[4]{b}e+\sqrt{2}\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-6\sqrt[4]{a}\left(\sqrt{\right.}}{}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

output
```
(24*b^(3/4)*d*x + 12*b^(3/4)*e*x^2 + 8*b^(3/4)*f*x^3 + 6*a^(1/4)*(Sqrt[2]*
Sqrt[b]*d + 2*a^(1/4)*b^(1/4)*e + Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*x)/a^(1/4)] - 6*a^(1/4)*(Sqrt[2]*Sqrt[b]*d - 2*a^(1/4)*b^(1/4)*e +
Sqrt[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(-(
a^(1/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] + 3*Sqrt[2]*(-(a^(1/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 6*b^(3/4)*c*Log[a + b*x^4])/(24
*b^(7/4))
```

3.488.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$$
$$\Big\downarrow \text{ 2370}$$
$$\int \left(\frac{x^3(c+ex^2)}{a+bx^4}+\frac{x^4(d+fx^2)}{a+bx^4}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}b^{7/4}}-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}b^{7/4}}-\frac{\sqrt{a}e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}+\frac{c\log\left(a+bx^4\right)}{4b}+\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

output
```
(d*x)/b + (e*x^2)/(2*b) + (f*x^3)/(3*b) - (Sqrt[a]*e*ArcTan[(Sqrt[b]*x^2)/
Sqrt[a]])/(2*b^(3/2)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d + Sqrt[a
]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a^(1/
4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/(4*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) + (c*Log
[a + b*x^4])/(4*b)
```

3.488.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

3.488.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.23

method	result
risch	$\frac{f x^3}{3b}+\frac{e x^2}{2b}+\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b+a)} \frac{(\_R^3 bc-\_R^2 af-\_R ae-ad)\ln(x-\_R)}{\_R^3}}{4b^2}$
default	$\frac{\frac{1}{3}f x^3+\frac{1}{2}e x^2+dx}{b}+\frac{-\frac{d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8}-\frac{ae\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}-\frac{af\sqrt{2}}{}}{b}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*f*x^3/b+1/2*e*x^2/b+d*x/b+1/4/b^2*sum((_R^3*b*c-_R^2*a*f-_R*a*e-a*d)/_
R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.488.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.10 (sec) , antiderivative size = 219615, normalized size of antiderivative = 684.16

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.488.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.488.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.95

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx = \frac{2\,fx^3+3\,ex^2+6\,dx}{6\,b}$$

$$+\frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c-abd+a^{\frac{3}{2}}\sqrt{b}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c+abd-a^{\frac{3}{2}}\sqrt{b}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}a^{\frac{5}{4}}b^{\frac{5}{4}}d\right.}{}}{8\,b}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/6*(2*f*x^3 + 3*e*x^2 + 6*d*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*
c - a*b*d + a^(3/2)*sqrt(b)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*c + a*b*d
 - a^(3/2)*sqrt(b)*f)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a
))/(a^(3/4)*b^(5/4)) - 2*(sqrt(2)*a^(5/4)*b^(5/4)*d + sqrt(2)*a^(7/4)*b^(3
/4)*f - 2*a^(3/2)*b*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) - 2
*(sqrt(2)*a^(5/4)*b^(5/4)*d + sqrt(2)*a^(7/4)*b^(3/4)*f + 2*a^(3/2)*b*e)*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqr
t(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/b
```

3.488.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.95

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$$

$$=\frac{c\log\left(|bx^4+a|\right)}{4\,b}+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2e-(ab^3)^{\frac{1}{4}}b^2d-(ab^3)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}$$

$$+\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2e-(ab^3)^{\frac{1}{4}}b^2d-(ab^3)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4}$$

$$-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2d-(ab^3)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4}$$

$$+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2d-(ab^3)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4}+\frac{2\,b^2fx^3+3\,b^2ex^2+6\,b^2dx}{6\,b^3}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*c*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e - (a*b^
3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/b^4 + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e - (a*b^3)^(
1/4)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4
))/(a/b)^(1/4))/b^4 - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 + 1/8*sqrt(2)*((a*b^3)^(1
/4)*b^2*d - (a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
b^4 + 1/6*(2*b^2*f*x^3 + 3*b^2*e*x^2 + 6*b^2*d*x)/b^3
```

3.488.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 838, normalized size of antiderivative = 2.61

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{a^4\,f^3+2\,a^3\,b\,c\,e\,f+a^3\,b\,d^2\,f-a^3\,b\,d\,e^2+a^2\,b^2\,c^2\,d}{b^2}\right.\right.$$
$$+\mathrm{root}\big(256\,b^7\,z^4-256\,b^6\,c\,z^3+64\,a\,b^4\,d\,f\,z^2+32\,a\,b^4\,e^2\,z^2+96\,b^5\,c^2\,z^2-32\,a\,b^3\,c\,d\,f\,z-16\,a^2\,b^2\,e\,f^2\,z+16\,a\,b$$
$$\left.-\frac{x\,(a^3\,c\,f^2-2\,a^3\,d\,e\,f+a^3\,e^3+b\,a^2\,c^2\,e-b\,a^2\,c\,d^2)}{b}\right)\mathrm{root}\big(256\,b^7\,z^4-256\,b^6\,c\,z^3$$
$$+64\,a\,b^4\,d\,f\,z^2+32\,a\,b^4\,e^2\,z^2+96\,b^5\,c^2\,z^2-32\,a\,b^3\,c\,d\,f\,z-16\,a^2\,b^2\,e\,f^2\,z+16\,a\,b^3\,d^2\,e\,z$$
$$-16\,a\,b^3\,c\,e^2\,z-16\,b^4\,c^3\,z-4\,a^2\,b\,d\,e^2\,f+4\,a^2\,b\,c\,e\,f^2+4\,a\,b^2\,c^2\,d\,f-4\,a\,b^2\,c\,d^2\,e$$
$$\left.+2\,a^2\,b\,d^2\,f^2+2\,a\,b^2\,c^2\,e^2+a^2\,b\,e^4+a\,b^2\,d^4+a^3\,f^4+b^3\,c^4,z,k\big)\right)+\frac{e\,x^2}{2\,b}+\frac{f\,x^3}{3\,b}+\frac{d\,x}{b}$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x)
```

output
```
symsum(log((a^4*f^3 + a^2*b^2*c^2*d - a^3*b*d*e^2 + a^3*b*d^2*f + 2*a^3*b*
c*e*f)/b^2 + root(256*b^7*z^4 - 256*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^
4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*
b^3*d^2*e*z - 16*a*b^3*c*e^2*z - 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*
c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^
2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*(root(256*b^7*z^4
 - 256*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*e^2*z^2 + 96*b^5*c^2*z^2 -
32*a*b^3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c*e^2*
z - 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4
*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4
 + a^3*f^4 + b^3*c^4, z, k)*(16*a^2*b^2*d - 16*a^2*b^2*e*x) - (8*a^2*b^3*c
*d + 8*a^3*b^2*e*f)/b^2 + (x*(4*a^3*b*f^2 - 4*a^2*b^2*d^2 + 8*a^2*b^2*c*e)
)/b) - (x*(a^3*e^3 + a^3*c*f^2 - 2*a^3*d*e*f - a^2*b*c*d^2 + a^2*b*c^2*e))
/b)*root(256*b^7*z^4 - 256*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*e^2*z^2
 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e
*z - 16*a*b^3*c*e^2*z - 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 +
 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a
^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k), k, 1, 4) + (e*x^2)/(2*b)
+ (f*x^3)/(3*b) + (d*x)/b
```

3.488. $\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4} \, dx$

3.489 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$

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3.489.1 Optimal result

Integrand size = 25, antiderivative size = 318

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx = -\frac{af-bx(c+dx+ex^2)}{4ab\,(a+bx^4)}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} - \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{3/4}}$$

output
```
1/4*(-a*f+b*x*(e*x^2+d*x+c))/a/b/(b*x^4+a)+1/4*d*arctan(x^2*b^(1/2)/a^(1/2
))/a^(3/2)/b^(1/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))
*(-e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*
x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-e*a^(1/2)+3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^
(1/2)+1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*c*b^(1/2))/a^
(7/4)/b^(3/4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+
3*c*b^(1/2))/a^(7/4)/b^(3/4)*2^(1/2)
```

3.489.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$$
$$= \frac{-\frac{8a(af-bx(c+x(d+ex)))}{a+bx^4} - 2\sqrt[4]{a}\sqrt[4]{b}\left(3\sqrt{2}\sqrt{b}c+4\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2\sqrt[4]{a}\sqrt[4]{b}\left(3\sqrt{2}\sqrt{b}\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

output
```
((-8*a*(a*f - b*x*(c + x*(d + e*x))))/(a + b*x^4) - 2*a^(1/4)*b^(1/4)*(3*S
qrt[2]*Sqrt[b]*c + 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(
1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
] + Sqrt[2]*b^(1/4)*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*a^(1/4)*Sqrt[b]*c
 - a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*
a^2*b)
```

3.489.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$$
$$\Big\downarrow \; 2393$$
$$-\frac{\int -\frac{ex^2+2dx+3c}{bx^4+a}dx}{4a} - \frac{af-bx(c+dx+ex^2)}{4ab(a+bx^4)}$$
$$\Big\downarrow \; 25$$
$$\frac{\int \frac{ex^2+2dx+3c}{bx^4+a}dx}{4a} - \frac{af-bx(c+dx+ex^2)}{4ab(a+bx^4)}$$

$$\downarrow 2415$$

$$\frac{\int \left(\frac{2dx}{bx^4+a}+\frac{ex^2+3c}{bx^4+a}\right)dx}{4a}-\frac{af-bx(c+dx+ex^2)}{4ab\,(a+bx^4)}$$

$$\downarrow 2009$$

$$\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(3\sqrt{b}c-\sqrt{}\right.}{}}{4a}$$

$$\frac{af-bx(c+dx+ex^2)}{4ab\,(a+bx^4)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

output
```
-1/4*(a*f - b*x*(c + d*x + e*x^2))/(a*b*(a + b*x^4)) + ((d*ArcTan[(Sqrt[b]
*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((3*Sqrt[b]*c
+ Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b
^(3/4)) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/
4)*b^(3/4)))/(4*a)
```

3.489.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

3.489. $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.489.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.27

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{cx}{4a}-\frac{f}{4b}}{b\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(\_R^2 e+2\_R d+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+d\left(\frac{x^2}{4a(b\,x^4+a)}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/a*e*x^3+1/4*d/a*x^2+1/4*c/a*x-1/4*f/b)/(b*x^4+a)+1/16/b/a*sum((_R^2*e
+2*_R*d+3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.489.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.02 (sec) , antiderivative size = 124301, normalized size of antiderivative = 390.88

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.489.6 Sympy [A] (verification not implemented)

Time = 43.91 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.63

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$$
$$= \operatorname{RootSum}\left(65536t^4a^7b^3+t^2\cdot\left(3072a^4b^2ce+2048a^4b^2d^2\right)+t\left(128a^3bde^2-1152a^2b^2c^2d\right)+a^2e^4+18abc^2\right.$$
$$+\frac{-af+bcx+bdx^2+bex^3}{4a^2b+4ab^2x^4}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a
*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t
*log(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304
*_t**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*
b**2*d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t
*a**4*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d
**2*e + 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5
+ 120*a**2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a
*b**2*c**3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 -
 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*
a*b**2*c**2*d**4 + 729*b**3*c**6))))) + (-a*f + b*c*x + b*d*x**2 + b*e*x**3
)/(4*a**2*b + 4*a*b**2*x**4)
```

3.489.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.96

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx = \frac{bex^3+bdx^2+bcx-af}{4\,(ab^2x^4+a^2b)}$$
$$+\frac{\frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-4\sqrt{a}\sqrt{b}d\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{}}}}{32\,a}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b*e*x^3 + b*d*x^2 + b*c*x - a*f)/(a*b^2*x^4 + a^2*b) + 1/32*(sqrt(2)*
(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sq
rt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x
^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)
*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e - 4*sqrt(a)*sqrt(b)*d)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)
*c + sqrt(2)*a^(3/4)*b^(1/4)*e + 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqr
t(sqrt(a)*sqrt(b))*b^(3/4)))/a
```

3.489.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.98

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx = \frac{bex^3+bdx^2+bcx-af}{4\,(bx^4+a)ab} + \frac{\sqrt{2}\Big(2\,\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\Big(2\,\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3} - \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(b*e*x^3 + b*d*x^2 + b*c*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*
sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt
(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/
32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c -
(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

3.489.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.50

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^2}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\right.\right.\right.$$

$$-\frac{9\,b^2\,c^2\,e-12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}+\frac{x\,\left(2\,b^2\,d^3-3\,b^2\,c\,d\,e\right)}{16\,a^3}\Bigg)\,\mathrm{root}\Big(65536\,a^7\,b^3\,z^4$$

$$+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e$$

$$\left.+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\Big)\right)+\frac{\frac{d\,x^2}{4\,a}-\frac{f}{4\,b}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x)
```

output
```
symsum(log((x*(2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3) - (9*b^2*c^2*e - 12*b^2*
c*d^2 + a*b*e^3)/(64*a^3) - root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2
+ 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b
*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)*(root
(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^
2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a
*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3
*c^2 - 4*a^2*b^2*e^2))/(16*a^3) + (b^2*d*e)/a))*root(65536*a^7*b^3*z^4 + 3
072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^
3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 +
a^2*e^4, z, k), k, 1, 4) + ((d*x^2)/(4*a) - f/(4*b) + (e*x^3)/(4*a) + (c*x
)/(4*a))/(a + b*x^4)
```

3.490 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx$

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3.490.1 Optimal result

Integrand size = 28, antiderivative size = 310

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx =& -\frac{c+dx+ex^2+fx^3}{4b(a+bx^4)}+\frac{e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}}\\&-\frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}}\\&+\frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}}\\&-\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}\\&+\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{3/4}b^{7/4}}\end{aligned}$$

output
```
1/4*(-f*x^3-e*x^2-d*x-c)/b/(b*x^4+a)+1/4*e*arctan(x^2*b^(1/2)/a^(1/2))/b^(
3/2)/a^(1/2)-1/32*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-3*f
*a^(1/2)+d*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)+1/32*ln(a^(1/4)*b^(1/4)*x*2^(1
/2)+a^(1/2)+x^2*b^(1/2))*(-3*f*a^(1/2)+d*b^(1/2))/a^(3/4)/b^(7/4)*2^(1/2)+
1/16*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(3*f*a^(1/2)+d*b^(1/2))/a^(3/4)/
b^(7/4)*2^(1/2)+1/16*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(3*f*a^(1/2)+d*b^
(1/2))/a^(3/4)/b^(7/4)*2^(1/2)
```

3.490.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx$$
$$= \frac{-\frac{8b^{3/4}(c+x(d+x(e+fx)))}{a+bx^4} - \frac{2\left(\sqrt{2}\sqrt{b}d+4\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\left(\sqrt{2}\sqrt{b}d-4\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f\right)\arctan\left(1+\right.}{a^{3/4}}}{32b^{7/4}}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

output
```
((-8*b^(3/4)*(c + x*(d + x*(e + f*x))))/(a + b*x^4) - (2*(Sqrt[2]*Sqrt[b]*
d + 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/a^(3/4) + (2*(Sqrt[2]*Sqrt[b]*d - 4*a^(1/4)*b^(1/4)*e + 3*Sq
rt[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[
2]*(-(Sqrt[b]*d) + 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/a^(3/4) + (Sqrt[2]*(Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4))/(32*b^(7/4))
```

3.490.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2363, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^2}dx$$
$$\Big\downarrow \; 2363$$
$$\frac{\int \frac{3fx^2+2ex+d}{bx^4+a}dx}{4b}-\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}$$
$$\Big\downarrow \; 2415$$
$$\frac{\int \left(\frac{2ex}{bx^4+a}+\frac{3fx^2+d}{bx^4+a}\right)dx}{4b}-\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}d-3\right.}{}}{4b}$$

$$\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

output
```
-1/4*(c + d*x + e*x^2 + f*x^3)/(b*(a + b*x^4)) + ((e*ArcTan[(Sqrt[b]*x^2)/
Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*d + 3*Sqrt[a]*f)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*d + 3*Sqrt
[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)
) - ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(
3/4)))/(4*b)
```

3.490.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.490.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.26

method	result
risch	$\frac{-\frac{f x^3}{4b}-\frac{e x^2}{4b}-\frac{dx}{4b}-\frac{c}{4b}}{b x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}(\_Z^4 b+a)}\frac{(3f\_R^2+2e\_R+d)\ln(x-\_R)}{\_R^3}}{16b^2}$
default	$\frac{-\frac{f x^3}{4b}-\frac{e x^2}{4b}-\frac{dx}{4b}-\frac{c}{4b}}{b x^4+a}+\frac{\frac{d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{e\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{\sqrt{ab}}+\frac{3f\sqrt{2}}{\ }}{4b}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*f*x^3/b-1/4*e*x^2/b-1/4*d*x/b-1/4*c/b)/(b*x^4+a)+1/16/b^2*sum((3*_R^
2*f+2*_R*e+d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.490.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.58 (sec) , antiderivative size = 122993, normalized size of antiderivative = 396.75

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.490.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

output
```
Timed out
```

3.490.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = -\frac{fx^3+ex^2+dx+c}{4\,(b^2x^4+ab)}$$
$$+\frac{\frac{\sqrt{2}\left(\sqrt{b}d-3\,\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{b}d-3\,\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}d+3\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f-4\,\sqrt{a}\sqrt{b}e\right.}{a^{\frac{3}{4}}\sqrt{\sqrt{a}}}}{32\,b}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^4 + a*b) + 1/32*(sqrt(2)*(sqrt(b)*d
 - 3*sqrt(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(
3/4)*b^(3/4)) - sqrt(2)*(sqrt(b)*d - 3*sqrt(a)*f)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(sqrt(2)*a^(1/4)*b^(3
/4)*d + 3*sqrt(2)*a^(3/4)*b^(1/4)*f - 4*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt
(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4
)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(sqrt(2)*a^(1/4)*b^(3/4)*d + 3*sqrt(2
)*a^(3/4)*b^(1/4)*f + 4*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x
 - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*s
qrt(b))*b^(3/4)))/b
```

3.490.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx$$
$$= -\frac{fx^3+ex^2+dx+c}{4\,(bx^4+a)b} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2e+(ab^3)^{\frac14}\,b^2d+3\,(ab^3)^{\frac34}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16\,ab^4} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2e+(ab^3)^{\frac14}\,b^2d+3\,(ab^3)^{\frac34}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16\,ab^4} + \frac{\sqrt{2}\left((ab^3)^{\frac14}\,b^2d-3\,(ab^3)^{\frac34}\,f\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,ab^4} - \frac{\sqrt{2}\left((ab^3)^{\frac14}\,b^2d-3\,(ab^3)^{\frac34}\,f\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,ab^4}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*(f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)*b) + 1/16*sqrt(2)*(2*sqrt(2)*s
qrt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/16*sqrt(2)*(2*sqrt
(2)*sqrt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/32*sqrt(2)*((
a*b^3)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(a*b^4) - 1/32*sqrt(2)*((a*b^3)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f
)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)
```

3.490.9 Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.80

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{x\,(2\,e^3-3\,d\,e\,f)}{16\,b}-\frac{3\,b\,d^2\,f-4\,b\,d\,e^2+27\,a\,f^3}{64\,b^2}\right.\right.$$

$$-\mathrm{root}\big(65536\,a^3\,b^7\,z^4+3072\,a^2\,b^4\,d\,f\,z^2+2048\,a^2\,b^4\,e^2\,z^2+1152\,a^2\,b^2\,e\,f^2\,z-128\,a\,b^3\,d^2\,e\,z-48\,a\,b\,d\,e^2\,f+1$$

$$+3072\,a^2\,b^4\,d\,f\,z^2+2048\,a^2\,b^4\,e^2\,z^2+1152\,a^2\,b^2\,e\,f^2\,z-128\,a\,b^3\,d^2\,e\,z-48\,a\,b\,d\,e^2\,f$$

$$\left.+18\,a\,b\,d^2\,f^2+16\,a\,b\,e^4+81\,a^2\,f^4+b^2\,d^4,z,k\big)\right)-\frac{\frac{c}{4\,b}+\frac{e\,x^2}{4\,b}+\frac{f\,x^3}{4\,b}+\frac{d\,x}{4\,b}}{b\,x^4+a}$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x)
```

output
```
symsum(log((x*(2*e^3 - 3*d*e*f))/(16*b) - (27*a*f^3 - 4*b*d*e^2 + 3*b*d^2*
f)/(64*b^2) - root(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^2*b^4
*e^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f + 18*
a*b*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*(3*a*e*f + (b*d^2*x
)/4 - (9*a*f^2*x)/4 + 4*root(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 20
48*a^2*b^4*e^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e
^2*f + 18*a*b*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*a*b^2*d -
 8*root(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e^2*z^2 +
1152*a^2*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b*d^2*f^2
 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*a*b^2*e*x))*root(65536*a^3*b^7
*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e^2*z^2 + 1152*a^2*b^2*e*f^2*z
- 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b*d^2*f^2 + 16*a*b*e^4 + 81*a^
2*f^4 + b^2*d^4, z, k), k, 1, 4) - (c/(4*b) + (e*x^2)/(4*b) + (f*x^3)/(4*b
) + (d*x)/(4*b))/(a + b*x^4)
```

3.491 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3} dx$

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3.491.1 Optimal result

Integrand size = 25, antiderivative size = 351

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3} dx = \frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2} + \frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}} - \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{11/4}b^{3/4}}$$

output
```
1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(b*x^4+a)+1/8*(-a*f+b*x*(e*x^2+d*x+c))/a/b/
(b*x^4+a)^2+3/16*d*arctan(x^2*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)-1/256*ln(-a
^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*e*a^(1/2)+21*c*b^(1/2))/
a^(11/4)/b^(3/4)*2^(1/2)+1/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^
(1/2))*(-5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)*2^(1/2)+1/128*arctan(-
1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2))/a^(11/4)/b^(3/4)*2
^(1/2)+1/128*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*e*a^(1/2)+21*c*b^(1/2)
)/a^(11/4)/b^(3/4)*2^(1/2)
```

3.491.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx$$
$$= \frac{\frac{8ax(7c+x(6d+5ex))}{a+bx^4} - \frac{32a^2(af-bx(c+x(d+ex)))}{b(a+bx^4)^2} - \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}c+24\sqrt[4]{a}\sqrt[4]{b}d+5\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

output
```
((8*a*x*(7*c + x*(6*d + 5*e*x)))/(a + b*x^4) - (32*a^2*(a*f - b*x*(c + x*(
d + e*x))))/(b*(a + b*x^4)^2) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1
/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/b^(3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*
Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqr
t[2]*(-21*a^(1/4)*Sqrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3
/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(2
56*a^3)
```

3.491.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2393, 25, 2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx$$

$$\Big\downarrow \; 2393$$

$$-\frac{\int -\frac{5ex^2+6dx+7c}{(bx^4+a)^2}dx}{8a} - \frac{af-bx(c+dx+ex^2)}{8ab\,(a+bx^4)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{5ex^2+6dx+7c}{(bx^4+a)^2}dx}{8a}-\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}$$

$\big\downarrow$ 2394

$$\frac{\frac{x(7c+6dx+5ex^2)}{4a(a+bx^4)}-\frac{\int -\frac{5ex^2+12dx+21c}{bx^4+a}dx}{4a}}{8a}-\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\int \frac{5ex^2+12dx+21c}{bx^4+a}dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a+bx^4)}}{8a}-\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}$$

$\big\downarrow$ 2415

$$\frac{\frac{\int \left(\frac{12dx}{bx^4+a}+\frac{5ex^2+21c}{bx^4+a}\right)dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a+bx^4)}}{8a}-\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}$$

$\big\downarrow$ 2009

$$\frac{\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(5\sqrt{a}e+21\sqrt{b}c)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(5\sqrt{a}e+21\sqrt{b}c)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{(21\sqrt{b}c-5\sqrt{a}e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{(21\sqrt{b}c-5\sqrt{a}e)\log\left(\sqrt{2}\right.}{4\sqrt{2}a^{3}}}{4a}}{8a}$$

$$\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

output
```
-1/8*(a*f - b*x*(c + d*x + e*x^2))/(a*b*(a + b*x^4)^2) + ((x*(7*c + 6*d*x
+ 5*e*x^2))/(4*a*(a + b*x^4)) + ((6*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt
[a]*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((21*
Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4
)))/(4*a))/(8*a)
```

3.491. $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx$

3.491.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.491.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.71 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.33

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\frac{(5\_R^2e+12\_Rd+21c)\ln(x-\_R)}{\_R^3}}{128a^2b}$
default	$c\left(\frac{x}{8a(b\,x^4+a)^2}+\frac{\frac{7x}{32a(b\,x^4+a)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+d\left(\frac{}{8a\ldots}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/32*b*e/a^2*x^7+3/16*b*d/a^2*x^6+7/32*b*c/a^2*x^5+9/32/a*e*x^3+5/16*d/a*
x^2+11/32*c/a*x-1/8*f/b)/(b*x^4+a)^2+1/128/a^2/b*sum((5*_R^2*e+12*_R*d+21*
c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.491.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.94 (sec) , antiderivative size = 124838, normalized size of antiderivative = 355.66

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.491.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

output
```
Timed out
```

3.491.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx = \frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5+9\,abex^3+10\,abdx^2+11\,abcx-4\,a^2f}{32\,(a^2b^3x^8+2\,a^3b^2x^4+a^4b)} + \frac{\frac{\sqrt{2}\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-24\right.}{a^{\frac{3}{4}}}}{256\,a^2}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 + 9*a*b*e*x^3 + 10*a*b*d*x^2
 + 11*a*b*c*x - 4*a^2*f)/(a^2*b^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sq
rt(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*l
og(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) +
2*(21*sqrt(2)*a^(1/4)*b^(3/4)*c + 5*sqrt(2)*a^(3/4)*b^(1/4)*e - 24*sqrt(a)
*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqr
t(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2
)*a^(1/4)*b^(3/4)*c + 5*sqrt(2)*a^(3/4)*b^(1/4)*e + 24*sqrt(a)*sqrt(b)*d)*
arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sq
rt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^2
```

3.491.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx$$
$$=\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$
$$-\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$
$$+\frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5+9\,abex^3+10\,abdx^2+11\,abcx-4\,a^2f}{32\left(bx^4+a\right)^2a^2b}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/
(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b
^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^
(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sq
rt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*
c*x^5 + 9*a*b*e*x^3 + 10*a*b*d*x^2 + 11*a*b*c*x - 4*a^2*f)/((b*x^4 + a)^2*
a^2*b)
```

3.491.9 Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.37

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(125\,a\,e^3-3024\,b\,c\,d^2+2205\,b\,c^2\,e-1728\,b\,d^3\,x+\mathrm{root}(268435456\,a^{11}\,b^3\,z^4+6881280\,a^6\,b^2\right.}{}\right.\right.$$
$$+6881280\,a^6\,b^2\,c\,e\,z^2+4718592\,a^6\,b^2\,d^2\,z^2-2709504\,a^3\,b^2\,c^2\,d\,z+153600\,a^4\,b\,d\,e^2\,z$$
$$\left.-60480\,a\,b\,c\,d^2\,e+22050\,a\,b\,c^2\,e^2+20736\,a\,b\,d^4+625\,a^2\,e^4+194481\,b^2\,c^4,z,k)\right)$$
$$+\frac{\frac{5\,d\,x^2}{16\,a}-\frac{f}{8\,b}+\frac{9\,e\,x^3}{32\,a}+\frac{11\,c\,x}{32\,a}+\frac{7\,b\,c\,x^5}{32\,a^2}+\frac{3\,b\,d\,x^6}{16\,a^2}+\frac{5\,b\,e\,x^7}{32\,a^2}}{a^2+2\,a\,b\,x^4+b^2\,x^8}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x)
```

output
```
symsum(log(-(b*(125*a*e^3 - 3024*b*c*d^2 + 2205*b*c^2*e - 1728*b*d^3*x + 3
44064*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*
b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c
*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4,
 z, k)^2*a^5*b^2*c - 3200*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*
e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d
*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e
^4 + 194481*b^2*c^4, z, k)*a^3*b*e^2*x + 2520*b*c*d*e*x + 56448*root(26843
5456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 27
09504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a
*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^2*b^2*c
^2*x - 196608*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718
592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 604
80*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*
b^2*c^4, z, k)^2*a^5*b^2*d*x + 15360*root(268435456*a^11*b^3*z^4 + 6881280
*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153
600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4
+ 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435
456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 270
9504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050...
```

3.492 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$

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3.492.1 Optimal result

Integrand size = 28, antiderivative size = 340

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx =& -\frac{c+dx+ex^2+fx^3}{8b(a+bx^4)^2}+\frac{x(d+2ex+3fx^2)}{32ab(a+bx^4)}\\&+\frac{e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}}-\frac{3\left(\sqrt{b}d+\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}}\\&+\frac{3\left(\sqrt{b}d+\sqrt{a}f\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}}\\&-\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}}\\&+\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{128\sqrt{2}a^{7/4}b^{7/4}}\end{aligned}$$

output
```
1/8*(-f*x^3-e*x^2-d*x-c)/b/(b*x^4+a)^2+1/32*x*(3*f*x^2+2*e*x+d)/a/b/(b*x^4
+a)+1/16*e*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-3/256*ln(-a^(1/4)*b
^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-f*a^(1/2)+d*b^(1/2))/a^(7/4)/b^(7/
4)*2^(1/2)+3/256*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-f*a^(
1/2)+d*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+3/128*arctan(-1+b^(1/4)*x*2^(1/2)/
a^(1/4))*(f*a^(1/2)+d*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)+3/128*arctan(1+b^(1
/4)*x*2^(1/2)/a^(1/4))*(f*a^(1/2)+d*b^(1/2))/a^(7/4)/b^(7/4)*2^(1/2)
```

3.492.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$$

$$= \frac{\frac{8b^{3/4}x(d+x(2e+3fx))}{a(a+bx^4)} - \frac{32b^{3/4}(c+x(d+x(e+fx)))}{(a+bx^4)^2} - \frac{2\left(3\sqrt{2}\sqrt{b}d+8\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{2\left(3\sqrt{2}\sqrt{b}d-8\sqrt[4]{a}\sqrt[4]{\cdots}\right.}{\cdots}}{2\cdots}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

output
```
((8*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)) - (32*b^(3/4)*(c + x*
(d + x*(e + f*x))))/(a + b*x^4)^2 - (2*(3*Sqrt[2]*Sqrt[b]*d + 8*a^(1/4)*b^
(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^
(7/4) + (2*(3*Sqrt[2]*Sqrt[b]*d - 8*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*
f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (3*Sqrt[2]*(-(Sqrt[b
]*d) + Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/
a^(7/4) + (3*Sqrt[2]*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(256*b^(7/4))
```

3.492.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2363, 2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^3}dx$$

$$\Big\downarrow \text{ 2363}$$

$$\frac{\int \frac{3fx^2+2ex+d}{\left(bx^4+a\right)^2}dx}{8b} - \frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}$$

$$\Big\downarrow \text{ 2394}$$

$$\frac{\frac{x(d+2ex+3fx^2)}{4a(a+bx^4)}-\frac{\int -\frac{3fx^2+4ex+3d}{bx^4+a}dx}{4a}}{8b}-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{3fx^2+4ex+3d}{bx^4+a}dx}{4a}+\frac{x(d+2ex+3fx^2)}{4a(a+bx^4)}}{8b}-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{\int \left(\frac{4ex}{bx^4+a}+\frac{3fx^2+3d}{bx^4+a}\right)dx}{4a}+\frac{x(d+2ex+3fx^2)}{4a(a+bx^4)}}{8b}-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}\right.}{4\sqrt{2}a^{3/4}\cdots}}{4a}}{8b}$$

$$\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

output
```
-1/8*(c + d*x + e*x^2 + f*x^3)/(b*(a + b*x^4)^2) + ((x*(d + 2*e*x + 3*f*x^
2))/(4*a*(a + b*x^4)) + ((2*e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt
[b]) - (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - (3*(Sqrt[b]*d - Sqr
t[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(3/4)) + (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)))/(4*a))/(8*b)
```

3.492. $\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^3}\,dx$

3.492.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.492.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.63 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\frac{3 f x^{7}}{32 a}+\frac{e x^{6}}{16 a}+\frac{d x^{5}}{32 a}-\frac{f x^{3}}{32 b}-\frac{e x^{2}}{16 b}-\frac{3 d x}{32 b}-\frac{c}{8 b}}{\left(b x^{4}+a \right)^{2}}+\frac{\sum_{\_R =\operatorname{RootOf}\left(\_Z^{4} b +a \right)}\frac{\left(3 f \_R^{2}+4 e \_R +3 d \right) \ln \left(x -\_R \right)}{\_R^{3}}}{128 a \,b^{2}}$
default	$\frac{\frac{3 f x^{7}}{32 a}+\frac{e x^{6}}{16 a}+\frac{d x^{5}}{32 a}-\frac{f x^{3}}{32 b}-\frac{e x^{2}}{16 b}-\frac{3 d x}{32 b}-\frac{c}{8 b}}{\left(b x^{4}+a \right)^{2}}+\frac{\frac{3 d \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8 a}+\frac{2 e}{}}{}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/32*f/a*x^7+1/16/a*e*x^6+1/32*d/a*x^5-1/32*f*x^3/b-1/16*e*x^2/b-3/32*d*x
/b-1/8*c/b)/(b*x^4+a)^2+1/128/a/b^2*sum((3*_R^2*f+4*_R*e+3*d)/_R^3*ln(x-_R
),_R=RootOf(_Z^4*b+a))
```

3.492.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.28 (sec) , antiderivative size = 124542, normalized size of antiderivative = 366.30

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.492.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

output
```
Timed out
```

3.492.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.01

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = \frac{3\,bfx^7+2\,bex^6+bdx^5-afx^3-2\,aex^2-3\,adx-4\,ac}{32\,(ab^3x^8+2\,a^2b^2x^4+a^3b)}$$

$$+\frac{\frac{3\sqrt{2}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{3\sqrt{2}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}d+3\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f-8\sqrt{a}\sqrt{b}\right.}{a^{\frac{3}{4}}\sqrt{\sqrt{a}}}}{256\,ab}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(3*b*f*x^7 + 2*b*e*x^6 + b*d*x^5 - a*f*x^3 - 2*a*e*x^2 - 3*a*d*x - 4*
a*c)/(a*b^3*x^8 + 2*a^2*b^2*x^4 + a^3*b) + 1/256*(3*sqrt(2)*(sqrt(b)*d - s
qrt(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*
b^(3/4)) - 3*sqrt(2)*(sqrt(b)*d - sqrt(a)*f)*log(sqrt(b)*x^2 - sqrt(2)*a^(
1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)
*d + 3*sqrt(2)*a^(3/4)*b^(1/4)*f - 8*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)
*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*s
qrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*d + 3*sqrt(2)
*a^(3/4)*b^(1/4)*f + 8*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x
- sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sq
rt(b))*b^(3/4)))/(a*b)
```

3.492.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.98

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = \frac{3\,bfx^7+2\,bex^6+bdx^5-afx^3-2\,aex^2-3\,adx-4\,ac}{32\,(bx^4+a)^2ab} + \frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ab}b^2e+3\,(ab^3)^{\frac{1}{4}}\,b^2d+3\,(ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4} + \frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ab}b^2e+3\,(ab^3)^{\frac{1}{4}}\,b^2d+3\,(ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4} + \frac{3\,\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2d-(ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4} - \frac{3\,\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2d-(ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/32*(3*b*f*x^7 + 2*b*e*x^6 + b*d*x^5 - a*f*x^3 - 2*a*e*x^2 - 3*a*d*x - 4*
a*c)/((b*x^4 + a)^2*a*b) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e + 3*(a
*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^
2*e + 3*(a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/256*sqrt(2)*((a*b^3)^(1/4
)*b^2*d - (a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
^2*b^4) - 3/256*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*log(x^2 -
sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
```

3.492.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.53

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(268435456\,a^7\,b^7\,z^4+589824\,a^4\,b^4\,d\,f\,z^2+524288\,a^4\,b^4\,e^2\,z^2+18432\,a^3\,b^2\,e\,f^2\,z-18432\,a^2\,b\right.\right.\right.$$
$$-\frac{3\,(9\,b\,d^2\,f-16\,b\,d\,e^2+9\,a\,f^3)}{32768\,a^3\,b^2}+\frac{x\,(8\,e^3-9\,d\,e\,f)}{4096\,a^3\,b}\Big)\,\mathrm{root}\big(268435456\,a^7\,b^7\,z^4$$
$$+589824\,a^4\,b^4\,d\,f\,z^2+524288\,a^4\,b^4\,e^2\,z^2+18432\,a^3\,b^2\,e\,f^2\,z-18432\,a^2\,b^3\,d^2\,e\,z$$
$$\left.-576\,a\,b\,d\,e^2\,f+162\,a\,b\,d^2\,f^2+256\,a\,b\,e^4+81\,a^2\,f^4+81\,b^2\,d^4,z,k\big)\right)$$
$$-\frac{\frac{c}{8\,b}-\frac{d\,x^5}{32\,a}-\frac{e\,x^6}{16\,a}+\frac{e\,x^2}{16\,b}-\frac{3\,f\,x^7}{32\,a}+\frac{f\,x^3}{32\,b}+\frac{3\,d\,x}{32\,b}}{a^2+2\,a\,b\,x^4+b^2\,x^8}$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x)
```

output
```
symsum(log((x*(8*e^3 - 9*d*e*f))/(4096*a^3*b) - (3*(9*a*f^3 - 16*b*d*e^2 +
 9*b*d^2*f))/(32768*a^3*b^2) - root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4
*d*f*z^2 + 524288*a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*
d^2*e*z - 576*a*b*d*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 8
1*b^2*d^4, z, k)*(root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d*f*z^2 + 52
4288*a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2*e*z - 576
*a*b*d*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^2*d^4, z,
 k)*((3*b^2*d)/2 - 2*b^2*e*x) + (3*e*f)/(32*a) + (x*(144*a*b^2*d^2 - 144*a
^2*b*f^2))/(4096*a^3*b)))*root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d*f*
z^2 + 524288*a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2*e
*z - 576*a*b*d*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^2
*d^4, z, k), k, 1, 4) - (c/(8*b) - (d*x^5)/(32*a) - (e*x^6)/(16*a) + (e*x^
2)/(16*b) - (3*f*x^7)/(32*a) + (f*x^3)/(32*b) + (3*d*x)/(32*b))/(a^2 + b^2
*x^8 + 2*a*b*x^4)
```

3.493 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$

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3.493.1 Optimal result

Integrand size = 25, antiderivative size = 382

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx = \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} - \frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3} + \frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} - \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{15/4}b^{3/4}}$$

output
```
1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(b*x^4+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c
)/a^3/(b*x^4+a)+1/12*(-a*f+b*x*(e*x^2+d*x+c))/a/b/(b*x^4+a)^3+5/32*d*arcta
n(x^2*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*b^(1/2))*(-15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/
2)+1/1024*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-15*e*a^(1/2)
+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/
a^(1/4))*(15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)*2^(1/2)+1/512*arctan
(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(15*e*a^(1/2)+77*c*b^(1/2))/a^(15/4)/b^(3/4)
*2^(1/2)
```

3.493.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.99

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{8ax(77c+15x(4d+3ex))}{a+bx^4} + \frac{32a^2x(11c+x(10d+9ex))}{(a+bx^4)^2} - \frac{256a^3(af-bx(c+x(d+ex)))}{b(a+bx^4)^3} - \frac{6\sqrt[4]{a}\left(77\sqrt{2}\sqrt{b}c+80\sqrt[4]{a}\sqrt[4]{b}d+15\sqrt{2}\sqrt{a}e\right)\arctan\left(\right.}{b^{3/4}}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```

output
```
((8*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10
*d + 9*e*x)))/(a + b*x^4)^2 - (256*a^3*(a*f - b*x*(c + x*(d + e*x))))/(b*(
a + b*x^4)^3) - (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d +
15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (
6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a
]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^
(1/4)*Sqrt[b]*c + 15*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/b^(3/4) + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^4)
```

3.493.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2393, 25, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}dx$$

$$\Big\downarrow \text{2393}$$

$$-\frac{\int -\frac{9ex^2+10dx+11c}{(bx^4+a)^3}dx}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{9ex^2+10dx+11c}{(bx^4+a)^3}dx}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}-\frac{\int -\frac{45ex^2+60dx+77c}{(bx^4+a)^2}dx}{8a}}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{45ex^2+60dx+77c}{(bx^4+a)^2}dx}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x(77c+60dx+45ex^2)}{4a(a+bx^4)}-\frac{\int -\frac{3(15ex^2+40dx+77c)}{bx^4+a}dx}{4a}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{15ex^2+40dx+77c}{bx^4+a}dx}{4a}+\frac{x(77c+60dx+45ex^2)}{4a(a+bx^4)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{\frac{3\int \left(\frac{40dx}{bx^4+a}+\frac{15ex^2+77c}{bx^4+a}\right)dx}{4a}+\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a+bx^4\right)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}-\frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\text{lo}}{4}\right.}{4a}}{8a}}{12a}$$

$$\frac{af-bx\left(c+dx+ex^2\right)}{12ab\left(a+bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```

output
```
-1/12*(a*f - b*x*(c + d*x + e*x^2))/(a*b*(a + b*x^4)^3) + ((x*(11*c + 10*d
*x + 9*e*x^2))/(8*a*(a + b*x^4)^2) + ((x*(77*c + 60*d*x + 45*e*x^2))/(4*a*
(a + b*x^4)) + (3*((20*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
- ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((77*Sqrt[b]*c -
 15*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*
Sqrt[2]*a^(3/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))))/(4*a
))/(8*a))/(12*a)
```

3.493.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.493.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.64 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4b+a)}\left(15\_R^2e+40\_R\right.}{512a^3b}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\right.}{8}$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+5/32*d/a^3*b^2*x^10+77/384*c/a^3*b^2*x^9+21/64*b*e/
a^2*x^7+5/12*b*d/a^2*x^6+33/64*b*c/a^2*x^5+113/384/a*e*x^3+11/32*d/a*x^2+5
1/128*c/a*x-1/12*f/b)/(b*x^4+a)^3+1/512/a^3/b*sum((15*_R^2*e+40*_R*d+77*c)
/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.493.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.78 (sec) , antiderivative size = 125011, normalized size of antiderivative = 327.25

$$\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^4} \, dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.493.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^4} \, dx = \text{Timed out}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

output
```
Timed out
```

3.493.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.05

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$$
$$= \frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9+126\,ab^2ex^7+160\,ab^2dx^6+198\,ab^2cx^5+113\,a^2bex^3+132\,a^2bdx^2+153}{384\,(a^3b^4x^{12}+3\,a^4b^3x^8+3\,a^5b^2x^4+a^6b)}$$
$$+\frac{\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+15\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e\right.}{}}{1024\,a^3}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 + 126*a*b^2*e*x^7 + 16
0*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153*
a^2*b*c*x - 32*a^3*f)/(a^3*b^4*x^12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*
b) + 1/1024*(sqrt(2)*(77*sqrt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*sqrt(b)*c
- 15*sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^
(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1
/4)*e - 80*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^
(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/
4)) + 2*(77*sqrt(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e + 80*
sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^3
```

3.493.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.01

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$$
$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77(ab^3)^{\frac{1}{4}}b^2c+15(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3}$$
$$+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77(ab^3)^{\frac{1}{4}}b^2c+15(ab^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512a^4b^3}$$
$$+\frac{\sqrt{2}\left(77(ab^3)^{\frac{1}{4}}b^2c-15(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024a^4b^3}$$
$$-\frac{\sqrt{2}\left(77(ab^3)^{\frac{1}{4}}b^2c-15(ab^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024a^4b^3}$$
$$+\frac{45b^3ex^{11}+60b^3dx^{10}+77b^3cx^9+126ab^2ex^7+160ab^2dx^6+198ab^2cx^5+113a^2bex^3+132a^2bdx^2+1}{384(bx^4+a)^3a^3b}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a
*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*
b^2*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))
/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b
^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/10
24*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^
10 + 77*b^3*c*x^9 + 126*a*b^2*e*x^7 + 160*a*b^2*d*x^6 + 198*a*b^2*c*x^5 +
113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153*a^2*b*c*x - 32*a^3*f)/((b*x^4 + a)
^3*a^3*b)
```

3.493.9 Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 879, normalized size of antiderivative = 2.30

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\frac{b\left(3375\,a\,e^3-123200\,b\,c\,d^2+88935\,b\,c^2\,e-64000\,b\,d^3\,x+\mathrm{root}(68719476736\,a^{15}\,b^3\,z^4+1211\right.}{}\right.\right.$$
$$+1211105280\,a^8\,b^2\,c\,e\,z^2+838860800\,a^8\,b^2\,d^2\,z^2-485703680\,a^4\,b^2\,c^2\,d\,z$$
$$+18432000\,a^5\,b\,d\,e^2\,z-7392000\,a\,b\,c\,d^2\,e+2668050\,a\,b\,c^2\,e^2+2560000\,a\,b\,d^4$$
$$\left.+35153041\,b^2\,c^4+50625\,a^2\,e^4,z,k\right)\right)$$
$$+\frac{\frac{11\,d\,x^2}{32\,a}-\frac{f}{12\,b}+\frac{113\,e\,x^3}{384\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{15\,b^2\,e\,x^{11}}{128\,a^3}+\frac{33\,b\,c\,x^5}{64\,a^2}+\frac{5\,b\,d\,x^6}{12\,a^2}+\frac{21\,b\,e\,x^7}{64\,a^2}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x)
```

output
```
symsum(log(-(b*(3375*a*e^3 - 123200*b*c*d^2 + 88935*b*c^2*e - 64000*b*d^3*
x + 20185088*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 +
838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e
^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153
041*b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2*c - 115200*root(68719476736*a
^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485
703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 26
68050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z,
 k)*a^4*b*e^2*x + 92400*b*c*d*e*x + 3035648*root(68719476736*a^15*b^3*z^4
+ 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b
^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^3*b^2*
c^2*x - 10485760*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^
2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b
*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 3
5153041*b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x + 614400*root(6871947
6736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2
 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*
e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e
^4, z, k)*a^4*b*d*e))/(2097152*a^9))*root(68719476736*a^15*b^3*z^4 + 12...
```

3.494 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$

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3.494.1 Optimal result

Integrand size = 28, antiderivative size = 380

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx = &-\frac{c+dx+ex^2+fx^3}{12b\,(a+bx^4)^3}+\frac{x(d+2ex+3fx^2)}{96ab\,(a+bx^4)^2}\\&+\frac{x(7d+12ex+15fx^2)}{384a^2b\,(a+bx^4)}+\frac{e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}\\&-\frac{\left(7\sqrt{b}d+5\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{11/4}b^{7/4}}\\&+\frac{\left(7\sqrt{b}d+5\sqrt{a}f\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{11/4}b^{7/4}}\\&-\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}}\\&+\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{512\sqrt{2}a^{11/4}b^{7/4}}\end{aligned}$$

output
```
1/12*(-f*x^3-e*x^2-d*x-c)/b/(b*x^4+a)^3+1/96*x*(3*f*x^2+2*e*x+d)/a/b/(b*x^
4+a)^2+1/384*x*(15*f*x^2+12*e*x+7*d)/a^2/b/(b*x^4+a)+1/32*e*arctan(x^2*b^(
1/2)/a^(1/2))/a^(5/2)/b^(3/2)-1/1024*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*b^(1/2))*(-5*f*a^(1/2)+7*d*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/1024*l
n(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-5*f*a^(1/2)+7*d*b^(1/2)
)/a^(11/4)/b^(7/4)*2^(1/2)+1/512*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(5*f
*a^(1/2)+7*d*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)+1/512*arctan(1+b^(1/4)*x*2^
(1/2)/a^(1/4))*(5*f*a^(1/2)+7*d*b^(1/2))/a^(11/4)/b^(7/4)*2^(1/2)
```

3.494.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{32b^{3/4}x(d+x(2e+3fx))}{a(a+bx^4)^2} + \frac{8b^{3/4}x(7d+3x(4e+5fx))}{a^2(a+bx^4)} - \frac{256b^{3/4}(c+x(d+x(e+fx)))}{(a+bx^4)^3} - \frac{6\left(7\sqrt{2}\sqrt{b}d+16\sqrt[4]{a}\sqrt[4]{b}e+5\sqrt{2}\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{a^{11/4}}}{}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

output
```
((32*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)^2) + (8*b^(3/4)*x*(7*
d + 3*x*(4*e + 5*f*x)))/(a^2*(a + b*x^4)) - (256*b^(3/4)*(c + x*(d + x*(e
+ f*x))))/(a + b*x^4)^3 - (6*(7*Sqrt[2]*Sqrt[b]*d + 16*a^(1/4)*b^(1/4)*e +
 5*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) +
(6*(7*Sqrt[2]*Sqrt[b]*d - 16*a^(1/4)*b^(1/4)*e + 5*Sqrt[2]*Sqrt[a]*f)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) + (3*Sqrt[2]*(-7*Sqrt[b]*d +
 5*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(1
1/4) + (3*Sqrt[2]*(7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*x + Sqrt[b]*x^2])/a^(11/4))/(3072*b^(7/4))
```

3.494.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2363, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}dx$$

$$\Big\downarrow \text{ 2363}$$

$$\frac{\int \frac{3fx^2+2ex+d}{(bx^4+a)^3}dx}{12b}-\frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3}$$

$$\Big\downarrow \text{ 2394}$$

$$\frac{\frac{x(d+2ex+3fx^2)}{8a(a+bx^4)^2}-\frac{\int -\frac{15fx^2+12ex+7d}{(bx^4+a)^2}dx}{8a}}{12b}-\frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{15fx^2+12ex+7d}{(bx^4+a)^2}dx}{8a}+\frac{x(d+2ex+3fx^2)}{8a(a+bx^4)^2}}{12b}-\frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3}$$

$$\Big\downarrow \text{ 2394}$$

$$\frac{\frac{\frac{x(7d+12ex+15fx^2)}{4a(a+bx^4)}-\frac{\int -\frac{3(5fx^2+8ex+7d)}{bx^4+a}dx}{4a}}{8a}+\frac{x(d+2ex+3fx^2)}{8a(a+bx^4)^2}}{12b}-\frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{3\int \frac{5fx^2+8ex+7d}{bx^4+a}dx}{4a}+\frac{x(7d+12ex+15fx^2)}{4a(a+bx^4)}}{8a}+\frac{x(d+2ex+3fx^2)}{8a(a+bx^4)^2}}{12b}-\frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3}$$

$$\Big\downarrow \text{ 2415}$$

$$\frac{\frac{\frac{3\int \left(\frac{8ex}{bx^4+a}+\frac{5fx^2+7d}{bx^4+a}\right)dx}{4a}+\frac{x(7d+12ex+15fx^2)}{4a(a+bx^4)}}{8a}+\frac{x(d+2ex+3fx^2)}{8a(a+bx^4)^2}}{12b}-\frac{c+dx+ex^2+fx^3}{12b(a+bx^4)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}f+7\sqrt{b}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}f+7\sqrt{b}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{2}\ldots\right.}{4\sqrt{2}a^{3/\ldots}}\ldots}{4a}\ldots}{8a}\ldots}{12b}$$

$$\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

output
```
-1/12*(c + d*x + e*x^2 + f*x^3)/(b*(a + b*x^4)^3) + ((x*(d + 2*e*x + 3*f*x
^2))/(8*a*(a + b*x^4)^2) + ((x*(7*d + 12*e*x + 15*f*x^2))/(4*a*(a + b*x^4)
) + (3*((4*e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - ((7*Sqrt[b
]*d + 5*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*b^(3/4)) + ((7*Sqrt[b]*d + 5*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^
(3/4)) + ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))))/(4*a))/(8*a))/(12*b)
```

3.494.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

3.494. $\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^4}\,dx$

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.494.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\frac{5bf\,x^{11}}{128a^2}+\frac{be\,x^{10}}{32a^2}+\frac{7bd\,x^9}{384a^2}+\frac{7f\,x^7}{64a}+\frac{e\,x^6}{12a}+\frac{3d\,x^5}{64a}-\frac{5f\,x^3}{384b}-\frac{e\,x^2}{32b}-\frac{7dx}{128b}-\frac{c}{12b}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4 b+a)}\frac{(5f\,\_R^2+8e\,\_R+7d)\ln(x-\_R}{\_R^3}}{512a^2b^2}$
default	$\frac{\frac{5bf\,x^{11}}{128a^2}+\frac{be\,x^{10}}{32a^2}+\frac{7bd\,x^9}{384a^2}+\frac{7f\,x^7}{64a}+\frac{e\,x^6}{12a}+\frac{3d\,x^5}{64a}-\frac{5f\,x^3}{384b}-\frac{e\,x^2}{32b}-\frac{7dx}{128b}-\frac{c}{12b}}{(b\,x^4+a)^3}+\frac{\frac{7d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right.\right.}{8a}}{}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(5/128/a^2*b*f*x^11+1/32*b*e/a^2*x^10+7/384*b*d/a^2*x^9+7/64*f/a*x^7+1/12/
a*e*x^6+3/64*d/a*x^5-5/384*f*x^3/b-1/32*e*x^2/b-7/128*d*x/b-1/12*c/b)/(b*x
^4+a)^3+1/512/a^2/b^2*sum((5*_R^2*f+8*_R*e+7*d)/_R^3*ln(x-_R),_R=RootOf(_Z
^4*b+a))
```

3.494.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.92 (sec) , antiderivative size = 125996, normalized size of antiderivative = 331.57

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.494.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

output
```
Timed out
```

3.494.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.04

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$$
$$= \frac{15\,b^2fx^{11}+12\,b^2ex^{10}+7\,b^2dx^9+42\,abfx^7+32\,abex^6+18\,abdx^5-5\,a^2fx^3-12\,a^2ex^2-21\,a^2dx-32\,a^2}{384\,(a^2b^4x^{12}+3\,a^3b^3x^8+3\,a^4b^2x^4+a^5b)}$$
$$+\frac{\frac{\sqrt{2}\left(7\,\sqrt{b}d-5\,\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(7\,\sqrt{b}d-5\,\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(7\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}d+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f-16\,\sqrt{}\right.}{a^{\frac{3}{4}}\sqrt{}}}{1024\,a^2b}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/384*(15*b^2*f*x^11 + 12*b^2*e*x^10 + 7*b^2*d*x^9 + 42*a*b*f*x^7 + 32*a*b
*e*x^6 + 18*a*b*d*x^5 - 5*a^2*f*x^3 - 12*a^2*e*x^2 - 21*a^2*d*x - 32*a^2*c
)/(a^2*b^4*x^12 + 3*a^3*b^3*x^8 + 3*a^4*b^2*x^4 + a^5*b) + 1/1024*(sqrt(2)
*(7*sqrt(b)*d - 5*sqrt(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x +
 sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(7*sqrt(b)*d - 5*sqrt(a)*f)*log(sqrt
(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(7*sq
rt(2)*a^(1/4)*b^(3/4)*d + 5*sqrt(2)*a^(3/4)*b^(1/4)*f - 16*sqrt(a)*sqrt(b)
*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a
)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(7*sqrt(2)*a^(1/4)
*b^(3/4)*d + 5*sqrt(2)*a^(3/4)*b^(1/4)*f + 16*sqrt(a)*sqrt(b)*e)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(
a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^2*b)
```

3.494.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.99

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$$
$$= \frac{\sqrt{2}\left(8\sqrt{2}\sqrt{ab}b^2e+7\left(ab^3\right)^{\frac{1}{4}}b^2d+5\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3b^4}$$
$$+ \frac{\sqrt{2}\left(8\sqrt{2}\sqrt{ab}b^2e+7\left(ab^3\right)^{\frac{1}{4}}b^2d+5\left(ab^3\right)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3b^4}$$
$$+ \frac{\sqrt{2}\left(7\left(ab^3\right)^{\frac{1}{4}}b^2d-5\left(ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^3b^4}$$
$$- \frac{\sqrt{2}\left(7\left(ab^3\right)^{\frac{1}{4}}b^2d-5\left(ab^3\right)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^3b^4}$$
$$+ \frac{15\,b^2fx^{11}+12\,b^2ex^{10}+7\,b^2dx^9+42\,abfx^7+32\,abex^6+18\,abdx^5-5\,a^2fx^3-12\,a^2ex^2-21\,a^2dx-32}{384\left(bx^4+a\right)^3a^2b}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d + 5*(a*b^
3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^3*b^4) + 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d
 + 5*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a^3*b^4) + 1/1024*sqrt(2)*(7*(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4
)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) - 1/1024*sqrt(
2)*(7*(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a^3*b^4) + 1/384*(15*b^2*f*x^11 + 12*b^2*e*x^10 + 7*b^2
*d*x^9 + 42*a*b*f*x^7 + 32*a*b*e*x^6 + 18*a*b*d*x^5 - 5*a^2*f*x^3 - 12*a^2
*e*x^2 - 21*a^2*d*x - 32*a^2*c)/((b*x^4 + a)^3*a^2*b)
```

3.494.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 888, normalized size of antiderivative = 2.34

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$$
$$= \frac{\frac{3\,d\,x^5}{64\,a}-\frac{c}{12\,b}+\frac{e\,x^6}{12\,a}-\frac{e\,x^2}{32\,b}+\frac{7\,f\,x^7}{64\,a}-\frac{5\,f\,x^3}{384\,b}-\frac{7\,d\,x}{128\,b}+\frac{7\,b\,d\,x^9}{384\,a^2}+\frac{b\,e\,x^{10}}{32\,a^2}+\frac{5\,b\,f\,x^{11}}{128\,a^2}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\frac{125\,a\,f^3-448\,b\,d\,e^2+245\,b\,d^2\,f-512\,b\,e^3\,x+\mathrm{root}(68719476736\,a^{11}\,b^7\,z^4+36700160\,a^6\,b^4\right.\right.$$
$$+36700160\,a^6\,b^4\,d\,f\,z^2+33554432\,a^6\,b^4\,e^2\,z^2+409600\,a^4\,b^2\,e\,f^2\,z-802816\,a^3\,b^3\,d^2\,e\,z$$
$$\left.-8960\,a\,b\,d\,e^2\,f+2450\,a\,b\,d^2\,f^2+4096\,a\,b\,e^4+625\,a^2\,f^4+2401\,b^2\,d^4,z,k)\right)$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x)
```

output
```
((3*d*x^5)/(64*a) - c/(12*b) + (e*x^6)/(12*a) - (e*x^2)/(32*b) + (7*f*x^7)
/(64*a) - (5*f*x^3)/(384*b) - (7*d*x)/(128*b) + (7*b*d*x^9)/(384*a^2) + (b
*e*x^10)/(32*a^2) + (5*b*f*x^11)/(128*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4
+ 3*a*b^2*x^8) + symsum(log(-(125*a*f^3 - 448*b*d*e^2 + 245*b*d^2*f - 512*
b*e^3*x + 1835008*root(68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2
 + 33554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*
e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2
401*b^2*d^4, z, k)^2*a^5*b^4*d + 560*b*d*e*f*x + 25088*root(68719476736*a^
11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 + 409600*
a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2
*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4, z, k)*a^2*b^3*d^2*x - 20
97152*root(68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*
a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a
*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4,
 z, k)^2*a^5*b^4*e*x - 12800*root(68719476736*a^11*b^7*z^4 + 36700160*a^6*
b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a
^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*
a^2*f^4 + 2401*b^2*d^4, z, k)*a^3*b^2*f^2*x + 40960*root(68719476736*a^11*
b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 + 409600*a^4
*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2...
```

3.494. $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$

3.495 $\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

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3.495.1 Optimal result

Integrand size = 30, antiderivative size = 418

$$\begin{aligned}\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx &= \frac{2acx\sqrt{a+bx^4}}{21b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}-\frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}\\&+\frac{1}{63}x^5(9c+7ex^2)\sqrt{a+bx^4}+\frac{fx^4(a+bx^4)^{3/2}}{10b}-\frac{(8af-15bdx^2)(a+bx^4)^{3/2}}{120b^2}\\&-\frac{a^2d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}+\frac{2a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}\\&-\frac{a^{7/4}\left(5\sqrt{b}c+7\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/10*f*x^4*(b*x^4+a)^(3/2)/b-1/120*(-15*b*d*x^2+8*a*f)*(b*x^4+a)^(3/2)/b^2
-1/16*a^2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+2/21*a*c*x*(b*x^4
+a)^(1/2)/b-1/16*a*d*x^2*(b*x^4+a)^(1/2)/b+2/45*a*e*x^3*(b*x^4+a)^(1/2)/b+
1/63*x^5*(7*e*x^2+9*c)*(b*x^4+a)^(1/2)-2/15*a^2*e*x*(b*x^4+a)^(1/2)/b^(3/2
)/(a^(1/2)+x^2*b^(1/2))+2/15*a^(9/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/
2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)-1/105*a^(7/4)*(cos(2*arctan(b^(1/4)*x
/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arcta
n(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(7*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^2*
b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)
```

3.495.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.66 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.48

$$\int x^4\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$

$$=\frac{\sqrt{a+bx^4}\left(720bcx(a+bx^4)+560bex^3(a+bx^4)+315bdx^2(a+2bx^4)+168f(a+bx^4)\left(-2a+3bx^4\right)-\frac{31}{}\right.}{5040b^2}$$

input
```
Integrate[x^4*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(Sqrt[a + b*x^4]*(720*b*c*x*(a + b*x^4) + 560*b*e*x^3*(a + b*x^4) + 315*b*
d*x^2*(a + 2*b*x^4) + 168*f*(a + b*x^4)*(-2*a + 3*b*x^4) - (315*a^(3/2)*Sq
rt[b]*d*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a] - (720*a*b*c*x
*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] - (5
60*a*b*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*
x^4)/a]))/(5040*b^2)
```

3.495.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(x^4\sqrt{a+bx^4}\left(c+ex^2\right)+x^5\sqrt{a+bx^4}\left(d+fx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}-\frac{a^2d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2ex\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{af\left(a+bx^4\right)^{3/2}}{15b^2}+\frac{1}{63}x^5\sqrt{a+bx^4}\left(9c+7ex^2\right)+\frac{2acx\sqrt{a+bx^4}}{21b}+\frac{dx^2\left(a+bx^4\right)^{3/2}}{8b}-\frac{adx^2\sqrt{a+bx^4}}{16b}+\frac{2aex^3\sqrt{a+bx^4}}{45b}+\frac{fx^4\left(a+bx^4\right)^{3/2}}{10b}$$

input
```
Int[x^4*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(2*a*c*x*Sqrt[a + b*x^4])/(21*b) - (a*d*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a
*e*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*e*x*Sqrt[a + b*x^4])/(15*b^(3/2)*(
Sqrt[a] + Sqrt[b]*x^2)) + (x^5*(9*c + 7*e*x^2)*Sqrt[a + b*x^4])/63 - (a*f*
(a + b*x^4)^(3/2))/(15*b^2) + (d*x^2*(a + b*x^4)^(3/2))/(8*b) + (f*x^4*(a
+ b*x^4)^(3/2))/(10*b) - (a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(1
6*b^(3/2)) + (2*a^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(
7/4)*Sqrt[a + b*x^4]) - (a^(7/4)*(5*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(7/4)*Sqrt[a + b*x^4])
```

3.495.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] && !PolyQ[Pq, x^(n/2)]
```

3.495.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\left(-504b^2 f\,x^8-560b^2 e\,x^7-630b^2 d\,x^6-720b^2 c\,x^5-168abf\,x^4-224abe\,x^3-315x^2abd-480abcx+336a^2 f\right)\sqrt{b\,x^4+a}}{5040b^2}-\frac{a^2\left(\frac{80c\sqrt{1-i}}{}\right.}{}$
default	$-\frac{f\left(b\,x^4+a\right)^{\frac{3}{2}}\left(-3b\,x^4+2a\right)}{30b^2}+e\left(\frac{x^7\sqrt{b\,x^4+a}}{9}+\frac{2a\,x^3\sqrt{b\,x^4+a}}{45b}-\frac{2ia^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\right.\right.}{15b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right.$
elliptic	$\frac{f\,x^8\sqrt{b\,x^4+a}}{10}+\frac{e\,x^7\sqrt{b\,x^4+a}}{9}+\frac{d\,x^6\sqrt{b\,x^4+a}}{8}+\frac{c\,x^5\sqrt{b\,x^4+a}}{7}+\frac{af\,x^4\sqrt{b\,x^4+a}}{30b}+\frac{2ae\,x^3\sqrt{b\,x^4+a}}{45b}+\frac{ad\,x^2\sqrt{b\,x^4+a}}{16b}+2ac$

input
```
int(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5040*(-504*b^2*f*x^8-560*b^2*e*x^7-630*b^2*d*x^6-720*b^2*c*x^5-168*a*b*
f*x^4-224*a*b*e*x^3-315*a*b*d*x^2-480*a*b*c*x+336*a^2*f)/b^2*(b*x^4+a)^(1/
2)-1/840*a^2/b*(80*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)+112*I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)
*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2
)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I))+105/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

3.495.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.51

$$\int x^4\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx =$$

$$-\frac{1344\,a^2\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-315\,a^2\sqrt{b}dx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+192}{}$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/10080*(1344*a^2*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)
/x), -1) - 315*a^2*sqrt(b)*d*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^
2 - a) + 192*(5*a*b*c - 7*a^2*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin(
(-a/b)^(1/4)/x), -1) - 2*(504*b^2*f*x^9 + 560*b^2*e*x^8 + 630*b^2*d*x^7 +
720*b^2*c*x^6 + 168*a*b*f*x^5 + 224*a*b*e*x^4 + 315*a*b*d*x^3 + 480*a*b*c*
x^2 - 336*a^2*f*x - 672*a^2*e)*sqrt(b*x^4 + a))/(b^2*x)
```

3.495.6 Sympy [A] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.60

$$\int x^4\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$

$$=\frac{a^{\frac{3}{2}}dx^2}{16b\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}dx^6}{16\sqrt{1+\frac{bx^4}{a}}}$$
$$+\frac{\sqrt{a}ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}-\frac{a^2d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}$$
$$+f\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right)+\frac{bdx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

output
```
a**(3/2)*d*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**5*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a
)*d*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*e*x**7*gamma(7/4)*hyper((-1/2,
7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) - a**2*d*asinh(sq
rt(b)*x**2/sqrt(a))/(16*b**(3/2)) + f*Piecewise((-a**2*sqrt(a + b*x**4)/(1
5*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b,
 0)), (sqrt(a)*x**8/8, True)) + b*d*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/a))
```

3.495.7 Maxima [F]

$$\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int\sqrt{bx^4+a}(fx^3+ex^2+dx+c)x^4\,dx$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

3.495.8 Giac [F]

$$\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int\sqrt{bx^4+a}(fx^3+ex^2+dx+c)x^4\,dx$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

3.495.9 Mupad [F(-1)]

Timed out.

$$\int x^4(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int x^4\,\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^4*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x^4*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.496 $\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

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3.496.1 Optimal result

Integrand size = 30, antiderivative size = 394

$$\begin{aligned}
&\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx \\
&= \frac{2adx\sqrt{a+bx^4}}{21b} - \frac{aex^2\sqrt{a+bx^4}}{16b} + \frac{2afx^3\sqrt{a+bx^4}}{45b} - \frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} \\
&\quad + \frac{1}{63}x^5(9d+7fx^2)\sqrt{a+bx^4} + \frac{(4c+3ex^2)(a+bx^4)^{3/2}}{24b} - \frac{a^2e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}} \\
&\quad + \frac{2a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}} \\
&\quad - \frac{a^{7/4}\left(5\sqrt{b}d+7\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
1/24*(3*e*x^2+4*c)*(b*x^4+a)^(3/2)/b-1/16*a^2*e*arctanh(x^2*b^(1/2)/(b*x^4
+a)^(1/2))/b^(3/2)+2/21*a*d*x*(b*x^4+a)^(1/2)/b-1/16*a*e*x^2*(b*x^4+a)^(1/
2)/b+2/45*a*f*x^3*(b*x^4+a)^(1/2)/b+1/63*x^5*(7*f*x^2+9*d)*(b*x^4+a)^(1/2)
-2/15*a^2*f*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))+2/15*a^(9/4)*f
*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)
))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^
(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)-1
/105*a^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(7*
f*a^(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/
2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)
```

3.496.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.59 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.55

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$$

$$=\frac{\sqrt{a+bx^4}\left(168\sqrt{b}c(a+bx^4)+144\sqrt{b}dx(a+bx^4)+112\sqrt{b}fx^3(a+bx^4)+63e\left(\sqrt{b}x^2(a+2bx^4)-\frac{a^{3/2}\text{arc}}{\sqrt{}}\right.\right.}{1008b^{3/2}}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(Sqrt[a + b*x^4]*(168*Sqrt[b]*c*(a + b*x^4) + 144*Sqrt[b]*d*x*(a + b*x^4)
+ 112*Sqrt[b]*f*x^3*(a + b*x^4) + 63*e*(Sqrt[b]*x^2*(a + 2*b*x^4) - (a^(3/
2)*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]) - (144*a*Sqrt[b]*d
*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] -
(112*a*Sqrt[b]*f*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt
[1 + (b*x^4)/a]))/(1008*b^(3/2))
```

3.496.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(x^3\sqrt{a+bx^4}\left(c+ex^2\right)+x^4\sqrt{a+bx^4}\left(d+fx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}-\frac{a^2e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{c\left(a+bx^4\right)^{3/2}}{6b}+\frac{1}{63}x^5\sqrt{a+bx^4}\left(9d+7fx^2\right)+\frac{2adx\sqrt{a+bx^4}}{21b}+\frac{ex^2\left(a+bx^4\right)^{3/2}}{8b}-\frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{2afx^3\sqrt{a+bx^4}}{45b}$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(2*a*d*x*Sqrt[a + b*x^4])/(21*b) - (a*e*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a
*f*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*f*x*Sqrt[a + b*x^4])/(15*b^(3/2)*(
Sqrt[a] + Sqrt[b]*x^2)) + (x^5*(9*d + 7*f*x^2)*Sqrt[a + b*x^4])/63 + (c*(a
 + b*x^4)^(3/2))/(6*b) + (e*x^2*(a + b*x^4)^(3/2))/(8*b) - (a^2*e*ArcTanh[
(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) + (2*a^(9/4)*f*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^4]) - (a^(7/4)*(5*Sqrt
[b]*d + 7*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(7/4)
*Sqrt[a + b*x^4])
```

3.496.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] && !PolyQ[Pq, x^(n/2)]
```

3.496.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.68

method	result
risch	$\frac{(560bf\,x^7+630be\,x^6+720bd\,x^5+840bc\,x^4+224af\,x^3+315ae\,x^2+480adx+840ac)\sqrt{b\,x^4+a}}{5040b}-\frac{a^2\left(\frac{80d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right.}{}$
default	$f\left(\frac{x^7\sqrt{b\,x^4+a}}{9}+\frac{2a\,x^3\sqrt{b\,x^4+a}}{45b}-\frac{2ia^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{15b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+e\left(\frac{x^2(b\,x^4+a)^{\frac{3}{2}}}{8b}-\right.$
elliptic	$\frac{f\,x^7\sqrt{b\,x^4+a}}{9}+\frac{e\,x^6\sqrt{b\,x^4+a}}{8}+\frac{d\,x^5\sqrt{b\,x^4+a}}{7}+\frac{c\,x^4\sqrt{b\,x^4+a}}{6}+\frac{2af\,x^3\sqrt{b\,x^4+a}}{45b}+\frac{ae\,x^2\sqrt{b\,x^4+a}}{16b}+\frac{2adx\sqrt{b\,x^4+a}}{21b}+ac$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5040*(560*b*f*x^7+630*b*e*x^6+720*b*d*x^5+840*b*c*x^4+224*a*f*x^3+315*a*
e*x^2+480*a*d*x+840*a*c)/b*(b*x^4+a)^(1/2)-1/840*a^2/b*(80*d/(I/a^(1/2)*b^
(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/
2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+112*I*f*a^(1/2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))
^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+105/2*e*ln(x^2*b^(1/2)
+(b*x^4+a)^(1/2))/b^(1/2))
```

3.496.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.52

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx =$$

$$-\frac{1344\,a^2\sqrt{b}fx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-315\,a^2\sqrt{b}ex\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+192}{}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/10080*(1344*a^2*sqrt(b)*f*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)
/x), -1) - 315*a^2*sqrt(b)*e*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^
2 - a) + 192*(5*a*b*d - 7*a^2*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin(
(-a/b)^(1/4)/x), -1) - 2*(560*b^2*f*x^8 + 630*b^2*e*x^7 + 720*b^2*d*x^6 +
840*b^2*c*x^5 + 224*a*b*f*x^4 + 315*a*b*e*x^3 + 480*a*b*d*x^2 + 840*a*b*c*
x - 672*a^2*f)*sqrt(b*x^4 + a))/(b^2*x)
```

3.496.6 Sympy [A] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.54

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{a^{\frac{3}{2}}ex^2}{16b\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}ex^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} - \frac{a^2e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + c\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + \frac{bex^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

output
```
a**(3/2)*e*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*d*x**5*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a
)*e*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*f*x**7*gamma(7/4)*hyper((-1/2,
7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) - a**2*e*asinh(sq
rt(b)*x**2/sqrt(a))/(16*b**(3/2)) + c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0))
, ((a + b*x**4)**(3/2)/(6*b), True)) + b*e*x**10/(8*sqrt(a)*sqrt(1 + b*x**
4/a))
```

3.496.7 Maxima [F]

$$\int x^3 \left(c+dx+ex^2+fx^3\right) \sqrt{a+bx^4}\, dx = \int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^3\, dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^4 + a)^(3/2)*c/b + integrate((f*x^6 + e*x^5 + d*x^4)*sqrt(b*x^4 +
 a), x)
```

3.496.8 Giac [F]

$$\int x^3 \left(c+dx+ex^2+fx^3\right) \sqrt{a+bx^4}\, dx = \int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^3\, dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^3, x)
```

3.496.9 Mupad [F(-1)]

Timed out.

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int x^3\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^3*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x^3*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.497 $\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

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3.497.1 Optimal result

Integrand size = 30, antiderivative size = 369

$$\int x^2\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$
$$=\frac{2aex\sqrt{a+bx^4}}{21b}-\frac{afx^2\sqrt{a+bx^4}}{16b}+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$+\frac{1}{35}x^3\left(7c+5ex^2\right)\sqrt{a+bx^4}+\frac{\left(4d+3fx^2\right)\left(a+bx^4\right)^{3/2}}{24b}-\frac{a^2f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}$$
$$-\frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{a^{5/4}\left(21\sqrt{b}c-5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/24*(3*f*x^2+4*d)*(b*x^4+a)^(3/2)/b-1/16*a^2*f*arctanh(x^2*b^(1/2)/(b*x^4
+a)^(1/2))/b^(3/2)+2/21*a*e*x*(b*x^4+a)^(1/2)/b-1/16*a*f*x^2*(b*x^4+a)^(1/
2)/b+1/35*x^3*(5*e*x^2+7*c)*(b*x^4+a)^(1/2)+2/5*a*c*x*(b*x^4+a)^(1/2)/b^(1
/2)/(a^(1/2)+x^2*b^(1/2))-2/5*a^(5/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1
/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/105*a^(5/4)*(cos(2*arctan(b^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arct
an(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-5*e*a^(1/2)+21*c*b^(1/2))*(a^(1/2)+x
^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1
/2)
```

3.497.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.69 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.49

$$\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{1}{336}\sqrt{a+bx^4}\left(\frac{56d(a+bx^4)}{b}+\frac{48ex(a+bx^4)}{b}+\frac{21fx^2(a+2bx^4)}{b}-\frac{21a^{3/2}f\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{3/2}\sqrt{1+\frac{bx^4}{a}}}-\frac{48aex\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{b\sqrt{1+\frac{bx^4}{a}}}+\frac{112cx^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)$$

input
```
Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(Sqrt[a + b*x^4]*((56*d*(a + b*x^4))/b + (48*e*x*(a + b*x^4))/b + (21*f*x^
2*(a + 2*b*x^4))/b - (21*a^(3/2)*f*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(b^(3/2
)*Sqrt[1 + (b*x^4)/a]) - (48*a*e*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*
x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (112*c*x^3*Hypergeometric2F1[-1/2, 3/4
, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/336
```

3.497.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)\,dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(x^2\sqrt{a+bx^4}\left(c+ex^2\right)+x^3\sqrt{a+bx^4}\left(d+fx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}c-5\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}-\frac{a^2f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}+\frac{1}{35}x^3\sqrt{a+bx^4}\left(7c+5ex^2\right)+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\left(a+bx^4\right)^{3/2}}{6b}+\frac{2aex\sqrt{a+bx^4}}{21b}+\frac{fx^2\left(a+bx^4\right)^{3/2}}{8b}-\frac{afx^2\sqrt{a+bx^4}}{16b}$$

input
```
Int[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(2*a*e*x*Sqrt[a + b*x^4])/(21*b) - (a*f*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a
*c*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^3*(7*c + 5*
e*x^2)*Sqrt[a + b*x^4])/35 + (d*(a + b*x^4)^(3/2))/(6*b) + (f*x^2*(a + b*x
^4)^(3/2))/(8*b) - (a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3
/2)) - (2*a^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sq
rt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sq
rt[a + b*x^4]) + (a^(5/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(105*b^(5/4)*Sqrt[a + b*x^4])
```

3.497.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.497.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.71

method	result
risch	$\frac{(210bf\,x^6+240be\,x^5+280bd\,x^4+336bc\,x^3+105x^2af+160aex+280ad)\sqrt{b\,x^4+a}}{1680b} - \frac{a\left(\frac{80ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}} - 336\right.}{}$
default	$f\left(\frac{x^2(b\,x^4+a)^{\frac{3}{2}}}{8b}-\frac{a\,x^2\sqrt{b\,x^4+a}}{16b}-\frac{a^2\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{16b^{\frac{3}{2}}}\right)+e\left(\frac{x^5\sqrt{b\,x^4+a}}{7}+\frac{2ax\sqrt{b\,x^4+a}}{21b}-\frac{2a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+}}{21b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right.$
elliptic	$\frac{f\,x^6\sqrt{b\,x^4+a}}{8}+\frac{e\,x^5\sqrt{b\,x^4+a}}{7}+\frac{d\,x^4\sqrt{b\,x^4+a}}{6}+\frac{c\,x^3\sqrt{b\,x^4+a}}{5}+\frac{af\,x^2\sqrt{b\,x^4+a}}{16b}+\frac{2aex\sqrt{b\,x^4+a}}{21b}+\frac{ad\sqrt{b\,x^4+a}}{6b}-\frac{2a^2e\sqrt{1}}{}$

input
```
int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1680*(210*b*f*x^6+240*b*e*x^5+280*b*d*x^4+336*b*c*x^3+105*a*f*x^2+160*a*
e*x+280*a*d)/b*(b*x^4+a)^(1/2)-1/840*a/b*(80*a*e/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-336*I*b^(1/2)*c*a^(1/2)/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-El
lipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+105/2*a*f*ln(x^2*b^(1/2)+(b*x^4+a)
^(1/2))/b^(1/2))
```

3.497.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.52

$$\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$$
$$= \frac{1344\,ab^{\frac{3}{2}}cx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+105\,a^2\sqrt{b}fx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)-64\,(21\,a}{}$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3360*(1344*a*b^(3/2)*c*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x),
 -1) + 105*a^2*sqrt(b)*f*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 -
a) - 64*(21*a*b*c + 5*a*b*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/
b)^(1/4)/x), -1) + 2*(210*b^2*f*x^7 + 240*b^2*e*x^6 + 280*b^2*d*x^5 + 336*
b^2*c*x^4 + 105*a*b*f*x^3 + 160*a*b*e*x^2 + 280*a*b*d*x + 672*a*b*c)*sqrt(
b*x^4 + a))/(b^2*x)
```

3.497.6 Sympy [A] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.57

$$\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{a^{\frac{3}{2}}fx^2}{16b\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}fx^6}{16\sqrt{1+\frac{bx^4}{a}}} - \frac{a^2f\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + d\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + \frac{bfx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

output
```
a**(3/2)*f*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**3*gamma(3/4)*hype
r((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*
e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(9/4)) + 3*sqrt(a)*f*x**6/(16*sqrt(1 + b*x**4/a)) - a**2*f*asinh(sqrt
(b)*x**2/sqrt(a))/(16*b**(3/2)) + d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)),
((a + b*x**4)**(3/2)/(6*b), True)) + b*f*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/
a))
```

3.497.7 Maxima [F]

$$\int x^2\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^2\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

3.497.8 Giac [F]

$$\int x^2\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int\sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)x^2\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

3.497.9 Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int x^2\,\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^2*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x^2*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.498 $\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

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3.498.1 Optimal result

Integrand size = 28, antiderivative size = 354

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{2afx\sqrt{a+bx^4}}{21b}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{35}x^3(7d+5fx^2)\sqrt{a+bx^4}+\frac{e(a+bx^4)^{3/2}}{6b}+\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} - \frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}} + \frac{a^{5/4}\left(21\sqrt{b}d-5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/6*e*(b*x^4+a)^(3/2)/b+1/4*a*c*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/
2)+2/21*a*f*x*(b*x^4+a)^(1/2)/b+1/4*c*x^2*(b*x^4+a)^(1/2)+1/35*x^3*(5*f*x^
2+7*d)*(b*x^4+a)^(1/2)+2/5*a*d*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1
/2))-2/5*a^(5/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)
/(b*x^4+a)^(1/2)+1/105*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(-5*f*a^(1/2)+21*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a
)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

3.498.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.60

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$$
$$=\frac{\sqrt{a+bx^4}\left(14ae\sqrt{1+\frac{bx^4}{a}}+12afx\sqrt{1+\frac{bx^4}{a}}+21bcx^2\sqrt{1+\frac{bx^4}{a}}+14bex^4\sqrt{1+\frac{bx^4}{a}}+12bfx^5\sqrt{1+\frac{bx^4}{a}}+\right.}{}$$

input
```
Integrate[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(Sqrt[a + b*x^4]*(14*a*e*Sqrt[1 + (b*x^4)/a] + 12*a*f*x*Sqrt[1 + (b*x^4)/a
] + 21*b*c*x^2*Sqrt[1 + (b*x^4)/a] + 14*b*e*x^4*Sqrt[1 + (b*x^4)/a] + 12*b
*f*x^5*Sqrt[1 + (b*x^4)/a] + 21*Sqrt[a]*Sqrt[b]*c*ArcSinh[(Sqrt[b]*x^2)/Sq
rt[a]] - 12*a*f*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)] + 28*b*d
*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]))/(84*b*Sqrt[1 + (b*x
^4)/a])
```

3.498.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(x\sqrt{a+bx^4}\left(c+ex^2\right)+x^2\sqrt{a+bx^4}\left(d+fx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}d-5\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{ac\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{1}{35}x^3\sqrt{a+bx^4}\left(7d+5fx^2\right)+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\left(a+bx^4\right)^{3/2}}{6b}+\frac{2afx\sqrt{a+bx^4}}{21b}$$

input
```
Int[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(2*a*f*x*Sqrt[a + b*x^4])/(21*b) + (c*x^2*Sqrt[a + b*x^4])/4 + (2*a*d*x*Sq
rt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^3*(7*d + 5*f*x^2)*
Sqrt[a + b*x^4])/35 + (e*(a + b*x^4)^(3/2))/(6*b) + (a*c*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (2*a^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)*(21*Sqrt[b]*d - 5*
Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(5/4)*Sqrt[a +
b*x^4])
```

3.498.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.498.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.72

method	result
risch	$\frac{\left(60 b f \,x^{5}+70 b e \,x^{4}+84 b d \,x^{3}+105 c b \,x^{2}+40 a f x +70 a e \right) \sqrt{b \,x^{4}+a}}{420 b}-\frac{a \left(\frac{20 a f \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}-\frac{84 i \sqrt{b}\, d \sqrt{a}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}} \cdots}{\cdots} \cdots\right)}{\cdots}$
default	$f \left(\frac{x^{5} \sqrt{b \,x^{4}+a}}{7}+\frac{2 a x \sqrt{b \,x^{4}+a}}{21 b}-\frac{2 a^{2} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{21 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}\right)+\frac{e \left(b \,x^{4}+a \right)^{\frac{3}{2}}}{6 b}+d \left(\frac{x^{3} \sqrt{b \,x^{4}+a}}{5}+\frac{2 i a^{\frac{3}{2}} \cdots}{\cdots} \cdots\right)$
elliptic	$\frac{f \,x^{5} \sqrt{b \,x^{4}+a}}{7}+\frac{e \,x^{4} \sqrt{b \,x^{4}+a}}{6}+\frac{d \,x^{3} \sqrt{b \,x^{4}+a}}{5}+\frac{c \,x^{2} \sqrt{b \,x^{4}+a}}{4}+\frac{2 a f x \sqrt{b \,x^{4}+a}}{21 b}+\frac{a e \sqrt{b \,x^{4}+a}}{6 b}-\frac{2 a^{2} f \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b} \cdots}{\cdots}} \cdots}{21 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \cdots}} \cdots$

input
```
int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/420*(60*b*f*x^5+70*b*e*x^4+84*b*d*x^3+105*b*c*x^2+40*a*f*x+70*a*e)/b*(b*
x^4+a)^(1/2)-1/210*a/b*(20*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-84*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*b^(1/2))^(
1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2
)*b^(1/2))^(1/2),I))-105/2*b^(1/2)*c*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2)))
```

3.498.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.48

$$\int x(c + dx + ex^2 + fx^3)\sqrt{a + bx^4}\,dx$$
$$= \frac{336\,a\sqrt{b}dx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1) + 105\,a\sqrt{b}cx\log\left(-2\,bx^4 - 2\,\sqrt{bx^4+a}\sqrt{b}x^2 - a\right) - 16\,(21\,ad}{}$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/840*(336*a*sqrt(b)*d*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -
1) + 105*a*sqrt(b)*c*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) -
 16*(21*a*d + 5*a*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)
/x), -1) + 2*(60*b*f*x^6 + 70*b*e*x^5 + 84*b*d*x^4 + 105*b*c*x^3 + 40*a*f*
x^2 + 70*a*e*x + 168*a*d)*sqrt(b*x^4 + a))/(b*x)
```

3.498.6 Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.45

$$\int x(c + dx + ex^2 + fx^3)\sqrt{a + bx^4}\,dx = \frac{\sqrt{a}cx^2\sqrt{1 + \frac{bx^4}{a}}}{4} + \frac{\sqrt{a}dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$
$$+ \frac{\sqrt{a}fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$
$$+ \frac{ac\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} + e\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b = 0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*c*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*d*x**3*gamma(3/4)*hyper((-1/
2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*f*x**5
*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(
9/4)) + a*c*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + e*Piecewise((sqrt(a)
*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
```

3.498.7 Maxima [F]

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(a*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^
2))/sqrt(b) + 2*sqrt(b*x^4 + a)*a/((b - (b*x^4 + a)/x^4)*x^2))*c + integra
te(sqrt(b*x^4 + a)*(f*x^4 + e*x^3 + d*x^2), x)
```

3.498.8 Giac [F]

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x, x)
```

3.498.9 Mupad [F(-1)]

Timed out.

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int x\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.499 $\int (c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

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3.499.1 Optimal result

Integrand size = 27, antiderivative size = 331

$$\int (c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{15}x(5c+3ex^2)\sqrt{a+bx^4}+\frac{f(a+bx^4)^{3/2}}{6b}+\frac{ad\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}-\frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{a^{3/4}\left(5\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$

output
```
1/6*f*(b*x^4+a)^(3/2)/b+1/4*a*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/
2)+1/4*d*x^2*(b*x^4+a)^(1/2)+1/15*x*(3*e*x^2+5*c)*(b*x^4+a)^(1/2)+2/5*a*e*
x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-2/5*a^(5/4)*e*(cos(2*arcta
n(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(
sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^
4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/15*a^(3/4)*(
cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))
*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*e*a^(1/2)+5*c*
b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b
^(3/4)/(b*x^4+a)^(1/2)
```

3.499.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.52

$$\int \left(c+dx+ex^2+fx^3\right) \sqrt{a+bx^4}\, dx$$
$$= \frac{\sqrt{a+bx^4}\left(2af\sqrt{1+\frac{bx^4}{a}}+3bdx^2\sqrt{1+\frac{bx^4}{a}}+2bfx^4\sqrt{1+\frac{bx^4}{a}}+3\sqrt{a}\sqrt{b}d\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+12bcx\,\text{Hyperge}\right.}{12b\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(Sqrt[a + b*x^4]*(2*a*f*Sqrt[1 + (b*x^4)/a] + 3*b*d*x^2*Sqrt[1 + (b*x^4)/a
] + 2*b*f*x^4*Sqrt[1 + (b*x^4)/a] + 3*Sqrt[a]*Sqrt[b]*d*ArcSinh[(Sqrt[b]*x
^2)/Sqrt[a]] + 12*b*c*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)] +
4*b*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]))/(12*b*Sqrt[1 +
 (b*x^4)/a])
```

3.499.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right) dx$$
$$\Big\downarrow \text{ 2424}$$
$$\int \left(\sqrt{a+bx^4}\left(c+ex^2\right)+x\sqrt{a+bx^4}\left(d+fx^2\right)\right) dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{ad\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{1}{15}x\sqrt{a+bx^4}\left(5c+3ex^2\right)+\frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\left(a+bx^4\right)^{3/2}}{6b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(d*x^2*Sqrt[a + b*x^4])/4 + (2*a*e*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a]
+ Sqrt[b]*x^2)) + (x*(5*c + 3*e*x^2)*Sqrt[a + b*x^4])/15 + (f*(a + b*x^4)^
(3/2))/(6*b) + (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) -
(2*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a +
b*x^4]) + (a^(3/4)*(5*Sqrt[b]*c + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

3.499.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

3.499.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.73

method	result
risch	$\frac{\left(10 b f \,x^{4}+12 b e \,x^{3}+15 b d \,x^{2}+20 b c x +10 a f \right) \sqrt{b \,x^{4}+a}}{60 b}+\frac{a \left(\frac{20 c \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}+\frac{12 i e \sqrt{a}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, (}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b}}\right.}{30}$
default	$c \left(\frac{x \sqrt{b \,x^{4}+a}}{3}+\frac{2 a \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{3 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}\right)+\frac{f \left(b \,x^{4}+a \right)^{\frac{3}{2}}}{6 b}+e \left(\frac{x^{3} \sqrt{b \,x^{4}+a}}{5}+\frac{2 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}}{}}}{5 \sqrt{}}\right.$
elliptic	$\frac{f \,x^{4} \sqrt{b \,x^{4}+a}}{6}+\frac{e \,x^{3} \sqrt{b \,x^{4}+a}}{5}+\frac{d \,x^{2} \sqrt{b \,x^{4}+a}}{4}+\frac{c x \sqrt{b \,x^{4}+a}}{3}+\frac{a f \sqrt{b \,x^{4}+a}}{6 b}+\frac{2 a c \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{3 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}+$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(10*b*f*x^4+12*b*e*x^3+15*b*d*x^2+20*b*c*x+10*a*f)/b*(b*x^4+a)^(1/2)+
1/30*a*(20*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2
))^(1/2),I)+12*I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ellipt
icF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
))+15/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

3.499.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.49

$$\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$
$$=\frac{48\,a\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+15\,a\sqrt{b}dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+16\,(5\,bc-3}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/120*(48*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1
) + 15*a*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 1
6*(5*b*c - 3*a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x)
, -1) + 2*(10*b*f*x^5 + 12*b*e*x^4 + 15*b*d*x^3 + 20*b*c*x^2 + 10*a*f*x +
24*a*e)*sqrt(b*x^4 + a))/(b*x)
```

3.499.6 Sympy [A] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.47

$$\int (c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{\sqrt{a}cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}dx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{\sqrt{a}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ad\,\text{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} + f\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a
)/(4*gamma(5/4)) + sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*e*x**3*ga
mma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4
)) + a*d*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + f*Piecewise((sqrt(a)*x*
*4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
```

3.499.7 Maxima [F]

$$\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)
```

3.499.8 Giac [F]

$$\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}\left(fx^3+ex^2+dx+c\right)dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)
```

3.499.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx=\int \sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.500 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx$

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3.500.8 Giac [F] . . . 3915
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3.500.1 Optimal result

Integrand size = 30, antiderivative size = 345

$$\begin{aligned}\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx &= \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}(2c+ex^2)\sqrt{a+bx^4}+\frac{1}{15}x(5d+3fx^2)\sqrt{a+bx^4}\\
&+\frac{a\text{earctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}-\frac{1}{2}\sqrt{a}c\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\\
&-\frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}\\
&+\frac{a^{3/4}\left(5\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/4*a*e*arctanh(x^2*b^(1/2
)/(b*x^4+a)^(1/2))/b^(1/2)+1/4*(e*x^2+2*c)*(b*x^4+a)^(1/2)+1/15*x*(3*f*x^2
+5*d)*(b*x^4+a)^(1/2)+2/5*a*f*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/
2))-2/5*a^(5/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/
(b*x^4+a)^(1/2)+1/15*a^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),
1/2*2^(1/2))*(3*f*a^(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a
^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

3.500.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx$$
$$= \frac{3a^{3/2}e\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+3\sqrt{b}\left((2c+ex^2)\left(a+bx^4\right)-2\sqrt{a}c\sqrt{a+bx^4}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+12a\sqrt{b}}{12\sqrt{b}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]
```

output
```
(3*a^(3/2)*e*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*Sqrt[b
]*((2*c + e*x^2)*(a + b*x^4) - 2*Sqrt[a]*c*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a
+ b*x^4]/Sqrt[a]]) + 12*a*Sqrt[b]*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F
1[-1/2, 1/4, 5/4, -((b*x^4)/a)] + 4*a*Sqrt[b]*f*x^3*Sqrt[1 + (b*x^4)/a]*Hy
pergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/(12*Sqrt[b]*Sqrt[a + b*x^4]
)
```

3.500.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)}{x}dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(\frac{\sqrt{a+bx^4}\left(c+ex^2\right)}{x}+\sqrt{a+bx^4}\left(d+fx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}-\frac{1}{2}\sqrt{a}c\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{ae\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{1}{4}\sqrt{a+bx^4}\left(2c+ex^2\right)+\frac{1}{15}x\sqrt{a+bx^4}\left(5d+3fx^2\right)+\frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]
```

output
```
(2*a*f*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*
x^2)*Sqrt[a + b*x^4])/4 + (x*(5*d + 3*f*x^2)*Sqrt[a + b*x^4])/15 + (a*e*Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*c*ArcTanh[Sqr
t[a + b*x^4]/Sqrt[a]])/2 - (2*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(3/4)*(5*Sqrt[b]*d + 3*Sqrt[a]*f)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

3.500.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.500.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.79

method	result
elliptic	$\frac{f\,x^{3}\sqrt{b\,x^{4}+a}}{5}+\frac{e\,x^{2}\sqrt{b\,x^{4}+a}}{4}+\frac{dx\sqrt{b\,x^{4}+a}}{3}+\frac{c\sqrt{b\,x^{4}+a}}{2}+\frac{2ad\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{ae\ln\left(2x^{2}\sqrt{b}+2\sqrt{b}\right.}{4\sqrt{b}}$
default	$d\left(\frac{x\sqrt{b\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+f\left(\frac{x^{3}\sqrt{b\,x^{4}+a}}{5}+\frac{2ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/5*f*x^3*(b*x^4+a)^(1/2)+1/4*e*x^2*(b*x^4+a)^(1/2)+1/3*d*x*(b*x^4+a)^(1/2
)+1/2*c*(b*x^4+a)^(1/2)+2/3*a*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*a*e*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))
/b^(1/2)+2/5*I*a^(3/2)*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
-1/2*a^(1/2)*c*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

3.500.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

3.500.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.59

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx = -\frac{\sqrt{a}c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{\sqrt{a}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}ex^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{\sqrt{a}fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ac}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{ae\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} + \frac{\sqrt{b}cx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x,x)
```

output
```
-sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*d*x*gamma(1/4)*hyper(
(-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*e*
x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (
7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a*c/(2*sqrt(b)*x**2*sqrt
(a/(b*x**4) + 1)) + a*e*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*
c*x**2/(2*sqrt(a/(b*x**4) + 1))
```

3.500.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

3.500.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

3.500.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx=\int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x, x)
```

3.501 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx$

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3.501.1 Optimal result

Integrand size = 30, antiderivative size = 341

$$\begin{aligned}
\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx
&= \frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{(3c-ex^2)\sqrt{a+bx^4}}{3x} + \frac{1}{4}(2d+fx^2)\sqrt{a+bx^4} \\
&\quad + \frac{af\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} - \frac{1}{2}\sqrt{a}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) \\
&\quad - \frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} \\
&\quad + \frac{\sqrt[4]{a}\left(3\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
-1/2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/4*a*f*arctanh(x^2*b^(1/2
)/(b*x^4+a)^(1/2))/b^(1/2)-1/3*(-e*x^2+3*c)*(b*x^4+a)^(1/2)/x+1/4*(f*x^2+2
*d)*(b*x^4+a)^(1/2)+2*c*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-2*
a^(1/4)*b^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a
)^(1/2)+1/3*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(e*a^(1/2)+3*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*
b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^4+a)^(1/2)
```

3.501.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.36 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \frac{-4\sqrt{b}c\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)+x\left(\sqrt{a}f\sqrt{a+bx^4}\,\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+\sqrt{b}\sqrt{1+\frac{bx}{a}}\right.}{4\sqrt{b}x}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]
```

output
```
(-4*Sqrt[b]*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^4)
/a)] + x*(Sqrt[a]*f*Sqrt[a + b*x^4]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + Sqrt[
b]*Sqrt[1 + (b*x^4)/a]*((2*d + f*x^2)*Sqrt[a + b*x^4] - 2*Sqrt[a]*d*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]]) + 4*Sqrt[b]*e*x*Sqrt[a + b*x^4]*Hypergeometric
2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)]))/(4*Sqrt[b]*x*Sqrt[1 + (b*x^4)/a])
```

3.501.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)}{x^2}dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(\frac{\sqrt{a+bx^4}\left(c+ex^2\right)}{x^2}+\frac{\sqrt{a+bx^4}\left(d+fx^2\right)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{1}{2}\sqrt{a}d\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{af\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}-\frac{\sqrt{a+bx^4}\left(3c-ex^2\right)}{3x}+\frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}+\frac{1}{4}\sqrt{a+bx^4}\left(2d+fx^2\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]
```

output
```
(2*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((3*c - e*x^2)*S
qrt[a + b*x^4])/(3*x) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/4 + (a*f*ArcTanh[(
Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*d*ArcTanh[Sqrt[a + b
*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

3.501.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.501.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.80

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{x}+\frac{f\,x^2\sqrt{b\,x^4+a}}{4}+\frac{e x\sqrt{b\,x^4+a}}{3}+\frac{d\sqrt{b\,x^4+a}}{2}+\frac{2ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{af\ln\left(2x^2\sqrt{b}+2\sqrt{b}\right.}{4\sqrt{b}}$
risch	$-\frac{c\sqrt{b\,x^4+a}}{x}+\frac{2ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{f\,x^2\sqrt{b\,x^4+a}}{4}+\frac{af\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{4\sqrt{b}}+\frac{ex\sqrt{b\,x^4+a}}{3}+\frac{d\sqrt{b}}{}$
default	$e\left(\frac{x\sqrt{b\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(\frac{x^2\sqrt{b\,x^4+a}}{4}+\frac{a\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{4\sqrt{b}}\right)+d\left(\frac{\sqrt{b\,x^4+a}}{2}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-c*(b*x^4+a)^(1/2)/x+1/4*f*x^2*(b*x^4+a)^(1/2)+1/3*e*x*(b*x^4+a)^(1/2)+1/2
*d*(b*x^4+a)^(1/2)+2/3*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)+1/4*a*f*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))/b^(1
/2)+2*I*b^(1/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^
(1/2)*d*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

3.501.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

3.501.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.92 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{a}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{\sqrt{a}ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}fx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{ad}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{af\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} + \frac{\sqrt{b}dx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**2,x)
```

output
```
sqrt(a)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x*gamma(3/4)) - sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*e
*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamm
a(5/4)) + sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/4 + a*d/(2*sqrt(b)*x**2*sqrt(a
/(b*x**4) + 1)) + a*f*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*d*
x**2/(2*sqrt(a/(b*x**4) + 1))
```

3.501.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

3.501.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

3.501.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^2}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^2,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^2, x)
```

3.502 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx$

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3.502.1 Optimal result

Integrand size = 30, antiderivative size = 342

$$\begin{aligned}&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx\\&=\frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2}-\frac{(3d-fx^2)\sqrt{a+bx^4}}{3x}\\&\quad+\frac{1}{2}\sqrt{b}c\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\\&\quad-\frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}\\&\quad+\frac{\sqrt[4]{a}\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/2*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*c*arctanh(x^2*b^(1/2)/
(b*x^4+a)^(1/2))*b^(1/2)-1/2*(-e*x^2+c)*(b*x^4+a)^(1/2)/x^2-1/3*(-f*x^2+3*
d)*(b*x^4+a)^(1/2)/x+2*d*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-2
*a^(1/4)*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+
a)^(1/2)+1/3*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1
/2))*(f*a^(1/2)+3*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2
*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^4+a)^(1/2)
```

3.502.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx$$
$$= \frac{-ac+aex^2-bcx^4+bex^6+\sqrt{a}\sqrt{b}cx^2\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)-\sqrt{a}ex^2\sqrt{a+bx^4}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-2a\cdots}{2x^2\sqrt{\cdots}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]
```

output
```
(-(a*c) + a*e*x^2 - b*c*x^4 + b*e*x^6 + Sqrt[a]*Sqrt[b]*c*x^2*Sqrt[1 + (b*
x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - Sqrt[a]*e*x^2*Sqrt[a + b*x^4]*Arc
Tanh[Sqrt[a + b*x^4]/Sqrt[a]] - 2*a*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric
2F1[-1/2, -1/4, 3/4, -((b*x^4)/a)] + 2*a*f*x^3*Sqrt[1 + (b*x^4)/a]*Hyperge
ometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/(2*x^2*Sqrt[a + b*x^4])
```

3.502.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}(c+dx+ex^2+fx^3)}{x^3}\,dx$$

$$\Big\downarrow \text{ 2372}$$

$$\int \left(\frac{\sqrt{a+bx^4}(c+ex^2)}{x^3}+\frac{\sqrt{a+bx^4}(d+fx^2)}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}+\frac{1}{2}\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}\sqrt{a}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{\sqrt{a+bx^4}(c-ex^2)}{2x^2}-\frac{\sqrt{a+bx^4}(3d-fx^2)}{3x}+\frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]
```

output
```
(2*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((c - e*x^2)*Sqr
t[a + b*x^4])/(2*x^2) - ((3*d - f*x^2)*Sqrt[a + b*x^4])/(3*x) + (Sqrt[b]*c
*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (Sqrt[a]*e*ArcTanh[Sqrt[a + b
*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

3.502.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.502. $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx$

3.502.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(2dx+c)}{2x^{2}}+\frac{2af\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{fx\sqrt{b\,x^{4}+a}}{3}+\frac{e\sqrt{b\,x^{4}+a}}{2}+\frac{\sqrt{b}\,c\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2}-$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{2x^{2}}-\frac{d\sqrt{b\,x^{4}+a}}{x}+\frac{fx\sqrt{b\,x^{4}+a}}{3}+\frac{e\sqrt{b\,x^{4}+a}}{2}+\frac{2af\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{\sqrt{b}\,c\ln\left(2x^{2}\sqrt{b}+2\sqrt{b}\right.}{2}$
default	$f\left(\frac{x\sqrt{b\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+e\left(\frac{\sqrt{b\,x^{4}+a}}{2}-\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{2}\right)+d\left(-\frac{\sqrt{b\,x}}{x}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^4+a)^(1/2)*(2*d*x+c)/x^2+2/3*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/
a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)
*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/3*f*x*(b*x^4+a)^(1/2)+1/2*e*(b
*x^4+a)^(1/2)+1/2*b^(1/2)*c*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/2*a^(1/2)*e*
ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+2*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*
(I/a^(1/2)*b^(1/2))^(1/2),I))
```

3.502.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

3.502.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx = -\frac{\sqrt{a}c}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{a}e\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{\sqrt{a}fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{ae}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{\sqrt{b}c\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} + \frac{\sqrt{b}ex^2}{2\sqrt{\frac{a}{bx^4}+1}} - \frac{bcx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**3,x)
```

output
```
-sqrt(a)*c/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*d*gamma(-1/4)*hyper((-1/2
, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - sqrt(a)*e*as
inh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*f*x*gamma(1/4)*hyper((-1/2, 1/4),
(5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a*e/(2*sqrt(b)*x**2*sqr
t(a/(b*x**4) + 1)) + sqrt(b)*c*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)*e*x
**2/(2*sqrt(a/(b*x**4) + 1)) - b*c*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

3.502.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

3.502.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

3.502.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^3}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^3,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^3, x)
```

3.503 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx$

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3.503.1 Optimal result

Integrand size = 30, antiderivative size = 357

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx \\
&= -\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3}-\frac{(d-fx^2)\sqrt{a+bx^4}}{2x^2} \\
&\quad+\frac{1}{2}\sqrt{b}d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}\sqrt{a}f\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) \\
&\quad-\frac{2\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} \\
&\quad+\frac{\sqrt[4]{b}\left(\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
-1/2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*d*arctanh(x^2*b^(1/2)/
(b*x^4+a)^(1/2))*b^(1/2)-2*e*(b*x^4+a)^(1/2)/x-1/3*(-3*e*x^2+c)*(b*x^4+a)^
(1/2)/x^3-1/2*(-f*x^2+d)*(b*x^4+a)^(1/2)/x^2+2*e*x*b^(1/2)*(b*x^4+a)^(1/2)
/(a^(1/2)+x^2*b^(1/2))-2*a^(1/4)*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b
^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+1/3*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1
/4)*x/a^(1/4))),1/2*2^(1/2))*(3*e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))
*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^4+a)^(1/2)
```

3.503.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.57

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx$$
$$= \frac{-2ac\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{bx^4}{a}\right)+3x\Big(-ad+afx^2-bdx^4+bfx^6+\sqrt{a}\sqrt{b}dx^2\sqrt{1}}{6}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]
```

output
```
(-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^4)/a
)] + 3*x*(-(a*d) + a*f*x^2 - b*d*x^4 + b*f*x^6 + Sqrt[a]*Sqrt[b]*d*x^2*Sqr
t[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - Sqrt[a]*f*x^2*Sqrt[a + b
*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]] - 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hyper
geometric2F1[-1/2, -1/4, 3/4, -((b*x^4)/a)]))/(6*x^3*Sqrt[a + b*x^4])
```

3.503.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}(c+dx+ex^2+fx^3)}{x^4}\,dx$$

$$\Big\downarrow \; 2372$$

$$\int \left(\frac{\sqrt{a+bx^4}(c+ex^2)}{x^4}+\frac{\sqrt{a+bx^4}(d+fx^2)}{x^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}+\frac{1}{2}\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}\sqrt{a}f\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{\sqrt{a+bx^4}(c-3ex^2)}{3x^3}-\frac{\sqrt{a+bx^4}(d-fx^2)}{2x^2}-\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]
```

output
```
(-2*e*Sqrt[a + b*x^4])/x + (2*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt
[b]*x^2) - ((c - 3*e*x^2)*Sqrt[a + b*x^4])/(3*x^3) - ((d - f*x^2)*Sqrt[a +
 b*x^4])/(2*x^2) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 -
(Sqrt[a]*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*e*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*c
 + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a
 + b*x^4])
```

3.503. $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx$

3.503.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.503.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\sqrt{b x^4+a}\,(6e x^2+3dx+2c)}{6x^3}+\frac{2bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+\frac{f\sqrt{b x^4+a}}{2}+\frac{2i\sqrt{b}\,e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\Big(}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}}$
elliptic	$-\frac{c\sqrt{b x^4+a}}{3x^3}-\frac{\sqrt{b x^4+a}\,d}{2x^2}-\frac{e\sqrt{b x^4+a}}{x}+\frac{f\sqrt{b x^4+a}}{2}+\frac{2bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+\frac{\sqrt{b}\,d\ln\left(2x^2\sqrt{b}+2\sqrt{b x}\right.}{2}$
default	$f\left(\frac{\sqrt{b x^4+a}}{2}-\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^4+a}}{x^2}\right)}{2}\right)+c\left(-\frac{\sqrt{b x^4+a}}{3x^3}+\frac{2b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}\right)+e\left(-\sqrt{b}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^4+a)^(1/2)*(6*e*x^2+3*d*x+2*c)/x^3+2/3*b*c/(I/a^(1/2)*b^(1/2))^(
1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*f*(b*x^4+a)^(1/2)+
2*I*b^(1/2)*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1
/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*b^(1/2
)*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))-1/2*a^(1/2)*f*ln((2*a+2*a^(1/2)*(b*x^4
+a)^(1/2))/x^2)
```

3.503.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

3.503.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.90 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx=\frac{\sqrt{a}c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}-\frac{\sqrt{a}d}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt{a}f\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{af}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{\sqrt{b}d\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2}+\frac{\sqrt{b}fx^2}{2\sqrt{\frac{a}{bx^4}+1}}-\frac{bdx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**4,x)
```

output
```
sqrt(a)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**3*gamma(1/4)) - sqrt(a)*d/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*e*
gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gam
ma(3/4)) - sqrt(a)*f*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a*f/(2*sqrt(b)*x**2
*sqrt(a/(b*x**4) + 1)) + sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)
*f*x**2/(2*sqrt(a/(b*x**4) + 1)) - b*d*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

3.503.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

3.503.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

3.503.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx=\int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^4}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^4,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^4, x)
```

3.504 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx$

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3.504.1 Optimal result

Integrand size = 30, antiderivative size = 329

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = -\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)\sqrt{a+bx^4}+\frac{2\sqrt{b}fx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}+\frac{1}{2}\sqrt{b}e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)$$
$$-\frac{bc\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}$$

output
```
-1/4*b*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)+1/2*e*arctanh(x^2*b^(1/2
)/(b*x^4+a)^(1/2))*b^(1/2)-1/12*(3*c/x^4+4*d/x^3+6*e/x^2+12*f/x)*(b*x^4+a)
^(1/2)+2*f*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-2*a^(1/4)*b^(1/
4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(
1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^
2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+1/3*b
^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a
^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*f*a^(1
/2)+d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(
1/2)/a^(1/4)/(b*x^4+a)^(1/2)
```

3.504.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.53

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx =$$
$$-\frac{\sqrt{1+\frac{bx^4}{a}}\left(3ac\sqrt{1+\frac{bx^4}{a}}+6aex^2\sqrt{1+\frac{bx^4}{a}}-6\sqrt{a}\sqrt{b}ex^4\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+3bcx^4\operatorname{arctanh}\left(\sqrt{1+\frac{bx^4}{a}}\right)+\right.}{12x^4\sqrt{a+}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]
```

output
```
-1/12*(Sqrt[1 + (b*x^4)/a]*(3*a*c*Sqrt[1 + (b*x^4)/a] + 6*a*e*x^2*Sqrt[1 +
 (b*x^4)/a] - 6*Sqrt[a]*Sqrt[b]*e*x^4*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*b
*c*x^4*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*d*x*Hypergeometric2F1[-3/4, -1/2
, 1/4, -((b*x^4)/a)] + 12*a*f*x^3*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*
x^4)/a)]))/(x^4*Sqrt[a + b*x^4])
```

3.504.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2364, 27, 2371, 798, 73, 221, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}(c+dx+ex^2+fx^3)}{x^5}\,dx$$
$$\Big\downarrow \text{2364}$$
$$-2b\int -\frac{12fx^3+6ex^2+4dx+3c}{12x\sqrt{bx^4+a}}dx-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{6}b\int \frac{12fx^3+6ex^2+4dx+3c}{x\sqrt{bx^4+a}}dx-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$
$$\Big\downarrow \text{2371}$$

$$\frac{1}{6}b\left(3c\int \frac{1}{x\sqrt{bx^4+a}}dx+\int \frac{12fx^2+6ex+4d}{\sqrt{bx^4+a}}dx\right)-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$\big\downarrow$ 798

$$\frac{1}{6}b\left(\frac{3}{4}c\int \frac{1}{x^4\sqrt{bx^4+a}}dx^4+\int \frac{12fx^2+6ex+4d}{\sqrt{bx^4+a}}dx\right)-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$\big\downarrow$ 73

$$\frac{1}{6}b\left(\frac{3c\int \frac{1}{\frac{x^8}{b}-\frac{a}{b}}d\sqrt{bx^4+a}}{2b}+\int \frac{12fx^2+6ex+4d}{\sqrt{bx^4+a}}dx\right)-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$\big\downarrow$ 221

$$\frac{1}{6}b\left(\int \frac{12fx^2+6ex+4d}{\sqrt{bx^4+a}}dx-\frac{3c\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$\big\downarrow$ 2424

$$\frac{1}{6}b\left(\int \left(\frac{6ex}{\sqrt{bx^4+a}}+\frac{12fx^2+4d}{\sqrt{bx^4+a}}\right)dx-\frac{3c\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$\big\downarrow$ 2009

$$\frac{1}{6}b\left(\frac{2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}-\frac{12\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{}{(\sqrt{}}}}{b^{3/4}}\right.$$
$$\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]
```

3.504. $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx$

output
```
-1/12*(((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*Sqrt[a + b*x^4]) + (
b*((12*f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (3*e*ArcTa
nh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/Sqrt[b] - (3*c*ArcTanh[Sqrt[a + b*x^4]/
Sqrt[a]])/(2*Sqrt[a]) - (12*a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (2*(Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*b^(3/4)*Sqrt[a + b*x^4])))/6
```

3.504.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

3.504. $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx$

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.504.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{b x^4+a}\,(12 f x^3+6 e x^2+4 d x+3 c)}{12 x^4}+\frac{b\left(\frac{4 d \sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+\frac{3 e\ln\left(x^2\sqrt{b}+\sqrt{b x^4+a}\right)}{\sqrt{b}}-\frac{3 c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^4+a}}{x^2}\right)}{2\sqrt{a}}\right.}{6}$
elliptic	$-\frac{c\sqrt{b x^4+a}}{4 x^4}-\frac{d\sqrt{b x^4+a}}{3 x^3}-\frac{e\sqrt{b x^4+a}}{2 x^2}-\frac{f\sqrt{b x^4+a}}{x}+\frac{2 b d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+\frac{\sqrt{b}\,e\ln\left(2x^2\sqrt{b}+2\sqrt{b x^4+a}\right)}{2}$
default	$d\left(-\frac{\sqrt{b x^4+a}}{3 x^3}+\frac{2 b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}\right)+f\left(-\frac{\sqrt{b x^4+a}}{x}+\frac{2 i\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x^4+a)^(1/2)*(12*f*x^3+6*e*x^2+4*d*x+3*c)/x^4+1/6*b*(4*d/(I/a^(1/
2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3*e*ln(x^
2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)-3/2*c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+
a)^(1/2))/x^2)+12*I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)))
```

3.504.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

3.504.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.64

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}e}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}f\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{\sqrt{b}e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} - \frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}} - \frac{bex^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**5,x)
```

output
```
sqrt(a)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**3*gamma(1/4)) - sqrt(a)*e/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*f*
gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gam
ma(3/4)) - sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*x**2) + sqrt(b)*e*asinh(sqrt(
b)*x**2/sqrt(a))/2 - b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) - b*e*x
**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

3.504.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
1/8*(b*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/sqrt(a
) - 2*sqrt(b*x^4 + a)/x^4)*c + integrate(sqrt(b*x^4 + a)*(f*x^2 + e*x + d)
/x^4, x)
```

3.504.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

3.504.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^5}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^5,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^5, x)
```

3.505 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx$

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3.505.1 Optimal result

Integrand size = 30, antiderivative size = 360

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx \\
&= -\frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{5ax} \\
&\quad+\frac{2b^{3/2}cx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{2}\sqrt{b}f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}} \\
&\quad-\frac{2b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} \\
&\quad+\frac{b^{3/4}\left(3\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
-1/4*b*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)+1/2*f*arctanh(x^2*b^(1/2
)/(b*x^4+a)^(1/2))*b^(1/2)-1/60*(12*c/x^5+15*d/x^4+20*e/x^3+30*f/x^2)*(b*x
^4+a)^(1/2)-2/5*b*c*(b*x^4+a)^(1/2)/a/x+2/5*b^(3/2)*c*x*(b*x^4+a)^(1/2)/a/
(a^(1/2)+x^2*b^(1/2))-2/5*b^(5/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))
^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)+1/15*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^
(1/4)*x/a^(1/4))),1/2*2^(1/2))*(5*e*a^(1/2)+3*c*b^(1/2))*(a^(1/2)+x^2*b^(1
/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
```

3.505.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.50

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\left(12ac\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{bx^4}{a}\right)+5x\left(3ad\sqrt{1+\frac{bx^4}{a}}+6afx^2\sqrt{1+\frac{bx^4}{a}}-6\sqrt{}\right.\right.}{60ax^5\sqrt{}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]
```

output
```
-1/60*(Sqrt[a + b*x^4]*(12*a*c*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^
4)/a)] + 5*x*(3*a*d*Sqrt[1 + (b*x^4)/a] + 6*a*f*x^2*Sqrt[1 + (b*x^4)/a] -
6*Sqrt[a]*Sqrt[b]*f*x^4*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*b*d*x^4*ArcTanh
[Sqrt[1 + (b*x^4)/a]] + 4*a*e*x*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^
4)/a)])))/(a*x^5*Sqrt[1 + (b*x^4)/a])
```

3.505.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}(c+dx+ex^2+fx^3)}{x^6}\,dx$$

$$\Big\downarrow \text{2364}$$

$$-2b\int -\frac{30fx^3+20ex^2+15dx+12c}{60x^2\sqrt{bx^4+a}}dx-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{30}b\int \frac{30fx^3+20ex^2+15dx+12c}{x^2\sqrt{bx^4+a}}dx-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

$$\Big\downarrow \text{2372}$$

$$\frac{1}{30}b\int \left(\frac{20ex^2+12c}{x^2\sqrt{bx^4+a}}+\frac{30fx^2+15d}{x\sqrt{bx^4+a}}\right)dx-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{30}b\left(\frac{2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+3\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{(}}{a^3}\right.$$
$$\frac{1}{60}\sqrt{a+bx^4}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]
```

output
```
-1/60*(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*Sqrt[a + b*x^4]
) + (b*((-12*c*Sqrt[a + b*x^4])/(a*x) + (12*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(
a*(Sqrt[a] + Sqrt[b]*x^2)) + (15*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])
/Sqrt[b] - (15*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (12*b^(1/
4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) +
(2*(3*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(S
qrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a
^(3/4)*b^(1/4)*Sqrt[a + b*x^4])))/30
```

3.505.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.505. $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx$

3.505.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(24bc\,x^4+30af\,x^3+20ae\,x^2+15adx+12ac)}{60x^5a}+\frac{b\left(\frac{20ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{12i\sqrt{b}\,c\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\cdots}}{\sqrt{\cdots}}\right.}{\cdots}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{5x^5}-\frac{d\sqrt{b\,x^4+a}}{4x^4}-\frac{e\sqrt{b\,x^4+a}}{3x^3}-\frac{f\sqrt{b\,x^4+a}}{2x^2}-\frac{2bc\sqrt{b\,x^4+a}}{5ax}+\frac{2be\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{\sqrt{b}\,f\,\mathrm{l}\cdots}{\cdots}$
default	$e\left(-\frac{\sqrt{b\,x^4+a}}{3x^3}+\frac{2b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(-\frac{(b\,x^4+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\,x^2\sqrt{b\,x^4+a}}{2a}+\frac{\sqrt{b}\,\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+\cdots}\right.}{2}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(b*x^4+a)^(1/2)*(24*b*c*x^4+30*a*f*x^3+20*a*e*x^2+15*a*d*x+12*a*c)/x
^5/a+1/30*b/a*(20*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/
2)*b^(1/2))^(1/2),I)+12*I*b^(1/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2
)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I))+15*a*f*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)-15/2*a^(1/2)*d*
ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

3.505.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

3.505.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.05 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\ -\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}f}{2x^2\sqrt{1+\frac{bx^4}{a}}} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{\sqrt{b}f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} - \frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}} - \frac{bfx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**6,x)
```

output
```
sqrt(a)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**5*gamma(-1/4)) + sqrt(a)*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*f/(2*x**2*sqrt(1
 + b*x**4/a)) - sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*x**2) + sqrt(b)*f*asinh(
sqrt(b)*x**2/sqrt(a))/2 - b*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) -
b*f*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

3.505.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

3.505.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

3.505.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx=\int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^6}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^6,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^6, x)
```

3.506 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx$

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3.506.1 Optimal result

Integrand size = 30, antiderivative size = 352

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx \\
&= -\frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{6ax^2} \\
&\quad -\frac{2bd\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}dx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{be\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}} \\
&\quad -\frac{2b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} \\
&\quad +\frac{b^{3/4}\left(3\sqrt{b}d+5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
-1/4*b*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/60*(10*c/x^6+12*d/x^5+
15*e/x^4+20*f/x^3)*(b*x^4+a)^(1/2)-1/6*b*c*(b*x^4+a)^(1/2)/a/x^2-2/5*b*d*(
b*x^4+a)^(1/2)/a/x+2/5*b^(3/2)*d*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))
-2/5*b^(5/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(
a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*
x^4+a)^(1/2)+1/15*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2
*2^(1/2))*(5*f*a^(1/2)+3*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1
/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
```

3.506.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.41

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx =$$
$$-\frac{\sqrt{a+bx^4}\left(12adx\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{bx^4}{a}\right)+5\left(\sqrt{1+\frac{bx^4}{a}}(2ac+3aex^2+2bcx^4)+3be\right.\right.}{60ax^6\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]
```

output
```
-1/60*(Sqrt[a + b*x^4]*(12*a*d*x*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*
x^4)/a)] + 5*(Sqrt[1 + (b*x^4)/a]*(2*a*c + 3*a*e*x^2 + 2*b*c*x^4) + 3*b*e*
x^6*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*f*x^3*Hypergeometric2F1[-3/4, -1/2,
 1/4, -((b*x^4)/a)])))/(a*x^6*Sqrt[1 + (b*x^4)/a])
```

3.506.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)}{x^7}\,dx$$

$$\Big\downarrow \text{ 2364}$$

$$-2b\int -\frac{20fx^3+15ex^2+12dx+10c}{60x^3\sqrt{bx^4+a}}dx-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{30}b\int \frac{20fx^3+15ex^2+12dx+10c}{x^3\sqrt{bx^4+a}}dx-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

$$\Big\downarrow \text{ 2372}$$

$$\frac{1}{30}b\int \left(\frac{15ex^2+10c}{x^3\sqrt{bx^4+a}}+\frac{20fx^2+12d}{x^2\sqrt{bx^4+a}}\right)dx-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{30}b\left(\frac{2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+3\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{(}}{a}\right.$$

$$\frac{1}{60}\sqrt{a+bx^4}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]
```

output
```
-1/60*(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*Sqrt[a + b*x^4]
) + (b*((-5*c*Sqrt[a + b*x^4])/(a*x^2) - (12*d*Sqrt[a + b*x^4])/(a*x) + (1
2*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (15*e*ArcTanh
[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (12*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) + (2*(3*Sqrt[b]*d + 5*Sqrt
[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*b^(1/4)*Sqrt[a +
b*x^4])))/30
```

3.506. $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx$

3.506.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.506.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(24bd\,x^5+10bc\,x^4+20af\,x^3+15ae\,x^2+12adx+10ac)}{60x^6a}+\frac{b\left(\frac{20af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{12i\sqrt{b}\,d\sqrt{a}\sqrt{1-\frac{i\sqrt{b}}{}}}{}\right.}{}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{6x^6}-\frac{d\sqrt{b\,x^4+a}}{5x^5}-\frac{e\sqrt{b\,x^4+a}}{4x^4}-\frac{f\sqrt{b\,x^4+a}}{3x^3}-\frac{bc\sqrt{b\,x^4+a}}{6a\,x^2}-\frac{2bd\sqrt{b\,x^4+a}}{5ax}+\frac{2bf\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
default	$-\frac{c(b\,x^4+a)^{\frac{3}{2}}}{6a\,x^6}+f\left(-\frac{\sqrt{b\,x^4+a}}{3x^3}+\frac{2b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+e\left(-\frac{(b\,x^4+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\sqrt{b\,x^4+}}{x^2}\right.}{4\sqrt{a}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(b*x^4+a)^(1/2)*(24*b*d*x^5+10*b*c*x^4+20*a*f*x^3+15*a*e*x^2+12*a*d*
x+10*a*c)/x^6/a+1/30*b/a*(20*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+12*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*
x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1
/2)*b^(1/2))^(1/2),I))-15/2*a^(1/2)*e*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x
^2))
```

3.506.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.47

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = -\frac{48\sqrt{a}bdx^6\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)-15\sqrt{a}bex^6\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-16\,(3\,bd-5\,af)\sqrt{\cdots}}{\cdots}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
-1/120*(48*sqrt(a)*b*d*x^6*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)),
 -1) - 15*sqrt(a)*b*e*x^6*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x
^4) - 16*(3*b*d - 5*a*f)*sqrt(a)*x^6*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/
a)^(1/4)), -1) + 2*(24*b*d*x^5 + 10*b*c*x^4 + 20*a*f*x^3 + 15*a*e*x^2 + 12
*a*d*x + 10*a*c)*sqrt(b*x^4 + a))/(a*x^6)
```

3.506.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.72 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.54

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{6a} - \frac{be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**7,x)
```

output
```
sqrt(a)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**5*gamma(-1/4)) + sqrt(a)*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(b)*c*sqrt(a/(b*x**4
) + 1)/(6*x**4) - sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*c*sqr
t(a/(b*x**4) + 1)/(6*a) - b*e*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))
```

3.506.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/6*(b*x^4 + a)^(3/2)*c/(a*x^6) + integrate(sqrt(b*x^4 + a)*(f*x^2 + e*x
+ d)/x^6, x)
```

3.506.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)
```

3.506.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx=\int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^7}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^7,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^7, x)
```

3.507 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx$

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3.507.1 Optimal result

Integrand size = 30, antiderivative size = 375

$$\begin{aligned}
\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx &= -\frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{21ax^3} \\
&\quad -\frac{bd\sqrt{a+bx^4}}{6ax^2}-\frac{2be\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}ex\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{bf\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}} \\
&\quad -\frac{2b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} \\
&\quad -\frac{b^{5/4}\left(5\sqrt{b}c-21\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
-1/4*b*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/420*(60*c/x^7+70*d/x^6
+84*e/x^5+105*f/x^4)*(b*x^4+a)^(1/2)-2/21*b*c*(b*x^4+a)^(1/2)/a/x^3-1/6*b*
d*(b*x^4+a)^(1/2)/a/x^2-2/5*b*e*(b*x^4+a)^(1/2)/a/x+2/5*b^(3/2)*e*x*(b*x^4
+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-2/5*b^(5/4)*e*(cos(2*arctan(b^(1/4)*x/a^
(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b
^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+
x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)-1/105*b^(5/4)*(cos(2*arctan(
b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(si
n(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-21*e*a^(1/2)+5*c*b^(1/2))*(a
^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x
^4+a)^(1/2)
```

3.507.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.39

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx =$$
$$-\frac{\sqrt{a+bx^4}\left(35x\left(\sqrt{1+\frac{bx^4}{a}}(2ad+3afx^2+2bdx^4)+3bfx^6\text{arctanh}\left(\sqrt{1+\frac{bx^4}{a}}\right)\right)+60ac\,\text{Hypergeomet}\right.}{420ax^7\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]
```

output
```
-1/420*(Sqrt[a + b*x^4]*(35*x*(Sqrt[1 + (b*x^4)/a]*(2*a*d + 3*a*f*x^2 + 2*
b*d*x^4) + 3*b*f*x^6*ArcTanh[Sqrt[1 + (b*x^4)/a]]) + 60*a*c*Hypergeometric
2F1[-7/4, -1/2, -3/4, -((b*x^4)/a)] + 84*a*e*x^2*Hypergeometric2F1[-5/4, -
1/2, -1/4, -((b*x^4)/a)]))/(a*x^7*Sqrt[1 + (b*x^4)/a])
```

3.507.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)}{x^8}dx$$

$$\Big\downarrow \text{ 2364}$$

$$-2b\int -\frac{105fx^3+84ex^2+70dx+60c}{420x^4\sqrt{bx^4+a}}dx-\frac{1}{420}\sqrt{a+bx^4}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{210}b\int \frac{105fx^3+84ex^2+70dx+60c}{x^4\sqrt{bx^4+a}}dx-\frac{1}{420}\sqrt{a+bx^4}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

$$\Big\downarrow \text{ 2372}$$

$$\frac{1}{210}b\int \left(\frac{84ex^2+60c}{x^4\sqrt{bx^4+a}}+\frac{105fx^2+70d}{x^3\sqrt{bx^4+a}}\right)dx-\frac{1}{420}\sqrt{a+bx^4}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{210}b\left(-\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-21\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{a+bx^4}}-\frac{84\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x\right.}{}\right.$$

$$\frac{1}{420}\sqrt{a+bx^4}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]
```

output
```
-1/420*(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*Sqrt[a + b*x^
4]) + (b*((-20*c*Sqrt[a + b*x^4])/(a*x^3) - (35*d*Sqrt[a + b*x^4])/(a*x^2)
 - (84*e*Sqrt[a + b*x^4])/(a*x) + (84*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(a*(Sqr
t[a] + Sqrt[b]*x^2)) - (105*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]
) - (84*b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt
[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a
 + b*x^4]) - (2*b^(1/4)*(5*Sqrt[b]*c - 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(a^(5/4)*Sqrt[a + b*x^4])))/210
```

3.507.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.507.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(168be\,x^6+70bd\,x^5+40bc\,x^4+105af\,x^3+84ae\,x^2+70adx+60ac)}{420x^7a}+\frac{b\left(-\frac{20bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{84i\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{7x^7}-\frac{d\sqrt{b\,x^4+a}}{6x^6}-\frac{e\sqrt{b\,x^4+a}}{5x^5}-\frac{f\sqrt{b\,x^4+a}}{4x^4}-\frac{2bc\sqrt{b\,x^4+a}}{21a\,x^3}-\frac{bd\sqrt{b\,x^4+a}}{6a\,x^2}-\frac{2be\sqrt{b\,x^4+a}}{5ax}-\frac{2b^2c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1\cdots}}{21a\sqrt{\cdots}}$
default	$-\frac{d\left(b\,x^4+a\right)^{\frac{3}{2}}}{6a\,x^6}+c\left(-\frac{\sqrt{b\,x^4+a}}{7x^7}-\frac{2b\sqrt{b\,x^4+a}}{21a\,x^3}-\frac{2b^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(-\frac{\left(b\,x^4+a\right)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\ln\cdots}{\cdots}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/420*(b*x^4+a)^(1/2)*(168*b*e*x^6+70*b*d*x^5+40*b*c*x^4+105*a*f*x^3+84*a
*e*x^2+70*a*d*x+60*a*c)/x^7/a+1/210*b/a*(-20*b*c/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+84*I*b^(1/2)*e*a^(1/2)/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2
)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ell
ipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-105/2*a^(1/2)*f*ln((2*a+2*a^(1/2)*(
b*x^4+a)^(1/2))/x^2))
```

3.507.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.46

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx =$$

$$-\frac{336\sqrt{a}bex^7\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-105\sqrt{a}bfx^7\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-16\,(5\,bc+21\,b\cdots}{\cdots}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-1/840*(336*sqrt(a)*b*e*x^7*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4))
, -1) - 105*sqrt(a)*b*f*x^7*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)
/x^4) - 16*(5*b*c + 21*b*e)*sqrt(a)*x^7*(-b/a)^(3/4)*elliptic_f(arcsin(x*(
-b/a)^(1/4)), -1) + 2*(168*b*e*x^6 + 70*b*d*x^5 + 40*b*c*x^4 + 105*a*f*x^3
 + 84*a*e*x^2 + 70*a*d*x + 60*a*c)*sqrt(b*x^4 + a))/(a*x^7)
```

3.507.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.51

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{6a} - \frac{bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**8,x)
```

output
```
sqrt(a)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**7*gamma(-3/4)) + sqrt(a)*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - sqrt(b)*d*sqrt(a/(b*x*
*4) + 1)/(6*x**4) - sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d*s
qrt(a/(b*x**4) + 1)/(6*a) - b*f*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))
```

3.507.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

3.507.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

3.507.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^8}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^8,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^8, x)
```

3.508 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx$

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3.508.1 Optimal result

Integrand size = 30, antiderivative size = 400

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx \\
&= -\frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)\sqrt{a+bx^4}-\frac{bc\sqrt{a+bx^4}}{16ax^4}-\frac{2bd\sqrt{a+bx^4}}{21ax^3} \\
&\quad -\frac{be\sqrt{a+bx^4}}{6ax^2}-\frac{2bf\sqrt{a+bx^4}}{5ax}+\frac{2b^{3/2}fx\sqrt{a+bx^4}}{5a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{b^2c\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}} \\
&\quad -\frac{2b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} \\
&\quad -\frac{b^{5/4}\left(5\sqrt{b}d-21\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
1/16*b^2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/840*(105*c/x^8+120*d
/x^7+140*e/x^6+168*f/x^5)*(b*x^4+a)^(1/2)-1/16*b*c*(b*x^4+a)^(1/2)/a/x^4-2
/21*b*d*(b*x^4+a)^(1/2)/a/x^3-1/6*b*e*(b*x^4+a)^(1/2)/a/x^2-2/5*b*f*(b*x^4
+a)^(1/2)/a/x+2/5*b^(3/2)*f*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-2/5*
b^(5/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/
2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a
)^(1/2)-1/105*b^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(
1/2))*(-21*f*a^(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2
)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^4+a)^(1/2)
```

3.508.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.36

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\left(30a^3d\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},-\frac{bx^4}{a}\right)+7x\left(6a^3fx\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},\right.\right.\right.}{210a^3x^7\sqrt{1+}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]
```

output
```
-1/210*(Sqrt[a + b*x^4]*(30*a^3*d*Hypergeometric2F1[-7/4, -1/2, -3/4, -((b
*x^4)/a)] + 7*x*(6*a^3*f*x*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^4)/a
)] + 5*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*(a^2*e + b^2*c*x^6*Hypergeometric2F
1[3/2, 3, 5/2, 1 + (b*x^4)/a])))))/(a^3*x^7*Sqrt[1 + (b*x^4)/a])
```

3.508.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)}{x^9}\,dx$$

$$\Big\downarrow \text{2364}$$

$$-2b\int -\frac{168fx^3+140ex^2+120dx+105c}{840x^5\sqrt{bx^4+a}}dx-\frac{1}{840}\sqrt{a+bx^4}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{420}b\int \frac{168fx^3+140ex^2+120dx+105c}{x^5\sqrt{bx^4+a}}dx-\frac{1}{840}\sqrt{a+bx^4}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{2372}$$

$$\frac{1}{420}b\int \left(\frac{140ex^2+105c}{x^5\sqrt{bx^4+a}}+\frac{168fx^2+120d}{x^4\sqrt{bx^4+a}}\right)dx-\frac{1}{840}\sqrt{a+bx^4}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{420}b\left(-\frac{4\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}d-21\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{a+bx^4}}-\frac{168\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{\right.}}{}\right.$$

$$\frac{1}{840}\sqrt{a+bx^4}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]
```

output
```
-1/840*(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*Sqrt[a + b
*x^4]) + (b*((-105*c*Sqrt[a + b*x^4])/(4*a*x^4) - (40*d*Sqrt[a + b*x^4])/(
a*x^3) - (70*e*Sqrt[a + b*x^4])/(a*x^2) - (168*f*Sqrt[a + b*x^4])/(a*x) +
(168*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (105*b*c*A
rcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) - (168*b^(1/4)*f*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) - (4*b^(1/4)*(5*Sqr
t[b]*d - 21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(5/4)*S
qrt[a + b*x^4])))/420
```

3.508.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.508. $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx$

3.508.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(672bf\,x^{7}+280be\,x^{6}+160bd\,x^{5}+105bc\,x^{4}+336af\,x^{3}+280ae\,x^{2}+240adx+210ac)}{1680x^{8}a}-\frac{b^{2}\left(\frac{80d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.}{}$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{8x^{8}}-\frac{d\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{e\sqrt{b\,x^{4}+a}}{6x^{6}}-\frac{f\sqrt{b\,x^{4}+a}}{5x^{5}}-\frac{bc\sqrt{b\,x^{4}+a}}{16a\,x^{4}}-\frac{2bd\sqrt{b\,x^{4}+a}}{21a\,x^{3}}-\frac{be\sqrt{b\,x^{4}+a}}{6a\,x^{2}}-\frac{2bf\sqrt{b\,x^{4}+a}}{5ax}-\frac{2d\,b^{2}}{}$
default	$-\frac{e\left(b\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}+d\left(-\frac{\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{2b\sqrt{b\,x^{4}+a}}{21a\,x^{3}}-\frac{2b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+c\left(-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{2}}}{8a\,x^{8}}+\frac{b(b}{1}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(b*x^4+a)^(1/2)*(672*b*f*x^7+280*b*e*x^6+160*b*d*x^5+105*b*c*x^4+3
36*a*f*x^3+280*a*e*x^2+240*a*d*x+210*a*c)/x^8/a-1/840/a*b^2*(80*d/(I/a^(1/
2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-336*I*f*a
^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-105/2*c/a^(1/2)*l
n((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

3.508.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.49

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx =$$
$$-\frac{1344\,a^{\frac{3}{2}}bfx^{8}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-105\,\sqrt{a}b^{2}cx^{8}\log\left(-\frac{bx^{4}+2\sqrt{bx^{4}+a}\sqrt{a}+2\,a}{x^{4}}\right)-64\,(5\,abd+2}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
-1/3360*(1344*a^(3/2)*b*f*x^8*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4
)), -1) - 105*sqrt(a)*b^2*c*x^8*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) +
2*a)/x^4) - 64*(5*a*b*d + 21*a*b*f)*sqrt(a)*x^8*(-b/a)^(3/4)*elliptic_f(ar
csin(x*(-b/a)^(1/4)), -1) + 2*(672*a*b*f*x^7 + 280*a*b*e*x^6 + 160*a*b*d*x
^5 + 105*a*b*c*x^4 + 336*a^2*f*x^3 + 280*a^2*e*x^2 + 240*a^2*d*x + 210*a^2
*c)*sqrt(b*x^4 + a))/(a^2*x^8)
```

3.508.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.04 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} - \frac{ac}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3\sqrt{b}c}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}}c}{16ax^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{6a} + \frac{b^2c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16a^{\frac{3}{2}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**9,x)
```

output
```
sqrt(a)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**7*gamma(-3/4)) + sqrt(a)*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - a*c/(8*sqrt(b)*x**10*s
qrt(a/(b*x**4) + 1)) - 3*sqrt(b)*c/(16*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b
)*e*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*c/(16*a*x**2*sqrt(a/(b*x**4)
+ 1)) - b**(3/2)*e*sqrt(a/(b*x**4) + 1)/(6*a) + b**2*c*asinh(sqrt(a)/(sqrt
(b)*x**2))/(16*a**(3/2))
```

3.508.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-1/32*(b^2*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^
(3/2) + 2*((b*x^4 + a)^(3/2)*b^2 + sqrt(b*x^4 + a)*a*b^2)/((b*x^4 + a)^2*a
 - 2*(b*x^4 + a)*a^2 + a^3))*c + integrate(sqrt(b*x^4 + a)*(f*x^2 + e*x +
d)/x^8, x)
```

3.508.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

3.508.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^9}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^9,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^9, x)
```

3.509 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx$

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3.509.8 Giac [F] . . . 3975
3.509.9 Mupad [F(-1)] . . . 3975

3.509.1 Optimal result

Integrand size = 30, antiderivative size = 425

$$\begin{aligned}&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx\\
&=-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\sqrt{a+bx^4}-\frac{2bc\sqrt{a+bx^4}}{45ax^5}-\frac{bd\sqrt{a+bx^4}}{16ax^4}\\
&\quad-\frac{2be\sqrt{a+bx^4}}{21ax^3}-\frac{bf\sqrt{a+bx^4}}{6ax^2}+\frac{2b^2c\sqrt{a+bx^4}}{15a^2x}-\frac{2b^{5/2}cx\sqrt{a+bx^4}}{15a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}\\
&\quad+\frac{b^2d\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{2b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^4}}\\
&\quad-\frac{b^{7/4}\left(7\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/16*b^2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/504*(56*c/x^9+63*d/x
^8+72*e/x^7+84*f/x^6)*(b*x^4+a)^(1/2)-2/45*b*c*(b*x^4+a)^(1/2)/a/x^5-1/16*
b*d*(b*x^4+a)^(1/2)/a/x^4-2/21*b*e*(b*x^4+a)^(1/2)/a/x^3-1/6*b*f*(b*x^4+a)
^(1/2)/a/x^2+2/15*b^2*c*(b*x^4+a)^(1/2)/a^2/x-2/15*b^(5/2)*c*x*(b*x^4+a)^(
1/2)/a^2/(a^(1/2)+x^2*b^(1/2))+2/15*b^(9/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^
2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-1/105*b^(7/4)*(cos(2*arctan(b^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(
2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(5*e*a^(1/2)+7*c*b^(1/2))*(a^(1/
2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a
)^(1/2)
```

3.509.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.35

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx = -\frac{\sqrt{a+bx^4}\left(14a^3c\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{1}{2},-\frac{5}{4},-\frac{bx^4}{a}\right)+3x^2\left(6a^3e\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},\right.\right.\right.}{126a^3x^9\sqrt{1+}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^10,x]
```

output
```
-1/126*(Sqrt[a + b*x^4]*(14*a^3*c*Hypergeometric2F1[-9/4, -1/2, -5/4, -((b
*x^4)/a)] + 3*x^2*(6*a^3*e*Hypergeometric2F1[-7/4, -1/2, -3/4, -((b*x^4)/a
)] + 7*x*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*(a^2*f + b^2*d*x^6*Hypergeometric
2F1[3/2, 3, 5/2, 1 + (b*x^4)/a]))))/(a^3*x^9*Sqrt[1 + (b*x^4)/a])
```

3.509.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}(c+dx+ex^2+fx^3)}{x^{10}}dx$$

$$\Big\downarrow \text{2364}$$

$$-2b\int -\frac{84fx^3+72ex^2+63dx+56c}{504x^6\sqrt{bx^4+a}}dx-\frac{1}{504}\sqrt{a+bx^4}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{252}b\int \frac{84fx^3+72ex^2+63dx+56c}{x^6\sqrt{bx^4+a}}dx-\frac{1}{504}\sqrt{a+bx^4}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$$\Big\downarrow \text{2372}$$

$$\frac{1}{252}b\int \left(\frac{72ex^2+56c}{x^6\sqrt{bx^4+a}}+\frac{84fx^2+63d}{x^5\sqrt{bx^4+a}}\right)dx-\frac{1}{504}\sqrt{a+bx^4}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{252}b\left(-\frac{12b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+7\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}}+\frac{168b^{5/4}c\left(\sqrt{a}+\sqrt{\right.}\right.}{}\right.$$

$$\frac{1}{504}\sqrt{a+bx^4}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^10,x]
```

output
```
-1/504*(((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*Sqrt[a + b*x^4
]) + (b*((-56*c*Sqrt[a + b*x^4])/(5*a*x^5) - (63*d*Sqrt[a + b*x^4])/(4*a*x
^4) - (24*e*Sqrt[a + b*x^4])/(a*x^3) - (42*f*Sqrt[a + b*x^4])/(a*x^2) + (1
68*b*c*Sqrt[a + b*x^4])/(5*a^2*x) - (168*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a
^2*(Sqrt[a] + Sqrt[b]*x^2)) + (63*b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4
*a^(3/2)) + (168*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(
7/4)*Sqrt[a + b*x^4]) - (12*b^(3/4)*(7*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a + b*x^4])))/252
```

3.509.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.509. $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx$

3.509.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.62 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(-672b^{2}c\,x^{8}+840abf\,x^{7}+480aeb\,x^{6}+315x^{5}dba+224abc\,x^{4}+840a^{2}f\,x^{3}+720a^{2}e\,x^{2}+630a^{2}dx+560a^{2}c)}{5040x^{9}a^{2}}-\frac{b^{2}\left(\frac{80ae\sqrt{1-}}{}\right.}{}$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{9x^{9}}-\frac{d\sqrt{b\,x^{4}+a}}{8x^{8}}-\frac{e\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{f\sqrt{b\,x^{4}+a}}{6x^{6}}-\frac{2bc\sqrt{b\,x^{4}+a}}{45a\,x^{5}}-\frac{bd\sqrt{b\,x^{4}+a}}{16a\,x^{4}}-\frac{2be\sqrt{b\,x^{4}+a}}{21a\,x^{3}}-\frac{bf\sqrt{b\,x^{4}+a}}{6a\,x^{2}}+\frac{2b^{2}c}{1}$
default	$-\frac{f(b\,x^{4}+a)^{\frac{3}{2}}}{6a\,x^{6}}+e\left(-\frac{\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{2b\sqrt{b\,x^{4}+a}}{21a\,x^{3}}-\frac{2b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+d\left(-\frac{(b\,x^{4}+a)^{\frac{3}{2}}}{8a\,x^{8}}+\frac{b(b}{1}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/5040*(b*x^4+a)^(1/2)*(-672*b^2*c*x^8+840*a*b*f*x^7+480*a*b*e*x^6+315*a*
b*d*x^5+224*a*b*c*x^4+840*a^2*f*x^3+720*a^2*e*x^2+630*a^2*d*x+560*a^2*c)/x
^9/a^2-1/840*b^2/a^2*(80*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(
I/a^(1/2)*b^(1/2))^(1/2),I)+112*I*b^(1/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1
/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4
+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)
*b^(1/2))^(1/2),I))-105/2*a^(1/2)*d*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2
))
```

3.509.5 Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.49

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx$$
$$=\frac{1344\sqrt{a}b^2cx^9\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)+315\sqrt{a}b^2dx^9\log\left(-\frac{bx^4+2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-192\left(7b^2c-5\right.}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="fricas")
```

output
```
1/10080*(1344*sqrt(a)*b^2*c*x^9*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1
/4)), -1) + 315*sqrt(a)*b^2*d*x^9*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a)
+ 2*a)/x^4) - 192*(7*b^2*c - 5*a*b*e)*sqrt(a)*x^9*(-b/a)^(3/4)*elliptic_f(
arcsin(x*(-b/a)^(1/4)), -1) + 2*(672*b^2*c*x^8 - 840*a*b*f*x^7 - 480*a*b*e
*x^6 - 315*a*b*d*x^5 - 224*a*b*c*x^4 - 840*a^2*f*x^3 - 720*a^2*e*x^2 - 630
*a^2*d*x - 560*a^2*c)*sqrt(b*x^4 + a))/(a^2*x^9)
```

3.509.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.20 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.58

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} - \frac{ad}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3\sqrt{b}d}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}}d}{16ax^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{6a} + \frac{b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16a^{\frac{3}{2}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**10,x)
```

output
```
sqrt(a)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**9*gamma(-5/4)) + sqrt(a)*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) - a*d/(8*sqrt(b)*x**10*s
qrt(a/(b*x**4) + 1)) - 3*sqrt(b)*d/(16*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b
)*f*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*d/(16*a*x**2*sqrt(a/(b*x**4)
+ 1)) - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(6*a) + b**2*d*asinh(sqrt(a)/(sqrt
(b)*x**2))/(16*a**(3/2))
```

3.509.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^{10}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

3.509.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^{10}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^10,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

3.509.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^{10}}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{10}}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^10,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^10, x)
```

3.510 $\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

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3.510.1 Optimal result

Integrand size = 30, antiderivative size = 476

$$\int x^4\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}-\frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^5(117c+77ex^2)\sqrt{a+bx^4}}{3003}-\frac{adx^2(a+bx^4)^{3/2}}{48b}+\frac{1}{143}x^5\left(13c+11ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{fx^4(a+bx^4)^{5/2}}{14b}-\frac{(12af-35bdx^2)\left(a+bx^4\right)^{5/2}}{420b^2}-\frac{a^3d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}+$$

output
```
-1/48*a*d*x^2*(b*x^4+a)^(3/2)/b+1/143*x^5*(11*e*x^2+13*c)*(b*x^4+a)^(3/2)+
1/14*f*x^4*(b*x^4+a)^(5/2)/b-1/420*(-35*b*d*x^2+12*a*f)*(b*x^4+a)^(5/2)/b^
2-1/32*a^3*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+4/77*a^2*c*x*(b*
x^4+a)^(1/2)/b-1/32*a^2*d*x^2*(b*x^4+a)^(1/2)/b+4/195*a^2*e*x^3*(b*x^4+a)^
(1/2)/b+2/3003*a*x^5*(77*e*x^2+117*c)*(b*x^4+a)^(1/2)-4/65*a^3*e*x*(b*x^4+
a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))+4/65*a^(13/4)*e*(cos(2*arctan(b^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*a
rctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a
^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)-2/5005*a^(11/4)*(cos(
2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ell
ipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(77*e*a^(1/2)+65*c*b^
(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(
7/4)/(b*x^4+a)^(1/2)
```

3.510.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.88 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.47

$$\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{\sqrt{a+bx^4}\left(43680bcx(a+bx^4)^2+36960bex^3(a+bx^4)^2+6864f(a+bx^4)^2(-2a+5bx^4)+50\right.}{}$$

input
```
Integrate[x^4*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(Sqrt[a + b*x^4]*(43680*b*c*x*(a + b*x^4)^2 + 36960*b*e*x^3*(a + b*x^4)^2
+ 6864*f*(a + b*x^4)^2*(-2*a + 5*b*x^4) + 5005*b*d*x^2*(3*a^2 + 14*a*b*x^4
 + 8*b^2*x^8) - (15015*a^(5/2)*Sqrt[b]*d*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/S
qrt[1 + (b*x^4)/a] - (43680*a^2*b*c*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -(
(b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] - (36960*a^2*b*e*x^3*Hypergeometric2F1[-3
/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/(480480*b^2)
```

3.510.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+bx^4\right)^{3/2}\left(c+dx+ex^2+fx^3\right)dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(x^4\left(a+bx^4\right)^{3/2}\left(c+ex^2\right)+x^5\left(a+bx^4\right)^{3/2}\left(d+fx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}e+65\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}+\frac{4a^{13/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}}-\frac{a^3d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3ex\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2cx\sqrt{a+bx^4}}{77b}-\frac{a^2dx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2ex^3\sqrt{a+bx^4}}{195b}-\frac{af\left(a+bx^4\right)^{5/2}}{35b^2}+\frac{1}{143}x^5\left(a+bx^4\right)^{3/2}\left(13c+11ex^2\right)+\frac{2ax^5\sqrt{a+bx^4}\left(117c+77ex^2\right)}{3003}+\frac{dx^2\left(a+bx^4\right)^{5/2}}{12b}-\frac{adx^2\left(a+bx^4\right)^{3/2}}{48b}+\frac{fx^4\left(a+bx^4\right)^{5/2}}{14b}$$

input
```
Int[x^4*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(4*a^2*c*x*Sqrt[a + b*x^4])/(77*b) - (a^2*d*x^2*Sqrt[a + b*x^4])/(32*b) +
(4*a^2*e*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*e*x*Sqrt[a + b*x^4])/(65*b^
(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*c + 77*e*x^2)*Sqrt[a + b*x^
4])/3003 - (a*d*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^5*(13*c + 11*e*x^2)*(a
+ b*x^4)^(3/2))/143 - (a*f*(a + b*x^4)^(5/2))/(35*b^2) + (d*x^2*(a + b*x^4
)^(5/2))/(12*b) + (f*x^4*(a + b*x^4)^(5/2))/(14*b) - (a^3*d*ArcTanh[(Sqrt[
b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3/2)) + (4*a^(13/4)*e*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^4]) - (2*a^(11/4)*(65*Sqrt[
b]*c + 77*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5005*b^(7/4
)*Sqrt[a + b*x^4])
```

3.510.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.510.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(-34320b^3 f x^{12}-36960b^3 e x^{11}-40040b^3 d x^{10}-43680b^3 c x^9-54912x^8 a b^2 f-61600a b^2 e x^7-70070a b^2 d x^6-81120a b^2 c x^5-6864a}{480480b^2}$
default	$-\frac{f\sqrt{b x^4+a}\,(-5b x^4+2a)(b^2x^8+2ab x^4+a^2)}{70b^2}+e\left(\frac{b x^{11}\sqrt{b x^4+a}}{13}+\frac{5a x^7\sqrt{b x^4+a}}{39}+\frac{4a^2x^3\sqrt{b x^4+a}}{195b}-\frac{4ia^{\frac{7}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{}}{}\right.$
elliptic	$\frac{bf x^{12}\sqrt{b x^4+a}}{14}+\frac{be x^{11}\sqrt{b x^4+a}}{13}+\frac{bd x^{10}\sqrt{b x^4+a}}{12}+\frac{bc x^9\sqrt{b x^4+a}}{11}+\frac{4af x^8\sqrt{b x^4+a}}{35}+\frac{5ae x^7\sqrt{b x^4+a}}{39}+\frac{7ad x^6\sqrt{b x^4+}}{48}$

input
```
int(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/480480*(-34320*b^3*f*x^12-36960*b^3*e*x^11-40040*b^3*d*x^10-43680*b^3*c
*x^9-54912*a*b^2*f*x^8-61600*a*b^2*e*x^7-70070*a*b^2*d*x^6-81120*a*b^2*c*x
^5-6864*a^2*b*f*x^4-9856*a^2*b*e*x^3-15015*a^2*b*d*x^2-24960*a^2*b*c*x+137
28*a^3*f)/b^2*(b*x^4+a)^(1/2)-1/80080/b*a^3*(4160*c/(I/a^(1/2)*b^(1/2))^(1
/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4
+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+4928*I*e*a^(1/2)/(I/a^(
1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x
^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+5005/2*d*ln(x^2*b^(1/2)+(b*x^4
+a)^(1/2))/b^(1/2))
```

3.510.5 Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.55

$$\int x^4\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx =$$

$$-\frac{59136\,a^3\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid-1)-15015\,a^3\sqrt{b}dx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+768}{}$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/960960*(59136*a^3*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/
4)/x), -1) - 15015*a^3*sqrt(b)*d*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b
)*x^2 - a) + 768*(65*a^2*b*c - 77*a^3*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f
(arcsin((-a/b)^(1/4)/x), -1) - 2*(34320*b^3*f*x^13 + 36960*b^3*e*x^12 + 40
040*b^3*d*x^11 + 43680*b^3*c*x^10 + 54912*a*b^2*f*x^9 + 61600*a*b^2*e*x^8
+ 70070*a*b^2*d*x^7 + 81120*a*b^2*c*x^6 + 6864*a^2*b*f*x^5 + 9856*a^2*b*e*
x^4 + 15015*a^2*b*d*x^3 + 24960*a^2*b*c*x^2 - 13728*a^3*f*x - 29568*a^3*e)
*sqrt(b*x^4 + a))/(b^2*x)
```

3.510.6 Sympy [A] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 462, normalized size of antiderivative = 0.97

$$\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{a^{\frac{5}{2}}dx^2}{32b\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{17a^{\frac{3}{2}}dx^6}{96\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bcx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{11\sqrt{a}bdx^{10}}{48\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bex^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}-\frac{a^3d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}}+af\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right)+bf\left(\begin{cases}\frac{4a^3\sqrt{a+bx^4}}{105b^3}-\frac{2a^2x^4\sqrt{a+bx^4}}{105b^2}+\frac{ax^8\sqrt{a+bx^4}}{70b}+\frac{x^{12}\sqrt{a+bx^4}}{14} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^{12}}{12} & \text{otherwise}\end{cases}\right)+\frac{b^2dx^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x**4*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

output
```
a**(5/2)*d*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**5*gamma(5/4)*hyp
er((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(
3/2)*d*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*e*x**7*gamma(7/4)*hyper((-1
/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*c
*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(13/4)) + 11*sqrt(a)*b*d*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*
x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(15/4)) - a**3*d*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*f*Pi
ecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b)
 + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b*f*Piec
ewise((4*a**3*sqrt(a + b*x**4)/(105*b**3) - 2*a**2*x**4*sqrt(a + b*x**4)/(
105*b**2) + a*x**8*sqrt(a + b*x**4)/(70*b) + x**12*sqrt(a + b*x**4)/14, Ne
(b, 0)), (sqrt(a)*x**12/12, True)) + b**2*d*x**14/(12*sqrt(a)*sqrt(1 + b*x
**4/a))
```

3.510.7 Maxima [F]

$$\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int (bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x^4\,dx$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

3.510.8 Giac [F]

$$\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int (bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x^4\,dx$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^4, x)
```

3.510.9 Mupad [F(-1)]

Timed out.

$$\int x^4(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int x^4\,\left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^4*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x^4*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.511 $\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}\,dx$

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3.511.1 Optimal result

Integrand size = 30, antiderivative size = 452

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}-\frac{4a^3fx\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^5(117d+77fx^2)\sqrt{a+bx^4}}{3003}-\frac{aex^2(a+bx^4)^{3/2}}{48b}+\frac{1}{143}x^5(13d+11fx^2)\left(a+bx^4\right)^{3/2}+\frac{(6c+5ex^2)\left(a+bx^4\right)^{5/2}}{60b}-\frac{a^3e\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}+\frac{4a^{13/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)$$

output
```
-1/48*a*e*x^2*(b*x^4+a)^(3/2)/b+1/143*x^5*(11*f*x^2+13*d)*(b*x^4+a)^(3/2)+
1/60*(5*e*x^2+6*c)*(b*x^4+a)^(5/2)/b-1/32*a^3*e*arctanh(x^2*b^(1/2)/(b*x^4
+a)^(1/2))/b^(3/2)+4/77*a^2*d*x*(b*x^4+a)^(1/2)/b-1/32*a^2*e*x^2*(b*x^4+a)
^(1/2)/b+4/195*a^2*f*x^3*(b*x^4+a)^(1/2)/b+2/3003*a*x^5*(77*f*x^2+117*d)*(
b*x^4+a)^(1/2)-4/65*a^3*f*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))+
4/65*a^(13/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*
(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b
*x^4+a)^(1/2)-2/5005*a^(11/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4)))
,1/2*2^(1/2))*(77*f*a^(1/2)+65*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)
/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)
```

3.511.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.75 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.53

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx = \frac{\sqrt{a+bx^4}\left(6864\sqrt{b}c(a+bx^4)^2+6240\sqrt{b}dx(a+bx^4)^2+5280\sqrt{b}fx^3(a+bx^4)^2+715e\left(\sqrt{b}x^2\right.\right.}{}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(Sqrt[a + b*x^4]*(6864*Sqrt[b]*c*(a + b*x^4)^2 + 6240*Sqrt[b]*d*x*(a + b*x
^4)^2 + 5280*Sqrt[b]*f*x^3*(a + b*x^4)^2 + 715*e*(Sqrt[b]*x^2*(3*a^2 + 14*
a*b*x^4 + 8*b^2*x^8) - (3*a^(5/2)*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 +
 (b*x^4)/a]) - (6240*a^2*Sqrt[b]*d*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((
b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] - (5280*a^2*Sqrt[b]*f*x^3*Hypergeometric2F
1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/(68640*b^(3/2))
```

3.511.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^4\right)^{3/2}\left(c+dx+ex^2+fx^3\right)dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(x^3\left(a+bx^4\right)^{3/2}\left(c+ex^2\right)+x^4\left(a+bx^4\right)^{3/2}\left(d+fx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}f+65\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}+\frac{4a^{13/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}}-\frac{a^3e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3fx\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}+\frac{c\left(a+bx^4\right)^{5/2}}{10b}+\frac{1}{143}x^5\left(a+bx^4\right)^{3/2}\left(13d+11fx^2\right)+\frac{2ax^5\sqrt{a+bx^4}(117d+77fx^2)}{3003}+\frac{ex^2\left(a+bx^4\right)^{5/2}}{12b}-\frac{aex^2\left(a+bx^4\right)^{3/2}}{48b}$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(4*a^2*d*x*Sqrt[a + b*x^4])/(77*b) - (a^2*e*x^2*Sqrt[a + b*x^4])/(32*b) +
(4*a^2*f*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*f*x*Sqrt[a + b*x^4])/(65*b^
(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*d + 77*f*x^2)*Sqrt[a + b*x^
4])/3003 - (a*e*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^5*(13*d + 11*f*x^2)*(a
+ b*x^4)^(3/2))/143 + (c*(a + b*x^4)^(5/2))/(10*b) + (e*x^2*(a + b*x^4)^(5
/2))/(12*b) - (a^3*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3/2))
+ (4*a^(13/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[
a + b*x^4]) - (2*a^(11/4)*(65*Sqrt[b]*d + 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1
/4)*x)/a^(1/4)], 1/2])/(5005*b^(7/4)*Sqrt[a + b*x^4])
```

3.511.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.511.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.70

method	result
risch	$\frac{(36960b^2fx^{11}+40040b^2ex^{10}+43680b^2dx^9+48048b^2cx^8+61600abfx^7+70070aebx^6+81120x^5dba+96096abcx^4+9856a^2fx^3+1501}{480480b}$
default	$f\left(\frac{bx^{11}\sqrt{bx^4+a}}{13}+\frac{5ax^7\sqrt{bx^4+a}}{39}+\frac{4a^2x^3\sqrt{bx^4+a}}{195b}-\frac{4ia^{\frac{7}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{65b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}}\right)+$
elliptic	$\frac{bfx^{11}\sqrt{bx^4+a}}{13}+\frac{bex^{10}\sqrt{bx^4+a}}{12}+\frac{bdx^9\sqrt{bx^4+a}}{11}+\frac{bcx^8\sqrt{bx^4+a}}{10}+\frac{5afx^7\sqrt{bx^4+a}}{39}+\frac{7aex^6\sqrt{bx^4+a}}{48}+\frac{13adx^5\sqrt{bx^4+}}{77}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/480480*(36960*b^2*f*x^11+40040*b^2*e*x^10+43680*b^2*d*x^9+48048*b^2*c*x^
8+61600*a*b*f*x^7+70070*a*b*e*x^6+81120*a*b*d*x^5+96096*a*b*c*x^4+9856*a^2
*f*x^3+15015*a^2*e*x^2+24960*a^2*d*x+48048*a^2*c)/b*(b*x^4+a)^(1/2)-1/8008
0/b*a^3*(4160*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*
(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)+4928*I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(E
llipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1
/2),I))+5005/2*e*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

3.511.5 Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.56

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx =$$
$$-\frac{59136\,a^3\sqrt{b}fx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid-1)-15015\,a^3\sqrt{b}ex\log\left(-2\,bx^4+2\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+76}{}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/960960*(59136*a^3*sqrt(b)*f*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/
4)/x), -1) - 15015*a^3*sqrt(b)*e*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b
)*x^2 - a) + 768*(65*a^2*b*d - 77*a^3*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f
(arcsin((-a/b)^(1/4)/x), -1) - 2*(36960*b^3*f*x^12 + 40040*b^3*e*x^11 + 43
680*b^3*d*x^10 + 48048*b^3*c*x^9 + 61600*a*b^2*f*x^8 + 70070*a*b^2*e*x^7 +
 81120*a*b^2*d*x^6 + 96096*a*b^2*c*x^5 + 9856*a^2*b*f*x^4 + 15015*a^2*b*e*
x^3 + 24960*a^2*b*d*x^2 + 48048*a^2*b*c*x - 29568*a^3*f)*sqrt(b*x^4 + a))/
(b^2*x)
```

3.511.6 Sympy [A] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.88

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{a^{\frac{5}{2}}ex^2}{32b\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{17a^{\frac{3}{2}}ex^6}{96\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bdx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{11\sqrt{a}bex^{10}}{48\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bfx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}-\frac{a^3e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}}+ac\left(\begin{cases}\frac{\sqrt{a}x^4}{4}&\text{for }b=0\\\frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b}&\text{otherwise}\end{cases}\right)+bc\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10}&\text{for }b\neq0\\\frac{\sqrt{a}x^8}{8}&\text{otherwise}\end{cases}\right)+\frac{b^2ex^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

output
```
a**(5/2)*e*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*d*x**5*gamma(5/4)*hyp
er((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(
3/2)*e*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*f*x**7*gamma(7/4)*hyper((-1
/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*d
*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(13/4)) + 11*sqrt(a)*b*e*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*
x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(15/4)) - a**3*e*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*c*Pi
ecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b
*c*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(
30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**
2*e*x**14/(12*sqrt(a)*sqrt(1 + b*x**4/a))
```

3.511.7 Maxima [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x^3\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/10*(b*x^4 + a)^(5/2)*c/b + integrate((b*f*x^10 + b*e*x^9 + b*d*x^8 + a*f
*x^6 + a*e*x^5 + a*d*x^4)*sqrt(b*x^4 + a), x)
```

3.511.8 Giac [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x^3\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^3, x)
```

3.511.9 Mupad [F(-1)]

Timed out.

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int x^3\,\left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^3*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x^3*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.512 $\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

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3.512.1 Optimal result

Integrand size = 30, antiderivative size = 427

$$\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^3(77c+45ex^2)\sqrt{a+bx^4}}{1155}-\frac{afx^2(a+bx^4)^{3/2}}{48b}+\frac{1}{99}x^3\left(11c+9ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{(6d+5fx^2)\left(a+bx^4\right)^{5/2}}{60b}-\frac{a^3f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}\left(77\sqrt{b}c-15\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}$$

output
```
-1/48*a*f*x^2*(b*x^4+a)^(3/2)/b+1/99*x^3*(9*e*x^2+11*c)*(b*x^4+a)^(3/2)+1/
60*(5*f*x^2+6*d)*(b*x^4+a)^(5/2)/b-1/32*a^3*f*arctanh(x^2*b^(1/2)/(b*x^4+a
)^(1/2))/b^(3/2)+4/77*a^2*e*x*(b*x^4+a)^(1/2)/b-1/32*a^2*f*x^2*(b*x^4+a)^(
1/2)/b+2/1155*a*x^3*(45*e*x^2+77*c)*(b*x^4+a)^(1/2)+4/15*a^2*c*x*(b*x^4+a)
^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-4/15*a^(9/4)*c*(cos(2*arctan(b^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arct
an(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1
/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+2/1155*a^(9/4)*(cos(2*ar
ctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellipti
cF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-15*e*a^(1/2)+77*c*b^(1/
2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4
)/(b*x^4+a)^(1/2)
```

3.512.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.48

$$\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx = \frac{\sqrt{a+bx^4}\left(\frac{528d(a+bx^4)^2}{b}+\frac{480ex(a+bx^4)^2}{b}+\frac{55f\left(\sqrt{b}x^2(3a^2+14abx^4+8b^2x^8)-\frac{3a^{5/2}\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)}{b^{3/2}}-\frac{480a^{\cdots}}{\cdots}\cdots\right)}{5280}$$

input
```
Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(Sqrt[a + b*x^4]*((528*d*(a + b*x^4)^2)/b + (480*e*x*(a + b*x^4)^2)/b + (5
5*f*(Sqrt[b]*x^2*(3*a^2 + 14*a*b*x^4 + 8*b^2*x^8) - (3*a^(5/2)*ArcSinh[(Sq
rt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]))/b^(3/2) - (480*a^2*e*x*Hypergeo
metric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (1760*a
*c*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a
]))/5280
```

3.512.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^4\right)^{3/2} \left(c+dx+ex^2+fx^3\right) \, dx$$

$$\Big\downarrow \; 2372$$

$$\int \left(x^2 \left(a+bx^4\right)^{3/2} \left(c+ex^2\right)+x^3 \left(a+bx^4\right)^{3/2} \left(d+fx^2\right)\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}c-15\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{a^3f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{2ax^3\sqrt{a+bx^4}\left(77c+45ex^2\right)}{1155}+\frac{1}{99}x^3\left(a+bx^4\right)^{3/2}\left(11c+9ex^2\right)+\frac{d\left(a+bx^4\right)^{5/2}}{10b}-\frac{afx^2\left(a+bx^4\right)^{3/2}}{48b}+\frac{fx^2\left(a+bx^4\right)^{5/2}}{12b}$$

input
```
Int[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(4*a^2*e*x*Sqrt[a + b*x^4])/(77*b) - (a^2*f*x^2*Sqrt[a + b*x^4])/(32*b) +
(4*a^2*c*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^
3*(77*c + 45*e*x^2)*Sqrt[a + b*x^4])/1155 - (a*f*x^2*(a + b*x^4)^(3/2))/(4
8*b) + (x^3*(11*c + 9*e*x^2)*(a + b*x^4)^(3/2))/99 + (d*(a + b*x^4)^(5/2))
/(10*b) + (f*x^2*(a + b*x^4)^(5/2))/(12*b) - (a^3*f*ArcTanh[(Sqrt[b]*x^2)/
Sqrt[a + b*x^4]])/(32*b^(3/2)) - (4*a^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(9/4)*(77*Sqrt[b]*c - 15*
Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*b^(5/4)*Sqrt[a +
 b*x^4])
```

3.512.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.512.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.73

method	result
risch	$\frac{(9240b^2 f\,x^{10}+10080b^2 e\,x^9+11088b^2 d\,x^8+12320b^2 c\,x^7+16170abf\,x^6+18720abe\,x^5+22176abd\,x^4+27104abc\,x^3+3465a^2 f\,x^2+5760a}{110880b}$
default	$f\left(\frac{b\,x^{10}\sqrt{b\,x^4+a}}{12}+\frac{7a\,x^6\sqrt{b\,x^4+a}}{48}+\frac{a^2x^2\sqrt{b\,x^4+a}}{32b}-\frac{a^3\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{32b^{\frac{3}{2}}}\right)+e\left(\frac{b\,x^9\sqrt{b\,x^4+a}}{11}+\frac{13a\,x^5\sqrt{b\,x^4+a}}{77}+\right.$
elliptic	$\frac{bf\,x^{10}\sqrt{b\,x^4+a}}{12}+\frac{be\,x^9\sqrt{b\,x^4+a}}{11}+\frac{bd\,x^8\sqrt{b\,x^4+a}}{10}+\frac{bc\,x^7\sqrt{b\,x^4+a}}{9}+\frac{7af\,x^6\sqrt{b\,x^4+a}}{48}+\frac{13ae\,x^5\sqrt{b\,x^4+a}}{77}+\frac{ad\,x^4\sqrt{b\,x^4+a}}{5}$

input
```
int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/110880*(9240*b^2*f*x^10+10080*b^2*e*x^9+11088*b^2*d*x^8+12320*b^2*c*x^7+
16170*a*b*f*x^6+18720*a*b*e*x^5+22176*a*b*d*x^4+27104*a*b*c*x^3+3465*a^2*f
*x^2+5760*a^2*e*x+11088*a^2*d)/b*(b*x^4+a)^(1/2)-1/18480*a^2/b*(960*a*e/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-492
8*I*b^(1/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1
/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1155/2*a*f
*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

3.512.5 Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.58

$$\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx = \frac{59136\,a^2b^{\frac{3}{2}}cx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+3465\,a^3\sqrt{b}fx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2\right.}{}$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/221760*(59136*a^2*b^(3/2)*c*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4
)/x), -1) + 3465*a^3*sqrt(b)*f*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*
x^2 - a) - 768*(77*a^2*b*c + 15*a^2*b*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f
(arcsin((-a/b)^(1/4)/x), -1) + 2*(9240*b^3*f*x^11 + 10080*b^3*e*x^10 + 110
88*b^3*d*x^9 + 12320*b^3*c*x^8 + 16170*a*b^2*f*x^7 + 18720*a*b^2*e*x^6 + 2
2176*a*b^2*d*x^5 + 27104*a*b^2*c*x^4 + 3465*a^2*b*f*x^3 + 5760*a^2*b*e*x^2
 + 11088*a^2*b*d*x + 29568*a^2*b*c)*sqrt(b*x^4 + a))/(b^2*x)
```

3.512.6 Sympy [A] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.93

$$\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx = \frac{a^{\frac{5}{2}}fx^2}{32b\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{17a^{\frac{3}{2}}fx^6}{96\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bcx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{a}bex^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)} + \frac{11\sqrt{a}bfx^{10}}{48\sqrt{1+\frac{bx^4}{a}}} - \frac{a^3f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}} + ad\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + bd\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right) + \frac{b^2fx^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

output
```
a**(5/2)*f*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**3*gamma(3/4)*hyp
er((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**(3/2
)*e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(9/4)) + 17*a**(3/2)*f*x**6/(96*sqrt(1 + b*x**4/a)) + sqrt(a)*b*c*x
**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*ga
mma(11/4)) + sqrt(a)*b*e*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**
4*exp_polar(I*pi)/a)/(4*gamma(13/4)) + 11*sqrt(a)*b*f*x**10/(48*sqrt(1 + b
*x**4/a)) - a**3*f*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*d*Piecewi
se((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*d*Pi
ecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b)
 + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*f*x
**14/(12*sqrt(a)*sqrt(1 + b*x**4/a))
```

3.512.7 Maxima [F]

$$\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x^2\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

3.512.8 Giac [F]

$$\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x^2\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

3.512.9 Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int x^2\left(b\,x^4+a\right)^{3/2}\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^2*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x^2*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.513 $\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

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3.513.1 Optimal result

Integrand size = 28, antiderivative size = 409

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2ax^3(77d+45fx^2)\sqrt{a+bx^4}}{1155}+\frac{1}{8}cx^2\left(a+bx^4\right)^{3/2}+\frac{1}{99}x^3(11d+9fx^2)\left(a+bx^4\right)^{3/2}+\frac{e(a+bx^4)^{5/2}}{10b}+\frac{3a^2c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}-\frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}\left(77\sqrt{b}d-15\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/8*c*x^2*(b*x^4+a)^(3/2)+1/99*x^3*(9*f*x^2+11*d)*(b*x^4+a)^(3/2)+1/10*e*(
b*x^4+a)^(5/2)/b+3/16*a^2*c*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+4
/77*a^2*f*x*(b*x^4+a)^(1/2)/b+3/16*a*c*x^2*(b*x^4+a)^(1/2)+2/1155*a*x^3*(4
5*f*x^2+77*d)*(b*x^4+a)^(1/2)+4/15*a^2*d*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2
)+x^2*b^(1/2))-4/15*a^(9/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4)))
,1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1
/2)/b^(3/4)/(b*x^4+a)^(1/2)+2/1155*a^(9/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(-15*f*a^(1/2)+77*d*b^(1/2))*(a^(1/2)+x^2*b^(1/
2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

3.513.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.71 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.48

$$\int x\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx = \frac{\sqrt{a+bx^4}\left(\frac{264e\left(a+bx^4\right)^2}{b}+\frac{240fx\left(a+bx^4\right)^2}{b}+165c\left(5ax^2+2bx^6+\frac{3a^{5/2}\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}\left(a+bx^4\right)}\right)-\cdots\right.}{2640}$$

input
```
Integrate[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(Sqrt[a + b*x^4]*((264*e*(a + b*x^4)^2)/b + (240*f*x*(a + b*x^4)^2)/b + 16
5*c*(5*a*x^2 + 2*b*x^6 + (3*a^(5/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x
^2)/Sqrt[a]])/(Sqrt[b]*(a + b*x^4))) - (240*a^2*f*x*Hypergeometric2F1[-3/2
, 1/4, 5/4, -((b*x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (880*a*d*x^3*Hypergeo
metric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/2640
```

3.513.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^4\right)^{3/2}\left(c+dx+ex^2+fx^3\right)dx$$

$$\Big\downarrow \; 2372$$

$$\int \left(x\left(a+bx^4\right)^{3/2}\left(c+ex^2\right)+x^2\left(a+bx^4\right)^{3/2}\left(d+fx^2\right)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}d-15\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{3a^2c\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{1}{8}cx^2\left(a+bx^4\right)^{3/2}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{1}{99}x^3\left(a+bx^4\right)^{3/2}\left(11d+9fx^2\right)+\frac{2ax^3\sqrt{a+bx^4}\left(77d+45fx^2\right)}{1155}+\frac{e\left(a+bx^4\right)^{5/2}}{10b}$$

input
```
Int[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(4*a^2*f*x*Sqrt[a + b*x^4])/(77*b) + (3*a*c*x^2*Sqrt[a + b*x^4])/16 + (4*a
^2*d*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^3*(7
7*d + 45*f*x^2)*Sqrt[a + b*x^4])/1155 + (c*x^2*(a + b*x^4)^(3/2))/8 + (x^3
*(11*d + 9*f*x^2)*(a + b*x^4)^(3/2))/99 + (e*(a + b*x^4)^(5/2))/(10*b) + (
3*a^2*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)*
d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) +
(2*a^(9/4)*(77*Sqrt[b]*d - 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(1155*b^(5/4)*Sqrt[a + b*x^4])
```

3.513.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.513.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.06 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.73

method	result
risch	$\frac{\left(5040 b^{2} f\,x^{9}+5544 b^{2} e\,x^{8}+6160 b^{2} d\,x^{7}+6930 b^{2} c\,x^{6}+9360 a b f\,x^{5}+11088 a b e\,x^{4}+13552 x^{3} a b d+17325 a b c\,x^{2}+2880 a^{2} f x+5544 a^{2} e\right) \sqrt{b\,x}}{55440 b}$
default	$f \left(\frac{b\,x^{9} \sqrt{b\,x^{4}+a}}{11}+\frac{13 a\,x^{5} \sqrt{b\,x^{4}+a}}{77}+\frac{4 a^{2} x \sqrt{b\,x^{4}+a}}{77 b}-\frac{4 a^{3} \sqrt{1-\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{77 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+\frac{e \left(b\,x^{4}+a\right)^{\frac{5}{2}}}{10 b}+d \Bigg($
elliptic	$\frac{b f\,x^{9} \sqrt{b\,x^{4}+a}}{11}+\frac{b e\,x^{8} \sqrt{b\,x^{4}+a}}{10}+\frac{b d\,x^{7} \sqrt{b\,x^{4}+a}}{9}+\frac{b c\,x^{6} \sqrt{b\,x^{4}+a}}{8}+\frac{13 a f\,x^{5} \sqrt{b\,x^{4}+a}}{77}+\frac{a e\,x^{4} \sqrt{b\,x^{4}+a}}{5}+\frac{11 a d\,x^{3} \sqrt{b\,x^{4}+a}}{45}$

input
```
int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/55440*(5040*b^2*f*x^9+5544*b^2*e*x^8+6160*b^2*d*x^7+6930*b^2*c*x^6+9360*
a*b*f*x^5+11088*a*b*e*x^4+13552*a*b*d*x^3+17325*a*b*c*x^2+2880*a^2*f*x+554
4*a^2*e)/b*(b*x^4+a)^(1/2)-1/9240*a^2/b*(480*a*f/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-2464*I*b^(1/2)*d*a^(1/2)/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-E
llipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3465/2*b^(1/2)*c*ln(x^2*b^(1/2)+(
b*x^4+a)^(1/2)))
```

3.513.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.55

$$\int x(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}\,dx = \frac{29568\,a^2\sqrt{b}dx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+10395\,a^2\sqrt{b}cx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x\right.}{}$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/110880*(29568*a^2*sqrt(b)*d*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4
)/x), -1) + 10395*a^2*sqrt(b)*c*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)
*x^2 - a) - 384*(77*a^2*d + 15*a^2*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(ar
csin((-a/b)^(1/4)/x), -1) + 2*(5040*b^2*f*x^10 + 5544*b^2*e*x^9 + 6160*b^2
*d*x^8 + 6930*b^2*c*x^7 + 9360*a*b*f*x^6 + 11088*a*b*e*x^5 + 13552*a*b*d*x
^4 + 17325*a*b*c*x^3 + 2880*a^2*f*x^2 + 5544*a^2*e*x + 14784*a^2*d)*sqrt(b
*x^4 + a))/(b*x)
```

3.513.6 Sympy [A] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.97

$$\begin{aligned}
\int x(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}\,dx &= \frac{a^{\frac{3}{2}}cx^2\sqrt{1+\frac{bx^4}{a}}}{4}+\frac{a^{\frac{3}{2}}cx^2}{16\sqrt{1+\frac{bx^4}{a}}} \\
&+\frac{a^{\frac{3}{2}}dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{a^{\frac{3}{2}}fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{3\sqrt{a}bcx^6}{16\sqrt{1+\frac{bx^4}{a}}} \\
&+\frac{\sqrt{a}bdx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bfx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)} \\
&+\frac{3a^2c\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}}+ae\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) \\
&+be\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right)+\frac{b^2cx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}
\end{aligned}$$

input
```
integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*c*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*c*x**2/(16*sqrt(1 + b*x**4
/a)) + a**(3/2)*d*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*gamma(7/4)) + a**(3/2)*f*x**5*gamma(5/4)*hyper((-1/2, 5/4)
, (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*c*x**6/(1
6*sqrt(1 + b*x**4/a)) + sqrt(a)*b*d*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11
/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*f*x**9*gamma(9
/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4))
+ 3*a**2*c*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*e*Piecewise((sqrt(
a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*e*Piecewise((
-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*s
qrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*c*x**10/(8*s
qrt(a)*sqrt(1 + b*x**4/a))
```

3.513.7 Maxima [F]

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/32*(3*a^2*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 +
a)/x^2))/sqrt(b) + 2*(3*sqrt(b*x^4 + a)*a^2*b/x^2 - 5*(b*x^4 + a)^(3/2)*a^
2/x^6)/(b^2 - 2*(b*x^4 + a)*b/x^4 + (b*x^4 + a)^2/x^8))*c + integrate((b*f
*x^8 + b*e*x^7 + b*d*x^6 + a*f*x^4 + a*e*x^3 + a*d*x^2)*sqrt(b*x^4 + a), x
)
```

3.513.8 Giac [F]

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x, x)
```

3.513.9 Mupad [F(-1)]

Timed out.

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int x\left(b\,x^4+a\right)^{3/2}\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.514 $\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

3.514.1 Optimal result . . . 4004
3.514.2 Mathematica [C] (verified) . . . 4005
3.514.3 Rubi [A] (verified) . . . 4006
3.514.4 Maple [C] (verified) . . . 4007
3.514.5 Fricas [A] (verification not implemented) . . . 4008
3.514.6 Sympy [A] (verification not implemented) . . . 4009
3.514.7 Maxima [F] . . . 4010
3.514.8 Giac [F] . . . 4010
3.514.9 Mupad [F(-1)] . . . 4010

3.514.1 Optimal result

Integrand size = 27, antiderivative size = 382

$$\begin{aligned}
&\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{3}{16}adx^2\sqrt{a+bx^4}\\
&+\frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2}{105}ax\left(15c+7ex^2\right)\sqrt{a+bx^4}+\frac{1}{8}dx^2\left(a+bx^4\right)^{3/2}\\
&+\frac{1}{63}x\left(9c+7ex^2\right)\left(a+bx^4\right)^{3/2}+\frac{f\left(a+bx^4\right)^{5/2}}{10b}+\frac{3a^2d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}\\
&-\frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}\\
&+\frac{2a^{7/4}\left(15\sqrt{b}c+7\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
1/8*d*x^2*(b*x^4+a)^(3/2)+1/63*x*(7*e*x^2+9*c)*(b*x^4+a)^(3/2)+1/10*f*(b*x
^4+a)^(5/2)/b+3/16*a^2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+3/16
*a*d*x^2*(b*x^4+a)^(1/2)+2/105*a*x*(7*e*x^2+15*c)*(b*x^4+a)^(1/2)+4/15*a^2
*e*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-4/15*a^(9/4)*e*(cos(2*a
rctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellipt
icE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((
b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+2/105*a^(7
/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1
/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(7*e*a^(1/2)
+15*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(
1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

3.514.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.46

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{1}{240}\sqrt{a+bx^4}\left(\frac{24f\left(a+bx^4\right)^2}{b}\right.$$
$$\left.+15d\left(5ax^2+2bx^6+\frac{3a^{5/2}\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}\left(a+bx^4\right)}\right)+\frac{240acx\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}+\frac{80aex}{}\right.$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(Sqrt[a + b*x^4]*((24*f*(a + b*x^4)^2)/b + 15*d*(5*a*x^2 + 2*b*x^6 + (3*a^
(5/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(a + b*
x^4))) + (240*a*c*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[
1 + (b*x^4)/a] + (80*a*e*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a
)])/Sqrt[1 + (b*x^4)/a]))/240
```

3.514.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^{3/2} \left(c+dx+ex^2+fx^3\right) \, dx$$

$$\Big\downarrow \ 2424$$

$$\int \left(\left(a+bx^4\right)^{3/2} \left(c+ex^2\right)+x\left(a+bx^4\right)^{3/2} \left(d+fx^2\right)\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+15\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{3a^2d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{63}x\left(a+bx^4\right)^{3/2}\left(9c+7ex^2\right)+\frac{2}{105}ax\sqrt{a+bx^4}\left(15c+7ex^2\right)+\frac{1}{8}dx^2\left(a+bx^4\right)^{3/2}+\frac{3}{16}adx^2\sqrt{a+bx^4}+\frac{f\left(a+bx^4\right)^{5/2}}{10b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(3*a*d*x^2*Sqrt[a + b*x^4])/16 + (4*a^2*e*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(
Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x*(15*c + 7*e*x^2)*Sqrt[a + b*x^4])/105 + (
d*x^2*(a + b*x^4)^(3/2))/8 + (x*(9*c + 7*e*x^2)*(a + b*x^4)^(3/2))/63 + (f
*(a + b*x^4)^(5/2))/(10*b) + (3*a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4
]])/(16*Sqrt[b]) - (4*a^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4)*(15*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3/4)*Sqrt[a + b*x^4])
```

3.514.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.514.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.75

method	result
risch	$\frac{\left(504 b^{2} f\,x^{8}+560 b^{2} e\,x^{7}+630 b^{2} d\,x^{6}+720 b^{2} c\,x^{5}+1008 a b f\,x^{4}+1232 a b e\,x^{3}+1575 x^{2} a b d+2160 a b c x+504 a^{2} f\right) \sqrt{b\,x^{4}+a}}{5040 b}+\frac{a^{2}\left(\frac{480 c\sqrt{1-}}{}\right.}{}$
default	$c\left(\frac{b\,x^{5} \sqrt{b\,x^{4}+a}}{7}+\frac{3 a x \sqrt{b\,x^{4}+a}}{7}+\frac{4 a^{2} \sqrt{1-\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{7 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+\frac{f\left(b\,x^{4}+a\right)^{\frac{5}{2}}}{10 b}+e\left(\frac{b\,x^{7} \sqrt{b\,x^{4}+a}}{9}+\frac{11 a}{}\right.$
elliptic	$\frac{b f\,x^{8} \sqrt{b\,x^{4}+a}}{10}+\frac{b e\,x^{7} \sqrt{b\,x^{4}+a}}{9}+\frac{b d\,x^{6} \sqrt{b\,x^{4}+a}}{8}+\frac{b c\,x^{5} \sqrt{b\,x^{4}+a}}{7}+\frac{a f\,x^{4} \sqrt{b\,x^{4}+a}}{5}+\frac{11 a e\,x^{3} \sqrt{b\,x^{4}+a}}{45}+\frac{5 a d\,x^{2} \sqrt{b\,x^{4}+a}}{16}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5040*(504*b^2*f*x^8+560*b^2*e*x^7+630*b^2*d*x^6+720*b^2*c*x^5+1008*a*b*f
*x^4+1232*a*b*e*x^3+1575*a*b*d*x^2+2160*a*b*c*x+504*a^2*f)/b*(b*x^4+a)^(1/
2)+1/840*a^2*(480*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)+224*I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)
*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))
^(1/2),I))+315/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

3.514.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.56

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^{3/2}\,dx = \frac{2688\,a^2\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+945\,a^2\sqrt{b}dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-\right.}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/10080*(2688*a^2*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/
x), -1) + 945*a^2*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2
 - a) + 384*(15*a*b*c - 7*a^2*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin(
(-a/b)^(1/4)/x), -1) + 2*(504*b^2*f*x^9 + 560*b^2*e*x^8 + 630*b^2*d*x^7 +
720*b^2*c*x^6 + 1008*a*b*f*x^5 + 1232*a*b*e*x^4 + 1575*a*b*d*x^3 + 2160*a*
b*c*x^2 + 504*a^2*f*x + 1344*a^2*e)*sqrt(b*x^4 + a))/(b*x)
```

3.514.6 Sympy [A] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.03

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx = \frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}dx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}dx^2}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}bcx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}bdx^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{3a^2d\,\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}} + af\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + bf\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right) + \frac{b^2dx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(5/4)) + a**(3/2)*d*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*d*x**2
/(16*sqrt(1 + b*x**4/a)) + a**(3/2)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (
7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*c*x**5*gamma(5
/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) +
3*sqrt(a)*b*d*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**7*gamma(7/4)*h
yper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + 3*a
**2*d*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*f*Piecewise((sqrt(a)*x*
*4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*f*Piecewise((-a**2
*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a
 + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*d*x**10/(8*sqrt(a
)*sqrt(1 + b*x**4/a))
```

3.514.7 Maxima [F]

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^{3/2}\,dx = \int (bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c), x)
```

3.514.8 Giac [F]

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^{3/2}\,dx = \int (bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c), x)
```

3.514.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^{3/2}\,dx = \int (b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)\,dx$$

input
```
int((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

3.515 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x} dx$

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3.515.1 Optimal result

Integrand size = 30, antiderivative size = 403

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x} dx = \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a(8c+3ex^2)\sqrt{a+bx^4} + \frac{2}{105}ax(15d+7fx^2)\sqrt{a+bx^4} + \frac{1}{24}(4c+3ex^2)(a+bx^4)^{3/2} + \frac{1}{63}x(9d+7fx^2)(a+bx^4)^{3/2} + \frac{3a^2e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} - \frac{1}{2}a^{3/2}c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} + \frac{2a^{7/4}\left(15\sqrt{b}d\right.}{}$$

output
```
1/24*(3*e*x^2+4*c)*(b*x^4+a)^(3/2)+1/63*x*(7*f*x^2+9*d)*(b*x^4+a)^(3/2)-1/
2*a^(3/2)*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))+3/16*a^2*e*arctanh(x^2*b^(1/2
)/(b*x^4+a)^(1/2))/b^(1/2)+1/16*a*(3*e*x^2+8*c)*(b*x^4+a)^(1/2)+2/105*a*x*
(7*f*x^2+15*d)*(b*x^4+a)^(1/2)+4/15*a^2*f*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/
2)+x^2*b^(1/2))-4/15*a^(9/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(
1/2)/b^(3/4)/(b*x^4+a)^(1/2)+2/105*a^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(7*f*a^(1/2)+15*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2)
)*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

3.515.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.52 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x}\,dx = \frac{1}{16}e\sqrt{a+bx^4}\left(5ax^2+2bx^6+\frac{3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1+\frac{bx^4}{a}}}\right)$$
$$+\frac{1}{6}c\left(\sqrt{a+bx^4}\,(4a+bx^4)-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+\frac{adx\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right.}{\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]
```

output
```
(e*Sqrt[a + b*x^4]*(5*a*x^2 + 2*b*x^6 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x^2)/S
qrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^4)/a])))/16 + (c*(Sqrt[a + b*x^4]*(4*a + b
*x^4) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]))/6 + (a*d*x*Sqrt[a + b
*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]
 + (a*f*x^3*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)
])/(3*Sqrt[1 + (b*x^4)/a])
```

3.515.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}\,(c+dx+ex^2+fx^3)}{x}\,dx$$
$$\Big\downarrow \text{2372}$$
$$\int \left(\frac{(a+bx^4)^{3/2}\,(c+ex^2)}{x}+(a+bx^4)^{3/2}\,(d+fx^2)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+15\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3a^2e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{16}a\sqrt{a+bx^4}(8c+3ex^2)+\frac{1}{24}\left(a+bx^4\right)^{3/2}(4c+3ex^2)+\frac{2}{105}ax\sqrt{a+bx^4}(15d+7fx^2)+\frac{1}{63}x\left(a+bx^4\right)^{3/2}(9d+7fx^2)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]
```

output
```
(4*a^2*f*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (a*(8*c
 + 3*e*x^2)*Sqrt[a + b*x^4])/16 + (2*a*x*(15*d + 7*f*x^2)*Sqrt[a + b*x^4])
/105 + ((4*c + 3*e*x^2)*(a + b*x^4)^(3/2))/24 + (x*(9*d + 7*f*x^2)*(a + b*
x^4)^(3/2))/63 + (3*a^2*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt
[b]) - (a^(3/2)*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (4*a^(9/4)*f*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(7
/4)*(15*Sqrt[b]*d + 7*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(105*b^(3/4)*Sqrt[a + b*x^4])
```

3.515.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.515.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.86

method	result
elliptic	$\frac{bf\,x^7\sqrt{b\,x^4+a}}{9}+\frac{be\,x^6\sqrt{b\,x^4+a}}{8}+\frac{bd\,x^5\sqrt{b\,x^4+a}}{7}+\frac{bc\,x^4\sqrt{b\,x^4+a}}{6}+\frac{11af\,x^3\sqrt{b\,x^4+a}}{45}+\frac{5ae\,x^2\sqrt{b\,x^4+a}}{16}+\frac{3adx\sqrt{b\,x^4+a}}{7}$
default	$d\left(\frac{b\,x^5\sqrt{b\,x^4+a}}{7}+\frac{3ax\sqrt{b\,x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(\frac{b\,x^7\sqrt{b\,x^4+a}}{9}+\frac{11a\,x^3\sqrt{b\,x^4+a}}{45}+\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/9*b*f*x^7*(b*x^4+a)^(1/2)+1/8*b*e*x^6*(b*x^4+a)^(1/2)+1/7*b*d*x^5*(b*x^4
+a)^(1/2)+1/6*b*c*x^4*(b*x^4+a)^(1/2)+11/45*a*f*x^3*(b*x^4+a)^(1/2)+5/16*a
*e*x^2*(b*x^4+a)^(1/2)+3/7*a*d*x*(b*x^4+a)^(1/2)+2/3*a*c*(b*x^4+a)^(1/2)+4
/7*a^2*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)+3/16*a^2*e*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))/b^(1/2)+4/15*I*a^(5
/2)*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1
/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^(3/2)*c*arct
anh(a^(1/2)/(b*x^4+a)^(1/2))
```

3.515.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x, x)
```

3.515.6 Sympy [A] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx = -\frac{a^{\frac{3}{2}}c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{a^{\frac{3}{2}}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}ex^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}ex^2}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}bdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}bex^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bfx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{a^2c}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a^2e\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}} + \frac{a\sqrt{b}cx^2}{2\sqrt{\frac{a}{bx^4}+1}} + bc\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + \frac{b^2ex^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x,x)
```

output
```
-a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*d*x*gamma(1/4)*hype
r((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**(3/2)
*e*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*e*x**2/(16*sqrt(1 + b*x**4/a)) + a
**(3/2)*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi
)/a)/(4*gamma(7/4)) + sqrt(a)*b*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,
), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*e*x**6/(16*sqrt(
1 + b*x**4/a)) + sqrt(a)*b*f*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b
*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + a**2*c/(2*sqrt(b)*x**2*sqrt(a/(
b*x**4) + 1)) + 3*a**2*e*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*sqrt
(b)*c*x**2/(2*sqrt(a/(b*x**4) + 1)) + b*c*Piecewise((sqrt(a)*x**4/4, Eq(b,
 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b**2*e*x**10/(8*sqrt(a)*sqrt(1
+ b*x**4/a))
```

3.515.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

3.515.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

3.515.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x, x)
```

3.516 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2} dx$

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3.516.1 Optimal result

Integrand size = 30, antiderivative size = 404

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2} dx = \frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{35}x(5ae+21bcx^2)\sqrt{a+bx^4} + \frac{1}{16}a(8d+3fx^2)\sqrt{a+bx^4} - \frac{(7c-ex^2)(a+bx^4)^{3/2}}{7x} + \frac{1}{24}(4d+3fx^2)(a+bx^4)^{3/2} + \frac{3a^2f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} - \frac{1}{2}a^{3/2}d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} + \frac{2a^{5/4}(21}{}$$

output
```
-1/7*(-e*x^2+7*c)*(b*x^4+a)^(3/2)/x+1/24*(3*f*x^2+4*d)*(b*x^4+a)^(3/2)-1/2
*a^(3/2)*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))+3/16*a^2*f*arctanh(x^2*b^(1/2)
/(b*x^4+a)^(1/2))/b^(1/2)+2/35*x*(21*b*c*x^2+5*a*e)*(b*x^4+a)^(1/2)+1/16*a
*(3*f*x^2+8*d)*(b*x^4+a)^(1/2)+12/5*a*c*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)
+x^2*b^(1/2))-12/5*a^(5/4)*b^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2)
)^2)^(1/2)/(b*x^4+a)^(1/2)+2/35*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(5*e*a^(1/2)+21*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*(
(b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^4+a)^(1/2)
```

3.516.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.46 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.55

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \frac{1}{16}f\sqrt{a+bx^4}\left(5ax^2 + 2bx^6 + \frac{3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{1}{6}d\left(\sqrt{a+bx^4}(4a+bx^4)-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right) - \frac{ac\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]
```

output
```
(f*Sqrt[a + b*x^4]*(5*a*x^2 + 2*b*x^6 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x^2)/S
qrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^4)/a])))/16 + (d*(Sqrt[a + b*x^4]*(4*a + b
*x^4) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]))/6 - (a*c*Sqrt[a + b*x
^4]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^4)/a)])/(x*Sqrt[1 + (b*x^4)/
a]) + (a*e*x*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a
)])/Sqrt[1 + (b*x^4)/a]
```

3.516.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^2}\,dx$$

$$\Big\downarrow \text{ 2372}$$

$$\int \left(\frac{(a+bx^4)^{3/2}(c+ex^2)}{x^2}+\frac{(a+bx^4)^{3/2}(d+fx^2)}{x}\right)dx$$

$$\downarrow 2009$$

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+21\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}d\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3a^2f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}-\frac{\left(a+bx^4\right)^{3/2}\left(7c-ex^2\right)}{7x}+\frac{2}{35}x\sqrt{a+bx^4}\left(5ae+21bcx^2\right)+\frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{16}a\sqrt{a+bx^4}\left(8d+3fx^2\right)+\frac{1}{24}\left(a+bx^4\right)^{3/2}\left(4d+3fx^2\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]
```

output
```
(12*a*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + (2*x*(5*a
*e + 21*b*c*x^2)*Sqrt[a + b*x^4])/35 + (a*(8*d + 3*f*x^2)*Sqrt[a + b*x^4])
/16 - ((7*c - e*x^2)*(a + b*x^4)^(3/2))/(7*x) + ((4*d + 3*f*x^2)*(a + b*x^
4)^(3/2))/24 + (3*a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b
]) - (a^(3/2)*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*
c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4
)*(21*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(S
qrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3
5*b^(1/4)*Sqrt[a + b*x^4])
```

3.516.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.516. $\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^2}\,dx$

3.516.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.40 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.86

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{x}+\frac{bf\,x^6\sqrt{b\,x^4+a}}{8}+\frac{be\,x^5\sqrt{b\,x^4+a}}{7}+\frac{bd\,x^4\sqrt{b\,x^4+a}}{6}+\frac{bc\,x^3\sqrt{b\,x^4+a}}{5}+\frac{5af\,x^2\sqrt{b\,x^4+a}}{16}+\frac{3aex\sqrt{b\,x^4+a}}{7}+$
default	$e\left(\frac{b\,x^5\sqrt{b\,x^4+a}}{7}+\frac{3ax\sqrt{b\,x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(\frac{3a^2\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{16\sqrt{b}}+\frac{b\,x^6\sqrt{b\,x}}{8}\right.$
risch	$-\frac{ac\sqrt{b\,x^4+a}}{x}+\frac{4a^2e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{bf\,x^6\sqrt{b\,x^4+a}}{8}+\frac{5af\,x^2\sqrt{b\,x^4+a}}{16}+\frac{3a^2f\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{16\sqrt{b}}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a*c*(b*x^4+a)^(1/2)/x+1/8*b*f*x^6*(b*x^4+a)^(1/2)+1/7*b*e*x^5*(b*x^4+a)^(
1/2)+1/6*b*d*x^4*(b*x^4+a)^(1/2)+1/5*b*c*x^3*(b*x^4+a)^(1/2)+5/16*a*f*x^2*
(b*x^4+a)^(1/2)+3/7*a*e*x*(b*x^4+a)^(1/2)+2/3*a*d*(b*x^4+a)^(1/2)+4/7*a^2*
e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
+3/16*a^2*f*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))/b^(1/2)+12/5*I*a^(3/2)*b^(
1/2)*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1
/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^(3/2)*d*arctanh(a^(
1/2)/(b*x^4+a)^(1/2))
```

3.516.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^2}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^2, x)
```

3.516.6 Sympy [A] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{a^{\frac{3}{2}}ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}fx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}fx^2}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bcx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}bex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}bfx^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{a^2d}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a^2f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}} + \frac{a\sqrt{b}dx^2}{2\sqrt{\frac{a}{bx^4}+1}} + bd\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + \frac{b^2fx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**2,x)
```

output
```
a**(3/2)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x*gamma(3/4)) - a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2
)*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(5/4)) + a**(3/2)*f*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*f*x**2/(16*sq
rt(1 + b*x**4/a)) + sqrt(a)*b*c*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*e*x**5*gamma(5/4)*hy
per((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt
(a)*b*f*x**6/(16*sqrt(1 + b*x**4/a)) + a**2*d/(2*sqrt(b)*x**2*sqrt(a/(b*x*
*4) + 1)) + 3*a**2*f*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*sqrt(b)*
d*x**2/(2*sqrt(a/(b*x**4) + 1)) + b*d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0))
, ((a + b*x**4)**(3/2)/(6*b), True)) + b**2*f*x**10/(8*sqrt(a)*sqrt(1 + b*
x**4/a))
```

3.516. $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx$

3.516.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

3.516.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

3.516.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^2}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^2,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^2, x)
```

3.517 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3}\,dx$

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3.517.1 Optimal result

Integrand size = 30, antiderivative size = 406

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3}\,dx = \frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}(2ae+3bcx^2)\sqrt{a+bx^4}$$
$$+\frac{2}{35}x(5af+21bdx^2)\sqrt{a+bx^4}-\frac{(3c-ex^2)(a+bx^4)^{3/2}}{6x^2}-\frac{(7d-fx^2)(a+bx^4)^{3/2}}{7x}$$
$$+\frac{3}{4}a\sqrt{b}c\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}a^{3/2}e\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\Big(2\,\ldots}{5\sqrt{a+bx^4}}$$

output
```
-1/6*(-e*x^2+3*c)*(b*x^4+a)^(3/2)/x^2-1/7*(-f*x^2+7*d)*(b*x^4+a)^(3/2)/x-1
/2*a^(3/2)*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))+3/4*a*c*arctanh(x^2*b^(1/2)/
(b*x^4+a)^(1/2))*b^(1/2)+1/4*(3*b*c*x^2+2*a*e)*(b*x^4+a)^(1/2)+2/35*x*(21*
b*d*x^2+5*a*f)*(b*x^4+a)^(1/2)+12/5*a*d*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)
+x^2*b^(1/2))-12/5*a^(5/4)*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2)
)^2)^(1/2)/(b*x^4+a)^(1/2)+2/35*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(5*f*a^(1/2)+21*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*(
(b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^4+a)^(1/2)
```

3.517.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.48

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^3}\,dx = \frac{-3ac\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},-\frac{bx^4}{a}\right)+x\left(ex\sqrt{\right.}\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]
```

output
```
(-3*a*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] +
 x*(e*x*Sqrt[1 + (b*x^4)/a]*(Sqrt[a + b*x^4]*(4*a + b*x^4) - 3*a^(3/2)*Arc
Tanh[Sqrt[a + b*x^4]/Sqrt[a]]) - 6*a*d*Sqrt[a + b*x^4]*Hypergeometric2F1[-
3/2, -1/4, 3/4, -((b*x^4)/a)] + 6*a*f*x^2*Sqrt[a + b*x^4]*Hypergeometric2F
1[-3/2, 1/4, 5/4, -((b*x^4)/a)]))/(6*x^2*Sqrt[1 + (b*x^4)/a])
```

3.517.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{3/2}\left(c+dx+ex^2+fx^3\right)}{x^3}dx$$

$$\Big\downarrow \text{ 2372}$$

$$\int \left(\frac{\left(a+bx^4\right)^{3/2}\left(c+ex^2\right)}{x^3}+\frac{\left(a+bx^4\right)^{3/2}\left(d+fx^2\right)}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+21\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}e\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3}{4}a\sqrt{b}c\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{\left(a+bx^4\right)^{3/2}\left(3c-ex^2\right)}{6x^2}+\frac{1}{4}\sqrt{a+bx^4}\left(2ae+3bcx^2\right)-\frac{\left(a+bx^4\right)^{3/2}\left(7d-fx^2\right)}{7x}+\frac{2}{35}x\sqrt{a+bx^4}\left(5af+21bdx^2\right)+\frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]
```

output
```
(12*a*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*a*e +
 3*b*c*x^2)*Sqrt[a + b*x^4])/4 + (2*x*(5*a*f + 21*b*d*x^2)*Sqrt[a + b*x^4]
)/35 - ((3*c - e*x^2)*(a + b*x^4)^(3/2))/(6*x^2) - ((7*d - f*x^2)*(a + b*x
^4)^(3/2))/(7*x) + (3*a*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/
4 - (a^(3/2)*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*d
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)
*(21*Sqrt[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sq
rt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35
*b^(1/4)*Sqrt[a + b*x^4])
```

3.517.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.517.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.31 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.85

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{2x^2}-\frac{ad\sqrt{b\,x^4+a}}{x}+\frac{bf\,x^5\sqrt{b\,x^4+a}}{7}+\frac{be\,x^4\sqrt{b\,x^4+a}}{6}+\frac{bd\,x^3\sqrt{b\,x^4+a}}{5}+\frac{bc\,x^2\sqrt{b\,x^4+a}}{4}+\frac{3afx\sqrt{b\,x^4+a}}{7}+\frac{2ae}{}$
default	$f\left(\frac{b\,x^5\sqrt{b\,x^4+a}}{7}+\frac{3ax\sqrt{b\,x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+e\left(\frac{b\,x^4\sqrt{b\,x^4+a}}{6}+\frac{2a\sqrt{b\,x^4+a}}{3}-\frac{a^{\frac{3}{2}}\,l}{}\right.$
risch	$-\frac{a\sqrt{b\,x^4+a}\,(2dx+c)}{2x^2}+\frac{4a^2f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{bf\,x^5\sqrt{b\,x^4+a}}{7}+\frac{3afx\sqrt{b\,x^4+a}}{7}-\frac{e\sqrt{b\,x^4+a}\,(-b\,x^4+}{6}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*c*(b*x^4+a)^(1/2)/x^2-a*d*(b*x^4+a)^(1/2)/x+1/7*b*f*x^5*(b*x^4+a)^(
1/2)+1/6*b*e*x^4*(b*x^4+a)^(1/2)+1/5*b*d*x^3*(b*x^4+a)^(1/2)+1/4*b*c*x^2*(
b*x^4+a)^(1/2)+3/7*a*f*x*(b*x^4+a)^(1/2)+2/3*a*e*(b*x^4+a)^(1/2)+4/7*a^2*f
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+
3/4*a*b^(1/2)*c*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+12/5*I*a^(3/2)*b^(1/2)
*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^(3/2)*e*arctanh(a^(1/2)
/(b*x^4+a)^(1/2))
```

3.517.5 Fricas [F]

$$\int\frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^3}\,dx=\int\frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^3, x)
```

3.517.6 Sympy [A] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3}\,dx = -\frac{a^{\frac{3}{2}}c}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}e\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{a^{\frac{3}{2}}fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}bcx^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}bcx^2}{2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bdx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}bfx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{a^2e}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}c\,\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{a\sqrt{b}ex^2}{2\sqrt{\frac{a}{bx^4}+1}} + be\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**3,x)
```

output
```
-a**(3/2)*c/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*d*gamma(-1/4)*hyper((-1
/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**(3/2)*e
*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*f*x*gamma(1/4)*hyper((-1/2, 1/
4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*c*x**2*sq
rt(1 + b*x**4/a)/4 - sqrt(a)*b*c*x**2/(2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*d
*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(7/4)) + sqrt(a)*b*f*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4
*exp_polar(I*pi)/a)/(4*gamma(9/4)) + a**2*e/(2*sqrt(b)*x**2*sqrt(a/(b*x**4
) + 1)) + 3*a*sqrt(b)*c*asinh(sqrt(b)*x**2/sqrt(a))/4 + a*sqrt(b)*e*x**2/(
2*sqrt(a/(b*x**4) + 1)) + b*e*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a +
b*x**4)**(3/2)/(6*b), True))
```

3.517.7 Maxima [F]

$$\int \frac{(c + dx + ex^2 + fx^3)\,(a + bx^4)^{3/2}}{x^3}\,dx = \int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

3.517.8 Giac [F]

$$\int \frac{(c + dx + ex^2 + fx^3)\,(a + bx^4)^{3/2}}{x^3}\,dx = \int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

3.517.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx + ex^2 + fx^3)\,(a + bx^4)^{3/2}}{x^3}\,dx = \int \frac{(b\,x^4 + a)^{3/2}\,(f\,x^3 + e\,x^2 + d\,x + c)}{x^3}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^3,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^3, x)
```

3.518 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx$

3.518.1 Optimal result . . . 4029
3.518.2 Mathematica [C] (verified) . . . 4030
3.518.3 Rubi [A] (verified) . . . 4030
3.518.4 Maple [C] (verified) . . . 4032
3.518.5 Fricas [F] . . . 4032
3.518.6 Sympy [A] (verification not implemented) . . . 4033
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3.518.8 Giac [F] . . . 4034
3.518.9 Mupad [F(-1)] . . . 4034

3.518.1 Optimal result

Integrand size = 30, antiderivative size = 408

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx = \frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2(9ae-5bcx^2)\sqrt{a+bx^4}}{15x} + \frac{1}{4}(2af+3bdx^2)\sqrt{a+bx^4} - \frac{(5c-3ex^2)(a+bx^4)^{3/2}}{15x^3} - \frac{(3d-fx^2)(a+bx^4)^{3/2}}{6x^2} + \frac{3}{4}a\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{1}{2}a^{3/2}f\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\right.}{5\sqrt{a+bx^4}}$$

output
```
-1/15*(-3*e*x^2+5*c)*(b*x^4+a)^(3/2)/x^3-1/6*(-f*x^2+3*d)*(b*x^4+a)^(3/2)/
x^2-1/2*a^(3/2)*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))+3/4*a*d*arctanh(x^2*b^(
1/2)/(b*x^4+a)^(1/2))*b^(1/2)-2/15*(-5*b*c*x^2+9*a*e)*(b*x^4+a)^(1/2)/x+1/
4*(3*b*d*x^2+2*a*f)*(b*x^4+a)^(1/2)+12/5*a*e*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^
(1/2)+x^2*b^(1/2))-12/5*a^(5/4)*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)
*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^
(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+2/15*a^(3/4)*b^(1/4)*(cos(2*arctan(b^(1/4)
*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arc
tan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(9*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^
2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)
```

3.518.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.48

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^4}\,dx = \frac{-2ac\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{bx^4}{a}\right)-3adx\sqrt{\cdots}}{\cdots}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]
```

output
```
(-2*a*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] -
 3*a*d*x*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)]
+ x^2*(f*x*Sqrt[1 + (b*x^4)/a]*(Sqrt[a + b*x^4]*(4*a + b*x^4) - 3*a^(3/2)*
ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) - 6*a*e*Sqrt[a + b*x^4]*Hypergeometric2F
1[-3/2, -1/4, 3/4, -((b*x^4)/a)]))/(6*x^3*Sqrt[1 + (b*x^4)/a])
```

3.518.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}\,(c+dx+ex^2+fx^3)}{x^4}\,dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(\frac{(a+bx^4)^{3/2}\,(c+ex^2)}{x^4}+\frac{(a+bx^4)^{3/2}\,(d+fx^2)}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}f\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3}{4}a\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{2\sqrt{a+bx^4}(9ae-5bcx^2)}{15x}-\frac{(a+bx^4)^{3/2}(5c-3ex^2)}{15x^3}-\frac{(a+bx^4)^{3/2}(3d-fx^2)}{6x^2}+\frac{1}{4}\sqrt{a+bx^4}(2af+3bdx^2)+\frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]
```

output
```
(12*a*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*(9*a*e
 - 5*b*c*x^2)*Sqrt[a + b*x^4])/(15*x) + ((2*a*f + 3*b*d*x^2)*Sqrt[a + b*x^
4])/4 - ((5*c - 3*e*x^2)*(a + b*x^4)^(3/2))/(15*x^3) - ((3*d - f*x^2)*(a +
 b*x^4)^(3/2))/(6*x^2) + (3*a*Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x
^4]])/4 - (a^(3/2)*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(
1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a
^(3/4)*b^(1/4)*(5*Sqrt[b]*c + 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
 + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(15*Sqrt[a + b*x^4])
```

3.518.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.518.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.84

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{3x^3}-\frac{ad\sqrt{b\,x^4+a}}{2x^2}-\frac{ae\sqrt{b\,x^4+a}}{x}+\frac{bf\,x^4\sqrt{b\,x^4+a}}{6}+\frac{be\,x^3\sqrt{b\,x^4+a}}{5}+\frac{bd\,x^2\sqrt{b\,x^4+a}}{4}+\frac{bcx\sqrt{b\,x^4+a}}{3}+\frac{2af\sqrt{b\,x}}{3}$
default	$f\left(\frac{b\,x^4\sqrt{b\,x^4+a}}{6}+\frac{2a\sqrt{b\,x^4+a}}{3}-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2}\right)+c\left(-\frac{a\sqrt{b\,x^4+a}}{3x^3}+\frac{bx\sqrt{b\,x^4+a}}{3}+\frac{4ab\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i}{}}}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{}}\right.$
risch	$-\frac{a\sqrt{b\,x^4+a}\,(6e\,x^2+3dx+2c)}{6x^3}+\frac{4abc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{f\sqrt{b\,x^4+a}\,(-b\,x^4+2a)}{6}+\frac{be\,x^3\sqrt{b\,x^4+a}}{5}-\frac{3i}{}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*c*(b*x^4+a)^(1/2)/x^3-1/2*a*d*(b*x^4+a)^(1/2)/x^2-a*e*(b*x^4+a)^(1/
2)/x+1/6*b*f*x^4*(b*x^4+a)^(1/2)+1/5*b*e*x^3*(b*x^4+a)^(1/2)+1/4*b*d*x^2*(
b*x^4+a)^(1/2)+1/3*b*c*x*(b*x^4+a)^(1/2)+2/3*a*f*(b*x^4+a)^(1/2)+4/3*a*b*c
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+
3/4*a*b^(1/2)*d*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+12/5*I*a^(3/2)*e*b^(1/
2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^(3/2)*f*arctanh(a^(1/2)
/(b*x^4+a)^(1/2))
```

3.518.5 Fricas [F]

$$\int\frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^4}\,dx=\int\frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^4, x)
```

3.518.6 Sympy [A] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}}d}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{\sqrt{a}bcx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}bdx^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}bdx^2}{2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{a^2f}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{a\sqrt{b}fx^2}{2\sqrt{\frac{a}{bx^4}+1}} + bf\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**4,x)
```

output
```
a**(3/2)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**3*gamma(1/4)) - a**(3/2)*d/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)
*e*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*
gamma(3/4)) - a**(3/2)*f*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*b*c*x*g
amma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/
4)) + sqrt(a)*b*d*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*d*x**2/(2*sqrt(1 +
 b*x**4/a)) + sqrt(a)*b*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**
4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**2*f/(2*sqrt(b)*x**2*sqrt(a/(b*x**
4) + 1)) + 3*a*sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/4 + a*sqrt(b)*f*x**2/
(2*sqrt(a/(b*x**4) + 1)) + b*f*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a +
 b*x**4)**(3/2)/(6*b), True))
```

3.518.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

3.518.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

3.518.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^4}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^4,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^4, x)
```

3.519 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx$

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3.519.1 Optimal result

Integrand size = 30, antiderivative size = 386

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx = \frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$+\frac{3}{4}b(c+ex^2)\sqrt{a+bx^4}+\frac{2}{15}bx(5d+9fx^2)\sqrt{a+bx^4}-\frac{1}{12}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)(a+bx^4)^{3/2}$$
$$+\frac{3}{4}a\sqrt{b}e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{3}{4}\sqrt{a}bc\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)$$
$$-\frac{12a^{5/4}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}$$
$$+\frac{2a^{3/4}\sqrt[4]{b}\left(5\sqrt{b}d+9\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15\sqrt{a+bx^4}}$$

output
```
-1/12*(3*c/x^4+4*d/x^3+6*e/x^2+12*f/x)*(b*x^4+a)^(3/2)-3/4*b*c*arctanh((b*
x^4+a)^(1/2)/a^(1/2))*a^(1/2)+3/4*a*e*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))
*b^(1/2)+3/4*b*(e*x^2+c)*(b*x^4+a)^(1/2)+2/15*b*x*(9*f*x^2+5*d)*(b*x^4+a)^
(1/2)+12/5*a*f*x*b^(1/2)*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5*a^(5/4
)*b^(1/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4
)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2
)+2/15*a^(3/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(9*f*a^(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)
+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)
```

3.519.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.42

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx = \frac{\sqrt{a+bx^4}\left(-10a^3d\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{bx^4}{a}\right)+3x\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]
```

output
```
(Sqrt[a + b*x^4]*(-10*a^3*d*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a
)] + 3*x*(-5*a^3*e*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] - 10*a
^3*f*x*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^4)/a)] + b*c*x^2*(a + b*x
^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))
/(30*a^2*x^3*Sqrt[1 + (b*x^4)/a])
```

3.519.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{3/2}\left(c+dx+ex^2+fx^3\right)}{x^5}\,dx$$

$$\Big\downarrow \; 2364$$

$$-6b\int -\frac{\left(12fx^3+6ex^2+4dx+3c\right)\sqrt{bx^4+a}}{12x}dx-\frac{1}{12}\left(a+bx^4\right)^{3/2}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}b\int \frac{\left(12fx^3+6ex^2+4dx+3c\right)\sqrt{bx^4+a}}{x}dx-\frac{1}{12}\left(a+bx^4\right)^{3/2}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$$\Big\downarrow \; 2372$$

$$\frac{1}{2}b\int \left(\frac{\sqrt{bx^4+a}\left(6ex^2+3c\right)}{x}+\left(12fx^2+4d\right)\sqrt{bx^4+a}\right)dx-$$
$$\frac{1}{12}\left(a+bx^4\right)^{3/2}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}b\left(\frac{4a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{24a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)}{\ldots}\right.$$
$$\frac{1}{12}\left(a+bx^4\right)^{3/2}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]
```

output
```
-1/12*(((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*(a + b*x^4)^(3/2)) +
 (b*((24*a*f*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (3*(
c + e*x^2)*Sqrt[a + b*x^4])/2 + (4*x*(5*d + 9*f*x^2)*Sqrt[a + b*x^4])/15 +
 (3*a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (3*Sqrt[a]*c
*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (24*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (4*a^(3/4)*(5*Sqrt[b]*d
+ 9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]
*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a
 + b*x^4])))/2
```

3.519. $\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^5}\,dx$

3.519.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.519.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.89

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{4x^4}-\frac{ad\sqrt{b\,x^4+a}}{3x^3}-\frac{ae\sqrt{b\,x^4+a}}{2x^2}-\frac{af\sqrt{b\,x^4+a}}{x}+\frac{bf\,x^3\sqrt{b\,x^4+a}}{5}+\frac{be\,x^2\sqrt{b\,x^4+a}}{4}+\frac{bdx\sqrt{b\,x^4+a}}{3}+\frac{bc\sqrt{b\,x^4+a}}{2}$
default	$d\left(-\frac{a\sqrt{b\,x^4+a}}{3x^3}+\frac{bx\sqrt{b\,x^4+a}}{3}+\frac{4ab\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(-\frac{a\sqrt{b\,x^4+a}}{x}+\frac{\sqrt{b\,x^4+a}\,b\,x^3}{5}+\frac{12i}{}\right.$
risch	$-\frac{a\sqrt{b\,x^4+a}\,(12f\,x^3+6e\,x^2+4dx+3c)}{12x^4}+\frac{4bda\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{bf\,x^3\sqrt{b\,x^4+a}}{5}+\frac{12i\sqrt{b}\,f\,a^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{5\sqrt{}}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*c*(b*x^4+a)^(1/2)/x^4-1/3*a*d*(b*x^4+a)^(1/2)/x^3-1/2*a*e*(b*x^4+a)
^(1/2)/x^2-a*f*(b*x^4+a)^(1/2)/x+1/5*b*f*x^3*(b*x^4+a)^(1/2)+1/4*b*e*x^2*(
b*x^4+a)^(1/2)+1/3*b*d*x*(b*x^4+a)^(1/2)+1/2*b*c*(b*x^4+a)^(1/2)+4/3*b*d*a
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+
3/4*a*e*b^(1/2)*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+12/5*I*a^(3/2)*f*b^(1/
2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/4*a^(1/2)*b*c*arctanh(a^(1/
2)/(b*x^4+a)^(1/2))
```

3.519.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^5, x)
```

3.519.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.48 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^5}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}}e}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}\,bc\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{\sqrt{a}\,bdx\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}\,bex^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}\,bex^2}{2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}\,bfx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}c}{2x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}e\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{b^{\frac{3}{2}}cx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**5,x)
```

output
```
a**(3/2)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**3*gamma(1/4)) - a**(3/2)*e/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)
*f*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*
gamma(3/4)) - 3*sqrt(a)*b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/4 + sqrt(a)*b*d*
x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(5/4)) + sqrt(a)*b*e*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*e*x**2/(2*sqrt(
1 + b*x**4/a)) + sqrt(a)*b*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*
x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/
(4*x**2) + a*sqrt(b)*c/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*e*asinh
(sqrt(b)*x**2/sqrt(a))/4 + b**(3/2)*c*x**2/(2*sqrt(a/(b*x**4) + 1))
```

3.519. $\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^5}\,dx$

3.519.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^5}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
1/8*(3*sqrt(a)*b*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a
))) + 4*sqrt(b*x^4 + a)*b - 2*sqrt(b*x^4 + a)*a/x^4)*c + integrate((b*f*x^
6 + b*e*x^5 + b*d*x^4 + a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a)/x^4, x)
```

3.519.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^5}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

3.519.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^5}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^5}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^5,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^5, x)
```

3.520 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx$

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3.520.1 Optimal result

Integrand size = 30, antiderivative size = 387

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx = \frac{12b^{3/2}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b(9c-5ex^2)\sqrt{a+bx^4}}{15x} + \frac{3}{4}b(d+fx^2)\sqrt{a+bx^4} - \frac{1}{60}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)(a+bx^4)^{3/2} + \frac{3}{4}a\sqrt{b}f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}bd\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\right.}{5\sqrt{a+bx^4}}$$

output
```
-1/60*(12*c/x^5+15*d/x^4+20*e/x^3+30*f/x^2)*(b*x^4+a)^(3/2)-3/4*b*d*arctan
h((b*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+3/4*a*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^(
1/2))*b^(1/2)-2/15*b*(-5*e*x^2+9*c)*(b*x^4+a)^(1/2)/x+3/4*b*(f*x^2+d)*(b*x
^4+a)^(1/2)+12/5*b^(3/2)*c*x*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5*a^
(1/4)*b^(5/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*
(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^
(1/2)+2/15*a^(1/4)*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/
2*2^(1/2))*(5*e*a^(1/2)+9*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(
1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)
```

3.520.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.43

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx = \frac{\sqrt{a+bx^4}\left(-6a^3c\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},-\frac{bx^4}{a}\right)-10a\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]
```

output
```
(Sqrt[a + b*x^4]*(-6*a^3*c*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a
)] - 10*a^3*e*x^2*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] - 15*a^
3*f*x^3*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] + 3*b*d*x^5*(a +
b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]
))/(30*a^2*x^5*Sqrt[1 + (b*x^4)/a])
```

3.520.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^6}\,dx$$

$$\Big\downarrow \text{ 2364}$$

$$-6b\int -\frac{(30fx^3+20ex^2+15dx+12c)\sqrt{bx^4+a}}{60x^2}dx-\frac{1}{60}(a+bx^4)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{10}b\int \frac{(30fx^3+20ex^2+15dx+12c)\sqrt{bx^4+a}}{x^2}dx-\frac{1}{60}(a+bx^4)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

$$\Big\downarrow \text{ 2372}$$

$$\frac{1}{10}b\int \left(\frac{\sqrt{bx^4+a}\,(20ex^2+12c)}{x^2}+\frac{(30fx^2+15d)\sqrt{bx^4+a}}{x}\right)dx-\frac{1}{60}\left(a+bx^4\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{10}b\left(\frac{4\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{24\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x\right.}{}\right.$$
$$\frac{1}{60}\left(a+bx^4\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]
```

output
```
-1/60*(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*(a + b*x^4)^(3/
2)) + (b*((24*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - (4*(9
*c - 5*e*x^2)*Sqrt[a + b*x^4])/(3*x) + (15*(d + f*x^2)*Sqrt[a + b*x^4])/2
+ (15*a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (15*Sqrt[a
]*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (24*a^(1/4)*b^(1/4)*c*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (4*a^(1/4)*(9*Sqrt[b]*c +
 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a +
 b*x^4])))/10
```

3.520.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.520.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.50 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.89

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{5x^5}-\frac{ad\sqrt{b\,x^4+a}}{4x^4}-\frac{ae\sqrt{b\,x^4+a}}{3x^3}-\frac{af\sqrt{b\,x^4+a}}{2x^2}-\frac{7bc\sqrt{b\,x^4+a}}{5x}+\frac{bf\,x^2\sqrt{b\,x^4+a}}{4}+\frac{bex\sqrt{b\,x^4+a}}{3}+\frac{bd\sqrt{b\,x^4+a}}{2}$
default	$e\left(-\frac{a\sqrt{b\,x^4+a}}{3x^3}+\frac{bx\sqrt{b\,x^4+a}}{3}+\frac{4ab\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(\frac{b\,x^2\sqrt{b\,x^4+a}}{4}+\frac{3a\sqrt{b}\,\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4\right.}}{4}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(84bc\,x^4+30af\,x^3+20ae\,x^2+15adx+12ac)}{60x^5}+\frac{4bea\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{bf\,x^2\sqrt{b\,x^4+a}}{4}+\frac{3\sqrt{b}}{}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a*c*(b*x^4+a)^(1/2)/x^5-1/4*a*d*(b*x^4+a)^(1/2)/x^4-1/3*a*e*(b*x^4+a)
^(1/2)/x^3-1/2*a*f*(b*x^4+a)^(1/2)/x^2-7/5*b*c*(b*x^4+a)^(1/2)/x+1/4*b*f*x
^2*(b*x^4+a)^(1/2)+1/3*b*e*x*(b*x^4+a)^(1/2)+1/2*b*d*(b*x^4+a)^(1/2)+4/3*b
*e*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)+3/4*a*f*b^(1/2)*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+12/5*I*b^(3/2)*c*a
^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1
/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/4*a^(1/2)*b*d*arctanh(a
^(1/2)/(b*x^4+a)^(1/2))
```

3.520.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^6}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^6, x)
```

3.520.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.65 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}}f}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bc\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{\sqrt{a}bex\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}bfx^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}bfx^2}{2\sqrt{1+\frac{bx^4}{a}}} - \frac{a\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}d}{2x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{b^{\frac{3}{2}}dx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**6,x)
```

output
```
a**(3/2)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**5*gamma(-1/4)) + a**(3/2)*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4
,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**(3/2)*f/(2*x**2*sqr
t(1 + b*x**4/a)) + sqrt(a)*b*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x
**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*d*asinh(sqrt(a)/(sqr
t(b)*x**2))/4 + sqrt(a)*b*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4
*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*f*x**2*sqrt(1 + b*x**4/a)/4
 - sqrt(a)*b*f*x**2/(2*sqrt(1 + b*x**4/a)) - a*sqrt(b)*d*sqrt(a/(b*x**4) +
 1)/(4*x**2) + a*sqrt(b)*d/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*f*a
sinh(sqrt(b)*x**2/sqrt(a))/4 + b**(3/2)*d*x**2/(2*sqrt(a/(b*x**4) + 1))
```

3.520. $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx$

3.520.7 Maxima [F]

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^6} \, dx = \int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^6} \, dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

3.520.8 Giac [F]

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^6} \, dx = \int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^6} \, dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

3.520.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^6} \, dx = \int \frac{(b\,x^4 + a)^{3/2}\,(f\,x^3 + e\,x^2 + d\,x + c)}{x^6} \, dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^6,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^6, x)
```

3.521 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx$

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3.521.1 Optimal result

Integrand size = 30, antiderivative size = 392

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \frac{12b^{3/2}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2} - \frac{2b(9d-5fx^2)\sqrt{a+bx^4}}{15x} - \frac{1}{60}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)(a+bx^4)^{3/2} + \frac{1}{2}b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}be\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12\sqrt[4]{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\,a\right.}{5\sqrt{a+bx^4}}$$

output
```
-1/60*(10*c/x^6+12*d/x^5+15*e/x^4+20*f/x^3)*(b*x^4+a)^(3/2)+1/2*b^(3/2)*c*
arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))-3/4*b*e*arctanh((b*x^4+a)^(1/2)/a^(1/
2))*a^(1/2)-1/4*b*(-3*e*x^2+2*c)*(b*x^4+a)^(1/2)/x^2-2/15*b*(-5*f*x^2+9*d)
*(b*x^4+a)^(1/2)/x+12/5*b^(3/2)*d*x*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-
12/5*a^(1/4)*b^(5/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*
x^4+a)^(1/2)+2/15*a^(1/4)*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(5*f*a^(1/2)+9*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+
a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)
```

3.521.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.42

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \frac{\sqrt{a+bx^4}\left(-5a^3c\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{bx^4}{a}\right)-6a^3\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]
```

output
```
(Sqrt[a + b*x^4]*(-5*a^3*c*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^4)/a
)] - 6*a^3*d*x*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)] - 10*a^3*
f*x^3*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] + 3*b*e*x^6*(a + b*
x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))
/(30*a^2*x^6*Sqrt[1 + (b*x^4)/a])
```

3.521.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^7}dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(20fx^3+15ex^2+12dx+10c)\sqrt{bx^4+a}}{60x^3}dx-\frac{1}{60}(a+bx^4)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}b\int \frac{(20fx^3+15ex^2+12dx+10c)\sqrt{bx^4+a}}{x^3}dx-\frac{1}{60}(a+bx^4)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

$$\Big\downarrow \text{2372}$$

$$\frac{1}{10}b\int\left(\frac{\sqrt{bx^4+a}(15ex^2+10c)}{x^3}+\frac{(20fx^2+12d)\sqrt{bx^4+a}}{x^2}\right)dx-\frac{1}{60}\left(a+bx^4\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

$\Big\downarrow$ 2009

$$\frac{1}{10}b\left(\frac{4\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+9\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{24\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x\right.}{}\right.$$
$$\frac{1}{60}\left(a+bx^4\right)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]
```

output
```
-1/60*(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*(a + b*x^4)^(3/
2)) + (b*((24*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - (5*(2
*c - 3*e*x^2)*Sqrt[a + b*x^4])/(2*x^2) - (4*(9*d - 5*f*x^2)*Sqrt[a + b*x^4
])/(3*x) + 5*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - (15*Sqrt[a
]*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (24*a^(1/4)*b^(1/4)*d*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (4*a^(1/4)*(9*Sqrt[b]*d +
 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a +
 b*x^4])))/10
```

3.521.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.521.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.88

method	result
elliptic	$-\frac{ac\sqrt{b\,x^{4}+a}}{6x^{6}}-\frac{ad\sqrt{b\,x^{4}+a}}{5x^{5}}-\frac{ae\sqrt{b\,x^{4}+a}}{4x^{4}}-\frac{af\sqrt{b\,x^{4}+a}}{3x^{3}}-\frac{2bc\sqrt{b\,x^{4}+a}}{3x^{2}}-\frac{7bd\sqrt{b\,x^{4}+a}}{5x}+\frac{bfx\sqrt{b\,x^{4}+a}}{3}+\frac{be\sqrt{b\,x^{4}+a}}{2}+$
default	$c\left(\frac{b^{\frac{3}{2}}\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2}-\frac{a\sqrt{b\,x^{4}+a}}{6x^{6}}-\frac{2b\sqrt{b\,x^{4}+a}}{3x^{2}}\right)+f\left(-\frac{a\sqrt{b\,x^{4}+a}}{3x^{3}}+\frac{bx\sqrt{b\,x^{4}+a}}{3}+\frac{4ab\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+}}\right.$
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(84bd\,x^{5}+40bc\,x^{4}+20af\,x^{3}+15ae\,x^{2}+12adx+10ac\right)}{60x^{6}}+\frac{4bfa\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{bfx\sqrt{b\,x^{4}+a}}{3}-$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*c*(b*x^4+a)^(1/2)/x^6-1/5*a*d*(b*x^4+a)^(1/2)/x^5-1/4*a*e*(b*x^4+a)
^(1/2)/x^4-1/3*a*f*(b*x^4+a)^(1/2)/x^3-2/3*b*c*(b*x^4+a)^(1/2)/x^2-7/5*b*d
*(b*x^4+a)^(1/2)/x+1/3*b*f*x*(b*x^4+a)^(1/2)+1/2*b*e*(b*x^4+a)^(1/2)+4/3*b
*f*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)+1/2*b^(3/2)*c*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+12/5*I*b^(3/2)*d*a^(
1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/4*a^(1/2)*e*b*arctanh(a^(
1/2)/(b*x^4+a)^(1/2))
```

3.521.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^7, x)
```

3.521.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.25 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}bc}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bd\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{\sqrt{a}bfx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{a\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}e}{2x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} + \frac{b^{\frac{3}{2}}ex^2}{2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^2cx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**7,x)
```

output
```
a**(3/2)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**5*gamma(-1/4)) + a**(3/2)*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4
,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*c/(2*x**2*sq
rt(1 + b*x**4/a)) + sqrt(a)*b*d*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*
x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*e*asinh(sqrt(a)/(sq
rt(b)*x**2))/4 + sqrt(a)*b*f*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**
4*exp_polar(I*pi)/a)/(4*gamma(5/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(6*
x**4) - a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*e/(2*x**2*sq
rt(a/(b*x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*c*asinh
(sqrt(b)*x**2/sqrt(a))/2 + b**(3/2)*e*x**2/(2*sqrt(a/(b*x**4) + 1)) - b**2
*c*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

3.521. $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx$

3.521.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/12*(3*b^(3/2)*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^
4 + a)/x^2)) + 6*sqrt(b*x^4 + a)*b/x^2 + 2*(b*x^4 + a)^(3/2)/x^6)*c + inte
grate((b*f*x^6 + b*e*x^5 + b*d*x^4 + a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a
)/x^6, x)
```

3.521.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)
```

3.521.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^7}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^7}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^7,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^7, x)
```

3.522 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8} dx$

3.522.1 Optimal result . . . 4056
3.522.2 Mathematica [C] (verified) . . . 4057
3.522.3 Rubi [A] (verified) . . . 4057
3.522.4 Maple [C] (verified) . . . 4059
3.522.5 Fricas [F] . . . 4060
3.522.6 Sympy [C] (verification not implemented) . . . 4060
3.522.7 Maxima [F] . . . 4062
3.522.8 Giac [F] . . . 4062
3.522.9 Mupad [F(-1)] . . . 4062

3.522.1 Optimal result

Integrand size = 30, antiderivative size = 412

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8} dx = -\frac{12be\sqrt{a+bx^4}}{5x} + \frac{12b^{3/2}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2b(5c-21ex^2)\sqrt{a+bx^4}}{35x^3} - \frac{b(2d-3fx^2)\sqrt{a+bx^4}}{4x^2} - \frac{1}{420}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)(a+bx^4)^{3/2} + \frac{1}{2}b^{3/2}d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}bf\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12\sqrt[4]{a}b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2 a\right.}{5\sqrt{a+bx^4}}$$

output
```
-1/420*(60*c/x^7+70*d/x^6+84*e/x^5+105*f/x^4)*(b*x^4+a)^(3/2)+1/2*b^(3/2)*
d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))-3/4*b*f*arctanh((b*x^4+a)^(1/2)/a^(
1/2))*a^(1/2)-12/5*b*e*(b*x^4+a)^(1/2)/x-2/35*b*(-21*e*x^2+5*c)*(b*x^4+a)^
(1/2)/x^3-1/4*b*(-3*f*x^2+2*d)*(b*x^4+a)^(1/2)/x^2+12/5*b^(3/2)*e*x*(b*x^4
+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5*a^(1/4)*b^(5/4)*e*(cos(2*arctan(b^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*a
rctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a
^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1/2)+2/35*b^(5/4)*(cos(2*arctan(b^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(
2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(21*e*a^(1/2)+5*c*b^(1/2))*(a^(1
/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^4+
a)^(1/2)
```

3.522.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.40

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx = \frac{\sqrt{a+bx^4}\left(-30a^3c\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{bx^4}{a}\right)+7\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]
```

output
```
(Sqrt[a + b*x^4]*(-30*a^3*c*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4)/
a)] + 7*x*(-5*a^3*d*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^4)/a)] - 6*
a^3*e*x*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)] + 3*b*f*x^6*(a +
 b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a
])))/(210*a^2*x^7*Sqrt[1 + (b*x^4)/a])
```

3.522.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^8}\,dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(105fx^3+84ex^2+70dx+60c)\sqrt{bx^4+a}}{420x^4}dx-\frac{1}{420}(a+bx^4)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{70}b\int \frac{(105fx^3+84ex^2+70dx+60c)\sqrt{bx^4+a}}{x^4}dx-\frac{1}{420}(a+bx^4)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

$$\Big\downarrow \; 2372$$

$$\frac{1}{70}b\int \left(\frac{\sqrt{bx^4+a}(84ex^2+60c)}{x^4}+\frac{(105fx^2+70d)\sqrt{bx^4+a}}{x^3}\right)dx-\frac{1}{420}(a+bx^4)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{70}b\left(\frac{4\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{168\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{\right.}\right.}{}\right.$$
$$\frac{1}{420}(a+bx^4)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]
```

output
```
-1/420*(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*(a + b*x^4)^(
3/2)) + (b*((-168*e*Sqrt[a + b*x^4])/x + (168*Sqrt[b]*e*x*Sqrt[a + b*x^4])
/(Sqrt[a] + Sqrt[b]*x^2) - (4*(5*c - 21*e*x^2)*Sqrt[a + b*x^4])/x^3 - (35*
(2*d - 3*f*x^2)*Sqrt[a + b*x^4])/(2*x^2) + 35*Sqrt[b]*d*ArcTanh[(Sqrt[b]*x
^2)/Sqrt[a + b*x^4]] - (105*Sqrt[a]*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2
- (168*a^(1/4)*b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a +
 b*x^4] + (4*b^(1/4)*(5*Sqrt[b]*c + 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(a^(1/4)*Sqrt[a + b*x^4])))/70
```

3.522.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.522.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(588 b e\,x^{6}+280 b d\,x^{5}+180 b c\,x^{4}+105 a f\,x^{3}+84 a e\,x^{2}+70 a d x+60 a c\right)}{420 x^{7}}+\frac{b\left(\frac{40 b c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+35 f\right.}{}$
elliptic	$-\frac{a c\sqrt{b\,x^{4}+a}}{7 x^{7}}-\frac{a d\sqrt{b\,x^{4}+a}}{6 x^{6}}-\frac{a e\sqrt{b\,x^{4}+a}}{5 x^{5}}-\frac{a f\sqrt{b\,x^{4}+a}}{4 x^{4}}-\frac{3 b c\sqrt{b\,x^{4}+a}}{7 x^{3}}-\frac{2 b d\sqrt{b\,x^{4}+a}}{3 x^{2}}-\frac{7 b e\sqrt{b\,x^{4}+a}}{5 x}+\frac{b f\sqrt{b\,x^{4}+a}}{2}+$
default	$d\left(\frac{b^{\frac{3}{2}}\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2}-\frac{a\sqrt{b\,x^{4}+a}}{6 x^{6}}-\frac{2 b\sqrt{b\,x^{4}+a}}{3 x^{2}}\right)+c\left(-\frac{a\sqrt{b\,x^{4}+a}}{7 x^{7}}-\frac{3 b\sqrt{b\,x^{4}+a}}{7 x^{3}}+\frac{4 b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/420*(b*x^4+a)^(1/2)*(588*b*e*x^6+280*b*d*x^5+180*b*c*x^4+105*a*f*x^3+84
*a*e*x^2+70*a*d*x+60*a*c)/x^7+1/70*b*(40*b*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+35*f*(b*x^4+a)^(1/2)+35*b^(1/2
)*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))+168*I*b^(1/2)*e*a^(1/2)/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I
/a^(1/2)*b^(1/2))^(1/2),I))-105/2*a^(1/2)*f*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1
/2))/x^2))
```

3.522.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^8}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^8, x)
```

3.522.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.32 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}bd}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}be\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} - \frac{a\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{a\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}f}{2x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} + \frac{b^{\frac{3}{2}}fx^2}{2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^2dx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**8,x)
```

output
```
a**(3/2)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**7*gamma(-3/4)) + a**(3/2)*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/
4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*c*gamma(-3
/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/
4)) - sqrt(a)*b*d/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*gamma(-1/4)*hy
per((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*s
qrt(a)*b*f*asinh(sqrt(a)/(sqrt(b)*x**2))/4 - a*sqrt(b)*d*sqrt(a/(b*x**4) +
 1)/(6*x**4) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*f/(2*
x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*
d*asinh(sqrt(b)*x**2/sqrt(a))/2 + b**(3/2)*f*x**2/(2*sqrt(a/(b*x**4) + 1))
 - b**2*d*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

3.522. $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx$

3.522.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^8}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

3.522.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^8}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

3.522.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^8}\,dx=\int \frac{\left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)}{x^8}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^8,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^8, x)
```

3.523 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx$

3.523.1 Optimal result . . . 4063
3.523.2 Mathematica [C] (verified) . . . 4064
3.523.3 Rubi [A] (verified) . . . 4064
3.523.4 Maple [C] (verified) . . . 4068
3.523.5 Fricas [F] . . . 4068
3.523.6 Sympy [C] (verification not implemented) . . . 4069
3.523.7 Maxima [F] . . . 4070
3.523.8 Giac [F] . . . 4070
3.523.9 Mupad [F(-1)] . . . 4070

3.523.1 Optimal result

Integrand size = 30, antiderivative size = 377

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = -\frac{1}{560}b\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\sqrt{a+bx^4} + \frac{12b^{3/2}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{1}{840}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)(a+bx^4)^{3/2} + \frac{1}{2}b^{3/2}e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3b^2c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{12\sqrt[4]{a}b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\right.}{5\sqrt{a+bx^4}}$$

output
```
-1/840*(105*c/x^8+120*d/x^7+140*e/x^6+168*f/x^5)*(b*x^4+a)^(3/2)+1/2*b^(3/
2)*e*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))-3/16*b^2*c*arctanh((b*x^4+a)^(1/
2)/a^(1/2))/a^(1/2)-1/560*b*(105*c/x^4+160*d/x^3+280*e/x^2+672*f/x)*(b*x^4
+a)^(1/2)+12/5*b^(3/2)*f*x*(b*x^4+a)^(1/2)/(a^(1/2)+x^2*b^(1/2))-12/5*a^(1
/4)*b^(5/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/(b*x^4+a)^(1
/2)+2/35*b^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b
^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))
*(21*f*a^(1/2)+5*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*
b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^4+a)^(1/2)
```

3.523.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.46

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\left(240a^2dx\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{bx^4}{a}\right)+7\left(15c\left(a(2a+5bx^4)\sqrt{1+\frac{bx^4}{a}}+3b^2x^8\text{ar}\right.\right.\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]
```

output
```
-1/1680*(Sqrt[a + b*x^4]*(240*a^2*d*x*Hypergeometric2F1[-7/4, -3/2, -3/4,
-((b*x^4)/a)] + 7*(15*c*(a*(2*a + 5*b*x^4)*Sqrt[1 + (b*x^4)/a] + 3*b^2*x^8
*ArcTanh[Sqrt[1 + (b*x^4)/a]]) + 40*a^2*e*x^2*Hypergeometric2F1[-3/2, -3/2
, -1/2, -((b*x^4)/a)] + 48*a^2*f*x^3*Hypergeometric2F1[-3/2, -5/4, -1/4, -
((b*x^4)/a)])))/(a*x^8*Sqrt[1 + (b*x^4)/a])
```

3.523.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2364, 27, 2364, 27, 2371, 798, 73, 221, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^9}\,dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(168fx^3+140ex^2+120dx+105c)\sqrt{bx^4+a}}{840x^5}dx - \frac{1}{840}(a+bx^4)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{140}b\int \frac{(168fx^3+140ex^2+120dx+105c)\sqrt{bx^4+a}}{x^5}dx-\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{ 2364}$$

$$\frac{1}{140}b\left(-2b\int -\frac{672fx^3+280ex^2+160dx+105c}{4x\sqrt{bx^4+a}}dx-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\right)-\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{140}b\left(\frac{1}{2}b\int \frac{672fx^3+280ex^2+160dx+105c}{x\sqrt{bx^4+a}}dx-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\right)-\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{ 2371}$$

$$\frac{1}{140}b\left(\frac{1}{2}b\left(105c\int \frac{1}{x\sqrt{bx^4+a}}dx+\int \frac{672fx^2+280ex+160d}{\sqrt{bx^4+a}}dx\right)-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\right)-\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{140}b\left(\frac{1}{2}b\left(\frac{105}{4}c\int \frac{1}{x^4\sqrt{bx^4+a}}dx^4+\int \frac{672fx^2+280ex+160d}{\sqrt{bx^4+a}}dx\right)-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\right)-\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{140}b\left(\frac{1}{2}b\left(\frac{105c\int \frac{1}{\frac{x^8}{b}-\frac{a}{b}}d\sqrt{bx^4+a}}{2b}+\int \frac{672fx^2+280ex+160d}{\sqrt{bx^4+a}}dx\right)-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\right)-\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{ 221}$$

3.523. $\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^9}dx$

$$\frac{1}{140}b\left(\frac{1}{2}b\left(\int\frac{672fx^2+280ex+160d}{\sqrt{bx^4+a}}dx-\frac{105c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672}{x}\right.\right.$$
$$\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$\downarrow$ 2424

$$\frac{1}{140}b\left(\frac{1}{2}b\left(\int\left(\frac{280ex}{\sqrt{bx^4+a}}+\frac{672fx^2+160d}{\sqrt{bx^4+a}}\right)dx-\frac{105c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}\right.\right.$$
$$\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$\downarrow$ 2009

$$\frac{1}{140}b\left(\frac{1}{2}b\left(\frac{16\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}-\frac{672\sqrt[4]{a}f\left(\sqrt{a}+\right.}{}\right.\right.$$
$$\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]
```

output
```
-1/840*(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*(a + b*x^4
)^(3/2)) + (b*(-1/4*(((105*c)/x^4 + (160*d)/x^3 + (280*e)/x^2 + (672*f)/x)
*Sqrt[a + b*x^4]) + (b*((672*f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt
[b]*x^2)) + (140*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/Sqrt[b] - (105*
c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (672*a^(1/4)*f*(Sqrt[a]
+ Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (16*(5*Sqrt[b]
*d + 21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*b^(3/
4)*Sqrt[a + b*x^4])))/2))/140
```

3.523. $\int\frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx$

3.523.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.523. $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx$

3.523.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(2352bf\,x^{7}+1120be\,x^{6}+720bd\,x^{5}+525bc\,x^{4}+336af\,x^{3}+280ae\,x^{2}+240adx+210ac)}{1680x^{8}}+\frac{b^{2}\left(\frac{160d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.}{}$
elliptic	$-\frac{ac\sqrt{b\,x^{4}+a}}{8x^{8}}-\frac{ad\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{ae\sqrt{b\,x^{4}+a}}{6x^{6}}-\frac{af\sqrt{b\,x^{4}+a}}{5x^{5}}-\frac{5bc\sqrt{b\,x^{4}+a}}{16x^{4}}-\frac{3bd\sqrt{b\,x^{4}+a}}{7x^{3}}-\frac{2be\sqrt{b\,x^{4}+a}}{3x^{2}}-\frac{7bf\sqrt{b\,x^{4}+a}}{5x}$
default	$e\left(\frac{b^{\frac{3}{2}}\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2}-\frac{a\sqrt{b\,x^{4}+a}}{6x^{6}}-\frac{2b\sqrt{b\,x^{4}+a}}{3x^{2}}\right)+d\left(-\frac{a\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{3b\sqrt{b\,x^{4}+a}}{7x^{3}}+\frac{4b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(b*x^4+a)^(1/2)*(2352*b*f*x^7+1120*b*e*x^6+720*b*d*x^5+525*b*c*x^4
+336*a*f*x^3+280*a*e*x^2+240*a*d*x+210*a*c)/x^8+1/280*b^2*(160*d/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)
^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+672*I*f*a^
(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1
/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+140*e*ln(x^2*b^(1/
2)+(b*x^4+a)^(1/2))/b^(1/2)-105/2*c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1
/2))/x^2))
```

3.523.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^9}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^9, x)
```

3.523.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.50 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \frac{a^{\frac{3}{2}} d\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, -\frac{1}{2} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{a^{\frac{3}{2}} f\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a} bd\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a} be}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a} bf\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^2 c}{8\sqrt{b} x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3a\sqrt{b} c}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b} e\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}} c\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}} c}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}} e\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}} e \operatorname{asinh}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right)}{2} - \frac{3b^2 c \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b} x^2}\right)}{16\sqrt{a}} - \frac{b^2 e x^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**9,x)
```

output
```
a**(3/2)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**7*gamma(-3/4)) + a**(3/2)*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/
4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*d*gamma(-3
/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/
4)) - sqrt(a)*b*e/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*gamma(-1/4)*hy
per((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**
2*c/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*c/(16*x**6*sqrt(a
/(b*x**4) + 1)) - a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*c*s
qrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*c/(16*x**2*sqrt(a/(b*x**4) + 1)) -
 b**(3/2)*e*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*e*asinh(sqrt(b)*x**2/sqrt(a)
)/2 - 3*b**2*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)) - b**2*e*x**2/(2
*sqrt(a)*sqrt(1 + b*x**4/a))
```

3.523.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
1/32*(3*b^2*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/s
qrt(a) - 2*(5*(b*x^4 + a)^(3/2)*b^2 - 3*sqrt(b*x^4 + a)*a*b^2)/((b*x^4 + a
)^2 - 2*(b*x^4 + a)*a + a^2))*c + integrate((b*f*x^6 + b*e*x^5 + b*d*x^4 +
 a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a)/x^8, x)
```

3.523.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

3.523.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^9}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^9,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^9, x)
```

3.524 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx$

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3.524.2 Mathematica [C] (verified) . . . 4072
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3.524.4 Maple [C] (verified) . . . 4075
3.524.5 Fricas [F] . . . 4075
3.524.6 Sympy [C] (verification not implemented) . . . 4076
3.524.7 Maxima [F] . . . 4077
3.524.8 Giac [F] . . . 4077
3.524.9 Mupad [F(-1)] . . . 4077

3.524.1 Optimal result

Integrand size = 30, antiderivative size = 405

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx = -\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680} - \frac{4b^2c\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2} + \frac{1}{2}b^{3/2}f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3b^2d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{4b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\ldots}{\sqrt[4]{\ldots}}\right.\right.}{15a^{3/4}\sqrt{a+bx^4}}$$

output
```
-1/504*(56*c/x^9+63*d/x^8+72*e/x^7+84*f/x^6)*(b*x^4+a)^(3/2)+1/2*b^(3/2)*f
*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))-3/16*b^2*d*arctanh((b*x^4+a)^(1/2)/a
^(1/2))/a^(1/2)-1/1680*b*(224*c/x^5+315*d/x^4+480*e/x^3+840*f/x^2)*(b*x^4+
a)^(1/2)-4/15*b^2*c*(b*x^4+a)^(1/2)/a/x+4/15*b^(5/2)*c*x*(b*x^4+a)^(1/2)/a
/(a^(1/2)+x^2*b^(1/2))-4/15*b^(9/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2
))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)+2/105*b^(7/4)*(cos(2*arctan(b^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan
(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(15*e*a^(1/2)+7*c*b^(1/2))*(a^(1/2)+x^2*
b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
```

3.524.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.43

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx =$$
$$-\frac{\sqrt{a+bx^4}\left(112a^2c\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2},-\frac{5}{4},-\frac{bx^4}{a}\right)+3x\left(48a^2ex\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\right.\right.\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]
```

output
```
-1/1008*(Sqrt[a + b*x^4]*(112*a^2*c*Hypergeometric2F1[-9/4, -3/2, -5/4, -(
(b*x^4)/a)] + 3*x*(48*a^2*e*x*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4
)/a)] + 7*(3*a*d*(2*a + 5*b*x^4)*Sqrt[1 + (b*x^4)/a] + 9*b^2*d*x^8*ArcTanh
[Sqrt[1 + (b*x^4)/a]] + 8*a^2*f*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -(
(b*x^4)/a)]))))/(a*x^9*Sqrt[1 + (b*x^4)/a])
```

3.524.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2364, 27, 2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^{10}}\,dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(84fx^3+72ex^2+63dx+56c)\sqrt{bx^4+a}}{504x^6}dx-\frac{1}{504}(a+bx^4)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{84}b\int \frac{\left(84fx^3+72ex^2+63dx+56c\right)\sqrt{bx^4+a}}{x^6}dx-\frac{1}{504}\left(a+bx^4\right)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$\big\downarrow$ 2364

$$\frac{1}{84}b\left(-2b\int -\frac{840fx^3+480ex^2+315dx+224c}{20x^2\sqrt{bx^4+a}}dx-\frac{1}{20}\sqrt{a+bx^4}\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\right)-\frac{1}{504}\left(a+bx^4\right)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$\big\downarrow$ 27

$$\frac{1}{84}b\left(\frac{1}{10}b\int \frac{840fx^3+480ex^2+315dx+224c}{x^2\sqrt{bx^4+a}}dx-\frac{1}{20}\sqrt{a+bx^4}\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\right)-\frac{1}{504}\left(a+bx^4\right)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$\big\downarrow$ 2372

$$\frac{1}{84}b\left(\frac{1}{10}b\int \left(\frac{480ex^2+224c}{x^2\sqrt{bx^4+a}}+\frac{840fx^2+315d}{x\sqrt{bx^4+a}}\right)dx-\frac{1}{20}\sqrt{a+bx^4}\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\right)-\frac{1}{504}\left(a+bx^4\right)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$\big\downarrow$ 2009

$$\frac{1}{84}b\left(\frac{1}{10}b\left(\frac{16\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}e+7\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{224\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{}\right.}{}\right.\right.$$

$$\frac{1}{504}\left(a+bx^4\right)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

input

```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]
```

3.524. $\int \frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^{10}}\,dx$

output
```
-1/504*(((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*(a + b*x^4)^(3
/2)) + (b*(-1/20*(((224*c)/x^5 + (315*d)/x^4 + (480*e)/x^3 + (840*f)/x^2)*
Sqrt[a + b*x^4]) + (b*((-224*c*Sqrt[a + b*x^4])/(a*x) + (224*Sqrt[b]*c*x*S
qrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (420*f*ArcTanh[(Sqrt[b]*x^2)
/Sqrt[a + b*x^4]])/Sqrt[b] - (315*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*S
qrt[a]) - (224*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)
*Sqrt[a + b*x^4]) + (16*(7*Sqrt[b]*c + 15*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])))/10))/84
```

3.524.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.524. $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx$

3.524.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(1344 b^{2} c\,x^{8}+3360 a b f\,x^{7}+2160 a e b\,x^{6}+1575 x^{5} d b a+1232 a b c\,x^{4}+840 a^{2} f\,x^{3}+720 a^{2} e\,x^{2}+630 a^{2} d x+560 a^{2} c\right)}{5040 x^{9} a}+\frac{b^{2}\left(\frac{480 a}{\ }\right.}{}$
elliptic	$-\frac{a c\sqrt{b\,x^{4}+a}}{9 x^{9}}-\frac{a d\sqrt{b\,x^{4}+a}}{8 x^{8}}-\frac{a e\sqrt{b\,x^{4}+a}}{7 x^{7}}-\frac{a f\sqrt{b\,x^{4}+a}}{6 x^{6}}-\frac{11 b c\sqrt{b\,x^{4}+a}}{45 x^{5}}-\frac{5 b d\sqrt{b\,x^{4}+a}}{16 x^{4}}-\frac{3 b e\sqrt{b\,x^{4}+a}}{7 x^{3}}-\frac{2 b f\sqrt{b\,x^{4}+a}}{3 x^{2}}$
default	$f\left(\frac{b^{\frac{3}{2}}\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2}-\frac{a\sqrt{b\,x^{4}+a}}{6 x^{6}}-\frac{2 b\sqrt{b\,x^{4}+a}}{3 x^{2}}\right)+e\left(-\frac{a\sqrt{b\,x^{4}+a}}{7 x^{7}}-\frac{3 b\sqrt{b\,x^{4}+a}}{7 x^{3}}+\frac{4 b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/5040*(b*x^4+a)^(1/2)*(1344*b^2*c*x^8+3360*a*b*f*x^7+2160*a*b*e*x^6+1575
*a*b*d*x^5+1232*a*b*c*x^4+840*a^2*f*x^3+720*a^2*e*x^2+630*a^2*d*x+560*a^2*
c)/x^9/a+1/840/a*b^2*(480*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*b^(1/2))^(1/2),I)+224*I*b^(1/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(
1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2
)*b^(1/2))^(1/2),I))+420*a*f*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)-315/2
*a^(1/2)*d*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

3.524.5 Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{10}}\,dx=\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)}{x^{10}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^10, x)
```

3.524.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.79 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}bf}{2x^2\sqrt{1+\frac{bx^4}{a}}} - \frac{a^2d}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3a\sqrt{b}d}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}d}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} - \frac{3b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}} - \frac{b^2fx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**10,x)
```

output
```
a**(3/2)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**9*gamma(-5/4)) + a**(3/2)*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/
4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*c*gamma(-5
/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-
1/4)) + sqrt(a)*b*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*f/(2*x**2*sqrt(1 + b*x**4/a))
 - a**2*d/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*d/(16*x**6*s
qrt(a/(b*x**4) + 1)) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2
)*d*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d/(16*x**2*sqrt(a/(b*x**4) +
1)) - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*f*asinh(sqrt(b)*x**2/sq
rt(a))/2 - 3*b**2*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)) - b**2*f*x*
*2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

3.524.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{10}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

3.524.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{10}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

3.524.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{10}}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^10,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^10, x)
```

3.525 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx$

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3.525.1 Optimal result

Integrand size = 30, antiderivative size = 399

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx = -\frac{b\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\sqrt{a+bx^4}}{1680} - \frac{b^2c\sqrt{a+bx^4}}{10ax^2} - \frac{4b^2d\sqrt{a+bx^4}}{15ax} + \frac{4b^{5/2}dx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)(a+bx^4)^{3/2}}{2520} - \frac{3b^2e\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{4b^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} + \frac{2b^{7/4}\left(7\sqrt{b}d+15\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}$$

output
```
-1/2520*(252*c/x^10+280*d/x^9+315*e/x^8+360*f/x^7)*(b*x^4+a)^(3/2)-3/16*b^
2*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/1680*b*(168*c/x^6+224*d/x^5
+315*e/x^4+480*f/x^3)*(b*x^4+a)^(1/2)-1/10*b^2*c*(b*x^4+a)^(1/2)/a/x^2-4/1
5*b^2*d*(b*x^4+a)^(1/2)/a/x+4/15*b^(5/2)*d*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^
2*b^(1/2))-4/15*b^(9/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2
*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/
a^(3/4)/(b*x^4+a)^(1/2)+2/105*b^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/2*2^(1/2))*(15*f*a^(1/2)+7*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b
*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)
```

3.525.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.43

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\left(63\sqrt{1+\frac{bx^4}{a}}(8b^2cx^8+2a^2(4c+5ex^2)+abx^4(16c+25ex^2))+945b^2ex^{10}\operatorname{arctanh}\left(\sqrt{1+\frac{bx^4}{a}}\right)\right.}{5040ax^{1}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]
```

output
```
-1/5040*(Sqrt[a + b*x^4]*(63*Sqrt[1 + (b*x^4)/a]*(8*b^2*c*x^8 + 2*a^2*(4*c
 + 5*e*x^2) + a*b*x^4*(16*c + 25*e*x^2)) + 945*b^2*e*x^10*ArcTanh[Sqrt[1 +
 (b*x^4)/a]] + 560*a^2*d*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/a
)] + 720*a^2*f*x^3*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4)/a)]))/(a*
x^10*Sqrt[1 + (b*x^4)/a])
```

3.525.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2364, 27, 2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^{11}}dx$$

$$\Big\downarrow \text{ 2364}$$

$$-6b\int -\frac{(360fx^3+315ex^2+280dx+252c)\sqrt{bx^4+a}}{2520x^7}dx-\frac{(a+bx^4)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{420}b\int \frac{(360fx^3+315ex^2+280dx+252c)\sqrt{bx^4+a}}{x^7}dx-\frac{(a+bx^4)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

$$\Big\downarrow \text{ 2364}$$

$$\frac{1}{420}b\left(-2b\int -\frac{480fx^3+315ex^2+224dx+168c}{4x^3\sqrt{bx^4+a}}dx-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\right)-\frac{(a+bx^4)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{420}b\left(\frac{1}{2}b\int \frac{480fx^3+315ex^2+224dx+168c}{x^3\sqrt{bx^4+a}}dx-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\right)-\frac{(a+bx^4)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

$$\Big\downarrow \text{ 2372}$$

$$\frac{1}{420}b\left(\frac{1}{2}b\int \left(\frac{315ex^2+168c}{x^3\sqrt{bx^4+a}}+\frac{480fx^2+224d}{x^2\sqrt{bx^4+a}}\right)dx-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\right)-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

$\Big\downarrow$ 2009

$$\frac{1}{420}b\left(\frac{1}{2}b\left(\frac{16\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}f+7\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{224\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{\right.}\right.}{}\right.\right.$$

$$\frac{\left(a+bx^4\right)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]
```

output
```
-1/2520*(((252*c)/x^10 + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7)*(a + b*x
^4)^(3/2)) + (b*(-1/4*(((168*c)/x^6 + (224*d)/x^5 + (315*e)/x^4 + (480*f)/
x^3)*Sqrt[a + b*x^4]) + (b*((-84*c*Sqrt[a + b*x^4])/(a*x^2) - (224*d*Sqrt[
a + b*x^4])/(a*x) + (224*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b
]*x^2)) - (315*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (224*b^(1
/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*
EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) +
 (16*(7*Sqrt[b]*d + 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])
/(a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])))/2))/420
```

3.525.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.525.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(1344 b^{2} d\,x^{9}+504 b^{2} c\,x^{8}+2160 a b f\,x^{7}+1575 a e b\,x^{6}+1232 x^{5} d b a+1008 a b c\,x^{4}+720 a^{2} f\,x^{3}+630 a^{2} e\,x^{2}+560 a^{2} d x+504 a^{2} c\right)}{5040 x^{10} a}$
default	$f\left(-\frac{a\sqrt{b\,x^{4}+a}}{7 x^{7}}-\frac{3 b\sqrt{b\,x^{4}+a}}{7 x^{3}}+\frac{4 b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+e\left(-\frac{3 b^{2}\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{16\sqrt{a}}-\frac{a\sqrt{b\,x^{4}}}{8 x^{8}}\right.$
elliptic	$-\frac{a c\sqrt{b\,x^{4}+a}}{10 x^{10}}-\frac{a d\sqrt{b\,x^{4}+a}}{9 x^{9}}-\frac{a e\sqrt{b\,x^{4}+a}}{8 x^{8}}-\frac{a f\sqrt{b\,x^{4}+a}}{7 x^{7}}-\frac{b c\sqrt{b\,x^{4}+a}}{5 x^{6}}-\frac{11 b d\sqrt{b\,x^{4}+a}}{45 x^{5}}-\frac{5 b e\sqrt{b\,x^{4}+a}}{16 x^{4}}-\frac{3 b f\sqrt{b\,x^{4}+a}}{7 x^{3}}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/5040*(b*x^4+a)^(1/2)*(1344*b^2*d*x^9+504*b^2*c*x^8+2160*a*b*f*x^7+1575*
a*b*e*x^6+1232*a*b*d*x^5+1008*a*b*c*x^4+720*a^2*f*x^3+630*a^2*e*x^2+560*a^
2*d*x+504*a^2*c)/x^10/a+1/840/a*b^2*(480*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+224*I*b^(1/2)*d*a^(1/2)/(I/a^(
1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x
^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ellipt
icE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-315/2*a^(1/2)*e*ln((2*a+2*a^(1/2)*(b*x
^4+a)^(1/2))/x^2))
```

3.525.5 Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.54

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx =$$

$$-\frac{2688\sqrt{a}b^2dx^{10}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)-945\sqrt{a}b^2ex^{10}\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-384\left(7b^2d-\right.}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="fricas")
```

output
```
-1/10080*(2688*sqrt(a)*b^2*d*x^10*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^
(1/4)), -1) - 945*sqrt(a)*b^2*e*x^10*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(
a) + 2*a)/x^4) - 384*(7*b^2*d - 15*a*b*f)*sqrt(a)*x^10*(-b/a)^(3/4)*ellipt
ic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(1344*b^2*d*x^9 + 504*b^2*c*x^8 + 216
0*a*b*f*x^7 + 1575*a*b*e*x^6 + 1232*a*b*d*x^5 + 1008*a*b*c*x^4 + 720*a^2*f
*x^3 + 630*a^2*e*x^2 + 560*a^2*d*x + 504*a^2*c)*sqrt(b*x^4 + a))/(a*x^10)
```

3.525.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.67 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^2e}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{10x^8} - \frac{3a\sqrt{b}e}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{5x^4} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}e}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{5}{2}}c\sqrt{\frac{a}{bx^4}+1}}{10a} - \frac{3b^2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**11,x)
```

output
```
a**(3/2)*d*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**9*gamma(-5/4)) + a**(3/2)*f*gamma(-7/4)*hyper((-7/4, -1/2), (-3/
4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*d*gamma(-5
/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-
1/4)) + sqrt(a)*b*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**2*e/(8*sqrt(b)*x**10*sqrt(a/(b*x**4)
+ 1)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*e/(16*x**
6*sqrt(a/(b*x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(3
/2)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*e/(16*x**2*sqrt(a/(b*x**4)
+ 1)) - b**(5/2)*c*sqrt(a/(b*x**4) + 1)/(10*a) - 3*b**2*e*asinh(sqrt(a)/(s
qrt(b)*x**2))/(16*sqrt(a))
```

3.525.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{11}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="maxima")
```

output
```
-1/10*(b*x^4 + a)^(5/2)*c/(a*x^10) + integrate((b*f*x^6 + b*e*x^5 + b*d*x^
4 + a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a)/x^10, x)
```

3.525.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{11}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^11, x)
```

3.525.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{11}}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^11,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^11, x)
```

3.526 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx$

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3.526.2 Mathematica [C] (verified) . . . 4087
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3.526.8 Giac [F] . . . 4093
3.526.9 Mupad [F(-1)] . . . 4093

3.526.1 Optimal result

Integrand size = 30, antiderivative size = 424

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = -\frac{b\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\sqrt{a+bx^4}}{18480}$$
$$-\frac{4b^2c\sqrt{a+bx^4}}{77ax^3}-\frac{b^2d\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2e\sqrt{a+bx^4}}{15ax}+\frac{4b^{5/2}ex\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)(a+bx^4)^{3/2}}{3960}-\frac{3b^2f\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}$$
$$-\frac{4b^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$
$$-\frac{2b^{9/4}\left(15\sqrt{b}c-77\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}$$

output
```
-1/3960*(360*c/x^11+396*d/x^10+440*e/x^9+495*f/x^8)*(b*x^4+a)^(3/2)-3/16*b
^2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/18480*b*(1440*c/x^7+1848*d
/x^6+2464*e/x^5+3465*f/x^4)*(b*x^4+a)^(1/2)-4/77*b^2*c*(b*x^4+a)^(1/2)/a/x
^3-1/10*b^2*d*(b*x^4+a)^(1/2)/a/x^2-4/15*b^2*e*(b*x^4+a)^(1/2)/a/x+4/15*b^
(5/2)*e*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-4/15*b^(9/4)*e*(cos(2*ar
ctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellipti
cE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b
*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)-2/1155*b^(9
/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1
/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-77*e*a^(1/
2)+15*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)
^(1/2)/a^(5/4)/(b*x^4+a)^(1/2)
```

3.526.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.41

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx =$$
$$-\frac{\sqrt{a+bx^4}\left(720a^2c\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{3}{2},-\frac{7}{4},-\frac{bx^4}{a}\right)+11x\left(9\sqrt{1+\frac{bx^4}{a}}(8b^2dx^8+2a^2(4d+5fx^2)\right.\right.}{7920a}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]
```

output
```
-1/7920*(Sqrt[a + b*x^4]*(720*a^2*c*Hypergeometric2F1[-11/4, -3/2, -7/4, -
((b*x^4)/a)] + 11*x*(9*Sqrt[1 + (b*x^4)/a]*(8*b^2*d*x^8 + 2*a^2*(4*d + 5*f
*x^2) + a*b*x^4*(16*d + 25*f*x^2)) + 135*b^2*f*x^10*ArcTanh[Sqrt[1 + (b*x^
4)/a]] + 80*a^2*e*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/a)])))/(
a*x^11*Sqrt[1 + (b*x^4)/a])
```

3.526.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2364, 27, 2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^{12}}dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(495fx^3+440ex^2+396dx+360c)\sqrt{bx^4+a}}{3960x^8}dx-\frac{(a+bx^4)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{660}b\int \frac{(495fx^3+440ex^2+396dx+360c)\sqrt{bx^4+a}}{x^8}dx-\frac{(a+bx^4)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

$$\Big\downarrow \text{2364}$$

$$\frac{1}{660}b\left(-2b\int -\frac{3465fx^3+2464ex^2+1848dx+1440c}{28x^4\sqrt{bx^4+a}}dx-\frac{1}{28}\sqrt{a+bx^4}\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\right)-\frac{(a+bx^4)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{660}b\left(\frac{1}{14}b\int \frac{3465fx^3+2464ex^2+1848dx+1440c}{x^4\sqrt{bx^4+a}}dx-\frac{1}{28}\sqrt{a+bx^4}\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\right)-\frac{(a+bx^4)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

$$\Big\downarrow \text{2372}$$

$$\frac{1}{660}b\left(\frac{1}{14}b\int\left(\frac{2464ex^2+1440c}{x^4\sqrt{bx^4+a}}+\frac{3465fx^2+1848d}{x^3\sqrt{bx^4+a}}\right)dx-\frac{1}{28}\sqrt{a+bx^4}\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\right)-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

$\downarrow$ 2009

$$\frac{1}{660}b\left(\frac{1}{14}b\left(-\frac{16\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}c-77\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{a+bx^4}}-\frac{2464\sqrt[4]{b}e\ldots}{\ldots}\right.\right.$$

$$-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]
```

output
```
-1/3960*(((360*c)/x^11 + (396*d)/x^10 + (440*e)/x^9 + (495*f)/x^8)*(a + b*
x^4)^(3/2)) + (b*(-1/28*(((1440*c)/x^7 + (1848*d)/x^6 + (2464*e)/x^5 + (34
65*f)/x^4)*Sqrt[a + b*x^4]) + (b*((-480*c*Sqrt[a + b*x^4])/(a*x^3) - (924*
d*Sqrt[a + b*x^4])/(a*x^2) - (2464*e*Sqrt[a + b*x^4])/(a*x) + (2464*Sqrt[b
]*e*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (3465*f*ArcTanh[Sqrt[
a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (2464*b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) - (16*b^(1/4)*(15*Sqrt[b]*c -
77*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(5/4)*Sqrt[a + b
*x^4])))/14))/660
```

3.526.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.526.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.20 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(14784b^2e\,x^{10}+5544b^2d\,x^9+2880b^2c\,x^8+17325abf\,x^7+13552aeb\,x^6+11088x^5dba+9360abc\,x^4+6930a^2f\,x^3+6160a^2e\,x^2}{55440x^{11}a}$
default	$f\left(-\frac{3b^2\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{16\sqrt{a}}-\frac{a\sqrt{b\,x^4+a}}{8x^8}-\frac{5b\sqrt{b\,x^4+a}}{16x^4}\right)-\frac{d(b^2x^8+2ab\,x^4+a^2)\sqrt{b\,x^4+a}}{10x^{10}a}+c\left(-\frac{a\sqrt{b\,x^4+a}}{11x^{11}}-\frac{13b\sqrt{}}{7}\right.$
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{11x^{11}}-\frac{ad\sqrt{b\,x^4+a}}{10x^{10}}-\frac{ae\sqrt{b\,x^4+a}}{9x^9}-\frac{af\sqrt{b\,x^4+a}}{8x^8}-\frac{13bc\sqrt{b\,x^4+a}}{77x^7}-\frac{bd\sqrt{b\,x^4+a}}{5x^6}-\frac{11be\sqrt{b\,x^4+a}}{45x^5}-\frac{5bf\sqrt{b\,x^4+a}}{16x^4}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/55440*(b*x^4+a)^(1/2)*(14784*b^2*e*x^10+5544*b^2*d*x^9+2880*b^2*c*x^8+1
7325*a*b*f*x^7+13552*a*b*e*x^6+11088*a*b*d*x^5+9360*a*b*c*x^4+6930*a^2*f*x
^3+6160*a^2*e*x^2+5544*a^2*d*x+5040*a^2*c)/x^11/a+1/9240/a*b^2*(-480*b*c/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+24
64*I*b^(1/2)*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(
1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3465/2*a^
(1/2)*f*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

3.526.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.54

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^{12}}\, dx =$$

$$-\frac{29568\sqrt{a}b^2ex^{11}\left(-\frac{b}{a}\right)^{\frac{3}{4}} E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right) | -1\right) - 10395\sqrt{a}b^2fx^{11}\log\left(-\frac{bx^4 - 2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right) - 384\,(15\,b}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="fricas")
```

output
```
-1/110880*(29568*sqrt(a)*b^2*e*x^11*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a
)^(1/4)), -1) - 10395*sqrt(a)*b^2*f*x^11*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*s
qrt(a) + 2*a)/x^4) - 384*(15*b^2*c + 77*b^2*e)*sqrt(a)*x^11*(-b/a)^(3/4)*e
lliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(14784*b^2*e*x^10 + 5544*b^2*d*x
^9 + 2880*b^2*c*x^8 + 17325*a*b*f*x^7 + 13552*a*b*e*x^6 + 11088*a*b*d*x^5
+ 9360*a*b*c*x^4 + 6930*a^2*f*x^3 + 6160*a^2*e*x^2 + 5544*a^2*d*x + 5040*a
^2*c)*sqrt(b*x^4 + a))/(a*x^11)
```

3.526.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.99 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\ -\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\ -\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\ -\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\ -\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} - \frac{a^2f}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{10x^8} - \frac{3a\sqrt{b}f}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{5x^4} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}f}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{5}{2}}d\sqrt{\frac{a}{bx^4}+1}}{10a} - \frac{3b^2f\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**12,x)
```

output
```
a**(3/2)*c*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*p
i)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*e*gamma(-9/4)*hyper((-9/4, -1/2), (
-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*c*gamma
(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamm
a(-3/4)) + sqrt(a)*b*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp
_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - a**2*f/(8*sqrt(b)*x**10*sqrt(a/(b*x
**4) + 1)) - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*f/(1
6*x**6*sqrt(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/(5*x**4) -
b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*f/(16*x**2*sqrt(a/(b*x
**4) + 1)) - b**(5/2)*d*sqrt(a/(b*x**4) + 1)/(10*a) - 3*b**2*f*asinh(sqrt(
a)/(sqrt(b)*x**2))/(16*sqrt(a))
```

3.526. $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx$

3.526.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{12}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12, x)
```

3.526.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{12}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12, x)
```

3.526.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{12}}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^12,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^12, x)
```

3.527 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx$

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3.527.2 Mathematica [C] (verified) . . . 4095
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3.527.1 Optimal result

Integrand size = 30, antiderivative size = 449

$$\begin{aligned}\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx = &-\frac{b\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\sqrt{a+bx^4}}{18480}\\
&-\frac{b^2c\sqrt{a+bx^4}}{32ax^4}-\frac{4b^2d\sqrt{a+bx^4}}{77ax^3}-\frac{b^2e\sqrt{a+bx^4}}{10ax^2}-\frac{4b^2f\sqrt{a+bx^4}}{15ax}\\
&+\frac{4b^{5/2}fx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)(a+bx^4)^{3/2}}{1980}\\
&+\frac{b^3c\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}}-\frac{4b^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}\\
&-\frac{2b^{9/4}\left(15\sqrt{b}d-77\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/1980*(165*c/x^12+180*d/x^11+198*e/x^10+220*f/x^9)*(b*x^4+a)^(3/2)+1/32*
b^3*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/18480*b*(1155*c/x^8+1440*
d/x^7+1848*e/x^6+2464*f/x^5)*(b*x^4+a)^(1/2)-1/32*b^2*c*(b*x^4+a)^(1/2)/a/
x^4-4/77*b^2*d*(b*x^4+a)^(1/2)/a/x^3-1/10*b^2*e*(b*x^4+a)^(1/2)/a/x^2-4/15
*b^2*f*(b*x^4+a)^(1/2)/a/x+4/15*b^(5/2)*f*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2
*b^(1/2))-4/15*b^(9/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*
2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a
^(3/4)/(b*x^4+a)^(1/2)-2/1155*b^(9/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/2*2^(1/2))*(-77*f*a^(1/2)+15*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*(
(b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^4+a)^(1/2)
```

3.527.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.33

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\left(90a^5d\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{3}{2},-\frac{7}{4},-\frac{bx^4}{a}\right)+11x\left(10a^5fx\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2}\right.\right.\right.}{990a^4x^{11}\sqrt{1+}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]
```

output
```
-1/990*(Sqrt[a + b*x^4]*(90*a^5*d*Hypergeometric2F1[-11/4, -3/2, -7/4, -((
b*x^4)/a)] + 11*x*(10*a^5*f*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4
)/a)] + 9*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*(a^3*e - b^3*c*x^10*Hypergeome
tric2F1[5/2, 4, 7/2, 1 + (b*x^4)/a]))))/(a^4*x^11*Sqrt[1 + (b*x^4)/a])
```

3.527.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2364, 27, 2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{3/2} \left(c+dx+ex^2+fx^3\right)}{x^{13}} \, dx$$

$$\Big\downarrow \text{ 2364}$$

$$-6b\int -\frac{\left(220fx^3+198ex^2+180dx+165c\right)\sqrt{bx^4+a}}{1980x^9}dx-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{330}b\int \frac{\left(220fx^3+198ex^2+180dx+165c\right)\sqrt{bx^4+a}}{x^9}dx-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}$$

$$\Big\downarrow \text{ 2364}$$

$$\frac{1}{330}b\left(-2b\int -\frac{2464fx^3+1848ex^2+1440dx+1155c}{56x^5\sqrt{bx^4+a}}dx-\frac{1}{56}\sqrt{a+bx^4}\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\right)-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{330}b\left(\frac{1}{28}b\int \frac{2464fx^3+1848ex^2+1440dx+1155c}{x^5\sqrt{bx^4+a}}dx-\frac{1}{56}\sqrt{a+bx^4}\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\right)-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}$$

$$\Big\downarrow \text{ 2372}$$

$$\frac{1}{330}b\left(\frac{1}{28}b\int\left(\frac{1848ex^2+1155c}{x^5\sqrt{bx^4+a}}+\frac{2464fx^2+1440d}{x^4\sqrt{bx^4+a}}\right)dx-\frac{1}{56}\sqrt{a+bx^4}\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\right)-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}$$

$\downarrow$ 2009

$$\frac{1}{330}b\left(\frac{1}{28}b\left(-\frac{16\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}d-77\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{a+bx^4}}-\frac{2464\sqrt[4]{b}f\ldots}{}\right.\right.-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]
```

output
```
-1/1980*(((165*c)/x^12 + (180*d)/x^11 + (198*e)/x^10 + (220*f)/x^9)*(a + b
*x^4)^(3/2)) + (b*(-1/56*(((1155*c)/x^8 + (1440*d)/x^7 + (1848*e)/x^6 + (2
464*f)/x^5)*Sqrt[a + b*x^4]) + (b*((-1155*c*Sqrt[a + b*x^4])/(4*a*x^4) - (
480*d*Sqrt[a + b*x^4])/(a*x^3) - (924*e*Sqrt[a + b*x^4])/(a*x^2) - (2464*f
*Sqrt[a + b*x^4])/(a*x) + (2464*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] +
 Sqrt[b]*x^2)) + (1155*b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) -
 (2464*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[
b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a
+ b*x^4]) - (16*b^(1/4)*(15*Sqrt[b]*d - 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(a^(5/4)*Sqrt[a + b*x^4])))/28))/330
```

3.527.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.527.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.37 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(29568 b^{2} f\,x^{11}+11088 b^{2} e\,x^{10}+5760 b^{2} d\,x^{9}+3465 b^{2} c\,x^{8}+27104 a b f\,x^{7}+22176 a e b\,x^{6}+18720 x^{5} d b a+16170 a b c\,x^{4}+12320 a^{2}\right.}{110880 x^{12} a}$
default	$c\left(-\frac{a\sqrt{b\,x^{4}+a}}{12 x^{12}}-\frac{7 b\sqrt{b\,x^{4}+a}}{48 x^{8}}-\frac{b^{2}\sqrt{b\,x^{4}+a}}{32 a\,x^{4}}+\frac{b^{3}\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{32 a^{\frac{3}{2}}}\right)-\frac{e\left(b^{2} x^{8}+2 a b\,x^{4}+a^{2}\right)\sqrt{b\,x^{4}+a}}{10 x^{10} a}+d\left(-\frac{a\sqrt{b\,x}}{11 x}\right.$
elliptic	$-\frac{a c\sqrt{b\,x^{4}+a}}{12 x^{12}}-\frac{a d\sqrt{b\,x^{4}+a}}{11 x^{11}}-\frac{a e\sqrt{b\,x^{4}+a}}{10 x^{10}}-\frac{a f\sqrt{b\,x^{4}+a}}{9 x^{9}}-\frac{7 b c\sqrt{b\,x^{4}+a}}{48 x^{8}}-\frac{13 b d\sqrt{b\,x^{4}+a}}{77 x^{7}}-\frac{b e\sqrt{b\,x^{4}+a}}{5 x^{6}}-\frac{11 b f\sqrt{b\,x^{4}+a}}{45 x^{5}}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/110880*(b*x^4+a)^(1/2)*(29568*b^2*f*x^11+11088*b^2*e*x^10+5760*b^2*d*x^
9+3465*b^2*c*x^8+27104*a*b*f*x^7+22176*a*b*e*x^6+18720*a*b*d*x^5+16170*a*b
*c*x^4+12320*a^2*f*x^3+11088*a^2*e*x^2+10080*a^2*d*x+9240*a^2*c)/x^12/a-1/
18480*b^3/a*(960*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/
2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-4928*I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)
*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))
^(1/2),I))-1155/2*c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

3.527.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx =$$

$$-\frac{59136\,a^{\frac{3}{2}}b^2fx^{12}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right) - 3465\,\sqrt{a}b^3cx^{12}\log\left(-\frac{bx^4+2\,\sqrt{bx^4+a}\sqrt{a}+2\,a}{x^4}\right) - 768\,(15\,ab^2 \cdots}{\cdots}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="fricas")
```

output
```
-1/221760*(59136*a^(3/2)*b^2*f*x^12*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a
)^(1/4)), -1) - 3465*sqrt(a)*b^3*c*x^12*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sq
rt(a) + 2*a)/x^4) - 768*(15*a*b^2*d + 77*a*b^2*f)*sqrt(a)*x^12*(-b/a)^(3/4
)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(29568*a*b^2*f*x^11 + 11088*a
*b^2*e*x^10 + 5760*a*b^2*d*x^9 + 3465*a*b^2*c*x^8 + 27104*a^2*b*f*x^7 + 22
176*a^2*b*e*x^6 + 18720*a^2*b*d*x^5 + 16170*a^2*b*c*x^4 + 12320*a^3*f*x^3
+ 11088*a^3*e*x^2 + 10080*a^3*d*x + 9240*a^3*c)*sqrt(b*x^4 + a))/(a^2*x^12
)
```

3.527.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.77 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} - \frac{a^2c}{12\sqrt{b}x^{14}\sqrt{\frac{a}{bx^4}+1}} - \frac{11a\sqrt{b}c}{48x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{10x^8} - \frac{17b^{\frac{3}{2}}c}{96x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{5x^4} - \frac{b^{\frac{5}{2}}c}{32ax^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{5}{2}}e\sqrt{\frac{a}{bx^4}+1}}{10a} + \frac{b^3c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{32a^{\frac{3}{2}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**13,x)
```

output
```
a**(3/2)*d*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*p
i)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*f*gamma(-9/4)*hyper((-9/4, -1/2), (
-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*d*gamma
(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamm
a(-3/4)) + sqrt(a)*b*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp
_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - a**2*c/(12*sqrt(b)*x**14*sqrt(a/(b*
x**4) + 1)) - 11*a*sqrt(b)*c/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*e
*sqrt(a/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*c/(96*x**6*sqrt(a/(b*x**4) +
 1)) - b**(3/2)*e*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5/2)*c/(32*a*x**2*sq
rt(a/(b*x**4) + 1)) - b**(5/2)*e*sqrt(a/(b*x**4) + 1)/(10*a) + b**3*c*asin
h(sqrt(a)/(sqrt(b)*x**2))/(32*a**(3/2))
```

3.527. $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx$

3.527.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{13}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="maxima")
```

output
```
-1/192*(3*b^3*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))
/a^(3/2) + 2*(3*(b*x^4 + a)^(5/2)*b^3 + 8*(b*x^4 + a)^(3/2)*a*b^3 - 3*sqrt
(b*x^4 + a)*a^2*b^3)/((b*x^4 + a)^3*a - 3*(b*x^4 + a)^2*a^2 + 3*(b*x^4 + a
)*a^3 - a^4))*c + integrate((b*f*x^6 + b*e*x^5 + b*d*x^4 + a*f*x^2 + a*e*x
 + a*d)*sqrt(b*x^4 + a)/x^12, x)
```

3.527.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{13}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^13, x)
```

3.527.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{13}}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^13,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^13, x)
```

3.528 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}} \, dx$

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3.528.1 Optimal result

Integrand size = 30, antiderivative size = 474

$$\begin{aligned}\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}} \, dx &= -\frac{b\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\sqrt{a+bx^4}}{240240} \\
&- \frac{4b^2c\sqrt{a+bx^4}}{195ax^5} - \frac{b^2d\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2e\sqrt{a+bx^4}}{77ax^3} - \frac{b^2f\sqrt{a+bx^4}}{10ax^2} \\
&+ \frac{4b^3c\sqrt{a+bx^4}}{65a^2x} - \frac{4b^{7/2}cx\sqrt{a+bx^4}}{65a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)(a+bx^4)^{3/2}}{8580} \\
&+ \frac{b^3d\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}} + \frac{4b^{13/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65a^{7/4}\sqrt{a+bx^4}} \\
&- \frac{2b^{11/4}\left(77\sqrt{b}c+65\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/8580*(660*c/x^13+715*d/x^12+780*e/x^11+858*f/x^10)*(b*x^4+a)^(3/2)+1/32
*b^3*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/240240*b*(12320*c/x^9+15
015*d/x^8+18720*e/x^7+24024*f/x^6)*(b*x^4+a)^(1/2)-4/195*b^2*c*(b*x^4+a)^(
1/2)/a/x^5-1/32*b^2*d*(b*x^4+a)^(1/2)/a/x^4-4/77*b^2*e*(b*x^4+a)^(1/2)/a/x
^3-1/10*b^2*f*(b*x^4+a)^(1/2)/a/x^2+4/65*b^3*c*(b*x^4+a)^(1/2)/a^2/x-4/65*
b^(7/2)*c*x*(b*x^4+a)^(1/2)/a^2/(a^(1/2)+x^2*b^(1/2))+4/65*b^(13/4)*c*(cos
(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*El
lipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2)
)*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-2/5005
*b^(11/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(65*e*
a^(1/2)+77*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2
))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)
```

3.528.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.32

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\Big(110a^5c\,\text{Hypergeometric2F1}\left(-\frac{13}{4},-\frac{3}{2},-\frac{9}{4},-\frac{bx^4}{a}\right)+13x^2\Big(10a^5e\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\right.}{1430a^4x^{13}\sqrt{1}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]
```

output
```
-1/1430*(Sqrt[a + b*x^4]*(110*a^5*c*Hypergeometric2F1[-13/4, -3/2, -9/4, -
((b*x^4)/a)] + 13*x^2*(10*a^5*e*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*
x^4)/a)] + 11*x*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*(a^3*f - b^3*d*x^10*Hype
rgeometric2F1[5/2, 4, 7/2, 1 + (b*x^4)/a]))))/(a^4*x^13*Sqrt[1 + (b*x^4)/a
])
```

3.528.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2364, 27, 2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^{14}}dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(858fx^3+780ex^2+715dx+660c)\sqrt{bx^4+a}}{8580x^{10}}dx-\frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{(858fx^3+780ex^2+715dx+660c)\sqrt{bx^4+a}}{x^{10}}dx}{1430}-\frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}$$

$$\Big\downarrow \text{2364}$$

$$\frac{b\left(-2b\int -\frac{24024fx^3+18720ex^2+15015dx+12320c}{168x^6\sqrt{bx^4+a}}dx-\frac{1}{168}\sqrt{a+bx^4}\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\right)}{1430}-\frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{1}{84}b\int \frac{24024fx^3+18720ex^2+15015dx+12320c}{x^6\sqrt{bx^4+a}}dx-\frac{1}{168}\sqrt{a+bx^4}\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\right)}{1430}-\frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}$$

$$\Big\downarrow \text{2372}$$

$$\frac{b\left(\frac{1}{84}b\int \left(\frac{18720ex^2+12320c}{x^6\sqrt{bx^4+a}}+\frac{24024fx^2+15015d}{x^5\sqrt{bx^4+a}}\right)dx-\frac{1}{168}\sqrt{a+bx^4}\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\right)}{1430}-\frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}$$

$$\downarrow 2009$$

$$b\left(\frac{1}{84}b\left(-\frac{48b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(65\sqrt{a}e+77\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{7/4}\sqrt{a+bx^4}}+\frac{7392b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(\right.}{a^{7/4}\sqrt{a+bx^4}}\right.\right.$$

$$\frac{\left(a+bx^4\right)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]
```

output
```
-1/8580*(((660*c)/x^13 + (715*d)/x^12 + (780*e)/x^11 + (858*f)/x^10)*(a +
b*x^4)^(3/2)) + (b*(-1/168*(((12320*c)/x^9 + (15015*d)/x^8 + (18720*e)/x^7
 + (24024*f)/x^6)*Sqrt[a + b*x^4]) + (b*((-2464*c*Sqrt[a + b*x^4])/(a*x^5)
 - (15015*d*Sqrt[a + b*x^4])/(4*a*x^4) - (6240*e*Sqrt[a + b*x^4])/(a*x^3)
- (12012*f*Sqrt[a + b*x^4])/(a*x^2) + (7392*b*c*Sqrt[a + b*x^4])/(a^2*x) -
 (7392*b^(3/2)*c*x*Sqrt[a + b*x^4])/(a^2*(Sqrt[a] + Sqrt[b]*x^2)) + (15015
*b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) + (7392*b^(5/4)*c*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(7/4)*Sqrt[a + b*x^4]) - (48*b^(3/4
)*(77*Sqrt[b]*c + 65*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
a^(7/4)*Sqrt[a + b*x^4]))/84))/1430
```

3.528.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

3.528. $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx$

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.528.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.65 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(-29568b^3c\,x^{12}+48048x^{11}a\,b^2f+24960a\,b^2e\,x^{10}+15015x^9a\,b^2d+9856a\,b^2c\,x^8+96096a^2bf\,x^7+81120a^2be\,x^6+70070x^5b}{480480x^{13}a^2}$
default	$d\left(-\frac{a\sqrt{b\,x^4+a}}{12x^{12}}-\frac{7b\sqrt{b\,x^4+a}}{48x^8}-\frac{b^2\sqrt{b\,x^4+a}}{32a\,x^4}+\frac{b^3\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{32a^{\frac{3}{2}}}\right)-\frac{f\,(b^2x^8+2ab\,x^4+a^2)\sqrt{b\,x^4+a}}{10x^{10}a}+e\left(-\frac{a\sqrt{b\,x}}{11a}\right.$
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{13x^{13}}-\frac{ad\sqrt{b\,x^4+a}}{12x^{12}}-\frac{ae\sqrt{b\,x^4+a}}{11x^{11}}-\frac{af\sqrt{b\,x^4+a}}{10x^{10}}-\frac{5bc\sqrt{b\,x^4+a}}{39x^9}-\frac{7bd\sqrt{b\,x^4+a}}{48x^8}-\frac{13be\sqrt{b\,x^4+a}}{77x^7}-\frac{bf\sqrt{b\,x^4+a}}{5x^6}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/480480*(b*x^4+a)^(1/2)*(-29568*b^3*c*x^12+48048*a*b^2*f*x^11+24960*a*b^
2*e*x^10+15015*a*b^2*d*x^9+9856*a*b^2*c*x^8+96096*a^2*b*f*x^7+81120*a^2*b*
e*x^6+70070*a^2*b*d*x^5+61600*a^2*b*c*x^4+48048*a^3*f*x^3+43680*a^3*e*x^2+
40040*a^3*d*x+36960*a^3*c)/x^13/a^2-1/80080/a^2*b^3*(4160*a*e/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+4928*I*b^(1/2
)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2
))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-5005/2*a^(1/2)*d*ln(
(2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

3.528.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.54

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx = \frac{59136\sqrt{a}b^3cx^{13}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)+15015\sqrt{a}b^3d}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="fricas")
```

output
```
1/960960*(59136*sqrt(a)*b^3*c*x^13*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)
^(1/4)), -1) + 15015*sqrt(a)*b^3*d*x^13*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sq
rt(a) + 2*a)/x^4) - 768*(77*b^3*c - 65*a*b^2*e)*sqrt(a)*x^13*(-b/a)^(3/4)*
elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(29568*b^3*c*x^12 - 48048*a*b^2
*f*x^11 - 24960*a*b^2*e*x^10 - 15015*a*b^2*d*x^9 - 9856*a*b^2*c*x^8 - 9609
6*a^2*b*f*x^7 - 81120*a^2*b*e*x^6 - 70070*a^2*b*d*x^5 - 61600*a^2*b*c*x^4
- 48048*a^3*f*x^3 - 43680*a^3*e*x^2 - 40040*a^3*d*x - 36960*a^3*c)*sqrt(b*
x^4 + a))/(a^2*x^13)
```

3.528.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.57 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{13}{4},-\frac{1}{2}\\-\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{13}\Gamma\left(-\frac{9}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} - \frac{a^2d}{12\sqrt{b}x^{14}\sqrt{\frac{a}{bx^4}+1}} - \frac{11a\sqrt{b}d}{48x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{10x^8} - \frac{17b^{\frac{3}{2}}d}{96x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{5x^4} - \frac{b^{\frac{5}{2}}d}{32ax^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{5}{2}}f\sqrt{\frac{a}{bx^4}+1}}{10a} + \frac{b^3d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{32a^{\frac{3}{2}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**14,x)
```

output
```
a**(3/2)*c*gamma(-13/4)*hyper((-13/4, -1/2), (-9/4,), b*x**4*exp_polar(I*p
i)/a)/(4*x**13*gamma(-9/4)) + a**(3/2)*e*gamma(-11/4)*hyper((-11/4, -1/2),
 (-7/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**11*gamma(-7/4)) + sqrt(a)*b*c*ga
mma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*g
amma(-5/4)) + sqrt(a)*b*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) - a**2*d/(12*sqrt(b)*x**14*sqrt(a/
(b*x**4) + 1)) - 11*a*sqrt(b)*d/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b
)*f*sqrt(a/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*d/(96*x**6*sqrt(a/(b*x**4
) + 1)) - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5/2)*d/(32*a*x**2
*sqrt(a/(b*x**4) + 1)) - b**(5/2)*f*sqrt(a/(b*x**4) + 1)/(10*a) + b**3*d*a
sinh(sqrt(a)/(sqrt(b)*x**2))/(32*a**(3/2))
```

3.528.7 Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{14}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14, x)
```

3.528.8 Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{14}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14, x)
```

3.528.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{14}}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^14,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^14, x)
```

3.529 $\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

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3.529.1 Optimal result

Integrand size = 30, antiderivative size = 361

$$\begin{aligned}
\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx
&= \frac{cx\sqrt{a+bx^4}}{3b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}\\
&\quad-\frac{(4af-3bdx^2)\sqrt{a+bx^4}}{12b^2}-\frac{ad\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}\\
&\quad+\frac{3a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}\\
&\quad-\frac{a^{3/4}\left(5\sqrt{b}c+9\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
-1/4*a*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/3*c*x*(b*x^4+a)^(1
/2)/b+1/5*e*x^3*(b*x^4+a)^(1/2)/b+1/6*f*x^4*(b*x^4+a)^(1/2)/b-1/12*(-3*b*d
*x^2+4*a*f)*(b*x^4+a)^(1/2)/b^2-3/5*a*e*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)
+x^2*b^(1/2))+3/5*a^(5/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1
/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2
)/b^(7/4)/(b*x^4+a)^(1/2)-1/30*a^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/
a^(1/4))),1/2*2^(1/2))*(9*e*a^(1/2)+5*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b
*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)
```

3.529.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.59

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{-20a^2f+20abcx+15abdx^2+12abex^3-10abfx^4+20b^2cx^5+15b^2dx^6+12b^2ex^7+10b^2fx^8-15a\sqrt{b}d\cdots}{\cdots}$$

input
```
Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(-20*a^2*f + 20*a*b*c*x + 15*a*b*d*x^2 + 12*a*b*e*x^3 - 10*a*b*f*x^4 + 20*
b^2*c*x^5 + 15*b^2*d*x^6 + 12*b^2*e*x^7 + 10*b^2*f*x^8 - 15*a*Sqrt[b]*d*Sq
rt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*a*b*c*x*Sqrt[1 +
 (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*a*b*e*x^3*
Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^
2*Sqrt[a + b*x^4])
```

3.529.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}} \, dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(\frac{x^4 (c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^5 (d+fx^2)}{\sqrt{a+bx^4}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}-\frac{ad\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{af\sqrt{a+bx^4}}{3b^2}+\frac{cx\sqrt{a+bx^4}}{3b}+\frac{dx^2\sqrt{a+bx^4}}{4b}+\frac{ex^3\sqrt{a+bx^4}}{5b}+\frac{fx^4\sqrt{a+bx^4}}{6b}$$

input
```
Int[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
-1/3*(a*f*Sqrt[a + b*x^4])/b^2 + (c*x*Sqrt[a + b*x^4])/(3*b) + (d*x^2*Sqrt
[a + b*x^4])/(4*b) + (e*x^3*Sqrt[a + b*x^4])/(5*b) + (f*x^4*Sqrt[a + b*x^4
])/(6*b) - (3*a*e*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) -
 (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*e*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(
3/4)*(5*Sqrt[b]*c + 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(30*b^(7/4)*Sqrt[a + b*x^4])
```

3.529.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.529.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.18 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\left(-10bf\,x^{4}-12be\,x^{3}-15bd\,x^{2}-20bcx+20af\right)\sqrt{b\,x^{4}+a}}{60b^{2}}-\frac{a\left(\frac{10c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{18ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}}{}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right.}{30b}$
default	$-\frac{f\sqrt{b\,x^{4}+a}\,\left(-b\,x^{4}+2a\right)}{6b^{2}}+e\left(\frac{x^{3}\sqrt{b\,x^{4}+a}}{5b}-\frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+d\left(\frac{x^{2}\sqrt{b}}{4}\right.$
elliptic	$\frac{f\,x^{4}\sqrt{b\,x^{4}+a}}{6b}+\frac{e\,x^{3}\sqrt{b\,x^{4}+a}}{5b}+\frac{d\,x^{2}\sqrt{b\,x^{4}+a}}{4b}+\frac{cx\sqrt{b\,x^{4}+a}}{3b}-\frac{af\sqrt{b\,x^{4}+a}}{3b^{2}}-\frac{ac\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-$

input
```
int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-10*b*f*x^4-12*b*e*x^3-15*b*d*x^2-20*b*c*x+20*a*f)/b^2*(b*x^4+a)^(1
/2)-1/30*a/b*(10*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/
2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)+18*I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(
EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(
1/2),I))+15/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

3.529.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.45

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx =$$
$$-\frac{72\,a\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-15\,a\sqrt{b}dx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+8\,(5\,bc-}{}$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/120*(72*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -
1) - 15*a*sqrt(b)*d*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) +
8*(5*b*c - 9*a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x)
, -1) - 2*(10*b*f*x^5 + 12*b*e*x^4 + 15*b*d*x^3 + 20*b*c*x^2 - 20*a*f*x -
36*a*e)*sqrt(b*x^4 + a))/(b^2*x)
```

3.529.6 Sympy [A] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.49

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}dx^2\sqrt{1+\frac{bx^4}{a}}}{4b}-\frac{ad\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}$$
$$+f\left(\begin{cases}-\frac{a\sqrt{a+bx^4}}{3b^2}+\frac{x^4\sqrt{a+bx^4}}{6b} & \text{for } b\neq 0\\ \frac{x^8}{8\sqrt{a}} & \text{otherwise}\end{cases}\right)$$
$$+\frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}+\frac{ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*d*asinh(sqrt(b)*x**2/sqrt(a))/
(4*b**(3/2)) + f*Piecewise((-a*sqrt(a + b*x**4)/(3*b**2) + x**4*sqrt(a + b
*x**4)/(6*b), Ne(b, 0)), (x**8/(8*sqrt(a)), True)) + c*x**5*gamma(5/4)*hyp
er((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) +
e*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(11/4))
```

3.529.7 Maxima [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^4}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a), x)
```

3.529.8 Giac [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^4}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a), x)
```

3.529.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{x^4\,(f\,x^3+e\,x^2+d\,x+c)}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
```

output
```
int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

3.530 $\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

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3.530.1 Optimal result

Integrand size = 30, antiderivative size = 336

$$\begin{aligned}
&\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx \\
&= \frac{dx\sqrt{a+bx^4}}{3b}+\frac{fx^3\sqrt{a+bx^4}}{5b}-\frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{(2c+ex^2)\sqrt{a+bx^4}}{4b} \\
&\quad-\frac{a e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+\frac{3a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}} \\
&\quad-\frac{a^{3/4}\left(5\sqrt{b}d+9\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
-1/4*a*e*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/3*d*x*(b*x^4+a)^(1
/2)/b+1/5*f*x^3*(b*x^4+a)^(1/2)/b+1/4*(e*x^2+2*c)*(b*x^4+a)^(1/2)/b-3/5*a*
f*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))+3/5*a^(5/4)*f*(cos(2*arc
tan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Elliptic
E(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*
x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)-1/30*a^(3/4)
*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)
))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(9*f*a^(1/2)+5*
d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)
/b^(7/4)/(b*x^4+a)^(1/2)
```

3.530.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.63

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{30\sqrt{b}c(a+bx^4)+20\sqrt{b}dx(a+bx^4)+15\sqrt{b}ex^2(a+bx^4)+12\sqrt{b}fx^3(a+bx^4)-15ae\sqrt{a+bx^4}\operatorname{arctanh}\Big(}{}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(30*Sqrt[b]*c*(a + b*x^4) + 20*Sqrt[b]*d*x*(a + b*x^4) + 15*Sqrt[b]*e*x^2*
(a + b*x^4) + 12*Sqrt[b]*f*x^3*(a + b*x^4) - 15*a*e*Sqrt[a + b*x^4]*ArcTan
h[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*a*Sqrt[b]*d*x*Sqrt[1 + (b*x^4)/a]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*a*Sqrt[b]*f*x^3*Sqrt[1 +
 (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^(3/2)*Sq
rt[a + b*x^4])
```

3.530.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(\frac{x^3(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^4(d+fx^2)}{\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}-\frac{ae\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{c\sqrt{a+bx^4}}{2b}+\frac{dx\sqrt{a+bx^4}}{3b}+\frac{ex^2\sqrt{a+bx^4}}{4b}+\frac{fx^3\sqrt{a+bx^4}}{5b}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(c*Sqrt[a + b*x^4])/(2*b) + (d*x*Sqrt[a + b*x^4])/(3*b) + (e*x^2*Sqrt[a +
b*x^4])/(4*b) + (f*x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*f*x*Sqrt[a + b*x^4])/
(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) - (a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4)*(5*Sqrt[b]*d + 9*Sqrt[a]*f)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

3.530.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.530.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\left(12 f \,x^{3}+15 e \,x^{2}+20 d x +30 c \right) \sqrt{b \,x^{4}+a}}{60 b}-\frac{a \left(\frac{10 d \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}+\frac{18 i f \sqrt{a}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right.}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}\, \sqrt{b}}\right.}{30 b}$
default	$f \left(\frac{x^{3} \sqrt{b \,x^{4}+a}}{5 b}-\frac{3 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-E\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{5 b^{\frac{3}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}\right)+e \left(\frac{x^{2} \sqrt{b \,x^{4}+a}}{4 b}-\frac{a \ln\left(x^{2} \sqrt{b}+\sqrt{b \,x^{4}+\right.}{4 b^{\frac{3}{2}}}\right.$
elliptic	$\frac{f \,x^{3} \sqrt{b \,x^{4}+a}}{5 b}+\frac{e \,x^{2} \sqrt{b \,x^{4}+a}}{4 b}+\frac{d x \sqrt{b \,x^{4}+a}}{3 b}+\frac{c \sqrt{b \,x^{4}+a}}{2 b}-\frac{a d \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{3 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}-\frac{a e \ln\left(2 x^{2} \sqrt{b}+2 \sqrt{b}\right.}{4 b^{\frac{3}{2}}}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(12*f*x^3+15*e*x^2+20*d*x+30*c)/b*(b*x^4+a)^(1/2)-1/30*a/b*(10*d/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+18*I*
f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+15/2*e*ln(x^2*
b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2))
```

3.530.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.46

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx =$$
$$-\frac{72\,a\sqrt{b}fx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-15\,a\sqrt{b}ex\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+8\,(5\,bd-}{120}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/120*(72*a*sqrt(b)*f*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -
1) - 15*a*sqrt(b)*e*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) +
8*(5*b*d - 9*a*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x)
, -1) - 2*(12*b*f*x^4 + 15*b*e*x^3 + 20*b*d*x^2 + 30*b*c*x - 36*a*f)*sqrt(
b*x^4 + a))/(b^2*x)
```

3.530.6 Sympy [A] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.46

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}ex^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{ae\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + c\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right) + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*e*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*e*asinh(sqrt(b)*x**2/sqrt(a))/
(4*b**(3/2)) + c*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)
/(2*b), True)) + d*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + f*x**7*gamma(7/4)*hyper((1/2, 7/4),
(11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

3.530.7 Maxima [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^3}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^4 + a)*c/b + integrate((f*x^6 + e*x^5 + d*x^4)/sqrt(b*x^4 + a
), x)
```

3.530.8 Giac [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^3}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^3/sqrt(b*x^4 + a), x)
```

3.530.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{x^3\,(f\,x^3+e\,x^2+d\,x+c)}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
```

output
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

3.531 $\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

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3.531.2 Mathematica [C] (verified) . . . 4123
3.531.3 Rubi [A] (verified) . . . 4123
3.531.4 Maple [C] (verified) . . . 4125
3.531.5 Fricas [A] (verification not implemented) . . . 4125
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3.531.7 Maxima [F] . . . 4126
3.531.8 Giac [F] . . . 4127
3.531.9 Mupad [F(-1)] . . . 4127

3.531.1 Optimal result

Integrand size = 30, antiderivative size = 308

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{ex\sqrt{a+bx^4}}{3b}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{(2d+fx^2)\sqrt{a+bx^4}}{4b}-\frac{af\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}} - \frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(3\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}$$

output
```
-1/4*a*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/3*e*x*(b*x^4+a)^(1
/2)/b+1/4*(f*x^2+2*d)*(b*x^4+a)^(1/2)/b+c*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/
2)+x^2*b^(1/2))-a^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2
*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/
b^(3/4)/(b*x^4+a)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(-e*a^(1/2)+3*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4
+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

3.531.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.63

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{6\sqrt{b}d(a+bx^4)+4\sqrt{b}ex(a+bx^4)+3\sqrt{b}fx^2(a+bx^4)-3af\sqrt{a+bx^4}\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-4a\sqrt{b}ex\sqrt{1+\cdots}}{12b^{3/2}\sqrt{a+\cdots}}$$

input
```
Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(6*Sqrt[b]*d*(a + b*x^4) + 4*Sqrt[b]*e*x*(a + b*x^4) + 3*Sqrt[b]*f*x^2*(a
+ b*x^4) - 3*a*f*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] -
4*a*Sqrt[b]*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*
x^4)/a)] + 4*b^(3/2)*c*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4,
 7/4, -((b*x^4)/a)])/(12*b^(3/2)*Sqrt[a + b*x^4])
```

3.531.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c+dx+ex^2+fx^3\right)}{\sqrt{a+bx^4}}\,dx$$
$$\Big\downarrow \text{2372}$$
$$\int \left(\frac{x^2\left(c+ex^2\right)}{\sqrt{a+bx^4}}+\frac{x^3\left(d+fx^2\right)}{\sqrt{a+bx^4}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}c-\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}-\frac{af\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\sqrt{a+bx^4}}{2b}+\frac{ex\sqrt{a+bx^4}}{3b}+\frac{fx^2\sqrt{a+bx^4}}{4b}$$

input
```
Int[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(d*Sqrt[a + b*x^4])/(2*b) + (e*x*Sqrt[a + b*x^4])/(3*b) + (f*x^2*Sqrt[a +
b*x^4])/(4*b) + (c*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) -
(a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*c*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*
(3*Sqrt[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5
/4)*Sqrt[a + b*x^4])
```

3.531.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.531.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.75

method	result
risch	$\frac{(3f\,x^2+4ex+6d)\sqrt{b\,x^4+a}}{12b}-\frac{\frac{2ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{6i\sqrt{b}\,c\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}}{6b}$
default	$f\left(\frac{x^2\sqrt{b\,x^4+a}}{4b}-\frac{a\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{4b^{\frac{3}{2}}}\right)+e\left(\frac{x\sqrt{b\,x^4+a}}{3b}-\frac{a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{d\sqrt{b\,x^4+a}}{2b}+$
elliptic	$\frac{f\,x^2\sqrt{b\,x^4+a}}{4b}+\frac{ex\sqrt{b\,x^4+a}}{3b}+\frac{d\sqrt{b\,x^4+a}}{2b}-\frac{ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{af\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{4b^{\frac{3}{2}}}+\frac{ic\sqrt{a}}{}$

input
```
int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*f*x^2+4*e*x+6*d)/b*(b*x^4+a)^(1/2)-1/6/b*(2*a*e/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b
*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-6*I*b^(1/2)*c*a^(1/
2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+3/2*a*f*ln(x^2*b^(1/2)+(b*x^4
+a)^(1/2))/b^(1/2))
```

3.531.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.48

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{24\,b^{\frac{3}{2}}cx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)+3\,a\sqrt{b}fx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)-8\,(3\,bc+be)\sqrt{}}{24\,b^2x}$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(24*b^(3/2)*c*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) +
 3*a*sqrt(b)*f*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) - 8*(3*
b*c + b*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) +
 2*(3*b*f*x^3 + 4*b*e*x^2 + 6*b*d*x + 12*b*c)*sqrt(b*x^4 + a))/(b^2*x)
```

3.531.6 Sympy [A] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.51

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}fx^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{af\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + d\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right) + \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*f*asinh(sqrt(b)*x**2/sqrt(a))/
(4*b**(3/2)) + d*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)
/(2*b), True)) + c*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + e*x**5*gamma(5/4)*hyper((1/2, 5/4),
(9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

3.531.7 Maxima [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^2}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a), x)
```

3.531.8 Giac [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^2}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a), x)
```

3.531.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{x^2\,(f\,x^3+e\,x^2+d\,x+c)}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
```

output
```
int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

3.532 $\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}} dx$

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3.532.1 Optimal result

Integrand size = 28, antiderivative size = 299

$$\begin{aligned}\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx &= \frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}\\ &-\frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}\\ &+\frac{\sqrt[4]{a}\left(3\sqrt{b}d-\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/2*c*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+1/2*e*(b*x^4+a)^(1/2)/b
+1/3*f*x*(b*x^4+a)^(1/2)/b+d*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2
))-a^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^
(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^
4+a)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(-f*a^(1/2)+3*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+
x^2*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

3.532.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.54

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{3ae+2afx+3bex^4+2bfx^5+3\sqrt{b}c\sqrt{a+bx^4}\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-2afx\sqrt{1+\frac{bx^4}{a}}\text{Hypergeometric2F1}\left(\frac{1}{4}\right.}{6b\sqrt{a+bx^4}}$$

input
```
Integrate[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(3*a*e + 2*a*f*x + 3*b*e*x^4 + 2*b*f*x^5 + 3*Sqrt[b]*c*Sqrt[a + b*x^4]*Arc
Tanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 2*a*f*x*Sqrt[1 + (b*x^4)/a]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*d*x^3*Sqrt[1 + (b*x^4)/a]*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(6*b*Sqrt[a + b*x^4])
```

3.532.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$\Big\downarrow \ 2372$$
$$\int \left(\frac{x(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^2(d+fx^2)}{\sqrt{a+bx^4}}\right)dx$$
$$\Big\downarrow \ 2009$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}d-\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}$$

input
```
Int[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(e*Sqrt[a + b*x^4])/(2*b) + (f*x*Sqrt[a + b*x^4])/(3*b) + (d*x*Sqrt[a + b*
x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a
+ b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

3.532.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.532.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(2fx+3e)\sqrt{b\,x^4+a}}{6b}-\frac{\frac{af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}-\frac{3i\sqrt{b}\,d\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}-3\sqrt{b}}{3b}$
default	$f\left(\frac{x\sqrt{b\,x^4+a}}{3b}-\frac{a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+\frac{e\sqrt{b\,x^4+a}}{2b}+\frac{id\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}\sqrt{b}}$
elliptic	$\frac{fx\sqrt{b\,x^4+a}}{3b}+\frac{e\sqrt{b\,x^4+a}}{2b}-\frac{af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{c\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{id\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+}}{\sqrt{}}$

input
```
int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*f*x+3*e)/b*(b*x^4+a)^(1/2)-1/3/b*(a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3*I*b^(1/2)*d*a^(1/2)/(I/a^(1/
2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Elliptic
E(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/2*b^(1/2)*c*ln(x^2*b^(1/2)+(b*x^4+a)^(
1/2)))
```

3.532.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.44

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$

$$= \frac{12\sqrt{b}dx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-4\sqrt{b}(3d+f)x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+3\sqrt{b}cx\log\left(\right.}{12bx}$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(12*sqrt(b)*d*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) -
 4*sqrt(b)*(3*d + f)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1)
 + 3*sqrt(b)*c*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*sqr
t(b*x^4 + a)*(2*f*x^2 + 3*e*x + 6*d))/(b*x)
```

3.532.6 Sympy [A] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.43

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = e\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}+\frac{fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
e*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True))
+ c*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + d*x**3*gamma(3/4)*hyper((1/2
, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + f*x**5*
gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*
gamma(9/4))
```

3.532.7 Maxima [F]

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*c*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^2
))/sqrt(b) + integrate((f*x^4 + e*x^3 + d*x^2)/sqrt(b*x^4 + a), x)
```

3.532.8 Giac [F]

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x/sqrt(b*x^4 + a), x)
```

3.532.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{x\,(f\,x^3+e\,x^2+d\,x+c)}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
```

output
```
int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

3.533 $\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx$

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3.533.1 Optimal result

Integrand size = 27, antiderivative size = 276

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \frac{f\sqrt{a+bx^4}}{2b}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}$$

output
```
1/2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)+1/2*f*(b*x^4+a)^(1/2)/b
+e*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-a^(1/4)*e*(cos(2*arctan
(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(s
in(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4
+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/2*a^(1/4)*(co
s(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*E
llipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2
))*(e+c*b^(1/2)/a^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)
/(b*x^4+a)^(1/2)
```

3.533.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.54

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \frac{f\sqrt{a+bx^4}}{2b}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}} + \frac{ex^3\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4],x]
```

output
```
(f*Sqrt[a + b*x^4])/(2*b) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*
Sqrt[b]) + (c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*
x^4)/a)])/Sqrt[a + b*x^4] + (e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1
/2, 3/4, 7/4, -((b*x^4)/a)])/(3*Sqrt[a + b*x^4])
```

3.533.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \ 2424$$

$$\int \left(\frac{c+ex^2}{\sqrt{a+bx^4}}+\frac{x(d+fx^2)}{\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\sqrt{a+bx^4}}{2b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4],x]
```

output
```
(f*Sqrt[a + b*x^4])/(2*b) + (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt
[b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1
/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*
EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) +
 (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

3.533.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.533.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.75

method	result
default	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{f\sqrt{b\,x^4+a}}{2b}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}+\frac{d\ln}{}$
risch	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{f\sqrt{b\,x^4+a}}{2b}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}+\frac{d\ln}{}$
elliptic	$\frac{f\sqrt{b\,x^4+a}}{2b}+\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
+1/2*f*(b*x^4+a)^(1/2)/b+I*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/
2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2
))^(1/2),I))+1/2*d*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))/b^(1/2)
```

3.533.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.49

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{4\,a\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+a\sqrt{b}dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+4\,(bc-ae)\sqrt{b}x}{4\,abx}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(4*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) +
 a*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 4*(b*c
- a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 2*s
qrt(b*x^4 + a)*(a*f*x + 2*a*e))/(a*b*x)
```

3.533.6 Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = f\left(\begin{cases} \frac{x^4}{4\sqrt{a}} & \text{for}\ b=0 \\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise} \end{cases}\right)+\frac{d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True))
+ d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1
/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gam
ma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gam
ma(7/4))
```

3.533.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

3.533.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

3.533.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \int \frac{f\,x^3+e\,x^2+d\,x+c}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(1/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(1/2), x)
```

3.534 $\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx$

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3.534.1 Optimal result

Integrand size = 30, antiderivative size = 285

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = \frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}$$

output
```
-1/2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)+1/2*e*arctanh(x^2*b^(1/2)/
(b*x^4+a)^(1/2))/b^(1/2)+f*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))
-a^(1/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)
*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1
/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+
a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1
/2))*(a^(1/2)+x^2*b^(1/2))*(f+d*b^(1/2)/a^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b
^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

3.534.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.56

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = \frac{e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{dx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}} + \frac{fx^3\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]),x]
```

output
```
(e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) + (d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4] + (f*x^3*Sqrt[1 + (b*x^4)
/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(3*Sqrt[a + b*x^4])
```

3.534.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2371, 798, 73, 221, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \text{ 2371}$$

$$c\int \frac{1}{x\sqrt{bx^4+a}}dx + \int \frac{fx^2+ex+d}{\sqrt{bx^4+a}}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4}c\int \frac{1}{x^4\sqrt{bx^4+a}}dx^4 + \int \frac{fx^2+ex+d}{\sqrt{bx^4+a}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{c\int \frac{1}{\frac{x^8}{b}-\frac{a}{b}}d\sqrt{bx^4+a}}{2b}+\int \frac{fx^2+ex+d}{\sqrt{bx^4+a}}dx$$

$$\Big\downarrow \text{221}$$

$$\int \frac{fx^2+ex+d}{\sqrt{bx^4+a}}dx-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{ex}{\sqrt{bx^4+a}}+\frac{fx^2+d}{\sqrt{bx^4+a}}\right)dx-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]),x]
```

output
```
(f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a
]])/(2*Sqrt[a]) - (a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(
3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

3.534. $\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx$

3.534.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.534. $\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx$

3.534.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.76

method	result
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{e\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{if\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i}{}}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
default	$\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{if\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}+\frac{e\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4}\right.}{2\sqrt{b}}$

input
```
int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
+1/2*e*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))/b^(1/2)+I*f*a^(1/2)/(I/a^(1/2)*
b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ell
ipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*c/a^(1/2)*arctanh(a^(1/2)/(b*x^
4+a)^(1/2))
```

3.534.5 Fricas [F]

$$\int\frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx=\int\frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}\,x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^5 + a*x), x)
```

3.534.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.44

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = \frac{e\,\text{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} - \frac{c\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(1/2),x)
```

output
```
e*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - c*asinh(sqrt(a)/(sqrt(b)*x**2)
)/(2*sqrt(a)) + d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(
I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + f*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4
,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

3.534.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

3.534.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

3.534.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(1/2)), x)
```

3.535 $\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx$

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3.535.1 Optimal result

Integrand size = 30, antiderivative size = 309

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

output
```
-1/2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)+1/2*f*arctanh(x^2*b^(1/2)/
(b*x^4+a)^(1/2))/b^(1/2)-c*(b*x^4+a)^(1/2)/a/x+c*x*b^(1/2)*(b*x^4+a)^(1/2)
/a/(a^(1/2)+x^2*b^(1/2))-b^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^
2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a
)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

3.535.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.51

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = \frac{f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{d\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{c\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt{a+bx^4}} + \frac{ex\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]
```

output
```
(f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (d*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1
[-1/4, 1/2, 3/4, -((b*x^4)/a)])/(x*Sqrt[a + b*x^4]) + (e*x*Sqrt[1 + (b*x^4
)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

3.535.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(\frac{c+ex^2}{x^2\sqrt{a+bx^4}}+\frac{d+fx^2}{x\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{d\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]
```

output
```
-((c*Sqrt[a + b*x^4])/(a*x)) + (Sqrt[b]*c*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] +
 Sqrt[b]*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) -
(d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*c*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + Sqr
t[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2
]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a
 + b*x^4])
```

3.535.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.535. $\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx$

3.535.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.76

method	result
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{a x}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{f\ln\left(2x^{2}\sqrt{b}+2\sqrt{b\,x^{4}+a}\right)}{2\sqrt{b}}+\frac{ic\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+}}$
default	$\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{f\ln\left(x^{2}\sqrt{b}+\sqrt{b\,x^{4}+a}\right)}{2\sqrt{b}}-\frac{d\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{2\sqrt{a}}+c\left(-\frac{\sqrt{b\,x^{4}+a}}{a x}+\frac{i\sqrt{b}\,\sqrt{1-}}{}\right.$
risch	$-\frac{c\sqrt{b\,x^{4}+a}}{a x}+\frac{\frac{a e\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{i\sqrt{b}\,c\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{a f\ln\left(x^{2}\sqrt{b}-\right.}{2\sqrt{}}}{a}$

input
```
int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-c*(b*x^4+a)^(1/2)/a/x+e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*b^(1/2))^(1/2),I)+1/2*f*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))/b^(1/2)+
I*c/a^(1/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*d/a^(1/2
)*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

3.535.5 Fricas [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}\,x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^6 + a*x^2), x)
```

3.535.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.41

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = \frac{f\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} - \frac{d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(1/2),x)
```

output
```
f*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*gamma(-1/4)*hyper((-1/4, 1/2
), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - d*asinh(sq
rt(a)/(sqrt(b)*x**2))/(2*sqrt(a)) + e*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,
), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

3.535.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

3.535.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^2}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

3.535.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^2\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(1/2)), x)
```

3.536 $\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx$

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3.536.1 Optimal result

Integrand size = 30, antiderivative size = 300

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx &= -\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{e\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\\&\quad-\frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}\\&\quad+\frac{\left(\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/2*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/2*c*(b*x^4+a)^(1/2)/a/x^
2-d*(b*x^4+a)^(1/2)/a/x+d*x*b^(1/2)*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2)
)-b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4
)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4
+a)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(f
*a^(1/2)+d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))
^2)^(1/2)/a^(3/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

3.536.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.49

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{2ax^2} - \frac{e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{d\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt{a+bx^4}} + \frac{fx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]
```

output
```
-1/2*(c*Sqrt[a + b*x^4])/(a*x^2) - (e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2
*Sqrt[a]) - (d*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*
x^4)/a)])/(x*Sqrt[a + b*x^4]) + (f*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1
[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

3.536.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \; 2372$$

$$\int \left(\frac{c+ex^2}{x^3\sqrt{a+bx^4}}+\frac{d+fx^2}{x^2\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{e\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]
```

output
```
-1/2*(c*Sqrt[a + b*x^4])/(a*x^2) - (d*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*d*
x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (e*ArcTanh[Sqrt[a + b*x^4
]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b
]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])
```

3.536.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.536.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.75

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{2a\,x^2}-\frac{d\sqrt{b\,x^4+a}}{ax}+\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{i\sqrt{b}\,d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\right.\right.}{\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
risch	$-\frac{\sqrt{b\,x^4+a}\,(2dx+c)}{2a\,x^2}+\frac{\frac{af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{i\sqrt{b}\,d\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sqrt{a}}{}}{a}$
default	$\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2\sqrt{a}}+d\left(-\frac{\sqrt{b\,x^4+a}}{ax}+\frac{i\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\right.\right.}{\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*(b*x^4+a)^(1/2)/a/x^2-d*(b*x^4+a)^(1/2)/a/x+f/(I/a^(1/2)*b^(1/2))^(
1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+I*b^(1/2)/a^(1/2)*d/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-El
lipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*e/a^(1/2)*arctanh(a^(1/2)/(b*x
^4+a)^(1/2))
```

3.536.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx =$$
$$-\frac{4\sqrt{a}bdx^2\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid-1)-\sqrt{a}bex^2\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-4\left(bd-af\right)\sqrt{a}x^2\left(-\right.}{4\,abx^2}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(4*sqrt(a)*b*d*x^2*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1
) - sqrt(a)*b*e*x^2*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) -
4*(b*d - a*f)*sqrt(a)*x^2*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)),
-1) + 2*sqrt(b*x^4 + a)*(2*b*d*x + b*c))/(a*b*x^2)
```

3.536.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} - \frac{e\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(1/2),x)
```

output
```
-sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(2*a) + d*gamma(-1/4)*hyper((-1/4, 1/2), (
3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - e*asinh(sqrt(a
)/(sqrt(b)*x**2))/(2*sqrt(a)) + f*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b
*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

3.536.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

3.536.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

3.536.9 Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.39

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = \frac{f\,x\,\sqrt{\frac{b\,x^4}{a}+1}\,{}_2\mathrm{F}_1\!\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\sqrt{b\,x^4+a}} - \frac{c\,\sqrt{b\,x^4+a}}{2\,a\,x^2} - \frac{d\,\sqrt{\frac{a}{b\,x^4}+1}\,{}_2\mathrm{F}_1\!\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{a}{b\,x^4}\right)}{3\,x\,\sqrt{b\,x^4+a}} - \frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(1/2)),x)
```

output
```
(f*x*((b*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(b*x^4)/a))/(a + b*
x^4)^(1/2) - (c*(a + b*x^4)^(1/2))/(2*a*x^2) - (d*(a/(b*x^4) + 1)^(1/2)*hy
pergeom([1/2, 3/4], 7/4, -a/(b*x^4)))/(3*x*(a + b*x^4)^(1/2)) - (e*atanh((
a + b*x^4)^(1/2)/a^(1/2)))/(2*a^(1/2))
```

3.537 $\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$

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3.537.1 Optimal result

Integrand size = 30, antiderivative size = 323

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx &= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\\&\quad-\frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}\\&\quad-\frac{\sqrt[4]{b}\left(\sqrt{b}c-3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*c*(b*x^4+a)^(1/2)/a/x^
3-1/2*d*(b*x^4+a)^(1/2)/a/x^2-e*(b*x^4+a)^(1/2)/a/x+e*x*b^(1/2)*(b*x^4+a)^
(1/2)/a/(a^(1/2)+x^2*b^(1/2))-b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1
/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)-1/6*b^(1/4)*(cos(2*arctan(b^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan
(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-3*e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^
(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^4+a)^(1/2)
```

3.537.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$$
$$= \frac{-2ac\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{bx^4}{a}\right)-3x\left(ad+bdx^4+\sqrt{a}fx^2\sqrt{a+bx^4}\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right.\right.}{6ax^3\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]),x]
```

output
```
(-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)
] - 3*x*(a*d + b*d*x^4 + Sqrt[a]*f*x^2*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*
x^4]/Sqrt[a]] + 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -((b*x^4)/a)]))/(6*a*x^3*Sqrt[a + b*x^4])
```

3.537.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$$
$$\Big\downarrow \; 2372$$
$$\int \left(\frac{c+ex^2}{x^4\sqrt{a+bx^4}}+\frac{d+fx^2}{x^3\sqrt{a+bx^4}}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-3\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{f\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]),x]
```

output
```
-1/3*(c*Sqrt[a + b*x^4])/(a*x^3) - (d*Sqrt[a + b*x^4])/(2*a*x^2) - (e*Sqrt
[a + b*x^4])/(a*x) + (Sqrt[b]*e*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x
^2)) - (f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*e*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) - (b^(1/4)*(
Sqrt[b]*c - 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/
4)*Sqrt[a + b*x^4])
```

3.537.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.537.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(6e\,x^{2}+3dx+2c)}{6a\,x^{3}}+\frac{-\frac{bc\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{3i\sqrt{b}\,e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}}{3a}$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{3a\,x^{3}}-\frac{d\sqrt{b\,x^{4}+a}}{2a\,x^{2}}-\frac{e\sqrt{b\,x^{4}+a}}{ax}-\frac{cb\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{i\sqrt{b}\,e\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{}\right.\right.}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}}}$
default	$-\frac{f\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{2\sqrt{a}}+c\left(-\frac{\sqrt{b\,x^{4}+a}}{3a\,x^{3}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+e\left(-\frac{\sqrt{b\,x^{4}+a}}{ax}+\frac{i\sqrt{b}\,\sqrt{1-}}{}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^4+a)^(1/2)*(6*e*x^2+3*d*x+2*c)/a/x^3+1/3/a*(-b*c/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)
/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3*I*b^(1/2)*e*a^
(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/
2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/2*a^(1/2)*f*ln((2*a+2*a^
(1/2)*(b*x^4+a)^(1/2))/x^2))
```

3.537.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx =$$
$$-\frac{12\,\sqrt{a}ex^3\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-4\,\sqrt{a}(c+3\,e)x^3\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-3\,\sqrt{a}}{12\,ax^3}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(12*sqrt(a)*e*x^3*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1
) - 4*sqrt(a)*(c + 3*e)*x^3*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4))
, -1) - 3*sqrt(a)*f*x^3*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4
) + 2*sqrt(b*x^4 + a)*(6*e*x^2 + 3*d*x + 2*c))/(a*x^3)
```

3.537.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.41

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} - \frac{f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(1/2),x)
```

output
```
-sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(2*a) + c*gamma(-3/4)*hyper((-3/4, 1/2), (
1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + e*gamma(-1/
4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma
(3/4)) - f*asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a))
```

3.537.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

3.537.8 Giac [F]

$$\int \frac{c + dx + ex^2 + fx^3}{x^4\sqrt{a + bx^4}}\,dx = \int \frac{fx^3 + ex^2 + dx + c}{\sqrt{bx^4 + ax^4}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

3.537.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3}{x^4\sqrt{a + bx^4}}\,dx = \int \frac{f\,x^3 + e\,x^2 + d\,x + c}{x^4\,\sqrt{b\,x^4 + a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(1/2)), x)
```

3.538 $\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx$

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3.538.1 Optimal result

Integrand size = 30, antiderivative size = 346

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{bc\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}\left(\sqrt{b}d-3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}$$

output
```
1/4*b*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/4*c*(b*x^4+a)^(1/2)/a/x
^4-1/3*d*(b*x^4+a)^(1/2)/a/x^3-1/2*e*(b*x^4+a)^(1/2)/a/x^2-f*(b*x^4+a)^(1/
2)/a/x+f*x*b^(1/2)*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-b^(1/4)*f*(cos(
2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ell
ipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))
*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)-1/6*b^(
1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(
1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-3*f*a^(1/
2)+d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1
/2)/a^(5/4)/(b*x^4+a)^(1/2)
```

3.538.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}\left(3ac\sqrt{1+\frac{bx^4}{a}}+6aex^2\sqrt{1+\frac{bx^4}{a}}-3bcx^4\text{arctanh}\left(\sqrt{1+\frac{bx^4}{a}}\right)+4adx\,\text{Hypergeometric2F1}\left(\cdots\right.\right.}{12a^2x^4\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]
```

output
```
-1/12*(Sqrt[a + b*x^4]*(3*a*c*Sqrt[1 + (b*x^4)/a] + 6*a*e*x^2*Sqrt[1 + (b*
x^4)/a] - 3*b*c*x^4*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*d*x*Hypergeometric2
F1[-3/4, 1/2, 1/4, -((b*x^4)/a)] + 12*a*f*x^3*Hypergeometric2F1[-1/4, 1/2,
 3/4, -((b*x^4)/a)]))/(a^2*x^4*Sqrt[1 + (b*x^4)/a])
```

3.538.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(\frac{c+ex^2}{x^5\sqrt{a+bx^4}}+\frac{d+fx^2}{x^4\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-3\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}+\frac{b\text{carctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]
```

output
```
-1/4*(c*Sqrt[a + b*x^4])/(a*x^4) - (d*Sqrt[a + b*x^4])/(3*a*x^3) - (e*Sqrt
[a + b*x^4])/(2*a*x^2) - (f*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*f*x*Sqrt[a +
 b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a
]])/(4*a^(3/2)) - (b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(
3/4)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*d - 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])
```

3.538.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.538.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(12f\,x^3+6e\,x^2+4dx+3c)}{12a\,x^4}-\frac{b\left(\frac{2d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{6if\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right.}{6a}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{4a\,x^4}-\frac{d\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{e\sqrt{b\,x^4+a}}{2a\,x^2}-\frac{f\sqrt{b\,x^4+a}}{ax}-\frac{bd\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{i\sqrt{b}\,f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+}}{\sqrt{}}$
default	$d\left(-\frac{\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(-\frac{\sqrt{b\,x^4+a}}{ax}+\frac{i\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x^4+a)^(1/2)*(12*f*x^3+6*e*x^2+4*d*x+3*c)/a/x^4-1/6*b/a*(2*d/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-6*I*f
*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/2*c/a^(1/2)*l
n((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

3.538.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.43

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx =$$
$$-\frac{24\,a^{\frac{3}{2}}fx^4\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-3\sqrt{a}bcx^4\log\left(-\frac{bx^4+2\sqrt{bx^4+a}\sqrt{a}+2\,a}{x^4}\right)-8\,(ad+3\,af)\sqrt{a}x^4(}{24\,a^2x^4}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(24*a^(3/2)*f*x^4*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1
) - 3*sqrt(a)*b*c*x^4*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4)
- 8*(a*d + 3*a*f)*sqrt(a)*x^4*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4
)), -1) + 2*(12*a*f*x^3 + 6*a*e*x^2 + 4*a*d*x + 3*a*c)*sqrt(b*x^4 + a))/(a
^2*x^4)
```

3.538.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4ax^2} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{f\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} + \frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**5/(b*x**4+a)**(1/2),x)
```

output
```
-sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*e*sqrt(a/(b*x**4) + 1
)/(2*a) + d*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*sqrt(a)*x**3*gamma(1/4)) + f*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,),
b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) + b*c*asinh(sqrt(a)/(sq
rt(b)*x**2))/(4*a**(3/2))
```

3.538.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^5}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*c*(2*sqrt(b*x^4 + a)*b/((b*x^4 + a)*a - a^2) + b*log((sqrt(b*x^4 + a)
 - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^(3/2)) + integrate((f*x^2 + e*x
 + d)/(sqrt(b*x^4 + a)*x^4), x)
```

3.538.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^5), x)
```

3.538.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^5\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^5*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x^5*(a + b*x^4)^(1/2)), x)
```

3.539 $\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx$

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3.539.1 Optimal result

Integrand size = 30, antiderivative size = 377

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}-\frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{bd\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}+\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}}-\frac{b^{3/4}\left(9\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}}$$

output
```
1/4*b*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/5*c*(b*x^4+a)^(1/2)/a/x
^5-1/4*d*(b*x^4+a)^(1/2)/a/x^4-1/3*e*(b*x^4+a)^(1/2)/a/x^3-1/2*f*(b*x^4+a)
^(1/2)/a/x^2+3/5*b*c*(b*x^4+a)^(1/2)/a^2/x-3/5*b^(3/2)*c*x*(b*x^4+a)^(1/2)
/a^2/(a^(1/2)+x^2*b^(1/2))+3/5*b^(5/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(
1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-1/30*b^(3/4)*(cos(2*arctan(b^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arct
an(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(5*e*a^(1/2)+9*c*b^(1/2))*(a^(1/2)+x^2
*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2
)
```

3.539.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.36

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}\left(12ac\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},-\frac{bx^4}{a}\right)+5x\left(3a(d+2fx^2)\sqrt{1+\frac{bx^4}{a}}-3bdx^4\text{arctanh}\right.\right.}{60a^2x^5\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]
```

output
```
-1/60*(Sqrt[a + b*x^4]*(12*a*c*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^4
)/a)] + 5*x*(3*a*(d + 2*f*x^2)*Sqrt[1 + (b*x^4)/a] - 3*b*d*x^4*ArcTanh[Sqr
t[1 + (b*x^4)/a]] + 4*a*e*x*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)
])))/(a^2*x^5*Sqrt[1 + (b*x^4)/a])
```

3.539.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \; 2372$$

$$\int \left(\frac{c+ex^2}{x^6\sqrt{a+bx^4}}+\frac{d+fx^2}{x^5\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}}+\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}}+\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}-\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]
```

output
```
-1/5*(c*Sqrt[a + b*x^4])/(a*x^5) - (d*Sqrt[a + b*x^4])/(4*a*x^4) - (e*Sqrt
[a + b*x^4])/(3*a*x^3) - (f*Sqrt[a + b*x^4])/(2*a*x^2) + (3*b*c*Sqrt[a + b
*x^4])/(5*a^2*x) - (3*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a^2*(Sqrt[a] + Sqrt[
b]*x^2)) + (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) + (3*b^(5/4)
*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a + b*x^4]) -
(b^(3/4)*(9*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(30*a^(7/4)*Sqrt[a + b*x^4])
```

3.539.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.539.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(-36bc\,x^4+30af\,x^3+20ae\,x^2+15adx+12ac)}{60a^2x^5}-\frac{b\left(\frac{10ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{18i\sqrt{b}\,c\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1-\cdots}}{\cdots}\cdots\right)}{30\cdots}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{5a\,x^5}-\frac{d\sqrt{b\,x^4+a}}{4a\,x^4}-\frac{e\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{f\sqrt{b\,x^4+a}}{2a\,x^2}+\frac{3bc\sqrt{b\,x^4+a}}{5a^2x}-\frac{be\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{3ib^{\frac{3}{2}}c\cdots}{\cdots}$
default	$e\left(-\frac{\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)-\frac{f\sqrt{b\,x^4+a}}{2a\,x^2}+d\left(-\frac{\sqrt{b\,x^4+a}}{4a\,x^4}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{4a^{\frac{3}{2}}}\right)-\cdots$

input
```
int((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(b*x^4+a)^(1/2)*(-36*b*c*x^4+30*a*f*x^3+20*a*e*x^2+15*a*d*x+12*a*c)/
a^2/x^5-1/30*b/a^2*(10*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)+18*I*b^(1/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^
(1/2))^(1/2),I))-15/2*a^(1/2)*d*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2))
```

3.539.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx$$
$$=\frac{72\sqrt{a}bcx^5\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)+15\sqrt{a}bdx^5\log\left(-\frac{bx^4+2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-8(9bc-5ae)\sqrt{a}x^5\cdots}{120a^2x^5}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/120*(72*sqrt(a)*b*c*x^5*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)),
-1) + 15*sqrt(a)*b*d*x^5*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^
4) - 8*(9*b*c - 5*a*e)*sqrt(a)*x^5*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)
^(1/4)), -1) + 2*(36*b*c*x^4 - 30*a*f*x^3 - 20*a*e*x^2 - 15*a*d*x - 12*a*c
)*sqrt(b*x^4 + a))/(a^2*x^5)
```

3.539.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.43

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4ax^2} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**6/(b*x**4+a)**(1/2),x)
```

output
```
-sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*f*sqrt(a/(b*x**4) + 1
)/(2*a) + c*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**4*exp_polar(I*pi)
/a)/(4*sqrt(a)*x**5*gamma(-1/4)) + e*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + b*d*asinh(sqrt(a
)/(sqrt(b)*x**2))/(4*a**(3/2))
```

3.539.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^6}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

3.539.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^6}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

3.539.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = \int \frac{f\,x^3+e\,x^2+d\,x+c}{x^6\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^6*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x^6*(a + b*x^4)^(1/2)), x)
```

3.540 $\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

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3.540.1 Optimal result

Integrand size = 30, antiderivative size = 365

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{b^2}+\frac{ex\sqrt{a+bx^4}}{3b^2}+\frac{fx^2\sqrt{a+bx^4}}{4b^2}+\frac{3cx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{3af\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{5/2}}-\frac{3\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\left(9\sqrt{b}c-5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}$$

output
```
-3/4*a*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(5/2)+1/2*x*(-b*d*x^3-b*c*
x^2+a*f*x+a*e)/b^2/(b*x^4+a)^(1/2)+d*(b*x^4+a)^(1/2)/b^2+1/3*e*x*(b*x^4+a)
^(1/2)/b^2+1/4*f*x^2*(b*x^4+a)^(1/2)/b^2+3/2*c*x*(b*x^4+a)^(1/2)/b^(3/2)/(
a^(1/2)+x^2*b^(1/2))-3/2*a^(1/4)*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^
2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)+1/12*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(-5*e*a^(1/2)+9*c*b^(1/2))*(a^(1/2)+x^2*b^(1
/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^4+a)^(1/2)
```

3.540.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.60

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{12a\sqrt{b}d+10a\sqrt{b}ex+9a\sqrt{b}fx^2+12b^{3/2}cx^3+6b^{3/2}dx^4+4b^{3/2}ex^5+3b^{3/2}}{}$$

input
```
Integrate[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(12*a*Sqrt[b]*d + 10*a*Sqrt[b]*e*x + 9*a*Sqrt[b]*f*x^2 + 12*b^(3/2)*c*x^3
+ 6*b^(3/2)*d*x^4 + 4*b^(3/2)*e*x^5 + 3*b^(3/2)*f*x^6 - 9*a^(3/2)*f*Sqrt[1
 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - 10*a*Sqrt[b]*e*x*Sqrt[1 + (
b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*b^(3/2)*c*x^
3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(12*
b^(5/2)*Sqrt[a + b*x^4])
```

3.540.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2367, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2367}$$

$$\frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}}-\frac{\int \frac{-2ab^2fx^5-2ab^2ex^4-4ab^2dx^3-3ab^2cx^2+2a^2bfx+a^2be}{\sqrt{bx^4+a}}dx}{2ab^3}$$

$$\Big\downarrow \text{ 2424}$$

$$\frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}}-\frac{\int \left(\frac{-2ab^2ex^4-3ab^2cx^2+a^2be}{\sqrt{bx^4+a}}+\frac{x(-2ab^2fx^4-4ab^2dx^2+2a^2bf)}{\sqrt{bx^4+a}}\right)dx}{2ab^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{x(ae+afx-bcx^2-bdx^3)}{2b^2\sqrt{a+bx^4}}-}{-\frac{a^{5/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}c-5\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt{a+bx^4}}+\frac{3a^{5/4}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt{a+bx^4}}}{2ab}$$

input
```
Int[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*b^2*Sqrt[a + b*x^4]) - (-2*a*b*d*
Sqrt[a + b*x^4] - (2*a*b*e*x*Sqrt[a + b*x^4])/3 - (a*b*f*x^2*Sqrt[a + b*x^
4])/2 - (3*a*b^(3/2)*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + (3*a^2
*Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 + (3*a^(5/4)*b^(5/4)*
c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] - (a^(5/4)*b^(3
/4)*(9*Sqrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
6*Sqrt[a + b*x^4]))/(2*a*b^3)
```

3.540.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.540. $\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

3.540.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.07 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.83

method	result
elliptic	$-\frac{2b\left(\frac{c\,x^3}{4b^2}-\frac{af\,x^2}{4b^3}-\frac{aex}{4b^3}-\frac{ad}{4b^3}\right)}{\sqrt{(x^4+\frac{a}{b})b}}+\frac{f\,x^2\sqrt{b\,x^4+a}}{4b^2}+\frac{ex\sqrt{b\,x^4+a}}{3b^2}+\frac{d\sqrt{b\,x^4+a}}{2b^2}-\frac{5ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}-$
default	$f\left(\frac{x^6}{4b\sqrt{b\,x^4+a}}+\frac{3a\,x^2}{4b^2\sqrt{b\,x^4+a}}-\frac{3a\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{4b^{\frac{5}{2}}}\right)+e\left(\frac{ax}{2b^2\sqrt{(x^4+\frac{a}{b})b}}+\frac{x\sqrt{b\,x^4+a}}{3b^2}-\frac{5a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b}}\right.$
risch	$\frac{(3f\,x^2+4ex+6d)\sqrt{b\,x^4+a}}{12b^2}+\frac{aex}{2b^2\sqrt{(x^4+\frac{a}{b})b}}-\frac{5ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}-\frac{c\,x^3}{2b\sqrt{(x^4+\frac{a}{b})b}}+\frac{3ic\sqrt{a}\sqrt{1-\frac{i\sqrt{b}}{\sqrt{}}}}{2b}$

input
```
int(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/4*c/b^2*x^3-1/4*a*f/b^3*x^2-1/4/b^3*a*e*x-1/4*a*d/b^3)/((x^4+a/b)*
b)^(1/2)+1/4*f*x^2*(b*x^4+a)^(1/2)/b^2+1/3*e*x*(b*x^4+a)^(1/2)/b^2+1/2*d*(
b*x^4+a)^(1/2)/b^2-5/6/b^2*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/4*a*f/b^(5/2)*ln(2*x^2*b^(1/2)+2*(b*x^4+a)
^(1/2))+3/2*I*c/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

3.540.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.66

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{36\,(b^2cx^5+abcx)\sqrt{b}(-\frac{a}{b})^{\frac{3}{4}}\,E(\arcsin\left(\frac{(-\frac{a}{b})^{\frac{1}{4}}}{x}\right)\,|-1)-4\,((9\,b^2c+5\,b^2e)x^5}{}$$

input
```
integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(36*(b^2*c*x^5 + a*b*c*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/
b)^(1/4)/x), -1) - 4*((9*b^2*c + 5*b^2*e)*x^5 + (9*a*b*c + 5*a*b*e)*x)*sqr
t(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 9*(a*b*f*x^5 +
a^2*f*x)*sqrt(b)*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(3*
b^2*f*x^7 + 4*b^2*e*x^6 + 6*b^2*d*x^5 + 12*b^2*c*x^4 + 9*a*b*f*x^3 + 10*a*
b*e*x^2 + 12*a*b*d*x + 18*a*b*c)*sqrt(b*x^4 + a))/(b^4*x^5 + a*b^3*x)
```

3.540.6 Sympy [A] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.55

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = d\left(\begin{cases} \frac{a}{b^2\sqrt{a+bx^4}} + \frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + f\left(\frac{3\sqrt{a}x^2}{4b^2\sqrt{1+\frac{bx^4}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} + \frac{x^6}{4\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{ex^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**6*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
d*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x**4)), Ne(b
, 0)), (x**8/(8*a**(3/2)), True)) + f*(3*sqrt(a)*x**2/(4*b**2*sqrt(1 + b*x
**4/a)) - 3*a*asinh(sqrt(b)*x**2/sqrt(a))/(4*b**(5/2)) + x**6/(4*sqrt(a)*b
*sqrt(1 + b*x**4/a))) + c*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**
4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + e*x**9*gamma(9/4)*hyper((3
/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(13/4))
```

3.540.7 Maxima [F]

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^6}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```

3.540.8 Giac [F]

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^6}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```

3.540.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^6\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.541 $\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

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3.541.1 Optimal result

Integrand size = 30, antiderivative size = 343

$$\begin{aligned}\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx &= \frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} \\ &+ \frac{e\sqrt{a+bx^4}}{b^2}+\frac{fx\sqrt{a+bx^4}}{3b^2}+\frac{3dx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} \\ &- \frac{3\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} \\ &+ \frac{\sqrt[4]{a}\left(9\sqrt{b}d-5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/2*c*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/2*x*(-b*e*x^3-b*d*x^2
-b*c*x+a*f)/b^2/(b*x^4+a)^(1/2)+e*(b*x^4+a)^(1/2)/b^2+1/3*f*x*(b*x^4+a)^(1
/2)/b^2+3/2*d*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))-3/2*a^(1/4)*
d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4
)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b
^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)+
1/12*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-5
*f*a^(1/2)+9*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1
/2))^2)^(1/2)/b^(9/4)/(b*x^4+a)^(1/2)
```

3.541.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.51

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{6ae+5afx-3bcx^2+6bdx^3+3bex^4+2bfx^5+3\sqrt{a}\sqrt{b}c\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}(}{}$$

input
```
Integrate[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(6*a*e + 5*a*f*x - 3*b*c*x^2 + 6*b*d*x^3 + 3*b*e*x^4 + 2*b*f*x^5 + 3*Sqrt[
a]*Sqrt[b]*c*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - 5*a*f*x*
Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 6*b*d
*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(
6*b^2*Sqrt[a + b*x^4])
```

3.541.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2367, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2367}$$

$$\frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}}-\frac{\int \frac{-2abfx^4-4abex^3-3abdx^2-2abcx+a^2f}{\sqrt{bx^4+a}}dx}{2ab^2}$$

$$\Big\downarrow \text{2424}$$

$$\frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}}-\frac{\int \left(\frac{x(-4abex^2-2abc)}{\sqrt{bx^4+a}}+\frac{-2abfx^4-3abdx^2+a^2f}{\sqrt{bx^4+a}}\right)dx}{2ab^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(af-bcx-bdx^2-bex^3)}{2b^2\sqrt{a+bx^4}} - \frac{-\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}d-5\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{3a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}\right.\right.}{\sqrt{a+bx^4}}}{2ab^2}$$

input
```
Int[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*b^2*Sqrt[a + b*x^4]) - (-2*a*e*Sq
rt[a + b*x^4] - (2*a*f*x*Sqrt[a + b*x^4])/3 - (3*a*Sqrt[b]*d*x*Sqrt[a + b*
x^4])/(Sqrt[a] + Sqrt[b]*x^2) - a*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
 b*x^4]] + (3*a^(5/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/S
qrt[a + b*x^4] - (a^(5/4)*(9*Sqrt[b]*d - 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(6*b^(1/4)*Sqrt[a + b*x^4]))/(2*a*b^2)
```

3.541.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

3.541. $\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

3.541.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.82

method	result
elliptic	$-\frac{2b\left(\frac{d\,x^{3}}{4b^{2}}+\frac{c\,x^{2}}{4b^{2}}-\frac{a f x}{4b^{3}}-\frac{a e}{4b^{3}}\right)}{\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{f x\sqrt{b\,x^{4}+a}}{3b^{2}}+\frac{e\sqrt{b\,x^{4}+a}}{2b^{2}}-\frac{5af\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{c\ln\left(2x^{2}\sqrt{b}+2\sqrt{b\,x\right.}}{2b^{\frac{3}{2}}}$
default	$f\left(\frac{ax}{2b^{2}\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{x\sqrt{b\,x^{4}+a}}{3b^{2}}-\frac{5a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+\frac{e\left(b\,x^{4}+2a\right)}{2\sqrt{b\,x^{4}+a}\,b^{2}}+d\left(-\frac{x^{3}}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\right.$
risch	$\frac{(2fx+3e)\sqrt{b\,x^{4}+a}}{6b^{2}}+\frac{afx}{2b^{2}\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{5af\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{d\,x^{3}}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{3id\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1}}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}$

input
```
int(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/4*d*x^3/b^2+1/4*c/b^2*x^2-1/4*a*f/b^3*x-1/4*a*e/b^3)/((x^4+a/b)*b)
^(1/2)+1/3*f*x*(b*x^4+a)^(1/2)/b^2+1/2*e*(b*x^4+a)^(1/2)/b^2-5/6/b^2*a*f/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/
2*c/b^(3/2)*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+3/2*I*d/b^(3/2)*a^(1/2)/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-El
lipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

3.541.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.62

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx=\frac{18\,(bdx^5+adx)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-2\,((9\,bd+5\,bf)x^5+(}{}$$

input
```
integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/12*(18*(b*d*x^5 + a*d*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(
1/4)/x), -1) - 2*((9*b*d + 5*b*f)*x^5 + (9*a*d + 5*a*f)*x)*sqrt(b)*(-a/b)^
(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 3*(b*c*x^5 + a*c*x)*sqrt(b)
*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(2*b*f*x^6 + 3*b*e*
x^5 + 6*b*d*x^4 - 3*b*c*x^3 + 5*a*f*x^2 + 6*a*e*x + 9*a*d)*sqrt(b*x^4 + a)
)/(b^3*x^5 + a*b^2*x)
```

3.541.6 Sympy [A] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.50

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = c\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + e\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}} + \frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + \frac{dx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{fx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**5*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
c*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b
*x**4/a))) + e*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b
*x**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + d*x**7*gamma(7/4)*hyper((
3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + f
*x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(3/2)*gamma(13/4))
```

3.541.7 Maxima [F]

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx=\int \frac{(fx^3+ex^2+dx+c)x^5}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*c*(2*x^2/(sqrt(b*x^4 + a)*b) + log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(
sqrt(b) + sqrt(b*x^4 + a)/x^2))/b^(3/2)) + integrate((f*x^8 + e*x^7 + d*x^
6)/(b*x^4 + a)^(3/2), x)
```

3.541.8 Giac [F]

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx=\int \frac{(fx^3+ex^2+dx+c)x^5}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^5/(b*x^4 + a)^(3/2), x)
```

3.541.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx=\int \frac{x^5\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.542 $\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

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3.542.1 Optimal result

Integrand size = 30, antiderivative size = 314

$$\begin{aligned}\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx &= -\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}}\\&+\frac{f\sqrt{a+bx^4}}{b^2}+\frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}\\&-\frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}\\&+\frac{\left(\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/2*d*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)-1/2*x*(f*x^3+e*x^2+d*x+
c)/b/(b*x^4+a)^(1/2)+f*(b*x^4+a)^(1/2)/b^2+3/2*e*x*(b*x^4+a)^(1/2)/b^(3/2)
/(a^(1/2)+x^2*b^(1/2))-3/2*a^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2)
)^2)^(1/2)/b^(7/4)/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/
a^(1/4))),1/2*2^(1/2))*(3*e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x
^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(7/4)/(b*x^4+a)^(1/2)
```

3.542.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.53

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{2af-bcx-bdx^2+2bex^3+bfx^4+\sqrt{a}\sqrt{b}d\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+bcx}{}$$

input
```
Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(2*a*f - b*c*x - b*d*x^2 + 2*b*e*x^3 + b*f*x^4 + Sqrt[a]*Sqrt[b]*d*Sqrt[1
+ (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + b*c*x*Sqrt[1 + (b*x^4)/a]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 2*b*e*x^3*Sqrt[1 + (b*x^4)/
a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(2*b^2*Sqrt[a + b*x^4])
```

3.542.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2367, 25, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2367}$$

$$-\frac{\int -\frac{4abfx^3+3abex^2+2abdx+abc}{\sqrt{bx^4+a}}dx}{2ab^2}-\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4abfx^3+3abex^2+2abdx+abc}{\sqrt{bx^4+a}}dx}{2ab^2}-\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2424}$$

$$\frac{\int \left(\frac{3abex^2+abc}{\sqrt{bx^4+a}}+\frac{x(4abfx^2+2abd)}{\sqrt{bx^4+a}}\right)dx}{2ab^2}-\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt{a+bx^4}}-\frac{3a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{\sqrt{a+bx^4}}}{2ab^2}$$

$$\frac{x\left(c+dx+ex^2+fx^3\right)}{2b\sqrt{a+bx^4}}$$

input
```
Int[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(x*(c + d*x + e*x^2 + f*x^3))/(b*Sqrt[a + b*x^4]) + (2*a*f*Sqrt[a + b
*x^4] + (3*a*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + a*Sqrt
[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - (3*a^(5/4)*b^(1/4)*e*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (a^(3/4)*b^(1/4)*(Sqr
t[b]*c + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*Sqrt[a +
 b*x^4]))/(2*a*b^2)
```

3.542.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.542.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.84

method	result
elliptic	$-\frac{2b\left(\frac{e\,x^3}{4b^2}+\frac{d\,x^2}{4b^2}+\frac{cx}{4b^2}-\frac{af}{4b^3}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{f\sqrt{b\,x^4+a}}{2b^2}+\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}+\frac{3ie\sqrt{a}\,\sqrt{}}{}$
default	$\frac{f\left(b\,x^4+2a\right)}{2\sqrt{b\,x^4+a}\,b^2}+e\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\frac{x^2}{2b\sqrt{b\,x^4+}}\right.$
risch	$\frac{f\sqrt{b\,x^4+a}}{2b^2}+\frac{be\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+bd\left(-\frac{x^2}{2b\sqrt{b\,x^4+a}}+\frac{\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4}\right.}{2b^{\frac{3}{2}}}\right.}{b}$

input
```
int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/4*e/b^2*x^3+1/4*d*x^2/b^2+1/4*c/b^2*x-1/4*a*f/b^3)/((x^4+a/b)*b)^(
1/2)+1/2*f*(b*x^4+a)^(1/2)/b^2+1/2*c/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1
/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*d/b^(3/2)*ln(2*x^2*b^(1/2)+2*(b*
x^4+a)^(1/2))+3/2*I/b^(3/2)*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/
2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2
),I))
```

3.542.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.71

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{6\left(abex^5+a^2ex\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+2\left((b^2c-3\,abe)x^5+\right.}{}$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(6*(a*b*e*x^5 + a^2*e*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)
^(1/4)/x), -1) + 2*((b^2*c - 3*a*b*e)*x^5 + (a*b*c - 3*a^2*e)*x)*sqrt(b)*(
-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (a*b*d*x^5 + a^2*d*x)
*sqrt(b)*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(a*b*f*x^5
+ 2*a*b*e*x^4 - a*b*d*x^3 - a*b*c*x^2 + 2*a^2*f*x + 3*a^2*e)*sqrt(b*x^4 +
a))/(a*b^3*x^5 + a^2*b^2*x)
```

3.542.6 Sympy [A] (verification not implemented)

Time = 7.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.55

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + f\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}}+\frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + \frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
d*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b
*x**4/a))) + f*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b
*x**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + c*x**5*gamma(5/4)*hyper((
5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x
**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/2)*gamma(11/4))
```

3.542.7 Maxima [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^4}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)
```

3.542.8 Giac [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^4}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)
```

3.542.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^4\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.543 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

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3.543.8 Giac [F] 4200
3.543.9 Mupad [F(-1)] 4200

3.543.1 Optimal result

Integrand size = 30, antiderivative size = 302

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{-c-dx-ex^2-fx^3}{2b\sqrt{a+bx^4}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} - \frac{3\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}$$

output
```
1/2*e*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/2*(-f*x^3-e*x^2-d*x-c
)/b/(b*x^4+a)^(1/2)+3/2*f*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/2))-
3/2*a^(1/4)*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x
^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*
(3*f*a^(1/2)+d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1
/2))^2)^(1/2)/a^(1/4)/b^(7/4)/(b*x^4+a)^(1/2)
```

3.543.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.60

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{-\sqrt{b}c-\sqrt{b}dx-\sqrt{b}ex^2+2\sqrt{b}fx^3+\sqrt{a}e\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+\sqrt{b}dx$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(-(Sqrt[b]*c) - Sqrt[b]*d*x - Sqrt[b]*e*x^2 + 2*Sqrt[b]*f*x^3 + Sqrt[a]*e*
Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + Sqrt[b]*d*x*Sqrt[1 +
(b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 2*Sqrt[b]*f*x^
3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(2*b
^(3/2)*Sqrt[a + b*x^4])
```

3.543.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2363, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2363}$$

$$\frac{\int \frac{3fx^2+2ex+d}{\sqrt{bx^4+a}}dx}{2b}-\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2424}$$

$$\frac{\int \left(\frac{2ex}{\sqrt{bx^4+a}}+\frac{3fx^2+d}{\sqrt{bx^4+a}}\right)dx}{2b}-\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+}{2b}$$

$$\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(c + d*x + e*x^2 + f*x^3)/(b*Sqrt[a + b*x^4]) + ((3*f*x*Sqrt[a + b*x^
4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/Sqrt[b] - (3*a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(3/4)*Sqrt[a + b*x^4]))/(2*b)
```

3.543.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.543. $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

3.543.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.82

method	result
elliptic	$-\frac{2b\left(\frac{f x^3}{4b^2}+\frac{e x^2}{4b^2}+\frac{dx}{4b^2}+\frac{c}{4b^2}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b x^4+a}}+\frac{e\ln\left(2x^2\sqrt{b}+2\sqrt{b x^4+a}\right)}{2b^{\frac{3}{2}}}+\frac{3if\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1}}{}$
default	$f\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b x^4+a}}\right)+e\left(-\frac{x^2}{2b\sqrt{b x^4+a}}+\frac{\ln\left(x^2\sqrt{b}+\sqrt{}\right.}{2b^{}}\right.$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/4*f*x^3/b^2+1/4*e/b^2*x^2+1/4*d*x/b^2+1/4*c/b^2)/((x^4+a/b)*b)^(1/
2)+1/2*d/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))
^(1/2),I)+1/2/b^(3/2)*e*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))+3/2*I*f/b^(3/2
)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))
^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

3.543.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.71

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{6\left(abfx^5+a^2fx\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)+2\left(\left(b^2d-3\,abf\right)x^5+\right.}{}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(6*(a*b*f*x^5 + a^2*f*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)
^(1/4)/x), -1) + 2*((b^2*d - 3*a*b*f)*x^5 + (a*b*d - 3*a^2*f)*x)*sqrt(b)*(
-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (a*b*e*x^5 + a^2*e*x)
*sqrt(b)*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(2*a*b*f*x^
4 - a*b*e*x^3 - a*b*d*x^2 - a*b*c*x + 3*a^2*f)*sqrt(b*x^4 + a))/(a*b^3*x^5
 + a^2*b^2*x)
```

3.543.6 Sympy [A] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.52

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = c\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + e\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
c*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), Tru
e)) + e*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt
(1 + b*x**4/a))) + d*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + f*x**7*gamma(7/4)*hyper((3/2, 7/4
), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4))
```

3.543.7 Maxima [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^3}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*c/(sqrt(b*x^4 + a)*b) + integrate((f*x^6 + e*x^5 + d*x^4)/(b*x^4 + a)
^(3/2), x)
```

3.543.8 Giac [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^3}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^(3/2), x)
```

3.543.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^3\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.544 $\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

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3.544.1 Optimal result

Integrand size = 30, antiderivative size = 333

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = -\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}} - \frac{d\sqrt{a+bx^4}}{2ab} - \frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/2*f*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)-1/2*x*(-b*d*x^3-b*c*x^2
+a*f*x+a*e)/a/b/(b*x^4+a)^(1/2)-1/2*d*(b*x^4+a)^(1/2)/a/b-1/2*c*x*(b*x^4+a
)^(1/2)/a/b^(1/2)/(a^(1/2)+x^2*b^(1/2))+1/2*c*(cos(2*arctan(b^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^
2*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)-1/4*(cos(2*arctan(b^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*
arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^
2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/b^(5/4)/(b*x^
4+a)^(1/2)
```

3.544.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.50

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{-3a\sqrt{b}(d+x(e+fx))+3a^{3/2}f\sqrt{1+\frac{bx^4}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+3a\sqrt{b}ex\sqrt{1+\frac{b}{}}}{}$$

input
```
Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(-3*a*Sqrt[b]*(d + x*(e + f*x)) + 3*a^(3/2)*f*Sqrt[1 + (b*x^4)/a]*ArcSinh[
(Sqrt[b]*x^2)/Sqrt[a]] + 3*a*Sqrt[b]*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometri
c2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b^(3/2)*c*x^3*Sqrt[1 + (b*x^4)/a]*Hy
pergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b^(3/2)*Sqrt[a + b*x^4]
)
```

3.544.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2367, 25, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{2367}$$

$$-\frac{\int -\frac{-2b^2dx^3-b^2cx^2+2abfx+abe}{\sqrt{bx^4+a}}dx}{2ab^2}-\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-2b^2dx^3-b^2cx^2+2abfx+abe}{\sqrt{bx^4+a}}dx}{2ab^2}-\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2424}$$

$$\frac{\int \left(\frac{abe-b^2cx^2}{\sqrt{bx^4+a}}+\frac{x(2abf-2b^2dx^2)}{\sqrt{bx^4+a}}\right)dx}{2ab^2}-\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\sqrt[4]{a}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{\sqrt{a+bx^4}}}{2ab^2}$$

$$\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}}$$

input
```
Int[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(a*b*Sqrt[a + b*x^4]) + (-(b*d*
Sqrt[a + b*x^4]) - (b^(3/2)*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) +
 a*Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] + (a^(1/4)*b^(5/4)*c*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] - (a^(1/4)*b^(3/4)
*(Sqrt[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*Sqrt[
a + b*x^4]))/(2*a*b^2)
```

3.544.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.544.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.75

method	result
elliptic	$-\frac{2b\left(-\frac{c x^3}{4ba}+\frac{f x^2}{4b^2}+\frac{ex}{4b^2}+\frac{d}{4b^2}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{f\ln\left(2x^2\sqrt{b}+2\sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}-\frac{ic\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\cdots}}{2\cdots}$
default	$f\left(-\frac{x^2}{2b\sqrt{b\,x^4+a}}+\frac{\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}\right)+e\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)-\frac{d}{2b\sqrt{b\,x^4\cdots}}$

input
```
int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/4/b/a*c*x^3+1/4*f*x^2/b^2+1/4*e/b^2*x+1/4*d/b^2)/((x^4+a/b)*b)^(1
/2)+1/2/b*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)+1/2*f/b^(3/2)*ln(2*x^2*b^(1/2)+2*(b*x^4+a)^(1/2))-1/2*I*c/a^(1/
2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

3.544.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.60

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx =$$

$$-\frac{2\left(b^2cx^5+abcx\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-2\left(\left(b^2c+b^2e\right)x^5+\left(abc+abe\right)x\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\cdots}{4(\cdots}$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*(b^2*c*x^5 + a*b*c*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b
)^(1/4)/x), -1) - 2*((b^2*c + b^2*e)*x^5 + (a*b*c + a*b*e)*x)*sqrt(b)*(-a/
b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - (a*b*f*x^5 + a^2*f*x)*sq
rt(b)*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(a*b*f*x^3 + a
*b*e*x^2 + a*b*d*x + a*b*c)*sqrt(b*x^4 + a))/(a*b^3*x^5 + a^2*b^2*x)
```

3.544.6 Sympy [A] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.47

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = d\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + f\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
d*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), Tru
e)) + f*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt
(1 + b*x**4/a))) + c*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + e*x**5*gamma(5/4)*hyper((5/4, 3/2
), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

3.544.7 Maxima [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)
```

3.544.8 Giac [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)
```

3.544.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^2\left(f\,x^3+e\,x^2+d\,x+c\right)}{\left(b\,x^4+a\right)^{3/2}}\,dx$$

input
```
int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.545 $\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

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3.545.1 Optimal result

Integrand size = 28, antiderivative size = 303

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx=-\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}}-\frac{e\sqrt{a+bx^4}}{2ab}-\frac{dx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}-\frac{\left(\sqrt{b}d-\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{5/4}\sqrt{a+bx^4}}$$

output
```
-1/2*x*(-b*e*x^3-b*d*x^2-b*c*x+a*f)/a/b/(b*x^4+a)^(1/2)-1/2*e*(b*x^4+a)^(1
/2)/a/b-1/2*d*x*(b*x^4+a)^(1/2)/a/b^(1/2)/(a^(1/2)+x^2*b^(1/2))+1/2*d*(cos
(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*El
lipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2)
)*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2
)-1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^
(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-f*a^(1/2
)+d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/
2)/a^(3/4)/b^(5/4)/(b*x^4+a)^(1/2)
```

3.545.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.38

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{-3ae-3afx+3bcx^2+3afx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{6ab\sqrt{a+bx^4}}$$

input
```
Integrate[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(-3*a*e - 3*a*f*x + 3*b*c*x^2 + 3*a*f*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric
2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*d*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeom
etric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b*Sqrt[a + b*x^4])
```

3.545.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2367, 25, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2367}$$

$$-\frac{\int -\frac{-2bex^3-bdx^2+af}{\sqrt{bx^4+a}}dx}{2ab}-\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-2bex^3-bdx^2+af}{\sqrt{bx^4+a}}dx}{2ab}-\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 2424}$$

$$\frac{\int \left(\frac{af-bdx^2}{\sqrt{bx^4+a}}-\frac{2bex^3}{\sqrt{bx^4+a}}\right)dx}{2ab}-\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{\sqrt{a+bx^4}}}{2ab}$$

$$\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}$$

input
```
Int[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(a*b*Sqrt[a + b*x^4]) + (-(e*Sq
rt[a + b*x^4]) - (Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + (
a^(1/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt
[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4]
 - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(2*b^(1/4)*Sqrt[a + b*x^4]))/(2*a*b)
```

3.545.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.545.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.75

method	result
elliptic	$-\frac{2b\left(-\frac{d\,x^3}{4ab}-\frac{c\,x^2}{4ba}+\frac{fx}{4b^2}+\frac{e}{4b^2}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{id\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\right.\right.}{2\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
default	$f\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)-\frac{e}{2b\sqrt{b\,x^4+a}}+d\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+i}}{2}\right.$

input
```
int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/4/a/b*d*x^3-1/4/b/a*c*x^2+1/4*f*x/b^2+1/4*e/b^2)/((x^4+a/b)*b)^(1
/2)+1/2*f/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)-1/2*I*d/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ellipt
icF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
))
```

3.545.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.49

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{(b^2dx^4+abd)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-\left((b^2d+abf)x^4+abd-\right.}{2\,(a}$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((b^2*d*x^4 + a*b*d)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(
1/4)), -1) - ((b^2*d + a*b*f)*x^4 + a*b*d + a^2*f)*sqrt(a)*(-b/a)^(3/4)*el
liptic_f(arcsin(x*(-b/a)^(1/4)), -1) + (b^2*d*x^3 + b^2*c*x^2 - a*b*f*x -
a*b*e)*sqrt(b*x^4 + a))/(a*b^3*x^4 + a^2*b^2)
```

3.545.6 Sympy [A] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.44

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = e\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}} + \frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
e*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), Tru
e)) + c*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + d*x**3*gamma(3/4)*hyper((3/
4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + f*x**
5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*gamma(9/4))
```

3.545.7 Maxima [F]

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*c*x^2/(sqrt(b*x^4 + a)*a) + integrate((f*x^4 + e*x^3 + d*x^2)/(b*x^4 +
 a)^(3/2), x)
```

3.545.8 Giac [F]

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x/(b*x^4 + a)^(3/2), x)
```

3.545.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

output
```
int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

3.546 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx$

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3.546.1 Optimal result

Integrand size = 27, antiderivative size = 275

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = -\frac{ex\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}} + \frac{e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}$$

output
```
1/2*(-a*f+b*x*(e*x^2+d*x+c))/a/b/(b*x^4+a)^(1/2)-1/2*e*x*(b*x^4+a)^(1/2)/a
/b^(1/2)/(a^(1/2)+x^2*b^(1/2))+1/2*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2)
)^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^
(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-e*a^(1/2)+c*b^(1/2))*(a^(1/2)+x^2*b^(1/2)
)*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/b^(3/4)/(b*x^4+a)^(1/2
)
```

3.546.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = \frac{-3af+3bcx+3bdx^2+3bcx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)+2}{6ab\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x]
```

output
```
(-3*a*f + 3*b*c*x + 3*b*d*x^2 + 3*b*c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric
2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeom
etric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b*Sqrt[a + b*x^4])
```

3.546.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2393, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2393}$$

$$-\frac{\int -\frac{c-ex^2}{\sqrt{bx^4+a}}dx}{2a}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{c-ex^2}{\sqrt{bx^4+a}}dx}{2a}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{1512}$$

$$\frac{\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\int \frac{1}{\sqrt{bx^4+a}}dx+\frac{\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\int \frac{1}{\sqrt{bx^4+a}}dx+\frac{e\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{e\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}+\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}}{2a}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{x\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}\right)}{\sqrt{b}}}{2a}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(a*f - b*x*(c + d*x + e*x^2))/(a*b*Sqrt[a + b*x^4]) + ((e*(-((x*Sqrt[
a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sq
rt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4])))/Sqrt[b] + ((c - (Sqrt[a]*e)/Sqr
t[b])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*
EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a +
 b*x^4]))/(2*a)
```

3.546.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

3.546.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.84

method	result
elliptic	$-\frac{2b\left(-\frac{e\,x^{3}}{4ba}-\frac{d\,x^{2}}{4ab}-\frac{cx}{4ba}+\frac{f}{4b^{2}}\right)}{\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{ie\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,(F(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i)-E(x\sqrt{}}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}$
default	$c\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)-\frac{f}{2b\sqrt{b\,x^{4}+a}}+e\,(\frac{x^{3}}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}}{\sqrt{a}}}}{2\sqrt{a}}$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/4/b/a*e*x^3-1/4/a/b*d*x^2-1/4/b/a*c*x+1/4*f/b^2)/((x^4+a/b)*b)^(1
/2)+1/2*c/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)-1/2*I/a^(1/2)*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ellipt
icF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
))
```

3.546.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.47

$$\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^{3/2}}\,dx = \frac{(bex^4+ae)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)-\left((bc+be)x^4+ac+ae\right)\sqrt{a}(}{2\left(ab^2x^4+a\right.}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((b*e*x^4 + a*e)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)
), -1) - ((b*c + b*e)*x^4 + a*c + a*e)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arc
sin(x*(-b/a)^(1/4)), -1) + (b*e*x^3 + b*d*x^2 + b*c*x - a*f)*sqrt(b*x^4 +
a))/(a*b^2*x^4 + a^2*b)
```

3.546.6 Sympy [A] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.48

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{\left(a+bx^4\right)^{3/2}}\,dx &= f\left(\begin{cases}-\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)\\ &+\frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{dx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}\end{aligned}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), Tru
e)) + c*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*a**(3/2)*gamma(5/4)) + d*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + e*x**3*g
amma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*
gamma(7/4))
```

3.546.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)
```

3.546.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)
```

3.546.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = \int \frac{f\,x^3+e\,x^2+d\,x+c}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2), x)
```

3.547 $\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx$

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3.547.1 Optimal result

Integrand size = 30, antiderivative size = 323

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx &= \frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}} \\ &+ \frac{c\sqrt{a+bx^4}}{2a^2} - \frac{fx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} \\ &+ \frac{f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} \\ &+ \frac{\left(\sqrt{b}d-\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)+1/2*x*(-b*c*x^3+a*f*x^2+a*
e*x+a*d)/a^2/(b*x^4+a)^(1/2)+1/2*c*(b*x^4+a)^(1/2)/a^2-1/2*f*x*(b*x^4+a)^(
1/2)/a/b^(1/2)/(a^(1/2)+x^2*b^(1/2))+1/2*f*(cos(2*arctan(b^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b
^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)
*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arc
tan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-f*a^(1/2)+d*b^(1/2))*(a^(1/2)+x^2*b
^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(5/4)/b^(3/4)/(b*x^4+a
)^(1/2)
```

3.547.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.39

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx=\frac{3c\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^4}{a}\right)+x\left(3d+3ex+3d\sqrt{1+\frac{bx^4}{a}}\,\text{Hyperg}\right.}{6a\sqrt{}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]
```

output
```
(3*c*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a] + x*(3*d + 3*e*x + 3*d
*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*f*
x^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)]))/(
6*a*Sqrt[a + b*x^4])
```

3.547.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2368, 25, 2371, 798, 73, 221, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{x\left(ad+aex+afx^2-bcx^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{\int -\frac{\frac{2b^2cx^4}{a}-bfx^3+bdx+2bc}{x\sqrt{bx^4+a}}dx}{2ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{2b^2cx^4}{a}-bfx^3+bdx+2bc}{x\sqrt{bx^4+a}}dx}{2ab}+\frac{x\left(ad+aex+afx^2-bcx^3\right)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2371}$$

$$\frac{\int \frac{\frac{2b^2cx^3}{a}-bfx^2+bd}{\sqrt{bx^4+a}}dx+2bc\int \frac{1}{x\sqrt{bx^4+a}}dx}{2ab}+\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 798

$$\frac{\int \frac{\frac{2b^2cx^3}{a}-bfx^2+bd}{\sqrt{bx^4+a}}dx+\frac{1}{2}bc\int \frac{1}{x^4\sqrt{bx^4+a}}dx^4}{2ab}+\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 73

$$\frac{\int \frac{\frac{2b^2cx^3}{a}-bfx^2+bd}{\sqrt{bx^4+a}}dx+c\int \frac{1}{\frac{x^8}{b}-\frac{a}{b}}d\sqrt{bx^4+a}}{2ab}+\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 221

$$\frac{\int \frac{\frac{2b^2cx^3}{a}-bfx^2+bd}{\sqrt{bx^4+a}}dx-\frac{bc\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{\sqrt{a}}}{2ab}+\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 2424

$$\frac{\int \left(\frac{2b^2cx^3}{a\sqrt{bx^4+a}}+\frac{bd-bfx^2}{\sqrt{bx^4+a}}\right)dx-\frac{bc\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{\sqrt{a}}}{2ab}+\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 2009

$$\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}+$$
$$\frac{\frac{\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}d}{\sqrt{a}}-f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{\sqrt{a+bx^4}}}{2ab}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]
```

output
```
(x*(a*d + a*e*x + a*f*x^2 - b*c*x^3))/(2*a^2*Sqrt[a + b*x^4]) + ((b*c*Sqrt
[a + b*x^4])/a - (Sqrt[b]*f*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - (
b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/Sqrt[a] + (a^(1/4)*b^(1/4)*f*(Sqrt[a
] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (a^(1/4)*b^(1/4)*((Sqr
t[b]*d)/Sqrt[a] - f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*Sqrt[a +
b*x^4]))/(2*a*b)
```

3.547. $\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx$

3.547.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.547. $\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx$

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.547.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.78

method	result
elliptic	$-\frac{2b\left(-\frac{f\,x^3}{4ab}-\frac{x^2 e}{4ab}-\frac{dx}{4ab}-\frac{c}{4ba}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{if\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-E\left(x\cdots\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
default	$d\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+f\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\cdots\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\cdots}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/4/a/b*f*x^3-1/4/a/b*x^2*e-1/4/a/b*d*x-1/4/b/a*c)/((x^4+a/b)*b)^(1
/2)+1/2*d/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)-1/2*I*f/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ellipt
icF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
))-1/2*c/a^(3/2)*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

3.547.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.59

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx=\frac{2\left(abfx^4+a^2f\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-2\left(\left(abd+abf\right)x^4+a^2d\cdots\right.}{\cdots}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*(a*b*f*x^4 + a^2*f)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)
^(1/4)), -1) - 2*((a*b*d + a*b*f)*x^4 + a^2*d + a^2*f)*sqrt(a)*(-b/a)^(3/4
)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + (b^2*c*x^4 + a*b*c)*sqrt(a)*log
(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(a*b*f*x^3 + a*b*e*x^
2 + a*b*d*x + a*b*c)*sqrt(b*x^4 + a))/(a^2*b^2*x^4 + a^3*b)
```

3.547.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.64 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.89

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx = c\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right)+\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}+\frac{fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(3/2),x)
```

output
```
c*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b
*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a)
+ 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9
/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a*
*(9/2) + 4*a**(7/2)*b*x**4)) + d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(
1 + b*x**4/a)) + f*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

3.547.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x), x)
```

3.547.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x), x)
```

3.547.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x\,(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)), x)
```

3.548 $\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx$

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3.548.1 Optimal result

Integrand size = 30, antiderivative size = 344

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx = \frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}}+\frac{d\sqrt{a+bx^4}}{2a^2} - \frac{c\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}cx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{3\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

output
```
-1/2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)+1/2*x*(-b*d*x^3-b*c*x^2+a*
f*x+a*e)/a^2/(b*x^4+a)^(1/2)+1/2*d*(b*x^4+a)^(1/2)/a^2-c*(b*x^4+a)^(1/2)/a
^2/x+3/2*c*x*b^(1/2)*(b*x^4+a)^(1/2)/a^2/(a^(1/2)+x^2*b^(1/2))-3/2*b^(1/4)
*c*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/
4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*
b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)
+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(
1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(e*a^(1/2)+
3*c*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/
2)/a^(7/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

3.548.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.36

$$\int \frac{c+dx+ex^2+fx^3}{x^2\left(a+bx^4\right)^{3/2}}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^4}{a}\right)-2c\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\right.}{2ax\sqrt{}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x]
```

output
```
(d*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a] - 2*c*Sqrt[1 + (b*x^4)
/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^4)/a)] + x^2*(e + f*x + e*Sqr
t[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)]))/(2*a*x*S
qrt[a + b*x^4])
```

3.548.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2368, 25, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^2\left(a+bx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{\int -\frac{\frac{2b^2dx^5}{a}+\frac{b^2cx^4}{a}+bex^2+2bdx+2bc}{x^2\sqrt{bx^4+a}}dx}{2ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{2b^2dx^5}{a}+\frac{b^2cx^4}{a}+bex^2+2bdx+2bc}{x^2\sqrt{bx^4+a}}dx}{2ab}+\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2372}$$

$$\frac{\int \left(\frac{\frac{b^2cx^4}{a}+bex^2+2bc}{x^2\sqrt{bx^4+a}}+\frac{\frac{2b^2dx^4}{a}+2bd}{x\sqrt{bx^4+a}}\right)dx}{2ab}+\frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+bx^4}}-\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}}{2ab}$$

$$\frac{x(ae+afx-bcx^2-bdx^3)}{2a^2\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x]
```

output
```
(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a^2*Sqrt[a + b*x^4]) + ((b*d*Sqrt
[a + b*x^4])/a - (2*b*c*Sqrt[a + b*x^4])/(a*x) + (3*b^(3/2)*c*x*Sqrt[a + b
*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]
)/Sqrt[a] - (3*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)
*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[a + b*x^4]))/(2*a*b)
```

3.548.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.548. $\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx$

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.548.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.78

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{a^2x}-\frac{2b\left(\frac{c\,x^3}{4a^2}-\frac{x^2f}{4ab}-\frac{xe}{4ab}-\frac{d}{4ab}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{3i\sqrt{b}\,c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\left(x\right.\right.}{2a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x}}$
default	$e\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{f\,x^2}{2a\sqrt{b\,x^4+a}}+d\left(\frac{1}{2a\sqrt{b\,x^4+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2a^{\frac{3}{2}}}\right)$
risch	$-\frac{c\sqrt{b\,x^4+a}}{a^2x}+\frac{a^2e\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+b^2c\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(F\right.}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{}}\right.}{a^2}$

input
```
int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-c*(b*x^4+a)^(1/2)/a^2/x-2*b*(1/4/a^2*c*x^3-1/4/a/b*x^2*f-1/4/a/b*x*e-1/4/
a/b*d)/((x^4+a/b)*b)^(1/2)+1/2/a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellipti
cF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/2*I*b^(1/2)*c/a^(3/2)/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/
(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a
^(1/2)*b^(1/2))^(1/2),I))-1/2*d/a^(3/2)*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

3.548.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.62

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx =$$

$$-\frac{6\,(b^2cx^5+abcx)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)-2\,((3\,b^2c-abe)x^5+(3\,abc-a^2e)x)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(a}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(6*(b^2*c*x^5 + a*b*c*x)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b
/a)^(1/4)), -1) - 2*((3*b^2*c - a*b*e)*x^5 + (3*a*b*c - a^2*e)*x)*sqrt(a)*
(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - (b^2*d*x^5 + a*b*d*x
)*sqrt(a)*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(3*b^2*c
*x^4 - a*b*f*x^3 - a*b*e*x^2 - a*b*d*x + 2*a*b*c)*sqrt(b*x^4 + a))/(a^2*b^
2*x^5 + a^3*b*x)
```

3.548.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.08 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.85

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx = d\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right)+\frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}+\frac{ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{fx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(3/2),x)
```

output
```
d*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b
*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a)
+ 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9
/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a*
*(9/2) + 4*a**(7/2)*b*x**4)) + c*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + e*x*gamma(1/4)*hyper((
1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + f*x
**2/(2*a**(3/2)*sqrt(1 + b*x**4/a))
```

3.548.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\left(a+bx^4\right)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\left(bx^4+a\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

3.548.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\left(a+bx^4\right)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\left(bx^4+a\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

3.548.9 Mupad [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.39

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx = \frac{d}{2\,a\,\sqrt{b\,x^4+a}} - \frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} + \frac{f\,x^2}{2\,a\,\sqrt{b\,x^4+a}} - \frac{c\,\left(\frac{a}{b\,x^4}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x^4}\right)}{7\,x\,(b\,x^4+a)^{3/2}} + \frac{e\,x\,\left(\frac{b\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{(b\,x^4+a)^{3/2}}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x)
```

output
```
d/(2*a*(a + b*x^4)^(1/2)) - (d*atanh((a + b*x^4)^(1/2)/a^(1/2)))/(2*a^(3/2
)) + (f*x^2)/(2*a*(a + b*x^4)^(1/2)) - (c*(a/(b*x^4) + 1)^(3/2)*hypergeom(
[3/2, 7/4], 11/4, -a/(b*x^4)))/(7*x*(a + b*x^4)^(3/2)) + (e*x*((b*x^4)/a +
 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(b*x^4)/a))/(a + b*x^4)^(3/2)
```

3.549 $\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx$

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3.549.1 Optimal result

Integrand size = 30, antiderivative size = 367

$$\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx = \frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}}+\frac{e\sqrt{a+bx^4}}{2a^2} -\frac{c\sqrt{a+bx^4}}{2a^2x^2}-\frac{d\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}dx\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} -\frac{3\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} +\frac{\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

output
```
-1/2*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)+1/2*x*(-b*e*x^3-b*d*x^2-b*
c*x+a*f)/a^2/(b*x^4+a)^(1/2)+1/2*e*(b*x^4+a)^(1/2)/a^2-1/2*c*(b*x^4+a)^(1/
2)/a^2/x^2-d*(b*x^4+a)^(1/2)/a^2/x+3/2*d*x*b^(1/2)*(b*x^4+a)^(1/2)/a^2/(a^
(1/2)+x^2*b^(1/2))-3/2*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4
))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)
^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1
/4))),1/2*2^(1/2))*(f*a^(1/2)+3*d*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a
)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

3.549.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.38

$$\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx = \frac{-ac+afx^3-2bcx^4+aex^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^4}{a}\right)-2adx\sqrt{1}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x]
```

output
```
(-(a*c) + a*f*x^3 - 2*b*c*x^4 + a*e*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1
+ (b*x^4)/a] - 2*a*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/
4, -((b*x^4)/a)] + a*f*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2,
 5/4, -((b*x^4)/a)])/(2*a^2*x^2*Sqrt[a + b*x^4])
```

3.549.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2368, 25, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{\int -\frac{\frac{2b^2ex^6}{a}+\frac{b^2dx^5}{a}+bfx^3+2bex^2+2bdx+2bc}{x^3\sqrt{bx^4+a}}dx}{2ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{2b^2ex^6}{a}+\frac{b^2dx^5}{a}+bfx^3+2bex^2+2bdx+2bc}{x^3\sqrt{bx^4+a}}dx}{2ab}+\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2372}$$

$$\frac{\int \left(\frac{\frac{b^2dx^4}{a}+bfx^2+2bd}{x^2\sqrt{bx^4+a}}+\frac{\frac{2b^2ex^6}{a}+2bex^2+2bc}{x^3\sqrt{bx^4+a}}\right)dx}{2ab}+\frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}}$$

$\downarrow$ 2009

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+bx^4}}-\frac{3b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}}{2ab}$$

$$\frac{x(af-bcx-bdx^2-bex^3)}{2a^2\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x]
```

output
```
(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a^2*Sqrt[a + b*x^4]) + ((b*e*Sqrt
[a + b*x^4])/a - (b*c*Sqrt[a + b*x^4])/(a*x^2) - (2*b*d*Sqrt[a + b*x^4])/(
a*x) + (3*b^(3/2)*d*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*e*
ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/Sqrt[a] - (3*b^(5/4)*d*(Sqrt[a] + Sqrt[b
]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*d
+ Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[a +
b*x^4]))/(2*a*b)
```

3.549.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.549.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.77

method	result
elliptic	$-\frac{c\sqrt{b x^4+a}}{2a^2x^2}-\frac{d\sqrt{b x^4+a}}{a^2x}-\frac{2b\left(\frac{d x^3}{4a^2}+\frac{c x^2}{4a^2}-\frac{x f}{4ab}-\frac{e}{4ba}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+\frac{3i\sqrt{b}\,d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1-\cdots}}{2\cdots}$
default	$f\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}\right)+e\left(\frac{1}{2a\sqrt{b x^4+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^4+a}}{x^2}\right)}{2a^{\frac{3}{2}}}\right)+d\left(-\frac{\cdots}{2a^2\cdots}\right.$
risch	$-\frac{\sqrt{b x^4+a}\,(2dx+c)}{2a^2x^2}+\frac{a^2 f\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}\right)+b^2 d\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\cdots}}{2b^{\frac{3}{2}}\cdots}\cdots\right.}{a^2}$

input
```
int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*(b*x^4+a)^(1/2)/a^2/x^2-d*(b*x^4+a)^(1/2)/a^2/x-2*b*(1/4*d/a^2*x^3+
1/4/a^2*c*x^2-1/4/a/b*x*f-1/4/b/a*e)/((x^4+a/b)*b)^(1/2)+1/2*f/a/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)
^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/2*I*b^(1
/2)*d/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1
/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2/a^(3/2)*e*arct
anh(a^(1/2)/(b*x^4+a)^(1/2))
```

3.549.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.63

$$\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx =$$

$$-\frac{6\left(b^2dx^6+abdx^2\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-2\left(\left(3\,b^2d-abf\right)x^6+\left(3\,abd-a^2f\right)x^2\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}\ldots}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(6*(b^2*d*x^6 + a*b*d*x^2)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(
-b/a)^(1/4)), -1) - 2*((3*b^2*d - a*b*f)*x^6 + (3*a*b*d - a^2*f)*x^2)*sqrt
(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - (b^2*e*x^6 + a*b
*e*x^2)*sqrt(a)*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(3
*b^2*d*x^5 + 2*b^2*c*x^4 - a*b*f*x^3 - a*b*e*x^2 + 2*a*b*d*x + a*b*c)*sqrt
(b*x^4 + a))/(a^2*b^2*x^6 + a^3*b*x^2)
```

3.549.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.47 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.86

$$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx = c\left(-\frac{1}{2a\sqrt{b}x^4\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right) + e\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} + \frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right) + \frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}+\frac{fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(3/2),x)
```

output
```
c*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqrt(a/(b*x*
*4) + 1))) + e*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)
 + a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1
 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**
4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/
a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)) + d*gamma(-1/4)*hyper((-1/4, 3/2
), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + f*x*gamma
(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamm
a(5/4))
```

3.549.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)
```

3.549.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)
```

3.549.9 Mupad [B] (verification not implemented)

Time = 10.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.40

$$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx=\frac{e}{2\,a\,\sqrt{b\,x^4+a}}-\frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{2\,c\,(b\,x^4+a)-a\,c}{2\,a^2\,x^2\,\sqrt{b\,x^4+a}}-\frac{d\,\left(\frac{a}{b\,x^4}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x^4}\right)}{7\,x\,(b\,x^4+a)^{3/2}}+\frac{f\,x\,\left(\frac{b\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{(b\,x^4+a)^{3/2}}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x)
```

output
```
e/(2*a*(a + b*x^4)^(1/2)) - (e*atanh((a + b*x^4)^(1/2)/a^(1/2)))/(2*a^(3/2
)) - (2*c*(a + b*x^4) - a*c)/(2*a^2*x^2*(a + b*x^4)^(1/2)) - (d*(a/(b*x^4)
 + 1)^(3/2)*hypergeom([3/2, 7/4], 11/4, -a/(b*x^4)))/(7*x*(a + b*x^4)^(3/2
)) + (f*x*((b*x^4)/a + 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(b*x^4)/a))/(a
 + b*x^4)^(3/2)
```

3.550 $\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx$

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3.550.1 Optimal result

Integrand size = 30, antiderivative size = 387

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx &= -\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}}+\frac{f\sqrt{a+bx^4}}{2a^2}\\
&-\frac{c\sqrt{a+bx^4}}{3a^2x^3}-\frac{d\sqrt{a+bx^4}}{2a^2x^2}-\frac{e\sqrt{a+bx^4}}{a^2x}+\frac{3\sqrt{b}ex\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}\\
&-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{3\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}}\\
&-\frac{\sqrt[4]{b}\left(5\sqrt{b}c-9\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*x*(b*f*x^3+b*e*x^2+b*d
*x+b*c)/a^2/(b*x^4+a)^(1/2)+1/2*f*(b*x^4+a)^(1/2)/a^2-1/3*c*(b*x^4+a)^(1/2
)/a^2/x^3-1/2*d*(b*x^4+a)^(1/2)/a^2/x^2-e*(b*x^4+a)^(1/2)/a^2/x+3/2*e*x*b^
(1/2)*(b*x^4+a)^(1/2)/a^2/(a^(1/2)+x^2*b^(1/2))-3/2*b^(1/4)*e*(cos(2*arcta
n(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(
sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^
4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-1/12*b^(1/4)*(
cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))
*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-9*e*a^(1/2)+5*c
*b^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/
a^(9/4)/(b*x^4+a)^(1/2)
```

3.550.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.35

$$\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx = \frac{-2ac\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{bx^4}{a}\right)-3x\left(ad+2bdx^4-afx^2\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x]
```

output
```
(-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^4)/a)
] - 3*x*(a*d + 2*b*d*x^4 - a*f*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*
x^4)/a] + 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -(
(b*x^4)/a)]))/(6*a^2*x^3*Sqrt[a + b*x^4])
```

3.550.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2368, 25, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx$$

$\Big\downarrow$ 2368

$$-\frac{\int -\frac{\frac{2b^2fx^7}{a}+\frac{b^2ex^6}{a}-\frac{b^2cx^4}{a}+2bfx^3+2bex^2+2bdx+2bc}{x^4\sqrt{bx^4+a}}dx}{2ab}-\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\frac{2b^2fx^7}{a}+\frac{b^2ex^6}{a}-\frac{b^2cx^4}{a}+2bfx^3+2bex^2+2bdx+2bc}{x^4\sqrt{bx^4+a}}dx}{2ab}-\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 2372

$$\frac{\int \left(\frac{\frac{b^2ex^6}{a}-\frac{b^2cx^4}{a}+2bex^2+2bc}{x^4\sqrt{bx^4+a}}+\frac{\frac{2b^2fx^6}{a}+2bfx^2+2bd}{x^3\sqrt{bx^4+a}}\right)dx}{2ab}-\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-9\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{3b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{a^{3/4}\sqrt{a+bx^4}}}{2ab}$$

$$\frac{x\left(bc+bdx+bex^2+bfx^3\right)}{2a^2\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x]
```

output
```
-1/2*(x*(b*c + b*d*x + b*e*x^2 + b*f*x^3))/(a^2*Sqrt[a + b*x^4]) + ((b*f*S
qrt[a + b*x^4])/a - (2*b*c*Sqrt[a + b*x^4])/(3*a*x^3) - (b*d*Sqrt[a + b*x^
4])/(a*x^2) - (2*b*e*Sqrt[a + b*x^4])/(a*x) + (3*b^(3/2)*e*x*Sqrt[a + b*x^
4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/S
qrt[a] - (3*b^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sq
rt[a + b*x^4]) - (b^(5/4)*(5*Sqrt[b]*c - 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4]))/(2*a*b)
```

3.550. $\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx^4)^{3/2}}\,dx$

3.550.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.550.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.80 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.77

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{3a^2x^3}-\frac{d\sqrt{b\,x^4+a}}{2a^2x^2}-\frac{e\sqrt{b\,x^4+a}}{a^2x}-\frac{2b\left(\frac{e\,x^3}{4a^2}+\frac{x^2d}{4a^2}+\frac{cx}{4a^2}-\frac{f}{4ab}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{5bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{3i\sqrt{b}\,e}{}$
default	$f\left(\frac{1}{2a\sqrt{b\,x^4+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2a^{\frac{3}{2}}}\right)+c\left(-\frac{bx}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^4+a}}{3a^2x^3}-\frac{5b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,\left(6e\,x^2+3dx+2c\right)}{6a^2x^3}-\frac{bcx}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{5bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{be\,x^3}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{b}\,e\sqrt{1-}}{}$

3.550. $\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx^4)^{3/2}}\,dx$

input
```
int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c*(b*x^4+a)^(1/2)/a^2/x^3-1/2*d*(b*x^4+a)^(1/2)/a^2/x^2-e*(b*x^4+a)^(
1/2)/a^2/x-2*b*(1/4/a^2*e*x^3+1/4/a^2*x^2*d+1/4/a^2*c*x-1/4/a/b*f)/((x^4+a
/b)*b)^(1/2)-5/6/a^2*b*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*b^(1/2))^(1/2),I)+3/2*I*b^(1/2)*e/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^
(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(
1/2))^(1/2),I))-1/2*f/a^(3/2)*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

3.550.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.56

$$\int \frac{c+dx+ex^2+fx^3}{x^4\left(a+bx^4\right)^{3/2}}\,dx =$$

$$-\frac{18\left(bex^7+aex^3\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)-2\left(\left(5\,bc+9\,be\right)x^7+\left(5\,ac+9\,ae\right)x^3\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} F(\varepsilon}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/12*(18*(b*e*x^7 + a*e*x^3)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b
/a)^(1/4)), -1) - 2*((5*b*c + 9*b*e)*x^7 + (5*a*c + 9*a*e)*x^3)*sqrt(a)*(-
b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - 3*(b*f*x^7 + a*f*x^3)*
sqrt(a)*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(9*b*e*x^6
 + 6*b*d*x^5 + 5*b*c*x^4 - 3*a*f*x^3 + 6*a*e*x^2 + 3*a*d*x + 2*a*c)*sqrt(b
*x^4 + a))/(a^2*b*x^7 + a^3*x^3)
```

3.550.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.87 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.83

$$\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx^4)^{3/2}}\,dx = d\left(-\frac{1}{2a\sqrt{b}x^4\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right) + f\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} + \frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right) + \frac{c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(3/2),x)
```

output
```
d*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqrt(a/(b*x*
*4) + 1))) + f*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)
 + a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1
 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**
4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/
a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)) + c*gamma(-3/4)*hyper((-3/4, 3/2
), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x**3*gamma(1/4)) + e*gamm
a(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x
*gamma(3/4))
```

3.550.7 Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx^4)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

3.550.8 Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx^4)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

3.550.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x^4(a+bx^4)^{3/2}}\,dx = \int \frac{f\,x^3+e\,x^2+d\,x+c}{x^4\,(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(3/2)), x)
```

3.551 $\int (gx)^m \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx$

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3.551.1 Optimal result

Integrand size = 30, antiderivative size = 269

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx = \frac{c(gx)^{1+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{g(1+m)} + \frac{d(gx)^{2+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2+m}{4},-p,\frac{6+m}{4},-\frac{bx^4}{a}\right)}{g^2(2+m)} + \frac{e(gx)^{3+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3+m}{4},-p,\frac{7+m}{4},-\frac{bx^4}{a}\right)}{g^3(3+m)} + \frac{f(gx)^{4+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{4+m}{4},-p,\frac{8+m}{4},-\frac{bx^4}{a}\right)}{g^4(4+m)}$$

output
```
c*(g*x)^(1+m)*(b*x^4+a)^p*hypergeom([-p, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/
g/(1+m)/((1+b*x^4/a)^p)+d*(g*x)^(2+m)*(b*x^4+a)^p*hypergeom([-p, 1/2+1/4*m
],[3/2+1/4*m],-b*x^4/a)/g^2/(2+m)/((1+b*x^4/a)^p)+e*(g*x)^(3+m)*(b*x^4+a)^
p*hypergeom([-p, 3/4+1/4*m],[7/4+1/4*m],-b*x^4/a)/g^3/(3+m)/((1+b*x^4/a)^p
)+f*(g*x)^(4+m)*(b*x^4+a)^p*hypergeom([-p, 1+1/4*m],[2+1/4*m],-b*x^4/a)/g^
4/(4+m)/((1+b*x^4/a)^p)
```

3.551.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.65

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \, dx$$
$$= x(gx)^m \left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(\frac{c\,\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{1+m} + x\left(\frac{d\,\text{Hypergeometric2F1}\left(\frac{2+m}{4},-p,\frac{6+m}{4},-\frac{bx^4}{a}\right)}{2+m} + x\left(\frac{e\,\text{Hypergeometric2F1}\left(\frac{3+m}{4},-p,\frac{7+m}{4},-\frac{bx^4}{a}\right)}{3+m} + \frac{fx\,\text{Hypergeometric2F1}\left(\frac{4+m}{4},-p,\frac{8+m}{4},-\frac{bx^4}{a}\right)}{4+m}\right)\right)\right)$$

input
```
Integrate[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

output
```
(x*(g*x)^m*(a + b*x^4)^p*((c*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4, -
((b*x^4)/a)])/(1 + m) + x*((d*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4,
-((b*x^4)/a)])/(2 + m) + x*((e*Hypergeometric2F1[(3 + m)/4, -p, (7 + m)/4,
 -((b*x^4)/a)])/(3 + m) + (f*x*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4,
 -((b*x^4)/a)])/(4 + m)))))/(1 + (b*x^4)/a)^p
```

3.551.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^m \left(a+bx^4\right)^p \left(c+dx+ex^2+fx^3\right) dx$$
$$\Big\downarrow \; 2372$$
$$\int \left(\left(c+ex^2\right)(gx)^m \left(a+bx^4\right)^p + \frac{\left(d+fx^2\right)(gx)^{m+1}\left(a+bx^4\right)^p}{g}\right) dx$$
$$\Big\downarrow \; 2009$$

$$\frac{c(gx)^{m+1}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{4},-p,\frac{m+5}{4},-\frac{bx^4}{a}\right)}{g(m+1)}+\frac{d(gx)^{m+2}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{4},-p,\frac{m+6}{4},-\frac{bx^4}{a}\right)}{g^2(m+2)}+\frac{e(gx)^{m+3}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+3}{4},-p,\frac{m+7}{4},-\frac{bx^4}{a}\right)}{g^3(m+3)}+\frac{f(gx)^{m+4}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+4}{4},-p,\frac{m+8}{4},-\frac{bx^4}{a}\right)}{g^4(m+4)}$$

input
```
Int[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

output
```
(c*(g*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4,
 -((b*x^4)/a)])/(g*(1 + m)*(1 + (b*x^4)/a)^p) + (d*(g*x)^(2 + m)*(a + b*x^
4)^p*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4, -((b*x^4)/a)])/(g^2*(2 +
m)*(1 + (b*x^4)/a)^p) + (e*(g*x)^(3 + m)*(a + b*x^4)^p*Hypergeometric2F1[(
3 + m)/4, -p, (7 + m)/4, -((b*x^4)/a)])/(g^3*(3 + m)*(1 + (b*x^4)/a)^p) +
(f*(g*x)^(4 + m)*(a + b*x^4)^p*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4,
 -((b*x^4)/a)])/(g^4*(4 + m)*(1 + (b*x^4)/a)^p)
```

3.551.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.551.4 Maple [F]

$$\int (gx)^m \left(f\,x^3+e\,x^2+dx+c\right) \left(b\,x^4+a\right)^p dx$$

input
```
int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

output
```
int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

3.551.5 Fricas [F]

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \, dx = \int \left(fx^3+ex^2+dx+c\right) \left(bx^4+a\right)^p (gx)^m \, dx$$

input
```
integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

3.551.6 Sympy [F(-1)]

Timed out.

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \, dx = \text{Timed out}$$

input
```
integrate((g*x)**m*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

output
```
Timed out
```

3.551.7 Maxima [F]

$$\int (gx)^m (c+dx+ex^2+fx^3) (a+bx^4)^p \, dx = \int (fx^3+ex^2+dx+c)(bx^4+a)^p (gx)^m \, dx$$

input
```
integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

3.551.8 Giac [F]

$$\int (gx)^m (c+dx+ex^2+fx^3) (a+bx^4)^p \, dx = \int (fx^3+ex^2+dx+c)(bx^4+a)^p (gx)^m \, dx$$

input
```
integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

3.551.9 Mupad [F(-1)]

Timed out.

$$\int (gx)^m (c+dx+ex^2+fx^3) (a+bx^4)^p \, dx = \int (g\,x)^m \left(b\,x^4+a\right)^p \left(f\,x^3+e\,x^2+d\,x+c\right) \, dx$$

input
```
int((g*x)^m*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int((g*x)^m*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3), x)
```

3.552 $\int (c+dx+ex^2+fx^3) (a+bx^4)^p \, dx$

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3.552.1 Optimal result

Integrand size = 25, antiderivative size = 143

$$\begin{aligned}\int (c+dx+ex^2+fx^3) (a+bx^4)^p \, dx &= \frac{f(a+bx^4)^{1+p}}{4b(1+p)} + \frac{cx(a+bx^4)^{1+p} \text{Hypergeometric2F1}\left(1,\frac{5}{4}+p,\frac{5}{4},-\frac{bx^4}{a}\right)}{a} \\ &+ \frac{dx^2(a+bx^4)^{1+p} \text{Hypergeometric2F1}\left(1,\frac{3}{2}+p,\frac{3}{2},-\frac{bx^4}{a}\right)}{2a} \\ &+ \frac{ex^3(a+bx^4)^{1+p} \text{Hypergeometric2F1}\left(1,\frac{7}{4}+p,\frac{7}{4},-\frac{bx^4}{a}\right)}{3a}\end{aligned}$$

output
```
1/4*f*(b*x^4+a)^(p+1)/b/(p+1)+c*x*(b*x^4+a)^(p+1)*hypergeom([1, 5/4+p],[5/
4],-b*x^4/a)/a+1/2*d*x^2*(b*x^4+a)^(p+1)*hypergeom([1, 3/2+p],[3/2],-b*x^4
/a)/a+1/3*e*x^3*(b*x^4+a)^(p+1)*hypergeom([1, 7/4+p],[7/4],-b*x^4/a)/a
```

3.552.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx=\frac{1}{12}\left(a+bx^4\right)^p\left(\frac{3f(a+bx^4)}{b(1+p)}+12cx\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+6dx^2\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^4}{a}\right)+4ex^3\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)\right)$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

output
```
((a + b*x^4)^p*((3*f*(a + b*x^4))/(b*(1 + p)) + (12*c*x*Hypergeometric2F1[
1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (6*d*x^2*Hypergeometric2F
1[1/2, -p, 3/2, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (4*e*x^3*Hypergeometric
2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p))/12
```

3.552.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^p\left(c+dx+ex^2+fx^3\right)dx$$

$$\Big\downarrow \text{ 2424}$$

$$\int \left(\left(c+ex^2\right)\left(a+bx^4\right)^p+x\left(d+fx^2\right)\left(a+bx^4\right)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$cx(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{2}dx^2(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)+\frac{f(a+bx^4)^{p+1}}{4b(p+1)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

output
```
(f*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (c*x*(a + b*x^4)^p*Hypergeometric2
F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (d*x^2*(a + b*x^4)^p*H
ypergeometric2F1[1/2, -p, 3/2, -((b*x^4)/a)])/(2*(1 + (b*x^4)/a)^p) + (e*x
^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(3*(1 + (b
*x^4)/a)^p)
```

3.552.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.552.4 Maple [F]

$$\int \left(f\,x^3+e\,x^2+dx+c\right)\left(b\,x^4+a\right)^p dx$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

output
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

3.552.5 Fricas [F]

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^p\,dx=\int (fx^3+ex^2+dx+c)(bx^4+a)^p\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

3.552.6 Sympy [A] (verification not implemented)

Time = 20.54 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^p\,dx=\frac{a^pcx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^pdx^2{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2}+\frac{a^pex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+f\left(\begin{cases}\frac{a^px^4}{4} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^4)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(a+bx^4) & \text{otherwise}\end{cases}}{4b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

output
```
a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(5/4)) + a**p*d*x**2*hyper((1/2, -p), (3/2,), b*x**4*exp_polar(I*pi)/
a)/2 + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*
pi)/a)/(4*gamma(7/4)) + f*Piecewise((a**p*x**4/4, Eq(b, 0)), (Piecewise(((
a + b*x**4)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**4), True))/(4*b),
True))
```

3.552.7 Maxima [F]

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx=\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

3.552.8 Giac [F]

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx=\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

3.552.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx=\int \left(b\,x^4+a\right)^p\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int((a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int((a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3), x)
```

3.553 $\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^p\,dx$

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3.553.1 Optimal result

Integrand size = 28, antiderivative size = 175

$$\begin{aligned}\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^p\,dx &= \frac{c(a+bx^4)^{1+p}}{4b(1+p)}+\frac{1}{5}dx^5(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)\\ &\quad+\frac{1}{6}ex^6(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^4}{a}\right)\\ &\quad+\frac{1}{7}fx^7(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^4}{a}\right)\end{aligned}$$

output
```
1/4*c*(b*x^4+a)^(p+1)/b/(p+1)+1/5*d*x^5*(b*x^4+a)^p*hypergeom([5/4, -p],[9
/4],-b*x^4/a)/((1+b*x^4/a)^p)+1/6*e*x^6*(b*x^4+a)^p*hypergeom([3/2, -p],[5
/2],-b*x^4/a)/((1+b*x^4/a)^p)+1/7*f*x^7*(b*x^4+a)^p*hypergeom([7/4, -p],[1
1/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

3.553.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.83

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx$$
$$=\frac{(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(105c(a+bx^4)\left(1+\frac{bx^4}{a}\right)^p+84bd(1+p)x^5\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)\right.}{420}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

output
```
((a + b*x^4)^p*(105*c*(a + b*x^4)*(1 + (b*x^4)/a)^p + 84*b*d*(1 + p)*x^5*H
ypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 70*b*e*(1 + p)*x^6*Hypergeo
metric2F1[3/2, -p, 5/2, -((b*x^4)/a)] + 60*b*f*(1 + p)*x^7*Hypergeometric2
F1[7/4, -p, 11/4, -((b*x^4)/a)]))/(420*b*(1 + p)*(1 + (b*x^4)/a)^p)
```

3.553.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^4\right)^p\left(c+dx+ex^2+fx^3\right)dx$$
$$\Big\downarrow \text{2372}$$
$$\int \left(x^3\left(c+ex^2\right)\left(a+bx^4\right)^p+x^4\left(d+fx^2\right)\left(a+bx^4\right)^p\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{c\left(a+bx^4\right)^{p+1}}{4b(p+1)}+\frac{1}{5}dx^5\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)+\frac{1}{6}ex^6\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^4}{a}\right)+\frac{1}{7}fx^7\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^4}{a}\right)$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

output
```
(c*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (d*x^5*(a + b*x^4)^p*Hypergeometri
c2F1[5/4, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p) + (e*x^6*(a + b*x^
4)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^4)/a)])/(6*(1 + (b*x^4)/a)^p)
+ (f*x^7*(a + b*x^4)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^4)/a)])/(7*
(1 + (b*x^4)/a)^p)
```

3.553.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.553.4 Maple [F]

$$\int x^{3} \left(f\,x^{3}+e\,x^{2}+dx+c\right) \left(b\,x^{4}+a\right)^{p} dx$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

output
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

3.553.5 Fricas [F]

$$\int x^3 \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \, dx = \int \left(fx^3+ex^2+dx+c\right) \left(bx^4+a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)*(b*x^4 + a)^p, x)
```

3.553.6 Sympy [A] (verification not implemented)

Time = 47.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.82

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx$$
$$=\frac{a^p dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{a^p ex^6{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6}$$
$$+\frac{a^p fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},-p\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+c\left(\begin{cases}\frac{a^px^4}{4} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^4)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^4\right) & \text{otherwise}\end{cases}}{4b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

output
```
a**p*d*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(9/4)) + a**p*e*x**6*hyper((3/2, -p), (5/2,), b*x**4*exp_polar(I*p
i)/a)/6 + a**p*f*x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*gamma(11/4)) + c*Piecewise((a**p*x**4/4, Eq(b, 0)), (Piecewi
se(((a + b*x**4)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**4), True))/(4
*b), True))
```

3.553.7 Maxima [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx=\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^px^3\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
1/4*(b*x^4 + a)^(p + 1)*c/(b*(p + 1)) + integrate((f*x^6 + e*x^5 + d*x^4)*
(b*x^4 + a)^p, x)
```

3.553.8 Giac [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx=\int (fx^3+ex^2+dx+c)\left(bx^4+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*x^3, x)
```

3.553.9 Mupad [F(-1)]

Timed out.

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx=\int x^3\left(b\,x^4+a\right)^p\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^3*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x^3*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3), x)
```

3.554 $\int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx$

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3.554.8 Giac [A] (verification not implemented) 4265
3.554.9 Mupad [B] (verification not implemented) 4265

3.554.1 Optimal result

Integrand size = 22, antiderivative size = 8

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx = -\log(1-x)$$

output
```
-ln(1-x)
```

3.554.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5} dx = -\log(1-x)$$

input
```
Integrate[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

output
```
-Log[1 - x]
```

3.554.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+x^3+x^2+x+1}{1-x^5} \, dx$$

$$\Big\downarrow \; 2019$$

$$\int \frac{1}{1-x}dx$$

$$\Big\downarrow \; 16$$

$$-\log(1-x)$$

input
```
Int[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

output
```
-Log[1 - x]
```

3.554.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.554.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$-\ln(-1+x)$	7
norman	$-\ln(-1+x)$	7
risch	$-\ln(-1+x)$	7
parallelrisch	$-\ln(-1+x)$	7
meijerg	Expression too large to display	542

input
```
int((x^4+x^3+x^2+x+1)/(-x^5+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(-1+x)
```

3.554.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\log(x-1)$$

input
```
integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="fricas")
```

output
```
-log(x - 1)
```

3.554.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\log(x-1)$$

input
```
integrate((x**4+x**3+x**2+x+1)/(-x**5+1),x)
```

output
```
-log(x - 1)
```

3.554.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\log(x-1)$$

input
```
integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="maxima")
```

output
```
-log(x - 1)
```

3.554.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\log(|x-1|)$$

input
```
integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="giac")
```

output
```
-log(abs(x - 1))
```

3.554.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\ln(x-1)$$

input
```
int(-(x + x^2 + x^3 + x^4 + 1)/(x^5 - 1),x)
```

output
```
-log(x - 1)
```

3.555 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} dx$

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3.555.1 Optimal result

Integrand size = 35, antiderivative size = 10

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} dx = \frac{1}{2}\log(3+2x)$$

output
```
1/2*ln(3+2*x)
```

3.555.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} dx = \frac{1}{2}\log(3+2x)$$

input
```
Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6
),x]
```

output
```
Log[3 + 2*x]/2
```

3.555.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-32x^5+48x^4-72x^3+108x^2-162x+243}{729-64x^6}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{2x+3}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\log(2x+3)$$

input
```
Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6),x]
```

output
```
Log[3 + 2*x]/2
```

3.555.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.555.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{\ln\left(x+\frac{3}{2}\right)}{2}$
default	$\frac{\ln(2x+3)}{2}$
norman	$\frac{\ln(2x+3)}{2}$
risch	$\frac{\ln(2x+3)}{2}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-}{12\left(x^6\right)^{\frac{1}{6}}}$

input
```
int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x,method=_RETU
RNVERBOSE)
```

output
```
1/2*ln(x+3/2)
```

3.555.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{1}{2}\log\left(2\,x+3\right)$$

input
```
integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algor
ithm="fricas")
```

output
```
1/2*log(2*x + 3)
```

3.555.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{\log{(2x+3)}}{2}$$

input
```
integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729),x)
```

output
```
log(2*x + 3)/2
```

3.555.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{1}{2}\,\log\left(2\,x+3\right)$$

input
```
integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algor
ithm="maxima")
```

output
```
1/2*log(2*x + 3)
```

3.555.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{1}{2}\,\log\left(|2\,x+3|\right)$$

input
```
integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algor
ithm="giac")
```

output
```
1/2*log(abs(2*x + 3))
```

3.555.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{2}$$

input
```
int((162*x - 108*x^2 + 72*x^3 - 48*x^4 + 32*x^5 - 243)/(64*x^6 - 729),x)
```

output
```
log(x + 3/2)/2
```

3.556 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6} \, dx$

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3.556.9 Mupad [B] (verification not implemented) 4275

3.556.1 Optimal result

Integrand size = 35, antiderivative size = 10

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6} \, dx = -\frac{1}{2}\log(3-2x)$$

output
```
-1/2*ln(3-2*x)
```

3.556.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6} \, dx = -\frac{1}{2}\log(3-2x)$$

input
```
Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6
),x]
```

output
```
-1/2*Log[3 - 2*x]
```

3.556.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{32x^5+48x^4+72x^3+108x^2+162x+243}{729-64x^6}dx$$

$$\Big\downarrow \ 2019$$

$$\int \frac{1}{3-2x}dx$$

$$\Big\downarrow \ 16$$

$$-\frac{1}{2}\log(3-2x)$$

input
```
Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6),x]
```

output
```
-1/2*Log[3 - 2*x]
```

3.556.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.556.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result
parallelrisch	$-\frac{\ln\left(x-\frac{3}{2}\right)}{2}$
default	$-\frac{\ln(-3+2x)}{2}$
norman	$-\frac{\ln(-3+2x)}{2}$
risch	$-\frac{\ln(-3+2x)}{2}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\cdots\right.}{12\left(x^{6}\right)^{\frac{1}{6}}\cdots}$

input
```
int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x,method=_RETUR
NVERBOSE)
```

output
```
-1/2*ln(x-3/2)
```

3.556.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{1}{2}\,\log\left(2\,x-3\right)$$

input
```
integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algori
thm="fricas")
```

output
```
-1/2*log(2*x - 3)
```

3.556.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243 + 162x + 108x^2 + 72x^3 + 48x^4 + 32x^5}{729 - 64x^6} \, dx = -\frac{\log(2x - 3)}{2}$$

input
```
integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729),x)
```

output
```
-log(2*x - 3)/2
```

3.556.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243 + 162x + 108x^2 + 72x^3 + 48x^4 + 32x^5}{729 - 64x^6} \, dx = -\frac{1}{2} \log(2x - 3)$$

input
```
integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algori
thm="maxima")
```

output
```
-1/2*log(2*x - 3)
```

3.556.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{243 + 162x + 108x^2 + 72x^3 + 48x^4 + 32x^5}{729 - 64x^6} \, dx = -\frac{1}{2} \log(|2x - 3|)$$

input
```
integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algori
thm="giac")
```

output
```
-1/2*log(abs(2*x - 3))
```

3.556.9 Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{\ln\left(x-\frac{3}{2}\right)}{2}$$

input
```
int(-(162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5 + 243)/(64*x^6 - 729),x)
```

output
```
-log(x - 3/2)/2
```

3.557 $\int \frac{81+36x^2+16x^4}{729-64x^6} dx$

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3.557.2 Mathematica [B] (verified) 4276
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3.557.8 Giac [B] (verification not implemented) 4279
3.557.9 Mupad [B] (verification not implemented) 4280

3.557.1 Optimal result

Integrand size = 22, antiderivative size = 10

$$\int \frac{81+36x^2+16x^4}{729-64x^6} dx = \frac{1}{6}\text{arctanh}\left(\frac{2x}{3}\right)$$

output
```
1/6*arctanh(2/3*x)
```

3.557.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{81+36x^2+16x^4}{729-64x^6} dx = -\frac{1}{12}\log(3-2x)+\frac{1}{12}\log(3+2x)$$

input
```
Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
```

output
```
-1/12*Log[3 - 2*x] + Log[3 + 2*x]/12
```

3.557.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2019, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{16x^4+36x^2+81}{729-64x^6}dx$$

$$\Big\downarrow \; 2019$$

$$\int \frac{1}{9-4x^2}dx$$

$$\Big\downarrow \; 219$$

$$\frac{1}{6}\text{arctanh}\left(\frac{2x}{3}\right)$$

input
```
Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
```

output
```
ArcTanh[(2*x)/3]/6
```

3.557.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.557.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.

Time = 1.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

method	result
parallelrisch	$-\frac{\ln\left(x-\frac{3}{2}\right)}{12}+\frac{\ln\left(x+\frac{3}{2}\right)}{12}$
default	$-\frac{\ln(-3+2x)}{12}+\frac{\ln(2x+3)}{12}$
norman	$-\frac{\ln(-3+2x)}{12}+\frac{\ln(2x+3)}{12}$
risch	$-\frac{\ln(-3+2x)}{12}+\frac{\ln(2x+3)}{12}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\cdots\right.}{36(x^6)^{\frac{1}{6}}}$

input
```
int((16*x^4+36*x^2+81)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-1/12*ln(x-3/2)+1/12*ln(x+3/2)
```

3.557.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx = \frac{1}{12}\,\log\left(2\,x+3\right)-\frac{1}{12}\,\log\left(2\,x-3\right)$$

input
```
integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

3.557.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx=-\frac{\log\left(x-\frac{3}{2}\right)}{12}+\frac{\log\left(x+\frac{3}{2}\right)}{12}$$

input
```
integrate((16*x**4+36*x**2+81)/(-64*x**6+729),x)
```

output
```
-log(x - 3/2)/12 + log(x + 3/2)/12
```

3.557.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx=\frac{1}{12}\log(2x+3)-\frac{1}{12}\log(2x-3)$$

input
```
integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

3.557.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx=\frac{1}{12}\log\left(\left|x+\frac{3}{2}\right|\right)-\frac{1}{12}\log\left(\left|x-\frac{3}{2}\right|\right)$$

input
```
integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/12*log(abs(x + 3/2)) - 1/12*log(abs(x - 3/2))
```

3.557.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx=\frac{\operatorname{atanh}\left(\frac{2x}{3}\right)}{6}$$

input
```
int(-(36*x^2 + 16*x^4 + 81)/(64*x^6 - 729),x)
```

output
```
atanh((2*x)/3)/6
```

3.558 $\int \frac{81+54x-24x^3-16x^4}{729-64x^6} dx$

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3.558.9 Mupad [B] (verification not implemented) 4285

3.558.1 Optimal result

Integrand size = 25, antiderivative size = 24

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
-1/9*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)
```

3.558.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6} \, dx = \frac{\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
```

output
```
ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])
```

3.558.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2019, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-16x^4-24x^3+54x+81}{729-64x^6}dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{4x^2-6x+9}dx$$
$$\Big\downarrow \text{1083}$$
$$-2\int \frac{1}{-(8x-6)^2-108}d(8x-6)$$
$$\Big\downarrow \text{217}$$
$$\frac{\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
```

output
```
ArcTan[(-6 + 8*x)/(6*Sqrt[3])]/(3*Sqrt[3])
```

3.558.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.558.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{9}$
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{9}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\mathrm{ar}\right.}{36(x^6)^{\frac{1}{6}}}$

input
```
int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
1/9*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

3.558.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx=\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)$$

input
```
integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

3.558.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{81 + 54x - 24x^3 - 16x^4}{729 - 64x^6}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729),x)
```

output
```
sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/9
```

3.558.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81 + 54x - 24x^3 - 16x^4}{729 - 64x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x - 3)\right)$$

input
```
integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

3.558.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81 + 54x - 24x^3 - 16x^4}{729 - 64x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x - 3)\right)$$

input
```
integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

3.558.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(4\,x-3)}{9}\right)}{9}$$

input
```
int(-(54*x - 24*x^3 - 16*x^4 + 81)/(64*x^6 - 729),x)
```

output
```
(3^(1/2)*atan((3^(1/2)*(4*x - 3))/9))/9
```

3.559 $\int \frac{3-2x}{729-64x^6} \, dx$

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3.559.2 Mathematica [A] (verified) 4286
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3.559.9 Mupad [B] (verification not implemented) 4290

3.559.1 Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{3-2x}{729-64x^6} \, dx = \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{162\sqrt{3}} + \frac{1}{486}\log(3+2x) - \frac{1}{972}\log\left(9-6x+4x^2\right)$$

output
```
1/486*ln(3+2*x)-1/972*ln(4*x^2-6*x+9)+1/486*arctan(1/9*(3+4*x)*3^(1/2))*3^
(1/2)
```

3.559.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{3-2x}{729-64x^6} \, dx = \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{162\sqrt{3}} + \frac{1}{486}\log(3+2x) - \frac{1}{972}\log\left(9-6x+4x^2\right)$$

input
```
Integrate[(3 - 2*x)/(729 - 64*x^6),x]
```

output
```
ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6
*x + 4*x^2]/972
```

3.559.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-2x}{729-64x^6} \, dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{32x^5+48x^4+72x^3+108x^2+162x+243}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{3-4x}{486 \left(4x^2-6x+9\right)}+\frac{1}{54 \left(4x^2+6x+9\right)}+\frac{1}{243(2x+3)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log(2x+3)$$

input
```
Int[(3 - 2*x)/(729 - 64*x^6),x]
```

output
```
ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6
*x + 4*x^2]/972
```

3.559.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.559.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(4x^2-6x+9)}{972}+\frac{\ln(2x+3)}{486}+\frac{\sqrt{3}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{486}$
risch	$\frac{\ln(2x+3)}{486}-\frac{\ln(4x^2-6x+9)}{972}+\frac{\arctan\left(\frac{(4x+3)\sqrt{3}}{9}\right)\sqrt{3}}{486}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\mathrm{ar}\right.}{972(x^6)^{\frac{1}{6}}}$

input
```
int((3-2*x)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-1/972*ln(4*x^2-6*x+9)+1/486*ln(2*x+3)+1/486*3^(1/2)*arctan(1/18*(8*x+6)*3
^(1/2))
```

3.559.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) - \frac{1}{972}\,\log\left(4\,x^2-6\,x+9\right)+\frac{1}{486}\,\log\left(2\,x+3\right)$$

input
```
integrate((3-2*x)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
 1/486*log(2*x + 3)
```

3.559.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{\log\left(x+\frac{3}{2}\right)}{486} - \frac{\log\left(4x^2-6x+9\right)}{972} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{486}$$

input
```
integrate((3-2*x)/(-64*x**6+729),x)
```

output
```
log(x + 3/2)/486 - log(4*x**2 - 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9
+ sqrt(3)/3)/486
```

3.559.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) - \frac{1}{972}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{486}\,\log\left(2\,x+3\right)$$

input
```
integrate((3-2*x)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
 1/486*log(2*x + 3)
```

3.559.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) - \frac{1}{972}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{486}\,\log\left(|2\,x+3|\right)$$

input
```
integrate((3-2*x)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
 1/486*log(abs(2*x + 3))
```

3.559.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{486} - \frac{\ln\left(x^2-\frac{3\,x}{2}+\frac{9}{4}\right)}{972} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{1327104\,\left(\frac{x}{884736}+\frac{1}{884736}\right)}-\frac{\sqrt{3}\,x}{7962624\,\left(\frac{x}{884736}+\frac{1}{884736}\right)}\right)}{486}$$

input
```
int((2*x - 3)/(64*x^6 - 729),x)
```

output
```
log(x + 3/2)/486 - log(x^2 - (3*x)/2 + 9/4)/972 - (3^(1/2)*atan(3^(1/2)/(1
327104*(x/884736 + 1/884736)) - (3^(1/2)*x)/(7962624*(x/884736 + 1/884736)
)))/486
```

3.560 $\int \frac{3+2x}{729-64x^6} \, dx$

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3.560.2 Mathematica [A] (verified) 4291
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3.560.7 Maxima [A] (verification not implemented) 4294
3.560.8 Giac [A] (verification not implemented) 4294
3.560.9 Mupad [B] (verification not implemented) 4295

3.560.1 Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{3+2x}{729-64x^6} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}} - \frac{1}{486}\log(3-2x) + \frac{1}{972}\log\left(9+6x+4x^2\right)$$

output
```
-1/486*ln(3-2*x)+1/972*ln(4*x^2+6*x+9)-1/486*arctan(1/9*(3-4*x)*3^(1/2))*3
^(1/2)
```

3.560.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{3+2x}{729-64x^6} \, dx = \frac{1}{972}\left(2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) - 2\log(3-2x) + \log\left(9+6x+4x^2\right)\right)$$

input
```
Integrate[(3 + 2*x)/(729 - 64*x^6),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 2*Log[3 - 2*x] + Log[9 + 6*x +
 4*x^2])/972
```

3.560.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+3}{729-64x^6}dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{-32x^5+48x^4-72x^3+108x^2-162x+243}dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{4x+3}{486\left(4x^2+6x+9\right)}+\frac{1}{54\left(4x^2-6x+9\right)}-\frac{1}{243(2x-3)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}+\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log(3-2x)$$

input
```
Int[(3 + 2*x)/(729 - 64*x^6),x]
```

output
```
-1/162*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/486 + Log[9 +
6*x + 4*x^2]/972
```

3.560.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.560.4 Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(-3+2x)}{486}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{486}+\frac{\ln(4x^2+6x+9)}{972}$
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{486}-\frac{\ln(-3+2x)}{486}+\frac{\ln(4x^2+6x+9)}{972}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\mathrm{ar}\right.}{972(x^6)^{\frac{1}{6}}}$

input
```
int((2*x+3)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-1/486*ln(-3+2*x)+1/486*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/972*ln(4*x^
2+6*x+9)
```

3.560.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3+2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) + \frac{1}{972}\,\log\left(4\,x^2+6\,x+9\right) - \frac{1}{486}\,\log\left(2\,x-3\right)$$

input
```
integrate((3+2*x)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
 1/486*log(2*x - 3)
```

3.560.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{3+2x}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{486} + \frac{\log\left(4x^2+6x+9\right)}{972} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{486}$$

input
```
integrate((3+2*x)/(-64*x**6+729),x)
```

output
```
-log(x - 3/2)/486 + log(4*x**2 + 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9
 - sqrt(3)/3)/486
```

3.560.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3+2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) + \frac{1}{972}\,\log\left(4\,x^2+6\,x+9\right) - \frac{1}{486}\,\log\left(2\,x-3\right)$$

input
```
integrate((3+2*x)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
 1/486*log(2*x - 3)
```

3.560.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{3+2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) + \frac{1}{972}\,\log\left(4\,x^2+6\,x+9\right) - \frac{1}{486}\,\log\left(|2\,x-3|\right)$$

input
```
integrate((3+2*x)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
 1/486*log(abs(2*x - 3))
```

3.560.9 Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{3+2x}{729-64x^6}\,dx = \frac{\ln\left(x^2+\frac{3\,x}{2}+\frac{9}{4}\right)}{972} - \frac{\ln\left(x-\frac{3}{2}\right)}{486} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{1327104\left(\frac{x}{884736}-\frac{1}{884736}\right)}+\frac{\sqrt{3}\,x}{7962624\left(\frac{x}{884736}-\frac{1}{884736}\right)}\right)}{486}$$

input
```
int(-(2*x + 3)/(64*x^6 - 729),x)
```

output
```
log((3*x)/2 + x^2 + 9/4)/972 - log(x - 3/2)/486 - (3^(1/2)*atan(3^(1/2)/(1
327104*(x/884736 - 1/884736)) + (3^(1/2)*x)/(7962624*(x/884736 - 1/884736)
)))/486
```

3.561 $\int \frac{9-6x+4x^2}{729-64x^6} \, dx$

3.561.1 Optimal result 4296
3.561.2 Mathematica [A] (verified) 4296
3.561.3 Rubi [A] (verified) 4297
3.561.4 Maple [A] (verified) 4298
3.561.5 Fricas [A] (verification not implemented) 4298
3.561.6 Sympy [A] (verification not implemented) 4299
3.561.7 Maxima [A] (verification not implemented) 4299
3.561.8 Giac [A] (verification not implemented) 4299
3.561.9 Mupad [B] (verification not implemented) 4300

3.561.1 Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{9-6x+4x^2}{729-64x^6} \, dx = \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{54\sqrt{3}} - \frac{1}{324}\log(3-2x) + \frac{1}{108}\log(3+2x) - \frac{1}{324}\log\left(9+6x+4x^2\right)$$

output
```
-1/324*ln(3-2*x)+1/108*ln(3+2*x)-1/324*ln(4*x^2+6*x+9)+1/162*arctan(1/9*(3
+4*x)*3^(1/2))*3^(1/2)
```

3.561.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{9-6x+4x^2}{729-64x^6} \, dx = \frac{1}{324}\left(2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right) - \log(3-2x) + 3\log(3+2x) - \log\left(9+6x+4x^2\right)\right)$$

input
```
Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

output
```
(2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - Log[3 - 2*x] + 3*Log[3 + 2*x] -
 Log[9 + 6*x + 4*x^2])/324
```

3.561.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2-6x+9}{729-64x^6}\,dx$$

$$\Big\downarrow \ 2019$$

$$\int \frac{1}{-16x^4-24x^3+54x+81}dx$$

$$\Big\downarrow \ 2462$$

$$\int \left(\frac{3-2x}{81\left(4x^2+6x+9\right)}-\frac{1}{162(2x-3)}+\frac{1}{54(2x+3)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{54\sqrt{3}}-\frac{1}{324}\log\left(4x^2+6x+9\right)-\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(2x+3)$$

input
```
Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

output
```
ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/324 + Log[3 + 2*
x]/108 - Log[9 + 6*x + 4*x^2]/324
```

3.561.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.561.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(-3+2x)}{324}+\frac{\ln(2x+3)}{108}-\frac{\ln(4x^2+6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{162}$
risch	$-\frac{\ln(-3+2x)}{324}-\frac{\ln(4x^2+6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{162}+\frac{\ln(2x+3)}{108}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\mathrm{ar}\right.}{324(x^6)^{\frac{1}{6}}}$

input
```
int((4*x^2-6*x+9)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-1/324*ln(-3+2*x)+1/108*ln(2*x+3)-1/324*ln(4*x^2+6*x+9)+1/162*3^(1/2)*arct
an(1/18*(8*x+6)*3^(1/2))
```

3.561.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx=\frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{324}\log\left(4x^2+6x+9\right)+\frac{1}{108}\log\left(2x+3\right)-\frac{1}{324}\log\left(2x-3\right)$$

input
```
integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
 1/108*log(2*x + 3) - 1/324*log(2*x - 3)
```

3.561.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{324}+\frac{\log\left(x+\frac{3}{2}\right)}{108}-\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{324}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{162}$$

input
```
integrate((4*x**2-6*x+9)/(-64*x**6+729),x)
```

output
```
-log(x - 3/2)/324 + log(x + 3/2)/108 - log(x**2 + 3*x/2 + 9/4)/324 + sqrt(
3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/162
```

3.561.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{324}\log\left(4x^2+6x+9\right)+\frac{1}{108}\log\left(2x+3\right)-\frac{1}{324}\log\left(2x-3\right)$$

input
```
integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
 1/108*log(2*x + 3) - 1/324*log(2*x - 3)
```

3.561.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-\frac{1}{324}\log\left(4x^2+6x+9\right)+\frac{1}{108}\log\left(|2x+3|\right)-\frac{1}{324}\log\left(|2x-3|\right)$$

input
```
integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
 1/108*log(abs(2*x + 3)) - 1/324*log(abs(2*x - 3))
```

3.561.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{108} - \frac{\ln\left(x-\frac{3}{2}\right)}{324} - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)$$

input
```
int(-(4*x^2 - 6*x + 9)/(64*x^6 - 729),x)
```

output
```
log(x + 3/2)/108 - log(x - 3/2)/324 - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1
/2)*1i)/324 + 1/324) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/324 - 1
/324)
```

3.562 $\int \frac{9+6x+4x^2}{729-64x^6} \, dx$

3.562.1 Optimal result 4301
3.562.2 Mathematica [A] (verified) 4301
3.562.3 Rubi [A] (verified) 4302
3.562.4 Maple [A] (verified) 4303
3.562.5 Fricas [A] (verification not implemented) 4303
3.562.6 Sympy [A] (verification not implemented) 4304
3.562.7 Maxima [A] (verification not implemented) 4304
3.562.8 Giac [A] (verification not implemented) 4304
3.562.9 Mupad [B] (verification not implemented) 4305

3.562.1 Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{9+6x+4x^2}{729-64x^6} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}} - \frac{1}{108}\log(3-2x) + \frac{1}{324}\log(3+2x) + \frac{1}{324}\log\left(9-6x+4x^2\right)$$

output
```
-1/108*ln(3-2*x)+1/324*ln(3+2*x)+1/324*ln(4*x^2-6*x+9)-1/162*arctan(1/9*(3
-4*x)*3^(1/2))*3^(1/2)
```

3.562.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{9+6x+4x^2}{729-64x^6} \, dx = \frac{1}{324}\left(2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) - 3\log(3-2x) + \log(3+2x) + \log\left(9-6x+4x^2\right)\right)$$

input
```
Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + Log[3 + 2*x]
+ Log[9 - 6*x + 4*x^2])/324
```

3.562.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+6x+9}{729-64x^6}dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{-16x^4+24x^3-54x+81}dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{2x+3}{81\left(4x^2-6x+9\right)}-\frac{1}{54(2x-3)}+\frac{1}{162(2x+3)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}+\frac{1}{324}\log\left(4x^2-6x+9\right)-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(2x+3)$$

input
```
Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

output
```
-1/54*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/108 + Log[3 + 2
*x]/324 + Log[9 - 6*x + 4*x^2]/324
```

3.562.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /; !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.562.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(-3+2x)}{108}+\frac{\ln(4x^2-6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{162}+\frac{\ln(2x+3)}{324}$
risch	$\frac{\ln(16x^2-24x+36)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{162}-\frac{\ln(-3+2x)}{108}+\frac{\ln(2x+3)}{324}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\mathrm{ar}\ldots\right.}{324(x^6)^{\frac{1}{6}}}$

input
```
int((4*x^2+6*x+9)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-1/108*ln(-3+2*x)+1/324*ln(4*x^2-6*x+9)+1/162*3^(1/2)*arctan(1/18*(8*x-6)*
3^(1/2))+1/324*ln(2*x+3)
```

3.562.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right)+\frac{1}{324}\log\left(2x+3\right)-\frac{1}{108}\log\left(2x-3\right)$$

input
```
integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
 1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

3.562.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{108} + \frac{\log\left(x+\frac{3}{2}\right)}{324} + \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{324} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{162}$$

input
```
integrate((4*x**2+6*x+9)/(-64*x**6+729),x)
```

output
```
-log(x - 3/2)/108 + log(x + 3/2)/324 + log(x**2 - 3*x/2 + 9/4)/324 + sqrt(
3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/162
```

3.562.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{324}\log\left(4x^2-6x+9\right) + \frac{1}{324}\log\left(2x+3\right) - \frac{1}{108}\log\left(2x-3\right)$$

input
```
integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
 1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

3.562.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{324}\log\left(4x^2-6x+9\right) + \frac{1}{324}\log\left(|2x+3|\right) - \frac{1}{108}\log\left(|2x-3|\right)$$

input
```
integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
 1/324*log(abs(2*x + 3)) - 1/108*log(abs(2*x - 3))
```

3.562.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{324} - \frac{\ln\left(x-\frac{3}{2}\right)}{108} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)$$

input
```
int(-(6*x + 4*x^2 + 9)/(64*x^6 - 729),x)
```

output
```
log(x + 3/2)/324 - log(x - 3/2)/108 - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1
/2)*1i)/324 - 1/324) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/324 + 1
/324)
```

3.563 $\int \frac{27-8x^3}{729-64x^6}\,dx$

3.563.1 Optimal result 4306
3.563.2 Mathematica [A] (verified) 4306
3.563.3 Rubi [A] (verified) 4307
3.563.4 Maple [A] (verified) 4309
3.563.5 Fricas [A] (verification not implemented) 4310
3.563.6 Sympy [A] (verification not implemented) 4310
3.563.7 Maxima [A] (verification not implemented) 4310
3.563.8 Giac [A] (verification not implemented) 4311
3.563.9 Mupad [B] (verification not implemented) 4311

3.563.1 Optimal result

Integrand size = 17, antiderivative size = 50

$$\int \frac{27-8x^3}{729-64x^6}\,dx=-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}+\frac{1}{54}\log(3+2x)-\frac{1}{108}\log\left(9-6x+4x^2\right)$$

output
```
1/54*ln(3+2*x)-1/108*ln(4*x^2-6*x+9)-1/54*arctan(1/9*(3-4*x)*3^(1/2))*3^(1
/2)
```

3.563.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{27-8x^3}{729-64x^6}\,dx=\frac{\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)}{18\sqrt{3}}+\frac{1}{54}\log(3+2x)-\frac{1}{108}\log\left(9-6x+4x^2\right)$$

input
```
Integrate[(27 - 8*x^3)/(729 - 64*x^6),x]
```

output
```
ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - Log[9 - 6*
x + 4*x^2]/108
```

3.563.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1386, 750, 16, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{27-8x^3}{729-64x^6}\,dx$$

$$\Big\downarrow \text{1386}$$

$$\int \frac{1}{8x^3+27}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{27}\int \frac{2(3-x)}{4x^2-6x+9}dx+\frac{1}{27}\int \frac{1}{2x+3}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{27}\int \frac{2(3-x)}{4x^2-6x+9}dx+\frac{1}{54}\log(2x+3)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{27}\int \frac{3-x}{4x^2-6x+9}dx+\frac{1}{54}\log(2x+3)$$

$$\Big\downarrow \text{1142}$$

$$\frac{2}{27}\left(\frac{9}{4}\int \frac{1}{4x^2-6x+9}dx-\frac{1}{8}\int -\frac{2(3-4x)}{4x^2-6x+9}dx\right)+\frac{1}{54}\log(2x+3)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{27}\left(\frac{9}{4}\int \frac{1}{4x^2-6x+9}dx+\frac{1}{4}\int \frac{3-4x}{4x^2-6x+9}dx\right)+\frac{1}{54}\log(2x+3)$$

$$\Big\downarrow \text{1083}$$

$$\frac{2}{27}\left(\frac{1}{4}\int \frac{3-4x}{4x^2-6x+9}dx-\frac{9}{2}\int \frac{1}{-(8x-6)^2-108}d(8x-6)\right)+\frac{1}{54}\log(2x+3)$$

$$\Big\downarrow \text{217}$$

$$\frac{2}{27}\left(\frac{1}{4}\int \frac{3-4x}{4x^2-6x+9}dx+\frac{1}{4}\sqrt{3}\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)\right)+\frac{1}{54}\log(2x+3)$$

$$\Big\downarrow \text{1103}$$

$$\frac{2}{27}\left(\frac{1}{4}\sqrt{3}\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)-\frac{1}{8}\log\left(4x^2-6x+9\right)\right)+\frac{1}{54}\log(2x+3)$$

input
```
Int[(27 - 8*x^3)/(729 - 64*x^6),x]
```

output
```
Log[3 + 2*x]/54 + (2*((Sqrt[3]*ArcTan[(-6 + 8*x)/(6*Sqrt[3])])/4 - Log[9 -
 6*x + 4*x^2]/8))/27
```

3.563.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

3.563.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln\left(4x^{2}-6x+9\right)}{108}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{54}+\frac{\ln\left(2x+3\right)}{54}$
risch	$\frac{\ln\left(2x+3\right)}{54}-\frac{\ln\left(4x^{2}-6x+9\right)}{108}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(-\frac{3}{2}+2x\right)\sqrt{3}}{9}\right)}{54}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\mathrm{ar}\right.}{108\left(x^{6}\right)^{\frac{1}{6}}}$

input
```
int((-8*x^3+27)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-1/108*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/54*ln(2
*x+3)
```

3.563.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{1}{108}\log\left(4x^2-6x+9\right) + \frac{1}{54}\log\left(2x+3\right)$$

input
```
integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) +
1/54*log(2*x + 3)
```

3.563.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{\log\left(x+\frac{3}{2}\right)}{54} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{108} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

input
```
integrate((-8*x**3+27)/(-64*x**6+729),x)
```

output
```
log(x + 3/2)/54 - log(x**2 - 3*x/2 + 9/4)/108 + sqrt(3)*atan(4*sqrt(3)*x/9
 - sqrt(3)/3)/54
```

3.563.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{1}{108}\log\left(4x^2-6x+9\right) + \frac{1}{54}\log\left(2x+3\right)$$

input
```
integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) +
1/54*log(2*x + 3)
```

3.563.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{1}{108}\log\left(x^2-\frac{3}{2}x+\frac{9}{4}\right) + \frac{1}{54}\log\left(\left|x+\frac{3}{2}\right|\right)$$

input
```
integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(x^2 - 3/2*x + 9/4)
+ 1/54*log(abs(x + 3/2))
```

3.563.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{54} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)$$

input
```
int((8*x^3 - 27)/(64*x^6 - 729),x)
```

output
```
log(x + 3/2)/54 - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 + 1/108)
 + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 - 1/108)
```

3.564 $\int \frac{27+36x+24x^2+8x^3}{729-64x^6} \, dx$

3.564.1 Optimal result 4312
3.564.2 Mathematica [A] (verified) 4312
3.564.3 Rubi [A] (verified) 4313
3.564.4 Maple [A] (verified) 4314
3.564.5 Fricas [A] (verification not implemented) 4314
3.564.6 Sympy [A] (verification not implemented) 4315
3.564.7 Maxima [A] (verification not implemented) 4315
3.564.8 Giac [A] (verification not implemented) 4315
3.564.9 Mupad [B] (verification not implemented) 4316

3.564.1 Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}} - \frac{1}{18}\log(3-2x) + \frac{1}{36}\log\left(9-6x+4x^2\right)$$

output
```
-1/18*ln(3-2*x)+1/36*ln(4*x^2-6*x+9)-1/54*arctan(1/9*(3-4*x)*3^(1/2))*3^(1
/2)
```

3.564.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6} \, dx = \frac{\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)}{18\sqrt{3}} - \frac{1}{18}\log(3-2x) + \frac{1}{36}\log\left(9-6x+4x^2\right)$$

input
```
Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

output
```
ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + Log[9 - 6*
x + 4*x^2]/36
```

3.564.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{8x^3+24x^2+36x+27}{729-64x^6}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{-8x^3+24x^2-36x+27}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{2x}{9\left(4x^2-6x+9\right)}-\frac{1}{9(2x-3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)$$

input
```
Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

output
```
-1/18*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/18 + Log[9 - 6*
x + 4*x^2]/36
```

3.564.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.564.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(-3+2x)}{18}+\frac{\ln(4x^2-6x+9)}{36}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{54}$
risch	$\frac{\ln(16x^2-24x+36)}{36}+\frac{\sqrt{3}\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{54}-\frac{\ln(-3+2x)}{18}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\mathrm{ar}\cdots\right.}{108(x^6)^{\frac{1}{6}}}$

input
```
int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-1/18*ln(-3+2*x)+1/36*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(
1/2))
```

3.564.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log\left(2x-3\right)$$

input
```
integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1
/18*log(2*x - 3)
```

3.564.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{18}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{36}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

input
```
integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729),x)
```

output
```
-log(x - 3/2)/18 + log(x**2 - 3*x/2 + 9/4)/36 + sqrt(3)*atan(4*sqrt(3)*x/9
 - sqrt(3)/3)/54
```

3.564.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log\left(2x-3\right)$$

input
```
integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1
/18*log(2*x - 3)
```

3.564.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log\left(|2x-3|\right)$$

input
```
integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1
/18*log(abs(2*x - 3))
```

3.564.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = -\frac{\ln\left(x-\frac{3}{2}\right)}{18} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)$$

input
```
int(-(36*x + 24*x^2 + 8*x^3 + 27)/(64*x^6 - 729),x)
```

output
```
log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 + 1/36) - log(x - (3^(1/2)
*3i)/4 - 3/4)*((3^(1/2)*1i)/108 - 1/36) - log(x - 3/2)/18
```

3.565 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$

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3.565.1 Optimal result

Integrand size = 35, antiderivative size = 110

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx = -\frac{1}{2916(3+2x)}-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}-\frac{\log(3-2x)}{17496}+\frac{5\log(3+2x)}{17496}-\frac{\log(9-6x+4x^2)}{17496}-\frac{\log(9+6x+4x^2)}{17496}$$

output
```
-1/2916/(3+2*x)-1/17496*ln(3-2*x)+5/17496*ln(3+2*x)-1/17496*ln(4*x^2-6*x+9
)-1/17496*ln(4*x^2+6*x+9)-1/26244*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/87
48*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

3.565.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx = \frac{-\frac{18}{3+2x}+2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+6\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-3\log(3-2x)+15\log(3+2x)-3\log(9-6x+4x^2)}{52488}$$

input
```
Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6
)^2,x]
```

output
```
(-18/(3 + 2*x) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt[3]*ArcT
an[(3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + 15*Log[3 + 2*x] - 3*Log[9 - 6
*x + 4*x^2] - 3*Log[9 + 6*x + 4*x^2])/52488
```

3.565.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-32x^5+48x^4-72x^3+108x^2-162x+243}{(729-64x^6)^2}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{(2x+3)^2(-32x^5+48x^4-72x^3+108x^2-162x+243)}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{3-2x}{4374(4x^2-6x+9)}+\frac{3-2x}{4374(4x^2+6x+9)}-\frac{1}{8748(2x-3)}+\frac{5}{8748(2x+3)}+\frac{1}{1458(2x+3)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{2916\sqrt{3}}-\frac{\log(4x^2-6x+9)}{17496}-\frac{\log(4x^2+6x+9)}{17496}-\frac{1}{2916(2x+3)}-\frac{\log(3-2x)}{17496}+\frac{5\log(2x+3)}{17496}$$

input
```
Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)^2,x]
```

output
```
-1/2916*1/(3 + 2*x) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) + ArcTa
n[(3 + 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) - Log[3 - 2*x]/17496 + (5*Log[3 +
2*x])/17496 - Log[9 - 6*x + 4*x^2]/17496 - Log[9 + 6*x + 4*x^2]/17496
```

3.565.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.565.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{1}{5832\left(x+\frac{3}{2}\right)}+\frac{5\ln\left(2x+3\right)}{17496}-\frac{\ln\left(4x^{2}+6x+9\right)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{8748}-\frac{\ln\left(-3+2x\right)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(-\frac{3}{2}+2x\right)\sqrt{3}}{9}\right)}{26244}-$
default	$-\frac{\ln\left(-3+2x\right)}{17496}-\frac{\ln\left(4x^{2}-6x+9\right)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(8x-6\right)\sqrt{3}}{18}\right)}{26244}-\frac{1}{2916\left(2x+3\right)}+\frac{5\ln\left(2x+3\right)}{17496}-\frac{\ln\left(4x^{2}+6x+9\right)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{(8}{}\right.}{8748}$
meijerg	$-\frac{\left(-1\right)^{\frac{5}{6}}\left(\frac{4x\left(-1\right)^{\frac{1}{6}}}{6-\frac{128x^{6}}{243}}-\frac{5x\left(-1\right)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right.}{}\right.}{6\left(x^{6}\right)^{\frac{1}{6}}}\right.}{8748}$

input
```
int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x,method=_RE
TURNVERBOSE)
```

output
```
-1/5832/(x+3/2)+5/17496*ln(2*x+3)-1/17496*ln(4*x^2+6*x+9)+1/8748*3^(1/2)*a
rctan(2/9*(2*x+3/2)*3^(1/2))-1/17496*ln(-3+2*x)+1/26244*3^(1/2)*arctan(2/9
*(-3/2+2*x)*3^(1/2))-1/17496*ln(4*x^2-6*x+9)
```

3.565.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$$

$$= \frac{6\sqrt{3}(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+2\sqrt{3}(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-3(2x+3)\log\left(4x^2+6x\right.}{52488\,(2x+}$$

input
```
integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, alg
orithm="fricas")
```

output
```
1/52488*(6*sqrt(3)*(2*x + 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(2*
x + 3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 3*(2*x + 3)*log(4*x^2 + 6*x + 9) -
3*(2*x + 3)*log(4*x^2 - 6*x + 9) + 15*(2*x + 3)*log(2*x + 3) - 3*(2*x + 3)
*log(2*x - 3) - 18)/(2*x + 3)
```

3.565.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{17496}+\frac{5\log\left(x+\frac{3}{2}\right)}{17496}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{17496}-\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{26244}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{8748}-\frac{1}{5832x+8748}$$

input
```
integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729)**2,
x)
```

output
```
-log(x - 3/2)/17496 + 5*log(x + 3/2)/17496 - log(x**2 - 3*x/2 + 9/4)/17496
 - log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)
/26244 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/8748 - 1/(5832*x + 8748)
```

3.565.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{1}{2916(2x+3)}-\frac{1}{17496}\log\left(4x^2+6x+9\right) - \frac{1}{17496}\log\left(4x^2-6x+9\right)+\frac{5}{17496}\log\left(2x+3\right)-\frac{1}{17496}\log\left(2x-3\right)$$

input
```
integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, alg
orithm="maxima")
```

output
```
1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/1
7496*log(4*x^2 - 6*x + 9) + 5/17496*log(2*x + 3) - 1/17496*log(2*x - 3)
```

3.565.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{1}{2916(2x+3)}-\frac{1}{17496}\log\left(4x^2+6x+9\right)-\frac{1}{17496}\log\left(4x^2-6x+9\right) + \frac{5}{17496}\log\left(|2x+3|\right)-\frac{1}{17496}\log\left(|2x-3|\right)$$

input
```
integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, alg
orithm="giac")
```

output
```
1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/1
7496*log(4*x^2 - 6*x + 9) + 5/17496*log(abs(2*x + 3)) - 1/17496*log(abs(2*
x - 3))
```

3.565.9 Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{5\ln\left(x+\frac{3}{2}\right)}{17496} - \frac{\ln\left(x-\frac{3}{2}\right)}{17496} - \frac{1}{5832\left(x+\frac{3}{2}\right)} - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right) - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right)$$

input
```
int(-(162*x - 108*x^2 + 72*x^3 - 48*x^4 + 32*x^5 - 243)/(64*x^6 - 729)^2,x
)
```

output
```
(5*log(x + 3/2))/17496 - log(x - 3/2)/17496 - 1/(5832*(x + 3/2)) - log(x -
 (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/17496 + 1/17496) + log(x + (3^(1/2)*3
i)/4 + 3/4)*((3^(1/2)*1i)/17496 - 1/17496) - log(x - (3^(1/2)*3i)/4 - 3/4)
*((3^(1/2)*1i)/52488 + 1/17496) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*
1i)/52488 - 1/17496)
```

3.566 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2} dx$

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3.566.1 Optimal result

Integrand size = 35, antiderivative size = 110

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2} dx = \frac{1}{2916(3-2x)} - \frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}} + \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{8748\sqrt{3}} - \frac{5\log(3-2x)}{17496} + \frac{\log(3+2x)}{17496} + \frac{\log(9-6x+4x^2)}{17496} + \frac{\log(9+6x+4x^2)}{17496}$$

output
```
1/2916/(3-2*x)-5/17496*ln(3-2*x)+1/17496*ln(3+2*x)+1/17496*ln(4*x^2-6*x+9)
+1/17496*ln(4*x^2+6*x+9)-1/8748*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/2624
4*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

3.566.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2} dx = \frac{6\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) + 2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right) + 3\left(\frac{6}{3-2x} - 5\log(3-2x) + \log(3+2x) + \log(9-6x+4x^2)\right.}{52488}$$

input
```
Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6
)^2,x]
```

output
```
(6*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*
Sqrt[3])] + 3*(6/(3 - 2*x) - 5*Log[3 - 2*x] + Log[3 + 2*x] + Log[9 - 6*x +
 4*x^2] + Log[9 + 6*x + 4*x^2]))/52488
```

3.566.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{32x^5+48x^4+72x^3+108x^2+162x+243}{(729-64x^6)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(3-2x)^2(32x^5+48x^4+72x^3+108x^2+162x+243)}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{2x+3}{4374(4x^2-6x+9)}+\frac{2x+3}{4374(4x^2+6x+9)}-\frac{5}{8748(2x-3)}+\frac{1}{1458(2x-3)^2}+\frac{1}{8748(2x+3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\log(4x^2-6x+9)}{17496}+\frac{\log(4x^2+6x+9)}{17496}+\frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(2x+3)}{17496}$$

input
```
Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)^2,x]
```

output
```
1/(2916*(3 - 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) + ArcTan
[(3 + 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) - (5*Log[3 - 2*x])/17496 + Log[3 +
2*x]/17496 + Log[9 - 6*x + 4*x^2]/17496 + Log[9 + 6*x + 4*x^2]/17496
```

3.566.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.566.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{1}{5832\left(x-\frac{3}{2}\right)}-\frac{5\ln(-3+2x)}{17496}+\frac{\ln(2x+3)}{17496}+\frac{\ln(4x^2+6x+9)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{26244}+\frac{\ln(4x^2-6x+9)}{17496}+\frac{\sqrt{3}\,\arctan}{}$
default	$-\frac{1}{2916(-3+2x)}-\frac{5\ln(-3+2x)}{17496}+\frac{\ln(4x^2-6x+9)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{8748}+\frac{\ln(2x+3)}{17496}+\frac{\ln(4x^2+6x+9)}{17496}+\frac{\sqrt{3}\,\arctan(}{262}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right.}{}}{6\left(x^6\right)^{\frac{1}{6}}}}{8748}$

input
```
int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x,method=_RET
URNVERBOSE)
```

output
```
-1/5832/(x-3/2)-5/17496*ln(-3+2*x)+1/17496*ln(2*x+3)+1/17496*ln(4*x^2+6*x+
9)+1/26244*3^(1/2)*arctan(2/9*(2*x+3/2)*3^(1/2))+1/17496*ln(4*x^2-6*x+9)+1
/8748*3^(1/2)*arctan(2/9*(-3/2+2*x)*3^(1/2))
```

3.566.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{2\sqrt{3}(2x-3)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+6\sqrt{3}(2x-3)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3(2x-3)\log\left(4x^2+6x\right.}{52488\,(2x-}$$

input
```
integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algo
rithm="fricas")
```

output
```
1/52488*(2*sqrt(3)*(2*x - 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 6*sqrt(3)*(2*
x - 3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(2*x - 3)*log(4*x^2 + 6*x + 9) +
3*(2*x - 3)*log(4*x^2 - 6*x + 9) + 3*(2*x - 3)*log(2*x + 3) - 15*(2*x - 3)
*log(2*x - 3) - 18)/(2*x - 3)
```

3.566.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$$
$$= -\frac{5\log\left(x-\frac{3}{2}\right)}{17496}+\frac{\log\left(x+\frac{3}{2}\right)}{17496}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{17496}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{17496}$$
$$+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{8748}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{26244}-\frac{1}{5832x-8748}$$

input
```
integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729)**2,x
)
```

output
```
-5*log(x - 3/2)/17496 + log(x + 3/2)/17496 + log(x**2 - 3*x/2 + 9/4)/17496
 + log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)
/8748 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/26244 - 1/(5832*x - 8748)
```

3.566.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{1}{2916(2x-3)}+\frac{1}{17496}\log\left(4x^2+6x+9\right) + \frac{1}{17496}\log\left(4x^2-6x+9\right)+\frac{1}{17496}\log(2x+3)-\frac{5}{17496}\log(2x-3)$$

input
```
integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algo
rithm="maxima")
```

output
```
1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/1
7496*log(4*x^2 - 6*x + 9) + 1/17496*log(2*x + 3) - 5/17496*log(2*x - 3)
```

3.566.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{1}{2916(2x-3)}+\frac{1}{17496}\log\left(4x^2+6x+9\right)+\frac{1}{17496}\log\left(4x^2-6x+9\right) + \frac{1}{17496}\log(|2x+3|)-\frac{5}{17496}\log(|2x-3|)$$

input
```
integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algo
rithm="giac")
```

output
```
1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/1
7496*log(4*x^2 - 6*x + 9) + 1/17496*log(abs(2*x + 3)) - 5/17496*log(abs(2*
x - 3))
```

3.566.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{\ln\left(x+\frac{3}{2}\right)}{17496} - \frac{5\ln\left(x-\frac{3}{2}\right)}{17496} - \frac{1}{5832\left(x-\frac{3}{2}\right)} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right) - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right)$$

input
```
int((162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5 + 243)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/17496 - (5*log(x - 3/2))/17496 - 1/(5832*(x - 3/2)) - log(x -
 (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/17496 - 1/17496) + log(x + (3^(1/2)*3
i)/4 - 3/4)*((3^(1/2)*1i)/17496 + 1/17496) - log(x - (3^(1/2)*3i)/4 + 3/4)
*((3^(1/2)*1i)/52488 - 1/17496) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*
1i)/52488 + 1/17496)
```

3.567 $\int \frac{81+36x^2+16x^4}{\left(729-64x^6\right)^2} \, dx$

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3.567.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{81+36x^2+16x^4}{\left(729-64x^6\right)^2} \, dx = \frac{1}{17496(3-2x)}-\frac{1}{17496(3+2x)}-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{2x}{3}\right)}{8748}$$

output
```
1/17496/(3-2*x)-1/17496/(3+2*x)+1/8748*arctanh(2/3*x)-1/39366*arctan(1/9*(
3-4*x)*3^(1/2))*3^(1/2)+1/39366*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

3.567.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.51

$$\int \frac{81+36x^2+16x^4}{\left(729-64x^6\right)^2} \, dx = \frac{\frac{36x}{9-4x^2}+3\sqrt{3}\arctan\left(\frac{1}{3}\left(-i+\sqrt{3}\right)x\right)+4i\sqrt{3}\operatorname{arctanh}\left(\frac{1}{3}\left(1-i\sqrt{3}\right)x\right)+\left(-3+\frac{2}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}\right)\operatorname{arctanh}\left(\frac{1}{3}(x\right.}{157464}$$

input
```
Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

output
```
((36*x)/(9 - 4*x^2) + 3*Sqrt[3]*ArcTan[((-I + Sqrt[3])*x)/3] + (4*I)*Sqrt[
3]*ArcTanh[((1 - I*Sqrt[3])*x)/3] + (-3 + 2/Sqrt[(1 + I*Sqrt[3])/6])*ArcTa
nh[(x + I*Sqrt[3]*x)/3] - 9*Log[3 - 2*x] + 9*Log[3 + 2*x])/157464
```

3.567.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2019, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{16x^4+36x^2+81}{(729-64x^6)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(9-4x^2)^2(16x^4+36x^2+81)}dx$$

$$\Big\downarrow \text{1484}$$

$$\int \left(\frac{1}{4374(4x^2-6x+9)}+\frac{1}{4374(4x^2+6x+9)}-\frac{1}{1458(4x^2-9)}+\frac{1}{8748(2x-3)^2}+\frac{1}{8748(2x+3)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\text{arctanh}\left(\frac{2x}{3}\right)}{8748}+\frac{1}{17496(3-2x)}-\frac{1}{17496(2x+3)}$$

input
```
Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

output
```
1/(17496*(3 - 2*x)) - 1/(17496*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/
(13122*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(13122*Sqrt[3]) + ArcTanh[
(2*x)/3]/8748
```

3.567.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.567.4 Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{x}{17496\left(x^{2}-\frac{9}{4}\right)}-\frac{\ln\left(-3+2x\right)}{17496}+\frac{\ln\left(2x+3\right)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,x}{9}\right)}{39366}+\frac{\sqrt{3}\,\arctan\left(\frac{8x^{3}\sqrt{3}}{81}+\frac{4\sqrt{3}\,x}{9}\right)}{39366}$
default	$-\frac{1}{17496\left(-3+2x\right)}-\frac{\ln\left(-3+2x\right)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(8x-6\right)\sqrt{3}}{18}\right)}{39366}-\frac{1}{17496\left(2x+3\right)}+\frac{\ln\left(2x+3\right)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(8x+6\right)\sqrt{3}}{18}\right)}{39366}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^{6}}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\cdots\right)}{\cdots}\cdots\right)}{6\left(x^{6}\right)^{\frac{1}{6}}}\cdots\right)}{26244}$

input
```
int((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/17496*x/(x^2-9/4)-1/17496*ln(-3+2*x)+1/17496*ln(2*x+3)+1/39366*3^(1/2)*
arctan(2/9*3^(1/2)*x)+1/39366*3^(1/2)*arctan(8/81*x^3*3^(1/2)+4/9*3^(1/2)*
x)
```

3.567.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \frac{4\sqrt{3}(4x^2-9)\arctan\left(\frac{4}{81}\sqrt{3}(2x^3+9x)\right)+4\sqrt{3}(4x^2-9)\arctan\left(\frac{2}{9}\sqrt{3}x\right)+9(4x^2-9)\log(2x+3)-\cdots}{157464(4x^2-9)}$$

input
```
integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/157464*(4*sqrt(3)*(4*x^2 - 9)*arctan(4/81*sqrt(3)*(2*x^3 + 9*x)) + 4*sqr
t(3)*(4*x^2 - 9)*arctan(2/9*sqrt(3)*x) + 9*(4*x^2 - 9)*log(2*x + 3) - 9*(4
*x^2 - 9)*log(2*x - 3) - 36*x)/(4*x^2 - 9)
```

3.567.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = -\frac{x}{17496x^2-39366} + \frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{9}\right)+2\,\mathrm{atan}\left(\frac{8\sqrt{3}x^3}{81}+\frac{4\sqrt{3}x}{9}\right)\right)}{78732} - \frac{\log\left(x-\frac{3}{2}\right)}{17496} + \frac{\log\left(x+\frac{3}{2}\right)}{17496}$$

input
```
integrate((16*x**4+36*x**2+81)/(-64*x**6+729)**2,x)
```

output
```
-x/(17496*x**2 - 39366) + sqrt(3)*(2*atan(2*sqrt(3)*x/9) + 2*atan(8*sqrt(3
)*x**3/81 + 4*sqrt(3)*x/9))/78732 - log(x - 3/2)/17496 + log(x + 3/2)/1749
6
```

3.567.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{81+36x^2+16x^4}{\left(729-64x^6\right)^2}\,dx = \frac{1}{39366}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{1}{39366}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(4\,x^2-9)} + \frac{1}{17496}\,\log\left(2\,x+3\right) - \frac{1}{17496}\,\log\left(2\,x-3\right)$$

input
```
integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9
*sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(2*x + 3) - 1/1749
6*log(2*x - 3)
```

3.567.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78

$$\int \frac{81+36x^2+16x^4}{\left(729-64x^6\right)^2}\,dx = \frac{1}{39366}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{1}{39366}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(4\,x^2-9)} + \frac{1}{17496}\,\log\left(|2\,x+3|\right) - \frac{1}{17496}\,\log\left(|2\,x-3|\right)$$

input
```
integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9
*sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(abs(2*x + 3)) - 1
/17496*log(abs(2*x - 3))
```

3.567.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{2x}{3}\right)}{8748} + \frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{8\sqrt{3}\,x^3}{81}+\frac{4\sqrt{3}\,x}{9}\right)+2\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{9}\right)\right)}{78732} - \frac{x}{17496\left(x^2-\frac{9}{4}\right)}$$

input
```
int((36*x^2 + 16*x^4 + 81)/(64*x^6 - 729)^2,x)
```

output
```
atanh((2*x)/3)/8748 + (3^(1/2)*(2*atan((4*3^(1/2)*x)/9 + (8*3^(1/2)*x^3)/8
1) + 2*atan((2*3^(1/2)*x)/9)))/78732 - x/(17496*(x^2 - 9/4))
```

3.568 $\int \frac{81+54x-24x^3-16x^4}{\left(729-64x^6\right)^2}\,dx$

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3.568.2 Mathematica [A] (verified) 4335
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3.568.1 Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{81+54x-24x^3-16x^4}{\left(729-64x^6\right)^2}\,dx = \frac{x}{4374\left(9-6x+4x^2\right)} - \frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}} - \frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} - \frac{\log\left(9-6x+4x^2\right)}{157464} + \frac{\log\left(9+6x+4x^2\right)}{52488}$$

output
```
1/4374*x/(4*x^2-6*x+9)-1/26244*ln(3-2*x)+1/78732*ln(3+2*x)-1/157464*ln(4*x
^2-6*x+9)+1/52488*ln(4*x^2+6*x+9)-1/13122*arctan(1/9*(3-4*x)*3^(1/2))*3^(1
/2)
```

3.568.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{81+54x-24x^3-16x^4}{\left(729-64x^6\right)^2}\,dx = \frac{\frac{36x}{9-6x+4x^2} + 12\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) - 6\log(3-2x) + 2\log(3+2x) - \log\left(9-6x+4x^2\right) + 3\log\left(9+6x+4\right.}{157464}$$

input
```
Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

output
```
((36*x)/(9 - 6*x + 4*x^2) + 12*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*
Log[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x
^2])/157464
```

3.568.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-16x^4-24x^3+54x+81}{(729-64x^6)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(4x^2-6x+9)^2(-16x^4-24x^3+54x+81)}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{39-4x}{78732(4x^2-6x+9)}+\frac{4x+3}{26244(4x^2+6x+9)}+\frac{3-x}{729(4x^2-6x+9)^2}-\frac{1}{13122(2x-3)}+\frac{1}{39366(2x+3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}+\frac{x}{4374(4x^2-6x+9)}-\frac{\log(4x^2-6x+9)}{157464}+\frac{\log(4x^2+6x+9)}{52488}-\frac{\log(3-2x)}{26244}+\frac{\log(2x+3)}{78732}$$

input
```
Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

output
```
x/(4374*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(4374*Sqrt[3])
- Log[3 - 2*x]/26244 + Log[3 + 2*x]/78732 - Log[9 - 6*x + 4*x^2]/157464 +
Log[9 + 6*x + 4*x^2]/52488
```

3.568.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.568.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

method	result
default	$-\frac{\ln(-3+2x)}{26244}+\frac{x}{17496x^2-26244x+39366}-\frac{\ln(4x^2-6x+9)}{157464}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{13122}+\frac{\ln(2x+3)}{78732}+\frac{\ln(4x^2+6x+9)}{52488}$
risch	$\frac{x}{17496x^2-26244x+39366}-\frac{\ln(-3+2x)}{26244}-\frac{\ln(64x^2-96x+144)}{157464}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{13122}+\frac{\ln(2x+3)}{78732}+\frac{\ln(4x^2+6x+9)}{52488}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{26244}$

input
```
int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/26244*ln(-3+2*x)+1/17496*x/(x^2-3/2*x+9/4)-1/157464*ln(4*x^2-6*x+9)+1/1
3122*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/78732*ln(2*x+3)+1/52488*ln(4*x
^2+6*x+9)
```

3.568.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.37

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx$$
$$= \frac{12\sqrt{3}(4x^2-6x+9)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3(4x^2-6x+9)\log(4x^2+6x+9)-(4x^2-6x+9)\log}{157464(4x^2-6x}$$

input
```
integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/157464*(12*sqrt(3)*(4*x^2 - 6*x + 9)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(
4*x^2 - 6*x + 9)*log(4*x^2 + 6*x + 9) - (4*x^2 - 6*x + 9)*log(4*x^2 - 6*x
+ 9) + 2*(4*x^2 - 6*x + 9)*log(2*x + 3) - 6*(4*x^2 - 6*x + 9)*log(2*x - 3)
 + 36*x)/(4*x^2 - 6*x + 9)
```

3.568.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{x}{17496x^2-26244x+39366} - \frac{\log\left(x-\frac{3}{2}\right)}{26244} + \frac{\log\left(x+\frac{3}{2}\right)}{78732} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{157464} + \frac{\log(4x^2+6x+9)}{52488} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{13122}$$

input
```
integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729)**2,x)
```

output
```
x/(17496*x**2 - 26244*x + 39366) - log(x - 3/2)/26244 + log(x + 3/2)/78732
 - log(x**2 - 3*x/2 + 9/4)/157464 + log(4*x**2 + 6*x + 9)/52488 + sqrt(3)*
atan(4*sqrt(3)*x/9 - sqrt(3)/3)/13122
```

3.568.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{1}{13122}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)+\frac{x}{4374\,(4\,x^2-6\,x+9)} + \frac{1}{52488}\,\log\left(4\,x^2+6\,x+9\right)-\frac{1}{157464}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{78732}\,\log\left(2\,x+3\right)-\frac{1}{26244}\,\log\left(2\,x-3\right)$$

input
```
integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9)
 + 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*
log(2*x + 3) - 1/26244*log(2*x - 3)
```

3.568.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{1}{13122}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)+\frac{x}{4374\,(4\,x^2-6\,x+9)} + \frac{1}{52488}\,\log\left(4\,x^2+6\,x+9\right)-\frac{1}{157464}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{78732}\,\log\left(|2\,x+3|\right)-\frac{1}{26244}\,\log\left(|2\,x-3|\right)$$

input
```
integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9)
 + 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*
log(abs(2*x + 3)) - 1/26244*log(abs(2*x - 3))
```

3.568.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{78732} - \frac{\ln\left(x-\frac{3}{2}\right)}{26244} + \frac{\ln\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{52488} + \frac{x}{17496\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3i}{4}\right)\left(\frac{1}{157464}+\frac{\sqrt{3}\,1i}{26244}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3i}{4}\right)\left(-\frac{1}{157464}+\frac{\sqrt{3}\,1i}{26244}\right)$$

input
```
int((54*x - 24*x^3 - 16*x^4 + 81)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/78732 - log(x - 3/2)/26244 + log((3*x)/2 + x^2 + 9/4)/52488 +
 x/(17496*(x^2 - (3*x)/2 + 9/4)) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)
*1i)/26244 + 1/157464) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/26244
 - 1/157464)
```

3.569 $\int \frac{3-2x}{(729-64x^6)^2}\,dx$

3.569.1 Optimal result 4341
3.569.2 Mathematica [A] (verified) 4341
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3.569.9 Mupad [B] (verification not implemented) 4346

3.569.1 Optimal result

Integrand size = 15, antiderivative size = 148

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = -\frac{1}{708588(3+2x)}+\frac{3-x}{708588\left(9-6x+4x^2\right)}+\frac{x}{236196\left(9+6x+4x^2\right)} -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{\log(3-2x)}{4251528} +\frac{\log(3+2x)}{472392}-\frac{\log\left(9-6x+4x^2\right)}{944784}+\frac{\log\left(9+6x+4x^2\right)}{8503056}$$

output
```
-1/708588/(3+2*x)+1/708588*(3-x)/(4*x^2-6*x+9)+1/236196*x/(4*x^2+6*x+9)-1/
4251528*ln(3-2*x)+1/472392*ln(3+2*x)-1/944784*ln(4*x^2-6*x+9)+1/8503056*ln
(4*x^2+6*x+9)-1/4251528*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/472392*arcta
n(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

3.569.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \frac{\frac{1944x}{243+162x+108x^2+72x^3+48x^4+32x^5}+2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+18\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-2\log(3-2x)+18\log(3+2}{8503056}$$

input
```
Integrate[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

output
```
((1944*x)/(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5) + 2*Sqrt[3]*A
rcTan[(-3 + 4*x)/(3*Sqrt[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] -
 2*Log[3 - 2*x] + 18*Log[3 + 2*x] - 9*Log[9 - 6*x + 4*x^2] + Log[9 + 6*x +
 4*x^2])/8503056
```

3.569.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-2x}{(729-64x^6)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(3-2x)(32x^5+48x^4+72x^3+108x^2+162x+243)^2}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{7-6x}{708588(4x^2-6x+9)}+\frac{2x+33}{2125764(4x^2+6x+9)}-\frac{x}{39366(4x^2-6x+9)^2}+\frac{x+3}{39366(4x^2+6x+9)^2}-\frac{1}{21257\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{3-x}{708588(4x^2-6x+9)}+\frac{x}{236196(4x^2+6x+9)}-\frac{\log(4x^2-6x+9)}{944784}+\frac{\log(4x^2+6x+9)}{8503056}-\frac{1}{708588(2x+3)}-\frac{\log(3-2x)}{4251528}+\frac{\log(2x+3)}{472392}$$

input
```
Int[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

output
```
-1/708588*1/(3 + 2*x) + (3 - x)/(708588*(9 - 6*x + 4*x^2)) + x/(236196*(9
+ 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) + ArcTan
[(3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - Log[3 - 2*x]/4251528 + Log[3 +
2*x]/472392 - Log[9 - 6*x + 4*x^2]/944784 + Log[9 + 6*x + 4*x^2]/8503056
```

3.569.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.569.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.70

method	result
risch	$\frac{x}{139968x^5+209952x^4+314928x^3+472392x^2+708588x+1062882}+\frac{\ln(16x^2+24x+36)}{8503056}+\frac{\arctan\left(\frac{(4x+3)\sqrt{3}}{9}\right)\sqrt{3}}{472392}-\frac{\ln(16x^2-24x}{944784}$
default	$-\frac{\ln(-3+2x)}{4251528}-\frac{\frac{x}{4}-\frac{3}{4}}{708588\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)}-\frac{\ln(4x^2-6x+9)}{944784}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{4251528}-\frac{1}{708588(2x+3)}+\frac{\ln(2x+3)}{472392}+\frac{}{944784x}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{708588}$

input
```
int((3-2*x)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
1/139968*x/(x^5+3/2*x^4+9/4*x^3+27/8*x^2+81/16*x+243/32)+1/8503056*ln(16*x
^2+24*x+36)+1/472392*arctan(1/9*(4*x+3)*3^(1/2))*3^(1/2)-1/944784*ln(16*x^
2-24*x+36)+1/4251528*3^(1/2)*arctan(1/9*(4*x-3)*3^(1/2))+1/472392*ln(2*x+3
)-1/4251528*ln(-3+2*x)
```

3.569.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(116) = 232$.

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.73

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx$$
$$= \frac{18\sqrt{3}(32x^5+48x^4+72x^3+108x^2+162x+243)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+2\sqrt{3}(32x^5+48x^4+72x^3}{}$$

input
```
integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/8503056*(18*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*a
rctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x
^2 + 162*x + 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + (32*x^5 + 48*x^4 + 72*x^
3 + 108*x^2 + 162*x + 243)*log(4*x^2 + 6*x + 9) - 9*(32*x^5 + 48*x^4 + 72*
x^3 + 108*x^2 + 162*x + 243)*log(4*x^2 - 6*x + 9) + 18*(32*x^5 + 48*x^4 +
72*x^3 + 108*x^2 + 162*x + 243)*log(2*x + 3) - 2*(32*x^5 + 48*x^4 + 72*x^3
 + 108*x^2 + 162*x + 243)*log(2*x - 3) + 1944*x)/(32*x^5 + 48*x^4 + 72*x^3
 + 108*x^2 + 162*x + 243)
```

3.569.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \frac{x}{139968x^5+209952x^4+314928x^3+472392x^2+708588x+1062882} - \frac{\log\left(x-\frac{3}{2}\right)}{4251528} + \frac{\log\left(x+\frac{3}{2}\right)}{472392} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{8503056} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{4251528} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{472392}$$

input
```
integrate((3-2*x)/(-64*x**6+729)**2,x)
```

output
```
x/(139968*x**5 + 209952*x**4 + 314928*x**3 + 472392*x**2 + 708588*x + 1062
882) - log(x - 3/2)/4251528 + log(x + 3/2)/472392 - log(x**2 - 3*x/2 + 9/4
)/944784 + log(x**2 + 3*x/2 + 9/4)/8503056 + sqrt(3)*atan(4*sqrt(3)*x/9 -
sqrt(3)/3)/4251528 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392
```

3.569.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.71

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right) + \frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right) + \frac{x}{4374\,(32\,x^5+48\,x^4+72\,x^3+108\,x^2+162\,x+243)} + \frac{1}{8503056}\log\left(4\,x^2+6\,x+9\right) - \frac{1}{944784}\log\left(4\,x^2-6\,x+9\right) + \frac{1}{472392}\log\left(2\,x+3\right) - \frac{1}{4251528}\log\left(2\,x-3\right)$$

input
```
integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 16
2*x + 243) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9
) + 1/472392*log(2*x + 3) - 1/4251528*log(2*x - 3)
```

3.569.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.75

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right) + \frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right) + \frac{x}{4374\,(4\,x^2+6\,x+9)(4\,x^2-6\,x+9)(2\,x+3)} + \frac{1}{8503056}\log\left(4\,x^2+6\,x+9\right) - \frac{1}{944784}\log\left(4\,x^2-6\,x+9\right) + \frac{1}{472392}\log\left(|2\,x+3|\right) - \frac{1}{4251528}\log\left(|2\,x-3|\right)$$

input
```
integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*
x + 3)) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) +
 1/472392*log(abs(2*x + 3)) - 1/4251528*log(abs(2*x - 3))
```

3.569.9 Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{472392} - \frac{\ln\left(x-\frac{3}{2}\right)}{4251528} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right) - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) + \frac{x}{139968\left(x^5+\frac{3x^4}{2}+\frac{9x^3}{4}+\frac{27x^2}{8}+\frac{81x}{16}+\frac{243}{32}\right)}$$

input
```
int(-(2*x - 3)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/472392 - log(x - 3/2)/4251528 - log(x - (3^(1/2)*3i)/4 - 3/4)
*((3^(1/2)*1i)/8503056 + 1/944784) - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/
2)*1i)/944784 - 1/8503056) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/8
503056 - 1/944784) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 +
1/8503056) + x/(139968*((81*x)/16 + (27*x^2)/8 + (9*x^3)/4 + (3*x^4)/2 + x
^5 + 243/32))
```

3.570 $\int \frac{3+2x}{(729-64x^6)^2}\,dx$

3.570.1 Optimal result 4347
3.570.2 Mathematica [A] (verified) 4347
3.570.3 Rubi [A] (verified) 4348
3.570.4 Maple [A] (verified) 4349
3.570.5 Fricas [B] (verification not implemented) 4350
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3.570.7 Maxima [A] (verification not implemented) 4351
3.570.8 Giac [A] (verification not implemented) 4352
3.570.9 Mupad [B] (verification not implemented) 4352

3.570.1 Optimal result

Integrand size = 15, antiderivative size = 146

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx = \frac{1}{708588(3-2x)}+\frac{x}{236196(9-6x+4x^2)}-\frac{3+x}{708588(9+6x+4x^2)} -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}-\frac{\log(3-2x)}{472392} +\frac{\log(3+2x)}{4251528}-\frac{\log(9-6x+4x^2)}{8503056}+\frac{\log(9+6x+4x^2)}{944784}$$

output
```
1/708588/(3-2*x)+1/236196*x/(4*x^2-6*x+9)+1/708588*(-3-x)/(4*x^2+6*x+9)-1/
472392*ln(3-2*x)+1/4251528*ln(3+2*x)-1/8503056*ln(4*x^2-6*x+9)+1/944784*ln
(4*x^2+6*x+9)-1/472392*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/4251528*arcta
n(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

3.570.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx = \frac{\frac{1944x}{243-162x+108x^2-72x^3+48x^4-32x^5}+18\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-18\log(3-2x)+2\log(3+2}{8503056}$$

input
```
Integrate[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

output
```
((1944*x)/(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5) + 18*Sqrt[3]*
ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] -
 18*Log[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 9*Log[9 + 6*x +
 4*x^2])/8503056
```

3.570.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+3}{(729-64x^6)^2}\,dx$$

$$\Big\downarrow \ 2019$$

$$\int \frac{1}{(2x+3)\left(-32x^5+48x^4-72x^3+108x^2-162x+243\right)^2}dx$$

$$\Big\downarrow \ 2462$$

$$\int \left(\frac{33-2x}{2125764\left(4x^2-6x+9\right)}+\frac{6x+7}{708588\left(4x^2+6x+9\right)}+\frac{3-x}{39366\left(4x^2-6x+9\right)^2}+\frac{x}{39366\left(4x^2+6x+9\right)^2}-\frac{}{23619}\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{x}{236196\left(4x^2-6x+9\right)}-\frac{x+3}{708588\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{8503056}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{708588(3-2x)}-\frac{\log(3-2x)}{472392}+\frac{\log(2x+3)}{4251528}$$

input
```
Int[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

output
```
1/(708588*(3 - 2*x)) + x/(236196*(9 - 6*x + 4*x^2)) - (3 + x)/(708588*(9 +
 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) + ArcTan[(
3 + 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) - Log[3 - 2*x]/472392 + Log[3 + 2*
x]/4251528 - Log[9 - 6*x + 4*x^2]/8503056 + Log[9 + 6*x + 4*x^2]/944784
```

3.570.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.570.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{x}{139968\left(x^5-\frac{3}{2}x^4+\frac{9}{4}x^3-\frac{27}{8}x^2+\frac{81}{16}x-\frac{243}{32}\right)}-\frac{\ln(-3+2x)}{472392}+\frac{\ln(4x^2+6x+9)}{944784}+\frac{\sqrt{3}\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{4251528}+\frac{\ln(2x+3)}{4251528}-\ln$
default	$-\frac{1}{708588(-3+2x)}-\frac{\ln(-3+2x)}{472392}+\frac{x}{944784x^2-1417176x+2125764}-\frac{\ln(4x^2-6x+9)}{8503056}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392}+\frac{\ln(2x+3)}{4251528}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right.\right.}{6(x^6)^{\frac{1}{6}}}\right.}{708588}$

input
```
int((2*x+3)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/139968*x/(x^5-3/2*x^4+9/4*x^3-27/8*x^2+81/16*x-243/32)-1/472392*ln(-3+2
*x)+1/944784*ln(4*x^2+6*x+9)+1/4251528*3^(1/2)*arctan(2/9*(2*x+3/2)*3^(1/2
))+1/4251528*ln(2*x+3)-1/8503056*ln(36*x^2-54*x+81)+1/472392*3^(1/2)*arcta
n(2/27*(6*x-9/2)*3^(1/2))
```

3.570.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(116) = 232$.

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.76

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx$$
$$= \frac{2\sqrt{3}(32x^5-48x^4+72x^3-108x^2+162x-243)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+18\sqrt{3}(32x^5-48x^4+72x^3}{}$$

input
```
integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/8503056*(2*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*ar
ctan(1/9*sqrt(3)*(4*x + 3)) + 18*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x
^2 + 162*x - 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + 9*(32*x^5 - 48*x^4 + 72*
x^3 - 108*x^2 + 162*x - 243)*log(4*x^2 + 6*x + 9) - (32*x^5 - 48*x^4 + 72*
x^3 - 108*x^2 + 162*x - 243)*log(4*x^2 - 6*x + 9) + 2*(32*x^5 - 48*x^4 + 7
2*x^3 - 108*x^2 + 162*x - 243)*log(2*x + 3) - 18*(32*x^5 - 48*x^4 + 72*x^3
 - 108*x^2 + 162*x - 243)*log(2*x - 3) - 1944*x)/(32*x^5 - 48*x^4 + 72*x^3
 - 108*x^2 + 162*x - 243)
```

3.570.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.85

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx$$
$$= -\frac{x}{139968x^5-209952x^4+314928x^3-472392x^2+708588x-1062882} - \frac{\log\left(x-\frac{3}{2}\right)}{472392} + \frac{\log\left(x+\frac{3}{2}\right)}{4251528} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{8503056} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{944784} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{4251528}$$

input
```
integrate((3+2*x)/(-64*x**6+729)**2,x)
```

output
```
-x/(139968*x**5 - 209952*x**4 + 314928*x**3 - 472392*x**2 + 708588*x - 106
2882) - log(x - 3/2)/472392 + log(x + 3/2)/4251528 - log(x**2 - 3*x/2 + 9/
4)/8503056 + log(x**2 + 3*x/2 + 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 -
 sqrt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/4251528
```

3.570.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\begin{aligned}\int \frac{3+2x}{(729-64x^6)^2}\,dx =& \frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)\\&+\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right)\\&-\frac{x}{4374\,(32\,x^5-48\,x^4+72\,x^3-108\,x^2+162\,x-243)}\\&+\frac{1}{944784}\log\left(4\,x^2+6\,x+9\right)-\frac{1}{8503056}\log\left(4\,x^2-6\,x+9\right)\\&+\frac{1}{4251528}\log\left(2\,x+3\right)-\frac{1}{472392}\log\left(2\,x-3\right)\end{aligned}$$

input
```
integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 16
2*x - 243) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9
) + 1/4251528*log(2*x + 3) - 1/472392*log(2*x - 3)
```

3.570.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx = \frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374(4x^2+6x+9)(4x^2-6x+9)(2x-3)} + \frac{1}{944784}\log\left(4x^2+6x+9\right) - \frac{1}{8503056}\log\left(4x^2-6x+9\right) + \frac{1}{4251528}\log\left(|2x+3|\right) - \frac{1}{472392}\log\left(|2x-3|\right)$$

input
```
integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*
x - 3)) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9) +
 1/4251528*log(abs(2*x + 3)) - 1/472392*log(abs(2*x - 3))
```

3.570.9 Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{4251528} - \frac{\ln\left(x-\frac{3}{2}\right)}{472392} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right) - \frac{x}{139968\left(x^5-\frac{3x^4}{2}+\frac{9x^3}{4}-\frac{27x^2}{8}+\frac{81x}{16}-\frac{243}{32}\right)}$$

input
```
int((2*x + 3)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/4251528 - log(x - 3/2)/472392 - log(x - (3^(1/2)*3i)/4 - 3/4)
*((3^(1/2)*1i)/944784 + 1/8503056) - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/
2)*1i)/8503056 - 1/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/9
44784 - 1/8503056) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/8503056 +
 1/944784) - x/(139968*((81*x)/16 - (27*x^2)/8 + (9*x^3)/4 - (3*x^4)/2 + x
^5 - 243/32))
```

3.570. $\int \frac{3+2x}{(729-64x^6)^2} \, dx$

3.571 $\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx$

3.571.1 Optimal result 4354
3.571.2 Mathematica [A] (verified) 4354
3.571.3 Rubi [A] (verified) 4355
3.571.4 Maple [A] (verified) 4356
3.571.5 Fricas [A] (verification not implemented) 4357
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3.571.7 Maxima [A] (verification not implemented) 4358
3.571.8 Giac [A] (verification not implemented) 4358
3.571.9 Mupad [B] (verification not implemented) 4359

3.571.1 Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = \frac{1}{472392(3-2x)}-\frac{1}{157464(3+2x)}+\frac{3+4x}{236196(9+6x+4x^2)} -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}-\frac{\log(3-2x)}{354294} +\frac{\log(3+2x)}{118098}-\frac{\log(9-6x+4x^2)}{944784}-\frac{5\log(9+6x+4x^2)}{2834352}$$

output
```
1/472392/(3-2*x)-1/157464/(3+2*x)+1/236196*(3+4*x)/(4*x^2+6*x+9)-1/354294*
ln(3-2*x)+1/118098*ln(3+2*x)-1/944784*ln(4*x^2-6*x+9)-5/2834352*ln(4*x^2+6
*x+9)-1/1417176*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/157464*arctan(1/9*(3
+4*x)*3^(1/2))*3^(1/2)
```

3.571.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = \frac{\frac{648x}{81+54x-24x^3-16x^4}+2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+18\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-8\log(3-2x)+24\log(3+2x)-3\log(9}{2834352}$$

input
```
Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

output
```
((648*x)/(81 + 54*x - 24*x^3 - 16*x^4) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sq
rt[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 8*Log[3 - 2*x] + 24*L
og[3 + 2*x] - 3*Log[9 - 6*x + 4*x^2] - 5*Log[9 + 6*x + 4*x^2])/2834352
```

3.571.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2-6x+9}{(729-64x^6)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(4x^2-6x+9)(-16x^4-24x^3+54x+81)^2}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{21-10x}{708588(4x^2+6x+9)}+\frac{3-2x}{236196(4x^2-6x+9)}+\frac{1}{4374(4x^2+6x+9)^2}-\frac{1}{177147(2x-3)}+\frac{1}{59049(2x+3)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{4x+3}{236196(4x^2+6x+9)}-\frac{\log(4x^2-6x+9)}{944784}-\frac{5\log(4x^2+6x+9)}{2834352}+\frac{1}{472392(3-2x)}-\frac{1}{157464(2x+3)}-\frac{\log(3-2x)}{354294}+\frac{\log(2x+3)}{118098}$$

input
```
Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

output
```
1/(472392*(3 - 2*x)) - 1/(157464*(3 + 2*x)) + (3 + 4*x)/(236196*(9 + 6*x +
 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) + ArcTan[(3 + 4*
x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/354294 + Log[3 + 2*x]/11809
8 - Log[9 - 6*x + 4*x^2]/944784 - (5*Log[9 + 6*x + 4*x^2])/2834352
```

3.571.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.571.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{x}{69984\left(x^{4}+\frac{3}{2}x^{3}-\frac{27}{8}x-\frac{81}{16}\right)}-\frac{5\ln\left(16x^{2}+24x+36\right)}{2834352}+\frac{\arctan\left(\frac{(4x+3)\sqrt{3}}{9}\right)\sqrt{3}}{157464}-\frac{\ln(-3+2x)}{354294}+\frac{\ln(2x+3)}{118098}+\frac{\sqrt{3}\,\arctan\left(\frac{2(-}{}\right.}{14171}$
default	$-\frac{1}{472392(-3+2x)}-\frac{\ln(-3+2x)}{354294}-\frac{\ln\left(4x^{2}-6x+9\right)}{944784}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{1417176}-\frac{1}{157464(2x+3)}+\frac{\ln(2x+3)}{118098}-\frac{-3x-}{708588(x^{2}+}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^{6}}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right.}{}\right.}{6\left(x^{6}\right)^{\frac{1}{6}}}\right.}{236196}$

input
```
int((4*x^2-6*x+9)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/69984*x/(x^4+3/2*x^3-27/8*x-81/16)-5/2834352*ln(16*x^2+24*x+36)+1/15746
4*arctan(1/9*(4*x+3)*3^(1/2))*3^(1/2)-1/354294*ln(-3+2*x)+1/118098*ln(2*x+
3)+1/1417176*3^(1/2)*arctan(2/9*(-3/2+2*x)*3^(1/2))-1/944784*ln(4*x^2-6*x+
9)
```

3.571.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx$$
$$= \frac{18\sqrt{3}(16x^4+24x^3-54x-81)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+2\sqrt{3}(16x^4+24x^3-54x-81)\arctan\left(\frac{1}{9}\sqrt{3}\right.}{}$$

input
```
integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/2834352*(18*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*
x + 3)) + 2*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
- 3)) - 5*(16*x^4 + 24*x^3 - 54*x - 81)*log(4*x^2 + 6*x + 9) - 3*(16*x^4 +
 24*x^3 - 54*x - 81)*log(4*x^2 - 6*x + 9) + 24*(16*x^4 + 24*x^3 - 54*x - 8
1)*log(2*x + 3) - 8*(16*x^4 + 24*x^3 - 54*x - 81)*log(2*x - 3) - 648*x)/(1
6*x^4 + 24*x^3 - 54*x - 81)
```

3.571.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = -\frac{x}{69984x^4+104976x^3-236196x-354294} - \frac{\log\left(x-\frac{3}{2}\right)}{354294} + \frac{\log\left(x+\frac{3}{2}\right)}{118098} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784} - \frac{5\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{2834352} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{1417176} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{157464}$$

input
```
integrate((4*x**2-6*x+9)/(-64*x**6+729)**2,x)
```

output
```
-x/(69984*x**4 + 104976*x**3 - 236196*x - 354294) - log(x - 3/2)/354294 +
log(x + 3/2)/118098 - log(x**2 - 3*x/2 + 9/4)/944784 - 5*log(x**2 + 3*x/2
+ 9/4)/2834352 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/1417176 + sqrt(3)
*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/157464
```

3.571.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\begin{aligned}\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx &= \frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) \\ &\quad+\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) \\ &\quad-\frac{x}{4374\,(16x^4+24x^3-54x-81)} \\ &\quad-\frac{5}{2834352}\log\left(4x^2+6x+9\right)-\frac{1}{944784}\log\left(4x^2-6x+9\right) \\ &\quad+\frac{1}{118098}\log\left(2x+3\right)-\frac{1}{354294}\log\left(2x-3\right)\end{aligned}$$

input
```
integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 + 24*x^3 - 54*x - 81) - 5/283435
2*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/118098*log(2*x
+ 3) - 1/354294*log(2*x - 3)
```

3.571.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx &= \frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) \\ &\quad+\frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) \\ &\quad-\frac{x}{4374\,(4x^2+6x+9)(2x+3)(2x-3)} \\ &\quad-\frac{5}{2834352}\log\left(4x^2+6x+9\right)-\frac{1}{944784}\log\left(4x^2-6x+9\right) \\ &\quad+\frac{1}{118098}\log\left(|2x+3|\right)-\frac{1}{354294}\log\left(|2x-3|\right)\end{aligned}$$

input
```
integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(2*x + 3)*(2*x - 3))
- 5/2834352*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/11809
8*log(abs(2*x + 3)) - 1/354294*log(abs(2*x - 3))
```

3.571.9 Mupad [B] (verification not implemented)

Time = 11.76 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{118098} - \frac{\ln\left(x-\frac{3}{2}\right)}{354294} - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right) + \frac{x}{69984\left(-x^4-\frac{3x^3}{2}+\frac{27x}{8}+\frac{81}{16}\right)}$$

input
```
int((4*x^2 - 6*x + 9)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/118098 - log(x - 3/2)/354294 - log(x - (3^(1/2)*3i)/4 + 3/4)*
((3^(1/2)*1i)/314928 + 5/2834352) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2
)*1i)/314928 - 5/2834352) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/28
34352 + 1/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/2834352 -
1/944784) + x/(69984*((27*x)/8 - (3*x^3)/2 - x^4 + 81/16))
```

3.572 $\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx$

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3.572.1 Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = \frac{1}{157464(3-2x)}-\frac{1}{472392(3+2x)}-\frac{3-4x}{236196(9-6x+4x^2)} -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}-\frac{\log(3-2x)}{118098} +\frac{\log(3+2x)}{354294}+\frac{5\log(9-6x+4x^2)}{2834352}+\frac{\log(9+6x+4x^2)}{944784}$$

output
```
1/157464/(3-2*x)-1/472392/(3+2*x)+1/236196*(-3+4*x)/(4*x^2-6*x+9)-1/118098
*ln(3-2*x)+1/354294*ln(3+2*x)+5/2834352*ln(4*x^2-6*x+9)+1/944784*ln(4*x^2+
6*x+9)-1/157464*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/1417176*arctan(1/9*(
3+4*x)*3^(1/2))*3^(1/2)
```

3.572.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx$$
$$= \frac{\frac{648x}{81-54x+24x^3-16x^4}+18\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-24\log(3-2x)+8\log(3+2x)+5\log(9}{2834352}$$

input
```
Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

output
```
((648*x)/(81 - 54*x + 24*x^3 - 16*x^4) + 18*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*S
qrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 24*Log[3 - 2*x] + 8*L
og[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/2834352
```

3.572.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+6x+9}{\left(729-64x^6\right)^2}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{\left(4x^2+6x+9\right)\left(-16x^4+24x^3-54x+81\right)^2}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{2x+3}{236196\left(4x^2+6x+9\right)}+\frac{10x+21}{708588\left(4x^2-6x+9\right)}+\frac{1}{4374\left(4x^2-6x+9\right)^2}-\frac{1}{59049(2x-3)}+\frac{1}{78732(2x-3)^2}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{472392\sqrt{3}}-\frac{3-4x}{236196\left(4x^2-6x+9\right)}+\frac{5\log\left(4x^2-6x+9\right)}{2834352}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{157464(3-2x)}-\frac{1}{472392(2x+3)}-\frac{\log(3-2x)}{118098}+\frac{\log(2x+3)}{354294}$$

input
```
Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

output
```
1/(157464*(3 - 2*x)) - 1/(472392*(3 + 2*x)) - (3 - 4*x)/(236196*(9 - 6*x +
 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) + ArcTan[(3 + 4*x
)/(3*Sqrt[3])]/(472392*Sqrt[3]) - Log[3 - 2*x]/118098 + Log[3 + 2*x]/35429
4 + (5*Log[9 - 6*x + 4*x^2])/2834352 + Log[9 + 6*x + 4*x^2]/944784
```

3.572.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.572.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{x}{69984\left(x^4-\frac{3}{2}x^3+\frac{27}{8}x-\frac{81}{16}\right)}+\frac{\ln(16x^2+24x+36)}{944784}+\frac{\arctan\left(\frac{(4x+3)\sqrt{3}}{9}\right)\sqrt{3}}{1417176}-\frac{\ln(-3+2x)}{118098}+\frac{5\ln(36x^2-54x+81)}{2834352}+\frac{\sqrt{3}\ \mathrm{arc}}{}$
default	$-\frac{1}{157464(-3+2x)}-\frac{\ln(-3+2x)}{118098}+\frac{3x-\frac{9}{4}}{708588x^2-1062882x+1594323}+\frac{5\ln(4x^2-6x+9)}{2834352}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{157464}-\frac{1}{472392(}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{236196}$

input
```
int((4*x^2+6*x+9)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/69984*x/(x^4-3/2*x^3+27/8*x-81/16)+1/944784*ln(16*x^2+24*x+36)+1/141717
6*arctan(1/9*(4*x+3)*3^(1/2))*3^(1/2)-1/118098*ln(-3+2*x)+5/2834352*ln(36*
x^2-54*x+81)+1/157464*3^(1/2)*arctan(2/27*(6*x-9/2)*3^(1/2))+1/354294*ln(2
*x+3)
```

3.572.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = \frac{2\sqrt{3}(16\,x^4-24\,x^3+54\,x-81)\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+18\sqrt{3}(16\,x^4-24\,x^3+54\,x-81)\arctan\left(\frac{1}{9}\sqrt{3}\right.}{}$$

input
```
integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/2834352*(2*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
 + 3)) + 18*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
- 3)) + 3*(16*x^4 - 24*x^3 + 54*x - 81)*log(4*x^2 + 6*x + 9) + 5*(16*x^4 -
 24*x^3 + 54*x - 81)*log(4*x^2 - 6*x + 9) + 8*(16*x^4 - 24*x^3 + 54*x - 81
)*log(2*x + 3) - 24*(16*x^4 - 24*x^3 + 54*x - 81)*log(2*x - 3) - 648*x)/(1
6*x^4 - 24*x^3 + 54*x - 81)
```

3.572.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\begin{aligned}\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = &-\frac{x}{69984x^4-104976x^3+236196x-354294}-\frac{\log\left(x-\frac{3}{2}\right)}{118098}\\&+\frac{\log\left(x+\frac{3}{2}\right)}{354294}+\frac{5\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{2834352}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{944784}\\&+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{157464}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{1417176}\end{aligned}$$

input
```
integrate((4*x**2+6*x+9)/(-64*x**6+729)**2,x)
```

output
```
-x/(69984*x**4 - 104976*x**3 + 236196*x - 354294) - log(x - 3/2)/118098 +
log(x + 3/2)/354294 + 5*log(x**2 - 3*x/2 + 9/4)/2834352 + log(x**2 + 3*x/2
 + 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/157464 + sqrt(3)*
atan(4*sqrt(3)*x/9 + sqrt(3)/3)/1417176
```

3.572.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = \frac{1}{1417176}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{1}{157464}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(16\,x^4-24\,x^3+54\,x-81)} + \frac{1}{944784}\,\log\left(4\,x^2+6\,x+9\right) + \frac{5}{2834352}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{354294}\,\log\left(2\,x+3\right) - \frac{1}{118098}\,\log\left(2\,x-3\right)$$

input
```
integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 - 24*x^3 + 54*x - 81) + 1/944784
*log(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/354294*log(2*x
+ 3) - 1/118098*log(2*x - 3)
```

3.572.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = \frac{1}{1417176}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{1}{157464}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(4\,x^2-6\,x+9)(2\,x+3)(2\,x-3)} + \frac{1}{944784}\,\log\left(4\,x^2+6\,x+9\right) + \frac{5}{2834352}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{354294}\,\log\left(|2\,x+3|\right) - \frac{1}{118098}\,\log\left(|2\,x-3|\right)$$

input
```
integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x + 3)*(2*x - 3))
+ 1/944784*log(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/35429
4*log(abs(2*x + 3)) - 1/118098*log(abs(2*x - 3))
```

3.572.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{354294} - \frac{\ln\left(x-\frac{3}{2}\right)}{118098} - \frac{x}{69984\left(x^4-\frac{3x^3}{2}+\frac{27x}{8}-\frac{81}{16}\right)} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right)$$

input
```
int((6*x + 4*x^2 + 9)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/354294 - log(x - 3/2)/118098 - x/(69984*((27*x)/8 - (3*x^3)/2
 + x^4 - 81/16)) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/314928 - 5/
2834352) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/314928 + 5/2834352)
 - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/2834352 - 1/944784) + log(x
 + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/2834352 + 1/944784)
```

3.573 $\int \frac{27-8x^3}{(729-64x^6)^2}\,dx$

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3.573.1 Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx=\frac{x}{4374(27+8x^3)}-\frac{7\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}-\frac{\log(3-2x)}{157464}+\frac{7\log(3+2x)}{472392}-\frac{7\log(9-6x+4x^2)}{944784}+\frac{\log(9+6x+4x^2)}{314928}$$

output
```
1/4374*x/(8*x^3+27)-1/157464*ln(3-2*x)+7/472392*ln(3+2*x)-7/944784*ln(4*x^
2-6*x+9)+1/314928*ln(4*x^2+6*x+9)-7/472392*arctan(1/9*(3-4*x)*3^(1/2))*3^(
1/2)+1/157464*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)
```

3.573.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx$$
$$=\frac{\frac{216x}{27+8x^3}+14\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+6\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-6\log(3-2x)+14\log(3+2x)-7\log(9-6x+4}{944784}$$

input
```
Integrate[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

output
```
((216*x)/(27 + 8*x^3) + 14*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt
[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 6*Log[3 - 2*x] + 14*Log[3 + 2*x] - 7*L
og[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784
```

3.573.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {1388, 931, 27, 1020, 750, 16, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{27-8x^3}{\left(729-64x^6\right)^2}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{\left(27-8x^3\right)\left(8x^3+27\right)^2}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{x}{4374\left(8x^3+27\right)}-\frac{\int -\frac{8\left(135-16x^3\right)}{\left(27-8x^3\right)\left(8x^3+27\right)}dx}{34992}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{135-16x^3}{\left(27-8x^3\right)\left(8x^3+27\right)}dx}{4374}+\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{\frac{3}{2}\int \frac{1}{27-8x^3}dx+\frac{7}{2}\int \frac{1}{8x^3+27}dx}{4374}+\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \text{750}$$

$$\frac{\frac{7}{2}\left(\frac{1}{27}\int \frac{2(3-x)}{4x^2-6x+9}dx+\frac{1}{27}\int \frac{1}{2x+3}dx\right)+\frac{3}{2}\left(\frac{1}{27}\int \frac{2(x+3)}{4x^2+6x+9}dx+\frac{1}{27}\int \frac{1}{3-2x}dx\right)}{4374}+\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{7}{2}\left(\frac{1}{27}\int \frac{2(3-x)}{4x^2-6x+9}dx+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{1}{27}\int \frac{2(x+3)}{4x^2+6x+9}dx-\frac{1}{54}\log(3-2x)\right)}{4374}+\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{7}{2}\left(\frac{2}{27}\int \frac{3-x}{4x^2-6x+9}dx+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{2}{27}\int \frac{x+3}{4x^2+6x+9}dx-\frac{1}{54}\log(3-2x)\right)}{4374}+\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{7}{2}\left(\frac{2}{27}\left(\frac{9}{4}\int \frac{1}{4x^2-6x+9}dx-\frac{1}{8}\int -\frac{2(3-4x)}{4x^2-6x+9}dx\right)+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{2}{27}\left(\frac{9}{4}\int \frac{1}{4x^2+6x+9}dx+\frac{1}{8}\int \frac{2(4x+3)}{4x^2+6x+9}dx\right)-\frac{1}{54}\text{lo}}{4374}$$
$$\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{7}{2}\left(\frac{2}{27}\left(\frac{9}{4}\int \frac{1}{4x^2-6x+9}dx+\frac{1}{4}\int \frac{3-4x}{4x^2-6x+9}dx\right)+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{2}{27}\left(\frac{9}{4}\int \frac{1}{4x^2+6x+9}dx+\frac{1}{4}\int \frac{4x+3}{4x^2+6x+9}dx\right)-\frac{1}{54}\log}{4374}$$
$$\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{7}{2}\left(\frac{2}{27}\left(\frac{1}{4}\int \frac{3-4x}{4x^2-6x+9}dx-\frac{9}{2}\int \frac{1}{-(8x-6)^2-108}d(8x-6)\right)+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{2}{27}\left(\frac{1}{4}\int \frac{4x+3}{4x^2+6x+9}dx-\frac{9}{2}\int \frac{1}{-(8x+6)^2-10}\right.\right.}{4374}$$
$$\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{7}{2}\left(\frac{2}{27}\left(\frac{1}{4}\int \frac{3-4x}{4x^2-6x+9}dx+\frac{1}{4}\sqrt{3}\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)\right)+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{2}{27}\left(\frac{1}{4}\int \frac{4x+3}{4x^2+6x+9}dx+\frac{1}{4}\sqrt{3}\arctan\left(\frac{8x+6}{6\sqrt{3}}\right)\right)\right.}{4374}$$
$$\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{7}{2}\left(\frac{2}{27}\left(\frac{1}{4}\sqrt{3}\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)-\frac{1}{8}\log\left(4x^2-6x+9\right)\right)+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{2}{27}\left(\frac{1}{4}\sqrt{3}\arctan\left(\frac{8x+6}{6\sqrt{3}}\right)+\frac{1}{8}\log\left(4x^2+\right.\right.\right.}{4374}$$
$$\frac{x}{4374\left(8x^3+27\right)}$$

input
```
Int[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

3.573. $\int \frac{27-8x^3}{(729-64x^6)^2}\,dx$

output
```
x/(4374*(27 + 8*x^3)) + ((7*(Log[3 + 2*x]/54 + (2*((Sqrt[3]*ArcTan[(-6 + 8
*x)/(6*Sqrt[3])])/4 - Log[9 - 6*x + 4*x^2]/8))/27))/2 + (3*(-1/54*Log[3 -
2*x] + (2*((Sqrt[3]*ArcTan[(6 + 8*x)/(6*Sqrt[3])])/4 + Log[9 + 6*x + 4*x^2
]/8))/27))/2)/4374
```

3.573.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

3.573. $\int \frac{27-8x^3}{(729-64x^6)^2}\,dx$

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.573.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

method	result
risch	$\frac{x}{34992x^3+118098}-\frac{\ln(-3+2x)}{157464}+\frac{7\ln(2x+3)}{472392}-\frac{7\ln(4x^2-6x+9)}{944784}+\frac{7\sqrt{3}\,\arctan\left(\frac{2\left(-\frac{3}{2}+2x\right)\sqrt{3}}{9}\right)}{472392}+\frac{\ln(4x^2+6x+9)}{314928}+\frac{\sqrt{3}}{}$
default	$-\frac{\ln(-3+2x)}{157464}-\frac{-\frac{3x}{4}-\frac{9}{8}}{118098\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)}-\frac{7\ln(4x^2-6x+9)}{944784}+\frac{7\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392}-\frac{1}{78732(2x+3)}+\frac{7\ln(2x+3)}{472392}+\frac{\ln(4x}{3}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{78732}$

input
```
int((-8*x^3+27)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
1/34992*x/(x^3+27/8)-1/157464*ln(-3+2*x)+7/472392*ln(2*x+3)-7/944784*ln(4*
x^2-6*x+9)+7/472392*3^(1/2)*arctan(2/9*(-3/2+2*x)*3^(1/2))+1/314928*ln(4*x
^2+6*x+9)+1/157464*3^(1/2)*arctan(2/9*(2*x+3/2)*3^(1/2))
```

3.573.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.16

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx$$
$$= \frac{6\sqrt{3}(8x^3+27)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+14\sqrt{3}(8x^3+27)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3\,(8x^3+27)\log\left(4x\right.}{94478}$$

input
```
integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/944784*(6*sqrt(3)*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x + 3)) + 14*sqrt(3
)*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(8*x^3 + 27)*log(4*x^2 +
6*x + 9) - 7*(8*x^3 + 27)*log(4*x^2 - 6*x + 9) + 14*(8*x^3 + 27)*log(2*x +
 3) - 6*(8*x^3 + 27)*log(2*x - 3) + 216*x)/(8*x^3 + 27)
```

3.573.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.97

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{x}{34992x^3+118098} - \frac{\log\left(x-\frac{3}{2}\right)}{157464} + \frac{7\log\left(x+\frac{3}{2}\right)}{472392} - \frac{7\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928} + \frac{7\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{157464}$$

input
```
integrate((-8*x**3+27)/(-64*x**6+729)**2,x)
```

output
```
x/(34992*x**3 + 118098) - log(x - 3/2)/157464 + 7*log(x + 3/2)/472392 - 7*
log(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2 + 9/4)/314928 + 7*sqrt(3
)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sq
rt(3)/3)/157464
```

3.573.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{x}{4374\,(8x^3+27)} + \frac{1}{314928}\log\left(4x^2+6x+9\right) - \frac{7}{944784}\log\left(4x^2-6x+9\right) + \frac{7}{472392}\log\left(2x+3\right) - \frac{1}{157464}\log\left(2x-3\right)$$

input
```
integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x +
 9) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(2*x + 3) - 1/157464*log
(2*x - 3)
```

3.573.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{x}{4374\,(8x^3+27)} + \frac{1}{314928}\log\left(4x^2+6x+9\right) - \frac{7}{944784}\log\left(4x^2-6x+9\right) + \frac{7}{472392}\log\left(|2x+3|\right) - \frac{1}{157464}\log\left(|2x-3|\right)$$

input
```
integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x +
 9) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(abs(2*x + 3)) - 1/15746
4*log(abs(2*x - 3))
```

3.573.9 Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{7\ln\left(x+\frac{3}{2}\right)}{472392} - \frac{\ln\left(x-\frac{3}{2}\right)}{157464} + \frac{x}{34992\left(x^3+\frac{27}{8}\right)} - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3i}{4}\right)\left(-\frac{1}{314928}+\frac{\sqrt{3}\,1i}{314928}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3i}{4}\right)\left(\frac{1}{314928}+\frac{\sqrt{3}\,1i}{314928}\right) - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3i}{4}\right)\left(\frac{7}{944784}+\frac{\sqrt{3}\,7i}{944784}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3i}{4}\right)\left(-\frac{7}{944784}+\frac{\sqrt{3}\,7i}{944784}\right)$$

input
```
int(-(8*x^3 - 27)/(64*x^6 - 729)^2,x)
```

output
```
(7*log(x + 3/2))/472392 - log(x - 3/2)/157464 + x/(34992*(x^3 + 27/8)) - l
og(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/314928 - 1/314928) + log(x + (3
^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/314928 + 1/314928) - log(x - (3^(1/2)*3i
)/4 - 3/4)*((3^(1/2)*7i)/944784 + 7/944784) + log(x + (3^(1/2)*3i)/4 - 3/4
)*((3^(1/2)*7i)/944784 - 7/944784)
```

3.574 $\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2} dx$

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3.574.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2} dx = \frac{1}{26244(3-2x)} - \frac{3-2x}{26244(9-6x+4x^2)} - \frac{11\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{7\log(3-2x)}{157464} + \frac{\log(3+2x)}{472392} + \frac{17\log(9-6x+4x^2)}{944784} + \frac{\log(9+6x+4x^2)}{314928}$$

output
```
1/26244/(3-2*x)+1/26244*(-3+2*x)/(4*x^2-6*x+9)-7/157464*ln(3-2*x)+1/472392
*ln(3+2*x)+17/944784*ln(4*x^2-6*x+9)+1/314928*ln(4*x^2+6*x+9)-11/472392*ar
ctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)-1/472392*arctan(1/9*(3+4*x)*3^(1/2))*3^(
1/2)
```

3.574.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2} dx = \frac{\frac{216x}{27-36x+24x^2-8x^3} + 22\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right) - 42\log(3-2x) + 2\log(3+2x) + 17\log(}{944784}$$

input
```
Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

output
```
((216*x)/(27 - 36*x + 24*x^2 - 8*x^3) + 22*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sq
rt[3])] - 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 42*Log[3 - 2*x] + 2*Lo
g[3 + 2*x] + 17*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784
```

3.574.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{8x^3+24x^2+36x+27}{(729-64x^6)^2}dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{(-8x^3+24x^2-36x+27)^2(8x^3+24x^2+36x+27)}dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{x}{39366(4x^2+6x+9)}+\frac{17x+3}{118098(4x^2-6x+9)}+\frac{2x+3}{4374(4x^2-6x+9)^2}-\frac{7}{78732(2x-3)}+\frac{1}{236196(2x+3)}\right.$$
$$\Big\downarrow \text{2009}$$
$$-\frac{11\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{3-2x}{26244(4x^2-6x+9)}+\frac{17\log(4x^2-6x+9)}{944784}+\frac{\log(4x^2+6x+9)}{314928}+\frac{1}{26244(3-2x)}-\frac{7\log(3-2x)}{157464}+\frac{\log(2x+3)}{472392}$$

input
```
Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

output
```
1/(26244*(3 - 2*x)) - (3 - 2*x)/(26244*(9 - 6*x + 4*x^2)) - (11*ArcTan[(3
- 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(157
464*Sqrt[3]) - (7*Log[3 - 2*x])/157464 + Log[3 + 2*x]/472392 + (17*Log[9 -
 6*x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^2]/314928
```

3.574.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.574.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{x}{34992\left(x^{3}-3x^{2}+\frac{9}{2}x-\frac{27}{8}\right)}-\frac{7\ln\left(-3+2x\right)}{157464}+\frac{\ln\left(4x^{2}+6x+9\right)}{314928}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{472392}+\frac{17\ln\left(484x^{2}-726x+1089\right)}{944784}+$
default	$-\frac{1}{26244\left(-3+2x\right)}-\frac{7\ln\left(-3+2x\right)}{157464}+\frac{\frac{9x}{4}-\frac{27}{8}}{118098x^{2}-177147x+\frac{531441}{2}}+\frac{17\ln\left(4x^{2}-6x+9\right)}{944784}+\frac{11\sqrt{3}\,\arctan\left(\frac{\left(8x-6\right)\sqrt{3}}{18}\right)}{472392}+\frac{\ln(2x+}{47239}$
meijerg	$-\frac{\left(-1\right)^{\frac{5}{6}}\left(\frac{4x\left(-1\right)^{\frac{1}{6}}}{6-\frac{128x^{6}}{243}}-\frac{5x\left(-1\right)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right)}{3}\right.}{}\right.}{6\left(x^{6}\right)^{\frac{1}{6}}}\right.}{78732}$

input
```
int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/34992*x/(x^3-3*x^2+9/2*x-27/8)-7/157464*ln(-3+2*x)+1/314928*ln(4*x^2+6*
x+9)-1/472392*3^(1/2)*arctan(2/9*(2*x+3/2)*3^(1/2))+17/944784*ln(484*x^2-7
26*x+1089)+11/472392*3^(1/2)*arctan(2/99*(22*x-33/2)*3^(1/2))+1/472392*ln(
2*x+3)
```

3.574.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.43

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = -\frac{2\sqrt{3}(8x^3-24x^2+36x-27)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-22\sqrt{3}(8x^3-24x^2+36x-27)\arctan\left(\frac{1}{9}\sqrt{3}\right.}{}$$

input
```
integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
-1/944784*(2*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x
+ 3)) - 22*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x -
3)) - 3*(8*x^3 - 24*x^2 + 36*x - 27)*log(4*x^2 + 6*x + 9) - 17*(8*x^3 - 24
*x^2 + 36*x - 27)*log(4*x^2 - 6*x + 9) - 2*(8*x^3 - 24*x^2 + 36*x - 27)*lo
g(2*x + 3) + 42*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*x - 3) + 216*x)/(8*x^3
- 24*x^2 + 36*x - 27)
```

3.574.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = &-\frac{x}{34992x^3-104976x^2+157464x-118098}-\frac{7\log\left(x-\frac{3}{2}\right)}{157464}\\ &+\frac{\log\left(x+\frac{3}{2}\right)}{472392}+\frac{17\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928}\\ &+\frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{472392}\end{aligned}$$

input
```
integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729)**2,x)
```

output
```
-x/(34992*x**3 - 104976*x**2 + 157464*x - 118098) - 7*log(x - 3/2)/157464
+ log(x + 3/2)/472392 + 17*log(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x
/2 + 9/4)/314928 + 11*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 - sqr
t(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392
```

3.574.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = -\frac{1}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{11}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(8\,x^3-24\,x^2+36\,x-27)} + \frac{1}{314928}\,\log\left(4\,x^2+6\,x+9\right) + \frac{17}{944784}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{472392}\,\log\left(2\,x+3\right) - \frac{7}{157464}\,\log\left(2\,x-3\right)$$

input
```
integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
-1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan
(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(8*x^3 - 24*x^2 + 36*x - 27) + 1/314928
*log(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(2*x
+ 3) - 7/157464*log(2*x - 3)
```

3.574.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = -\frac{1}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{11}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(4\,x^2-6\,x+9)(2\,x-3)} + \frac{1}{314928}\,\log\left(4\,x^2+6\,x+9\right) + \frac{17}{944784}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{472392}\,\log\left(|2\,x+3|\right) - \frac{7}{157464}\,\log\left(|2\,x-3|\right)$$

input
```
integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
-1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan
(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x - 3)) + 1/31492
8*log(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(abs
(2*x + 3)) - 7/157464*log(abs(2*x - 3))
```

3.574.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{472392} - \frac{7\,\ln\left(x-\frac{3}{2}\right)}{157464} - \frac{x}{34992\,\left(x^3-3\,x^2+\frac{9\,x}{2}-\frac{27}{8}\right)} + \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) - \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{17}{944784}+\frac{\sqrt{3}\,11\mathrm{i}}{944784}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{17}{944784}+\frac{\sqrt{3}\,11\mathrm{i}}{944784}\right)$$

input
```
int((36*x + 24*x^2 + 8*x^3 + 27)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/472392 - (7*log(x - 3/2))/157464 - x/(34992*((9*x)/2 - 3*x^2
+ x^3 - 27/8)) + log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 + 1/31
4928) - log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 - 1/314928) - l
og(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*11i)/944784 - 17/944784) + log(x +
(3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*11i)/944784 + 17/944784)
```

3.575 $\int \frac{x(27-2x^3)}{729-64x^6} dx$

3.575.1 Optimal result 4380
3.575.2 Mathematica [A] (verified) 4380
3.575.3 Rubi [A] (verified) 4381
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3.575.1 Optimal result

Integrand size = 18, antiderivative size = 99

$$\int \frac{x(27-2x^3)}{729-64x^6} dx = -\frac{5\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{96\sqrt{3}} - \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{32\sqrt{3}} - \frac{1}{96}\log(3-2x) - \frac{5}{288}\log(3+2x) + \frac{5}{576}\log\left(9-6x+4x^2\right) + \frac{1}{192}\log\left(9+6x+4x^2\right)$$

output
```
-1/96*ln(3-2*x)-5/288*ln(3+2*x)+5/576*ln(4*x^2-6*x+9)+1/192*ln(4*x^2+6*x+9
)-5/288*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)-1/96*arctan(1/9*(3+4*x)*3^(1/2
))*3^(1/2)
```

3.575.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{x(27-2x^3)}{729-64x^6} dx = \frac{1}{576}\left(10\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) - 6\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right) - 6\log(3-2x) - 10\log(3+2x) + 5\log\left(9-6x+4x^2\right) + 3\log\left(9+6x+4x^2\right)\right)$$

input
```
Integrate[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```

output
```
(10*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*Sqrt[3]*ArcTan[(3 + 4*x)/(3
*Sqrt[3])] - 6*Log[3 - 2*x] - 10*Log[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3
*Log[9 + 6*x + 4*x^2])/576
```

3.575.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1835, 27, 821, 16, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(27-2x^3)}{729-64x^6}dx$$

$$\Big\downarrow \text{1835}$$

$$3\int \frac{x}{8(27-8x^3)}dx+5\int \frac{x}{8(8x^3+27)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{8}\int \frac{x}{27-8x^3}dx+\frac{5}{8}\int \frac{x}{8x^3+27}dx$$

$$\Big\downarrow \text{821}$$

$$\frac{5}{8}\left(\frac{1}{18}\int \frac{2x+3}{4x^2-6x+9}dx-\frac{1}{18}\int \frac{1}{2x+3}dx\right)+\frac{3}{8}\left(\frac{1}{18}\int \frac{1}{3-2x}dx-\frac{1}{18}\int \frac{3-2x}{4x^2+6x+9}dx\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{5}{8}\left(\frac{1}{18}\int \frac{2x+3}{4x^2-6x+9}dx-\frac{1}{36}\log(2x+3)\right)+\frac{3}{8}\left(-\frac{1}{18}\int \frac{3-2x}{4x^2+6x+9}dx-\frac{1}{36}\log(3-2x)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{5}{8}\left(\frac{1}{18}\left(\frac{9}{2}\int \frac{1}{4x^2-6x+9}dx+\frac{1}{4}\int -\frac{2(3-4x)}{4x^2-6x+9}dx\right)-\frac{1}{36}\log(2x+3)\right)+\frac{3}{8}\left(\frac{1}{18}\left(\frac{1}{4}\int \frac{2(4x+3)}{4x^2+6x+9}dx-\frac{9}{2}\int \frac{1}{4x^2+6x+9}dx\right)-\frac{1}{36}\log(3-2x)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{8}\left(\frac{1}{18}\left(\frac{9}{2}\int \frac{1}{4x^2-6x+9}dx-\frac{1}{2}\int \frac{3-4x}{4x^2-6x+9}dx\right)-\frac{1}{36}\log(2x+3)\right)+\frac{3}{8}\left(\frac{1}{18}\left(\frac{1}{2}\int \frac{4x+3}{4x^2+6x+9}dx-\frac{9}{2}\int \frac{1}{4x^2+6x+9}dx\right)-\frac{1}{36}\log(3-2x)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{5}{8}\left(\frac{1}{18}\left(-\frac{1}{2}\int \frac{3-4x}{4x^2-6x+9}dx-9\int \frac{1}{-(8x-6)^2-108}d(8x-6)\right)-\frac{1}{36}\log(2x+3)\right)+\frac{3}{8}\left(\frac{1}{18}\left(\frac{1}{2}\int \frac{4x+3}{4x^2+6x+9}dx+9\int \frac{1}{-(8x+6)^2-108}d(8x+6)\right)-\frac{1}{36}\log(3-2x)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{5}{8}\left(\frac{1}{18}\left(\frac{1}{2}\sqrt{3}\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)-\frac{1}{2}\int \frac{3-4x}{4x^2-6x+9}dx\right)-\frac{1}{36}\log(2x+3)\right)+\frac{3}{8}\left(\frac{1}{18}\left(\frac{1}{2}\int \frac{4x+3}{4x^2+6x+9}dx-\frac{1}{2}\sqrt{3}\arctan\left(\frac{8x+6}{6\sqrt{3}}\right)\right)-\frac{1}{36}\log(3-2x)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{5}{8}\left(\frac{1}{18}\left(\frac{1}{2}\sqrt{3}\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)+\frac{1}{4}\log\left(4x^2-6x+9\right)\right)-\frac{1}{36}\log(2x+3)\right)+\frac{3}{8}\left(\frac{1}{18}\left(\frac{1}{4}\log\left(4x^2+6x+9\right)-\frac{1}{2}\sqrt{3}\arctan\left(\frac{8x+6}{6\sqrt{3}}\right)\right)-\frac{1}{36}\log(3-2x)\right)$$

input
```
Int[(x*(27 - 2*x^3))/(729 - 64*x^6),x]
```

output
```
(5*(-1/36*Log[3 + 2*x] + ((Sqrt[3]*ArcTan[(-6 + 8*x)/(6*Sqrt[3])])/2 + Log
[9 - 6*x + 4*x^2]/4)/18))/8 + (3*(-1/36*Log[3 - 2*x] + (-1/2*(Sqrt[3]*ArcT
an[(6 + 8*x)/(6*Sqrt[3])]) + Log[9 + 6*x + 4*x^2]/4)/18))/8
```

3.575.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.575. $\int \frac{x(27-2x^3)}{729-64x^6}\,dx$

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1835
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[-(e/2 + c*(d/(2*q)))   Int[(f
*x)^m/(q - c*x^n), x], x] + Simp[(e/2 - c*(d/(2*q)))   Int[(f*x)^m/(q + c*x
^n), x], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.575.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.77

method	result
default	$-\frac{\ln(-3+2x)}{96}+\frac{5\ln\left(4x^2-6x+9\right)}{576}+\frac{5\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{288}-\frac{5\ln(2x+3)}{288}+\frac{\ln\left(4x^2+6x+9\right)}{192}-\frac{\sqrt{3}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{96}$
risch	$-\frac{\ln(-3+2x)}{96}-\frac{5\ln(2x+3)}{288}+\frac{5\ln\left(4x^2-6x+9\right)}{576}+\frac{5\sqrt{3}\arctan\left(\frac{2\left(-\frac{3}{2}+2x\right)\sqrt{3}}{9}\right)}{288}+\frac{\ln\left(4x^2+6x+9\right)}{192}-\frac{\sqrt{3}\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{}}{9}\right.}{96}$
meijerg	$\frac{x^5\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}+\sqrt{3}\,\text{arc}\right.}{288\left(x^6\right)^{\frac{5}{6}}}$

3.575. $\int \frac{x(27-2x^3)}{729-64x^6}\,dx$

input
```
int(x*(-2*x^3+27)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-1/96*ln(-3+2*x)+5/576*ln(4*x^2-6*x+9)+5/288*3^(1/2)*arctan(1/18*(8*x-6)*3
^(1/2))-5/288*ln(2*x+3)+1/192*ln(4*x^2+6*x+9)-1/96*3^(1/2)*arctan(1/18*(8*
x+6)*3^(1/2))
```

3.575.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right) + \frac{1}{192}\log\left(4\,x^2+6\,x+9\right)+\frac{5}{576}\log\left(4\,x^2-6\,x+9\right) - \frac{5}{288}\log\left(2\,x+3\right)-\frac{1}{96}\log\left(2\,x-3\right)$$

input
```
integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
-1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqr
t(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9)
- 5/288*log(2*x + 3) - 1/96*log(2*x - 3)
```

3.575.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{96}-\frac{5\log\left(x+\frac{3}{2}\right)}{288}+\frac{5\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{576} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{192}+\frac{5\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{288}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{96}$$

input
```
integrate(x*(-2*x**3+27)/(-64*x**6+729),x)
```

output
```
-log(x - 3/2)/96 - 5*log(x + 3/2)/288 + 5*log(x**2 - 3*x/2 + 9/4)/576 + lo
g(x**2 + 3*x/2 + 9/4)/192 + 5*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/288
- sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/96
```

3.575.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(4x^2+6x+9\right)+\frac{5}{576}\log\left(4x^2-6x+9\right)-\frac{5}{288}\log\left(2x+3\right)-\frac{1}{96}\log\left(2x-3\right)$$

input
```
integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
-1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqr
t(3)*(4*x - 3)) + 1/192*log(4*x^2 + 6*x + 9) + 5/576*log(4*x^2 - 6*x + 9)
- 5/288*log(2*x + 3) - 1/96*log(2*x - 3)
```

3.575.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{1}{96}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+\frac{5}{288}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{192}\log\left(x^2+\frac{3}{2}x+\frac{9}{4}\right)+\frac{5}{576}\log\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)-\frac{5}{288}\log\left(\left|x+\frac{3}{2}\right|\right)-\frac{1}{96}\log\left(\left|x-\frac{3}{2}\right|\right)$$

input
```
integrate(x*(-2*x^3+27)/(-64*x^6+729),x, algorithm="giac")
```

output
```
-1/96*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 5/288*sqrt(3)*arctan(1/9*sqr
t(3)*(4*x - 3)) + 1/192*log(x^2 + 3/2*x + 9/4) + 5/576*log(x^2 - 3/2*x + 9
/4) - 5/288*log(abs(x + 3/2)) - 1/96*log(abs(x - 3/2))
```

3.575.9 Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{x(27-2x^3)}{729-64x^6}\,dx = -\frac{\ln\left(x-\frac{3}{2}\right)}{96} - \frac{5\,\ln\left(x+\frac{3}{2}\right)}{288} + \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{192}+\frac{\sqrt{3}\,1\mathrm{i}}{192}\right) - \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{192}+\frac{\sqrt{3}\,1\mathrm{i}}{192}\right) - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{576}+\frac{\sqrt{3}\,5\mathrm{i}}{576}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{576}+\frac{\sqrt{3}\,5\mathrm{i}}{576}\right)$$

input
```
int((x*(2*x^3 - 27))/(64*x^6 - 729),x)
```

output
```
log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/192 + 1/192) - (5*log(x + 3/2)
)/288 - log(x - 3/2)/96 - log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/192
- 1/192) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*5i)/576 - 5/576) + log(
x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*5i)/576 + 5/576)
```

3.576 $\int \frac{(cx)^m\left(d+ex^n+fx^{2n}+gx^{3n}\right)}{a+bx^n}\,dx$

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3.576.1 Optimal result

Integrand size = 36, antiderivative size = 162

$$\begin{aligned}\int \frac{(cx)^m\left(d+ex^n+fx^{2n}+gx^{3n}\right)}{a+bx^n}\,dx &= \frac{(bf-ag)x^{1+n}(cx)^m}{b^2(1+m+n)}+\frac{gx^{1+2n}(cx)^m}{b(1+m+2n)}+\frac{\left(b^2e-abf+a^2g\right)(cx)^{1+m}}{b^3c(1+m)}\\&\quad+\frac{\left(b^3d-ab^2e+a^2bf-a^3g\right)(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{ab^3c(1+m)}\end{aligned}$$

output
```
(-a*g+b*f)*x^(1+n)*(c*x)^m/b^2/(1+m+n)+g*x^(1+2*n)*(c*x)^m/b/(1+m+2*n)+(a^
2*g-a*b*f+b^2*e)*(c*x)^(1+m)/b^3/c/(1+m)+(-a^3*g+a^2*b*f-a*b^2*e+b^3*d)*(c
*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/b^3/c/(1+m)
```

3.576.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.80

$$\int \frac{(cx)^m\left(d+ex^n+fx^{2n}+gx^{3n}\right)}{a+bx^n}\,dx = \frac{x(cx)^m\left(\frac{b^2e-abf+a^2g}{1+m}+\frac{b(bf-ag)x^n}{1+m+n}+\frac{b^2gx^{2n}}{1+m+2n}+\frac{\left(b^3d-ab^2e+a^2bf-a^3g\right)\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(1+m)}\right)}{b^3}$$

input
```
Integrate[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x]
```

output
```
(x*(c*x)^m*((b^2*e - a*b*f + a^2*g)/(1 + m) + (b*(b*f - a*g)*x^n)/(1 + m +
 n) + (b^2*g*x^(2*n))/(1 + m + 2*n) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)
*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*(1 + m))
))/b^3
```

3.576.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m \left(d+ex^n+fx^{2n}+gx^{3n}\right)}{a+bx^n} \, dx$$

$$\Big\downarrow \text{2383}$$

$$\int \left(\frac{(cx)^m \left(a^2g-abf+b^2e\right)}{b^3}+\frac{(cx)^m \left(a^3(-g)+a^2bf-ab^2e+b^3d\right)}{b^3 \left(a+bx^n\right)}+\frac{x^n(cx)^m(bf-ag)}{b^2}+\frac{gx^{2n}(cx)^m}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cx)^{m+1} \left(a^2g-abf+b^2e\right)}{b^3c(m+1)}+\frac{(cx)^{m+1} \left(a^3(-g)+a^2bf-ab^2e+b^3d\right) \text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{ab^3c(m+1)}+\frac{x^{n+1}(cx)^m(bf-ag)}{b^2(m+n+1)}+\frac{gx^{2n+1}(cx)^m}{b(m+2n+1)}$$

input
```
Int[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x]
```

output
```
((b*f - a*g)*x^(1 + n)*(c*x)^m)/(b^2*(1 + m + n)) + (g*x^(1 + 2*n)*(c*x)^m
)/(b*(1 + m + 2*n)) + ((b^2*e - a*b*f + a^2*g)*(c*x)^(1 + m))/(b^3*c*(1 +
m)) + ((b^3*d - a*b^2*e + a^2*b*f - a^3*g)*(c*x)^(1 + m)*Hypergeometric2F1
[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b^3*c*(1 + m))
```

3.576.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2383
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

3.576.4 Maple [F]

$$\int \frac{(cx)^m\,(d+e\,x^n+f\,x^{2n}+g\,x^{3n})}{a+b\,x^n}dx$$

input
```
int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x)
```

output
```
int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x)
```

3.576.5 Fricas [F]

$$\int \frac{(cx)^m\,(d+ex^n+fx^{2n}+gx^{3n})}{a+bx^n}\,dx = \int \frac{(gx^{3\,n}+fx^{2\,n}+ex^n+d)(cx)^m}{bx^n+a}\,dx$$

input
```
integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="fr
icas")
```

output
```
integral((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a), x)
```

3.576.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.50 (sec) , antiderivative size = 860, normalized size of antiderivative = 5.31

$$\int \frac{(cx)^m\,(d+ex^n+fx^{2n}+gx^{3n})}{a+bx^n}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n),x)
```

output
```
a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*c**m*d*m*x**(m + 1)*lerchphi(b*x**n*exp
_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 + 1/n))
 + a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*c**m*d*x**(m + 1)*lerchphi(b*x**n*ex
p_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 + 1/n)
) + a**(-m/n - 4 - 1/n)*a**(m/n + 3 + 1/n)*c**m*g*m*x**(m + 3*n + 1)*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 3 + 1/n)*gamma(m/n + 3 + 1/n)/(n**2
*gamma(m/n + 4 + 1/n)) + 3*a**(-m/n - 4 - 1/n)*a**(m/n + 3 + 1/n)*c**m*g*x
**(m + 3*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 3 + 1/n)*gamma
(m/n + 3 + 1/n)/(n*gamma(m/n + 4 + 1/n)) + a**(-m/n - 4 - 1/n)*a**(m/n + 3
 + 1/n)*c**m*g*x**(m + 3*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n
+ 3 + 1/n)*gamma(m/n + 3 + 1/n)/(n**2*gamma(m/n + 4 + 1/n)) + a**(-m/n - 3
 - 1/n)*a**(m/n + 2 + 1/n)*c**m*f*m*x**(m + 2*n + 1)*lerchphi(b*x**n*exp_p
olar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n**2*gamma(m/n + 3 +
 1/n)) + 2*a**(-m/n - 3 - 1/n)*a**(m/n + 2 + 1/n)*c**m*f*x**(m + 2*n + 1)*
lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/
(n*gamma(m/n + 3 + 1/n)) + a**(-m/n - 3 - 1/n)*a**(m/n + 2 + 1/n)*c**m*f*x
**(m + 2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma
(m/n + 2 + 1/n)/(n**2*gamma(m/n + 3 + 1/n)) + a**(-m/n - 2 - 1/n)*a**(m/n
+ 1 + 1/n)*c**m*e*m*x**(m + n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m
/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n**2*gamma(m/n + 2 + 1/n)) + a**(-m...
```

3.576.7 Maxima [F]

$$\int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{a + bx^n} \, dx = \int \frac{(gx^{3n} + fx^{2n} + ex^n + d)(cx)^m}{bx^n + a} \, dx$$

input
```
integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="ma
xima")
```

output
```
(b^3*c^m*d - a*b^2*c^m*e + a^2*b*c^m*f - a^3*c^m*g)*integrate(x^m/(b^4*x^n
 + a*b^3), x) + ((m^2 + m*(n + 2) + n + 1)*b^2*c^m*g*x*e^(m*log(x) + 2*n*l
og(x)) + ((m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*b^2*c^m*e - (m^2 + m*(3*n
+ 2) + 2*n^2 + 3*n + 1)*a*b*c^m*f + (m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*
a^2*c^m*g)*x*x^m + ((m^2 + 2*m*(n + 1) + 2*n + 1)*b^2*c^m*f - (m^2 + 2*m*(
n + 1) + 2*n + 1)*a*b*c^m*g)*x*e^(m*log(x) + n*log(x)))/((m^3 + 3*m^2*(n +
 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n + 1)*b^3)
```

3.576.8 Giac [F]

$$\int \frac{(cx)^m (d+ex^n+fx^{2n}+gx^{3n})}{a+bx^n}\,dx = \int \frac{(gx^{3n}+fx^{2n}+ex^n+d)(cx)^m}{bx^n+a}\,dx$$

input
```
integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n),x, algorithm="gi
ac")
```

output
```
integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/(b*x^n + a), x)
```

3.576.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (d+ex^n+fx^{2n}+gx^{3n})}{a+bx^n}\,dx = \int \frac{(c\,x)^m\,(d+e\,x^n+f\,x^{2\,n}+g\,x^{3\,n})}{a+b\,x^n}\,dx$$

input
```
int(((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n),x)
```

output
```
int(((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n), x)
```

3.577 $\int (c+dx^{-1+n}) (a+bx^n)^3 \, dx$

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3.577.1 Optimal result

Integrand size = 19, antiderivative size = 84

$$\int (c+dx^{-1+n}) (a+bx^n)^3 \, dx = a^3cx + \frac{3a^2bcx^{1+n}}{1+n} + \frac{3ab^2cx^{1+2n}}{1+2n} + \frac{b^3cx^{1+3n}}{1+3n} + \frac{d(a+bx^n)^4}{4bn}$$

output
```
a^3*c*x+3*a^2*b*c*x^(1+n)/(1+n)+3*a*b^2*c*x^(1+2*n)/(1+2*n)+b^3*c*x^(1+3*n
)/(1+3*n)+1/4*d*(a+b*x^n)^4/b/n
```

3.577.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int (c+dx^{-1+n}) (a+bx^n)^3 \, dx = \frac{x(c+dx^{-1+n}) \left(4a^3cx + \frac{12a^2bcx^{1+n}}{1+n} + \frac{12ab^2cx^{1+2n}}{1+2n} + \frac{4b^3cx^{1+3n}}{1+3n} + \frac{d(a+bx^n)^4}{bn}\right)}{4(cx+dx^n)}$$

input
```
Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

output
```
(x*(c + d*x^(-1 + n))*(4*a^3*c*x + (12*a^2*b*c*x^(1 + n))/(1 + n) + (12*a*
b^2*c*x^(1 + 2*n))/(1 + 2*n) + (4*b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b
*x^n)^4)/(b*n)))/(4*(c*x + d*x^n))
```

3.577.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2430, 775, 793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^3 \left(c+dx^{n-1}\right) \, dx$$

$$\Big\downarrow \text{ 2430}$$

$$c\int (bx^n+a)^3dx+d\int x^{n-1}(bx^n+a)^3dx$$

$$\Big\downarrow \text{ 775}$$

$$c\int \left(3a^2bx^n+3ab^2x^{2n}+b^3x^{3n}+a^3\right)dx+d\int x^{n-1}(bx^n+a)^3dx$$

$$\Big\downarrow \text{ 793}$$

$$c\int \left(3a^2bx^n+3ab^2x^{2n}+b^3x^{3n}+a^3\right)dx+\frac{d(a+bx^n)^4}{4bn}$$

$$\Big\downarrow \text{ 2009}$$

$$c\left(a^3x+\frac{3a^2bx^{n+1}}{n+1}+\frac{3ab^2x^{2n+1}}{2n+1}+\frac{b^3x^{3n+1}}{3n+1}\right)+\frac{d(a+bx^n)^4}{4bn}$$

input
```
Int[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

output
```
(d*(a + b*x^n)^4)/(4*b*n) + c*(a^3*x + (3*a^2*b*x^(1 + n))/(1 + n) + (3*a*
b^2*x^(1 + 2*n))/(1 + 2*n) + (b^3*x^(1 + 3*n))/(1 + 3*n))
```

3.577.3.1 Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2430
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Simp[A   Int[(a + b*x^n)^p, x], x] + Simp[B   Int[x^m*(a + b*x^n)^p, x], x]
 /; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

3.577.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

method	result
risch	$a^3cx+\frac{b^3d\,x^{4n}}{4n}+\frac{b^2(nbcx+3adn+ad)x^{3n}}{n(1+3n)}+\frac{3ab(2nbcx+2adn+ad)x^{2n}}{2n(1+2n)}+\frac{a^2(3nbcx+adn+ad)x^n}{n(1+n)}$
norman	$a^3cx+\frac{a^3d\,\mathrm{e}^{n\ln(x)}}{n}+\frac{a\,b^2d\,\mathrm{e}^{3n\ln(x)}}{n}+\frac{b^3cx\,\mathrm{e}^{3n\ln(x)}}{1+3n}+\frac{b^3d\,\mathrm{e}^{4n\ln(x)}}{4n}+\frac{3d\,a^2b\,\mathrm{e}^{2n\ln(x)}}{2n}+\frac{3a\,b^2cx\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{3a^2bc}{1}$
parallelrisch	$\frac{6x\,x^nx^{-1+n}a^2bd+4a^3cxn+66x\,x^nx^{-1+n}a^2bd\,n^2+36x\,x^nx^{-1+n}a^2bdn+36x^{2n}a\,b^2c\,n^3x+6x\,x^{3n}x^{-1+n}b^3d\,n^3+11x\,x^{3n}x^{-1+}}{}$

input
```
int((c+d*x^(-1+n))*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*c*x+1/4*b^3*d/n*(x^n)^4+b^2*(b*c*n*x+3*a*d*n+a*d)/n/(1+3*n)*(x^n)^3+3/
2*a*b*(2*b*c*n*x+2*a*d*n+a*d)/n/(1+2*n)*(x^n)^2+a^2*(3*b*c*n*x+a*d*n+a*d)/
n/(1+n)*x^n
```

3.577.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(82) = 164.

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.63

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)^3\,dx$$
$$=\frac{4\left(6\,a^3cn^4+11\,a^3cn^3+6\,a^3cn^2+a^3cn\right)x+\left(6\,b^3dn^3+11\,b^3dn^2+6\,b^3dn+b^3d\right)x^{4\,n}+4\left(6\,ab^2dn^3+11\,ab^2\right.}{}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/4*(4*(6*a^3*c*n^4 + 11*a^3*c*n^3 + 6*a^3*c*n^2 + a^3*c*n)*x + (6*b^3*d*n
^3 + 11*b^3*d*n^2 + 6*b^3*d*n + b^3*d)*x^(4*n) + 4*(6*a*b^2*d*n^3 + 11*a*b
^2*d*n^2 + 6*a*b^2*d*n + a*b^2*d + (2*b^3*c*n^3 + 3*b^3*c*n^2 + b^3*c*n)*x
)*x^(3*n) + 6*(6*a^2*b*d*n^3 + 11*a^2*b*d*n^2 + 6*a^2*b*d*n + a^2*b*d + 2*
(3*a*b^2*c*n^3 + 4*a*b^2*c*n^2 + a*b^2*c*n)*x)*x^(2*n) + 4*(6*a^3*d*n^3 +
11*a^3*d*n^2 + 6*a^3*d*n + a^3*d + 3*(6*a^2*b*c*n^3 + 5*a^2*b*c*n^2 + a^2*
b*c*n)*x)*x^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

3.577.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1340 vs. $2(75) = 150$.

Time = 0.78 (sec) , antiderivative size = 1340, normalized size of antiderivative = 15.95

$$\int \left(c + dx^{-1+n}\right) (a + bx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((c+d*x**(-1+n))*(a+b*x**n)**3,x)
```

output
```
Piecewise((a**3*c*x - a**3*d/x + 3*a**2*b*c*log(x) - 3*a**2*b*d/(2*x**2) -
 3*a*b**2*c/x - a*b**2*d/x**3 - b**3*c/(2*x**2) - b**3*d/(4*x**4), Eq(n, -
1)), (a**3*c*x - 2*a**3*d/sqrt(x) + 6*a**2*b*c*sqrt(x) - 3*a**2*b*d/x + 3*
a*b**2*c*log(x) - 2*a*b**2*d/x**(3/2) - 2*b**3*c/sqrt(x) - b**3*d/(2*x**2)
, Eq(n, -1/2)), (a**3*c*x - 3*a**3*d/x**(1/3) + 9*a**2*b*c*x**(2/3)/2 - 9*
a**2*b*d/(2*x**(2/3)) + 9*a*b**2*c*x**(1/3) - 3*a*b**2*d/x + b**3*c*log(x)
 - 3*b**3*d/(4*x**(4/3)), Eq(n, -1/3)), ((a + b)**3*(c*x + d*log(x)), Eq(n
, 0)), (24*a**3*c*n**4*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**3*c*n
**3*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*c*n**2*x/(24*n**4 + 44
*n**3 + 24*n**2 + 4*n) + 4*a**3*c*n*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n)
+ 24*a**3*d*n**3*x*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**
3*d*n**2*x*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*d*n*x*
x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 4*a**3*d*x*x**(n - 1)/(24
*n**4 + 44*n**3 + 24*n**2 + 4*n) + 72*a**2*b*c*n**3*x*x**n/(24*n**4 + 44*n
**3 + 24*n**2 + 4*n) + 60*a**2*b*c*n**2*x*x**n/(24*n**4 + 44*n**3 + 24*n**
2 + 4*n) + 12*a**2*b*c*n*x*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a
**2*b*d*n**3*x*x**n*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 66*a*
*2*b*d*n**2*x*x**n*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a**
2*b*d*n*x*x**n*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*a**2*b*d
*x*x**n*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a*b**2*c*n*...
```

3.577.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int (c+dx^{-1+n}) (a+bx^n)^3 \, dx = a^3cx + \frac{b^3dx^{4n}}{4n} + \frac{ab^2dx^{3n}}{n} + \frac{3a^2bdx^{2n}}{2n} + \frac{b^3cx^{3n+1}}{3n+1} + \frac{3ab^2cx^{2n+1}}{2n+1} + \frac{3a^2bcx^{n+1}}{n+1} + \frac{a^3dx^n}{n}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
a^3*c*x + 1/4*b^3*d*x^(4*n)/n + a*b^2*d*x^(3*n)/n + 3/2*a^2*b*d*x^(2*n)/n
+ b^3*c*x^(3*n + 1)/(3*n + 1) + 3*a*b^2*c*x^(2*n + 1)/(2*n + 1) + 3*a^2*b*
c*x^(n + 1)/(n + 1) + a^3*d*x^n/n
```

3.577.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(82) = 164$.

Time = 0.29 (sec) , antiderivative size = 392, normalized size of antiderivative = 4.67

$$\int \left(c + dx^{-1+n}\right)\left(a + bx^n\right)^3 \, dx$$
$$= \frac{24\,a^3cn^4x + 8\,b^3cn^3xx^{3\,n} + 36\,ab^2cn^3xx^{2\,n} + 72\,a^2bcn^3xx^n + 44\,a^3cn^3x + 6\,b^3dn^3x^{4\,n} + 24\,ab^2dn^3x^{3\,n} + 12}{}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
1/4*(24*a^3*c*n^4*x + 8*b^3*c*n^3*x*x^(3*n) + 36*a*b^2*c*n^3*x*x^(2*n) + 7
2*a^2*b*c*n^3*x*x^n + 44*a^3*c*n^3*x + 6*b^3*d*n^3*x^(4*n) + 24*a*b^2*d*n^
3*x^(3*n) + 12*b^3*c*n^2*x*x^(3*n) + 36*a^2*b*d*n^3*x^(2*n) + 48*a*b^2*c*n
^2*x*x^(2*n) + 24*a^3*d*n^3*x^n + 60*a^2*b*c*n^2*x*x^n + 24*a^3*c*n^2*x +
11*b^3*d*n^2*x^(4*n) + 44*a*b^2*d*n^2*x^(3*n) + 4*b^3*c*n*x*x^(3*n) + 66*a
^2*b*d*n^2*x^(2*n) + 12*a*b^2*c*n*x*x^(2*n) + 44*a^3*d*n^2*x^n + 12*a^2*b*
c*n*x*x^n + 4*a^3*c*n*x + 6*b^3*d*n*x^(4*n) + 24*a*b^2*d*n*x^(3*n) + 36*a^
2*b*d*n*x^(2*n) + 24*a^3*d*n*x^n + b^3*d*x^(4*n) + 4*a*b^2*d*x^(3*n) + 6*a
^2*b*d*x^(2*n) + 4*a^3*d*x^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

3.577.9 Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.37

$$\int \left(c + dx^{-1+n}\right)\left(a + bx^n\right)^3 \, dx = a^3\,c\,x + \frac{a^3\,d\,x^n}{n} + \frac{b^3\,d\,x^{4\,n}}{4\,n} + \frac{b^3\,c\,x\,x^{3\,n}}{3\,n+1} + \frac{3\,a^2\,b\,d\,x^{2\,n}}{2\,n} + \frac{a\,b^2\,d\,x^{3\,n}}{n} + \frac{3\,a\,b^2\,c\,x\,x^{2\,n}}{2\,n+1} + \frac{3\,a^2\,b\,c\,x\,x^n}{n+1}$$

input
```
int((c + d*x^(n - 1))*(a + b*x^n)^3,x)
```

output
```
a^3*c*x + (a^3*d*x^n)/n + (b^3*d*x^(4*n))/(4*n) + (b^3*c*x*x^(3*n))/(3*n +
 1) + (3*a^2*b*d*x^(2*n))/(2*n) + (a*b^2*d*x^(3*n))/n + (3*a*b^2*c*x*x^(2*
n))/(2*n + 1) + (3*a^2*b*c*x*x^n)/(n + 1)
```

3.578 $\int \left(c+dx^{-1+n}\right) (a+bx^n)^2 \, dx$

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3.578.1 Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \left(c+dx^{-1+n}\right) (a+bx^n)^2 \, dx = a^2cx+\frac{2abcx^{1+n}}{1+n}+\frac{b^2cx^{1+2n}}{1+2n}+\frac{d(a+bx^n)^3}{3bn}$$

output
```
a^2*c*x+2*a*b*c*x^(1+n)/(1+n)+b^2*c*x^(1+2*n)/(1+2*n)+1/3*d*(a+b*x^n)^3/b/
n
```

3.578.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.97

$$\int \left(c+dx^{-1+n}\right) (a+bx^n)^2 \, dx = \frac{a^3d(1+3n+2n^2)+3a^2b(1+3n+2n^2)(cnx+dx^n)+3ab^2(1+2n)x^n(2cnx+d(1+n)x^n)+b^3(1+n)x}{3bn(1+n)(1+2n)}$$

input
```
Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
```

output
```
(a^3*d*(1 + 3*n + 2*n^2) + 3*a^2*b*(1 + 3*n + 2*n^2)*(c*n*x + d*x^n) + 3*a
*b^2*(1 + 2*n)*x^n*(2*c*n*x + d*(1 + n)*x^n) + b^3*(1 + n)*x^(2*n)*(3*c*n*
x + d*(1 + 2*n)*x^n))/(3*b*n*(1 + n)*(1 + 2*n))
```

3.578.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2430, 775, 793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 \left(c+dx^{n-1}\right) \, dx$$

$$\Big\downarrow \text{2430}$$

$$c\int (bx^n+a)^2dx+d\int x^{n-1}(bx^n+a)^2dx$$

$$\Big\downarrow \text{775}$$

$$c\int \left(2abx^n+b^2x^{2n}+a^2\right)dx+d\int x^{n-1}(bx^n+a)^2dx$$

$$\Big\downarrow \text{793}$$

$$c\int \left(2abx^n+b^2x^{2n}+a^2\right)dx+\frac{d(a+bx^n)^3}{3bn}$$

$$\Big\downarrow \text{2009}$$

$$c\left(a^2x+\frac{2abx^{n+1}}{n+1}+\frac{b^2x^{2n+1}}{2n+1}\right)+\frac{d(a+bx^n)^3}{3bn}$$

input
```
Int[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
```

output
```
(d*(a + b*x^n)^3)/(3*b*n) + c*(a^2*x + (2*a*b*x^(1 + n))/(1 + n) + (b^2*x^
(1 + 2*n))/(1 + 2*n))
```

3.578.3.1 Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2430
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Simp[A   Int[(a + b*x^n)^p, x], x] + Simp[B   Int[x^m*(a + b*x^n)^p, x], x]
 /; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

3.578.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

method	result
risch	$a^2cx+\frac{b^2d\,x^{3n}}{3n}+\frac{b(nbcx+2adn+ad)x^{2n}}{n(1+2n)}+\frac{a(2nbcx+adn+ad)x^n}{n(1+n)}$
norman	$a^2cx+\frac{a^2d\,\mathrm{e}^{n\ln(x)}}{n}+\frac{abd\,\mathrm{e}^{2n\ln(x)}}{n}+\frac{b^2cx\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{b^2d\,\mathrm{e}^{3n\ln(x)}}{3n}+\frac{2abcx\,\mathrm{e}^{n\ln(x)}}{1+n}$
parallelrisch	$\frac{2x\,x^{2n}x^{-1+n}b^2d\,n^2+3x\,x^{2n}x^{-1+n}b^2dn+3x^{2n}b^2c\,n^2x+6x\,x^nx^{-1+n}abd\,n^2+x\,x^{2n}x^{-1+n}b^2d+3x^{2n}b^2cnx+9x\,x^nx^{-1+n}abdn-}{3n(1+2n}$

input
```
int((c+d*x^(-1+n))*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*c*x+1/3*b^2*d/n*(x^n)^3+b*(b*c*n*x+2*a*d*n+a*d)/n/(1+2*n)*(x^n)^2+a*(2
*b*c*n*x+a*d*n+a*d)/n/(1+n)*x^n
```

3.578.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(59) = 118.

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.62

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)^2\,dx$$
$$=\frac{3\left(2\,a^2cn^3+3\,a^2cn^2+a^2cn\right)x+\left(2\,b^2dn^2+3\,b^2dn+b^2d\right)x^{3\,n}+3\left(2\,abdn^2+3\,abdn+abd+\left(b^2cn^2+b^2cn\right.\right.}{3\left(2\,n^3+3\,n^2+n\right)}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/3*(3*(2*a^2*c*n^3 + 3*a^2*c*n^2 + a^2*c*n)*x + (2*b^2*d*n^2 + 3*b^2*d*n
+ b^2*d)*x^(3*n) + 3*(2*a*b*d*n^2 + 3*a*b*d*n + a*b*d + (b^2*c*n^2 + b^2*c
*n)*x)*x^(2*n) + 3*(2*a^2*d*n^2 + 3*a^2*d*n + a^2*d + 2*(2*a*b*c*n^2 + a*b
*c*n)*x)*x^n)/(2*n^3 + 3*n^2 + n)
```

3.578.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(53) = 106$.

Time = 0.48 (sec) , antiderivative size = 598, normalized size of antiderivative = 9.80

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^2\,dx$$
$$= \begin{cases} a^2cx - \frac{a^2d}{x} + 2abc\log(x) - \frac{abd}{x^2} - \frac{b^2c}{x} - \frac{b^2d}{3x^3} \\ a^2cx - \frac{2a^2d}{\sqrt{x}} + 4abc\sqrt{x} - \frac{2abd}{x} + b^2c\log(x) - \frac{2b^2d}{3x^{\frac{3}{2}}} \\ (a+b)^2\,(cx+d\log(x)) \\ \frac{6a^2cn^3x}{6n^3+9n^2+3n} + \frac{9a^2cn^2x}{6n^3+9n^2+3n} + \frac{3a^2cnx}{6n^3+9n^2+3n} + \frac{6a^2dn^2xx^{n-1}}{6n^3+9n^2+3n} + \frac{9a^2dnxx^{n-1}}{6n^3+9n^2+3n} + \frac{3a^2dxx^{n-1}}{6n^3+9n^2+3n} + \frac{12abcn^2xx^n}{6n^3+9n^2+3n} + \frac{6abcnxx^n}{6n^3+9n^2+3} \end{cases}$$

input
```
integrate((c+d*x**(-1+n))*(a+b*x**n)**2,x)
```

output
```
Piecewise((a**2*c*x - a**2*d/x + 2*a*b*c*log(x) - a*b*d/x**2 - b**2*c/x -
b**2*d/(3*x**3), Eq(n, -1)), (a**2*c*x - 2*a**2*d/sqrt(x) + 4*a*b*c*sqrt(x
) - 2*a*b*d/x + b**2*c*log(x) - 2*b**2*d/(3*x**(3/2)), Eq(n, -1/2)), ((a +
 b)**2*(c*x + d*log(x)), Eq(n, 0)), (6*a**2*c*n**3*x/(6*n**3 + 9*n**2 + 3*
n) + 9*a**2*c*n**2*x/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*c*n*x/(6*n**3 + 9*n*
*2 + 3*n) + 6*a**2*d*n**2*x*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 9*a**2*d*
n*x*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*d*x*x**(n - 1)/(6*n**3 + 9
*n**2 + 3*n) + 12*a*b*c*n**2*x*x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*c*n*x*
x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*d*n**2*x*x**n*x**(n - 1)/(6*n**3 + 9*
n**2 + 3*n) + 9*a*b*d*n*x*x**n*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 3*a*b*
d*x*x**n*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*c*n**2*x*x**(2*n)/(6*
n**3 + 9*n**2 + 3*n) + 3*b**2*c*n*x*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 2*b
**2*d*n**2*x*x**(2*n)*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*d*n*x*x*
*(2*n)*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + b**2*d*x*x**(2*n)*x**(n - 1)/(
6*n**3 + 9*n**2 + 3*n), True))
```

3.578.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^2\,dx = a^2cx + \frac{b^2dx^{3n}}{3n} + \frac{abdx^{2n}}{n} + \frac{b^2cx^{2n+1}}{2n+1} + \frac{2abcx^{n+1}}{n+1} + \frac{a^2dx^n}{n}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
a^2*c*x + 1/3*b^2*d*x^(3*n)/n + a*b*d*x^(2*n)/n + b^2*c*x^(2*n + 1)/(2*n +
 1) + 2*a*b*c*x^(n + 1)/(n + 1) + a^2*d*x^n/n
```

3.578.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(59) = 118.

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.21

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^2\,dx$$
$$= \frac{6a^2cn^3x + 3b^2cn^2xx^{2n} + 12abcn^2xx^n + 9a^2cn^2x + 2b^2dn^2x^{3n} + 6abdn^2x^{2n} + 3b^2cnxx^{2n} + 6a^2dn^2x^n \cdots}{3(2n^3 + 3n^2 + \cdots}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
1/3*(6*a^2*c*n^3*x + 3*b^2*c*n^2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 9*a^2*c*
n^2*x + 2*b^2*d*n^2*x^(3*n) + 6*a*b*d*n^2*x^(2*n) + 3*b^2*c*n*x*x^(2*n) +
6*a^2*d*n^2*x^n + 6*a*b*c*n*x*x^n + 3*a^2*c*n*x + 3*b^2*d*n*x^(3*n) + 9*a*
b*d*n*x^(2*n) + 9*a^2*d*n*x^n + b^2*d*x^(3*n) + 3*a*b*d*x^(2*n) + 3*a^2*d*
x^n)/(2*n^3 + 3*n^2 + n)
```

3.578.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^2\,dx = a^2\,c\,x + \frac{a^2\,d\,x^n}{n} + \frac{b^2\,d\,x^{3n}}{3n} + \frac{b^2\,c\,x\,x^{2n}}{2n+1} + \frac{a\,b\,d\,x^{2n}}{n} + \frac{2\,a\,b\,c\,x\,x^n}{n+1}$$

input
```
int((c + d*x^(n - 1))*(a + b*x^n)^2,x)
```

output
```
a^2*c*x + (a^2*d*x^n)/n + (b^2*d*x^(3*n))/(3*n) + (b^2*c*x*x^(2*n))/(2*n +
 1) + (a*b*d*x^(2*n))/n + (2*a*b*c*x*x^n)/(n + 1)
```

3.578. $\int \left(c+dx^{-1+n}\right) (a+bx^n)^2 \, dx$

3.579 $\int (c+dx^{-1+n}) (a+bx^n) \, dx$

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3.579.9 Mupad [B] (verification not implemented) 4408

3.579.1 Optimal result

Integrand size = 17, antiderivative size = 41

$$\int (c+dx^{-1+n}) (a+bx^n) \, dx = acx+\frac{adx^n}{n}+\frac{bdx^{2n}}{2n}+\frac{bcx^{1+n}}{1+n}$$

output
```
a*c*x+a*d*x^n/n+1/2*b*d*x^(2*n)/n+b*c*x^(1+n)/(1+n)
```

3.579.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int (c+dx^{-1+n}) (a+bx^n) \, dx = \frac{2a(cnx+dx^n)+bx^n\left(\frac{2cnx}{1+n}+dx^n\right)}{2n}$$

input
```
Integrate[(c + d*x^(-1 + n))*(a + b*x^n),x]
```

output
```
(2*a*(c*n*x + d*x^n) + b*x^n*((2*c*n*x)/(1 + n) + d*x^n))/(2*n)
```

3.579.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2430, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)\left(c+dx^{n-1}\right)\,dx$$

$$\Big\downarrow \ 2430$$

$$c\int (bx^n+a)\,dx+d\int x^{n-1}(bx^n+a)\,dx$$

$$\Big\downarrow \ 802$$

$$c\int (bx^n+a)\,dx+d\int \left(ax^{n-1}+bx^{2n-1}\right)dx$$

$$\Big\downarrow \ 2009$$

$$c\left(ax+\frac{bx^{n+1}}{n+1}\right)+d\left(\frac{ax^n}{n}+\frac{bx^{2n}}{2n}\right)$$

input
```
Int[(c + d*x^(-1 + n))*(a + b*x^n),x]
```

output
```
d*((a*x^n)/n + (b*x^(2*n))/(2*n)) + c*(a*x + (b*x^(1 + n))/(1 + n))
```

3.579.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2430
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Simp[A   Int[(a + b*x^n)^p, x], x] + Simp[B   Int[x^m*(a + b*x^n)^p, x], x]
 /; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

3.579.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result	size
risch	$acx+\frac{bd\,x^{2n}}{2n}+\frac{(nbcx+adn+ad)x^n}{n(1+n)}$	43
norman	$acx+\frac{ad\,\mathrm{e}^{n\ln(x)}}{n}+\frac{bcx\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{bd\,\mathrm{e}^{2n\ln(x)}}{2n}$	45
parallelrisch	$\frac{x\,x^n x^{-1+n}bdn+x\,x^n x^{-1+n}bd+2x^n bcnx+2x\,x^{-1+n}adn+2acx\,n^2+2x\,x^{-1+n}ad+2acxn}{2n(1+n)}$	81

input
```
int((c+d*x^(-1+n))*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/2*b*d/n*(x^n)^2+(b*c*n*x+a*d*n+a*d)/n/(1+n)*x^n
```

3.579.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx=\frac{2\,(acn^2+acn)x+(bdn+bd)x^{2\,n}+2\,(bcnx+adn+ad)x^n}{2\,(n^2+n)}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(2*(a*c*n^2 + a*c*n)*x + (b*d*n + b*d)*x^(2*n) + 2*(b*c*n*x + a*d*n +
a*d)*x^n)/(n^2 + n)
```

3.579.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(36) = 72.

Time = 0.30 (sec) , antiderivative size = 180, normalized size of antiderivative = 4.39

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx$$
$$=\begin{cases} acx-\frac{ad}{x}+bc\log(x)-\frac{bd}{2x^2} & \text{for } n=-1\\ (a+b)\,(cx+d\log(x)) & \text{for } n=0\\ \frac{2acn^2x}{2n^2+2n}+\frac{2acnx}{2n^2+2n}+\frac{2adnxx^{n-1}}{2n^2+2n}+\frac{2adxx^{n-1}}{2n^2+2n}+\frac{2bcnxx^n}{2n^2+2n}+\frac{bdnxx^nx^{n-1}}{2n^2+2n}+\frac{bdxx^nx^{n-1}}{2n^2+2n} & \text{otherwise}\end{cases}$$

input
```
integrate((c+d*x**(-1+n))*(a+b*x**n),x)
```

output
```
Piecewise((a*c*x - a*d/x + b*c*log(x) - b*d/(2*x**2), Eq(n, -1)), ((a + b)
*(c*x + d*log(x)), Eq(n, 0)), (2*a*c*n**2*x/(2*n**2 + 2*n) + 2*a*c*n*x/(2*
n**2 + 2*n) + 2*a*d*n*x*x**(n - 1)/(2*n**2 + 2*n) + 2*a*d*x*x**(n - 1)/(2*
n**2 + 2*n) + 2*b*c*n*x*x**n/(2*n**2 + 2*n) + b*d*n*x*x**n*x**(n - 1)/(2*n
**2 + 2*n) + b*d*x*x**n*x**(n - 1)/(2*n**2 + 2*n), True))
```

3.579.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx = acx+\frac{bdx^{2n}}{2n}+\frac{bcx^{n+1}}{n+1}+\frac{adx^n}{n}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="maxima")
```

output
```
a*c*x + 1/2*b*d*x^(2*n)/n + b*c*x^(n + 1)/(n + 1) + a*d*x^n/n
```

3.579.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx = \frac{2\,acn^2x+2\,bcnxx^n+2\,acnx+bdnx^{2n}+2\,adnx^n+bdx^{2n}+2\,adx^n}{2\,(n^2+n)}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="giac")
```

output
```
1/2*(2*a*c*n^2*x + 2*b*c*n*x*x^n + 2*a*c*n*x + b*d*n*x^(2*n) + 2*a*d*n*x^n
 + b*d*x^(2*n) + 2*a*d*x^n)/(n^2 + n)
```

3.579.9 Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx = a\,c\,x + \frac{a\,d\,x^n}{n} + \frac{b\,d\,x^{2\,n}}{2\,n} + \frac{b\,c\,x\,x^n}{n+1}$$

input
```
int((c + d*x^(n - 1))*(a + b*x^n),x)
```

output
```
a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x*x^n)/(n + 1)
```

3.580 $\int \left(c+dx^{-1+n}\right) dx$

3.580.1 Optimal result 4409
3.580.2 Mathematica [A] (verified) 4409
3.580.3 Rubi [A] (verified) 4410
3.580.4 Maple [A] (verified) 4410
3.580.5 Fricas [A] (verification not implemented) 4411
3.580.6 Sympy [A] (verification not implemented) 4411
3.580.7 Maxima [A] (verification not implemented) 4411
3.580.8 Giac [A] (verification not implemented) 4412
3.580.9 Mupad [B] (verification not implemented) 4412

3.580.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \left(c+dx^{-1+n}\right) dx = cx+\frac{dx^n}{n}$$

output
```
c*x+d*x^n/n
```

3.580.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right) dx = cx+\frac{dx^n}{n}$$

input
```
Integrate[c + d*x^(-1 + n),x]
```

output
```
c*x + (d*x^n)/n
```

3.580.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^{n-1}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$cx+\frac{dx^n}{n}$$

input
```
Int[c + d*x^(-1 + n),x]
```

output
```
c*x + (d*x^n)/n
```

3.580.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.580.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$cx+\frac{d\,x^{n}}{n}$	13
parts	$cx+\frac{d\,x^{n}}{n}$	13
risch	$cx+\frac{dx\,x^{-1+n}}{n}$	16
parallelrisch	$cx+\frac{dx\,x^{-1+n}}{n}$	16
norman	$cx+\frac{dx\,\mathrm{e}^{(-1+n)\ln(x)}}{n}$	18

input
```
int(c+d*x^(-1+n),x,method=_RETURNVERBOSE)
```

output
```
c*x+d*x^n/n
```

3.580.5 Fricas [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \left(c+dx^{-1+n}\right) dx = \frac{cnx+dxx^{n-1}}{n}$$

input
```
integrate(c+d*x^(-1+n),x, algorithm="fricas")
```

output
```
(c*n*x + d*x*x^(n - 1))/n
```

3.580.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right) dx = cx+d\left(\begin{cases} \frac{x^n}{n} & \text{for } n\neq 0 \\ \log\left(x\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(c+d*x**(-1+n),x)
```

output
```
c*x + d*Piecewise((x**n/n, Ne(n, 0)), (log(x), True))
```

3.580.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right) dx = cx+\frac{dx^n}{n}$$

input
```
integrate(c+d*x^(-1+n),x, algorithm="maxima")
```

output
```
c*x + d*x^n/n
```

3.580.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right) dx = cx + \frac{dx^n}{n}$$

input
```
integrate(c+d*x^(-1+n),x, algorithm="giac")
```

output
```
c*x + d*x^n/n
```

3.580.9 Mupad [B] (verification not implemented)

Time = 11.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right) dx = c\,x + \frac{d\,x^n}{n}$$

input
```
int(c + d*x^(n - 1),x)
```

output
```
c*x + (d*x^n)/n
```

3.581 $\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx$

3.581.1 Optimal result . . . 4413
3.581.2 Mathematica [A] (verified) . . . 4413
3.581.3 Rubi [A] (verified) . . . 4414
3.581.4 Maple [F] . . . 4415
3.581.5 Fricas [F] . . . 4415
3.581.6 Sympy [A] (verification not implemented) . . . 4415
3.581.7 Maxima [F] . . . 4416
3.581.8 Giac [F] . . . 4416
3.581.9 Mupad [B] (verification not implemented) . . . 4416

3.581.1 Optimal result

Integrand size = 19, antiderivative size = 42

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx=\frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

output
```
c*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a+d*ln(a+b*x^n)/b/n
```

3.581.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx=\frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

input
```
Integrate[(c + d*x^(-1 + n))/(a + b*x^n),x]
```

output
```
(c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a
+ b*x^n])/(b*n)
```

3.581.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2430, 778, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^{n-1}}{a+bx^n}dx$$

$$\Big\downarrow \text{2430}$$

$$c\int \frac{1}{bx^n+a}dx+d\int \frac{x^{n-1}}{bx^n+a}dx$$

$$\Big\downarrow \text{778}$$

$$d\int \frac{x^{n-1}}{bx^n+a}dx+\frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}$$

$$\Big\downarrow \text{792}$$

$$\frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

input
```
Int[(c + d*x^(-1 + n))/(a + b*x^n),x]
```

output
```
(c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a
+ b*x^n])/(b*n)
```

3.581.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2430
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Simp[A   Int[(a + b*x^n)^p, x], x] + Simp[B   Int[x^m*(a + b*x^n)^p, x], x]
 /; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

3.581.4 Maple [F]

$$\int \frac{c+d\,x^{-1+n}}{a+b\,x^{n}}dx$$

input
```
int((c+d*x^(-1+n))/(a+b*x^n),x)
```

output
```
int((c+d*x^(-1+n))/(a+b*x^n),x)
```

3.581.5 Fricas [F]

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx=\int \frac{dx^{n-1}+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^(n - 1) + c)/(b*x^n + a), x)
```

3.581.6 Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx=\frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}cx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)}+d\left(\begin{cases}\frac{x^n}{an} & \text{for } b=0\\ \tilde{\infty}x^n & \text{for } n=0\\ \frac{\log(an+bnx^n)}{bn} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((c+d*x**(-1+n))/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(-1 - 1/n)*c*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamm
a(1/n)/(n**2*gamma(1 + 1/n)) + d*Piecewise((x**n/(a*n), Eq(b, 0)), (zoo*x*
*n, Eq(n, 0)), (log(a*n + b*n*x**n)/(b*n), True))
```

3.581.7 Maxima [F]

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \int \frac{dx^{n-1}+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="maxima")
```

output
```
d*log(x)/b + integrate((b*c*x - a*d)/(b^2*x*x^n + a*b*x), x)
```

3.581.8 Giac [F]

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \int \frac{dx^{n-1}+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^(n - 1) + c)/(b*x^n + a), x)
```

3.581.9 Mupad [B] (verification not implemented)

Time = 11.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \frac{c\,x\,{}_2\mathrm{F}_1\left(1,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a} + \frac{d\,\ln\left(a+b\,x^n\right)}{b\,n}$$

input
```
int((c + d*x^(n - 1))/(a + b*x^n),x)
```

output
```
(c*x*hypergeom([1, 1/n], 1/n + 1, -(b*x^n)/a))/a + (d*log(a + b*x^n))/(b*n
)
```

3.582 $\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx$

3.582.1 Optimal result 4417
3.582.2 Mathematica [A] (verified) 4417
3.582.3 Rubi [A] (verified) 4418
3.582.4 Maple [F] 4419
3.582.5 Fricas [F] 4419
3.582.6 Sympy [C] (verification not implemented) 4420
3.582.7 Maxima [F] 4421
3.582.8 Giac [F] 4421
3.582.9 Mupad [B] (verification not implemented) 4421

3.582.1 Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx=-\frac{d}{bn(a+bx^n)}+\frac{cx\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}$$

output
```
-d/b/n/(a+b*x^n)+c*x*hypergeom([2, 1/n],[1+1/n],-b*x^n/a)/a^2
```

3.582.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx=-\frac{d}{abn+b^2nx^n}+\frac{cx\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}$$

input
```
Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^2,x]
```

output
```
-(d/(a*b*n + b^2*n*x^n)) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -
((b*x^n)/a)])/a^2
```

3.582.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2430, 778, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^{n-1}}{(a+bx^n)^2}dx$$
$$\Big\downarrow \text{2430}$$
$$c\int \frac{1}{(bx^n+a)^2}dx+d\int \frac{x^{n-1}}{(bx^n+a)^2}dx$$
$$\Big\downarrow \text{778}$$
$$d\int \frac{x^{n-1}}{(bx^n+a)^2}dx+\frac{cx\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}$$
$$\Big\downarrow \text{793}$$
$$\frac{cx\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}-\frac{d}{bn\,(a+bx^n)}$$

input
```
Int[(c + d*x^(-1 + n))/(a + b*x^n)^2,x]
```

output
```
-(d/(b*n*(a + b*x^n))) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((
b*x^n)/a)])/a^2
```

3.582.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2430
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Simp[A   Int[(a + b*x^n)^p, x], x] + Simp[B   Int[x^m*(a + b*x^n)^p, x], x]
 /; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

3.582.4 Maple [F]

$$\int \frac{c+d\,x^{-1+n}}{\left(a+b\,x^{n}\right)^{2}}dx$$

input
```
int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
```

3.582.5 Fricas [F]

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx = \int \frac{dx^{n-1}+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d*x^(n - 1) + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

3.582.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.87 (sec) , antiderivative size = 388, normalized size of antiderivative = 8.82

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx = c\left(\frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}nx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} + \frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}nx\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} - \frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{\frac{1}{n}}a^{-2-\frac{1}{n}}bnxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} - \frac{a^{\frac{1}{n}}a^{-2-\frac{1}{n}}bxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)}\right) + d\left(\begin{cases}\tilde{\infty}\log(x) & \text{for } a=0\wedge b=0\wedge n=0\\ -\frac{xx^{-2n}x^{n-1}}{b^2n} & \text{for } a=0\\ \frac{\tilde{\infty}xx^{n-1}}{n} & \text{for } b=-ax^{-n}\\ \frac{\log(x)}{(a+b)^2} & \text{for } n=0\\ \frac{xx^{n-1}}{a^2n+abnx^n} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((c+d*x**(-1+n))/(a+b*x**n)**2,x)
```

output
```
c*(a*a**(1/n)*a**(-2 - 1/n)*n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)
*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) + a*a**(1
/n)*a**(-2 - 1/n)*n*x*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamm
a(1 + 1/n)) - a*a**(1/n)*a**(-2 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*pi)/a
, 1, 1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))
+ a**(1/n)*a**(-2 - 1/n)*b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) - a**
(1/n)*a**(-2 - 1/n)*b*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*ga
mma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + d*Piecewi
se((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(n - 1)/(b**2*n*x*
*(2*n)), Eq(a, 0)), (zoo*x*x**(n - 1)/n, Eq(b, -a/x**n)), (log(x)/(a + b)*
*2, Eq(n, 0)), (x*x**(n - 1)/(a**2*n + a*b*n*x**n), True))
```

3.582.7 Maxima [F]

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^2} \, dx = \int \frac{dx^{n-1} + c}{(bx^n + a)^2} \, dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
c*(n - 1)*integrate(1/(a*b*n*x^n + a^2*n), x) + (b*c*x - a*d)/(a*b^2*n*x^n
 + a^2*b*n)
```

3.582.8 Giac [F]

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^2} \, dx = \int \frac{dx^{n-1} + c}{(bx^n + a)^2} \, dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^(n - 1) + c)/(b*x^n + a)^2, x)
```

3.582.9 Mupad [B] (verification not implemented)

Time = 10.89 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^2} \, dx = \frac{c\,x\,{}_2F_1\left(2, \frac{1}{n};\ \frac{1}{n} + 1;\ -\frac{b\,x^n}{a}\right)}{a^2} - \frac{a\,d}{b\,\left(a^2\,n + a\,b\,n\,x^n\right)}$$

input
```
int((c + d*x^(n - 1))/(a + b*x^n)^2,x)
```

output
```
(c*x*hypergeom([2, 1/n], 1/n + 1, -(b*x^n)/a))/a^2 - (a*d)/(b*(a^2*n + a*b
*n*x^n))
```

3.583 $\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx$

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3.583.2 Mathematica [A] (verified) 4422
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3.583.4 Maple [F] 4424
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3.583.6 Sympy [F(-1)] 4424
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3.583.9 Mupad [B] (verification not implemented) 4425

3.583.1 Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx = -\frac{d}{2bn\,(a+bx^n)^2} + \frac{cx\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3}$$

output
```
-1/2*d/b/n/(a+b*x^n)^2+c*x*hypergeom([3, 1/n],[1+1/n],-b*x^n/a)/a^3
```

3.583.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx = \frac{-a^3d+2bcnx(a+bx^n)^2\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{2a^3bn\,(a+bx^n)^2}$$

input
```
Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^3,x]
```

output
```
(-(a^3*d) + 2*b*c*n*x*(a + b*x^n)^2*Hypergeometric2F1[3, n^(-1), 1 + n^(-1
), -((b*x^n)/a)])/(2*a^3*b*n*(a + b*x^n)^2)
```

3.583.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2430, 778, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^{n-1}}{(a+bx^n)^3}dx$$

$$\Big\downarrow \text{2430}$$

$$c\int \frac{1}{(bx^n+a)^3}dx+d\int \frac{x^{n-1}}{(bx^n+a)^3}dx$$

$$\Big\downarrow \text{778}$$

$$d\int \frac{x^{n-1}}{(bx^n+a)^3}dx+\frac{cx\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3}$$

$$\Big\downarrow \text{793}$$

$$\frac{cx\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3}-\frac{d}{2bn\left(a+bx^n\right)^2}$$

input
```
Int[(c + d*x^(-1 + n))/(a + b*x^n)^3,x]
```

output
```
-1/2*d/(b*n*(a + b*x^n)^2) + (c*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/a^3
```

3.583.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2430
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Simp[A   Int[(a + b*x^n)^p, x], x] + Simp[B   Int[x^m*(a + b*x^n)^p, x], x]
 /; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

3.583.4 Maple [F]

$$\int \frac{c+d\,x^{-1+n}}{\left(a+b\,x^{n}\right)^{3}}dx$$

input
```
int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

output
```
int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

3.583.5 Fricas [F]

$$\int \frac{c+dx^{-1+n}}{\left(a+bx^{n}\right)^{3}}\,dx=\int \frac{dx^{n-1}+c}{\left(bx^{n}+a\right)^{3}}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d*x^(n - 1) + c)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n +
a^3), x)
```

3.583.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^{-1+n}}{\left(a+bx^{n}\right)^{3}}\,dx=\text{Timed out}$$

input
```
integrate((c+d*x**(-1+n))/(a+b*x**n)**3,x)
```

output
```
Timed out
```

3.583.7 Maxima [F]

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx = \int \frac{dx^{n-1}+c}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
(2*n^2 - 3*n + 1)*c*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b^2
*c*(2*n - 1)*x*x^n + a*b*c*(3*n - 1)*x - a^2*d*n)/(a^2*b^3*n^2*x^(2*n) + 2
*a^3*b^2*n^2*x^n + a^4*b*n^2)
```

3.583.8 Giac [F]

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx = \int \frac{dx^{n-1}+c}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((d*x^(n - 1) + c)/(b*x^n + a)^3, x)
```

3.583.9 Mupad [B] (verification not implemented)

Time = 10.81 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx = \frac{c\,x\,{}_2\mathrm{F}_1\left(3,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a^3} - \frac{d}{2\,b\,\left(a^2\,n+b^2\,n\,x^{2\,n}+2\,a\,b\,n\,x^n\right)}$$

input
```
int((c + d*x^(n - 1))/(a + b*x^n)^3,x)
```

output
```
(c*x*hypergeom([3, 1/n], 1/n + 1, -(b*x^n)/a))/a^3 - d/(2*b*(a^2*n + b^2*n
*x^(2*n) + 2*a*b*n*x^n))
```

3.584 $\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx$

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3.584.8 Giac [F] . . . 4431
3.584.9 Mupad [F(-1)] . . . 4431

3.584.1 Optimal result

Integrand size = 38, antiderivative size = 305

$$\begin{aligned}\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx &= \frac{d(cx)^{1+m}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{c(1+m)\sqrt{a+bx^n}}\\ &+\frac{ex^{1+n}(cx)^m\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m+n}{n},\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{(1+m+n)\sqrt{a+bx^n}}\\ &+\frac{fx^{1+2n}(cx)^m\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m+2n}{n},\frac{1+m+3n}{n},-\frac{bx^n}{a}\right)}{(1+m+2n)\sqrt{a+bx^n}}\\ &+\frac{gx^{1+3n}(cx)^m\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m+3n}{n},\frac{1+m+4n}{n},-\frac{bx^n}{a}\right)}{(1+m+3n)\sqrt{a+bx^n}}\end{aligned}$$

output
```
d*(c*x)^(1+m)*hypergeom([1/2, (1+m)/n],[(1+m+n)/n],-b*x^n/a)*(1+b*x^n/a)^(
1/2)/c/(1+m)/(a+b*x^n)^(1/2)+e*x^(1+n)*(c*x)^m*hypergeom([1/2, (1+m+n)/n],
[(1+m+2*n)/n],-b*x^n/a)*(1+b*x^n/a)^(1/2)/(1+m+n)/(a+b*x^n)^(1/2)+f*x^(1+2
*n)*(c*x)^m*hypergeom([1/2, (1+m+2*n)/n],[(1+m+3*n)/n],-b*x^n/a)*(1+b*x^n/
a)^(1/2)/(1+m+2*n)/(a+b*x^n)^(1/2)+g*x^(1+3*n)*(c*x)^m*hypergeom([1/2, (1+
m+3*n)/n],[(1+m+4*n)/n],-b*x^n/a)*(1+b*x^n/a)^(1/2)/(1+m+3*n)/(a+b*x^n)^(1
/2)
```

3.584.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.68

$$\int \frac{(cx)^m (d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx$$
$$= \frac{x(cx)^m\sqrt{1+\frac{bx^n}{a}}\left(\frac{d\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{1+m}+x^n\left(\frac{e\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m+n}{n},\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{1+m+n}+x^n\left(\right.\right.\right.}{\sqrt{a+bx^n}}$$

input
```
Integrate[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/Sqrt[a + b*x^n],x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^n)/a]*((d*Hypergeometric2F1[1/2, (1 + m)/n, (1 +
m + n)/n, -((b*x^n)/a)])/(1 + m) + x^n*((e*Hypergeometric2F1[1/2, (1 + m +
 n)/n, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n) + x^n*((f*Hypergeometri
c2F1[1/2, (1 + m + 2*n)/n, (1 + m + 3*n)/n, -((b*x^n)/a)])/(1 + m + 2*n) +
 (g*x^n*Hypergeometric2F1[1/2, (1 + m + 3*n)/n, (1 + m + 4*n)/n, -((b*x^n)
/a)])/(1 + m + 3*n)))))/Sqrt[a + b*x^n]
```

3.584.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx$$
$$\Big\downarrow \text{2383}$$
$$\int \left(\frac{d(cx)^m}{\sqrt{a+bx^n}}+\frac{ex^n(cx)^m}{\sqrt{a+bx^n}}+\frac{fx^{2n}(cx)^m}{\sqrt{a+bx^n}}+\frac{gx^{3n}(cx)^m}{\sqrt{a+bx^n}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{d(cx)^{m+1}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{c(m+1)\sqrt{a+bx^n}}+\frac{ex^{n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+n+1}{n},\frac{m+2n+1}{n},-\frac{bx^n}{a}\right)}{(m+n+1)\sqrt{a+bx^n}}+\frac{fx^{2n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2n+1}{n},\frac{m+3n+1}{n},-\frac{bx^n}{a}\right)}{(m+2n+1)\sqrt{a+bx^n}}+\frac{gx^{3n+1}(cx)^m\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3n+1}{n},\frac{m+4n+1}{n},-\frac{bx^n}{a}\right)}{(m+3n+1)\sqrt{a+bx^n}}$$

input
```
Int[((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/Sqrt[a + b*x^n],x]
```

output
```
(d*(c*x)^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m)/n, (1
+ m + n)/n, -((b*x^n)/a)])/(c*(1 + m)*Sqrt[a + b*x^n]) + (e*x^(1 + n)*(c*x
)^m*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + n)/n, (1 + m + 2*n
)/n, -((b*x^n)/a)])/((1 + m + n)*Sqrt[a + b*x^n]) + (f*x^(1 + 2*n)*(c*x)^m
*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 2*n)/n, (1 + m + 3*n)
/n, -((b*x^n)/a)])/((1 + m + 2*n)*Sqrt[a + b*x^n]) + (g*x^(1 + 3*n)*(c*x)^
m*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 3*n)/n, (1 + m + 4*n
)/n, -((b*x^n)/a)])/((1 + m + 3*n)*Sqrt[a + b*x^n])
```

3.584.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2383
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

3.584. $\int \frac{(cx)^m(d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx$

3.584.4 Maple [F]

$$\int \frac{(cx)^m (d + e\,x^n + f\,x^{2n} + g\,x^{3n})}{\sqrt{a + b\,x^n}}dx$$

input
```
int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x)
```

output
```
int((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x)
```

3.584.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{\sqrt{a + bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorit
hm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.584.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.81 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.12

$$\int \frac{(cx)^m (d + ex^n + fx^{2n} + gx^{3n})}{\sqrt{a + bx^n}} dx$$
$$= \frac{a^{\frac{m}{n}+\frac{1}{n}} a^{-\frac{m}{n}-\frac{1}{2}-\frac{1}{n}} c^m d x^{m+1} \Gamma\left(\frac{m}{n}+\frac{1}{n}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\right|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$
$$+ \frac{a^{-\frac{m}{n}-\frac{7}{2}-\frac{1}{n}} a^{\frac{m}{n}+3+\frac{1}{n}} c^m g x^{m+3n+1} \Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{n}+3+\frac{1}{n}\\ \frac{m}{n}+4+\frac{1}{n}\end{matrix}\right|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+4+\frac{1}{n}\right)}$$
$$+ \frac{a^{-\frac{m}{n}-\frac{5}{2}-\frac{1}{n}} a^{\frac{m}{n}+2+\frac{1}{n}} c^m f x^{m+2n+1} \Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{n}+2+\frac{1}{n}\\ \frac{m}{n}+3+\frac{1}{n}\end{matrix}\right|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)}$$
$$+ \frac{a^{-\frac{m}{n}-\frac{3}{2}-\frac{1}{n}} a^{\frac{m}{n}+1+\frac{1}{n}} c^m e x^{m+n+1} \Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{n}+1+\frac{1}{n}\\ \frac{m}{n}+2+\frac{1}{n}\end{matrix}\right|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$

input
```
integrate((c*x)**m*(d+e*x**n+f*x**(2*n)+g*x**(3*n))/(a+b*x**n)**(1/2),x)
```

output
```
a**(m/n + 1/n)*a**(-m/n - 1/2 - 1/n)*c**m*d*x**(m + 1)*gamma(m/n + 1/n)*hy
per((1/2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma
(m/n + 1 + 1/n)) + a**(-m/n - 7/2 - 1/n)*a**(m/n + 3 + 1/n)*c**m*g*x**(m +
 3*n + 1)*gamma(m/n + 3 + 1/n)*hyper((1/2, m/n + 3 + 1/n), (m/n + 4 + 1/n,
), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 4 + 1/n)) + a**(-m/n - 5/2 - 1
/n)*a**(m/n + 2 + 1/n)*c**m*f*x**(m + 2*n + 1)*gamma(m/n + 2 + 1/n)*hyper(
(1/2, m/n + 2 + 1/n), (m/n + 3 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma
(m/n + 3 + 1/n)) + a**(-m/n - 3/2 - 1/n)*a**(m/n + 1 + 1/n)*c**m*e*x**(m +
 n + 1)*gamma(m/n + 1 + 1/n)*hyper((1/2, m/n + 1 + 1/n), (m/n + 2 + 1/n,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 2 + 1/n))
```

3.584. $\int \frac{(cx)^m (d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}} dx$

3.584.7 Maxima [F]

$$\int \frac{(cx)^m (d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx = \int \frac{(gx^{3n}+fx^{2n}+ex^n+d)(cx)^m}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n + a), x)
```

3.584.8 Giac [F]

$$\int \frac{(cx)^m (d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx = \int \frac{(gx^{3n}+fx^{2n}+ex^n+d)(cx)^m}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^m*(d+e*x^n+f*x^(2*n)+g*x^(3*n))/(a+b*x^n)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(c*x)^m/sqrt(b*x^n + a), x)
```

3.584.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (d+ex^n+fx^{2n}+gx^{3n})}{\sqrt{a+bx^n}}\,dx = \int \frac{(c\,x)^m\,(d+e\,x^n+f\,x^{2\,n}+g\,x^{3\,n})}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n)^(1/2),x)
```

output
```
int(((c*x)^m*(d + e*x^n + f*x^(2*n) + g*x^(3*n)))/(a + b*x^n)^(1/2), x)
```

3.585 $\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx$

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3.585.1 Optimal result

Integrand size = 58, antiderivative size = 45

$$\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx=-\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

output
```
-2*(a*g+2*a*h*x^(1/4*n)-b*f*x^(1/2*n))/a/n/(a+b*x^n)^(1/2)
```

3.585.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx=\frac{2bfx^{n/2}-2a\left(g+2hx^{n/4}\right)}{an\sqrt{a+bx^n}}$$

input
```
Integrate[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n) + b*h*x
^(-1 + (5*n)/4))/(a + b*x^n)^(3/2),x]
```

output
```
(2*b*f*x^(n/2) - 2*a*(g + 2*h*x^(n/4)))/(a*n*Sqrt[a + b*x^n])
```

3.585.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2029, 2356}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-ahx^{\frac{n}{4}-1}+bfx^{\frac{n}{2}-1}+bgx^{n-1}+bhx^{\frac{5n}{4}-1}}{(a+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{ 2029}$$

$$\int \frac{x^{\frac{n}{4}-1}\left(-ah+bfx^{n/4}+bgx^{3n/4}+bhx^n\right)}{(a+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{ 2356}$$

$$-\frac{2\left(ag+2ahx^{n/4}-bfx^{n/2}\right)}{an\sqrt{a+bx^n}}$$

input
```
Int[(-(a*h*x^(-1 + n/4)) + b*f*x^(-1 + n/2) + b*g*x^(-1 + n) + b*h*x^(-1 +
 (5*n)/4))/(a + b*x^n)^(3/2),x]
```

output
```
(-2*(a*g + 2*a*h*x^(n/4) - b*f*x^(n/2)))/(a*n*Sqrt[a + b*x^n])
```

3.585.3.1 Defintions of rubi rules used

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2356
```
Int[((x_)^(m_.)*((e_) + (h_.)*(x_)^(n_.) + (f_.)*(x_)^(q_.) + (g_.)*(x_)^(r
_.)))/((a_) + (c_.)*(x_)^(n_.))^(3/2), x_Symbol] :> Simp[-(2*a*g + 4*a*h*x^
(n/4) - 2*c*f*x^(n/2))/(a*c*n*Sqrt[a + c*x^n]), x] /; FreeQ[{a, c, e, f, g,
 h, m, n}, x] && EqQ[q, n/4] && EqQ[r, 3*(n/4)] && EqQ[4*m - n + 4, 0] && E
qQ[c*e + a*h, 0]
```

3.585.4 Maple [F]

$$\int \frac{-ah\,x^{-1+\frac{n}{4}}+bf\,x^{-1+\frac{n}{2}}+bg\,x^{-1+n}+bh\,x^{-1+\frac{5n}{4}}}{(a+b\,x^{n})^{\frac{3}{2}}}dx$$

input
```
int((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+
b*x^n)^(3/2),x)
```

output
```
int((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n))/(a+
b*x^n)^(3/2),x)
```

3.585.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx=\frac{2\sqrt{bx^4x^{n-4}+a}\left(bfx^2x^{\frac{1}{2}n-2}-2\,ahxx^{\frac{1}{4}n-1}-ag\right)}{abnx^4x^{n-4}+a^2n}$$

input
```
integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n
))/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x^4*x^(n - 4) + a)*(b*f*x^2*x^(1/2*n - 2) - 2*a*h*x*x^(1/4*n - 1)
 - a*g)/(a*b*n*x^4*x^(n - 4) + a^2*n)
```

3.585.6 Sympy [A] (verification not implemented)

Time = 137.42 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.60

$$\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx=bg\left(\begin{cases}\frac{\log(x)}{a^{\frac{3}{2}}} & \text{for } b=0\wedge n=0\\ \frac{xx^{n-1}}{a^{\frac{3}{2}}n} & \text{for } b=0\\ \frac{\log(x)}{(a+b)^{\frac{3}{2}}} & \text{for } n=0\\ -\frac{2}{bn\sqrt{a+bx^n}} & \text{otherwise}\end{cases}\right)+\frac{2\sqrt{b}f}{an\sqrt{\frac{ax^{-n}}{b}+1}}-\frac{hx^{\frac{n}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{a}n\Gamma\left(\frac{5}{4}\right)}+\frac{bhx^{\frac{5n}{4}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{a^{\frac{3}{2}}n\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((-a*h*x**(-1+1/4*n)+b*f*x**(-1+1/2*n)+b*g*x**(-1+n)+b*h*x**(-1+5
/4*n))/(a+b*x**n)**(3/2),x)
```

output
```
b*g*Piecewise((log(x)/a**(3/2), Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(a**(3
/2)*n), Eq(b, 0)), (log(x)/(a + b)**(3/2), Eq(n, 0)), (-2/(b*n*sqrt(a + b*
x**n)), True)) + 2*sqrt(b)*f/(a*n*sqrt(a/(b*x**n) + 1)) - h*x**(n/4)*gamma
(1/4)*hyper((1/4, 3/2), (5/4,), b*x**n*exp_polar(I*pi)/a)/(sqrt(a)*n*gamma
(5/4)) + b*h*x**(5*n/4)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**n*exp_po
lar(I*pi)/a)/(a**(3/2)*n*gamma(9/4))
```

3.585.7 Maxima [F]

$$\int \frac{-ahx^{-1+\frac{n}{4}} + bfx^{-1+\frac{n}{2}} + bgx^{-1+n} + bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx = \int \frac{bhx^{\frac{5}{4}n-1} + bgx^{n-1} + bfx^{\frac{1}{2}n-1} - ahx^{\frac{1}{4}n-1}}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n
))/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(
1/4*n - 1))/(b*x^n + a)^(3/2), x)
```

3.585.8 Giac [F]

$$\int \frac{-ahx^{-1+\frac{n}{4}} + bfx^{-1+\frac{n}{2}} + bgx^{-1+n} + bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx = \int \frac{bhx^{\frac{5}{4}n-1} + bgx^{n-1} + bfx^{\frac{1}{2}n-1} - ahx^{\frac{1}{4}n-1}}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-a*h*x^(-1+1/4*n)+b*f*x^(-1+1/2*n)+b*g*x^(-1+n)+b*h*x^(-1+5/4*n
))/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*h*x^(5/4*n - 1) + b*g*x^(n - 1) + b*f*x^(1/2*n - 1) - a*h*x^(
1/4*n - 1))/(b*x^n + a)^(3/2), x)
```

3.585.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-ahx^{-1+\frac{n}{4}}+bfx^{-1+\frac{n}{2}}+bgx^{-1+n}+bhx^{-1+\frac{5n}{4}}}{(a+bx^n)^{3/2}}\,dx=\int \frac{b\,f\,x^{\frac{n}{2}-1}-a\,h\,x^{\frac{n}{4}-1}+b\,h\,x^{\frac{5\,n}{4}-1}+b\,g\,x^{n-1}}{(a+b\,x^n)^{3/2}}\,dx$$

input
```
int((b*f*x^(n/2 - 1) - a*h*x^(n/4 - 1) + b*h*x^((5*n)/4 - 1) + b*g*x^(n -
1))/(a + b*x^n)^(3/2),x)
```

output
```
int((b*f*x^(n/2 - 1) - a*h*x^(n/4 - 1) + b*h*x^((5*n)/4 - 1) + b*g*x^(n -
1))/(a + b*x^n)^(3/2), x)
```

3.586 $\int (cx)^m (d+ex+fx^2+gx^3) (a+bx^n)^p \, dx$

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3.586.1 Optimal result

Integrand size = 30, antiderivative size = 273

$$\begin{aligned}
&\int (cx)^m (d+ex+fx^2+gx^3) (a+bx^n)^p \, dx \\
&= \frac{d(cx)^{1+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{c(1+m)} \\
&\quad + \frac{e(cx)^{2+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{2+m}{n},-p,\frac{2+m+n}{n},-\frac{bx^n}{a}\right)}{c^2(2+m)} \\
&\quad + \frac{f(cx)^{3+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3+m}{n},-p,\frac{3+m+n}{n},-\frac{bx^n}{a}\right)}{c^3(3+m)} \\
&\quad + \frac{g(cx)^{4+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{4+m}{n},-p,\frac{4+m+n}{n},-\frac{bx^n}{a}\right)}{c^4(4+m)}
\end{aligned}$$

output
```
d*(c*x)^(1+m)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/c/
(1+m)/((1+b*x^n/a)^p)+e*(c*x)^(2+m)*(a+b*x^n)^p*hypergeom([-p, (2+m)/n],[(
2+m+n)/n],-b*x^n/a)/c^2/(2+m)/((1+b*x^n/a)^p)+f*(c*x)^(3+m)*(a+b*x^n)^p*hy
pergeom([-p, (3+m)/n],[(3+m+n)/n],-b*x^n/a)/c^3/(3+m)/((1+b*x^n/a)^p)+g*(c
*x)^(4+m)*(a+b*x^n)^p*hypergeom([-p, (4+m)/n],[(4+m+n)/n],-b*x^n/a)/c^4/(4
+m)/((1+b*x^n/a)^p)
```

3.586.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.65

$$\int (cx)^m\left(d+ex+fx^2+gx^3\right)(a+bx^n)^p\,dx$$
$$=x(cx)^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(\frac{d\,\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{1+m}+x\left(\frac{e\,\text{Hypergeometric2F1}\left(\frac{2+m}{n},-p,\frac{2+m+n}{n},-\frac{bx^n}{a}\right)}{2+m}+x\left(\frac{f\,\text{Hypergeometric2F1}\left(\frac{3+m}{n},-p,\frac{3+m+n}{n},-\frac{bx^n}{a}\right)}{3+m}+\frac{gx\,\text{Hypergeometric2F1}\left(\frac{4+m}{n},-p,\frac{4+m+n}{n},-\frac{bx^n}{a}\right)}{4+m}\right)\right)\right)$$

input
```
Integrate[(c*x)^m*(d + e*x + f*x^2 + g*x^3)*(a + b*x^n)^p,x]
```

output
```
(x*(c*x)^m*(a + b*x^n)^p*((d*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/
n, -((b*x^n)/a)])/(1 + m) + x*((e*Hypergeometric2F1[(2 + m)/n, -p, (2 + m
+ n)/n, -((b*x^n)/a)])/(2 + m) + x*((f*Hypergeometric2F1[(3 + m)/n, -p, (3
 + m + n)/n, -((b*x^n)/a)])/(3 + m) + (g*x*Hypergeometric2F1[(4 + m)/n, -p
, (4 + m + n)/n, -((b*x^n)/a)])/(4 + m)))))/(1 + (b*x^n)/a)^p
```

3.586.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m(a+bx^n)^p\left(d+ex+fx^2+gx^3\right)dx$$
$$\Big\downarrow \text{2383}$$
$$\int\left(\frac{g(cx)^{m+3}(a+bx^n)^p}{c^3}+\frac{f(cx)^{m+2}(a+bx^n)^p}{c^2}+d(cx)^m(a+bx^n)^p+\frac{e(cx)^{m+1}(a+bx^n)^p}{c}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{g(cx)^{m+4}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+4}{n},-p,\frac{m+n+4}{n},-\frac{bx^n}{a}\right)}{c^4(m+4)}+\frac{f(cx)^{m+3}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+3}{n},-p,\frac{m+n+3}{n},-\frac{bx^n}{a}\right)}{c^3(m+3)}+\frac{e(cx)^{m+2}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2}{n},-p,\frac{m+n+2}{n},-\frac{bx^n}{a}\right)}{c^2(m+2)}+\frac{d(cx)^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{c(m+1)}$$

input
```
Int[(c*x)^m*(d + e*x + f*x^2 + g*x^3)*(a + b*x^n)^p,x]
```

output
```
(d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n
)/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*(c*x)^(2 + m)*(a +
b*x^n)^p*Hypergeometric2F1[(2 + m)/n, -p, (2 + m + n)/n, -((b*x^n)/a)])/(c
^2*(2 + m)*(1 + (b*x^n)/a)^p) + (f*(c*x)^(3 + m)*(a + b*x^n)^p*Hypergeomet
ric2F1[(3 + m)/n, -p, (3 + m + n)/n, -((b*x^n)/a)])/(c^3*(3 + m)*(1 + (b*x
^n)/a)^p) + (g*(c*x)^(4 + m)*(a + b*x^n)^p*Hypergeometric2F1[(4 + m)/n, -p
, (4 + m + n)/n, -((b*x^n)/a)])/(c^4*(4 + m)*(1 + (b*x^n)/a)^p)
```

3.586.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2383
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

3.586.4 Maple [F]

$$\int (cx)^m\left(g\,x^3+f\,x^2+ex+d\right)(a+b\,x^n)^p\,dx$$

input
```
int((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x)
```

output
```
int((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x)
```

3.586.5 Fricas [F]

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right) (a+bx^n)^p \, dx = \int \left(gx^3+fx^2+ex+d\right)(bx^n+a)^p(cx)^m \, dx$$

input
```
integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

3.586.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 104.02 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.11

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right) (a+bx^n)^p \, dx = \frac{a^{\frac{m}{n}+\frac{1}{n}} a^{-\frac{m}{n}+p-\frac{1}{n}} c^m d x^{m+1} \Gamma\left(\frac{m}{n}+\frac{1}{n}\right) {}_2F_1\left(\left.\begin{matrix} -p, \frac{m}{n}+\frac{1}{n} \\ \frac{m}{n}+1+\frac{1}{n} \end{matrix}\right| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)} + \frac{a^{\frac{m}{n}+\frac{2}{n}} a^{-\frac{m}{n}+p-\frac{2}{n}} c^m e x^{m+2} \Gamma\left(\frac{m}{n}+\frac{2}{n}\right) {}_2F_1\left(\left.\begin{matrix} -p, \frac{m}{n}+\frac{2}{n} \\ \frac{m}{n}+1+\frac{2}{n} \end{matrix}\right| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{2}{n}\right)} + \frac{a^{\frac{m}{n}+\frac{3}{n}} a^{-\frac{m}{n}+p-\frac{3}{n}} c^m f x^{m+3} \Gamma\left(\frac{m}{n}+\frac{3}{n}\right) {}_2F_1\left(\left.\begin{matrix} -p, \frac{m}{n}+\frac{3}{n} \\ \frac{m}{n}+1+\frac{3}{n} \end{matrix}\right| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{3}{n}\right)} + \frac{a^{\frac{m}{n}+\frac{4}{n}} a^{-\frac{m}{n}+p-\frac{4}{n}} c^m g x^{m+4} \Gamma\left(\frac{m}{n}+\frac{4}{n}\right) {}_2F_1\left(\left.\begin{matrix} -p, \frac{m}{n}+\frac{4}{n} \\ \frac{m}{n}+1+\frac{4}{n} \end{matrix}\right| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{4}{n}\right)}$$

input
```
integrate((c*x)**m*(g*x**3+f*x**2+e*x+d)*(a+b*x**n)**p,x)
```

output
```
a**(m/n + 1/n)*a**(-m/n + p - 1/n)*c**m*d*x**(m + 1)*gamma(m/n + 1/n)*hype
r((-p, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/
n + 1 + 1/n)) + a**(m/n + 2/n)*a**(-m/n + p - 2/n)*c**m*e*x**(m + 2)*gamma
(m/n + 2/n)*hyper((-p, m/n + 2/n), (m/n + 1 + 2/n,), b*x**n*exp_polar(I*pi
)/a)/(n*gamma(m/n + 1 + 2/n)) + a**(m/n + 3/n)*a**(-m/n + p - 3/n)*c**m*f*
x**(m + 3)*gamma(m/n + 3/n)*hyper((-p, m/n + 3/n), (m/n + 1 + 3/n,), b*x**
n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1 + 3/n)) + a**(m/n + 4/n)*a**(-m/n +
p - 4/n)*c**m*g*x**(m + 4)*gamma(m/n + 4/n)*hyper((-p, m/n + 4/n), (m/n +
1 + 4/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1 + 4/n))
```

3.586.7 Maxima [F]

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right) (a+bx^n)^p \, dx = \int \left(gx^3+fx^2+ex+d\right)(bx^n+a)^p(cx)^m \, dx$$

input
```
integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

3.586.8 Giac [F]

$$\int (cx)^m \left(d+ex+fx^2+gx^3\right) (a+bx^n)^p \, dx = \int \left(gx^3+fx^2+ex+d\right)(bx^n+a)^p(cx)^m \, dx$$

input
```
integrate((c*x)^m*(g*x^3+f*x^2+e*x+d)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((g*x^3 + f*x^2 + e*x + d)*(b*x^n + a)^p*(c*x)^m, x)
```

3.586.9 Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(d + ex + fx^2 + gx^3\right) (a + bx^n)^p \, dx = \int (c\,x)^m \, (a + b\,x^n)^p \, \left(g\,x^3 + f\,x^2 + e\,x + d\right) \, dx$$

input
```
int((c*x)^m*(a + b*x^n)^p*(d + e*x + f*x^2 + g*x^3),x)
```

output
```
int((c*x)^m*(a + b*x^n)^p*(d + e*x + f*x^2 + g*x^3), x)
```

3.587 $\int (cx)^m (a+bx^n)^p (d+ex^n+fx^{2n}+gx^{3n})\,dx$

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3.587.1 Optimal result

Integrand size = 36, antiderivative size = 297

$$\begin{aligned}\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right)\,dx &= \frac{d(cx)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{c(1+m)}\\&+\frac{ex^{1+n}(cx)^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m+n}{n},-p,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{1+m+n}\\&+\frac{fx^{1+2n}(cx)^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m+2n}{n},-p,\frac{1+m+3n}{n},-\frac{bx^n}{a}\right)}{1+m+2n}\\&+\frac{gx^{1+3n}(cx)^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m+3n}{n},-p,\frac{1+m+4n}{n},-\frac{bx^n}{a}\right)}{1+m+3n}\end{aligned}$$

output
```
d*(c*x)^(1+m)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/c/
(1+m)/((1+b*x^n/a)^p)+e*x^(1+n)*(c*x)^m*(a+b*x^n)^p*hypergeom([-p, (1+m+n)
/n],[(1+m+2*n)/n],-b*x^n/a)/(1+m+n)/((1+b*x^n/a)^p)+f*x^(1+2*n)*(c*x)^m*(a
+b*x^n)^p*hypergeom([-p, (1+m+2*n)/n],[(1+m+3*n)/n],-b*x^n/a)/(1+m+2*n)/((
1+b*x^n/a)^p)+g*x^(1+3*n)*(c*x)^m*(a+b*x^n)^p*hypergeom([-p, (1+m+3*n)/n],
[(1+m+4*n)/n],-b*x^n/a)/(1+m+3*n)/((1+b*x^n/a)^p)
```

3.587.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.69

$$\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right) dx$$
$$= x(cx)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(\frac{d\,\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{1+m} + x^n\left(\frac{e\,\text{Hypergeometric2F1}\left(\frac{1+m+n}{n},-p,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{1+m+n} + x^n\left(\frac{f\,\text{Hypergeometric2F1}\left(\frac{1+m+2n}{n},-p,\frac{1+m+3n}{n},-\frac{bx^n}{a}\right)}{1+m+2n} + \frac{gx^n\,\text{Hypergeometric2F1}\left(\frac{1+m+3n}{n},-p,\frac{1+m+4}{n}\right.}{1+m+3n}\right.\right.\right.$$

input
```
Integrate[(c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x]
```

output
```
(x*(c*x)^m*(a + b*x^n)^p*((d*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/
n, -((b*x^n)/a)])/(1 + m) + x^n*((e*Hypergeometric2F1[(1 + m + n)/n, -p, (
1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n) + x^n*((f*Hypergeometric2F1[(1
+ m + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/(1 + m + 2*n) + (g*x^n*H
ypergeometric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)])/(1 +
 m + 3*n)))))/(1 + (b*x^n)/a)^p
```

3.587.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right) dx$$
$$\Big\downarrow \text{2383}$$
$$\int \left(d(cx)^m (a+bx^n)^p+ex^n(cx)^m (a+bx^n)^p+fx^{2n}(cx)^m (a+bx^n)^p+gx^{3n}(cx)^m (a+bx^n)^p\right) dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{d(cx)^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{c(m+1)}+\frac{ex^{n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+n+1}{n},-p,\frac{m+2n+1}{n},-\frac{bx^n}{a}\right)}{m+n+1}+\frac{fx^{2n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+2n+1}{n},-p,\frac{m+3n+1}{n},-\frac{bx^n}{a}\right)}{m+2n+1}+\frac{gx^{3n+1}(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+3n+1}{n},-p,\frac{m+4n+1}{n},-\frac{bx^n}{a}\right)}{m+3n+1}$$

input
```
Int[(c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x]
```

output
```
(d*(c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n
)/n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p) + (e*x^(1 + n)*(c*x)^m*(
a + b*x^n)^p*Hypergeometric2F1[(1 + m + n)/n, -p, (1 + m + 2*n)/n, -((b*x^
n)/a)])/((1 + m + n)*(1 + (b*x^n)/a)^p) + (f*x^(1 + 2*n)*(c*x)^m*(a + b*x^
n)^p*Hypergeometric2F1[(1 + m + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)]
)/((1 + m + 2*n)*(1 + (b*x^n)/a)^p) + (g*x^(1 + 3*n)*(c*x)^m*(a + b*x^n)^p
*Hypergeometric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)])/((
1 + m + 3*n)*(1 + (b*x^n)/a)^p)
```

3.587.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2383
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]
```

3.587.4 Maple [F]

$$\int (cx)^m (a+b\,x^n)^p \left(d+e\,x^n+f\,x^{2n}+g\,x^{3n}\right)dx$$

input
```
int((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x)
```

output
```
int((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x)
```

3.587.5 Fricas [F]

$$\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right) dx = \int (gx^{3n}+fx^{2n}+ex^n+d)(bx^n+a)^p(cx)^m \, dx$$

input
```
integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="
fricas")
```

output
```
integral((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^m, x)
```

3.587.6 Sympy [F(-1)]

Timed out.

$$\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right) dx = \text{Timed out}$$

input
```
integrate((c*x)**m*(a+b*x**n)**p*(d+e*x**n+f*x**(2*n)+g*x**(3*n)),x)
```

output
```
Timed out
```

3.587.7 Maxima [F]

$$\int (cx)^m (a+bx^n)^p \left(d+ex^n+fx^{2n}+gx^{3n}\right) dx = \int (gx^{3n}+fx^{2n}+ex^n+d)(bx^n+a)^p(cx)^m \, dx$$

input
```
integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="
maxima")
```

output
```
integrate((g*x^(3*n) + f*x^(2*n) + e*x^n + d)*(b*x^n + a)^p*(c*x)^m, x)
```

3.587.8 Giac [F(-2)]

Exception generated.

$$\int (cx)^m (a + bx^n)^p \left(d + ex^n + fx^{2n} + gx^{3n}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(a+b*x^n)^p*(d+e*x^n+f*x^(2*n)+g*x^(3*n)),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2,0,6,4,0,2,4,4,1,0,0,0]%%%}+%%%{4,[2,0,6,4,0,2,3,4,1,0
,0,0]%%%}
```

3.587.9 Mupad [F(-1)]

Timed out.

$$\int (cx)^m (a + bx^n)^p \left(d + ex^n + fx^{2n} + gx^{3n}\right) dx$$
$$= \int (c\,x)^m (a + b\,x^n)^p \left(d + e\,x^n + f\,x^{2\,n} + g\,x^{3\,n}\right) dx$$

input
```
int((c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)),x)
```

output
```
int((c*x)^m*(a + b*x^n)^p*(d + e*x^n + f*x^(2*n) + g*x^(3*n)), x)
```

3.588 $\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx$

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3.588.1 Optimal result

Integrand size = 35, antiderivative size = 162

$$\begin{aligned}\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx &= \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\,(a+bx^n)} \\ &- \frac{(bd(2-n)-af(2+n))x^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{a^2bn(2+n)} \\ &+ \frac{(ae-bc(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2bn}\end{aligned}$$

output
```
x*(b*c-a*e+(-a*f+b*d)*x^(1/2*n))/a/b/n/(a+b*x^n)-(b*d*(2-n)-a*f*(2+n))*x^(
1+1/2*n)*hypergeom([1, 1/2+1/n],[3/2+1/n],-b*x^n/a)/a^2/b/n/(2+n)+(a*e-b*c
*(1-n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b/n
```

3.588.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx = \frac{x\left(\frac{2afx^{n/2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-\frac{bx^n}{a}\right)}{2+n}+ae\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},\right.\right.}{}$$

input
```
Integrate[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]
```

output
```
(x*((2*a*f*x^(n/2)*Hypergeometric2F1[1, 1/2 + n^(-1), 3/2 + n^(-1), -((b*x
^n)/a)])/(2 + n) + a*e*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/
a)] + (2*(b*d - a*f)*x^(n/2)*Hypergeometric2F1[2, 1/2 + n^(-1), 3/2 + n^(-
1), -((b*x^n)/a)])/(2 + n) + (b*c - a*e)*Hypergeometric2F1[2, n^(-1), 1 +
n^(-1), -((b*x^n)/a)]))/(a^2*b)
```

3.588.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2431, 1748, 778, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \ 2431$$

$$\frac{\int \frac{2(ae-bc(1-n))-(bd(2-n)-af(n+2))x^{n/2}}{bx^n+a}dx}{2abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{abn\left(a+bx^n\right)}$$

$$\Big\downarrow \ 1748$$

$$\frac{2(ae-bc(1-n))\int \frac{1}{bx^n+a}dx-(bd(2-n)-af(n+2))\int \frac{x^{n/2}}{bx^n+a}dx}{2abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{abn\left(a+bx^n\right)}$$

$$\Big\downarrow \ 778$$

$$\frac{\frac{2x(ae-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}-(bd(2-n)-af(n+2))\int \frac{x^{n/2}}{bx^n+a}dx}{2abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{abn\left(a+bx^n\right)}$$

$$\Big\downarrow \ 888$$

$$\frac{\frac{2x(ae-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}-\frac{2x^{\frac{n+2}{2}}(bd(2-n)-af(n+2))\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{a(n+2)}}{2abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{abn\left(a+bx^n\right)}$$

input
```
Int[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]
```

output
```
(x*(b*c - a*e + (b*d - a*f)*x^(n/2)))/(a*b*n*(a + b*x^n)) + ((-2*(b*d*(2 -
 n) - a*f*(2 + n))*x^((2 + n)/2)*Hypergeometric2F1[1, (1 + 2/n)/2, (3 + 2/
n)/2, -((b*x^n)/a)])/(a*(2 + n)) + (2*(a*e - b*c*(1 - n))*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a)/(2*a*b*n)
```

3.588.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1748
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Simp[d
   Int[1/(a + c*x^(2*n)), x], x] + Simp[e   Int[x^n/(a + c*x^(2*n)), x], x]
 /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (
PosQ[a*c] || !IntegerQ[n])
```

rule 2431
```
Int[(P3_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{A = Coeff[P3, x
^(n/2), 0], B = Coeff[P3, x^(n/2), 1], C = Coeff[P3, x^(n/2), 2], D = Coeff
[P3, x^(n/2), 3]}, Simp[-(x*(b*A - a*C + (b*B - a*D)*x^(n/2))*(a + b*x^n)^(
p + 1))/(a*b*n*(p + 1)), x] - Simp[1/(2*a*b*n*(p + 1))   Int[(a + b*x^n)^(p
 + 1)*Simp[2*a*C - 2*b*A*(n*(p + 1) + 1) + (a*D*(n + 2) - b*B*(n*(2*p + 3)
+ 2))*x^(n/2), x], x], x]] /; FreeQ[{a, b, n}, x] && PolyQ[P3, x^(n/2), 3]
&& ILtQ[p, -1]
```

3.588. $\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2} dx$

3.588.4 Maple [F]

$$\int \frac{c+d\,x^{\frac{n}{2}}+e\,x^{n}+f\,x^{\frac{3n}{2}}}{\left(a+b\,x^{n}\right)^{2}}dx$$

input
```
int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
```

3.588.5 Fricas [F]

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx=\int \frac{fx^{\frac{3}{2}\,n}+dx^{\frac{1}{2}\,n}+ex^n+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="fric
as")
```

output
```
integral((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n
+ a^2), x)
```

3.588.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx=\text{Timed out}$$

input
```
integrate((c+d*x**(1/2*n)+e*x**n+f*x**(3/2*n))/(a+b*x**n)**2,x)
```

output
```
Timed out
```

3.588.7 Maxima [F]

$$\int \frac{c + dx^{n/2} + ex^n + fx^{3n/2}}{(a + bx^n)^2} \, dx = \int \frac{fx^{\frac{3}{2}n} + dx^{\frac{1}{2}n} + ex^n + c}{(bx^n + a)^2} \, dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="maxi
ma")
```

output
```
((b*d - a*f)*x*x^(1/2*n) + (b*c - a*e)*x)/(a*b^2*n*x^n + a^2*b*n) + integr
ate(1/2*(2*b*c*(n - 1) + 2*a*e + (a*f*(n + 2) + b*d*(n - 2))*x^(1/2*n))/(a
*b^2*n*x^n + a^2*b*n), x)
```

3.588.8 Giac [F]

$$\int \frac{c + dx^{n/2} + ex^n + fx^{3n/2}}{(a + bx^n)^2} \, dx = \int \frac{fx^{\frac{3}{2}n} + dx^{\frac{1}{2}n} + ex^n + c}{(bx^n + a)^2} \, dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="giac
")
```

output
```
integrate((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b*x^n + a)^2, x)
```

3.588.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx^{n/2} + ex^n + fx^{3n/2}}{(a + bx^n)^2} \, dx = \int \frac{c + e\,x^n + d\,x^{n/2} + f\,x^{\frac{3n}{2}}}{(a + b\,x^n)^2} \, dx$$

input
```
int((c + e*x^n + d*x^(n/2) + f*x^((3*n)/2))/(a + b*x^n)^2,x)
```

output
```
int((c + e*x^n + d*x^(n/2) + f*x^((3*n)/2))/(a + b*x^n)^2, x)
```

3.589 $\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

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3.589.1 Optimal result

Integrand size = 46, antiderivative size = 24

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

output
```
x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

3.589.2 Mathematica [A] (verified)

Time = 7.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

input
```
Integrate[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c +
d*x^2]),x]
```

output
```
x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

3.589.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2(ad+bc)+ac+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{2023}$$

$$x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

input
```
Int[(a*c + 2*(b*c + a*d)*x^2 + 3*b*d*x^4)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
),x]
```

output
```
x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
```

3.589.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.589.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
gosper	$x\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}$	21
default	$x\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}$	21
risch	$x\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}$	21
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,x\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$	62

input
```
int((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

3.589.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\sqrt{bx^2+a}\sqrt{dx^2+c}x$$

input
```
integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),
x, algorithm="fricas")
```

output
```
sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x
```

3.589.6 Sympy [F]

$$\int \frac{ac+2(bc+ad)x^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{ac+2adx^2+2bcx^2+3bdx^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((a*c+2*(a*d+b*c)*x**2+3*b*d*x**4)/(b*x**2+a)**(1/2)/(d*x**2+c)**
(1/2),x)
```

output
```
Integral((a*c + 2*a*d*x**2 + 2*b*c*x**2 + 3*b*d*x**4)/(sqrt(a + b*x**2)*sq
rt(c + d*x**2)), x)
```

3.589.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{ac + 2(bc + ad)x^2 + 3bdx^4}{\sqrt{a + bx^2}\sqrt{c + dx^2}}\,dx = \sqrt{bx^2 + a}\sqrt{dx^2 + c}x$$

input
```
integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),
x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x
```

3.589.8 Giac [F]

$$\int \frac{ac + 2(bc + ad)x^2 + 3bdx^4}{\sqrt{a + bx^2}\sqrt{c + dx^2}}\,dx = \int \frac{3\,bdx^4 + 2\,(bc + ad)x^2 + ac}{\sqrt{bx^2 + a}\sqrt{dx^2 + c}}\,dx$$

input
```
integrate((a*c+2*(a*d+b*c)*x^2+3*b*d*x^4)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),
x, algorithm="giac")
```

output
```
integrate((3*b*d*x^4 + 2*(b*c + a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*sqrt(d*x^
2 + c)), x)
```

3.589.9 Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{ac + 2(bc + ad)x^2 + 3bdx^4}{\sqrt{a + bx^2}\sqrt{c + dx^2}}\,dx = x\,\sqrt{b\,x^2 + a}\,\sqrt{d\,x^2 + c}$$

input
```
int((a*c + 2*x^2*(a*d + b*c) + 3*b*d*x^4)/((a + b*x^2)^(1/2)*(c + d*x^2)^(
1/2)),x)
```

output
```
x*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)
```

3.590 $\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx$

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3.590.1 Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} - \frac{\arctan\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1+x^4}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

output
```
1/4*arctan(2^(1/4)*x/(x^4+1)^(1/4))*2^(3/4)-1/4*arctan(1/2*(x^4+1)^(1/4)*2
^(3/4))*2^(3/4)+1/4*arctanh(2^(1/4)*x/(x^4+1)^(1/4))*2^(3/4)+1/4*arctanh(1
/2*(x^4+1)^(1/4)*2^(3/4))*2^(3/4)
```

3.590.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = \frac{1}{4}x^4\operatorname{AppellF1}\left(1,\frac{1}{4},1,2,-x^4,x^4\right) + \frac{2\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right) - \log\left(1-\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right) + \log\left(1+\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[4]{2}}$$

input
```
Integrate[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

output
```
(x^4*AppellF1[1, 1/4, 1, 2, -x^4, x^4])/4 + (2*ArcTan[(2^(1/4)*x)/(1 + x^4
)^(1/4)] - Log[1 - (2^(1/4)*x)/(1 + x^4)^(1/4)] + Log[1 + (2^(1/4)*x)/(1 +
 x^4)^(1/4)])/(4*2^(1/4))
```

3.590.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2438, 902, 756, 216, 219, 946, 73, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+1}{(1-x^4)\sqrt[4]{x^4+1}}dx$$

$$\Big\downarrow \text{2438}$$

$$\int \frac{1}{(1-x^4)\sqrt[4]{x^4+1}}dx+\int \frac{x^3}{(1-x^4)\sqrt[4]{x^4+1}}dx$$

$$\Big\downarrow \text{902}$$

$$\int \frac{1}{1-\frac{2x^4}{x^4+1}}d\frac{x}{\sqrt[4]{x^4+1}}+\int \frac{x^3}{(1-x^4)\sqrt[4]{x^4+1}}dx$$

$$\Big\downarrow \text{756}$$

$$\int \frac{x^3}{(1-x^4)\sqrt[4]{x^4+1}}dx+\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{2}x^2}{\sqrt{x^4+1}}}d\frac{x}{\sqrt[4]{x^4+1}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{2}x^2}{\sqrt{x^4+1}}+1}d\frac{x}{\sqrt[4]{x^4+1}}$$

$$\Big\downarrow \text{216}$$

$$\int \frac{x^3}{(1-x^4)\sqrt[4]{x^4+1}}dx+\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{2}x^2}{\sqrt{x^4+1}}}d\frac{x}{\sqrt[4]{x^4+1}}+\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}$$

$$\Big\downarrow \text{219}$$

$$\int \frac{x^3}{(1-x^4)\sqrt[4]{x^4+1}}dx+\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{4}\int \frac{1}{(1-x^4)\sqrt[4]{x^4+1}}dx^4+\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}$$

$$\Big\downarrow \text{73}$$

$$\int \frac{x^8}{2-x^{16}}d\sqrt[4]{x^4+1}+\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{2}-x^8}d\sqrt[4]{x^4+1}-\frac{1}{2}\int \frac{1}{x^8+\sqrt{2}}d\sqrt[4]{x^4+1}+\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{2}-x^8}d\sqrt[4]{x^4+1}+\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\arctan\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\arctan\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{x^4+1}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

input
```
Int[(1 + x^3)/((1 - x^4)*(1 + x^4)^(1/4)),x]
```

output
```
ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) - ArcTan[(1 + x^4)^(1/4)/2
^(1/4)]/(2*2^(1/4)) + ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) + A
rcTanh[(1 + x^4)^(1/4)/2^(1/4)]/(2*2^(1/4))
```

3.590.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.590. $\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx$

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2438
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)
*(x_)^(n_))^(q_.), x_Symbol] :> Simp[A   Int[(a + b*x^n)^p*(c + d*x^n)^q, x
], x] + Simp[B   Int[x^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, A, B, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]
```

3.590. $\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx$

3.590.4 Maple [F(-1)]

Timed out.

$$\int \frac{x^3+1}{(-x^4+1)(x^4+1)^{\frac{1}{4}}}dx$$

input
```
int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x)
```

output
```
int((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x)
```

3.590.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (residue poly has multiple non-linear fac
tors)
```

3.590.6 Sympy [F]

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = -\int \left(-\frac{x}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\right)dx - \int \frac{x^2}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\,dx - \int \frac{1}{x^3\sqrt[4]{x^4+1}-x^2\sqrt[4]{x^4+1}+x\sqrt[4]{x^4+1}-\sqrt[4]{x^4+1}}\,dx$$

input
```
integrate((x**3+1)/(-x**4+1)/(x**4+1)**(1/4),x)
```

output
```
-Integral(-x/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 +
1)**(1/4) - (x**4 + 1)**(1/4)), x) - Integral(x**2/(x**3*(x**4 + 1)**(1/4)
 - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1)**(1/4) - (x**4 + 1)**(1/4)), x) -
 Integral(1/(x**3*(x**4 + 1)**(1/4) - x**2*(x**4 + 1)**(1/4) + x*(x**4 + 1
)**(1/4) - (x**4 + 1)**(1/4)), x)
```

3.590.7 Maxima [F]

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = \int -\frac{x^3+1}{(x^4+1)^{\frac{1}{4}}(x^4-1)}\,dx$$

input
```
integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="maxima")
```

output
```
-integrate((x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)
```

3.590.8 Giac [F]

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = \int -\frac{x^3+1}{(x^4+1)^{\frac{1}{4}}(x^4-1)}\,dx$$

input
```
integrate((x^3+1)/(-x^4+1)/(x^4+1)^(1/4),x, algorithm="giac")
```

output
```
integrate(-(x^3 + 1)/((x^4 + 1)^(1/4)*(x^4 - 1)), x)
```

3.590.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^3}{(1-x^4)\sqrt[4]{1+x^4}}\,dx = \int -\frac{x^3+1}{(x^4-1)\,(x^4+1)^{1/4}}\,dx$$

input
```
int(-(x^3 + 1)/((x^4 - 1)*(x^4 + 1)^(1/4)),x)
```

output
```
int(-(x^3 + 1)/((x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

3.591 $\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} \left(ac-bdx^{2n}\right) dx$

3.591.1 Optimal result 4463
3.591.2 Mathematica [A] (verified) 4463
3.591.3 Rubi [A] (verified) 4464
3.591.4 Maple [F] 4464
3.591.5 Fricas [B] (verification not implemented) 4465
3.591.6 Sympy [F(-1)] 4465
3.591.7 Maxima [F] 4465
3.591.8 Giac [B] (verification not implemented) 4466
3.591.9 Mupad [B] (verification not implemented) 4466

3.591.1 Optimal result

Integrand size = 48, antiderivative size = 28

$$\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} \left(ac-bdx^{2n}\right) dx = x(a+bx^n)^{-1/n} (c+dx^n)^{-1/n}$$

output
```
x/((a+b*x^n)^(1/n))/((c+d*x^n)^(1/n))
```

3.591.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^{\frac{-1-n}{n}} (c+dx^n)^{\frac{-1-n}{n}} \left(ac-bdx^{2n}\right) dx = x(a+bx^n)^{-1/n} (c+dx^n)^{-1/n}$$

input
```
Integrate[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(
2*n)),x]
```

output
```
x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

3.591.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {2437}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^{\frac{-n-1}{n}}(c+dx^n)^{\frac{-n-1}{n}}\left(ac-bdx^{2n}\right)dx$$

$$\Big\downarrow \text{ 2437}$$

$$x(a+bx^n)^{-1/n}(c+dx^n)^{-1/n}$$

input
```
Int[(a + b*x^n)^((-1 - n)/n)*(c + d*x^n)^((-1 - n)/n)*(a*c - b*d*x^(2*n)),
x]
```

output
```
x/((a + b*x^n)^n^(-1)*(c + d*x^n)^n^(-1))
```

3.591.3.1 Defintions of rubi rules used

rule 2437
```
Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) +
 (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[e*x*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(p + 1)/(a*c)), x] /; FreeQ[{a, b, c, d, e, g, n, p}, x] && EqQ[n2, 2*n] &
& EqQ[n*(p + 1) + 1, 0] && EqQ[a*c*g - b*d*e*(2*n*(p + 1) + 1), 0]
```

3.591.4 Maple [F]

$$\int (a+b\,x^n)^{\frac{-1-n}{n}}(c+d\,x^n)^{\frac{-1-n}{n}}\left(ac-bd\,x^{2n}\right)dx$$

input
```
int((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x)
```

output
```
int((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x)
```

3.591.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(28) = 56$.

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}\left(ac-bdx^{2n}\right)\,dx=\frac{bdxx^{2\,n}+acx+(bc+ad)xx^n}{(bx^n+a)^{\frac{n+1}{n}}(dx^n+c)^{\frac{n+1}{n}}}$$

input
```
integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, a
lgorithm="fricas")
```

output
```
(b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)/((b*x^n + a)^((n + 1)/n)*(d*x^
n + c)^((n + 1)/n))
```

3.591.6 Sympy [F(-1)]

Timed out.

$$\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}\left(ac-bdx^{2n}\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*x**n)**((-1-n)/n)*(c+d*x**n)**((-1-n)/n)*(a*c-b*d*x**(2*n))
,x)
```

output
```
Timed out
```

3.591.7 Maxima [F]

$$\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}\left(ac-bdx^{2n}\right)\,dx=\int -\frac{bdx^{2\,n}-ac}{(bx^n+a)^{\frac{n+1}{n}}(dx^n+c)^{\frac{n+1}{n}}}\,dx$$

input
```
integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, a
lgorithm="maxima")
```

output
```
-integrate((b*d*x^(2*n) - a*c)/((b*x^n + a)^((n + 1)/n)*(d*x^n + c)^((n +
1)/n)), x)
```

3.591.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(28) = 56$.

Time = 0.33 (sec) , antiderivative size = 228, normalized size of antiderivative = 8.14

$$\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}(ac-bdx^{2n})\,dx$$
$$= bdxx^{2n}e^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}$$
$$+ bcxx^ne^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}$$
$$+ adxx^ne^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}$$
$$+ acxe^{\left(-\frac{n\log(bx^n+a)+\log(bx^n+a)}{n}-\frac{n\log(dx^n+c)+\log(dx^n+c)}{n}\right)}$$

input
```
integrate((a+b*x^n)^((-1-n)/n)*(c+d*x^n)^((-1-n)/n)*(a*c-b*d*x^(2*n)),x, a
lgorithm="giac")
```

output
```
b*d*x*x^(2*n)*e^(-(n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c
) + log(d*x^n + c))/n) + b*c*x*x^n*e^(-(n*log(b*x^n + a) + log(b*x^n + a))
/n - (n*log(d*x^n + c) + log(d*x^n + c))/n) + a*d*x*x^n*e^(-(n*log(b*x^n +
 a) + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n + c))/n) + a*c*x*e
^(-(n*log(b*x^n + a) + log(b*x^n + a))/n - (n*log(d*x^n + c) + log(d*x^n +
 c))/n)
```

3.591.9 Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.39

$$\int (a+bx^n)^{\frac{-1-n}{n}}(c+dx^n)^{\frac{-1-n}{n}}(ac-bdx^{2n})\,dx = \frac{\frac{a\,c\,x}{(a+b\,x^n)^{\frac{n+1}{n}}}+\frac{x\,x^n\,(a\,d+b\,c)}{(a+b\,x^n)^{\frac{n+1}{n}}}+\frac{b\,d\,x\,x^{2\,n}}{(a+b\,x^n)^{\frac{n+1}{n}}}}{(c+d\,x^n)^{\frac{n+1}{n}}}$$

input
```
int((a*c - b*d*x^(2*n))/((a + b*x^n)^((n + 1)/n)*(c + d*x^n)^((n + 1)/n)),
x)
```

output
```
((a*c*x)/(a + b*x^n)^((n + 1)/n) + (x*x^n*(a*d + b*c))/(a + b*x^n)^((n + 1
)/n) + (b*d*x*x^(2*n))/(a + b*x^n)^((n + 1)/n))/(c + d*x^n)^((n + 1)/n)
```

3.592 $\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p (ac-bdx^{2n}) \, dx$

3.592.1 Optimal result 4467
3.592.2 Mathematica [A] (verified) 4467
3.592.3 Rubi [A] (verified) 4468
3.592.4 Maple [C] (warning: unable to verify) 4468
3.592.5 Fricas [B] (verification not implemented) 4469
3.592.6 Sympy [F(-2)] 4469
3.592.7 Maxima [A] (verification not implemented) 4470
3.592.8 Giac [B] (verification not implemented) 4470
3.592.9 Mupad [B] (verification not implemented) 4471

3.592.1 Optimal result

Integrand size = 45, antiderivative size = 45

$$\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p (ac-bdx^{2n}) \, dx = -\frac{(hx)^{-n(1+p)} (a+bx^n)^{1+p} (c+dx^n)^{1+p}}{hn(1+p)}$$

output
```
-(a+b*x^n)^(p+1)*(c+d*x^n)^(p+1)/h/n/(p+1)/((h*x)^(n*(p+1)))
```

3.592.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int (hx)^{-1-n-np} (a+bx^n)^p (c+dx^n)^p (ac-bdx^{2n}) \, dx = -\frac{(hx)^{-n-np} (a+bx^n)^{1+p} (c+dx^n)^{1+p}}{hn+hnp}$$

input
```
Integrate[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2
*n)),x]
```

output
```
-(((h*x)^(-n - n*p)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n + h*n*p)
)
```

3.592.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2388}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (hx)^{n(-p)-n-1} (a+bx^n)^p (c+dx^n)^p \left(ac-bdx^{2n}\right) \, dx$$

$$\Big\downarrow \text{ 2388}$$

$$-\frac{(hx)^{-n(p+1)} (a+bx^n)^{p+1} (c+dx^n)^{p+1}}{hn(p+1)}$$

input
```
Int[(h*x)^(-1 - n - n*p)*(a + b*x^n)^p*(c + d*x^n)^p*(a*c - b*d*x^(2*n)),x
]
```

output
```
-(((a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(h*n*(1 + p)*(h*x)^(n*(1 + p))
))
```

3.592.3.1 Defintions of rubi rules used

rule 2388
```
Int[((h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(
n_.))^(p_.)*((e_) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[e*(h*x)^(m + 1)*(
a + b*x^n)^(p + 1)*((c + d*x^n)^(p + 1)/(a*c*h*(m + 1))), x] /; FreeQ[{a, b
, c, d, e, g, h, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[m + n*(p + 1) + 1, 0]
&& EqQ[a*c*g*(m + 1) - b*d*e*(m + 2*n*(p + 1) + 1), 0] && NeQ[m, -1]
```

3.592.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.07

$$-\frac{(a+b\,x^n)^p\,\mathrm{e}^{-\frac{(np+n+1)\left(-i\pi\,\mathrm{csgn}(ihx)^3+i\pi\,\mathrm{csgn}(ihx)^2\,\mathrm{csgn}(ih)+i\pi\,\mathrm{csgn}(ihx)^2\,\mathrm{csgn}(ix)-i\pi\,\mathrm{csgn}(ihx)\,\mathrm{csgn}(ih)\,\mathrm{csgn}(ix)+2\ln(x)+2\ln(h)\right)}{2}}\,(bd\,x^{2n}+c}{n\,(1+p)}$$

input
```
int((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x)
```

output
```
-(a+b*x^n)^p*exp(-1/2*(n*p+n+1)*(-I*Pi*csgn(I*h*x)^3+I*Pi*csgn(I*h*x)^2*cs
gn(I*h)+I*Pi*csgn(I*h*x)^2*csgn(I*x)-I*Pi*csgn(I*h*x)*csgn(I*h)*csgn(I*x)+
2*ln(x)+2*ln(h)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*x/n/(1+p)*(c+d*x^n)^p
```

3.592.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.64

$$\int (hx)^{-1-n-np}(a+bx^n)^p(c+dx^n)^p\left(ac-bdx^{2n}\right)\,dx = -\frac{\left(bdxx^{2n}e^{(-(np+n+1)\log(h)-(np+n+1)\log(x))}+acxe^{(-(np+n+1)\log(h)-(np+n+1)\log(x))}+(bc+ad)xx^ne^{(-(np+n+1)}\right.}{np+n}$$

input
```
integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, al
gorithm="fricas")
```

output
```
-(b*d*x*x^(2*n)*e^(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x)) + a*c*x*e
^(-(n*p + n + 1)*log(h) - (n*p + n + 1)*log(x)) + (b*c + a*d)*x*x^n*e^(-(n
*p + n + 1)*log(h) - (n*p + n + 1)*log(x)))*(b*x^n + a)^p*(d*x^n + c)^p/(n
*p + n)
```

3.592.6 Sympy [F(-2)]

Exception generated.

$$\int (hx)^{-1-n-np}(a+bx^n)^p(c+dx^n)^p\left(ac-bdx^{2n}\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x)**(-n*p-n-1)*(a+b*x**n)**p*(c+d*x**n)**p*(a*c-b*d*x**(2*n))
,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.592.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.71

$$\int (hx)^{-1-n-np}(a+bx^n)^p(c+dx^n)^p(ac-bdx^{2n})\,dx$$
$$=-\frac{(bdx^{2n}+ac+(bc+ad)x^n)h^{-np-n-1}e^{(-np\log(x)+p\log(bx^n+a)+p\log(dx^n+c)-n\log(x))}}{n(p+1)}$$

input
```
integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, al
gorithm="maxima")
```

output
```
-(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n)*h^(-n*p - n - 1)*e^(-n*p*log(x) + p
*log(b*x^n + a) + p*log(d*x^n + c) - n*log(x))/(n*(p + 1))
```

3.592.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(46) = 92.

Time = 0.35 (sec) , antiderivative size = 237, normalized size of antiderivative = 5.27

$$\int (hx)^{-1-n-np}(a+bx^n)^p(c+dx^n)^p(ac-bdx^{2n})\,dx =$$
$$-\frac{(bx^n+a)^p(dx^n+c)^pbdxx^{2n}e^{(-np\log(h)-np\log(x)-n\log(h)-n\log(x)-\log(h)-\log(x))}+(bx^n+a)^p(dx^n+c)^pbcxx^n e}{}$$

input
```
integrate((h*x)^(-n*p-n-1)*(a+b*x^n)^p*(c+d*x^n)^p*(a*c-b*d*x^(2*n)),x, al
gorithm="giac")
```

output
```
-((b*x^n + a)^p*(d*x^n + c)^p*b*d*x*x^(2*n)*e^(-n*p*log(h) - n*p*log(x) -
n*log(h) - n*log(x) - log(h) - log(x)) + (b*x^n + a)^p*(d*x^n + c)^p*b*c*x
*x^n*e^(-n*p*log(h) - n*p*log(x) - n*log(h) - n*log(x) - log(h) - log(x))
+ (b*x^n + a)^p*(d*x^n + c)^p*a*d*x*x^n*e^(-n*p*log(h) - n*p*log(x) - n*lo
g(h) - n*log(x) - log(h) - log(x)) + (b*x^n + a)^p*(d*x^n + c)^p*a*c*x*e^(
-n*p*log(h) - n*p*log(x) - n*log(h) - n*log(x) - log(h) - log(x)))/(n*p +
n)
```

3.592.9 Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.76

$$\int (hx)^{-1-n-np}(a+bx^n)^p (c+dx^n)^p \left(ac-bdx^{2n}\right) dx$$
$$= -(c+d\,x^n)^p \left(\frac{a\,c\,x\,(a+b\,x^n)^p}{n\,(h\,x)^{n+n\,p+1}\,(p+1)} + \frac{x\,x^n\,(a\,d+b\,c)\,(a+b\,x^n)^p}{n\,(h\,x)^{n+n\,p+1}\,(p+1)} + \frac{b\,d\,x\,x^{2\,n}\,(a+b\,x^n)^p}{n\,(h\,x)^{n+n\,p+1}\,(p+1)}\right)$$

input
```
int(((a*c - b*d*x^(2*n))*(a + b*x^n)^p*(c + d*x^n)^p)/(h*x)^(n + n*p + 1),
x)
```

output
```
-(c + d*x^n)^p*((a*c*x*(a + b*x^n)^p)/(n*(h*x)^(n + n*p + 1)*(p + 1)) + (x
*x^n*(a*d + b*c)*(a + b*x^n)^p)/(n*(h*x)^(n + n*p + 1)*(p + 1)) + (b*d*x*x
^(2*n)*(a + b*x^n)^p)/(n*(h*x)^(n + n*p + 1)*(p + 1)))
```

3.593 $\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1$

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3.593.1 Optimal result

Integrand size = 69, antiderivative size = 31

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx = \frac{ex(a+bx^n)^{1+p} (c+dx^n)^{1+p}}{ac}$$

output
```
e*x*(a+b*x^n)^(p+1)*(c+d*x^n)^(p+1)/a/c
```

3.593.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx = \frac{ex(a+bx^n)^{1+p} (c+dx^n)^{1+p}}{ac}$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^
n)/(a*c) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)),x]
```

output
```
(e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

3.593.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.014$, Rules used = {2436}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p (c+dx^n)^p \left(\frac{bde(2np+2n+1)x^{2n}}{ac}+\frac{e(np+n+1)x^n(ad+bc)}{ac}+e\right) dx$$
$$\Big\downarrow \; 2436$$
$$\frac{ex(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ac}$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + n + n*p)*x^n)/(a*
c) + (b*d*e*(1 + 2*n + 2*n*p)*x^(2*n))/(a*c)),x]
```

output
```
(e*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c)
```

3.593.3.1 Defintions of rubi rules used

rule 2436
```
Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(p_.)*((e_) +
 (f_.)*(x_)^(n_.) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Simp[e*x*(a + b*x^n)^(
p + 1)*((c + d*x^n)^(p + 1)/(a*c)), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}
, x] && EqQ[n2, 2*n] && EqQ[a*c*f - e*(b*c + a*d)*(n*(p + 1) + 1), 0] && Eq
Q[a*c*g - b*d*e*(2*n*(p + 1) + 1), 0]
```

3.593.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

$$\frac{(a+b\,x^n)^p\,(bd\,x^{2n}+ad\,x^n+bc\,x^n+ac)\,ex(c+d\,x^n)^p}{ac}$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(2*n*p+
2*n+1)*x^(2*n)/a/c),x)
```

output
```
(a+b*x^n)^p*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c*(c+d*x^n)^p
```

3.593.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx = \frac{(bdexx^{2n}+acex+(bc+ad)exx^n)(bx^n+a)^p(dx^n+c)^p}{ac}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(
2*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="fricas")
```

output
```
(b*d*e*x*x^(2*n) + a*c*e*x + (b*c + a*d)*e*x*x^n)*(b*x^n + a)^p*(d*x^n + c
)^p/(a*c)
```

3.593.6 Sympy [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+n+1)*x**n/a/c+b*
d*e*(2*n*p+2*n+1)*x**(2*n)/a/c),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.593.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx = \frac{(bdexx^{2n}+acex+(bce+ade)xx^n)e^{(p\log(bx^n+a)+p\log(dx^n+c))}}{ac}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(
2*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="maxima")
```

output
```
(b*d*e*x*x^(2*n) + a*c*e*x + (b*c*e + a*d*e)*x*x^n)*e^(p*log(b*x^n + a) +
p*log(d*x^n + c))/(a*c)
```

3.593.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(31) = 62.

Time = 0.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.58

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx$$
$$= \frac{(bx^n+a)^p(dx^n+c)^p bdexx^{2n}+(bx^n+a)^p(dx^n+c)^p bcexx^n+(bx^n+a)^p(dx^n+c)^p adexx^n+(bx^n+a)^p(}{ac}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+n+1)*x^n/a/c+b*d*e*(
2*n*p+2*n+1)*x^(2*n)/a/c),x, algorithm="giac")
```

output
```
((b*x^n + a)^p*(d*x^n + c)^p*b*d*e*x*x^(2*n) + (b*x^n + a)^p*(d*x^n + c)^p
*b*c*e*x*x^n + (b*x^n + a)^p*(d*x^n + c)^p*a*d*e*x*x^n + (b*x^n + a)^p*(d*
x^n + c)^p*a*c*e*x)/(a*c)
```

3.593.9 Mupad [B] (verification not implemented)

Time = 10.78 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+n+np)x^n}{ac}+\frac{bde(1+2n+2np)x^{2n}}{ac}\right) dx$$
$$= (c+d\,x^n)^p \left(e\,x\,(a+b\,x^n)^p+\frac{e\,x\,x^n\,(a\,d+b\,c)\,(a+b\,x^n)^p}{a\,c}+\frac{b\,d\,e\,x\,x^{2\,n}\,(a+b\,x^n)^p}{a\,c}\right)$$

input
```
int((a + b*x^n)^p*(c + d*x^n)^p*(e + (e*x^n*(a*d + b*c)*(n + n*p + 1))/(a*
c) + (b*d*e*x^(2*n)*(2*n + 2*n*p + 1))/(a*c)),x)
```

output
```
(c + d*x^n)^p*(e*x*(a + b*x^n)^p + (e*x*x^n*(a*d + b*c)*(a + b*x^n)^p)/(a*
c) + (b*d*e*x*x^(2*n)*(a + b*x^n)^p)/(a*c))
```

3.594 $\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)}{ac(1+m)}\right.$

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3.594.1 Optimal result

Integrand size = 86, antiderivative size = 45

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \frac{e(hx)^{1+m} (a+bx^n)^{1+p} (c+dx^n)^{1+p}}{ach(1+m)}$$

output
```
e*(h*x)^(1+m)*(a+b*x^n)^(p+1)*(c+d*x^n)^(p+1)/a/c/h/(1+m)
```

3.594.2 Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \frac{ex(hx)^m (a+bx^n)^{1+p} (c+dx^n)^{1+p}}{ac(1+m)}$$

input
```
Integrate[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m +
 n + n*p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*x^(2*n))/(a*c*
(1 + m))),x]
```

output
```
(e*x*(h*x)^m*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*(1 + m))
```

3.594.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.012$, Rules used = {2387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(\frac{bdex^{2n}(m+2np+2n+1)}{ac(m+1)}+\frac{ex^n(m+np+n+1)(ad+bc)}{ac(m+1)}+e\right)dx$$

$$\Big\downarrow \text{ 2387}$$

$$\frac{e(hx)^{m+1}(a+bx^n)^{p+1}(c+dx^n)^{p+1}}{ach(m+1)}$$

input
```
Int[(h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + ((b*c + a*d)*e*(1 + m + n + n
*p)*x^n)/(a*c*(1 + m)) + (b*d*e*(1 + m + 2*n + 2*n*p)*x^(2*n))/(a*c*(1 + m
))),x]
```

output
```
(e*(h*x)^(1 + m)*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + p))/(a*c*h*(1 + m))
```

3.594.3.1 Defintions of rubi rules used

rule 2387
```
Int[((h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(
n_.))^(p_.)*((e_) + (f_.)*(x_)^(n_.) + (g_.)*(x_)^(n2_.)), x_Symbol] :> Sim
p[e*(h*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(p + 1)/(a*c*h*(m + 1)))
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[
a*c*f*(m + 1) - e*(b*c + a*d)*(m + n*(p + 1) + 1), 0] && EqQ[a*c*g*(m + 1)
- b*d*e*(m + 2*n*(p + 1) + 1), 0] && NeQ[m, -1]
```

3.594.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

$$\frac{(a+b\,x^n)^p\,x^m h^m \mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(ihx)m(\mathrm{csgn}(ihx)-\mathrm{csgn}(ix))(-\mathrm{csgn}(ihx)+\mathrm{csgn}(ih))}{2}}\left(bd\,x^{2n}+ad\,x^n+bc\,x^n+ac\right)ex(c+d\,x^n)^p}{ac\,(1+m)}$$

input
```
int((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a/c/(1+
m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x)
```

output
```
(a+b*x^n)^p*x^m*h^m*exp(1/2*I*Pi*csgn(I*h*x)*m*(csgn(I*h*x)-csgn(I*x))*(-c
sgn(I*h*x)+csgn(I*h)))*(b*d*(x^n)^2+a*d*x^n+b*c*x^n+a*c)*e*x/a/c/(1+m)*(c+
d*x^n)^p
```

3.594.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e + \frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \frac{\left(bdexx^{2n}e^{(m\log(h)+m\log(x))} + acexe^{(m\log(h)+m\log(x))} + (bc+ad)exx^n e^{(m\log(h)+m\log(x))}\right)(bx^n+a)^p(dx^n+c)^p}{acm+ac}$$

input
```
integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a
/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="fricas")
```

output
```
(b*d*e*x*x^(2*n)*e^(m*log(h) + m*log(x)) + a*c*e*x*e^(m*log(h) + m*log(x))
 + (b*c + a*d)*e*x*x^n*e^(m*log(h) + m*log(x)))*(b*x^n + a)^p*(d*x^n + c)^
p/(a*c*m + a*c)
```

3.594.6 Sympy [F(-2)]

Exception generated.

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e + \frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((h*x)**m*(a+b*x**n)**p*(c+d*x**n)**p*(e+(a*d+b*c)*e*(n*p+m+n+1)*
x**n/a/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x**(2*n)/a/c/(1+m)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.594.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.04

$$\int (hx)^m (a + bx^n)^p (c + dx^n)^p \left(e + \frac{(bc + ad)e(1 + m + n + np)x^n}{ac(1 + m)} + \frac{bde(1 + m + 2n + 2np)x^{2n}}{ac(1 + m)}\right) dx = \frac{\left(aceh^m xx^m + bdeh^m xe^{(m\log(x)+2n\log(x))} + (bceh^m + adeh^m)xe^{(m\log(x)+n\log(x))}\right)e^{(p\log(bx^n+a)+p\log(dx^n+c))}}{ac(m+1)}$$

input
```
integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a
/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="maxima")
```

output
```
(a*c*e*h^m*x*x^m + b*d*e*h^m*x*e^(m*log(x) + 2*n*log(x)) + (b*c*e*h^m + a*
d*e*h^m)*x*e^(m*log(x) + n*log(x)))*e^(p*log(b*x^n + a) + p*log(d*x^n + c)
)/(a*c*(m + 1))
```

3.594.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(45) = 90$.

Time = 0.46 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.44

$$\int (hx)^m (a + bx^n)^p (c + dx^n)^p \left(e + \frac{(bc + ad)e(1 + m + n + np)x^n}{ac(1 + m)} + \frac{bde(1 + m + 2n + 2np)x^{2n}}{ac(1 + m)}\right) dx = \frac{(bx^n + a)^p(dx^n + c)^p bdexx^{2n}e^{(m\log(h)+m\log(x))} + (bx^n + a)^p(dx^n + c)^p bcexx^n e^{(m\log(h)+m\log(x))} + (bx^n + a)}{acm + ac}$$

input
```
integrate((h*x)^m*(a+b*x^n)^p*(c+d*x^n)^p*(e+(a*d+b*c)*e*(n*p+m+n+1)*x^n/a
/c/(1+m)+b*d*e*(2*n*p+m+2*n+1)*x^(2*n)/a/c/(1+m)),x, algorithm="giac")
```

output
```
((b*x^n + a)^p*(d*x^n + c)^p*b*d*e*x*x^(2*n)*e^(m*log(h) + m*log(x)) + (b*
x^n + a)^p*(d*x^n + c)^p*b*c*e*x*x^n*e^(m*log(h) + m*log(x)) + (b*x^n + a)
^p*(d*x^n + c)^p*a*d*e*x*x^n*e^(m*log(h) + m*log(x)) + (b*x^n + a)^p*(d*x^
n + c)^p*a*c*e*x*e^(m*log(h) + m*log(x)))/(a*c*m + a*c)
```

3.594.9 Mupad [B] (verification not implemented)

Time = 11.69 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.36

$$\int (hx)^m (a+bx^n)^p (c+dx^n)^p \left(e+\frac{(bc+ad)e(1+m+n+np)x^n}{ac(1+m)} + \frac{bde(1+m+2n+2np)x^{2n}}{ac(1+m)}\right) dx = (c+d\,x^n)^p \left(\frac{e\,x\,(h\,x)^m\,(a+b\,x^n)^p}{m+1} + \frac{e\,x\,x^n\,(h\,x)^m\,(a\,d+b\,c)\,(a+b\,x^n)^p}{a\,c\,(m+1)} + \frac{b\,d\,e\,x\,x^{2\,n}\,(h\,x)^m\,(a+b\,x^n)^p}{a\,c\,(m+1)}\right)$$

input
```
int((h*x)^m*(a + b*x^n)^p*(c + d*x^n)^p*(e + (e*x^n*(a*d + b*c)*(m + n + n
*p + 1))/(a*c*(m + 1)) + (b*d*e*x^(2*n)*(m + 2*n + 2*n*p + 1))/(a*c*(m + 1
))),x)
```

output
```
(c + d*x^n)^p*((e*x*(h*x)^m*(a + b*x^n)^p)/(m + 1) + (e*x*x^n*(h*x)^m*(a*d
 + b*c)*(a + b*x^n)^p)/(a*c*(m + 1)) + (b*d*e*x*x^(2*n)*(h*x)^m*(a + b*x^n
)^p)/(a*c*(m + 1)))
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 4481

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
            fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
            gamma,loggamma,digamma,zeta,polylog,LambertW,
            elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
      return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
      m1 = expnType(expn.args[0])
      m2 = expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
      return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
   elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
      m1 = max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
   elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
      m1 = max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   elif is_appell_function(expn.func):
      m1 = max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   elif isinstance(expn,RootSum):
      m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
      return max(7,m1)
   elif str(expn).find("Integral") != -1:
      m1 = max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```